Chapter 1

1.1 Exercises

1. This is an example of inductive reasoning because you are using the specific pattern of adding 2 each time to guess the next number.

2. This is an example of inductive reasoning because you are using the observed pattern of 61-minute intervals to hypothesize that this pattern will continue.

3. This is an example of deductive reasoning because you are applying the general formula for perimeter of a square to a specific square.

4. This is an example of deductive reasoning because you are applying the general policy for re-stocking fees to a specific item.

5. This is an example of a deductive argument because a specific conclusion, “you can expect it to be ready in ten days,” is drawn from the two given premises.

6. This is an example of a deductive argument because the specific conclusion, “you will feel better,” is drawn from the two given premises.

7. This is an example of inductive reasoning because you are reasoning from a specific pattern to the conclusion that “It will also rain tomorrow.”

8. This is an example of inductive reasoning because you are reasoning from the specific pattern of all boys (so far) to the conclusion that the next child will also be a boy.

9. This represents deductive reasoning since you are moving from a general rule (formula for area of a circle) to a specific result (a search area of over 500,000 square miles).

10. This represents deductive reasoning since you are moving from a general rule (subtraction property of equality) to a specific result (the subtraction of 13 from both sides).

11. This is a deductive argument where you are reasoning from the two given premises. The first, “If you build it, they will come” is a general statement and the conclusion, “they will come” is specific.

12. This is deductive reasoning because (as in Exercise 11) you are reasoning from the given premises, the first being general, to a specific conclusion “Socrates is mortal.”

13. This is an example of inductive reasoning because you are reasoning from a specific pattern of all previous attendees to a conclusion that the next one “I” will also be accepted into graduate school.

14. This is an example of inductive reasoning because you are reasoning from a specific historical pattern where the summer flowers have alternated their colorings from summer to summer to a specific conclusion about the color of this summer’s flowers.

15. This is an example of inductive reasoning because you are reasoning from a specific pattern to a general rule to obtain the next number in the sequence.

16. This is an example of inductive reasoning because you are reasoning from a specific pattern to a generalization as to the reason why the fleas no longer jump on command.

17. Writing exercise; answers will vary.

18. Writing exercise; answers will vary.

19. Each number in the list is obtained by adding 3 to the previous number. The most probable next term is $18+3=21$.

20. The probable next term is 38 because each term in the list (after the first) can be obtained by adding 5 to the previous term.

21. Each number in the list is obtained by multiplying the previous number by 4. The most probable next term is $4\times 768=3072$.

22. The probable next term is 1 because each term following the first can be obtained by taking one half of the previous term (or dividing the preceding term by 2).

23. Beginning with the third term, each number in the sequence is the sum of the two previous terms.

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$9 = (3+6)$
$15 = (6+9)$
$24 = (9+15)$
$39 = (15+24)$
The most probable next term is
$63 = (24+39)$.

24. $\frac{11}{13}$ is the probable next term. The apparent pattern is to add 2 to both numerator and denominator.

25. The numerators and denominators are consecutive counting numbers. The probable next term is $\frac{11}{12}$.

26. The next term is probably 36, which is 6^2. Notice that the other numbers can be computed as 1^2, 2^2, 3^2, 4^2, 5^2.

27. The most probable next term is $6^3 = 216$. Observe the sequence:
$1 = 1^3$
$8 = 2^3$
$27 = 3^3$
$64 = 4^3$
$125 = 5^3$
This sequence is made up of the cubes of each counting number.

28. The next term is probably 56. Note that each term (after the first term) may be computed by adding successively 4, 6, 8, 10, and 12 to each preceding term. Thus, it follows that a probable next term would be $42+14=56$.

29. The probable next term is 52. Note that each term (after the first) may be computed by adding successively 3, 5, 7, 9, and 11 to each preceding term. Thus, it follows that a probable next term would be
$39+13=52$.

30. The probable next term is 7. Note that each term after the first may be obtained by subtracting successively 6, 5, 4, and 3. Thus it follows that the next term would be
$9-2=7$.

31. The probable next term is 5 since the sequence of numbers seems to add one more 5 each time the 5's precede the number 3.

32. The probable next term is 2 since the sequence of numbers seems to add one more 2 each time the 2's follow the number 8.

33. There are many possibilities. One such list is 10, 20, 30, 40, 50, … .

34. There are many possibilities. One such list is 48, 38, 29, 21, 14.

35.
$(9\times 9)+7 = 88$
$(98\times 9)+6 = 888$
$(987\times 9)+5 = 8888$
$(9876\times 9)+4 = 88{,}888$
Observe that on the left, the pattern suggests that the digit 5 will be appended to the first number. Thus, we get (98,765 × 9) which is added to 3. On the right, the pattern suggests appending another digit 8 to obtain 888,888. Therefore,
$(98{,}765\times 9)+3 = 888{,}885+3 = 888{,}888$
By computation, the conjecture is verified.

36.
$(1\times 9)+2 = 11$
$(12\times 9)+3 = 111$
$(123\times 9)+4 = 1111$
$(1234\times 9)+5 = 11{,}111$
Observe that on the left, the pattern suggests that the digit 5 will be appended to the first number. Thus, $(12{,}345\times 9)+6 = 111{,}111$ which can be verified using a calculator to compute the left side of the equation.

37.
$3367\times 3 = 10{,}101$
$3367\times 6 = 20{,}202$
$3367\times 9 = 30{,}303$
$3367\times 12 = 40{,}404$
Observe that on the left, the pattern suggests that 3367 will be multiplied by the next multiple of 3, which is 15. On the right, the pattern suggests the result 50,505. The pattern suggests the following equation:
$3367\times 15 = 50{,}505$.
Multiply 3367×15 to verify the conjecture.

38.
$15{,}873\times 7 = 111{,}111$
$15{,}873\times 14 = 222{,}222$
$15{,}873\times 21 = 333{,}333$
$15{,}873\times 28 = 444{,}444$
Observe that on the left, the pattern suggests that 15,873 will be multiplied by the next multiple of 7, which is 35. On the right, the pattern suggests the result 555,555. The pattern suggests the following equation:

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$15,873 \times 35 = 555,555.$
Multiply to verify the conjecture.

39.
$$34 \times 34 = 1156$$
$$334 \times 334 = 111,556$$
$$3334 \times 3334 = 11,115,556$$
The pattern suggests the following equation:
$33,334 \times 33,334 = 1,111,155,556.$
Multiply $33,334 \times 33,334$ to verify the conjecture.

40.
$$11 \times 11 = 121$$
$$111 \times 111 = 12,321$$
$$1111 \times 1111 = 1,234,321$$
The pattern suggests the following equation:
$11,111 \times 11,111 = 123,454,321.$
Multiply $11,111 \times 11,111$ to verify the conjecture.

41.
$$3 = \frac{3(2)}{2}$$
$$3+6 = \frac{6(3)}{2}$$
$$3+6+9 = \frac{9(4)}{2}$$
$$3+6+9+12 = \frac{12(5)}{2}$$
The pattern suggests the following equation:
$$3+6+9+12+15 = \frac{15(6)}{2}.$$
Since both the left and right sides equal 45, the conjecture is verified.

42.
$$2 = 4-2$$
$$2+4 = 8-2$$
$$2+4+8 = 16-2$$
$$2+4+8+16 = 32-2$$
This pattern suggests the equation
$2+4+8+16+32 = 64-2.$
Since both the left and right sides equal 62, the conjecture is verified.

43.
$$5(6) = 6(6-1)$$
$$5(6)+5(36) = 6(36-1)$$
$$5(6)+5(36)+5(216) = 6(216-1)$$
$$5(6)+5(36)+5(216)+5(1296) = 6(1296-1)$$
Observe that the last equation may be written as:
$$5(6^1)+5(6^2)+5(6^3)+5(6^4) = 6(6^4-1).$$
Thus, the next equation would likely be:
$$5(6)+5(36)+5(216)+5(1296)+5(6^5) = 6(6^5-1)$$
or,
$$5(6)+5(36)+5(216)+5(1296)+5(7776) = 6(7776-1).$$

44.
$$3 = \frac{3(3-1)}{2}$$
$$3+9 = \frac{3(9-1)}{2}$$
$$3+9+27 = \frac{3(27-1)}{2}$$
$$3+9+27+81 = \frac{3(81-1)}{2}$$
Observe that the last equation may be written as:
$$3^1+3^2+3^3+3^4 = \frac{3(3^4-1)}{2}.$$
Thus, the next equation would likely be:
$$3^1+3^2+3^3+3^4+3^5 = \frac{3(3^5-1)}{2}, \text{ or}$$
$$3+9+27+81+243 = \frac{3(243-1)}{2}$$
$$363 = \frac{3(242)}{2} = \frac{726}{2} = 363.$$
Thus, the conjecture is verified.

45.
$$\frac{1}{2} = 1-\frac{1}{2}$$
$$\frac{1}{2}+\frac{1}{4} = 1-\frac{1}{4}$$
$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8} = 1-\frac{1}{8}$$
$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16} = 1-\frac{1}{16}$$
Observe that the last equation may be written as
$$\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4} = 1-\frac{1}{2^4}.$$
The next equation would be
$$\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5} = 1-\frac{1}{2^5}, \text{ or}$$
$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32} = 1-\frac{1}{32}.$$
Using the common denominator 32 for each fraction, the left and right side add (in each case) to $\frac{31}{32}$. The conjecture is, therefore, verified.

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46.
$$\frac{1}{1\cdot 2}=\frac{1}{2}$$
$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}=\frac{2}{3}$$
$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}=\frac{3}{4}$$
$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}=\frac{4}{5}$$
This pattern suggests the following equation:
$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}=\frac{5}{6}.$$
This is verified by:
$$\begin{aligned}&\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}\\ &=\frac{30}{60}+\frac{10}{60}+\frac{5}{60}+\frac{3}{60}+\frac{2}{60}\\ &=\frac{30+10+5+3+2}{60}\\ &=\frac{50}{60}\\ &=\frac{5}{6}\end{aligned}$$

47. 1 + 2 + 3 + … +200
Pairing and adjoining the first term to the last term, the second term to the second-to-last term, etc., we have:
1 + 200 = 201, 2 + 199 = 201,
3 + 198 = 201, …
There are 100 of these sums. Therefore, $100\times 201 = 20{,}100$.

