Chapter 1

Algebra and Problem Solving

Exercise Set 1.1

1. A letter that can be any one of a set of numbers is called a *variable*.

2. A letter representing a specific number that never changes is called a *constant*.

3. When $x = 10$, the *value* of the expression $4x$ is 40.

4. In a^b, the letter a is called the *base* and the letter b is called the *exponent*.

5. When all variables in a variable expression are replaced by numbers and a result is calculated, we say that we are *evaluating* the expression.

6. To calculate $4+12\div 3\cdot 2$, the first operation that we perform is *division*.

7. A number that can be written in the form a/b, where a and b are integers (with $b \neq 0$) is said to be a *rational* number.

8. A real number that cannot be written as a quotient of two integers is an example of an *irrational* number.

9. Division can be used to show that $\frac{7}{40}$ can be written as a *terminating* decimal.

10. Division can be used to show that $\frac{13}{7}$ can be written as a *repeating* decimal.

11. Five less than some number
Let n represent the number; $n - 5$

12. Let n represent the number; $n + 10$, or $10 + n$

13. Twice a number
Let x represent the number; $2x$

14. Let t represent the number: $8t$

15. Twenty-nine percent of some number
Let x represent the number; $0.29x$, or $\frac{29}{100}x$

16. Let x represent the number; $0.13x$, or $\frac{13}{100}x$

17. Six less than half of a number
Let y represent the number; $\frac{1}{2}y - 6$

18. Let y represent the number; $2y + 3$

19. Seven more than ten percent of some number
Let s represent the number; $0.1s + 7$, or $\frac{10}{100}s + 7$

20. Let s represent the number; $0.06s - 4$, or $\frac{6}{100}s - 4$

21. One less than the product of two numbers
Let m and n represent the numbers; $mn - 1$

22. Let m and n represent the number; $m - n + 1$

23. Ninety miles per every four gallons of gas
We have
$$90 \div 4, \text{ or } \frac{90}{4}.$$

24. $100 \div 60$, or $\frac{100}{60}$

25. The area of a square is given by $A = s^2$. We substitute $s = 6$ and solve.
$$A = s^2 = 6^2 = 36 \text{ ft}^2$$

26. $A = s^2 = 12^2 = 144 \text{ ft}^2$

27. The area of a square is given by $A = s^2$. We substitute $s = 0.5$ and solve.
$$A = s^2 = 0.5^2 = 0.25 \text{ m}^2$$

28. $A = s^2 = 2.5^2 = 6.25 \text{ m}^2$

29. The area of a triangle is given by $A = \frac{1}{2}bh$. We substitute $b = 5$, $h = 7$ and solve.
$$A = \frac{1}{2}bh = \frac{1}{2}(5)(7) = 17.5 \text{ ft}^2$$

30. $A = \frac{1}{2}bh = \frac{1}{2}(2.9)(2.1) = 3.045 \text{ m}^2$

31. The area of a triangle is given by $A = \frac{1}{2}bh$. We substitute $b = 7$, $h = 3.2$ and solve.
$$A = \frac{1}{2}bh = \frac{1}{2}(7)(3.2) = 11.2 \text{ ft}^2$$

32. $A = \frac{1}{2}bh = \frac{1}{2}(3.6)(4) = 7.2 \text{ ft}^2$

33. Substitute and carry out the operations indicated.
$$\begin{aligned} 3(x-7)+2 &= 3(10-7)+2 \\ &= 3(3)+2 \\ &= 9+2 \\ &= 11 \end{aligned}$$

34.
$$\begin{aligned} 5+(2x-3) &= 5+(2\cdot 8-3) \\ &= 5+(16-3) \\ &= 5+13 \\ &= 18 \end{aligned}$$

35. Substitute and carry out the operations indicated.
$$\begin{aligned}12+3(n+2)^2 &= 12+3(1+2)^2\\ &= 12+3(3)^2\\ &= 12+3(9)\\ &= 12+27\\ &= 39\end{aligned}$$

36. $$\begin{aligned}(n-10)^2-8 &= (15-10)^2-8\\ &= (5)^2-8\\ &= 25-8\\ &= 17\end{aligned}$$

37. Substitute and carry out the operations indicated.
$$\begin{aligned}4x+y &= 4\cdot 2+3\\ &= 8+3\\ &= 11\end{aligned}$$

38. $$\begin{aligned}8a-b &= 8\cdot 5-7\\ &= 40-7\\ &= 33\end{aligned}$$

39. Substitute and carry out the operations indicated.
$$\begin{aligned}20+r^2-s &= 20+(5)^2-10\\ &= 20+25-10\\ &= 35\end{aligned}$$

40. $$\begin{aligned}m^3+7-n &= 2^3+7-8\\ &= 8+7-8\\ &= 7\end{aligned}$$

41. Substitute and carry out the operations indicated.
$$\begin{aligned}2c\div 3b &= 2\cdot 6\div 3\cdot 2\\ &= 12\div 3\cdot 2\\ &= 4\cdot 2\\ &= 8\end{aligned}$$

42. $$\begin{aligned}3z\div 2y &= 3\cdot 6\div 2\cdot 1\\ &= 18\div 2\cdot 1\\ &= 9\cdot 1\\ &= 9\end{aligned}$$

43. $3n^2p-3pn^2 = 3\cdot 5^2\cdot 9-3\cdot 9\cdot 5^2$

Observe that $3\cdot 5^2\cdot 9$ and $3\cdot 9\cdot 5^2$ represent the same number, so their difference is 0.

44. $$\begin{aligned}2a^3b-2b^2 &= 2\cdot 3^3\cdot 7-2\cdot 7^2\\ &= 2\cdot 27\cdot 7-2\cdot 49\\ &= 54\cdot 7-98\\ &= 378-98\\ &= 280\end{aligned}$$

45. Substitute and carry out the operations indicated.
$$\begin{aligned}5x\div(2+x-y) &= 5\cdot 6\div(2+6-2)\\ &= 5\cdot 6\div(8-2)\\ &= 5\cdot 6\div 6\\ &= 30\div 6\\ &= 5\end{aligned}$$

46. $$\begin{aligned}3(m+2n)\div m &= 3(7+2\cdot 0)\div 7\\ &= 3(7+0)\div 7\\ &= 3\cdot 7\div 7\\ &= 21\div 7\\ &= 3\end{aligned}$$

47. Substitute and carry out the operations indicated.
$$\begin{aligned}[10-(a-b)]^2 &= [10-(7-2)]^2\\ &= [10-5]^2\\ &= 5^2\\ &= 25\end{aligned}$$

48. $$\begin{aligned}[17-(x+y)]^2 &= [17-(4+1)]^2\\ &= [17-5]^2\\ &= 12^2\\ &= 144\end{aligned}$$

49. Substitute and carry out the operations indicated.
$$\begin{aligned}[5(r+s)]^2 &= [5(1+2)]^2\\ &= [5(3)]^2\\ &= 15^2\\ &= 225\end{aligned}$$

50. $$\begin{aligned}[3(a-b)]^2 &= [3(7-5)]^2\\ &= [3(2)]^2\\ &= 6^2\\ &= 36\end{aligned}$$

51. Substitute and carry out the operations indicated.
$$\begin{aligned}x^2-[3(x-y)]^2 &= 6^2-[3(6-4)]^2\\ &= 6^2-[3(2)]^2\\ &= 6^2-6^2\\ &= 0\end{aligned}$$

52. $$\begin{aligned}m^2-[2(m-n)]^2 &= 7^2-[2(7-5)]^2\\ &= 7^2-[2(2)]^2\\ &= 7^2-4^2\\ &= 49-16\\ &= 33\end{aligned}$$

53. Substitute and carry out the operations indicated.
$$\begin{aligned}(m-2n)^2-2(m+n) &= (8-2\cdot 1)^2-2(8+1)\\ &= (8-2)^2-2(9)\\ &= 6^2-2(9)\\ &= 36-2(9)\\ &= 36-18\\ &= 18\end{aligned}$$

54. $$\begin{aligned}(r-s)^2-3(2r-s) &= (11-3)^2-3(2\cdot 11-3)\\ &= 8^2-3(22-3)\\ &= 8^2-3(19)\\ &= 64-3(19)\\ &= 64-57\\ &= 7\end{aligned}$$

55. List the letters in the set: {a, l, g, e, b, r}

56. {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

57. List the numbers in the set: {1, 3, 5, 7, … }

58. {2, 4, 6, 8, … }

59. List the numbers in the set: {10, 20, 30, 40, … }

60. {5, 10, 15, 20, … }

61. Specify the conditions under which a number is in the set: $\{x|x$ is an even number between 9 and $99\}$

62. $\{n|n$ is a multiple of 5 between 7 and $79\}$

63. Specify the conditions under which a number is in the set: $\{x|x$ is whole number less than $5\}$

64. $\{x|x$ is an integer greater than -4 and less than $3\}$

65. Specify the conditions under which a number is in the set: $\{x|x$ is an odd number between 10 and $20\}$

66. $\{x|x$ is an even number between 23 and $33\}$

67. **a.** 0 and 6 are whole numbers.
b. –3, 0, and 6 are integers.
c. –8.7, –3, 0, $\frac{2}{3}$, and 6 are rational numbers.
d. $\sqrt{7}$ is an irrational number.
e. All of the given numbers are real numbers.

68. **a.** 0, 3
b. –4, 0, 3
c. $-\frac{9}{2}$, –4, –1.2, 0, 3
d. $\sqrt{5}$
e. All of the given numbers are real numbers.

69. **a.** 0 and 8 are whole numbers.
b. –17, 0, and 8 are integers.
c. –17, –0.01, 0, $\frac{5}{4}$, and 8 are rational numbers.
d. $\sqrt{77}$ is an irrational number.
e. All of the given numbers are real numbers.

70. **a.** 0, 1
b. –5, 0, 1
c. –6.08, –5, 0, 1, $\frac{99}{2}$
d. $\sqrt{17}$
e. All of the given numbers are real numbers.

71. Since 196 is a natural number, the statement is true.

72. Since every member of the set of natural numbers is also a member of the set of whole numbers, the statement is true.

73. Since every whole number is an integer, the statement is true.

74. Since $\sqrt{8}$ is not a rational number, the statement is false.

75. Since $\frac{2}{3}$ is not an integer, the statement is false.

76. Since every member of the set of irrational numbers is also a member of the set of real numbers, the statement is true.

77. Since $\sqrt{10}$ is an irrational number, and every member of the set of irrational numbers is a member of the set of real numbers, the statement is true.

78. Since 4.3 is not an integer, the statement is true.

79. Since the set of integers includes some numbers that are not natural numbers, the statement is true.

80. Since every member of the set of rational numbers is also a member of the set of real numbers, the statement is true.

81. Since the set of rational numbers includes some numbers that are not integers, the statement is false.

82. Since $\frac{8}{15}$ is not an irrational number, the statement is false.

83. *Writing Exercise*. Rational numbers can be expressed as p/q, where p and q are integers and $q \neq 0$. The set of integers is a subset of the set of rational numbers in which p is a multiple of q.

84. *Writing Exercise*. Charlie is not following the rules for the order of operations. He is performing the operations as if the expression had been written as $(15-4+1)\div(2\cdot 3)$.

85. *Writing Exercise*. The statement is true because every member of the set {2, 4, 6} is also a member of the set {2, 4, 6}.

86. *Writing Exercise*. Mia's statement is true since 6 is an integer. Giovanni writes that the *set that contains* 6 is an integer. This is not true. If Giovanni had written $\{6\} \subseteq \mathbb{Z}$, his statement would have been true.

87. The quotient of the sum of two numbers and their difference
Let a and b represent the numbers. Then we have $\frac{a+b}{a-b}$.

88. Let m and n represent the numbers; $3(m^3+n^3)$

89. Half of the difference of the squares of two numbers
Let r and s represent the numbers. Then we have

$$\frac{1}{2}(r^2-s^2), \text{ or } \frac{r^2-s^2}{2}.$$

90. Let x and y represent the numbers; $(x-y)(x+y)$

91. The only whole number that is not also a natural number is 0. Using roster notation to name the set, we have {0}.

92. {–1, –2, –3, …}

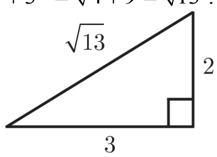
93. List the numbers in the set: {5, 10, 15, 20, …}

94. {3, 6, 9, 12, …}

95. List the numbers in the set: {1, 3, 5, 7, …}

96. {…, –4, –2, 0, 2, 4, …}

97. Recall from geometry that when a right triangle has legs of length 2 and 3, the length of the hypotenuse is $\sqrt{2^2+3^2}=\sqrt{4+9}=\sqrt{13}$. We draw such a triangle:



Exercise Set 1.2

1. It is true that the sum of the two negative number is always negative.

2. The product of two negative numbers is positive, so the statement is false.

3. It is true that the product of a negative number and a positive number is always negative.

4. The statement is false. Consider –2 + 5 = 3, for example.

5. The statement is false. Consider –5+2 = –3, for example.

6. If a and b are negative with $a < b$, then a is farther than b from 0 on the number line, so it is true that $|a|>|b|$.

7. The statement is false. Let $a = 2$ and $b = 6$. Then $2 < 6$ and $|2|<|6|$.

8. It is true that the commutative law of addition states that for all real numbers a and b, $a + b$ and $b + a$ are equivalent.

9. It is true that the associative law of multiplication states that for all real numbers a, b, and c, $(ab)c$ is equivalent to $a(bc)$.

10. It is the commutative law of multiplication, not the distributive law, that states that the order in which two numbers are multiplied does not change the result, so the statement is false.

11. $|-10|=10$ –10 is 10 units from 0

12. 3

13. $|7|=7$ 7 is 7 units from 0

14. 13

15. $|-46.8|=46.8$ –46.8 is 46.8 units from 0

16. 36.9

17. $|0|=0$ 0 is 0 units from itself

18. $3\frac{3}{4}$

19. $\left|1\frac{7}{8}\right|=1\frac{7}{8}$ $1\frac{7}{8}$ is $1\frac{7}{8}$ units from 0

20. 7.24

21. $|-4.21|=4.21$ –4.21 is 4.21 units from 0

22. 5.309

23. $-5 \le -4$
–5 is less than or equal to –4, a true statement since –5 is to the left of –4.

24. –2 is less than or equal to –8; false

25. $-9 > 1$
–9 is greater than 1, a false statement since –9 is to the left of 1.

26. –9 is less than 1; true

27. $0 \ge -5$
0 is greater than or equal to –5, a true statement since –5 is to the left of 0.

28. 9 is less than or equal to 9; true

29. $-8 < -3$
–8 is less than –3, a true statement since –8 is to the left of –3.

30. 7 is greater than or equal to –8; true

31. $-4 \ge -4$
–4 is greater than or equal to –4. Since –4 = –4 is true, $-4 \ge -4$ is true.

32. 2 is less than 2; false

33. $-5 < -5$
–5 is less than –5, a false statement since –5 does not lie to the left of itself.

34. –2 is greater than –12; true

35. 4 + 8
Two positive numbers: Add the numbers getting 12. The answer is positive 12.

36. 12

37. (–3) + (–9)
Two negative numbers: Add the absolute values, getting 12. The answer is negative, –12.

38. –14

39. –5.3 + 2.8
A negative number and a positive number: The absolute values are 5.3 and 2.8. Subtract 2.8 from 5.3 to get 2.5. The negative number is farther from 0, so the answer is negative, –2.5.

40. 3.6

41. $\frac{2}{7}+\left(-\frac{3}{5}\right)=\frac{10}{35}+\left(-\frac{21}{35}\right)$
A positive and negative number. The absolute values

are $\frac{10}{35}$ and $\frac{21}{35}$. Subtract $\frac{10}{35}$ from $\frac{21}{35}$ to get $\frac{11}{35}$. The negative number is farther from 0, so the answer is negative, $-\frac{11}{35}$.

42. $-\frac{1}{40}$

43. $-3.26 + (-5.8)$
Two negative numbers: Add the absolute values, getting 9.06. The answer is negative, -9.06.

44. -9.6

45. $-\frac{1}{9}+\frac{2}{3}=-\frac{1}{9}+\frac{6}{9}$
A negative and positive number. The absolute values are $\frac{1}{9}$ and $\frac{6}{9}$. Subtract $\frac{1}{9}$ from $\frac{6}{9}$ to get $\frac{5}{9}$. The positive number is farther from 0, so the answer is positive, $\frac{5}{9}$.

46. $\frac{3}{10}$

47. $-6.25 + 0$
One number is zero: The sum is the other number, -6.25.

48. -3.69

49. $4.19 + (-4.19)$
A negative and a positive number: The numbers have the same absolute value, 4.19, so the answer is 0.

50. 0

51. $-18.3 + 22.1$
A negative and a positive number; The absolute values are 18.3 and 22.1. Subtract 18.3 from 22.1 to get 3.8. The positive number is farther from 0, so the answer is positive, 3.8.

52. -6.6

53. The opposite of 2.37 is -2.37, because $2.37 + (-2.37) = 0$.

54. -6.98

55. The opposite of -56 is 56, because $-56 + 56 = 0$.

56. 11

57. The opposite of 0 is 0, because $0 + 0 = 0$.

58. $2\frac{1}{3}$

59. If $x = 8$, then $-x = -8$. (The opposite of 8 is -8.)

60. -12

61. If $x=-\frac{1}{10}$, then $-x=-\left(-\frac{1}{10}\right)=\frac{1}{10}$. (The opposite of $-\frac{1}{10}$ is $\frac{1}{10}$.)

62. $\frac{8}{3}$

63. If $x = -4.67$, then $-x = -(-4.67) = 4.67$. (The opposite of -4.67 is 4.67.)

64. -3.14

65. If $x = 0$, then $-x = 0$. (The opposite of 0 is 0.)

66. 7

67. $10-4=10+(-4)$ Change the sign and add.
$=6$

68. 8

69. $4-10=4+(-10)$ Change the sign and add.
$=-6$

70. -8

71. $-5-(-12)=-5+12$ Change the sign and add.
$=7$

72. 4

73. $-5 - 14 = -5 + (-14) = -19$

74. -17

75. $2.7 - 5.8 = 2.7 + (-5.8) = -3.1$

76. -0.5

77. $-\frac{3}{5}-\frac{1}{2}=-\frac{3}{5}+\left(-\frac{1}{2}\right)$
$=-\frac{6}{10}+\left(-\frac{5}{10}\right)$ Finding a common denominator
$=-\frac{11}{10}$

78. $-\frac{13}{15}$

79. $-31 - (-31) = 0$
A negative and a positive number: The numbers have the same absolute value, 31, so the answer is 0.

80. 0

81. $0-(-5.37)=0+5.37$ Change the sign and add.
$=5.37$

82. -9.09

83. $(-3)8$
Two numbers with unlike signs: Multiply their absolute values, getting 24. The answer is negative, -24.

84. -45

85. $(-2)(-11)$
Two numbers with the same sign: Multiply their absolute values, getting 22. The answer is positive, 22.

86. 42

87. (4.2)(–5)
Two numbers with unlike signs: Multiply their absolute values, getting 21. The answer is negative, –21.

88. –28

89. $\frac{3}{7}(-1)$
Two numbers with unlike signs: Multiply their absolute values, getting $\frac{3}{7}$. The answer is negative, $-\frac{3}{7}$.

90. $-\frac{2}{5}$

91. $(-17.45)\cdot 0 = 0$

92. 0

93. $-\frac{2}{3}\left(\frac{3}{4}\right)$
Two numbers with unlike signs: Multiply their absolute values, getting $\frac{1}{2}$. The answer is negative, $-\frac{1}{2}$.

94. $-\frac{1}{4}$

95. $\frac{-28}{-7}$
Two numbers with same sign: Divide their absolute values, getting 4. The answer is positive, 4.

96. 3

97. $\frac{-100}{25}$
Two numbers with unlike signs: Divide their absolute values getting 4. The answer is negative, –4.

98. –10

99. $\frac{73}{-1}$
Two numbers with unlike signs: Divide their absolute values, getting 73. The answer is negative, –73.

100. –62

101. $\frac{0}{-7} = 0$

102. 0

103. The reciprocal of 8 is $\frac{1}{8}$, because $8\cdot\frac{1}{8} = 1$.

104. $-\frac{1}{7}$

105. The reciprocal of $-\frac{5}{7}$ is $-\frac{7}{5}$, because $-\frac{5}{7}\cdot\left(-\frac{7}{5}\right) = 1$.

106. $\frac{3}{4}$

107. Does not exist

108. $-\frac{10}{9}$

109. $\frac{3}{5} \div \frac{6}{7}$
$= \frac{3}{5}\cdot\frac{7}{6}$ Multiplying by the reciprocal of $\frac{6}{7}$
$= \frac{21}{30}$, or $\frac{7}{10}$

110. $\frac{2}{3} \div \frac{5}{6} = \frac{2}{3}\cdot\frac{6}{5} = \frac{12}{15} = \frac{4}{5}$

111. $\left(-\frac{3}{5}\right) \div \frac{1}{2}$
$= -\frac{3}{5}\cdot\frac{2}{1}$ Multiplying by the reciprocal of $\frac{1}{2}$
$= -\frac{6}{5}$

112. $\left(-\frac{4}{7}\right) \div \frac{1}{3} = -\frac{4}{7}\cdot\frac{3}{1} = -\frac{12}{7}$

113. $\left(-\frac{2}{9}\right) \div (-8)$
$= -\frac{2}{9}\cdot\left(-\frac{1}{8}\right)$ Multiplying by the reciprocal of -8
$= \frac{2}{72}$, or $\frac{1}{36}$

114. $\left(-\frac{2}{11}\right) \div (-6) = -\frac{2}{11}\cdot\left(-\frac{1}{6}\right) = \frac{2}{66}$, or $\frac{1}{33}$

115. $-\frac{12}{7} \div \left(-\frac{12}{7}\right)$
This is a number divided by itself so the quotient is 1. We would also do this exercise as follows.
$-\frac{12}{7} \div \left(-\frac{12}{7}\right) = -\frac{12}{7}\cdot\left(-\frac{7}{12}\right)$ Multiplying by the reciprocal of $-\frac{12}{7}$
$= 1$

116. $\left(-\frac{2}{7}\right) \div (-1) = -\frac{2}{7}\cdot(-1) = \frac{2}{7}$

117. $-4^2 = -(4\cdot 4) = -16$ Squaring 4 and then taking the opposite

118. $(-4)^2 = (-4)(-4) = 16$

119. $-(-3)^2 - (-3)(-3) = -9$ Squaring (-3) and then taking the opposite

120. $-(-2)^2 = -(-2)(-2) = -4$

121. $(2-5)^2 = (-3)^2$ Working within the
$= 9$ parentheses first

122. $2^2 - 5^2 = 4 - 25 = -21$

123. $9-\left(8-3\cdot 2^3\right)=9-(8-3\cdot 8)$ Working within the parentheses first
$=9-(8-24)$
$=9-(-16)$
$=9+16$
$=25$

124. $19-\left(4+2\cdot 3^2\right)=19-(4+2\cdot 9)=19-(4+18)$
$=19-22=-3$

125. $\dfrac{5\cdot 2-4^2}{27-2^4}=\dfrac{5\cdot 2-16}{27-16}=\dfrac{10-16}{11}=\dfrac{-6}{11}$, or $-\dfrac{6}{11}$

126. $\dfrac{7\cdot 3-5^2}{9+4\cdot 2}=\dfrac{7\cdot 3-25}{9+8}=\dfrac{21-25}{17}=\dfrac{-4}{17}$, or $-\dfrac{4}{17}$

127. $\dfrac{3^4-(5-3)^4}{8-2^3}=\dfrac{3^4-2^4}{8-8}=\dfrac{81-16}{0}$
Since division by 0 is undefined, this quotient is undefined.

