Chapter 1: Probability Models in Electrical and Computer Engineering

1.1 (a) Sample space

$S_1 = \{H, T\}$ $\quad S_2 = \{1, 2, 3, 4, 5, 6\}$ $\quad S_3 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(b) $p_H = p_T = \frac{1}{2}$ if both sides equally likely (fair coin)

$p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = \frac{1}{6}$ if die fair

$p_0 = p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = p_8 = p_9 = \frac{1}{10}$ balls identical

1.2 (a) $S = \{HH, HT, TH, TT\}$ $\quad S_{URN} = \{0, 1, 2, 3\}$

4 equiprobable outcomes

4 identical numbered balls
or 2 draws from urn with 2 identical balls

(b)

toss 1

toss 2	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

pair of tosses results in 36 equiprobable outcomes

$\Rightarrow$ Urn with 36 identical numbered balls
or 2 draws from urn with 6 balls

(c) with replacement:
Same as (b) with 52×52 equiprobable pairs
$\Rightarrow$ urn with $52 \times 52 = 2704$ identical numbered balls
or 2 draws from urn with 52 balls

without replacement:
52×51 equiprobable pairs
$\Rightarrow$ urn with $52 \times 51 = 2652$ identical numbered balls
or 2 draws from urn with 52 identical balls without replacement

1.3 (a) Equivalent to toss of a biased coin : black = heads ; white = tail
white dots much more frequent than black dots so $p_H << 1$.

(b) Binary signal ⇒ 2 outcomes in each transmission
Each outcome can assigned to head or tails
p_H depends on probability of signal outcomes

(c) Device is either working ("heads") or not ("tails")
p_H depends on device and testing schedule

(d) Joe is either online ("heads") or not ("tails")
p_H depends on when observation is made

(e) Received bits equals transmitted bit ("heads") or not ("tails")
p_H depends on properties of transmission channel

1.4 (a) $S_{Lisa} = \{00, 01, 10\}$ $S_{Homer} = \{10, 11\}$ $S_{Bart} = \{00, 10\}$

(b) Lisa: $p_{00} = p_{01} = p_{10} = \frac{1}{3}$

Homer: ball 00 & ball 10 read as 10 $\quad p_{10} = \frac{2}{3}$ $p_{11} = \frac{1}{3}$
ball 01 reads as 11

Bart : ball 00 & ball 01 read as 00 $\quad p_{00} = \frac{2}{3}$ $p_{10} = \frac{1}{3}$
ball 10 reads as 10

1.5 (a) Toss coin: Heads ⇒ "1" Tails ⇒ Do 2nd toss
Heads ⇒ "2" Tails ⇒ Do 3rd toss
Heads ⇒ "3" Tails ⇒ "4"

(b) Urn with 8 identical balls with labels : {1, 1, 1, 1, 2, 2, 3, 4}

(c) Draw Card : if Ace reject outcome and restart experiment
if Not Ace output # assigned to the card
where 24 cards assigned "1" ~ 24/48
12 cards " "2" ~ 12/48
6 " " "3" ~ 6/48
6 " " "4" ~ 6/48

1.6 a) In the first draw the outcome can be black (b) or white (w). If the first draw is black, then the second outcome can be b or w. However if the first draw is white, then the run only contains black balls so the second outcome must be b. Therefore $S = \{bb, bw, wb\}$.

b) In this case all outcomes can be b or w. Therefore $S = \{bb, bw, wb, ww\}$.

c) In part a) the outcome ww cannot occur so $f_{ww} = 0$. In part b) let N be a larger number of repetitions of the experiment. The number of times the first outcome is w is approximately $N/3$ since the run has one white ball and two black balls. Of these $N/3$ outcomes approximately $1/2$ are also white in the second draw. Thus $N/9$ if the outcome result is ww, and thus $f_{ww} = \frac{1}{9}$.

d) In the first experiment, the outcome of the first draw affects the probability of the outcomes in the second draw. In the second experiment, the outcome of the first draw does not affect the probability of the outcomes in the second draw.

1.7 When the experiment is performed, either A occurs or it doesn't (i.e. B occurs); thus $N_A(n) + N_B(n) = n$ in n repetitions of the experiment, and

$$f_A(n) + f_B(n) = \frac{N_A(n)}{n} + \frac{N_B(n)}{n} = 1.$$

Thus $f_B(n) = 1 - f_A(n)$.

1.8 If A, B, or C occurs, then D occurs. Furthermore since A, B, or C cannot occur simultaneously, in n repetitions of the experiment we have

$$N_D(n) = N_A(n) + N_B(n) + N_C(n)$$

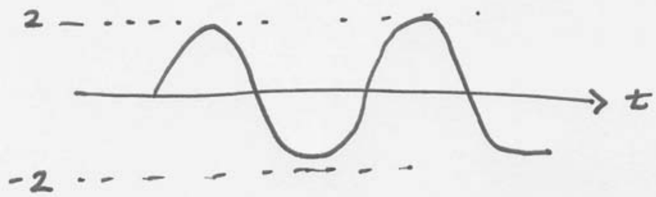
and dividing both sides by n

$$f_D(n) = f_A(n) + f_B(n) + f_C(n).$$

1.9

$$\begin{aligned}
< X >_n &= \frac{1}{n}\sum_{j=1}^{n} X(j) \qquad n > 0 \\
&= \frac{n-1}{n}\frac{1}{n-1}\left\{\sum_{j=1}^{n-1} X(j) + X(n)\right\} \\
&= \left(1 - \frac{1}{n}\right) < X >_{n-1} + \frac{1}{n}X(n) \\
&= < X >_{n-1} + \frac{X(n) - < X >_{n-1}}{n}
\end{aligned}$$

1.10



(a) Sample values equally likely to be in positive and negative amplitude regions
Symmetry of function in positive & negative range
$\Rightarrow$ longterm average of samples $= 0$

(b) "Voltage positive" $\Leftrightarrow$ half samples $\Leftrightarrow P[>0] = \frac{1}{2}$
"Voltage <-2" does not occur $\Rightarrow P[<-2] = 0$

(c) The observed frequencies can change if τ is a rational number times 2π
for example: $\tau = 2\pi$ gives only one observed value
$1 = 2\cos 2\pi = 2\cos 4\pi = 2\cos 6\pi = \cdots$

1.11 By "random" we mean "unpredictable",
but we also mean "occurring as repetition of an identical random experiment."
We may also mean "equiprobable outcomes".
If "random" means all 3 of above attributes then we expect long-term relative frequencies of integer to be $\frac{1}{10}$.
We then also expect long term relative frequencies of each possible pair of integers to be $\frac{1}{100}$.