48. 1 + 2 + 3 + … + 400
Pairing and adjoining the first term to the last term, the second term to the second-to-last term, etc., we have:
1 + 400 = 401, 2 + 399 = 401,
3 + 398 = 401, …
There are 200 of these sums. Therefore, $200\times 401 = 80{,}200$.

49. 1 + 2 + 3 + … + 800
Pairing and adjoining the first term to the last term, the second term to the second-to-last term, etc., we have:
1 + 800 = 801, 2 + 799 = 801,
3 + 798 = 801, …
There are 400 of these sums. Therefore, $400\times 801 = 320{,}400$.

50. 1 + 2 + 3 + … + 2000
Observe that there would be 1000 pairs (1 + 2000 = 2001, 2 + 1999 = 2001, …).
Thus, $1000\times 2001 = 2{,}001{,}000$.

51. 1 + 2 + 3 + … + 175
Note that there are an odd number of terms. So consider omitting, for the moment, the last term and take 1 + 174 = 175, 2 + 173 = 175, 3 + 172 = 175, etc. There are $\frac{174}{2}=87$ of these pairs in addition to the last term. Thus, $(87\times 175)+175$, or $88\times 175 = 15{,}400$.

52. Writing exercise; answers will vary.

53.
$$\begin{aligned}2+4+6+...+100 &= 2(1+2+3+\cdots+50)\\ &= 2[25(1+50)]\\ &= 2(1275)\\ &= 2550\end{aligned}$$

54.
$$\begin{aligned}4+8+12+...+200 &= 4(1+2+3+\cdots+50)\\ &= 4[25(1+50)]\\ &= 4(1275)\\ &= 5100\end{aligned}$$

55. These are the number of chimes a clock rings, starting with 12 o'clock, if the clock rings the number of hours on the hour and 1 chime on the half-hour. The next most probable number is the number of chimes at 3:30, which is 1.

56. E, which represents the first letter of eleven (One, Two, Three, and so on)

57. **(a)** Here are three examples.

623	841	584
–326	–148	–485
297	693	99

(b) In each result, the middle digit is always 9, and the sum of the first and third digits is always 9 (considering 0 as the first digit if the difference has only two digits).

(c) Writing exercise; answers will vary.

58. **(a)-(e)** Answers will vary.

(f) Try several numbers. The last result will always be half of the number added in step (b).

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59. $142,857 \times 1 = 142,857$
$142,857 \times 2 = 285,714$
$142,857 \times 3 = 428,571$
$142,857 \times 4 = 571,428$
$142,857 \times 5 = 714,285$
$142,857 \times 6 = 857,142$

Each result consists of the same six digits, but in a different order. But $142,857 \times 7 = 999,999$. Thus, the pattern doesn't continue.

60. Count the chords and record the results.

No. of points	No. of chords	No. of chords added
2	1	1
3	3	2
4	6	3
5	10	4
6	15	5

By the pattern, 6 more chords would be added for a total of 21 chords.

1.2 Exercises

1. If we choose any term after the first term, and subtract the preceding term, the common difference is 10. Therefore, this is an arithmetic sequence. The next term in the sequence is $46 + 10 = 56$.

2. If we choose any term after the first term, and subtract the preceding term, the common difference is 8. Therefore, this is an arithmetic sequence. The next term in the sequence is $40 + 8 = 48$.

3. If any term after the first is multiplied by 3, the following term is obtained. Therefore, this is a geometric sequence. The next term in the sequence is $405 \cdot 3 = 1215$.

4. If any term after the first is multiplied by 6, the following term is obtained. Therefore, this is a geometric sequence. The next term in the sequence is $2592 \cdot 6 = 15,552$.

5. There is neither a common difference nor a common ratio. This is neither an arithmetic nor a geometric sequence.

6. There is neither a common difference nor a common ratio. This is neither an arithmetic nor a geometric sequence.

7. If any term after the first is multiplied by $\frac{1}{2}$, the following term is obtained. Therefore, this is a geometric sequence. The next term in the sequence is $16 \cdot \frac{1}{2} = 8$.

8. If any term after the first is multiplied by $\frac{1}{4}$, the following term is obtained. Therefore, this is a geometric sequence. The next term in the sequence is $16 \cdot \frac{1}{4} = 4$.

9. There is neither a common difference nor a common ratio. This is neither an arithmetic nor a geometric sequence.

10. There is neither a common difference nor a common ratio. This is neither an arithmetic nor a geometric sequence.

11. If we choose any term after the first term, and subtract the preceding term, the common difference is 2. Therefore, this is an arithmetic sequence. The next term in the sequence is $20 + 2 = 22$.

12. If we choose any term after the first term, and subtract the preceding term, the common difference is 40. Therefore, this is an arithmetic sequence. The next term in the sequence is $170 + 40 = 210$.

13. 1 4 11 22 37 56 <u>79</u>
3 7 11 15 19 <u>23</u>
4 4 4 4 (4)

Each line represents the difference of the two numbers above it. The number 23 is found from adding the predicted difference, (4), in line three to 19 in line 2. And 79 is found by adding 23, in line two, to 56 in line one. Thus, our next term in the sequence is 79.

14. 3 14 31 54 83 118 <u>159</u>
11 17 23 29 35 <u>41</u>
6 6 6 6 (6)

The number 41 is found from adding a predicted difference, (6), to 35. And 159 is found from adding 41 to 118. Thus, our next term in the sequence is 159.

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15. 6 20 50 102 182 296 $\underline{450}$
14 30 52 80 114 $\underline{154}$
16 22 28 34 $\underline{40}$
6 6 6 (6)
Thus, our next term in the sequence is
154 + 296 = 450.

16. 1 11 35 79 149 251 $\underline{391}$
10 24 44 70 102 $\underline{140}$
14 20 26 32 $\underline{38}$
6 6 6 (6)
Thus, our next term in the sequence is
140 + 251 = 391.

17. 0 12 72 240 600 1260 2352 $\underline{4032}$
12 60 168 360 660 1092 $\underline{1680}$
48 108 192 300 432 $\underline{588}$
60 84 108 132 $\underline{156}$
24 24 24 (24)
Thus, our next term in the sequence is
$1680+2352=4032.$

18. 2 57 220 575 1230 2317 $\underline{3992}$
55 163 355 655 1087 $\underline{1675}$
108 192 300 432 $\underline{588}$
84 108 132 $\underline{156}$
24 24 (24)
Thus, our next term in the sequence is
$1675+2317=3992.$

19. 5 34 243 1022 3121 7770 16799 $\underline{32758}$
29 209 779 2099 4649 9029 $\underline{15959}$
180 570 1320 2550 4380 $\underline{6930}$
390 750 1230 1830 $\underline{2550}$
360 480 600 $\underline{720}$
120 120 (120)
Thus, our next term in the sequence is
$15959+16799=32,758.$

20. 3 19 165 771 2503 6483 14409 $\underline{28675}$
16 146 606 1732 3980 7926 $\underline{14266}$
130 460 1126 2248 3946 $\underline{6340}$
330 666 1122 1698 $\underline{2394}$
336 456 576 $\underline{696}$
120 120 (120)
Thus, our next term in the sequence is
$14,266+14,409=28,675.$

21. 1 2 4 8 16 31 (57) 99
1 2 4 8 15 $\underline{26}$ $\underline{42}$
1 2 4 7 11 $\underline{16}$
1 2 3 4 $\underline{5}$
1 1 1 (1)
The next term of the sequence is 57.
Following this pattern, we predict that the number of regions determined by 8 points is 99. Use $n=8$ in the formula

$$\frac{n^4-6n^3+23n^2-18n+24}{24}.$$

$$\frac{8^4-6\times8^3+23\times8^2-18\times8+24}{24}$$
$$=\frac{4096-3072+1472-144+24}{24}$$
$$=\frac{2376}{24}$$
$$=99$$

Thus, the result agrees with our prediction.

22. **(a)** Continuing the pattern, the next addition fact would be 16 + 16 = 32.

(b) The next four facts would be
32 + 32 = 64; 64 + 64 = 128;
128 + 128 = 256; 256 + 256 = 512.

23. By the pattern, the next equation is
$(4321\times9)-1=38,888.$
To verify, calculate left side and compare,
$38,889-1=38,888.$

24.
$$(1\times8)+1=9$$
$$(12\times8)+2=98$$
$$(1238\times8)+3=987$$
By the pattern, the next equation is
$(1234\times8)+4=9876.$
To verify, $9872+4=9876.$

25. $999,999\times2=1,999,998$
$999,999\times3=2,999,997$
By the pattern, the next equation is
$999,999\times4=3,999,996.$
To verify, multiply the left side to get
$3,999,996=3,999,996.$

26.
$$101\times101=10,201$$
$$10,101\times10,101=102,030,201$$
by the pattern, the next equation is
$1,010,101\times1,010,101=1,020,304,030,201.$
To verify, multiply the left side to get
$1,020,304,030,201=1,020,304,030,201.$

27.
$$3^2-1^2=2^3$$
$$6^2-3^2=3^3$$
$$10^2-6^2=4^3$$
$$15^2-10^2=5^3$$
Following this pattern, we see that the next equation will start with 21^2 since $15+6=21.$ This equation will be

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$21^2 - 15^2 = 6^3$. The left side is $441 - 225 = 216$. The right side also equals 216.

28. $1 = 1^2$

$1 + 2 + 1 = 2^2$

$1 + 2 + 3 + 2 + 1 = 3^2$

$1 + 2 + 3 + 4 + 3 + 2 + 1 = 4^2$

By the pattern, the next equation is

$1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 5^{2.}$

To verify, 25 = 25.

