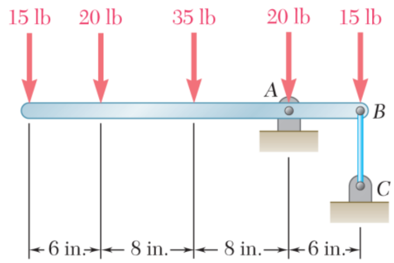
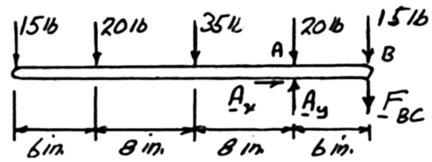


PROBLEM 4.1

For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the tension in cable *BC*.

SOLUTION

Free-Body Diagram:



(*a*) Reaction at *A*: $\Sigma F_x = 0{:}\quad A_x = 0$

$$+\circlearrowleft \Sigma M_B = 0{:}\quad (15\text{ lb})(28\text{ in.}) + (20\text{ lb})(22\text{ in.}) + (35\text{ lb})(14\text{ in.}) + (20\text{ lb})(6\text{ in.}) - A_y(6\text{ in.}) = 0$$

$$A_y = +245\text{ lb} \qquad \mathbf{A} = 245\text{ lb}\uparrow \blacktriangleleft$$

(*b*) Tension in *BC*:

$$+\circlearrowleft \Sigma M_A = 0{:}\quad (15\text{ lb})(22\text{ in.}) + (20\text{ lb})(16\text{ in.}) + (35\text{ lb})(8\text{ in.}) - (15\text{ lb})(6\text{ in.}) - F_{BC}(6\text{ in.}) = 0$$

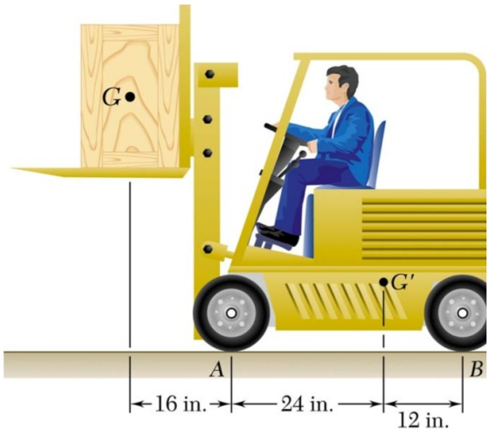
$$F_{BC} = +140.0\text{ lb} \qquad F_{BC} = 140.0\text{ lb} \blacktriangleleft$$

Check:

$$+\uparrow \Sigma F_y = 0{:}\quad -15\text{ lb} - 20\text{ lb} = 35\text{ lb} - 20\text{ lb} + A - F_{BC} = 0$$

$$-105\text{ lb} + 245\text{ lb} - 140.0 = 0$$

$$0 = 0 \quad \text{(Checks)}$$

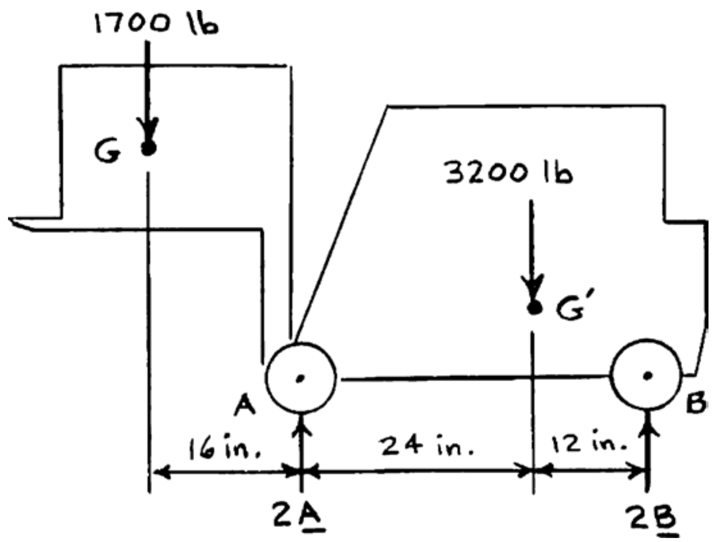


PROBLEM 4.2

A 3200-lb forklift truck is used to lift a 1700-lb crate. Determine the reaction at each of the two (*a*) front wheels *A*, (*b*) rear wheels *B*.

SOLUTION

Free-Body Diagram:



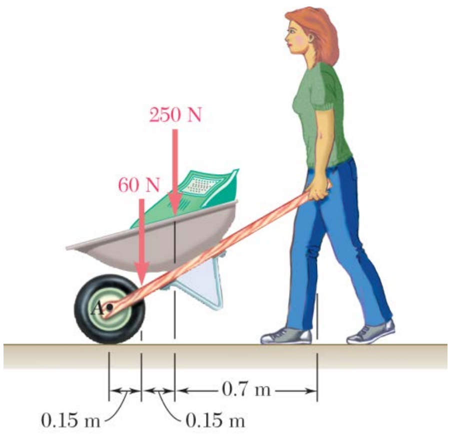
(*a*) Front wheels: $+\circlearrowleft \Sigma M_B = 0$: $(1700 \text{ lb})(52 \text{ in.}) + (3200 \text{ lb})(12 \text{ in.}) - 2A(36 \text{ in.}) = 0$

$A = +1761.11 \text{ lb}$ $\qquad$ $\mathbf{A} = 1761 \text{ lb} \uparrow$ ◀

(*b*) Rear wheels: $+\uparrow \Sigma F_y = 0$: $-1700 \text{ lb} - 3200 \text{ lb} + 2(1761.11 \text{ lb}) + 2B = 0$

$B = +688.89 \text{ lb}$ $\qquad$ $\mathbf{B} = 689 \text{ lb} \uparrow$ ◀

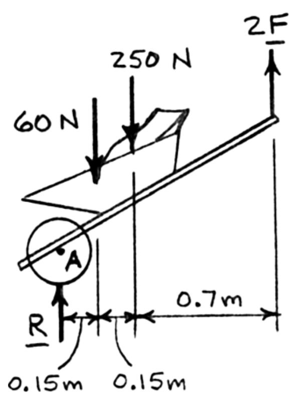
PROBLEM 4.3



A gardener uses a 60-N wheelbarrow to transport a 250-N bag of fertilizer. What force must she exert on each handle?

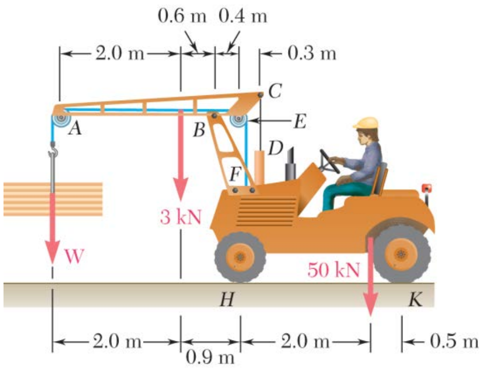
SOLUTION

Free-Body Diagram:



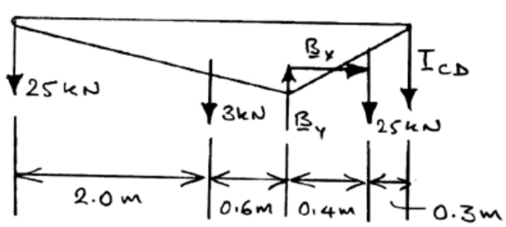
$$+\circlearrowleft \Sigma M_A = 0: \quad (2F)(1\text{ m}) - (60\text{ N})(0.15\text{ m}) - (250\text{ N})(0.3\text{ m}) = 0$$

$$\mathbf{F} = 42.0\text{ N}\uparrow \blacktriangleleft$$

	**PROBLEM 4.4** A load of lumber weighing $W = 25$ kN is being raised as shown by a mobile crane. Knowing that the tension is 25 kN in all portions of cable *AEF* and that the weight of boom *ABC* is 3 kN, determine (*a*) the tension in rod *CD*, (*b*) the reaction at pin *B*.

SOLUTION

Free-Body Diagram: (boom)



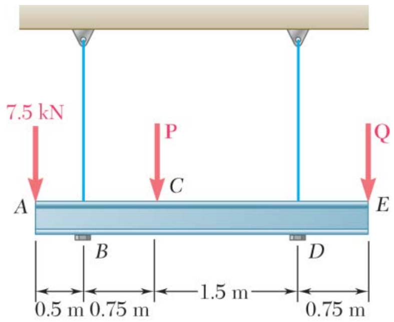
(*a*) $+\circlearrowleft\ \Sigma M_B = 0$: $(25\text{ kN})(2.6\text{ m}) + (3\text{ kN})(0.6\text{ m}) - (25\text{ kN})(0.4\text{ m}) - T_{CD}(0.7\text{ m}) = 0$

$T_{CD} = 81.143$ kN or $T_{CD} = 81.1$ kN ◀

(*b*) $\xrightarrow{+}\ \Sigma F_x = 0$: $B_x = 0$ so that $B = B_y$

$+\uparrow\ \Sigma F_y = 0$: $(-25 - 3 - 25 - 81.143)\text{kN} + B = 0$

$B = 134.143$ kN or $\mathbf{B} = 134.1\text{ kN}\uparrow$ ◀

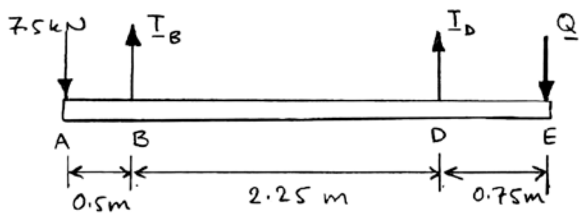


PROBLEM 4.5

Three loads are applied as shown to a light beam supported by cables attached at B and D. Neglecting the weight of the beam, determine the range of values of Q for which neither cable becomes slack when $P = 0$.

SOLUTION

Free-Body Diagram:



For $Q_{\min}$, $T_D = 0$

$+\circlearrowleft \Sigma M_B = 0$: $\quad (7.5 \text{ kN})(0.5 \text{ m}) - Q_{\min}(3 \text{ m}) = 0$

$$Q_{\min} = 1.250 \text{ kN}$$

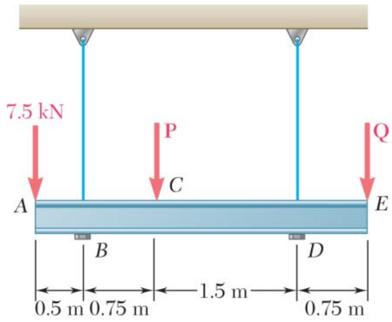
For $Q_{\max}$, $T_B = 0$

$+\circlearrowleft \Sigma M_D = 0$: $\quad (7.5 \text{ kN})(2.75 \text{ m}) - Q_{\max}(0.75 \text{ m}) = 0$

$$Q_{\max} = 27.5 \text{ kN}$$

Therefore: $\quad 1.250 \text{ kN} \le Q \le 27.5 \text{ kN}$ ◀

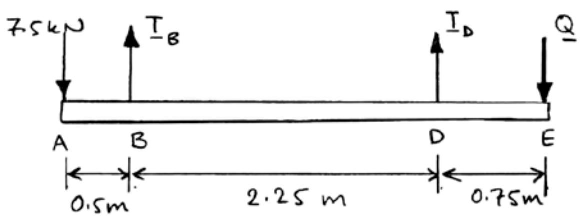
PROBLEM 4.6



Three loads are applied as shown to a light beam supported by cables attached at B and D. Knowing that the maximum allowable tension in each cable is 12 kN and neglecting the weight of the beam, determine the range of values of Q for which the loading is safe when $P = 0$.

SOLUTION

Free-Body Diagram:



$+\circlearrowleft \Sigma M_D = 0$: $\quad (7.5 \text{ kN})(2.75 \text{ m}) - T_B(2.25 \text{ m}) - Q(0.75 \text{ m}) = 0$

$$Q = 27.5 - 3T_B \qquad (1)$$

$+\circlearrowleft \Sigma M_B = 0$: $\quad (7.5 \text{ kN})(0.5 \text{ m}) + T_D(2.25 \text{ m}) - Q(3 \text{ m}) = 0$

$$Q = 1.25 + (0.75\, T_D) \qquad (2)$$

For the loading to be safe, cables must not be slack and tension must not exceed 12 kN.

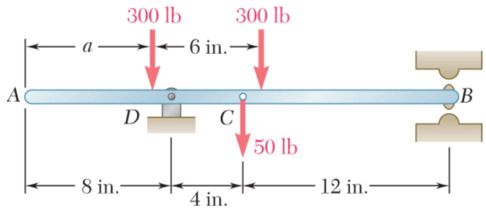
Thus, making $0 \leq T_B \leq 12$ kN in (1), we have

$$-8.50 \text{ kN} \leq Q \leq 27.5 \text{ kN} \qquad (3)$$

And making $0 \leq T_D \leq 12$ kN in (2), we have

$$1.25 \leq Q \leq 10.25 \text{ kN} \qquad (4)$$

(3) and (4) now give: $\quad 1.250 \text{ kN} \leq Q \leq 10.25 \text{ kN}$ ◀

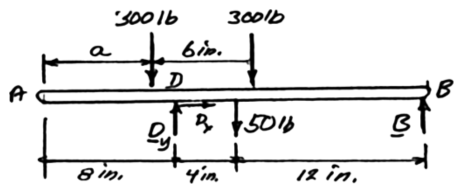


PROBLEM 4.7

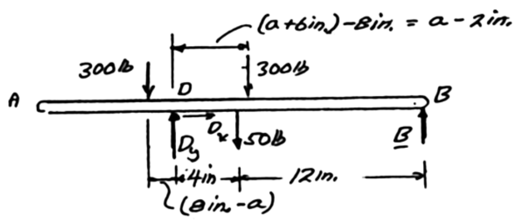
For the beam and loading shown, determine the range of the distance *a* for which the reaction at *B* does not exceed 100 lb downward or 200 lb upward.

SOLUTION

Assume *B* is positive when directed $\uparrow$.



Sketch showing distance from *D* to forces.



$$+\circlearrowleft \Sigma M_D = 0: \quad (300\text{ lb})(8\text{ in.} - a) - (300\text{ lb})(a - 2\text{ in.}) - (50\text{ lb})(4\text{ in.}) + 16B = 0$$

$$-600a + 2800 + 16B = 0$$

$$a = \frac{(2800 + 16B)}{600} \qquad (1)$$

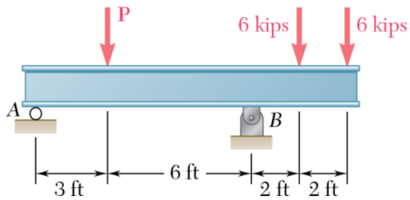
For $B = 100\text{ lb}\downarrow = -100\text{ lb}$, Eq. (1) yields:

$$a \geq \frac{[2800 + 16(-100)]}{600} = \frac{1200}{600} = 2\text{ in.} \qquad a \geq 2.00\text{ in.} \;\triangleleft$$

For $B = 200\uparrow = +200\text{ lb}$, Eq. (1) yields:

$$a \leq \frac{[2800 + 16(200)]}{600} = \frac{6000}{600} = 10\text{ in.} \qquad a \leq 10.00\text{ in.} \;\triangleleft$$

Required range: $2.00\text{ in.} \leq a \leq 10.00\text{ in.}$ ◀

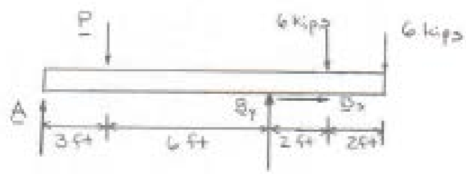


PROBLEM 4.8

For the beam of Sample Prob. 4.2, determine the range of values of P for which the beam will be safe, knowing that the maximum allowable value of each of the reactions is 25 kips and that the reaction at A must be directed upward.

SOLUTION

Free-Body Diagram:



$+\circlearrowleft \Sigma M_A = 0$: $\quad -P(3\text{ ft}) + B_y(9\text{ ft}) - (6\text{ kips})(11\text{ft}) - (6\text{ kips})(13\text{ ft}) = 0$

$$Q = (3B_y - 48)\text{ kips} \qquad (1)$$

$+\circlearrowleft \Sigma M_B = 0$: $\quad -A_y(9\text{ ft}) + P(6\text{ ft}) - (6\text{ kips})(2\text{ ft}) - (6\text{ kips})(4\text{ ft}) = 0$

$$P = (1.5A_y + 6)\text{ kips} \qquad (2)$$

For the loading to meet the design criteria, the reactions must not exceed 25 kips.

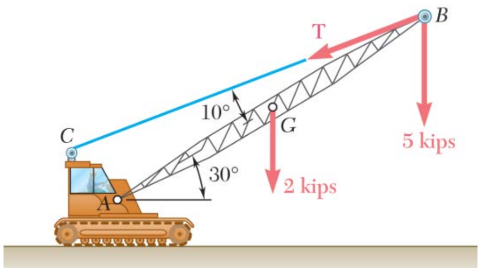
Thus, making $B_y \le 25$ kips in (1), we have

$$P \le 27.0\text{ kips} \qquad (3)$$

And making $0 \le A_y \le 25$ kips in (2), we have

$$6.00 \le P \le 43.5\text{ kips} \qquad (4)$$

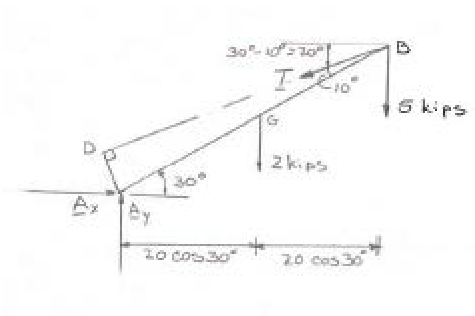
(3) and (4) now give: $\quad 6.00\text{ kips} \le P \le 27.0\text{ kips}$ ◀



PROBLEM 4.9

The 40-ft boom AB weighs 2 kips; the distance from the axle A to the center of gravity G of the boom is 20 ft. For the position shown, determine (*a*) the tension T in the cable, (*b*) the reaction at A.

SOLUTION



(*a*)

$$AD = (AB)\sin 10^\circ = (40\text{ ft})\sin 10^\circ$$

$$AD = 6.9459\text{ ft}$$

$$+\circlearrowleft \Sigma M_A = 0\text{:}\quad T(AD) - 2(20\cos 30^\circ) - 5(40\cos 30^\circ) = 0$$

$$T = 29.924\text{ kips}$$

$T = 29.9\text{ kips}$ ◀

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0\text{:}\quad A_x - (29.924)\cos 20^\circ = 0$$

$$A_x = +28.119\text{ kips}$$

$$+\uparrow \Sigma F_y = 0\text{:}\quad A_y - (29.924)\sin 20^\circ - 2 - 5 = 0$$

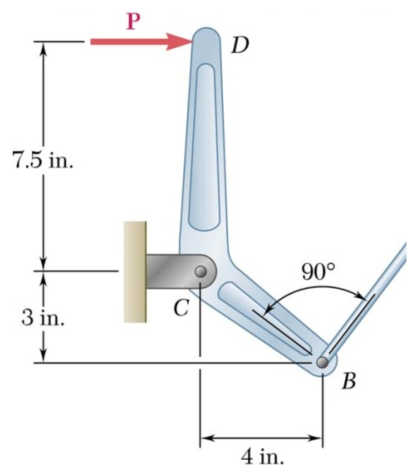
$$A_y = 17.2346\text{ kips}$$

$$\mathbf{A} = \sqrt{28.119^2 + 17.2346^2} = 32.980\text{ kips}$$

$$\alpha = \tan^{-1}\left(\frac{17.2346}{28.119}\right)$$

$$= 31.5°$$

$\mathbf{A} = 33.0\text{ kips}\ \measuredangle\ 31.5°$ ◀

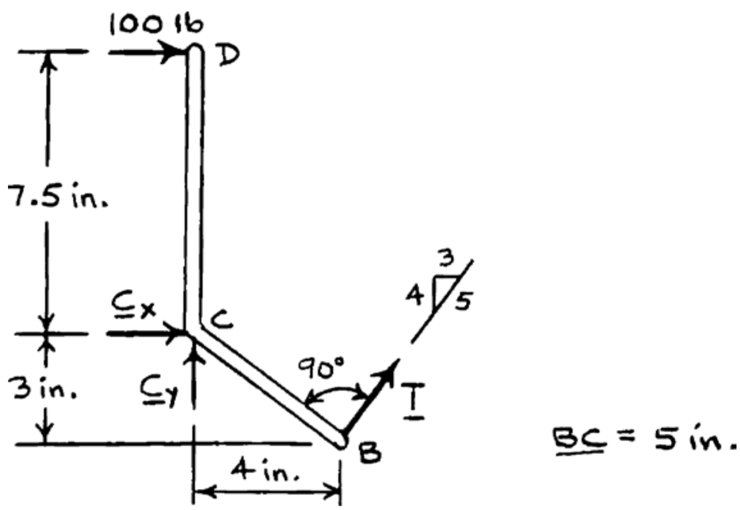


PROBLEM 4.10

The lever *BCD* is hinged at *C* and attached to a control rod at *B*. If $P = 100$ lb, determine (*a*) the tension in rod *AB*, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:

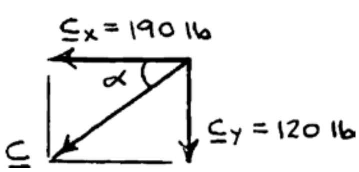


(*a*) $$+\circlearrowleft \Sigma M_C = 0: \quad T(5\text{ in.}) - (100\text{ lb})(7.5\text{ in.}) = 0$$

$$T = 150.0\text{ lb} \blacktriangleleft$$

(*b*) $$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 100\text{ lb} + \frac{3}{5}(150.0\text{ lb}) = 0$$

$$C_x = -190\text{ lb} \qquad \mathbf{C}_x = 190\text{ lb} \longleftarrow$$

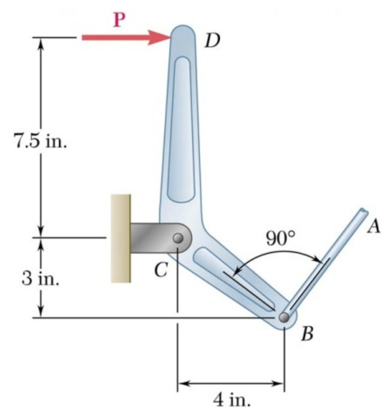


$$+\uparrow \Sigma F_y = 0: \quad C_y + \frac{4}{5}(150.0\text{ lb}) = 0$$

$$C_y = -120\text{ lb} \qquad \mathbf{C}_y = 120\text{ lb} \downarrow$$

$$\alpha = 32.3° \qquad C = 225\text{ lb}$$

$$\mathbf{C} = 225\text{ lb} \;\swarrow\; 32.3° \blacktriangleleft$$

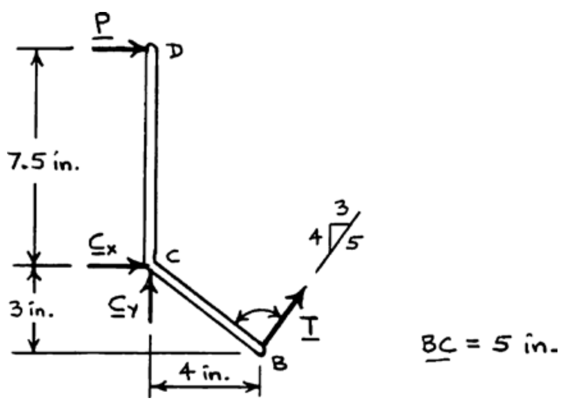


PROBLEM 4.11

The lever *BCD* is hinged at *C* and attached to a control rod at *B*. Determine the maximum force **P** that can be safely applied at *D* if the maximum allowable value of the reaction at *C* is 250 lb.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_C = 0: \quad T(5\text{ in.}) - P(7.5\text{ in.}) = 0$$

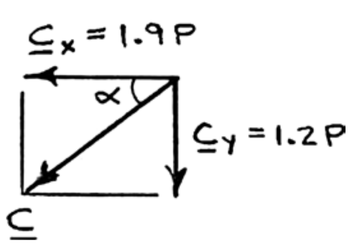
$$T = 1.5P$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P + C_x + \frac{3}{5}(1.5P) = 0$$

$$C_x = -1.9P \qquad \mathbf{C}_x = 1.9P \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + \frac{4}{5}(1.5P) = 0$$

$$C_y = -1.2P \qquad \mathbf{C}_y = 1.2P \downarrow$$



$$C = \sqrt{C_x^2 + C_y^2}$$

$$= \sqrt{(1.9P)^2 + (1.2P)^2}$$

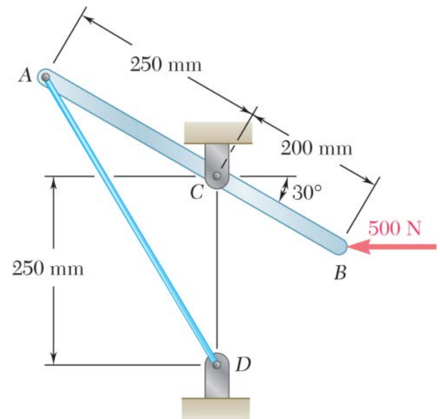
$$C = 2.2472P$$

For $C = 250$ lb,

$$250\text{ lb} = 2.2472P$$

$$P = 111.2\text{ lb}$$

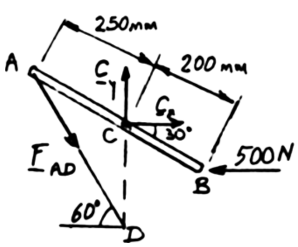
$\mathbf{P} = 111.2$ lb $\longrightarrow$ ◀



PROBLEM 4.12

A lever *AB* is hinged at *C* and attached to a control cable at *A*. If the lever is subjected to a 500-N horizontal force at *B*, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

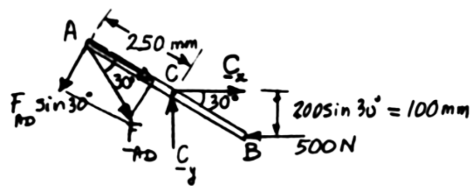
SOLUTION



Triangle *ACD* is isosceles with $\angle C = 90° + 30° = 120°$ $\angle A = \angle D = \dfrac{1}{2}(180° - 120°) = 30°$.

Thus, *DA* forms angle of 60° with the horizontal axis.

(*a*) We resolve $\mathbf{F}_{AD}$ into components along *AB* and perpendicular to *AB*.



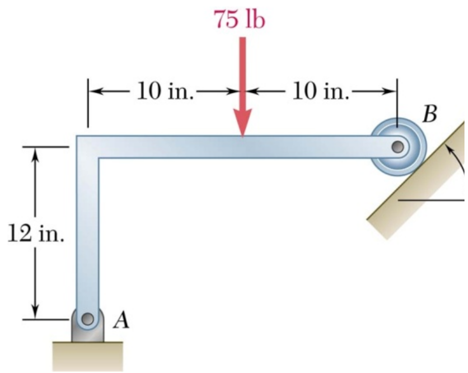
$$+\circlearrowleft \Sigma M_C = 0: \quad (F_{AD}\sin 30°)(250 \text{ mm}) - (500 \text{ N})(100 \text{ mm}) = 0 \qquad F_{AD} = 400 \text{ N} \blacktriangleleft$$

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad -(400 \text{ N})\cos 60° + C_x - 500 \text{ N} = 0 \qquad C_x = +300 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad -(400 \text{ N})\sin 60° + C_y = 0 \qquad C_y = +346.4 \text{ N}$$

$$\mathbf{C} = 458 \text{ N} \measuredangle 49.1° \blacktriangleleft$$



PROBLEM 4.13

Determine the reactions at A and B when (a) $\alpha = 0$, (b) $\alpha = 90°$, (c) $\alpha = 30°$.

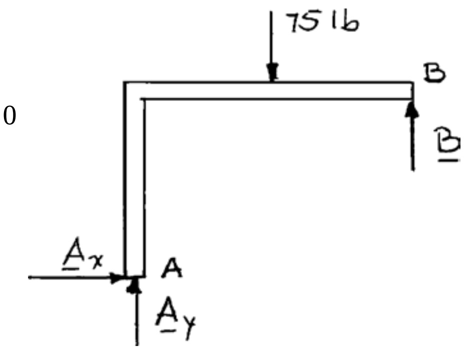
SOLUTION

(a) $\alpha = 0$

$$+\circlearrowleft \Sigma M_A = 0: \quad B(20 \text{ in.}) - 75 \text{ lb}(10 \text{ in.}) = 0$$
$$B = 37.5 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 75 \text{ lb} + 37.5 \text{ lb} = 0$$
$$A_y = 37.5 \text{ lb}$$



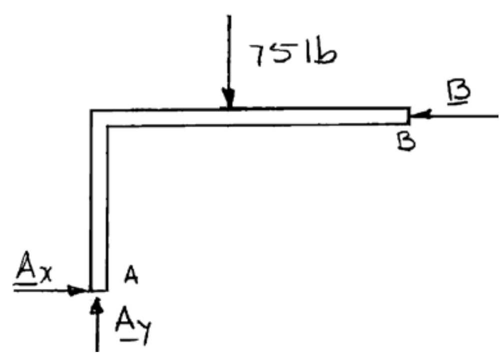
$\mathbf{A} = \mathbf{B} = 37.5 \text{ lb} \uparrow$ ◀

(b) $\alpha = 90°$

$$+\circlearrowleft \Sigma M_A = 0: \quad B(12 \text{ in.}) - 75 \text{ lb}(10 \text{ in.}) = 0$$
$$B = 62.5 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B = 0$$
$$A_x = 62.5 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 75 \text{ lb} = 0$$
$$A_y = 75 \text{ lb}$$



$$A = \sqrt{A_x^2 + A_y^2}$$
$$= \sqrt{(62.5 \text{ lb})^2 + (75 \text{ lb})^2}$$
$$= 97.6 \text{ lb}$$

$$\tan\theta = \frac{75}{62.5}$$
$$\theta = 50.2°$$

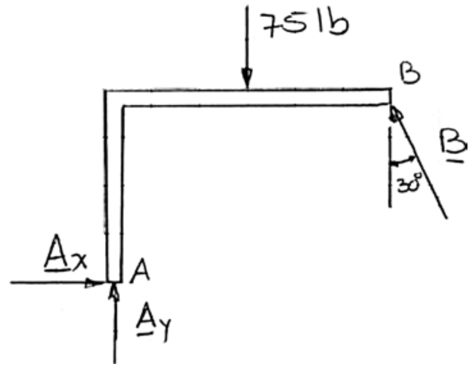
$\mathbf{A} = 97.6 \text{ lb} \measuredangle 50.2°$; $\mathbf{B} = 62.5 \text{ lb} \longleftarrow$ ◀

SOLUTION Continued

$(c)\qquad \alpha = 30°$

$$+\circlearrowright \Sigma M_A = 0: \quad (B\cos 30°)(20\text{ in.}) + (B\sin 30°)(12\text{ in.}) - (75\text{ lb})(10\text{ in.}) = 0$$

$$B = 32.161\text{ lb}$$



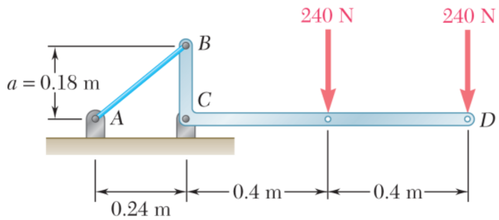
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (32.161)\sin 30° = 0$$
$$A_x = 16.0805\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + (32.161)\cos 30° - 75 = 0$$
$$A_y = 47.148\text{ lb}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(16.0805)^2 + (47.148)^2} = 49.8\text{ lb}$$

$$\tan\theta = \frac{47.148}{16.0805}$$
$$\theta = 71.2°$$

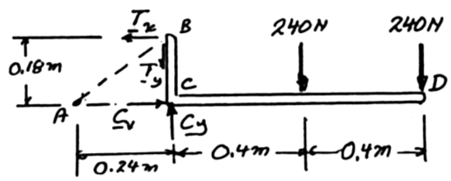
$\mathbf{A} = 49.8\text{ lb} \measuredangle 71.2°;\ \mathbf{B} = 32.2\text{ lb} \measuredangle 60.0°$ ◀



PROBLEM 4.14

The bracket *BCD* is hinged at *C* and attached to a control cable at *B*. For the loading shown, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION



At *B*:

$$\frac{T_y}{T_x} = \frac{0.18 \text{ m}}{0.24 \text{ m}}$$

$$T_y = \frac{3}{4} T_x \qquad (1)$$

(*a*)

$$+\circlearrowleft \Sigma M_C = 0: \quad T_x(0.18 \text{ m}) - (240 \text{ N})(0.4 \text{ m}) - (240 \text{ N})(0.8 \text{ m}) = 0$$

$$T_x = +1600 \text{ N}$$

From Eq. (1):

$$T_y = \frac{3}{4}(1600 \text{ N}) = 1200 \text{ N}$$

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{1600^2 + 1200^2} = 2000 \text{ N} \qquad T = 2.00 \text{ kN} \blacktriangleleft$$

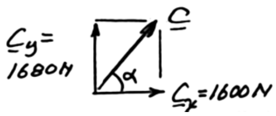
(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - T_x = 0$$

$$C_x - 1600 \text{ N} = 0 \quad C_x = +1600 \text{ N} \qquad \mathbf{C}_x = 1600 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - T_y - 240 \text{ N} - 240 \text{ N} = 0$$

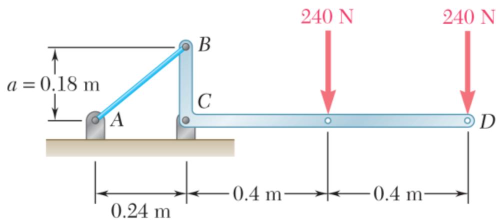
$$C_y - 1200 \text{ N} - 480 \text{ N} = 0$$



$$C_y = +1680 \text{ N} \qquad \mathbf{C}_y = 1680 \text{ N} \uparrow$$

$$\alpha = 46.4°$$

$$C = 2320 \text{ N} \qquad \mathbf{C} = 2.32 \text{ kN} \measuredangle 46.4° \blacktriangleleft$$

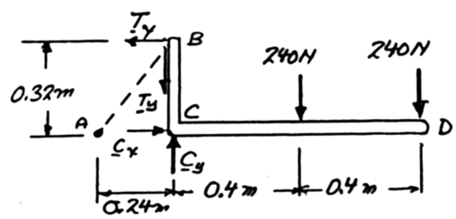


PROBLEM 4.15

Solve Problem 4.14, assuming that $a = 0.32$ m.

