Chapter 3

3-1 What can best explain each of the following statements?

(a) Concrete cylinders under compression may fail only around the midheight

When compression test on a concrete cylinder is conducted, both ends of the concrete cylinder may be confined by the testing machine in some test programs. Therefore, the strength on both ends is higher than that on the midheight.

(b) The age of the concrete influences compressive strength.

Concrete strength increases with age, as illustrated in Figs. 3-5 to 3-7. ACI Committee 209 has proposed the following equation to represent the rate of strength gain for concrete made from Type I cement and moist-cured at 70^0 F: $\boldsymbol{f'_{c(t)} = f'_{c(28)}\left(\frac{t}{4+0.85t}\right)}$, in which $\boldsymbol{f'_{c(t)}}$ is the compressive strength at age *t*. For type III cement, the coefficients 4 and 0.85 become 2.3 and 0.92, respectively. Concrete cured under temperatures other than 70^0 F may set faster or slower than indicated by these equations, as shown in Fig. 3-6.

3-2 A group of 85 tests on a given type of concrete had a mean strength of 5655 psi and a standard deviation of 472 psi. Does this concrete satisfy the strength requirement for 5000-psi concrete?

From Eq. 3-3a:
$f'_c = f_{cr} - 1.34s$
Using $f_{cr} = 5655\, psi$
(for design) $f'_c = 5655 - 1.34(472) = 5022.52\, psi$

From Eq. 3-3b:
$f'_c = f_{cr} - 2.33s + 500$
Using $f_{cr} = 5655\, psi$
(for design) $f'_c = 5655 - 2.33(472) + 500 = 5055.24\, psi$

The concrete satisfies the strength requirement for 5000 psi concrete, since both of specified compressive strengths $\boldsymbol{f'_c}$ calculated exceed 5000 psi.

3-3 The concrete containing Type III cement in a structure is cured for 7 days at 60^0 F, followed by 14 days at 35^0 F. Use the maturity concept to estimate its strength as a function of the 28-day strength under standard curing.

$°\mathrm{C} = \frac{5}{9}(°\mathrm{F} - 32)$, so $60°\mathrm{F} = 15.6°\mathrm{C}$ and $35°\mathrm{F} = 1.7°\mathrm{C}$

From Eq. (3-6):

$M = \sum_{i=1}^{n}(T_i + 10)(t_i) = (15.6 + 10)(7) + (1.7 + 10)(14) = 343\ C$ days

From Fig. 3-8, the compressive strength will be around 0.87 times the 28-day strength under standard curing conditions.

3-4 Use Fig. 3.12a to estimate the compressive strength σ_2 for biaxially loaded concrete subjected to

(a) $\sigma_1 = 0.2\, f_t$ in tension, $\sigma_2 = 0.91f'_c$

(b) $\sigma_1 = 0.6\, f'_c$ in compression, $\sigma_2 = 1.6f'_c$

(c) $\sigma_1 = 0.4\, f'_t$ in tension, $\sigma_2 = 0.80f'_c$

3-5 The concrete in the core of a spiral column is subjected to a uniform confining stress σ_3 of 500 psi. What will the unconfined compressive strength f'_c be? The compressive strength σ_1 is 6000 psi.

From Eq. 3-16:

$\sigma_1 = f'_c + 4.1\sigma_3$

$6000 = f'_c + 4.1(500)$

$f'_c = 6000 - 4.1(500) = 3950$ psi

3-6 What are plastic and carbonation shrinkage of concrete?

Plastic shrinkage:

Bleed water arriving at an unprotected concrete surface will be subject to evaporation if the rate of evaporation is greater than the rate of arrival of water at the surface, then there will be a net reduction in water content of the surface concrete, and plastic shrinkage, i.e. drying shrinkage whilst the concrete is still plastic, will occur. The restraint of the mass of concrete will cause tensile strains to be set up in the near-surface region, and as the concrete has near zero tensile strength, plastic shrinkage cracking may result.

Carbonation shrinkage:

Carbonation shrinkage is a phenomenon very recently recognized. CO_2 present in the atmosphere reacts in the presence of water with hydrated cement. $Ca(OH)_2$ gets converted to calcium carbonate and also some other cement compounds are decomposed. Such a complete decomposition of calcium compound in hydrated cement is chemically possible even at the low pressure of CO_2 in normal atmosphere.

$$CO_2 + C_a(OH)_2 \rightarrow C_aCO_3 + H_2O$$

3-7 What factors can reduce concrete drying shrinkage?

Some of the measures that can be taken to reduce the drying shrinkage of concrete include:

- Use the minimum water content (consistent with placing and finishing requirements)
- Use highest possible volume fraction of good quality aggregate and maximum possible aggregate size
- Use Shrinkage Limited Cement (Type SL) where available
- Avoid using admixture known to increase drying shrinkage, e.g. those containing calcium chloride
- Ensure concrete is properly placed, compacted and cured

3-8 **A structure is made from concrete containing Type I cement. The average ambient relative humidity is 85 percent. The concrete was moist-cured for 7 days. $f'_c = 5000$ psi.**

(a) **Compute the unrestrained shrinkage strain of a rectangular beam with cross-sectional dimensions 12 in x 24 in. at 3 years after the concrete was placed.**

Compute the unrestrained shrinkage strain of a rectangular beam with cross-sectional dimensions 12 in x 24 in. at 3 years after the concrete was placed.