29. $2^2 - 1^2 = 2 + 1$

$3^2 - 2^2 = 3 + 2$

$4^2 - 3^2 = 4 + 3$

Following this pattern, we see that the next equation will be $5^2 - 4^2 = 5 + 4$.

To verify, the left side is $25 - 16 = 9$. The right side also equals 9.

30. $1^2 + 1 = 2^2 - 2$

$2^2 + 2 = 3^2 - 3$

$3^2 + 3 = 4^2 - 4$

By the pattern, the next equation is

$4^2 + 4 = 5^2 - 5$.

To verify, evaluate each side of the equation to get

$16 + 4 = 25 - 5$

$20 = 20$

31. $1 = 1 \times 1$

$1 + 5 = 2 \times 3$

$1 + 5 + 9 = 3 \times 5$

The last term on the left side is 4 more than the previous last term. The first factor on the right side is the next counting number; the second factor is the next odd number. Thus, the probable next equation is

$1 + 5 + 9 + 13 = 4 \times 7$.

To verify, calculate both sides to arrive at 28 = 28.

32. $1 + 2 = 3$

$4 + 5 + 6 = 7 + 8$

$9 + 10 + 11 + 12 = 13 + 14 + 15$

By the pattern, the next equation is

$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$.

To verify, calculate both sides to arrive at 90 = 90.

33. $1 + 2 + 3 + \ldots + 300$

$$S = \frac{300(300+1)}{2} = \frac{90300}{2} = 45{,}150$$

34. $1 + 2 + 3 + \ldots + 500$

$$S + \frac{500(500+1)}{2} = \frac{250500}{2} = 125,250$$

35. $1 + 2 + 3 + \ldots + 675$

$$S = \frac{675(675+1)}{2} = \frac{456300}{2} = 228{,}150$$

36. $1 + 2 + 3 + \ldots + 825$

$$S = \frac{825(825+1)}{2} = \frac{681450}{2} = 340,725$$

37. $1 + 3 + 5 + 7 + \ldots + 101$

Note that

$n = \dfrac{1+101}{2} = 51$ terms, so that

$S = 51^2 = 2601$.

38. $1 + 3 + 5 + 7 + \ldots + 49$

Note that

$n = \dfrac{1+49}{2} = 25$ terms, so that

$S = 25^2 = 625$.

39. $1 + 3 + 5 + \ldots + 999$

Observe that

$n = \dfrac{1+999}{2} = 500$ terms, so that

$S = 500^2 = 250,000$.

40. $1 + 3 + 5 + 7 + \ldots + 301$

Observe that

$n = \dfrac{1+301}{2} = 151$ terms, so that

$S = 151^2 = 22,801$.

41. Since each term in the second series is twice that of the first series, we might expect the sum to be twice as large or

$$S = 2 \times \frac{n(n+1)}{2} = n(n+1).$$

42. Writing exercise; answers will vary.

43. Writing exercise; answers will vary.

44. Writing exercise; answers will vary.

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45.

Figurate Number	1st	2nd	3rd	4th	5th	6th	7th	8th
Triangular	1	3	6	10	15	21	28	36
Square	1	4	9	16	25	36	49	64
Pentagonal	1	5	12	22	35	51	70	92
Hexagonal	1	6	15	28	45	66	91	120
Heptagonal	1	7	18	34	55	81	112	148
Octagonal	1	8	21	40	65	96	133	176

46. The first five hexagonal numbers are
$1 = 1$
$6 = 1 + 5$
$15 = 1 + 5 + 9$
$28 = 1 + 5 + 9 + 13$
$45 = 1 + 5 + 9 + 13 + 17.$

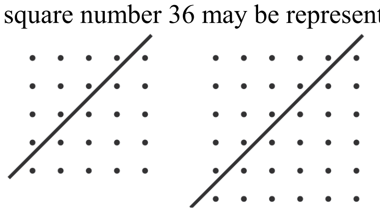
47. $8(1) + 1 = 9 = 3^2;\ 8(3) + 1 = 25 = 5^2;\ 8(6) + 1 = 49 = 7^2;\ 8(10) + 1 = 81 = 9^2$

48. The triangular numbers are
1, 3, 6, 10, 15, … .
$1 \div 3 = 0,$ remainder 1
$3 \div 3 = 1,$ remainder 0
$6 \div 3 = 2,$ remainder 0
$10 \div 3 = 3,$ remainder 1
$15 \div 3 = 5,$ remainder 0
$21 \div 3 = 7,$ remainder 0
The pattern of remainders is 1, 0, 0, 1, 0, 0, … .

49. The square numbers are 1, 4, 9, 25, 36, … .
$1 \div 4 = 0,$ remainder 1
$4 \div 4 = 1,$ remainder 0
$9 \div 4 = 2,$ remainder 1
$16 \div 4 = 4,$ remainder 0
$25 \div 4 = 6,$ remainder 1
$36 \div 4 = 9,$ remainder 0
The pattern of remainders is 1, 0, 1, 0, 1, 0, … .

50. Creating the first ten, or so, pentagonal numbers, we have: 1, 5, 12, 22, 35, 51, 70, 92, 117, 145, … . Dividing each by 5 (n = 5) generates the following sequence of remainders: 1, 0, 2, 2, 0, 1, 0, 2, 2, 0, … . Creating the first twelve, or so, hexagonal numbers, we have: 1, 6, 15, 28, 45, 66, 91, 120, 153, 190, 231, 276, … . Dividing each by 6 (n = 6) generates the following sequence of remainders: 1, 0, 3, 4, 3, 0, 1, 0, 3, 4, 3, 0, … . That is, in each case, we have generated a repeating sequence of numbers.

51. The square number 25 may be represented by the sum of the two triangular numbers 10 and 15. The square number 36 may be represented by the sum of the two triangular numbers 15 and 21.



52. **(a)** For $n = 2$ to $n = 8$, the fractions $\frac{n\text{th square number}}{(n+1)\text{th square number}}$

are $\frac{2^2}{3^2} = \frac{4}{9}$, $\frac{3^2}{4^2} = \frac{9}{16}$, $\frac{4^2}{5^2} = \frac{16}{25}$, $\frac{5^2}{6^2} = \frac{25}{36}$, $\frac{6^2}{7^2} = \frac{36}{49}$, $\frac{7^2}{8^2} = \frac{49}{64}$, $\frac{8^2}{9^2} = \frac{64}{81}$.

(b) For $n = 2$ to $n = 8$, the fractions $\frac{n\text{th triangular number}}{(n+1)\text{th triangular number}}$ are

$\frac{3}{6}$, $\frac{6}{10}$, $\frac{10}{15}$, $\frac{15}{21}$, $\frac{21}{28}$, $\frac{28}{36}$, and $\frac{36}{45}$.

(c) The fraction formed by two consecutive square numbers is always reduced to lowest terms, while the fraction formed by two consecutive triangular numbers is never reduced to lowest terms.

53. To find the sixteenth square number, use

$S_n = n^2$ with $n = 16$.

$S_{16} = 16^2 = 256$

54. To find the eleventh triangular number, use

$T_n = \frac{n(n+1)}{2}$ with $n = 11$.

$T_{11} = \frac{11(12)}{2} = 66$

55. To find the ninth pentagonal number, use

$P_n = \frac{n(3n-1)}{2}$ with $n = 9$.

$P_9 = \frac{9(26)}{2} = 117$

56. To find the seventh hexagonal number, use

$H_n = \frac{n(4n-2)}{2}$ with $n = 7$.

$H_7 = \frac{7(26)}{2} = 91$

57. To find the tenth heptagonal number, use

$Hp_n = \frac{n(5n-3)}{2}$ with $n = 10$.

$Hp_{10} = \frac{10(47)}{2} = 235$

58. To find the twelfth octagonal number, use

$O_n = \frac{n(6n-4)}{2}$ with $n = 12$.

$O_{12} = \frac{12(68)}{2} = 408$

59. Since each coefficient in parentheses appears to step up by 1, we would predict:

$N_n = \frac{n(7n-5)}{2}$.

$N_6 = \frac{6(37)}{2} = 111$

This verifies our prediction for $n = 6$.

60. Using Exercise 59 to find the tenth nonagonal number, we have

$N_n = \frac{n(7n-5)}{2}$ with $n = 10$.

$N_{10} = \frac{10(65)}{2} = 325$

61. The triangular numbers are
1, 3, 6, 10, 15, 21, 28, 36, 45, … .
Adding consecutive triangular numbers, for example,
1 + 3 = 4, 3 + 6 = 9, 6 + 10 = 16, … , will give square numbers.

62. The triangular numbers are
1, 3, 6, 10, 15, 21, 28, 36, 45, … .
The squares of each of these numbers are
1, 9, 36, 100, 225, 441, 784, 1296, … ,
Adding consecutive squares, for example,
1 + 9 = 10, 9 + 36 = 45, etc.,
will give triangular numbers.

63. In each case, you get a perfect cube number. That is, if we take the 2nd and 3rd triangular numbers 3 and 6, $6^2 - 3^2 = 36 - 9 = 27$ which is the perfect cube number 3^3.

64. $T_{n-1} = \frac{(n-1)(n)}{2}$ with $n = 3$.

$T_2 = \frac{2(3)}{2} = 3$

Multiply by 3: $3 \times 3 = 9$

Add $n = 3$: $9 + 3 = 12$

The result is the nth pentagonal number.

65. This sequence has a common difference of 4, so it is an arithmetic sequence with

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$n = 11$
$a_1 = 2$
$d = 4.$
Using the formula,
$a_n = a_1 + (n-1)d$
$a_{11} = 2 + (11-1) \cdot 4$
$a_{11} = 2 + 10 \cdot 4$
$a_{11} = 42$
The eleventh term in the sequence is 42.

66. This sequence has a common difference of 10, so it is an arithmetic sequence with
$n = 16$
$a_1 = 5$
$d = 10.$
Using the formula,
$a_n = a_1 + (n-1)d$
$a_{16} = 5 + (16-1) \cdot 10$
$a_{16} = 5 + 15 \cdot 10$
$a_{16} = 155$
The sixteenth term in the sequence is 155.