PROBLEM 4.14 The bracket *BCD* is hinged at *C* and attached to a control cable at *B*. For the loading shown, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION



(*a*) At *B*:

$$\frac{T_y}{T_x} = \frac{0.32\text{ m}}{0.24\text{ m}}$$

$$T_y = \frac{4}{3}T_x$$

$$+\circlearrowleft \Sigma M_C = 0: \quad T_x(0.32\text{ m}) - (240\text{ N})(0.4\text{ m}) - (240\text{ N})(0.8\text{ m}) = 0$$

$$T_x = 900\text{ N}$$

From Eq. (1):

$$T_y = \frac{4}{3}(900\text{ N}) = 1200\text{ N}$$

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{900^2 + 1200^2} = 1500\text{ N} \qquad T = 1.500\text{ kN} \blacktriangleleft$$

(*b*)

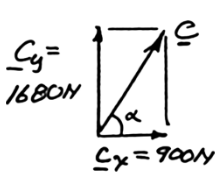
$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - T_x = 0$$

$$C_x - 900\text{ N} = 0 \quad C_x = +900\text{ N} \qquad \mathbf{C}_x = 900\text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - T_y - 240\text{ N} - 240\text{ N} = 0$$

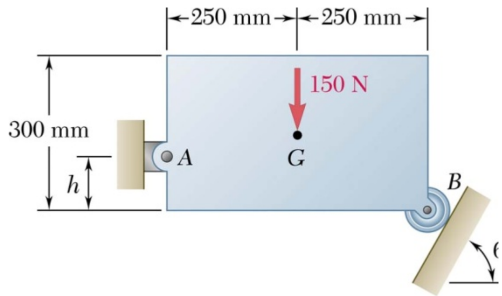
$$C_y - 1200\text{ N} - 480\text{ N} = 0$$

$$C_y = +1680\text{ N} \qquad \mathbf{C}_y = 1680\text{ N} \uparrow$$



$$\alpha = 61.8°$$

$$C = 1906\text{ N} \qquad \mathbf{C} = 1.906\text{ kN} \measuredangle 61.8° \blacktriangleleft$$

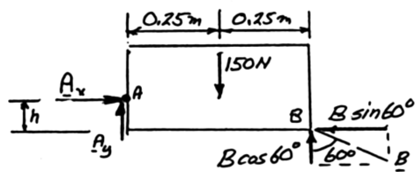


PROBLEM 4.16

Determine the reactions at A and B when (*a*) $h = 0$, (*b*) $h = 200$ mm.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad (B\cos 60°)(0.5 \text{ m}) - (B\sin 60°)h - (150 \text{ N})(0.25 \text{ m}) = 0$$

$$B = \frac{37.5}{0.25 - 0.866h} \qquad (1)$$

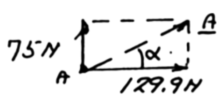
(*a*) When $h = 0$,

From Eq. (1): $B = \dfrac{37.5}{0.25} = 150$ N $\qquad$ $\mathbf{B} = 150.0 \text{ N} \measuredangle 30.0°$ ◀

$\xrightarrow{+} \Sigma F_y = 0: \quad A_x - B\sin 60° = 0$

$A_x = (150)\sin 60° = 129.9$ N $\qquad$ $\mathbf{A}_x = 129.9 \text{ N} \longrightarrow$

$+\uparrow \Sigma F_y = 0: \quad A_y - 150 + B\cos 60° = 0$



$A_y = 150 - (150)\cos 60° = 75$ N $\qquad$ $\mathbf{A}_y = 75 \text{ N} \uparrow$

$\alpha = 30°$

$A = 150.0$ N $\qquad$ $\mathbf{A} = 150.0 \text{ N} \measuredangle 30.0°$ ◀

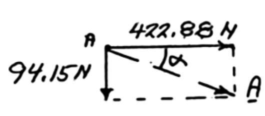
(*b*) When $h = 200$ mm $= 0.2$ m,

From Eq. (1): $B = \dfrac{37.5}{0.25 - 0.866(0.2)} = 488.3$ N $\qquad$ $\mathbf{B} = 488 \text{ N} \measuredangle 30.0°$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B\sin 60° = 0$

$A_x = (488.3)\sin 60° = 422.88$ N $\qquad$ $\mathbf{A}_x = 422.88 \text{ N} \longrightarrow$

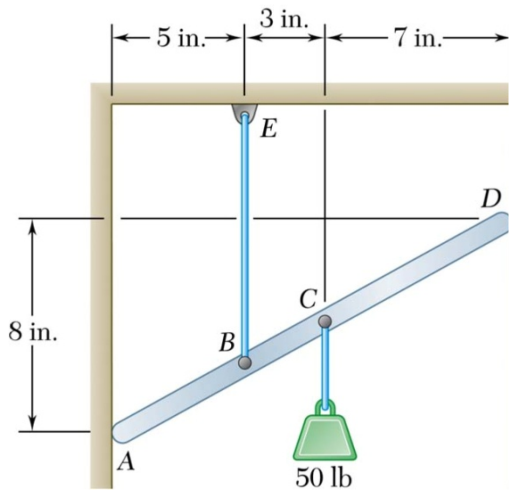
$+\uparrow \Sigma F_y = 0: \quad A_y - 150 + B\cos 60° = 0$



$A_y = 150 - (488.3)\cos 60° = -94.15$ N $\qquad$ $\mathbf{A}_y = 94.15 \text{ N} \downarrow$

$\alpha = 12.55°$

$A = 433.2$ N $\qquad$ $\mathbf{A} = 433 \text{ N} \measuredangle 12.55°$ ◀

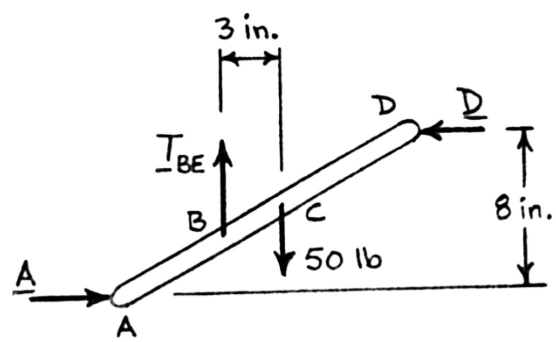


PROBLEM 4.17

A light bar AD is suspended from a cable BE and supports a 50-lb block at C. The ends A and D of the bar are in contact with frictionless vertical walls. Determine the tension in cable BE and the reactions at A and D.

SOLUTION

Free-Body Diagram:



$\Sigma F_x = 0: \quad A = D$

$\Sigma F_y = 0: \qquad T_{BE} = 50.0 \text{ lb}$ ◀

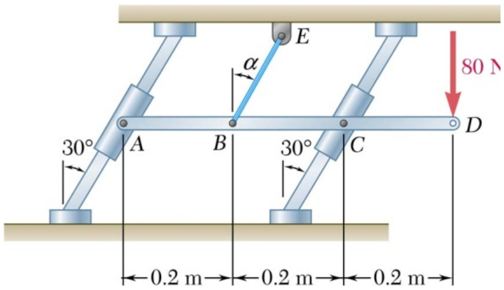
We note that the forces shown form two couples.

$$+\circlearrowleft \Sigma M = 0: \quad A(8 \text{ in.}) - (50 \text{ lb})(3 \text{ in.}) = 0$$

$$A = 18.75 \text{ lb}$$

$\mathbf{A} = 18.75 \text{ lb} \longrightarrow \qquad \mathbf{D} = 18.75 \text{ lb} \longleftarrow$ ◀

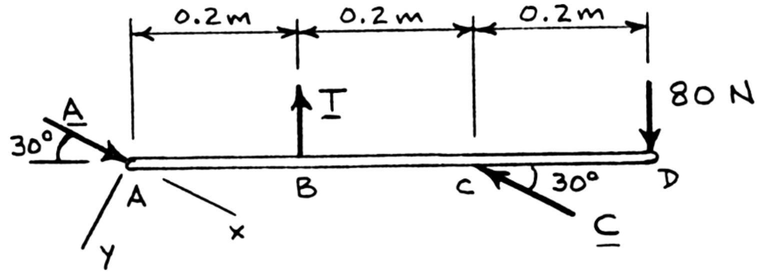
PROBLEM 4.18



Bar *AD* is attached at *A* and *C* to collars that can move freely on the rods shown. If the cord *BE* is vertical ($\alpha = 0$), determine the tension in the cord and the reactions at *A* and *C*.

SOLUTION

Free-Body Diagram:



$$+\swarrow \Sigma F_y = 0: \quad -T\cos 30° + (80\text{ N})\cos 30° = 0$$

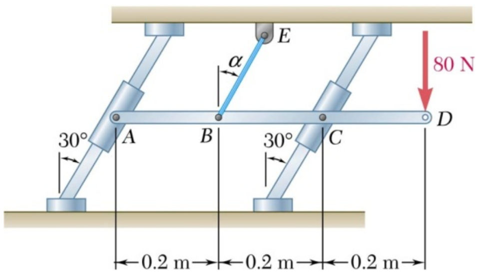
$$T = 80\text{ N} \qquad\qquad T = 80.0\text{ N} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (A\sin 30°)(0.4\text{ m}) - (80\text{ N})(0.2\text{ m}) - (80\text{ N})(0.2\text{ m}) = 0$$

$$A = +160\text{ N} \qquad \mathbf{A} = 160.0\text{ N} \; 30.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (80\text{ N})(0.2\text{ m}) - (80\text{ N})(0.6\text{ m}) + (C\sin 30°)(0.4\text{ m}) = 0$$

$$C = +160\text{ N} \qquad \mathbf{C} = 160.0\text{ N} \; 30.0° \blacktriangleleft$$



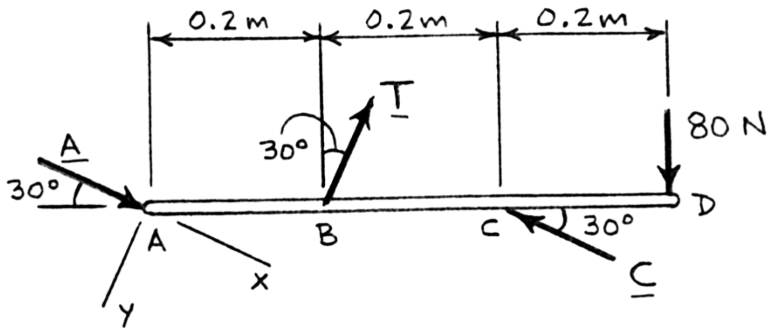
PROBLEM 4.19

Solve Problem 4.18 if the cord BE is parallel to the rods ($\alpha = 30°$).

PROBLEM 4.18 Bar *AD* is attached at *A* and *C* to collars that can move freely on the rods shown. If the cord *BE* is vertical ($\alpha = 0$), determine the tension in the cord and the reactions at *A* and *C*.

SOLUTION

Free-Body Diagram:



$$+\swarrow \Sigma F_y = 0: \quad -T + (80\text{ N})\cos 30° = 0$$

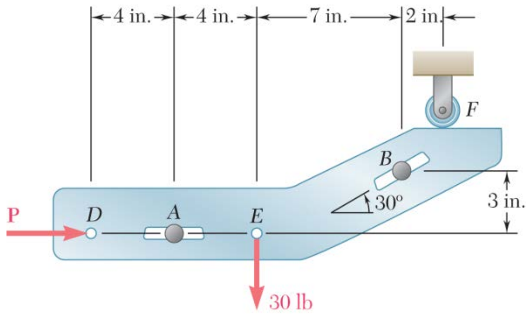
$$T = 69.282\text{ N} \qquad\qquad T = 69.3\text{ N} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -(69.282\text{ N})\cos 30°(0.2\text{ m})$$
$$-(80\text{ N})(0.2\text{ m}) + (A\sin 30°)(0.4\text{ m}) = 0$$

$$A = +140.000\text{ N} \qquad \mathbf{A} = 140.0\text{ N} \;\nwarrow 30.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad +(69.282\text{ N})\cos 30°(0.2\text{ m})$$
$$-(80\text{ N})(0.6\text{ m}) + (C\sin 30°)(0.4\text{ m}) = 0$$

$$C = +180.000\text{ N} \qquad \mathbf{C} = 180.0\text{ N} \;\searrow 30.0° \blacktriangleleft$$

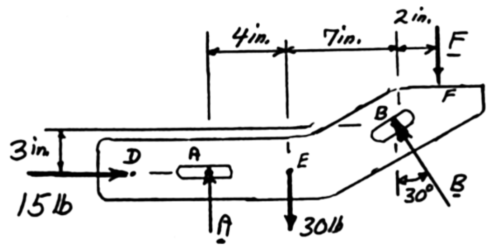


PROBLEM 4.20

Two slots have been cut in plate *DEF*, and the plate has been placed so that the slots fit two fixed, frictionless pins *A* and *B*. Knowing that $P = 15$ lb, determine (*a*) the force each pin exerts on the plate, (*b*) the reaction at *F*.

SOLUTION

Free-Body Diagram:



(*a*) $\xrightarrow{+} \Sigma F_x = 0$: $15 \text{ lb} - B\sin 30° = 0$ **B** $= 30.0$ lb $\searrow$ $60.0°$ ◀

(*b*) $+\circlearrowleft \Sigma M_A = 0$: $-(30 \text{ lb})(4 \text{ in.}) + B\sin 30°(3 \text{ in.}) + B\cos 30°(11 \text{ in.}) - F(13 \text{ in.}) = 0$

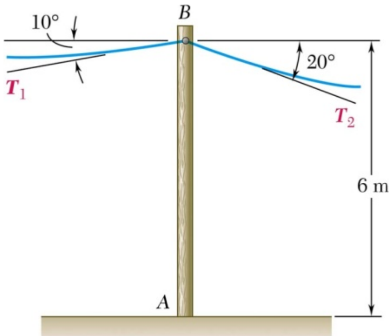
$$-120 \text{ lb}\cdot\text{in.} + (30 \text{ lb})\sin 30°(3 \text{ in.}) + (30 \text{ lb})\cos 30°(11 \text{ in.}) - F(13 \text{ in.}) = 0$$

$F = +16.2145$ lb **F** $= 16.21$ lb $\downarrow$ ◀

(*a*) $+\uparrow \Sigma F_y = 0$: $A - 30 \text{ lb} + B\cos 30° - F = 0$

$$A - 30 \text{ lb} + (30 \text{ lb})\cos 30° - 16.2145 \text{ lb} = 0$$

$A = +20.23$ lb **A** $= 20.2$ lb $\uparrow$ ◀

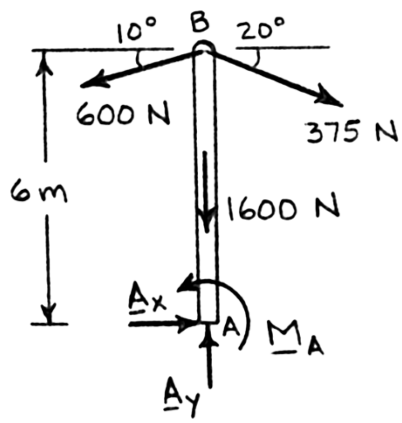


PROBLEM 4.21

A 6-m telephone pole weighing 1600 N is used to support the ends of two wires. The wires form the angles shown with the horizontal axis and the tensions in the wires are, respectively, $T_1 = 600\text{ N}$ and $T_2 = 375\text{ N}$. Determine the reaction at the fixed end *A*.

SOLUTION

Free-Body Diagram:



$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + (375\text{ N})\cos 20° - (600\text{ N})\cos 10° = 0$$

$$A_x = +238.50\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1600\text{ N} - (600\text{ N})\sin 10° - (375\text{ N})\sin 20° = 0$$

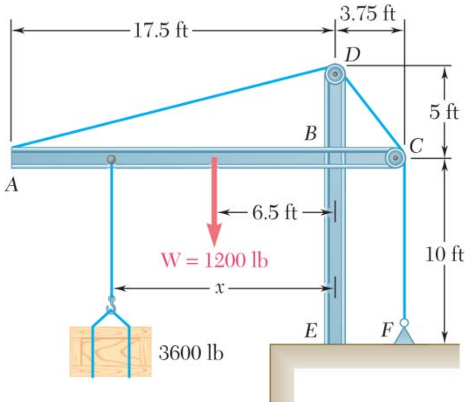
$$A_y = +1832.45\text{ N}$$

$$A = \sqrt{238.50^2 + 1832.45^2}$$

$$\theta = \tan^{-1}\frac{1832.45}{238.50} \qquad \mathbf{A} = 1848\text{ N} \measuredangle 82.6° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A + (600\text{ N})\cos 10°(6\text{ m}) - (375\text{ N})\cos 20°(6\text{ m}) = 0$$

$$M_A = -1431.00\text{ N}\cdot\text{m} \qquad \mathbf{M}_A = 1431\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

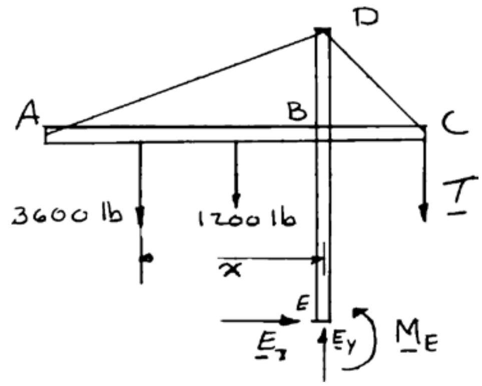


PROBLEM 4.22

The rig shown consists of a 1200-lb horizontal member *ABC* and a vertical member *DBE* welded together at *B*. The rig is being used to raise a 3600-lb crate at a distance x = 12 ft from the vertical member *DBE*. If the tension in the cable is 4 kips, determine the reaction at *E*, assuming that the cable is (*a*) anchored at *F* as shown in the figure, (*b*) attached to the vertical member at a point located 1 ft above *E*.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft M_E = 0: \quad M_E + (3600\text{ lb})x + (1200\text{ lb})(6.5\text{ ft}) - T(3.75\text{ ft}) = 0$$

$$M_E = 3.75T - 3600x - 7800 \qquad (1)$$

(*a*) For $x = 12$ ft and $T = 4000$ lbs,

$$M_E = 3.75(4000) - 3600(12) - 7800$$
$$= 36{,}000\text{ lb}\cdot\text{ft}$$

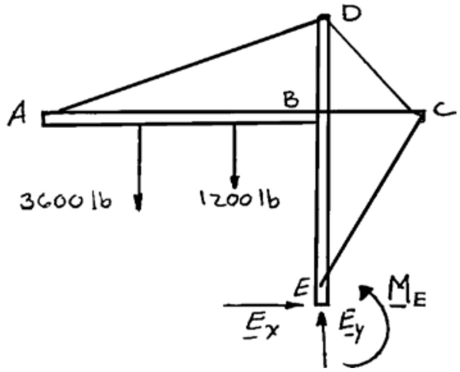
$$\xrightarrow{+} \Sigma F_x = 0 \quad \therefore \quad E_x = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad E_y - 3600\text{ lb} - 1200\text{ lb} - 4000 = 0$$

$$E_y = 8800\text{ lb}$$

$$\mathbf{E} = 8.80\text{ kips}\uparrow; \ \mathbf{M}_E = 36.0\text{ kip}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

SOLUTION Continued



(*b*) $+\circlearrowleft \Sigma M_E = 0$: $M_E + (3600 \text{ lb})(12 \text{ ft}) + (1200 \text{ lb})(6.5 \text{ ft}) = 0$

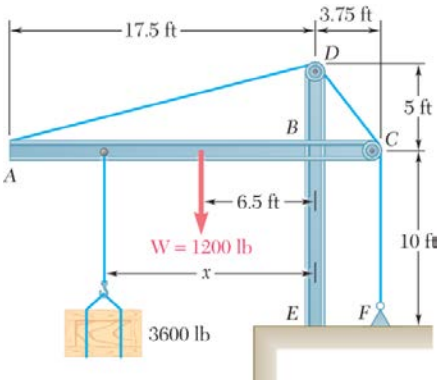
$$M_E = -51{,}000 \text{ lb} \cdot \text{ft}$$

$$\xrightarrow{+} \Sigma F_x = 0 \quad \therefore \quad E_x = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad E_y - 3600 \text{ lb} - 1200 \text{ lb} = 0$$

$$E_y = 4800 \text{ lb}$$

$\mathbf{E} = 4.80 \text{ kips} \uparrow$; $\mathbf{M}_E = 51.0 \text{ kip} \cdot \text{ft} \circlearrowright$ ◀

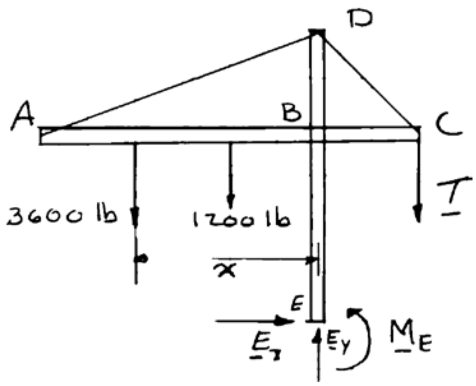


PROBLEM 4.23

For the rig and crate of Prob. 4.22, and assuming that cable is anchored at F as shown, determine (a) the required tension in cable $ADCF$ if the maximum value of the couple at E as x varies from 1.5 to 17.5 ft is to be as small as possible, (b) the corresponding maximum value of the couple.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft M_E = 0: \quad M_E + (3600 \text{ lb})x + (1200 \text{ lb})(6.5 \text{ ft}) - T(3.75 \text{ ft}) = 0$$

$$M_E = 3.75T - 3600x - 7800 \qquad (1)$$

For $x = 1.5$ ft, Eq. (1) becomes

$$(M_E)_1 = 3.75T - 3600(1.5) - 7800 \qquad (2)$$

For $x = 17.5$ ft, Eq. (1) becomes

$$(M_E)_2 = 3.75T - 3600(17.5) - 7800$$

(a) For smallest max value of $|M_E|$, we set

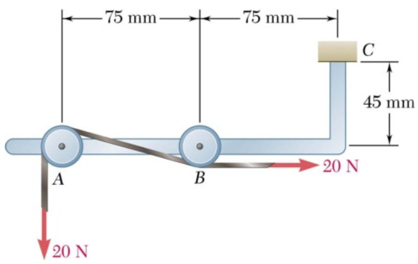
$$(M_E)_1 = ^{-}(M_E)_2$$

$$3.75T - 13,200 = -3.75T + 70,800 \qquad \mathbf{T} = 11.20 \text{ kips} \blacktriangleleft$$

(b) From Equation (2), then

$$M_E = 3.75(11.20) - 13.20 \qquad |M_E| = 28.8 \text{ kip} \cdot \text{ft} \blacktriangleleft$$

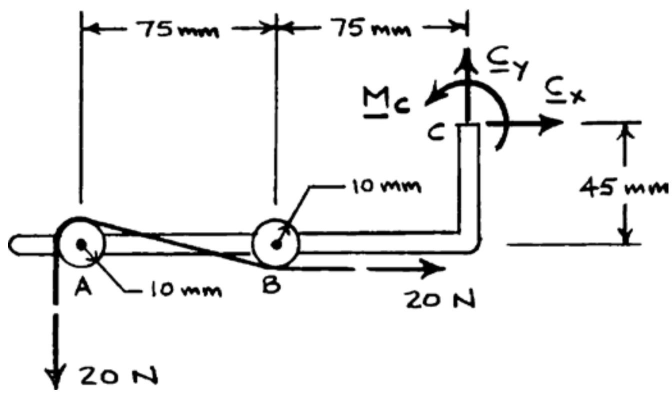
PROBLEM 4.24



A tension of 20 N is maintained in a tape as it passes through the support system shown. Knowing that the radius of each pulley is 10 mm, determine the reaction at C.

SOLUTION

Free-Body Diagram:



$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (20\text{ N}) = 0 \qquad C_x = -20\text{ N}$

$+\uparrow \Sigma F_y = 0: \quad C_y - (20\text{ N}) = 0 \qquad C_y = +20\text{ N}$

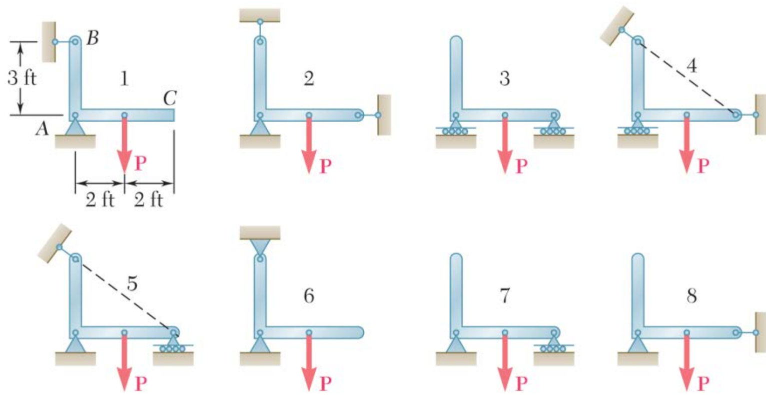
$\mathbf{C} = 28.3\text{ N} \searrow 45.0°$ ◀

$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (20\text{ N})(0.160\text{ m}) + (20\text{ N})(0.055\text{ m}) = 0$

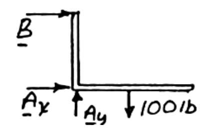
$M_C = -4.30\text{ N}\cdot\text{m} \qquad \mathbf{M}_C = 4.30\text{ N}\cdot\text{m} \circlearrowright$ ◀

PROBLEM 4.25

The bracket *ABC* can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown and, wherever possible, compute the reactions, assuming that the magnitude of the force **P** is 100 lb.



SOLUTION



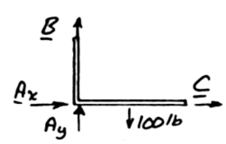
1. Three non-concurrent, non-parallel reactions:

   (*a*) Bracket: complete constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained

$$\mathbf{A} = 120.2\text{ lb} \measuredangle 56.3°, \quad \mathbf{B} = 66.7\text{ lb} \longleftarrow$$

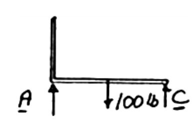


2. Four concurrent, reactions (through *A*):

   (*a*) Bracket: improper constraint

   (*b*) Reactions: indeterminate

   (*c*) No equilibrium $(\Sigma M_A \neq 0)$



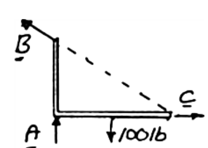
3. Two reactions:

   (*a*) Bracket: partial constraint

   (*b*) Reactions: indeterminate

   (*c*) Equilibrium maintained

$$\mathbf{A} = 50.0\text{ lb}\uparrow, \quad \mathbf{C} = 50.0\text{ lb}\uparrow$$



4. Three non-concurrent, non-parallel reactions:

   (*a*) Bracket: complete constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained

$$\mathbf{A} = 50.0\text{ lb}\uparrow, \quad \mathbf{B} = 83.3\text{ lb} \measuredangle 36.9°, \quad \mathbf{C} = 66.7\text{ lb} \longrightarrow$$

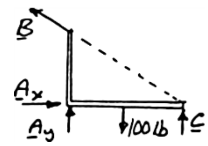
SOLUTION Continued

5. Four non-concurrent, non-parallel reactions:

   (*a*) Bracket: complete constraint

   (*b*) Reactions: indeterminate

   (*c*) Equilibrium maintained $(\Sigma M_C = 0)\ \mathbf{A}_y = 50.0 \text{ lb}\uparrow$

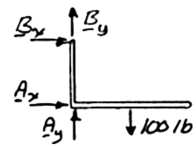


6. Four non-concurrent, non-parallel reactions:

   (*a*) Bracket: complete constraint

   (*b*) Reactions: indeterminate

   (*c*) Equilibrium maintained



$$\mathbf{A}_x = 66.7 \text{ lb} \longrightarrow \quad \mathbf{B}_x = 66.7 \text{ lb} \longleftarrow$$

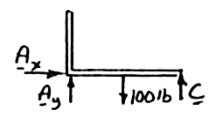
$$(\mathbf{A}_y + \mathbf{B}_y = 100.0 \text{ lb}\uparrow)$$

7. Three non-concurrent, non-parallel reactions:

   (*a*) Bracket: complete constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained



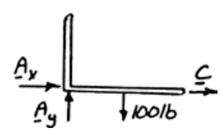
$$\mathbf{A} = \mathbf{C} = 50.0 \text{ lb}\uparrow$$

8. Three concurrent, reactions (through *A*)

   (*a*) Bracket: improper constraint

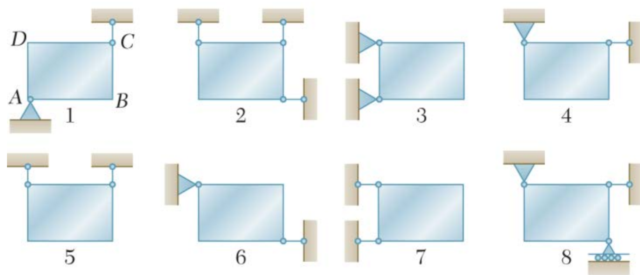
   (*b*) Reactions: indeterminate

   (*c*) No equilibrium $(\Sigma M_A \neq 0)$

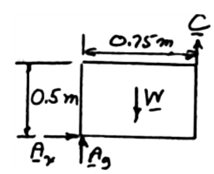


PROBLEM 4.26

Eight identical 500×750-mm rectangular plates, each of mass $m = 40$ kg, are held in a vertical plane as shown. All connections consist of frictionless pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions.



SOLUTION



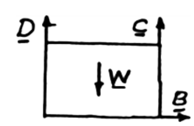
1. Three non-concurrent, non-parallel reactions:

(*a*) Plate: completely constrained

(*b*) Reactions: determinate

(*c*) Equilibrium maintained

$$\mathbf{A} = \mathbf{C} = 196.2 \text{ N} \uparrow$$



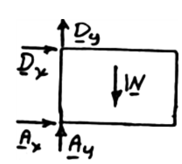
2. Three non-concurrent, non-parallel reactions:

(*a*) Plate: completely constrained

(*b*) Reactions: determinate

(*c*) Equilibrium maintained

$$\mathbf{B} = 0, \quad \mathbf{C} = \mathbf{D} = 196.2 \text{ N} \uparrow$$



3. Four non-concurrent, non-parallel reactions:

(*a*) Plate: completely constrained

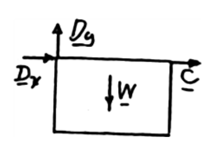
(*b*) Reactions: indeterminate

(*c*) Equilibrium maintained

$$\mathbf{A}_x = 294 \text{ N} \longrightarrow, \quad \mathbf{D}_x = 294 \text{ N} \longleftarrow$$

$$(\mathbf{A}_y + \mathbf{D}_y = 392 \text{ N} \uparrow)$$

4. Three concurrent reactions (through *D*):



(*a*) Plate: improperly constrained

(*b*) Reactions: indeterminate

(*c*) No equilibrium $\qquad (\Sigma M_D \neq 0)$

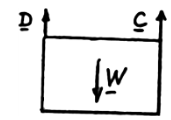
SOLUTION Continued

5. Two reactions:

   (*a*) Plate: partial constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained



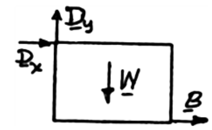
$$\mathbf{C} = \mathbf{D} = 196.2 \text{ N} \uparrow$$

6. Three non-concurrent, non-parallel reactions:

   (*a*) Plate: completely constrained

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained



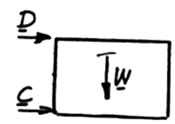
$$\mathbf{B} = 294 \text{ N} \longrightarrow, \quad \mathbf{D} = 491 \text{ N} \searrow 53.1°$$

7. Two reactions:

   (*a*) Plate: improperly constrained

   (*b*) Reactions determined by dynamics

   (*c*) No equilibrium $(\Sigma F_y \neq 0)$

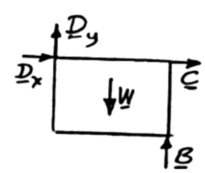


8. Four non-concurrent, non-parallel reactions:

   (*a*) Plate: completely constrained

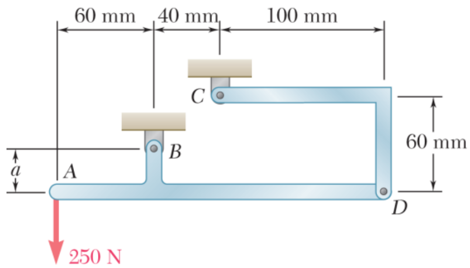
   (*b*) Reactions: indeterminate

   (*c*) Equilibrium maintained



$$\mathbf{B} = \mathbf{D}_y = 196.2 \text{ N} \uparrow$$

$$(\mathbf{C} + \mathbf{D}_x = 0)$$



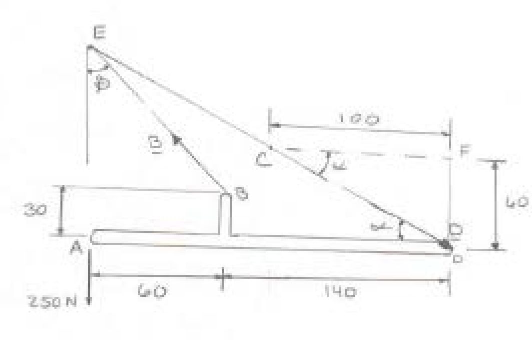
PROBLEM 4.27

Determine the reactions at B and C when $a = 30$ mm.

SOLUTION

Since CD is a two-force member, the force it exerts on member ABD is directed along DC.

Free-Body Diagram of ABD: (Three-Force member)



The reaction at B must pass through E, where **D** and the 250-N load intersect.

Triangle CFD:

$$\tan\alpha = \frac{60}{100} = 0.6$$

$$\alpha = 30.964°$$

Triangle EAD:

$$AE = 200\tan\alpha = 120 \text{ mm}$$

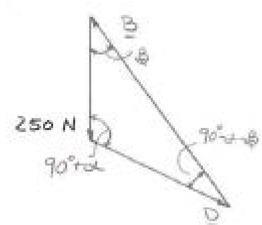
$$GE = AE - AG = 120 - 30 = 90 \text{ mm}$$

Triangle EGB:

$$\tan\beta = \frac{GB}{GE} = \frac{60}{90}$$

$$\beta = 33.690°$$

<ins>Force triangle</ins>



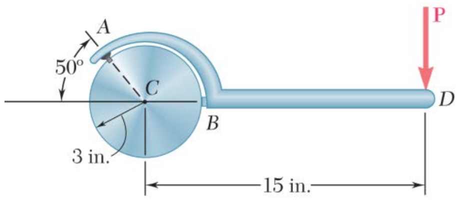
$$\frac{B}{\sin 120.964°} = \frac{D}{\sin 33.690°} = \frac{250 \text{ N}}{\sin 25.346°}$$

$$B = 500.7 \text{ N}$$

$$D = 323.9 \text{ N}$$

$\mathbf{B} = 501 \text{ N} \measuredangle\ 56.3°$ ◀

$\mathbf{C} = \mathbf{D} = 324 \text{ N} \measuredangle\ 31.0°$ ◀



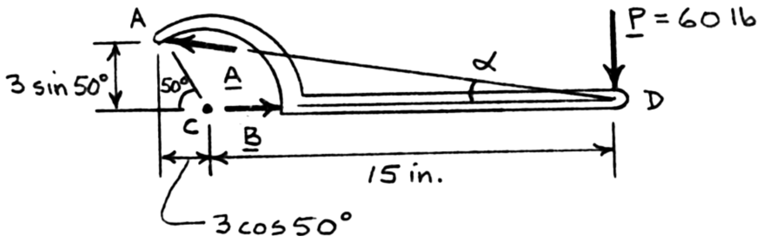
PROBLEM 4.28

The spanner shown is used to rotate a shaft. A pin fits in a hole at *A*, while a flat, frictionless surface rests against the shaft at *B*. If a 60-lb force **P** is exerted on the spanner at *D*, find the reactions at *A* and *B*.

SOLUTION

Free-Body Diagram:

(Three-force body)



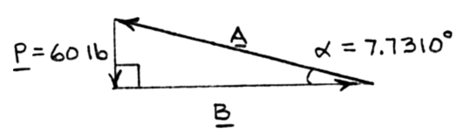
The line of action of **A** must pass through *D*, where **B** and **P** intersect.

$$\tan\alpha = \frac{3\sin 50°}{3\cos 50° + 15}$$
$$= 0.135756$$
$$\alpha = 7.7310°$$

$$A = \frac{60 \text{ lb}}{\sin 7.7310°}$$
$$= 446.02 \text{ lb}$$

$$B = \frac{60 \text{ lb}}{\tan 7.7310°}$$
$$= 441.97 \text{ lb}$$

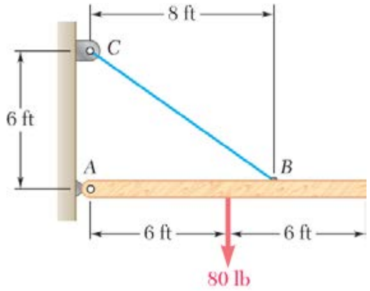
Force triangle



A = 446 lb ⦪ 7.73° ◀

B = 442 lb ⟶ ◀

PROBLEM 4.29

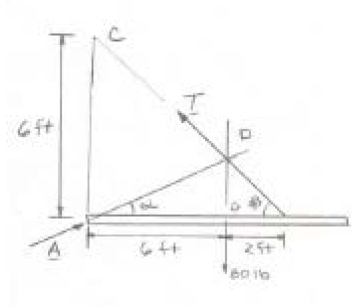


A 12-ft wooden beam weighing 80 lb is supported by a pin and bracket at *A* and by cable *BC*. Find the reaction at *A* and the tension in the cable.

SOLUTION

Since *CB* is a two-force member, the force it exerts on member *AB* is directed along *CB*.

Free-Body Diagram of *AB*: (Three-Force member)



The reaction at *B* must pass through *E*, where **T** and the 80-lb load intersect.

Triangle *CFD*:

$$DG = \frac{BG}{AB}(AC)$$

$$DG = \frac{2}{8}(6\text{ ft}) = 1.50\text{ ft}$$

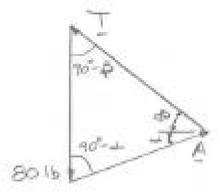
Triangle *EAD*:

$$\tan\alpha = \frac{DG}{AE} = \frac{1.50}{6}$$

$$\alpha = 14.0362^\circ$$

Triangle *EGB*: Force triangle

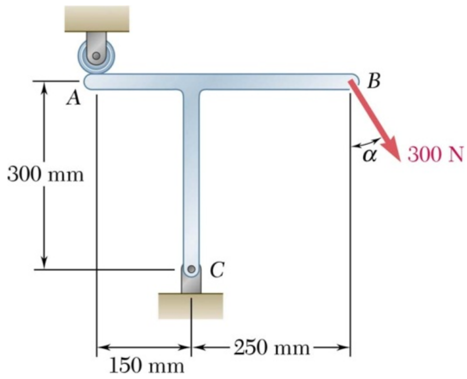
$$\frac{A}{\sin 53.13^\circ} = \frac{T}{\sin 75.964^\circ} = \frac{80\text{ lb}}{\sin 50.906^\circ}$$



$\mathbf{A} = 82.5\text{ lb} \measuredangle 14.04^\circ$ ◀

$T = 100.0\text{ lb}$ ◀

PROBLEM 4.30

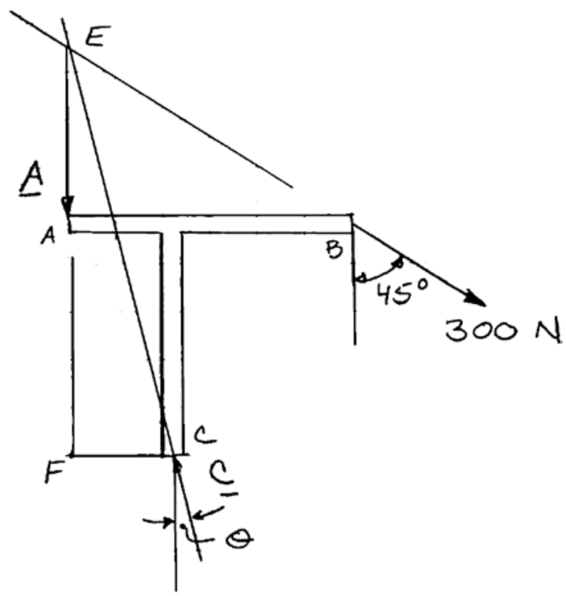


A T-shaped bracket supports a 300-N load as shown. Determine the reactions at A and C when $\alpha = 45°$.

