Problem 10RE

step 1 of 1

To find the area of the parallelogram spanned by the given vectors

(A) given vectors are

$\vec{v} = -\hat{i} + \hat{j} \qquad \vec{w} = \hat{k}$

Area of $\|gm$ spanned by $\vec{v}, \vec{w} = \mathrm{P}\vec{v} \times \vec{w}\mathrm{P}$

$\vec{v} \times \vec{w} = \hat{i} + \hat{j}$

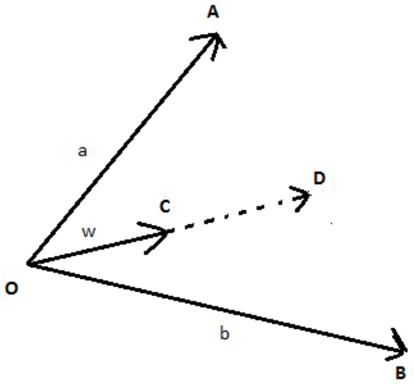
$\Rightarrow$ Area $= \mathrm{P}\vec{v} \times \vec{w}\mathrm{P} = \mathrm{P}\hat{i} \times \hat{j}\mathrm{P} = \sqrt{2} = 1.414$ Sq. units

Problem 11RE

Step-by-step solution

step 1 of 1

Consider the following figure:



Let the vector $\mathbf{w}$ **be the length of** $\mathbf{OC}$ **and it is** $t(\mathbf{OM})$ **where** $0 \le t \le 1$.

The length of $\mathbf{OM}$ **is** $\mathbf{OM} = s\mathbf{a} + (1-s)\mathbf{b}$ **where** $0 \le s \le 1$.

The value of the vector is,

$$\begin{aligned}\mathbf{w} &= t(\mathbf{OM}) \\ &= t(s\mathbf{a} + (1-s)\mathbf{b}) \\ &= (ts)\mathbf{a} + t(1-s)\mathbf{b}\end{aligned}$$

Conversely,

If $\mathbf{w} = (ts)\mathbf{a} + t(1-s)\mathbf{b}$, $(t \cdot s) \ge 0, (t(1-s)) \ge 0 \text{ and } (t \cdot s) + (t(1-s)) \le 1$ **then**

$t = (t \cdot s) + t(1-s)$ **Where** $0 \le t \le 1$ **and** $0 \le s \le 1$.

So, C **belongs to the triangle.**

Hence, the triangle in space, whose vertices are the origin and the end, points of vectors $\mathbf{a}$ and $\mathbf{b}$ **i.e.** $\mathbf{v} = (ts)\mathbf{a} + t(1-s)\mathbf{b}$ **where** $ts \ge 0,\ t(1-s) \ge 0 \text{ and } ts + t(1-s) \le 1$.

Problem 12RE

Step-by-step solution

step 1 of 2

Firstly show that, if the three vectors a, b, c lie in the same plane through the origin then

There exists α, β, γ **not all zeros such that** $\alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c} = 0$.

The vector equation of the plane passing through the point $\mathbf{r}_0 = (x_0, y_0, z_0)$ **containing the vectors** $\mathbf{a}$ **and b is** $\mathbf{r} = \mathbf{r}_0 + s\,\mathbf{a} + t\,\mathbf{b}$ **where** ***s*** **and** ***t*** **are parameters and which all not zero.**

Since the vector $\mathbf{r}_0$ **is origin** $\langle 0,0,0 \rangle$.

Hence the required equation of the plane is $\mathbf{r} = s\,\mathbf{a} + t\,\mathbf{b}$.

The vector c also contained in the plane $\mathbf{r} = s\,\mathbf{a} + t\,\mathbf{b}$ **this implies**

$\mathbf{c} = s\,\mathbf{a} + t\,\mathbf{b}$ Replace $\mathbf{r}$ by $\mathbf{c}$

$s\,\mathbf{a} + t\,\mathbf{b} - \mathbf{c} = \mathbf{0}$

$\alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c} = 0 \quad \text{Where } \alpha = s, \beta = t, \gamma = -1$

This shows that $\alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c} = 0$.

step 2 of 2

Show that if there exists α, β, γ not all zeros such that $\alpha\mathbf{a}+\beta\mathbf{b}+\gamma\mathbf{c}=0$ then the three vectors a, b, c lie in the same plane.

$$\alpha\mathbf{a}+\beta\mathbf{b}+\gamma\mathbf{c}=0$$

$$\mathbf{c}=\frac{\alpha}{\gamma}\mathbf{a}+\frac{\beta}{\gamma}\mathbf{b} \quad \text{Since } \gamma \neq 0$$

That means the vector c can be written as linear combination of other two vectors.

This shows that the three vectors a, b and c lie in a same plane.

Problem 13RE

step 1 of 1

Given that $a_1, b_2, a_3, b_1, b_2, b_3 \in R$

To prove $(a_1b_1+a_2b_2+a_3b_3)^2 \leq (a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$

Let $\vec{A}$ be a vector such that $\vec{A}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

Let $\vec{B}$ be a vector such that $\vec{B}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

Problem 14RE

step 1 of 1

Given that $\hat{u}, \hat{v}, \hat{w}$ are unit vectors and they are orthogonal to each other

To show if $\vec{a}=\alpha\hat{u}+\beta\hat{v}+\gamma\hat{w}$ then $\alpha=\vec{a}.\hat{u}$, $\beta=\vec{a}.\hat{v}$, $\gamma=\vec{a}.\hat{w}$

Since $\hat{u}, \hat{v}, \hat{w}$ are unit vectors we have $\|\hat{u}\|=1,\ \|\hat{v}\|=1,\ \|\hat{w}\|=1$