1. Compute the humidity modification factor from Eq. (3-30a):

   $\gamma_{rh} = 3.00 - 0.03 \times RH = 3.00 - 0.03 \times 85 = 0.45$

2. Use Eq. (3.31) to compute the volume/surface area ratio modification factor:

   Volume per foot of beam = $12 \times 12 \times 24 = 3456 \text{ in}^3$

   Surface area per foot on beam = $2[(12 \times 12) + (24 \times 12)] = 864 \text{ in}^3$

   $\gamma_{VS} = 1.2^{-0.12V/S} = 1.2^{-0.48} = 0.92$

3. Use Eq. (3-29) to compute the ultimate shrinkage strain:

   $(\varepsilon_{sh})_u = \gamma_{rh} \times \gamma_{VS} \times 780 = 0.45 \times 0.92 \times 780 \times 10^{-6} = 323 \times 10^{-6}$ strain

4. Use Eq. (3-28) to compute the shrinkage strain after 3 years:

   $t = 3 \times 365 - 7 = 1088$ days

   $$(\varepsilon_{sh})_t = \frac{t}{35 + t}(\varepsilon_{sh})_u = \frac{1088}{35 + 1088} \times 323 \times 10^{-6} = 313 \times 10^{-6}$$

(b) **Compute the axial shortening of a 25 in. x 25 in. x 11 ft plain concrete column at age of 2 years. A compression load of 450 kips was applied to the column at age 21 days.**

1. Compute the ultimate shrinkage strain coefficient C_u using Eq. (3-36)-(3-39).

   $\lambda_{rh} = 1.27 - 0.0067 \times RH = 1.27 - 0.0067 \times 85 = 0.7$

   $\lambda_{to} = 1.25 \times t_o^{-0.118} = 1.25 \times 21^{-0.118} = 0.87$

   $\lambda_{VS} = 0.67 \times \left[1 + 1.13^{-0.54V/S}\right]$,

   in which: $V = 25 \text{ in.} \times 25 \text{ in.} \times 11 \text{ in.} \times 12 \frac{\text{in.}}{\text{ft}} = 82500 \text{ in}^3$

   $S = 4 \text{ sides} \times 25 \text{ in.} \times 11 \text{ in.} \times 12 \frac{\text{in.}}{\text{ft}} = 13200 \text{ in}^2$

   $\lambda_{VS} = 0.67 \times \left[1 + 1.13^{-0.54 \times 82500/13200}\right] = 1.11$

   $C_u = 2.35 \times \lambda_{rh} \times \lambda_{to} \times \lambda_{VS} = 2.35 \times 0.7 \times 0.87 \times 1.11 = 1.59$

2. Compute the creep coefficient for the time since loading C_t using Eq. (3-35)

   $t = 2 \times 365 - 21 = 709$ days

   $C_t = \frac{t^{0.6}}{10+t^{0.6}} C_u = \frac{709^{0.6}}{10+709^{0.6}} \times 1.59 = 1.33$

3. Compute the total stress-dependent ratio strain $\varepsilon_c(\text{total})$

   First, calculate the creep strain since the load was applied:

   $f_{cm} = 1.2 \times f'_c = 1.2 \times 5000 = 6000$ psi

   $E_c(28) = 57000\sqrt{f_{cm}} = 57000 \times \sqrt{6000} = 4.42 \times 10^6$ psi

   $$\varepsilon_{cc}(t, t_o) = \frac{\sigma_c(t_o)}{E_c(28)} C_t = \frac{450000 / 25 \text{ in.} \times 25 \text{ in.}}{4.42 \times 10^6 \text{psi}} \times 1.33 = 0.22 \times 10^{-3} \text{ strain}$$

   Then, calculate the initial strain when the load is applied:

   $$f'_c(t_o) = f'_c(28) \times \frac{t_o}{4 + 0.85 t_o} = 5000 \times \frac{21}{4 + 0.85 \times 21} = 4805.5 \text{ psi}$$

   $f_{cm}(t_o) = 1.2 f'_c(t_o) = 1.2 \times 4805.5 = 5766.6$ psi

   $E_c(t_o) = 57000\sqrt{f_{cm}(t_o)} = 57000 \times \sqrt{5766.6} = 4.33 \times 10^6$ psi

   $$\varepsilon_c(t_o) = \frac{\sigma_c(t_o)}{E_c(28)} = \frac{450000 / 25 \text{ in.} \times 25 \text{ in.}}{4.33 \times 10^6 \text{psi}} = 0.17 \times 10^{-3} \text{ strain}$$

   Thus, $\varepsilon_c(\text{total}) = \varepsilon_c(t_o) + \varepsilon_{cc}(t, t_o) = 0.17 \times 10^{-3} + 0.22 \times 10^{-3} = 0.39 \times 10^{-3}$

4. Compute the axial shortening

   The column is 11 ft long, so the total expected shortening due to stress-dependent strain is

   $$\Delta l = l \times \varepsilon_c(\text{total}) = 11\, ft \times 12 \frac{\text{in.}}{\text{ft}} \times 0.39 \times 10^{-3} = 0.051 \text{ in.}$$