128. $\dfrac{4^3-(7-4)^2}{3^2-7}=\dfrac{4^3-3^2}{9-7}=\dfrac{64-9}{2}=\dfrac{55}{2}$

129. $\dfrac{(2-3)^3-5|2-4|}{7-2\cdot 5^2}=\dfrac{(-1)^3-5|-2|}{7-2\cdot 25}=\dfrac{-1-5(2)}{7-50}$
$=\dfrac{-1-10}{-43}=\dfrac{-11}{-43}=\dfrac{11}{43}$

130. $\dfrac{8\div 4\cdot 6\left|4^2-5^2\right|}{9-4+11-4^2}=\dfrac{8\div 4\cdot 6|16-25|}{9-4+11-16}=\dfrac{8\div 4\cdot 6|-9|}{5+11-16}$
$=\dfrac{8\div 4\cdot 6\cdot 9}{16-16}=\dfrac{2\cdot 6\cdot 9}{0}$
Since division by 0 is undefined, this quotient is undefined.

131. $\left|2^2-7\right|^3+4=|4-7|^3+4=|-3|^3+4=3^3+4$
$=27+4=31$

132. $|-2-3|\cdot 4^2-3=|-5|\cdot 4^2-3=5\cdot 4^2-3=5\cdot 16-3$
$=80-3=77$

133. $32-(-5)^2+15\div(-3)\cdot 2$
$=32-25+15\div(-3)\cdot 2$ Evaluating the exponential expression
$=32-25-5\cdot 2$ Dividing
$=32-25-10$ Multiplying
$=-3$ Subtracting

134. $43-(-9+2)^2+18\div 6\cdot(-2)$
$=43-(-7)^2+18\div 6\cdot(-2)$
$=43-49+18\div 6\cdot(-2)$
$=43-49+3\cdot(-2)$
$=43-49-6$
$=-12$

135. Using the commutative law of addition, we have
$6+xy=xy+6$.
Using the commutative law of multiplication, we have
$6+xy=6+yx$.

136. $4a+7b=7b+4a$
or $4a+7b=a\cdot 4+7b$
or $4a+7b=4a+b\cdot 7$
or $4a+7b=a\cdot 4+b\cdot 7$

137. Using the commutative law of multiplication, we have
$-9(ab)=(ab)(-9)$
or $-9(ab)=-9(ba)$

138. $(7x)y=y(7x)$
or $(7x)y=(x\cdot 7)y$

139. $(3x)y$
$=3(xy)$ Associative law of multiplication

140. $(-7a)b$

141. $(3y+4)+10$
$=3y+(4+10)$ Associative law of addition

142. $(x+2y)+5$

143. $7(x+1)=7\cdot x+7\cdot 1$ Using the distributive law
$=7x+7$

144. $3a+15$

145. $5(m-n)=5\cdot m-5\cdot n$ Using the distributive law
$=5m-5n$

146. $6s-6t$

147. $-5(2a+3b)$
$=-5\cdot 2a+(-5)\cdot 3b$
$=-10a-15b$

148. $-6c-10d$

149. $9a(b-c+d)$
$=9a\cdot b-9a\cdot c+9a\cdot d$
$=9ab-9ac+9ad$

150. $5xy-5xz+5xw$

151. $5x+50=5\cdot x+5\cdot 10=5(x+10)$

152. $5(d+6)$

153. $9p-3=3\cdot 3p-3\cdot 1=3(3p-1)$

154. $3(5x-1)$

155. $7x-21y+14z=7\cdot x-7\cdot 3y+7\cdot 2z=7(x-3y+2z)$

156. $3(2y-3x-w)$

157. $255-34b=17\cdot 15-17\cdot 2b=17(15-2b)$

158. $13(t-11)$

159. $xy+x=x\cdot y+x\cdot 1=x(y+1)$

160. $b(a+1)$

161. *Writing Exercise*. The sum has the same sign as the number that is farther from 0.

162. *Writing Exercise*. "Five is less than x" can be translated to $5 < x$. "Five less than x" is an expression translated to $x - 5$.

163. *Writing Exercise*. The quotient 7/0 is defined to be the number that gives a result of 7 when multiplied by 0. There is no such number, so we say the quotient is undefined.

164. *Writing Exercise*. When we factor the expression $2x + 4$, one factor is 2.

165. $(8-5)^3 + 9 = 36$

166. $2 \cdot (7 + 3^2 \cdot 5) = 104$

167. $5 \cdot 2^3 \div (3-4)^4 = 40$

168. $(2-7) \cdot 2^2 + 9 = -11$

169.
$$\begin{aligned}
&17 - \sqrt{11-(3+4)} \div [-5-(-6)]^2 \\
&= 17 - \sqrt{11-7} \div [-5+6]^2 \\
&= 17 - \sqrt{4} \div [1]^2 \\
&= 17 - 2 \div 1 \\
&= 17 - 2 \\
&= 15
\end{aligned}$$

170.
$$\begin{aligned}
&15 - 1 + \sqrt{5^2-(3+1)^2}\,(-1) \\
&= 15 - 1 + \sqrt{5^2-(4)^2}\,(-1) \\
&= 15 - 1 + \sqrt{25-16}\,(-1) \\
&= 15 - 1 + \sqrt{9}\,(-1) \\
&= 15 - 1 + 3(-1) \\
&= 15 - 1 - 3 \\
&= 11
\end{aligned}$$

171. Any value of a such that $a \leq -6.2$ satisfies the given conditions. The largest of these values is –6.2.

172.
$$\begin{aligned}
&5(a+bc) \\
&= (a+bc)5 && \text{Commutative law of multiplication} \\
&= a \cdot 5 + (bc)5 && \text{Distributive law} \\
&= a \cdot 5 + (cb)5 && \text{Commutative law of multiplication} \\
&= a \cdot 5 + c(b \cdot 5) && \text{Associative law of multiplication} \\
&= c(b \cdot 5) + a \cdot 5 && \text{Commutative law of addition}
\end{aligned}$$

173. *Writing Exercise*. No; 5 – 3 = 2, but 3 – 5 = –2; also $8 \div 2 = 4$, but $2 \div 8 = 0.25$.

174. *Writing Exercise*. No; (12 – 6) – 2 = 6 – 2 = 4, but 12 – (6 – 2) = 12 – 4 = 8; also $(12 \div 6) \div 2 = 2 \div 2 = 1$, but $12 \div (6 \div 2) = 12 \div 3 = 4$.

175. a. Let t represent the temperature at midnight, in °F; $-16 - 5 = t$; $t = -21$. The temperature at midnight was –21°F.

b. Let x represent the temperature outside Ethan's jet, in °F; $42 - 3.5(20) = x$; $x = -28$. The temperature outside Ethan's jet is –28°F.

Exercise Set 1.3

1. Two equations are *equivalent* if they have the same solutions.

2. An equation in x of the form $ax = b$ is a *linear* equation.

3. A *contradiction* is an equation that is never true.

4. An *identity* is an equation that is always true.

5. By the distributive law, the expression $2(x + 7)$ is equivalent to the expression $2x + 14$, so they are equivalent expressions.

6. $2(x + 7) = 11$ and $2x + 14 = 11$ are equations and they have the same solution, $-\frac{3}{2}$, so they are equivalent equations.

7. $4x - 9 = 7$ and $4x = 16$ are equations and they have the same solution, 4, so they are equivalent equations.

8. Combining like terms in the expression $5x - 9 - x$, we get the expression $4x - 9$, so these are equivalent expressions.

9. Combining like terms in the expression $8t + 5 - 2t + 1$, we get the expression $6t + 6$, so these are equivalent expressions.

10. $5t - 2 + t = 8$ and $6t = 10$ are equations and they have the same solution, $\frac{5}{3}$, so they are equivalent equations.

11. $3t = 21$ and $t + 4 = 11$
Each equation has only one solution, the number 7. Thus, the equations are equivalent.

12. For $t = 9$, the first equation is true, but the second is false. They are not equivalent.

13. $12 - x = 3$ and $2x = 20$
When x is replaced by 9, the first equation is true, but the second equation is false. Thus the equations are not equivalent.

14. Each equation has only one solution, the number 4, so they are equivalent.

15. $5x = 2x$ and $\frac{4}{x} = 3$
When x is replaced by 0, the first equation is true, but the second equation is not defined. Thus the equations are not equivalent.

16. For $x = 3$, the first equation is true but the second is not defined. They are not equivalent.

17.
$$\begin{aligned}
x - 2.9 &= 13.4 \\
x - 2.9 + 2.9 &= 13.4 + 2.9 && \text{Adding princple; adding 2.9} \\
x + 0 &= 13.4 + 2.9 && \text{Law of opposites} \\
x &= 16.3
\end{aligned}$$

Check: $x - 2.9 = 13.4$

$$\begin{array}{c|c} 16.3 - 2.9 & 13.4 \end{array}$$

$13.4 \stackrel{?}{=} 13.4$ TRUE

The solution is 16.3.

18. $y + 4.3 = 11.2$
$y = 6.9$

19. $8t = 72$
$\frac{1}{8} \cdot 8t = \frac{1}{8} \cdot 72$ Multiplication principle; multiplying by $\frac{1}{8}$, the reciprocal of 8
$1t = 9$
$t = 9$
Check: $8t = 72$
$8 \cdot 9 \mid 72$
$72 \stackrel{?}{=} 72$ TRUE
The solution is 9.

20. $9t = 63$
$t = 7$

21. $\frac{2}{3}x = 30$
$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 30$
$1x = 45$
$x = 45$
Check: $\frac{2}{3}x = 30$
$\frac{2}{3} \cdot 45 \mid 30$
$30 \stackrel{?}{=} 30$ TRUE
The solution is 45.

22. $\frac{5}{4}x = -80$
$x = \frac{4}{5}(-80)$
$x = -64$

23. $4a + 25 = 9$
$4a + 25 - 25 = 9 - 25$
$4a = -16$
$\frac{1}{4} \cdot 4a = \frac{1}{4}(-16)$
$1a = -4$
$a = -4$
Check: $4a + 25 = 9$
$4(-4) + 25 \mid 9$
$-16 + 25 \mid$
$9 \stackrel{?}{=} 9$ TRUE
The solution is –4.

24. $5a - 11 = 24$
$5a = 35$
$a = 7$

25. $2y - 8 = 9$
$2y - 8 + 8 = 9 + 8$
$2y = 17$
$\frac{1}{2} \cdot 2y = \frac{1}{2} \cdot 17$
$1y = \frac{17}{2}$
$y = \frac{17}{2}$
Check: $2y - 8 = 9$
$2\left(\frac{17}{2}\right) - 8 \mid 9$
$17 - 8 \mid 9$
$9 \stackrel{?}{=} 9$ TRUE
The solution is $\frac{17}{2}$.

26. $3y + 4 = 2$
$3y = -2$
$y = -\frac{2}{3}$

27. $9t^2 + t^2 = (9 + 1)t^2 = 10t^2$

28. $8a^2$

29. $16a - a = (16 - 1)a = 15a$

30. $10t$

31. $n - 8n = (1 - 8)n = -7n$

32. $-2p$

33. $5x - 3x + 8x = (5 - 3 + 8)x = 10x$

34. $-6x$

35. $18p - 12 + 3p + 8$
$= 18p + 3p - 12 + 8$ Commutative law of addition
$= (18 + 3)p + (-12 + 8)$
$= 21p - 4$

36. $5y + 13$

37. $-7t^2 + 3t + 5t^3 - t^3 + 2t^2 - t$
$= (-7 + 2)t^2 + (3 - 1)t + (5 - 1)t^3$
$= -5t^2 + 2t + 4t^3$

38. $-12n + 6n^2 + 5n^3$

39. $2x + 3(5x - 7)$
$= 2x + 15x - 21$
$= 17x - 21$

40. $5x + 4(x + 11) = 5x + 4x + 44 = 9x + 44$

41. $7a - (2a + 5)$
$= 7a - 2a - 5$
$= 5a - 5$

42. $x - (5x + 9) = x - 5x - 9 = -4x - 9$

43. $m - (6m - 2)$
$= m - 6m + 2$
$= -5m + 2$

44. $5a - (4a - 3) = 5a - 4a + 3 = a + 3$

45. $3d - 7 - (5 - 2d)$
$= 3d - 7 - 5 + 2d$
$= 5d - 12$

46. $8x - 9 - (7 - 5x) = 8x - 9 - 7 + 5x = 13x - 16$

47. $2(x-3)+4(7-x)$
$=2x-6+28-4x$
$=-2x+22$

48. $3(y+6)+5(2-4y)=3y+18+10-20y=-17y+28$

49. $3p-4-2(p+6)$
$=3p-4-2p-12$
$=p-16$

50. $8c-1-3(2c+1)=8c-1-6c-3=2c-4$

51. $-2(a-5)-[7-3(2a-5)]$
$=-2a+10-[7-6a+15]$
$=-2a+10-[22-6a]$
$=-2a+10-22+6a$
$=4a-12$

52. $-3(b+2)-[9-5(8b-1)]$
$=-3b-6-[9-40b+5]$
$=-3b-6-[14-40b]$
$=-3b-6-14+40b$
$=37b-20$

53. $5\{-2x+3[2-4(5x+1)]\}$
$=5\{-2x+3[2-20x-4]\}$
$=5\{-2x+3[-20x-2]\}$
$=5\{-2x-60x-6\}$
$=5\{-62x-6\}$
$=-310x-30$

54. $7\{-7x+8[5-3(4x+6)]\}$
$=7\{-7x+8[5-12x-18]\}$
$=7\{-7x+8[-12x-13]\}$
$=7\{-7x-96x-104\}$
$=7\{-103x-104\}$
$=-721x-728$

55. $8y-\{6[2(3y-4)-(7y+1)]+12\}$
$=8y-\{6[6y-8-7y-1]+12\}$
$=8y-\{6[-y-9]+12\}$
$=8y-\{-6y-54+12\}$
$=8y-\{-6y-42\}$
$=8y+6y+42$
$=14y+42$

56. $2y+\{7[3(2y-5)-(8y+7)]+9\}$
$=2y+\{7[6y-15-8y-7]+9\}$
$=2y+\{7[-2y-22]+9\}$
$=2y+\{-14y-154+9\}$
$=2y+\{-14y-145\}$
$=2y-14y-145$
$=-12y-145$

57. $4x+5x=63$
$9x=63$
$\frac{1}{9}\cdot 9x=\frac{1}{9}\cdot 63$
$x=7$

Check: $4x+5x=63$

$4\cdot 7+5\cdot 7$	63
$28+35$	

$63\stackrel{?}{=}63$ TRUE

The solution is 7.

58. $3x-7x=60$
$-4x=60$
$x=-15$

59. $\frac{1}{4}y-\frac{2}{3}y=5$
$\frac{3}{12}y-\frac{8}{12}y=5$
$-\frac{5}{12}y=5$
$\left(-\frac{12}{5}\right)\left(-\frac{5}{12}y\right)=\left(-\frac{12}{5}\right)\cdot 5$
$1y=-12$
$y=-12$

Check: $\frac{1}{4}y-\frac{2}{3}y=5$

$\frac{1}{4}(-12)-\frac{2}{3}(-12)$	5
$-3+8$	5

$5\stackrel{?}{=}5$ TRUE

The solution is –12.

60. $\frac{3}{5}t-\frac{1}{2}t=3$
$\frac{1}{10}t=3$
$t=\frac{10}{1}\cdot 3=30$

61. $4(t-3)-t=6$
$4t-12-t=6$
$3t-12=6$
$3t-12+12=6+12$
$3t=18$
$\frac{1}{3}\cdot 3t=\frac{1}{3}\cdot 18$
$t=6$

Check: $4(t-3)-t=6$

$4(6-3)-6$	6
$4(3)-6$	
$12-6$	

$6\stackrel{?}{=}6$ TRUE

The solution is 6.

62. $2(t+5)+t=4$
$2t+10+t=4$
$3t=-6$
$t=-2$

63. $3(x+4)=7x$
$3x+12=7x$
$3x+12-3x=7x-3x$
$12=4x$
$\frac{1}{4}\cdot 12=\frac{1}{4}\cdot 4x$
$3=x$

Check: $3(x+4)=7x$

$$\begin{array}{r|l} 3(3+4) & 7\cdot 3 \\ 3\cdot 7 & \\ \end{array}$$
$21 \stackrel{?}{=} 21$ TRUE

The solution is 3.

64. $$\begin{aligned} 3(y+5) &= 8y \\ 3y+15 &= 8y \\ 15 &= 5y \\ 3 &= y \end{aligned}$$

65. $$\begin{aligned} 70 &= 10(3t-2) \\ 70 &= 30t-20 \\ 70+20 &= 30t-20+20 \\ 90 &= 30t \\ \frac{1}{30}\cdot 90 &= \frac{1}{30}\cdot 30t \\ 3 &= t \end{aligned}$$

Check: $70=10(3t-2)$

$$\begin{array}{r|l} 70 & 10(3\cdot 3-2) \\ & 10(9-2) \\ & 10\cdot 7 \end{array}$$
$70 \stackrel{?}{=} 70$ TRUE

The solution is 3.

66. $$\begin{aligned} 27 &= 9(5y-2) \\ 27 &= 45y-18 \\ 45 &= 45y \\ 1 &= y \end{aligned}$$

67. $$\begin{aligned} 1.8(2-n) &= 9 \\ 3.6-1.8n &= 9 \\ 3.6-1.8n-3.6 &= 9-3.6 \\ -1.8n &= 5.4 \\ \left(-\frac{1}{1.8}\right)(-1.8n) &= \left(-\frac{1}{1.8}\right)\cdot 5.4 \\ n &= -3 \end{aligned}$$

Check: $1.8(2-n)=9$

$$\begin{array}{r|l} 1.8(2-(-3)) & 9 \\ 1.8(2+3) & \\ 1.8(5) & \end{array}$$
$9 \stackrel{?}{=} 9$ TRUE

The solution is –3.

68. $$\begin{aligned} 2.1(3-x) &= 8.4 \\ 6.3-2.1x &= 8.4 \\ -2.1x &= 2.1 \\ x &= -1 \end{aligned}$$

69. $$\begin{aligned} 5y-(2y-10) &= 25 \\ 5y-2y+10 &= 25 \\ 3y+10 &= 25 \\ 3y+10-10 &= 25-10 \\ 3y &= 15 \\ \frac{1}{3}\cdot 3y &= \frac{1}{3}\cdot 15 \\ y &= 5 \end{aligned}$$

Check: $5y-(2y-10)=25$

$$\begin{array}{r|l} 5\cdot 5-(2\cdot 5-10) & 25 \\ 25-(10-10) & \\ 25-0 & \end{array}$$
$25 \stackrel{?}{=} 25$ TRUE

The solution is 5.

70. $$\begin{aligned} 8x-(3x-5) &= 40 \\ 8x-3x+5 &= 40 \\ 5x &= 35 \\ x &= 7 \end{aligned}$$

71. $$\begin{aligned} \frac{9}{10}y-\frac{7}{10} &= \frac{21}{5} \\ \frac{9}{10}y-\frac{7}{10}+\frac{7}{10} &= \frac{21}{5}+\frac{7}{10} \\ \frac{9}{10}y &= \frac{42}{10}+\frac{7}{10} \\ \frac{9}{10}y &= \frac{49}{10} \\ \frac{10}{9}\cdot\frac{9}{10}y &= \frac{10}{9}\cdot\frac{49}{10} \\ y &= \frac{49}{9} \end{aligned}$$

Check: $\frac{9}{10}y-\frac{7}{10}=\frac{21}{5}$

$$\begin{array}{r|l} \frac{9}{10}\cdot\frac{49}{9}-\frac{7}{10} & \frac{21}{5} \\ \frac{49}{10}-\frac{7}{10} & \\ \frac{42}{10} & \end{array}$$
$\frac{21}{5} \stackrel{?}{=} \frac{21}{5}$ TRUE

The solution is $\frac{49}{9}$.

72. $$\begin{aligned} \frac{4}{5}t-\frac{3}{10} &= \frac{2}{5} \\ \frac{4}{5}t &= \frac{7}{10} \\ t &= \frac{5}{4}\cdot\frac{7}{10}=\frac{7}{8} \end{aligned}$$

73. $$\begin{aligned} 7r-2+5r &= 6r+6-4r \\ 12r-2 &= 2r+6 \\ 12r-2-2r &= 2r+6-2r \\ 10r-2 &= 6 \\ 10r-2+2 &= 6+2 \\ 10r &= 8 \\ \frac{1}{10}\cdot 10r &= \frac{1}{10}\cdot 8 \\ r &= \frac{8}{10} \\ r &= \frac{4}{5} \end{aligned}$$

Check: $7r-2+5r=6r+6-4r$

$$\begin{array}{r|l} 7\cdot\frac{4}{5}-2+5\cdot\frac{4}{5} & 6\cdot\frac{4}{5}+6-4\cdot\frac{4}{5} \\ \frac{28}{5}-2+4 & \frac{24}{5}+6-\frac{16}{5} \end{array}$$
$\frac{38}{5} \stackrel{?}{=} \frac{38}{5}$ TRUE

The solution is $\frac{4}{5}$.