And $\hat{u}, \hat{v}, \hat{w}$ are orthogonal to each other we have $\langle\hat{u},\hat{v}\rangle=0, \langle\hat{v},\hat{w}\rangle=0, \langle\hat{w},\hat{u}\rangle=0$

$$\begin{aligned}
&\vec{a}=\alpha\hat{u}+\beta\hat{v}+\gamma\hat{w}\\
&\Rightarrow \vec{a}\cdot\vec{u}=(\alpha\hat{u}+\beta\hat{v}+\gamma\hat{w})\cdot\hat{u}\\
&\qquad =\alpha\hat{u}\cdot\hat{u}+\beta\hat{v}\cdot\hat{u}+\gamma\hat{w}\cdot\hat{u}\\
&\qquad =\alpha\times 1+\beta(0)+\gamma(0) \qquad\qquad \text{(using given)}\\
&\qquad =\alpha
\end{aligned}$$

$$\Rightarrow \alpha=\vec{a}\cdot\hat{u}$$

Problem 15RE

Step-by-step solution

step 1 of 1

If the matrices are $n\times n$ matrices and if $A=[a_{ij}], B=[b_{ij}]$ then the product $AB=C$ has following entries

$$C_{ij}=\sum_{k=1}^{n}a_{ik}b_{kj}$$

Which is the dot product of i^{th} row of *A* and j^{th} column of *B*.

Problem 16RE

Step-by-step solution

step 1 of 1

Consider the matrices $A=\begin{bmatrix}2&1&2\\4&0&1\\1&3&0\end{bmatrix}$ and $B=\begin{bmatrix}3&0&5\\1&2&1\\0&3&1\end{bmatrix}$.

The value of the product AB is,

$$\begin{aligned}
AB&=\begin{bmatrix}2&1&2\\4&0&1\\1&3&0\end{bmatrix}\begin{bmatrix}3&0&5\\1&2&1\\0&3&1\end{bmatrix}\\
&=\begin{bmatrix}2\cdot3+1\cdot1+2\cdot0 & 2\cdot0+1\cdot2+2\cdot3 & 2\cdot5+1\cdot1+2\cdot1\\4\cdot3+0\cdot1+1\cdot0 & 4\cdot0+0\cdot2+1\cdot3 & 4\cdot5+0\cdot1+1\cdot1\\1\cdot3+3\cdot1+0\cdot0 & 1\cdot0+3\cdot2+0\cdot3 & 1\cdot5+3\cdot1+0\cdot1\end{bmatrix}\\
&=\begin{bmatrix}7&8&13\\12&3&21\\6&6&8\end{bmatrix}
\end{aligned}$$

Hence, the product AB is $\boxed{\begin{bmatrix}7&8&13\\12&3&21\\6&6&8\end{bmatrix}}$.

Problem 17RE

step 1 of 1

Given that $\vec{a}, \vec{b}$ be two vectors in the plane and

$\vec{a} = (a_1, a_2), \vec{b} = (b_1, b_2)$ and λ be real number

To show that the area of parallelogram determined by a and $b + \lambda a$ is same as that determined by a and b. and sketch and also relate this property to a known property of determinants

Problem 18RE

step 1 of 1

Given the vertices of a parallelepiped they are

Let $\vec{a} = (0,1,0) \quad \vec{b} = (1,1,1) \quad \vec{c} = (0,2,0) \quad \vec{d} = (3,1,2)$

Now we have to find the volume of the parallelepiped determined by the given vertices

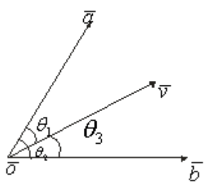
Sides of a parallelepiped are given by $\overrightarrow{AB} = \vec{b} - \vec{a} = (1,0,1)$

$$\overrightarrow{AC} = \vec{c} - \vec{a} = (0,1,0)$$

$$\overrightarrow{AD} = \vec{d} - \vec{a} = (3,0,2)$$

Problem 19RE

step 1 of 1



Problem 1RE

Step-by-step solution

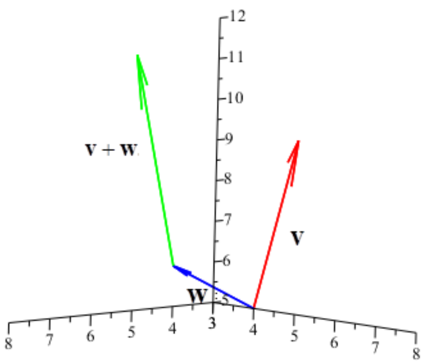
step 1 of 6

The vectors in this problem are $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}, \mathbf{w} = \mathbf{i} - \mathbf{j} + \mathbf{k}.$

Evaluate $\mathbf{v} + \mathbf{w}.$

$$\begin{aligned} \mathbf{v} + \mathbf{w} &= (3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}) + (\mathbf{i} - \mathbf{j} + \mathbf{k}) \\ &= (3+1)\mathbf{i} + (4-1)\mathbf{j} + (5+1)\mathbf{k} \\ &= 4\mathbf{i} + 3\mathbf{j} + 6\mathbf{k} \end{aligned}$$

The vectors $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}, \mathbf{w} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{v} + \mathbf{w} = 4\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ can be plotted as follows.

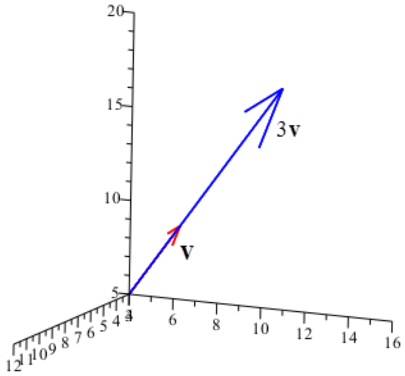


step 2 of 6

Evaluate $3\mathbf{v}.$

$$\begin{aligned} 3\mathbf{v} &= 3(3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}) \\ &= 9\mathbf{i} + 12\mathbf{j} + 15\mathbf{k} \end{aligned}$$

The vectors $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}, 3\mathbf{v} = 9\mathbf{i} + 12\mathbf{j} + 15\mathbf{k}$ can be plotted as follows.