SOLUTION

Free-Body Diagram:

(Three-force body)



The line of action of **C** must pass through E, where **A** and the 300-N force intersect.

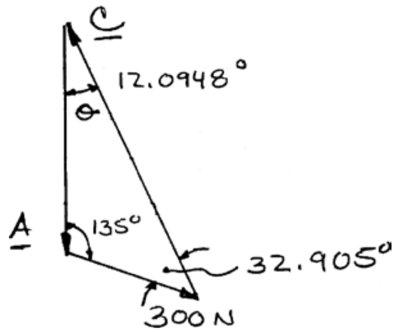
Triangle ABE is isosceles: $EA = AB = 400\text{ mm}$

In triangle CEF:

$$\tan\theta = \frac{CF}{EF} = \frac{CF}{EA + AF} = \frac{150\text{ mm}}{700\text{ mm}} \qquad \theta = 12.0948°$$

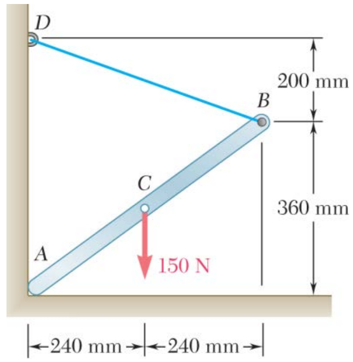
SOLUTION Continued

Force Triangle



Law of sines: $$\frac{A}{\sin 32.905^\circ} = \frac{C}{\sin 135^\circ} = \frac{300 \text{ N}}{\sin 12.0948^\circ}$$

$\mathbf{A} = 778 \text{ N} \downarrow$; $\mathbf{C} = 1012 \text{ N} \searrow 77.9^\circ$ ◀

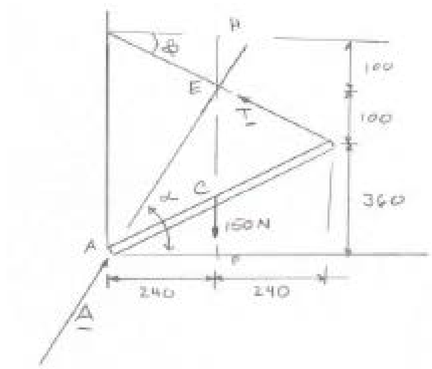


PROBLEM 4.31

One end of rod *AB* rests in the corner *A* and the other end is attached to cord *BD*. If the rod supports a 150-N load at its midpoint *C*, find the reaction at *A* and the tension in the cord.

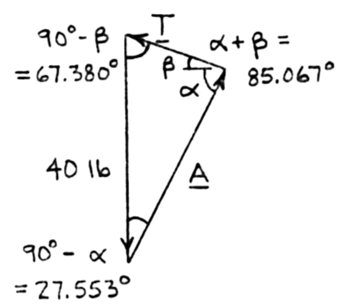
SOLUTION

Free-Body Diagram: (Three-force body) Dimensions in mm



The line of action of reaction at *A* must pass through *E*, where **T** and the 150-N load intersect.

Force triangle



$$\tan\alpha = \frac{EF}{AF} = \frac{460}{240}$$

$$\alpha = 62.447°$$

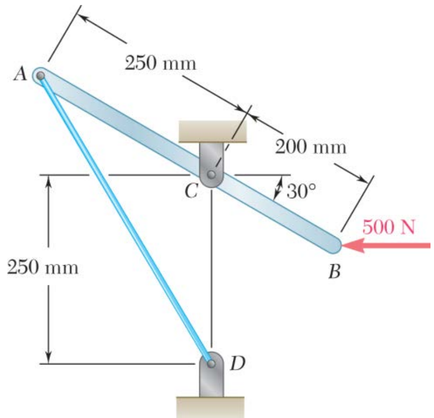
$$\tan\beta = \frac{EH}{DH} = \frac{100}{240}$$

$$\beta = 22.620°$$

$$\frac{A}{\sin 67.380°} = \frac{T}{\sin 27.553°} = \frac{150\text{ N}}{\sin 85.067°}$$

A = 139.0 N ∠ 62.4° ◀

T = 69.6 N ◀



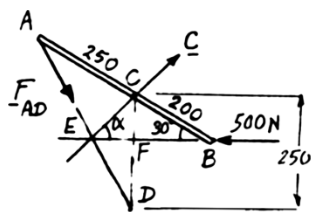
PROBLEM 4.32

Using the method of Section 4.2B, solve Problem 4.12.

PROBLEM 4.12 A lever *AB* is hinged at *C* and attached to a control cable at *A*. If the lever is subjected to a 500-N horizontal force at *B*, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:
(Three-Force body)



Dimensions in mm

Reaction at C must pass through E, where $\mathbf{F}_{AD}$ and 500-N force intersect.

Since $AC = CD = 250$ mm, triangle ACD is isosceles.

We have $$\angle C = 90° + 30° = 120°$$

and $$\angle A = \angle D = \frac{1}{2}(180° - 120°) = 30°$$

On the other hand, from triangle *BCF*:

$$CF = (BC)\sin 30° = 200 \sin 30° = 100 \text{ mm}$$
$$FD = CD - CF = 250 - 100 = 150 \text{ mm}$$

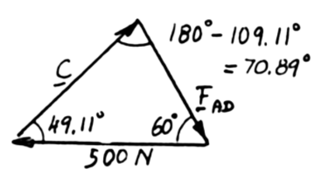
From triangle *EFD*, and since $\angle D = 30°$:

$$EF = (FD)\tan 30° = 150 \tan 30° = 86.60 \text{ mm}$$

From triangle *EFC*: $$\tan\alpha = \frac{CF}{EF} = \frac{100 \text{ mm}}{86.60 \text{ mm}}$$

$$\alpha = 49.11°$$

<u>Force triangle</u>

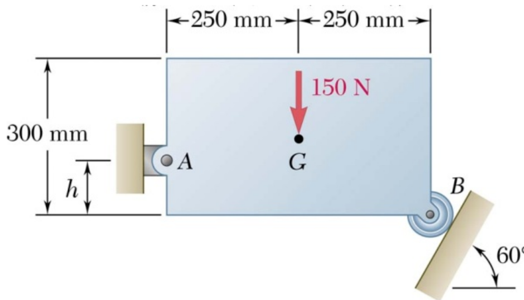


<u>Law of sines</u> $$\frac{F_{AD}}{\sin 49.11°} = \frac{C}{\sin 60°} = \frac{500 \text{ N}}{\sin 70.89°}$$

$$F_{AD} = 400 \text{ N}, \quad C = 458 \text{ N}$$

(*a*) $F_{AD} = 400$ N ◀

(*b*) $\mathbf{C} = 458 \text{ N} \measuredangle 49.1°$ ◀



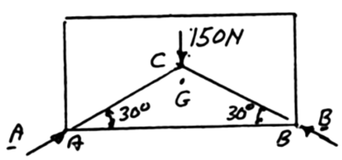
PROBLEM 4.33

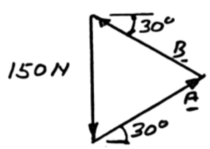
Using the method of Section 4.2B, solve Problem 4.16.

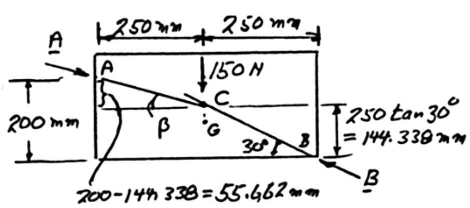
PROBLEM 4.16 Determine the reactions at A and B when (a) $h = 0$, (b) $h = 200$ mm.

SOLUTION

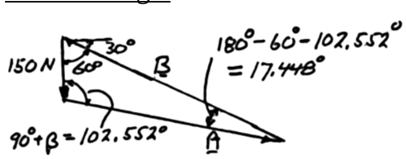
Free-Body Diagram:







Force Triangle



(a) $\underline{h = 0}$

Reaction **A** must pass through C where the 150-N weight and **B** intersect.

Force triangle is equilateral.

$\mathbf{A} = 150.0 \text{ N} \ 30.0°$ ◀

$\mathbf{B} = 150.0 \text{ N} \ 30.0°$ ◀

(b) $\underline{h = 200 \text{ mm}}$

$$\tan\beta = \frac{55.662}{250}$$

$$\beta = 12.5521°$$

Law of sines:

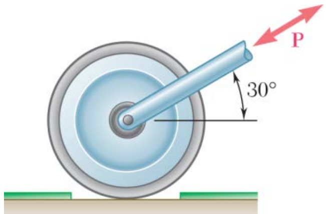
$$\frac{150 \text{ N}}{\sin 17.4480°} = \frac{A}{\sin 60°} = \frac{B}{\sin 102.552°}$$

$$A = 433.24 \text{ N}$$

$$B = 488.31 \text{ N}$$

$\mathbf{A} = 433 \text{ N} \ 12.55°$ ◀

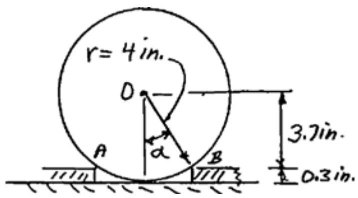
$\mathbf{B} = 488 \text{ N} \ 30.0°$ ◀



PROBLEM 4.34

A 40-lb roller, of diameter 8 in., which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 0.3 in., determine the force **P** required to move the roller onto the tiles if the roller is (*a*) pushed to the left, (*b*) pulled to the right.

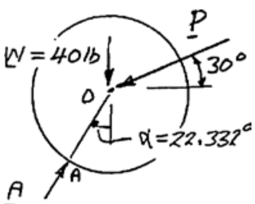
SOLUTION



Geometry: For each case as roller comes into contact with tile,

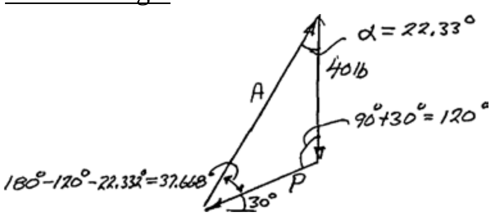
$$\alpha = \cos^{-1}\frac{3.7 \text{ in.}}{4 \text{ in.}}$$

$$\alpha = 22.332°$$



(*a*) Roller pushed to left (three-force body): Forces must pass through *O*.

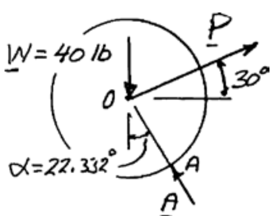
Force Triangle



Law of sines: $\frac{40 \text{ lb}}{\sin 37.668°} = \frac{P}{\sin 22.332°}$; $P = 24.87$ lb

P = 24.9 lb 30.0° ◀

(*b*) Roller pulled to right (three-force body): Forces must pass through *O*.

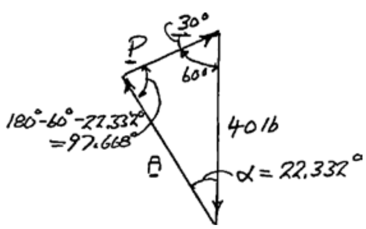


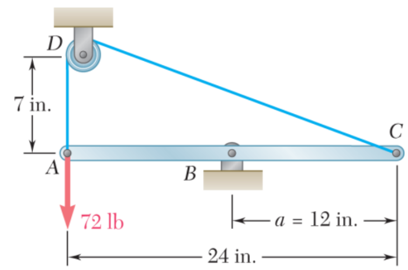
Law of sines:

$\frac{40 \text{ lb}}{\sin 97.668°} = \frac{P}{\sin 22.332°}$; $P = 15.3361$ lb

P = 15.34 lb 30.0° ◀

Force Triangle



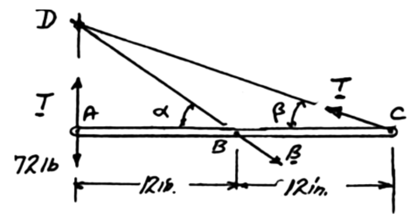


PROBLEM 4.35

Member *ABC* is supported by a pin and bracket at *B* and by an inextensible cord attached at *A* and *C* and passing over a frictionless pulley at *D*. The tension may be assumed to be the same in portions *AD* and *CD* of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at *B*.

SOLUTION

Free-Body Diagram:



Reaction at *B* must pass through *D*.

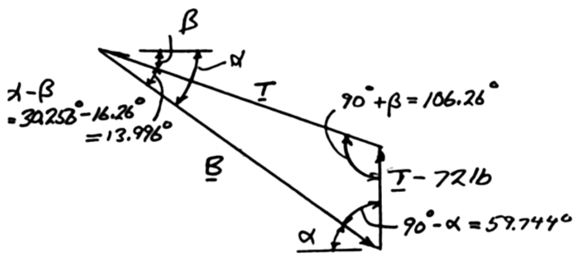
$$\tan\alpha = \frac{7\text{ in.}}{12\text{ in.}}$$

$$\alpha = 30.256°$$

$$\tan\beta = \frac{7\text{ in.}}{24\text{ in.}}$$

$$\beta = 16.26°$$

<u>Force Triangle</u>



Law of sines:

$$\frac{T}{\sin 59.744°} = \frac{T - 72\text{ lb}}{\sin 13.996°} = \frac{B}{\sin 106.26}$$

$$T(\sin 13.996°) = (T - 72\text{ lb})(\sin 59.744°)$$

$$T(0.24185) = (T - 72)(0.86378)$$

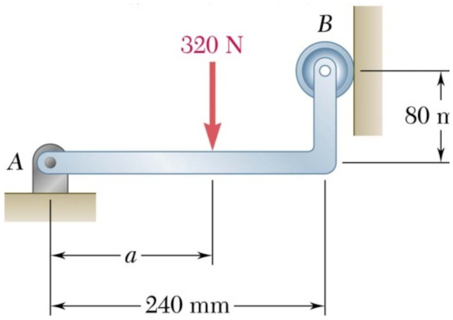
$$T = 100.00\text{ lb}$$

$T = 100.0$ lb ◀

$$B = (100\text{ lb})\frac{\sin 106.26°}{\sin 59.744°}$$

$$= 111.14\text{ lb}$$

$\mathbf{B} = 111.1$ lb ◣ 30.3° ◀

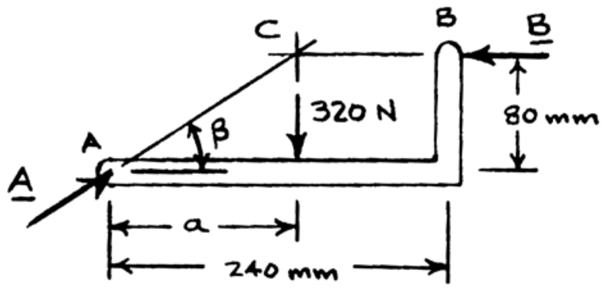


PROBLEM 4.36

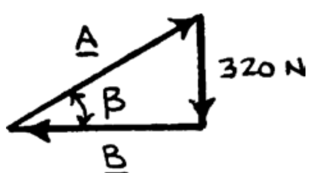
Determine the reactions at A and B when $a = 150$ mm.

SOLUTION

Free-Body Diagram:



<u>Force triangle</u>



$$\tan \beta = \frac{80 \text{ mm}}{a} = \frac{80 \text{ mm}}{150 \text{ mm}}$$

$$\beta = 28.072°$$

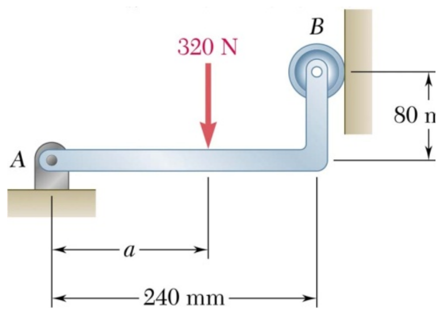
$$A = \frac{320 \text{ N}}{\sin 28.072°}$$

A = 680 N ∠ 28.1° ◀

$$B = \frac{320 \text{ N}}{\tan 28.072°}$$

B = 600 N ⟵ ◀

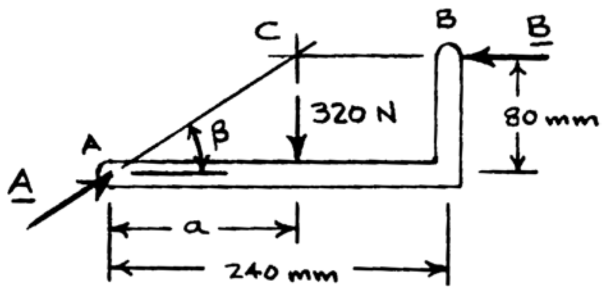
PROBLEM 4.37



Determine the value of a for which the magnitude of the reaction at B is equal to 800 N.

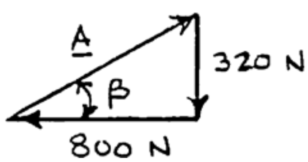
SOLUTION

Free-Body Diagram:



<u>Force triangle</u>

$$\tan\beta = \frac{80\text{ mm}}{a} \qquad a = \frac{80\text{ mm}}{\tan\beta} \qquad (1)$$



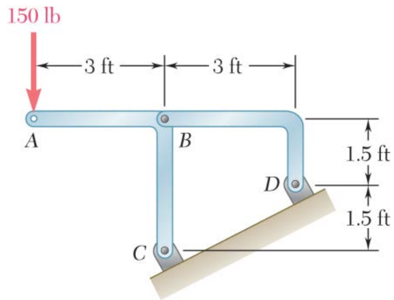
From force triangle:

$$\tan\beta = \frac{320\text{ N}}{800\text{ N}} = 0.4$$

From Eq. (1):

$$a = \frac{80\text{ mm}}{0.4}$$

$a = 200$ mm ◀



PROBLEM 4.38

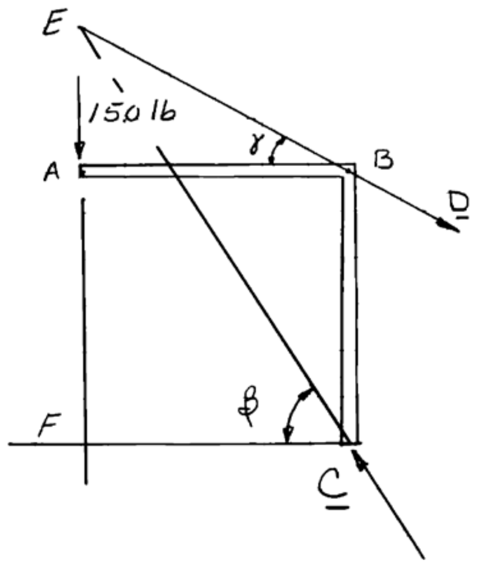
For the frame and loading shown, determine the reactions at *C* and *D*.

SOLUTION

Since *BD* is a two-force member, the reaction at *D* must pass through Points *B* and *D*.

Free-Body Diagram:

(Three-force body)



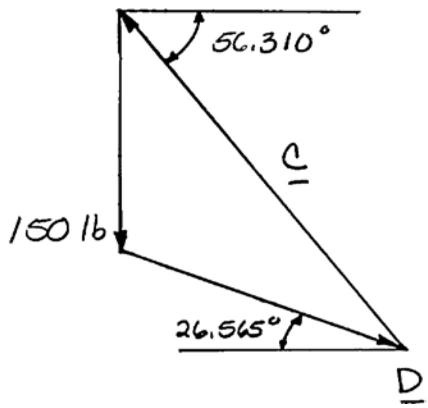
Reaction at *C* must pass through *E*, where the reaction at *D* and the 150-lb load intersect.

Triangle *CEF*: $\tan\beta = \dfrac{4.5 \text{ ft}}{3 \text{ ft}} \qquad \beta = 56.310°$

Triangle *ABE*: $\tan\gamma = \dfrac{1}{2} \qquad \gamma = 26.565°$

SOLUTION Continued

Force Triangle

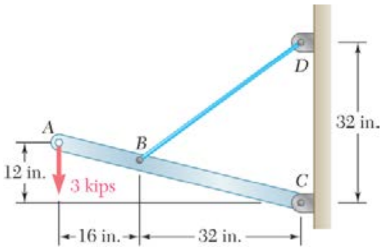


Law of sines: $$\frac{150 \text{ lb}}{\sin 29.745°} = \frac{C}{\sin 116.565°} = \frac{D}{\sin 33.690°}$$

$C = 270.42$ lb,

$D = 167.704$ lb

C = 270 lb ⦣ 56.3°; **D** = 167.7 lb ⦣ 26.6° ◀



PROBLEM 4.39

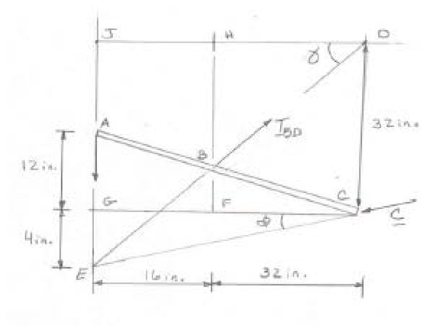
For the boom and loading shown, determine (*a*) the tension in cord *BD*, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:

Three-force body: 3-kip load and *T* intersect at *E*.

Geometry:



$$\frac{BF}{AG} = \frac{CF}{CG}; \quad \frac{BF}{12} = \frac{32}{48}$$

$$BF = 8 \text{ in.}$$

$$BH = 32 - BF = 32 - 8 = 24 \text{ in.}$$

$$\frac{JE}{BH} = \frac{DJ}{DH}; \quad \frac{JE}{24} = \frac{48}{32}; \quad JE = 36 \text{ in.}$$

$$EG = JE - JG = 36 - 32 = 4 \text{ in.}$$

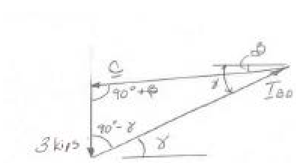
$$\tan\alpha = \frac{4 \text{ in.}}{48 \text{ in.}}$$

$$\alpha = 4.7636°$$

$$\tan\beta = \frac{24 \text{ in.}}{32 \text{ in.}}$$

$$\beta = 36.870°$$

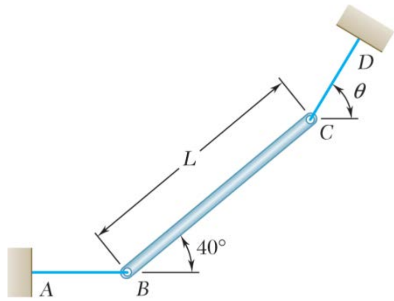
<u>Force Triangle</u>



Law of sines: $$\frac{T_{BD}}{\sin 94.764°} = \frac{3 \text{ kips}}{\sin 32.106°} = \frac{C}{\sin 53.13°}$$

(*a*) $T_{BD} = 5.63$ kips ◀

(*b*) **C** = 4.52 kips 4.76° ◀



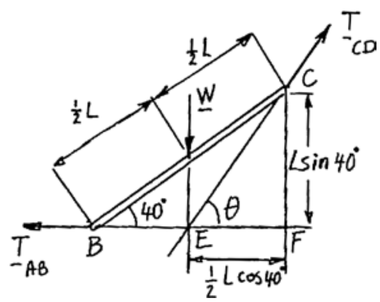
PROBLEM 4.40

A slender rod BC of length L and weight W is held by two cables as shown. Knowing that cable AB is horizontal and that the rod forms an angle of 40° with the horizontal, determine (*a*) the angle θ that cable CD forms with the horizontal, (*b*) the tension in each cable.

SOLUTION

Free-Body Diagram:

(Three-force body)



(*a*) The line of action of $\mathbf{T}_{CD}$ must pass through E, where $\mathbf{T}_{AB}$ and $\mathbf{W}$ intersect.

$$\tan\theta = \frac{CF}{EF}$$

$$= \frac{L\sin 40^\circ}{\frac{1}{2}L\cos 40^\circ}$$

$$= 2\tan 40^\circ$$

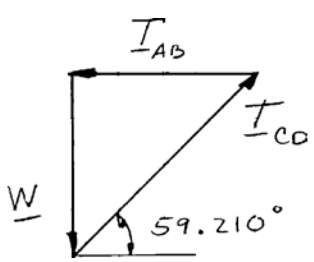
$$= 59.210^\circ$$

$$\theta = 59.2^\circ \blacktriangleleft$$

(b) <u>Force Triangle</u>

$$T_{AB} = W\tan 30.790^\circ$$

$$= 0.59588W$$



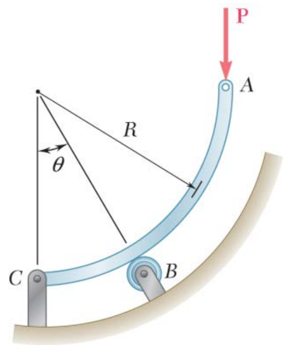
$$T_{AB} = 0.596W \blacktriangleleft$$

$$T_{CD} = \frac{W}{\cos 30.790^\circ}$$

$$= 1.16408W$$

$$T_{CD} = 1.164W \blacktriangleleft$$

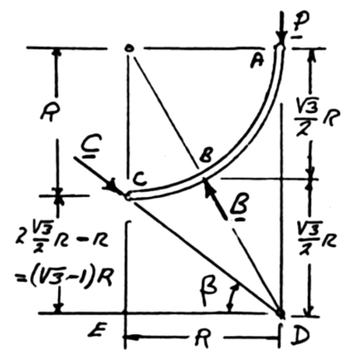
PROBLEM 4.41



Knowing that $\theta = 30°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION

Free-Body Diagram:
(Three-force body)

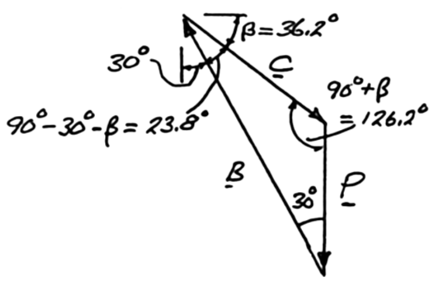


Reaction at *C* must pass through *D* where force **P** and reaction at *B* intersect.

In Δ *CDE*:

$$\tan\beta = \frac{(\sqrt{3}-1)R}{R}$$
$$= \sqrt{3}-1$$
$$\beta = 36.2°$$

<u>Force Triangle</u>

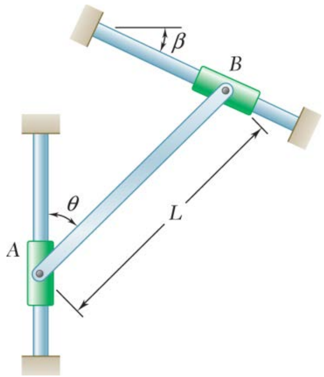


Law of sines:

$$\frac{P}{\sin 23.8°} = \frac{B}{\sin 126.2°} = \frac{C}{\sin 30°}$$
$$B = 2.00P$$
$$C = 1.239P$$

(*a*) $\mathbf{B} = 2P$ 60.0° ◀

(*b*) $\mathbf{C} = 1.239P$ 36.2° ◀



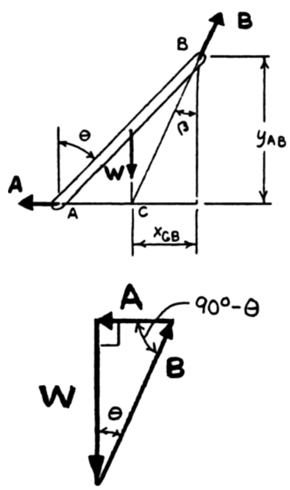
PROBLEM 4.42

A slender rod of length L is attached to collars that can slide freely along the guides shown. Knowing that the rod is in equilibrium, derive an expression for the angle θ in terms of the angle β.

SOLUTION

As shown in the free-body diagram of the slender rod AB, the three forces intersect at C. From the force geometry:

Free-Body Diagram:



$$\tan\beta = \frac{x_{GB}}{y_{AB}}$$

where

$$y_{AB} = L\cos\theta$$

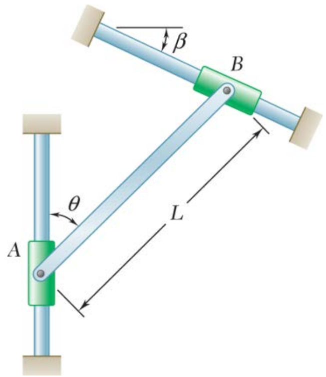
and

$$x_{GB} = \frac{1}{2}L\sin\theta$$

$$\tan\beta = \frac{\frac{1}{2}L\sin\theta}{L\cos\theta}$$

$$= \frac{1}{2}\tan\theta$$

or $\tan\theta = 2\tan\beta$ ◀



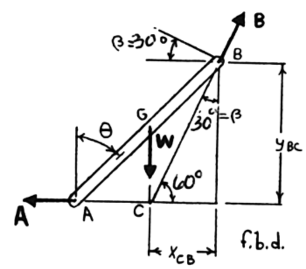
PROBLEM 4.43

An 8-kg slender rod of length L is attached to collars that can slide freely along the guides shown. Knowing that the rod is in equilibrium and that $\beta = 30°$, determine (*a*) the angle θ that the rod forms with the vertical, (*b*) the reactions at A and B.

SOLUTION

(*a*) As shown in the free-body diagram of the slender rod AB, the three forces intersect at C. From the geometry of the forces:

Free-Body Diagram:



$$\tan\beta = \frac{x_{CB}}{y_{BC}}$$

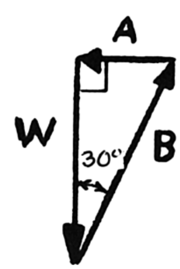
where

$$x_{CB} = \frac{1}{2}L\sin\theta$$

and

$$y_{BC} = L\cos\theta$$

$$\tan\beta = \frac{1}{2}\tan\theta$$



or $\quad \tan\theta = 2\tan\beta$

For $\quad \beta = 30°$

$$\tan\theta = 2\tan 30°$$
$$= 1.15470$$
$$\theta = 49.107°$$

or $\quad \theta = 49.1°$ ◀

(*b*) $\quad W = mg = (8\text{ kg})(9.81\text{ m/s}^2) = 78.480\text{ N}$

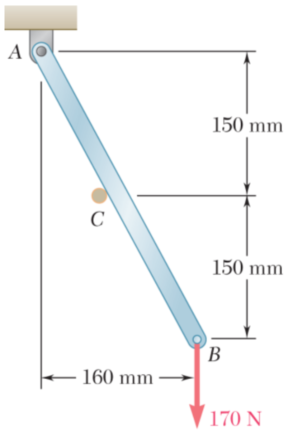
From force triangle: $\quad A = W\tan\beta$

$$= (78.480\text{ N})\tan 30°$$
$$= 45.310\text{ N}$$

or $\quad \mathbf{A} = 45.3\text{ N} \longleftarrow$ ◀

and $\quad B = \dfrac{W}{\cos\beta} = \dfrac{78.480\text{ N}}{\cos 30°} = 90.621\text{ N}$

or $\quad \mathbf{B} = 90.6\text{ N} \measuredangle\ 60.0°$ ◀

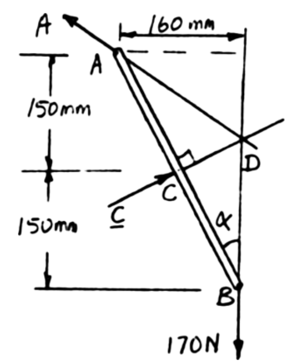


PROBLEM 4.44

Rod *AB* is supported by a pin and bracket at *A* and rests against a frictionless peg at *C*. Determine the reactions at *A* and *C* when a 170-N vertical force is applied at *B*.

SOLUTION

Free-Body Diagram:
(Three-force body)



The reaction at *A* must pass through *D* where **C** and the 170-N force intersect.

$$\tan\alpha = \frac{160 \text{ mm}}{300 \text{ mm}}$$

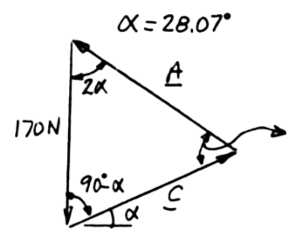
$$\alpha = 28.07^\circ$$

We note that triangle *ABD* is isosceles (since $AC = BC$) and, therefore,

$$\angle CAD = \alpha = 28.07^\circ$$

Also, since $CD \perp CB$, reaction **C** forms angle $\alpha = 28.07^\circ$ with the horizontal axis.

Force triangle



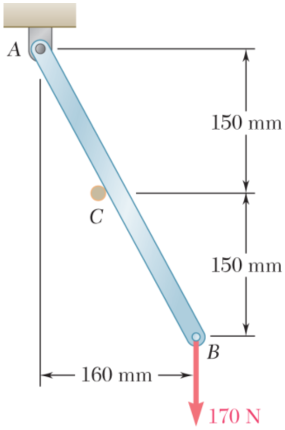
We note that **A** forms angle 2α with the vertical axis. Thus, **A** and **C** form angle

$$180^\circ - (90^\circ - \alpha) - 2\alpha = 90^\circ - \alpha$$

Force triangle is isosceles, and we have

$$A = 170 \text{ N}$$
$$C = 2(170 \text{ N})\sin\alpha$$
$$= 160.0 \text{ N}$$

$\mathbf{A} = 170.0$ N $\searrow$ 33.9°; $\mathbf{C} = 160.0$ N $\measuredangle$ 28.1° ◀



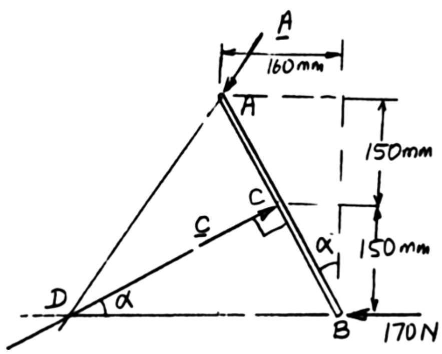
PROBLEM 4.45

Solve Problem 4.44, assuming that the 170-N force applied at *B* is horizontal and directed to the left.

PROBLEM 4.44 Rod *AB* is supported by a pin and bracket at *A* and rests against a frictionless peg at *C*. Determine the reactions at *A* and *C* when a 170-N vertical force is applied at *B*.

SOLUTION

Free-Body Diagram: (Three-Force body)



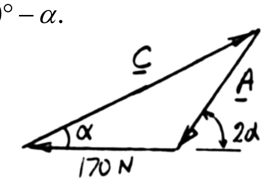
The reaction at *A* must pass through *D*, where **C** and the 170-N force intersect.

$$\tan\alpha = \frac{160 \text{ mm}}{300 \text{ mm}}$$

$$\alpha = 28.07^\circ$$

We note that triangle *ADB* is isosceles (since $AC = BC$). Therefore $\angle A = \angle B = 90^\circ - \alpha$.

Also $\angle ADB = 2\alpha$



Force triangle

The angle between **A** and **C** must be $2\alpha - \alpha = \alpha$

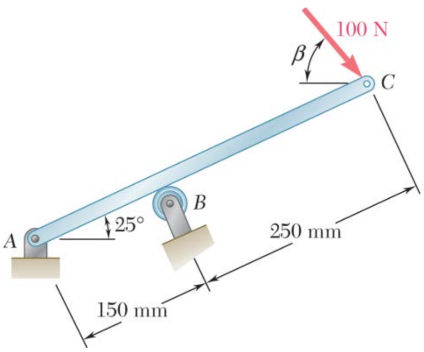
Thus, force triangle is isosceles and

$$\alpha = 28.07^\circ$$

$$A = 170.0 \text{ N}$$

$$C = 2(170 \text{ N})\cos\alpha = 300 \text{ N}$$

$\mathbf{A} = 170.0 \text{ N} \measuredangle 56.1^\circ \quad \mathbf{C} = 300 \text{ N} \measuredangle 28.1^\circ$ ◀



PROBLEM 4.46

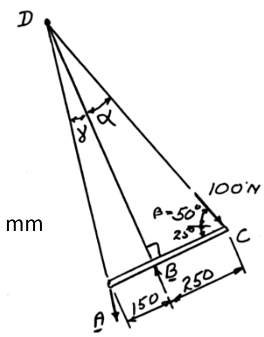
Determine the reactions at A and B when $\beta = 50°$.

SOLUTION

Free-Body Diagram: (Three-force body)

Reaction **A** must pass through Point D where the 100-N force and **B** intersect.

