Chapter1.3

Problem 1E

If

$$A = \begin{bmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{bmatrix}$$

Compute

(a) $2A$ **(b)** $A + B$
(c) $2A - 3B$ **(d)** $(2A)^T - (3B)^T$
(e) AB **(f)** BA
(g) $A^T B^T$ **(h)** $(BA)^T$

Step-by-step solution

step 1 of 8

Given $A = \begin{bmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

And $B = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{bmatrix}$

step 2 of 8

(a)
$$2A = 2\begin{bmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 6 & 2 & 8 \\ -4 & 0 & 2 \\ 2 & 4 & 4 \end{bmatrix}$$

step 3 of 8

(b) $$A+B=\begin{bmatrix}3&1&4\\-2&0&1\\1&2&2\end{bmatrix}+\begin{bmatrix}1&0&2\\-3&1&1\\2&-4&1\end{bmatrix}$$
$$=\begin{bmatrix}4&1&6\\-5&1&2\\3&-2&3\end{bmatrix}$$

step 4 of 8

(c) $$2A-3B=2\begin{bmatrix}3&1&4\\-2&0&1\\1&2&2\end{bmatrix}-3\begin{bmatrix}1&0&2\\-3&1&1\\2&-4&1\end{bmatrix}$$
$$=\begin{bmatrix}6&2&8\\-4&0&2\\2&4&4\end{bmatrix}-\begin{bmatrix}3&0&6\\-9&3&3\\6&-12&3\end{bmatrix}$$
$$=\begin{bmatrix}3&2&2\\5&-3&-1\\-4&16&1\end{bmatrix}$$

step 5 of 8

(d) $$(2A)^T-(3B)^T=\begin{bmatrix}6&2&8\\-4&0&2\\2&4&4\end{bmatrix}^T-\begin{bmatrix}3&0&6\\-9&3&3\\6&-12&3\end{bmatrix}^T$$
$$=\begin{bmatrix}6&-4&2\\2&0&4\\8&2&4\end{bmatrix}-\begin{bmatrix}3&-9&6\\0&3&-12\\6&3&3\end{bmatrix}$$
$$=\begin{bmatrix}3&5&-4\\2&-3&16\\2&-1&1\end{bmatrix}$$

step 6 of 8

(f) Consider $$BA=\begin{bmatrix}1&0&2\\-3&1&1\\2&-4&1\end{bmatrix}\begin{bmatrix}3&1&4\\-2&0&1\\1&2&2\end{bmatrix}$$
$$=\begin{bmatrix}5&5&8\\-10&-1&-9\\15&4&6\end{bmatrix}$$

step 7 of 8

(g) Consider $A^T \cdot B^T = \begin{bmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix}^T \begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{bmatrix}^T$

$$= \begin{bmatrix} 3 & -2 & 1 \\ 1 & 0 & 2 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 2 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -10 & 15 \\ 5 & -1 & 4 \\ 8 & -9 & 6 \end{bmatrix}$$

step 8 of 8

(h) Consider $(BA)^T = \left(\begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix} \right)^T$

$$= \begin{bmatrix} 5 & 5 & 8 \\ -10 & -1 & -9 \\ 15 & 4 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -10 & 15 \\ 5 & -1 & 4 \\ 8 & -9 & 6 \end{bmatrix}$$

Problem 2E

For each of the pairs of matrices that follow, determine whether it is possible to multiply the first matrix times the second. If it is possible, perform the multiplication.

(a) $\begin{pmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$

(c) $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 4 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$

(e) $\begin{bmatrix} 4 & 6 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$

(f) $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 4 & 5 \end{bmatrix}$

Step-by-step solution

step 1 of 6

(a) Consider $\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{bmatrix}$

Since the number of columns of the first matrix is equal to the number of rows of second matrix. So, it is possible two multiply the matrices.

$$\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 15 & 19 \\ 4 & 0 \end{bmatrix}$$

step 2 of 6

(b) Consider $\begin{bmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

Since the number of columns of the first matrix (2) is not equal to the number of rows (1) of the second matrix. So, we cannot multiply these two matrices.

step 3 of 6

(c) Consider $\begin{bmatrix}1 & 4 & 3\\0 & 1 & 4\\0 & 0 & 2\end{bmatrix}\begin{bmatrix}3 & 2\\1 & 1\\4 & 5\end{bmatrix}$

Since the number of columns of the first matrix (3) is equal to the number of rows (3) of the second matrix, it is possible to multiply two matrices.

$$\begin{bmatrix}1 & 4 & 3\\0 & 1 & 4\\0 & 0 & 2\end{bmatrix}\begin{bmatrix}3 & 2\\1 & 1\\4 & 5\end{bmatrix}=\begin{bmatrix}19 & 21\\17 & 21\\8 & 10\end{bmatrix}$$

step 4 of 6

(d) Consider $\begin{bmatrix}4 & 6\\2 & 1\end{bmatrix}\begin{bmatrix}3 & 1 & 5\\4 & 1 & 6\end{bmatrix}$

Since the number of columns of the first matrix (2) is equal to the number of rows of the second matrix, it is possible to multiply two matrices.

