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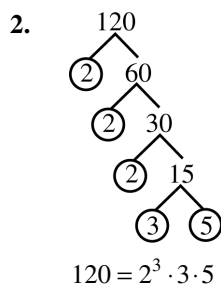
# Chapter 1

## Numerical Pathways

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### Check Points 1.1

1. The statement given in part (b) is true.
  - a. False, 8 does not divide 48,324 because 8 does not divide 324.
  - b. True, 6 divides 48,324 because both 2 and 3 divide 48,324. 2 divides 48,324 because the last digit is 4. 3 divides 48,324 because the sum of the digits, 21, is divisible by 3.
  - c. False, 4 *does* divide 48,324 because the last two digits form 24 which is divisible by 4.



3.  $225 = 3^2 \cdot 5^2$   
 $825 = 3 \cdot 5^2 \cdot 11$   
Greatest Common Divisor:  $3 \cdot 5^2 = 75$
4.  $192 = 2^6 \cdot 3$   
 $288 = 2^5 \cdot 3^2$   
Greatest Common Divisor:  $2^5 \cdot 3 = 96$   
The largest number of people that can be placed in each singing group is 96.
5.  $18 = 2 \cdot 3^2$   
 $30 = 2 \cdot 3 \cdot 5$   
Least common multiple is:  $90 = 2 \cdot 3^2 \cdot 5$
6.  $40 = 2^3 \cdot 5$   
 $60 = 2^2 \cdot 3 \cdot 5$   
Least common multiple is:  $120 = 2^3 \cdot 3 \cdot 5$   
It will be 120 minutes, or 2 hours, until both movies begin again at the same time.  
The time will be 5:00 PM.

### Concept and Vocabulary Check 1.1

1. prime
2. composite
3. greatest common divisor

4. least common multiple
5. false; Changes to make the statement true will vary.
6. true
7. false; Changes to make the statement true will vary.
8. false; Changes to make the statement true will vary.

**Exercise Set 1.1**

1. 6944
  - a. Yes. The last digit is four.
  - b. No. The sum of the digits is 23, which is not divisible by 3.
  - c. Yes. The last two digits form 44, which is divisible by 4.
  - d. No. The number does not end in 0 or 5.
  - e. No. The number is not divisible by both 2 and 3.
  - f. Yes. The last three digits form 944, which is divisible by 8.
  - g. No. The sum of the digits is 23, which is not divisible by 9.
  - h. No. The number does not end in 0.
  - i. No. The number is not divisible by both 3 and 4.
2. 7245
  - a. No. The last digit is five.
  - b. Yes. The sum of the digits is 18, which is divisible by 3.
  - c. No. The last two digits form 45, which is not divisible by 4.
  - d. Yes. The number ends with 5.
  - e. No. The number is not divisible by both 2 and 3.
  - f. No. The last 3 digits form 245, which is not divisible by 8.
  - g. Yes. The sum of the digits is 18, which is divisible by 9.
  - h. No. The number does not end in 0.
  - i. No. The number is not divisible by both 3 and 4.
3. 21,408
  - a. Yes. The last digit is eight.
  - b. Yes. The sum of the digits is 15, which is divisible by 3.

- c. Yes. The last two digits form 08, which is divisible by 4.
- d. No. The number does not end in 0 or 5.
- e. Yes. The number is divisible by both 2 and 3.
- f. Yes. The last three digits form 408, which is divisible by 8.
- g. No. The sum of the digits is 15, which is not divisible by 9.
- h. No. The number does not end in 0.
- i. Yes. The number is divisible by both 3 and 4.

**4.** 25,025

- a. No. The last digit is 5.
- b. No. The sum of the digits is 14, which is not divisible by 3.
- c. No. The last two digits form 25 which is not divisible by 4.
- d. Yes. The last digit is 5.
- e. No. The number is not divisible by 2 and 3.
- f. No. The last three digits form 025 which is not divisible by 8.
- g. No. The sum of the digits is 14, which is not divisible by 9.
- h. No. The number does not end in 0.
- i. No. The number is not divisible by 3 and 4.

**5.** 26,428

- a. Yes. The last digit is 8.
- b. No. The sum of the digits is 22, which is not divisible by 3.
- c. Yes. The last 2 digits form 28, which is divisible by 4.
- d. No. The last digit is eight.
- e. No. The number is not divisible by both two and three.
- f. No. The last three digits form 428, which is not divisible by 8.
- g. No. The sum of the digits is 22, which is not divisible by 9.
- h. No. The number does not end in 0.
- i. No. The number is not divisible by 3 and 4.

**6.** 89,001

- a. No. The last digit is one.
- b. Yes. The sum of the digits is 18, which is divisible by 3.

- c. No. The last two digits form 01, which is not divisible by 4.
- d. No. The last digit is one.
- e. No. The number is not divisible by two and three.
- f. No. The last three digits form 001, which is not divisible by 8.
- g. Yes. The sum of the digits is 18, which is divisible by 9.
- h. No. The number does not end in 0.
- i. No. The number is not divisible by 3 and 4.

**7.** 374,832

- a. Yes. The last digit is 2.
- b. Yes. The sum of the digits is 27, which is divisible by 3.
- c. Yes. The last two digits form 32, which is divisible by 4.
- d. No. The last digit is two.
- e. Yes. The number is divisible by 2 and 3.
- f. Yes. The last 3 digits form 832, which is divisible by 8.
- g. Yes. The sum of the digits is 27, which is divisible by 9.
- h. No. The last digit is 2.
- i. Yes. The number is divisible by both 3 and 4.

**8.** 347,712

- a. Yes. The last digit is 2.
- b. Yes. The sum of the digits is 24, which is divisible by 3.
- c. Yes. The last two digits form 12, which is divisible by 4.
- d. No. The last digit is 2.
- e. Yes. The number is divisible by both 2 and 3.
- f. Yes. The last 3 digits form 712, which is divisible by 8.
- g. No. The sum of the digits is 24, which is not divisible by 9.
- h. No. The last digit is 2.
- i. Yes. The number is divisible by both 3 and 4.

**9.** 6,126,120

- a. Yes. The last digit is 0.
- b. Yes. The sum of the digits is 18, which is divisible by 3.
- c. Yes. The last two digits form 20, which is divisible by 4.

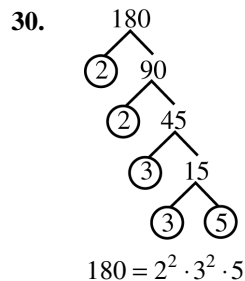
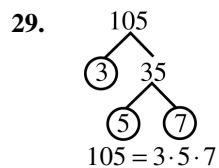
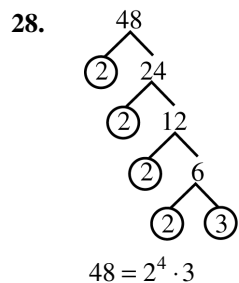
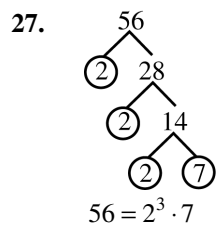
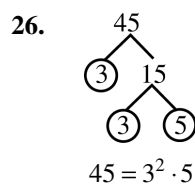
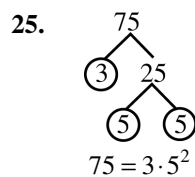
- d. Yes. The last digit is 0.
  - e. Yes. The number is divisible by both 2 and 3.
  - f. Yes. The last 3 digits form 120, which is divisible by 8.
  - g. Yes. The sum of the digits is 18, which is divisible by 9.
  - h. Yes. The last digit is 0.
  - i. Yes. The number is divisible by both 3 and 4.
10. 5,941,221
- a. No. The last digit is 1.
  - b. Yes. The sum of the digits is 24, which is divisible by 3.
  - c. No. The last two digits form 21, which is not divisible by 4.
  - d. No. The last digit is 1.
  - e. No. The number is not divisible by both 2 and 3.
  - f. No. The last 3 digits form 221, which is not divisible by 8.
  - g. No. The sum of the digits is 24, which is not divisible by 9.
  - h. No. The last digit is 1.
  - i. No. The number is not divisible by both 3 and 4.
11. True.  $5958 \div 3 = 1986$   
The sum of the digits is 27, which is divisible by 3.
12. True.  $8142 \div 3 = 2714$   
The sum of the digits is 15, which is divisible by 3.
13. True.  $10,612 \div 4 = 2653$   
The last two digits form 12, which is divisible by 4.
14. True.  $15,984 \div 4 = 3996$   
The last two digits form 84, which is divisible by 4.
15. False
16. False
17. True.  $104,538 \div 6 = 17,423$   
The number is divisible by both 2 and 3.
18. True.  $163,944 \div 6 = 27,324$   
The number is divisible by both 2 and 3.
19. True.  $20,104 \div 8 = 2513$   
The last three digits form 104, which is divisible by 8.
20. True.  $28,096 \div 8 = 3512$   
The last three digits form 96, which is divisible by 8.

21. False

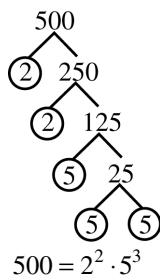
22. False

23. True.  $517,872 \div 12 = 43,156$   
The number is divisible by both 3 and 4.

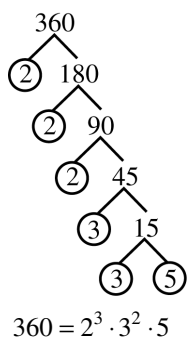
24. True.  $785,172 \div 12 = 65,431$   
The number is divisible by both 3 and 4.



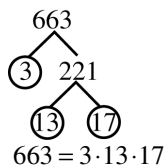
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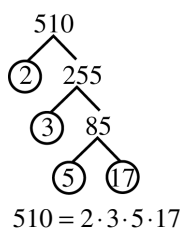
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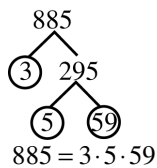
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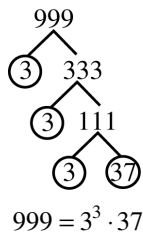
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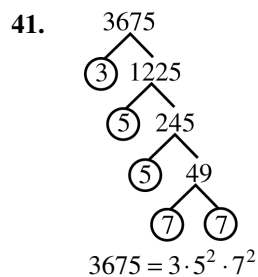
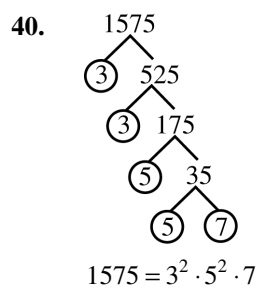
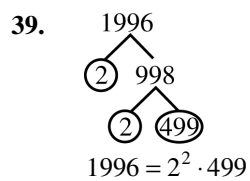
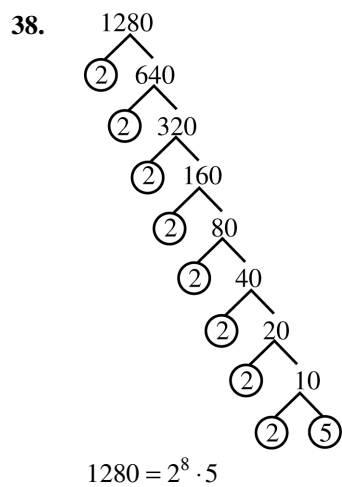
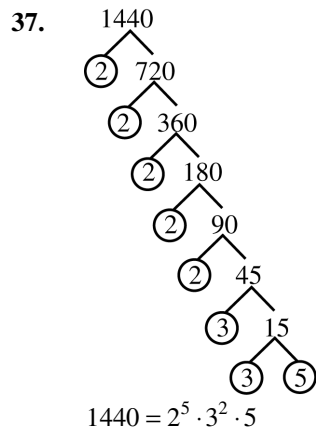


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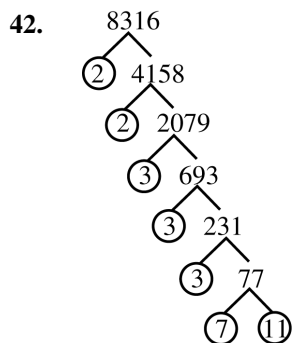


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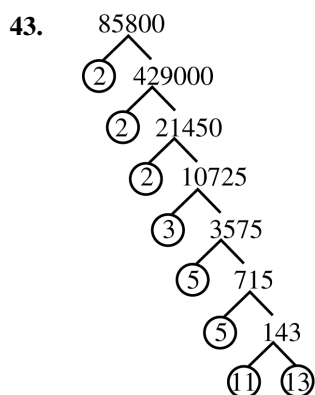




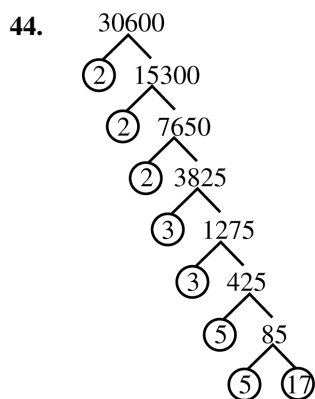




$$8316 = 2^2 \cdot 3^3 \cdot 7 \cdot 11$$



$$85,800 = 2^3 \cdot 3 \cdot 5^2 \cdot 11 \cdot 13$$



$$30,600 = 2^3 \cdot 3^2 \cdot 5^2 \cdot 17$$

45.  $42 = 2 \cdot 3 \cdot 7$

$$56 = 2^3 \cdot 7$$

Greatest Common Divisor:  $2 \cdot 7 = 14$

46.  $25 = 5^2$

$$70 = 2 \cdot 5 \cdot 7$$

Greatest Common Divisor: 5

47.  $16 = 2^4$

$$42 = 2 \cdot 3 \cdot 7$$

Greatest Common Divisor: 2

- 48.**  $66 = 2 \cdot 3 \cdot 11$   
 $90 = 2 \cdot 3^2 \cdot 5$   
Greatest Common Divisor:  $2 \cdot 3 = 6$
- 49.**  $60 = 2^2 \cdot 3 \cdot 5$   
 $108 = 2^2 \cdot 3^3$   
Greatest Common Divisor:  $2^2 \cdot 3 = 12$
- 50.**  $96 = 2^5 \cdot 3$   
 $212 = 2^2 \cdot 53$   
Greatest Common Divisor:  $2^2 = 4$
- 51.**  $72 = 2^3 \cdot 3^2$   
 $120 = 2^3 \cdot 3 \cdot 5$   
Greatest Common Divisor:  $2^3 \cdot 3 = 24$
- 52.**  $220 = 2^2 \cdot 5 \cdot 11$   
 $400 = 2^4 \cdot 5^2$   
Greatest Common Divisor:  $2^2 \cdot 5 = 20$
- 53.**  $324 = 2 \cdot 3^2 \cdot 19$   
 $380 = 2^2 \cdot 5 \cdot 19$   
Greatest Common Divisor:  $2 \cdot 19 = 38$
- 54.**  $224 = 2^5 \cdot 7$   
 $430 = 2 \cdot 5 \cdot 43$   
Greatest Common Divisor: 2
- 55.**  $240 = 2^4 \cdot 3 \cdot 5$   
 $285 = 3 \cdot 5 \cdot 19$   
Greatest Common Divisor:  $3 \cdot 5 = 15$
- 56.**  $150 = 2 \cdot 3 \cdot 5^2$   
 $480 = 2^5 \cdot 3 \cdot 5$   
Greatest Common Divisor:  $2 \cdot 3 \cdot 5 = 30$
- 57.**  $42 = 2 \cdot 3 \cdot 7$   
 $56 = 2^3 \cdot 7$   
Least Common Multiple:  $2^3 \cdot 3 \cdot 7 = 168$
- 58.**  $25 = 5^2$   
 $70 = 2 \cdot 5 \cdot 7$   
Least Common Multiple:  $2 \cdot 5^2 \cdot 7 = 350$
- 59.**  $16 = 2^4$   
 $42 = 2 \cdot 3 \cdot 7$   
Least Common Multiple:  $2^4 \cdot 3 \cdot 7 = 336$

**60.**  $66 = 2 \cdot 3 \cdot 11$   
 $90 = 2 \cdot 3^2 \cdot 5$   
 Least Common Multiple =  $2 \cdot 3^2 \cdot 5 \cdot 11 = 990$

**61.**  $60 = 2^2 \cdot 3 \cdot 5$   
 $108 = 2^2 \cdot 3^3$   
 Least Common Multiple:  $2^2 \cdot 3^3 \cdot 5 = 540$

**62.**  $96 = 2^5 \cdot 3$   
 $212 = 2^2 \cdot 53$   
 Least Common Multiple =  $2^5 \cdot 3 \cdot 53 = 5088$

**63.**  $72 = 2^3 \cdot 3^2$   
 $120 = 2^3 \cdot 3 \cdot 5$   
 Least Common Multiple:  $2^3 \cdot 3^2 \cdot 5 = 360$

**64.**  $220 = 2^2 \cdot 5 \cdot 11$   
 $400 = 2^4 \cdot 5^2$   
 Least Common Multiple =  $2^4 \cdot 5^2 \cdot 11 = 4400$

**65.**  $342 = 2 \cdot 3^2 \cdot 19$   
 $380 = 2^2 \cdot 5 \cdot 19$   
 Least Common Multiple:  $2^2 \cdot 3^2 \cdot 5 \cdot 19 = 3420$

**66.**  $224 = 2^5 \cdot 7$   
 $430 = 2 \cdot 5 \cdot 43$   
 Least Common Multiple  
 $= 2^5 \cdot 5 \cdot 7 \cdot 43$   
 $= 48,160$

**67.**  $240 = 2^4 \cdot 3 \cdot 5$   
 $285 = 3 \cdot 5 \cdot 19$   
 Least Common Multiple  
 $= 2^4 \cdot 3 \cdot 5 \cdot 19$   
 $= 4560$

**68.**  $150 = 2 \cdot 3 \cdot 5^2$   
 $480 = 2^5 \cdot 3 \cdot 5$   
 Least Common Multiple  
 $= 2^5 \cdot 3 \cdot 5^2$   
 $= 2400$

**69.**  $d = 8$   
 $9 \mid 12,348$

70.  $d = 7$   
 $9 \overline{)23,427}$
71.  $d = 6$   
 $8 \overline{)76,523,456}$
72.  $d = 4$   
 $8 \overline{)88,888,824}$
73.  $d = 2, 6$   
 $4 \overline{)963,232}$  and  $4 \overline{)963,236}$
74.  $d = 2, 6$   
 $4 \overline{)752,672}$  and  $4 \overline{)752,676}$
75. 28 is a perfect number.  
 $28 = 1 + 2 + 4 + 7 + 14$
76. 6 is a perfect number.  
 $6 = 1 + 2 + 3$
77. 20 is not a perfect number.  
 $20 \neq 1 + 2 + 4 + 5 + 10$
78. 50 is not a perfect number.  
 $50 \neq 1 + 2 + 5 + 10 + 25$
79. 41 is not an emirp because 14 is not prime.
80. 43 is not an emirp because 34 is not prime.
81. 107 is an emirp because 701 is also prime.
82. 113 is an emirp because 311 is also prime.
83. 13 is not a Germain prime because  $2(13) + 1 = 27$  is not prime.
84. 11 is a Germain prime because  $2(11) + 1 = 23$  is prime.
85. 241 is not a Germain prime because  $2(241) + 1 = 483$  is not prime.
86. 97 is not a Germain prime because  $2(97) + 1 = 195$  is not prime.
87. The GCD of 24 and 27 is 3.  
The LCM of 24 and 27 is 216.  
 $3 \times 216 = 648$   
 $24 \times 27 = 648$   
The product of the greatest common divisor and least common multiple of two numbers equals the product of the two numbers.

- 88.** The GCD of 48 and 72 is 24.  
 The LCM of 48 and 72 is 144.  
 $24 \times 144 = 3456$   
 $48 \times 72 = 3456$   
 The product of the greatest common divisor and least common multiple of two numbers equals the product of the two numbers.
- 89.** The numbers are the prime numbers less than 100.
- 90. a.** Multiples of 18: 18, 36, 54, 72, 90, 108, 126, 144, 162, 180, 198, 216  
 Multiples of 12: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168, 180, 192, 204, 216  
 Common multiples of 12 and 18: 36, 72, 108, 144, 180, 216  
 Over a 216-year period, the two species will share the forest 6 times (once every 36 years).
- b.** The least common multiple of 17 and 13 is 221.  
 The two species will share the forest once every 221 years.
- c.** By having a prime number as the length of its life cycle, the species will share the forest with other species less often.
- 91.**  $300 = 2^2 \cdot 3 \cdot 5^2$   
 $144 = 2^4 \cdot 3^2$   
 Greatest Common Divisor:  $2^2 \cdot 3 = 12$   
 There would be 25 groups with 12 bottles of water each. There would be 12 groups with 12 cans of food each.
- 92.**  $180 = 2^2 \cdot 3^2 \cdot 5$   
 $144 = 2^4 \cdot 3^2$   
 Greatest Common Divisor:  $2^2 \cdot 3^2 = 36$   
 There would be 5 all-male groups of 36.  
 There would be 4 all-female groups of 36.
- 93.**  $310 = 2 \cdot 5 \cdot 31$   
 $460 = 2^2 \cdot 5 \cdot 23$   
 Greatest Common Divisor:  $2 \cdot 5 = 10$   
 There would be 31 groups of 10 five-dollar bills. There would be 46 groups of 10 ten-dollar bills.
- 94.**  $360 = 2^3 \cdot 3^2 \cdot 5$   
 $432 = 2^4 \cdot 3^3$   
 Greatest Common Divisor:  $2^3 \cdot 3^2 = 72$   
 There would be 5 groups of 72 football cards.  
 There would be 6 groups of 72 baseball cards.
- 95.**  $6 = 2 \cdot 3$   
 $10 = 2 \cdot 5$   
 Least Common Multiple is:  $2 \cdot 3 \cdot 5 = 30$   
 It will be 30 more nights until both have the evening off, or July 1.
- 96.**  $40 = 2^3 \cdot 5$   
 $100 = 2^2 \cdot 5^2$   
 Least common multiple is:  $2^3 \cdot 5^2 = 200$   
 It will be 200 minutes until each movie starts at the same time, or 3 hours and 20 minutes. The time would be 3:20 PM.

97.  $15 = 3 \cdot 5$

$18 = 2 \cdot 3^2$

Least Common Multiple is:  $2 \cdot 3^2 \cdot 5 = 90$

It takes 90 minutes.

98.  $40 = 2^3 \cdot 5$

$45 = 3^2 \cdot 5$

Least common multiple is:  $2^3 \cdot 3^2 \cdot 5 = 360$

It will take 360 seconds or 6 minutes.

99. Ex: 1020 is divisible by 4 but not by 8.

100. a.  $\text{GCD} = 2^{14} \cdot 3^{25} \cdot 5^{30}$

b.  $\text{LCM} = 2^{17} \cdot 3^{37} \cdot 5^{31}$

101. 53

His age was prime 6 years ago at the age of 47. His age will be prime again in 6 years at the age of 59.

102.  $85 + 15 = 100 = 2^2 \cdot 5^2$

$100 + 15 = 115 = 5 \cdot 23$

$\text{LCM} = 2^2 \cdot 5 \cdot 23 = 2300$

The films will begin at the same time

$2300 \text{ min} \left( = 38\frac{1}{3} \text{ hr} \right)$  after noon (today), or at 2:20 A.M. on the third day.

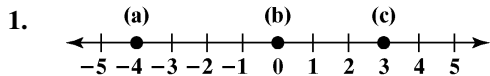
103. 2 and 3 are the only two consecutive prime numbers whose difference is odd. Since all other primes are odd, the difference between any other two consecutive primes is even.

104. Yes, since 96 is divisible by 4, then 67,234,096 is divisible by 4.

105. No, since  $1 + 2 + 5 + 4 + 1 + 7 + 5 + 0 = 25$  is not divisible by 3, then 12,541,750 is not divisible by 3.

106. Yes, since  $4 + 8 + 2 + 0 + 1 + 6 + 5 + 1 = 27$  is divisible by 9, then 48,201,651 is divisible by 9.

### Check Points 1.2



2. a.  $6 > -7$  because 6 is to the right of  $-7$  on the number line.

b.  $-8 < -1$  because  $-8$  is to the left of  $-1$  on the number line.

c.  $-25 < -2$  because  $-25$  is to the left of  $-2$  on the number line.

d.  $-14 < 0$  because  $-14$  is to the left of 0 on the number line.

3. a.  $|-8| = 8$  because  $-8$  is 8 units from 0.

b.  $|6| = 6$  because 6 is 6 units from 0.

c.  $-|8| = -8$  because 8 is 8 units from 0 and the negative of 8 is  $-8$ .

4. a.  $30 - (-7) = 30 + 7 = 37$

b.  $-14 - (-10) = -14 + 10 = -4$

c.  $-14 - 10 = -24$

5. a.  $\overbrace{3}^{\text{eats 5 servings}} - \overbrace{(-5)}^{\text{frequently stressed}} = 3 + 5 = 8 \text{ years}$

b.  $\overbrace{-1}^{\text{less than 6 to 8 hours of sleep}} - \overbrace{(-15)}^{\text{smokes cigarettes}} = -1 + 15 = 14 \text{ years}$

6. a.  $(-5)^2 = (-5)(-5) = 25$

b.  $-5^2 = -(5 \cdot 5) = -25$

c.  $(-4)^3 = (-4)(-4)(-4) = -64$

d.  $(-3)^4 = (-3)(-3)(-3)(-3) = 81$

7.  $7^2 - 48 \div 4^2 \cdot 5 + 2$   
 $= 49 - 48 \div 16 \cdot 5 + 2$   
 $= 49 - 3 \cdot 5 + 2$   
 $= 49 - 15 + 2$   
 $= 34 + 2$   
 $= 36$

8.  $(-8)^2 - (10 - 13)^2(-2)$   
 $= (-8)^2 - (-3)^2(-2)$   
 $= 64 - (9)(-2)$   
 $= 64 - (-18)$   
 $= 64 + (+18)$   
 $= 82$

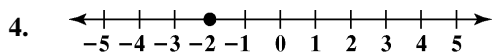
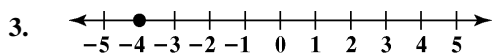
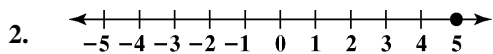
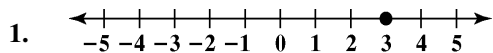
### Concept and Vocabulary Check 1.2

1.  $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

2. left

3. the distance from 0 to  $a$
4. additive inverses
5. false; Changes to make the statement true will vary.
6. true
7. true
8. false; Changes to make the statement true will vary.

**Exercise Set 1.2**



5.  $-2 < 7$  because  $-2$  is to the left of  $7$  on the number line.
6.  $-1 < 13$  because  $-1$  is to the left of  $13$  on the number line.
7.  $-13 < -2$  because  $-13$  is to the left of  $-2$  on the number line.
8.  $-1 > -13$  because  $-1$  is to the right of  $-13$  on the number line.
9.  $8 > -50$  because  $8$  is to the right of  $-50$  on the number line.
10.  $7 > -9$  because  $7$  is to the right of  $-9$  on the number line.
11.  $-100 < 0$  because  $-100$  is to the left of  $0$  on the number line.
12.  $0 > -300$  because  $0$  is to the right of  $-300$  on the number line.
13.  $|-14| = 14$  because  $-14$  is 14 units from 0.
14.  $|-16| = 16$  because  $-16$  is 16 units from 0.
15.  $|14| = 14$  because  $14$  is 14 units from 0.
16.  $|16| = 16$  because  $16$  is 16 units from 0.
17.  $|-300,000| = 300,000$  because  $-300,000$  is 300,000 units from 0.
18.  $|-1,000,000| = 1,000,000$  because  $-1,000,000$  is 1,000,000 units from 0.