67. This sequence has a common difference of 20, so it is an arithmetic sequence with
$n = 21$
$a_1 = 19$
$d = 20.$
Using the formula,
$a_n = a_1 + (n-1)d$
$a_{21} = 19 + (21-1) \cdot 20$
$a_{21} = 19 + 20 \cdot 20$
$a_{21} = 419$
The 21st term in the sequence is 419.

68. This sequence has a common difference of 30, so it is an arithmetic sequence with
$n = 36$
$a_1 = 8$
$d = 30.$
Using the formula,
$a_n = a_1 + (n-1)d$
$a_{36} = 8 + (36-1) \cdot 30$
$a_{36} = 8 + 35 \cdot 30$
$a_{36} = 1058$
The 36th term in the sequence is 1058.

69. This sequence has a common difference of $\frac{1}{2}$, so it is an arithmetic sequence with
$n = 101$
$a_1 = \frac{1}{2}$
$d = \frac{1}{2}.$
Using the formula,
$a_n = a_1 + (n-1)d$
$a_{101} = \frac{1}{2} + (101-1) \cdot \frac{1}{2}$
$a_{101} = \frac{1}{2} + 100 \cdot \frac{1}{2}$
$a_{101} = \frac{101}{2}$
The 101st term in the sequence is $\frac{101}{2}$.

70. This sequence has a common difference of 0.75, so it is an arithmetic sequence with
$n = 151$
$a_1 = 0.75$
$d = 0.75.$
Using the formula,
$a_n = a_1 + (n-1)d$
$a_{151} = 0.75 + (151-1) \cdot 0.75$
$a_{151} = 0.75 + 150 \cdot 0.75$
$a_{151} = 113.25$
The 151st term in the sequence is 113.25.

71. This sequence has a common ration of 2, so it is a geometric sequence with
$n = 11$
$a_1 = 2$
$r = 2.$
Using the formula,
$a_n = a_1 \cdot r^{n-1}$
$a_{11} = 2 \cdot 2^{11-1}$
$a_{11} = 2 \cdot 2^{10}$
$a_{11} = 2048$
The eleventh term in the sequence is 2048.

72. This sequence has a common ration of 4, so it is a geometric sequence with
$n = 9$
$a_1 = 1$
$r = 4.$
Using the formula,
$a_n = a_1 \cdot r^{n-1}$
$a_9 = 1 \cdot 4^{9-1}$
$a_9 = 1 \cdot 4^8$
$a_9 = 65,536$
The ninth term in the sequence is 65,536.

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73. This sequence has a common ration of $\frac{1}{2}$, so it is a geometric sequence with

$n = 12$
$a_1 = 1$
$r = \frac{1}{2}$.

Using the formula,

$$a_n = a_1 \cdot r^{n-1}$$
$$a_{12} = 1 \cdot \left(\frac{1}{2}\right)^{12-1}$$
$$a_{12} = 1 \cdot \left(\frac{1}{2}\right)^{11}$$
$$a_{12} = \frac{1}{2048}$$

The 12th term in the sequence is $\frac{1}{2048}$.

74. This sequence has a common ration of $\frac{1}{3}$, so it is a geometric sequence with

$n = 10$
$a_1 = 1$
$r = \frac{1}{3}$.

Using the formula,

$$a_n = a_1 \cdot r^{n-1}$$
$$a_{10} = 1 \cdot \left(\frac{1}{3}\right)^{10-1}$$
$$a_{11} = 1 \cdot \left(\frac{1}{3}\right)^{9}$$
$$a_{11} = \frac{1}{19,683}$$

The 10th term in the sequence is $\frac{1}{19,683}$.

75. This sequence has a common ration of $\frac{1}{4}$, so it is a geometric sequence with

$n = 8$
$a_1 = 40$
$r = \frac{1}{4}$.

Using the formula,

$$a_n = a_1 \cdot r^{n-1}$$
$$a_8 = 40 \cdot \left(\frac{1}{4}\right)^{8-1}$$
$$a_8 = 40 \cdot \left(\frac{1}{4}\right)^{7}$$
$$a_8 = \frac{40}{16384}$$
$$a_8 = \frac{5}{2048}$$

The 8th term in the sequence is $\frac{5}{2048}$.

76. This sequence has a common ration of $\frac{2}{10} = \frac{1}{5}$, so it is a geometric sequence with

$n = 9$
$a_1 = 10$
$r = \frac{1}{5}$.

Using the formula,

$$a_n = a_1 \cdot r^{n-1}$$
$$a_9 = 10 \cdot \left(\frac{1}{5}\right)^{9-1}$$
$$a_9 = 10 \cdot \left(\frac{1}{5}\right)^{8}$$
$$a_9 = \frac{10}{390,625}$$
$$a_9 = \frac{2}{78,125}$$

The 9th term in the sequence is $\frac{2}{78,125}$.

77. Starting with 481, we complete the described process:

$$\begin{array}{r} 841 \\ -148 \\ \hline 693 \end{array} \qquad \begin{array}{r} 963 \\ -369 \\ \hline 594 \end{array} \qquad \begin{array}{r} 954 \\ -459 \\ \hline 495 \end{array}$$

Continuing the process will lead to 495 again, so the Kaprekar constant is 495.

78. **(a)** Start with 45986 and complete the process.

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$$45986 \rightarrow \begin{array}{r} 98654 \\ -45689 \\ \hline 52965 \end{array} \quad \begin{array}{r} 96552 \\ -25569 \\ \hline 70983 \end{array} \quad \begin{array}{r} 98730 \\ -03789 \\ \hline 97941 \end{array}$$

$$\begin{array}{r} 99441 \\ -14499 \\ \hline 84942 \end{array} \quad \begin{array}{r} 98442 \\ -24489 \\ \hline 73953 \end{array} \quad \begin{array}{r} 97533 \\ -33579 \\ \hline 63954 \end{array}$$

We see that 63954 is reached first.

(b) Answers will vary. For example, start with 12345.

$$\begin{array}{r} 54321 \\ -12345 \\ \hline 41976 \end{array} \quad \begin{array}{r} 97641 \\ -14679 \\ \hline 82962 \end{array}$$

In this case, 82962 is reached first.

79. To find any entry within the body of the triangle, add the two entries immediately to the left and to the right in the row above it. For example, in the sixth row, 10 = 4 + 6. The next three rows of Pascal's triangle are as follows.

1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

80. The first eight sums are 1, 2, 4, 8, 16, 32, 64, and 128. The pattern is that in each successive row, the sum is twice that of the previous row. A reasonable prediction is that the sum in the ninth row is 256, which is correct.

$$1+8+28+56+70+56+28+8+1=256$$

81. The sums along the diagonals are 1, 1, 2, 3, 5, and 8. These are the first six terms of the Fibonacci sequence.

82. The first four powers of 11, beginning with 11^0, are 1, 11, 121, and 1331. These correspond to the first four rows of Pascal's triangle. The fifth row of Pascal's triangle corresponds to 11^4, which is equal to 14,641.

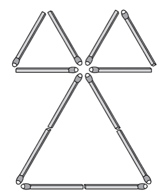
1.3 Exercises

1. The man runs from the first floor to the third floor in 20 seconds. This is a rate of 10 seconds per floor. Running from the first floor up to the sixth floor involves a change of five floors. Thus we have

$$\frac{10 \text{ sec}}{\text{floor}} \cdot 5 \text{ floors} = 50 \text{ seconds.}$$

2. Sally saves 36 cents per day. In order for her savings to be an integral multiple of dollars, it must be an integral multiple of 100 cents, which means it must be a multiple of $2 \cdot 2 \cdot 5 \cdot 5$ (the prime factorization of 100). The prime factorization of 36 is $2 \cdot 2 \cdot 3 \cdot 3$, so we need two factors of 5. Since $5 \cdot 5 = 25$, it will take 25 days for Sally to have a savings of $2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 = 900$ cents, or \$9.00.

3.



4.

1	3	2	4
2	4	3	1
3	1	4	2
4	2	1	3

5. With the integer assignments described, 1 could represent B, I, P, or W, and 6 could represent G, N, or U. Thus there are twelve possible two-letter sequences, as shown.

BG IG PG WG

BN **IN** PN WN

BU IU PU WU

The only sequence forming a word is IN.

6. The repeating sequence PROBLEMSOLVING contains 14 letters. Note that P is in positions 1, 15, 29, and so on, each having a remainder of 1 when divided by 14. Thus to find the letter in the 2012th position, we can divide 2012 by 14 to get a remainder of 10. Since L is the 10th letter in the sequence, it is in the 2012th position.

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7. We know that the ages of the children differ by 5 years, so we can guess the ages of the children, and then check to see that the sum of the squares of their ages is 97, and verify that when those squares are written one after the other as a four digit number, they form another square number (which will be the mother's age).

$$1^2 + 6^2 = 37$$
$$2^2 + 7^2 = 53$$
$$3^2 + 8^2 = 73$$
$$4^2 + 9^2 = 97$$
$$1681 = 41^2$$

Thus the mother is 41 years old.

8. We know that all three alarms go off at 12:34 p.m., so the next time they will go off simultaneously must be a multiple of all three intervals (20 min, 30 min, and 45 min). The least common multiple of all three is 180, so the next time will be 180 minutes (3 hours) later, or at 3:34 p.m.

9. Each Monday evening, the teacher drops off the shirts that he wore during the week (7 shirts). He also picks up the 7 shirts he had worn the previous week (which are now clean). But he is also wearing a shirt at the dry cleaners. This accounts for 15 shirts, so he must own at least 15 shirts.

10. You should choose the envelope labeled \$30. This way, you will know that you actually hold one of the envelopes containing only one kind of bill. Looking at one of the bills in that envelope will tell you its contents. Then use the fact that all the envelopes are incorrectly labeled to determine the contents of the remaining two envelopes.

11. When 8 girls leave, twice as many boys as girls remain. Because we started with an equal number of boys and girls, 8 is half the number of boys in the classroom. Thus, there are 16 boys, so there are 16 girls, for a total of 32 students.