74. $$\begin{aligned} 9m-15-2m &= 6m-1-m \\ 7m &= 15+5m-1 \\ 2m &= 14 \\ m &= 7 \end{aligned}$$

75. $$\begin{aligned} \frac{2}{3}(x-2)-1 &= \frac{1}{4}(x-3) \\ \frac{2}{3}x-\frac{4}{3}-1 &= \frac{1}{4}x-\frac{3}{4} \\ \frac{2}{3}x-\frac{7}{3} &= \frac{1}{4}x-\frac{3}{4} \\ \frac{2}{3}x-\frac{7}{3}-\frac{1}{4}x &= \frac{1}{4}x-\frac{3}{4}-\frac{1}{4}x \\ \frac{5}{12}x-\frac{7}{3} &= -\frac{3}{4} \\ \frac{5}{12}x-\frac{7}{3}+\frac{7}{3} &= -\frac{3}{4}+\frac{7}{3} \\ \frac{5}{12}x &= \frac{19}{12} \\ \frac{12}{5}\cdot\frac{5}{12}x &= \frac{12}{5}\cdot\frac{19}{12} \\ x &= \frac{19}{5} \end{aligned}$$

The check is left to the student. The solution is $\frac{19}{5}$.

76. $$\begin{aligned} \frac{1}{4}(6t+48)-20 &= -\frac{1}{3}(4t-72) \\ \frac{3}{2}t+12-20 &= -\frac{4}{3}t+24 \\ \frac{3}{2}t-8 &= -\frac{4}{3}t+24 \\ \frac{17}{6}t &= 32 \\ t &= \frac{6}{17}\cdot 32 = \frac{192}{17} \end{aligned}$$

77. $$\begin{aligned} 2(t-5)-3(2t-7) &= 12-5(3t+1) \\ 2t-10-6t+21 &= 12-15t-5 \\ -4t+11 &= -15t+7 \\ -4t+11+15t &= -15t+7+15t \\ 11t+11 &= 7 \\ 11t+11-11 &= 7-11 \\ 11t &= -4 \\ \frac{1}{11}\cdot 11t &= \frac{1}{11}(-4) \\ t &= -\frac{4}{11} \end{aligned}$$

Check:

$$\begin{array}{r|l} \multicolumn{2}{c}{2(t-5)-3(2t-7)=12-5(3t+1)} \\ \hline 2\left(-\frac{4}{11}-5\right)-3\left(2\left(-\frac{4}{11}\right)-7\right) & 12-5\left(3\left(-\frac{4}{11}\right)+1\right) \\ 2\left(-\frac{59}{11}\right)-3\left(-\frac{8}{11}-7\right) & 12-5\left(-\frac{12}{11}+1\right) \\ -\frac{118}{11}-3\left(-\frac{85}{11}\right) & 12-5\left(-\frac{1}{11}\right) \\ -\frac{118}{11}+\frac{255}{11} & 12+\frac{5}{11} \end{array}$$

$$\frac{137}{11} \stackrel{?}{=} \frac{137}{11} \quad \text{TRUE}$$

The solution is $-\frac{4}{11}$.

78. $$\begin{aligned} 4t+8-6(2t-1) &= 3(4t-3)-7(t-2) \\ 4t+8-12t+6 &= 12t-9-7t+14 \\ -8t+14 &= 5t+5 \\ 9 &= 13t \\ \frac{9}{13} &= t \end{aligned}$$

79. $$\begin{aligned} 3[2-4(x-1)] &= 3-4(x+2) \\ 3[2-4x+4] &= 3-4x-8 \\ 3[6-4x] &= -4x-5 \\ 18-12x &= -4x-5 \\ 18-12x+12x &= -4x-5+12x \\ 18 &= 8x-5 \\ 18+5 &= 8x-5+5 \\ 23 &= 8x \\ \frac{1}{8}\cdot 23 &= \frac{1}{8}(8x) \\ \frac{23}{8} &= x \end{aligned}$$

Check: $3[2-4(x-1)]=3-4(x+2)$

$$\begin{array}{r|l} 3\left[2-4\left(\frac{23}{8}-1\right)\right] & 3-4\left(\frac{23}{8}+2\right) \\ 3\left[2-4\left(\frac{15}{8}\right)\right] & 3-4\left(\frac{39}{8}\right) \\ 3\left[2-\frac{15}{2}\right] & 3-\frac{39}{2} \\ 3\left(-\frac{11}{2}\right) & \frac{6}{2}-\frac{39}{2} \end{array}$$

$$-\frac{33}{2} \stackrel{?}{=} -\frac{33}{2} \quad \text{TRUE}$$

The solution is $\frac{23}{8}$.

80. $$\begin{aligned} 5+2(x-3) &= 2[5-4(x+2)] \\ 5+2x-6 &= 2[5-4x-8] \\ 2x-1 &= 2[-4x-3] \\ 2x-1 &= -8x-6 \\ 10x &= -5 \\ x &= -\frac{1}{2} \end{aligned}$$

81. $$\begin{aligned} 2x+2 &= 2(x+1) \\ 2x+2 &= 2x+2 \\ 2x+2-2x &= 2x+2-2x \\ 2 &= 2 \end{aligned}$$

All real numbers are solutions. The solution set is the set of all real numbers. The equation is an identity.

82. $$\begin{aligned} x+2 &= x+3 \\ 2 &= 3 \end{aligned}$$

Since the original equation is equivalent to the false equation 2 = 3, there is no solution. The solution set is $\varnothing$. The equation is a contradiction.

83. $$\begin{aligned} 7x-2-3x &= 4x \\ 4x-2 &= 4x \\ 4x-2-4x &= 4x-4x \\ -2 &= 0 \end{aligned}$$

Since the original equation is equivalent to the false equation –2 = 0, there is no solution. The solution set is $\varnothing$. The equation is a contradiction.

84. $3t+5+t=5+4t$
$4t+5=5+4t$
$5=5$
All real numbers are solutions. The solution set is the set of all real numbers. The equation is an identity.

85.
$$\begin{aligned}2+9x&=3(4x+1)-1\\2+9x&=12x+3-1\\2+9x&=12x+2\\2+9x-2&=12x+2-2\\9x&=12x\\9x-9x&=12x-9x\\0&=3x\\\frac{1}{3}\cdot 0&=\frac{1}{3}\cdot 3x\\0&=x\end{aligned}$$
The solution set is $\{0\}$. The equation is a conditional equation.

86. $4+7x=7(x+1)$
$4+7x=7x+7$
$4=7$ False equation

The solution set is $\varnothing$. The equation is a contradiction.

87.
$$\begin{aligned}3x-(8-x)&=6x-2(x+4)\\3x-8+x&=6x-2x-8\\4x-8&=4x-8\\4x-8-4x&=4x-8-4x\\-8&=-8\end{aligned}$$
All real numbers are solutions. The solution set is the set of all real numbers. The equation is an identity.

88.
$$\begin{aligned}\tfrac{1}{3}(2x-7)&=\tfrac{1}{2}(x+3)\\\tfrac{2}{3}x-\tfrac{7}{3}&=\tfrac{1}{2}x+\tfrac{3}{2}\\\tfrac{1}{6}x&=\tfrac{23}{6}\\x&=23\end{aligned}$$
The solution set is $\{23\}$. The equation is a conditional equation.

89. $-9t+2=-9t-7(6\div 2(49)+8)$

Observe that $-7(6\div 2(49)+8)$ is a negative number. Then on the left side we have $-9t$ plus a positive number and on the right side we have $-9t$ plus a negative number. This is a contradiction, so the solution set is $\varnothing$.

90. $-9t+2=2-9t-5\left(8\div 4\left(1+3^4\right)\right)$

Observe that $5\left(8\div 4\left(1+3^4\right)\right)$ is a positive number. Then the constant term on the right side of the equation is 2 minus a positive number. That is, it is a number less than 2. Thus, on the left we have $-9t+2$ and on the right we have $-9t$ plus a number less than 2. This is a contradiction, so the solution set is $\varnothing$.

91.
$$\begin{aligned}2\{9-3[-2x-4]\}&=12x+42\\2\{9+6x+12\}&=12x+42\\2\{21+6x\}&=12x+42\\42+12x&=12x+42\\42+12x-12x&=12x+42-12x\\42&=42\end{aligned}$$
The original equation is equivalent to the equation 42 = 42, which is true for all real numbers. Thus the solution set is the set of all real numbers. The equation is an identity.

92.
$$\begin{aligned}3\{7-2[7x-4]\}&=-40x+45\\3\{7-14x+8\}&=-40x+45\\3\{15-14x\}&=-40x+45\\45-42x&=-40x+45\\0&=2x\\0&=x\end{aligned}$$
The solution set is $\{0\}$. The equation is conditional.

93. *Writing Exercise*. The statement, "The equation has no solution," says that there is no number that makes the equation true. The statement, "The solution of the equation is zero" says that there is a number, the number zero, that makes the equation true.

94. *Writing Exercise*. Most would not consider this to be the best approach, since it introduces fractions that must be added to complete the solution.

95. *Writing Exercise*.
$3x+6y+4x+2y$
$=3x+4x+6y+2y$ Commutative law of addition
$=(3+4)x+(6+2)y$ Distributive law
$=7x+8y$

96. *Writing Exercise*. Equivalent expressions have the same value for all possible replacements. Any replacement that does not make any of the expressions undefined can be substituted for the variable. Equivalent equations have the same solution(s). True equations result only when a solution is substituted for the variable.

97.
$$\begin{aligned}-0.00458y+1.7787&=13.002y-1.005\\-13.00658y+1.7787&=-1.005\\-13.00658y&=-2.7837\\y&=\frac{-2.7837}{-13.00658}\\0.2140224409&\approx y\end{aligned}$$
The check is left to the student. The solution is approximately 0.2140224409.

98.
$$\begin{aligned}4.23x-17.898&=-1.65x-42.454\\5.88x&=-24.556\\x&\approx -4.176190476\end{aligned}$$

99.
$$\begin{aligned} 6x-\{5x-[7x-(4x-(3x+1))]\} &= 6x+5 \\ 6x-\{5x-[7x-(4x-3x-1)]\} &= 6x+5 \\ 6x-\{5x-[7x-(x-1)]\} &= 6x+5 \\ 6x-\{5x-[7x-x+1]\} &= 6x+5 \\ 6x-\{5x-[6x+1]\} &= 6x+5 \\ 6x-\{5x-6x-1\} &= 6x+5 \\ 6x-\{-x-1\} &= 6x+5 \\ 6x+x+1 &= 6x+5 \\ 7x+1 &= 6x+5 \\ x &= 4 \end{aligned}$$

100.
$$\begin{aligned} 8x-\{3x-[2x-(5x-(7x-1))]\} &= 8x+7 \\ 8x-\{3x-[2x-(-2x+1)]\} &= 8x+7 \\ 8x-\{3x-[4x-1]\} &= 8x+7 \\ 8x-\{-x+1\} &= 8x+7 \\ 9x-1 &= 8x+7 \\ x &= 8 \end{aligned}$$

101.
$$\begin{aligned} 23-2\{4+3[x-1]\}+5\{x-2(x+3)\} & \\ = 7\{x-2[5-(2x+3)]\} & \\ 23-2\{4+3x-3\}+5\{x-2x-6\} & \\ = 7\{x-2[5-2x-3]\} & \\ 23-2\{3x+1\}+5\{-x-6\} &= 7\{x-2[-2x+2]\} \\ 23-6x-2-5x-30 &= 7\{x+4x-4\} \\ -11x-9 &= 7\{5x-4\} \\ -11x-9 &= 35x-28 \\ -9 &= 46x-28 \\ 19 &= 46x \\ \frac{19}{46} &= x \end{aligned}$$

The check is left to the student. The solution is $\frac{19}{46}$.

102.
$$\begin{aligned} 17-3\{5+2[x-2]\}+4\{x-3(x+7)\} & \\ = 9\{x+3[2+3(4-x)]\} & \\ 17-3\{5+2x-4\}+4\{x-3x-21\} & \\ = 9\{x+3[2+12-3x]\} & \\ 17-3\{1+2x\}+4\{-2x-21\} &= 9\{x+3[14-3x]\} \\ 17-3-6x-8x-84 &= 9\{x+42-9x\} \\ -14x-70 &= 9\{-8x+42\} \\ -14x-70 &= -72x+378 \\ 58x-70 &= 378 \\ 58x &= 448 \\ x &= \frac{448}{58}, \text{ or } \frac{224}{29} \end{aligned}$$

103. *Writing Exercise*. Answers may vary. One such equation is $\frac{2}{5}x+\frac{1}{10}=\frac{9}{5}x-\frac{3}{10}$. If we first multiply both sides by 10, we avoid having to add fractions when we use the addition principle.

104. a. $y = 500 + 0.1x$

b.
$$\begin{aligned} y &= 500+0.1x \\ 900 &= 500+0.1x \\ 400 &= 0.1x \\ 4000 &= x \end{aligned}$$
Jasmine's sales were \$4000 that week.

Mid-Chapter Review

1. $3x-2(x-1)=3x-2x+2$
$\quad = x+2$

2.
$$\begin{aligned} 3x-2(x-1) &= 6x \\ 3x-2x+2 &= 6x \\ x+2 &= 6x \\ 2 &= 5x \\ \frac{2}{5} &= x \end{aligned}$$

3. Five less than three times a number
Let n represent the number; $3n - 5$

4.
$$\begin{aligned} 2a \div 3x - a + x &= 2(3) \div 3(5) - 3 + 5 \\ &= 6 \div 3(5) - 3 + 5 \\ &= 2(5) - 3 + 5 \\ &= 10 - 3 + 5 \\ &= 12 \end{aligned}$$

5. The area of a triangle is given by $A=\frac{1}{2}bh$. We substitute $b=\frac{1}{2}$, $h=3$ and solve.
$$A=\frac{1}{2}bh=\frac{1}{2}\left(\frac{1}{2}\right)(3)=\frac{3}{4}\text{ ft}^2$$

6. $\frac{1}{2}-\left(-\frac{1}{3}\right)=\frac{5}{6}$

7. $-32 \div (-0.8) = \frac{-32}{-0.8} = 40$
Two numbers with same sign: Divide their absolute values, getting 40. The answer is positive, 40.

8. 2.52

9. $\left(\frac{3}{10}\right)\left(-\frac{2}{5}\right)=-\frac{6}{50}$, or $-\frac{3}{25}$
Two numbers with unlike signs: Multiply their absolute values, getting $\frac{3}{25}$. The answer is negative, $-\frac{3}{25}$.

10.
$$\begin{aligned} 8-2^3 \div 4 \cdot (-2)+1-2 &= 8-8 \div 4 \cdot (-2)+1-2 \\ &= 8-2 \cdot (-2)+1-2 \\ &= 8+4+1-2 \\ &= 11 \end{aligned}$$

11. $(x+3)+y$
$= x+(3+y)$ Associative law of addition

12. $3x-5-x+12=2x+7$

13. $4t-(3t-1)=4t-3t+1$
$\quad = t+1$

14.
$$\begin{aligned} 8x+2[x-(x-1)] &= 8x+2[x-x+1] \\ &= 8x+2[1] \\ &= 8x+2 \end{aligned}$$

15. $$\begin{aligned} &-(p-4)-[3-(9-2p)]+p \\ &= -p+4-[3-9+2p]+p \\ &= -p+4-[-6+2p]+p \\ &= -p+4+6-2p+p \\ &= -2p+10 \end{aligned}$$

16. $$\begin{aligned} 2x-6 &= 3x+5 \\ 2x-6-2x &= 3x+5-2x \\ -6 &= x+5 \\ -6-5 &= x+5-5 \\ -11 &= x \end{aligned}$$

17. $$\begin{aligned} 5-(t-2) &= 6 \\ 5-t+2 &= 6 \\ 7-t &= 6 \\ -t &= -1 \\ t &= 1 \end{aligned}$$

18. $$\begin{aligned} 6(y-1)-2(y+1) &= 4(y-2) \\ 6y-6-2y-2 &= 4y-8 \\ 4y-8 &= 4y-8 \\ -8 &= -8 \end{aligned}$$

All real numbers are solutions. The solution set is the set of all real numbers. The equation is an identity.

19. $$\begin{aligned} 3(x-1)-2(2x+1) &= 5(x-1) \\ 3x-3-4x-2 &= 5x-5 \\ -x-5 &= 5x-5 \\ -x-5+x &= 5x-5+x \\ -5 &= 6x-5 \\ -5+5 &= 6x-5+5 \\ 0 &= 6x \\ \frac{1}{6}\cdot 0 &= \frac{1}{6}\cdot 6x \\ 0 &= x \end{aligned}$$

20. $$\begin{aligned} \tfrac{1}{3}t-2 &= \tfrac{1}{6}+t \\ -2 &= \tfrac{1}{6}+\tfrac{4}{3}t \\ -\tfrac{13}{6} &= \tfrac{4}{3}t \\ -\tfrac{13}{4} &= t \end{aligned}$$

Exercise Set 1.4

1. The five steps of the problem solving process:
   1. Familiarize
   2. Translate
   3. Carry out
   4. Check
   5. State

2. Carry out

3. *State* Give the answer clearly.

4. Translate

5. *Familiarize* Read the problem carefully.

6. Check

7. ***Familiarize***. We want to find two numbers. We can let x represent the first number and note that the second number is 9 more than the first. Also the sum of the numbers is 91.
   Translate. The second number can be named $x + 9$. We translate to an equation:
   $$\begin{array}{ccccc} \underbrace{\text{First number}} & \text{plus} & \underbrace{\text{second number}} & \text{is} & 91. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ x & + & (x+9) & = & 91 \end{array}$$

8. Let x and $x + 6$ represent the numbers;
   $x + (x + 6) = 88$

9. ***Familiarize***. Let t = the time, in hours, it will take Noah to paddle 8 mi. We will use the formula Distance = Speed × Time. Noah's speed paddling against the current is (4.6 – 2.1) mph.
   Translate. We substitute in the formula
   $8 = (4.6 - 2.1)t$.

10. Let t = the time, in hours, it will take the plane to travel 725 km into the wind. The speed of the plane flying into the wind is 390 – 65 km/h. Then we have
    $725 = (390 - 65)t$.

11. ***Familiarize***. There are three angle measures involved, and we want to find all three. We can let x represent the smallest angle measure and note that the second is one more than x and the third is one more than the second, or two more than x. We also note that the sum of the three angle measures must be 180°.
    Translate. The three angle measures are x, $x + 1$, and $x + 2$. We translate to an equation:
    $$\begin{array}{ccccccc} \underbrace{\text{First}} & \text{plus} & \underbrace{\text{second}} & \text{plus} & \underbrace{\text{third}} & \text{is} & 180^\circ. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ x & + & (x+1) & + & (x+2) & = & 180 \end{array}$$

12. Let c = the cost without the 15% discount. Then the discounted price was c – 15%c, or $c - 0.15c$. Then we have $c - 0.15c = 272$.

13. ***Familiarize***. Since the escalator's speed is 105 ft/min and Dominik's walking speed is 100 ft/min, Dominik will move at a speed of 100 + 105 ft/min on the escalator. Let t = the time, in minutes, it takes to reach the top.
    Translate. We will use the formula Distance = Speed × Time.
    $$\begin{array}{ccccc} \underbrace{\text{Distance}} & = & \underbrace{\text{Speed}} & \times & \text{Time} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 205 & = & (100+105) & \times & t \end{array}$$

14. Let t represent the time, in seconds, that is takes Alida to walk the length of the sidewalk. Alida will move at a speed of 6 + 4, or 10 ft/sec. Then we have $912 = (6 + 4)t$.

15. ***Familiarize***. Let w represent the wholesale price, in dollars. Then the wholesale price raised by 50% is $w + 0.5w$.

Translate.

$$\begin{array}{ccccc} \underbrace{\text{Wholesale price raised 50\%}} & \text{plus} & \underbrace{\$1.50} & \text{is} & \underbrace{\text{selling price.}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ w+0.5w & + & 1.50 & = & 22.50 \end{array}$$

16. Let b = the original amount of the bill. Then the discounted price was $b - 5\%b$, or $b - 0.05b$. Then we have

$b - 0.05b = 142.50.$

17. ***Familiarize***. Let t = the time, in minutes, required for the plane to reach 29,000 ft. Since Distance = Speed × Time, the plane will travel $3500 \times t$ ft in t min. Note that the plane starts at an altitude of 8000 ft.
Translate.

$$\begin{array}{ccccc} \underbrace{\text{Current attitude}} & \text{plus} & \underbrace{\text{distance climbed}} & \text{is} & \underbrace{\text{29,000 ft.}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 8000 & + & 3500t & = & 29{,}000 \end{array}$$

18. ***Familiarize***. Let x represent the measure of the second angle. Then the first angle is four times x, and the third angle is 5° more than twice x. The sum of the three angles is 180°.
Translate. The first angle is $4x$, the second angle is x, and the third angle is $2x + 5$. Translate to an equation:

$$\begin{array}{ccccccc} \underbrace{\text{First}} & \text{plus} & \underbrace{\text{second}} & \text{plus} & \underbrace{\text{third}} & \text{is} & 180^\circ. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4x & + & x & + & (2x+5) & = & 180 \end{array}$$

19. ***Familiarize***. Note that each odd integer is 2 more than the one preceding it. If we let n represent the first odd integer then $n + 2$ represents the next odd integer and $(n + 2) + 2$, or $n + 4$ is the third odd integer.
Translate.

$$\begin{array}{ccccccc} \underbrace{\text{First}} & \text{plus} & \underbrace{\text{twice the second}} & \text{plus} & \underbrace{\text{three times the third}} & \text{is} & 70. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ n & + & 2(n+2) & + & 3(n+4) & = & 70 \end{array}$$

20. Let x represent the first even integer. Then the second even integer is $x + 2$. We have

$2x + 3(x + 2) = 76.$

21. ***Familiarize***. The perimeter of an equilateral triangle is 3 times the length of a side. Let s = the length of a side of the smaller triangle. Then $2s$ = the length of a side of the larger triangle. The sum of the two perimeters is 90 cm.
Translate.

$$\begin{array}{ccccc} \underbrace{\text{Perimeter of smaller triangle}} & \text{plus} & \underbrace{\text{perimeter of larger triangle}} & \text{is} & \underbrace{\text{90 cm.}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 3s & + & 3\cdot 2s & = & 90 \end{array}$$

22. Let x represent the longer length; $x+\frac{2}{3}x=10$

23. ***Familiarize***. Let c represent the number of calls Cody will need on his next shift if he is to average 3 calls per shift. We find the average by adding the number of calls on each of the 5 shifts and then dividing by the number of addends.
Translate.

$$\begin{array}{ccc} \underbrace{\text{Average number of calls per shift}} & \text{is} & \underbrace{3.} \\ \downarrow & \downarrow & \downarrow \\ \dfrac{5+2+1+3+c}{5} & = & 3 \end{array}$$

24. Let x represent the score on the next test;

$$\frac{93+89+72+80+96+x}{6}=88$$

25. ***Familiarize***. Let p represent the price Tony paid for his graphing calculator.
Translate.

$$\begin{array}{ccccc} \underbrace{\text{Price Tess paid}} & \text{is} & \underbrace{\text{price Tony paid}} & \text{less} & \$13. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 124 & = & p & - & 13 \end{array}$$

Carry out. We solve the equation.