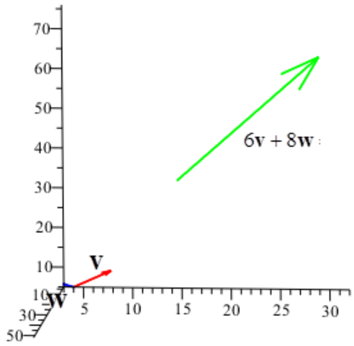


step 3 of 6

Evaluate $6\mathbf{v}+8\mathbf{w}.$

$$\begin{aligned}6\mathbf{v}+8\mathbf{w}&=6(3\mathbf{i}+4\mathbf{j}+5\mathbf{k})+8(\mathbf{i}-\mathbf{j}+\mathbf{k})\\&=(18+8)\mathbf{i}+(24-8)\mathbf{j}+(30+8)\mathbf{k}\\&=26\mathbf{i}+16\mathbf{j}+38\mathbf{k}\end{aligned}$$

The vectors $\mathbf{v}=3\mathbf{i}+4\mathbf{j}+5\mathbf{k}, \mathbf{w}=\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $6\mathbf{v}+8\mathbf{w}=26\mathbf{i}+16\mathbf{j}+38\mathbf{k}$ can be plotted as follows.

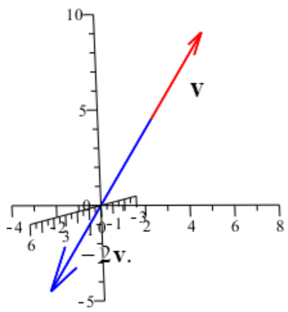


step 4 of 6

Evaluate $-2\mathbf{v}.$

$$\begin{aligned}-2\mathbf{v}&=-2(3\mathbf{i}+4\mathbf{j}+5\mathbf{k})\\&=(-6)\mathbf{i}+(-8)\mathbf{j}+(-10)\mathbf{k}\end{aligned}$$

The vectors $\mathbf{v}=3\mathbf{i}+4\mathbf{j}+5\mathbf{k}$ and $-2\mathbf{v}=(-6)\mathbf{i}+(-8)\mathbf{j}+(-10)\mathbf{k}$ can be plotted as follows.



step 5 of 6

Evaluate $\mathbf{v}\cdot\mathbf{w}.$

$$\begin{aligned}\mathbf{v}\cdot\mathbf{w}&=(3\mathbf{i}+4\mathbf{j}+5\mathbf{k})\cdot(\mathbf{i}-\mathbf{j}+\mathbf{k})\\&=3-4+5\\&=4\end{aligned}$$

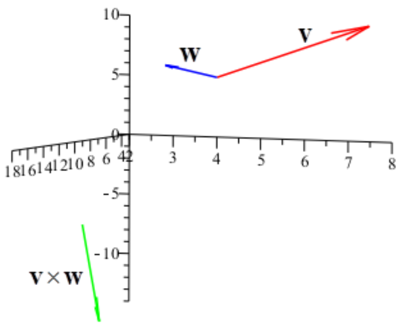
The obtained quantity is a real number and it cannot be plotted.

step 6 of 6

Evaluate $\mathbf{v}\times\mathbf{w}.$

$$\begin{aligned}\mathbf{v}\times\mathbf{w} &= (3\mathbf{i}+4\mathbf{j}+5\mathbf{k})\times(\mathbf{i}-\mathbf{j}+\mathbf{k})\\ &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3 & 4 & 5\\ 1 & -1 & 1\end{vmatrix}\\ &= (4+5)\mathbf{i}+(5-3)\mathbf{j}+(-3-4)\mathbf{k}\\ &= 9\mathbf{i}+2\mathbf{j}-7\mathbf{k}\end{aligned}$$

The vectors $\mathbf{v}=3\mathbf{i}+4\mathbf{j}+5\mathbf{k}, \mathbf{w}=\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{v}\times\mathbf{w}=9\mathbf{i}+2\mathbf{j}-7\mathbf{k}$ can be plotted as follows.



Problem 20RE

Step-by-step solution

step 1 of 1

Consider the two vectors $\|\mathbf{b}\|\mathbf{a}+\|\mathbf{a}\|\mathbf{b}$ and $\|\mathbf{b}\|\mathbf{a}-\|\mathbf{a}\|\mathbf{b}$.

If the two vectors are orthogonal then their dot product is zero.

To prove the orthogonal, we need to prove $(\|\mathbf{b}\|\mathbf{a}-\|\mathbf{a}\|\mathbf{b})\cdot(\|\mathbf{b}\|\mathbf{a}+\|\mathbf{a}\|\mathbf{b})=0$.

The dot product of the two vectors is,

$$\begin{aligned}(\|\mathbf{b}\|\mathbf{a}-\|\mathbf{a}\|\mathbf{b})\cdot(\|\mathbf{b}\|\mathbf{a}+\|\mathbf{a}\|\mathbf{b}) &= (\|\mathbf{b}\|\mathbf{a})^2+\|\mathbf{b}\|\mathbf{a}\|\mathbf{a}\|\mathbf{b}-\|\mathbf{b}\|\mathbf{a}\|\mathbf{a}\|\mathbf{b}-(\|\mathbf{a}\|\mathbf{b})^2\\ &= (\|\mathbf{b}\|\mathbf{a})^2-(\|\mathbf{a}\|\mathbf{b})^2 \quad ......(1)\end{aligned}$$

But, the value of $(\|\mathbf{a}\|)^2$ is,

$$\begin{aligned}(\|\mathbf{a}\|)^2 &= \left(\sqrt{\mathbf{a}\cdot\mathbf{a}}\right)^2\\ &= \mathbf{a}\cdot\mathbf{a}\\ &= |a|^2\end{aligned}$$

The equation (1) becomes,

$$\begin{aligned}(\|\mathbf{b}\|\mathbf{a}-\|\mathbf{a}\|\mathbf{b})\cdot(\|\mathbf{b}\|\mathbf{a}+\|\mathbf{a}\|\mathbf{b}) &= (\|\mathbf{b}\|\mathbf{a})^2-(\|\mathbf{a}\|\mathbf{b})^2\\ &= \left(|\mathbf{b}|^2\mathbf{a}^2\right)-\left(|\mathbf{a}|^2\mathbf{b}^2\right) \quad \left(\text{Since,} \|\mathbf{b}\|^2=|\mathbf{b}|^2 \text{ and } \|\mathbf{a}\|^2=|\mathbf{a}|^2\right)\\ &= \mathbf{b}^2\mathbf{a}^2-\mathbf{a}^2\mathbf{b}^2 \quad \left(\text{Since,} |\mathbf{a}|^2=\mathbf{a}^2 \text{ and } |\mathbf{b}|^2=\mathbf{b}^2\right)\\ &= 0\end{aligned}$$

Hence, the two vectors $\|\mathbf{b}\|\mathbf{a}+\|\mathbf{a}\|\mathbf{b}$ and $\|\mathbf{b}\|\mathbf{a}-\|\mathbf{a}\|\mathbf{b}$ are orthogonal.