19.  $-7 + (-5) = -12$

20.  $-3 + (-4) = -7$

21.  $12 + (-8) = 4$

22.  $13 + (-5) = 8$

23.  $6 + (-9) = -3$

24.  $3 + (-11) = -8$

25.  $-9 + (+4) = -5$

26.  $-7 + (+3) = -4$

27.  $-9 + (-9) = -18$

28.  $-13 + (-13) = -26$

29.  $9 + (-9) = 0$

30.  $13 + (-13) = 0$

31.  $13 - 8 = 5$

32.  $14 - 3 = 11$

33.  $8 - 15 = 8 + (-15) = -7$

34.  $9 - 20 = 9 + (-20) = -11$

35.  $4 - (-10) = 4 + 10 = 14$

36.  $3 - (-17) = 3 + 17 = 20$

37.  $-6 - (-17) = -6 + 17 = 11$

38.  $-4 - (-19) = -4 + 19 = 15$

39.  $-12 - (-3) = -12 + 3 = -9$

40.  $-19 - (-2) = -19 + 2 = -17$

41.  $-11 - 17 = -11 + (-17) = -28$

42.  $-19 - 21 = -19 + (-21) = -40$

43.  $6(-9) = -54$

44.  $5(-7) = -35$

45.  $(-7)(-3) = 21$

**46.**  $(-8)(-5) = 40$

**47.**  $(-2)(6) = -12$

**48.**  $(-3)(10) = -30$

**49.**  $(-13)(-1) = 13$

**50.**  $(-17)(-1) = 17$

**51.**  $0(-5) = 0$

**52.**  $0(-8) = 0$

**53.**  $5^2 = 5 \cdot 5 = 25$

**54.**  $6^2 = 6 \cdot 6 = 36$

**55.**  $(-5)^2 = (-5) \cdot (-5) = 25$

**56.**  $(-6)^2 = (-6)(-6) = 36$

**57.**  $4^3 = 4 \cdot 4 \cdot 4 = 64$

**58.**  $2^3 = 2 \cdot 2 \cdot 2 = 8$

**59.**  $(-5)^3 = (-5)(-5)(-5) = 25(-5) = -125$

**60.**  $(-4)^3 = (-4)(-4)(-4) = 16(-4) = -64$

**61.**  $(-5)^4 = (-5)(-5)(-5)(-5) = 625$

**62.**  $(-4)^4 = (-4)(-4)(-4)(-4) = 256$

**63.**  $-3^4 = -[3 \cdot 3 \cdot 3 \cdot 3] = -81$

**64.**  $-1^4 = -[1 \cdot 1 \cdot 1 \cdot 1] = -1$

**65.**  $(-3)^4 = (-3)(-3)(-3)(-3) = 81$

**66.**  $(-1)^4 = (-1)(-1)(-1)(-1) = 1$

**67.**  $\frac{-12}{4} = -3$

**68.**  $\frac{-40}{5} = -8$

**69.**  $\frac{21}{-3} = -7$

$$70. \frac{60}{-6} = -10$$

$$71. \frac{-90}{-3} = 30$$

$$72. \frac{-66}{-6} = 11$$

$$73. \frac{0}{-7} = 0$$

$$74. \frac{0}{-8} = 0$$

$$75. \frac{-7}{0} \text{ is undefined}$$

$$76. \frac{0}{0} \text{ is undefined.}$$

$$77. (-480) \div 24 = \frac{-480}{24} = -20$$

$$78. (-300) \div 12 = \frac{-300}{12} = -25$$

$$79. (465) \div (-15) = \frac{465}{-15} = -31$$

$$80. (-594) \div (-18) = \frac{-594}{-18} = 33$$

$$81. 7 + 6 \cdot 3 = 7 + 18 = 25$$

$$82. -5 + (-3) \cdot 8 = -5 + (-24) = -29$$

$$83. (-5) - 6(-3) = -5 + 18 = 13$$

$$84. -8(-3) - 5(-6) = 24 - (-30) = 24 + 30 = 54$$

$$\begin{aligned} 85. \quad 6 - 4(-3) - 5 &= 6 - (-12) - 5 \\ &= 6 + 12 - 5 \\ &= 18 - 5 \\ &= 13 \end{aligned}$$

$$\begin{aligned} 86. \quad 3 - 7(-1) - 6 &= 3 - (-7) - 6 \\ &= 3 + 7 - 6 \\ &= 10 - 6 \\ &= 4 \end{aligned}$$

$$\begin{aligned} 87. \quad 3 - 5(-4 - 2) &= 3 - 5(-6) \\ &= 3 - (-30) \\ &= 3 + 30 \\ &= 33 \end{aligned}$$

$$\begin{aligned} 88. \quad 3 - 9(-1 - 6) &= 3 - 9(-7) \\ &= 3 - (-63) \\ &= 3 + 63 \\ &= 66 \end{aligned}$$

$$89. \quad (2 - 6)(-3 - 5) = (-4)(-8) = 32$$

$$\begin{aligned} 90. \quad 9 - 5(6 - 4) - 10 &= 9 - 5(2) - 10 \\ &= 9 - 10 - 10 \\ &= -1 - 10 \\ &= -11 \end{aligned}$$

$$\begin{aligned} 91. \quad 3(-2)^2 - 4(-3)^2 &= 3(4) - 4(9) \\ &= 12 - 36 \\ &= -24 \end{aligned}$$

$$\begin{aligned} 92. \quad 5(-3)^2 - 2(-2)^3 &= 5(9) - 2(-8) \\ &= 45 - (-16) \\ &= 45 + 16 \\ &= 61 \end{aligned}$$

$$\begin{aligned} 93. \quad (2 - 6)^2 - (3 - 7)^2 &= (-4)^2 - (-4)^2 \\ &= 16 - 16 \\ &= 0 \end{aligned}$$

$$\begin{aligned} 94. \quad (4 - 6)^2 - (5 - 9)^3 &= (-2)^2 - (-4)^3 \\ &= 4 - (-64) \\ &= 4 + 64 \\ &= 68 \end{aligned}$$

$$\begin{aligned} 95. \quad 6(3 - 5)^3 - 2(1 - 3)^3 &= 6(-2)^3 - 2(-2)^3 \\ &= 6(-8) - 2(-8) \\ &= -48 + 16 \\ &= -32 \end{aligned}$$

$$\begin{aligned} 96. \quad -3(-6 + 8)^3 - 5(-3 + 5)^3 &= -3(2)^3 - 5(2)^3 \\ &= -3(8) - 5(8) \\ &= -24 - 40 \\ &= -64 \end{aligned}$$

$$\begin{aligned} 97. \quad 8^2 - 16 \div 2^2 \cdot 4 - 3 &= 64 - 16 \div 4 \cdot 4 - 3 \\ &= 64 - 4 \cdot 4 - 3 \\ &= 64 - 16 - 3 \\ &= 45 \end{aligned}$$

$$\begin{aligned} 98. \quad 10^2 - 100 \div 5^2 \cdot 2 - (-3) \\ &= 10^2 - 100 \div 25 \cdot 2 - (-3) \\ &= 100 - 4 \cdot 2 + 3 \\ &= 100 - 8 + 3 \\ &= 92 + 3 \\ &= 95 \end{aligned}$$

$$\begin{aligned} 99. \quad 24 \div [3^2 \div (8 - 5)] - (-6) \\ &= 24 \div [9 \div 3] - (-6) \\ &= 24 \div 3 + 6 \\ &= 8 + 6 \\ &= 14 \end{aligned}$$

$$\begin{aligned} 100. \quad 30 \div [5^2 \div (7 - 12)] - (-9) \\ &= 30 \div [25 \div (-5)] - (-9) \\ &= 30 \div [-5] + 9 \\ &= -6 + 9 \\ &= 3 \end{aligned}$$

$$\begin{aligned} 101. \quad 8 - 3[-2(2 - 5) - 4(8 - 6)] \\ &= 8 - 3[-2(-3) - 4(2)] \\ &= 8 - 3[6 - 8] \\ &= 8 - 3[-2] \\ &= 8 + 6 \\ &= 14 \end{aligned}$$

$$\begin{aligned} 102. \quad 8 - 3[-2(5 - 7) - 5(4 - 2)] \\ &= 8 - 3[-2(-2) - 5(2)] \\ &= 8 - 3[4 - 10] \\ &= 8 - 3[-6] \\ &= 8 + 18 \\ &= 26 \end{aligned}$$

$$\begin{aligned} 103. \quad -2^2 + 4[16 \div (3 - 5)] \\ &= -4 + 4[16 \div (-2)] \\ &= -4 + 4[-8] \\ &= -4 - 32 \\ &= -36 \end{aligned}$$

$$\begin{aligned} 104. \quad & -3^2 + 2[20 \div (7 - 11)] \\ & = -9 + 2[20 \div (-4)] \\ & = -9 + 2[-5] \\ & = -9 - 10 \\ & = -19 \end{aligned}$$

$$\begin{aligned} 105. \quad & 4|10 - (8 - 20)| \\ & = 4|10 - (-12)| \\ & = 4|10 + 12| \\ & = 4|22| \\ & = 88 \end{aligned}$$

$$\begin{aligned} 106. \quad & -5|7 - (20 - 8)| \\ & = -5|7 - (12)| \\ & = -5|-5| \\ & = -5(5) \\ & = -25 \end{aligned}$$

$$\begin{aligned} 107. \quad & [-5^2 + (6 - 8)^3 - (-4)] - [|-2|^3 + 1 - 3^2] \\ & = [-5^2 + (-2)^3 - (-4)] - [2^3 + 1 - 3^2] \\ & = [-25 - 8 - (-4)] - [8 + 1 - 9] \\ & = [-25 - 8 + 4] - [8 + 1 - 9] \\ & = [-33 + 4] - [0] \\ & = -29 - 0 \\ & = -29 \end{aligned}$$

$$\begin{aligned} 108. \quad & [-4^2 + (7 - 10)^3 - (-27)] - [|-2|^5 + 1 - 5^2] \\ & = [-4^2 + (-3)^3 - (-27)] - [2^5 + 1 - 5^2] \\ & = [-16 - 27 - (-27)] - [32 + 1 - 25] \\ & = [-16 - 27 + 27] - [32 + 1 - 25] \\ & = -16 - 8 \\ & = -24 \end{aligned}$$

$$\begin{aligned}
 109. \quad & \frac{12 \div 3 \cdot 5 | 2^2 + 3^2 |}{7 + 3 - 6^2} \\
 &= \frac{12 \div 3 \cdot 5 | 4 + 9 |}{7 + 3 - 36} \\
 &= \frac{12 \div 3 \cdot 5 | 13 |}{10 - 36} \\
 &= \frac{12 \div 3 \cdot 5 \cdot 13}{-26} \\
 &= \frac{4 \cdot 5 \cdot 13}{-26} \\
 &= \frac{260}{-26} \\
 &= -10
 \end{aligned}$$

$$\begin{aligned}
 110. \quad & \frac{-3 \cdot 5^2 + 89}{(5-6)^2 - 2 | 3-7 |} \\
 &= \frac{-3 \cdot 25 + 89}{(-1)^2 - 2 | -4 |} \\
 &= \frac{-75 + 89}{1 - 2(4)} \\
 &= \frac{14}{1-8} \\
 &= \frac{14}{-7} \\
 &= -2
 \end{aligned}$$

$$111. \quad -10 - (-2)^3 = -10 - (-8) = -10 + 8 = -2$$

$$112. \quad -100 - (-5)^3 = -100 - (-125) = -100 + 125 = 25$$

$$113. \quad [2(7-10)]^2 = [2(-3)]^2 = [-6]^2 = 36$$

$$114. \quad [2(9-11)]^4 = [2(-2)]^4 = [-4]^4 = 256$$

$$\begin{aligned}
 115. \quad & \text{The difference in elevation is} \\
 & 20,320 - (-282) \\
 &= 20,320 + 282 \\
 &= 20,602 \text{ feet}
 \end{aligned}$$

$$\begin{aligned}
 116. \quad & \text{The difference in elevation is} \\
 & 19,321 - (-436) \\
 &= 19,321 + 436 \\
 &= 19,757 \text{ feet}
 \end{aligned}$$

$$117. \quad 10 + (-15) = -5$$

shrink by 5 years

**118.**  $2 + (-5) = -3$

shrink by 3 years

**119.**  $-5 + (-6) = -11$

shrink by 11 years

**120.**  $-1 + (-15) = -16$

shrink by 16 years

**121.**  $5 + (-5) = 0$

no change

**122.**  $5 + (-5) = 0$

no change

**123.**  $10 - (-15) = 10 + 15 = 25$

25 years

**124.**  $10 - (-6) = 10 + 6 = 16$

16 years

**125.**  $-5 - (-6) = -5 + 6 = 1$

1 year

**126.**  $-1 - (-5) = -1 + 5 = 4$

4 years

**127. a.**  $1991 - 1863 = 128$

In 2001, there was a \$128 billion surplus.

**b.**  $2304 - 3603 = -1299$

In 2011, there was a \$1299 billion deficit.

**c.**  $128 - (-1299) = 128 + 1299 = 1427$

The difference is \$1427 billion.

**128. a.**  $1991 - 1863 = 128$

In 2001, there was a \$128 billion surplus.

**b.**  $2105 - 3518 = -1413$

In 2009, there was a \$1413 billion deficit.

**c.**  $128 - (-1413) = 128 + 1413 = 1541$

The difference is \$1541 billion.

**129.** 2007 deficit:  $2568 - 2729 = -161$

2011 deficit:  $2304 - 3603 = -1299$

The difference between the 2007 deficit and the 2011 deficit was  $-161 - (-1299) = -161 + 1299 = 1138$  or \$1138 billion.

**130.** 2007 deficit:  $2568 - 2729 = -161$

2009 deficit:  $2105 - 3518 = -1413$

The difference between the 2007 deficit and the 2009 deficit was  $-161 - (-1413) = -161 + 1413 = 1252$  or \$1252 billion.



131.  $8 - 2 \cdot (3 - 4) = 10$

132.  $(8 - 2) \cdot 3 - 4 = 14$

133.  $-7$

134.  $-36$

135.  $150$

**Check Points 1.3**

1.  $72 = 2^3 \cdot 3^2$

$90 = 2 \cdot 5 \cdot 3^2$

Greatest Common Divisor is  $2 \cdot 3^2$  or 18.

$$\frac{72}{90} = \frac{72 \div 18}{90 \div 18} = \frac{4}{5}$$

2.  $2\frac{5}{8} = \frac{8 \cdot 2 + 5}{8} = \frac{16 + 5}{8} = \frac{21}{8}$

3.  $\frac{5}{3} = 1\frac{2}{3}$

4. a.  $\frac{3}{8} = 0.375$

$$\begin{array}{r}
 0.375 \\
 8 \overline{) 3.000} \\
 \underline{24} \phantom{00} \\
 60 \phantom{00} \\
 \underline{56} \phantom{00} \\
 40 \phantom{00} \\
 \underline{40} \phantom{00} \\
 0
 \end{array}$$

b.  $\frac{5}{11} = 0.\overline{45}$

$$\begin{array}{r}
 0.4545 \dots \\
 11 \overline{) 5.0000} \\
 \underline{44} \phantom{0000} \\
 60 \phantom{0000} \\
 \underline{55} \phantom{0000} \\
 50 \phantom{0000} \\
 \underline{44} \phantom{0000} \\
 60 \phantom{0000} \\
 \underline{55} \phantom{0000} \\
 5
 \end{array}$$

5. a.  $0.9 = \frac{9}{10}$

b.  $0.86 = \frac{86}{100} = \frac{86 \div 2}{100 \div 2} = \frac{43}{50}$

c.  $0.053 = \frac{53}{1000}$

6.  $n = 0.\overline{2}$   
 $n = 0.22222\ldots$   
 $10n = 2.22222\ldots$

$$\begin{array}{r} 10n = 2.2222\ldots \\ -n = 0.2222\ldots \\ \hline 9n = 2.0 \\ n = \frac{2}{9} \end{array}$$

7.  $n = 0.\overline{79}$   
 $n = 0.7979\ldots$   
 $100n = 79.7979\ldots$

$$\begin{array}{r} 100n = 79.7979\ldots \\ -n = 0.7979\ldots \\ \hline 99n = 79 \\ n = \frac{79}{99} \end{array}$$

8. a.  $\frac{4}{11} \cdot \frac{2}{3} = \frac{8}{33}$

b.  $\left(-\frac{3}{7}\right)\left(-\frac{14}{4}\right) = \frac{42}{28} = \frac{42 \div 14}{28 \div 14} = \frac{3}{2}$  or  $1\frac{1}{2}$

c.  $\left(3\frac{2}{5}\right)\left(1\frac{1}{2}\right) = \frac{17}{5} \cdot \frac{3}{2} = \frac{51}{10}$  or  $5\frac{1}{10}$

9. a.  $\frac{9}{11} \div \frac{5}{4} = \frac{9}{11} \cdot \frac{4}{5} = \frac{36}{55}$

b.  $-\frac{8}{15} \div \frac{2}{5} = -\frac{8}{15} \cdot \frac{5}{2} = -\frac{40}{30} = -\frac{4}{3}$  or  $-1\frac{1}{3}$

c.  $3\frac{3}{8} \div 2\frac{1}{4} = \frac{27}{8} \div \frac{9}{4} = \frac{27}{8} \cdot \frac{4}{9} = \frac{108}{72} = \frac{3}{2}$  or  $1\frac{1}{2}$

$$10. \text{ a. } \frac{5}{12} + \frac{3}{12} = \frac{5+3}{12} = \frac{8}{12} = \frac{2}{3}$$

$$\text{b. } \frac{7}{4} - \frac{1}{4} = \frac{7-1}{4} = \frac{6}{4} = \frac{3}{2} \text{ or } 1\frac{1}{2}$$

$$\begin{aligned} \text{c. } -3\frac{3}{8} - \left(-1\frac{1}{8}\right) &= -\frac{27}{8} - \left(-\frac{9}{8}\right) \\ &= -\frac{27}{8} + \frac{9}{8} \\ &= \frac{-27+9}{8} \\ &= \frac{-18}{8} \\ &= -\frac{9}{4} \\ &\text{or } -2\frac{1}{4} \end{aligned}$$

$$11. \frac{1}{5} + \frac{3}{4} = \frac{1}{5} \cdot \frac{4}{4} + \frac{3}{4} \cdot \frac{5}{5} = \frac{4}{20} + \frac{15}{20} = \frac{19}{20}$$

$$12. \frac{3}{10} - \frac{7}{12} = \frac{3}{10} \cdot \frac{6}{6} - \frac{7}{12} \cdot \frac{5}{5} = \frac{18}{60} - \frac{35}{60} = -\frac{17}{60}$$

$$\begin{aligned} 13. & \left(-\frac{1}{2}\right)^2 - \left(\frac{7}{10} - \frac{8}{15}\right)^2 (-18) \\ &= \left(-\frac{1}{2}\right)^2 - \left(\frac{21}{30} - \frac{16}{30}\right)^2 (-18) \\ &= \left(-\frac{1}{2}\right)^2 - \left(\frac{5}{30}\right)^2 (-18) \\ &= \left(-\frac{1}{2}\right)^2 - \left(\frac{1}{6}\right)^2 (-18) \\ &= \frac{1}{4} - \frac{1}{36} (-18) \\ &= \frac{1}{4} + \frac{18}{36} \\ &= \frac{1}{4} + \frac{2}{4} \\ &= \frac{3}{4} \end{aligned}$$

$$14. \text{ First, find the sum: } \frac{1}{3} + \frac{1}{2} = \frac{1}{3} \cdot \frac{2}{2} + \frac{1}{2} \cdot \frac{3}{3} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

$$\text{Next, divide by 2: } \frac{5}{6} \div \frac{2}{1} = \frac{5}{6} \cdot \frac{1}{2} = \frac{5}{12}$$

15. Amount of eggs needed

$$= \frac{\text{desired serving size}}{\text{recipe serving size}} \times \text{eggs in recipe}$$
$$= \frac{7 \cancel{\text{dozen}}}{5 \cancel{\text{dozen}}} \times 2 \text{ eggs}$$
$$= \frac{14}{5} \text{ eggs}$$
$$= 2\frac{4}{5} \text{ eggs}$$
$$\approx 3 \text{ eggs}$$

### Concept and Vocabulary Check 1.3

1. rational numbers; integers; zero
2. an improper fraction; the numerator is greater than the denominator
3. terminate/stop; have repeating digits
4. reciprocal/multiplicative inverse
5. false; Changes to make the statement true will vary.
6. false; Changes to make the statement true will vary.
7. true
8. false; Changes to make the statement true will vary.

### Exercise Set 1.3

1.  $10 = 2 \cdot 5$   
 $15 = 3 \cdot 5$   
Greatest Common Divisor is 5.  
 $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$
2.  $18 = 2 \cdot 3^2$   
 $45 = 3^2 \cdot 5$   
Greatest Common Divisor is  $3^2$  or 9.  
 $\frac{18}{45} = \frac{18 \div 9}{45 \div 9} = \frac{2}{5}$
3.  $15 = 3 \cdot 5$   
 $18 = 2 \cdot 3^2$   
Greatest Common Divisor is 3.  
 $\frac{15}{18} = \frac{15 \div 3}{18 \div 3} = \frac{5}{6}$

4.  $16 = 2^4$

$$64 = 2^6$$

Greatest Common Divisor is  $2^4$  or 16.

$$\frac{16}{64} = \frac{16 \div 16}{64 \div 16} = \frac{1}{4}$$

5.  $24 = 2^3 \cdot 3$

$$42 = 2 \cdot 3 \cdot 7$$

Greatest Common Divisor is  $2 \cdot 3$  or 6.

$$\frac{24}{42} = \frac{24 \div 6}{42 \div 6} = \frac{4}{7}$$

6.  $32 = 2^5$

$$80 = 2^4 \cdot 5$$

Greatest Common Divisor is  $2^4$  or 16.

$$\frac{32}{80} = \frac{32 \div 16}{80 \div 16} = \frac{2}{5}$$

7.  $60 = 2^2 \cdot 3 \cdot 5$

$$108 = 2^2 \cdot 3^3$$

Greatest Common Divisor is  $2^2 \cdot 3$  or 12.

$$\frac{60}{108} = \frac{60 \div 12}{108 \div 12} = \frac{5}{9}$$

8.  $112 = 2^4 \cdot 7$

$$128 = 2^7$$

Greatest Common Divisor is  $2^4$  or 16.

$$\frac{112}{128} = \frac{112 \div 16}{128 \div 16} = \frac{7}{8}$$

9.  $342 = 2 \cdot 3^2 \cdot 19$

$$380 = 2^2 \cdot 5 \cdot 19$$

Greatest Common Divisor is  $2 \cdot 19$  or 38.

$$\frac{342}{380} = \frac{342 \div 38}{380 \div 38} = \frac{9}{10}$$

10.  $210 = 2 \cdot 3 \cdot 5 \cdot 7$

$$252 = 2^2 \cdot 3^2 \cdot 7$$

Greatest Common Divisor is  $2 \cdot 3 \cdot 7$  or 42.

$$\frac{210}{252} = \frac{210 \div 42}{252 \div 42} = \frac{5}{6}$$

11.  $308 = 2^2 \cdot 7 \cdot 11$

$$418 = 2 \cdot 11 \cdot 19$$

Greatest Common Divisor is  $2 \cdot 11$  or 22.

$$\frac{308}{418} = \frac{308 \div 22}{418 \div 22} = \frac{14}{19}$$

12.  $144 = 2^4 \cdot 3^2$

$300 = 2^2 \cdot 3 \cdot 5^2$

Greatest Common Divisor is  $2^2 \cdot 3$  or 12.

$$\frac{144}{300} = \frac{144 \div 12}{300 \div 12} = \frac{12}{25}$$

13.  $2\frac{3}{8} = \frac{8 \cdot 2 + 3}{8} = \frac{16 + 3}{8} = \frac{19}{8}$

14.  $2\frac{7}{9} = \frac{9 \cdot 2 + 7}{9} = \frac{18 + 7}{9} = \frac{25}{9}$

15.  $-7\frac{3}{5} = -\frac{5 \cdot 7 + 3}{5} = -\frac{35 + 3}{5} = -\frac{38}{5}$

16.  $-6\frac{2}{5} = -\frac{5 \cdot 6 + 2}{5} = -\frac{30 + 2}{5} = -\frac{32}{5}$

17.  $12\frac{7}{16} = \frac{16 \cdot 12 + 7}{16} = \frac{192 + 7}{16} = \frac{199}{16}$

18.  $11\frac{5}{16} = \frac{16 \cdot 11 + 5}{16} = \frac{176 + 5}{16} = \frac{181}{16}$

19.  $\frac{23}{5} = 4\frac{3}{5}$

20.  $\frac{47}{8} = 5\frac{7}{8}$

21.  $-\frac{76}{9} = -8\frac{4}{9}$

22.  $-\frac{59}{9} = -6\frac{5}{9}$

23.  $\frac{711}{20} = 35\frac{11}{20}$

24.  $\frac{788}{25} = 31\frac{13}{25}$

25.  $\frac{3}{4} = 0.75$

$$\begin{array}{r} 0.75 \\ 4 \overline{) 3.00} \\ \underline{28} \phantom{00} \\ 20 \phantom{00} \\ \underline{20} \phantom{00} \\ 0 \end{array}$$

$$26. \quad \frac{3}{5} = 0.6$$

$$\begin{array}{r} 0.6 \\ 5 \overline{) 3.0} \\ \underline{30} \\ 0 \end{array}$$

$$27. \quad \frac{7}{20} = 0.35$$

$$\begin{array}{r} 0.35 \\ 20 \overline{) 7.00} \\ \underline{60} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

$$28. \quad \frac{3}{20} = 0.15$$

$$\begin{array}{r} 0.15 \\ 20 \overline{) 3.00} \\ \underline{20} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

$$29. \quad \frac{7}{8} = 0.875$$

$$\begin{array}{r} 0.875 \\ 8 \overline{) 7.000} \\ \underline{64} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$$30. \quad \frac{5}{16} = 0.3125$$

$$\begin{array}{r} 0.3125 \\ 16 \overline{) 5.0000} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{32} \\ 80 \end{array}$$

31.  $\frac{9}{11} = 0.\overline{81}$

$$\begin{array}{r} 0.8181... \\ 11 \overline{) 9.0000} \\ \underline{88} \phantom{00} \\ 20 \phantom{00} \\ \underline{11} \phantom{00} \\ 90 \phantom{00} \\ \underline{88} \phantom{00} \\ 20 \phantom{00} \\ \underline{11} \phantom{00} \\ 9 \phantom{00} \end{array}$$

32.  $\frac{3}{11} = 0.\overline{27}$

$$\begin{array}{r} 0.2727... \\ 11 \overline{) 3.0000} \\ \underline{22} \phantom{00} \\ 80 \phantom{00} \\ \underline{77} \phantom{00} \\ 30 \phantom{00} \\ \underline{22} \phantom{00} \\ 80 \phantom{00} \\ \underline{77} \phantom{00} \\ 3 \phantom{00} \end{array}$$

33.  $\frac{22}{7} = 3.\overline{142857}$

$$\begin{array}{r} 3.142857... \\ 7 \overline{) 22.000000} \\ \underline{21} \phantom{000000} \\ 10 \phantom{000000} \\ \underline{7} \phantom{000000} \\ 30 \phantom{000000} \\ \underline{28} \phantom{000000} \\ 20 \phantom{000000} \\ \underline{14} \phantom{000000} \\ 60 \phantom{000000} \\ \underline{56} \phantom{000000} \\ 40 \phantom{000000} \\ \underline{35} \phantom{000000} \\ 50 \phantom{000000} \\ \underline{49} \phantom{000000} \\ 10 \phantom{000000} \end{array}$$



34.  $\frac{20}{3} = 6.\overline{6}$

$$\begin{array}{r} 6.66... \\ 3 \overline{) 20.00} \\ \underline{18} \phantom{00} \\ 20 \phantom{00} \\ \underline{18} \phantom{00} \\ 20 \phantom{00} \\ \underline{18} \phantom{00} \\ 2 \phantom{00} \end{array}$$

35.  $\frac{2}{7} = 0.\overline{285714}$

$$\begin{array}{r} 0.2857142... \\ 7 \overline{) 2.000000} \\ \underline{14} \phantom{000000} \\ 60 \phantom{000000} \\ \underline{56} \phantom{000000} \\ 40 \phantom{000000} \\ \underline{35} \phantom{000000} \\ 50 \phantom{000000} \\ \underline{49} \phantom{000000} \\ 10 \phantom{000000} \\ \underline{7} \phantom{000000} \\ 30 \phantom{000000} \\ \underline{28} \phantom{000000} \\ 20 \phantom{000000} \\ \underline{14} \phantom{000000} \\ 6 \phantom{000000} \end{array}$$

36.  $\frac{5}{7} = 0.\overline{714285}$

$$\begin{array}{r} 0.7142857... \\ 7 \overline{) 5.000000} \\ \underline{49} \phantom{000000} \\ 10 \phantom{000000} \\ \underline{7} \phantom{000000} \\ 30 \phantom{000000} \\ \underline{28} \phantom{000000} \\ 20 \phantom{000000} \\ \underline{14} \phantom{000000} \\ 60 \phantom{000000} \\ \underline{56} \phantom{000000} \\ 40 \phantom{000000} \\ \underline{35} \phantom{000000} \\ 50 \phantom{000000} \\ \underline{49} \phantom{000000} \\ 1 \phantom{000000} \end{array}$$

37.  $0.3 = \frac{3}{10}$

$$38. \quad 0.9 = \frac{9}{10}$$

$$39. \quad 0.4 = \frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$$

$$40. \quad 0.6 = \frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

$$41. \quad 0.39 = \frac{39}{100}$$

$$42. \quad 0.59 = \frac{59}{100}$$

$$43. \quad 0.82 = \frac{82}{100} = \frac{82 \div 2}{100 \div 2} = \frac{41}{50}$$

$$44. \quad 0.64 = \frac{64}{100}$$

$$64 = 2^6$$

$$100 = 2^2 \cdot 5^2$$

Greatest Common Divisor is  $2^2$  or 4.

$$\frac{64}{100} = \frac{64 \div 4}{100 \div 4} = \frac{16}{25}$$

$$45. \quad 0.725 = \frac{725}{1000}$$

$$725 = 5^2 \cdot 29$$

$$1000 = 2^3 \cdot 5^3$$

Greatest Common Divisor is  $5^2$  or 25.