12. Let x = the first digit and y = the second digit in the two-digit number. When we put a 6 as the right-most digit, the three-digit number is $xy6$.

$$\begin{array}{r} x \quad y \quad 6 \\ + \qquad\quad 6 \\ \hline x(y+1)\,2 \end{array}$$

and $x(y + 1) = 76$

so $x = 7$

$y + 1 = 6$

$y = 5.$

The original number is 75.

13. In each row, multiply the first digit by the last digit. The result is the two-digit number between them.

$3 \times 8 = 24$

$7 \times 3 = 21$

$8 \times 5 = 40$

$4 \times 9 = 36$

The x is the digit in the ones place in the result $8 \times 5 = 40,$ so x is 0.

14. Check the integers greater than 1, starting with 2. The only proper divisor of 2 is 1, so 2 is not abundant. The same is true for any prime number, so 3, 5, 7, and 11 are not abundant. The proper divisors of the next several composite numbers are:

4:	1, 2	1 + 2 = 3
6:	1, 2, 3	1 + 2 + 3 = 6
8:	1, 2, 4	1 + 2 + 4 = 7
10:	1, 2, 5	1 + 2 + 5 = 8
12:	1, 2, 3, 4, 6	1 + 2 + 3 + 4 + 6 = 16

The smallest abundant integer is 12.

15. Let X = the number of schools that took part in the race. Then there were $3x$ participants in the race. Erin finished in the middle position, so $3x$ is an odd number that is divisible by 3. Also, $3x > 28$, since Iliana finished 28th. The smallest odd number that is divisible by 3 and greater than 28 is 33. This must be the number of participants in the race because if there were 39 participants, then the middle position would have been 20, and Erin would have finished after Katelyn. So $3x = 33$, and $x = 11$. 11 schools sent participants to the cross-country meet.

16. Statement C must be true, because otherwise there would be more than 11 fish.

$3 + 3 + 3 + 3 = 12$

The other statements could be true, but the only statement that must be true is C.

17. Draw several squares, and for each square draw a different line that goes through the center of the square. Any line through the center of the square will divide the square

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into two halves that have the same size and shape. There are infinitely many lines that can be drawn through the center of a square, so there are infinitely many ways to cut a square in half.

18. Only one person is telling the truth. If we assume Max is truthful, then he is innocent, and Sam and Brett are lying. By their statements, since Max is known to be innocent, Sam must not be innocent, so Sam broke the window. If we assume Sam is truthful, then Max and Brett are lying. Brett's statement is the same as Sam's statement, so Sam must be lying as well. This is a contradiction, so Sam is not the truthful person. Similarly, if we assume Brett is the only truthful person, we get a contradiction. Therefore, Max is telling the truth, and Sam broke the window.

19. The key to this problem is to think about the way the books are placed on the shelf. When they are placed in alphabetical order from left to right, that means volume A is on the far left and volume Z is on the far right. But think about where the covers for these volumes are. The front cover for volume A is on its right side, touching the back cover of volume B. The back cover for volume Z is on its left side, touching the front cover for volume Y. So the bookworm starts with the front cover of volume A $\left(\frac{1}{4}\text{ inch}\right)$ and eats through the entire volumes B through Y (24 books, 2 inches each, 48 inches total), then finishes by eating the back cover of volume Z $\left(\frac{1}{4}\text{ inch}\right)$. So the total amount eaten is 48.5 inches.

20. Write the possible arrangements of cards:

?	K	Q
K	Q	Q
Q	Q	?
Q	Q	Q
S	H	?
S	S	H
?	S	S
S	S	S

Rows with a question mark indicate that not enough information is given. Look at rows 1-4. Row 4 is not correct because there is no king. So row 2 must be correct. Now look at rows 5-8. Row 8 is not correct because there is no heart. So row 6 must be correct. Thus, the cards are king of spades, queen of spades, and queen of hearts.

21. If we let n = the number of cats she has, then we can interpret her response as $n=\left(\frac{5}{6}n+7\right)$ and solve for n.

$$n=\left(\frac{5}{6}n+7\right)$$
$$6n=5n+42$$
$$n=42$$

22. Let x = the number of pencils Bob starts with. Then, after giving away $\frac{4}{5}x$ pencils to Barbara he would have $x-\frac{4}{5}x$ pencils left. He then gives $\frac{2}{3}$ of these, or $\frac{2}{3}\left(x-\frac{4}{5}x\right)$ pencils to Bonnie. This leads to an equation $\left(x-\frac{4}{5}x\right)-\left(\frac{2}{3}\left(x-\frac{4}{5}x\right)\right)=10.$ Solving for x yields:

$$\left(x-\frac{4}{5}x\right)-\left(\frac{2}{3}\left(x-\frac{4}{5}x\right)\right)=10$$
$$x-\frac{4}{5}x-\frac{2}{3}x+\frac{8}{15}x=10$$
$$15\left(x-\frac{4}{5}x-\frac{2}{3}x+\frac{8}{15}x\right)=15\cdot 10$$
$$15x-12x-10x+8x=150$$
$$x=150$$

23. Let f = the number of gallons in a full tank. Then

$$\frac{1}{8}f+15=\frac{3}{4}f$$
$$8\left(\frac{1}{8}f+15\right)=8\left(\frac{3}{4}f\right)$$
$$f+120=6f$$
$$120=5f$$
$$f=24\text{ gallons}$$

Thus a full tank is 24 gallons. The van started with $\frac{1}{8}$ of a full tank or $\frac{1}{8}\cdot 24=3$ gallons. Then 15 gallons were added for a total of (3 + 15 = 18) gallons. So (24 – 18) = 6 gallons are needed to fill the tank.

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24. Let f = the number of gallons in a full tank. Then

$$\begin{aligned}\frac{1}{4}f+6&=\frac{5}{8}f\\ 8\left(\frac{1}{4}f+6\right)&=8\left(\frac{5}{8}f\right)\\ 2f+48&=5f\\ 48&=3f\\ f&=16\text{ gallons}\end{aligned}$$

25. 18, 38, 24, 46, 42
8, 24, 8, 24, 8
By trial and error we might notice that if we multiply the two digits of each of the numbers in the first row, we get the corresponding number in the second row of numbers (8 and 24, which repeat).

26. I put the ring in the box and put my lock on the box. I send you the box. You put your lock on, as well, and send it back to me. I then remove my lock with my key and send you the box (with your lock still on) back to you, so you can remove your lock with your key and get the ring.

27. Given the sequence 16, 80, 48, 64, A, B, C, D, where each term is the arithmetic mean of the previous two terms: e.g. $48=\frac{16+80}{2}$. We therefore know that

$\text{A}=\frac{48+64}{2}=56;\ \text{B}=\frac{64+56}{2}=60;$

$\text{C}=\frac{56+60}{2}=58;\ \text{D}=\frac{60+58}{2}=59.$

28. Let n = the value of the "certain number." Then what Cindy did was $(n-9)\div 3$ to arrive at 43. Solve the corresponding equation for n:

$$\begin{aligned}\frac{(n-9)}{3}&=43\\ n-9&=129\\ n&=138\end{aligned}$$

If she followed the teacher's instructions she would have gotten:

$$\begin{aligned}\frac{(138-3)}{9}&=\frac{135}{9}\\ &=15.\end{aligned}$$

29. Visualize (or create unfolded box strip) with "1" on top, and folding "2," "3," and "4" around the middle. Option A satisfies this result.

30. OHIO is another state such that each capitalized letter has vertical symmetry, and it has horizontal symmetry.

31. Choose a sock from the box labeled *red and green socks.* Since it is mislabeled, it contains only *red* socks or only *green* socks, determined by the sock you choose. If the sock is green, relabel this box *green socks.* Since the other two boxes were mislabeled, switch the remaining label to the other box and place the label that says *red and green socks* on the unlabeled box. No other choice guarantees a correct relabeling, since you can remove only one sock.

32. One strategy is to set up an equation as a model of the problem and solve. Let x = year of birth. Then

$x+2016-(x+10)-(x+50)+(2016-x)$
$=80,$

where present age is found by $2016-x.$ Solving the equation we get:

$$\begin{aligned}x+2016-x-10-x-50+2016-x&=80\\ -2x+4032-60&=80\\ -2x+3972&=80\\ -2x&=-3892\\ x&=1946\end{aligned}$$

Thus, Mr. Green's current age is 2016 – 1946 = 70 years.

(Note: this solution assumes that "this year" is 2016, but the answer of 70 years will be true regardless of the current year.)

33. The total number of dots on each die is 1 + 2 + 3 + 4 + 5 + 6 = 21. Thus the top die has (21 – dots showing), unseen dots, or 21 – (1 + 2 + 3) = 21 – 6 = 15. The middle die has $21-(4+6)=21-10=11.$ The bottom die has $21-(5+1)=21-6=15$ dots not shown. The total is $15+11+15=41$ dots not shown. This is option D.
Alternatively, since each die has 21 dots, there are $21\times 3=63$ total dots. Thus, there are $63-22=41$ unseen dots.

34. Careful! Since you are the bus driver, the answer is *your* age.

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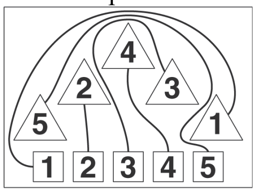
35. One example of a solution follows.

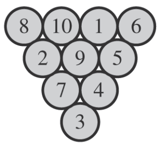


36. By trial and error, the following arrangement will work:

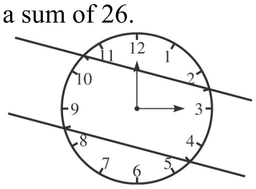
9	7	2	14	11	5	4	12	...

...	13	3	6	10	15	1	8

37. Use trial and error. One possible solution is as follows.



38. Use trial and error. The following figure represents a solution where each region has a sum of 26.



39. For units column assume that 1 is borrowed from the *a* digit. This suggests that $b = 9$ since $12 - 9 = 3$. To arrive at 7 in the tens column, we know that 8 must be subtracted from 15. Thus, $a = 6$ (remember that we borrowed one from that column, also). We borrowed one from the 7, as well, so that $c = 6 - 4 = 2$ in the hundreds column. Thus, $a + b + c = 6 + 9 + 2 = 17$.
This is represented by option D.