$$\begin{bmatrix}4 & 6\\2 & 1\end{bmatrix}\begin{bmatrix}3 & 1 & 5\\4 & 1 & 6\end{bmatrix}=\begin{bmatrix}36 & 10 & 56\\10 & 3 & 16\end{bmatrix}$$

step 5 of 6

(e) Consider $\begin{bmatrix}4 & 6 & 1\\2 & 1 & 1\end{bmatrix}\begin{bmatrix}3 & 2 & 5\\4 & 1 & 6\end{bmatrix}$

Since the number of columns of the first matrix is 3 not equal to the number of rows (2) of the second matrix, it is not possible to multiply two matrices.

step 6 of 6

(f) Consider $\begin{bmatrix}2\\-1\\3\end{bmatrix}\begin{bmatrix}3 & 2 & 4 & 5\end{bmatrix}$

Since the number of columns of the first matrix is equal to the numbers of rows of the second matrix (1), it possible to multiply two matrices.

$$\begin{bmatrix}2\\-1\\3\end{bmatrix}\begin{bmatrix}3 & 2 & 4 & 5\end{bmatrix}=\begin{bmatrix}6 & 4 & 8 & 10\\-3 & -2 & -4 & -5\\9 & 6 & 12 & 15\end{bmatrix}$$

Problem 3E

For which of the pairs in Exercise 2 is it possible to multiply the second matrix times the first, and what would the dimension of the product matrix be?

Reference:Exercise 2:

For each of the pairs of matrices that follow, determine whether it is possible to multiply the first matrix times the second. If it is possible, perform the multiplication.

(a) $\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 4 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$

(e) $\begin{bmatrix} 4 & 6 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$

(f) $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 4 & 5 \end{bmatrix}$

Step-by-step solution

step 1 of 6

(a) Consider $\begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{bmatrix}$

Since the number of columns of the first matrix is 2, which is equal to the number of rows of the second matrix multiplication is possible and its order is 3×3.

Then $\begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 10 & 4 \\ -3 & 5 & 7 \\ 10 & 20 & 6 \end{bmatrix}$

step 2 of 6

(b) Consider $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{bmatrix}$

Since the number of columns of the first matrix is 2, which is equal to the number of rows of the second matrix, the matrix multiplication exists. And its order is 1×2.

Then $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 40 & -28 \end{bmatrix}$

step 3 of 6

(c) Consider $\begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$

Since the number of columns of the first matrix is 2, which is not equal to the number of rows of the second matrix 3, it is not possible to multiply these two matrices.

step 4 of 6

(d) Consider $\begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix}$

Since the number of columns of the first matrix is 3, which is not equal to the number of rows of the second matrix 2, it is not possible to multiply these two matrices.

step 5 of 6

(e) Consider $\begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}\begin{bmatrix} 4 & 6 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

Since the number of columns of the first matrix is 3, which is not equal to the number of rows of the second matrix 2, it is not possible to multiply these two matrices.

step 6 of 6

(f) Consider $\begin{bmatrix} 3 & 2 & 4 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

Since the number of columns of the first matrix is 4, which is not equal to the number of rows of the second matrix 3. So, it is not possible to multiply these two matrices.

Problem 4E

Write each of the following systems of equations as a matrix equation.

(a) $3x_1 + 2x_2 = 1$
$2x_1 - 3x_2 = 5$

(b) $x_1 + x_2 = 5$
$2x_1 + x_2 - x_3 = 6$
$3x_1 - 2x_2 + 2x_3 = 7$

(c) $2x_1 + x_2 + x_3 = 4$
$x_1 - x_2 + 2x_3 = 2$
$3x_1 - 2x_2 - x_3 = 0$

Step-by-step solution

step 1 of 6

(a) Given system is $3x_1 + 2x_2 = 1$
$2x_1 - 3x_2 = 5$

step 2 of 6

Matrix equation of the system is

$$\boxed{\begin{bmatrix} 3 & 2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}}$$

step 3 of 6

(b) Given system is $x_1 + x_2 = 5$
$2x_1 + x_2 - x_3 = 6$
$3x_1 - 2x_2 + 2x_3 = 7$

step 4 of 6

Matrix equation of the system is

$$\boxed{\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ 3 & -2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}}$$

step 5 of 6

(c) Given system is $2x_1 + x_2 + x_3 = 4$

$$x_1 - x_2 + 2x_3 = 2$$

$$3x_1 - 2x_2 - x_3 = 0$$

step 6 of 6

Matrix equation of the system is

$$\boxed{\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & -2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}}$$

Problem 5E

If

$$A = \begin{pmatrix} 3 & 4 \\ 1 & 1 \\ 2 & 7 \end{pmatrix}$$

verify that

(a) 5A = 3A + 2A

(b) 6A = 3(2A)

(c) $(A^T)^T = A$

Step-by-step solution

step 1 of 6

Given $A = \begin{bmatrix} 3 & 4 \\ 1 & 1 \\ 2 & 7 \end{bmatrix}$

step 2 of 6

(a) Consider $5A = 5\begin{bmatrix} 3 & 4 \\ 1 & 1 \\ 2 & 7 \end{bmatrix}$

$$= \begin{bmatrix} 15 & 20 \\ 5 & 5 \\ 10 & 35 \end{bmatrix}$$

step 3 of 6

Consider $3A + 2A = 3\begin{bmatrix} 3 & 4 \\ 1 & 1 \\ 2 & 7 \end{bmatrix} + 2\begin{bmatrix} 3 & 4 \\ 1 & 1 \\ 2 & 7 \end{bmatrix}$

$$= \begin{bmatrix} 9 & 12 \\ 3 & 3 \\ 6 & 21 \end{bmatrix} + \begin{bmatrix} 6 & 8 \\ 2 & 2 \\ 4 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 20 \\ 5 & 5 \\ 10 & 35 \end{bmatrix}$$