$$\frac{725}{1000} = \frac{725 \div 25}{1000 \div 25} = \frac{29}{40}$$

$$46. \quad 0.625 = \frac{625}{1000}$$

$$625 = 5^4$$

$$1000 = 2^3 \cdot 5^3$$

Greatest Common Divisor is  $5^3$  or 125.

$$\frac{625 \div 125}{1000 \div 125} = \frac{5}{8}$$

$$47. \quad 0.5399 = \frac{5399}{10,000}$$

$$48. \quad 0.7006 = \frac{7006}{10,000}$$

$$7006 = 2 \cdot 31 \cdot 113$$

$$10,000 = 2^4 \cdot 5^4$$

Greatest Common Divisor is 2.

$$\frac{7006}{10,000} = \frac{7006 \div 2}{10,000 \div 2} = \frac{3503}{5000}$$

$$49. \quad n = 0.777\ldots$$

$$10n = 7.777\ldots$$

$$10n = 7.777\ldots$$

$$\begin{array}{r} -n = 0.777\ldots \\ \hline 9n = 7 \end{array}$$

$$n = \frac{7}{9}$$

$$50. \quad n = 0.1111\ldots$$

$$10n = 1.1111\ldots$$

$$10n = 1.1111\ldots$$

$$\begin{array}{r} -n = 0.1111\ldots \\ \hline 9n = 1 \end{array}$$

$$n = \frac{1}{9}$$

$$51. \quad n = 0.999\ldots$$

$$10n = 9.999\ldots$$

$$10n = 9.999\ldots$$

$$\begin{array}{r} -n = 0.999\ldots \\ \hline 9n = 9 \end{array}$$

$$n = 1$$

$$52. \quad n = 0.\overline{3}\ldots$$

$$10n = 3.333\ldots$$

$$10n = 3.3333\ldots$$

$$\begin{array}{r} -n = 0.3333\ldots \\ \hline 9n = 3.0 \end{array}$$

$$n = \frac{3}{9} \text{ or } \frac{1}{3}$$

$$\begin{aligned} 53. \quad n &= 0.3636\ldots \\ 100n &= 36.3636\ldots \end{aligned}$$

$$\begin{array}{r} 100n = 36.3636\ldots \\ -n = 0.3636\ldots \\ \hline 99n = 36 \end{array}$$

$$n = \frac{36}{99} \text{ or } \frac{4}{11}$$

$$\begin{aligned} 54. \quad n &= 0.8181\ldots \\ 100n &= 81.8181\ldots \end{aligned}$$

$$\begin{array}{r} 100n = 81.8181\ldots \\ -n = 0.8181\ldots \\ \hline 99n = 81 \end{array}$$

$$n = \frac{81}{99} \text{ or } \frac{9}{11}$$

$$\begin{aligned} 55. \quad n &= 0.257257\ldots \\ 1000n &= 257.257257\ldots \end{aligned}$$

$$\begin{array}{r} 1000n = 257.257257\ldots \\ -n = .257257\ldots \\ \hline 999n = 257 \end{array}$$

$$n = \frac{257}{999}$$

$$\begin{aligned} 56. \quad n &= 0.529529\ldots \\ 1000n &= 529.529529\ldots \end{aligned}$$

$$\begin{array}{r} 1000n = 529.529529\ldots \\ -n = 0.529529\ldots \\ \hline 999n = 529 \end{array}$$

$$n = \frac{529}{999}$$

$$57. \quad \frac{3}{8} \cdot \frac{7}{11} = \frac{3 \cdot 7}{8 \cdot 11} = \frac{21}{88}$$

$$58. \quad \frac{5}{8} \cdot \frac{3}{11} = \frac{5 \cdot 3}{8 \cdot 11} = \frac{15}{88}$$

$$59. \quad \left(-\frac{1}{10}\right)\left(\frac{7}{12}\right) = \frac{(-1)(7)}{10 \cdot 12} = \frac{-7}{120} = -\frac{7}{120}$$

$$60. \quad \left(-\frac{1}{8}\right)\left(\frac{5}{9}\right) = \frac{(-1)(5)}{8 \cdot 9} = \frac{-5}{72} = -\frac{5}{72}$$

$$61. \left(-\frac{2}{3}\right)\left(-\frac{9}{4}\right) = \frac{(-2)(-9)}{3 \cdot 4} = \frac{18}{12} = \frac{3}{2}$$

$$62. \left(-\frac{5}{4}\right)\left(-\frac{6}{7}\right) = \frac{(-5)(-6)}{4 \cdot 7} = \frac{30}{28} = \frac{15}{14}$$

$$63. \left(3\frac{3}{4}\right)\left(1\frac{3}{5}\right) = \frac{15}{4} \cdot \frac{8}{5} = \frac{120}{20} = \frac{6}{1} = 6$$

$$64. \left(2\frac{4}{5}\right)\left(1\frac{1}{4}\right) = \frac{14}{5} \cdot \frac{5}{4} = \frac{70}{20} = \frac{7}{2} \text{ or } 3\frac{1}{2}$$

$$65. \frac{5}{4} \div \frac{3}{8} = \frac{5}{4} \cdot \frac{8}{3} = \frac{5 \cdot 8}{4 \cdot 3} = \frac{40}{12} = \frac{10}{3}$$

$$66. \frac{5}{8} \div \frac{4}{3} = \frac{5}{8} \cdot \frac{3}{4} = \frac{5 \cdot 3}{8 \cdot 4} = \frac{15}{32}$$

$$\begin{aligned} 67. -\frac{7}{8} \div \frac{15}{16} &= -\frac{7}{8} \cdot \frac{16}{15} \\ &= \frac{(-7)(16)}{8 \cdot 15} \\ &= \frac{-112}{120} \\ &= -\frac{14}{15} \end{aligned}$$

$$\begin{aligned} 68. -\frac{13}{20} \div \frac{4}{5} &= -\frac{13}{20} \cdot \frac{5}{4} \\ &= \frac{(-13)(5)}{20 \cdot 4} \\ &= \frac{-65}{80} \\ &= -\frac{65}{80} \\ &= -\frac{13}{16} \end{aligned}$$

$$69. 6\frac{3}{5} \div 1\frac{1}{10} = \frac{33}{5} \div \frac{11}{10} = \frac{33}{5} \cdot \frac{10}{11} = \frac{330}{55} = \frac{6}{1} = 6$$

$$70. 1\frac{3}{4} \div 2\frac{5}{8} = \frac{7}{4} \div \frac{21}{8} = \frac{7}{4} \cdot \frac{8}{21} = \frac{56}{84} = \frac{2}{3}$$

$$71. \frac{2}{11} + \frac{3}{11} = \frac{2+3}{11} = \frac{5}{11}$$

$$72. \frac{5}{13} + \frac{2}{13} = \frac{5+2}{13} = \frac{7}{13}$$

$$73. \frac{5}{6} - \frac{1}{6} = \frac{5-1}{6} = \frac{4}{6} = \frac{2}{3}$$

$$74. \frac{7}{12} - \frac{5}{12} = \frac{7-5}{12} = \frac{2}{12} = \frac{1}{6}$$

$$75. \frac{7}{12} - \left(-\frac{1}{12}\right) = \frac{7}{12} + \frac{1}{12} = \frac{7+1}{12} = \frac{8}{12} = \frac{2}{3}$$

$$76. \frac{5}{16} - \left(-\frac{5}{16}\right) = \frac{5}{16} + \frac{5}{16} = \frac{5+5}{16} = \frac{10}{16} = \frac{5}{8}$$

$$\begin{aligned} 77. \frac{1}{2} + \frac{1}{5} &= \left(\frac{1}{2}\right)\left(\frac{5}{5}\right) + \left(\frac{1}{5}\right)\left(\frac{2}{2}\right) \\ &= \frac{5}{10} + \frac{2}{10} \\ &= \frac{5+2}{10} \\ &= \frac{7}{10} \end{aligned}$$

$$\begin{aligned} 78. \frac{1}{3} + \frac{1}{5} &= \left(\frac{1}{3}\right)\left(\frac{5}{5}\right) + \left(\frac{1}{5}\right)\left(\frac{3}{3}\right) \\ &= \frac{5}{15} + \frac{3}{15} \\ &= \frac{5+3}{15} \\ &= \frac{8}{15} \end{aligned}$$

$$\begin{aligned} 79. \frac{3}{4} + \frac{3}{20} &= \left(\frac{3}{4}\right)\left(\frac{5}{5}\right) + \frac{3}{20} \\ &= \frac{15}{20} + \frac{3}{20} \\ &= \frac{15+3}{20} \\ &= \frac{18}{20} \\ &= \frac{9}{10} \end{aligned}$$

$$80. \frac{2}{5} + \frac{2}{15} = \left(\frac{2}{5}\right)\left(\frac{3}{3}\right) + \frac{2}{15} = \frac{6}{15} + \frac{2}{15} = \frac{6+2}{15} = \frac{8}{15}$$

$$\begin{aligned}
 81. \quad \frac{5}{24} + \frac{7}{30} &= \left(\frac{5}{24}\right)\left(\frac{5}{5}\right) + \left(\frac{7}{30}\right)\left(\frac{4}{4}\right) \\
 &= \frac{25}{120} + \frac{28}{120} \\
 &= \frac{25+28}{120} \\
 &= \frac{53}{120}
 \end{aligned}$$

$$\begin{aligned}
 82. \quad \frac{7}{108} + \frac{55}{144} \\
 108 &= 2^2 \cdot 3^3 \\
 144 &= 2^4 \cdot 3^2 \\
 \text{Least Common Multiple: } 2^4 \cdot 3^3 &= 432 \\
 \frac{7}{108} + \frac{55}{144} &= \left(\frac{7}{108}\right)\left(\frac{4}{4}\right) + \left(\frac{55}{144}\right)\left(\frac{3}{3}\right) \\
 &= \frac{28}{432} + \frac{165}{432} \\
 &= \frac{28+165}{432} = \frac{193}{432}
 \end{aligned}$$

$$\begin{aligned}
 83. \quad \frac{13}{18} - \frac{2}{9} &= \frac{13}{18} - \frac{2}{9}\left(\frac{2}{2}\right) \\
 &= \frac{13}{18} - \frac{4}{18} \\
 &= \frac{13-4}{18} \\
 &= \frac{9}{18} \\
 &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 84. \quad \frac{13}{15} - \frac{2}{45} &= \left(\frac{13}{15}\right)\left(\frac{3}{3}\right) - \frac{2}{45} = \frac{39}{45} - \frac{2}{45} = \frac{39-2}{45} \\
 &= \frac{37}{45}
 \end{aligned}$$

$$\begin{aligned}
 85. \quad \frac{4}{3} - \frac{3}{4} &= \frac{4}{3}\left(\frac{4}{4}\right) - \frac{3}{4}\left(\frac{3}{3}\right) \\
 &= \frac{16}{12} - \frac{9}{12} \\
 &= \frac{16-9}{12} \\
 &= \frac{7}{12}
 \end{aligned}$$

$$86. \quad \frac{3}{2} - \frac{2}{3} = \frac{3}{2} \left( \frac{3}{3} \right) - \frac{2}{3} \left( \frac{2}{2} \right) = \frac{9}{6} - \frac{4}{6} = \frac{9-4}{6} = \frac{5}{6}$$

$$87. \quad \frac{1}{15} - \frac{27}{50}$$

$$15 = 3 \cdot 5$$

$$50 = 2 \cdot 5^2$$

Least Common Multiple is  $2 \cdot 3 \cdot 5^2 = 6 \cdot 25 = 150$

$$\begin{aligned} \frac{1}{15} \left( \frac{10}{10} \right) - \frac{27}{50} \left( \frac{3}{3} \right) &= \frac{10}{150} - \frac{81}{150} \\ &= \frac{10-81}{150} \\ &= -\frac{71}{150} \end{aligned}$$

$$\begin{aligned} 88. \quad \frac{4}{15} - \frac{1}{6} &= \frac{4}{15} \left( \frac{2}{2} \right) - \frac{1}{6} \left( \frac{5}{5} \right) \\ &= \frac{8}{30} - \frac{5}{30} \\ &= \frac{8-5}{30} \\ &= \frac{3}{30} \\ &= \frac{1}{10} \end{aligned}$$

$$89. \quad 2\frac{2}{3} + 1\frac{3}{4} = 2\frac{8}{12} + 1\frac{9}{12} = \frac{32}{12} + \frac{21}{12} = \frac{53}{12} \text{ or } 4\frac{5}{12}$$

$$90. \quad 2\frac{1}{8} + 3\frac{3}{4} = 2\frac{1}{8} + 3\frac{6}{8} = \frac{17}{8} + \frac{30}{8} = \frac{47}{8} \text{ or } 5\frac{7}{8}$$

$$91. \quad 3\frac{2}{3} - 2\frac{1}{2} = 3\frac{4}{6} - 2\frac{3}{6} = \frac{22}{6} - \frac{15}{6} = \frac{7}{6} \text{ or } 1\frac{1}{6}$$

$$92. \quad 3\frac{3}{4} - 2\frac{1}{3} = 3\frac{9}{12} - 2\frac{4}{12} = \frac{45}{12} - \frac{28}{12} = \frac{17}{12} \text{ or } 1\frac{5}{12}$$

$$\begin{aligned} 93. \quad -5\frac{2}{3} + 3\frac{1}{6} &= -5\frac{4}{6} + 3\frac{1}{6} = \frac{-34}{6} + \frac{19}{6} = \frac{-15}{6} = -\frac{5}{2} \\ &\text{or } -2\frac{1}{2} \end{aligned}$$

$$94. \quad -2\frac{1}{2} + 1\frac{3}{4} = -2\frac{2}{4} + 1\frac{3}{4} = \frac{-10}{4} + \frac{7}{4} = -\frac{3}{4}$$

$$95. \quad -1\frac{4}{7} - \left( -2\frac{5}{14} \right) = -1\frac{8}{14} + 2\frac{5}{14} = \frac{-22}{14} + \frac{33}{14} = \frac{11}{14}$$



$$96. -1\frac{4}{9} - \left(-2\frac{5}{18}\right) = -1\frac{8}{18} + 2\frac{5}{18} = \frac{-26}{18} + \frac{41}{18} = \frac{15}{18} = \frac{5}{6}$$

$$\begin{aligned} 97. \left(\frac{1}{2} - \frac{1}{3}\right) \div \frac{5}{8} &= \left[\left(\frac{1}{2}\right)\left(\frac{3}{3}\right) - \frac{1}{3}\left(\frac{2}{2}\right)\right] \div \frac{5}{8} \\ &= \left(\frac{3}{6} - \frac{2}{6}\right) \div \frac{5}{8} \\ &= \frac{1}{6} \div \frac{5}{8} \\ &= \frac{1}{6} \cdot \frac{8}{5} \\ &= \frac{1 \cdot 8}{6 \cdot 5} \\ &= \frac{8}{30} \\ &= \frac{4}{15} \end{aligned}$$

$$\begin{aligned} 98. \left(\frac{1}{2} + \frac{1}{4}\right) \div \left(\frac{1}{2} + \frac{1}{3}\right) &= \left(\frac{1}{2} \cdot \frac{2}{2} + \frac{1}{4}\right) \div \left(\frac{1}{2} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{2}{2}\right) \\ &= \left(\frac{2}{4} + \frac{1}{4}\right) \div \left(\frac{3}{6} + \frac{2}{6}\right) \\ &= \frac{3}{4} \div \frac{5}{6} \\ &= \frac{3}{4} \cdot \frac{6}{5} \\ &= \frac{3 \cdot 6}{4 \cdot 5} \\ &= \frac{18}{20} \\ &= \frac{9}{10} \end{aligned}$$

$$\begin{aligned} 99. -\frac{9}{4}\left(\frac{1}{2}\right) + \frac{3}{4} \div \frac{5}{6} &= -\frac{9}{8} + \frac{9}{10} \\ &= -\frac{9}{40} \end{aligned}$$

$$\begin{aligned} 100. \left[-\frac{4}{7} - \left(-\frac{2}{5}\right)\right] \left[-\frac{3}{8} + \left(-\frac{1}{9}\right)\right] &= \left[-\frac{6}{35}\right] \left[-\frac{35}{72}\right] \\ &= \frac{1}{12} \end{aligned}$$

$$\begin{aligned}
 101. \quad \frac{\frac{7}{9} - 3}{\frac{5}{6}} \div \frac{3}{2} + \frac{3}{4} &= \frac{-\frac{20}{9}}{\frac{5}{6}} \div \frac{3}{2} + \frac{3}{4} \\
 &= -\frac{8}{3} \times \frac{2}{3} + \frac{3}{4} \\
 &= -\frac{16}{9} + \frac{3}{4} \\
 &= -\frac{37}{36} \text{ or } -1\frac{1}{36}
 \end{aligned}$$

$$\begin{aligned}
 102. \quad \frac{\frac{17}{25}}{\frac{3}{5} - 4} \div \frac{1}{5} + \frac{1}{2} &= \frac{\frac{17}{25}}{-\frac{17}{5}} \div \frac{1}{5} + \frac{1}{2} \\
 &= -\frac{1}{5} \div \frac{1}{5} + \frac{1}{2} \\
 &= -1 + \frac{1}{2} \\
 &= -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 103. \quad \frac{1}{4} - 6(2+8) \div \left(-\frac{1}{3}\right) \left(-\frac{1}{9}\right) \\
 &= \frac{1}{4} - 6(10) \div 3 \\
 &= \frac{1}{4} - 60 \div 3 \\
 &= \frac{1}{4} - 20 \\
 &= -19\frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 104. \quad \frac{3}{4} - 4(2+7) \div \left(-\frac{1}{2}\right) \left(-\frac{1}{6}\right) \\
 &= \frac{3}{4} - 4(9) \div 3 \\
 &= \frac{3}{4} - 36 \div 3 \\
 &= \frac{3}{4} - 12 \\
 &= -11\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 105. \quad \frac{1}{4} + \frac{1}{3} &= \left(\frac{1}{4}\right)\left(\frac{3}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{4}{4}\right) \\
 &= \frac{3}{12} + \frac{4}{12} \\
 &= \frac{3+4}{12} \\
 &= \frac{7}{12} \\
 \frac{7}{12} \div 2 &= \frac{7}{12} \cdot \frac{1}{2} = \frac{7}{24}
 \end{aligned}$$

$$\begin{aligned}
 106. \quad \frac{2}{3} + \frac{5}{6} &= \left(\frac{2}{3}\right)\left(\frac{2}{2}\right) + \frac{5}{6} \\
 &= \frac{4}{6} + \frac{5}{6} \\
 &= \frac{4+5}{6} \\
 &= \frac{9}{6} \\
 &= \frac{3}{2} \\
 \frac{3}{2} \div 2 &= \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 107. \quad \frac{1}{2} + \frac{2}{3} &= \left(\frac{1}{2}\right)\left(\frac{3}{3}\right) + \left(\frac{2}{3}\right)\left(\frac{2}{2}\right) \\
 &= \frac{3}{6} + \frac{4}{6} \\
 &= \frac{3+4}{6} \\
 &= \frac{7}{6} \\
 \frac{7}{6} \div 2 &= \frac{7}{6} \cdot \frac{1}{2} = \frac{7}{12}
 \end{aligned}$$

$$\begin{aligned}
 108. \quad \frac{3}{5} + \frac{2}{3} &= \left(\frac{3}{5}\right)\left(\frac{3}{3}\right) + \left(\frac{2}{3}\right)\left(\frac{5}{5}\right) \\
 &= \frac{9}{15} + \frac{10}{15} \\
 &= \frac{9+10}{15} \\
 &= \frac{19}{15} \\
 \frac{19}{15} \div 2 &= \frac{19}{15} \cdot \frac{1}{2} = \frac{19}{30}
 \end{aligned}$$

$$\begin{aligned}
 109. \quad -\frac{2}{3} + \left(-\frac{5}{6}\right) &= \left(-\frac{2}{3}\right)\left(\frac{2}{2}\right) - \frac{5}{6} \\
 &= \frac{-4}{6} - \frac{5}{6} \\
 &= \frac{-4-5}{6} \\
 &= -\frac{9}{6} \\
 -\frac{9}{6} \div 2 &= -\frac{9}{6} \cdot \frac{1}{2} = -\frac{9}{12} = -\frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 110. \quad -4 + \left(-\frac{7}{2}\right) &= (-4)\left(\frac{2}{2}\right) - \left(\frac{7}{2}\right) \\
 &= -\frac{8}{2} - \frac{7}{2} \\
 &= -\frac{8+7}{2} \\
 &= -\frac{15}{2} \\
 -\frac{15}{2} \div 2 &= -\frac{15}{2} \cdot \frac{1}{2} = -\frac{15}{4}
 \end{aligned}$$

$$\begin{aligned}
 111. \quad \frac{13}{4} + \frac{13}{9} &= \frac{13 \cdot 9}{4 \cdot 9} + \frac{13 \cdot 4}{9 \cdot 4} \\
 &= \frac{117}{36} + \frac{52}{36} \\
 &= \frac{117+52}{36} \\
 &= \frac{169}{36}
 \end{aligned}$$

$$\begin{aligned}
 \frac{13}{4} \times \frac{13}{9} &= \frac{13 \cdot 13}{4 \cdot 9} \\
 &= \frac{169}{36}
 \end{aligned}$$

Both are equal to  $\frac{169}{36}$

$$\begin{aligned}
 112. \quad \frac{169}{30} + \frac{13}{15} &= \frac{169}{30} + \left(\frac{13}{15}\right)\left(\frac{2}{2}\right) \\
 &= \frac{169}{30} + \frac{26}{30} \\
 &= \frac{169+26}{30} \\
 &= \frac{195}{30} \\
 &= \frac{39}{6} \\
 &= \frac{13}{2}
 \end{aligned}$$

$$\begin{aligned}
 \frac{169}{30} \div \frac{13}{15} &= \frac{169}{30} \cdot \frac{15}{13} \\
 &= \frac{169 \cdot 15}{30 \cdot 13} \\
 &= \frac{169 \cdot \cancel{15}^1}{\cancel{30}_2 \cdot 13} \\
 &= \frac{\cancel{169}^{13} \cdot 1}{2 \cdot \cancel{13}_1} \\
 &= \frac{13 \cdot 1}{2 \cdot 1} \\
 &= \frac{13}{2}
 \end{aligned}$$

Both are equal to  $\frac{13}{2}$

$$\begin{aligned}
 113. \quad \frac{5}{2^2 \cdot 3^2} - \frac{1}{2 \cdot 3^2} &= \frac{5}{2^2 \cdot 3^2} - \frac{2}{2} \cdot \frac{1}{2 \cdot 3^2} \\
 &= \frac{5}{2^2 \cdot 3^2} - \frac{2}{2^2 \cdot 3^2} \\
 &= \frac{3}{2^2 \cdot 3^2} \\
 &= \frac{1}{2^2 \cdot 3}
 \end{aligned}$$

$$\begin{aligned}
 114. \quad \frac{7}{3^2 \cdot 5^2} - \frac{1}{3 \cdot 5^3} &= \frac{5}{5} \cdot \frac{7}{3^2 \cdot 5^2} - \frac{3}{3} \cdot \frac{1}{3 \cdot 5^3} \\
 &= \frac{35}{3^2 \cdot 5^3} - \frac{3}{3^2 \cdot 5^3} \\
 &= \frac{32}{3^2 \cdot 5^3}
 \end{aligned}$$

$$\begin{aligned}
 115. \quad & \frac{1}{2^4 \cdot 5^3 \cdot 7} + \frac{1}{2 \cdot 5^4} - \frac{1}{2^3 \cdot 5^2} \\
 &= \frac{5}{5} \cdot \frac{1}{2^4 \cdot 5^3 \cdot 7} + \frac{2^3 \cdot 7}{2^3 \cdot 7} \cdot \frac{1}{2 \cdot 5^4} - \frac{2 \cdot 5^2 \cdot 7}{2 \cdot 5^2 \cdot 7} \cdot \frac{1}{2^3 \cdot 5^2} \\
 &= \frac{5}{2^4 \cdot 5^4 \cdot 7} + \frac{56}{2^4 \cdot 5^4 \cdot 7} - \frac{350}{2^4 \cdot 5^4 \cdot 7} \\
 &= -\frac{289}{2^4 \cdot 5^4 \cdot 7}
 \end{aligned}$$

$$\begin{aligned}
 116. \quad & \frac{1}{2^3 \cdot 17^8} + \frac{1}{2 \cdot 17^9} - \frac{1}{2^2 \cdot 3 \cdot 17^8} \\
 &= \frac{3 \cdot 17}{3 \cdot 17} \cdot \frac{1}{2^3 \cdot 17^8} + \frac{2^2 \cdot 3}{2^2 \cdot 3} \cdot \frac{1}{2 \cdot 17^9} - \frac{2 \cdot 17}{2 \cdot 17} \cdot \frac{1}{2^2 \cdot 3 \cdot 17^8} \\
 &= \frac{51}{2^3 \cdot 3 \cdot 17^9} + \frac{12}{2^3 \cdot 3 \cdot 17^9} - \frac{34}{2^3 \cdot 3 \cdot 17^9} \\
 &= \frac{29}{2^3 \cdot 3 \cdot 17^9}
 \end{aligned}$$

$$\begin{aligned}
 117. \quad \text{a.} \quad & \frac{560}{1600} = \frac{7}{20} \\
 \text{b.} \quad & 0.35 = 35\% \\
 \text{c.} \quad & \frac{720}{1600} = \frac{9}{20} \\
 \text{d.} \quad & 0.45 = 45\% \\
 \text{e.} \quad & 45\% - 35\% = 10\%
 \end{aligned}$$

$$\begin{aligned}
 118. \quad \text{a.} \quad & \frac{400}{1600} = \frac{1}{4} \\
 \text{b.} \quad & 0.25 = 25\% \\
 \text{c.} \quad & \frac{304}{1600} = \frac{19}{100} \\
 \text{d.} \quad & 0.19 = 19\% \\
 \text{e.} \quad & 25\% - 19\% = 6\%
 \end{aligned}$$

- 119.** For each ingredient in the recipe, multiply the original quantity by  $\frac{8}{16}$  or  $\frac{1}{2}$ .

$$\frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3} \text{ cup butter}$$

$$5 \cdot \frac{1}{2} = \frac{5}{2} = 2\frac{1}{2} \text{ ounces unsweetened chocolate}$$

$$1\frac{1}{2} \cdot \frac{1}{2} = \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4} \text{ cup sugar}$$

$$2 \cdot \frac{1}{2} = 1 \text{ teaspoon vanilla}$$

$$2 \cdot \frac{1}{2} = 1 \text{ egg}$$

$$1 \cdot \frac{1}{2} = \frac{1}{2} \text{ cup flour}$$

- 120.** For each ingredient in the recipe, multiply the original quantity by  $\frac{12}{16}$  or  $\frac{3}{4}$ .

$$\frac{2}{3} \cdot \frac{3}{4} = \frac{1}{2} \text{ cup butter}$$

$$5 \cdot \frac{3}{4} = \frac{15}{4} = 3\frac{3}{4} \text{ ounces unsweetened chocolate}$$

$$1\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{2} \cdot \frac{3}{4} = \frac{9}{8} = 1\frac{1}{8} \text{ cups sugar}$$

$$2 \cdot \frac{3}{4} = \frac{3}{2} = 1\frac{1}{2} \text{ teaspoons vanilla}$$

$$2 \cdot \frac{3}{4} = \frac{3}{2} = 1\frac{1}{2} \text{ egg}$$

$$1 \cdot \frac{3}{4} = \frac{3}{4} \text{ cup flour}$$

- 121.** For each ingredient in the recipe, multiply the original quantity by  $\frac{20}{16}$  or  $\frac{5}{4}$ .

$$\frac{2}{3} \cdot \frac{5}{4} = \frac{5}{6} \text{ cup butter}$$

$$5 \cdot \frac{5}{4} = \frac{25}{4} = 6\frac{1}{4} \text{ ounces unsweetened chocolate}$$

$$1\frac{1}{2} \cdot \frac{5}{4} = \frac{3}{2} \cdot \frac{5}{4} = \frac{15}{8} = 1\frac{7}{8} \text{ cups sugar}$$

$$2 \cdot \frac{5}{4} = \frac{5}{2} = 2\frac{1}{2} \text{ teaspoons vanilla}$$

$$2 \cdot \frac{5}{4} = \frac{5}{2} = 2\frac{1}{2} \text{ eggs}$$

$$1 \cdot \frac{5}{4} = \frac{5}{4} = 1\frac{1}{4} \text{ cups flour}$$

- 122.** For each ingredient in the recipe, multiply the original quantity by  $\frac{24}{16}$  or  $\frac{3}{2}$ .

$$\frac{2}{3} \cdot \frac{3}{2} = 1 \text{ cup butter}$$

$$5 \cdot \frac{3}{2} = \frac{15}{2} = 7\frac{1}{2} \text{ ounces unsweetened chocolate}$$

$$1\frac{1}{2} \cdot \frac{3}{2} = \frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4} = 2\frac{1}{4} \text{ cups sugar}$$

$$2 \cdot \frac{3}{2} = 3 \text{ teaspoons vanilla}$$

$$2 \cdot \frac{3}{2} = 3 \text{ eggs}$$

$$1 \cdot \frac{3}{2} = \frac{3}{2} = 1\frac{1}{2} \text{ cups flour}$$

- 123.** Begin by dividing the 1 cup of butter by the quantity of butter needed for a 16-brownie batch:

$$1 \div \frac{2}{3} = 1 \times \frac{3}{2} = \frac{3}{2} = 1\frac{1}{2}$$

Thus, 1 cup of butter is enough for  $1\frac{1}{2}$  batches.