Dimensions in mm



In right Δ BCD:

$$\alpha = 90° - 75° = 15°$$

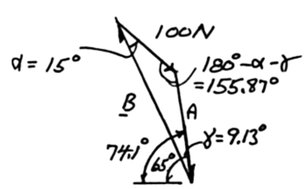
$$BD = 250 \tan 75° = 933.01 \text{ mm}$$

In right Δ ABD:

$$\tan\gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933.01 \text{ mm}}$$

$$\gamma = 9.1333°$$

Force Triangle

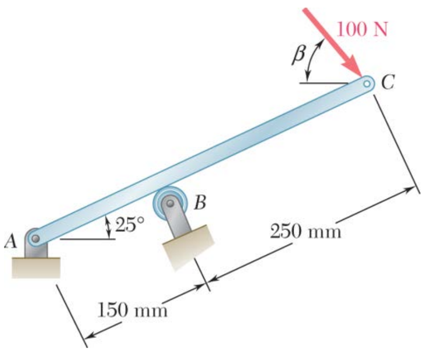


Law of sines:

$$\frac{100 \text{ N}}{\sin 9.1333°} = \frac{A}{\sin 15°} = \frac{B}{\sin 155.867°}$$

$$A = 163.1 \text{ N}; \quad B = 257.6 \text{ N}$$

$\mathbf{A} = 163.1$ N $\measuredangle$ 74.1° $\quad$ $\mathbf{B} = 258$ N $\measuredangle$ 65.0° ◀



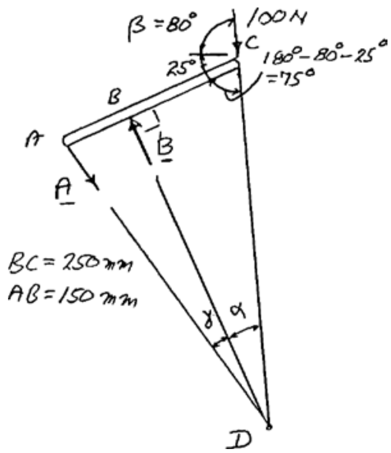
PROBLEM 4.47

Determine the reactions at A and B when $\beta = 80°$.

SOLUTION

Free-Body Diagram:

(Three-force body)



Reaction **A** must pass through D where the 100-N force and **B** intersect.

In right triangle BCD:

$$\alpha = 90° - 75° = 15°$$

$$BD = BC \tan 75° = 250 \tan 75°$$

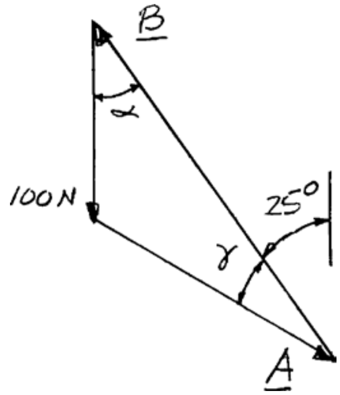
$$BD = 933.01 \text{ mm}$$

In right triangle ABD:

$$\tan \gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933.01 \text{ mm}} \qquad \gamma = 9.1333°$$

SOLUTION Continued

Force Triangle

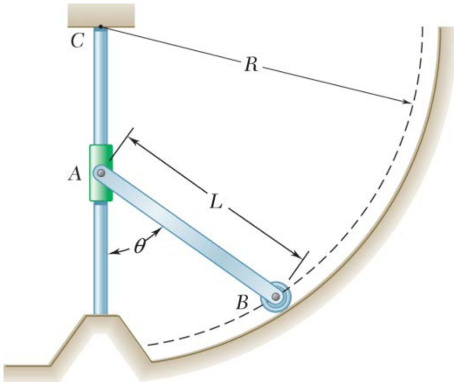


Law of sines:

$$\frac{100\text{ N}}{\sin 9.1333°} = \frac{A}{\sin 15°} = \frac{B}{\sin 155.867°}$$

A = 163.1 N ◥ 55.9° ◀

B = 258 N ◤ 65.0° ◀

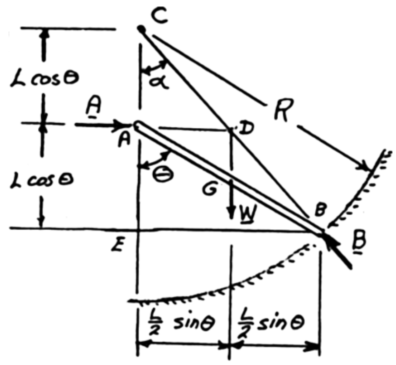


PROBLEM 4.48

A slender rod of length L and weight W is attached to a collar at A and is fitted with a small wheel at B. Knowing that the wheel rolls freely along a cylindrical surface of radius R, and neglecting friction, derive an equation in θ, L, and R that must be satisfied when the rod is in equilibrium.

SOLUTION

Free-Body Diagram (Three-force body)



Reaction **B** must pass through D where **B** and **W** intersect.

Note that ΔABC and ΔBGD are similar.

$$AC = AE = L\cos\theta$$

In ΔABC:

$$(CE)^2 + (BE)^2 = (BC)^2$$

$$(2L\cos\theta)^2 + (L\sin\theta)^2 = R^2$$

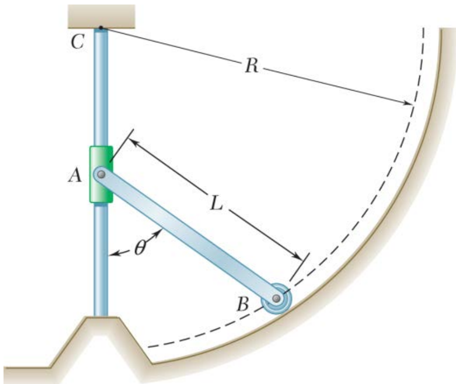
$$\left(\frac{R}{L}\right)^2 = 4\cos^2\theta + \sin^2\theta$$

$$\left(\frac{R}{L}\right)^2 = 4\cos^2\theta + 1 - \cos^2\theta$$

$$\left(\frac{R}{L}\right)^2 = 3\cos^2\theta + 1$$

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{R}{L}\right)^2 - 1\right] \blacktriangleleft$$

PROBLEM 4.49



Knowing that for the rod of Problem 4.48, L = 15 in., R = 20 in., and W = 10 lb, determine (a) the angle θ corresponding to equilibrium, (b) the reactions at A and B.

SOLUTION

See the solution to Problem 4.48 for the free-body diagram and analysis leading to the following equation:

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{R}{L}\right)^2 - 1\right]$$

For $L = 15$ in., $R = 20$ in., and $W = 10$ lb,

(a)

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{20\text{ in.}}{15\text{ in.}}\right)^2 - 1\right]; \quad \theta = 59.39°$$

$\theta = 59.4°$ ◀

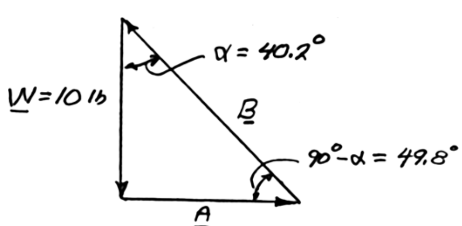
In ΔABC:

$$\tan\alpha = \frac{BE}{CE} = \frac{L\sin\theta}{2L\cos\theta} = \frac{1}{2}\tan\theta$$

$$\tan\alpha = \frac{1}{2}\tan 59.39° = 0.8452$$

$$\alpha = 40.2°$$

<u>Force Triangle</u>



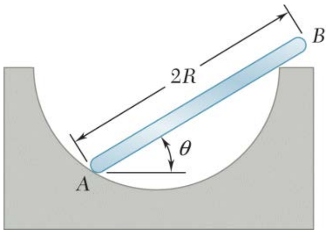
$$A = W\tan\alpha = (10\text{ lb})\tan 40.2° = 8.45\text{ lb}$$

$$B = \frac{W}{\cos\alpha} = \frac{(10\text{ lb})}{\cos 40.2°} = 13.09\text{ lb}$$

(b)

$\mathbf{A} = 8.45$ lb ⟶ ◀

$\mathbf{B} = 13.09$ lb ⦪ 49.8° ◀



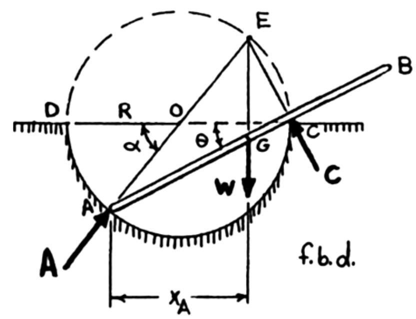
PROBLEM 4.50

A uniform rod *AB* of length 2*R* rests inside a hemispherical bowl of radius *R* as shown. Neglecting friction, determine the angle θ corresponding to equilibrium.

SOLUTION

Based on the F.B.D., the uniform rod *AB* is a three-force body. Point *E* is the point of intersection of the three forces. Since force *A* passes through *O*, the center of the circle, and since force **C** is perpendicular to the rod, triangle *ACE* is a right triangle inscribed in the circle. Thus, *E* is a point on the circle.

Note that the angle α of triangle *DOA* is the central angle corresponding to the inscribed angle θ of triangle *DCA*.



$$\alpha = 2\theta$$

The horizontal projections of $AE, (x_{AE})$, and $AG, (x_{AG})$, are equal.

$$x_{AE} = x_{AG} = x_A$$

or $$(AE)\cos 2\theta = (AG)\cos\theta$$

and $$(2R)\cos 2\theta = R\cos\theta$$

Now $$\cos 2\theta = 2\cos^2\theta - 1$$

then $$4\cos^2\theta - 2 = \cos\theta$$

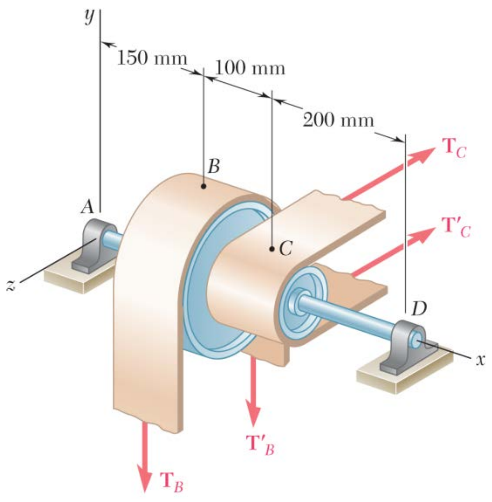
or $$4\cos^2\theta - \cos\theta - 2 = 0$$

Applying the quadratic equation,

$$\cos\theta = 0.84307 \quad \text{and} \quad \cos\theta = -0.59307$$

$$\theta = 32.534° \quad \text{and} \quad \theta = 126.375° \quad \text{(Discard)}$$

or $\theta = 32.5°$ ◀

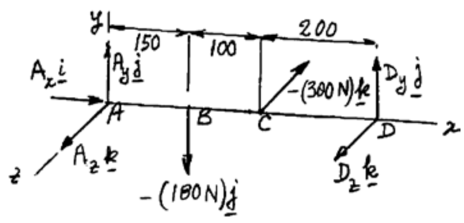


PROBLEM 4.51

Two transmission belts pass over a double-sheaved pulley that is attached to an axle supported by bearings at *A* and *D*. The radius of the inner sheave is 125 mm and the radius of the outer sheave is 250 mm. Knowing that when the system is at rest, the tension is 90 N in both portions of belt *B* and 150 N in both portions of belt *C*, determine the reactions at *A* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

We replace $\mathbf{T}_B$ and $\mathbf{T}'_B$ by their resultant $(-180\text{ N})\mathbf{j}$ and $\mathbf{T}_C$ and $\mathbf{T}'_C$ by their resultant $(-300\text{ N})\mathbf{k}$.



Dimensions in mm

We have five unknowns and six equations of equilibrium. Axle *AD* is free to rotate about the *x*-axis, but equilibrium is maintained $(\Sigma M_x = 0)$.

$$\Sigma\mathbf{M}_A = 0: \quad (150\mathbf{i})\times(-180\mathbf{j}) + (250\mathbf{i})\times(-300\mathbf{k}) + (450\mathbf{i})\times(D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-27\times10^3\mathbf{k} + 75\times10^3\mathbf{j} + 450D_y\mathbf{k} - 450D_z\mathbf{j} = 0$$

Equating coefficients of **j** and **k** to zero,

$$\mathbf{j}: \quad 75\times10^3 - 450D_z = 0 \qquad D_z = 166.7\text{ N}$$

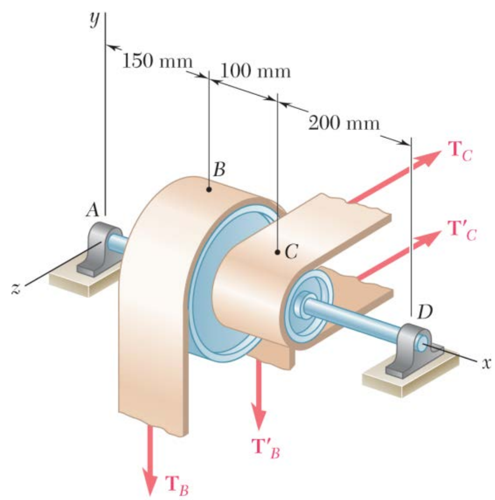
$$\mathbf{k}: \quad -27\times10^3 + 450D_y = 0 \qquad D_y = 60.0\text{ N}$$

$$\Sigma F_x = 0: \quad A_x = 0$$

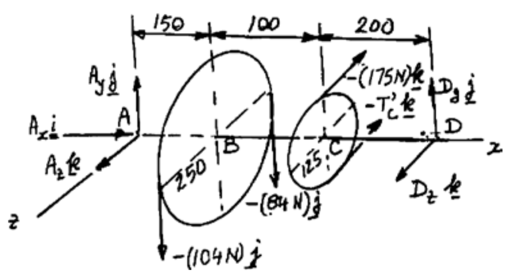
$$\Sigma F_y = 0: \quad A_y + D_y - 180\text{ N} = 0 \qquad A_y = 180 - 60 = 120.0\text{ N}$$

$$\Sigma F_z = 0: \quad A_z + D_z - 300\text{ N} = 0 \qquad A_z = 300 - 166.7 = 133.3\text{ N}$$

$$\mathbf{A} = (120.0\text{ N})\mathbf{j} + (133.3\text{ N})\mathbf{k}; \quad \mathbf{D} = (60.0\text{ N})\mathbf{j} + (166.7\text{ N})\mathbf{k} \blacktriangleleft$$

	**PROBLEM 4.52** Solve Problem 4.51, assuming that the pulley rotates at a constant rate and that T_B = 104 N, T'_B = 84 N, T_C = 175 N. **PROBLEM 4.51** Two transmission belts pass over a double-sheaved pulley that is attached to an axle supported by bearings at *A* and *D*. The radius of the inner sheave is 125 mm and the radius of the outer sheave is 250 mm. Knowing that when the system is at rest, the tension is 90 N in both portions of belt *B* and 150 N in both portions of belt *C*, determine the reactions at *A* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION



Dimensions in mm

We have six unknowns and six equations of equilibrium. —OK

$$\Sigma \mathbf{M}_A = 0: \quad (150\mathbf{i} + 250\mathbf{k}) \times (-104\mathbf{j}) + (150\mathbf{i} - 250\mathbf{k}) \times (-84\mathbf{j})$$
$$+ (250\mathbf{i} + 125\mathbf{j}) \times (-175\mathbf{k}) + (250\mathbf{i} - 125\mathbf{j}) \times (-T_C)$$
$$+ 450\mathbf{i} \times (D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-150(104 + 84)\mathbf{k} + 250(104 - 84)\mathbf{i} + 250(175 + T'_C)\,\mathbf{j} \;\; -125(175 - T'_C)$$
$$+ 450D_y\mathbf{k} - 450D_z\mathbf{j} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{i}: \quad 250(104 - 84) - 125(175 - T'_C) = 0 \qquad 175 = T'_C = 40 \qquad T'_C = 135;$$

$$\mathbf{j}: \quad 250(175 + 135) - 450D_z = 0 \qquad D_z = 172.2 \text{ N}$$

$$\mathbf{k}: \quad -150(104 + 84) + 450\,D_y = 0 \qquad D_y = 62.7 \text{ N}$$

SOLUTION Continued

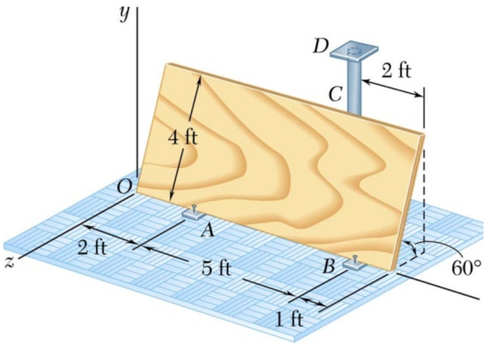
$$\Sigma F_x = 0: \qquad A_x = 0$$

$$\Sigma F_y = 0: \qquad A_y - 104 - 84 + 62.7 = 0 \qquad A_y = 125.3 \text{ N}$$

$$\Sigma F_z = 0: \qquad A_z - 175 - 135 + 172.2 = 0 \qquad A_z = 137.8 \text{ N}$$

$$\mathbf{A} = (125.3 \text{ N})\mathbf{j} + (137.8 \text{ N})\mathbf{k}; \quad \mathbf{D} = (62.7 \text{ N})\mathbf{j} + (172.2 \text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.53

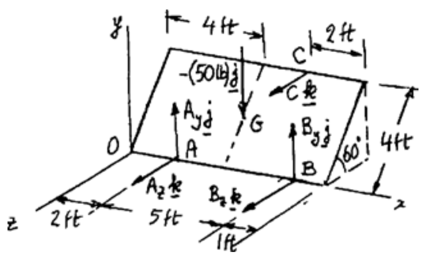


A 4 × 8-ft sheet of plywood weighing 40 lb has been temporarily propped against column *CD*. It rests at *A* and *B* on small wooden blocks and against protruding nails. Neglecting friction at all surfaces of contact, determine the reactions at *A*, *B*, and *C*.

SOLUTION

Free-Body Diagram:

We have five unknowns and six equations of equilibrium. Plywood sheet is free to move in x direction, but equilibrium is maintained $(\Sigma F_x = 0)$.



$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times (B_y\,\mathbf{j} + B_z\mathbf{k}) + \mathbf{r}_{C/A} \times C\,\mathbf{k} + \mathbf{r}_{G/A} \times (-40 \text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 4\sin 60^\circ & -4\cos 60^\circ \\ 0 & 0 & C \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2\sin 60^\circ & -2\cos 60^\circ \\ 0 & -40 & 0 \end{vmatrix} = 0$$

$$(4C\sin 60^\circ - 80\cos 60^\circ)\,\mathbf{i} + (-5B_z - 4C)\,\mathbf{j} + (5B_y - 80)\mathbf{k} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{i}: \quad 4C\sin 60^\circ - 80\cos 60^\circ = 0 \qquad C = 11.5470 \text{ lb}$$

$$\mathbf{j}: \quad -5B_z - 4C = 0 \qquad B_z = 9.2376 \text{ lb}$$

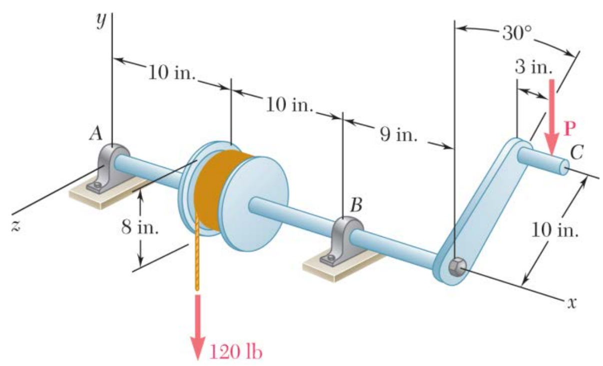
$$\mathbf{k}: \quad 5B_y - 80 = 0 \qquad B_y = 16.0000 \text{ lb}$$

$$\Sigma F_y = 0: \quad A_y + B_y - 40 = 0 \qquad A_y = 40 - 16.0000 = 24.000 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z + B_z + C = 0$$

$$A_z = 9.2376 - 11.5470 = -2.3094 \text{ lb}$$

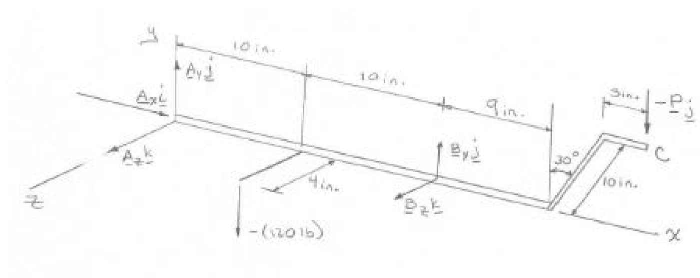
$$\mathbf{A} = (24.0 \text{ lb})\mathbf{j} - (2.31 \text{ lb})\mathbf{k}; \quad \mathbf{B} = (16.00 \text{ lb})\mathbf{j} - (9.24 \text{ lb})\mathbf{k}; \quad \mathbf{C} = (11.55 \text{ lb})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.54

A small winch is used to raise a 120-lb load. Find (*a*) the magnitude of the vertical force **P** that should be applied at *C* to maintain equilibrium in the position shown, (*b*) the reactions at *A* and *B*, assuming that the bearing at *B* does not exert any axial thrust.

SOLUTION



Dimensions in in.

We have six unknowns and six equations of equilibrium.

$$\mathbf{r}_C = (32\text{ in.})\mathbf{i} + (10\text{ in.})\cos 30^\circ\,\mathbf{j} - (10\text{ in.})\sin 30^\circ\,\mathbf{k}$$
$$= 32\mathbf{i} + 8.6603\mathbf{j} - 5\mathbf{k}$$

$$\Sigma\mathbf{M}_A = 0: \quad (10\mathbf{i} + 4\mathbf{k}) \times (-120\mathbf{j}) + (20\mathbf{i}) \times (B_y\mathbf{j} + B_z\mathbf{k}) + (32\mathbf{i} + 8.6603\mathbf{j} - 5\mathbf{k}) \times (-P\mathbf{j}) = 0$$
$$-1200\mathbf{k} + 480\mathbf{i} + 20B_y\mathbf{k} - 20B_z\mathbf{j} - 32P\mathbf{k} - 5P\mathbf{i} = 0$$

Equating the coefficients of the unit vectors to zero,

$\mathbf{i}$: $480 - 5P) = 0 \qquad P = 96.0$ lb; $\qquad$ (*a*) $P = 96.0$ lb ◀

$\mathbf{j}$: $20B_z = 0 \qquad B_z = 0$

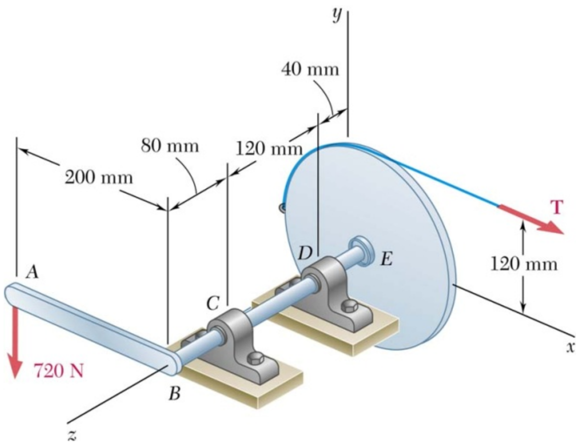
$\mathbf{k}$: $-1200 + 20B_y - 32(96.0) = 0 \qquad B_y = 213.6$ lb

$\Sigma F_x = 0: \qquad A_x = 0$

$\Sigma F_y = 0: \qquad A_y - 120 + 213.6 - 96.0 = 0 \qquad A_y = 2.40$ lb

$\Sigma F_z = 0: \qquad A_z + B_z = 0 \qquad A_z = -B_z = 0$

(*b*) $\mathbf{A} = (2.40\text{ lb})\mathbf{j}; \quad \mathbf{B} = (214\text{ lb})\mathbf{j}$ ◀

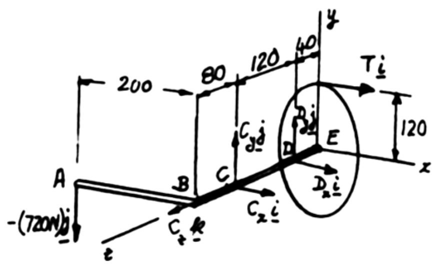


PROBLEM 4.55

A 200-mm lever and a 240-mm-diameter pulley are welded to the axle *BE* that is supported by bearings at *C* and *D*. If a 720-N vertical load is applied at *A* when the lever is horizontal, determine (*a*) the tension in the cord, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Dimensions in mm



We have six unknowns and six equations of equilibrium. —OK

$$\Sigma \mathbf{M}_C = 0: \quad (-120\mathbf{k}) \times (D_x\mathbf{i} + D_y\mathbf{j}) + (120\mathbf{j} - 160\mathbf{k}) \times T\mathbf{i} + (80\mathbf{k} - 200\mathbf{i}) \times (-720\mathbf{j}) = 0$$

..

Equating to zero the coefficients of the unit vectors:

$$\mathbf{k}: \quad -120T + 144 \times 10^3 = 0 \qquad (a)\quad T = 1200 \text{ N} \blacktriangleleft$$

$$\mathbf{i}: \quad 120D_y + 57.6 \times 10^3 = 0 \qquad D_y = -480 \text{ N}$$

$$\mathbf{j}: \quad -120D_x - 160(1200 \text{ N}) = 0 \qquad D_x = -1600 \text{ N}$$

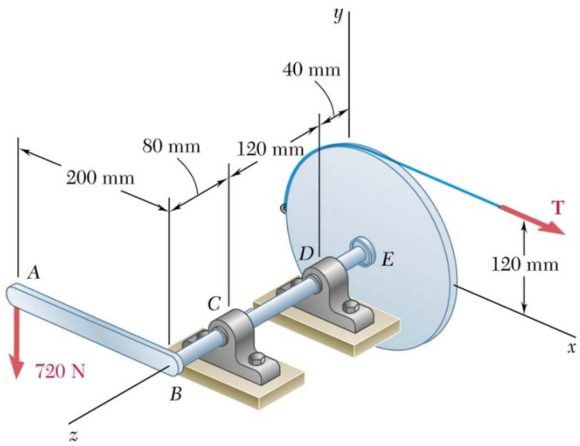
$$\Sigma F_x = 0: \quad C_x + D_x + T = 0 \qquad C_x = 1600 - 1200 = 400 \text{ N}$$

$$\Sigma F_y = 0: \quad C_y + D_y - 720 = 0 \qquad C_y = 480 + 720 = 1200 \text{ N}$$

$$\Sigma F_z = 0: \quad C_z = 0$$

(*b*) $\quad \mathbf{C} = (400 \text{ N})\mathbf{i} + (1200 \text{ N})\mathbf{j}; \quad \mathbf{D} = -(1600 \text{ N})\mathbf{i} - (480 \text{ N})\mathbf{j} \blacktriangleleft$

PROBLEM 4.56

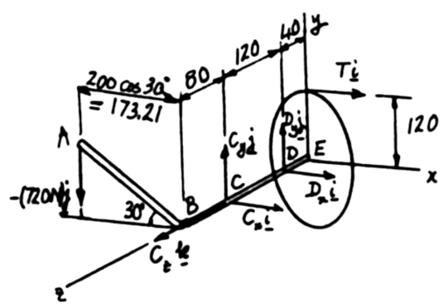


Solve Problem 4.55, assuming that the axle has been rotated clockwise in its bearings by 30° and that the 720-N load remains vertical.

PROBLEM 4.55 A 200-mm lever and a 240-mm-diameter pulley are welded to the axle *BE* that is supported by bearings at *C* and *D*. If a 720-N vertical load is applied at *A* when the lever is horizontal, determine (*a*) the tension in the cord, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Dimensions in mm



We have six unknowns and six equations of equilibrium.

(*a*) $\Sigma \mathbf{M}_C = 0: \quad (-120\mathbf{k}) \times (D_x\mathbf{i} + D_y\mathbf{j}) + (120\mathbf{j} - 160\mathbf{k}) \times T\mathbf{i} + (80\mathbf{k} - 173.21\mathbf{i}) \times (-720\mathbf{j}) = 0$

$$-120D_x\mathbf{j} + 120D_y\mathbf{i} - 120T\,\mathbf{k} - 160T\,\mathbf{j} + 57.6\times10^3\,\mathbf{i} + 124.71\times10^3\,\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors,

$\mathbf{k}: \quad -120T + 124.71\times10^3 = 0 \qquad T = 1039.2 \text{ N} \qquad\qquad T = 1039 \text{ N}$ ◀

$\mathbf{i}: \quad 120D_y + 57.6\times10^3 = 0 \qquad D_y = -480 \text{ N}$

$\mathbf{j}: \quad -120D_x - 160(1039.2) \qquad D_x = -1385.6 \text{ N}$

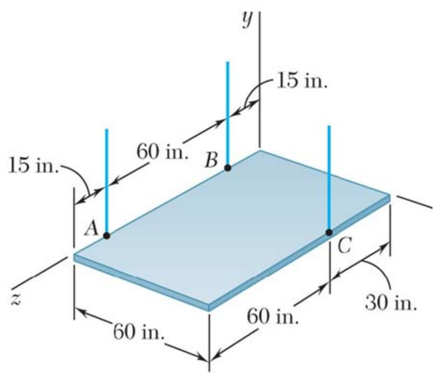
$\Sigma F_x = 0: \quad C_x + D_x + T = 0 \qquad C_x = 1385.6 - 1039.2 = 346.4$

$\Sigma F_y = 0: \quad C_y + D_y - 720 = 0 \qquad C_y = 480 + 720 = 1200 \text{ N}$

$\Sigma F_z = 0: \quad C_z = 0$

(*b*) $\mathbf{C} = (346 \text{ N})\mathbf{i} + (1200 \text{ N})\mathbf{j} \quad \mathbf{D} = -(1386 \text{ N})\mathbf{i} - (480 \text{ N})\mathbf{j}$ ◀

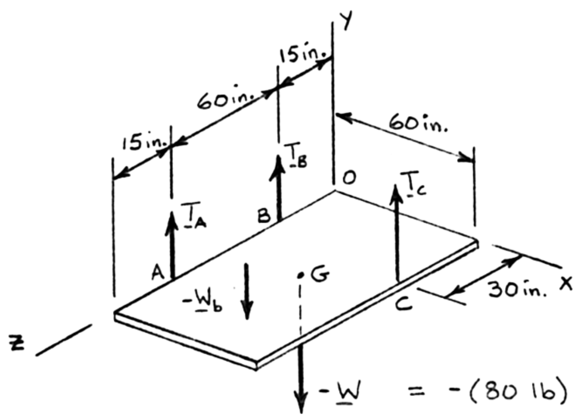
PROBLEM 4.57



The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the tension in each wire.

SOLUTION

Free-Body Diagram:



$$\Sigma \mathbf{M}_B = 0: \quad \mathbf{r}_{A/B} \times T_A \mathbf{j} + \mathbf{r}_{C/B} \times T_C \mathbf{j} + r_{G/B} \times (-80 \text{ lb})\mathbf{j} = 0$$

$$(60 \text{ in.})\mathbf{k} \times T_A \mathbf{j} + [(60 \text{ in.})\mathbf{i} + (15 \text{ in.})\mathbf{k}] \times T_C \mathbf{j} + [(30 \text{ in.})\mathbf{i} + (30 \text{ in.})\mathbf{k}] \times (-80 \text{ lb})\mathbf{j} = 0$$

$$-60T_A \mathbf{i} + 60T_C \mathbf{k} - 15T_C \mathbf{i} - 2400\mathbf{k} + 2400\mathbf{i} = 0$$

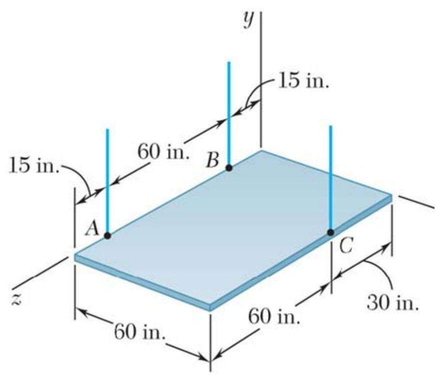
Equating to zero the coefficients of the unit vectors:

$\mathbf{i}$: $\quad 60T_A - 15(40) + 2400 = 0 \qquad T_A = 30.0 \text{ lb}$ ◀

$\mathbf{k}$: $\quad 60T_C - 2400 = 0 \qquad T_C = 40.0 \text{ lb}$ ◀

$\Sigma F_y = 0$: $\quad T_A + T_B + T_C - 80 \text{ lb} = 0$

$30 \text{ lb} + T_B + 40 \text{ lb} - 80 \text{ lb} = 0 \qquad T_B = 10.00 \text{ lb}$ ◀

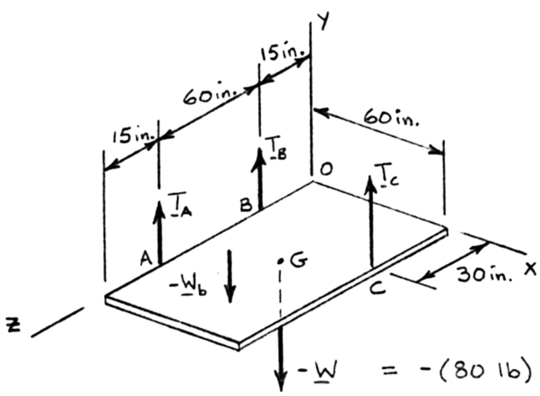


PROBLEM 4.58

The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the weight and location of the lightest block that should be placed on the plate if the tensions in the three wires are to be equal.

SOLUTION

Free-Body Diagram:



Let $-W_b\mathbf{j}$ be the weight of the block and x and z the block's coordinates.

Since tensions in wires are equal, let

$$T_A = T_B = T_C = T$$

$$\Sigma M_0 = 0: \quad (\mathbf{r}_A \times T\mathbf{j}) + (\mathbf{r}_B \times T\mathbf{j}) + (\mathbf{r}_C \times T\mathbf{j}) + \mathbf{r}_G \times (-W\mathbf{j}) + (x\mathbf{i} + z\mathbf{k}) \times (-W_b\mathbf{j}) = 0$$

or, $$(75\,\mathbf{k}) \times T\mathbf{j} + (15\,\mathbf{k}) \times T\mathbf{j} + (60\mathbf{i} + 30\mathbf{k}) \times T\mathbf{j} + (30\mathbf{i} + 45\mathbf{k}) \times (-W\mathbf{j}) + (x\mathbf{i} + z\mathbf{k}) \times (-W_b\mathbf{j}) = 0$$

or, $$-75T\mathbf{i} - 15T\,\mathbf{i} + 60T\,\mathbf{k} - 30T\,\mathbf{i} - 30W\,\mathbf{k} + 45W\,\mathbf{i} - W_b \times \mathbf{k} + W_b z\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -120T + 45W + W_b z = 0 \qquad (1)$$

$$\mathbf{k}: \quad 60T - 30W - W_b x = 0 \qquad (2)$$

Also, $$\Sigma F_y = 0: \quad 3T - W - W_b = 0 \qquad (3)$$

Eq. (1) + 40 Eq. (3): $$5W + (z - 40)W_b = 0 \qquad (4)$$

Eq. (2) – 20 Eq. (3): $$-10W - (x - 20)W_b = 0 \qquad (5)$$

SOLUTION Continued

Solving (4) and (5) for W_b/W and recalling of $0 \le x \le 60$ in., $0 \le z \le 90$ in.,

(4): $$\frac{W_b}{W} = \frac{5}{40 - z} \ge \frac{5}{40 - 0} = 0.125$$

(5): $$\frac{W_b}{W} = \frac{10}{20 - x} \ge \frac{10}{20 - 0} = 0.5$$

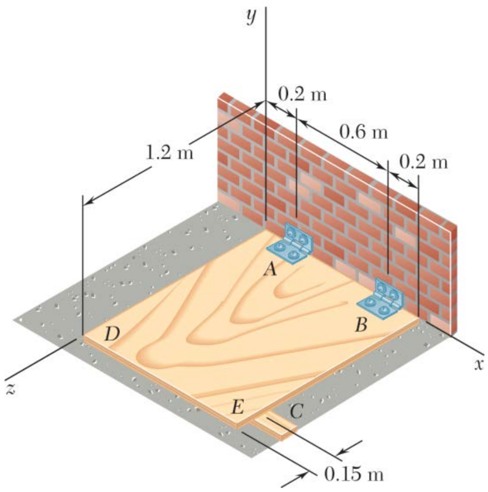
Thus, $(W_b)_{\min} = 0.5W = 0.5(80) = 40$ lb $\qquad (W_b)_{\min} = 40.0$ lb ◀

Making $W_b = 0.5W$ in (4) and (5):

$$5W + (z - 40)(0.5W) = 0 \qquad z = 30.0 \text{ in.} ◀$$

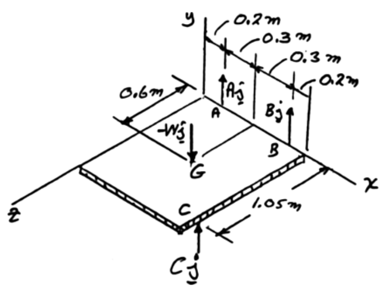
$$-10W - (x - 20)(0.5W) = 0 \qquad x = 0 \text{ in.} ◀$$

PROBLEM 4.59



An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at *A* and *B* and is maintained in a position slightly above the floor by a small block *C*. Determine the vertical component of the reaction (*a*) at *A*, (*b*) at *B*, (*c*) at *C*.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$
$$\mathbf{r}_{C/A} = 0.8\mathbf{i} + 1.05\mathbf{k}$$
$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18\text{ kg})9.81$$
$$W = 176.58\text{ N}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$(0.6\mathbf{i}) \times B\mathbf{j} + (0.8\mathbf{i} + 1.05\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$0.6B\mathbf{k} + 0.8C\mathbf{k} - 1.05C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad 1.05C + 0.6W = 0 \quad C = \left(\frac{0.6}{1.05}\right)176.58\text{ N} = 100.90\text{ N}$$

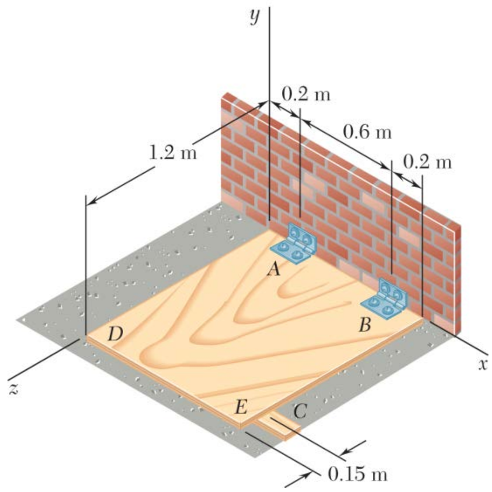
$$\mathbf{k}: \quad 0.6B + 0.8C - 0.3W = 0$$

$$0.6B + 0.8(100.90\text{ N}) - 0.3(176.58\text{ N}) = 0 \quad B = -46.24\text{ N}$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A - 46.24\text{ N} + 100.90\text{ N} + 176.58\text{ N} = 0 \quad A = 121.92\text{ N}$$

$$(a) \quad A = 121.9\text{ N} \quad (b) \quad B = -46.2\text{ N} \quad (c) \quad C = 100.9\text{ N} \blacktriangleleft$$

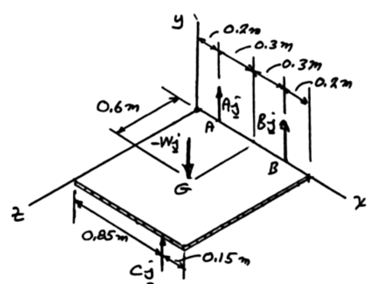


PROBLEM 4.60

Solve Problem 4.59, assuming that the small block C is moved and placed under edge DE at a point 0.15 m from corner E.