Hence $\boxed{5A = 3A + 2A}$

step 4 of 6

(b) Consider $3(2A)=3\left(2\begin{bmatrix}3 & 4\\1 & 1\\2 & 7\end{bmatrix}\right)$

$$=3\begin{bmatrix}6 & 8\\2 & 2\\4 & 14\end{bmatrix}$$

$$=\begin{bmatrix}18 & 24\\6 & 6\\12 & 42\end{bmatrix}$$

$$=\begin{bmatrix}6\times 3 & 6\times 4\\6\times 1 & 6\times 1\\6\times 2 & 6\times 7\end{bmatrix}$$

$$=6\begin{bmatrix}3 & 4\\1 & 1\\2 & 7\end{bmatrix}$$

$$=6A$$

Hence $\boxed{6A=3(2A)}$

step 5 of 6

(c) Consider $A^T=\begin{bmatrix}3 & 4\\1 & 1\\2 & 7\end{bmatrix}^T$

$$=\begin{bmatrix}3 & 1 & 2\\4 & 1 & 7\end{bmatrix}$$

step 6 of 6

Then $\left(A^T\right)^T=\begin{bmatrix}3 & 1 & 2\\4 & 1 & 7\end{bmatrix}^T$

$$=\begin{bmatrix}3 & 4\\1 & 1\\2 & 7\end{bmatrix}$$

$$=A$$

Hence $\boxed{\left(A^T\right)^T=A}$

Problem 6E

If

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \\ -2 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 4 \\ 1 & 6 \end{bmatrix}$$

Verify that

(a) $3(AB) = (3A)B = A(3B)$
(b) $(AB)^T = B^T A^T$

Step-by-step solution

step 1 of 8

Given matrices are

$$A = \begin{bmatrix} 4 & 1 & 6 \\ 2 & 3 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & 0 \\ -2 & 2 & -4 \end{bmatrix}$$

step 2 of 8

(a) Consider $A + B = \begin{bmatrix} 4 & 1 & 6 \\ 2 & 3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 0 \\ -2 & 2 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 5 & 4 & 6 \\ 0 & 5 & 1 \end{bmatrix}$$

step 3 of 8

And $B + A = \begin{bmatrix} 1 & 3 & 0 \\ -2 & 2 & -4 \end{bmatrix} + \begin{bmatrix} 4 & 1 & 6 \\ 2 & 3 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 5 & 4 & 6 \\ 0 & 5 & 1 \end{bmatrix}$$

$\boxed{\text{Therefore, } A + B = B + A}$

step 4 of 8

(b) Consider $(A + B) = \begin{bmatrix} 4 & 1 & 6 \\ 2 & 3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 0 \\ -2 & 2 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 5 & 4 & 6 \\ 0 & 5 & 1 \end{bmatrix}$$

step 5 of 8

Then $3\left(A+B\right)=3\begin{bmatrix}5 & 4 & 6\\ 0 & 5 & 1\end{bmatrix}$

$$=\begin{bmatrix}15 & 12 & 18\\ 0 & 15 & 3\end{bmatrix}$$

step 6 of 8

Consider $3A+3B=3\begin{bmatrix}4 & 1 & 6\\ 2 & 3 & 5\end{bmatrix}+3\begin{bmatrix}1 & 3 & 0\\ -2 & 2 & -4\end{bmatrix}$

$$=\begin{bmatrix}12 & 3 & 18\\ 6 & 9 & 15\end{bmatrix}+\begin{bmatrix}3 & 9 & 0\\ -6 & 6 & -12\end{bmatrix}$$

$$=\begin{bmatrix}15 & 12 & 18\\ 0 & 15 & 3\end{bmatrix}$$

$\boxed{\text{Henc } 3\left(A+B\right)=3A+3B}$

step 7 of 8

(c) Consider $\left(A+B\right)^T=\left(\begin{bmatrix}4 & 1 & 6\\ 2 & 3 & 5\end{bmatrix}+\begin{bmatrix}1 & 3 & 0\\ -2 & 2 & -4\end{bmatrix}\right)^T$

$$=\begin{bmatrix}5 & 4 & 6\\ 0 & 5 & 1\end{bmatrix}^T$$

$$=\begin{bmatrix}5 & 0\\ 4 & 5\\ 6 & 1\end{bmatrix}$$

step 8 of 8

Then $A^T+B^T=\begin{bmatrix}4 & 1 & 6\\ 2 & 3 & 5\end{bmatrix}^T+\begin{bmatrix}1 & 3 & 0\\ -2 & 2 & -4\end{bmatrix}^T$

$$=\begin{bmatrix}4 & 2\\ 1 & 3\\ 6 & 5\end{bmatrix}+\begin{bmatrix}1 & -2\\ 3 & 2\\ 0 & -4\end{bmatrix}$$

$$=\begin{bmatrix}5 & 0\\ 4 & 5\\ 6 & 1\end{bmatrix}$$

$\boxed{\text{Hence } \left(A+B\right)T=A^T+B^T}$

Problem 7E

If

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \\ -2 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 4 \\ 1 & 6 \end{bmatrix}$$