Since each batch makes 16 brownies,  $1\frac{1}{2}$  batches will make  $16 \times 1\frac{1}{2} = 24$  brownies. Thus, 1 cup of butter is enough for 24 brownies.

- 124.** Begin by dividing the 1 cup of sugar by the quantity of sugar needed for a 16-brownie batch:

$$1 \div 1\frac{1}{2} = 1 \div \frac{3}{2} = 1 \times \frac{2}{3} = \frac{2}{3}$$

Thus, 1 cup of sugar is enough for  $\frac{2}{3}$  batches.

Since each batch makes 16 brownies,  $\frac{2}{3}$  of a batch will make  $16 \times \frac{2}{3} = \frac{32}{3} = 10\frac{2}{3}$  brownies. Ignoring part of a brownie means that 1 cup of sugar is enough for 10 brownies.

- 125.**  $2\frac{2}{3} \cdot \frac{11}{8} = \frac{8}{3} \cdot \frac{11}{8} = \frac{88}{24} = \frac{11}{3}$  or  $3\frac{2}{3}$  cups of water.

- 126.**  $2\frac{2}{3} \cdot \frac{6}{8} = \frac{8}{3} \cdot \frac{6}{8} = \frac{48}{24} = 2$  cups of water.

- 127. a.** Strings D, E, G, A, and B are  $\frac{8}{9}$  of the length of the previous string.

**b.** There are black keys to the left of the keys for the notes D, E, G, A, and B.

- 128. a.** Strings F and c are not  $\frac{8}{9}$  of the length of the previous string.

**b.** There are no black keys to the left of the keys for the notes F and c.



129.  $24 - \frac{1}{16} - 7\frac{1}{2} = \frac{24}{1} - \frac{1}{16} - \frac{15}{2} = \frac{384}{16} - \frac{1}{16} - \frac{120}{16} = \frac{263}{16} = 16\frac{7}{16}$  in.

130.  $36 - \frac{1}{16} - 7\frac{1}{4} = \frac{36}{1} - \frac{1}{16} - \frac{29}{4} = \frac{576}{16} - \frac{1}{16} - \frac{116}{16} = \frac{459}{16} = 28\frac{11}{16}$  in.

131.  $1 - \frac{5}{12} - \frac{1}{4} = \frac{12}{12} - \frac{5}{12} - \frac{3}{12} = \frac{4}{12} = \frac{1}{3}$  ownership.

132.  $1 - \frac{1}{4} - \frac{2}{5} - \frac{1}{10} = \frac{20}{20} - \frac{5}{20} - \frac{8}{20} - \frac{2}{20} = \frac{5}{20} = \frac{1}{4}$

133. The total distance is their sum:  $\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20}$  miles.

The difference is the amount farther:  $\frac{3}{4} - \frac{2}{5} = \frac{15}{20} - \frac{8}{20} = \frac{7}{20}$  mile.

134. 40 hours at \$12 rate

6 hours at  $\left(\frac{3}{2}\right)$  \$12 rate or  $\left(\frac{3}{2}\right)$  \$12 =  $\frac{\$36}{2} = \$18$

$40 \cdot \$12 + 6 \cdot \$18 = \$480 + \$108 = \$588$

135.  $\frac{3}{5}$  of the total goes to relatives, so there is  $\frac{2}{5}$  of the estate left.  $\frac{1}{4}$  of that  $\frac{2}{5}$  goes for AIDS research:

$\frac{1}{4} \cdot \frac{2}{5} = \frac{2}{20} = \frac{1}{10}$

136.  $2\frac{3}{8} \cdot 16 = \frac{19}{8} \cdot \frac{16}{1} = \frac{304}{8} = 38$  miles

137. 1st measure:  $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{2}{8} + \frac{2}{8} + \frac{1}{8} + \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$

2nd measure:  $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

3rd measure:  $\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$

4th measure:  $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{2}{8} + \frac{2}{8} + \frac{1}{8} + \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$

The musical notation is a single staff in 3/4 time, starting with a treble clef and a key signature of one flat (B-flat). The melody consists of eighth and quarter notes. The lyrics are written below the staff, aligned with the notes: 'say' (quarter), 'does' (quarter), 'that' (quarter), 'Star-span-gled' (quarter), 'Ban-ner' (quarter), 'yet' (quarter), 'wave' (quarter), 'O'er' (quarter), 'the' (quarter).

138. Conjecture: The sums of  $\frac{2}{3}$ ,  $\frac{3}{4}$ , and  $\frac{4}{5}$  will be followed by a sum of  $\frac{5}{6}$ .

$$\begin{aligned}\text{Verification: } \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} &= \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} \\ &= \frac{30}{60} + \frac{10}{60} + \frac{5}{60} + \frac{3}{60} + \frac{2}{60} \\ &= \frac{50}{60} \\ &= \frac{5}{6}\end{aligned}$$

139. a.  $\frac{197}{800} = 0.24625$
- b.  $\frac{4539}{3125} = 1.45248$
- c.  $\frac{7}{6250} = 0.00112$

#### Check Points 1.4

1. a.  $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$
- b.  $\sqrt{60} = \sqrt{4 \cdot 15} = \sqrt{4} \cdot \sqrt{15} = 2\sqrt{15}$
- c.  $\sqrt{55}$  cannot be simplified.
2. a.  $\sqrt{3} \cdot \sqrt{10} = \sqrt{3 \cdot 10} = \sqrt{30}$
- b.  $\sqrt{10} \cdot \sqrt{10} = \sqrt{10 \cdot 10} = \sqrt{100} = 10$
- c.  $\sqrt{6} \cdot \sqrt{2} = \sqrt{6 \cdot 2} = \sqrt{12} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$
3. a.  $\frac{\sqrt{80}}{\sqrt{5}} = \sqrt{\frac{80}{5}} = \sqrt{16} = 4$
- b.  $\frac{\sqrt{48}}{\sqrt{6}} = \sqrt{\frac{48}{6}} = \sqrt{8} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$
4. a.  $8\sqrt{3} + 10\sqrt{3} = (8+10)\sqrt{3} = 18\sqrt{3}$
- b.  $4\sqrt{13} - 9\sqrt{13} = (4-9)\sqrt{13} = -5\sqrt{13}$
- c.  $7\sqrt{10} + 2\sqrt{10} - \sqrt{10} = (7+2-1)\sqrt{10} = 8\sqrt{10}$

5. a.  $\sqrt{3} + \sqrt{12} = \sqrt{3} + \sqrt{4} \cdot \sqrt{3} = \sqrt{3} + 2\sqrt{3} = 3\sqrt{3}$

b.  $4\sqrt{8} - 7\sqrt{18}$   
 $= 4\sqrt{4 \cdot 2} - 7\sqrt{9 \cdot 2}$   
 $= 4 \cdot 2\sqrt{2} - 7 \cdot 3\sqrt{2}$   
 $= 8\sqrt{2} - 21\sqrt{2}$   
 $= (8 - 21)\sqrt{2}$   
 $= -13\sqrt{2}$

6. a.  $\frac{25}{\sqrt{10}} = \frac{25}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{25\sqrt{10}}{\sqrt{100}} = \frac{25\sqrt{10}}{10} = \frac{5\sqrt{10}}{2}$

b.  $\sqrt{\frac{2}{7}} = \frac{\sqrt{2}}{\sqrt{7}} = \frac{\sqrt{2}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{14}}{\sqrt{49}} = \frac{\sqrt{14}}{7}$

c.  $\frac{5}{\sqrt{18}} = \frac{5}{\sqrt{18}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{36}} = \frac{5\sqrt{2}}{6}$

### Concept and Vocabulary Check 1.4

1. terminating; repeating
2.  $\pi$
3.  $\sqrt{n}$ ;  $n$
4. 49; 6;  $7\sqrt{6}$
5. coefficient
6. 8; 10;  $18\sqrt{3}$
7. 5; 4;  $9\sqrt{2}$
8. rationalizing the denominator
9.  $\sqrt{7}$
10.  $\sqrt{3}$
11. false; Changes to make the statement true will vary.
12. false; Changes to make the statement true will vary.
13. true
14. false; Changes to make the statement true will vary.

Exercise Set 1.4

1.  $\sqrt{9} = 3$  because  $3^2 = 9$ .
2.  $\sqrt{16} = 4$  because  $4^2 = 16$ .
3.  $\sqrt{25} = 5$  because  $5^2 = 25$ .
4.  $\sqrt{49} = 7$  because  $7^2 = 49$ .
5.  $\sqrt{64} = 8$  because  $8^2 = 64$ .
6.  $\sqrt{100} = 10$  because  $10^2 = 100$ .
7.  $\sqrt{121} = 11$  because  $11^2 = 121$ .
8.  $\sqrt{144} = 12$  because  $12^2 = 144$ .
9.  $\sqrt{169} = 13$  because  $13^2 = 169$ .
10.  $\sqrt{225} = 15$  because  $15^2 = 225$ .
11. a.  $\sqrt{173} \approx 13.2$   
b.  $\sqrt{173} \approx 13.15$   
c.  $\sqrt{173} \approx 13.153$
12. a.  $\sqrt{3176} \approx 56.4$   
b.  $\sqrt{3176} \approx 56.36$   
c.  $\sqrt{3176} \approx 56.356$
13. a.  $\sqrt{17,761} \approx 133.3$   
b.  $\sqrt{17,761} \approx 133.27$   
c.  $\sqrt{17,761} \approx 133.270$
14. a.  $\sqrt{779,264} \approx 882.8$   
b.  $\sqrt{779,264} \approx 882.76$   
c.  $\sqrt{779,264} \approx 882.759$
15. a.  $\sqrt{\pi} \approx 1.8$   
b.  $\sqrt{\pi} \approx 1.77$   
c.  $\sqrt{\pi} \approx 1.772$

16. a.  $\sqrt{2\pi} \approx 2.5$

b.  $\sqrt{2\pi} \approx 2.51$

c.  $\sqrt{2\pi} \approx 2.507$

17.  $\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$

18.  $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$

19.  $\sqrt{80} = \sqrt{16 \cdot 5} = \sqrt{16} \cdot \sqrt{5} = 4\sqrt{5}$

20.  $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$

21.  $\sqrt{250} = \sqrt{25 \cdot 10} = \sqrt{25} \cdot \sqrt{10} = 5\sqrt{10}$

22.  $\sqrt{192} = \sqrt{64 \cdot 3} = \sqrt{64} \cdot \sqrt{3} = 8\sqrt{3}$

23.  $7\sqrt{28} = 7\sqrt{4 \cdot 7}$   
 $= 7\sqrt{4} \cdot \sqrt{7}$   
 $= 7 \cdot 2 \cdot \sqrt{7}$   
 $= 14\sqrt{7}$

24.  $3\sqrt{52} = 3\sqrt{4 \cdot 13} = 3\sqrt{4} \cdot \sqrt{13} = 3 \cdot 2 \cdot \sqrt{13} = 6\sqrt{13}$

25.  $\sqrt{7} \cdot \sqrt{6} = \sqrt{7 \cdot 6} = \sqrt{42}$

26.  $\sqrt{19} \cdot \sqrt{3} = \sqrt{19 \cdot 3} = \sqrt{57}$

27.  $\sqrt{6} \cdot \sqrt{6} = \sqrt{6 \cdot 6} = \sqrt{36} = 6$

28.  $\sqrt{5} \cdot \sqrt{5} = \sqrt{5 \cdot 5} = \sqrt{25} = 5$

29.  $\sqrt{3} \cdot \sqrt{6} = \sqrt{3 \cdot 6}$   
 $= \sqrt{18}$   
 $= \sqrt{9 \cdot 2}$   
 $= \sqrt{9} \cdot \sqrt{2}$   
 $= 3\sqrt{2}$

30.  $\sqrt{12} \cdot \sqrt{2} = \sqrt{12 \cdot 2}$   
 $= \sqrt{24}$   
 $= \sqrt{4 \cdot 6}$   
 $= \sqrt{4} \cdot \sqrt{6}$   
 $= 2\sqrt{6}$

$$\begin{aligned}
 31. \quad \sqrt{2} \cdot \sqrt{26} &= \sqrt{2 \cdot 26} \\
 &= \sqrt{52} \\
 &= \sqrt{4 \cdot 13} \\
 &= \sqrt{4} \cdot \sqrt{13} \\
 &= 2\sqrt{13}
 \end{aligned}$$

$$\begin{aligned}
 32. \quad \sqrt{5} \cdot \sqrt{50} &= \sqrt{5 \cdot 50} \\
 &= \sqrt{250} \\
 &= \sqrt{25 \cdot 10} \\
 &= \sqrt{25} \cdot \sqrt{10} \\
 &= 5\sqrt{10}
 \end{aligned}$$

$$33. \quad \frac{\sqrt{54}}{\sqrt{6}} = \sqrt{\frac{54}{6}} = \sqrt{9} = 3$$

$$34. \quad \frac{\sqrt{75}}{\sqrt{3}} = \sqrt{\frac{75}{3}} = \sqrt{25} = 5$$

$$\begin{aligned}
 35. \quad \frac{\sqrt{90}}{\sqrt{2}} &= \sqrt{\frac{90}{2}} \\
 &= \sqrt{45} \\
 &= \sqrt{9 \cdot 5} \\
 &= \sqrt{9} \cdot \sqrt{5} \\
 &= 3\sqrt{5}
 \end{aligned}$$

$$36. \quad \frac{\sqrt{60}}{\sqrt{3}} = \sqrt{\frac{60}{3}} = \sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$$

$$\begin{aligned}
 37. \quad \frac{-\sqrt{96}}{\sqrt{2}} &= -\sqrt{\frac{96}{2}} \\
 &= -\sqrt{48} \\
 &= -\sqrt{16 \cdot 3} \\
 &= -\sqrt{16} \cdot \sqrt{3} \\
 &= -4\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 38. \quad \frac{-\sqrt{150}}{\sqrt{3}} &= -\sqrt{\frac{150}{3}} \\
 &= -\sqrt{50} \\
 &= -\sqrt{25 \cdot 2} \\
 &= -\sqrt{25} \cdot \sqrt{2} \\
 &= -5\sqrt{2}
 \end{aligned}$$

$$39. \quad 7\sqrt{3} + 6\sqrt{3} = (7+6)\sqrt{3} = 13\sqrt{3}$$

$$40. \quad 8\sqrt{5} + 11\sqrt{5} = (8+11)\sqrt{5} = 19\sqrt{5}$$

$$41. \quad 4\sqrt{13} - 6\sqrt{13} = (4-6)\sqrt{13} = -2\sqrt{13}$$

$$42. \quad 6\sqrt{17} - 8\sqrt{17} = (6-8)\sqrt{17} = -2\sqrt{17}$$

$$43. \quad \sqrt{5} + \sqrt{5} = 1\sqrt{5} + 1\sqrt{5} = (1+1)\sqrt{5} = 2\sqrt{5}$$

$$44. \quad \sqrt{3} + \sqrt{3} = 1\sqrt{3} + 1\sqrt{3} = (1+1)\sqrt{3} = 2\sqrt{3}$$

$$45. \quad 4\sqrt{2} - 5\sqrt{2} + 8\sqrt{2} = (4-5+8)\sqrt{2} = 7\sqrt{2}$$

$$46. \quad 6\sqrt{3} + 8\sqrt{3} - 16\sqrt{3} = (6+8-16)\sqrt{3} = -2\sqrt{3}$$

$$\begin{aligned} 47. \quad \sqrt{5} + \sqrt{20} &= 1\sqrt{5} + \sqrt{4} \cdot \sqrt{5} \\ &= 1\sqrt{5} + 2\sqrt{5} \\ &= (1+2)\sqrt{5} \\ &= 3\sqrt{5} \end{aligned}$$

$$\begin{aligned} 48. \quad \sqrt{3} + \sqrt{27} &= 1\sqrt{3} + \sqrt{9} \cdot \sqrt{3} \\ &= 1\sqrt{3} + 3\sqrt{3} \\ &= (1+3)\sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

$$\begin{aligned} 49. \quad \sqrt{50} - \sqrt{18} &= \sqrt{25} \cdot \sqrt{2} - \sqrt{9} \cdot \sqrt{2} \\ &= 5\sqrt{2} - 3\sqrt{2} \\ &= (5-3)\sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} 50. \quad \sqrt{63} - \sqrt{28} &= \sqrt{9} \cdot \sqrt{7} - \sqrt{4} \cdot \sqrt{7} \\ &= 3\sqrt{7} - 2\sqrt{7} \\ &= (3-2)\sqrt{7} \\ &= \sqrt{7} \end{aligned}$$

$$\begin{aligned} 51. \quad 3\sqrt{18} + 5\sqrt{50} &= 3\sqrt{9} \cdot \sqrt{2} + 5\sqrt{25} \cdot \sqrt{2} \\ &= 3 \cdot 3 \cdot \sqrt{2} + 5 \cdot 5\sqrt{2} \\ &= 9\sqrt{2} + 25\sqrt{2} \\ &= (9+25)\sqrt{2} \\ &= 34\sqrt{2} \end{aligned}$$

$$\begin{aligned}
 52. \quad 4\sqrt{12} + 2\sqrt{75} &= 4\sqrt{4} \cdot \sqrt{3} + 2\sqrt{25} \cdot \sqrt{3} \\
 &= 4 \cdot 2 \cdot \sqrt{3} + 2 \cdot 5 \cdot \sqrt{3} \\
 &= 8\sqrt{3} + 10\sqrt{3} \\
 &= (8+10)\sqrt{3} \\
 &= 18\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 53. \quad \frac{1}{4}\sqrt{12} - \frac{1}{2}\sqrt{48} &= \frac{1}{4}\sqrt{4} \cdot \sqrt{3} - \frac{1}{2}\sqrt{16} \cdot \sqrt{3} \\
 &= \frac{1}{4} \cdot 2 \cdot \sqrt{3} - \frac{1}{2} \cdot 4 \cdot \sqrt{3} \\
 &= \frac{1}{2}\sqrt{3} - \frac{4}{2}\sqrt{3} \\
 &= \left(\frac{1}{2} - \frac{4}{2}\right)\sqrt{3} \\
 &= -\frac{3}{2}\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 54. \quad \frac{1}{5}\sqrt{300} - \frac{2}{3}\sqrt{27} &= \frac{1}{5} \cdot \sqrt{100} \cdot \sqrt{3} - \frac{2}{3} \cdot \sqrt{9} \cdot \sqrt{3} \\
 &= \frac{1}{5} \cdot 10 \cdot \sqrt{3} - \frac{2}{3} \cdot 3 \cdot \sqrt{3} \\
 &= 2\sqrt{3} - 2\sqrt{3} \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 55. \quad 3\sqrt{75} + 2\sqrt{12} - 2\sqrt{48} \\
 &= 3 \cdot \sqrt{25} \cdot \sqrt{3} + 2 \cdot \sqrt{4} \cdot \sqrt{3} - 2 \cdot \sqrt{16} \cdot \sqrt{3} \\
 &= 3 \cdot 5 \cdot \sqrt{3} + 2 \cdot 2 \cdot \sqrt{3} - 2 \cdot 4 \cdot \sqrt{3} \\
 &= 15\sqrt{3} + 4\sqrt{3} - 8\sqrt{3} \\
 &= (15+4-8)\sqrt{3} \\
 &= 11\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 56. \quad 2\sqrt{72} + 3\sqrt{50} - \sqrt{128} \\
 &= 2 \cdot \sqrt{36} \cdot \sqrt{2} + 3 \cdot \sqrt{25} \cdot \sqrt{2} - \sqrt{64} \cdot \sqrt{2} \\
 &= 2 \cdot 6 \cdot \sqrt{2} + 3 \cdot 5 \cdot \sqrt{2} - 8 \cdot \sqrt{2} \\
 &= 12\sqrt{2} + 15\sqrt{2} - 8\sqrt{2} \\
 &= (12+15-8)\sqrt{2} \\
 &= 19\sqrt{2}
 \end{aligned}$$

$$57. \quad \frac{5}{\sqrt{3}} = \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{\sqrt{9}} = \frac{5\sqrt{3}}{3}$$

$$58. \quad \frac{12}{\sqrt{5}} = \frac{12}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{12\sqrt{5}}{\sqrt{25}} = \frac{12\sqrt{5}}{5}$$



$$59. \frac{21}{\sqrt{7}} = \frac{21}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{21\sqrt{7}}{\sqrt{49}} = \frac{21\sqrt{7}}{7} = 3\sqrt{7}$$

$$60. \frac{30}{\sqrt{5}} = \frac{30}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{30\sqrt{5}}{\sqrt{25}} = \frac{30\sqrt{5}}{5} = 6\sqrt{5}$$

$$\begin{aligned} 61. \frac{12}{\sqrt{30}} &= \frac{12\sqrt{30}}{\sqrt{30}\sqrt{30}} \\ &= \frac{12\sqrt{30}}{\sqrt{900}} \\ &= \frac{12\sqrt{30}}{30} \\ &= \frac{2\sqrt{30}}{5} \end{aligned}$$

$$62. \frac{15}{\sqrt{50}} = \frac{15}{\sqrt{50}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{15\sqrt{2}}{\sqrt{100}} = \frac{15\sqrt{2}}{10} = \frac{3\sqrt{2}}{2}$$

$$\begin{aligned} 63. \frac{15}{\sqrt{12}} &= \frac{15}{\sqrt{4 \cdot 3}} \\ &= \frac{15}{\sqrt{4}\sqrt{3}} \\ &= \frac{15}{2\sqrt{3}} \\ &= \frac{15\sqrt{3}}{2\sqrt{3}\sqrt{3}} \\ &= \frac{15\sqrt{3}}{2\sqrt{9}} \\ &= \frac{15\sqrt{3}}{2 \cdot 3} \\ &= \frac{15\sqrt{3}}{6} \\ &= \frac{5\sqrt{3}}{2} \end{aligned}$$

$$64. \frac{13}{\sqrt{40}} = \frac{13\sqrt{10}}{\sqrt{40}\sqrt{10}} = \frac{13\sqrt{10}}{\sqrt{400}} = \frac{13\sqrt{10}}{20}$$

$$65. \frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{\sqrt{25}} = \frac{\sqrt{10}}{5}$$

$$66. \frac{\sqrt{5}}{\sqrt{7}} = \frac{\sqrt{5}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{35}}{\sqrt{49}} = \frac{\sqrt{35}}{7}$$

$$\begin{aligned} 67. \quad & 3\sqrt{8} - \sqrt{32} + 3\sqrt{72} - \sqrt{75} \\ &= 6\sqrt{2} - 4\sqrt{2} + 18\sqrt{2} - 5\sqrt{3} \\ &= 20\sqrt{2} - 5\sqrt{3} \end{aligned}$$

$$\begin{aligned} 68. \quad & 3\sqrt{54} - 2\sqrt{24} - \sqrt{96} + 4\sqrt{63} \\ &= 9\sqrt{6} - 4\sqrt{6} - 4\sqrt{6} + 12\sqrt{7} \\ &= \sqrt{6} + 12\sqrt{7} \end{aligned}$$

$$\begin{aligned} 69. \quad & 3\sqrt{7} - 5\sqrt{14} \cdot \sqrt{2} = 3\sqrt{7} - 5\sqrt{28} \\ &= 3\sqrt{7} - 10\sqrt{7} \\ &= -7\sqrt{7} \end{aligned}$$

$$\begin{aligned} 70. \quad & 4\sqrt{2} - 8\sqrt{10} \cdot \sqrt{5} = 4\sqrt{2} - 8\sqrt{50} \\ &= 4\sqrt{2} - 40\sqrt{2} \\ &= -36\sqrt{2} \end{aligned}$$

$$\begin{aligned} 71. \quad & \frac{\sqrt{32}}{5} + \frac{\sqrt{18}}{7} = \frac{4\sqrt{2}}{5} + \frac{3\sqrt{2}}{7} \\ &= \frac{28\sqrt{2}}{35} + \frac{15\sqrt{2}}{35} \\ &= \frac{43\sqrt{2}}{35} \end{aligned}$$

$$\begin{aligned} 72. \quad & \frac{\sqrt{27}}{2} + \frac{\sqrt{75}}{7} = \frac{3\sqrt{3}}{2} + \frac{5\sqrt{3}}{7} \\ &= \frac{21\sqrt{3}}{14} + \frac{10\sqrt{3}}{14} \\ &= \frac{31\sqrt{3}}{14} \end{aligned}$$

$$\begin{aligned} 73. \quad & \frac{\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}} \\ &= \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{2}{\sqrt{6}} + \frac{3}{\sqrt{6}} \\ &= \frac{5}{\sqrt{6}} \\ &= \frac{5}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{5\sqrt{6}}{6} \end{aligned}$$

$$\begin{aligned}
 74. \quad & \frac{\sqrt{2}}{\sqrt{7}} + \frac{\sqrt{7}}{\sqrt{2}} \\
 &= \frac{\sqrt{2}}{\sqrt{7}} \cdot \frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{7}}{\sqrt{7}} \\
 &= \frac{2}{\sqrt{14}} + \frac{7}{\sqrt{14}} \\
 &= \frac{9}{\sqrt{14}} \\
 &= \frac{9}{\sqrt{14}} \cdot \frac{\sqrt{14}}{\sqrt{14}} \\
 &= \frac{9\sqrt{14}}{14}
 \end{aligned}$$

$$\begin{aligned}
 75. \quad & d(x) = \sqrt{\frac{3x}{2}} \\
 & d(72) = \sqrt{\frac{3(72)}{2}} \\
 & \quad = \sqrt{3(36)} \\
 & \quad = \sqrt{3} \cdot \sqrt{36} \\
 & \quad = 6\sqrt{3} \approx 10.4 \text{ miles}
 \end{aligned}$$

A passenger on the pool deck can see roughly 10.4 miles.

$$\begin{aligned}
 76. \quad & r(120) = \sqrt{\frac{3(120)}{2}} = \sqrt{180} \\
 & \quad = \sqrt{36 \cdot 5} = \sqrt{36} \cdot \sqrt{5} = 6\sqrt{5} \text{ miles}
 \end{aligned}$$

The captain can see  $6\sqrt{5} \approx 13.4$  miles.

$$\begin{aligned}
 77. \quad & v = \sqrt{20L}; L = 245 \\
 & \quad v = \sqrt{20 \cdot 245} = \sqrt{4900} = 70
 \end{aligned}$$

The motorist was traveling 70 miles per hour, so he was speeding.

$$\begin{aligned}
 78. \quad & v = \sqrt{20L}; L = 45 \\
 & \quad v = \sqrt{20 \cdot 45} = \sqrt{900} = 30
 \end{aligned}$$

The motorist was traveling 30 miles per hour, so she was not speeding.

$$79. \text{ a. } 41 \text{ in.}$$

$$\begin{aligned}
 \text{b. } \quad & h = 2.9\sqrt{x} + 20.1 \\
 & \quad = 2.9\sqrt{50} + 20.1 \\
 & \quad \approx 40.6 \text{ in.}
 \end{aligned}$$

The estimate from part (a) describes the median height obtained from the formula quite well.

80. a. 41 in.

b. 
$$h = 3.1\sqrt{x} + 19$$
$$= 3.1\sqrt{50} + 19$$
$$\approx 40.9 \text{ in.}$$

The estimate from part (a) describes the median height obtained from the formula quite well.

81. a. At birth we have  $x = 0$ .

$$y = 2.9\sqrt{x} + 36$$
$$= 2.9\sqrt{0} + 36$$
$$= 2.9(0) + 36$$
$$= 36$$

According to the model, the head circumference at birth is 36 cm.

b. At 9 months we have  $x = 9$ .

$$y = 2.9\sqrt{x} + 36$$
$$= 2.9\sqrt{9} + 36$$
$$= 2.9(3) + 36$$
$$= 44.7$$

According to the model, the head circumference at 9 months is 44.7 cm.

c. At 14 months we have  $x = 14$ .

$$y = 2.9\sqrt{x} + 36$$
$$= 2.9\sqrt{14} + 36$$
$$\approx 46.9$$

According to the model, the head circumference at 14 months is roughly 46.9 cm.

d. The model describes healthy children.

82. a. At birth we have  $x = 0$ .

$$y = 4\sqrt{x} + 35$$
$$= 4\sqrt{0} + 35$$
$$= 4(0) + 35$$
$$= 35$$

According to the model, the head circumference at birth is 35 cm.

b. At 9 months we have  $x = 9$ .

$$y = 4\sqrt{x} + 35$$
$$= 4\sqrt{9} + 35$$
$$= 4(3) + 35$$
$$= 47$$

According to the model, the head circumference at 9 months is 47 cm.

c. At 14 months we have  $x = 14$ .

$$y = 4\sqrt{x} + 35$$
$$= 4\sqrt{14} + 35$$
$$\approx 50$$

According to the model, the head circumference at 14 months is roughly 50 cm.

d. The model describes severe autistic children.