PROBLEM 4.59 An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at A and B and is maintained in a position slightly above the floor by a small block C. Determine the vertical component of the reaction (*a*) at A, (*b*) at B, (*c*) at C.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$
$$\mathbf{r}_{C/A} = 0.65\mathbf{i} + 1.2\mathbf{k}$$
$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18 \text{ kg})\, 9.81 \text{ m/s}^2$$
$$W = 176.58 \text{ N}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$0.6\mathbf{i} \times B\mathbf{j} + (0.65\mathbf{i} + 1.2\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$0.6B\mathbf{k} + 0.65C\mathbf{k} - 1.2C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -1.2C + 0.6W = 0 \qquad C = \left(\frac{0.6}{1.2}\right) 176.58 \text{ N} = 88.29 \text{ N}$$

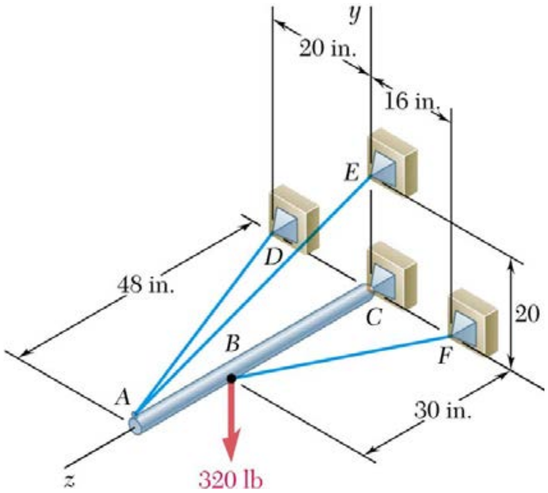
$$\mathbf{k}: \quad 0.6B + 0.65C - 0.3W = 0$$

$$0.6B + 0.65(88.29 \text{ N}) - 0.3(176.58 \text{ N}) = 0 \qquad B = -7.36 \text{ N}$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A - 7.36 \text{ N} + 88.29 \text{ N} - 176.58 \text{ N} = 0 \qquad A = 95.648 \text{ N}$$

$$(a) \quad A = 95.6 \text{ N} \quad (b) \quad B = -7.36 \text{ N} \quad (c) \quad C = 88.3 \text{ N} \blacktriangleleft$$

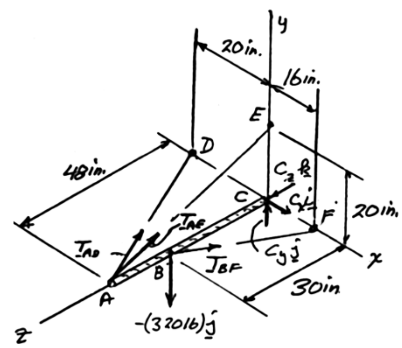


PROBLEM 4.61

A 48-in. boom is held by a ball-and-socket joint at C and by two cables BF and DAE; cable DAE passes around a frictionless pulley at A. For the loading shown, determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram:



Five unknowns and six equations of equilibrium, but equilibrium is maintained $(\Sigma M_{AC} = 0)$.

T = Tension in both parts of cable DAE.

$$\mathbf{r}_B = 30\mathbf{k}$$
$$\mathbf{r}_A = 48\mathbf{k}$$

$$\overrightarrow{AD} = -20\mathbf{i} - 48\mathbf{k} \qquad AD = 52 \text{ in.}$$
$$\overrightarrow{AE} = 20\mathbf{j} - 48\mathbf{k} \qquad AE = 52 \text{ in.}$$
$$\overrightarrow{BF} = 16\mathbf{i} - 30\mathbf{k} \qquad BF = 34 \text{ in.}$$

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD} = \frac{T}{52}(-20\mathbf{i} - 48\mathbf{k}) = \frac{T}{13}(-5\mathbf{i} - 12\mathbf{k})$$
$$\mathbf{T}_{AE} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{52}(20\mathbf{j} - 48\mathbf{k}) = \frac{T}{13}(5\mathbf{j} - 12\mathbf{k})$$
$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = \frac{T_{BF}}{34}(16\mathbf{i} - 30\mathbf{k}) = \frac{T_{BF}}{17}(8\mathbf{i} - 15\mathbf{k})$$

$$\Sigma\mathbf{M}_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times \mathbf{T}_{BF} + \mathbf{r}_B \times (-320 \text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix}\frac{T_{BF}}{17} + (30\mathbf{k}) \times (-320\mathbf{j}) = 0$$

Coefficient of $\mathbf{i}$: $\qquad -\frac{240}{13}T + 9600 = 0 \qquad T = 520 \text{ lb}$

SOLUTION Continued

Coefficient of **j**: $-\frac{240}{13}T + \frac{240}{17}T_{BD} = 0$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(520) \quad T_{BD} = 680 \text{ lb}$$

$\Sigma \mathbf{F} = 0$: $\mathbf{T}_{AD} + \mathbf{T}_{AE} + \mathbf{T}_{BF} - 320\mathbf{j} + C = 0$

Coefficient of **i**: $-\frac{20}{52}(520) + \frac{8}{17}(680) + C_x = 0$

$$-200 + 320 + C_x = 0 \qquad C_x = -120 \text{ lb}$$

Coefficient of **j**: $\frac{20}{52}(520) - 320 + C_y = 0$

$$200 - 320 + C_y = 0 \qquad C_y = 120 \text{ lb}$$

Coefficient of **k**: $-\frac{48}{52}(520) - \frac{48}{52}(520) - \frac{30}{34}(680) + C_z = 0$

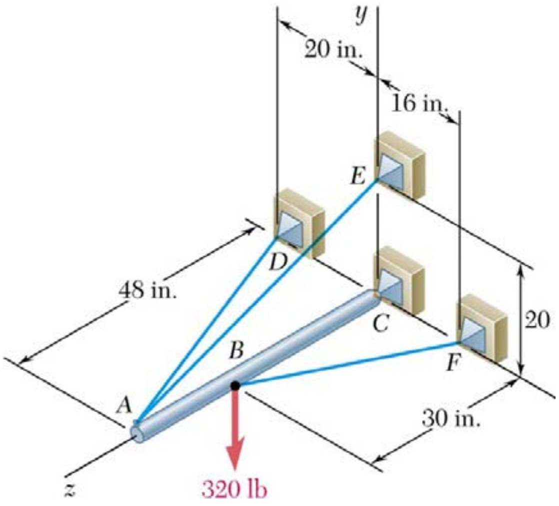
$$-480 - 480 - 600 + C_z = 0$$

$$C_z = 1560 \text{ lb}$$

Answers: $T_{DAE} = T$ $\qquad T_{DAE} = 520 \text{ lb}$ ◀

$T_{BD} = 680 \text{ lb}$ ◀

$\mathbf{C} = -(120.0 \text{ lb})\mathbf{i} + (120.0 \text{ lb})\mathbf{j} + (1560 \text{ lb})\mathbf{k}$ ◀



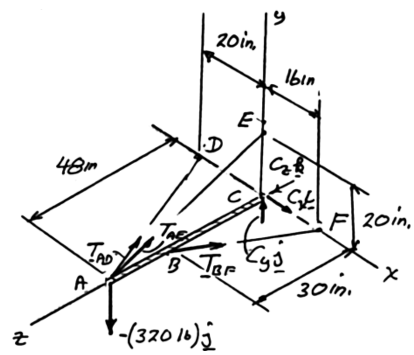
PROBLEM 4.62

Solve Problem 4.61, assuming that the 320-lb load is applied at *A*.

PROBLEM 4.61 A 48-in. boom is held by a ball-and-socket joint at *C* and by two cables *BF* and *DAE*; cable *DAE* passes around a frictionless pulley at *A*. For the loading shown, determine the tension in each cable and the reaction at *C*.

SOLUTION

Free-Body Diagram:



Five unknowns and six equations of equilibrium, but equilibrium is maintained $(\Sigma M_{AC} = 0)$.

T = tension in both parts of cable *DAE*.

$$\mathbf{r}_B = 30\mathbf{k}$$
$$\mathbf{r}_A = 48\mathbf{k}$$

$$\overrightarrow{AD} = -20\mathbf{i} - 48\mathbf{k} \qquad AD = 52 \text{ in.}$$
$$\overrightarrow{AE} = 20\mathbf{j} - 48\mathbf{k} \qquad AE = 52 \text{ in.}$$
$$\overrightarrow{BF} = 16\mathbf{i} - 30\mathbf{k} \qquad BF = 34 \text{ in.}$$

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD} = \frac{T}{52}(-20\mathbf{i} - 48\mathbf{k}) = \frac{T}{13}(-5\mathbf{i} - 12\mathbf{k})$$
$$\mathbf{T}_{AE} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{52}(20\mathbf{j} - 48\mathbf{k}) = \frac{T}{13}(5\mathbf{j} - 12\mathbf{k})$$
$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = \frac{T_{BF}}{34}(16\mathbf{i} - 30\mathbf{k}) = \frac{T_{BF}}{17}(8\mathbf{i} - 15\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times \mathbf{T}_{BF} + \mathbf{r}_A \times (-320 \text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix}\frac{T_{BF}}{17} + 48\mathbf{k} \times (-320\mathbf{j}) = 0$$

Coefficient of **i**: $\quad -\frac{240}{13}T + 15{,}360 = 0 \qquad T = 832 \text{ lb}$

SOLUTION Continued

Coefficient of **j**: $$-\frac{240}{13}T + \frac{240}{17}T_{BD} = 0$$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(832) \qquad T_{BD} = 1088 \text{ lb}$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AD} + \mathbf{T}_{AE} + \mathbf{T}_{BF} - 320\mathbf{j} + \mathbf{C} = 0$$

Coefficient of **i**: $$-\frac{20}{52}(832) + \frac{8}{17}(1088) + C_x = 0$$

$$-320 + 512 + C_x = 0 \qquad C_x = -192 \text{ lb}$$

Coefficient of **j**: $$\frac{20}{52}(832) - 320 + C_y = 0$$

$$320 - 320 + C_y = 0 \qquad C_y = 0$$

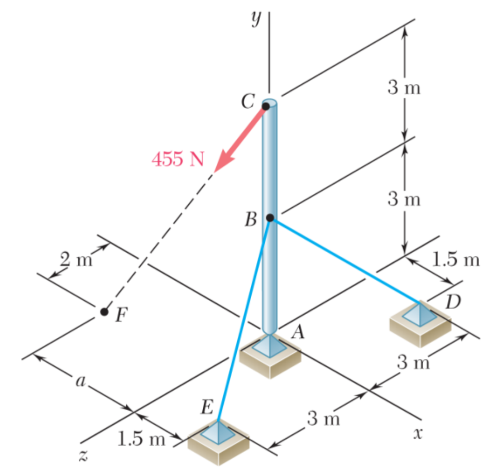
Coefficient of **k**: $$-\frac{48}{52}(832) - \frac{48}{52}(852) - \frac{30}{34}(1088) + C_z = 0$$

$$-768 - 768 - 960 + C_z = 0 \qquad C_z = 2496 \text{ lb}$$

Answers: $T_{DAE} = T$ $\qquad T_{DAE} = 832 \text{ lb}$ ◀

$T_{BD} = 1088 \text{ lb}$ ◀

$\mathbf{C} = -(192.0 \text{ lb})\mathbf{i} + (2496 \text{ lb})\mathbf{k}$ ◀

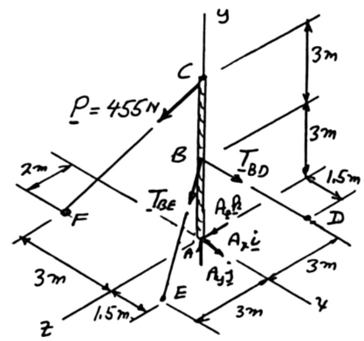


PROBLEM 4.63

The 6-m pole *ABC* is acted upon by a 455-N force as shown. The pole is held by a ball-and-socket joint at *A* and by two cables *BD* and *BE*. For $a = 3$ m, determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:



Five unknowns and six Eqs. of equilibrium, but equilibrium is maintained

$$(\Sigma M_{AC} = 0)$$

$$\mathbf{r}_B = 3\mathbf{j}$$
$$\mathbf{r}_C = 6\mathbf{j}$$

$$\overrightarrow{CF} = -3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k} \qquad CF = 7 \text{ m}$$
$$\overrightarrow{BD} = 1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k} \qquad BD = 4.5 \text{ m}$$
$$\overrightarrow{BE} = 1.5\mathbf{i} - 3\mathbf{j} + 3\mathbf{k} \qquad BE = 4.5 \text{ m}$$

$$\mathbf{P} = P\frac{\overrightarrow{CF}}{CE} = \frac{P}{7}(-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})$$

$$\mathbf{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{4.5}(1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}) = \frac{T_{BD}}{3}(\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$

$$\mathbf{T}_{BE} = T_{BE} = \frac{\overrightarrow{BE}}{BE} = \frac{T_{BD}}{3}(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_B \times \mathbf{T}_{BD} + \mathbf{r}_B \times \mathbf{T}_{BE} + \mathbf{r}_C \times \mathbf{P} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 1 & -2 & -2 \end{vmatrix} \frac{T_{BD}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 1 & -2 & 2 \end{vmatrix} \frac{T_{BE}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ -3 & -6 & 2 \end{vmatrix} \frac{P}{7} = 0$$

Coefficient of **i**:
$$-2T_{BD} + 2T_{BE} + \frac{12}{7}P = 0 \qquad (1)$$

Coefficient of **k**:
$$-T_{BD} - T_{BF} + \frac{18}{7}P = 0 \qquad (2)$$

SOLUTION Continued

Eq. (1) + 2 Eq. (2): $-4T_{BD} + \frac{48}{7}P = 0 \quad T_{BD} = \frac{12}{7}P$

Eq. (2): $-\frac{12}{7}P - T_{BE} + \frac{18}{7}P = 0 \quad T_{BE} = \frac{6}{7}P$

Since $P = 445 \text{ N} \quad T_{BD} = \frac{12}{7}(455)$ $\qquad T_{BD} = 780 \text{ N}$ ◀

$T_{BE} = \frac{6}{7}(455)$ $\qquad T_{BE} = 390 \text{ N}$ ◀

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{BD} + \mathbf{T}_{BE} + \mathbf{P} + \mathbf{A} = 0$$

Coefficient of **i**: $\frac{780}{3} + \frac{390}{3} - \frac{455}{7}(3) + A_x = 0$

$$260 + 130 - 195 + A_x = 0 \quad A_x = 195.0 \text{ N}$$

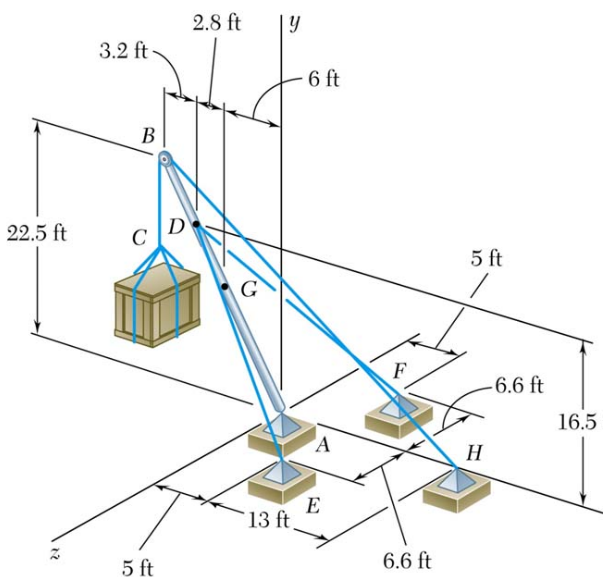
Coefficient of **j**: $-\frac{780}{3}(2) - \frac{390}{3}(2) - \frac{455}{7}(6) + A_y = 0$

$$-520 - 260 - 390 + A_y = 0 \quad A_y = 1170 \text{ N}$$

Coefficient of **k**: $-\frac{780}{3}(2) + \frac{390}{3}(2) + \frac{455}{7}(2) + A_z = 0$

$$-520 + 260 + 130 + A_z = 0 \quad A_z = +130.0 \text{ N}$$

$\mathbf{A} = -(195.0 \text{ N})\mathbf{i} + (1170 \text{ N})\mathbf{j} + (130.0 \text{ N})\mathbf{k}$ ◀

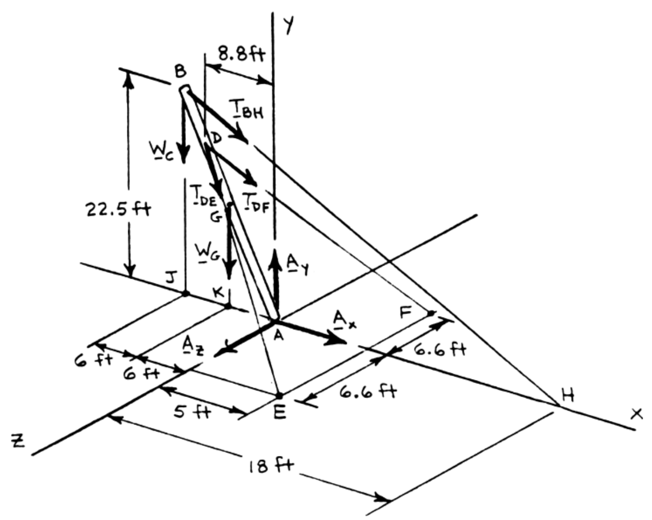


PROBLEM 4.64

A 600-lb crate hangs from a cable that passes over a pulley *B* and is attached to a support at *H*. The 200-lb boom *AB* is supported by a ball-and-socket joint at *A* and by two cables *DE* and *DF*. The center of gravity of the boom is located at *G*. Determine (*a*) the tension in cables *DE* and *DF*, (*b*) the reaction at *A*.

SOLUTION

Free-Body Diagram:



$$W_C = 600 \text{ lb}$$
$$W_G = 200 \text{ lb}$$

We have five unknowns $(T_{DE}, T_{DF}, A_x, A_y, A_z)$ and five equilibrium equations. The boom is free to spin about the AB axis, but equilibrium is maintained, since $\Sigma M_{AB} = 0$.

We have

$$\overline{BH} = (30 \text{ ft})\mathbf{i} - (22.5 \text{ ft})\mathbf{j} \qquad BH = 37.5 \text{ ft}$$

$$\overline{DE} = (13.8 \text{ ft})\mathbf{i} - \frac{8.8}{12}(22.5 \text{ ft})\mathbf{j} + (6.6 \text{ ft})\mathbf{k}$$

$$= (13.8 \text{ ft})\mathbf{i} - (16.5 \text{ ft})\mathbf{j} + (6.6 \text{ ft})\mathbf{k} \qquad DE = 22.5 \text{ ft}$$

$$\overline{DF} = (13.8 \text{ ft})\mathbf{i} - (16.5 \text{ ft})\mathbf{j} - (6.6 \text{ ft})\mathbf{k} \qquad DF = 22.5 \text{ ft}$$

SOLUTION Continued

Thus:

$$\mathbf{T}_{BH} = \mathbf{T}_{BH}\frac{\overrightarrow{BH}}{BH} = (600\text{ lb})\frac{30\mathbf{i}-22.5\mathbf{j}}{37.5} = (480\text{ lb})\mathbf{i} - (360\text{ lb})\mathbf{j}$$

$$\mathbf{T}_{DE} = \mathbf{T}_{DE}\frac{\overrightarrow{DE}}{DE} = \frac{T_{DE}}{22.5}(13.8\mathbf{i} - 16.5\mathbf{j} + 6.6\mathbf{k})$$

$$\mathbf{T}_{DF} = \mathbf{T}_{DF}\frac{\overrightarrow{DF}}{DF} = \frac{T_{DE}}{22.5}(13.8\mathbf{i} - 16.5\mathbf{j} - 6.6\mathbf{k})$$

(*a*) $\Sigma\mathbf{M}_A = 0$: $(\mathbf{r}_J \times \mathbf{W}_C) + (\mathbf{r}_K \times \mathbf{W}_G) + (\mathbf{r}_H \times \mathbf{T}_{BH}) + (\mathbf{r}_E \times \mathbf{T}_{DE}) + (\mathbf{r}_F \times \mathbf{T}_{DF}) = 0$

$$-(12\mathbf{i}) \times (-600\mathbf{j}) - (6\mathbf{i}) \times (-200\mathbf{j}) + (18\mathbf{i}) \times (480\mathbf{i} - 360\mathbf{j})$$

$$+\frac{T_{DE}}{22.5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 6.6 \\ 13.8 & -16.5 & 6.6 \end{vmatrix} + \frac{T_{DF}}{22.5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & -6.6 \\ 13.8 & -16.5 & -6.6 \end{vmatrix} = 0$$

or

$$7200\mathbf{k} + 1200\mathbf{k} - 6480\mathbf{k} + 4.84(T_{DE} - T_{DF})\mathbf{i}$$

$$+\frac{58.08}{22.5}(T_{DE} - T_{DF})\mathbf{j} - \frac{82.5}{22.5}(T_{DE} + T_{DF})\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors,

i or **j**: $T_{DE} - T_{DF} = 0 \qquad T_{DE} = T_{DF}$*

k: $7200 + 1200 - 6480 - \frac{82.5}{22.5}(2T_{DE}) = 0 \qquad T_{DE} = 261.82\text{ lb}$

$T_{DE} = T_{DF} = 262\text{ lb}$ ◀

(*b*) $\Sigma F_x = 0$: $A_x + 480 + 2\left(\frac{13.8}{22.5}\right)(261.82) = 0 \qquad A_x = -801.17\text{ lb}$

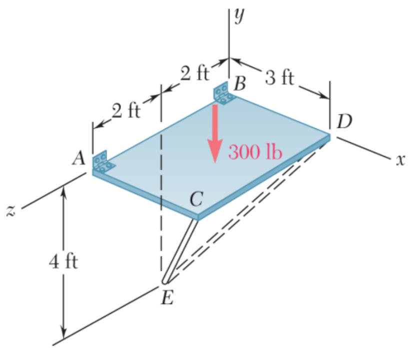
$\Sigma F_y = 0$: $A_y - 600 - 200 - 360 - 2\left(\frac{16.5}{22.5}\right)(261.82) = 0 \qquad A_y = 1544.00\text{ lb}$

$\Sigma F_z = 0$: $A_z = 0$

$\mathbf{A} = -(801\text{ lb})\mathbf{i} + (1544\text{ lb})\mathbf{j}$ ◀

*Remark: The fact is that $T_{DE} = T_{DF}$ could have been noted at the outset from the symmetry of structure with respect to *xy* plane.

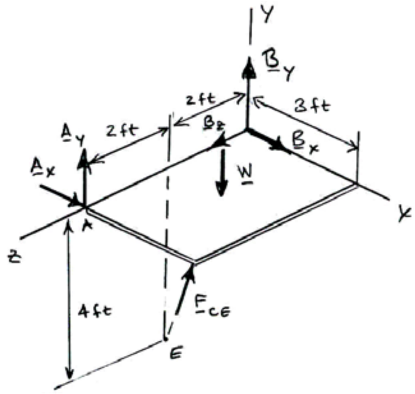
PROBLEM 4.65



The horizontal platform *ABCD* weighs 60 lb and supports a 240-lb load at its center. The platform is normally held in position by hinges at *A* and *B* and by braces *CE* and *DE*. If brace *DE* is removed, determine the reactions at the hinges and the force exerted by the remaining brace *CE*. The hinge at *A* does not exert any axial thrust.

SOLUTION

Free-Body Diagram:



Express forces, weight in terms of rectangular components:

$$\overrightarrow{EC} = (3\text{ ft})\mathbf{i} + (4\text{ ft})\mathbf{j} + (2\text{ ft})\mathbf{k}$$

$$\mathbf{F}_{CE} = F_{CE}\frac{\overrightarrow{EC}}{EC} = F_{CE}\frac{3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}}{\sqrt{(3)^2 + (4)^2 + (2)^2}}$$

$$= 0.55709 F_{CE}\,\mathbf{i} + 0.74278 F_{CE}\,\mathbf{j} + 0.37139 F_{CE}\,\mathbf{k}$$

$$\mathbf{W} = -(mg)\mathbf{j} = -(300\text{ lb})\mathbf{j}$$

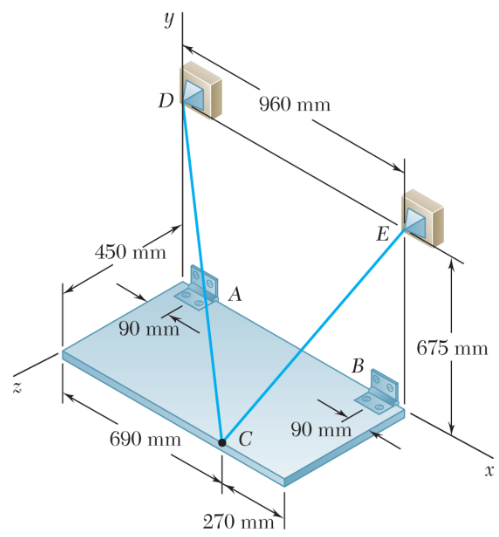
$$\Sigma\mathbf{M}_B = 0: \quad (4\text{ ft})\mathbf{k}\times\mathbf{A} + \left[(1.5\text{ ft})\mathbf{i} + (2\text{ ft})\mathbf{k}\right]\times(-300\text{ lb})\mathbf{j}$$

$$+\left[(3\text{ ft})\mathbf{i} + (4\text{ ft})\mathbf{k}\right]\times\mathbf{F}_{CE} = 0$$

or

$$-(4\text{ ft})A_y\mathbf{i} + (4\text{ ft})A_z\mathbf{j} - (1.5\text{ ft})(300\text{ lb})\mathbf{k} + (2\text{ ft})(300\text{ lb})\mathbf{i}$$

$$+\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 4 \\ 0.55709 & 0.74278 & 0.37139 \end{vmatrix} F_{CE}(\text{ft}) = 0$$

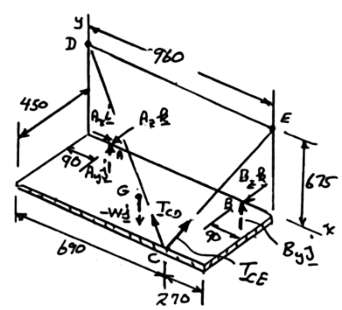


PROBLEM 4.66

A 100-kg uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE* that passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION

Dimensions in mm



$$\mathbf{r}_{B/A}(960-180)\mathbf{i} = 780\mathbf{i}$$

$$\mathbf{r}_{G/A} = \left(\frac{960}{2} - 90\right)\mathbf{i} + \frac{450}{2}\mathbf{k}$$

$$= 390\mathbf{i} + 225\mathbf{k}$$

$$\mathbf{r}_{C/A} = 600\mathbf{i} + 450\mathbf{k}$$

(*a*) T = Tension in cable *DCE*

$$\overrightarrow{CD} = -690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \qquad CD = 1065 \text{ mm}$$

$$\overrightarrow{CE} = 270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \qquad CE = 855 \text{ mm}$$

$$\mathbf{T}_{CD} = \frac{T}{1065}(-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{T}_{CE} = \frac{T}{855}(270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{W} = -mg\mathbf{i} = -(100 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(981 \text{ N})\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{C/A} \times \mathbf{T}_{CD} + \mathbf{r}_{C/A} \times \mathbf{T}_{CE} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ -690 & 675 & -450 \end{vmatrix} \frac{T}{1065} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

SOLUTION Continued

Coefficient of **i**: $$-(450)(675)\frac{T}{1065}-(450)(675)\frac{T}{855}+220.73\times10^{3}=0$$

$$T=344.64\text{ N}\quad T=345\text{ N} \blacktriangleleft$$

Coefficient of **j**: $$(-690\times450+600\times450)\frac{344.64}{1065}+(270\times450+600\times450)\frac{344.64}{855}-780B_z=0$$

$$B_z=185.516\text{ N}$$

Coefficient of **k**: $$(600)(675)\frac{344.64}{1065}+(600)(675)\frac{344.64}{855}-382.59\times10^{3}+780B_y=0\quad B_y=113.178\text{ N}$$

(*b*) $$\mathbf{B}=(113.2\text{ N})\mathbf{j}+(185.5\text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma\mathbf{F}=0:\quad \mathbf{A}+\mathbf{B}+\mathbf{T}_{CD}+\mathbf{T}_{CE}+\mathbf{W}=0$$

Coefficient of **i**: $$A_x-\frac{690}{1065}(344.64)+\frac{270}{855}(344.64)=0 \qquad A_x=114.454\text{ N}$$

Coefficient of **j**: $$A_y+113.178+\frac{675}{1065}(344.64)+\frac{675}{855}(344.64)-981=0\quad A_y=377.30\text{ N}$$

Coefficient of **k**: $$A_z+185.516-\frac{450}{1065}(344.64)-\frac{450}{855}(344.64)=0 \qquad A_z=141.496\text{ N}$$

$$\mathbf{A}=(114.4\text{ N})\mathbf{i}+(377\text{ N})\mathbf{j}+(141.5\text{ N})\mathbf{k} \blacktriangleleft$$

SOLUTION Continued

Setting the coefficients of the unit vectors equal to zero:

$\mathbf{k}$: $\quad (0.74278 F_{CE})(3 \text{ ft}) - (300 \text{ lb})(1.5 \text{ ft}) = 0$

$$F_{CE} = 201.94 \text{ lb}$$

or $\quad F_{CE} = 202 \text{ lb}$ ◀

$\mathbf{j}$: $\quad A_x(4 \text{ ft}) + [0.55709(201.94 \text{ lb})](4 \text{ ft}) - [0.37139(201.94 \text{ lb})](3 \text{ ft}) = 0$

$$A_x = -56.250 \text{ lb}$$

$\mathbf{i}$: $\quad -A_y(4 \text{ ft}) - [0.74278(201.94 \text{ lb})](4 \text{ ft}) + (300 \text{ lb})(2 \text{ ft}) = 0$

$$A_y = 0$$

$\Sigma F_x = 0$: $\quad -56.250 \text{ lb} + B_x + 0.55709(201.94 \text{ lb}) = 0$

$$B_x = -56.249 \text{ lb}$$

$\Sigma F_y = 0$: $\quad 0 + B_y - 300 \text{ lb} + 0.74278(201.94 \text{ lb}) = 0$

$$B_y = 150.003 \text{ lb}$$

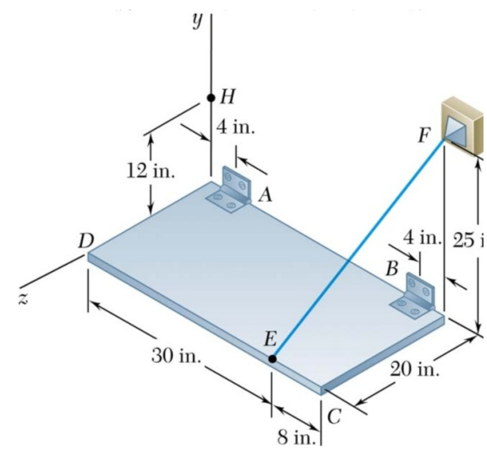
$\Sigma F_z = 0$: $\quad B_z + 0.371391(201.94 \text{ lb}) = 0$

$$B_z = -74.999 \text{ lb}$$

Therefore:

$\mathbf{A} = -(56.3 \text{ lb})\mathbf{i}$ ◀

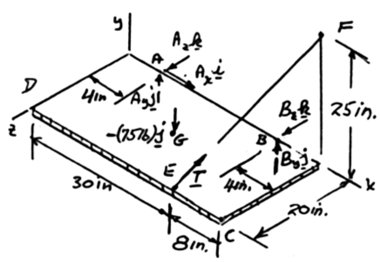
$\mathbf{B} = -(56.2 \text{ lb})\mathbf{i} + (150.0 \text{ lb})\mathbf{j} - (75.0 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.67

The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at A and B and by cable EF. Assuming that the hinge at B does not exert any axial thrust, determine (a) the tension in the cable, (b) the reactions at A and B.

SOLUTION



$$\mathbf{r}_{B/A} = (38-8)\mathbf{i} = 30\mathbf{i}$$

$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k}$$
$$= 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = \frac{38}{2}\mathbf{i} + 10\mathbf{k}$$
$$= 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{EF} = 8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k}$$

$$EF = 33 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{33}(8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{E/A} \times T + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + \mathbf{r}_{B/A} \times B = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -20 \end{vmatrix}\frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

(a) Coefficient of **i**: $\quad -(25)(20)\dfrac{T}{33} + 750 = 0:$ $\qquad T = 49.5 \text{ lb}$ ◀

Coefficient of **j**: $\quad (160+520)\dfrac{49.5}{33} - 30B_z = 0: \quad B_z = 34 \text{ lb}$

(b) Coefficient of **k**: $\quad (26)(25)\dfrac{49.5}{33} - 1425 + 30B_y = 0: \quad B_y = 15 \text{ lb}$ $\qquad \mathbf{B} = (15.00 \text{ lb})\mathbf{j} + (34.0 \text{ lb})\mathbf{k}$ ◀

SOLUTION Continued

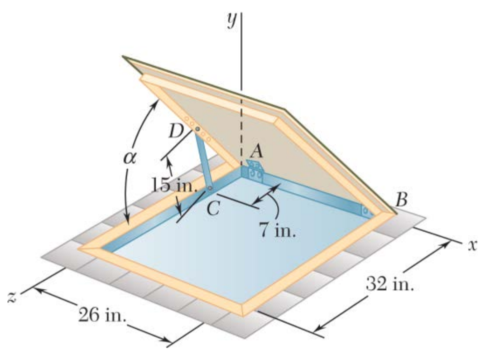
$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T} - (75 \text{ lb})\mathbf{j} = 0$$

Coefficient of **i**: $A_x + \frac{8}{33}(49.5) = 0 \quad A_x = -12.00 \text{ lb}$

Coefficient of **j**: $A_y + 15 + \frac{25}{33}(49.5) - 75 = 0 \quad A_y = 22.5 \text{ lb}$

Coefficient of **k**: $A_z + 34 - \frac{20}{33}(49.5) = 0 \quad A_z = -4.00 \text{ lb}$

$\mathbf{A} = -(12.00 \text{ lb})\mathbf{i} + (22.5 \text{ lb})\mathbf{j} - (4.00 \text{ lb})\mathbf{k}$ ◀

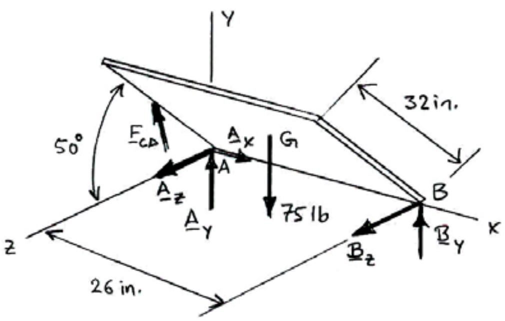


PROBLEM 4.68

The lid of a roof scuttle weighs 75 lb. It is hinged at corners A and B and maintained in the desired position by a rod CD pivoted at C; a pin at end D of the rod fits into one of several holes drilled in the edge of the lid. For the position shown, determine (a) the magnitude of the force exerted by rod CD, (b) the reactions at the hinges. Assume that the hinge at B does not exert any axial thrust.