$124 = p - 13$
$137 = p$ Adding 13 to both sides

Check. The price Tess paid, \$124, is \$13 less than \$137, so the answer checks.
State. Tony paid \$137 for his graphing calculator.

26. Let s represent the number of students in Rose's class. Solve $35 = s + 12$.
$s = 23$ students

27. ***Familiarize***. Let r represent the average monthly rent of an apartment in Charlotte, in dollars.
Translate.

$$\begin{array}{ccccc} \underbrace{\text{Rent in Greenville}} & \text{is} & \dfrac{4}{5} & \text{of} & \underbrace{\text{rent in Charlotte}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1100 & = & \dfrac{4}{5} & \cdot & r \end{array}$$

Carry out. We solve the equation.

$$1100=\frac{4}{5}r$$
$$\frac{5}{4}\cdot 1100=\frac{5}{4}\cdot\frac{4}{5}r$$
$$1375=r$$

Check. Since $\frac{4}{5}$ of \$1375 is \$1100, the answer checks.

State. The average monthly apartment rent in Charlotte is \$1375.

28. Let p represent the amount Anne paid for a haircut in Seattle.

Solve $75=\frac{3}{2}p$

$p = \$50$

29. ***Familiarize***. Let x represent the number of flu shots given by Mike. Then $x + 11$ represents the number of flu shots given by Vance.
Translate.

$$\begin{array}{ccc} \underbrace{\text{Total flu shots}} & \text{is} & \underbrace{53.} \\ \downarrow & \downarrow & \downarrow \\ x+(x+11) & = & 53 \end{array}$$

Carry out. We solve the equation.

$$\begin{aligned} x+(x+11)&=53 \\ 2x+11&=53 \\ 2x&=42 \quad \text{Subtracting 11 from both sides} \\ x&=21 \end{aligned}$$

If $x = 21$, then $x + 11 = 21 + 11$, or 32.
Check. 32 is 11 more than 21. Also, 21 + 32 = 53 flu shots. The answer checks.
State. Vance gave 32 flu shots.

30. Let c represent the number of calls Paul made.
Solve $256=(c-12)+c$
$c = 134$
$c - 12 = 134 - 12 = 122$
Meghan made 122 calls.

31. ***Familiarize***. Let w represent the width of the mirror, in cm. Then $3w$ represents the length. Recall that the formula for the perimeter P of a rectangle with length l and width w is $P = 2l + 2w$.
Translate.

$$\begin{array}{ccc} \underbrace{\text{Perimeter}} & \text{is} & \underbrace{120\text{ cm.}} \\ \downarrow & \downarrow & \downarrow \\ 2\cdot 3w+2\cdot w & = & 120 \end{array}$$

Carry out. We solve the equation.

$$\begin{aligned} 2\cdot 3w+2\cdot w&=120 \\ 6w+2w&=120 \\ 8w&=120 \\ w&=15 \end{aligned}$$

When $w = 15$, then $3w=3\cdot 15=45$.
Check. If the length is 45 cm and the width is 15 cm, then the length is three times the width. Also $P=2\cdot 45+2\cdot 15=90+30=120$ cm. The answer checks.
State. The length of the mirror is 45 cm, and the width is 15 cm.

32. Let w represent the width of the tile, in cm. Then $2w$ represents the length.
Solve $2\cdot 2w+2\cdot w=21.$
$w = 3.5$, so $2w = 2(3.5) = 7$
The length of the tile is 7 cm, and the width is 3.5 cm.

33. ***Familiarize***. Let l represent the length of the greenhouse, in meters. Then $\frac{1}{4}l$ represents the width.
Recall that the formula for the perimeter P, of a rectangle with length l and width w is $P = 2l + 2w$.
Translate.

$$\begin{array}{ccc} \underbrace{\text{Perimeter}} & \text{is} & \underbrace{130\text{ m.}} \\ \downarrow & \downarrow & \downarrow \\ 2l+2\cdot\frac{1}{4}l & = & 130 \end{array}$$

Carry out. We solve the equation.

$$\begin{aligned} 2l+2\cdot\frac{1}{4}l&=130 \\ 2l+\frac{1}{2}l&=130 \\ \frac{5}{2}l&=130 \\ \frac{2}{5}\cdot\frac{5}{2}l&=\frac{2}{5}\cdot 130 \\ l&=52 \end{aligned}$$

When $l = 52$, then $\frac{1}{4}l=13$.
Check. If the length is 52 m and the width is 13 m, then the width is $\frac{1}{4}$ of the length. Also, $P=2\cdot 52+2\cdot 13=104+26=130$ m. The answer checks.
State. The length of the greenhouse is 52 m, and the width is 13 m.

34. Let y = the length; then $\frac{1}{3}y=$ the width.
Solve $2\cdot y+2\cdot\frac{1}{3}y=32.$
$y = 12$, so the length is 12 m, and the width is $\frac{1}{3}\cdot 12$ m, or 4 m.

35. The Familiarize and Translate steps were done in Exercise 9.
$8 = (4.6 - 2.1)t$.
Carry out. We solve the equation.

$$\begin{aligned} 8&=(4.6-2.1)t \\ 8&=2.5t \\ \frac{8}{2.5}&=\frac{2.5}{2.5}t \\ 3.2&=t \end{aligned}$$

Check. At a speed of 2.5 mph, in 3.2 hr, Noah paddles 2.5(3.2), or 8 mi. Our answer checks.
State. Noah takes 3.2 hr to paddle 8 mi.

36. $205 = (100 + 105)t$
$t = 1$ min

37. The Familiarize and Translate steps were done in Exercise 11.
Carry out. We solve the equation.

$$\begin{aligned} x+(x+1)+(x+2)&=180 \\ 3x+3&=180 \\ 3x&=180-3 \\ 3x&=177 \\ \frac{1}{3}\cdot 3x&=\frac{1}{3}\cdot 177 \\ x&=59 \end{aligned}$$

When $x = 59$, then $x+1=59+1=60,$ and $x+2=59+2=61.$
Check. The angles, 59°, 60°, and 61° are consecutive integers. Also 59° + 60° + 61° = 180°. The answer checks.
State. The measures of the angles are 59°, 60°, and 61°.

38.
$$\begin{aligned} 4x+x+(2x+5)&=180\\ 7x+5&=180\\ 7x&=180-5\\ 7x&=175\\ \frac{1}{7}\cdot 7x&=\frac{1}{7}\cdot 175\\ x&=25 \end{aligned}$$
When $x = 25$, then $4x = 4\cdot 25 = 100$, and
$2x+5=2\cdot 25+5=55.$
The measures of the angles are 100°, 25°, and 55°.

39. The Familiarize and Translate steps were done in Exercise 16.
Carry out. We solve the equation.
$$\begin{aligned} c-0.05c&=142.50\\ 0.95c&=142.50\\ \frac{0.95c}{0.95}&=\frac{142.50}{0.95}\\ c&=150 \end{aligned}$$
Check. 5% of \$150 is \$7.50 and \$150 – \$7.50 = \$142.50.
The answer checks.
State. The cost would have been \$150 if the bill had not been paid promptly.

40. $c - 0.15c = 272$
$c = \$320$
The original cost of the order was \$320.

41. The Familiarize and Translate steps were done in Exercise 15.
Carry out. We solve the equation.
$$\begin{aligned} 1.5w+1.50&=22.50\\ 1.5w&=21\\ w&=\frac{1}{1.5}\cdot 21\\ w&=14 \end{aligned}$$
Check. If a wholesale price of \$14 is raised by 50%, we have \$14 + 0.5(\$14) = \$14 + \$7 = \$21. When \$1.50 is added to this figure, we have \$21 + \$1.50 = \$22.50.
The answer checks.
State. The wholesale price is \$14.

42.
$$\begin{aligned} 2n+3(n+2)&=76\\ 2n+3n+6&=76\\ 5n+6&=76\\ 5n&=70\\ n&=14 \end{aligned}$$
When $n = 14$, then $n + 2 = 14 + 2 = 16$. The integers are 14 and 16.

43. *Writing Exercise*. Answers may vary. One possibility is: Jason has $\frac{1}{3}$ of a pizza. He offers half to Beth. What part of the pizza did Beth receive?

44. *Writing Exercise*. Answers may vary. One possibility is: A garden is $16\frac{1}{2}$ ft long. Flowers are to be planted $1\frac{1}{3}$ ft apart. Find the number of rows of flowers.

45. *Writing Exercise*. The manner in which a guess or estimate is manipulated can give insight into the form of the equation to which the problem will be translated.

46. *Writing Exercise*. If a problem is not translated correctly in Step 2 (Translate), it is very probable that the solution of the equation is not a solution of the original problem.

47. ***Familiarize***. The average score on the first four tests is $\frac{83+91+78+81}{4}$, or 83.25. Let x = the number of points above this average that Tico scores on the next test. Then the score on the fifth test is $83.25 + x$.
Translate.
$$\begin{array}{ccccc} \underbrace{\text{Average score on 5 tests}} & \text{is} & 2 & \underbrace{\text{more than}} & \underbrace{\text{average score on 4 tests.}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \frac{83+91+78+81+(83.25+x)}{5} & = & 2 & + & 83.25 \end{array}$$
Carry out. Carry out some algebraic manipulation.
$$\begin{aligned} \frac{83+91+78+81+(83.25+x)}{5}&=2+83.25\\ \frac{416.25+x}{5}&=85.25\\ 416.25+x&=426.25\\ x&=10 \end{aligned}$$
Check. If Tico scores 10 points more than the average of the first four tests on the fifth test, his score will be 83.25 + 10, or 93.25. Then the five-test average will be $\frac{83+91+78+81+93.25}{5}$, or 85.25. This is 2 points above the four-test average, so the answer checks.
State. Tico must score 10 points above the four-test average in order to raise the average 2 points.

48. Let x = the height; then the sides are given by $x + 1$, $x + 2$, and $x + 3$, where $x + 2$ represents the base.
Solve $(x + 1) + (x + 2) + (x + 3) = 42$.
$x = 12$, so the height of the triangle is 12 in., and the base is 12 + 2, or 14 in. Then the area is
$\frac{1}{2}\cdot 14\text{ in.}\cdot 12\text{ in.}$, or 84 sq in.

49. ***Familiarize***. Let a = the number of animals adopted in 2011. From 2011 to 2012 the number of adoptions decreased 1.1%, so $a - 0.011a$, or $0.989a$. From 2012 to 2013 the number of adoptions decreased 3.6%, so $0.989a - 0.036(0.989a)$, or $0.964(0.989a)$. From 2013 to 2014 the numbers decreased 8.5%, so $0.964(0.989a) - 0.085(0.964)(0.989a)$, or $0.915(0.964)(0.989a)$. From 2014 to 2015, the numbers increased 11.0%, so the number of adoptions became $0.915(0.964)(0.989a) + 0.11(0.915)(0.964)(0.989a)$, or $1.11(0.915)(0.964)(0.989a)$.
Translate.
$$\begin{array}{ccc} \underbrace{\text{Adoptions in 2015}} & \text{were} & 2879. \\ \downarrow & \downarrow & \downarrow \\ 1.11(0.915)(0.964)(0.989a) & = & 2879 \end{array}$$
Carry out. We solve the equation.
$$\begin{aligned} 1.11(0.915)(0.964)(0.989a)&=2879\\ a&=\frac{2879}{1.11(0.915)(0.964)(0.989)}\\ a&\approx 2974 \end{aligned}$$

Check. If the number of adoptions in 2011 was 2974 , then in 2012 there were 0.989(2974), or 2941. In 2013 there were 0.964(2941), or 2835. In 2014 there were 0.915(2835), or 2594, and in 2015 there were 1.11(2594), or 2879. Our answer checks.
State. There were 2974 animal adoptions in 2011.

50. Let x represent the number by which the reduced salary would have to be multiplied in order to return it to the original salary S. Note that n% can be expressed as $0.01n$ in decimal notation.

$$\underbrace{\text{Reduced salary}}\quad \text{times} \quad \underbrace{\text{what number}} \quad \text{is} \quad \underbrace{\text{original salary.}}$$
$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$
$$S(1-0.01n) \qquad \cdot \qquad x \qquad = \qquad S$$

$$S(1-0.01n)(x)=S$$
$$x=\frac{1}{1-0.01n}, \text{ or } \frac{100}{100-n}$$

Connecting the Concepts

1. $2x-3=7$
$$2x=10$$
$$x=5$$

2. $2x-c=h$
$$2x=c+h$$
$$x=\frac{c+h}{2}$$

3. $8-3(y-7)=2y$
$$8-3y+21=2y$$
$$-3y+29=2y$$
$$29=5y$$
$$\frac{29}{5}=y$$

4. $8-n(y-c)=ay$
$$8-ny+nc=ay$$
$$8+nc=ay+ny$$
$$8+nc=y(a+n)$$
$$\frac{8+nc}{a+n}=y$$

5.
$$\frac{a}{3}-\frac{1}{2}=4a$$
$$6\left(\frac{a}{3}-\frac{1}{2}\right)=6(4a)$$
$$2a-3=24a$$
$$-3=22a$$
$$-\frac{3}{22}=a$$

6.
$$\frac{a}{3}-\frac{n}{2}=xa$$
$$6\left(\frac{a}{3}-\frac{n}{2}\right)=6(xa)$$
$$2a-3n=6xa$$
$$-3n=6xa-2a$$
$$-3n=a(6x-2)$$
$$\frac{-3n}{6x-2}=a \quad \text{or } a=\frac{3n}{2-6x}$$

Exercise Set 1.5

1. A formula is an *equation* that uses letters to represent a relationship between two or more quantities.

2. The formula $A=\pi r^2$ is used to calculate the *area* of a circle.

3. The formula $C=\pi d$ is used to calculate the *circumference* of a circle.

4. The formula $\underline{A=\frac{1}{2}bh}$ is used to calculate the area of a triangle of height h and base b.

5. The formula $\underline{A=bh}$ is used to calculate the area of a parallelogram of height h and base length b.

6. The formula $l=A/w$ can be used to determine the *length* of a rectangle, given its area and width.

7. In the formula for the area of a trapezoid, $A=\frac{h}{2}(b_1+b_2)$ the numbers 1 and 2 are referred to as *subscripts*.

8. When two or more terms on the same side of a formula contain the letter for which we are solving, we can *factor* so that the letter is only written once.

9.
$$E=wA$$
$$\frac{1}{w}\cdot E=\frac{1}{w}\cdot wA \quad \text{Multiplying both sides by } \frac{1}{w}$$
$$\frac{E}{w}=A \quad \text{Simplifying}$$

10. $a=\dfrac{F}{m}$

11.
$$d=rt$$
$$\frac{1}{t}\cdot d=\frac{1}{t}\cdot rt \quad \text{Multiplying both sides by } \frac{1}{t}$$
$$\frac{d}{t}=r \quad \text{Simplifying}$$

12. $E=\dfrac{P}{I}$

13.
$$V=lwh$$
$$\frac{1}{lw}\cdot V=\frac{1}{lw}\cdot lwh \quad \text{Multiplying both sides by } \frac{1}{lw}$$
$$\frac{V}{lw}=h \quad \text{Simplifying}$$

14. $r=\dfrac{I}{Pt}$

15.
$$L=\frac{k}{d^2}$$
$$d^2\cdot L=d^2\cdot\frac{k}{d^2} \quad \text{Multiplying both sides by } d^2$$
$$d^2L=k \quad \text{Simplifying}$$

16.
$$F=\frac{mv^2}{r}$$
$$F\cdot\frac{r}{v^2}=\frac{mv^2}{r}\cdot\frac{r}{v^2}$$
$$\frac{Fr}{v^2}=m$$

17. $G = w + 150n$

$G - w = 150n$ Subtracting w from both sides

$\frac{1}{150}(G - w) = \frac{1}{150} \cdot 150n$ Multiplying both sides by $\frac{1}{150}$

$\frac{G - w}{150} = n$ Simplifying

18. $P = b + 1.5t$

$P - b = 1.5t$

$\frac{P - b}{1.5} = t$, or

$\frac{2}{3}(P - b) = t$

19. $2w + 2h + l = p$

$l = p - 2w - 2h$ Adding $-2w - 2h$ to both sides

20. $2w + 2h + l = p$

$2w = p - 2h - l$

$w = \frac{p - 2h - l}{2}$

21. $2x + 3y = 4$

$3y = 4 - 2x$ Subtracting $2x$ from both sides

$\frac{1}{3} \cdot 3y = \frac{1}{3}(4 - 2x)$ Multiplying both sides by $\frac{1}{3}$

$y = \frac{4 - 2x}{3}$ Simplifying

or $y = -\frac{2}{3}x + \frac{4}{3}$

22. $3x - 7y = 2$

$-7y = -3x + 2$

$y = \frac{3x - 2}{7}$ or $y = \frac{3}{7}x - \frac{2}{7}$

23. $Ax + By = C$

$By = C - Ax$ Subtracting Ax from both sides

$\frac{1}{B} \cdot By = \frac{1}{B}(C - Ax)$ Multiplying both sides by $\frac{1}{B}$

$y = \frac{C - Ax}{B}$ Simplifying

24. $P = 2l + 2w$

$P - 2w = 2l$

$\frac{P - 2w}{2} = l$, or $\frac{P}{2} - w = l$

25. $C = \frac{5}{9}(F - 32)$

$\frac{9}{5} \cdot C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$ Multiplying both sides by $\frac{9}{5}$

$\frac{9}{5}C = F - 32$ Simplifying

$\frac{9}{5}C + 32 = F$

26. $T = \frac{3}{10}(I - 12{,}000)$

$\frac{10}{3}T = I - 12{,}000$

$\frac{10}{3}T + 12{,}000 = I$

27. $V = \frac{4}{3}\pi r^3$

$\frac{3}{4\pi} \cdot V = \frac{3}{4\pi} \cdot \frac{4}{3}\pi r^3$ Multiplying both sides by $\frac{3}{4\pi}$

$\frac{3V}{4\pi} = r^3$ Simplifying

28. $\pi = \frac{3V}{4r^3}$

29. $np + nm = t$

$n(p + m) = t$ Factoring

$n = \frac{t}{p + m}$ Dividing both sides by $p + m$

30. $ab + ac = d$

$a(b + c) = d$

$a = \frac{d}{b + c}$

31. $uv + wv = x$

$v(u + w) = x$ Factoring

$v = \frac{x}{u + w}$ Dividing both sides by $u + w$

32. $st + rt = n$

$t(s + r) = n$

$t = \frac{n}{s + r}$

33. $A = \frac{q_1 + q_2 + q_3}{n}$

$n \cdot A = n \cdot \frac{q_1 + q_2 + q_3}{n}$ Clearing the fraction

$nA = q_1 + q_2 + q_3$

$nA \cdot \frac{1}{A} = (q_1 + q_2 + q_3) \cdot \frac{1}{A}$ Multiplying both sides by $\frac{1}{A}$

$n = \frac{q_1 + q_2 + q_3}{A}$

34. $g = \frac{km_1m_2}{d^2}$

$d^2 g = km_1m_2$

$d^2 = \frac{km_1m_2}{g}$

35. $v = \frac{d_2 - d_1}{t}$

$t \cdot v = t \cdot \frac{d_2 - d_1}{t}$ Clearing the fraction

$tv = d_2 - d_1$

$tv \cdot \frac{1}{v} = (d_2 - d_1) \cdot \frac{1}{v}$ Multiplying both sides by $\frac{1}{v}$

$t = \frac{d_2 - d_1}{v}$

36. $v = \frac{s_2 - s_1}{m}$

$mv = s_2 - s_1$

$m = \frac{s_2 - s_1}{v}$

37. $v = \dfrac{d_2 - d_1}{t}$

$t \cdot v = t \cdot \dfrac{d_2 - d_1}{t}$ Clearing the fraction

$tv = d_2 - d_1$

$tv - d_2 = -d_1$ Subtracting d_2 from both sides

$-1 \cdot (tv - d_2) = -1(-d_1)$ Multiplying both sides by -1

$-tv + d_2 = d_1$,

or $d_2 - tv = d_1$

38. $v = \dfrac{s_2 - s_1}{m}$

$vm = s_2 - s_1$

$s_1 = s_2 - vm$

39. $bd = c + ba$

$bd - ba = c$ Adding $-ba$ to both sides

$b(d - a) = c$ Factoring

$b = \dfrac{c}{d - a}$ Dividing both sides by $d - a$

40. $st = n + sm$

$st - sm = n$

$s(t - m) = n$

$s = \dfrac{n}{t - m}$

41. $v - w = uvw$

$v = uvw + w$ Adding w to both sides

$v = w(uv + 1)$ Factoring

$\dfrac{v}{uv + 1} = w$ Dividing both sides by $uv + 1$

42. $p - q = qrs$

$p = qrs + q$

$p = q(rs + 1)$

$\dfrac{p}{rs + 1} = q$

43. $n - mk = mt^2$

$n = mt^2 + mk$ Adding mk to both sides

$n = m(t^2 + k)$ Factoring

$\dfrac{n}{t^2 + k} = m$ Dividing both sides by $t^2 + k$

44. $d - ct = ca^3$

$d = ca^3 + ct$

$d = c(a^3 + t)$

$\dfrac{d}{a^3 + t} = c$

45. ***Familiarize***. In Example 2, we find the formula for simple interest, $I = Prt$, when I is the interest, P is the principal, r is the interest rate, and t is the time, in years.
Translate. We want to find the interest rate, so we solve the formula for r.

$I = Prt$

$\dfrac{1}{Pt} \cdot I = \dfrac{1}{Pt} \cdot Prt$

$\dfrac{I}{Pt} = r$

Carry out. The model $r = \dfrac{I}{Pt}$ can be used to find the rate of interest at which an amount (the principal) must be invested in order to earn a given amount. We substitute \$2600 for P, $\frac{1}{2}$ for t (6 months = $\frac{1}{2}$ yr), and \$104 for I.

$\dfrac{I}{Pt} = r$

$\dfrac{\$104}{\$2600\left(\frac{1}{2}\right)} = r$

$\dfrac{104}{1300} = r$

$0.08 = r$

$8\% = r$

Check. Since $\$2600(0.08)\left(\frac{1}{2}\right) = \$104$, the answer checks.
State. The interest rate must be 8%.