83. The message is “Paige Fox is bad at math.”

$$\begin{aligned}
 84. \quad R_f \sqrt{1 - \left(\frac{v}{c}\right)^2} &= R_f \sqrt{1 - \left(\frac{0.8c}{c}\right)^2} \\
 &= R_f \sqrt{1 - (0.8)^2} \\
 &= R_f \sqrt{0.36} \\
 &= 0.6R_f
 \end{aligned}$$

If 100 weeks have passed for your friend on Earth, then you were gone for  $0.6(100) = 60$  weeks.

$$\begin{aligned}
 85. \quad R_f \sqrt{1 - \left(\frac{v}{c}\right)^2} &= R_f \sqrt{1 - \left(\frac{0.9c}{c}\right)^2} \\
 &= R_f \sqrt{1 - (0.9)^2} \\
 &= R_f \sqrt{0.19} \\
 &= 0.44R_f
 \end{aligned}$$

If 100 weeks have passed for your friend on Earth, then you were gone for  $0.44(100) = 44$  weeks.

86. Answers will vary.

$$\begin{aligned}
 87. \quad \sqrt{2} &\approx 1.4 \\
 \sqrt{2} &< 1.5
 \end{aligned}$$

$$\begin{aligned}
 88. \quad -\pi &\approx -3.14 \\
 -\pi &> -3.5
 \end{aligned}$$

$$\begin{aligned}
 89. \quad \frac{-3.14}{2} &= -1.5700 \\
 -\frac{\pi}{2} &\approx -1.5708 \\
 \frac{-3.14}{2} &> -\frac{\pi}{2}
 \end{aligned}$$

90. The square root is multiplied by  $\sqrt{2}$ .

$$\begin{aligned}
 91. \quad -\sqrt{47} &\approx -6.86 \\
 \text{Therefore } -\sqrt{47} &\text{ is between } -7 \text{ and } -6.
 \end{aligned}$$

$$\begin{aligned}
 92. \quad \sqrt{2} + \sqrt{\frac{1}{2}} &= \sqrt{2} + \frac{\sqrt{1}}{\sqrt{2}} = \sqrt{2} + \frac{\sqrt{1}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\
 &= \sqrt{2} + \frac{\sqrt{2}}{2} = \frac{2\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \\
 &= \frac{2\sqrt{2} + \sqrt{2}}{2} \\
 &= \frac{(2+1)\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}
 \end{aligned}$$

93. Answers will vary.

Example:  $(6 + \sqrt{2}) - (1 + \sqrt{2}) = 5$

### Check Points 1.5

1.  $\left\{-9, -1.3, 0, 0.\bar{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}\right\}$ 
  - a. natural numbers:  $\sqrt{9}$  because  $\sqrt{9} = 3$
  - b. whole numbers:  $0, \sqrt{9}$
  - c. integers:  $-9, 0, \sqrt{9}$
  - d. rational numbers:  $-9, -1.3, 0, 0.\bar{3}, \sqrt{9}$
  - e. Irrational numbers:  $\frac{\pi}{2}, \sqrt{10}$
  - f. real numbers: All numbers in this set.
2.
  - a. associative property of multiplication
  - b. commutative property of addition
  - c. distributive property of multiplication over addition
  - d. commutative property of multiplication
  - e. identity property of addition
  - f. inverse property of multiplication
3.
  - a. Yes, the natural numbers are closed with respect to multiplication.
  - b. No, the integers are not closed with respect to division. Example:  $3 \div 5 = 0.6$  which is not an integer.

### Concept and Vocabulary Check 1.5

1. rational; irrational
2. closure
3.  $ab = ba$
4.  $(a + b) + c = a + (b + c)$
5.  $a(b + c) = ab + ac$
6. identity
7. identity; 1
8. multiplicative inverse; multiplicative identity

9. false; Changes to make the statement true will vary. A sample change is: Some rational numbers are not integers.
10. false; Changes to make the statement true will vary. A sample change is: All whole numbers are integers.
11. true
12. false; Changes to make the statement true will vary. A sample change is: Some irrational numbers are negative.
13. false; Changes to make the statement true will vary. A sample change is: Subtraction is not commutative.
14. false; Changes to make the statement true will vary. A sample change is:  $(24 \div 6) \div 2 \neq 24 \div (6 \div 2)$
15. true
16. false; Changes to make the statement true will vary. A sample change is:  $2 \cdot a + 5 \neq 5 \cdot a + 2$

### Exercise Set 1.5

1.  $\left\{-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}\right\}$ 
  - a. Natural numbers:  $\sqrt{100}$  because  $\sqrt{100} = 10$
  - b. Whole numbers:  $0, \sqrt{100}$
  - c. Integers:  $-9, 0, \sqrt{100}$
  - d. Rational numbers:  $-9, -\frac{4}{5}, 0, 0.25, 9.2, \sqrt{100}$
  - e. Irrational numbers:  $\sqrt{3}$
  - f. Real numbers: All numbers in this set.
2.  $\{-7, -0.\bar{6}, 0, \sqrt{49}, \sqrt{50}\}$ 
  - a. Natural numbers:  $\sqrt{49}$  because  $\sqrt{49} = 7$
  - b. Whole numbers:  $0, \sqrt{49}$
  - c. Integers:  $-7, 0, \sqrt{49}$
  - d. Rational numbers:  $-7, -0.\bar{6}, 0, \sqrt{49}$
  - e. Irrational numbers:  $\sqrt{50}$
  - f. Real numbers: All numbers in this set.
3.  $\left\{-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}\right\}$ 
  - a. Natural numbers:  $\sqrt{64}$  because  $\sqrt{64} = 8$

- b. Whole numbers: 0 and  $\sqrt{64}$
  - c. Integers:  $-11, 0, \sqrt{64}$
  - d. Rational numbers:  $-11, -\frac{5}{6}, 0, 0.75, \sqrt{64}$
  - e. Irrational numbers:  $\sqrt{5}, \pi$
  - f. Real numbers: All numbers in this set.
4.  $\{-5, -0.\bar{3}, 0, \sqrt{2}, \sqrt{4}\}$
- a. Natural numbers:  $\sqrt{4}$  because  $\sqrt{4} = 2$
  - b. Whole numbers: 0 and  $\sqrt{4}$
  - c. Integers:  $-5, 0, \sqrt{4}$
  - d. Rational numbers:  $-5, -0.\bar{3}, 0, \sqrt{4}$
  - e. Irrational numbers:  $\sqrt{2}$
  - f. Real numbers: All numbers in this set.
5. 0 is the only whole number that is not a natural number.
6. Answers will vary. Example answer:  $-1$
7. Answers will vary. Example answer:  $0.5$
8. Answers will vary. Example answer:  $-\frac{2}{5}$
9. Answers will vary. Example answer:  $7$
10. Answers will vary. Example answer:  $5$
11. Answers will vary. Example answer:  $\sqrt{3}$
12. Answers will vary. Example answer:  $-200$
13.  $3 + (4 + 5) = 3 + (5 + 4)$
14.  $\sqrt{5} \cdot 4 = 4 \cdot \sqrt{5}$
15.  $9 \cdot (6 + 2) = 9 \cdot (2 + 6)$
16.  $(3 + 7) + 9 = 3 + (7 + 9)$
17.  $(4 \cdot 5) \cdot 3 = 4 \cdot (5 \cdot 3)$
18.  $3 \cdot (6 + 4) = 3 \cdot 6 + 3 \cdot 4$



19.  $7 \cdot (4 + 5) = 7 \cdot 4 + 7 \cdot 5$
20.  $2 \cdot (7 + 3) = 2 \cdot 7 + 2 \cdot 3$
21.  $5(6 + \sqrt{2}) = 5 \cdot 6 + 5 \cdot \sqrt{2} = 30 + 5\sqrt{2}$
22.  $4(3 + \sqrt{5}) = 4 \cdot 3 + 4 \cdot \sqrt{5} = 12 + 4\sqrt{5}$
23.  $\sqrt{7}(3 + \sqrt{2}) = \sqrt{7} \cdot 3 + \sqrt{7} \cdot \sqrt{2} = 3\sqrt{7} + \sqrt{14}$
24.  $\sqrt{6}(7 + \sqrt{5}) = \sqrt{6} \cdot 7 + \sqrt{6} \cdot \sqrt{5} = 7\sqrt{6} + \sqrt{30}$
25.  $\sqrt{3}(5 + \sqrt{3}) = \sqrt{3} \cdot 5 + \sqrt{3} \cdot \sqrt{3} = 5\sqrt{3} + \sqrt{9}$   
 $= 5\sqrt{3} + 3$
26.  $\sqrt{7}(9 + \sqrt{7}) = \sqrt{7} \cdot 9 + \sqrt{7} \cdot \sqrt{7}$   
 $= 9\sqrt{7} + \sqrt{49}$   
 $= 9\sqrt{7} + 7$
27.  $\sqrt{6}(\sqrt{2} + \sqrt{6}) = \sqrt{6} \cdot \sqrt{2} + \sqrt{6} \cdot \sqrt{6}$   
 $= \sqrt{12} + \sqrt{36}$   
 $= 2\sqrt{3} + 6$
28.  $\sqrt{10}(\sqrt{2} + \sqrt{10}) = \sqrt{10} \cdot \sqrt{2} + \sqrt{10} \cdot \sqrt{10}$   
 $= \sqrt{20} + \sqrt{100}$   
 $= 2\sqrt{5} + 10$
29. commutative property of addition
30. distributive property of multiplication over addition
31. associative property of addition
32. commutative property of multiplication
33. commutative property of addition
34. commutative property of multiplication
35. distributive property of multiplication over addition
36. distributive property of multiplication over addition
37. associative property of multiplication
38. Commutative property of multiplication.
39. Identity property of multiplication.
40. Identity property of addition.

- 41. Inverse property of addition.
- 42. Inverse property of multiplication.
- 43. Inverse property of multiplication.
- 44. Inverse property of addition.
- 45. Answers will vary.  
Example:  $1 - 2 = -1$
- 46. Answers will vary.  
Example:  $4 \div 8 = \frac{1}{2}$
- 47. Answers will vary.  
Example:  $-2 \div 8 = -\frac{1}{4}$
- 48. Answers will vary.  
Example:  $\sqrt{3} - \sqrt{3} = 0$
- 49. Answers will vary.  
Example:  $\sqrt{5}\sqrt{5} = \sqrt{25} = 5$
- 50. false
- 51. true
- 52. true
- 53. false
- 54. vampire
- 55. vampire
- 56. not a vampire
- 57. vampire
- 58. narcissistic;  $3^3 + 7^3 + 0^3 = 370$
- 59. narcissistic;  $3^3 + 7^3 + 1^3 = 371$
- 60. not narcissistic;  $3^3 + 7^3 + 2^3 = 374$ , not 372
- 61. narcissistic;  $9^4 + 4^4 + 7^4 + 4^4 = 9474$

62. a. distributive property

$$\begin{aligned}
 \text{b. } \frac{D(A+1)}{24} &= \frac{200(12+1)}{24} \\
 &= \frac{200(13)}{24} \\
 &= \frac{2600}{24} \\
 &\approx 108 \text{ mg} \\
 \frac{DA + D}{24} &= \frac{200 \cdot 12 + 200}{24} \\
 &= \frac{2400 + 200}{24} \\
 &= \frac{2600}{24} \\
 &\approx 108 \text{ mg}
 \end{aligned}$$

63.  $7 + 2(x+9)$

$$\begin{aligned}
 &= 7 + (2x+18) \quad \text{distributive property} \\
 &= 7 + (18+2x) \quad \text{commutative property of addition} \\
 &= (7+18) + 2x \quad \text{associative property} \\
 &= 25 + 2x \\
 &= 2x + 25 \quad \text{commutative property of addition}
 \end{aligned}$$

64.  $5(x+4) + 3x$

$$\begin{aligned}
 &= (5x+20) + 3x \quad \text{distributive property} \\
 &= (20+5x) + 3x \quad \text{commutative property of addition} \\
 &= 20 + (5x+3x) \quad \text{associative property} \\
 &= 20 + (5+3)x \quad \text{distributive property} \\
 &= 20 + 8x \\
 &= 8x + 20 \quad \text{commutative property of addition}
 \end{aligned}$$

65. Answers will vary. An example is: In a group of at least 367 people, at least two people will have the same birthday, but a subset of the group containing 50 people would not necessarily have two people with the same birthday.

### Check Points 1.6

1. a.  $19^0 = 1$

b.  $(3\pi)^0 = 1$

c.  $(-14)^0 = 1$

d.  $-14^0 = -1$

2. a.  $9^{-2} = \frac{1}{9^2} = \frac{1}{81}$

b.  $6^{-3} = \frac{1}{6^3} = \frac{1}{216}$

c.  $12^{-1} = \frac{1}{12}$

3. a.  $7.4 \times 10^9 = 7,400,000,000$

b.  $3.017 \times 10^{-6} = 0.000003017$

4. a.  $7,410,000,000 = 7.41 \times 10^9$

b.  $0.000000092 = 9.2 \times 10^{-8}$

5.  $410 \times 10^7 = (4.1 \times 10^2) \times 10^7$   
 $= 4.1 \times (10^2 \times 10^7)$   
 $= 4.1 \times 10^{2+7}$   
 $= 4.1 \times 10^9$

6.  $(1.3 \times 10^7) \times (4 \times 10^{-2}) = (1.3 \times 4) \times (10^7 \times 10^{-2})$   
 $= 5.2 \times 10^{7+(-2)}$   
 $= 5.2 \times 10^5$   
 $= 520,000$

7.  $\frac{6.9 \times 10^{-8}}{3 \times 10^{-2}} = \left( \frac{6.9}{3} \right) \times \left( \frac{10^{-8}}{10^{-2}} \right)$   
 $= 2.3 \times 10^{-8-(-2)}$   
 $= 2.3 \times 10^{-6}$   
 $= 0.0000023$

8. a.  $0.0036 \times 5,200,000$   
 $= 3.6 \times 10^{-3} \times 5.2 \times 10^6$   
 $= (3.6 \times 5.2) \times (10^{-3} \times 10^6)$   
 $= 18.72 \times 10^3$   
 $= 1.872 \times 10 \times 10^3$   
 $= 1.872 \times 10^4$

b. Based on part (a):  
 $0.0036 \times 5,200,000$   
 $= 1.872 \times 10^4$   
 $= 18,720$

9.  $\frac{2.6 \times 10^{12}}{3.12 \times 10^8} = \left( \frac{2.6}{3.12} \right) \times \left( \frac{10^{12}}{10^8} \right) \approx 0.83 \times 10^4 = \$8300$

**Concept and Vocabulary Check 1.6**

1. add
2. multiply
3. subtract
4. one
5. a number greater than or equal to 1 and less than 10;  
10 to an integer power
6. false; Changes to make the statement true will vary.
7. false; Changes to make the statement true will vary.
8. false; Changes to make the statement true will vary.
9. true
10. false; Changes to make the statement true will vary.

**Exercise Set 1.6**

1.  $2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$
2.  $3^3 \cdot 3^2 = 3^{3+2} = 3^5 = 243$
3.  $4 \cdot 4^2 = 4^1 \cdot 4^2 = 4^{1+2} = 4^3 = 64$
4.  $5 \cdot 5^2 = 5^1 \cdot 5^2 = 5^{1+2} = 5^3 = 125$
5.  $(2^2)^3 = 2^{2 \cdot 3} = 2^6 = 64$
6.  $(3^3)^2 = 3^{3 \cdot 2} = 3^6 = 729$
7.  $(1^4)^5 = 1^{4 \cdot 5} = 1^{20} = 1$
8.  $(1^3)^7 = 1^{3 \cdot 7} = 1^{21} = 1$
9.  $\frac{4^7}{4^5} = 4^{7-5} = 4^2 = 16$
10.  $\frac{6^7}{6^5} = 6^{7-5} = 6^2 = 36$
11.  $\frac{2^8}{2^4} = 2^{8-4} = 2^4 = 16$

12.  $\frac{3^8}{3^4} = 3^{8-4} = 3^4 = 81$

13.  $3^0 = 1$

14.  $9^0 = 1$

15.  $(-3)^0 = 1$

16.  $(-9)^0 = 1$

17.  $-3^0 = -1$

18.  $-9^0 = -1$

19.  $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

20.  $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

21.  $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

22.  $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

23.  $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

24.  $2^{-6} = \frac{1}{2^6} = \frac{1}{64}$

25.  $3^4 \cdot 3^{-2} = 3^{4+(-2)} = 3^2 = 9$

26.  $2^5 \cdot 2^{-2} = 2^{5+(-2)} = 2^3 = 8$

27.  $3^{-3} \cdot 3 = 3^{-3} \cdot 3^1 = 3^{-3+1} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

28.  $2^{-3} \cdot 2 = 2^{-3} \cdot 2^1 = 2^{-3+1} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

29.  $\frac{2^3}{2^7} = 2^{3-7} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

30.  $\frac{3^4}{3^7} = 3^{4-7} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

$$31. (x^5 x^3)^{-2} = (x^8)^{-2} = \frac{1}{(x^8)^2} = \frac{1}{x^{16}}$$

$$32. (x^2 x^4)^{-3} = (x^6)^{-3} = \frac{1}{(x^6)^3} = \frac{1}{x^{18}}$$

$$33. \frac{(x^3)^4}{(x^2)^7} = \frac{x^{12}}{x^{14}} = \frac{1}{x^2}$$

$$34. \frac{(x^2)^5}{(x^3)^4} = \frac{x^{10}}{x^{12}} = \frac{1}{x^2}$$

$$35. \left(\frac{x^5}{x^2}\right)^{-4} = (x^3)^{-4} = x^{-12} = \frac{1}{x^{12}}$$

$$36. \left(\frac{x^7}{x^2}\right)^{-3} = (x^5)^{-3} = x^{-15} = \frac{1}{x^{15}}$$

$$37. \frac{2x^5 \cdot 3x}{15x^6} = \frac{6x^6}{15x^6} = \frac{6}{15} = \frac{2}{5}$$

$$38. \frac{4x^7 \cdot 5x}{10x^8} = \frac{20x^8}{10x^8} = \frac{20}{10} = 2$$

$$39. (-2x^3 y^{-4})(3x^{-1} y) = -6x^2 y^{-3} = -\frac{6x^2}{y^3}$$

$$40. (-5x^4 y^{-3})(4x^{-1} y) = -20x^3 y^{-2} = -\frac{20x^3}{y^2}$$

$$41. \frac{30x^2 y^5}{-6x^8 y^{-3}} = -\frac{5y^8}{x^6}$$

$$42. \frac{24x^2 y^{13}}{-8x^5 y^{-2}} = -\frac{3y^{15}}{x^3}$$

$$43. 2.7 \times 10^2 = 270$$

$$44. 4.7 \times 10^3 = 4700$$

$$45. 9.12 \times 10^5 = 912,000$$

$$46. 8.14 \times 10^4 = 81,400$$

**47.**  $8 \times 10^7 = 8.0 \times 10^7 = 80,000,000$

**48.**  $7 \times 10^6 = 7.0 \times 10^6 = 7,000,000$

**49.**  $1 \times 10^5 = 1.0 \times 10^5 = 100,000$

**50.**  $1 \times 10^8 = 1.0 \times 10^8 = 100,000,000$

**51.**  $7.9 \times 10^{-1} = 0.79$

**52.**  $8.6 \times 10^{-1} = 0.86$

**53.**  $2.15 \times 10^{-2} = 0.0215$

**54.**  $3.14 \times 10^{-2} = 0.0314$

**55.**  $7.86 \times 10^{-4} = 0.000786$

**56.**  $4.63 \times 10^{-5} = 0.0000463$

**57.**  $3.18 \times 10^{-6} = 0.00000318$

**58.**  $5.84 \times 10^{-7} = 0.000000584$

**59.**  $370 = 3.7 \times 10^2$

**60.**  $530 = 5.3 \times 10^2$

**61.**  $3600 = 3.6 \times 10^3$

**62.**  $2700 = 2.7 \times 10^3$

**63.**  $32,000 = 3.2 \times 10^4$

**64.**  $64,000 = 6.4 \times 10^4$

**65.**  $220,000,000 = 2.2 \times 10^8$

**66.**  $370,000,000,000 = 3.7 \times 10^{11}$

**67.**  $0.027 = 2.7 \times 10^{-2}$

**68.**  $0.014 = 1.4 \times 10^{-2}$

**69.**  $0.0037 = 3.7 \times 10^{-3}$

**70.**  $0.00083 = 8.3 \times 10^{-4}$

**71.**  $0.00000293 = 2.93 \times 10^{-6}$

**72.**  $0.000000647 = 6.47 \times 10^{-7}$



$$73. \quad 820 \times 10^5 = (8.2 \times 10^2) \times 10^5 = 8.2 \times 10^7$$

$$74. \quad 630 \times 10^8 = (6.3 \times 10^2) \times 10^8 = 6.3 \times 10^{10}$$

$$75. \quad 0.41 \times 10^6 = (4.1 \times 10^{-1}) \times 10^6 = 4.1 \times 10^5$$

$$76. \quad 0.57 \times 10^9 = (5.7 \times 10^{-1}) \times 10^9 = 5.7 \times 10^8$$

$$77. \quad 2100 \times 10^{-9} = (2.1 \times 10^3) \times 10^{-9} = 2.1 \times 10^{-6}$$

$$78. \quad 97,000 \times 10^{-11} = (9.7 \times 10^4) \times 10^{-11} = 9.7 \times 10^{-7}$$

$$\begin{aligned} 79. \quad (2 \times 10^3)(3 \times 10^2) &= (2 \times 3) \times (10^{3+2}) \\ &= 6 \times 10^5 \\ &= 600,000 \end{aligned}$$

$$\begin{aligned} 80. \quad (5 \times 10^2)(4 \times 10^4) &= (5 \times 4) \times (10^{2+4}) \\ &= 20 \times 10^6 \\ &= 2 \times 10 \times 10^6 \\ &= 2 \times 10^7 \\ &= 20,000,000 \end{aligned}$$

$$\begin{aligned} 81. \quad (2 \times 10^9)(3 \times 10^{-5}) &= (2 \times 3) \times (10^{9-5}) \\ &= 6 \times 10^4 \\ &= 60,000 \end{aligned}$$

$$\begin{aligned} 82. \quad (4 \times 10^8)(2 \times 10^{-4}) &= (4 \times 2) \times (10^{8-4}) \\ &= 8 \times 10^4 \\ &= 80,000 \end{aligned}$$

$$\begin{aligned} 83. \quad (4.1 \times 10^2)(3 \times 10^{-4}) &= (4.1 \times 3) \times (10^{2-4}) \\ &= 12.3 \times 10^{-2} \\ &= 1.23 \times 10 \times 10^{-2} \\ &= 1.23 \times 10^{-1} \\ &= 0.123 \end{aligned}$$

$$\begin{aligned} 84. \quad (1.2 \times 10^3)(2 \times 10^{-5}) &= (1.2 \times 2) \times (10^{3-5}) \\ &= 2.4 \times 10^{-2} \\ &= 0.024 \end{aligned}$$

$$\begin{aligned} 85. \quad \frac{12 \times 10^6}{4 \times 10^2} &= \left( \frac{12}{4} \right) \times \left( \frac{10^6}{10^2} \right) \\ &= 3 \times 10^{6-2} \\ &= 3 \times 10^4 \\ &= 30,000 \end{aligned}$$

$$\begin{aligned} 86. \quad \frac{20 \times 10^{20}}{10 \times 10^{15}} &= \left( \frac{20}{10} \right) \times \left( \frac{10^{20}}{10^{15}} \right) \\ &= 2 \times 10^{20-15} \\ &= 2 \times 10^5 \\ &= 200,000 \end{aligned}$$

$$\begin{aligned} 87. \quad \frac{15 \times 10^4}{5 \times 10^{-2}} &= \left( \frac{15}{5} \right) \times \left( \frac{10^4}{10^{-2}} \right) \\ &= 3 \times 10^{4-(-2)} \\ &= 3 \times 10^6 \\ &= 3,000,000 \end{aligned}$$

$$\begin{aligned} 88. \quad \frac{18 \times 10^2}{9 \times 10^{-3}} &= \left( \frac{18}{9} \right) \times \left( \frac{10^2}{10^{-3}} \right) \\ &= 2 \times 10^{2-(-3)} \\ &= 2 \times 10^5 \\ &= 200,000 \end{aligned}$$

$$\begin{aligned} 89. \quad \frac{6 \times 10^3}{2 \times 10^5} &= \left( \frac{6}{2} \right) \times \left( \frac{10^3}{10^5} \right) \\ &= 3 \times 10^{3-5} \\ &= 3 \times 10^{-2} \\ &= 0.03 \end{aligned}$$

$$\begin{aligned} 90. \quad \frac{8 \times 10^4}{2 \times 10^7} &= \left( \frac{8}{2} \right) \times \left( \frac{10^4}{10^7} \right) \\ &= 4 \times 10^{4-7} \\ &= 4 \times 10^{-3} \\ &= 0.004 \end{aligned}$$

$$\begin{aligned} 91. \quad \frac{6.3 \times 10^{-6}}{3 \times 10^{-3}} &= \left( \frac{6.3}{3} \right) \times \left( \frac{10^{-6}}{10^{-3}} \right) \\ &= 2.1 \times 10^{-6-(-3)} \\ &= 2.1 \times 10^{-3} \\ &= 0.0021 \end{aligned}$$

$$\begin{aligned}
 92. \quad \frac{9.6 \times 10^{-7}}{3 \times 10^{-3}} &= \left( \frac{9.6}{3} \right) \times \left( \frac{10^{-7}}{10^{-3}} \right) \\
 &= 3.2 \times 10^{-7-(-3)} \\
 &= 3.2 \times 10^{-4} \\
 &= 0.00032
 \end{aligned}$$

$$\begin{aligned}
 93. \quad (82,000,000)(3,000,000,000) \\
 &= (8.2 \times 10^7)(3.0 \times 10^9) \\
 &= (8.2 \times 3.0) \times (10^{7+9}) \\
 &= 24.6 \times 10^{16} \\
 &= 2.46 \times 10 \times 10^{16} \\
 &= 2.46 \times 10^{17}
 \end{aligned}$$

$$\begin{aligned}
 94. \quad (94,000,000)(6,000,000,000) \\
 &= (9.4 \times 10^7)(6.0 \times 10^9) \\
 &= (9.4 \times 6.0) \times (10^{7+9}) \\
 &= 56.4 \times 10^{16} \\
 &= 5.64 \times 10 \times 10^{16} \\
 &= 5.64 \times 10^{17}
 \end{aligned}$$

$$\begin{aligned}
 95. \quad (0.0005)(6,000,000) \\
 &= (5.0 \times 10^{-4})(6.0 \times 10^6) \\
 &= (5.0 \times 6.0)(10^{-4+6}) \\
 &= 30 \times 10^2 \\
 &= 3 \times 10 \times 10^2 \\
 &= 3 \times 10^3
 \end{aligned}$$

$$\begin{aligned}
 96. \quad (0.000015)(0.004) &= (1.5 \times 10^{-5})(4.0 \times 10^{-3}) \\
 &= (1.5 \times 4.0) \times (10^{-5-3}) \\
 &= 6 \times 10^{-8}
 \end{aligned}$$

$$\begin{aligned}
 97. \quad \frac{9,500,000}{500} &= \frac{9.5 \times 10^6}{5 \times 10^2} \\
 &= \left( \frac{9.5}{5} \right) \times (10^{6-2}) \\
 &= 1.9 \times 10^4
 \end{aligned}$$

$$\begin{aligned}
 98. \quad \frac{30,000}{0.0005} &= \frac{3 \times 10^4}{5 \times 10^{-4}} \\
 &= \left(\frac{3}{5}\right) \times (10^{4-(-4)}) \\
 &= 0.6 \times 10^8 \\
 &= 6 \times 10^{-1} \times 10^8 \\
 &= 6 \times 10^7
 \end{aligned}$$

$$\begin{aligned}
 99. \quad \frac{0.00008}{200} &= \frac{8 \times 10^{-5}}{2 \times 10^2} \\
 &= \left(\frac{8}{2}\right) \times (10^{-5-2}) \\
 &= 4 \times 10^{-7}
 \end{aligned}$$

$$\begin{aligned}
 100. \quad \frac{0.0018}{0.0000006} &= \frac{1.8 \times 10^{-3}}{6 \times 10^{-7}} \\
 &= \left(\frac{1.8}{6}\right) \times (10^{-3-(-7)}) \\
 &= 0.3 \times 10^4 \\
 &= 3 \times 10^{-1} \times 10^4 \\
 &= 3 \times 10^3
 \end{aligned}$$

$$\begin{aligned}
 101. \quad \frac{480,000,000,000}{0.00012} &= \frac{4.8 \times 10^{11}}{1.2 \times 10^{-4}} \\
 &= \left(\frac{4.8}{1.2}\right) \times (10^{11-(-4)}) \\
 &= 4 \times 10^{15}
 \end{aligned}$$

$$\begin{aligned}
 102. \quad \frac{0.000000096}{16,000} &= \frac{9.6 \times 10^{-8}}{1.6 \times 10^4} \\
 &= \left(\frac{9.6}{1.6}\right) \times (10^{-8-4}) \\
 &= 6 \times 10^{-12}
 \end{aligned}$$

$$\begin{aligned}
 103. \quad \frac{2^4}{2^5} + \frac{3^3}{3^5} &= \frac{1}{2} + \frac{1}{3^2} \\
 &= \frac{1}{2} + \frac{1}{9} \\
 &= \frac{11}{18}
 \end{aligned}$$

$$\begin{aligned}
 104. \quad \frac{3^5}{3^6} + \frac{2^3}{2^6} &= \frac{1}{3} + \frac{1}{2^3} \\
 &= \frac{1}{3} + \frac{1}{8} \\
 &= \frac{11}{24}
 \end{aligned}$$

$$\begin{aligned}
 105. \quad \frac{2^6}{2^4} - \frac{5^4}{5^6} &= \frac{2^2}{1} - \frac{1}{5^2} \\
 &= 4 - \frac{1}{25} \\
 &= \frac{99}{25} \\
 &= 3\frac{24}{25}
 \end{aligned}$$

$$\begin{aligned}
 106. \quad \frac{5^6}{5^4} - \frac{2^4}{2^6} &= \frac{5^2}{1} - \frac{1}{2^2} \\
 &= 25 - \frac{1}{4} \\
 &= \frac{99}{4} \\
 &= 24\frac{3}{4}
 \end{aligned}$$

$$107. \quad \frac{(5 \times 10^3)(1.2 \times 10^{-4})}{(2.4 \times 10^2)} = 2.5 \times 10^{-3}$$

$$108. \quad \frac{(2 \times 10^2)(2.6 \times 10^{-3})}{(4 \times 10^3)} = 1.3 \times 10^{-4}$$

$$109. \quad \frac{(1.6 \times 10^4)(7.2 \times 10^{-3})}{(3.6 \times 10^8)(4 \times 10^{-3})} = 0.8 \times 10^{-4} = 8 \times 10^{-5}$$

$$110. \quad \frac{(1.2 \times 10^6)(8.7 \times 10^{-2})}{(2.9 \times 10^6)(3 \times 10^{-3})} = 1.2 \times 10^1$$

$$111. \quad \text{a. } 2.17 \times 10^{12}$$

$$\text{b. } 3.09 \times 10^8$$

$$\begin{aligned} \text{c. } \frac{2.17 \times 10^{12}}{3.09 \times 10^8} &= \frac{2.17}{3.09} \times \frac{10^{12}}{10^8} \\ &\approx 0.702 \times 10^4 \\ &= 7.02 \times 10^3 \\ &= 7020 \end{aligned}$$