40. Use a calculator to find the square root of each number. Only 329,476 has a square root, 574, without a decimal remainder. Thus, $574^2 = 329{,}476$.

41. If we let *D* = the total distance of the trip, and *x* = the distance traveled while asleep,

then
$$\begin{aligned} x + \frac{1}{2}x &= \frac{1}{2}D \\ 2x + x &= D \\ 3x &= D \\ x &= \frac{1}{3}D \end{aligned}$$

Thus, the distance traveled while asleep is $\frac{1}{3}$ of the total distance traveled.

42. Fill the big bucket. Pour into the small bucket. This leaves 4 gallons in the larger bucket. Empty the small bucket. Pour from the big bucket to fill up the small bucket. This leaves 1 gallon in the big bucket. Empty the small bucket. Pour 1 gallon from the big bucket to the small bucket. Fill up the big bucket. Pour into the small bucket. This leaves 5 gallons in the big bucket. Pour out the small bucket. This leaves exactly 5 gallons in the big bucket to take home. The above sequence is indicated by the following table.

Big bucket	7	4	4	1	1	0	7	5	5
Small bucket	0	3	0	3	0	1	1	3	0

43. Count systematically.

	No. of rows × No. of columns
15	1×1 rectangles
12	1×2 rectangles
9	1×3 rectangles
10	2×1 rectangles
8	2×2 rectangles
6	2×3 rectangles
5	3×1 rectangles
4	3×2 rectangles
3	3×3 rectangles
6	1×4 rectangles
4	2×4 rectangles
2	3×4 rectangles
3	1×5 rectangles
2	2×5 rectangles
1	3×5 rectangles
90	total rectangles

This gives a total of 90 rectangles.

44. The following represents a solution found by trial and error.

	3	5	
7	1	8	2
	4	6	

45. One strategy is to assume the car was driving near usual highway speed limits (55-

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75 mph). We begin by trying 55 mph. In two hours the car would have traveled 110 miles. Adding 110 miles to the odometer reading, 15951, we get $110+15951=16061$ miles, which is palindromic. Thus, the speed of the car was 55 miles per hour.

46. 11 round coins = \$15, so

1 round coin = $\$\frac{15}{11}.$

11 square coins = \$16, so

1 square coin = $\$\frac{16}{11}.$

11 triangular coins = \$17, so

1 triangular coin = $\$\frac{17}{11}.$

If r = the number of round coins,
s = the number of square coins, and
t = the number of triangular coins, we must solve the equation

$\frac{15}{11}r+\frac{16}{11}s+\frac{17}{11}t=11.$

Multiply both sides by 11:

$15r+16s+17t=121$

Using trial and error, we see that if $r = 7$, $s=1,$ and $t=0$, then

$15(7)+16(1)+17(0)=121.$

The customer should pay with 7 round coins, 1 square coin, and 0 triangular coins.

47. Similar to Example 5 in the text, we might examine the units place and tens place for repetitive powers of 7 in order to explore possible patterns.

$7^1 =$	07	$7^5 =$	16,807
$7^2 =$	49	$7^6 =$	117,649
$7^3 =$	343	$7^7 =$	823,543
$7^4 =$	2401	$7^8 =$	5,764,801

Since the final two digits cycle over four values, we might consider dividing the successive exponents by 4 and examining their remainders. (Note: We are using inductive reasoning when we assume that this pattern will continue and will apply when the exponent is 1997.) Dividing the exponent 1997 by 4, we get a remainder of 1. This is the same remainder we get when dividing the exponent 1 (on 7^1) and 5 (on 7^5). Thus, we expect that the last two digits for 7^{1997} would be 07 as well.

48. Similar to Example 5 in the text, we might examine the units place for repetitive powers of 3 in order to explore possible patterns.

$3^1 =$	3	$3^5 =$	243	$3^9 =$	19683
$3^2 =$	9	$3^6 =$	729	$3^{10} =$	59049
$3^3 =$	27	$3^7 =$	2187	$3^{11} =$	177147
$3^4 =$	81	$3^8 =$	6561	$3^{12} =$	531441

If we divide the exponent 324 by 4 (since the pattern of the units digit cycles after every 4th power), we get a remainder of 0. Noting that in the line of 3^4, where each exponent when divided by 4 yields a remainder of 0, we reason inductively that 3^{324} has the same units digit 1.

49. Start with a smaller problem.

1000	10,000	100,000	1,000,000
− 50	− 50	− 50	− 50
950	9950	99,950	999,950

Following this pattern, the number that is the result of $10^{50}-50$ has one zero, one 5, and $(50-2)=48$ 9's. Thus,

$(48\times 9)+5+0=432+5+0=437.$

50. At end of 1st day, the frog has a net progression of 1 foot; day 2: 2 feet; day 3: 3 feet; …; day 16: 16 feet (it crawls up 4 feet from 15 to 19 feet and then falls back 3 feet to 16 feet); on the 17th day it crawls up 4 feet from the 16-foot level, which takes it to the top before slipping back.

51. Similar to Example 5 in the text (and Exercise 47 above), we might examine the units place for repetitive powers of 7 in order to explore possible patterns.

$7^1 =$	7	$7^5 =$	16,807
$7^2 =$	49	$7^6 =$	117,649
$7^3 =$	343	$7^7 =$	823,543
$7^4 =$	2401	$7^8 =$	5,764,801

Since the units digit cycles over four values, we might consider dividing the successive exponents by 4 and examining their remainders. Divide the exponent 491 by 4 to get a quotient of 122 and a remainder of 3. Reasoning inductively, the units digit would be the same as that of 7^3 and 7^7, which is 3.

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52. Since her final amount was \$8 and this represents half of the remaining money after buying a train ticket and spending \$4, we reason that she had $2\times\$8+\$4=\$20$ after buying the train ticket. And the ticket cost the same or \$20 for a total of \$40 spent after the purchase of the book. Therefore, she started with \$10 (cost of book) + \$40 = \$50.

53. Joanie will want to use as many eight-cent stamps as possible. Since no multiple of 8 has 3 as its last digit, she will need an odd number of five-cent stamps. Working backward, find the largest multiple of 8 that is less than 153 and has 8 as its last digit. The number is 128, or $8\cdot 16.$ So Joanie should use 16 eight-cent stamps and 5 five-cent stamps, for a total of 21 stamps.

54. Counting directly we get 9 (1-unit squares) plus 4 (4-unit squares) plus 1 (9-unit square). This equals 14 squares in total.

55. To find the minimum number of socks to pull out, guess and check. There are two colors of socks. If you pull out 2 socks, you could have 2 of one color or 1 of each color. You must pull out more than 2 socks. If you pull out 3 socks, you might have 3 of one color or 1 of one color and 2 of the other. In either case, you have a matching pair, so 3 is the minimum number of socks to pull out.

56. To count the triangles, it helps to draw sketches of the figure several times. There are 5 triangles formed by two sides of the pentagon and a diagonal. There are 4 triangles formed with each side of the pentagon as a base, so there are $4\times 5=20$ triangles formed in this way. Each point of the star forms a small triangle, so there are 5 of these. Finally, there are 5 triangles formed with a diagonal as a base. In each, the other two sides are inside the pentagon. (None of these triangles has a side common to the pentagon.) Thus, the total number of triangles in the figure is $5+20+5+5=35.$

57. Use trial and error to find the smallest perfect number. Try making a chart such as the following one.

Number	Divisors other than itself	Sum
1	None	
2	1	1
3	1	1
4	1, 2	3
5	1	1
6	1, 2, 3	6

Six is the smallest perfect number.

58. Becky's mother named her third child Becky, since Becky is the only child left after Penny and Nichole have been named.

59. Working backward, we see that if the lily pad doubles its size each day so that it completely covers the pond on the twentieth day, the pond was half-covered on the previous (or nineteenth) day.

60. It is a *palindrome*, since it reads the same backwards as forwards.

61. From condition (2), we can figure that since the author is living now, the year must be 196_, since $9-3=6.$ Then, from condition (1), $23-(1+9+6)=7,$ so the year is 1967.

62. The only even prime number is 2. A homophone (similar sounding word) of "two" is "too." A synonym (a word with the same meaning) of "too" is "also." Finally, an anagram (rearrangement of the letters) of "also" is LAOS.

63. By Eve's statement, Adam must have \$2 more than Eve. But according to Adam, a loss of \$1 from Eve to Adam gives Adam twice the amount that Eve has. By trial and error, the counting numbers 5 and 7 are the first to satisfy both conditions. Thus Eve has \$5, and Adam has \$7.

64. The answer is 14, since the correct problem is as follows.

$$\begin{array}{r} 435 \\ 826 \\ +\ 147 \\ \hline 1408 \end{array}$$

65. The first digit in the answer cannot be 0, 2, 3, or 5, since these digits have already been used. It cannot be more than 3, since one of the factors is a number in the 30's, making it impossible to get a product over 45,000. Thus, the first digit of the answer must be 1. To find the first digit in the 3-digit factor, use estimation. Dividing a number between 15,000 and 16,000 by a number between 30 and 40 could give a result with a first digit of 3, 4, or 5. Since 3 and 5 have already been used, this first digit must be 4. Thus, the 3-digit factor is 402. We now have the following.

$$\begin{array}{r} \underline{4}\ 0\ 2 \\ \times\quad\ \ 3\ \ \\ \hline \underline{1}\ 5,\qquad \end{array}$$

To find the units digit of the 2-digit factor, use trial and error with the digits that have not yet been used: 6, 7, 8, and 9.

$36 \times 402 = 14,472$ (too small and reuses 2 and 4)

$37 \times 402 = 14,874$ (too small and reuses 4)

$38 \times 402 = 15,276$ (reuses 2)

$39 \times 402 = 15,678$ (correct)

The correct problem is as follows.

$$\begin{array}{r} 4\ 0\ 2 \\ \times\quad 3\ 9 \\ \hline 1\ 5,\ 6\ 7\ 8 \end{array}$$

Notice that a combination of strategies was used to solve this problem.