Problem 21RE

Step-by-step solution

step 1 of 1

Let $\mathbf{x}$ and $\mathbf{y}$ are two vectors in space then the triangle inequality is,

$\|\mathbf{x}\|-\|\mathbf{y}\|\le\|\mathbf{x}+\mathbf{y}\|\le\|\mathbf{x}\|+\|\mathbf{y}\|$.

To prove $\|\mathbf{v}-\mathbf{w}\|\ge|(\|\mathbf{v}\|-\|\mathbf{w}\|)|$, use the triangle inequality form.

Write the vector $\|\mathbf{v}\|$ as,

$$\begin{aligned}\|\mathbf{v}\| &= \|(\mathbf{v}-\mathbf{w})+\mathbf{w}\| \quad (\text{Add \& Subtract vector } \mathbf{w})\\ &\le \|(\mathbf{v}-\mathbf{w})\|+\|\mathbf{w}\| \quad (\text{From the Traingle inequality})\\ \|\mathbf{v}\|-\|\mathbf{w}\| &\le \|(\mathbf{v}-\mathbf{w})\| \quad (\text{Subtract } \|\mathbf{w}\| \text{ on both sides})\end{aligned}$$

Hence, we have proved i.e. $\boxed{\|(\mathbf{v}-\mathbf{w})\|\ge\|\mathbf{v}\|-\|\mathbf{w}\|}$.

Problem 22RE

Step-by-step solution

step 1 of 1

Consider the point (x_1, y_1) and the line $ax + by = c$.

To prove that the distance from the point to the line is $\frac{|ax_1 + by_1 - c|}{\sqrt{a^2 + b^2}}$, use the vector projection formula $\text{Proj}_v u = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2}|\mathbf{v}|$.

Rewrite the equation of line as,

$$
\begin{aligned}
ax + by &= c \\
by &= c - ax \quad (\text{Subtract } ax \text{ on both sides}) \\
y &= \frac{c}{b} - \frac{a}{b}x \quad (\text{Divide by } b \text{ on both sides})
\end{aligned}
$$

Hence, the slope of the equation of line is $m = -\frac{a}{b}$.

Problem 23RE

Step-by-step solution

step 1 of 1

To verify the direction of $\mathbf{b} \times \mathbf{c}$ given by the right –hand rule, by choosing $\mathbf{b}, \mathbf{c}$ to be two of the vectors $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$, choose the different of vectors for $\mathbf{b}, \mathbf{c}$ and then find the value of $\mathbf{b} \times \mathbf{c}$.

Choose the two different vectors as $\mathbf{b} = \mathbf{i}, \mathbf{c} = \mathbf{j}$.

The value of $\mathbf{b} \times \mathbf{c}$ is,

$$
\begin{aligned}
\mathbf{b} \times \mathbf{c} &= \mathbf{i} \times \mathbf{j} \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \\
&= \mathbf{i}(0-0) - \mathbf{j}(0-0) + \mathbf{k}(1-0) \\
&= \mathbf{k}
\end{aligned}
$$

Hence, the direction of $\mathbf{b} \times \mathbf{c}$ by choosing $\mathbf{b} = \mathbf{i}, \mathbf{c} = \mathbf{j}$ is $\boxed{\mathbf{i} \times \mathbf{j} = \mathbf{k}}$.

Problem 24RE

Step-by-step solution

step 1 of 1

a)

Consider the equation $\mathbf{a} \cdot \mathbf{b} = \mathbf{a}' \cdot \mathbf{b}$ and prove that $\mathbf{a} = \mathbf{a}'$.

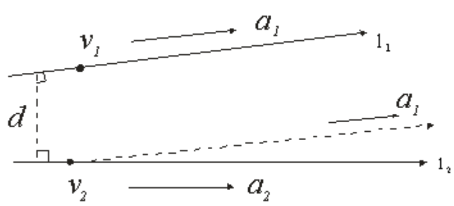
$$
\begin{aligned}
\mathbf{a} \cdot \mathbf{b} &= \mathbf{a}' \cdot \mathbf{b} \\
\mathbf{a} \cdot \mathbf{b} - \mathbf{a}' \cdot \mathbf{b} &= 0 \\
(\mathbf{a} - \mathbf{a}') \cdot \mathbf{b} &= 0 \qquad \text{Use distributive property} \\
\Rightarrow \quad \mathbf{a} - \mathbf{a}' &= \mathbf{0} \qquad \text{Since it is true for all } \mathbf{b} \\
\mathbf{a} - \mathbf{a}' + \mathbf{a}' &= \mathbf{0} + \mathbf{a}' \qquad \text{Add } \mathbf{a}' \text{ on both the sides} \\
\mathbf{a} + \mathbf{0} &= \mathbf{0} + \mathbf{a}' \\
\mathbf{a} &= \mathbf{a}'
\end{aligned}
$$

Hence proved!

