SUPPLEMENTAL TOPIC 1
EXERCISE SOLUTIONS

S1.1 Yes, X is a hypergeometric random variable with $N = 500$, $n = 60$, $a = 300$.

S1.2 Yes, X is a hypergeometric random variable with $N = 250$, $n = 5$, $a = 100$.

S1.3 No, X is a binomial random variable. There isn't a fixed population with units of two types.

S1.4 Yes, X is a hypergeometric random variable with $N = 10$ million, $n = 1000$, $a = (.4)(10{,}000{,}000) = 4{,}000{,}000$.

S1.5 $N = 100$, $n = 12$, $a = 40$.

S1.6 X is an integer between 0 and the smaller of (a, n), so in this case it's an integer between 0 and 12.

S1.7 $N = 16$, $n = 8$, $a = 5$.

S1.8 X is an integer between 0 and the smaller of (a, n), so in this case it's an integer between 0 and 5.

S1.9 X = number of winning numbers between 1 and 31; X is a hypergeometric random variable with $N = 51$, $n = 6$ and $a = 31$. The questions asks for $P(X = 6)$ =

$$\frac{\frac{a!}{k!(a-k)!}\,\frac{b!}{(n-k)!(b-n+k)!}}{\frac{N!}{n!(N-n)!}} = \frac{\frac{31!}{6!(31-6)!}\,\frac{20!}{(6-6)!(20-6+6)!}}{\frac{51!}{6!(51-6)!}} = .0409$$

S1.10 **a.** $N = 39$, $n = 5$, $a = 5$

b. $$P(X=0) = \frac{\frac{a!}{k!(a-k)!}\,\frac{b!}{(n-k)!(b-n+k)!}}{\frac{N!}{n!(N-n)!}} = \frac{\frac{5!}{0!(5-0)!}\,\frac{34!}{(5-0)!(34-5+0)!}}{\frac{39!}{5!(39-5)!}} = .4833$$

c. $$P(X=1) = \frac{\frac{a!}{k!(a-k)!}\,\frac{b!}{(n-k)!(b-n+k)!}}{\frac{N!}{n!(N-n)!}} = \frac{\frac{5!}{1!(5-1)!}\,\frac{34!}{(5-1)!(34-5+1)!}}{\frac{39!}{5!(39-5)!}} = .4027$$

d. $$P(X=2) = \frac{\frac{a!}{k!(a-k)!}\,\frac{b!}{(n-k)!(b-n+k)!}}{\frac{N!}{n!(N-n)!}} = \frac{\frac{5!}{2!(5-2)!}\,\frac{34!}{(5-2)!(34-5+2)!}}{\frac{39!}{5!(39-5)!}} = .1039$$

e. $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - .4833 - .4027 = .114$

S1.11 Define X = number of girls selected. X is a hypergeometric random variable with $N = 22$, $n = 4$ and $a = 10$.

$$P(X=4) = \frac{\frac{a!}{k!(a-k)!}\,\frac{b!}{(n-k)!(b-n+k)!}}{\frac{N!}{n!(N-n)!}} = \frac{\frac{10!}{4!(10-4)!}\,\frac{12!}{(4-4)!(12-4+4)!}}{\frac{22!}{4!(22-4)!}} = .0287$$

S1.12 The answer will differ for each student. In general, there must be a population of 25 with 15 of one type and 10 of another type, and 5 are drawn without replacement. X = number out of the 5 who are of the first type.

S1.13 **a.** Because sampling is without replacement, X is a hypergeometric random variable, although the population is so large compared to the sample that it is essentially a binomial random variable. $\mu = np =$

(1000)(.38) = 380 and σ = $\sqrt{np(1-p)\dfrac{(N-n)}{(N-1)}}$ = $\sqrt{1000(.38)(.62)\dfrac{8{,}000{,}000-1000}{7{,}999{,}999}}$ = 15.348. (If X is treated as binomial, σ = 15.349.)

b. X is a binomial random variable. $\mu = np = (100)(.25) = 25$ and $\sigma = \sqrt{np(1-p)} = \sqrt{100(.25)(.75)} = 4.33$.

c. X is a binomial random variable. $\mu = np = (200)(.20) = 40$ and $\sigma = \sqrt{np(1-p)} = \sqrt{100(.2)(.8)} = 5.66$.

d. X is a hypergeometric random variable. $\mu = np = (100)(.2) = 20$ and $\sigma = \sqrt{np(1-p)\dfrac{(N-n)}{(N-1)}}$ = $\sqrt{100(.2)(.8)\dfrac{(200-100)}{(200-1)}} = 2.836$.

S1.14 X = number of red candies Maria receives. P($X \le 5$) = essentially 0.

S1.15 **a.** $P(X=0) = \dfrac{\mu^k e^{-\mu}}{k!} = \dfrac{1^0 e^{-1}}{0!} = \dfrac{e^{-1}}{1} = .3679$.

b. $P(X \ge 1) = 1 - P(X = 0) = 1 - .3679 = .6321$.

c. $P(X=2) = \dfrac{\mu^k e^{-\mu}}{k!} = \dfrac{1^2 e^{-1}}{2!} = \dfrac{e^{-1}}{2} = .1839$

S1.16 $P(X=0) = \dfrac{\mu^k e^{-\mu}}{k!} = \dfrac{5^0 e^{-5}}{0!} = e^{-5} = .0067$.