46. Solve $I = Prt$ for P and substitute.

$P = \dfrac{I}{rt} = \dfrac{\$150}{0.04(2)} = \$1875$

47. ***Familiarize***. In the text, we find the formula for the area of a parallelogram, $A = bh$, where b is the base and h is the height.
Translate. We solve the formula for h.

$A = bh$

$\dfrac{1}{b} \cdot A = \dfrac{1}{b} \cdot bh$

$\dfrac{A}{b} = h$

Carry out. The model $h = \dfrac{A}{b}$ can be used to find the height of any parallelogram for which the area and base are known. We substitute 96 for A and 6 for b.

$h = \dfrac{A}{b}$

$h = \dfrac{96}{6}$

$h = 16$

Check. We repeat the calculation. The answer checks.
State. The height is 16 cm.

48. Solve $A = bh$ for b and substitute.

$b = \dfrac{A}{h} = \dfrac{84}{7} = 12$ cm

49. ***Familiarize and Translate***. In Example 4, the formula for body mass index is solved for W:

$W = \dfrac{IH^2}{704.5}$

Carry out. We substitute 30.8 for I and 74 for H (6 ft 2 in. is 74 in.) and calculate W.

$W = \dfrac{30.8(74)^2}{704.5} \approx 239$

Check. We could repeat the calculations or substitute in the original formula and then solve for W. The answer checks.
State. Arnold Schwarzenegger weighs about 239 lb.

50. See Example 4. Also note that 5 ft 8 in. is 68 in.

Solve $W = \dfrac{17.9(68)^2}{704.5} \approx 117$ lb

51. ***Familiarize and Translate***. We will use the model developed in Example 5, $m = \pi r^2 hD$ to find the weight of the salt. Then we will add 28 g, the weight of the empty canister, to find the weight of the filled canister.
Carry out. We substitute 4 for r, 13.6 for h, and 2.16 for D and calculate m.

$$\begin{aligned} m &= \pi r^2 hD \\ &= \pi(4)^2(13.6)(2.16) \\ &\approx 1476.6 \end{aligned}$$

Add the weight of the empty canister:
1476.6 + 28 = 1504.6
Check. We repeat the calculations. The answer checks.
State. The filled canister weighs about 1504.6 g.

52. We will use the model developed in Example 5, $m = \pi r^2 hD$ to find the weight of a gold coin. Then we will subtract 26.4 g, the weight of the silver coin found in Example 5, to determine how much more a gold coin would weigh.

$$m = \pi(2)^2(0.2)(19.3) \approx 48.5$$

Find the difference in weights:
$48.5 - 26.4 = 22.1 \approx 22$ g

53. ***Familiarize***. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where A is the area, h is the height, and b_1 and b_2 are the bases.
Translate. The unknown dimension is the height, so we solve the formula for h. We have $h = \dfrac{2A}{b_1 + b_2}$.
Carry out. We substitute.

$$h = \frac{2A}{b_1 + b_2}$$
$$h = \frac{2 \cdot 90}{8 + 12}$$
$$h = \frac{180}{20}$$
$$h = 9$$

Check. We repeat the calculation. The answer checks.
State. The unknown dimension, the height of the trapezoid, is 9 ft.

54. Solve $P = 2l + 2w$ for l and substitute.

$$l = \frac{P - 2w}{2} = \frac{76 - 2 \cdot 13}{2} = \frac{76 - 26}{2} = \frac{50}{2} = 25 \text{ ft}$$

55. Observe that 4% of \$1000 is \$40, so \$40 is the amount of simple interest that would be earned in 1 yr. Thus, it will take 1 yr for the investment to be worth \$1040.

56. $t = \dfrac{A - P}{Pr}$ (See Example 3)

$$t = \frac{1178 - 950}{950(0.03)} = 8 \text{ years}$$

57. ***Familiarize***. We use the formula given in the text, $f = \dfrac{2r + c}{2L}$.
Translate. The unknown quantity is the pipe length. We solve the formula for L.

$$f = \frac{2r + c}{2L}.$$
$$L = \frac{2r + c}{2f}$$

Carry out. We substitute 40 for b and 7.8 for r and calculate c.

$$L = \frac{2(1) + 13{,}500}{2(27.5)} \approx 246$$

Check. We repeat the calculations. The answer checks.
State. The pipe should be 246 inches long.

58.
$$L = \frac{E(H_i - H_f)}{H_i}$$
$$1470 = \frac{E(13 - 7)}{13}$$
$$E = 3185 \text{ mL}$$

59. ***Familiarize***. We will use the formula given in the text, $R = r + \dfrac{400(W - L)}{N}$.
Translate. We solve the formula for r.

$$R = r + \frac{400(W - L)}{N}$$
$$R - \frac{400(W - L)}{N} = r$$

Carry out. We substitute 1305 for R, 5 for w, 3 for L, and 5 + 3, or 8, for N and calculate r.

$$1305 - \frac{400(5 - 3)}{8} = r$$
$$1205 = r$$

Check. We can repeat the calculation or substitute in the original formula and then solve for r. The answer checks.
State. The average rating of Ulana's opponents was 1205.

60. Calculate $r = 1050 - \dfrac{400(2 - 5)}{2 + 5}$. (See Exercise 59.)

$r \approx 1221$

61. ***Familiarize***. We will use the formula given in the text, $K = 917 + 6(w + h - a)$.
Translate. We solve the formula for h.

$$\begin{aligned} K &= 917 + 6(w + h - a) \\ K &= 917 + 6w + 6h - 6a \\ K - 917 - 6w + 6a &= 6h \\ \frac{K - 917 - 6w + 6a}{6} &= h \end{aligned}$$

Carry out. We substitute 1901 for K, 120 for w, and 23 for a and calculate h.

$$\frac{1901-917-6\cdot 120+6\cdot 23}{6}=h$$
$$67=h$$

Check. We can repeat the calculation or substitute in the original formula and then solve for *h*. The answer checks.
State. Julie is 67 in., or 5 ft 7 in., tall.

62. First solve the given formula for *w*.

$$K=917+6(w+h-a)$$
$$K=917+6w+6h-6a$$
$$K-917-6h+6a=6w$$
$$\frac{K-917-6h+6a}{6}=w$$

Then substitute and calculate *w*. Note that 5 ft 4 in. is 64 in.

$$\frac{1901-917-6\cdot 64+6\cdot 31}{6}=w$$
$$131\text{ lb}=w$$

63. ***Familiarize***. We use the formula given in the text. $g=0.0778n+4.55s-2.2029$.
Translate. We solve the formula for *n*.

$$g=0.0778n+4.55s-2.2029$$
$$n=\frac{g-4.55s+2.2029}{0.0778}$$

Carry out. We substitute 3.0 for *g* and 1.02 for *s* and calculate *n*.

$$n=\frac{3.0-4.55(1.02)+2.2029}{0.0778}$$
$$n\approx 7.22$$

Check. We can repeat the calculations or substitute in the original formula and solve for *n*. The answer checks.
State. There should be 7.22 words per sentence.

64. First solve the given formula for *s*.

$$g=0.0778n+4.55s-2.2029$$
$$s=\frac{g-0.0778n+2.2029}{4.55}$$

Then we substitute and calculate *s*.

$$s=\frac{3.8-0.0778(5)+2.2029}{4.55}$$
$$\approx 1.23 \text{ syllables per word}$$

65. ***Familiarize***. We use the formula given in the text.

$$S=\frac{HR(W_i-W_n)}{1000}.$$

Translate. We solve the formula for *R*.

$$S=\frac{HR(W_i-W_n)}{1000}$$
$$R=\frac{1000S}{H(W_i-W_n)}$$

Carry out. We substitute 100 for W_i, 15 for W_n, 2000 for *H*, and \$20.40 for *S*, and calculate *R*.

$$R=\frac{1000(20.40)}{2000(100-15)}=0.12$$

Check. We can repeat the calculations or substitute in the original formula and solve for *R*. The answer checks.
State. The cost is \$0.12 per kWh.

66. First solve the given formula for W_n.

$$S=\frac{HR(W_i-W_n)}{1000}$$
$$W_n=W_i-\frac{1000S}{HR}$$

Then we substitute and calculate W_n.

$$W_n=150-\frac{1000(42.90)}{2600(0.15)}$$
$$=40\text{ watts}$$

67. ***Familiarize***. We use the formula given in the text.

$$r=\frac{tmap}{hs}$$

Translate. We solve the formula for *t*.

$$r=\frac{tmap}{hs}$$
$$t=\frac{rhs}{map}$$

Carry out. We substitute 30 for *s*, 4 for *h*, 0.05 for *m*, 100 for *a*, 0.15 for *p*, 3.2 for *r* and solve for *t*.

$$t=\frac{3.2\cdot 4\cdot 30}{0.05\cdot 100\cdot 0.15}$$
$$t=512$$

Check. We repeat the calculation or substitute in the original formula and solve for *t*. The answer checks.
State. The average daily blog traffic is 512 visits per day.

68. First solve the given formula for *h*.

$$r=\frac{tmap}{hs}$$
$$h=\frac{tmap}{rs}$$

Then we substitute and calculate *h*.

$$h=\frac{1200\cdot 0.04\cdot 150\cdot 0.14}{4.8\cdot 35}=6\text{ hrs}$$

69. ***Familiarize***. We will use Goiten's model, $I = 1.08(T/N)$. Note that 8 hr = 8 × 1 hr = 8 × 60 min = 480 min.
Translate. We solve the formula for *N*.

$$I=1.08\left(\frac{T}{N}\right)$$
$$N\cdot I=N(1.08)\left(\frac{T}{N}\right)$$
$$NI=1.08T$$
$$N=\frac{1.08T}{I}$$

Carry out. We substitute 480 for *T* and 15 for *I*.

$$N=\frac{1.08T}{I}$$
$$N=\frac{1.08(480)}{15}$$
$$N=34.56$$
$$N\approx 34 \quad \text{Rounding down}$$

Check. We repeat the calculations. The answer checks.
State. Dr. Cruz should schedule 34 appointments in one day.

70. Solve $I = 1.08\left(\frac{T}{N}\right)$ for T and substitute.

$$T = \frac{NI}{1.08} = \frac{25 \cdot 20}{1.08} \approx 463 \text{ min, or } 7.7 \text{ hr}$$

71. ***Familiarize***. We will use Thurnau's model, $P = 9.337da - 299$.
Translate. Since we want to find the diameter of the fetus' head, we solve for d.

$$\begin{aligned} P &= 9.337da - 299 \\ P + 299 &= 9.337da \\ \frac{P+299}{9.337a} &= d \end{aligned}$$

Carry out. Substitute 1614 for P and 24.1 for a in the formula and calculate:

$$\begin{aligned} \frac{1614+299}{9.337(24.1)} &= d \\ 8.5 &\approx d \end{aligned}$$

Check. We repeat the calculation. The answer checks.
State. The diameter of the fetus' head at 29 weeks is about 8.5 cm.

72. $P = 94.593c + 34.227a - 2134.616$

$$\begin{aligned} P - 34.227a + 2134.616 &= 94.593c \\ \frac{P - 34.227a + 2134.616}{94.593} &= c \\ \frac{1277 - 34.227(23.4) + 2134.616}{94.593} &= c \\ 27.6 \text{ cm} &\approx c \end{aligned}$$

73. *Writing Exercise*. Yes. The formula for the area of the parallelogram is $A = \frac{1}{2}h(b_1 + b_2)$. For a rectangle, $b_1 = b_2$. Simplifying the formula, we have, $A = \frac{1}{2}h(b_1 + b_1) = \frac{1}{2}h(2b_1) = hb_1$, which is the same as the usual formula for the area of a rectangle, $A = lw$.

74. *Writing Exercise*. Not necessarily; miscalculation of the age of the fetus at the time the ultrasonic image was taken could affect the result. Mismeasurement of the fetal head and/or abdomen could also affect the result.

75. *Writing Exercise*. The formulas will be used to predict fetal birth weight P, so it is most convenient to have P alone on one side.

76. *Writing Exercise*. Since N is the sum of the number of wins, losses, and draws, playing to a draw will help Heidi if she has more losses than wins because $W - L$ is negative and, the larger the value of N, the smaller the number that is subtracted from r to calculate her rating. Conversely, playing to a draw will hurt her if she has more wins than losses because $W - L$ is positive and, the larger the value of N, the smaller the number that is added to r to calculate her rating.

77. ***Familiarize***. First we find the volume of the ring. Note that the inner diameter is 2 cm, so the inner radius is 2/2 or 1 cm. Then the volume of the ring is the volume of a right circular cylinder with height 0.5 cm and radius
1 + 0.15, or 1.15 cm, less the volume of a right circular cylinder with height 0.5 cm and radius 1 cm. Recall that the formula for the volume of a right circular cylinder is $V = \pi r^2 h$. Then the volume of the ring is $\pi(1.15)^2(0.5) - \pi(1)^2(0.5) = 0.16125\pi \text{ cm}^3$.
Translate. To find the weight of the ring we will use the formula $D = \frac{m}{V}$. Solving for m, we get

$$\begin{aligned} D &= \frac{m}{V} \\ V \cdot D &= V \cdot \frac{m}{V} \\ V \cdot D &= m \end{aligned}$$

Carry out. We substitute in the formula $m = V \cdot D$

$$\begin{aligned} m &= 0.16125\pi(21.5) \\ m &\approx 10.9 \end{aligned}$$

Check. We repeat the calculations. The answer checks.
State The ring will weigh about 10.9 g.

78. $m = \pi r^2 hD$ (See Example 6.)

$$\begin{aligned} \frac{m}{\pi r^2 D} &= h \\ \frac{177.6}{\pi\left(\frac{1.85}{2}\right)^2(8.93)} &= h \\ 7.4 \text{ cm} &\approx h \end{aligned}$$

79. *Writing Exercise*. The trapezoids, positioned as shown, form a parallelogram with base $b_1 + b_2$ and height h. The area of a parallelogram is base × height, so the area of the given parallelogram is $(b_1 + b_2)h$, or $h(b_1 + b_2)$. Since the area of each trapezoid is one-half the area of the parallelogram, the area of a trapezoid is $\frac{1}{2} \cdot h(b_1 + b_2)$, or $\frac{h}{2}(b_1 + b_2)$.

80.

$$\begin{aligned} A &= 4lw + w^2 \\ A - w^2 &= 4lw \\ \frac{A - w^2}{4w} &= l \quad \text{Multiplying both sides by } \frac{1}{4w} \end{aligned}$$

81.

$$\begin{aligned} s &= v_i t + \frac{1}{2}at^2 \\ s - v_i t &= \frac{1}{2}at^2 \\ 2(s - v_i t) &= at^2 \\ \frac{2(s - v_i t)}{t^2} &= a, \text{ or} \\ \frac{2s - 2v_i t}{t^2} &= a \end{aligned}$$

82.

$$\begin{aligned} \frac{P_1V_1}{T_1} &= \frac{P_2V_2}{T_2} \\ P_1V_1T_2 &= P_2V_2T_1 \\ T_2 &= \frac{P_2V_2T_1}{P_1V_1} \end{aligned}$$

83. $b=\dfrac{h+w+p}{a+w+p+f}$

$b(a+w+p+f)=h+w+p$ Multiplying both sides by $a+w+p+f$

$ab+bw+bp+bf=h+w+p$

$bw-w=h+p-ab-bp-bf$

$w(b-1)=h+p-b(a+p+f)$

$w=\dfrac{h+p-b(a+p+f)}{b-1}$ Multiplying both sides by $\dfrac{1}{b-1}$

84. $m=\dfrac{\frac{d}{e}}{\frac{e}{f}}$

$\dfrac{me}{f}=\dfrac{d}{e}$ Multiplying both sides by $\dfrac{e}{f}$

$\dfrac{me^2}{f}=d$ Multiplying both sides by e

85. $\dfrac{b}{a-b}=c$

$b=c(a-b)$

$b+bc=ac$

$b(1+c)=ac$

$b=\dfrac{ac}{1+c}$

86. $\dfrac{a}{a+b}=c$

$a=c(a+b)$

$a=ac+bc$

$a-ac=bc$

$a(1-c)=bc$

$a=\dfrac{bc}{1-c}$

87. $s+\dfrac{s+t}{s-t}=\dfrac{1}{t}+\dfrac{s+t}{s-t}$

Observe that if we subtract $\dfrac{s+t}{s-t}$ from both sides we are left with an equivalent equation, $s=\dfrac{1}{t}$. We solve this equation for t.

$s=\dfrac{1}{t}$

$st=1$ Multiplying both sides by t

$t=\dfrac{1}{s}$ Multiplying both sides by $\dfrac{1}{s}$

Exercise Set 1.6

1. The power rule
2. Raising a quotient to a power
3. Raising a product to a power
4. The quotient rule
5. The product rule
6. The power rule
7. Raising a quotient to a power
8. Raising a product to a power
9. The quotient rule
10. The product rule
11. $6^4\cdot 6^7=6^{4+7}=6^{11}$
12. 3^{17}
13. $m^0\cdot m^8=m^{0+8}=m^8$
14. t^6
15. $5x^4\cdot 4x^3=5\cdot 4\cdot x^4\cdot x^3=20x^{4+3}=20x^7$
16. $6a^9$
17. $(-3a^2)(-8a^6)=(-3)(-8)a^2\cdot a^6=24a^{2+6}=24a^8$
18. $-24m^9$
19. $(m^5n^2)(m^3np^0)=(m^5m^3)(n^2n)(p^0)=m^{5+3}n^{2+1}\cdot 1$ $=m^8n^3$
20. x^7y^7
21. $\dfrac{t^8}{t^3}=t^{8-3}=t^5$
22. $\dfrac{a^{11}}{a^8}=a^{11-8}=a^3$
23. $\dfrac{15a^7}{3a^2}=\dfrac{15}{3}a^{7-2}=5a^5$
24. $3t^6$
25. $\dfrac{m^7n^9}{m^2n^5}=m^{7-2}\cdot n^{9-5}=m^5n^4$
26. mn^3
27. $\dfrac{32x^8y^5}{8x^2y}=\dfrac{32}{8}\cdot x^{8-2}\cdot y^{5-1}=4x^6y^4$
28. $5x^6y^6$
29. $\dfrac{28x^{10}y^9z^8}{-7x^2y^3z^2}=\dfrac{28}{-7}\cdot x^{10-2}\cdot y^{9-3}\cdot z^{8-2}=-4x^8y^6z^6$
30. $5x^6y^3z^2$
31. $-x^0=-(-2)^0=-(1)=-1$
32. $(-x)^0=(-(-2))^0=2^0=1$

33. $(4x)^0 = (4(-2))^0 = (-8)^0 = 1$

34. $4x^0 = 4(-2)^0 = 4 \cdot 1 = 4$

35. $t^{-9} = \frac{1}{t^9}$

36. $\frac{1}{m^2}$

37. $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

38. $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

39. $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$

40. $(-2)^{-4} = \frac{1}{(-2)^4} = \frac{1}{16}$

41. $-3^{-2} = -\frac{1}{3^2} = -\frac{1}{9}$

42. $-2^{-4} = -\frac{1}{2^4} = -\frac{1}{16}$

43. $-1^{-10} = -\frac{1}{1^{10}} = -1$

44. $-10^{-2} = -\frac{1}{10^2} = -\frac{1}{100}$

45. $\frac{1}{10^{-3}} = 10^3 = 1000$

46. $\frac{1}{2^{-4}} = 2^4 = 16$

47. $6x^{-1} = 6 \cdot \frac{1}{x} = \frac{6}{x}$

48. $9x^{-4} = 9 \cdot \frac{1}{x^4} = \frac{9}{x^4}$

49. $3a^8b^{-6} = 3a^8 \cdot \frac{1}{b^6} = \frac{3a^8}{b^6}$

50. $5a^{-7}b^4 = \frac{5b^4}{a^7}$

51. $\frac{2z^{-3}}{x^5} = \frac{2}{x^5} \cdot \frac{1}{z^3} = \frac{2}{x^5z^3}$

52. $\frac{5a^{-1}}{b} = \frac{5}{b} \cdot \frac{1}{a} = \frac{5}{ab}$

53. $\frac{3y^2}{z^{-4}} = 3y^2 \cdot z^4 = 3y^2z^4$

54. $\frac{t^{-6}}{7s^2} = \frac{1}{7s^2} \cdot \frac{1}{t^6} = \frac{1}{7s^2t^6}$

55. $\frac{ab^{-1}}{c^{-1}} = a \cdot \frac{1}{b} \cdot c = \frac{ac}{b}$

56. $\frac{x^{-3}y^4}{z^{-5}} = \frac{1}{x^3} \cdot y^4z^5 = \frac{y^4z^5}{x^3}$

57. $\frac{pq^{-2}r^{-3}}{2u^5v^{-4}} = \frac{p}{2u^5} \cdot \frac{1}{q^2} \cdot \frac{1}{r^3} \cdot v^4 = \frac{pv^4}{2q^2r^3u^5}$

58. $\frac{5a^{-3}bc^{-1}}{d^{-6}f^2} = \frac{5b}{f^2} \cdot \frac{1}{a^3} \cdot \frac{1}{c} \cdot d^6 = \frac{5bd^6}{a^3cf^2}$

59. $\frac{1}{x^3} = x^{-3}$

60. n^{-4}

61. $\frac{1}{(-10)^3} = (-10)^{-3}$

62. 12^{-5}

63. $8^{10} = \frac{1}{8^{-10}}$

64. $\frac{1}{(-6)^{-4}}$

65. $4x^2 = 4 \cdot \frac{1}{x^{-2}} = \frac{4}{x^{-2}}$

66. $\frac{-4}{y^{-5}}$, or $-\frac{4}{y^{-5}}$

67. $\frac{1}{(5y)^3} = (5y)^{-3}$

68. $(5x)^{-5}$

69. $\frac{1}{3y^4} = \frac{1}{3} \cdot \frac{1}{y^4} = \frac{1}{3} \cdot y^{-4} = \frac{y^{-4}}{3}$

70. $\frac{b^{-3}}{4}$

71. $6^{-3} \cdot 6^{-5} = 6^{-3+(-5)} = 6^{-8}$, or $\frac{1}{6^8}$

72. 4^{-3}, or $\frac{1}{4^3}$

73. $a \cdot a^{-8} = a^{1+(-8)} = a^{-7}$, or $\frac{1}{a^7}$

74. b^3

75. $x^{-7} \cdot x^2 \cdot x^5 = x^{-7+2+5} = x^0 = 1$

76. a

77. $\left(4mn^3\right)\left(-2m^3n^2\right) = 4(-2)\cdot m\cdot m^3\cdot n^3\cdot n^2$
$= -8m^{1+3}n^{3+2}$
$= -8m^4n^5$