66. Add the diagonal elements together to get 15. Each row, column, and other diagonal must also add to 15. This yields the following perfect square.

6	1	8
7	5	3
2	9	4

67. Notice that the first column has three given numbers. Thus, $34-(6+11+16)=1$ is the first number in the second row. (Note: You could use the diagonal to solve for missing number in the same manner.) Then, $34-(1+15+14)=4$ is in the second row, third column. The diagonal from upper left to lower right has three given numbers. Therefore, $34-(6+15+10)=3$ is in the fourth row, fourth column. Continue filling in the missing numbers until the magic square is completed.

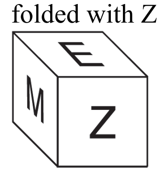
6	12	7	9
1	15	4	14
11	5	10	8
16	2	13	3

68. Solve this problem by looking for a pattern. 1/7 has a decimal representation of 0.142857…, where the group of 6 digits, 142857, is repeated indefinitely. When 100 is divided by 6, the remainder is 4, so the fourth digit of the repeating group, which is 8, is the 100th digit of the decimal representation.

69. 25 pitches: Game tied 0 to 0 going into the 9th inning. Each pitcher has pitched a minimum of 24 pitches (three per inning). The winning pitcher pitches 3 more (fly ball/out) pitches for a total of 27. The losing (visiting team) pitcher pitches 1 more (for a total of 25) which happens to be a home run, thus, losing the game by a score of 1-0. (Note: the same result occurs if the losing pitcher gives up one homerun in any inning.)

70. For three weighings, first balance four against four. Of the lighter four, balance two against the other two. Finally, of the lighter two, balance them one against the other. To find the bad coin in two weighings, divide the eight coins into groups of 3, 3, 2. Weigh the groups of three against each other on the scale. If the groups weigh the same, the fake coin is in the two left out and can be found in one additional weighing. If the two groups of three do not weigh the same, pick the lighter group. Choose any two of the coins and weigh them. If one of these is lighter, it is the fake; if they weigh the same, then the third coin is the fake.

71. Draw a sketch, visualize, or cut a piece of paper to build the cube. The cube may be folded with Z on the front.



Then, E is on top and M is on the left face. This places Q opposite the face marked Z. (D is on the bottom and X is on the right face.)

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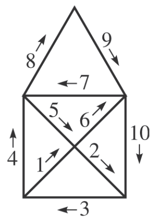
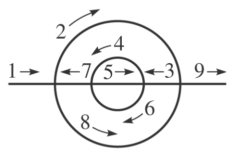
72. A solution, found by trial and error, is shown here.



73. A solution, found by trial and error, is shown here.



74. None. To see this result, set two books beside each other and note the location of page 1 of the 1st book and the location of the last page of the 2nd book.

75. Solve this problem by making a list. First, find the ways he can use pennies to make 15 cents.
15 pennies
10 pennies, 1 nickel
5 pennies, 1 dime
5 pennies, 2 nickels
Find additional ways he can use nickels.
3 nickels
1 nickel, 1 dime
There are 6 ways to make 15 cents, so there are 6 ways he can pay 15 cents for a chocolate mint.

76. A strategy would be to assume the first possible age for the teenage girl, 13, and check by adding 2. This gives 15, not a perfect square. Trying the next possible age, 14, adding 2 gives the perfect square 16. Check by subtracting 10 from 14 to get 4 and note that $4^2 = 16$. Thus, her age is 14 years, and her brother's age is $14 - 5 = 9$ years.

77. The triangle is a right triangle because $10^2 + 24^2 = 26^2$. The area of the triangle is $\frac{1}{2}(24)(10) = 120$ sq cm. The rectangle has the same area as the triangle, with width 3 cm, so the length of the rectangle is $\frac{120}{3} = 40$ cm. Then the perimeter of the rectangle is $2(40) + 2(3) = 80 + 6 = 86$ cm.

78. One strategy is to list all possible combinations of coins: 2 quarters; 5 dimes; 10 nickels; 1 quarter, 2 dimes, 1 nickel; 1 quarter, 1 dime, 3 nickels; 1 quarter, 5 nickels; 1 dime, 8 nickels; 2 dimes, 6 nickels; 3 dimes, 4 nickels; 4 dimes, 2 nickels; for a total of 10 ways.

79. Jessica is married to James or Dan. Since Jessica is married to the oldest person in the group, she is not married to James, who is younger than Cathy. So Jessica is married to Dan, and Cathy is married to James. Since Jessica is married to the oldest person, we know that Dan is 36. Since James is older than Jessica but younger than Cathy, we conclude that Cathy is 31, James is 30, and Jessica is 29.

80. Find the pattern in the last digit by repeatedly multiplying 7 by itself:

$$7^1 = 7$$
$$7^2 = 49$$
$$7^3 = 343$$
$$7^4 = 2401$$
$$7^5 = 16{,}807$$
$$7^6 = 117{,}649$$

The pattern in the last digit of each power of 7 is 7, 9, 3, 1 and then the pattern repeats. Divide the exponent 1783 by 4, the number of digits in the pattern: $1783 \div 4 = 445$ with a remainder of 3. The third digit in the pattern is 3, so the last digit of $49{,}327^{1783}$ is 3.

81. The maximum number of squares is 6. One possible array is as follows.

	X	X
X		X
X	X	

82. Webster does not have a combination of coins in his pocket that equals \$1, but that does not mean that he has less than \$1 in his pocket. We know he has pennies, dimes, and quarters in his pocket. To get the largest possible total value, start with 3 quarters (must be less than 4 quarters, otherwise he

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would have change for a dollar). Then add 4 dimes (because 5 dimes could be combined with 2 quarters to make change for a dollar). Finally, add 4 pennies (because 5 pennies could be combined with 3 quarters and 2 dimes to make change for a dollar). So Webster has \$1.19 in his pocket.

1.4 Exercises

Using a graphing calculator, such as the TI-84, we would enter the expressions as indicated on the left side of the equality then push [Enter] to arrive at the answer. When using scientific or other types of calculators some adjustments will have to be made. See observations related to the solutions for Exercise 8 and 13 below. It is a good idea to review your calculator handbook for related examples.

1. $39.7+(8.2-4.1)=43.8$

2. $2.8\times(3.2-1.1)=5.88$

3. $\sqrt{5.56440921}=2.3589$

4. $\sqrt{37.38711025}=6.1145$

5. $\sqrt[3]{418.508992}=7.48$

6. $\sqrt[3]{700.227072}=8.88$

7. $2.67^2=7.1289$

8. $3.49^3=42.508549$ Observe that for many calculators, the symbol "^" will be used to indicate the exponent prior to inserting the value 3.

9. $5.76^5\approx 6340.338097$

10. $1.48^6\approx 10.50921537$

Observe that when using a calculator, the numerator must be grouped in parentheses as must the denominator. This will make the last operation (the indicated) division.

11. $\dfrac{(14.32-8.1)}{(2\times 3.11)}=1$

12. $\dfrac{(12.3+18.276)}{(3\times 1.04)}=9.8$

13. $\sqrt[5]{1.35}\approx 1.061858759$. Observe that many scientific calculators have only the $\sqrt[2]{\ }$ function built into the calculator. For an index larger than 2, you might want to think of the nth root of a number b as equivalent to the exponential expression $b^{1/n}$. For example, $\sqrt[5]{1.35}=(1.35)^{1/5}$. Then use your exponentiation function (button) to calculate the 5th root of 1.35. Note that you will enter the exponent $\frac{1}{5}$ on the calculator in parentheses as $(1\div 5)$.

14. $\sqrt[6]{3.21}\approx 1.214555893$

15. $\dfrac{\pi}{\sqrt{2}}\approx 2.221441469$

16. $\dfrac{2\pi}{\sqrt{3}}\approx 3.627598728$

17. $\sqrt[4]{\dfrac{2143}{22}}\approx 3.141592653$

18. $\dfrac{12345679\times 72}{\sqrt[3]{27}}=296296296$

19. $\dfrac{\sqrt{2}}{\sqrt[3]{6}}\approx 0.7782717162$

20. $\dfrac{\sqrt[3]{12}}{\sqrt{3}}\approx 1.321802152$

21. Choose a five-digit number such as 73,468.
$$73468\times 9=661212$$
$$6+6+1+2+1+2=18$$
$$1+8=9$$
Choose a six-digit number such as 739,216.
$$739216\times 9=6652944$$
$$6+6+5+2+9+4+4=36$$
$$3+6=9$$
Yes, the same result holds.

22. Squaring the numbers 15, 25, 35, 45, 55, 65, 75, and 85, we get

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$15^2 = 225,$
$25^2 = 625,$
$35^2 = 1225,$
$45^2 = 2025,$
$55^2 = 3025,$
$65^2 = 4225,$
$75^2 = 5625,$
$85^2 = 7225.$

It appears that to square a two-digit number ending in 5, we can multiply the number in the tens position by the next counting number and place the two digits, 25, at the end of the result. Thus, we can get 95^2 by $9 \times 10 = 90$ followed by the two digits 25 or $95^2 = 9025$.

23. $(-3) \div (-8) = 0.375$
$(-5) \div (-4) = 1.25$
$(-2.7) \div (-4.3) \approx 0.6279069767$
Dividing a negative number by another negative number gives a <u>positive</u> number.

24. $5 \times (-4) = -20$
$-3 \times 8 = -24$
$2.7 \times (-4.3) = -11.61$
Multiplying a negative number and a positive number gives a <u>negative</u> product.

25. $5.6^0 = 1;\ \pi^0 = 1;\ 2^0 = 1;\ 120^0 = 1;$
Raising a nonzero number to the power 0 gives a result of <u>1</u>.

26. $1^2 = 1;\ 1^3 = 1;\ 1^{-3} = 1;\ 1^0 = 1;$
Raising 1 to any power gives a result of <u>1</u>.