Verify that

(a) $3(AB) = (3A)B = A(3B)$
(b) $(AB)^T = B^T A^T$

Step-by-step solution

step 1 of 7

Consider the vectors $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \\ -2 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 4 \\ 1 & 6 \end{bmatrix}.$

The objective is to verify $3(AB) = (3A)B = A(3B)$ and $(AB)^T = B^T A^T$.

step 2 of 7

(a)

Consider,

$$AB = \begin{bmatrix} 2 & 1 \\ 6 & 3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 & 8+6 \\ 12+3 & 24+18 \\ -4+4 & -8+24 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 14 \\ 15 & 42 \\ 0 & 16 \end{bmatrix}$$

step 3 of 7

Then $3(AB)=3\left(\begin{bmatrix}5 & 14\\15 & 42\\0 & 16\end{bmatrix}\right)$

$$=\begin{bmatrix}15 & 42\\45 & 126\\0 & 48\end{bmatrix}$$

step 4 of 7

Consider,

$$(3A)B=\left(3\begin{bmatrix}2 & 1\\6 & 3\\-2 & 4\end{bmatrix}\right)\begin{bmatrix}2 & 4\\1 & 6\end{bmatrix}$$

$$=\begin{bmatrix}6 & 3\\18 & 9\\-6 & 12\end{bmatrix}\begin{bmatrix}2 & 4\\1 & 6\end{bmatrix}$$

$$=\begin{bmatrix}15 & 42\\45 & 126\\0 & 48\end{bmatrix}$$

step 5 of 7

Consider,

$$A(3B)=\begin{bmatrix}2 & 1\\6 & 3\\-2 & 4\end{bmatrix}\left(3\begin{bmatrix}2 & 4\\1 & 6\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & 1\\6 & 3\\-2 & 4\end{bmatrix}\begin{bmatrix}6 & 12\\3 & 18\end{bmatrix}$$

$$=\begin{bmatrix}12+3 & 24+18\\36+9 & 72+54\\-12+12 & -24+72\end{bmatrix}$$

$$=\begin{bmatrix}15 & 42\\45 & 126\\0 & 48\end{bmatrix}$$

Hence, $\boxed{3(AB)=(2A)B=A(3B)}$.

step 6 of 7

(b)

Consider,

$$(AB)^T = \begin{bmatrix} 5 & 14 \\ 15 & 42 \\ 0 & 16 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & 15 & 0 \\ 14 & 42 & 16 \end{bmatrix} \qquad (1)$$

And

$$B^T = \begin{bmatrix} 2 & 4 \\ 1 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 1 \\ 4 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 1 \\ 6 & 3 \\ -2 & 4 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 6 & -2 \\ 1 & 3 & 4 \end{bmatrix}$$

step 7 of 7

Now,

$$B^T A^T = \begin{bmatrix} 2 & 1 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 2 & 6 & -2 \\ 1 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 & 12+3 & -4+4 \\ 8+6 & 24+18 & -8+24 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 15 & 0 \\ 14 & 42 & 16 \end{bmatrix} \qquad (2)$$

From $(1) \& (2)$,

$\boxed{(AB)^T = B^T A^T}$.

Problem 8E

If

$A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix}, C = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

Verify that

(a) $(A+B)+C = A+(B+C)$
(b) $(AB)C = A(BC)$
(c) $A(B+C) = AB+AC$
(d) $(A+B)C = AC+BC$

Step-by-step solution

step 1 of 10

Given $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix}, C = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

step 2 of 10

(a) Consider $A+B = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 5 \\ 1 & 7 \end{bmatrix}$$

step 3 of 10

And $B+C = \begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

step 4 of 10

Consider $(A+B)+C = \begin{bmatrix} 0 & 5 \\ 1 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 6 \\ 3 & 8 \end{bmatrix}$$

step 5 of 10

And $A+(B+C)=\begin{bmatrix}2&4\\1&3\end{bmatrix}+\begin{bmatrix}1&2\\2&5\end{bmatrix}$

$$=\begin{bmatrix}3&6\\3&8\end{bmatrix}$$

$\boxed{\text{Hence } (A+B)+C=A+(B+C)}$

step 6 of 10

(b) Consider $(AB)C=\left(\begin{bmatrix}2&4\\1&3\end{bmatrix}\begin{bmatrix}-2&1\\0&4\end{bmatrix}\right)\begin{bmatrix}3&1\\2&1\end{bmatrix}$

$$=\begin{bmatrix}-4&18\\-2&13\end{bmatrix}\begin{bmatrix}3&1\\2&1\end{bmatrix}$$

$$=\begin{bmatrix}24&14\\20&11\end{bmatrix}$$

$\boxed{\text{Hence } (AB)C=A(BC)}$

step 7 of 10

(c) Consider $A(B+C)=\begin{bmatrix}2&4\\1&3\end{bmatrix}\left(\begin{bmatrix}-2&1\\0&4\end{bmatrix}+\begin{bmatrix}3&1\\2&1\end{bmatrix}\right)$