SOLUTION

Free-Body Diagram:



Geometry:

Using triangle ACD and the law of sines

$$\frac{\sin\alpha}{7\text{ in.}} = \frac{\sin 50^\circ}{15\text{ in.}} \quad \text{or } \alpha = 20.946^\circ$$

$$\beta = 50^\circ + 20.946^\circ = 70.946^\circ$$

Expressing $\mathbf{F}_{CD}$ in terms of its rectangular coordinates:

$$\mathbf{F}_{CD} = F_{CD}\sin\beta\mathbf{j} + F_{CD}\cos\beta\mathbf{k}$$

$$= F_{CD}\sin 70.946^\circ\mathbf{j} + F_{CD}\cos 70.946^\circ\mathbf{k}$$

$$\mathbf{F}_{CD} = 0.94521F_{CD}\mathbf{j} + 0.32646F_{CD}\mathbf{k}$$

$\Sigma\mathbf{M}_B = 0$: $\quad (-26\text{ in.})\mathbf{i}\times\mathbf{A} + \left[(-13\text{ in.})\mathbf{i} + (16\text{ in.})\sin 50^\circ\mathbf{j} + (16\text{ in.})\cos 50^\circ\mathbf{k}\right]\times(-75\text{ lb})\mathbf{j}$

$$+\left[(-26\text{ in.})\mathbf{i} + (7\text{ in.})\mathbf{k}\right]\times\mathbf{F}_{CD} = 0$$

or $\quad -(26\text{ in.})A_y\mathbf{k} + (26\text{ in.})A_z\mathbf{j} + (13\text{ in.})(75\text{ lb})\mathbf{k} + (16\text{ in.})(75\text{ lb})\cos 50^\circ\mathbf{i}$

$$-(26\text{ in.})(0.94521F_{CD})\mathbf{k} + (26\text{ in.})(0.32646F_{CD})\mathbf{j} - (7\text{ in.})(0.94521F_{CD})\mathbf{i} = 0$$

SOLUTION Continued

(a) Setting the coefficients of the unit vectors to zero:

$$\mathbf{i}: \quad (75\text{ lb})\left[(16\text{ in.})\cos 50°\right] - (0.94521F_{CD})(7\text{ in.}) = 0$$

$$F_{CD} = 116.6\text{ lb} ◀$$

(b) $\Sigma F_x = 0$: $\quad A_x = 0$

$$\mathbf{k}: \quad -\left[0.94521(116.580\text{ lb})\right](26\text{ in.}) + (75\text{ lb})(13\text{ in.}) - A_y(26\text{ in.}) = 0$$

$$A_y = -72.693\text{ lb}$$

$$\mathbf{j}: \quad \left[0.32646(116.580\text{ lb})\right](26\text{ in.}) + A_z(26\text{ in.}) = 0$$

$$A_z = -38.059\text{ lb}$$

$$\Sigma F_y = 0: \quad -72.693\text{ lb} + 0.94521(116.580\text{ lb}) - 75\text{ lb} + B_y = 0$$

$$B_y = 37.500\text{ lb}$$

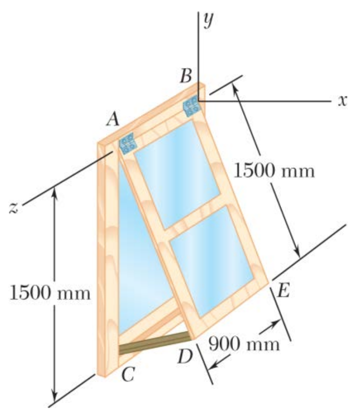
$$\Sigma F_z = 0: \quad -38.059\text{ lb} + 0.32646(116.580\text{ lb}) + B_z = 0$$

$$B_z = 0$$

Therefore:

$$\mathbf{A} = -(72.7\text{ lb})\mathbf{j} - (38.1\text{ lb})\mathbf{k} ◀$$

$$\mathbf{B} = (37.5\text{ lb})\mathbf{j} ◀$$



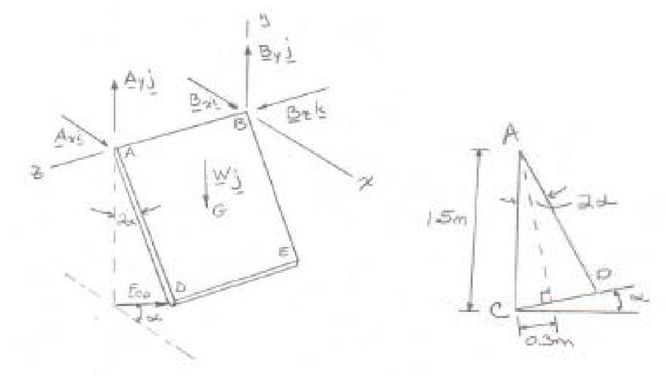
PROBLEM 4.69

A 10-kg storm window measuring 900 × 1500 mm is held by hinges at *A* and *B*. In the position shown, it is held away from the side of the house by a 600-mm stick *CD*. Assuming that the hinge at *A* does not exert any axial thrust, determine the magnitude of the force exerted by the stick and the components of the reactions at *A* and *B*.

SOLUTION

Free-Body Diagram: Since *CD* is a two-force member, F_{CD} is directed along *CD* and triangle *ACD* is isosceles. We have

$$\sin\alpha = \frac{0.3 \text{ m}}{1.5 \text{ m}} = 0.2; \quad \alpha = 11.5370^\circ \text{ and } 2\alpha = 23.074^\circ$$



$$\mathbf{W} = -(10 \text{ kg})(9.81 \text{ m/s}^2) = -(98.10 \text{ N})\mathbf{j}$$

$$\mathbf{r}_G = (0.75 \text{ m})\sin 23.074^\circ\,\mathbf{i} - (0.75 \text{ m})\cos 23.074^\circ\,\mathbf{j} + (0.45 \text{ m})\mathbf{k}$$
$$\mathbf{r}_G = (0.29394 \text{ m})\mathbf{i} - (0.690 \text{ m})\mathbf{j} + (0.45 \text{ m})\mathbf{k}$$
$$\mathbf{F}_{CD} = F_{CD}(\cos 11.5370^\circ\,\mathbf{i} + \sin 11.5370^\circ\,\mathbf{j})$$
$$\mathbf{F}_{CD} = F_{CD}(0.97980\mathbf{i} + 0.20\mathbf{j})$$

$$\Sigma M_B = 0: \quad \mathbf{r}_A \times \mathbf{A} + \mathbf{r}_G \times \mathbf{W} + \mathbf{r}_C \times \mathbf{F}_{CD} = 0$$

$$0.9\mathbf{k} \times (A_x\mathbf{i} + A_y\mathbf{j}) + (0.29394\mathbf{i} - 0.690\mathbf{j} + 0.45\mathbf{k}) \times (-98.10\mathbf{j})$$
$$+ (1.5\mathbf{j} + 0.9\mathbf{k}) \times F_{CD}(0.97980\mathbf{i} + 0.20\mathbf{j}) = 0$$

$$0.90A_x\mathbf{j} - 0.90A_y\mathbf{i} - 28.836\mathbf{k} + 44.145\mathbf{i} + 1.46970F_{CD}\,\mathbf{k} + 0.88182F_{CD}\mathbf{j} - 0.180F_{CD}\mathbf{i} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{k}: \quad -28.836 + 1.46970F_{CD} = 0; \quad F_{CD} = 19.6203 \text{ N}$$

SOLUTION Continued

$$\mathbf{j}: \quad -0.90A_x + 0.88182(19.6203\text{ N}) = 0; \qquad A_x = 19.2240\text{ N}$$

$$\mathbf{i}: \quad -0.90A_y + 44.145 - 0.180(19.6203\text{ N}) = 0; \qquad A_y = 45.123\text{ N}$$

$$F_{CD} = 19.62\text{ N} \blacktriangleleft$$

$$\mathbf{A} = -(19.22\text{N})\mathbf{i} + (45.1\text{ N})\mathbf{j} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad A_x + B_x + F_{CD}\cos 11.5370° = 0$$

$$-19.2240 + B_x + 19.6203\cos 11.5370° = 0$$

$$B_x = 0$$

$$\Sigma F_y = 0: \quad A_y + B_y + F_{CD}\sin 11.5370° - W = 0$$

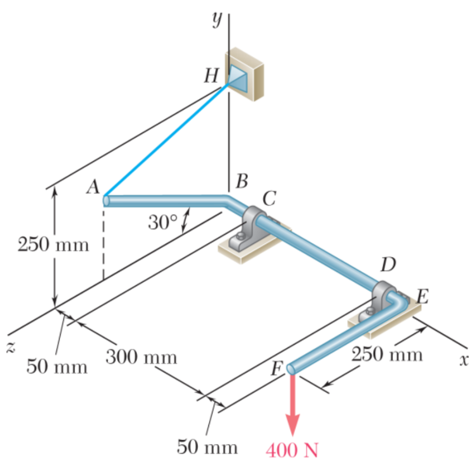
$$45.122 + B_y + 19.6230\sin 11.5370° - 98.1 = 0$$

$$B_y = 49.053\text{ N}$$

$$\Sigma F_z = 0: \quad B_z = 0$$

$$\mathbf{B} = +(49.1\text{ N})\mathbf{j} \blacktriangleleft$$

PROBLEM 4.70

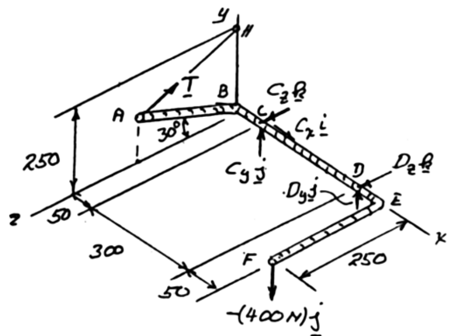


The bent rod $ABEF$ is supported by bearings at C and D and by wire AH. Knowing that portion AB of the rod is 250 mm long, determine (a) the tension in wire AH, (b) the reactions at C and D. Assume that the bearing at D does not exert any axial thrust.

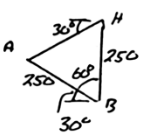
SOLUTION

Free-Body Diagram:

Dimensions in mm



ΔABH is equilateral.



$$\mathbf{r}_{H/C} = -50\mathbf{i} + 250\mathbf{j}$$
$$\mathbf{r}_{D/C} = 300\mathbf{i}$$
$$\mathbf{r}_{F/C} = 350\mathbf{i} + 250\mathbf{k}$$
$$\mathbf{T} = T(\sin 30°)\mathbf{j} - T(\cos 30°)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$$

$$\Sigma \mathbf{M}_C = 0: \quad \mathbf{r}_{H/C} \times \mathbf{T} + \mathbf{r}_D \times \mathbf{D} + \mathbf{r}_{F/C} \times (-400\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 300 & 0 & 0 \\ 0 & D_y & D_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} = 0$$

(a) Coefficient **i**: $\quad -216.5T + 100 \times 10^3 = 0$

$$T = 461.9 \text{ N} \qquad\qquad T = 462 \text{ N} \blacktriangleleft$$

(b) Coefficient of **j**: $\quad -43.3T - 300D_z = 0$

$$-43.3(461.9) - 300D_z = 0 \qquad D_z = -66.67 \text{ N}$$

SOLUTION Continued

Coefficient of **k**: $-25T + 300D_y - 140 \times 10^3 = 0$

$$-25(461.9) + 300D_y - 140 \times 10^3 = 0 \qquad D_y = 505.1 \text{ N}$$

$$\mathbf{D} = (505 \text{ N})\mathbf{j} - (66.7 \text{ N})\mathbf{k} \blacktriangleleft$$

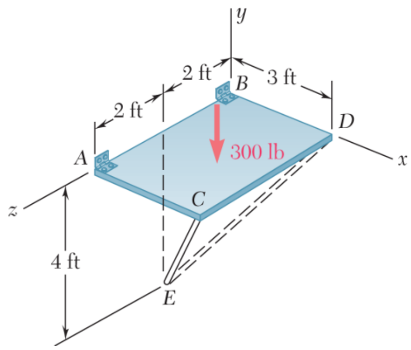
$$\Sigma \mathbf{F} = 0: \quad \mathbf{C} + \mathbf{D} + \mathbf{T} - 400\mathbf{j} = 0$$

Coefficient **i**: $C_x = 0 \qquad C_x = 0$

Coefficient **j**: $C_y + (461.9)0.5 + 505.1 - 400 = 0 \qquad C_y = -336 \text{ N}$

Coefficient **k**: $C_z - (461.9)0.866 - 66.67 = 0 \qquad C_z = 467 \text{ N} \qquad \mathbf{C} = -(336 \text{ N})\mathbf{j} + (467 \text{ N})\mathbf{k} \blacktriangleleft$

PROBLEM 4.71



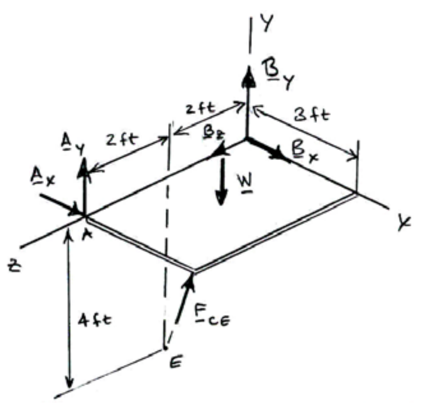
Solve Prob. 4.65, assuming that the hinge at B has been removed and that the hinge at A can exert an axial thrust, as well as couples about axes parallel to the x and y axes.

PROBLEM 4.65 The horizontal platform $ABCD$ weighs 60 lb and supports a 240-lb load at its center. The platform is normally held in position by hinges at A and B and by braces CE and DE. If brace DE is removed, determine the reactions at the hinges and the force exerted by the remaining brace CE. The hinge at A does not exert any axial thrust.

SOLUTION

Free-Body Diagram:

$B_x = B_y = B_z = 0$



Express forces, weight in terms of rectangular components:

$$\overrightarrow{EC} = (3\text{ ft})\mathbf{i} + (4\text{ ft})\mathbf{j} + (2\text{ ft})\mathbf{k}$$

$$\mathbf{F}_{CE} = F_{CE}\frac{\overrightarrow{EC}}{EC} = F_{CE}\frac{3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}}{\sqrt{(3)^2 + (4)^2 + (2)^2}}$$

$$= 0.55709F_{CE}\,\mathbf{i} + 0.74278F_{CE}\,\mathbf{j} + 0.37139F_{CE}\,\mathbf{k}$$

$$\mathbf{W} = -(mg)\mathbf{j} = -(300\text{ lb})\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0\text{:}\;\; \mathbf{r}_{G/A} \times (-W)\mathbf{j} + \mathbf{r}_{C/A} \times F_{CE} + (M_{A_x})\mathbf{i} + (M_{A_y})\mathbf{j} = 0$$

$$\Sigma\mathbf{M}_A = 0\text{:}\qquad (1.5\mathbf{i} - 2\mathbf{k}) \times (-300\mathbf{j}) + 3\mathbf{i} \times F_{CE}(0.55709\mathbf{i} + 0.74278\mathbf{j} + 0.37139\mathbf{k})$$
$$+ (M_{A_x})\mathbf{i} + (M_{A_y})\mathbf{j} = 0$$

or $$-450\mathbf{k} - 600\mathbf{i} + 2.2283F_{CE}\mathbf{k} - 1.11417F_{CE}\mathbf{j} + M_{A_x}\mathbf{i} + M_{A_y}\mathbf{j} = 0$$

SOLUTION Continued

Setting the coefficients of the unit vectors equal to zero:

$\mathbf{k}$: $\quad -450 + (2.2283F_{CE}) = 0$

$$F_{CE} = 201.95 \text{ lb}$$

or $\quad F_{CE} = 202 \text{ lb}$ ◀

$\mathbf{j}$: $\quad -1.11417(201.95) + (M_{A_y}) = 0$

$$M_{A_y} = 225.00 \text{ lb} \cdot \text{ft}$$

$\mathbf{i}$: $\quad -600 + M_{A_x} = 0$

$$M_{A_x} = 600 \text{ lb} \cdot \text{ft}$$

$\Sigma F_x = 0$: $\quad A_x + 0.55709(201.94 \text{ lb}) = 0$

$$A_x = -112.499 \text{ lb}$$

$\Sigma F_y = 0$: $\quad A_y - 300 \text{ lb} + 0.74278(201.94 \text{ lb}) = 0$

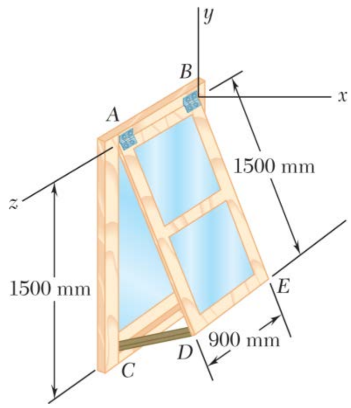
$$A_y = 150.003 \text{ lb}$$

$\Sigma F_z = 0$: $\quad A_z + 0.371391(201.94 \text{ lb}) = 0$

$$A_z = -74.999 \text{ lb}$$

Therefore: $\quad \mathbf{M}_A = (600 \text{ lb} \cdot \text{ft})\mathbf{i} + (225 \text{ lb} \cdot \text{ft})\mathbf{j}$ ◀

$\mathbf{A} = -(112.5 \text{ lb})\mathbf{i} + (150.0 \text{ lb})\mathbf{j} - (75.0 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.72

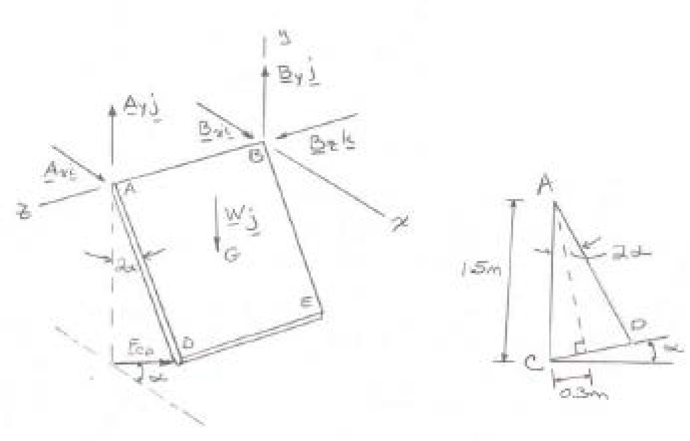
Solve Prob. 4.69, assuming that the hinge at A has been removed and that the hinge at B can exert couples about axes parallel to the x and y axes.

PROBLEM 4.69 A 10-kg storm window measuring 900 × 1500 mm is held by hinges at A and B. In the position shown, it is held away from the side of the house by a 600-mm stick CD. Assuming that the hinge at A does not exert any axial thrust, determine the magnitude of the force exerted by the stick and the components of the reactions at A and B.

SOLUTION

Free-Body Diagram: Since CD is a two-force member, F_{CD}

is directed along CD and triangle ACD is isosceles. We have



$$\mathbf{W} = -(10\text{ kg})(9.81\text{ m/s}^2) = -(98.10\text{ N})\mathbf{j}$$

$$\mathbf{r}_G = (0.75\text{ m})\sin 23.074^\circ\,\mathbf{i} - (0.75\text{ m})\cos 23.074^\circ\,\mathbf{j} + (0.45\text{ m})\mathbf{k}$$
$$\mathbf{r}_G = (0.29394\text{ m})\mathbf{i} - (0.690\text{ m})\mathbf{j} + (0.45\text{ m})\mathbf{k}$$

$$\mathbf{F}_{CD} = F_{CD}(\cos 11.5370^\circ\,\mathbf{i} + \sin 11.5370^\circ\,\mathbf{j})$$
$$\mathbf{F}_{CD} = F_{CD}(0.97980\mathbf{i} + 0.20\mathbf{j})$$

With hinge at A removed, hinge at B will develop two couples to prevent rotation about the x and z axes.

$$\Sigma M_B = 0:\quad (M_B)_x\mathbf{i} + (M_B)_y\mathbf{j} + \mathbf{r}_G \times \mathbf{W} + \mathbf{r}_C \times \mathbf{F}_{CD} = 0$$

$$(M_B)_x\mathbf{i} + (M_B)_y\mathbf{j} + (0.29394\mathbf{i} - 0.690\mathbf{j} + 0.45\mathbf{k}) \times (-98.10)\mathbf{j}$$
$$+ (1.5\mathbf{j} + 0.9\mathbf{k}) \times F_{CD}(0.97980\mathbf{i} + 0.20\mathbf{j}) = 0$$

$$(M_B)_x\mathbf{i} + (M_B)_y\mathbf{j} - 28.836\mathbf{k} + 44.145\mathbf{i} + 1.46970F_{CD}\,\mathbf{k} + 0.88182F_{CD}\mathbf{j} - 0.180F_{CD}\mathbf{i} = 0$$

SOLUTION Continued

Equating the coefficients of the unit vectors to zero,

$$\mathbf{k}: \quad -28.836 + 1.46970 F_{CD} = 0; \quad F_{CD} = 19.6203 \text{ N}$$

$$\mathbf{j}: \quad (M_B)_y + 0.88182(19.6203) = 0; \qquad (M_B)_y = -17.3016 \text{ N}\cdot\text{m}$$

$$\mathbf{i}: \quad (M_B)_x + 44.145 - 0.180(19.6203) = 0; \quad (M_B)_x = -40.613 \text{ N}\cdot\text{m}$$

$$F_{CD} = 19.62 \text{ N} \blacktriangleleft$$

$$\mathbf{M}_B = -(40.6 \text{ N}\cdot\text{m})\mathbf{i} - (17.30 \text{ N}\cdot\text{m})\mathbf{j} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad B_x + F_{CD}\cos 11.5370° = 0$$

$$B_x + 19.6203 \cos 11.5370° = 0$$

$$B_x = -19.22 \text{ N}$$

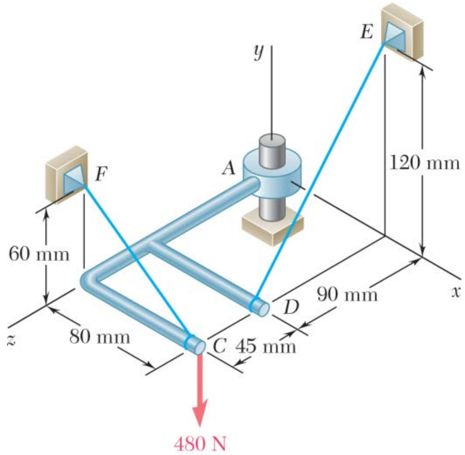
$$\Sigma F_y = 0: \quad B_y + F_{CD}\sin 11.5370° - W = 0$$

$$B_y + 19.6230 \sin 11.5370° - 98.1 = 0$$

$$B_y = 94.2 \text{ N}$$

$$\Sigma F_z = 0: \quad B_z = 0$$

$$\mathbf{B} = -(19.22 \text{ N})\mathbf{i} + (94.2 \text{ N})\mathbf{j} \blacktriangleleft$$

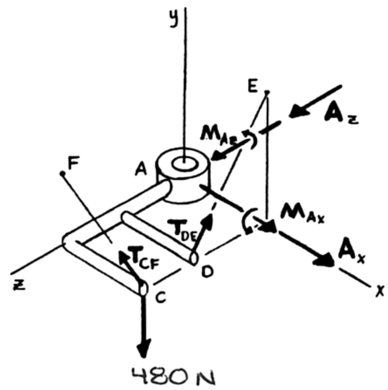


PROBLEM 4.73

The assembly shown is welded to collar *A* that fits on the vertical pin shown. The pin can exert couples about the *x* and *z* axes but does not prevent motion about or along the *y*-axis. For the loading shown, determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:



First note:

$$\mathbf{T}_{CF} = \boldsymbol{\lambda}_{CF} T_{CF} = \frac{-(0.08 \text{ m})\mathbf{i} + (0.06 \text{ m})\mathbf{j}}{\sqrt{(0.08)^2 + (0.06)^2} \text{ m}} T_{CF}$$

$$= T_{CF}(-0.8\mathbf{i} + 0.6\mathbf{j})$$

$$\mathbf{T}_{DE} = \boldsymbol{\lambda}_{DE} T_{DE} = \frac{(0.12 \text{ m})\mathbf{j} - (0.09 \text{ m})\mathbf{k}}{\sqrt{(0.12)^2 + (0.09)^2} \text{ m}} T_{DE}$$

$$= T_{DE}(0.8\mathbf{j} - 0.6\mathbf{k})$$

From F.B.D. of assembly:

$$\Sigma F_y = 0: \quad 0.6T_{CF} + 0.8T_{DE} - 480 \text{ N} = 0$$

or

$$0.6T_{CF} + 0.8T_{DE} = 480 \text{ N} \qquad (1)$$

$$\Sigma M_y = 0: \quad -(0.8T_{CF})(0.135 \text{ m}) + (0.6T_{DE})(0.08 \text{ m}) = 0$$

or

$$T_{DE} = 2.25T_{CF} \qquad (2)$$

Substituting Equation (2) into Equation (1),

$$0.6T_{CF} + 0.8[(2.25)T_{CF}] = 480 \text{ N}$$

$$T_{CF} = 200.00 \text{ N}$$

or $T_{CF} = 200 \text{ N}$ ◀

and from Equation (2): $T_{DE} = 2.25(200.00 \text{ N}) = 450.00$

or $T_{DE} = 450 \text{ N}$ ◀

SOLUTION Continued

From F.B.D. of assembly:

$$\Sigma F_z = 0: \quad A_z - (0.6)(450.00 \text{ N}) = 0 \quad A_z = 270.00 \text{ N}$$

$$\Sigma F_x = 0: \quad A_x - (0.8)(200.00 \text{ N}) = 0 \quad A_x = 160.000 \text{ N}$$

$$\text{or} \quad \mathbf{A} = (160.0 \text{ N})\mathbf{i} + (270 \text{ N})\mathbf{k} \blacktriangleleft$$

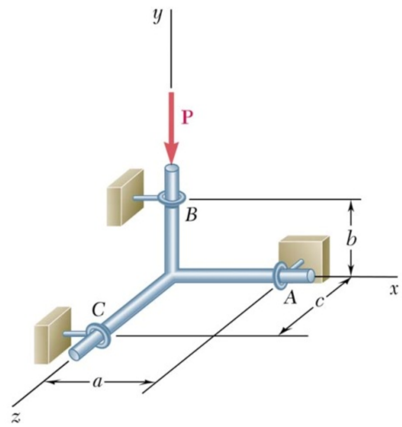
$$\Sigma M_x = 0: \quad M_{A_x} + (480 \text{ N})(0.135 \text{ m}) - [(200.00 \text{ N})(0.6)](0.135 \text{ m}) - [(450 \text{ N})(0.8)](0.09 \text{ m}) = 0$$

$$M_{A_x} = -16.2000 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0: \quad M_{A_z} - (480 \text{ N})(0.08 \text{ m}) + [(200.00 \text{ N})(0.6)](0.08 \text{ m}) + [(450 \text{ N})(0.8)](0.08 \text{ m}) = 0$$

$$M_{A_z} = 0$$

$$\text{or} \quad \mathbf{M}_A = -(16.20 \text{ N}\cdot\text{m})\mathbf{i} \blacktriangleleft$$

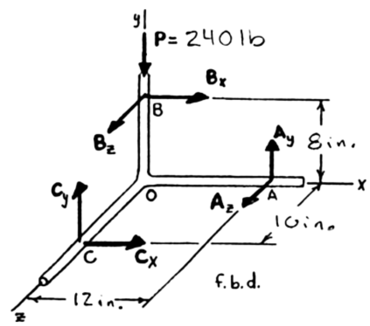


PROBLEM 4.74

Three rods are welded together to form a "corner" that is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ lb, $a = 12$ in., $b = 8$ in., and $c = 10$ in.

SOLUTION

From F.B.D. of weldment:



$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ C_x & C_y & 0 \end{vmatrix} = 0$$

$$(-12A_z\mathbf{j} + 12A_y\mathbf{k}) + (8B_z\mathbf{i} - 8B_x\mathbf{k}) + (-10C_y\mathbf{i} + 10C_x\mathbf{j}) = 0$$

From **i**-coefficient: $\quad 8B_z - 10C_y = 0$

or $\quad B_z = 1.25C_y \quad (1)$

j-coefficient: $\quad -12A_z + 10C_x = 0$

or $\quad C_x = 1.2A_z \quad (2)$

k-coefficient: $\quad 12A_y - 8B_x = 0$

or $\quad B_x = 1.5A_y \quad (3)$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} - \mathbf{P} = 0$$

or $\quad (B_x + C_x)\mathbf{i} + (A_y + C_y - 240 \text{ lb})\mathbf{j} + (A_z + B_z)\mathbf{k} = 0$

From **i**-coefficient: $\quad B_x + C_x = 0$

or $\quad C_x = -B_x \quad (4)$

j-coefficient: $\quad A_y + C_y - 240 \text{ lb} = 0$

or $\quad A_y + C_y = 240 \text{ lb} \quad (5)$

SOLUTION Continued

k-coefficient: $A_z + B_z = 0$

or
$$A_z = -B_z \tag{6}$$

Substituting C_x from Equation (4) into Equation (2),
$$-B_z = 1.2A_z \tag{7}$$

Using Equations (1), (6), and (7),
$$C_y = \frac{B_z}{1.25} = \frac{-A_z}{1.25} = \frac{1}{1.25}\left(\frac{B_x}{1.2}\right) = \frac{B_x}{1.5} \tag{8}$$

From Equations (3) and (8):
$$C_y = \frac{1.5A_y}{1.5} \quad \text{or} \quad C_y = A_y$$

and substituting into Equation (5),
$$2A_y = 240 \text{ lb}$$
$$A_y = C_y = 120 \text{ lb} \tag{9}$$

Using Equation (1) and Equation (9),
$$B_z = 1.25(120 \text{ lb}) = 150.0 \text{ lb}$$

Using Equation (3) and Equation (9),
$$B_x = 1.5(120 \text{ lb}) = 180.0 \text{ lb}$$

From Equation (4): $C_x = -180.0 \text{ lb}$

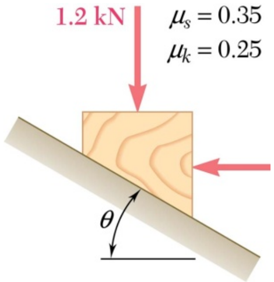
From Equation (6): $A_z = -150.0 \text{ lb}$

Therefore,

$\mathbf{A} = (120.0 \text{ lb})\mathbf{j} - (150.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = (180.0 \text{ lb})\mathbf{i} + (150.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{C} = -(180.0 \text{ lb})\mathbf{i} + (120.0 \text{ lb})\mathbf{j}$ ◀

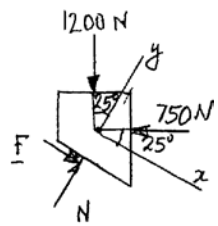


PROBLEM 4.75

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 25°$ and $P = 750$ N.

SOLUTION

<u>Assume equilibrium</u>:



$$\overset{+}{\searrow} \Sigma F_x = 0: \quad F + (1200 \text{ N})\sin 25° - (750 \text{ N})\cos 25° = 0$$

$$F = +172.6 \text{ N} \qquad \mathbf{F} = 172.6 \text{ N} \searrow$$

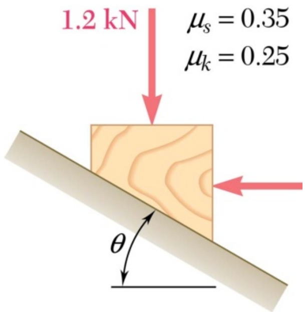
$$+\nearrow \Sigma F_y = 0: \quad N - (1200 \text{ N})\cos 25° - (750 \text{ N})\sin 25° = 0$$

$$N = 1404.5 \text{ N}$$

Maximum friction force: $F_m = \mu_s N = 0.35(1404.5 \text{ N}) = 491.6 \text{ N}$

Since $F < F_m$, <u>block is in equilibrium</u> ◀

Friction force: $\mathbf{F} = 172.6 \text{ N} \searrow 25.0°$ ◀

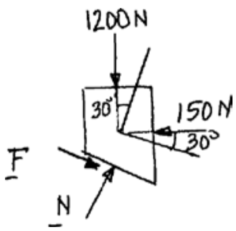


PROBLEM 4.76

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 30°$ and $P = 150$ N.

SOLUTION

<u>Assume equilibrium</u>:



$$\searrow^{+} \Sigma F_x = 0: \quad F + (1200 \text{ N})\sin 30° - (150 \text{ N})\cos 30° = 0$$

$$F = -470.1 \text{ N} \qquad\qquad \mathbf{F} = 470.1 \text{ N} \nwarrow$$

$$+\nearrow \Sigma F_y = 0: \quad N - (1200 \text{ N})\cos 30° - (150 \text{ N})\sin 30° = 0$$

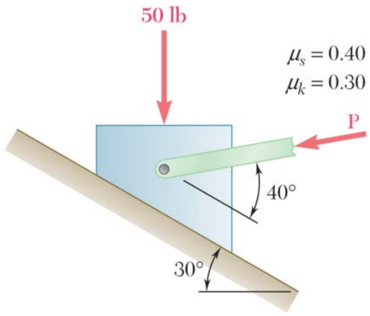
$$N = 1114.2 \text{ N}$$

(a) Maximum friction force:

$$F_m = \mu_s N$$
$$= 0.35(1114.2 \text{ N})$$
$$= 390.0 \text{ N}$$

Since **F** is $\nwarrow$ and $F > F_m$, <u>block moves down</u> ◀

(*b*) Actual friction force: $F = F_k = \mu_k N = 0.25(1114.2 \text{ N}) = 279$ N $\qquad$ $\mathbf{F} = 279 \text{ N} \searrow 30.0°$ ◀

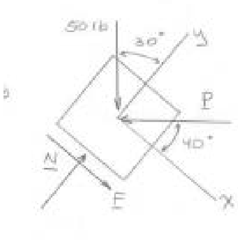


PROBLEM 4.77

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 120$ lb.

SOLUTION

<u>Assume equilibrium</u>:



$$\searrow^{+} \Sigma F_x = 0: \quad F + (50 \text{ lb})\sin 30° - (120 \text{ lb})\cos 40° = 0$$

$$F = +66.925 \text{ lb}$$

$$+\nearrow \Sigma F_y = 0: \quad N - (50 \text{ lb})\cos 30° - (120 \text{ lb})\sin 40° = 0$$

$$N = +120.436 \text{ lb}$$

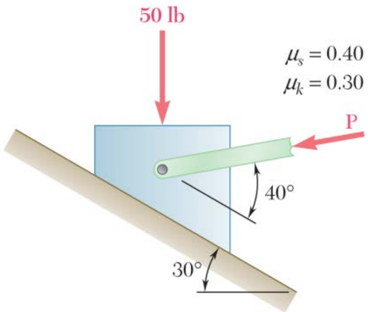
Maximum friction force:

$$F_m = \mu_s N$$
$$= 0.40(120.436 \text{ lb})$$
$$= 48.174 \text{ lb}$$

We note that $F > F_m$. Thus, <u>block moves up</u> ◀

Actual friction force:

$F = F_k = \mu_k N = 0.30(120.436 \text{ lb}) = 36.131$ lb, **F** $= 36.1$ lb $\nwarrow 30.0°$ ◀

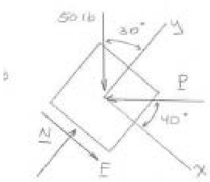


PROBLEM 4.78

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 80$ lb.

SOLUTION

Assume equilibrium:



$$\overset{+}{\searrow} \Sigma F_x = 0: \quad F + (50 \text{ lb})\sin 30° - (80 \text{ lb})\cos 40° = 0$$

$$F = +36.284 \text{ lb}$$

$$+\nearrow \Sigma F_y = 0: \quad N - (50 \text{ lb})\cos 30° - (80 \text{ lb})\sin 40° = 0$$

$$N = +94.724 \text{ lb}$$

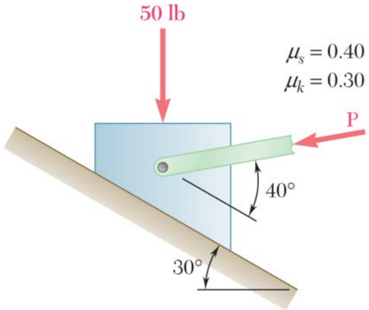
Maximum friction force:

$$\begin{aligned} F_m &= \mu_s N \\ &= 0.40(94.724 \text{ lb}) \\ &= 37.890 \text{ lb} \end{aligned}$$

We note that $F < F_m$. Thus, block is in equilibrium ◀

Thus $\mathbf{F} = 36.3 \text{ lb} \nwarrow 30.0°$ ◀

Note: $F_k = \mu_k N = 0.30(94.724 \text{ lb}) = 28.417$ lb, $F > F_K$. If block is originally in motion, it will keep moving with $F_K = 28.4$ lb.



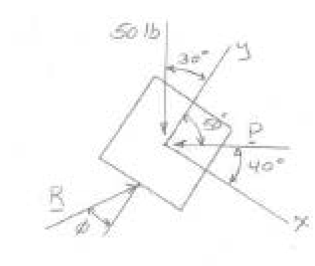
PROBLEM 4.79

Determine the smallest value of P required to (*a*) start the block up the incline, (*b*) keep it moving up.