Problem 25RE

step 1 of 1

(A)



Problem 26RE

step 1 of 1

Given equations are $Ax+By+Cz+D_1=0$ and $Ax+By+Cz+D_2=0$

To show two planes given by the above equations are parallel and the distance between them is $\dfrac{|D_1-D_2|}{\sqrt{A^2+B^2+C^2}}$

Let $P_1 \equiv Ax+By+Cz+D_1=0$

$P_2 \equiv Ax+By+Cz+D_2=0$

Take $\vec{N}_1 =$ Normal vector for $P_1 = A\hat{i}+B\hat{j}+C\hat{k}$

Take $\vec{N}_2 =$ Normal vector for $P_2 = A\hat{i}+B\hat{j}+C\hat{k}$

Problem 27RE

step 1 of 1

(A) Given the vertices of a triangle $(x_1,y_1),(x_2,y_2),(x_3,y_3)$

To prove that the area of the triangle in the plane with given vertices is the absolute value of $\dfrac{1}{2}\begin{vmatrix}1 & 1 & 1\\ x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\end{vmatrix}$

Let $A=(x_1,y_1)$ $\quad B=(x_2,y_2)$ $\quad C=(x_3,y_3)$

Area of $\Delta ABC = \dfrac{1}{2}\left(\vec{AB}\times\vec{AC}\right)$

$\vec{AB}=(x_2-x_1,y_2-y_1)$

$\vec{AC}=(x_3-x_1,y_3-y_1)$

Problem 28RE

Step-by-step solution

step 1 of 1

(a)

Consider the Cartesian coordinate $(0,3,4)$.

To find the cylindrical coordinates (r,θ,z)**, find the value** $r=\sqrt{x^2+y^2}$ and $\theta=\tan^{-1}\left(\dfrac{y}{x}\right)$

The value of r **is,**

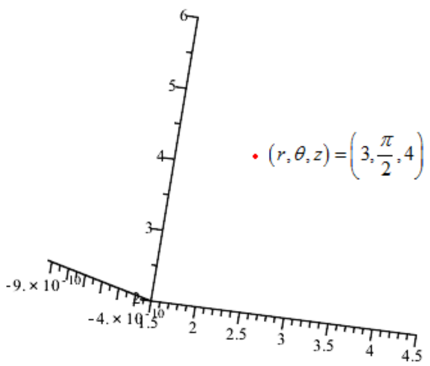
$$\begin{aligned} r &= \sqrt{x^2+y^2}\\ &= \sqrt{(0)^2+(3)^2}\\ &= 3 \end{aligned}$$

The value of θ **is,**

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{y}{x}\right)\\ &= \tan^{-1}\left(\frac{3}{0}\right)\\ &= \frac{\pi}{2} \end{aligned}$$

Hence, the cylindrical coordinates (r,θ,z) **are** $\boxed{(r,\theta,z)=\left(3,\frac{\pi}{2},4\right)}$.

The plot of cylindrical coordinates is as shown below.



Problem 29RE

Step-by-step solution

step 1 of 1

The objective is to convert the following points from cylindrical to Cartesian and spherical coordinates.

Problem 2RE

step 1 of 1

Given that $\vec{v} = 2\hat{j} + \hat{k}, \quad \vec{w} = -\hat{i} - \hat{k}$

Now we have to compute $\vec{v} + \vec{w}$, $3\vec{v}$, $6\vec{v} + 8\vec{w}$, $-2\vec{v}$, $\vec{v}.\vec{w}$ and $\vec{v} \times \vec{w}$

And to interpret the each operation geometrically by graphing the vectors

Computing $\vec{v} + \vec{w} = \left(2\hat{j} + \hat{k}\right) + \left(-\hat{i} - \hat{k}\right)$

$$= -\hat{i} + 2\hat{j}$$

$$3\vec{v} = 3\left(2\hat{j} + \hat{k}\right)$$

$$= 6\hat{j} + 3\hat{k}$$

Problem 30RE

Step-by-step solution

step 1 of 1

The objective is to convert the following points from spherical to cylindrical and Cartesian coordinates.

Problem 31RE

step 1 of 1

Now we have to rewrite the equation $z = x^2 - y^2$ by using cylindrical and spherical coordinates

Cylindrical Co-ordinates,

$x = r\cos\theta$

$y = r\sin\theta$

$\Rightarrow z = r^2\cos^2\theta - r^2\sin^2\theta$

i.e. $z = r^2\cos 2\theta$

Problem 32RE

Problem 32RE

Problem 33RE

step 1 of 1

Given that $x = (3,2,1,0)$, $y = (1,1,1,2)$

To verify Cauchy-Schwarz and Triangle inequalities

We know that Cauchy-Schwarz inequality in $\mathbb{R}^n$

Let x, y be vectors in $\mathbb{R}^n$. Then $|x \cdot y| \le \|x\| \|y\|$

For that

Consider

$$x.y = (3,2,1,0)(1,1,1,2)$$

$$= 3(1) + 2(1) + 1(1) + 0(2)$$

$$= 6$$

Problem 34RE

step 1 of 1

Given matrices are

$$A = \begin{bmatrix} 3 & 0 & 1 \\ 2 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

To multiply the matrices and check that does $AB = BA$

Problem 35RE

Step-by-step solution

step 1 of 1

a)

The matrices A and B are $n \times n$ matrices and x is a column matrix in $\mathbb{R}^n$ that is order of the matrix x is $n \times 1$.

Let $A = \left[a_{ij}\right]_{n\times n}, B = \left[b_{ij}\right]_{n\times n}, \mathbf{x} = \left[x_i\right]_{n\times 1}$.

Consider the left hand side of the equation $(AB)\mathbf{x} = A(B\mathbf{x})$ and simplify it.

$$\begin{aligned}(AB)\mathbf{x} &= \left(\left[a_{ij}\right]_{n\times n}\cdot\left[b_{ij}\right]_{n\times n}\right)\cdot\left[x_i\right]_{n\times 1}\\ &= \left[\sum_{k=1}^{n} a_{ik}b_{kj}\right]_{n\times n}\cdot\left[x_i\right]_{n\times 1}\\ &= \left[c_{ij}\right]_{n\times n}\cdot\left[x_i\right]_{n\times 1} \quad \text{where } \sum_{k=1}^{n} a_{ik}b_{kj} = c_{ij}\\ &= \left[\sum_{r=1}^{n}\left(c_{ir}x_{r1}\right)\right]_{n\times 1}\\ &= \left[\sum_{r=1}^{n}\left(\left(\sum_{r=1}^{n} a_{ir}b_{rj}\right)x_{r1}\right)\right]_{n\times 1} \qquad \text{......(1)}\end{aligned}$$

Problem 36RE

Problem 36RE

Problem 37RE

Step-by-step solution

step 1 of 1

The objective is to verify that a liner mapping T of $\mathbb{R}^n$ to $\mathbb{R}^n$ is determined by an $n \times n$ matrix.