$$=\begin{bmatrix}2&4\\1&3\end{bmatrix}\begin{bmatrix}1&2\\2&5\end{bmatrix}$$

$$=\begin{bmatrix}10&24\\7&17\end{bmatrix}$$

step 8 of 10

Consider $AB+AC=\begin{bmatrix}2&4\\1&3\end{bmatrix}\begin{bmatrix}-2&1\\0&4\end{bmatrix}+\begin{bmatrix}2&4\\1&3\end{bmatrix}\begin{bmatrix}3&1\\2&1\end{bmatrix}$

$$=\begin{bmatrix}-4&18\\-2&13\end{bmatrix}+\begin{bmatrix}14&6\\9&4\end{bmatrix}$$

$$=\begin{bmatrix}10&24\\7&17\end{bmatrix}$$

$\boxed{\text{Hence } A(BC+C)=AB+AC}$

step 9 of 10

(d) Consider $(A+B)C = \left(\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix}\right)\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 5 \\ 1 & 7 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 5 \\ 17 & 8 \end{bmatrix}$$

step 10 of 10

Consider $AC + BC = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 14 & 6 \\ 9 & 4 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ 8 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 5 \\ 17 & 8 \end{bmatrix}$$

$\boxed{\text{Hence } (A+B)C = AC + BC}$

Problem 9E

Let

$$A = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

(a) Write b as a linear combination of the column vectors a_1 and a_2.

(b) Use the result from part (a) to determine a solution of the linear system Ax = b. Does the system have any other solutions? Explain.

(c) Write c as a linear combination of the column vectors a_1 and a_2.

Step-by-step solution

step 1 of 5

Given $A = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 0 \end{bmatrix}, c = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$

step 2 of 5

(a) From A, the column vector a_1 and a_2 are

$$a_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, a_2 \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

step 3 of 5

Suppose $b = \alpha a_1 + \beta a_2$ then

$$\begin{bmatrix} 4 \\ 0 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{aligned} 4 &= \alpha + 2\beta \\ 0 &= \alpha - 2\beta \end{aligned}$$

Then $\beta = 1, \alpha = 2$

Hence $\boxed{b = 2a_1 + a_2}$

step 4 of 5

(b) From part (a), $b = 2a_1 + a_2$

b is a linear combination of column vector of A.

Hence the linear system $Ax = b$ is consistent.

The system has unique solution.

step 5 of 5

(c) Suppose $C = ra_1 + \delta a_2$

$$\Rightarrow \quad \begin{bmatrix} -3 \\ -2 \end{bmatrix} = r \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \delta \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{aligned} r + 2\delta &= -3 \\ r - 2\delta &= -2 \end{aligned}$$

Hence $r = \frac{-5}{2}$ and $\delta = -1/4$

$$\boxed{C = -\frac{5}{2}a_1 - \frac{1}{4}a_2}$$

Problem 10E

For each of the choices of A and b that follow, determine whether the system Ax = b is consistent by examining how b relates to the column vectors of A. Explain your answers in each case.

(a) $A = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$

(c) $A = \begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Step-by-step solution

step 1 of 4

For each of the choices of A and b that follow, determine whether the system Ax = b is consistent by examining how b relates to the column vectors of A.

step 2 of 4

(a)

Consider the following expression,

$$A = \begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix}, \quad \mathrm{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

In this, the equation Ax = b becomes

$$2x_1 + x_2 = 3$$
$$-2x_1 - x_2 = 1$$

Here, the second row of A is a multiple of the first row of A. So

If we multiply the second row of A by -1, we get

$$(-1)[-2x_1 - x_2] = (-1)[1]$$

Or

$$2x_1 + x_2 = 1$$

So,

$2x_1 + x_2 = 1$ and also $2x_1 + x_2 = 3$.

It is now possible for $2x_1 + x_2$ to be equal to 1 and 3 both, so the system of equations is inconsistent.

step 3 of 4

(b)

Consider the following expression,

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}, \quad \mathrm{b} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

In this, the equation Ax = b becomes

$$x_1 + 4x_2 = 5$$
$$2x_1 + 3x_2 = 5$$

Solve these equations by:

Multiply the first row of A to 2, so the equation becomes

$$2x_1 + 8x_2 = 10$$

Now, subtract row 2 from row 1, gives

$$\begin{array}{r} 2x_1 + 8x_2 = 10 \\ -(2x_1 + 3x_2 = 5) \\ \hline 5x_2 = 5 \end{array}$$

So $x_2 = 1$

Now put $x_2 = 1$ in first row and find the value of x_1, it gives

$$\begin{aligned} x_1 + 4(1) &= 5 \\ x_1 &= 5 - 4 \\ x_1 &= 1 \end{aligned}$$

So, a consistent solution $\boxed{x_1 = 1, x_2 = 1}$,

Hence, the system of solutions is consistent.

step 4 of 4

(c)

Consider the following expression,

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \\ 3 & 2 & 1 \end{pmatrix}, \quad \mathrm{b} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

In this, the equation Ax = b becomes

$$3x_1 + 2x_2 + x_3 = 1$$
$$3x_1 + 2x_2 + x_3 = 0$$
$$3x_1 + 2x_2 + x_3 = -1$$

Now,

$3x_1 + 2x_2 + x_3$ equals all values 1, 0 and -1.

It is now possible for $3x_1 + 2x_2 + x_3$ to be equal to all 1, 0 and -1, so the system of equations is inconsistent.