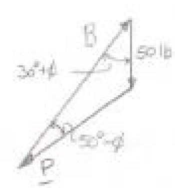
SOLUTION

(*a*) <u>To start block up the incline</u>:

$$\mu_s = 0.40$$
$$\phi_s = \tan^{-1} 0.40 = 21.80°$$



From force triangle:



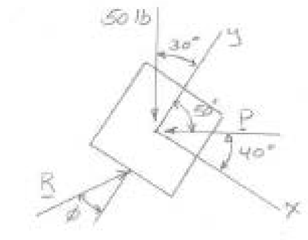
$$\frac{P}{\sin 51.80°} = \frac{50 \text{ lb}}{\sin 28.20°}$$

$P = 83.2$ lb ◀

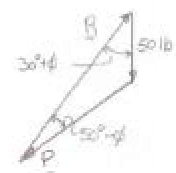
(*b*) <u>To keep block moving up:</u>

$$\mu_k = 0.30$$
$$\phi_k = \tan^{-1} 0.30 = 16.70°$$

SOLUTION Continued



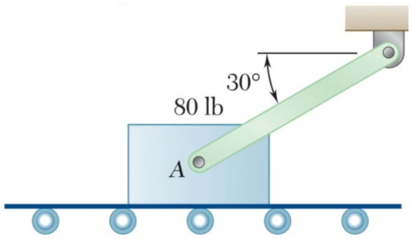
From force triangle:



$$\frac{P}{\sin 46.70°} = \frac{50 \text{ lb}}{\sin 33.30°}$$

$P = 66.3$ lb ◀

PROBLEM 4.80

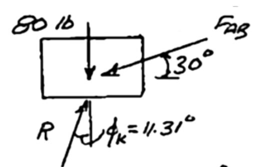


The 80-lb block is attached to link AB and rests on a moving belt. Knowing that $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the magnitude of the horizontal force **P** that should be applied to the belt to maintain its motion (*a*) to the right, (*b*) to the left.

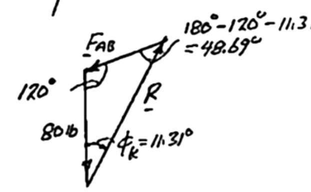
SOLUTION

We note that link AB is a two-force member, since there is motion between belt and block $\mu_k = 0.20$ and $\phi_k = \tan^{-1} 0.20 = 11.31°$

(*a*) Belt moves to right



Free body: Block

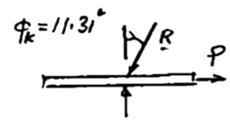


Force triangle:

$$\frac{R}{\sin 120°} = \frac{80 \text{ lb}}{\sin 48.69°}$$

$$R = 92.23 \text{ lb}$$

Free body: Belt

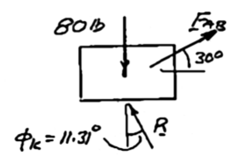


$$\xrightarrow{+} \Sigma F_x = 0: \quad P - (92.23 \text{ lb}) \sin 11.31°$$

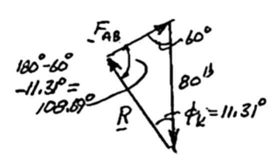
$$P = 18.089 \text{ lb}$$

$\mathbf{P} = 18.09 \text{ lb} \longrightarrow$ ◀

(*b*) Belt moves to left



Free body: Block

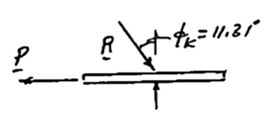


Force triangle:

$$\frac{R}{\sin 60°} = \frac{80 \text{ lb}}{\sin 108.69°}$$

$$R = 73.139 \text{ lb}$$

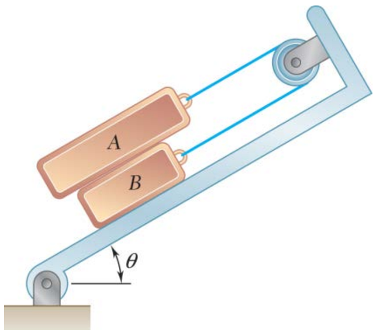
Free body: Belt



$$\xrightarrow{+} \Sigma F_x = 0: \quad (73.139 \text{ lb}) \sin 11.31° - P = 0$$

$$P = 14.344 \text{ lb}$$

$\mathbf{P} = 14.34 \text{ lb} \longleftarrow$ ◀



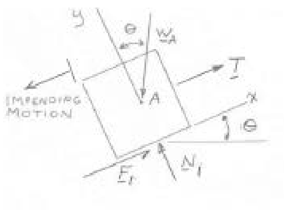
PROBLEM 4.81

The 50-lb block A and the 25-lb block B are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between the two blocks and zero between block B and the incline, determine the value of θ for which motion is impending.

SOLUTION

Since motion impends, $F = \mu_s N$ between A+B

<u>Free body: Block *A*</u>



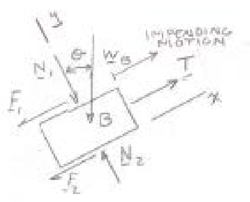
Impending motion: $\Sigma F_y = 0$: $\quad N_1 - 50\cos\theta = 0$

$N_1 = 50\cos\theta$

$$\Sigma F_x = 0: \quad T - 50\sin\theta + \mu_1 N_1 = 0$$

$$T = 50\sin\theta - \mu_1(50)\cos\theta \qquad (1)$$

<u>Free body: Block *B*</u>



Impending motion: $\Sigma F_y = 0$: $\quad N_2 - N_1 - 25\cos\theta = 0$

$$N_2 = 75\cos\theta$$

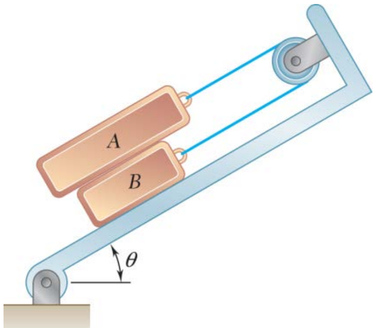
$$\Sigma F_x: \quad T - \mu_1 N_1 - \mu_2 N_2 - 25\sin\theta$$

$$T = \mu_1(50)\cos\theta + \mu_2(75)\cos\theta + 25\sin\theta \qquad (2)$$

Eq. (1)-Eq. (2): $\quad 0 = 25\sin\theta - \mu_1(100)\cos\theta - \mu_2(75)\cos\theta = 0$

Substituting in for $\mu_1 = 0.15,\ \mu_2 = 0,$ we have:

$$15\cos\theta = 25\sin\theta: \quad \tan\theta = \frac{15}{25}; \qquad \theta = 31.0° ◀$$



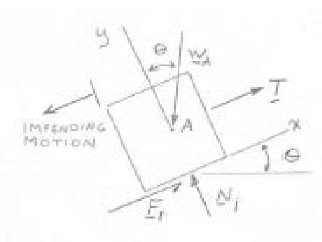
PROBLEM 4.82

The 50-lb block A and the 25-lb block B are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

Since motion impends, $F = \mu_s N$ between A+B

Free body: Block A



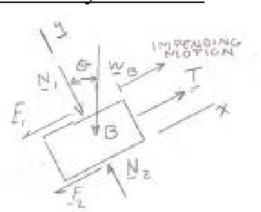
Impending motion: $\Sigma F_y = 0$: $N_1 - 50\cos\theta = 0$

$N_1 = 50\cos\theta$

$$\Sigma F_x = 0: \quad T - 50\sin\theta + \mu_1 N_1 = 0$$

$$T = 50\sin\theta - \mu_1(50)\cos\theta \qquad (1)$$

Free body: Block B



Impending motion: $\Sigma F_y = 0$: $N_2 - N_1 - 25\cos\theta = 0$

$$N_2 = 75\cos\theta$$

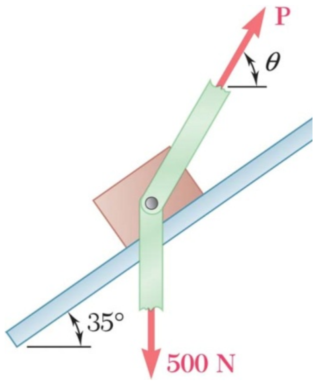
$$\Sigma F_x: \quad T - \mu_1 N_1 - \mu_2 N_2 - 25\sin\theta$$

$$T = \mu_1(50)\cos\theta + \mu_2(75)\cos\theta + 25\sin\theta \qquad (2)$$

Eq. (1)-Eq. (2): $\quad 0 = 25\sin\theta - \mu_1(100)\cos\theta - \mu_2(75)\cos\theta = 0$

Substituting in for $\mu_1 = \mu_2 = 0.15$, we have:

$$175(0.15)\cos\theta = 25\sin\theta: \quad \tan\theta = \frac{26.25}{25}; \qquad \theta = 46.4° \blacktriangleleft$$



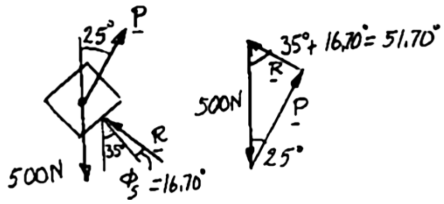
PROBLEM 4.83

The coefficients of friction between the block and the rail are $\mu_s = 0.30$ and $\mu_k = 0.25$. Knowing that $\theta = 65°$, determine the smallest value of P required (*a*) to start the block moving up the rail, (*b*) to keep it from moving down.

SOLUTION

(*a*) To start block up the rail:

$$\mu_s = 0.30$$
$$\phi_s = \tan^{-1} 0.30 = 16.70°$$

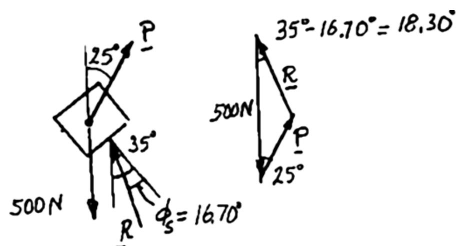


Force triangle:

$$\frac{P}{\sin 51.70°} = \frac{500 \text{ N}}{\sin (180° - 25° - 51.70°)}$$

$P = 403$ N ◀

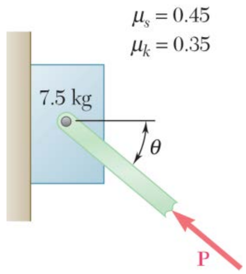
(*b*) To prevent block from moving down:



Force triangle:

$$\frac{P}{\sin 18.30°} = \frac{500 \text{ N}}{\sin (180° - 25° - 18.30°)}$$

$P = 229$ N ◀



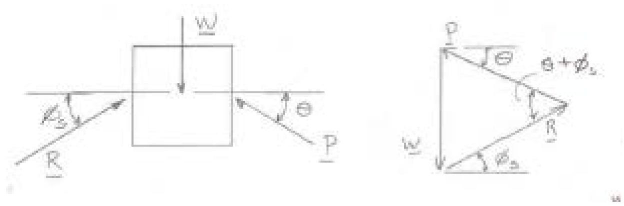
PROBLEM 4.84

Knowing that P = 100 N, determine the range of values of θ for which equilibrium of the 7.5-kg block is maintained.

SOLUTION

Free-body diagram of block and force triangle:

For motion impending downward, $W = (7.5\text{ kg})(9.81\text{ m/s}^2) = 73.575\text{ N}$



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.45) = 24.23°$$

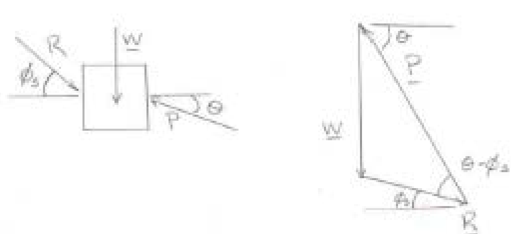
$$\frac{100\text{ N}}{\sin 65.77°} = \frac{73.575\text{ N}}{\sin(\theta + \phi_s)}$$

$$\sin(\theta + 24.23°) = 0.67093$$

$$\theta + 24.23° = 42.14°$$

$$\theta = 17.91°$$

For motion impending upward,



SOLUTION Continued

$$\frac{100\text{ N}}{\sin 114.23^\circ} = \frac{73.575\text{ N}}{\sin\left(\theta - \phi_s\right)}$$

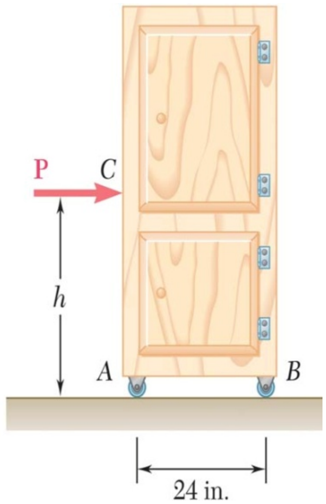
$$\sin\left(\theta - 24.23^\circ\right) = 0.67093$$

$$\theta - 24.23^\circ = 42.14^\circ$$

$$\theta = 66.37^\circ$$

$$17.91^\circ \le \theta \le 66.4^\circ \blacktriangleleft$$

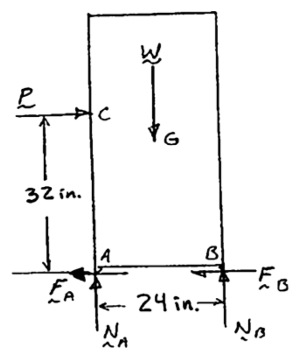
PROBLEM 4.85



A 120-lb cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. If $h = 32$ in., determine the magnitude of the force **P** required to move the cabinet to the right (*a*) if all casters are locked, (*b*) if the casters at *B* are locked and the casters at *A* are free to rotate, (*c*) if the casters at *A* are locked and the casters at *B* are free to rotate.

SOLUTION

FBD cabinet: *Note:* for tipping,



$$N_A = F_A = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad (12 \text{ in.})W - (32 \text{ in.})P_{\text{tip}} = 0$$

$$P_{\text{tip}} = 2.66667$$

(*a*) All casters locked. Impending slip:

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0$$

$$N_A + N_B = W \qquad W = 120 \text{ lb}$$

So $$F_A + F_B = \mu_s W \qquad \mu_s = 0.3$$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0$$

$$P = F_A + F_B = \mu_s W$$

$$P = 0.3(120 \text{ lb}) \qquad \text{or} \quad \mathbf{P} = 36.0 \text{ lb} \longrightarrow \blacktriangleleft$$

$$(P = 0.3W < P_{\text{tip}} \quad \text{OK})$$

(*b*) Casters at *A* free, so $$F_A = 0$$

Impending slip: $$F_B = \mu_s N_B$$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_B = 0$$

$$P = F_B = \mu_s N_B \qquad N_B = \frac{P}{\mu_s}$$

SOLUTION Continued

$$\circlearrowleft \Sigma M_A = 0: \quad (32 \text{ in.})P + (12 \text{ in.})W - (24 \text{ in.})N_B = 0$$

$$8P + 3W - 6\frac{P}{0.3} = 0 \quad P = 0.25W$$

$$(P = 0.25W < P_{\text{tip}} \quad \text{OK})$$

$$P = 0.25(120 \text{ lb}) \qquad \text{or} \quad \mathbf{P} = 30.0 \text{ lb} \longrightarrow \blacktriangleleft$$

(*c*) Casters at *B* free, so $F_B = 0$

Impending slip: $F_A = \mu_s N_A$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_A = 0 \quad P = F_A = \mu_s N_A$$

$$N_A = \frac{P}{\mu_s} = \frac{P}{0.3}$$

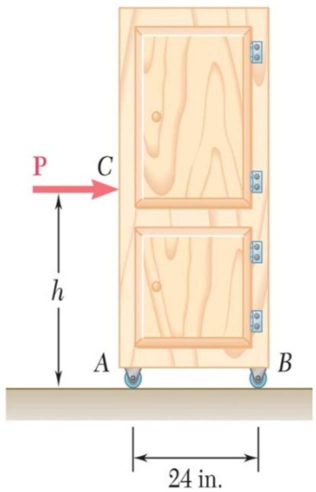
$$\circlearrowright \Sigma M_B = 0: \quad (12 \text{ in.})W - (32 \text{ in.})P - (24 \text{ in.})N_A = 0$$

$$3W - 8P - 6\frac{P}{0.3} = 0$$

$$P = 0.107143W = 12.8572$$

$$(P < P_{\text{tip}} \quad \text{OK})$$

$$\mathbf{P} = 12.86 \text{ lb} \longrightarrow \blacktriangleleft$$

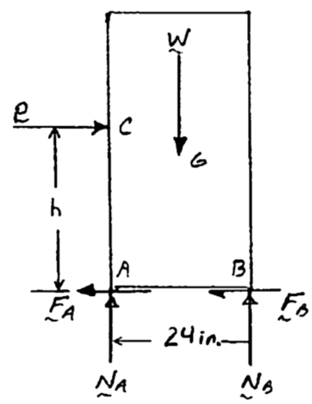


PROBLEM 4.86

A 120-lb cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Assuming that the casters at both *A* and *B* are locked, determine (*a*) the force **P** required to move the cabinet to the right, (*b*) the largest allowable value of *h* if the cabinet is not to tip over.

SOLUTION

FBD cabinet:



(*a*)

$$+\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0$$

$$N_A + N_B = W$$

Impending slip:

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

So

$$F_A + F_B = \mu_s W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - F_A - F_B = 0$$

$$P = F_A + F_B = \mu_s W$$

$$P = 0.3(120 \text{ lb}) = 141.26 \text{ N}$$

$W = 120$ lb

$\mu_s = 0.3$

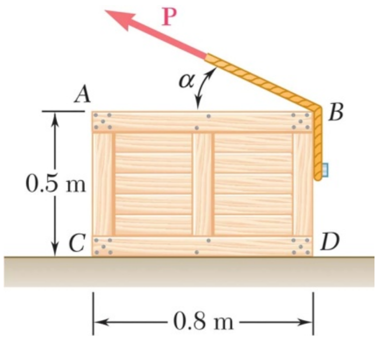
$\mathbf{P} = 36.0 \text{ lb} \longrightarrow$ ◀

(*b*) For tipping, $N_A = F_A = 0$

$$\circlearrowleft \Sigma M_B = 0: \quad hP - (12 \text{ in.})W = 0$$

$$h_{\max} = (12 \text{ in.})\frac{W}{P} = (12 \text{ in.})\frac{1}{\mu_s} = \frac{12 \text{ in.}}{0.3}$$

$h_{\max} = 40.0$ in. ◀



PROBLEM 4.87

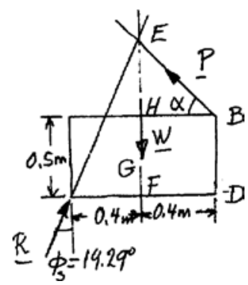
A 40-kg packing crate must be moved to the left along the floor without tipping. Knowing that the coefficient of static friction between the crate and the floor is 0.35, determine (*a*) the largest allowable value of α, (*b*) the corresponding magnitude of the force **P**.

SOLUTION

(*a*) Free-body diagram

If the crate is about to tip about *C*, contact between crate and ground is only at *C* and the reaction **R** is applied at *C*. As the crate is about to slide, **R** must form with the vertical an angle

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}$$
$$0.35 = 19.29°$$



Since the crate is a 3-force body. **P** must pass through *E* where **R** and **W** intersect.

$$EF = \frac{CF}{\tan\theta_s} = \frac{0.4\text{ m}}{0.35} = 1.1429\text{ m}$$

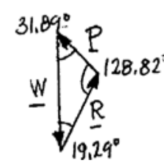
$$EH = EF - HF = 1.1429 - 0.5 = 0.6429\text{ m}$$

$$\tan\alpha = \frac{EH}{HB} = \frac{0.6429\text{ m}}{0.4\text{ m}}$$

$$\alpha = 58.11°$$

$\alpha = 58.1°$ ◀

(*b*) Force Triangle

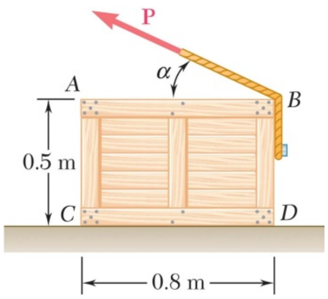


$$\frac{P}{\sin 19.29°} = \frac{W}{\sin 128.82°} \qquad P = 0.424\,W$$

$$P = 0.424(40\text{ kg})(9.81\text{ m/s}^2),$$

$P = 166.4$ N ◀

Note: After the crate starts moving, μ_s should be replaced by the lower value μ_k. This will yield a larger value of α.



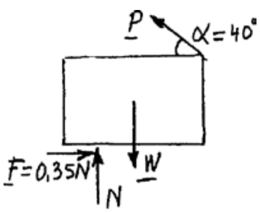
PROBLEM 4.88

A 40-kg packing crate is pulled by a rope as shown. The coefficient of static friction between the crate and the floor is 0.35. If $\alpha = 40°$, determine (*a*) the magnitude of the force **P** required to move the crate, (*b*) whether the crate will slide or tip.

SOLUTION

Force P for which sliding is impending

(We assume that crate does not tip)



$$W = (40\text{ kg})(9.81\text{ m/s}^2) = 392.4\text{ N}$$

$$+\!\uparrow \Sigma F_y = 0: \quad N - W + P\sin 40° = 0$$

$$N = W - P\sin 40° \tag{1}$$

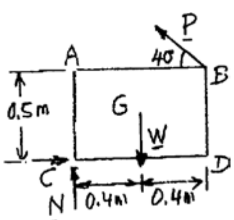
$$\xrightarrow{+} \Sigma F_x = 0: \quad 0.35\,N - P\cos 40° = 0$$

Substitute for N from Eq. (1): $0.35(W - P\sin 40°) - P\cos 40° = 0$

$$P = \frac{0.35W}{0.35\sin 40° + \cos 40°} \qquad P = 0.3532W \triangleleft$$

Force P for which crate rotates about C

(We assume that crate does not slide)

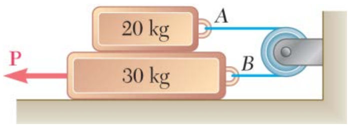


$$+\circlearrowleft \Sigma M_C = 0: \quad (P\sin 40°)(0.8\text{ m}) + (P\cos 40°)(0.5\text{ m})$$
$$-W(0.4\text{ m}) = 0$$

$$P = \frac{0.4W}{0.8\sin 40° + 0.5\cos 40°} = 0.4458W$$

(*b*) Crate will first slide ◀

(*a*) $P = 0.3532(392.4\text{ N})$ $\qquad P = 138.6\text{ N}$ ◀

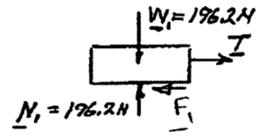


PROBLEM 4.89

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force **P** required to start the 30-kg block moving if cable AB (a) is attached as shown, (b) is removed.

SOLUTION

(a) <u>Free body: 20-kg block</u>

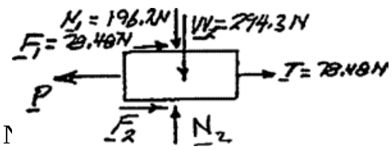


$$W_1 = (20\text{ kg})(9.81\text{ m/s}^2) = 196.2\text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2\text{ N}) = 78.48\text{ N}$$

$$\xrightarrow{+} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48\text{ N}$$

<u>Free body: 30-kg block</u>



$$\Sigma F = 0: \quad P - F_1 - F_2 - T = 0$$

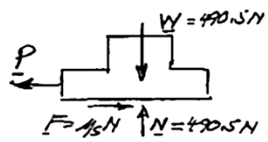
$$P = 78.48\text{ N} + 196.2\text{ N} + 78.48\text{ N} = 353.2\text{ N}$$

$$\xleftarrow{+}$$

$$\mathbf{P} = 353\text{ N} \longleftarrow \blacktriangleleft$$

(b) <u>Free body: Both blocks</u>

<u>Blocks move together</u>

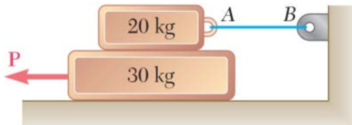


$$W = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N}$$

$$\xleftarrow{+} \Sigma F = 0: \quad P - F = 0$$

$$P = \mu_s N = 0.4(490.5\text{ N}) = 196.2\text{ N}$$

$$\mathbf{P} = 196.2\text{ N} \longleftarrow \blacktriangleleft$$

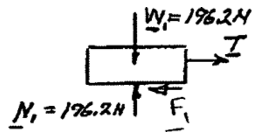


PROBLEM 4.90

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force **P** required to start the 30-kg block moving if cable AB (a) is attached as shown, (b) is removed.

SOLUTION

(a) Free body: 20-kg block

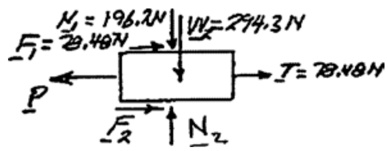


$$W_1 = (20\text{ kg})(9.81\text{ m/s}^2) = 196.2\text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2\text{ N}) = 78.48\text{ N}$$

$$\xrightarrow{+} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48\text{ N}$$

Free body: 30-kg block



$$W_2 = (30\text{ kg})(9.81\text{ m/s}^2) = 294.3\text{ N}$$

$$N_2 = 196.2\text{ N} + 294.3\text{ N} = 490.5\text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5\text{ N}) = 196.2\text{ N}$$

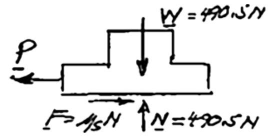
$$\xleftarrow{+} \Sigma F = 0: \quad P - F_1 - F_2 = 0$$

$$P = 78.48\text{ N} + 196.2\text{ N} = 274.7\text{ N}$$

$\mathbf{P} = 275\text{ N} \longleftarrow$ ◀

(b) Free body: Both blocks

Blocks move together



$$W = (50\text{ kg})(9.81\text{ m/s}^2)$$

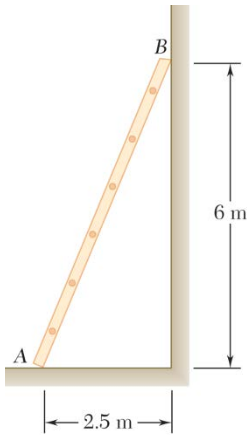
$$= 490.5\text{ N}$$

$$\xleftarrow{+} \Sigma F = 0: \quad P - F = 0$$

$$P = \mu_s N = 0.4(490.5\text{ N}) = 196.2\text{ N}$$

$\mathbf{P} = 196.2\text{ N} \longleftarrow$ ◀

PROBLEM 4.91



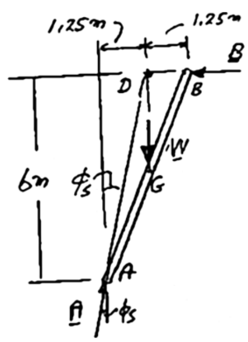
A 6.5-m ladder *AB* leans against a wall as shown. Assuming that the coefficient of static friction μ_s is zero at *B*, determine the smallest value of μ_s at *A* for which equilibrium is maintained.

SOLUTION

<u>Free body: Ladder</u>

Three-force body.

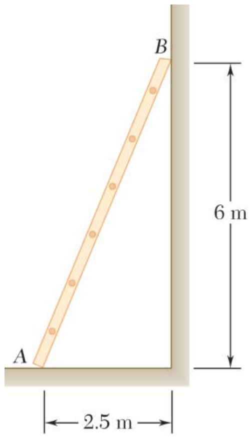
Line of action of **A** must pass through *D*, where **W** and **B** intersect.



At *A*:

$$\mu_s = \tan\phi_s = \frac{1.25 \text{ m}}{6 \text{ m}} = 0.2083$$

$$\mu_s = 0.208 \blacktriangleleft$$



PROBLEM 4.92

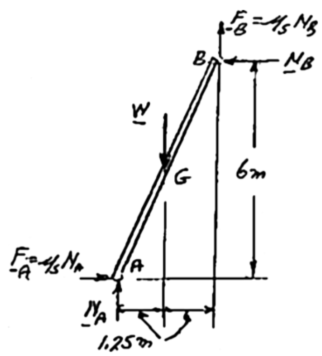
A 6.5-m ladder *AB* leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at *A* and *B*, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

<u>Free body: Ladder</u>

Motion impending:

$$F_A = \mu_s N_A$$
$$F_B = \mu_s N_B$$



$$+\circlearrowleft \Sigma M_A = 0: \quad W(1.25\text{ m}) - N_B(6\text{ m}) - \mu_s N_B(2.5\text{ m}) = 0$$

$$N_B = \frac{1.25W}{6 + 2.5\mu_s} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_A + \mu_s N_B - W = 0$$

$$N_A = W - \mu_s N_B$$
$$N_A = W - \frac{1.25\mu_s W}{6 + 2.5\mu_s} \qquad (2)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \mu_s N_A - N_B = 0$$

Substitute for N_A and N_B from Eqs. (1) and (2):

$$\mu_s W - \frac{1.25\mu_s^2 W}{6 + 2.5\mu_s} = \frac{1.25W}{6 + 2.5\mu_s}$$

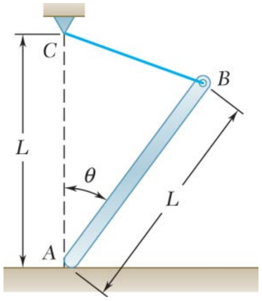
$$6\mu_s + 2.5\mu_s^2 - 1.25\mu_s^2 = 1.25$$

$$1.25\mu_s^2 + 6\mu_s - 1.25 = 0$$

$$\mu_s = 0.2$$

and $\quad \mu_s = -5 \quad$ (Discard)

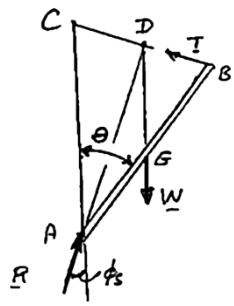
$$\mu_s = 0.200 \blacktriangleleft$$



PROBLEM 4.93

End A of a slender, uniform rod of length L and weight W bears on a surface as shown, while end B is supported by a cord BC. Knowing that the coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine (a) the largest value of θ for which motion is impending, (b) the corresponding value of the tension in the cord.

SOLUTION



Free-body diagram

Rod AB is a three-force body. Thus, line of action of $\mathbf{R}$ must pass through D, Where $\mathbf{W}$ and $\mathbf{T}$ intersect.

Since AG=GB, CD=DB and the median AD of the isosceles Triangle ABC bisects the angle θ.

(a) Thus,

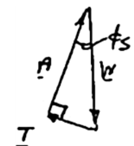
$$\phi_s = \frac{1}{2}\theta$$

Since motion impends,

$$\phi_s = \tan^{-1} 0.40 = 21.80°$$
$$\theta = 2\phi_s = 2(21.8°)$$

$\theta = 43.6°$ ◀

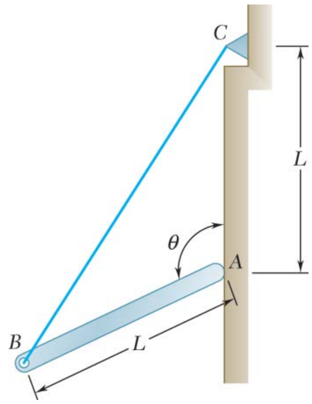
(b) Force triangle:



This is a right triangle.

$$T = W \sin\phi_s$$
$$= W \sin 21.8°$$

$T = 0.371W$ ◀



PROBLEM 4.94

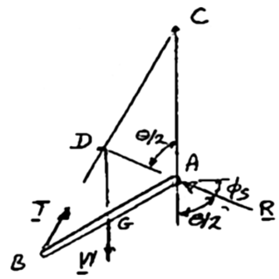
End A of a slender, uniform rod of length L and weight W bears on a surface as shown, while end B is supported by a cord BC. Knowing that the coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine (a) the largest value of θ for which motion is impending, (b) the corresponding value of the tension in the cord.

SOLUTION

Free-body diagram

Three-force body. Line of action of **R** must pass through D, where **T** and **R** intersect.

Motion impends:



$$\tan\phi_s = 0.4$$

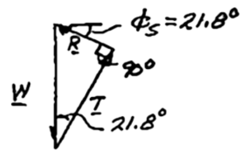
$$\phi_s = 21.80°$$

(a) Since $BG=GA$, it follows that $BD=DC$ and AD bisects $\angle BAC$

$$\frac{\theta}{2} + \phi_s = 90°$$

$$\frac{\theta}{2} + 21.8° = 90°$$

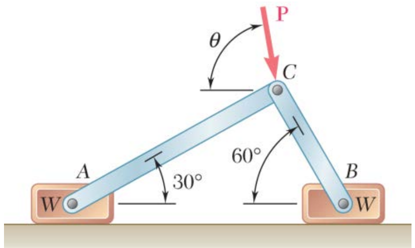
$\theta = 136.4°$ ◀



(b) Force triangle (right triangle):

$$T = W\cos 21.8°$$

$T = 0.928W$ ◀

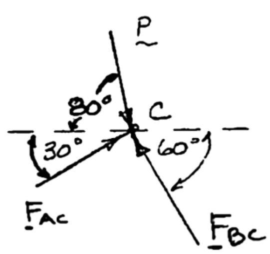


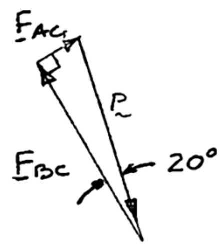
PROBLEM 4.95

Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $\theta = 80°$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD pin C:



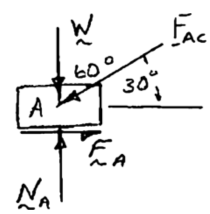


$$F_{AC} = P\sin 20° = 0.34202P$$
$$F_{BC} = P\cos 20° = 0.93969P$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W - F_{AC}\sin 30° = 0$$

or
$$N_A = W + 0.34202P\sin 30° = W + 0.171010P$$

FBD block A:



$$\rightarrow \Sigma F_x = 0: \quad F_A - F_{AC}\cos 30° = 0$$

or
$$F_A = 0.34202P\cos 30° = 0.29620P$$

For impending motion at A:
$$F_A = \mu_s N_A$$

Then
$$N_A = \frac{F_A}{\mu_s}: \quad W + 0.171010P = \frac{0.29620}{0.3}P$$

or
$$P = 1.22500W$$

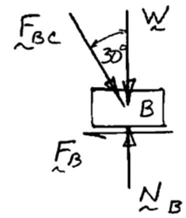
$$\uparrow \Sigma F_y = 0: \quad N_B - W - F_{BC}\cos 30° = 0$$

$$N_B = W + 0.93969P\cos 30° = W + 0.81380P$$

$$\rightarrow \Sigma F_x = 0: \quad F_{BC}\sin 30° - F_B = 0$$

$$F_B = 0.93969P\sin 30° = 0.46985P$$

SOLUTION Continued



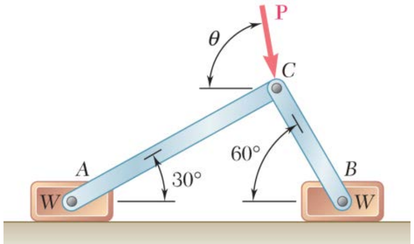
<u>FBD block *B*</u>:

For impending motion at *B*: $F_B = \mu_s N_B$

Then $N_B = \dfrac{F_B}{\mu_s}$: $W + 0.81380P = \dfrac{0.46985P}{0.3}$

or $P = 1.32914W$

Thus, maximum *P* for equilibrium $P_{\max} = 1.225W$ ◀



PROBLEM 4.96

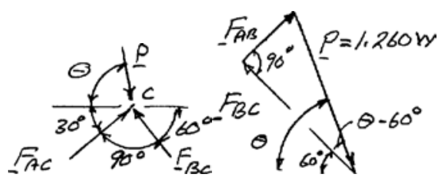
Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $P = 1.260W$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the range of values of θ, between 0 and 180°, for which equilibrium is maintained.

SOLUTION

AC and BC are two-force members

Free body: Joint C

Force triangle:



From Force triangle:

$$F_{AB} = P\sin(\theta - 60°) = 1.26W\sin(\theta - 60°) \tag{1}$$

$$F_{BC} = P\cos(\theta - 60°) = 1.26W\cos(\theta - 60°) \tag{2}$$

We shall, in turn, seek θ corresponding to impending motion of each block

For motion of A impending to left

from solution of Prob. 8.46: $F_{AC} = 0.419W$

EQ (1):

$$F_{AC} = 0.419W = 1.26W\sin(\theta - 60°)$$

$$\sin(\theta - 60°) = 0.33254$$

$$\theta - 60° = 19.423°$$

$$\theta = 79.42° \quad \triangleleft$$

For motion of B impending to right.

from solution of Prob. 8.46: $F_{BC} = 1.249W$

Eq. (2):

$$F_{BC} = 1.249W = 1.26W\cos(\theta - 60°)$$

$$\cos(\theta - 60°) = 0.99127$$

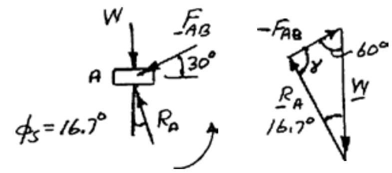
$$\theta - 60° = \pm 7.58°$$

$$\theta - 60° = +7.58° \qquad \theta = 67.6° \quad \triangleleft$$

$$\theta - 60° = -7.58° \qquad \theta = 52.4° \quad \triangleleft$$

SOLUTION Continued

For motion of A impending to right



$$\gamma = 180° - 60° - 16.7° = 103.3°$$

Law of sines:

$$\frac{-F_{AB}}{\sin 16.7°} = \frac{W}{\sin 103.3°}$$

$$F_{AB} = -0.29528W$$

Note: Direction of $+F_{AB}$ is kept same as in free body of Joint C.