Let A be any $n \times n$ matrix over a field $\mathbb{R}$.

Use the definition of linear transformation:

Let V_1 and V_2 be vector spaces. A linear transformation is function $T: V_1 \to V_2$ with the following properties:

(1)

For any $u, v \in V_1$ then $T(v+w) = T(v) + T(w)$

(2)

For any $u, v \in V_1$ then $T(kv) = kT(v)$

Problem 38RE

Step-by-step solution

step 1 of 1

Consider the vector $(3,-1,2)$ and line $\mathbf{v} = (2,-1,0) + t(2,3,0)$.

The objective is to find the equation of the plane.

Problem 39RE

Step-by-step solution

step 1 of 1

Consider the work done W is moving an object from $(0,0)$ to $(7,2)$ and subject to a constant force F is $W = \vec{F}\cdot\vec{r}$, where $\vec{r}$ is the vector with its head at (7,2) and tail at (0,0) and the units are feet and pounds.

(a)

Consider the vector $\vec{F} = 10\cos\theta\,\hat{i} + 10\sin\theta\,\hat{j}$.

The vector $\vec{r} = (7,2) - (0,0)$

$= (7,2)$ Find the value of W in terms of θ as follows:

$$\begin{aligned}w &= \vec{F}\cdot\vec{r}\\ &= \left(10\cos\theta\,\hat{i} + 10\sin\theta\,\hat{j}\right)\cdot\left(7\hat{i} + 2\hat{j}\right)\\ &= 70\cos\theta + 20\sin\theta\end{aligned}$$

Problem 3RE

Step-by-step solution

step 1 of 1

(a)

Consider the point $(-1,2,-1)$.

To find the equation of the line through $(-1,2,-1)$ in the direction of $\mathbf{j}$, apply the formula of the equation of the line passing the point $\mathbf{a}$ in the direction of the vector $\mathbf{v}$.

The equation of the line passing through the point $(-1,2,-1)$ in the direction of $\mathbf{j}$ is,

$$\begin{aligned}\mathbf{l}(t) &= \mathbf{a}+\mathbf{v}t \\ &= (-1,2,-1)+t(0,1,0) \\ &= (-1,2+t,-1)\end{aligned}$$

Hence, the equation of the line is $\boxed{\mathbf{l}(t)=-\mathbf{i}+(2+t)\mathbf{j}-\mathbf{k}}$.

Problem 40RE

Step-by-step solution

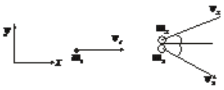
step 1 of 1

The objective is to find the angle and speed of the second marble.

Consider a particle with mass m that moves with velocity $\mathbf{v}$ and its momentum is,

$$\mathbf{p}=m\mathbf{v}$$

Sketch the graph of the mass and velocity as shown below:



Problem 41RE

Step-by-step solution

step 1 of 1

The objective is to prove that

$$\begin{vmatrix} x+2 & y & z \\ z & y+1 & 10 \\ 5 & 5 & 2 \end{vmatrix} = -\begin{vmatrix} y & x+2 & z \\ 1 & z-x-2 & 10-z \\ 5 & 5 & 2 \end{vmatrix}$$ For all x, y, z.

Consider,

$$L\cdot H\cdot S=\begin{vmatrix} x+2 & y & z \\ z & y+1 & 10 \\ 5 & 5 & 2 \end{vmatrix}$$

Apply the column operation as follows:

$$L\cdot H\cdot S=-\begin{vmatrix} y & x+2 & z \\ y+1 & z & 10 \\ 5 & 5 & 2 \end{vmatrix}$$

Apply the row operation as follows:

$$\begin{aligned}L\cdot H\cdot S &= -\begin{vmatrix} y & x+2 & z \\ 1 & z-x-2 & 10-z \\ 5 & 5 & 2 \end{vmatrix} \\ &= R.H.S.\end{aligned}$$

Hence $L\cdot H\cdot S=R\cdot H\cdot S$

That implies,

$$\begin{vmatrix} x+2 & y & z \\ z & y+1 & 10 \\ 5 & 5 & 2 \end{vmatrix} = -\begin{vmatrix} y & x+2 & z \\ 1 & z-x-2 & 10-z \\ 5 & 5 & 2 \end{vmatrix}$$

Hence proved.

Problem 42RE

step 1 of 3

$$\text{To prove } \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \neq 0$$

$$\text{Consider } \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

step 2 of 3

$$R_2 \leftarrow\!\!| R_2 - R_1$$
$$R_3 \leftarrow\!\!| R_3 - R_1$$
$$= \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$$

step 3 of 3

$$= 1(y-x)(z^2-x^2)-(z-x)(y^2-x^2)$$
$$= (y-x)(z-x)(z+x)-(y+x)$$
$$= (y-x)(z-x)(z-y)$$
$$\neq 0$$

If x, y, z are all different.

Hence proved

Problem 43RE

Step-by-step solution

step 1 of 1

The objective is to prove that

$$\begin{vmatrix} 66 & 628 & 246 \\ 88 & 435 & 24 \\ 2 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 68 & 627 & 247 \\ 86 & 436 & 23 \\ 2 & -1 & 1 \end{vmatrix}$$

Consider,

$$L \cdot H \cdot S = \begin{vmatrix} 66 & 628 & 246 \\ 88 & 435 & 24 \\ 2 & -1 & 1 \end{vmatrix}$$

Apply the row operation as follows:

$$R_1 \leftarrow, R_1 + R_3 \ , R_2 \leftarrow R_2 - R_3$$

$$L \cdot H \cdot S = \begin{vmatrix} 66+2 & 628-1 & 246+1 \\ 88-2 & 435-(-1) & 24-1 \\ 2 & -1 & 1 \end{vmatrix}$$
$$= \begin{vmatrix} 68 & 627 & 247 \\ 86 & 436 & 23 \\ 2 & -1 & 1 \end{vmatrix}$$
$$= R \cdot H \cdot S$$

Hence $L \cdot H \cdot S = R \cdot H \cdot S$

That implies,

$$\begin{vmatrix} 66 & 628 & 246 \\ 88 & 435 & 24 \\ 2 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 68 & 627 & 247 \\ 86 & 436 & 23 \\ 2 & -1 & 1 \end{vmatrix}$$

Hence proved.