78. $-18x^8y$

79. $\left(-7x^4y^{-5}\right)\left(-5x^{-6}y^8\right) = (-7)(-5)\cdot x^4\cdot x^{-6}\cdot y^{-5}\cdot y^8$
$= 35x^{4+(-6)}y^{-5+8}$
$= 35x^{-2}y^3$, or $\dfrac{35y^3}{x^2}$

80. $24u^{-10}v^6$, or $\dfrac{24v^6}{u^{10}}$

81. $\left(5a^{-2}b^{-3}\right)\left(2a^{-4}b\right) = 5\cdot 2\cdot a^{-2}\cdot a^{-4}\cdot b^{-3}\cdot b$
$= 10a^{-2+(-4)}b^{-3+1}$
$= 10a^{-6}b^{-2}$, or $\dfrac{10}{a^6b^2}$

82. $6a^{-4}b^{-9}$, or $\dfrac{6}{a^4b^9}$

83. $\dfrac{10^{-3}}{10^6} = 10^{-3-6} = 10^{-9}$, or $\dfrac{1}{10^9}$

84. 12^{-12}, or $\dfrac{1}{12^{12}}$

85. $\dfrac{2^{-7}}{2^{-5}} = 2^{-7-(-5)} = 2^{-7+5} = 2^{-2}$, or $\dfrac{1}{2^2}$, or $\dfrac{1}{4}$

86. 9^2, or 81

87. $\dfrac{y^4}{y^{-5}} = y^{4-(-5)} = y^{4+5} = y^9$

88. a^5

89. $\dfrac{24a^5b^3}{-8a^4b} = \dfrac{24}{-8}a^{5-4}b^{3-1} = -3ab^2$

90. $3m^3n^{-5}$, or $\dfrac{3m^3}{n^5}$

91. $\dfrac{15m^5n^3}{10m^{10}n^{-4}} = \dfrac{15}{10}m^{5-10}n^{3-(-4)} = \dfrac{3}{2}m^{-5}n^7$, or $\dfrac{3n^7}{2m^5}$

92. $-\dfrac{4}{3}x^9y^{-2}$, or $-\dfrac{4x^9}{3y^2}$

93. $\dfrac{-6x^{-2}y^4z^8}{-24x^{-5}y^6z^{-3}} = \dfrac{-6}{-24}x^{-2-(-5)}y^{4-6}z^{8-(-3)}$
$= \dfrac{1}{4}x^3y^{-2}z^{11}$, or $\dfrac{x^3z^{11}}{4y^2}$

94. $\dfrac{1}{4}a^{10}b^{-9}c^{-1}$, or $\dfrac{a^{10}}{4b^9c}$

95. $\left(x^4\right)^3 = x^{4\cdot 3} = x^{12}$

96. a^6

97. $\left(9^3\right)^{-4} = 9^{3(-4)} = 9^{-12}$, or $\dfrac{1}{9^{12}}$

98. 8^{-12}, or $\dfrac{1}{8^{12}}$

99. $\left(t^{-8}\right)^{-5} = t^{-8(-5)} = t^{40}$

100. x^{12}

101. $(-5xy)^2 = (-5)^2x^2y^2 = 25x^2y^2$

102. $-125a^3b^3$

103. $\left(-2a^{-2}b\right)^{-3} = (-2)^{-3}\cdot\left(a^{-2}\right)^{-3}\cdot b^{-3} = (-2)^{-3}a^6b^{-3}$
$= \dfrac{1}{(-2)^3}\cdot a^6\cdot\dfrac{1}{b^3} = -\dfrac{a^6}{8b^3}$

104. $(-4)^{-2}x^{-12}y^4$, or $\dfrac{y^4}{16x^{12}}$

105. $\left(\dfrac{m^2n^{-1}}{4}\right)^3 = \dfrac{m^6n^{-3}}{4^3} = \dfrac{m^6n^{-3}}{64}$, or $\dfrac{m^6}{64n^3}$

106. $\dfrac{9x^{10}}{y^{-8}}$, or $9x^{10}y^8$

107. $\dfrac{\left(2a^3\right)^3 4a^{-3}}{\left(a^2\right)^5} = \dfrac{2^3a^{3\cdot 3}4a^{-3}}{a^{2\cdot 5}} = \dfrac{8a^9 4a^{-3}}{a^{10}}$
$= 8\cdot 4a^{9+(-3)-10} = 32a^{-4}$, or $\dfrac{32}{a^4}$

108. $\dfrac{\left(3x^2\right)^3 2x^{-4}}{\left(x^4\right)^2} = \dfrac{27x^6 2x^{-4}}{x^8} = 54x^{-6}$, or $\dfrac{54}{x^6}$

109. $\left(8x^{-3}y^2\right)^{-4}\left(8x^{-3}y^2\right)^4 = \left(8x^{-3}y^2\right)^{-4+4} = \left(8x^{-3}y^2\right)^0 = 1$

110. $\left(2a^{-1}b^3\right)^{-2}\left(2a^{-1}b^3\right)^{-2} = \left(2a^{-1}b^3\right)^{-4}$
$= 2^{-4}a^4b^{-12}$, or $\dfrac{a^4}{16b^{12}}$

111. $\dfrac{\left(5a^3b\right)^2}{10a^2b} = \dfrac{5^2\left(a^3\right)^2b^2}{10a^2b} = \dfrac{25a^6b^2}{10a^2b}$
$= \dfrac{25}{10}a^{6-2}b^{2-1} = \dfrac{5}{2}a^4b$, or $\dfrac{5a^4b}{2}$

112. $\dfrac{\left(3x^3y^4\right)^3}{6xy^3} = \dfrac{27x^9y^{12}}{6xy^3} = \dfrac{9}{2}x^8y^9$, or $\dfrac{9x^8y^9}{2}$

113. $\left(\dfrac{2x^3y^{-2}}{3y^{-3}}\right)^3=\left(\dfrac{2}{3}x^3y^{-2+3}\right)^3=\left(\dfrac{2}{3}x^3y\right)^3=\dfrac{2^3}{3^3}\left(x^3\right)^3y^3$
$=\dfrac{8x^9y^3}{27}$

114. $\left(\dfrac{-4x^4y^{-2}}{5x^{-1}y^4}\right)^{-4}=\left(\dfrac{-4}{5}x^5y^{-6}\right)^{-4}=\dfrac{5^4x^{-20}y^{24}}{(-4)^4}$
$=\dfrac{625x^{-20}y^{24}}{256}$, or $\dfrac{625y^{24}}{256x^{20}}$

115. $\left(\dfrac{21x^5y^{-7}}{14x^{-2}y^{-6}}\right)^0=1$
(Any nonzero real number raised to the zero power is 1.)

116. $\left(\dfrac{6a^{-2}b^6}{8a^{-4}b^0}\right)^{-2}=\left(\dfrac{3}{4}a^2b^6\right)^{-2}=\left(\dfrac{4}{3}\right)^2a^{-4}b^{-12}$, or $\dfrac{16}{9a^4b^{12}}$

117. $\left(\dfrac{5x^0y^{-7}}{2x^{-2}y^4}\right)^{-2}=\left(\dfrac{5}{2}x^{0+2}y^{-7-4}\right)^{-2}=\left(\dfrac{5x^2y^{-11}}{2}\right)^{-2}$
$=\left(\dfrac{2}{5x^2y^{-11}}\right)^2=\dfrac{2^2}{5^2\left(x^2\right)^2\left(y^{-11}\right)^2}$
$=\dfrac{4}{25x^4y^{-22}}=\dfrac{4}{25}x^{-4}y^{22}$, or $\dfrac{4y^{22}}{25x^4}$

118. $\left(\dfrac{4a^3b^{-9}}{6a^{-2}b^5}\right)^0=1$
(Any nonzero real number raised to the zero power is 1.)

119. *Writing Exercise* For any even number n.
$(-1)^n=\underbrace{[(-1)(-1)][(-1)(-1)]\cdots[(-1)(-1)]}_{n/2\text{ pairs of factors}}$
$=\underbrace{1\cdot 1\cdots 1}_{n/2\text{ factors of }1}$
$=1$

120. *Writing Exercise*. Since $(-17)^{-8}=\dfrac{1}{(-17)^8}$ and $(-17)^8$ is positive, then $(-17)^{-8}$ is positive.

121. *Writing Exercise*. In the expression $3-(-2)^{-1}$, the first "–" after the 3 indicates subtraction. The second "–" inside the parentheses indicates the number opposite of two, or negative two. The third "–", found in the exponent, indicates that the expression $(-2)^{-1}=\dfrac{1}{(-2)}$.

122. *Writing Exercise*. Since $5^{-6}=\dfrac{1}{5^6}$, $4^{-9}=\dfrac{1}{4^9}$, and $5^6<4^9$, then $5^{-6}>4^{-9}$ is true.

123. $\dfrac{8a^{x-2}}{2a^{2x+2}}=\dfrac{8}{2}\cdot a^{x-2-(2x+2)}=4a^{x-2-2x-2}=4a^{-x-4}$

124. $\left[7y(7-8)^{-4}-8y(8-7)^{-2}\right]^{(-2)^2}$
$=\left[7y(7-8)^{-4}-8y(8-7)^{-2}\right]^4$
$=\left[7y(-1)^{-4}-8y(1)^{-2}\right]^4$
$=\left[\dfrac{7y}{(-1)^4}-\dfrac{8y}{1^2}\right]^4$
$=[7y-8y]^4$
$=[-y]^4$
$=y^4$

125. $\left\{\left[\left(8^{-a}\right)^{-2}\right]^b\right\}^{-c}\cdot\left[\left(8^0\right)^a\right]^c=8^{-2abc}\cdot 8^0=8^{-2abc}$

126. $\left(3^{a+2}\right)^a=3^{(a+2)a}=3^{a^2+2a}$

127. $\dfrac{-28x^{b+5}y^{4+c}}{7x^{b-5}y^{c-4}}=-4x^{b+5-(b-5)}y^{4+c-(c-4)}$
$=-4x^{b+5-b+5}y^{4+c-c+4}=-4x^{10}y^8$

128. $\dfrac{4x^{2a+3}y^{2b-1}}{2x^{a+1}y^{b+1}}=\dfrac{4}{2}\cdot x^{2a+3-(a+1)}y^{2b-1-(b+1)}$
$=2x^{2a+3-a-1}y^{2b-1-b-1}=2x^{a+2}y^{b-2}$

129. $\dfrac{3^{q+3}-3^2\left(3^q\right)}{3\left(3^{q+4}\right)}=\dfrac{3^{q+3}-3^{q+2}}{3^{q+5}}=\dfrac{3^{q+2}(3-1)}{3^{q+2}\left(3^3\right)}=\dfrac{2}{3^3}=\dfrac{2}{27}$

130. $\dfrac{25x^{a+b}y^{b-a}}{-5x^{a-b}y^{b+a}}=-5x^{a+b-(a-b)}y^{b-a-(b+a)}$
$=-5x^{a+b-a+b}y^{b-a-b-a}=-5x^{2b}y^{-2a}$

131. $\left[\left(\dfrac{a^{-2c}}{b^{7c}}\right)^{-3}\left(\dfrac{a^{4c}}{b^{-3c}}\right)^2\right]^{-a}=\left(\dfrac{a^{6c}}{b^{-21c}}\cdot\dfrac{a^{8c}}{b^{-6c}}\right)^{-a}=\left(\dfrac{a^{14c}}{b^{-27c}}\right)^{-a}$
$=\dfrac{a^{-14ac}}{b^{27ac}}$

132. Let s = side of second cube. Then $8s$ = side of first cube. Volume of first cube $=(8s)^3=512s^3$.
The first cube has volume 512 times as great as the second cube.

Exercise Set 1.7

1. The number 27×10^{16} *is not* written in scientific notation.
2. Very small number are represented in scientific notation using *negative* powers of 10.
3. The number 4.587×10^5 has *four* significant digits.
4. In a series of calculations, rounding should be done *at the very end*.

5. The length of an Olympic marathon, in centimeters, is a large number so its representation in scientific notation would include a positive power of 10.

6. The thickness of a cat's whiskers, in meters, is a small number, so its representation in scientific notation would include a negative power of 10.

7. The mass of a hydrogen atom, in grams, is a small number so its representation in scientific notation would include a negative power of 10.

8. The mass of a pickup truck, in grams, is a large number so its representation in scientific notation would include a positive power of 10.

9. The time between leap years, in seconds, is a large number so its representation in scientific notation would include a positive power of 10.

10. The time between a bird's heartbeats, in hours, is a small number so its representation in scientific notation would include a negative power of 10.

11. $64{,}000{,}000{,}000 = \dfrac{64{,}000{,}000{,}000}{10^{10}} \cdot 10^{10}$ Multiplying by 1 $(10^{10}/10^{10} = 1)$

$= 6.4 \times 10^{10}$ This is scientific notation.

12. 3.7×10^6

13. $0.0000013 = \dfrac{0.0000013}{10^6} \cdot 10^6$ Multiplying by 1: $10^6 / 10^6 = 1$

$= \dfrac{1.3}{10^6}$

$= 1.3 \times 10^{-6}$ This is scientific notation.

14. 7.8×10^{-5}

15. $0.00009 = \dfrac{0.0009}{10^5} \cdot 10^5$ Multiplying by 1: $10^5 / 10^5 = 1$

$= \dfrac{9}{10^5}$

$= 9 \times 10^{-5}$ This is scientific notation.

16. 6×10^{-8}

17. $803{,}000{,}000{,}000 = \dfrac{803{,}000{,}000{,}000}{10^{11}} \cdot 10^{11}$

$= 8.03 \times 10^{11}$

18. 3.09×10^{12}

19. $0.000000904 = \dfrac{0.000000904}{10^7} \cdot 10^7$

$= \dfrac{9.04}{10^7}$

$= 9.04 \times 10^{-7}$

20. 8.02×10^{-9}

21. $431{,}700{,}000{,}000 = \dfrac{431{,}700{,}000{,}000}{10^{11}} \cdot 10^{11}$

$= 4.317 \times 10^{11}$

22. 9.534×10^{11}

23. $4 \times 10^5 = 400{,}000$ Moving the decimal point 5 places to the right.

24. 0.000003

25. $1.2 \times 10^{-4} = 0.00012$ Moving the decimal point 4 places to the left.

26. 860,000,000

27. $3.76 \times 10^{-9} = 0.00000000376$ Moving the decimal point 9 places to the left.

28. 0.0427

29. $8.056 \times 10^{12} = 8{,}056{,}000{,}000{,}000$ Moving the decimal point 12 places to the right.

30. 50,020,000,000

31. $7.001 \times 10^{-5} = 0.00007001$ Moving the decimal point 5 places to the left.

32. 0.002049

33. $(3.4 \times 10^{-8})(2.6 \times 10^{15})$

$= (3.4 \times 2.6)(10^{-8} \times 10^{15})$

$= 8.84 \times 10^7$

$= 8.8 \times 10^7$ Rounding to 2 significant digits

34. $(1.8 \times 10^{20})(4.7 \times 10^{-12}) = 8.46 \times 10^8$

$= 8.5 \times 10^8$ (2 significant digits)

35. $(2.36 \times 10^6)(1.4 \times 10^{-11})$

$= (2.36 \times 1.4)(10^6 \times 10^{-11})$

$= 3.304 \times 10^{-5}$

$= 3.3 \times 10^{-5}$ Rounding to 2 significant digits

36. $(4.26 \times 10^{-6})(8.2 \times 10^{-6}) = 34.932 \times 10^{-12}$

$= 3.4932 \times 10^{-11} = 3.5 \times 10^{-11}$ (2 significant digits)

37. $(5.2 \times 10^6)(2.6 \times 10^4) = (5.2 \times 2.6)(10^6 \times 10^4)$

$= 13.52 \times 10^{10}$

$= (1.352 \times 10) \times 10^{10}$

$= 1.352 \times (10 \times 10^{10})$

$= 1.352 \times 10^{11}$

$= 1.4 \times 10^{11}$ (2 significant digits)

38. $(6.11 \times 10^3)(1.01 \times 10^{13}) = 6.1711 \times 10^{16}$

$= 6.17 \times 10^{16}$ (3 significant digits)

39. $(7.01\times10^{-5})(6.5\times10^{-7})$
$=(7.01\times6.5)(10^{-5}\times10^{-7})$
$=45.565\times10^{-12}$
$=(4.5565\times10)\times10^{-12}$
$=4.5565\times10^{-11}$
$=4.6\times10^{-11}$ (2 significant digits)

40. $(4.08\times10^{-10})(7.7\times10^{5})=31.416\times10^{-5}$
$=3.1416\times10^{-4}=3.1\times10^{-4}$ (2 significant digits)

41. $(2.0\times10^{6})(3.02\times10^{-6})$

Observe that $10^{6}\times10^{-6}=1,$ so the product is 2.0(3.02), or 6.04, or 6.0 rounded to two significant digits.

42. $(7.04\times10^{-9})(9.01\times10^{-7})=63.4304\times10^{-16}$
$=6.34304\times10^{-15}=6.34\times10^{-15}$ (3 significant digits)

43. $\dfrac{6.5\times10^{15}}{2.6\times10^{4}}=\dfrac{6.5}{2.6}\times\dfrac{10^{15}}{10^{4}}$
$=2.5\times10^{11}$ (2 significant digits)

44. 2.5×10^{13}

45. $\dfrac{9.4\times10^{-9}}{4.7\times10^{-2}}=\dfrac{9.4}{4.7}\times\dfrac{10^{-9}}{10^{-2}}$
$=2.0\times10^{-7}$ (2 significant digits)

46. $\dfrac{4.0\times10^{-6}}{8.0\times10^{-3}}=0.5\times10^{-3}$
$=(5.0\times10^{-1})\times10^{-3}$ (2 significant digits)
$=5.0\times10^{-4}$

47. $\dfrac{3.2\times10^{-7}}{8.0\times10^{8}}=\dfrac{3.2}{8.0}\times\dfrac{10^{-7}}{10^{8}}$
$=0.40\times10^{-15}$ (2 significant digits)
$=(4.0\times10^{-1})\times10^{-15}$
$=4.0\times10^{-16}$

48. 3.0×10^{11} (2 significant digits)

49. $\dfrac{9.36\times10^{-11}}{3.12\times10^{11}}=\dfrac{9.36}{3.12}\times\dfrac{10^{-11}}{10^{11}}$
$=3.00\times10^{-22}$ (3 significant digits)

50. 2.00×10^{10} (3 significant digits)

51. $\dfrac{6.12\times10^{19}}{3.06\times10^{-7}}=\dfrac{6.12}{3.06}\times\dfrac{10^{19}}{10^{-7}}$
$=2.00\times10^{26}$ (3 significant digits)

52. $\dfrac{4.7\times10^{-9}}{2.0\times10^{-9}}$

Observe that $\dfrac{10^{-9}}{10^{-9}}=1,$ so the quotient is $\dfrac{4.7}{2.0},$ or 2.35, or 2.4, rounded to two significant digits.

53. ***Familiarize***. Let s = the number of stars. We have 8 trillion cubic light-years.
Translate. We multiply.

$$s=\frac{0.025\text{ star}}{\text{light-year}^3}\cdot 8{,}000{,}000{,}000{,}000\text{ light-years}^3$$

Carry out.

$$s=\frac{0.025\text{ star}}{\text{light-year}^3}\cdot 8{,}000{,}000{,}000{,}000\text{ light-years}^3$$
$$=(2.5\times10^{-2})(8\times10^{12})\text{ stars}$$
$$=2\times10^{11}\text{ stars}$$

Check. Recheck the translation and calculations. The answer checks.

State. There are 2×10^{11} stars in the Milky Way.

54. Let b = the number of bacteria. We have 10 million bacteria per square centimeter. Recall $1\text{ cm}^2=1\times10^{-10}\text{ km}^2$. We convert from square centimeters to square kilometers:

$$\frac{1\times10^{7}\text{ bacteria}}{1\text{ cm}^2}=\frac{1\times10^{7}\text{ bacteria}}{1\times10^{-10}\text{ km}^2}=\frac{1\times10^{17}\text{ bacteria}}{\text{km}^2}$$

$$b=\frac{1\times10^{17}\text{ bacteria}}{\text{km}^2}\cdot14{,}000\text{ km}^2$$
$$=(1\times10^{17})(1.4\times10^{4})$$
$$=1.4\times10^{21}$$

There are 1.4×10^{21} bacteria in Hawaii's coral reef.

55. ***Familiarize***. We have a cylinder with diameter 4.0×10^{-10} in. and length 100 yd. We will use the formula for the volume of a cylinder $V=\pi r^2h.$ The radius is $\dfrac{4.0\times10^{-10}}{2},$ or 2.0×10^{-10} in. We convert 100 yd to inches:
$100\text{ yd}=100\times1\text{ yd}=100\times36\text{ in.}=3600\text{ in.},$ or 3.6×10^{3} in.
Translate. We substitute in the formula.

$$V=\pi r^2h$$
$$V=\pi(2.0\times10^{-10})^2(3.6\times10^{3})$$

Carry out. We do the calculation.

$$V=\pi(2.0\times10^{-10})^2(3.6\times10^{3})$$
$$=\pi\times4.0\times10^{-20}\times3.6\times10^{3}$$
$$=(\pi\times4.0\times3.6)\times(10^{-20}\times10^{3})$$
$$\approx45.2\times10^{-17}$$
$$\approx(4.52\times10)\times10^{-17}$$
$$\approx4.52\times10^{-16}$$
$$\approx4.5\times10^{-16}\quad\text{(2 significant digits)}$$

Check. Recheck the translation and the calculations. The answer checks.
State. The volume of a 100-yd carbon nanotube is about 4.5×10^{-16} in^3.

56. ***Familiarize***. We can think of the plastic as a rectangular solid whose length is the perimeter of the house and with width 8 mil, or $8\times\frac{1}{1000}$ in.

$=8\times 0.001\text{ in.}=0.008\text{ in.}=8\times 10^{-3}\text{ in.}$ and height 4 ft. The perimeter of the house is $2\cdot 32\text{ ft}+2\cdot 24\text{ ft}$, or 112 ft. We will convert the perimeter and the height to in.

$112\text{ ft}=112\times 1\text{ ft}=112\times 12\text{ in.}=1344\text{ in.}$, or 1.344×10^3 in.