Problem 11E

Let A be a 5 × 3 matrix. If

$b = a_1 + a_2 = a_2 + a_3$

then what can you conclude about the number of solutions of the linear system Ax = b? Explain.

Step-by-step solution

step 1 of 6

The objective is to find the number of solutions of the system $A\mathbf{x} = \mathbf{b}$ if A is 5×3 matrix and $\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 = \mathbf{a}_2 + \mathbf{a}_3$.

Write $A\mathbf{x} = \mathbf{b}$ as $\mathbf{b} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3$.

As $\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 = \mathbf{a}_2 + \mathbf{a}_3$, one solution is $x_1 = 1, x_2 = 1, x_3 = 0$ and another solution is $x_1 = 0, x_2 = 1, x_3 = 1$.

Therefore, there are at least two solutions.

As the intersection of five planes is either empty or a unique point or a line or a whole plane, so conclude that the set of solutions is either a line or a plane.

Therefore, the system $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

step 2 of 6

Let $\mathbf{a}_1 = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$ and $\mathbf{a}_2 = \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix}$ are arbitrary vectors.

Then,

$$\begin{aligned} \mathbf{b} &= \mathbf{a}_1 + \mathbf{a}_2 \\ &= \begin{bmatrix} a+A \\ b+B \\ c+C \\ d+D \\ e+E \end{bmatrix} \end{aligned}$$

step 3 of 6

Write $\mathbf{a}_3 = \mathbf{b} - \mathbf{a}_2$ but $\mathbf{b} - \mathbf{a}_2 = \mathbf{a}_1$ so

$$\mathbf{a}_3 = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

The system is:

$$\begin{aligned} ax_1 + Ax_2 + ax_3 &= a + A \\ bx_1 + Bx_2 + bx_3 &= b + B \\ cx_1 + Cx_2 + cx_3 &= c + C \\ dx_1 + Dx_2 + dx_3 &= d + D \\ ex_1 + Ex_2 + ex_3 &= e + E \end{aligned}$$

step 4 of 6

The augmented matrix for $A\mathbf{x} = \mathbf{b}$ is:

$$\left[\begin{array}{ccc|c} a & A & a & a+A \\ b & B & b & b+B \\ c & C & c & c+C \\ d & D & d & d+D \\ e & E & e & e+E \end{array}\right]$$

Either $a = b = c = d = e = 0$ or assume (by interchanging rows) that $a \neq 0.$

In this case the system is equivalent to either

$$\left[\begin{array}{ccc|c} 0 & A & 0 & A \\ 0 & B & 0 & B \\ 0 & C & 0 & C \\ 0 & D & 0 & D \\ 0 & E & 0 & E \end{array}\right] \text{ or } \left[\begin{array}{ccc|c} 1 & \frac{A}{a} & 1 & 1+\frac{A}{a} \\ 0 & B - A\frac{b}{a} & 0 & B - A\frac{b}{a} \\ 0 & C - A\frac{c}{a} & 0 & C - A\frac{c}{a} \\ 0 & D - A\frac{d}{a} & 0 & D - A\frac{d}{a} \\ 0 & E - A\frac{e}{a} & 0 & E - A\frac{e}{a} \end{array}\right]$$

step 5 of 6

For the first matrix either $A = B = C = D = E = 0$ or assume (by interchanging rows that $A \neq 0.$

In this case the system corresponding to the first matrix is equivalent to either

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{ or } \left[\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

And each corresponds to a system with infinitely many solutions (x_1 and x_3 are arbitrary in both cases).

step 6 of 6

For the second matrix above, $B - A\frac{b}{a} = C - A\frac{c}{a} = D - A\frac{d}{a} = E - A\frac{e}{a} = 0$ or assume that (by interchanging rows) that $B - A\frac{b}{a} \neq 0.$

In this case the system corresponding to the second matrix is equivalent to either

$$\left[\begin{array}{ccc|c} 1 & \frac{A}{a} & 1 & 1+\frac{A}{a} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{ or } \left[\begin{array}{ccc|c} 1 & \frac{A}{a} & 1 & 1+\frac{A}{a} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Thus, the system has infinitely many solutions (x_3 in particular is arbitrarily for both).

Problem 12E

Let A be a 3 × 4 matrix. If

$b = a_1 + a_2 + a_3 + a_4$

then what can you conclude about the number of solutions of the linear system Ax = b? Explain.

Step-by-step solution

step 1 of 2

Let A be a 3×4 matrix, and $\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4$, the system is $\mathbf{Ax} = \mathbf{b}$.

Where $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$, $\mathbf{A} = \begin{pmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \end{pmatrix}$

Where $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4$ are column vectors and each column vector are consists of three elements.

The column vectors can be written as,

$$\mathbf{a}_1 = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}, \mathbf{a}_2 = \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}, \mathbf{a}_3 = \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}, \mathbf{a}_4 = \begin{pmatrix} a_{14} \\ a_{24} \\ a_{34} \end{pmatrix}$$

The objective is to conclude about the number of solutions of the linear system $\mathbf{Ax} = \mathbf{b}$.

step 2 of 2

The system $\mathbf{Ax} = \mathbf{b}$ can be written as follows.

$$\begin{pmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4$$

That is $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 + x_4\mathbf{a}_4 = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4$

Equate corresponding terms.

$$x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 1$$

So, the solution is $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

There exists only one solution.