Problem 44RE

step 1 of 1

To prove $\begin{vmatrix} n & n+1 & n+2 \\ n+3 & n+4 & n+5 \\ n+6 & n+7 & n+8 \end{vmatrix}$ has the same value no matter what n is. And also to find its value

$$R_2 \leftarrow\!\!| R_2 - R_1$$
$$R_3 \leftarrow\!\!| R_3 - R_1$$

Problem 45RE

Step-by-step solution

step 1 of 1

Vectors are quantities having both magnitude and direction, while scalars are quantities having only magnitude.

Problem 46RE

Step-by-step solution

step 1 of 1

We shall find a 4×4 matrix *C* such that for every 4×4 matrix *A*, we have $CA=3A$

Problem 47RE

Step-by-step solution

step 1 of 1

The objective is to find the values of A^{-1}, B^{-1} and $(AB)^{-1}$ and to show that

$(AB)^{-1}\neq A^{-1}B^{-1}$ but $(AB)^{-1}=B^{-1}A^{-1}$.

Consider,

$A=\begin{bmatrix}1&1\\0&1\end{bmatrix}$ and $B=\begin{bmatrix}1&0\\2&1\end{bmatrix}$

Let A be a 2×2 matrix then the inverse of 2×2 matrix A is,

$$\begin{aligned}A^{-1}&=\frac{1}{ad-bc}\begin{bmatrix}d&-b\\c&a\end{bmatrix}\\&=\frac{1}{|A|}(adjA)\end{aligned}$$

Where, A is nonsingular matrix.

Problem 48RE

Step-by-step solution

step 1 of 1

Consider the matrix,

$$A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$$

The matrix A is said to be invertible, if its determinant is nonsingular that is $ad-bc\neq0,$ and the inverse of a matrix A is,

$$A^{-1}=\frac{1}{ad-bc}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}.$$

Problem 49RE

step 1 of 1

The volume of a tetrahedron with concurrent edges a,b,c is given by

$$V=\frac{1}{6}\vec{a}\cdot(\vec{b}\times\vec{c})$$

(A) to express the volume as a determinant $V=\frac{1}{6}\vec{a}\cdot(\vec{b}\times\vec{c})$

$$=\frac{1}{6}\begin{vmatrix}a_x&a_y&a_z\\b_x&b_y&b_z\\c_x&c_y&c_z\end{vmatrix}$$

Problem 4RE

Step-by-step solution

step 1 of 1

(a)

Consider the point $(0,1,0)$.

To find the equation of the line through $(0,1,0)$ in the direction of $3\mathbf{i}+\mathbf{k}$, apply the formula of the equation of the line passing the point $\mathbf{a}$ in the direction of the vector $\mathbf{v}$.

The equation of the line passing through the point $(0,1,0)$ in the direction of $3\mathbf{i}+\mathbf{k}$ is,

$$\begin{aligned}\mathbf{l}(t)&=\mathbf{a}+\mathbf{v}t\\&=(0,1,0)+t(3,0,1)\quad(\text{Since } 3\mathbf{i}+\mathbf{k} \text{ is } (3,0,1) \text{ in point form})\\&=(3t,1,t)\end{aligned}$$

Hence, the equation of the line is $\boxed{\mathbf{l}(t)=(3t)\mathbf{i}+\mathbf{j}+(t)\mathbf{k}}$.

Problem 50RE

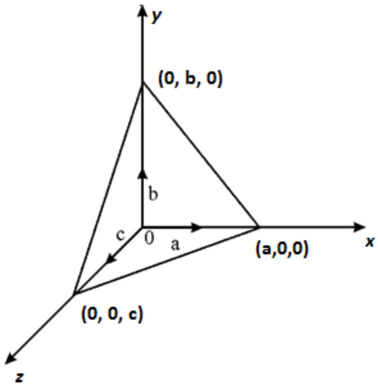
Step-by-step solution

step 1 of 1

A tetrahedron is a polyhedron and it has four triangular faces, six edges and four vertices.

(a)

The diagram of tetrahedron is as shown below:



A tetrahedron in the xyz coordinates with one vertex at $(0,0,0)$ and the three edges concurrent at $(0,0,0)$.

Problem 51RE

Problem 51RE

Problem 52RE

step 1 of 1

A line is given by

$x = 3t + 1$

$y = 16t - 2$

$z = -(t+2)$

Now we have to find the unit vector parallel to the given line

Problem 53RE

Problem 53RE

Problem 54RE

Step-by-step solution

step 1 of 1

Consider the two planes, $8x + y + z = 1$ and $x - y - z = 0$.

The normal vector $\mathbf{N}_1$ of the plane $8x + y + z = 1$ is $8\mathbf{i} + \mathbf{j} + \mathbf{z} = 1$.

The normal vector $\mathbf{N}_2$ of the plane $x - y - z = 0$ is $\mathbf{i} - \mathbf{j} - \mathbf{z} = 0$.

To find the unit vector parallel to both planes, first find the cross product, $\mathbf{N}_1 \times \mathbf{N}_2$ and use the formula, $\frac{\mathbf{N}_1 \times \mathbf{N}_2}{\|\mathbf{N}_1 \times \mathbf{N}_2\|}$.