$4\text{ ft}=4\times 1\text{ ft}=4\times 12\text{ in.}=48\text{ in.}$, or 4.8×10 in.

Recall that the formula for the volume of a rectangular solid is $V = lwh$.

$$V=lwh$$
$$V=(1.344\times 10^3)\times(4.8\times 10)\times(8\times 10^{-3})$$

Carry out. We do the calculation.

$$V=(1.344\times 10^3)\times(4.8\times 10)\times(8\times 10^{-3})$$
$$V=(1.344\times 4.8\times 8)\times(10^3\times 10\times 10^{-3})$$
$$\approx 51.6\times 10$$
$$\approx 5\times 10^2 \quad \text{(1 significant digit)}$$

State. The volume of the plastic is about $5\times 10^2\text{ in}^3$. (If we had used feet as the unit of length, the result would be about $3\times 10^{-1}\text{ ft}^3$.)

57. ***Familiarize***. Let p = the number of pages of information per person. Recall 1 exabyte is 10^{15} kilobytes. We convert from exabytes to kilobytes:

$$2.5\text{ exabytes}=2.5\times 1\text{ exabytes}$$
$$=2.5\times 10^{15}\text{ kilobytes}$$
$$=2.5\times 10^{15}\text{ kilobytes}$$

Translate. We divide.

$$p=\frac{2.5\times 10^{15}\text{ kilobytes}}{7.1\text{ billion people}}\cdot\frac{1\text{ page}}{2\text{ kilobytes}}$$

Carry out. We perform the calculations and write scientific notation for the answer.

$$p=\frac{2.5\times 10^{15}\text{ kilobytes}}{7.1\text{ billion people}}\cdot\frac{1\text{ page}}{2\text{ kilobytes}}$$
$$=\frac{2.5}{7.1}\times\frac{10^{15}}{10^9}\cdot\frac{1}{2}\frac{\text{pages}}{\text{person}}$$
$$\approx 1.76\times 10^5\text{ pages/person}$$
$$=1.8\times 10^5 \quad \text{(2 significant digits)}$$

Check. Recheck the translation and calculation. The answer checks.

State. Each person generates an average of 1.8×10^5 pages of information.

58. 50 gigabytes $=5\times 10$ gigabytes

Convert from terabytes to gigabytes.

10 terabytes $=10\times 10^3$ gigabytes $=10^4$ gigabytes

Divide.

$$\frac{10^4}{5\times 10^1}=2\times 10^2\text{ sec} \quad \text{(1 significant digit)}$$

59. ***Familiarize***. Let w = the weight of each sheet. 1 ream = 500 sheets.

Translate. We divide.

$$w=\frac{2.25}{500}$$

Carry out.

$$w=\frac{2.25}{5\times 10^2}$$
$$=0.450\times 10^{-2}$$
$$=4.50\times 10^{-3}$$

Check. Recheck the translation and calculations. The answer checks.

State. Each sheet of copier paper weighs 4.50×10^{-3} kg, or 4.50 g.

60. First we will find the number n of \$5 bills in \$4,540,000 worth of \$5 bills. Then we will find the weight w of a \$5 bill. Recall that 1 ton = 2000 lb.

$$n=\frac{4{,}540{,}000}{5}=\frac{4.54\times 10^6}{5}=0.908\times 10^6$$
$$=(9.08\times 10^{-1})\times 10^6=9.08\times 10^5$$
$$w=\frac{2000}{n}=\frac{2000}{9.08\times 10^5}=\frac{2\times 10^3}{9.08\times 10^5}$$
$$\approx 0.220\times 10^{-2}=(2.20\times 10^{-1})\times 10^{-2}$$
$$=2.20\times 10^{-3}\text{ lb} \quad \text{(3 significant digits)}$$

61. ***Familiarize***. We know that 1 light year = 5.88×10^{12} mi. Let y = the number of light years from one and of the Milky Way galaxy to the other.

Translate. The distance is $(5.88\times 10^{12})y$ mi. It is also given by 5.88×10^{17} mi. We write the equation:

$$(5.88\times 10^{12})y=5.88\times 10^{17}$$

Carry out. We solve the equation.

$$(5.88\times 10^{12})y=5.88\times 10^{17}$$
$$y=\frac{5.88\times 10^{17}}{5.88\times 10^{12}}$$
$$y=\frac{5.88}{5.88}\times\frac{10^{17}}{10^{12}}$$
$$y=1.00\times 10^5$$

Check. Since light travels 5.88×10^{12} mi in one light year, in 1.00×10^5 yr it will travel $(1.00\times 10^5)\times(5.88\times 10^{12})=5.88\times 10^{17}$ mi. The answer checks.

State. The distance from one end of the galaxy to the other is 1.00×10^5 light years.

62. Let y = the number of light years from Earth to Sirius.

Solve: $(5.88\times 10^{12})y=4.704\times 10^{13}$

$$y=8.00 \quad \text{(3 significant digits)}$$

63. ***Familiarize***. We are told that 1 Angstrom $=1\times 10^{-10}$ m, 1 parsec ≈ 3.26 light years, and 1 light year $=9.46\times 10^{15}$ m. Let a represent the number of Angstroms in one parsec.

Translate The length of one parsec is $a\times 10^{-10}$ m. It can also be expressed as 3.26 light years, or

$3.26\times 9.46\times 10^{15}$ m. Since these quantities represent the same number, we can write the equation.

$a\times 10^{-10}=3.26\times 9.46\times 10^{15}.$

Carry out. Solve the equation:

$$\begin{aligned}
a\times 10^{-10}&=3.26\times 9.46\times 10^{15}\\
a\times 10^{-10}\times\frac{1}{10^{-10}}&=3.26\times 9.46\times 10^{15}\times\frac{1}{10^{-10}}\\
a&=\frac{3.26\times 9.46\times 10^{15}}{10^{-10}}\\
&=(3.26\times 9.46)\times\frac{10^{15}}{10^{-10}}\\
&=30.8396\times 10^{25}\\
&=(3.08396\times 10)\times 10^{25}\\
&=3.08396\times\left(10\times 10^{25}\right)\\
&=3.08\times 10^{26}\quad\text{(Rounding to 3 significant digits)}
\end{aligned}$$

Check. We recheck the translation and calculation.

State. There are about 3.08×10^{26} Angstroms in one parsec.

64. $\dfrac{3.26\times 9.46\times 10^{15}}{10^{3}}=30.8396\times 10^{12}$

$=3.08\times 10^{13}$ (3 significant digits)

65. ***Familiarize***. We have a very long cylinder. Its length is the average distance from Earth to the sun, 1.5×10^{11} m, and the diameter of its base is 3 Å. We will use the formula for the volume of a cylinder, $V=\pi r^2h$ (See Example 8.)

Translate. We will express all distances in Angstroms.

Height (length): $1.5\times 10^{11}\text{ m}=\dfrac{1.5\times 10^{11}}{10^{-10}}$ Å, or

1.5×10^{21} Å

Diameter: 3 Å

The radius is half the diameter:

Radius: $\dfrac{1}{2}\times 3\text{ Å}=1.5\text{ Å}$

Now substitute into the formula (using 3.14 for π):

$V=\pi r^2h$

$V=3.14\times 1.5^2\times 1.5\times 10^{21}$

Carry out. Do the calculations.

$$\begin{aligned}
V&=3.14\times 1.5^2\times 1.5\times 10^{21}\\
&=10.5975\times 10^{21}\\
&=1.05975\times 10^{22}\\
&=1\times 10^{22}\quad\text{Rounding to 1 significant digit}
\end{aligned}$$

We can convert this result to cubic meters, if desired.

$1\text{ Å}=10^{-10}$ m, So 1 cu Å $=\left(10^{-10}\right)^3\text{ m}^3=10^{-30}\text{m}^3$.

Then 1×10^{22} cu Å $=1\times 10^{22}\text{ cu Å}\times\dfrac{10^{-30}\text{ m}^3}{1\text{ cu Å}}$

$=1\times 10^{22}\times 10^{-30}\times\dfrac{\text{cu Å}}{\text{cu Å}}\times\text{m}^3=1\times 10^{-8}\text{ m}^3.$

Check. We recheck the translation and the calculations.

State. The volume of the sunbeam is about 1×10^{22} cu Å, or 1×10^{-8} m^3.

66. Height: 1.5×10^{21} Å (See exercise 65.)

Radius: $\dfrac{5}{2}$ Å, or 2.5 Å

$V=\pi r^2h$

$V=3.14(2.5)^2\left(1.5\times 10^{21}\right)$

$V=29.4375\times 10^{21}$

$V=2.94375\times 10^{22}$ cu Å

$V=3\times 10^{22}$ cu Å (1 significant digit)

Converting to m^3 (See Exercise 65.), we have

$3\times 10^{22}\text{ cu Å}=3\times 10^{22}\text{ cu Å}\times\dfrac{10^{-30}\text{ m}^3}{1\text{ cu Å}}$

$=3\times 10^{-8}\text{ m}^3.$

67. ***Familiarize***. First we will find d, the number of drops in a pound. Then we will find b, the number of bacteria in a drop of U.S. mud.

Translate. To find d we convert 1 pound to drops:

$d=1\text{ lb}\cdot\dfrac{16\text{ oz}}{1\text{ lb}}\cdot\dfrac{6.0\text{ tsp}}{1\text{ oz}}\cdot\dfrac{60.0\text{ drops}}{1\text{ tsp}}.$

Then we divide to find b:

$b=\dfrac{4.55\times 10^{11}}{d}.$

Carry out. We do the calculations.

$d=1\text{ lb}\cdot\dfrac{16\text{ oz}}{1\text{ lb}}\cdot\dfrac{6.0\text{ tsp}}{1\text{ oz}}\cdot\dfrac{60.0\text{ drops}}{1\text{ tsp}}$

$=5760$ drops

Now we find b.

$b=\dfrac{4.55\times 10^{11}}{d}=\dfrac{4.55\times 10^{11}}{5.760\times 10^{3}}\approx 0.790\times 10^{8}$

$\approx\left(7.90\times 10^{-1}\right)\times 10^{8}=7.9\times 10^{7}$

(Our answer must have 2 significant digits.)

Check. If there are about 7.9×10^7 bacteria in a drop of U.S. mud, then in a pound there are about

$\dfrac{7.9\times 10^{7}}{1\text{ drop}}\cdot\dfrac{60.0\text{ drops}}{1\text{ tsp}}\cdot\dfrac{6.0\text{ tsp}}{1\text{ oz}}\cdot\dfrac{16\text{ oz}}{1\text{ lb}}$

$=\dfrac{45{,}504\times 10^{7}}{1\text{ lb}}\approx 4.55\times 10^{11}$ bacteria per pound. The answer checks.

State. About 7.9×10^7 bacteria live in a drop of U.S. mud.

68. 100 years $=100\times 52$, or 5200 weeks

13 weeks $=\dfrac{13}{5200}$, or 0.0025 of 100 years

$5.9\times 10^{14}\times 0.0025=5.9\times 10^{14}\times 2.5\times 10^{-3}$

$=14.75\times 10^{11}=1.475\times 10^{12}\approx 1.5\times 10^{12}$ mi

(2 significant digits)

69. ***Familiarize***. First we will find the distance C around Jupiter at the equator, in km. Then we will use the formula Speed × Time = Distance to find the speed s at which Jupiter's equator is spinning.
Translate. We will use the formula for the circumference of a circle to find the distance around Jupiter at the equator:

$$C = \pi d = \pi(1.43\times 10^5).$$

Then we find the speed s at which Jupiter's equator is spinning:

$$\underbrace{\text{Speed}} \times \underbrace{\text{Time}} = \underbrace{\text{Distance}}$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$s \times 10 = C$$

Carry out. First we find C.

$$C = \pi(1.43\times 10^5) \approx 4.49\times 10^5$$

Then we find s.

$$s\times 10 = C$$
$$s\times 10 = 4.49\times 10^5$$
$$s = \frac{4.49\times 10^5}{10}$$
$$s = 4.49\times 10^4$$

Check. At 4.49×10^4 km/h. in 10 hr, Jupiter's equator travels $4.49\times 10^4\times 10$, or 4.49×10^5 km. A circle with circumference 4.49×10^5 km has a diameter of $\frac{4.49\times 10^4}{\pi} \approx 1.43\times 10^5$ km. The answer checks.
State. Jupiter's equator spins at a speed of about 4.49×10^4 km/h.

70. Find the circumference of a circle with radius 9.3×10^7 miles:

$$C = 2\pi r = 2\pi(9.3\times 10^7) \approx 58\times 10^7$$
$$= 5.8\times 10^8 \text{ miles} \qquad \text{(2 significant digits)}$$

71. *Writing Exercise*. Answers may vary.
The height of a section of a laser's light beam is 10.0 mm. The radius is 1.00×10^2 mm. Find the volume of the light beam. (Use 3.14 for π.)

72. *Writing Exercise*. Answers may vary.
Scientific notation is convenient when writing very large or very small numbers. It is also convenient to use scientific notation when performing computations involving very large and/or very small numbers.

73. *Writing Exercise*. \$5 million in \$20 bills contains $\frac{5\times 10^6}{20} = 0.25\times 10^6 = 2.5\times 10^5$ bills. In Exercise 62 we found that a \$5 bill weighs about 2.20×10^{-3} lb. Assuming that a \$20 bill weighs the same as a \$5 bill, 2.5×10^5 bills would weigh $2.5\times 10^5\times 2.20\times 10^{-3} = 5.5\times 10^2$, or 550 lb. Thus, it is not possible that a criminal is carrying \$5 million in \$20 bills in a briefcase.

74. *Writing Exercise*. For 5^n, where n is an odd natural number, the one's digit will be 5. Since this is not the case with the given calculator readout, we know that the readout is an approximation.

75. ***Familiarize***. Let d = the average density, v = the volume and m = the mass.
Translate. We divide.

$$d = \frac{m}{v}$$

To convert the volume from km^3 to cm^3,

$$v = 1.08\times 10^{12}\ \text{km}^3\cdot\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)^3\cdot\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3$$

To convert the mass from kg to g

$$m = 5.976\times 10^{24}\ \text{kg}\cdot\frac{1000 \text{ g}}{1 \text{ kg}}$$

Carry out. We do the calculations.

$$v = 1.08\times 10^{12}\cdot(1000)^3\cdot(100)^3$$
$$= 1.08\times 10^{27}$$
$$m = 5.976\times 10^{24}\cdot 1000$$
$$= 5.976\times 10^{27}$$
$$d = \frac{m}{v} = \frac{5.976\times 10^{27}}{1.08\times 10^{27}}$$
$$= \frac{5.976}{1.08}\times\frac{10^{27}}{10^{27}}$$
$$= 5.53 \qquad \text{(3 significant digits)}$$

Check. We recheck the translation and calculations.
State. The average density of Earth is approximately $5.53\ \text{g/cm}^3$.

76. Weight of half is $\frac{1.2\times 10^{-9}}{2}$ or 6.0×10^{-10}.
Lower bound is $6.0\times 10^{-10} - 3.5\times 10^{-10} = 2.5\times 10^{-10}$.
Upper bound is $6.0\times 10^{-10} + 3.5\times 10^{-10} = 9.5\times 10^{-10}$.
The weight of each half is between 2.5×10^{-10} oz and 9.5×10^{-10} oz.

77. ***Familiarize***. Let r = the radius, v = the speed, and t = the time.
Translate. Convert the time 1 year to hours.

$$1 \text{ year} = 365 \text{ days} = 365\times 24 \text{ hr} = 8.76\times 10^3 \text{ hr}$$

Recall that 1 meter $\approx 6.21\times 10^{-4}$ miles. Convert the radius from meters to miles.

$$1.5\times 10^{11} \text{ m} = 1.5\times 10^{11}\times 6.21\times 10^{-4} \text{ mi}$$
$$= 9.315\times 10^7 \text{ mi}$$

To find the orbital speed, divide circumference by time.

$$v = \frac{C}{t} = \frac{2\pi r}{t}$$

Carry out. We do the calculations.

$$v = \frac{2\pi(9.315\times 10^7)}{8.76\times 10^3}$$
$$v \approx 6.7\times 10^4 \qquad \text{(2 significant digits)}$$

Check. We recheck the translation and calculations.
State. Earth's orbital speed around the sun is approximately 6.7×10^4 mph.

78. $\frac{4}{32}=0.125=\frac{1}{8}=1.25\times10^{-1}$

79. The larger number is the one in which the power of ten has the larger exponent. Since –90 is larger than –91, 8×10^{-90} is larger than 9×10^{-91}.

$$\begin{aligned}8\times10^{-90}-9\times10^{-91}&=10^{-90}\left(8-9\times10^{-1}\right)\\&=10^{-90}(8-0.9)\\&=7.1\times10^{-90}\end{aligned}$$

Thus, 8×10^{-90} is larger by 7.1×10^{-90}.

80. $$\begin{aligned}\frac{1}{8.00\times10^{-23}}&=\frac{1}{8.00}\times\frac{1}{10^{-23}}=0.125\times10^{23}\\&=1.25\times10^{22}\end{aligned}$$

81. $$\begin{aligned}(4096)^{0.05}(4096)^{0.2}&=4096^{0.25}\\&=\left(2^{12}\right)^{0.25}\\&=2^3\\&=8\end{aligned}$$

82. The unit's digit is the digit 3 raised to some power. Look for a pattern:

Exponent	1	2	3	4	5	6	7	8
Unit's digit	3	9	7	1	3	9	7	1

$(513)^{128}=\left(513^4\right)^{32}$, so the unit's digit is 1.

83. ***Familiarize***. Observe that there are 2^{n-1} grains of sand on the nth square of the chessboard. Let g represent this quantity. Recall that a chessboard has 64 squares. Note also that $2^{10}\approx10^3$.
Translate. We write the equation

$g=2^{n-1}$.

To find the number of grains of sand on the last (or 64th) square, substitute 64 for n: $g=2^{64-1}$.
Carry out. Recheck the translation and the calculations.

$$\begin{aligned}g&=2^{64-1}=2^{63}=2^3\left(2^{10}\right)^6\\&\approx2^3\left(10^3\right)^6\approx8\times10^{18}\end{aligned}$$

State. Approximately 8×10^{18} grains of sand are required for the last square.

84. Find the size of a Hubble-barn, in gallons with radius one Hubble, $10^{23}\text{ km}=10^{23}\times10^3\text{m}=10^{26}\text{ m}$.

$V=bh=\left(10^{-28}\right)\left(10^{26}\right)=10^{-2}\text{ m}^3$

Convert to gallons.

$$\begin{aligned}V&=10^{-2}\text{ m}^3\approx10^{-2}\times2.6417\times10^2\text{ gal}\\&\approx3\text{ gal (1 significant digit)}\end{aligned}$$

85. Answers will vary.

Chapter 1 Review

1. The correct choice is (e), since the equation $2x-1=9$ is equivalent to $2x=10$.

2. The correct choice is (g), since the expression $2x-1$ is equivalent to $5x-1-3x$.

3. The correct choice is (j), since the equation $\frac{3}{4}x=5$ is equivalent to $\frac{4}{3}\cdot\frac{3}{4}x=\frac{4}{3}\cdot5$.

4. The correct choice is (a) since the expression $\frac{3}{4}x-5$ is equivalent to $2+\frac{3}{4}x-7$.

5. The correct choice is (i), since the expression $2(x+7)$ is equivalent to $2x+14$.

6. The correct choice is (b), since the equation $2(x+7)=6$ is equivalent to $2x+14=6$.

7. The correct choice is (f), since the equation $4x-3+2x=5$ is equivalent to $6x-3=5$.

8. The correct choice is (c), since the expression $4x-3+2x$ is equivalent to $6x-3$.

9. The correct choice is (d), since the expression $6+2x$ is equivalent to $2(3+x)$.

10. The correct choice is (h), since the equation $6=2x$ is equivalent to $3=x$.

11. Eight less than the quotient of two numbers
Let x and y represent the numbers.

Then we have $\frac{x}{y}-8$.

12. Substitute and carry out the operations.

$$\begin{aligned}7x^2-5y\div zx&=7(-2)^2-5(3)\div(-5)(-2)\\&=7\cdot4-15\div(-5)\cdot(-2)\\&=28+3\cdot(-2)\\&=28-6\\&=22\end{aligned}$$

13. Name the set consisting of the first five odd natural numbers
$\{1, 3, 5, 7, 9\}$
or $\{x \mid x \text{ is an odd natural number less than } 10\}$

14. $A=\frac{1}{2}(50)(70)=1750\text{ cm}^2$

15. $|-19|=19$ –19 is 19 units from 0

16. $|0|=0$ 0 is 0 units from itself

17. $|6.08|=6.08$ 6.08 is 6.08 units from 0

18. $-2.3+(-8.7)$
Two negative numbers: Add the absolute values, getting 11. The answer is negative, –11.

19. $-\frac{3}{4}-\left(-\frac{4}{5}\right)=-\frac{3}{4}+\frac{4}{5}=-\frac{15}{20}+\frac{16}{20}=\frac{1}{20}$

20. $10+(-5.6)=4.4$

21. $12.3-16.1=12.3+(-16.1)=-3.8$

22. $(-12)(-8)$
Two numbers with the same sign: Multiply their absolute values, getting 96. The answer is positive, 96.

23. $\left(-\frac{2}{3}\right)\left(\frac{5}{8}\right)$
Two numbers with unlike signs: Multiply their absolute values to get $\frac{10}{24}$, or $\frac{5}{12}$. The answer is negative, $-\frac{5}{12}$.

24. $\frac{72.8}{-8}$
Two numbers with unlike signs. Divide their absolute values, getting 9.1. The answer is negative, –9.1.

25. $-7 \div \frac{4}{3} = -7 \cdot \left(\frac{3}{4}\right)$ Multiplying by the reciprocal of $\frac{4}{3}$
$= -\frac{21}{4}$

26. If $a = -6.28$, then $-a = -(-6.28) = 6.28$.

27. $12 + x = x + 12$ Using the commutative law of addition

28. $5x + y = x \cdot 5 + y$ Using the commutative law of multiplication
or $= y + 5x$ Using the commutative law of addition

29. $(4+a)+b = 4+(a+b)$ Using the associative law of addition

30. $x(yz) = (xy)z$ Using the associative law of multiplication

31. $12m + 4n - 2 = 2 \cdot 6m + 2 \cdot 2n - 2 \cdot 1$
$= 2(6m + 2n - 1)$

32. $3x^3 - 6x^2 + x^3 + 5$
$= (3+1)x^3 - 6x^2 + 5$
$= 4x^3 - 6x^2 + 5$

33. $7x - 4[2x + 3(5 - 4x)]$
$= 7x - 4[2x + 15 - 12x]$
$= 7x - 4[-10x + 15]$
$= 7x + 40x - 60$
$= 47x - 60$

34. $3(t+1) - t = 4$
$3t + 3 - t = 4$
$2t + 3 = 4$
$2t + 3 - 3 = 4 - 3$
$2t = 1$
$\frac{1}{2} \cdot 2t = \frac{1}{2} \cdot 1$
$t = \frac{1}{2}$
The solution is $\frac{1}{2}$.