\$7020 per American

112. a.  $2.20 \times 10^{12}$

b.  $3.08 \times 10^8$

$$\begin{aligned} \text{c. } \frac{2.20 \times 10^{12}}{3.08 \times 10^8} &= \frac{2.20}{3.08} \times \frac{10^{12}}{10^8} \\ &\approx 0.714 \times 10^4 \\ &= 7.14 \times 10^3 \\ &= 7140 \end{aligned}$$

\$7140 per American

$$\begin{aligned} 113. \quad 1340 \times 10^6 \cdot 7.90 &= 1.34 \times 10^9 \cdot 7.9 \\ &= 1.34 \cdot 7.9 \times 10^9 \\ &= 10.586 \times 10^9 \\ &= 1.0586 \times 10^{10} \end{aligned}$$

Box-office receipts were  $\$1.0586 \times 10^{10}$  in 2010.

$$\begin{aligned} 114. \quad 1380 \times 10^6 \cdot 6.40 &= 1.38 \times 10^9 \cdot 6.4 \\ &= 1.38 \cdot 6.4 \times 10^9 \\ &= 8.832 \times 10^9 \end{aligned}$$

Box-office receipts were  $\$8.832 \times 10^9$  in 2005.

$$\begin{aligned} 115. \quad 5.3 \times 10^{-23} \cdot 20,000 &= 5.3 \times 10^{-23} \cdot 2 \times 10^4 \\ &= 5.3 \cdot 2 \times 10^{-23} \cdot 10^4 \\ &= 10.6 \times 10^{-19} \\ &= 1.06 \times 10^1 \cdot 10^{-19} \\ &= 1.06 \times 10^{-18} \end{aligned}$$

The mass is  $1.06 \times 10^{-18}$  gram.

$$\begin{aligned} 116. \quad 1.67 \times 10^{-24} \cdot 80,000 &= 1.67 \times 10^{-24} \cdot 8 \times 10^4 \\ &= 1.67 \cdot 8 \times 10^{-24} \cdot 10^4 \\ &= 13.36 \times 10^{-20} \\ &= 1.336 \times 10^1 \cdot 10^{-20} \\ &= 1.336 \times 10^{-19} \end{aligned}$$

The mass is  $1.336 \times 10^{-19}$  gram.

117. false; Changes to make the statement true will vary. A sample change is:  $4^{-2} > 4^{-3}$ .

118. true

119. false; Changes to make the statement true will vary. A sample change is:  $5^2 \cdot 5^{-2} = 2^5 \cdot 2^{-5}$ .

120. false; Changes to make the statement true will vary. A sample change is:  $534.7 \neq 5347$ .

121. false; Changes to make the statement true will vary. A sample change is:  $\frac{8 \times 10^{30}}{4 \times 10^{-5}} = 2 \times 10^{30-(-5)} = 2 \times 10^{35}$ .

122. false; Changes to make the statement true will vary. A sample change is:  $(7 \times 10^5) + (2 \times 10^{-3}) = 700,000.002$ . 123. true

124. Answers will vary. Possible answer:

$$2.0 \times 10^0 = 2.0 \times 1 = 2$$

There is no advantage here since  $10^0 = 1$ .

$$\begin{aligned} 125. \quad 1 - (2^{-1} + 2^{-2}) &= 1 - \left( \frac{1}{2} + \frac{1}{4} \right) \\ &= \frac{4}{4} - \left( \frac{2}{4} + \frac{1}{4} \right) \\ &= \frac{4}{4} - \frac{3}{4} = \frac{1}{4} \end{aligned}$$

### Check Points 1.7

1.  $100, 100 + 20 = 120, 120 + 20 = 140, 140 + 20 = 160, 160 + 20 = 180, 180 + 20 = 200$   
100, 120, 140, 160, 180, and 200

2.  $8, 8 - 3 = 5, 5 - 3 = 2, 2 - 3 = -1, -1 - 3 = -4, -4 - 3 = -7$   
8, 5, 2, -1, -4, and -7

3.  $a_n = a_1 + (n-1)d$   
 $a_9 = 6 + (9-1)(-5)$   
 $= 6 + 8(-5)$   
 $= 6 - 40$   
 $= -34$

4. a.  $a_n = a_1 + (n-1)d$   
 $a_n = 16 + (n-1)(0.35)$   
 $= 16 + 0.35n - 0.35$   
 $= 0.35n + 15.65$

b.  $a_n = 0.35n + 15.65$   
 $= 0.35(21) + 15.65$   
 $= 23$

23% of the U.S. population is projected to be Latino in 2030.

$$5. \quad 12, \quad 12\left(-\frac{1}{2}\right) = -6, \quad -6\left(-\frac{1}{2}\right) = 3, \quad 3\left(-\frac{1}{2}\right) = -\frac{3}{2}, \\ -\frac{3}{2}\left(-\frac{1}{2}\right) = \frac{3}{4}, \quad \frac{3}{4}\left(-\frac{1}{2}\right) = -\frac{3}{8} \\ 12, \quad -6, \quad 3, \quad -\frac{3}{2}, \quad \frac{3}{4}, \quad -\frac{3}{8}$$

$$6. \quad a_n = a_1 r^{n-1} \text{ with } a_1 = 5, r = -3, \text{ and } n = 7 \\ a_7 = 5(-3)^{7-1} = 5(-3)^6 = 5(729) = 3645$$

$$7. \quad a_n = a_1 r^{n-1} \text{ with } a_1 = 3 \text{ and } r = \frac{6}{3} = 2. \text{ Thus } a_n = 3(2)^{n-1} \\ a_8 = 3(2)^{8-1} = 3(2)^7 = 3(128) = 384$$

### Concept and Vocabulary Check 1.7

1. arithmetic; common difference
2.  $a_n = a_1 + (n-1)d$  ;  
the first term; the common difference
3. geometric; common ratio
4.  $a_n = a_1 r^{n-1}$  ;  
the first term; the common ratio
5. false; Changes to make the statement true will vary. A sample change is: The common difference is  $-2$ .
6. false; Changes to make the statement true will vary. A sample change is: The sequence does not have a common difference and is therefore not an arithmetic sequence.
7. false; Changes to make the statement true will vary. A sample change is: The  $n$ th term is  $a_1 + (n-1)d$ .
8. false; Changes to make the statement true will vary. A sample change is: The sequence does not have a common ratio and is therefore not a geometric sequence.
9. false; Changes to make the statement true will vary. A sample change is: Adjacent terms of a geometric sequence have a common ratio.
10. false; Changes to make the statement true will vary. A sample change is: A sequence can be neither arithmetic nor geometric.
11. true

### Exercise Set 1.7

1.  $8, 8 + 2 = 10, 10 + 2 = 12, 12 + 2 = 14, 14 + 2 = 16, 16 + 2 = 18$   
 $8, 10, 12, 14, 16, \text{ and } 18$



2.  $5, 5 + 3 = 8, 8 + 3 = 11, 11 + 3 = 14, 14 + 3 = 17, 17 + 3 = 20$   
5, 8, 11, 14, 17, and 20
3.  $200, 200 + 20 = 220, 220 + 20 = 240, 240 + 20 = 260, 260 + 20 = 280, 280 + 20 = 300$   
200, 220, 240, 260, 280, and 300
4.  $300, 300 + 50 = 350, 350 + 50 = 400, 400 + 50 = 450, 450 + 50 = 500, 500 + 50 = 550$   
300, 350, 400, 450, 500, and 550
5.  $-7, -7 + 4 = -3, -3 + 4 = 1, 1 + 4 = 5, 5 + 4 = 9, 9 + 4 = 13$   
-7, -3, 1, 5, 9, and 13
6.  $-8, -8 + 5 = -3, -3 + 5 = 2, 2 + 5 = 7, 7 + 5 = 12, 12 + 5 = 17$   
-8, -3, 2, 7, 12, and 17
7.  $-400, -400 + 300 = -100, -100 + 300 = 200, 200 + 300 = 500, 500 + 300 = 800, 800 + 300 = 1100$   
-400, -100, 200, 500, 800, and 1100
8.  $-500, -500 + 400 = -100, -100 + 400 = 300, 300 + 400 = 700, 700 + 400 = 1100, 1100 + 400 = 1500$   
-500, -100, 300, 700, 1100, and 1500
9.  $7, 7 - 3 = 4, 4 - 3 = 1, 1 - 3 = -2, -2 - 3 = -5, -5 - 3 = -8$   
7, 4, 1, -2, -5, and -8
10.  $9, 9 - 5 = 4, 4 - 5 = -1, -1 - 5 = -6, -6 - 5 = -11, -11 - 5 = -16$   
9, 4, -1, -6, -11, and -16
11.  $200, 200 - 60 = 140, 140 - 60 = 80, 80 - 60 = 20, 20 - 60 = -40, -40 - 60 = -100$   
200, 140, 80, 20, -40, and -100
12.  $300, 300 - 90 = 210, 210 - 90 = 120, 120 - 90 = 30, 30 - 90 = -60, -60 - 90 = -150$   
300, 210, 120, 30, -60, and -150
13.  $\frac{5}{2}, \frac{5}{2} + \frac{1}{2} = \frac{6}{2} = 3, \frac{6}{2} + \frac{1}{2} = \frac{7}{2}, \frac{7}{2} + \frac{1}{2} = \frac{8}{2} = 4, \frac{8}{2} + \frac{1}{2} = \frac{9}{2}, \frac{9}{2} + \frac{1}{2} = \frac{10}{2} = 5$   
 $\frac{5}{2}, 3, \frac{7}{2}, 4, \frac{9}{2},$  and 5
14.  $\frac{3}{4}, \frac{3}{4} + \frac{1}{4} = 1, \frac{4}{4} + \frac{1}{4} = \frac{5}{4}, \frac{5}{4} + \frac{1}{4} = \frac{6}{4} = \frac{3}{2}, \frac{6}{4} + \frac{1}{4} = \frac{7}{4}, \frac{7}{4} + \frac{1}{4} = \frac{8}{4} = 2$   
 $\frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4},$  and 2
15.  $\frac{3}{2}, \frac{6}{4} + \frac{1}{4} = \frac{7}{4}, \frac{7}{4} + \frac{1}{4} = \frac{8}{4} = 2, \frac{8}{4} + \frac{1}{4} = \frac{9}{4}, \frac{9}{4} + \frac{1}{4} = \frac{10}{4} = \frac{5}{2}, \frac{10}{4} + \frac{1}{4} = \frac{11}{4}$   
 $\frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2},$  and  $\frac{11}{4}$
16.  $\frac{3}{2}, \frac{6}{4} - \frac{1}{4} = \frac{5}{4}, \frac{5}{4} - \frac{1}{4} = \frac{4}{4} = 1, \frac{4}{4} - \frac{1}{4} = \frac{3}{4}, \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}, \frac{2}{4} - \frac{1}{4} = \frac{1}{4}$   
 $\frac{3}{2}, \frac{5}{4}, 1, \frac{3}{4}, \frac{1}{2},$  and  $\frac{1}{4}$
17.  $4.25, 4.25 + 0.3 = 4.55, 4.55 + 0.3 = 4.85, 4.85 + 0.3 = 5.15, 5.15 + 0.3 = 5.45, 5.45 + 0.3 = 5.75$   
4.25, 4.55, 4.85, 5.15, 5.45, and 5.75
18.  $6.3, 6.3 + 0.25 = 6.55, 6.55 + 0.25 = 6.8, 6.8 + 0.25 = 7.05, 7.05 + 0.25 = 7.3, 7.3 + 0.25 = 7.55$   
6.3, 6.55, 6.8, 7.05, 7.3, and 7.55

- 19.**  $4.5, 4.5 - 0.75 = 3.75, 3.75 - 0.75 = 3, 3 - 0.75 = 2.25, 2.25 - 0.75 = 1.5, 1.5 - 0.75 = 0.75$   
 $4.5, 3.75, 3, 2.25, 1.5,$  and  $0.75$
- 20.**  $3.5, 3.5 - 1.75 = 1.75, 1.75 - 1.75 = 0, 0 - 1.75 = -1.75, -1.75 - 1.75 = -3.5, -3.5 - 1.75 = -5.25$   
 $3.5, 1.75, 0, -1.75, -3.5,$  and  $-5.25$
- 21.**  $a_1 = 13, d = 4$   
 $a_6 = 13 + (6-1)(4)$   
 $= 13 + 5(4)$   
 $= 13 + 20$   
 $= 33$
- 22.**  $a_1 = 9, d = 2$   
 $a_{16} = 9 + (16-1)(2)$   
 $= 9 + 15(2)$   
 $= 9 + 30$   
 $= 39$
- 23.**  $a_1 = 7, d = 5$   
 $a_{50} = 7 + (50-1)(5)$   
 $= 7 + 49(5)$   
 $= 7 + 245$   
 $= 252$
- 24.**  $a_1 = 8, d = 6$   
 $a_{60} = 8 + (60-1)(6)$   
 $= 8 + 59(6)$   
 $= 8 + 354$   
 $= 362$
- 25.**  $a_1 = -5, d = 9$   
 $a_9 = -5 + (9-1)(9)$   
 $= -5 + 8(9)$   
 $= -5 + 72$   
 $= 67$
- 26.**  $a_1 = -8, d = 10$   
 $a_{10} = -8 + (10-1)(10)$   
 $= -8 + 9(10)$   
 $= -8 + 90$   
 $= 82$
- 27.**  $a_1 = -40, d = 5$   
 $a_{200} = -40 + (200-1)(5)$   
 $= -40 + 199(5)$   
 $= -40 + 995$   
 $= 955$

28.  $a_1 = -60, d = 5$

$$\begin{aligned}a_{150} &= -60 + (150 - 1)(5) \\&= -60 + 149(5) \\&= -60 + 745 \\&= 685\end{aligned}$$

29.  $a_1 = 8, d = -10$

$$\begin{aligned}a_{10} &= 8 + (10 - 1)(-10) \\&= 8 + 9(-10) \\&= 8 - 90 \\&= -82\end{aligned}$$

30.  $a_1 = 10, d = -6$

$$\begin{aligned}a_{11} &= 10 + (11 - 1)(-6) \\&= 10 + 10(-6) \\&= 10 + (-60) \\&= -50\end{aligned}$$

31.  $a_1 = 35, d = -3$

$$\begin{aligned}a_{60} &= 35 + (60 - 1)(-3) \\&= 35 + 59(-3) \\&= 35 + (-177) \\&= -142\end{aligned}$$

32.  $a_1 = -32, d = 4$

$$\begin{aligned}a_{70} &= -32 + (70 - 1)(4) \\&= -32 + 69(4) \\&= -32 + 276 \\&= 244\end{aligned}$$

33.  $a_1 = 12, d = -5$

$$\begin{aligned}a_{12} &= 12 + (12 - 1)(-5) \\&= 12 + 11(-5) \\&= 12 + (-55) \\&= -43\end{aligned}$$

34.  $a_1 = -20, d = -4$

$$\begin{aligned}a_{20} &= -20 + (20 - 1)(-4) \\&= -20 + 19(-4) \\&= -20 + (-76) \\&= -96\end{aligned}$$

35.  $a_1 = -70, d = -2$

$$\begin{aligned}a_{90} &= -70 + (90 - 1)(-2) \\&= -70 + 89(-2) \\&= -70 + (-178) \\&= -248\end{aligned}$$

36.  $a_1 = 106, d = -12$

$$\begin{aligned}a_{80} &= 106 + (80 - 1)(-12) \\&= 106 + 79(-12) \\&= 106 + (-948) \\&= -842\end{aligned}$$

37.  $a_1 = 6, d = \frac{1}{2}$

$$\begin{aligned}a_{12} &= 6 + (12 - 1)\left(\frac{1}{2}\right) \\&= 6 + 11\left(\frac{1}{2}\right) \\&= \frac{12}{2} + \frac{11}{2} \\&= \frac{23}{2}\end{aligned}$$

38.  $a_1 = 8, d = \frac{1}{4}$

$$\begin{aligned}a_{14} &= 8 + (14 - 1)\left(\frac{1}{4}\right) \\&= 8 + 13\left(\frac{1}{4}\right) \\&= \frac{32}{4} + \frac{13}{4} \\&= \frac{45}{4}\end{aligned}$$

39.  $a_1 = 14, d = -0.25$

$$\begin{aligned}a_{50} &= 14 + (50 - 1)(-0.25) \\&= 14 + 49(-0.25) \\&= 14 + (-12.25) \\&= 1.75\end{aligned}$$

40.  $a_1 = -12, d = -0.5$

$$\begin{aligned}a_{110} &= -12 + (110 - 1)(-0.5) \\&= -12 + 109(-0.5) \\&= -12 + (-54.5) \\&= -66.5\end{aligned}$$

41.  $a_n = a_1 + (n-1)d$  with  $a_1 = 1$ ,  $d = 4$

$$a_n = 1 + (n-1)4$$

$$= 1 + 4n - 4$$

$$= 4n - 3$$

$$\text{Thus } a_{20} = 4(20) - 3 = 77.$$

42.  $a_n = a_1 + (n-1)d$  with  $a_1 = 2$ ,  $d = 5$

$$a_n = 2 + (n-1)5$$

$$= 2 + 5n - 5$$

$$= 5n - 3$$

$$\text{Thus } a_{20} = 5(20) - 3 = 97.$$

43.  $a_n = a_1 + (n-1)d$  with  $a_1 = 7$ ,  $d = -4$

$$a_n = 7 + (n-1)(-4)$$

$$= 7 - 4n + 4$$

$$= -4n + 11$$

$$\text{Thus } a_{20} = -4(20) + 11 = -69.$$

44.  $a_n = a_1 + (n-1)d$  with  $a_1 = 6$ ,  $d = -5$

$$a_n = 6 + (n-1)(-5)$$

$$= 6 - 5n + 5$$

$$= -5n + 11$$

$$\text{Thus } a_{20} = -5(20) + 11 = -89.$$

45.  $a_n = a_1 + (n-1)d$  with  $a_1 = 9$ ,  $d = 2$

$$a_n = 9 + (n-1)2$$

$$= 9 + 2n - 2$$

$$= 2n + 7$$

$$\text{Thus } a_{20} = 2(20) + 7 = 47.$$

46.  $a_n = a_1 + (n-1)d$  with  $a_1 = 6$ ,  $d = 3$

$$a_n = 6 + (n-1)3$$

$$= 6 + 3n - 3$$

$$= 3n + 3$$

$$\text{Thus } a_{20} = 3(20) + 3 = 63.$$

47.  $a_n = a_1 + (n-1)d$  with  $a_1 = -20$ ,  $d = -4$

$$a_n = -20 + (n-1)(-4)$$

$$= -20 - 4n + 4$$

$$= -4n - 16$$

$$\text{Thus } a_{20} = -4(20) - 16 = -96.$$

48.  $a_n = a_1 + (n-1)d$  with  $a_1 = -70$ ,  $d = -5$

$$a_n = -70 + (n-1)(-5)$$

$$= -70 - 5n + 5$$

$$= -5n - 65$$

$$\text{Thus } a_{20} = -5(20) - 65 = -165.$$

49.  $a_1 = 4, r = 2$   
 $4, 4 \cdot 2 = 8, 8 \cdot 2 = 16, 16 \cdot 2 = 32, 32 \cdot 2 = 64,$   
 $64 \cdot 2 = 128$   
 $4, 8, 16, 32, 64, 128$
50.  $a_1 = 2, r = 3$   
 $2, 2 \cdot 3 = 6, 6 \cdot 3 = 18, 18 \cdot 3 = 54, 54 \cdot 3 = 162,$   
 $162 \cdot 3 = 486$   
 $2, 6, 18, 54, 162, 486$
51.  $a_1 = 1000, r = 1$   
 $1000, 1000 \cdot 1 = 1000, 1000 \cdot 1 = 1000, \dots$   
 $1000, 1000, 1000, 1000, 1000, 1000$
52.  $a_1 = 5000, r = 1$   
 $5000, 5000 \cdot 1 = 5000, 5000 \cdot 1 = 5000, \dots$   
 $5000, 5000, 5000, 5000, 5000, 5000$
53.  $a_1 = 3, r = -2$   
 $3, 3(-2) = -6, -6(-2) = 12, 12(-2) = -24, -24(-2) = 48, 48(-2) = -96$   
 $3, -6, 12, -24, 48, -96$
54.  $a_1 = 2, r = -3$   
 $2, 2(-3) = -6, -6(-3) = 18, 18(-3) = -54,$   
 $-54(-3) = 162, 162(-3) = -486$   
 $2, -6, 18, -54, 162, -486$
55.  $a_1 = 10, r = -4$   
 $10, 10(-4) = -40, -40(-4) = 160,$   
 $160(-4) = -640, -640(-4) = 2560,$   
 $2560(-4) = -10,240$   
 $10, -40, 160, -640, 2560, \text{and } -10,240$
56.  $a_1 = 20, r = -4$   
 $20, 20(-4) = -80, -80(-4) = 320,$   
 $320(-4) = -1280, -1280(-4) = 5120,$   
 $5120(-4) = -20,480$   
 $20, -80, 320, -1280, 5120, \text{and } -20,480$
57.  $a_1 = 2000, r = -1$   
 $2000, 2000(-1) = -2000,$   
 $-2000(-1) = 2000, \dots$   
 $2000, -2000, 2000, -2000, 2000, -2000$
58.  $a_1 = 3000, r = -1$   
 $3000, 3000(-1) = -3000, -3000(-1) = 3000, \dots$   
 $3000, -3000, 3000, -3000, 3000, -3000$
59.  $a_1 = -2, r = -3$   
 $-2, -2(-3) = 6, 6(-3) = -18, -18(-3) = 54, 54(-3) = -162, -162(-3) = 486$   
 $-2, 6, -18, 54, -162, 486$

60.  $a_1 = -4, r = -2$

$$\begin{aligned} -4, -4(-2) = 8, 8(-2) = -16, -16(-2) = 32, \\ 32(-2) = -64, -64(-2) = 128 \\ -4, 8, -16, 32, -64, 128 \end{aligned}$$

61.  $a_1 = -6, r = -5$

$$\begin{aligned} -6, -6(-5) = 30, 30(-5) = -150, \\ -150(-5) = 750, 750(-5) = -3750, \\ -3750(-5) = 18,750 \\ -6, 30, -150, 750, -3750, 18750 \end{aligned}$$

62.  $a_1 = -8, r = -5$

$$\begin{aligned} -8, -8(-5) = 40, 40(-5) = -200, \\ -200(-5) = 1000, 1000(-5) = -5000, \\ -5000(-5) = 25,000 \\ -8, 40, -200, 1000, -5000, 25000 \end{aligned}$$

63.  $a_1 = \frac{1}{4}, r = 2$

$$\begin{aligned} \frac{1}{4}, \frac{1}{4} \cdot 2 = \frac{1}{2}, \frac{1}{2} \cdot 2 = 1, 1 \cdot 2 = 2, 2 \cdot 2 = 4, \\ 4 \cdot 2 = 8 \\ \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8 \end{aligned}$$

64.  $a_1 = \frac{1}{2}, r = 2$

$$\begin{aligned} \frac{1}{2}, \frac{1}{2} \cdot 2 = 1, 1 \cdot 2 = 2, 2 \cdot 2 = 4, 4 \cdot 2 = 8, 8 \cdot 2 = 16 \\ \frac{1}{2}, 1, 2, 4, 8, 16 \end{aligned}$$

65.  $a_1 = \frac{1}{4}, r = \frac{1}{2}$

$$\begin{aligned} \frac{1}{4}, \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}, \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}, \frac{1}{16} \cdot \frac{1}{2} = \frac{1}{32}, \\ \frac{1}{32} \cdot \frac{1}{2} = \frac{1}{64}, \frac{1}{64} \cdot \frac{1}{2} = \frac{1}{128} \\ \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128} \end{aligned}$$

66.  $a_1 = \frac{1}{5}, r = \frac{1}{2}$

$$\begin{aligned} \frac{1}{5}, \frac{1}{5} \cdot \frac{1}{2} = \frac{1}{10}, \frac{1}{10} \cdot \frac{1}{2} = \frac{1}{20}, \frac{1}{20} \cdot \frac{1}{2} = \frac{1}{40}, \\ \frac{1}{40} \cdot \frac{1}{2} = \frac{1}{80}, \frac{1}{80} \cdot \frac{1}{2} = \frac{1}{160} \\ \frac{1}{5}, \frac{1}{10}, \frac{1}{20}, \frac{1}{40}, \frac{1}{80}, \frac{1}{160} \end{aligned}$$

67.  $a_1 = -\frac{1}{16}, r = -4$

$$-\frac{1}{16}, -\frac{1}{16} \cdot (-4) = \frac{1}{4}, \frac{1}{4} \cdot (-4) = -1, \\ -1(-4) = 4, 4(-4) = -16, -16(-4) = 64 \\ -\frac{1}{16}, \frac{1}{4}, -1, 4, -16, 64$$

68.  $a_1 = -\frac{1}{8}, r = -2$

$$-\frac{1}{8}, -\frac{1}{8}(-2) = \frac{1}{4}, \frac{1}{4}(-2) = -\frac{1}{2}, -\frac{1}{2}(-2) = 1, \\ 1(-2) = -2, -2(-2) = 4 \\ -\frac{1}{8}, \frac{1}{4}, -\frac{1}{2}, 1, -2, 4$$

69.  $a_1 = 2, r = 0.1$

$$2, 2(0.1) = 0.2, 0.2(0.1) = 0.02, \\ 0.02(0.1) = 0.002, 0.002(0.1) = 0.0002, 0.0002(0.1) = 0.00002. \\ 2, 0.2, 0.02, 0.002, 0.0002, 0.00002$$

70.  $a_1 = -1000, r = 0.1$

$$-1000, -1000(0.1) = -100, -100(0.1) = -10, -10(0.1) = -1, -1(0.1) = -0.1, -0.1(0.1) = -0.01 \\ -1000, -100, -10, -1, -0.1, -0.01$$

71.  $a_1 = 4, r = 2$

$$a_7 = 4(2)^{7-1} \\ = 4(2)^6 \\ = 4(64) \\ = 256$$

72.  $a_1 = 4, r = 3$

$$a_5 = 4(3)^{5-1} \\ = 4(3)^4 \\ = 4(81) \\ = 324$$

73.  $a_1 = 2, r = 3$

$$a_{20} = 2(3)^{20-1} \\ = 2(3)^{19} \\ = 2,324,522,934 \\ \approx 2.32 \times 10^9$$