Problem 13E

Let Ax = b be a linear system whose augmented matrix (A|b) has reduced row echelon form

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

(a) Find all solutions to the system.

(b) If

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{a}_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 3 \end{bmatrix}$$

Determine b.

Step-by-step solution

step 1 of 3

a) Let $A\mathbf{x} = \mathbf{b}$ be a linear system whose reduced augmented matrix is,

$$\begin{pmatrix} 1 & 2 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Now, take x_1, x_2, x_3, x_4, x_5 be the solutions of the given system.

Write the reduced system into as

$$\begin{pmatrix} 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \\ 0 \\ 0 \end{pmatrix}$$

Simplify the above system.

$$\begin{aligned} x_1 + 2x_2 + 3x_4 + x_5 &= -2 \\ x_3 + 2x_4 + 4x_5 &= 5 \end{aligned}$$

step 2 of 3

Assume $x_4 = s, x_5 = t$ then $x_3 = 5 - 2x_4 - 4x_5$ and $x_3 = 5 - 2s - 4t$

And also assume that $x_2 = k$.

Substitute these value in the equation, $x_1 + 2x_2 + 3x_4 + x_5 = -2$.

$$\begin{aligned} x_1 + 2k + 3s + t &= -2 \\ x_1 &= -2 - 2k - 3s - t \end{aligned}$$

Now, the solution is $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -2-2k-3s-t \\ k \\ 5-2s-4t \\ s \\ t \end{pmatrix}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 5 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} k + \begin{pmatrix} -3 \\ 0 \\ -2 \\ 1 \\ 0 \end{pmatrix} s + \begin{pmatrix} -1 \\ 0 \\ -4 \\ 0 \\ 1 \end{pmatrix} t$$

Here s, t, k are real numbers.

step 3 of 3

b) Need to determine the matrix $\mathbf{b}$, when $\mathbf{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 4 \end{pmatrix}, \mathbf{a}_3 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 3 \end{pmatrix}$

The reduced augmented matrix is,

$$\begin{pmatrix} 1 & 2 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply the row operations as follows.

$R_4 \to 4R_1 + 3R_2$ gives

$$\begin{pmatrix} 1 & 2 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 8 & 3 & 18 & 16 & 7 \end{pmatrix}$$

$R_3 \to 3R_1 + R_2$ gives

$$\begin{pmatrix} 1 & 2 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & 2 & 4 & 5 \\ 3 & 3 & 1 & 11 & 7 & -1 \\ 4 & 8 & 3 & 18 & 16 & 7 \end{pmatrix}$$

$R_1 \to R_1 + 2R_2$ gives

$$\begin{pmatrix} 1 & 2 & 2 & 7 & 9 & 8 \\ 0 & 0 & 1 & 2 & 4 & 5 \\ 3 & 3 & 1 & 11 & 7 & -1 \\ 4 & 8 & 3 & 18 & 16 & 7 \end{pmatrix}$$

$R_1 \to R_1 - 3R_2$ gives

$$\begin{pmatrix} 1 & 2 & 2 & 7 & 9 & 8 \\ 1 & 2 & -1 & 1 & -3 & -7 \\ 3 & 3 & 1 & 11 & 7 & -1 \\ 4 & 8 & 3 & 18 & 16 & 7 \end{pmatrix}$$

Form the above resultant matrix,

$$\mathbf{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 4 \end{pmatrix}, \mathbf{a}_2 = \begin{pmatrix} 2 \\ 2 \\ 3 \\ 8 \end{pmatrix}, \mathbf{a}_3 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 3 \end{pmatrix}, \mathbf{a}_4 = \begin{pmatrix} 7 \\ 1 \\ 11 \\ 18 \end{pmatrix}, \mathbf{a}_5 = \begin{pmatrix} 9 \\ -3 \\ 7 \\ 16 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 8 \\ -7 \\ -1 \\ 7 \end{pmatrix}$$

Hence, the required column vector is $\boxed{\mathbf{b} = \begin{pmatrix} 8 \\ -7 \\ -1 \\ 7 \end{pmatrix}}$

Problem 14E

Let A be an m × n matrix. Explain why the matrix multiplications A^TA and AA^T are possible.

Step-by-step solution

step 1 of 1

Given that A is an $m \times n$ matrix.
Then A^T is a matrix of dimension $n \times m$.
Since the number of columns of A is equal to the number of rows of A^T, AA^T exists.
Since the number of columns of columns of A^T is equal to the number of rows of A, A^TA exists.
The dimension of AA^T is $m \times m$ and A^TA is of $n \times n$ dimension.

Problem 15E

A matrix A is said to be skew symmetric if $A^T = -A$. Show

that if a matrix is skew symmetric, then its diagonal entries must all be 0.