S1.17 X is a Poisson random variable with mean μ = 6(5) = 30.

S1.18 A Poisson random variable is discrete because it can take a countable number of values, not a range of values in a continuous interval.

S1.19 $P(X \ge 20) = 1 - P(X \le 19) = 1 - .9965 = .0035$.

S1.20 **a.** $P(X=0) = \dfrac{\mu^k e^{-\mu}}{k!} = \dfrac{3^0 e^{-3}}{0!} = e^{-3} = .0498$.

b. $P(X \ge 1) = 1 - P(X = 0) = 1 - .0498 = .9502$.

c. $P(X > 3) = 1 - P(X \le 2) = 1 - .4232 = .5768$.

S1.21 **a.** Poisson with mean = 1.

b. $P(X > 3) = 1 - P(X \le 2) = 1 - .9197 = .0803$.

S1.22 **a.** μ = 30 × 2 = 60.

b. μ = 4/2 = 2.

c. μ = 3 × 4 = 12.

d. μ = 7/5 = 1.4.

S1.23 **a.** Mean = variance = 4, standard deviation = 2.

b. Mean = variance = 0.25, standard deviation = 0.5.

c. Mean = variance = 25, standard deviation = 5.

S1.24 **a.** $e^{-4} = .0183$.

b. $e^{-0.25} = .7788$.

c. $e^{-25} = .1.4 \times 10^{-11}$.

S1.25 The mean and variance are equal for a Poisson random variable and in this case, the sample mean and variance are so far apart that it is not likely that the population mean and variance are equal.

S1.26 **a.** Yes, n is large, p is small and $np = 5$ is less than 7. $\mu = np = 5$.
b. No. Even though $np = 5$ is less than 7, n is not large and p is not small.
c. No. Even though n is large and p is small, $np = 100$ is much larger than 7 and is too large.
d. Yes, n is large, p is small and $np = 1$ is less than 7. $\mu = np = 1$.

S1.27 **a.** $n = 200$, $p = .01$.
b. X is approximately a Poisson random variable with $\mu = np = 2$, so the Poisson formula can be used to find probabilities related to X.
c. $P(X = 0) = (.99)^{200} = .1340$.
d. Approximate $P(X = 0) = e^{-2} = .1353$. The answers are very similar. There is about a 13.5% chance of no defective candies in a bag.

S1.28 **a.** $n = 403$, $p = .01$.
b. $\mu = np = 4.03$.
c. If X is at least 3 the hotel will not be oversold, so $X = 0, 1, 2, 3$.
d. $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 0.0178 + 0.0716 + 0.1443 + 0.1939 = .4276$.

S1.29 There are n =30 trials where a "trial" is one roll of the die. There are $k = 6$ possible outcomes on each trial, where the outcome is the number of dots on the top face. The outcomes are independent from one roll to the next. The probabilities associated with each face showing remain constant from one roll to the next; p_i = 1/6 for $i = 1, 2, ..., 6$.

S1.30 $P(X_1 = 5, X_2 = 5,...,X_6 = 5) = \dfrac{30!}{(5!)(5!)(5!)(5!)(5!)(5!)}\left(\dfrac{1}{6}\right)^{30}$

S1.31 Mean = $np_i = 30(1/6) = 5$, variance = $np_i(1 - p_i) = 30(1/6)(5/6) = 4.167$.

S1.32 No. The number of trials is random and is not fixed in advance.

S1.33 Although there are a fixed number of trials (200 cars) and 7 possible outcomes, it is unlikely that the outcomes are independent from one trial to the next because cars are likely to observe the lines before choosing one. Therefore, it is not a multinomial experiment. If traffic was so sparse that only one car arrived at a time, with no lines, then it would be a multinomial experiment.

S1.34 Yes, this is a multinomial experiment. There are 1000 "trials" with 3 possible outcomes on each one and the outcomes are independent from one person to the next.

S1.35 Binomial with $n = 1000$ and p = proportion of all voters who support Candidate A.

S1.36 **a.** $P(X_1 = 1, X_2 = 1, X_3 = 3) = \dfrac{5!}{(1!)(1!)(3!)}(.2)^1(.6)^1(.2)^3 = \dfrac{5\times 4}{1}(.12)(.008) = .0192$.

b. $P(X_1 = 2, X_2 = 0, X_3 = 3) = \dfrac{5!}{(2!)(0!)(3!)}(.2)^2(.6)^0(.2)^3 = \dfrac{5\times 4}{2}(.04)(.008) = .0032$

c. $P(X = 3) = P(X_1 = 0, X_2 = 2, X_3 = 3) + P(X_1 = 1, X_2 = 1, X_3 = 3) + P(X_1 = 2, X_2 = 0, X_3 = 3) = .0288 + .0192 + .0032 = .0512$.