74.  $a_1 = 2, r = 2$   
 $a_{20} = 2(2)^{20-1}$   
 $= 2(2)^{19}$   
 $= 1,048,576$

75.  $a_1 = 50, r = 1$   
 $a_{100} = 50(1)^{100-1}$   
 $= 50(1)^{99}$   
 $= 50$

76.  $a_1 = 60, r = 1$   
 $a_{200} = 60(1)^{200-1}$   
 $= 60(1)^{199}$   
 $= 60$

77.  $a_1 = 5, r = -2$   
 $a_7 = 5(-2)^{7-1}$   
 $= 5(-2)^6$   
 $= 320$

78.  $a_1 = 4, r = -3$   
 $a_4 = 4(-3)^{4-1}$   
 $= 4(-3)^3$   
 $= -108$

79.  $a_1 = 2, r = -1$   
 $a_{30} = 2(-1)^{30-1}$   
 $= 2(-1)^{29}$   
 $= -2$

80.  $a_1 = 6, r = -1$   
 $a_{40} = 6(-1)^{40-1}$   
 $= 6(-1)^{39}$   
 $= -6$

81.  $a_1 = -2, r = -3$   
 $a_6 = -2(-3)^{6-1}$   
 $= -2(-3)^5$   
 $= 486$

82.  $a_1 = -5, r = -2$   
 $a_5 = -5(-2)^{5-1}$   
 $= -5(-2)^4$   
 $= -80$

83.  $a_1 = 6, r = \frac{1}{2}$

$$a_8 = 6\left(\frac{1}{2}\right)^{8-1}$$

$$= 6\left(\frac{1}{2}\right)^7$$

$$= \frac{6}{128}$$

$$= \frac{3}{64}$$

84.  $a_1 = 12, r = \frac{1}{2}$

$$a_8 = 12\left(\frac{1}{2}\right)^{8-1}$$

$$= 12\left(\frac{1}{2}\right)^7$$

$$= \frac{12}{128}$$

$$= \frac{3}{32}$$

85.  $a_1 = 18, r = -\frac{1}{3}$

$$a_6 = 18\left(-\frac{1}{3}\right)^{6-1}$$

$$= 18\left(-\frac{1}{3}\right)^5$$

$$= -\frac{18}{243}$$

$$= -\frac{2}{27}$$

86.  $a_1 = 9, r = -\frac{1}{3}$

$$a_4 = 9\left(-\frac{1}{3}\right)^{4-1}$$

$$= 9\left(-\frac{1}{3}\right)^3$$

$$= -\frac{9}{27}$$

$$= -\frac{1}{3}$$

$$\begin{aligned}
 87. \quad a_1 &= 1000, r = -\frac{1}{2} \\
 a_{40} &= 1000 \left( -\frac{1}{2} \right)^{40-1} \\
 &= 1000 \left( -\frac{1}{2} \right)^{39} \\
 &\approx -1.82 \times 10^{-9}
 \end{aligned}$$

$$\begin{aligned}
 88. \quad a_1 &= 8000, r = -\frac{1}{2} \\
 a_{30} &= 8000 \left( -\frac{1}{2} \right)^{30-1} \\
 &= 8000 \left( -\frac{1}{2} \right)^{29} \\
 &\approx -0.000014901
 \end{aligned}$$

$$\begin{aligned}
 89. \quad a_1 &= 1,000,000, r = 0.1 \\
 a_8 &= 1,000,000(0.1)^{8-1} \\
 &= 1,000,000(0.1)^7 \\
 &= 0.1
 \end{aligned}$$

$$\begin{aligned}
 90. \quad a_1 &= 40,000, r = 0.1 \\
 a_8 &= 40,000(0.1)^{8-1} \\
 &= 40,000(0.1)^7 \\
 &= 0.004
 \end{aligned}$$

$$\begin{aligned}
 91. \quad a_n &= a_1 r^{n-1} \text{ with } a_1 = 3 \text{ and } r = \frac{12}{3} = 4. \\
 \text{Thus } a_n &= 3(4)^{n-1} \\
 a_7 &= 3(4)^{7-1} = 3(4)^6 = 3(4096) = 12,288
 \end{aligned}$$

$$\begin{aligned}
 92. \quad a_n &= a_1 r^{n-1} \text{ with } a_1 = 3 \text{ and } r = \frac{15}{3} = 5. \\
 \text{Thus } a_n &= 3(5)^{n-1} \\
 a_7 &= 3(5)^{7-1} = 3(5)^6 = 3(15,625) = 46,875
 \end{aligned}$$

$$\begin{aligned}
 93. \quad a_n &= a_1 r^{n-1} \text{ with } a_1 = 18 \text{ and } r = \frac{6}{18} = \frac{1}{3}. \\
 \text{Thus } a_n &= 18 \left( \frac{1}{3} \right)^{n-1} \\
 a_7 &= 18 \left( \frac{1}{3} \right)^{7-1} = 18 \left( \frac{1}{3} \right)^6 = 18 \left( \frac{1}{729} \right) = \frac{18}{729} = \frac{2}{81}
 \end{aligned}$$

94.  $a_n = a_1 r^{n-1}$  with  $a_1 = 12$  and  $r = \frac{6}{12} = \frac{1}{2}$ .

Thus  $a_n = 12 \left( \frac{1}{2} \right)^{n-1}$

$$a_7 = 12 \left( \frac{1}{2} \right)^{7-1} = 12 \left( \frac{1}{2} \right)^6 = 12 \left( \frac{1}{64} \right) = \frac{12}{64} = \frac{3}{16}$$

95.  $a_n = a_1 r^{n-1}$  with  $a_1 = 1.5$  and  $r = \frac{-3}{1.5} = -2$ .

Thus  $a_n = 1.5(-2)^{n-1}$

$$a_7 = 1.5(-2)^{7-1} = 1.5(-2)^6 = 1.5(64) = 96$$

96.  $a_n = a_1 r^{n-1}$  with  $a_1 = 5$  and  $r = \frac{-1}{5}$ .

Thus  $a_n = 5 \left( -\frac{1}{5} \right)^{n-1}$

$$a_7 = 5 \left( -\frac{1}{5} \right)^{7-1} = 5 \left( -\frac{1}{5} \right)^6 = 5 \left( \frac{1}{15,625} \right) = \frac{5}{15,625} = \frac{1}{3125}$$

97.  $a_n = a_1 r^{n-1}$  with  $a_1 = 0.0004$  and  $r = \frac{-0.004}{0.0004} = -10$ . Thus  $a_n = 0.0004(-10)^{n-1}$

$$a_7 = 0.0004(-10)^{7-1} = 0.0004(-10)^6 = 0.0004(1,000,000) = 400$$

98.  $a_n = a_1 r^{n-1}$  with  $a_1 = 0.0007$  and  $r = \frac{-0.007}{0.0007} = -10$ . Thus  $a_n = 0.0007(-10)^{n-1}$

$$a_7 = 0.0007(-10)^{7-1} = 0.0007(-10)^6 = 0.0007(1,000,000) = 700$$

99. The common difference of the arithmetic sequence is 4.

$$2 + 4 = 6, 6 + 4 = 10, 10 + 4 = 14,$$

$$14 + 4 = 18, 18 + 4 = 22$$

$$2, 6, 10, 14, 18, 22, \dots$$

100. The common difference of the arithmetic sequence is 5.

$$3 + 5 = 8, 8 + 5 = 13, 13 + 5 = 18, 18 + 5 = 23, 23 + 5 = 28$$

$$3, 8, 13, 18, 23, 28, \dots$$

101. The common ratio of the geometric sequence is 3.

$$5 \cdot 3 = 15, 15 \cdot 3 = 45, 45 \cdot 3 = 135, 5, 15, 45, 135, 405, 1215, \dots$$

$$135 \cdot 3 = 405, 405 \cdot 3 = 1215$$

102. The common ratio of the geometric sequence is 2.

$$15 \cdot 2 = 30, 30 \cdot 2 = 60, 60 \cdot 2 = 120,$$

$$120 \cdot 2 = 240, 240 \cdot 2 = 480$$

$$15, 30, 60, 120, 240, 480, \dots$$

- 103.** The common difference of the arithmetic sequence is 5.

$$-7 + 5 = -2, -2 + 5 = 3, 3 + 5 = 8,$$

$$8 + 5 = 13, 13 + 5 = 18.$$

$$-7, -2, 3, 8, 13, 18, \dots$$

- 104.** The common difference of the arithmetic sequence is 4.

$$-9 + 4 = -5, -5 + 4 = -1, -1 + 4 = 3, 3 + 4 = 7,$$

$$7 + 4 = 11$$

$$-9, -5, -1, 3, 7, 11, \dots$$

- 105.** The common ratio of the geometric sequence is  $\frac{1}{2}$ .

$$3 \cdot \frac{1}{2} = \frac{3}{2}, \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}, \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}, \frac{3}{8} \cdot \frac{1}{2} = \frac{3}{16}$$

$$\frac{3}{16} \cdot \frac{1}{2} = \frac{3}{32}$$

$$3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, \dots$$

- 106.** The common ratio of the geometric sequence is  $\frac{1}{2}$ .

$$6 \cdot \frac{1}{2} = 3, 3 \cdot \frac{1}{2} = \frac{3}{2}, \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}, \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8},$$

$$\frac{3}{8} \cdot \frac{1}{2} = \frac{3}{16}$$

$$6, 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \dots$$

- 107.** The common difference of the arithmetic sequence is  $\frac{1}{2}$ .

$$\frac{1}{2} + \frac{1}{2} = 1, 1 + \frac{1}{2} = \frac{3}{2}, \frac{3}{2} + \frac{1}{2} = 2, 2 + \frac{1}{2} = \frac{5}{2},$$

$$\frac{5}{2} + \frac{1}{2} = 3$$

$$\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \dots$$

- 108.** The common difference of the arithmetic sequence is  $\frac{1}{3}$ .

$$\frac{2}{3} + \frac{1}{3} = 1, 1 + \frac{1}{3} = \frac{4}{3}, \frac{4}{3} + \frac{1}{3} = \frac{5}{3}, \frac{5}{3} + \frac{1}{3} = 2,$$

$$2 + \frac{1}{3} = \frac{7}{3}$$

$$\frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2, \frac{7}{3}, \dots$$

- 109.** The common ratio of the geometric sequence is  $-1$ .

$$7(-1) = -7, -7(-1) = 7, 7(-1) = -7,$$

$$-7(-1) = 7, 7(-1) = -7$$

$$7, -7, 7, -7, 7, -7, \dots$$

110. The common ratio of the geometric sequence is  $-1$ .

$$6(-1) = -6, -6(-1) = 6, 6(-1) = -6, -6(-1) = 6, 6(-1) = -6 \\ 6, -6, 6, -6, 6, -6, \dots$$

111. The common difference of the arithmetic sequence is  $-14$ .

$$7 - 14 = -7, -7 - 14 = -21, -21 - 14 = -35, -35 - 14 = -49, -49 - 14 = -63 \\ 7, -7, -21, -35, -49, -63, \dots$$

112. The common difference of the arithmetic sequence is  $-12$ .

$$6 - 12 = -6, -6 - 12 = -18, -18 - 12 = -30, \\ -30 - 12 = -42, -42 - 12 = -54 \\ 6, -6, -18, -30, -42, -54, \dots$$

113. The common ratio of the geometric sequence is  $\sqrt{5}$ .

$$\sqrt{5} \cdot \sqrt{5} = 5, 5 \cdot \sqrt{5} = 5\sqrt{5}, 5\sqrt{5} \cdot \sqrt{5} = 25, \\ 25 \cdot \sqrt{5} = 25\sqrt{5}, 25\sqrt{5} \cdot \sqrt{5} = 125 \\ \sqrt{5}, 5, 5\sqrt{5}, 25, 25\sqrt{5}, 125, \dots$$

114. The common ratio of the geometric sequence is  $\sqrt{3}$ .

$$\sqrt{3} \cdot \sqrt{3} = 3, 3 \cdot \sqrt{3} = 3\sqrt{3}, 3\sqrt{3} \cdot \sqrt{3} = 9, \\ 9 \cdot \sqrt{3} = 9\sqrt{3}, 9\sqrt{3} \cdot \sqrt{3} = 27 \\ \sqrt{3}, 3, 3\sqrt{3}, 9, 9\sqrt{3}, 27, \dots$$

115. arithmetic; use  $S_n = \frac{n}{2}(a_1 + a_n)$

$$S_{10} = \frac{10}{2}(4 + 58) = 310$$

116. arithmetic; use  $S_n = \frac{n}{2}(a_1 + a_n)$

$$S_{10} = \frac{10}{2}(7 + 115) = 610$$

117. geometric; use  $S_n = \frac{a_1(1 - r^n)}{1 - r}$

$$S_{10} = \frac{2(1 - 3^{10})}{1 - 3} = 59,048$$

118. geometric; use  $S_n = \frac{a_1(1 - r^n)}{1 - r}$

$$S_{10} = \frac{3(1 - 2^{10})}{1 - 2} = 3069$$

119. geometric; use  $S_n = \frac{a_1(1 - r^n)}{1 - r}$

$$S_{10} = \frac{3(1 - (-2)^{10})}{1 - (-2)} = -1023$$

120. geometric; use  $S_n = \frac{a_1(1-r^n)}{1-r}$

$$S_{10} = \frac{4(1-(-3)^{10})}{1-(-3)} = -59,048$$

121. arithmetic; use  $S_n = \frac{n}{2}(a_1 + a_n)$

$$S_{10} = \frac{10}{2}(-10 + 26) = 80$$

122. arithmetic; use  $S_n = \frac{n}{2}(a_1 + a_n)$

$$S_{10} = \frac{10}{2}(-15 + 39) = 120$$

123.  $1 + 2 + 3 + 4 + \cdots + 100$

$$S_{100} = \frac{100}{2}(1 + 100) = 5050$$

124.  $2 + 4 + 6 + \cdots + 200$

$$S_{100} = \frac{100}{2}(2 + 200) = 10,100$$

125. a.  $a_n = a_1 + (n-1)d$

$$a_n = 18.4 + (n-1)0.6$$

$$= 18.4 + 0.6n - 0.6$$

$$= 0.6n + 17.8$$

b.  $a_n = 0.6n + 17.8$

$$= 0.6(30) + 17.8$$

$$= 35.8$$

The percentage is projected to be 35.8% in 2019.

126. a.  $a_n = a_1 + (n-1)d$

$$a_n = 24.4 + (n-1)0.3$$

$$= 24.4 + 0.3n - 0.3$$

$$= 0.3n + 24.1$$

b.  $a_n = 0.3n + 24.1$

$$= 0.3(30) + 24.1$$

$$= 33.1$$

The percentage is projected to be 33.1% in 2019.

127. Company A:  $a_{10} = 24000 + (10-1)1600 = 38,400$

Company B:  $b_{10} = 28000 + (10-1)1000 = 37,000$

Company A will pay \$1400 more in year 10.

128. Company A:  $a_{10} = 23000 + (10 - 1)1200 = 33,800$   
 Company B:  $b_{10} = 26000 + (10 - 1)800 = 33,200$   
 Company A will pay \$600 more in year 10.

129.  $a_1 = 1, r = 2$

$$\begin{aligned} a_{15} &= 1(2)^{15-1} \\ &= 2^{14} \\ &= 16,384 \end{aligned}$$

On the 15th day you will put aside \$16,384.

130.  $a_1 = 1, r = 2$

$$\begin{aligned} a_{30} &= 1(2)^{30-1} \\ &= 2^{29} \\ &= 536,870,912 \end{aligned}$$

On the 30th day you will put aside \$536,870,912.

131.  $a_7 = \$3,000,000(1.04)^{7-1}$   
 $\approx \$3,795,957$  salary in year 7.

132.  $a_6 = \$30,000(1.05)^{6-1}$   
 $\approx \$38,288$  salary in year 6.

133. a.  $r_{2000 \text{ to } 2001} = \frac{34.21}{33.87} \approx 1.01$

$$r_{2001 \text{ to } 2002} = \frac{34.55}{34.21} \approx 1.01$$

$$r_{2002 \text{ to } 2003} = \frac{34.90}{34.55} \approx 1.01$$

$$r_{2003 \text{ to } 2004} = \frac{35.25}{34.90} \approx 1.01$$

$$r_{2004 \text{ to } 2005} = \frac{35.60}{35.25} \approx 1.01$$

$$r_{2005 \text{ to } 2006} = \frac{36.00}{35.60} \approx 1.01$$

$$r_{2006 \text{ to } 2007} = \frac{36.36}{36.00} \approx 1.01$$

$$r_{2007 \text{ to } 2008} = \frac{36.72}{36.36} \approx 1.01$$

$$r_{2008 \text{ to } 2009} = \frac{37.09}{36.72} \approx 1.01$$

$$r_{2009 \text{ to } 2010} = \frac{37.25}{37.09} \approx 1.01$$

$r$  is approximately 1.01 for all but one division.

b.  $a_n = a_1 r^{n-1}$

$$a_n = 33.87(1.01)^{n-1}$$



- c. Since year 2020 is the 21th term, find  $a_{21}$ .

$$a_n = 33.87(1.01)^{n-1}$$

$$a_{21} = 33.87(1.01)^{21-1} \approx 41.33$$

The population of California will be approximately 41.33 million in 2020.

134. a.  $r_{2000 \text{ to } 2001} = \frac{21.27}{20.85} \approx 1.02$

$$r_{2001 \text{ to } 2002} = \frac{21.70}{21.27} \approx 1.02$$

$$r_{2002 \text{ to } 2003} = \frac{22.13}{21.70} \approx 1.02$$

$$r_{2003 \text{ to } 2004} = \frac{22.57}{22.13} \approx 1.02$$

$$r_{2004 \text{ to } 2005} = \frac{23.02}{22.57} \approx 1.02$$

$$r_{2005 \text{ to } 2006} = \frac{23.48}{23.02} \approx 1.02$$

$$r_{2006 \text{ to } 2007} = \frac{23.95}{23.48} \approx 1.02$$

$$r_{2007 \text{ to } 2008} = \frac{24.43}{23.95} \approx 1.02$$

$$r_{2008 \text{ to } 2009} = \frac{24.92}{24.43} \approx 1.02$$

$$r_{2009 \text{ to } 2010} = \frac{25.15}{24.92} \approx 1.01$$

$r$  is approximately 1.02 for all but one division.

b.  $a_n = a_1 r^{n-1}$

$$a_n = 20.85(1.02)^{n-1}$$

- c. Since year 2020 is the 21th term, find  $a_{21}$ .

$$a_n = 20.85(1.02)^{n-1}$$

$$a_{21} = 20.85(1.02)^{21-1} \approx 30.98$$

The population of Texas will be approximately 30.98 million in 2020.

135. Company A:

$$a_1 = 20,000, d = 1000$$

$$a_6 = 20,000 + (6-1)(1000)$$

$$= 20,000 + 5000$$

$$= 25,000$$

Company B:

$$a_1 = 20,000, r = 1.05$$

$$a_6 = 20,000(1.05)^{6-1}$$

$$= 20,000(1.05)^5$$

$$= 25,525.63$$

Company B will pay more in the sixth year.

136. The second option can be represented by  $a_{28} + a_{29} + a_{30}$  where  $a_n = a_1 r^{n-1}$  with  $a_1 = 0.01$  and  $r = 2$ .

$$a_n = a_1 r^{n-1} = 0.01(2)^{n-1}$$

$$a_{28} + a_{29} + a_{30}$$

$$= 0.01(2)^{28-1} + 0.01(2)^{29-1} + 0.01(2)^{30-1}$$

$$= 1,342,177.28 + 2,684,354.56 + 5,368,709.12$$

$$= 9,395,240.96$$

You should take the \$10,000,000 and the BMW because the second option yields only \$9,395,240.96.

## Chapter 1 Review Exercises

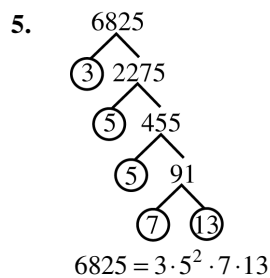
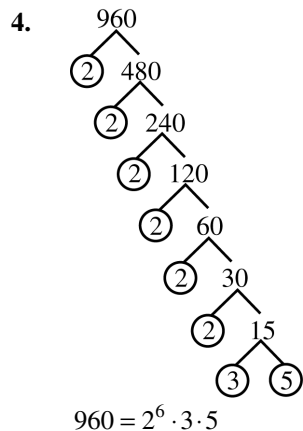
1. 238,632
  - 2: Yes; The last digit is 2.
  - 3: Yes; The sum of the digits is 24, which is divisible by 3.
  - 4: Yes; The last two digits form 32, which is divisible by 4.
  - 5: No; The last number does not end in 0 or 5.
  - 6: Yes; The number is divisible by both 2 and 3.
  - 8: Yes; The last three digits form 632, which is divisible by 8.
  - 9: No; The sum of the digits is 24, which is not divisible by 9.
  - 10: No; the last digit is not 0.
  - 12: Yes; The number is divisible by both 3 and 4.

The number is divisible by 2, 3, 4, 6, 8, 12.
2. 421,153,470
  - 2: Yes; The last digit is 0.
  - 3: Yes; The sum of the digits is 27, which is divisible by 3.
  - 4: No; The last two digits form 70, which is not divisible by 4.
  - 5: Yes; The number ends in 0.
  - 6: Yes; The number is divisible by both 2 and 3.
  - 8: No; The last three digits form 470, which is not divisible by 8.
  - 9: Yes; The sum of the digits is 27, which is divisible by 9.
  - 10: Yes; The number ends in 0.
  - 12: No; The number is not divisible by both 3 and 4.

The number is divisible by 2, 3, 5, 6, 9, 10.

3.

$$705 = 3 \cdot 5 \cdot 47$$



6.  $30 = 2 \cdot 3 \cdot 5$   
 $48 = 2^4 \cdot 3$   
 Greatest Common Divisor =  $2 \cdot 3 = 6$   
 Least Common Multiple =  $2^4 \cdot 3 \cdot 5 = 240$

7.  $36 = 2^2 \cdot 3^2$   
 $150 = 2 \cdot 3 \cdot 5^2$   
 Greatest Common Divisor =  $2 \cdot 3 = 6$   
 Least Common Multiple =  $2^2 \cdot 3^2 \cdot 5^2 = 900$

8.  $216 = 2^3 \cdot 3^3$   
 $254 = 2 \cdot 127$   
 Greatest Common Divisor = 2  
 Least Common Multiple =  $2^3 \cdot 3^3 \cdot 127$   
 $= 27,432$

9.  $24 = 2^3 \cdot 3$   
 $60 = 2^2 \cdot 3 \cdot 5$   
 Greatest Common Divisor =  $2^2 \cdot 3 = 12$   
 There can be 12 people placed on each team.

10.  $42 = 2 \cdot 3 \cdot 7$   
 $56 = 2^3 \cdot 7$   
 Least Common Multiple =  $2^3 \cdot 3 \cdot 7 = 168$   
 $168 \div 60 = 2.8$  or 2 hours and 48 minutes. They will begin again at 11:48 A.M.

11.  $-93 < 17$  because  $-93$  is to the left of 17 on the number line.

**12.**  $-2 > -200$  because  $-2$  is to the right of  $-200$  on the number line.

**13.**  $|-860| = 860$  because  $-860$  is 860 units from 0 on the number line.

**14.**  $|53| = 53$  because 53 is 53 units from 0 on the number line.

**15.**  $|0| = 0$  because 0 is 0 units from 0 on the number line.

**16.**  $8 + (-11) = -3$

**17.**  $-6 + (-5) = -11$

**18.**  $-7 - 8 = -7 + (-8) = -15$

**19.**  $-7 - (-8) = -7 + 8 = 1$

**20.**  $(-9)(-11) = 99$

**21.**  $5(-3) = -15$

**22.**  $\frac{-36}{-4} = 9$

**23.**  $\frac{20}{-5} = -4$

**24.**  $-40 \div 5 \cdot 2 = -8 \cdot 2 = -16$

**25.**  $-6 + (-2) \cdot 5 = -6 + (-10) = -16$

**26.**  $6 - 4(-3 + 2) = 6 - 4(-1) = 6 + 4 = 10$

**27.**  $28 \div (2 - 4^2) = 28 \div (2 - 16)$   
 $= 28 \div (-14)$   
 $= -2$

**28.**  $36 - 24 \div 4 \cdot 3 - 1 = 36 - 6 \cdot 3 - 1$   
 $= 36 - 18 - 1$   
 $= 18 - 1$   
 $= 17$

**29.**  $-57 - (-715) = -57 + 715 = \$658 \text{ billion}$

**30.**  $40 = 2^3 \cdot 5$

$$75 = 3 \cdot 5^2$$

Greatest Common Divisor is 5.

$$\frac{40}{75} = \frac{40 \div 5}{75 \div 5} = \frac{8}{15}$$

$$31. 36 = 2^2 \cdot 3^2$$

$$150 = 2 \cdot 3 \cdot 5^2$$

Greatest Common Divisor is  $2 \cdot 3$  or 6.

$$\frac{36}{150} = \frac{36 \div 6}{150 \div 6} = \frac{6}{25}$$

$$32. 165 = 3 \cdot 5 \cdot 11$$

$$180 = 2^2 \cdot 3^2 \cdot 5$$

Greatest Common Divisor is  $3 \cdot 5$  or 15.

$$\frac{165}{180} = \frac{165 \div 15}{180 \div 15} = \frac{11}{12}$$

$$33. 5\frac{9}{11} = \frac{11 \cdot 5 + 9}{11} = \frac{64}{11}$$

$$34. -3\frac{2}{7} = -\frac{7 \cdot 3 + 2}{7} = -\frac{23}{7}$$

$$35. \frac{27}{5} = 5\frac{2}{5}$$

$$36. -\frac{17}{9} = -1\frac{8}{9}$$

$$37. \frac{4}{5} = 0.8$$

$$\begin{array}{r} 0.8 \\ 5 \overline{) 4.0} \\ \underline{40} \\ 0 \end{array}$$

$$38. \frac{3}{7} = 0.428571$$

$$\begin{array}{r} 0.4285714 \\ 7 \overline{) 3.0000000} \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 2 \end{array}$$

39.  $\frac{5}{8} = 0.625$

$$\begin{array}{r} 0.625 \\ 8 \overline{) 5.000} \\ \underline{48} \phantom{00} \\ 20 \phantom{00} \\ \underline{16} \phantom{00} \\ 40 \phantom{00} \\ \underline{40} \\ 0 \end{array}$$

40.  $\frac{9}{16} = 0.5625$

$$\begin{array}{r} 0.5625 \\ 16 \overline{) 9.0000} \\ \underline{80} \phantom{000} \\ 100 \phantom{00} \\ \underline{96} \phantom{00} \\ 40 \phantom{00} \\ \underline{32} \phantom{00} \\ 80 \phantom{00} \\ \underline{80} \\ 0 \end{array}$$

41.  $0.6 = \frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$

42.  $0.68 = \frac{68}{100}$

$$68 = 2^2 \cdot 17$$

$$100 = 2^2 \cdot 5^2$$

Greatest Common Divisor is  $2^2$  or 4.

$$\frac{68 \div 4}{100 \div 4} = \frac{17}{25}$$

43.  $0.588 = \frac{588}{1000}$

$$588 = 2^2 \cdot 3 \cdot 7^2$$

$$1000 = 2^3 \cdot 5^3$$

Greatest Common Divisor is  $2^2$  or 4.

$$\frac{588 \div 4}{1000 \div 4} = \frac{147}{250}$$

$$44. \quad 0.0084 = \frac{84}{10,000}$$

$$84 = 2^2 \cdot 3 \cdot 7$$

$$10,000 = 2^4 \cdot 5^4$$

Greatest Common Divisor is  $2^2$  or 4.