35. $\frac{2}{3}n - \frac{5}{6} = \frac{8}{3}$
$\frac{2}{3}n - \frac{5}{6} + \frac{5}{6} = \frac{8}{3} + \frac{5}{6}$
$\frac{2}{3}n = \frac{16}{6} + \frac{5}{6}$
$\frac{2}{3}n = \frac{21}{6}$
$\frac{3}{2} \cdot \frac{2}{3}n = \frac{3}{2} \cdot \frac{21}{6}$
$n = \frac{21}{4}$
The solution is $\frac{21}{4}$.

36. $-9x + 4(2x - 3) = 5(2x - 3) + 7$
$-9x + 8x - 12 = 10x - 15 + 7$
$-x - 12 = 10x - 8$
$-x - 12 + x = 10x - 8 + x$
$-12 = 11x - 8$
$-12 + 8 = 11x - 8 + 8$
$-4 = 11x$
$\frac{1}{11}(-4) = \frac{1}{11} \cdot 11x$
$-\frac{4}{11} = x$
The solution is $-\frac{4}{11}$.

37. $3(x-4) + 2 = x + 2(x-5)$
$3x - 12 + 2 = x + 2x - 10$
$3x - 10 = 3x - 10$
$-10 = -10$
All real numbers are solutions. The solution set is the set of all real numbers. The equation is an identity.

38. $5t - (7 - t) = 4t + 2(9 + t)$
$5t - 7 + t = 4t + 18 + 2t$
$6t - 7 = 6t + 18$
$-7 = 18$ False equation
The solution set is $\varnothing$. The equation is a contradiction.

39. ***Familiarize***. Let x represent the number.
Translate.

$$\underbrace{\text{Twice the number}}_{\downarrow\; 2x} \quad \underset{\downarrow\; +}{\text{plus}} \quad \underbrace{15}_{\downarrow\; 15} \quad \underset{\downarrow\; =}{\text{is}} \quad \underset{\downarrow\; 21}{21.}$$

40. ***Familiarize***. Let x = one number, then $x - 19$ = the other number.
Translate.

$$\underbrace{\text{First number}}_{\downarrow\; x} \quad \underset{\downarrow\; +}{\text{plus}} \quad \underbrace{\text{second number}}_{\downarrow\; (x-19)} \quad \underset{\downarrow\; =}{\text{is}} \quad \underset{\downarrow\; 115}{115.}$$

Carry out. We solve the equation.
$x + x - 19 = 115$
$2x - 19 = 115$
$2x - 19 + 19 = 115 + 19$
$2x = 134$
$x = 67$
When $x = 67$, $x - 19 = 67 - 19 = 48$

Check. 48 is 19 less than 67. Also 48 + 67 = 115. The answer checks.
State. The smaller number is 48.

41. ***Familiarize***. Let x represent the measure of the second angle. Then the first angle is three times x, and the third angle is two times x. The sum of the three angles is 180°.
Translate. The first angle is $3x$, the second angle is x, and the third angle is $2x$. Translate to an equation:

$$\begin{array}{ccccccc} \underbrace{\text{First}} & \text{plus} & \underbrace{\text{second}} & \text{plus} & \underbrace{\text{third}} & \text{is} & 180^\circ. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 3x & + & x & + & 2x & = & 180 \end{array}$$

Carry out. We solve the equation.

$$3x + x + 2x = 180$$
$$6x = 180$$
$$x = 30$$

When $x = 30$, then $2x = 2\cdot 30 = 60$, and $3x = 3\cdot 30 = 90$.
Check. One angle, 90°, is 3 times the 30° angle, and the third angle, 60°, is two times the 30° angle. Also 60° + 30° + 90° = 180°. The answer checks.
State. The measures of the angles are 90°, 30°, and 60°.

42. $x = \dfrac{bc}{t}$

$tx = bc$ Multiplying both sides by t

$\dfrac{tx}{b} = c$ Multiplying both sides by $\dfrac{1}{b}$

43.
$$c = mx - rx$$
$$c = x(m - r)$$
$$\frac{c}{m-r} = x$$

44. ***Familiarize***. Recall the formula for the volume of a right circular cylinder is $V = \pi r^2 h$.
Translate. We solve the formula for h.

$$V = \pi r^2 h$$
$$h = \frac{V}{\pi r^2}$$

Carry out. We substitute 3.5 for r, 538.51 for V, and 3.14 for π and calculate.

$$h = \frac{538.51}{3.14(3.5)^2}$$
$$h = 14$$

Check. We can repeat the calculation or substitute into the original formula and then solve for h. The answer checks.
State. The height of the candle is 14 cm.

45.
$$\begin{aligned} &\left(-4mn^8\right)\left(7m^3n^2\right) \\ &= (-4)(7)m\cdot m^3\cdot n^8\cdot n^2 \\ &= -28m^{1+3}n^{8+2} \\ &= -28m^4n^{10} \end{aligned}$$

46. $\dfrac{12x^3y^8}{3x^2y^2} = \dfrac{12}{3}x^{3-2}y^{8-2} = 4xy^6$

47.
$$\begin{aligned} a^0 &= (-8)^0 = 1 \\ a^2 &= (-8)^2 = 64 \\ -a^2 &= -(-8)^2 = -64 \end{aligned}$$

48. $3^{-5}\cdot 3^7 = 3^{-5+7} = 3^2$, or 9

49. $\left(2t^4\right)^3 = 2^3\cdot\left(t^4\right)^3 = 8t^{12}$

50.
$$\begin{aligned} \left(-5a^{-3}b^2\right)^{-3} &= (-5)^{-3}\left(a^{-3}\right)^{-3}\left(b^2\right)^{-3} \\ &= -\frac{1}{5^3}\cdot a^9\cdot b^{-6} \\ &= -\frac{a^9}{125b^6} \end{aligned}$$

51. $\left(\dfrac{x^2y^3}{z^4}\right)^{-2} = \dfrac{\left(x^2\right)^{-2}\left(y^3\right)^{-2}}{\left(z^4\right)^{-2}} = \dfrac{x^{-4}y^{-6}}{z^{-8}} = \dfrac{z^8}{x^4y^6}$

52.
$$\begin{aligned} \left(\frac{3m^{-5}n}{9m^2n^{-2}}\right)^4 &= \left(\frac{1}{3}m^{-5-2}n^{1-(-2)}\right)^4 \\ &= \left(\frac{1}{3}m^{-7}n^3\right)^4 \\ &= \frac{m^{-28}n^{12}}{81} \\ &= \frac{n^{12}}{81m^{28}} \end{aligned}$$

53. $\dfrac{4(9-2\cdot 3)-3^2}{4^2-3^2} = \dfrac{4(9-6)-9}{16-9} = \dfrac{4(3)-9}{7} = \dfrac{12-9}{7} = \dfrac{3}{7}$

54.
$$\begin{aligned} &1-(2-5)^2+5\div 10\cdot 4^2 \\ &= 1-(-3)^2+5\div 10\cdot 16 \\ &= 1-9+\frac{1}{2}\cdot 16 \\ &= 1-9+8 \\ &= 0 \end{aligned}$$

55. $0.000307 = \dfrac{0.000307\times 10^4}{10^4} = \dfrac{3.07}{10^4} = 3.07\times 10^{-4}$

56.
$$\begin{aligned} &30{,}860{,}000{,}000{,}000 \\ &= \frac{30{,}860{,}000{,}000{,}000}{10^{13}}\times 10^{13} \\ &= 3.086\times 10^{13} \end{aligned}$$

57.
$$\begin{aligned} &\left(8.7\times 10^{-9}\right)\times\left(4.3\times 10^{15}\right) \\ &= (8.7\times 4.3)\left(10^{-9}\times 10^{15}\right) \\ &= 37.41\times 10^6 \\ &= (3.741\times 10)\times 10^6 \\ &= 3.7\times 10^7 \quad \text{(2 significant digits)} \end{aligned}$$

58. $\dfrac{1.2\times10^{-12}}{6.1\times10^{-7}}=\dfrac{1.2}{6.1}\times\dfrac{10^{-12}}{10^{-7}}$
$=0.1967\times10^{-5}$
$=(1.967\times10^{-1})\times10^{-5}$
$=1.967\times10^{-6}$
$\approx 2.0\times10^{-6}$ (2 significant digits)

59. ***Familiarize***. Recall the formula for the volume is $V = lwh$. We convert 1.2 m to mm.

$$1.2\text{ m}=1.2\text{ m}\times\frac{1000\text{ mm}}{1\text{ m}}=1200\text{ mm}$$

We convert 79 m to mm.

$$79\text{ m}=79\text{ m}\times\frac{1000\text{ mm}}{1\text{ m}}=79{,}000\text{ mm}$$

Translate. We substitute into the formula.

$V = lwh$
$V=(1200)\times(79{,}000)\times(0.00015)$

Carry out. We do the calculations.

$V=(1200)\times(79{,}000)\times(0.00015)$
$=14{,}220\text{ mm}^3$,
or $1.4\times10^4\text{ mm}^3$ (2 significant digits)
or $1.4\times10^{-5}\text{ m}^3$

Check. We recheck the translation and the calculations.

State. The volume of the sheet is about $1.4\times10^4\text{ mm}^3$, or or $1.4\times10^{-5}\text{ m}^3$.

60. *Writing Exercise*. To write an equation that has no solution, begin with a simple equation that is false for any value of x, such as $x = x + 1$. Then add or multiply by the same quantities on both sides of the equation to construct a more complicated equation with no solution.

61. *Writing Exercise*.
a. $-(-x)$ is positive when x is positive; the opposite of the opposite of a number is the number itself;
b. $-x^2$ is never positive; x^2 is always nonnegative, so the opposite of x^2 is always nonpositive;
c. $-x^3$ is positive when x is negative; x^3 is negative when x is negative, and the opposite of a negative number is positive;
d. $(-x)^2$ is positive when $x\neq0$; the square of any nonzero number is positive;
e. x^{-2} is positive when $x\neq0$; $x^{-2}=\dfrac{1}{x^2}$ and x^2 is positive when x is nonzero.

62. First we express the quotient, 3 parts per billion, as the fraction and then express as a percent.

$\dfrac{3}{1{,}000{,}000{,}000}$
$=\dfrac{3}{1{,}000{,}000{,}000}\cdot\dfrac{100}{100}$
$=\dfrac{300}{1{,}000{,}000{,}000}\%$
$=0.0000003\%$

63. $a+b(c-a^2)^0+(abc)^{-1}$
$=3+(-2)(-4-(3)^2)^0+(3(-2)(-4))^{-1}$
$=3+(-2)(1)+(24)^{-1}$
$=3-2+\dfrac{1}{24}$
$=1+\dfrac{1}{24}$
$=1\dfrac{1}{24}$, or $\dfrac{25}{24}$

64. ***Familiarize***. First we find the area of each pizza. Then we find the price per square inch of each pizza. Recall, the formula for the area of a circle is $A=\pi r^2$ and the radius is $\dfrac{d}{2}$.

Translate. For each pizza, the price per square inch is the cost of each pizza divided by the area of each pizza.

$$P=\frac{C}{A}$$

Carry out. We substitute in the formula $A=\pi r^2$. For the first pizza, which costs \$12, $r=\dfrac{13}{2}$, or 6.5. We use 3.14 for π.

$$A=3.14(6.5)^2=132.665$$

So the price per square inch is

$$P=\frac{12}{132.665}\approx0.09$$

For the second pizza, which costs \$15, $r=\dfrac{17}{2}$, or 8.5. We use 3.14 for π.

$$A=3.14(8.5)^2=226.865$$

So the price per square inch is

$$P=\frac{15}{226.865}\approx0.07$$

Check. Repeat the calculations. The answer checks.
State. The 17-inch pizza is about 9¢ per square inch, while the 13-inch pizza is about 7¢ per square inch. The 17-inch pizza is a better deal.

65. ***Familiarize and Translate***. The surface area of a cube is the sum of the areas of the six sides, each a square, or $S_a=6s^2$.
We can solve for the side s, by substituting.

$486=6s^2$
$81=s^2$
$9=s$

Recall the formula for the volume of a cube is $V=s^3$.
Carry out. We substitute.

$$V=9^3=729$$

Check. We repeat the calculations. The answer checks.
State. The volume of the cube is 729 cm^3.

66. $m = \dfrac{x}{y-z}$
$m(y-z) = x$
$my - mz = x$
$-mz = -my + x$
$z = \dfrac{-my+x}{-m}$, or $\dfrac{my-x}{m}$

67. $\dfrac{(3^{-2})^a \cdot (3^b)^{-2a}}{(3^{-2})^b \cdot (9^{-b})^{-3a}} = \dfrac{(3^{-2})^a \cdot (3^b)^{-2a}}{(3^{-2})^b \cdot \left[(3^2)^{-b}\right]^{-3a}} = \dfrac{3^{-2a} \cdot 3^{-2ab}}{3^{-2b} \cdot 3^{6ab}}$
$= 3^{-2a-2ab+2b-6ab}$
$= 3^{-2a+2b-8ab}$

68. $5x - 7(x+3) - 4 = 2(7-x) + \_\_\_$
$5x - 7x - 21 - 4 = 14 - 2x + \_\_\_$
$-2x - 25 = 14 - 2x + \_\_\_$
$-25 = 14 + \_\_\_$
$-39 = \_\_\_$

69. $20 - 7[3(2x+4) - 10] = 9 - 2(x-5) + \_\_\_$
$20 - 7[6x + 12 - 10] = 9 - 2x + 10 + \_\_\_$
$20 - 7[6x + 2] = 19 - 2x + \_\_\_$
$20 - 42x - 14 = 19 - 2x + \_\_\_$
$6 - 42x = 19 - 2x + \_\_\_$
$6 - 40x = 19 + \_\_\_$
$-40x = \_\_\_$

70. $a \cdot 2 + cb + cd + ad$
$= ad + a \cdot 2 + cb + cd$
$= a(d+2) + c(b+d)$

71. Answers will vary; $\sqrt{5}/4$ is one example.

Chapter 1 Test

1. Let m and n represent the numbers; $mn - 4$

2. $a^3 - 5b + b \div ac$
$= (-2)^3 - 5(6) + 6 \div (-2) \cdot (3)$
$= -8 - 5(6) + 6 \div (-2) \cdot 3$
$= -8 - 30 + (-3) \cdot 3$
$= -8 - 30 - 9$
$= -47$

3. We substitute 7.8 for b and 46.5 for h and multiply.
$A = \frac{1}{2} \cdot b \cdot h = \frac{1}{2}(7.8)(46.5) = 181.35$ sq m

4. –15 + (–16)
Two negative numbers: Add the absolute values, getting 31. The answer is negative, –31.

5. –7.5 + 3.8
A negative and positive number: The absolute values are 7.5 and 3.8. Subtract 3.8 from 7.5 to get 3.7. The negative number is farther from 0, so the answer is negative, –3.7.

6. 29.5 – 43.7 = 29.5 + (–43.7) = –14.2

7. –6.4(5.3)
Two numbers with unlike signs: Multiply their absolute values getting 33.92. The answer is negative, –33.92.

8. $-\frac{7}{6} - \left(-\frac{5}{4}\right) = -\frac{7}{6} + \frac{5}{4}$
$= -\frac{14}{12} + \frac{15}{12}$
$= \frac{1}{12}$

9. $-\frac{2}{7}\left(-\frac{5}{14}\right)$
Two numbers with the same sign: Multiply their absolute values getting $\frac{10}{98}$, or $\frac{5}{49}$. The answer is positive, $\frac{5}{49}$.

10. $\dfrac{-42.6}{-7.1}$
Two numbers with the same sign: Divide their absolute values, getting 6. The answer is positive, 6.

11. $\frac{2}{5} \div \left(-\frac{3}{10}\right) = \frac{2}{5} \cdot \left(-\frac{10}{3}\right)$
Two numbers with unlike signs: Divide their absolute values getting $\frac{4}{3}$. The answer is negative, $-\frac{4}{3}$.

12. $7 + (1-3)^2 - 9 \div 2^2 \cdot 6$
$= 7 + (-2)^2 - 9 \div 4 \cdot 6$
$= 7 + 4 - \frac{9}{4} \cdot 6$
$= 7 + 4 - \frac{27}{2}$
$= 11 - \frac{27}{2}$
$= -\frac{5}{2}$

13. Using the commutative law of addition, we have $3 + x = x + 3$.

14. $4y - 10 - 7y - 19 = (4-7)y - 10 - 19 = -3y - 29$

15. $10x - 7 = 38x + 49$
$-7 = 28x + 49$
$-56 = 28x$
$-2 = x$
The solution is –2.

16. $13t - (5-2t) = 5(3t-1)$
$13t - 5 + 2t = 15t - 5$
$15t - 5 = 15t - 5$
$-5 = -5$ True
All real numbers are solutions. The equation is an identity.

17. $2p = sp + t$
$2p - sp = t$
$p(2-s) = t$
$p = \dfrac{t}{2-s}$

18. ***Familiarize***. Let x = the number of points on the sixth test.

Translate

$$\underbrace{\text{Average score on 6 tests}}\quad \underbrace{\text{is}}\quad \underbrace{85.}$$
$$\downarrow \qquad \downarrow \qquad \downarrow$$
$$\frac{84+80+76+96+80+x}{6} = 85$$

Carry out. We solve the equation.

$$\frac{416+x}{6}=85$$
$$416+x=510$$
$$x=94$$

Check. $\frac{84+80+76+96+80+94}{6}$, or 85. The answer checks.

State. Linda must score 94 so her average will be 85.

19. ***Familiarize***. Let x represent the first odd integer. Then the second odd integer is $x + 2$ and the third is $x + 2 + 2$, or $x + 4$.

Translate.

$$\underbrace{\text{Four times the first}}\ \text{plus}\ \underbrace{\text{Three times the second}}\ \text{plus}\ \underbrace{\text{Two times the third}}\ \text{is}\ \underbrace{167.}$$
$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$
$$4x \quad + \quad 3(x+2) \quad + \quad 2(x+4) \quad = \quad 167$$

Carry out. We solve the equation.

$$4x+3(x+2)+2(x+4)=167$$
$$4x+3x+6+2x+8=167$$
$$9x+14=167$$
$$9x=153$$
$$x=17$$

$x+2=19$

$x+4=21$

Check. $4\cdot 17+3\cdot 19+2\cdot 21$, or 68 + 57 + 42, or 167. The answer checks.

State. The consecutive odd integers are 17, 19, and 21.

20. $3x-7-(4-5x)=3x-7-4+5x=8x-11$

21. $6b-[7-2(9b-1)]$
$=6b-[7-18b+2]$
$=6b-[9-18b]$
$=6b-9+18b$
$=24b-9$

22. $(7x^{-4}y^{-7})(-6x^{-6}y)$
$=(7)(-6)(x^{-4}x^{-6})(y^{-7}y)$
$=-42x^{-4+(-6)}y^{-7+1}$
$=-42x^{-10}y^{-6}$, or $-\frac{42}{x^{10}y^{6}}$

23. $-6^{-2}=-\frac{1}{6^2}$, or $-\frac{1}{36}$

24. $(-5x^{-1}y^3)^3=(-5)^3(x^{-1})^3(y^3)^3$
$=-125x^{-3}y^9$
$=-\frac{125y^9}{x^3}$

25. $\left(\frac{2x^3y^{-6}}{-4y^{-2}}\right)^{-2}=\left(-\frac{2}{4}x^3y^{-6-(-2)}\right)^{-2}=\left(-\frac{1}{2}x^3y^{-4}\right)^{-2}$
$=\left(-\frac{1}{2}\right)^{-2}(x^3)^{-2}(y^{-4})^{-2}$
$=(-2)^2x^{-6}y^8=\frac{4y^8}{x^6}$

26. $(7x^3y)^0=1$

27. $(9.05\times 10^{-3})(2.22\times 10^{-5})$
$=(9.05\times 2.22)\times(10^{-3}\times 10^{-5})$
$=20.091\times 10^{-8}$
$=(2.0091\times 10^1)\times 10^{-8}$
$=2.0091\times 10^{-7}$
$\approx 2.01\times 10^{-7}$ (3 significant digits)

28. $\frac{1.8\times 10^{-4}}{4.8\times 10^{-7}}=\frac{1.8}{4.8}\times\frac{10^{-4}}{10^{-7}}$
$=0.375\times 10^3$
$=(3.75\times 10^{-1})\times 10^3$
$=3.75\times 10^2$
$\approx 3.8\times 10^2$ (2 significant digits)

29. ***Familiarize***. Let n = the smallest number of neutrinos that could have the same mass as an alpha particle of mass 3.62×10^{-27} kg.

Translate. We divide.

$$n=\frac{3.62\times 10^{-27}\text{ kg}}{1.8\times 10^{-36}\text{ kg}}$$

Carry out. We perform the calculation and write scientific notation for the answer.

$$n=\frac{3.62\times 10^{-27}\text{ kg}}{1.8\times 10^{-36}\text{ kg}}$$
$$=\frac{3.62}{1.8}\times\frac{10^{-27}}{10^{-36}}$$
$$\approx 2.0\times 10^9 \quad \text{(2 significant digits)}$$

Check. We multiply the answer by the maximum mass of a neutrino.

$(2.0\times 10^9)(1.8\times 10^{-36})=3.6\times 10^{-27}\approx 3.62\times 10^{-27}$

The answer checks.

State. The smallest number of neutrinos that could have the same mass as an alpha particle of mass 3.62×10^{-27} kg is about 2.0×10^9 neutrinos.

30. $(2x^{3a}y^{b+1})^{3c}=2^{3c}\cdot(x^{3a})^{3c}\cdot(y^{b+1})^{3c}=8^cx^{9ac}y^{3bc+3c}$

31. $\frac{-27a^{x+1}}{3a^{x-2}}=\frac{-27}{3}\cdot a^{(x+1)-(x-2)}=-9a^3$

32.
$$-\frac{5x+2}{x+10}=1$$
$$-(5x+2)=1(x+10)$$
$$-5x-2=x+10$$
$$-2=6x+10$$
$$-12=6x$$
$$-2=x$$