27. $\frac{1}{7} \approx 0.1428571$
$\frac{1}{(-9)} \approx -0.1111111$
$\frac{1}{3} \approx 0.3333333$
$\frac{1}{(-8)} = -0.125$
The sign of the reciprocal of a number is <u>the same as</u> the sign of the number.

28. (5/0) = ERROR; (9/0) = ERROR;
(0/0) = ERROR
Dividing a number by 0 gives an <u>error message</u> on a calculator. (Division by 0 is not allowed.)

29. (0/8) = 0; (0/–2) = 0; $(0/\pi) = 0$
Zero divided by a nonzero number gives a quotient of <u>0</u>.

30. $\sqrt{-3};\ \sqrt{-4};\ \sqrt{-10}$
Taking the square root of a negative number gives an <u>error message</u> on a calculator.

31. $(-3) \times (-4) \times (-5) = -60$
$(-3) \times (-4) \times (-5) \times (-6) \times (-7) = -2520$
$(-3) \times (-4) \times (-5) \times (-6) \times (-7) \times (-8) \times (-9)$
$= -181440$
Multiplying an *odd* number of negative numbers gives a <u>negative</u> product.

32. $(-3) \times (-4) = 12$
$(-3) \times (-4) \times (-5) \times (-6) = 360$
$(-3) \times (-4) \times (-5) \times (-6) \times (-7) \times (-8)$
$= 20160$
Multiplying an *even* number of negative numbers gives a <u>positive</u> product.

33. Writing exercise; answers will vary.

34. Writing exercise; answers will vary.

35. Writing exercise; answers will vary.

36. Looking at Table 4 in Example 1, we see that Lucy's accumulated savings on the *n*th day is $\frac{2^{n-1}}{100}$ dollars. We can use a calculator to verify that this expression first exceeds one million when $n = 28$.

37. $563 \div 9 \approx 62.555556$. Since more than 62 are needed, we require 63 pages.

38. $408 \div 20 = 20.400$. Since more than 20 are needed, we require 21 drawers.

39. $800 \div 60 \approx 13.333$. Since more than 13 are needed, we require 14 containers.

40. $155 \div 24 \approx 6.45833$. Since more than 6 teachers are needed, 7 are required.

41. $140{,}000 \div 80 \approx 160{,}000 \div 80 = \$2000$; option B

42. $2009 \div 50 \approx 2000 \div 50 = 40$ hours for one way. Thus, when traveling both ways, the time is approximated by $2 \times 40 = 80$ hours; option C

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43. $230,058 \div 76.9 \approx \frac{230,000}{77} \approx 2987 \approx 3000$; option A

44. $90824.2 \div 88 \approx \left(\frac{90,000}{90}\right) = 1000$; option C

45. Approximating the numbers for ease of calculation we have
1400 yards $\div$ 100 catches = 14 yards/catch; option D

46. $(40.5 \text{ meters}) \times (13.5 \text{ meters})$
$\approx (40 \text{ meters}) \times (15 \text{ meters})$
$= 600$ square meters; option D

47. The graph shows that 6% came from other regions.

48. Add the percentages for Latin America and Asia: 55% + 27% = 82%

49. Twelve percent of the 37.5 million foreign born people amounts to about
$12\% \times 37.5 \text{ million} = 0.12 \times 37.5 \text{ million}$
$= 4.5 \text{ million}$

50. Since 55% of the foreign-born population was from Latin America, and 27% was from Asia, the difference was 28% of the 37.5 million foreign-born people.
$28\% \times 37.5 \text{ million} = 0.28 \times 37.5 \text{ million}$
$= 10.5 \text{ million}$

51. U.S. milk production was greater than 185 billion pounds in 2007, 2008, 2009, and 2010.

52. U.S. milk production was about the same in 2008 and 2009.

53. 2004: about 171 billion pounds;
2010: about 193 billion pounds

54. U.S. milk production increased about 22 billion pounds during these years, from about 171 billion pounds to about 193 billion pounds.

55. The greatest increase in the number of imported cars occurred from 2009 to 2010. The increase was about 1.5 million.

56. 2007: 7.2 million; 2008: 6.5 million;
2009: 4.3 million

57. The number of cars imported each year was decreasing.

58. Fewer than 6 million cars were imported into the United States during 2009 and 2010.

59. Writing assignment

60. Writing assignment

61. Writing assignment

62. Writing assignment

Chapter 1 Test

1. This is an example of inductive reasoning, since you are reasoning from a specific pattern to the general conclusion that she will again exceed her annual sales goal.

2. This is a deductive argument because you are reasoning from the stated general property to the specific result, 176^2 is a natural number.

3. One strategy would be to begin counting the smallest squares (10). Then count the two-square rectangles slanted from lower left to upper right (6) followed by the number of three-square rectangles in the same direction (2). There is the same number count for the rectangles slanted lower right to upper left, so double these results. Finally, count the number of larger squares (5).

Rectangle Type	Count
Small squares	10
Two-square rectangles	12
Three-square rectangles	4
Larger squares	5
Total number of rectangles	31

4. The specific pattern seems to indicate that the second factor in the product is a multiple of 17 and the digits on the right side of the equation increase by 1. If this pattern is correct, then the next term in the sequence would be
$65,359,477,124,183 \times 68$
$= 4,444,444,444,444,444$
since $4 \times 17 = 68$. This can be verified by

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multiplying $65,359,477,124,183 \times 68$ on your calculator.

5. 3 11 31 69 131 223 <u>351</u>
8 20 38 62 92 <u>128</u>
12 18 24 30 <u>36</u>
6 6 6 (6)
Thus, our next term in the sequence is $128 + 223 = 351$.

6. Using the method of Gauss, we have 1 + 250 = 251, 2 + 249 = 251, etc. There are $\frac{250}{2} = 125$ such pairs, so the sum can be calculated as $125 \times 251 = 31,375$.

7. The next predicted octagonal number is 65, since the next equation on the list would be $65 = 1 + 7 + 13 + 19 + 25$, where 25 = 19 + 6.

8. Beginning with the first five octagonal numbers and applying the method of successive differences, we get
1 8 21 40 65 96 133 176
7 13 19 25 31 37 43
6 6 6 6 (6) (6)
Dividing each octagonal number by 4 we get the following pattern of remainders: 1, 0, 1, 0, 1, 0, 1, 0, … .

9. After the first two terms (both of which are 1), we can find the next by adding the two previous terms. That is, to get the 3rd term, add 1 + 1 = 2; the 4th term, 1 + 2 = 3; the 5th term, 2 + 3 = 5; and so forth. The next term is 13 + 21 = 34.

10. To make the fraction as small as possible we want the smallest possible numerator (24) and the largest possible denominator (96). Thus, we get the fraction $\frac{24}{96}$, which reduces to $\frac{1}{4}$.

11. Examine the units place for repetitive powers of 9 in order to explore possible patterns.
$9^1 = 9$ $9^3 = 729$ $9^5 = 59049$
$9^2 = 81$ $9^4 = 6561$ $9^6 = 531441$
The pattern we see here is that odd powers of 9 will have a units digit of 9, and even powers of 9 will have a 1 for the units digit. Using this pattern, we reason inductively that 9^{1997} has a units digit of 9.

12. There are 5 smaller triangles representing the extremities of the inside star. There are 5 triangles outside (between) the extremities of the star. Each (outside) line segment forms the base of (5) isosceles triangles that have their apex at each point of the star. Using the line segment connecting two points of the star as a base, two triangles can be formed; one (outside) with a point on the star as an opposite vertex; and one (inside) with opposite vertex at the intersection of any two lines forming the star. There is a total of 10 of these isosceles triangles. This gives a complete total of 35 triangles.

13. Answers will vary. One possible solution is $1 + 2 + 3 - 4 + 5 + 6 + 78 + 9 + 0 = 100$.

14. Dr. Small is 36 inches tall, and he shrinks 2 inches per year. So he will disappear after 36/2 = 18 years. In 18 years, Ms. Tall will grow $18 \cdot \frac{2}{3} = 12$ inches, so she will be 96 + 12 = 108 inches tall, or 9 ft.

15. Observe the following patterns on successive powers of 11, 14, and 16 in order to determine the units value of each term in the sum $11^{11} + 14^{14} + 16^{16}$.

$11^1 = 11$	$14^1 = 14$	$16^1 = 16$
$11^2 = 121$	$14^2 = 196$	$16^2 = 256$
$11^3 = 1331$	$14^3 = 2744$	$16^3 = 4096$
	$14^4 = 38146$	

Thus, we would expect 11^{11} to have the same unit digit value of 1. Since powers of 14 have units digits which cycles between 4 and 6, we observe that division of the exponents by 2 yield remainders of 1 or 0. We might expect the same pattern to continue to 14^{14}. Division of the exponent by 2 gives a remainder of 0. We get the same remainder, 0, for all even powers on 14, and each of these numbers has a units digit of 6. The powers of 16 seem to all have the same unit value of 6. Thus, if we add the units digits 1 + 6 + 6 = 13, we see that the units digit of this sum is 3.

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16. Making the following observations

$9 \times 1 = 9$

$9 \times 2 = 18 \ \ (1 + 8 = 9)$

$9 \times 3 = 27 \ \ (2 + 7 = 9)$

$9 \times 4 = 36 \ \ (3 + 6 = 9)$

$9 \times 5 = 45 \ \ (4 + 5 = 9)$

suggests that the sum of the digits in the product will always be 9.

17. $\sqrt{98.16} \approx 9.907572861$ But answers may vary depending upon what calculator you are using.

18. $3.25^3 = 34.328125$

19. The ratio of made shots to those attempted is approximately $\frac{150}{210} = \frac{5}{7}$. So in 20 attempts, we would expect her to make about 14 shots; option B

20. **(a)** The unemployment rate decreased between 2003 and 2004, between 2004 and 2005, and between 2005 and 2006.

(b) Between 2007 and 2010, the unemployment rate was increasing.

(c) 2008: 5.8%; 2009: 9.3%; The unemployment rate increased by 3.5%.

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