Step-by-step solution

step 1 of 2

Given that A is a skew symmetric matrix.
i.e. $A^T = -A$
Let $A = \left[a_{ij}\right]_{n\times n}$
i.e. A is of dimension $n \times n$.
Then $A^T = \left[b_{ij}\right]_{n\times n}$, where $b_{ij} = a_{ji}$.
Since A is skew-symmetric

$$A^T = -A$$

step 2 of 2

$$\Rightarrow \quad \left[b_{ij}\right] = -\left[a_{ij}\right] \quad \text{for } 1 \le i, j \le n$$
$$\Rightarrow \quad b_{ij} = -a_{ij} \quad \text{for } 1 \le i, j \le n$$
$$\Rightarrow \quad a_{ji} = -a_{ij} \quad \text{for } 1 \le i, j \le n$$
$$\Rightarrow \quad b_{ji} = -a_{ij} \quad \text{for } 1 \le i, j \le n$$

When $i = j$, $a_{ii} + a_{ii} = 0$ for all i

$$\Rightarrow \quad 2a_{ii} = 0 \quad \text{for all } i$$
$$\Rightarrow \quad a_{ii} = 0 \quad \text{for all } i$$

$\boxed{\text{i.e. The diagonal entries of A must be zero.}}$

Problem 16E

In Application 2, suppose that we are searching the database of seven linear algebra books for the search words elementary, matrix, algebra. Form a search vector x, and then compute a vector y that represents the results of the search. Explain the significance of the entries of the vector y.

Application 2:

The growth of digital libraries on the Internet has led to dramatic improvements in thestorage and retrieval of information. Modern retrieval methods are based on matrixtheory and linear algebra.

In a typical situation, a database consists of a collection of documents and we wishto search the collection and find the documents that best match some particular searchconditions. Depending on the type of database, we could search for such items asresearch articles in journals, Web pages on the Internet, books in a library, or moviesin a film collection.

To see how the searches are done, let us assume that our database consists of mdocuments and that there are n dictionary words that can be used as keywords forsearches. Not all words are allowable, since it would not be practical to search forcommon words such as articles or prepositions. If the key dictionary words are orderedalphabetically, then we can represent the database by an m ? n matrix A. Eachdocument is represented by a column of the matrix. The first entry in the j th columnof A would be a number representing the relative frequency of the first key dictionaryword in the j th document. The entry a_{2j} represents the relative frequency of the secondword in the j th document, and so on. The list of keywords to be used in the searchis represented by a vector x in R^m. The ith entry of x is taken to be 1 if the ith wordin the list of keywords is on our search list; otherwise, we set $x_i = 0$. To carry out thesearch, we simply multiply A^T times x.

Step-by-step solution

step 1 of 1

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The given book titles are

B1: Applied Linear Algebra

B2: Elementary Linear Algebra

B3: Elementary Linear Algebra with Applications

B4: Linear Algebra and its Applications

B5: Linear Algebra with Applications

B6: Matrix Algebra with Applications

B7: Matrix Theory

The array representation for Database of linear Algebra Books are

	BOOKS						
Key words	B1	B2	B3	B4	B5	B6	B7
Algebra	1	1	1	1	1	1	0
Applications	1	0	1	1	1	1	0
Elementary	0	1	1	0	0	0	0
Linear	1	1	1	1	1	0	0
Matrix	0	0	0	0	0	1	1
Theory	0	0	0	0	0	0	1

If we are searching for are elementary, matrix, algebra, then the database matrix and search vector are respectively given by

$$A=\begin{bmatrix}1&1&1&1&1&1&0\\1&0&1&1&1&1&0\\0&1&1&0&0&0&0\\1&1&1&1&1&0&0\\0&0&0&0&0&1&1\\0&0&0&0&0&0&1\end{bmatrix}\qquad \mathbf{x}=\begin{pmatrix}1\\0\\1\\0\\1\\0\end{pmatrix}$$

If we set $\mathbf{y}=A^T\mathbf{x}$, then

$$\mathbf{y}=\begin{bmatrix}1&1&0&1&0&0\\1&0&1&1&0&0\\1&1&1&1&0&0\\1&1&0&1&0&0\\1&1&0&1&0&1\\1&1&0&0&1&0\\0&0&0&0&1&1\end{bmatrix}\begin{pmatrix}1\\0\\1\\0\\1\\0\end{pmatrix}=\begin{bmatrix}1\\2\\2\\1\\1\\2\\1\end{bmatrix}$$

The value of y_1 is the number of search word matches in the title of the first book, the value of y_2 is the number of matches in the second book title, and so on.

Problem 17E

Let A be a 2 × 2 matrix with $a_{11} \neq 0$ and let $\alpha = a_{21}/a_{11}$. Show that A can be factored into a product of the form

$$\begin{pmatrix}1&0\\\alpha&1\end{pmatrix}\begin{pmatrix}a_{11}&a_{12}\\0&b\end{pmatrix}$$

What is the value of b?

Step-by-step solution

step 1 of 3

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Given $a_{11} \neq 0$ and $\alpha = \dfrac{a_{21}}{a_{11}}$.

step 2 of 3

Consider $\begin{bmatrix} 1 & 0 \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ 0 & b \end{bmatrix}$

$$= \begin{bmatrix} a_{11} & a_{12} \\ \alpha a_{11} & \alpha a_{12} + b \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & \dfrac{a_{21}}{a_{11}} a_{12} + b \end{bmatrix}$$

step 3 of 3

Thus $\begin{bmatrix} 1 & 0 \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ 0 & b \end{bmatrix} = A$

Only when $\dfrac{a_{21}}{a_{11}} a_{12} + b = a_{22}$

$$\Rightarrow \boxed{b = a_{22} - \dfrac{a_{21}}{a_{11}} a_{12}}$$