$$\frac{84 \div 4}{10,000 \div 4} = \frac{21}{2500}$$

$$45. \quad \begin{array}{l} n = 0.555 \dots \\ 10n = 5.555 \dots \end{array}$$

$$\begin{array}{r} 10n = 5.555\dots \\ - \quad n = 0.555\dots \\ \hline 9n = 5 \end{array}$$

$$n = \frac{5}{9}$$

$$46. \quad \begin{array}{l} n = 0.3434 \dots \\ 100n = 34.3434 \dots \end{array}$$

$$\begin{array}{r} 100n = 34.3434\dots \\ - \quad n = 0.3434\dots \\ \hline 99n = 34 \end{array}$$

$$n = \frac{34}{99}$$

$$47. \quad \begin{array}{l} n = 0.113113 \dots \\ 1000n = 113.113113 \dots \end{array}$$

$$\begin{array}{r} 1000n = 113.113113\dots \\ - \quad n = 0.113113\dots \\ \hline 999n = 113 \end{array}$$

$$n = \frac{113}{999}$$

$$48. \quad \frac{3}{5} \cdot \frac{7}{10} = \frac{3 \cdot 7}{5 \cdot 10} = \frac{21}{50}$$

$$49. \quad \left(3\frac{1}{3}\right)\left(1\frac{3}{4}\right) = \frac{10}{3} \cdot \frac{7}{4} = \frac{70}{12} = \frac{35}{6} \text{ or } 5\frac{5}{6}$$

$$50. \quad \frac{4}{5} \div \frac{3}{10} = \frac{4}{5} \cdot \frac{10}{3} = \frac{4 \cdot 10}{5 \cdot 3} = \frac{40}{15} = \frac{8}{3}$$

$$51. \quad -1\frac{2}{3} \div 6\frac{2}{3} = -\frac{5}{3} \div \frac{20}{3} = -\frac{5}{3} \cdot \frac{3}{20} = -\frac{15}{60} = -\frac{1}{4}$$

$$52. \quad \frac{2}{9} + \frac{4}{9} = \frac{2+4}{9} = \frac{6}{9} = \frac{2}{3}$$

$$\begin{aligned} 53. \quad \frac{7}{9} + \frac{5}{12} &= \frac{7}{9} \cdot \frac{4}{4} + \frac{5}{12} \cdot \frac{3}{3} \\ &= \frac{28}{36} + \frac{15}{36} \\ &= \frac{28+15}{36} \\ &= \frac{43}{36} \end{aligned}$$

$$\begin{aligned} 54. \quad \frac{3}{4} - \frac{2}{15} &= \frac{3}{4} \cdot \frac{15}{15} - \frac{2}{15} \cdot \frac{4}{4} \\ &= \frac{45}{60} - \frac{8}{60} \\ &= \frac{45-8}{60} \\ &= \frac{37}{60} \end{aligned}$$

$$\begin{aligned} 55. \quad \frac{1}{3} + \frac{1}{2} \cdot \frac{4}{5} &= \frac{1}{3} + \frac{1 \cdot 4}{2 \cdot 5} \\ &= \frac{1}{3} + \frac{4}{10} \\ &= \frac{1}{3} + \frac{2}{5} \\ &= \frac{1}{3} \cdot \frac{5}{5} + \frac{2}{5} \cdot \frac{3}{3} \\ &= \frac{5}{15} + \frac{6}{15} \\ &= \frac{11}{15} \end{aligned}$$

$$\begin{aligned} 56. \quad \frac{3}{8} \left( \frac{1}{2} + \frac{1}{3} \right) &= \frac{3}{8} \left( \frac{1}{2} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{2}{2} \right) \\ &= \frac{3}{8} \left( \frac{3}{6} + \frac{2}{6} \right) \\ &= \frac{3}{8} \left( \frac{5}{6} \right) \\ &= \frac{15}{48} \\ &= \frac{5}{16} \end{aligned}$$



$$\begin{aligned}
 57. \quad \frac{1}{2} - \frac{2}{3} \div \frac{5}{9} + \frac{3}{10} &= \frac{1}{2} - \frac{2}{3} \times \frac{9}{5} + \frac{3}{10} \\
 &= \frac{1}{2} - \frac{6}{5} + \frac{3}{10} \\
 &= \frac{5}{10} - \frac{12}{10} + \frac{3}{10} \\
 &= -\frac{4}{10} \\
 &= -\frac{2}{5}
 \end{aligned}$$

$$\begin{aligned}
 58. \quad \left(\frac{1}{2} + \frac{1}{3}\right) \div \left(\frac{1}{4} - \frac{3}{8}\right) &= \left(\frac{3}{6} + \frac{2}{6}\right) \div \left(\frac{2}{8} - \frac{3}{8}\right) \\
 &= \left(\frac{5}{6}\right) \div \left(\frac{-1}{8}\right) \\
 &= \frac{5}{6} \times \frac{8}{-1} \\
 &= -\frac{20}{3} \\
 &= -6\frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 59. \quad \frac{1}{7} + \frac{1}{8} &= \frac{1}{7} \cdot \frac{8}{8} + \frac{1}{8} \cdot \frac{7}{7} \\
 &= \frac{8}{56} + \frac{7}{56} \\
 &= \frac{15}{56} \\
 \frac{15}{56} \div 2 &= \frac{15}{56} \cdot \frac{1}{2} = \frac{15}{112}
 \end{aligned}$$

$$\begin{aligned}
 60. \quad \frac{3}{4} + \frac{3}{5} &= \frac{3}{4} \cdot \frac{5}{5} + \frac{3}{5} \cdot \frac{4}{4} \\
 &= \frac{15}{20} + \frac{12}{20} \\
 &= \frac{27}{20} \\
 \frac{27}{20} \div 2 &= \frac{27}{20} \cdot \frac{1}{2} = \frac{27}{40}
 \end{aligned}$$

$$61. \quad 4\frac{1}{2} \cdot \frac{15}{6} = \frac{9}{2} \cdot \frac{15}{6} = \frac{135}{12} = \frac{45}{4} \text{ or } 11\frac{1}{4} \text{ pounds.}$$

$$\begin{aligned}
 62. \quad 1 - \left( \frac{1}{4} + \frac{1}{3} \right) &= 1 - \left( \frac{1}{4} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{4}{4} \right) \\
 &= 1 - \left( \frac{3}{12} + \frac{4}{12} \right) \\
 &= \frac{12}{12} - \frac{7}{12} \\
 &= \frac{5}{12}
 \end{aligned}$$

At the end of the second day,  $\frac{5}{12}$  of the tank is filled with gas.

$$63. \quad \sqrt{28} = \sqrt{4 \cdot 7} = \sqrt{4} \cdot \sqrt{7} = 2\sqrt{7}$$

$$64. \quad \sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$$

$$65. \quad \sqrt{150} = \sqrt{25 \cdot 6} = \sqrt{25} \cdot \sqrt{6} = 5\sqrt{6}$$

$$66. \quad \sqrt{300} = \sqrt{100 \cdot 3} = \sqrt{100} \cdot \sqrt{3} = 10\sqrt{3}$$

$$67. \quad \sqrt{6} \cdot \sqrt{8} = \sqrt{6 \cdot 8} = \sqrt{48} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$$

$$\begin{aligned}
 68. \quad \sqrt{10} \cdot \sqrt{5} &= \sqrt{10 \cdot 5} \\
 &= \sqrt{50} \\
 &= \sqrt{25} \cdot \sqrt{2} \\
 &= 5\sqrt{2}
 \end{aligned}$$

$$69. \quad \frac{\sqrt{24}}{\sqrt{2}} = \sqrt{\frac{24}{2}} = \sqrt{12} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$$

$$70. \quad \frac{\sqrt{27}}{\sqrt{3}} = \sqrt{\frac{27}{3}} = \sqrt{9} = 3$$

$$71. \quad \sqrt{5} + 4\sqrt{5} = 1\sqrt{5} + 4\sqrt{5} = (1+4)\sqrt{5} = 5\sqrt{5}$$

$$72. \quad 7\sqrt{11} - 13\sqrt{11} = (7-13)\sqrt{11} = -6\sqrt{11}$$

$$\begin{aligned}
 73. \quad \sqrt{50} + \sqrt{8} &= \sqrt{25} \cdot \sqrt{2} + \sqrt{4} \cdot \sqrt{2} \\
 &= 5\sqrt{2} + 2\sqrt{2} \\
 &= (5+2)\sqrt{2} \\
 &= 7\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 74. \quad \sqrt{3} - 6\sqrt{27} &= \sqrt{3} - 6\sqrt{9} \cdot \sqrt{3} \\
 &= \sqrt{3} - 6 \cdot 3\sqrt{3} \\
 &= 1\sqrt{3} - 18\sqrt{3} \\
 &= (1-18)\sqrt{3} \\
 &= -17\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 75. \quad 2\sqrt{18} + 3\sqrt{8} &= 2\sqrt{9} \cdot \sqrt{2} + 3\sqrt{4} \cdot \sqrt{2} \\
 &= 2 \cdot 3 \cdot \sqrt{2} + 3 \cdot 2 \cdot \sqrt{2} \\
 &= 6\sqrt{2} + 6\sqrt{2} \\
 &= (6+6)\sqrt{2} \\
 &= 12\sqrt{2}
 \end{aligned}$$

$$76. \quad \frac{30}{\sqrt{5}} = \frac{30}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{30\sqrt{5}}{\sqrt{25}} = \frac{30\sqrt{5}}{5} = 6\sqrt{5}$$

$$77. \quad \sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{\sqrt{9}} = \frac{\sqrt{6}}{3}$$

$$\begin{aligned}
 78. \quad W &= 4\sqrt{2x} \\
 &= 4\sqrt{2 \cdot 6} \\
 &= 4\sqrt{12} \\
 &= 8\sqrt{3} \approx 13.9 \text{ feet per second}
 \end{aligned}$$

$$79. \quad \left\{ -17, -\frac{9}{13}, 0, 0.75, \sqrt{2}, \pi, \sqrt{81} \right\}$$

a. Natural numbers:

$\sqrt{81}$  because  $\sqrt{81} = 9$

b. Whole numbers: 0,  $\sqrt{81}$

c. Integers:  $-17, 0, \sqrt{81}$

d. Rational numbers:

$-17, -\frac{9}{13}, 0, 0.75, \sqrt{81}$

e. Irrational numbers:  $\sqrt{2}, \pi$

f. Real numbers: All numbers in this set.

80. Answers will vary. Example:  $-3$

81. Answers will vary. Example:  $\frac{1}{2}$

**82.** Answers will vary: Example:  $\sqrt{2}$

**83.** Commutative property of addition

**84.** Associative property of multiplication

**85.** Distributive property of multiplication over addition.

**86.** Commutative property of multiplication

**87.** Commutative property of multiplication

**88.** Commutative property of addition

**89.** Inverse property of multiplication

**90.** Identity property of multiplication

**91.** Answers will vary. Example:  $2 \div 6 = \frac{1}{3}$

**92.** Answers will vary. Example:  $0 - 2 = -2$

**93.**  $6 \cdot 6^2 = 6^1 \cdot 6^2 = 6^{1+2} = 6^3 = 216$

**94.**  $2^3 \cdot 2^3 = 2^{3+3} = 2^6 = 64$

**95.**  $(2^2)^2 = 2^{2 \cdot 2} = 2^4 = 16$

**96.**  $(3^3)^2 = 3^{3 \cdot 2} = 3^6 = 729$

**97.**  $\frac{5^6}{5^4} = 5^{6-4} = 5^2 = 25$

**98.**  $7^0 = 1$

**99.**  $(-7)^0 = 1$

**100.**  $6^{-3} = \frac{1}{6^3} = \frac{1}{216}$

**101.**  $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

**102.**  $\frac{7^4}{7^6} = 7^{4-6} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

**103.**  $3^5 \cdot 3^{-2} = 3^{5-2} = 3^3 = 27$

**104.**  $4.6 \times 10^2 = 460$

**105.**  $3.74 \times 10^4 = 37,400$

$$106. 2.55 \times 10^{-3} = 0.00255$$

$$107. 7.45 \times 10^{-5} = 0.0000745$$

$$108. 7520 = 7.52 \times 10^3$$

$$109. 3,590,000 = 3.59 \times 10^6$$

$$110. 0.00725 = 7.25 \times 10^{-3}$$

$$111. 0.000000409 = 4.09 \times 10^{-7}$$

$$112. 420 \times 10^{11} = (4.2 \times 10^2) \times 10^{11} = 4.2 \times 10^{13}$$

$$113. 0.97 \times 10^{-4} = (9.7 \times 10^{-1}) \times 10^{-4} = 9.7 \times 10^{-5}$$

$$\begin{aligned} 114. (3 \times 10^7)(1.3 \times 10^{-5}) &= (3 \times 1.3) \times 10^{7-5} \\ &= 3.9 \times 10^2 \\ &= 390 \end{aligned}$$

$$\begin{aligned} 115. (5 \times 10^3)(2.3 \times 10^2) &= (5 \times 2.3) \times 10^{3+2} \\ &= 11.5 \times 10^5 \\ &= 1.15 \times 10 \times 10^5 \\ &= 1.15 \times 10^6 \\ &= 1,150,000 \end{aligned}$$

$$\begin{aligned} 116. \frac{6.9 \times 10^3}{3 \times 10^5} &= \left( \frac{6.9}{3} \right) \times 10^{3-5} \\ &= 2.3 \times 10^{-2} \\ &= 0.023 \end{aligned}$$

$$\begin{aligned} 117. \frac{2.4 \times 10^{-4}}{6 \times 10^{-6}} &= \left( \frac{2.4}{6} \right) \times 10^{-4-(-6)} \\ &= 0.4 \times 10^{-4+6} \\ &= 0.4 \times 10^2 \\ &= 40 \end{aligned}$$

$$\begin{aligned} 118. (60,000)(540,000) &= (6.0 \times 10^4)(5.4 \times 10^5) \\ &= (6.0 \times 5.4) \times 10^{4+5} \\ &= 32.4 \times 10^9 \\ &= 3.24 \times 10 \times 10^9 \\ &= 3.24 \times 10^{10} \end{aligned}$$

$$\begin{aligned}
 119. \quad (91,000)(0.0004) &= (9.1 \times 10^4)(4 \times 10^{-4}) \\
 &= (9.1 \times 4) \times 10^{4-4} \\
 &= 36.4 \times 10^0 \\
 &= 3.64 \times 10^1
 \end{aligned}$$

$$\begin{aligned}
 120. \quad \frac{8,400,000}{4000} &= \frac{8.4 \times 10^6}{4 \times 10^3} \\
 &= \left( \frac{8.4}{4} \right) \times 10^{6-3} \\
 &= 2.1 \times 10^3
 \end{aligned}$$

$$\begin{aligned}
 121. \quad \frac{0.000003}{0.00000006} &= \frac{3 \times 10^{-6}}{6 \times 10^{-8}} \\
 &= \left( \frac{3}{6} \right) \times 10^{-6-(-8)} \\
 &= 0.5 \times 10^2 \\
 &= 5 \times 10^{-1} \times 10^2 \\
 &= 5 \times 10^1
 \end{aligned}$$

$$122. \quad 1.3 \times 10^{12}$$

$$123. \quad 3.2 \times 10^7$$

$$124. \quad \frac{1.3 \times 10^{12}}{3.2 \times 10^7} \approx 0.40625 \times 10^5 = 40,625 \text{ years}$$

$$\begin{aligned}
 125. \quad 180(3.2 \times 10^4)(5 \times 10^6) &= (180 \times 3.2 \times 5) \times (10^4 \times 10^6) \\
 &= 2880 \times 10^{10} \\
 &= 2.88 \times 10^{13}
 \end{aligned}$$

$$\begin{aligned}
 126. \quad a_1 &= 7, d = 4 \\
 7, 7 + 4 &= 11, 11 + 4 = 15, 15 + 4 = 19, 19 + 4 = 23, 23 + 4 = 27 \\
 7, 11, 15, 19, 23, 27
 \end{aligned}$$

$$\begin{aligned}
 127. \quad a_1 &= -4, d = -5 \\
 -4, -4 - 5 &= -9, -9 - 5 = -14, -14 - 5 = -19, -19 - 5 = -24, -24 - 5 = -29 \\
 -4, -9, -14, -19, -24, -29
 \end{aligned}$$

$$128. \quad a_1 = \frac{3}{2}, d = -\frac{1}{2}$$

$$\frac{3}{2}, \frac{3}{2} - \frac{1}{2} = \frac{2}{2} = 1, \frac{2}{2} - \frac{1}{2} = \frac{1}{2}, \frac{1}{2} - \frac{1}{2} = 0, 0 - \frac{1}{2} = -\frac{1}{2}, -\frac{1}{2} - \frac{1}{2} = -1$$

$$\frac{3}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1$$

$$129. \quad a_1 = 5, d = 3$$

$$a_6 = 5 + (6-1)(3)$$

$$= 5 + 5(3)$$

$$= 5 + 15$$

$$= 20$$

$$130. \quad a_1 = -8, d = -2$$

$$a_{12} = -8 + (12-1)(-2)$$

$$= -8 + 11(-2)$$

$$= -8 + (-22)$$

$$= -30$$

$$131. \quad a_1 = 14, d = -4$$

$$a_{14} = 14 + (14-1)(-4)$$

$$= 14 + 13(-4)$$

$$= 14 + (-52)$$

$$= -38$$

$$132. \quad a_n = a_1 + (n-1)d \text{ with } a_1 = -7, d = 4$$

$$a_n = -7 + (n-1)4$$

$$= -7 + 4n - 4$$

$$= 4n - 11$$

Thus  $a_{20} = 4(20) - 11 = 69$ .

$$133. \quad a_n = a_1 + (n-1)d \text{ with } a_1 = 200, d = -20$$

$$a_n = 200 + (n-1)(-20)$$

$$= 200 - 20n + 20$$

$$= -20n + 220$$

Thus  $a_{20} = -20(20) + 220 = -180$ .

$$134. \quad a_1 = 3, r = 2$$

$$3, 3 \cdot 2 = 6, 6 \cdot 2 = 12, 12 \cdot 2 = 24,$$

$$24 \cdot 2 = 48, 48 \cdot 2 = 96$$

$$3, 6, 12, 24, 48, 96$$

$$135. \quad a_1 = \frac{1}{2}, r = \frac{1}{2}$$

$$\frac{1}{2}, \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}, \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}, \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}, \frac{1}{16} \cdot \frac{1}{2} = \frac{1}{32}, \frac{1}{32} \cdot \frac{1}{2} = \frac{1}{64}$$

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$$

$$136. \quad a_1 = 16, r = -\frac{1}{2}$$

$$16, 16\left(-\frac{1}{2}\right) = -8, -8\left(-\frac{1}{2}\right) = 4, 4\left(-\frac{1}{2}\right) = -2, -2\left(-\frac{1}{2}\right) = 1, 1\left(-\frac{1}{2}\right) = -\frac{1}{2}$$

$$16, -8, 4, -2, 1, -\frac{1}{2}$$

$$137. \quad a_1 = 2, r = 3$$

$$a_4 = 2(3)^{4-1}$$

$$= 2(3)^3$$

$$= 2(27)$$

$$= 54$$

$$138. \quad a_1 = 16, r = \frac{1}{2}$$

$$a_6 = 16\left(\frac{1}{2}\right)^{6-1}$$

$$= 16\left(\frac{1}{2}\right)^5$$

$$= \frac{16}{32}$$

$$= \frac{1}{2}$$

$$139. \quad a_1 = -3, r = 2$$

$$a_5 = -3(2)^{5-1}$$

$$= -3(2)^4$$

$$= -3(16)$$

$$= -48$$

$$140. \quad a_n = a_1 r^{n-1} \text{ with } a_1 = 1 \text{ and } r = \frac{2}{1} = 2. \text{ Thus } a_n = 2^{n-1}$$

$$a_8 = 2^{8-1} = 2^7 = 128$$



**141.**  $a_n = a_1 r^{n-1}$  with  $a_1 = 100$  and  $r = \frac{10}{100} = \frac{1}{10}$ . Thus  $a_n = 100 \left( \frac{1}{10} \right)^{n-1}$

$$\begin{aligned} a_8 &= 100 \left( \frac{1}{10} \right)^{8-1} \\ &= 100 \left( \frac{1}{10} \right)^7 \\ &= \frac{100}{10,000,000} \\ &= \frac{1}{100,000} \end{aligned}$$

**142.** The common difference in the arithmetic sequence is 5.

$$\begin{aligned} 4 + 5 &= 9, 9 + 5 = 14, 14 + 5 = 19, \\ 19 + 5 &= 24, 24 + 5 = 29 \\ 4, 9, 14, 19, 24, 29, \dots \end{aligned}$$

**143.** The common ratio in the geometric sequence is 3.

$$\begin{aligned} 2 \cdot 3 &= 6, 6 \cdot 3 = 18, 18 \cdot 3 = 54, 54 \cdot 3 = 162, 162 \cdot 3 = 486 \\ 2, 6, 18, 54, 162, 486, \dots \end{aligned}$$

**144.** The common ratio in the geometric sequence is  $\frac{1}{4}$ .

$$\begin{aligned} 1 \cdot \frac{1}{4} &= \frac{1}{4}, \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}, \frac{1}{16} \cdot \frac{1}{4} = \frac{1}{64}, \frac{1}{64} \cdot \frac{1}{4} = \frac{1}{256}, \frac{1}{256} \cdot \frac{1}{4} = \frac{1}{1024} \\ 1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}, \frac{1}{1024}, \dots \end{aligned}$$

**145.** The common difference in the arithmetic sequence is  $-7$ .

$$\begin{aligned} 0 - 7 &= -7, -7 - 7 = -14, -14 - 7 = -21, -21 - 7 = -28, -28 - 7 = -35 \\ 0, -7, -14, -21, -28, -35, \dots \end{aligned}$$

**146. a.** 
$$\begin{aligned} a_n &= 34.5 + (n-1)(-0.3) \\ &= 34.5 - 0.3n + 0.3 \\ &= 34.8 - 0.3n \end{aligned}$$

**b.** 
$$\begin{aligned} a_n &= 34.8 - 0.3n \\ a_{56} &= 34.8 - 0.3(56) \\ &= 18 \end{aligned}$$

The model projects wives will devote 18 hours per week to housework in 2020.

147. a. Divide each value by the previous value:

$$\frac{18.80}{18.44} \approx 1.02$$

$$\frac{18.44}{18.15} \approx 1.02$$

$$\frac{18.15}{17.86} \approx 1.02$$

$$\frac{17.86}{17.58} \approx 1.02$$

$$\frac{17.58}{17.30} \approx 1.02$$

$$\frac{17.30}{17.03} \approx 1.02$$

$$\frac{17.03}{16.76} \approx 1.02$$

$$\frac{16.76}{16.50} \approx 1.02$$

$$\frac{16.50}{16.24} \approx 1.02$$

$$\frac{16.24}{15.98} \approx 1.02$$

The population is increasing geometrically with  $r = 1.02$ .

b.  $a_n = 15.98(1.02)^n$

- c. 2080 is 8 decades after 2000 so  $n = 8$ .

$$a_n = 15.98(1.02)^{n-1}$$

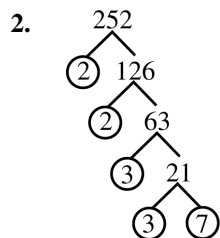
$$a_{31} = 15.98(1.02)^{31-1}$$

$$\approx 28.95$$

In 2030, the model predicts Florida's population will be 28.95 million.

## Chapter 1 Test

1. 391,248
  - 2: Yes; the last digit is 8.
  - 3: Yes; the sum of the digits is 27, which is divisible by 3.
  - 4: Yes; the last two digits form 48, which is divisible by 4.
  - 5: No; the number does not end in 0 or 5.
  - 6: Yes; the number is divisible by both 2 and 3.
  - 8: Yes; the last three digits form 248, which is divisible by 8.
  - 9: Yes; the sum of the digits is 27, which is divisible by 9.
  - 10: No; the number does not end in 0.
  - 12: Yes; the number is divisible by both 3 and 4.
- 391, 248 is divisible by 2, 3, 4, 6, 8, 9, 12.



$$252 = 2^2 \cdot 3^2 \cdot 7$$

3.  $48 = 2^4 \cdot 3$

$$72 = 2^3 \cdot 3^2$$

$$\text{Greatest Common Divisor} = 2^3 \cdot 3 = 24$$

$$\text{Least Common Multiple} = 2^4 \cdot 3^2 = 144$$

4.  $-6 - (5 - 12) = -6 - (-7) = -6 + 7 = 1$

5. 
$$\begin{aligned} (-3)(-4) \div (7 - 10) &= (-3)(-4) \div (-3) \\ &= 12 \div (-3) \\ &= -4 \end{aligned}$$

6. 
$$\begin{aligned} (6 - 8)^2 (5 - 7)^3 &= (-2)^2 (-2)^3 \\ &= 4(-8) \\ &= -32 \end{aligned}$$

7.  $\frac{7}{12} = 0.58\overline{3}$

$$\begin{array}{r} 0.5833\ldots \\ 12 \overline{) 7.0000} \\ \underline{60} \phantom{00} \\ 100 \phantom{00} \\ \underline{96} \phantom{00} \\ 40 \phantom{00} \\ \underline{36} \phantom{00} \\ 40 \phantom{00} \\ \underline{36} \phantom{00} \\ 4 \end{array}$$

8.  $n = 0.6464\ldots$

$$100n = 64.6464\ldots$$

$$100n = 64.6464\ldots$$

$$- \quad n = 0.6464\ldots$$

$$\hline 99n = 64$$

$$n = \frac{64}{99}$$

$$\begin{aligned}
 9. \quad \left(-\frac{3}{7}\right) \div \left(-2\frac{1}{7}\right) &= \left(-\frac{3}{7}\right) \div \left(-\frac{15}{7}\right) \\
 &= \left(-\frac{3}{7}\right) \cdot \left(-\frac{7}{15}\right) \\
 &= \frac{(-3)(-7)}{7 \cdot 15} \\
 &= \frac{21}{105} \\
 &= \frac{1}{5}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \frac{19}{24} - \frac{7}{40} &= \frac{19}{24} \cdot \frac{5}{5} - \frac{7}{40} \cdot \frac{3}{3} \\
 &= \frac{95}{120} - \frac{21}{120} \\
 &= \frac{74}{120} \\
 &= \frac{37}{60}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad \frac{1}{2} - 8\left(\frac{1}{4} + 1\right) &= \frac{1}{2} - 8\left(\frac{5}{4}\right) \\
 &= \frac{1}{2} - 10 \\
 &= \frac{1}{2} - \frac{20}{2} \\
 &= -\frac{19}{2}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad \frac{1}{2} + \frac{2}{3} &= \frac{1}{2} \cdot \frac{3}{3} + \frac{2}{3} \cdot \frac{2}{2} \\
 &= \frac{3}{6} + \frac{4}{6} \\
 &= \frac{7}{6} \\
 \frac{7}{6} \div 2 &= \frac{7}{6} \cdot \frac{1}{2} = \frac{7}{12}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \sqrt{10} \cdot \sqrt{5} &= \sqrt{10 \cdot 5} \\
 &= \sqrt{50} \\
 &= \sqrt{25 \cdot 2} \\
 &= \sqrt{25} \cdot \sqrt{2} \\
 &= 5\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \sqrt{50} + \sqrt{32} &= \sqrt{25} \cdot \sqrt{2} + \sqrt{16} \cdot \sqrt{2} \\
 &= 5\sqrt{2} + 4\sqrt{2} \\
 &= (5+4)\sqrt{2} \\
 &= 9\sqrt{2}
 \end{aligned}$$

$$15. \quad \frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{\sqrt{4}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

$$16. \quad \text{The rational numbers are:}$$

$$-7, -\frac{4}{5}, 0, 0.25, \sqrt{4}, \frac{22}{7}$$

17. Commutative property of addition

18. Distributive property of multiplication over addition

$$19. \quad 3^3 \cdot 3^2 = 3^{3+2} = 3^5 = 243$$

$$20. \quad \frac{4^6}{4^3} = 4^{6-3} = 4^3 = 64$$

$$21. \quad 8^{-2} = \frac{1}{8^2} = \frac{1}{64}$$

$$\begin{aligned}
 22. \quad (3 \times 10^8)(2.5 \times 10^{-5}) &= (3 \times 2.5) \times 10^{8-5} \\
 &= 7.5 \times 10^3 \\
 &= 7500
 \end{aligned}$$

$$\begin{aligned}
 23. \quad \frac{49,000}{0.007} &= \frac{4.9 \times 10^4}{7 \times 10^{-3}} \\
 &= \left( \frac{4.9}{7} \right) \times 10^{4-(-3)} \\
 &= 0.7 \times 10^7 \\
 &= 7 \times 10^{-1} \times 10^7 \\
 &= 7 \times 10^6
 \end{aligned}$$

$$24. \quad \$53.6 \times 10^9 = (\$5.36 \times 10^1) \times 10^9 = \$5.36 \times 10^{10}$$

$$25. \quad 307 \times 10^6 = (3.07 \times 10^2) \times 10^6 = 3.07 \times 10^8$$

$$\begin{aligned}
 26. \quad \frac{5.36 \times 10^{10}}{3.07 \times 10^8} &= \left( \frac{5.36}{3.07} \right) \times 10^2 \\
 &\approx 1.75 \times 10^2 \\
 &\approx \$175
 \end{aligned}$$

**27.**  $a_1 = 1, d = -5$

$$1, 1 - 5 = -4, -4 - 5 = -9,$$

$$-9 - 5 = -14, -14 - 5 = -19,$$

$$-19 - 5 = -24$$

$$1, -4, -9, -14, -19, -24$$

**28.**  $a_1 = -2, d = 3$

$$a_9 = -2 + (9-1)(3)$$

$$= -2 + 8(3)$$

$$= -2 + 24$$

$$= 22$$

**29.**  $a_1 = 16, r = \frac{1}{2}$

$$16, 16 \cdot \frac{1}{2} = 8, 8 \cdot \frac{1}{2} = 4, 4 \cdot \frac{1}{2} = 2, 2 \cdot \frac{1}{2} = 1, 1 \cdot \frac{1}{2} = \frac{1}{2}$$

$$16, 8, 4, 2, 1, \frac{1}{2}$$

**30.**  $a_1 = 5, r = 2$

$$a_7 = 5(2)^{7-1}$$

$$= 5(2)^6$$

$$= 5(64)$$

$$= 320$$