Eq. (1): $$F_{AB} = -0.29528W = 1.26W \sin(\theta - 60°)$$

$$\sin(\theta - 60°) = -0.23435$$

$$(\theta - 60°) = -13.553°$$

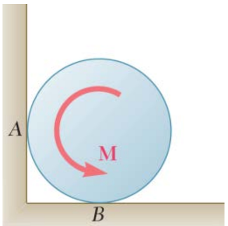
$\theta = 46.4°$ ◁

Summary:

A moves to right	No motion	B moves to right	No motion	A moves to left

θ: 46.4°, 52.4°, 67.6°, 79.4°

No motion for: $46.4° \le \theta \le 52.4°$ and $67.6° \le \theta \le 79.4°$ ◀

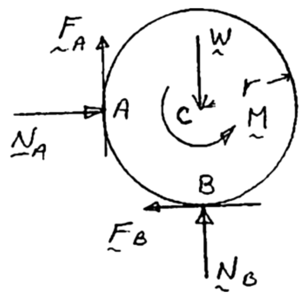


PROBLEM 4.97

The cylinder shown is of weight W and radius r, and the coefficient of static friction μ_s is the same at A and B. Determine the magnitude of the largest couple **M** that can be applied to the cylinder if it is not to rotate.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_s N_A$$
$$F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: \quad N_A - F_B = 0$$
$$N_A = F_B = \mu_s N_B$$
$$F_A = \mu_s N_A = \mu_s^2 N_B$$

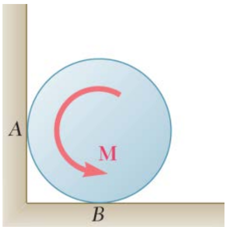
$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0$$
$$N_B + \mu_s^2 N_B = W$$

or
$$N_B = \frac{W}{1+\mu_s^2}$$

and
$$F_B = \frac{\mu_s W}{1+\mu_s^2}$$
$$F_A = \frac{\mu_s^2 W}{1+\mu^2}$$

$$\circlearrowleft \Sigma M_C = 0: \quad M - r(F_A + F_B) = 0$$
$$M = r\left(\mu_s + \mu_s^2\right)\frac{W}{1+\mu_s^2}$$

$$M_{\max} = Wr\mu_s \frac{1+\mu_s}{1+\mu_s^2} \blacktriangleleft$$

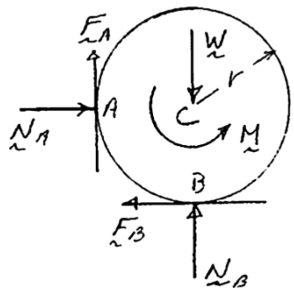


PROBLEM 4.98

The cylinder shown is of weight W and radius r. Express in terms W and r the magnitude of the largest couple **M** that can be applied to the cylinder if it is not to rotate, assuming the coefficient of static friction to be (*a*) zero at A and 0.30 at B, (*b*) 0.25 at A and 0.30 at B.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_A N_A$$
$$F_B = \mu_B N_B$$

$$\xrightarrow{} \Sigma F_x = 0: \quad N_A - F_B = 0$$
$$N_A = F_B = \mu_B N_B$$

$$F_A = \mu_A N_A = \mu_A \mu_B N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0$$
$$N_B(1 + \mu_A \mu_B) = W$$

or
$$N_B = \frac{1}{1+\mu_A \mu_B} W$$

and
$$F_B = \mu_B N_B = \frac{\mu_B}{1+\mu_A \mu_B} W$$

$$F_A = \mu_A \mu_B N_B = \frac{\mu_A \mu_B}{1+\mu_A \mu_B} W$$

$$\Sigma M_C = 0: \quad M - r(F_A + F_B) = 0$$

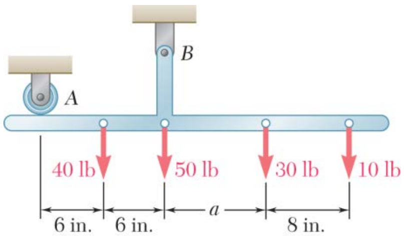
$$M = Wr\mu_B \frac{1+\mu_A}{1+\mu_A \mu_B}$$

(*a*) For $\mu_A = 0$ and $\mu_B = 0.30$:

$$M = 0.300Wr \blacktriangleleft$$

(*b*) For $\mu_A = 0.25$ and $\mu_B = 0.30$:

$$M = 0.349Wr \blacktriangleleft$$

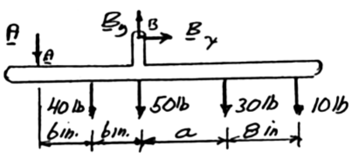


PROBLEM 4.99

A T-shaped bracket supports the four loads shown. Determine the reactions at A and B (a) if $a = 10$ in., (b) if $a = 7$ in.

SOLUTION

Free-Body Diagram:



$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$

$+\circlearrowleft \Sigma M_B = 0: \quad (40 \text{ lb})(6 \text{ in.}) - (30 \text{ lb})a - (10 \text{ lb})(a + 8 \text{ in.}) + (12 \text{ in.})A = 0$

$$A = \frac{(40a - 160)}{12} \qquad (1)$$

$+\circlearrowleft \Sigma M_A = 0: \quad -(40 \text{ lb})(6 \text{ in.}) - (50 \text{ lb})(12 \text{ in.}) - (30 \text{ lb})(a + 12 \text{ in.})$
$\quad - (10 \text{ lb})(a + 20 \text{ in.}) + (12 \text{ in.})B_y = 0$

$$B_y = \frac{(1400 + 40a)}{12}$$

Since $\quad B_x = 0, \quad B = \dfrac{(1400 + 40a)}{12} \qquad (2)$

(a) For $a = 10$ in.,

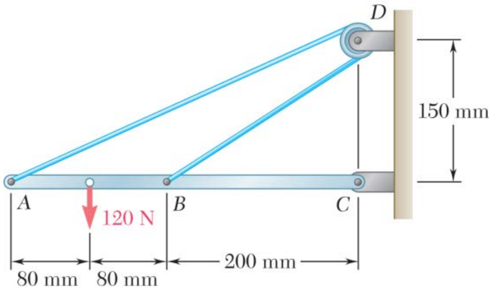
Eq. (1): $\quad A = \dfrac{(40 \times 10 - 160)}{12} = +20.0 \text{ lb} \qquad \mathbf{A} = 20.0 \text{ lb} \downarrow$ ◀

Eq. (2): $\quad B = \dfrac{(1400 + 40 \times 10)}{12} = +150.0 \text{ lb} \qquad \mathbf{B} = 150.0 \text{ lb} \uparrow$ ◀

(b) For $a = 7$ in.,

Eq. (1): $\quad A = \dfrac{(40 \times 7 - 160)}{12} = +10.00 \text{ lb} \qquad \mathbf{A} = 10.00 \text{ lb} \downarrow$ ◀

Eq. (2): $\quad B = \dfrac{(1400 + 40 \times 7)}{12} = +140.0 \text{ lb} \qquad \mathbf{B} = 140.0 \text{ lb} \uparrow$ ◀



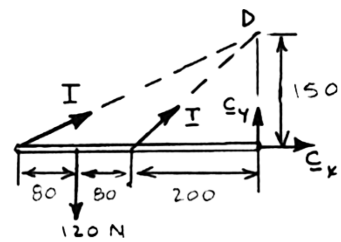
PROBLEM 4.100

Neglecting friction and the radius of the pulley, determine (*a*) the tension in cable *ADB*, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:

Dimensions in mm



Geometry:

Distance: $$AD = \sqrt{(0.36)^2 + (0.150)^2} = 0.39 \text{ m}$$

Distance: $$BD = \sqrt{(0.2)^2 + (0.15)^2} = 0.25 \text{ m}$$

Equilibrium for beam:

(*a*) $$+\circlearrowleft \Sigma M_C = 0: \quad (120 \text{ N})(0.28 \text{ m}) - \left(\frac{0.15}{0.39}T\right)(0.36 \text{ m}) - \left(\frac{0.15}{0.25}T\right)(0.2 \text{ m}) = 0$$

$$T = 130.000 \text{ N}$$

or $T = 130.0$ N ◀

(*b*) $$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + \left(\frac{0.36}{0.39}\right)(130.000 \text{ N}) + \left(\frac{0.2}{0.25}\right)(130.000 \text{ N}) = 0$$

$$C_x = -224.00 \text{ N}$$

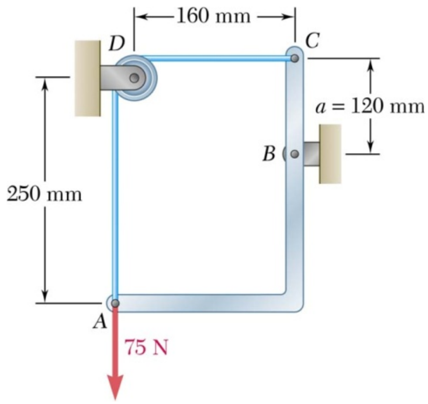
$$+\uparrow \Sigma F_y = 0: \quad C_y + \left(\frac{0.15}{0.39}\right)(130.00 \text{ N}) + \left(\frac{0.15}{0.25}\right)(130.00 \text{ N}) - 120 \text{ N} = 0$$

$$C_y = -8.0000 \text{ N}$$

Thus, $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-224)^2 + (-8)^2} = 224.14 \text{ N}$$

and $$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{8}{224} = 2.0454°$$

C = 224 N ↙ 2.05° ◀

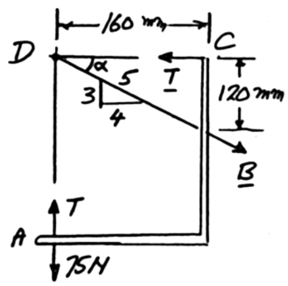


PROBLEM 4.101

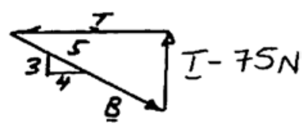
Member *ABC* is supported by a pin and bracket at *B* and by an inextensible cord attached at *A* and *C* and passing over a frictionless pulley at *D*. The tension may be assumed to be the same in portions *AD* and *CD* of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at *B*.

SOLUTION

Free-Body Diagram:



Force Triangle



Reaction at *B* must pass through *D*.

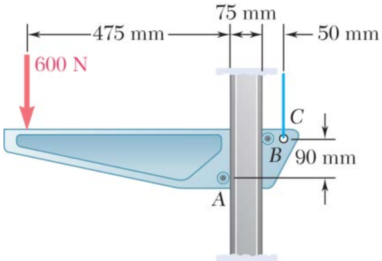
$$\tan\alpha = \frac{120}{160}; \quad \alpha = 36.9°$$

$$\frac{T}{4} = \frac{T - 75\text{ N}}{3} = \frac{B}{5}$$

$$3T = 4T - 300; \quad T = 300\text{ N}$$

$$B = \frac{5}{4}T = \frac{5}{4}(300\text{ N}) = 375\text{ N}$$

$\mathbf{B} = 375\text{ N} \searrow 36.9°$

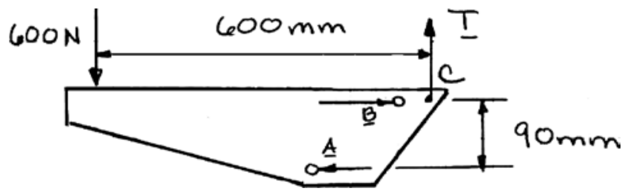


PROBLEM 4.102

A movable bracket is held at rest by a cable attached at C and by frictionless rollers at A and B. For the loading shown, determine (a) the tension in the cable, (b) the reactions at A and B.

SOLUTION

Free-Body Diagram:



(a) $+\uparrow \Sigma F_y = 0: \quad T - 600\text{ N} = 0$

$T = 600\text{ N}$ ◀

(b) $\xrightarrow{+} \Sigma F_x = 0: \quad B - A = 0 \qquad \therefore \quad B = A$

Note that the forces shown form two couples.

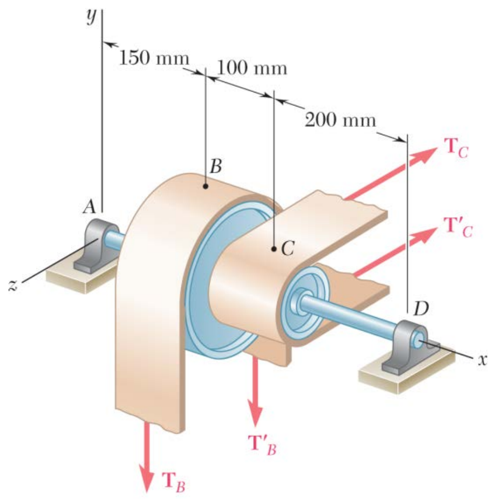
$$+\circlearrowleft \Sigma M = 0: \quad (600\text{ N})(600\text{ mm}) - A(90\text{ mm}) = 0$$

$$A = 4000\text{ N}$$

$$\therefore \quad B = 4000\text{ N}$$

$\mathbf{A} = 4.00\text{ kN} \longleftarrow; \ \mathbf{B} = 4.00\text{ kN} \longrightarrow$ ◀

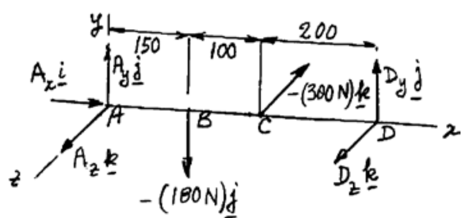
PROBLEM 4.103



Two transmission belts pass over a double-sheaved pulley that is attached to an axle supported by bearings at *A* and *D*. The radius of the inner sheave is 125 mm and the radius of the outer sheave is 250 mm. Knowing that when the system is at rest, the tension is 90 N in both portions of belt *B* and 150 N in both portions of belt *C*, determine the reactions at *A* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

We replace $\mathbf{T}_B$ and $\mathbf{T}'_B$ by their resultant $(-180\text{ N})\mathbf{j}$ and $\mathbf{T}_C$ and $\mathbf{T}'_C$ by their resultant $(-300\text{ N})\mathbf{k}$.



Dimensions in mm

We have five unknowns and six equations of equilibrium. Axle *AD* is free to rotate about the *x*-axis, but equilibrium is maintained $(\Sigma M_x = 0)$.

$$\Sigma \mathbf{M}_A = 0: \quad (150\mathbf{i}) \times (-180\mathbf{j}) + (250\mathbf{i}) \times (-300\mathbf{k}) + (450\mathbf{i}) \times (D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-27 \times 10^3\mathbf{k} + 75 \times 10^3\mathbf{j} + 450D_y\mathbf{k} - 450D_z\mathbf{j} = 0$$

Equating coefficients of **j** and **k** to zero,

$$\mathbf{j}: \quad 75 \times 10^3 - 450D_z = 0 \qquad D_z = 166.7\text{ N}$$

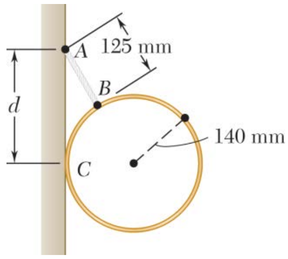
$$\mathbf{k}: \quad -27 \times 10^3 + 450D_y = 0 \qquad D_y = 60.0\text{ N}$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma F_y = 0: \quad A_y + D_y - 180\text{ N} = 0 \qquad A_y = 180 - 60 = 120.0\text{ N}$$

$$\Sigma F_z = 0: \quad A_z + D_z - 300\text{ N} = 0 \qquad A_z = 300 - 166.7 = 133.3\text{ N}$$

$$\mathbf{A} = (120.0\text{ N})\mathbf{j} + (133.3\text{ N})\mathbf{k}; \quad \mathbf{D} = (60.0\text{ N})\mathbf{j} + (166.7\text{ N})\mathbf{k} \blacktriangleleft$$



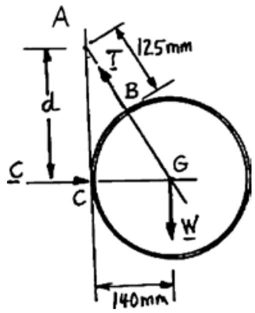
PROBLEM 4.104

A thin ring of mass 2 kg and radius r = 140 mm is held against a frictionless wall by a 125-mm string AB. Determine (a) the distance d, (b) the tension in the string, (c) the reaction at C.

SOLUTION

Free-Body Diagram:

(Three-force body)



The force T exerted at B must pass through the center G of the ring, since **C** and **W** intersect at that point.

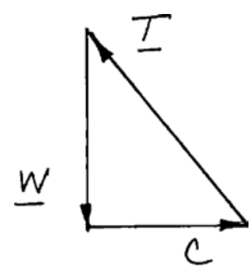
Thus, points A, B, and G are in a straight line.

(a) From triangle ACG:

$$d = \sqrt{(AG)^2 - (CG)^2}$$
$$= \sqrt{(265 \text{ mm})^2 - (140 \text{ mm})^2}$$
$$= 225.00 \text{ mm}$$

$d = 225 \text{ mm}$ ◀

Force Triangle



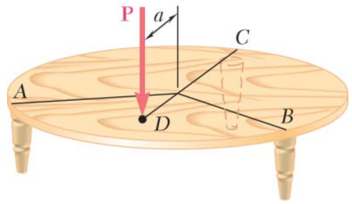
$$W = (2 \text{ kg})(9.81 \text{ m/s}^2) = 19.6200 \text{ N}$$

Law of sines:

$$\frac{T}{265 \text{ mm}} = \frac{C}{140 \text{ mm}} = \frac{19.6200 \text{ N}}{225.00 \text{ mm}}$$

(b) $T = 23.1 \text{ N}$ ◀

(c) $\mathbf{C} = 12.21 \text{ N} \longrightarrow$ ◀

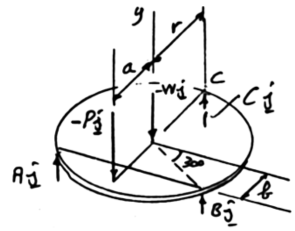


PROBLEM 4.105

The table shown weighs 30 lb and has a diameter of 4 ft. It is supported by three legs equally spaced around the edge. A vertical load **P** of magnitude 100 lb is applied to the top of the table at *D*. Determine the maximum value of *a* if the table is not to tip over. Show, on a sketch, the area of the table over which **P** can act without tipping the table.

SOLUTION

$$r = 2\text{ ft} \quad b = r\sin 30° = 1\text{ ft}$$



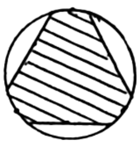
We shall sum moments about *AB*.

$$(b+r)C + (a-b)P - bW = 0$$

$$(1+2)C + (a-1)100 - (1)30 = 0$$

$$C = \frac{1}{3}[30 - (a-1)100]$$

If table is not to tip, $C \geq 0$.

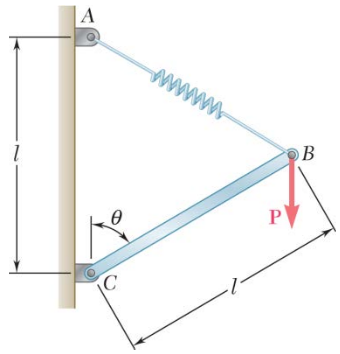


$$[30 - (a-1)100] \geq 0$$

$$30 \geq (a-1)100$$

$$a - 1 \leq 0.3 \quad a \leq 1.3\text{ ft} \qquad a = 1.300\text{ ft} \blacktriangleleft$$

Only ⊥ distance from *P* to *AB* matters. Same condition must be satisfied for each leg. *P* must be located in shaded area for no tipping.

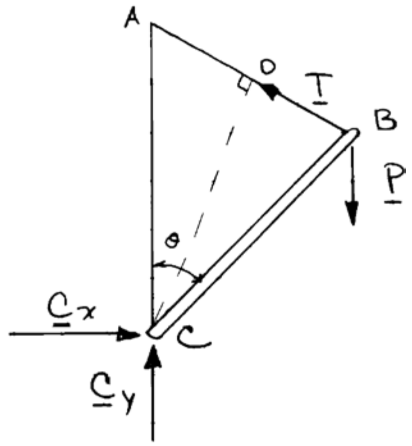


PROBLEM 4.106

A vertical load **P** is applied at end *B* of rod *BC*. The constant of the spring is *k*, and the spring is unstretched when $\theta = 60°$. (*a*) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position terms of *P*, *k*, and *l*. (*b*) Determine the value of θ corresponding to equilibrium if $P = \frac{1}{4}kl$.

SOLUTION

Free-Body Diagram:



(*a*) Triangle *ABC* is isosceles. We have

$$AB = 2(AD) = 2l\sin\left(\frac{\theta}{2}\right);\ CD = l\cos\left(\frac{\theta}{2}\right)$$

Elongation of spring: $x = (AB)_\theta - (AB)_\theta = 60°$

$$= 2l\sin\left(\frac{\theta}{2}\right) - 2l\sin 30°$$

$$T = kx = 2kl\left(\sin\frac{\theta}{2} - \frac{1}{2}\right)$$

$$\circlearrowleft \Sigma M_C = 0: \quad T\left(l\cos\frac{\theta}{2}\right) - P(l\sin\theta) = 0$$

SOLUTION Continued

$$2kl\left(\sin\frac{\theta}{2}-\frac{1}{2}\right)l\cos\frac{\theta}{2}-Pl\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)=0$$

$$\cos\frac{\theta}{2}=0 \quad \text{or} \quad 2(kl-P)\sin\frac{\theta}{2}-kl=0$$

$$\theta=180^\circ \text{ (trivial)} \qquad \sin\frac{\theta}{2}=\frac{\frac{1}{2}kl}{kl-P}$$

$$\theta=2\sin^{-1}\left[\frac{1}{2}kl/(kl-P)\right] \blacktriangleleft$$

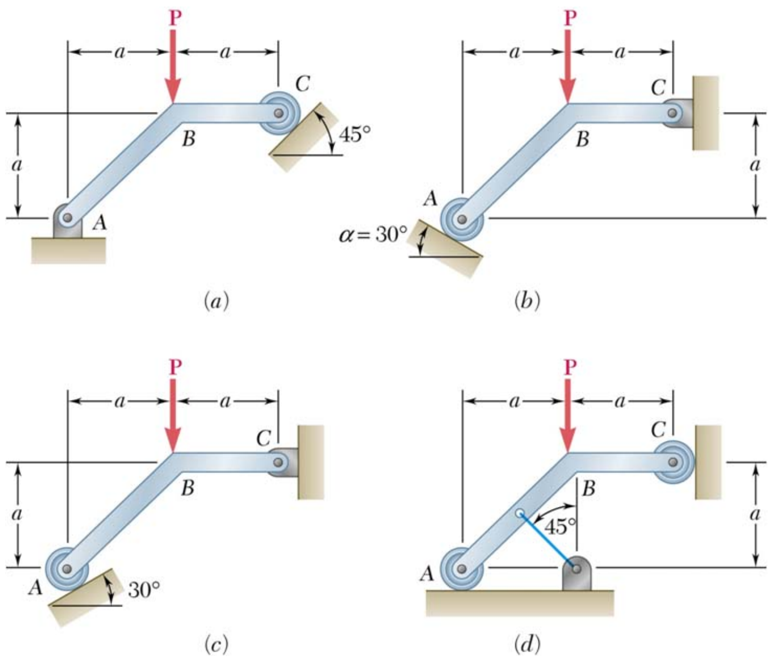
(b) For $P=\frac{1}{4}kl$,

$$\sin\frac{\theta}{2}=\frac{\frac{1}{2}kl}{\frac{3}{4}kl}=\frac{2}{3}$$

$$\frac{\theta}{2}=41.8^\circ \qquad \theta=83.6^\circ \blacktriangleleft$$

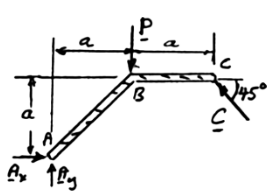
PROBLEM 4.107

A force **P** is applied to a bent rod *ABC*, which may be supported in four different ways as shown. In each case, if possible, determine the reactions at the supports.



SOLUTION

(*a*)



$$+\circlearrowleft \Sigma M_A = 0: \quad -P_a + (C\sin 45°)2a + (\cos 45°)a = 0$$

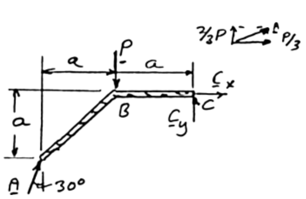
$$3\frac{C}{\sqrt{2}} = P \qquad C = \frac{\sqrt{2}}{3}P \qquad \mathbf{C} = 0.471P \searrow 45° ◀$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - \left(\frac{\sqrt{2}}{3}P\right)\frac{1}{\sqrt{2}} \qquad A_x = \frac{P}{3} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - P + \left(\frac{\sqrt{2}}{3}P\right)\frac{1}{\sqrt{2}} \qquad A_y = \frac{2P}{3} \uparrow$$

$$\mathbf{A} = 0.745P \measuredangle 63.4° ◀$$

(*b*)



$$+\circlearrowleft \Sigma M_C = 0: \quad +Pa - (A\cos 30°)2a + (A\sin 30°)a = 0$$

$$A(1.732 - 0.5) = P \quad A = 0.812P$$

$$\mathbf{A} = 0.812P \measuredangle 60.0° ◀$$

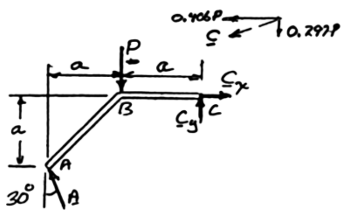
$$\xrightarrow{+} \Sigma F_x = 0: \quad (0.812P)\sin 30° + C_x = 0 \qquad C_x = -0.406P$$

$$+\uparrow \Sigma F_y = 0: \quad (0.812P)\cos 30° - P + C_y = 0 \qquad C_y = -0.297P$$

$$\mathbf{C} = 0.503P \nearrow 36.2° ◀$$

SOLUTION Continued

(c)



$$+\circlearrowleft \Sigma M_C = 0: \quad +Pa - (A\cos 30°)2a + (A\sin 30°)a = 0$$

$$A(1.732 + 0.5) = P \quad A = 0.448P$$

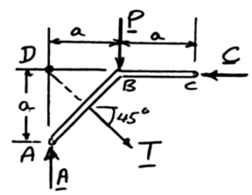
$$A = 0.448P \; \searrow 60.0° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -(0.448P)\sin 30° + C_x = 0 \qquad C_x = 0.224P \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad (0.448P)\cos 30° - P + C_y = 0 \quad C_y = 0.612P \uparrow$$

$$\mathbf{C} = 0.652P \; \nearrow 69.9° \blacktriangleleft$$

(d) Force **T** exerted by wire and reactions **A** and **C** all intersect at Point D.

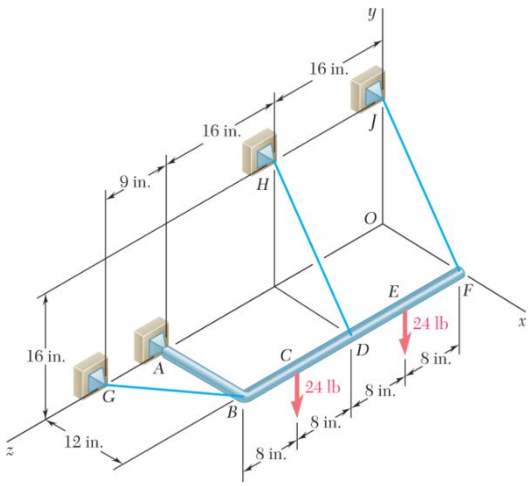


$$+\circlearrowleft \Sigma M_D = 0: \quad P_a = 0$$

Equilibrium is not maintained.

Rod is improperly constrained. ◀

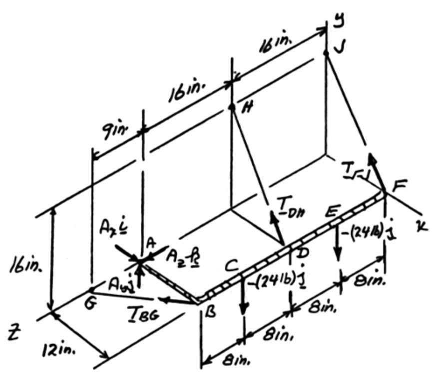
PROBLEM 4.108



The rigid L-shaped member *ABF* is supported by a ball-and-socket joint at *A* and by three cables. For the loading shown, determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{B/A} = 12\mathbf{i}$$
$$\mathbf{r}_{F/A} = 12\mathbf{j} - 8\mathbf{k}$$
$$\mathbf{r}_{D/A} = 12\mathbf{i} - 16\mathbf{k}$$
$$\mathbf{r}_{E/A} = 12\mathbf{i} - 24\mathbf{k}$$
$$\mathbf{r}_{F/A} = 12\mathbf{i} - 32\mathbf{k}$$
$$\overrightarrow{BG} = -12\mathbf{i} + 9\mathbf{k}$$
$$BG = 15 \text{ in.}$$
$$\boldsymbol{\lambda}_{BG} = -0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\overrightarrow{DH} = -12\mathbf{i} + 16\mathbf{j}; \quad DH = 20 \text{ in.}; \quad \lambda_{DH} = -0.6\mathbf{i} + 0.8\mathbf{j}$$
$$\overrightarrow{FJ} = -12\mathbf{i} + 16\mathbf{j}; \quad FJ = 20 \text{ in.}; \quad \lambda_{FJ} = -0.6\mathbf{i} + 0.8\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{B/A} \times \mathbf{T}_{BG}\lambda_{BG} + \mathbf{r}_{DH} \times \mathbf{T}_{DH}\lambda_{DH} + \mathbf{r}_{F/A} \times \mathbf{T}_{FJ}\lambda_{FJ}$$
$$+\mathbf{r}_{F/A} \times (-24\mathbf{j}) + \mathbf{r}_{E/A} \times (-24\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ -0.8 & 0 & 0.6 \end{vmatrix} T_{BG} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -16 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{DH} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -32 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{FJ}$$
$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -8 \\ 0 & -24 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -24 \\ 0 & -24 & 0 \end{vmatrix} = 0$$

Coefficient of **i**: $\quad +12.8T_{DH} + 25.6T_{FJ} - 192 - 576 = 0 \quad (1)$

Coefficient of **k**: $\quad +9.6T_{DH} + 9.6T_{FJ} - 288 - 288 = 0 \quad (2)$

$\frac{3}{4}$ Eq. (1) – Eq. (2): $\quad 9.6T_{FJ} = 0 \qquad T_{FJ} = 0$ ◀

SOLUTION Continued

From Eq. (1): $12.8T_{DH} - 268 = 0$ $\quad T_{DH} = 60 \text{ lb}$ ◀

Coefficient of **j:** $-7.2T_{BG} + (16 \times 0.6)(60.0 \text{ lb}) = 0$ $\quad T_{BG} = 80.0 \text{ lb}$ ◀

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + T_{BG}\boldsymbol{\lambda}_{BG} + T_{DH}\boldsymbol{\lambda}_{DH} + T_{FJ} - 24\mathbf{j} - 24\mathbf{j} = 0$$

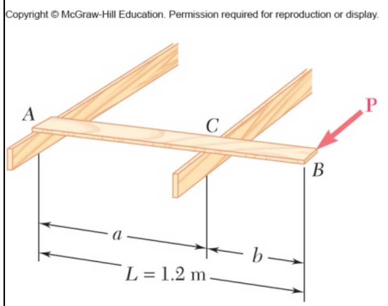
Coefficient of **i**: $A_x + (80)(-0.8) + (60.0)(-0.6) = 0$ $\quad A_x = 100.0 \text{ lb}$

Coefficient of **j**: $A_y + (60.0)(0.8) - 24 - 24 = 0$ $\quad A_y = 0$

Coefficient of **k:** $A_z + (80.0)(+0.6) = 0$ $\quad A_z = -48.0 \text{ lb}$

$$\mathbf{A} = (100.0 \text{ lb})\mathbf{i} - (48.0 \text{ lb})\mathbf{k}$$ ◀

Note: The value $A_y = 0$ can be confirmed by considering $\Sigma M_{BF} = 0$.



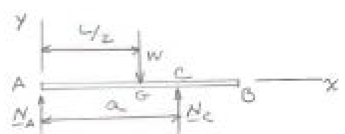
PROBLEM 4.109

A 1.2-m plank with a mass of 3 kg rests on two joists. Knowing that the coefficient of static friction between the plank and the joists is 0.30, determine the magnitude of the horizontal force required to move the plank when (*a*) *a* = 750 mm, (*b*) *a* = 900 mm.

SOLUTION

Free body member *AB*:

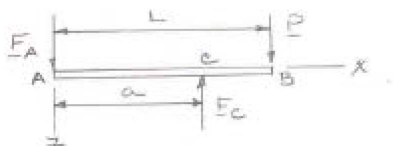
In the vertical plane,



$$+\circlearrowleft \Sigma M_A = 0: \quad N_C a - W\frac{L}{2} = 0 \quad \text{hence } N_C = W\frac{L}{2a} \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_A + N_C - W = 0 \quad \text{hence} \quad N_A = W\frac{2a-L}{2a} \quad (2)$$

In the horizontal plane,



$$+\circlearrowleft \Sigma M_A = 0: \quad F_C a - PL = 0 \quad \text{hence} \quad F_C = P\frac{L}{a} \quad (3)$$

$$+\downarrow \Sigma F_Z = 0: \quad F_A + P - F_C = 0 \quad \text{hence} \quad F_A = P\frac{L-a}{a} \quad (4)$$

(a) Substituting given data: $a = 0.75$ m, $L = 1.2$ m, $\mu_s = 0.30$, $W = (3\text{ kg})(9.81\text{ m/s}^2) = 29.430$ N

$$N_C = 0.8W, \quad N_A = 0.2W, \quad F_C = 1.6P, \quad F_A = 0.6P$$

To slip at *A*, $F_A = \mu_s N_A$ or $0.6P = 0.30(0.2W)$ $P = 0.1W$

To slip at *C*, $F_C = \mu_s N_C$ or $1.6P = 0.30(0.8W)$ $P = 0.15W$

The plank will slip at *A* $P = 2.94$ N ◀

PROBLEM 4.109 (Continued)

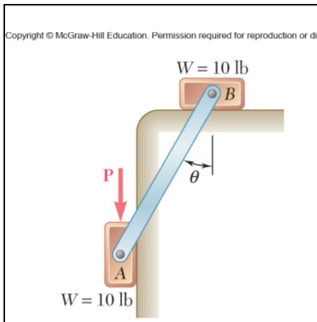
(b) Substituting given data: $a = 0.9$ m, $L = 1.2$ m, $\mu_s = 0.30$, $W = (3 \text{ kg})(9.81 \text{ m/s}^2) = 29.430$ N

$$N_C = \frac{2}{3}W, \quad N_A = \frac{1}{3}W, \quad F_C = \frac{4}{3}P, \quad F_A = \frac{1}{3}P$$

To slip at *A*, $F_A = \mu_s N_A$ or $\frac{1}{3}P = 0.30\left(\frac{1}{3}W\right)$ $P = 0.3W$

To slip at *C*, $F_C = \mu_s N_C$ or $\frac{4}{3}P = 0.30\left(\frac{2}{3}W\right)$ $P = 0.15W$

The plank will slip at C for $P = 0.15(29.43 \text{ N})$ $P = 4.41$ N ◀



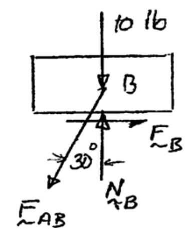
PROBLEM 4.110

Two 10-lb blocks A and B are connected by a slender rod of negligible weight. The coefficient of static friction is 0.30 between all surfaces of contact, and the rod forms an angle $\theta = 30°$. with the vertical. (a) Show that the system is in equilibrium when $P = 0$. (b) Determine the largest value of P for which equilibrium is maintained.

SOLUTION

<u>FBD block *B*:</u>

(*a*) Since $P = 2.69$ lb to initiate motion, equilibrium exists with $P = 0$ ◀



(*b*) For $P_{\max}$, motion impends at both surfaces:

Block *B*: $\uparrow \Sigma F_y = 0\colon \quad N_B - 10\text{ lb} - F_{AB}\cos 30° = 0$

$$N_B = 10\text{ lb} + \frac{\sqrt{3}}{2}F_{AB} \qquad (1)$$

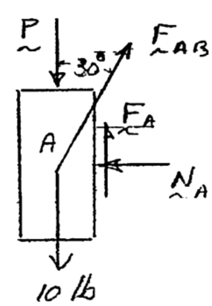
Impending motion: $F_B = \mu_s N_B = 0.3N_B$

$$\longrightarrow \Sigma F_x = 0\colon \quad F_B - F_{AB}\sin 30° = 0$$

$$F_{AB} = 2F_B = 0.6N_B \qquad (2)$$

Solving Eqs. (1) and (2): $N_B = 10\text{ lb} + \frac{\sqrt{3}}{2}(0.6N_B) = 20.8166\text{ lb}$

<u>FBD block *A*:</u>



Then $F_{AB} = 0.6N_B = 12.4900\text{ lb}$

Block *A*: $\longrightarrow \Sigma F_x = 0\colon \quad F_{AB}\sin 30° - N_A = 0$

$$N_A = \frac{1}{2}F_{AB} = \frac{1}{2}(12.4900\text{ lb}) = 6.2450\text{ lb}$$

Impending motion: $F_A = \mu_s N_A = 0.3(6.2450\text{ lb}) = 1.8735\text{ lb}$

$$\uparrow \Sigma F_y = 0\colon \quad F_A + F_{AB}\cos 30° - P - 10\text{ lb} = 0$$

$$P = F_A + \frac{\sqrt{3}}{2}F_{AB} - 10\text{ lb}$$

$$= 1.8735\text{ lb} + \frac{\sqrt{3}}{2}(12.4900\text{ lb}) - 10\text{ lb}$$

$$= 2.69\text{ lb}$$

$P = 2.69$ lb ◀