The value of $\mathbf{N}_1 \times \mathbf{N}_2$ is,

$$\begin{aligned}\mathbf{N}_1 \times \mathbf{N}_2 &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 1 & 1 \\ 1 & -1 & -1 \end{vmatrix} \\ &= \mathbf{i}(-1+1) - \mathbf{j}(-8-1) + \mathbf{k}(-8-1) \\ &= 9\mathbf{j} - 9\mathbf{k}\end{aligned}$$

The value of unit vector is,

$$\begin{aligned}\frac{\mathbf{N}_1 \times \mathbf{N}_2}{\|\mathbf{N}_1 \times \mathbf{N}_2\|} &= \frac{9\mathbf{j} - 9\mathbf{k}}{\sqrt{(9)^2 + (9)^2}} \\ &= \frac{9\mathbf{j} - 9\mathbf{k}}{\sqrt{162}} \\ &= \frac{9\mathbf{j} - 9\mathbf{k}}{9\sqrt{2}} \\ &= \frac{1}{\sqrt{2}}(\mathbf{j} - \mathbf{k})\end{aligned}$$

$= \frac{\sqrt{2}}{2}(\mathbf{j}-\mathbf{k})$

Hence, the unit vector parallel to both planes is $\boxed{\frac{\sqrt{2}}{2}(\mathbf{j}-\mathbf{k})}$.

Problem 55RE

Step-by-step solution

step 1 of 1

Given that $\hat{i}+2\hat{j}-\hat{k}$ & $\hat{k}$

Now we have to find the Unit vector orthogonal to $\hat{i}+2\hat{j}-\hat{k}$ & $\hat{k}$

Unit vector is along $\left(\hat{i}+2\hat{j}-\hat{k}\right)\times\hat{k}$

$=-\hat{j}+2\hat{i}$

Hence the unit vector orthogonal to $\hat{i}+2\hat{j}-\hat{k}$ & $\hat{k}$ **is**

$\frac{1}{\sqrt{5}}\left(2\hat{i}-\hat{j}\right)$

Problem 56RE

step 1 of 2

Given line is $x=2t-1,\ y=-t-1,\ z=t+2$ & vector $\hat{i}-\hat{j}$
Now we have to find the Unit vector orthogonal to the given line and the given vector

step 2 of 2

Vector P to line is $\left(2\hat{i}-\hat{j}+\hat{k}\right)$
$\Rightarrow$ unit vector is along
$\left(2\hat{i}-\hat{j}+\hat{k}\right)\times\left(\hat{i}-\hat{j}\right)$
$=\hat{k}+\hat{j}-2\hat{k}+\hat{j}$
$=\hat{i}+\hat{j}-\hat{k}$
$\Rightarrow$ unit vector is $\frac{\hat{i}+\hat{j}-\hat{k}}{\left|\hat{i}+\hat{j}-\hat{k}\right|}$

Hence the Unit vector orthogonal to the given line and the given vector
$=\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$

Problem 57RE

Step-by-step solution

step 1 of 1

Let $\mathbf{u}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ **be the required unit vector.**

To find the unit vector at angle, first write the statement at angle of 30° **to** $\mathbf{i}$ **and making equal angles with** $\mathbf{j}$ and $\mathbf{k}$ **in the below form i.e.,**

$\mathbf{u}\cdot\mathbf{i}=\|\mathbf{u}\|\,\|\mathbf{i}\|\cos\left(30^0\right)$

$x=\frac{\sqrt{3}}{2}$ **and** $\mathbf{u}\cdot\mathbf{j}=\mathbf{u}\cdot\mathbf{k}$, $y=z$ **…��?(1)**

Problem 5RE

Step-by-step solution

step 1 of 1

Consider the following points

$(2,1,-1)$, $(3,0,2)$, $(4,-3,1)$

Problem 6RE

Step-by-step solution

step 1 of 1

Consider the equation of a point-direction form of a line is,

$$x = x_1 + at,$$
$$y = y_1 + bt,$$
$$z = z_1 + ct,$$

Here, $a = (x_1, y_1, z_1)$ and $v = (a, b, c)$

Problem 7RE

Step-by-step solution

step 1 of 1

(a)

Consider the vectors $\mathbf{v} = -\mathbf{i} + \mathbf{j}$ **and** $\mathbf{w} = \mathbf{k}$.

To compute $\mathbf{v}\cdot\mathbf{w}$ **for the set of vectors, apply the formula that** $\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ **where the vectors are** $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ **and** $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$.

The value of $\mathbf{v}\cdot\mathbf{w}$ **is,**

$$\begin{aligned}\mathbf{v}\cdot\mathbf{w} &= (-\mathbf{i} + \mathbf{j} + 0\mathbf{k})\cdot(0\mathbf{i} + 0\mathbf{j} + \mathbf{k}) \\ &= (-1\cdot 0 + 1\cdot 0 + 0\cdot 1) \\ &= 0\end{aligned}$$

Hence, the value of $\mathbf{v}\cdot\mathbf{w}$ **is** $\boxed{0}$.

Problem 8RE

step 1 of 1

To compute $\vec{v}\times\vec{w}$ for the given set of vectors

(A) Given that $\vec{v} = -\hat{i} + \hat{j}$ $\quad\vec{w} = \hat{k}$

$$\vec{v}\times\vec{w} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ 0 & 0 & 1\end{vmatrix}$$

$$= \hat{i}(1-0) - \hat{j}(-1-0) + \hat{k}(0-0)$$
$$= \hat{i} + \hat{j}$$
$$\Rightarrow \vec{v}\times\vec{w} = \hat{i} + \hat{j}$$

Problem 9RE

step 1 of 1

To find the cosine of the angle between the given set of vectors

(A) Given vectors are $\vec{v} = -\hat{i} + \hat{j}$ $\quad\vec{w} = \hat{k}$

$$\|\vec{v}\| = \sqrt{(-1)^2 + 1^2} = \sqrt{2} \quad \|\vec{w}\| = \sqrt{1^2} = 1$$
$$\vec{v}\cdot\vec{w} = -1(0) + 0(1) + 0(1) = 0$$

we have that $\vec{v}\cdot\vec{w} = \|\vec{v}\|\|\vec{w}\|\cos\theta$ $(\theta = \text{angle between } \vec{v}, \vec{w})$

$$\Rightarrow 0 = \sqrt{2}\times 1\times\cos\theta$$
$$\Rightarrow \boxed{\cos\theta = 0}$$