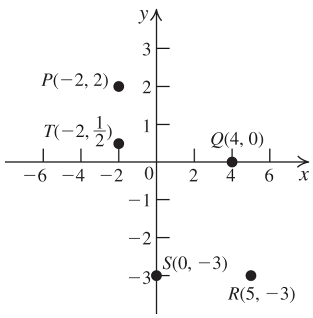
Chapter 1 Graphs and Functions

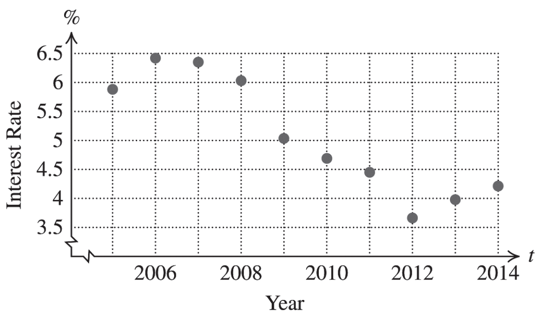
1.1 Graphs of Equations

1.1 Practice Problems

1.



2.



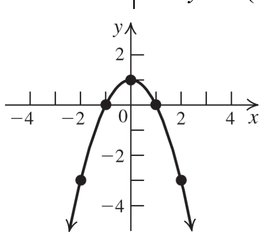
3. $(x_1, y_1) = (-5, 2);\ (x_2, y_2) = (-4, 1)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-4 - (-5))^2 + (1 - 2)^2}$$
$$= \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

4. $M = \left(\dfrac{5+6}{2}, \dfrac{-2+(-1)}{2}\right) = \left(\dfrac{11}{2}, -\dfrac{3}{2}\right)$

5. $y = -x^2 + 1$

x	$y = -x^2 + 1$	(x, y)
-2	$y = -(-2)^2 + 1$	$(-2, -3)$
-1	$y = -(-1)^2 + 1$	$(-1, 0)$
0	$y = -(0)^2 + 1$	$(0, 1)$
1	$y = -(1)^2 + 1$	$(1, 0)$
2	$y = -(2)^2 + 1$	$(2, -3)$



6. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = 2x^2 + 3x - 2 \Rightarrow$ $0 = (2x-1)(x+2) \Rightarrow x = \dfrac{1}{2}$ or $x = -2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y = 2(0)^2 + 3(0) - 2 \Rightarrow y = -2.$ The x-intercepts are $\dfrac{1}{2}$ and -2; the y-intercept is -2.

7. To test for symmetry with respect to the y-axis, replace x with $-x$ to determine if $(-x, y)$ satisfies the equation. $(-x)^2 - y^2 = 1 \Rightarrow x^2 - y^2 = 1$, which is the same as the original equation. So the graph is symmetric with respect to the y-axis.

8. $y = x^4 - 4x^2$

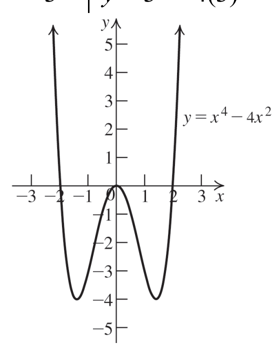
Test for all three symmetries:

x-axis: $-y = x^4 - 4x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.

y-axis: $y = (-x)^4 - 4(-x)^2 \Rightarrow y = x^4 - 4x^2$, which is the same as the original equation. So the graph is symmetric with respect to the y-axis.

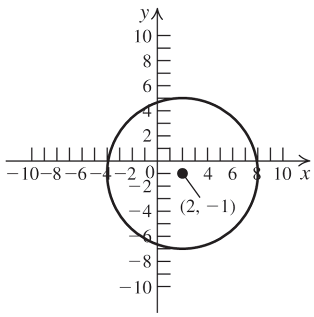
origin: $-y = (-x)^4 - 4(-x)^2 \Rightarrow -y = x^4 - 4x^2$, which is not the same as the original equation. So the graph is not symmetric with respect to the origin. Now, make a table of values. Since the graph is symmetric with respect to the y-axis, if (x, y) is on the graph, then so is $(-x, y)$.

x	$y = x^4 - 4x^2$	(x, y)	$(-x, y)$
0	$y = 0^4 - 4(0)^2$	$(0, 0)$	$(0, 0)$
1	$y = 1^4 - 4(1)^2$	$(1, -3)$	$(-1, -3)$
2	$y = 2^4 - 4(2)^2$	$(2, 0)$	$(-2, 0)$
3	$y = 3^4 - 4(3)^2$	$(3, 45)$	$(-3, 45)$



9. The standard form of the equation of a circle is
$(x-h)^2+(y-k)^2=r^2$
$(h, k) = (3, -6)$ and $r = 10$
The equation of the circle is
$(x-3)^2+(y+6)^2=100$.

10. $(x-2)^2+(y+1)^2=36 \Rightarrow (h,k)=(2,-1),\ r=6$
This is the equation of a circle with center $(2, -1)$ and radius 6.



11. $x^2+y^2+4x-6y-12=0 \Rightarrow$
$x^2+4x+y^2-6y=12$
Now complete the square:
$x^2+4x+4+y^2-6y+9=12+4+9 \Rightarrow$
$(x+2)^2+(y-3)^2=25$
This is a circle with center $(-2, 3)$ and radius 5.

1.1 Exercises
Concepts and Vocabulary

1. The graph of an equation in two variables such as x and y is the set of all ordered pairs (a, b) that satisfy the equation.

2. The distance between the points $P=(x_1, y_1)$ and $Q=(x_2, y_2)$ is given by the formula $d(P, Q)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

3. The coordinates of the midpoint $M=(x, y)$ of the line segment joining $P=(x_1, y_1)$ and $Q=(x_2, y_2)$ are given by $M=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

4. The standard form of the equation of a circle with center (h, k) and radius r is $(x-h)^2+(y-k)^2=r^2$.

5. False. A point with negative first coordinate and a positive second coordinate lies in quadrant II.

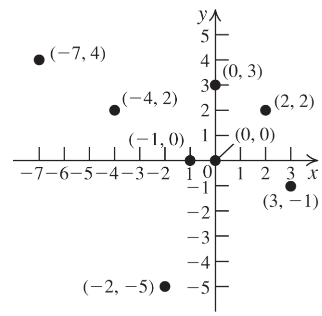
6. True

7. False. The center of the circle given by the equation $(x+3)^2+(y+4)^2=9$ is $(-3, -4)$.

8. True

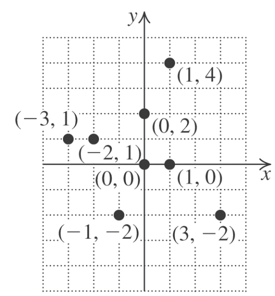
Building Skills

9.



(2, 2): Q I
(3, −1): Q IV
(−1, 0): x-axis
(−2, −5): Q III
(0, 0): origin
(−7, 4): Q II
(0, 3): y-axis
(−4, 2): Q II

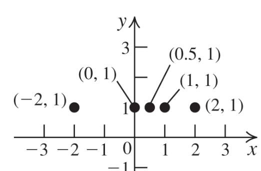
10.



(1,4): Q I
(3, −2): Q IV
(1, 0): x-axis
(−1, −2): Q III
(0, 0): origin
(−2, 1): Q II
(0, 2): y-axis
(−3, 1): Q II

11. a. Answers will vary. Sample answer: (–2, 0), (–1, 0), (0, 0), (1, 0), (2, 0). The y-coordinate is 0.

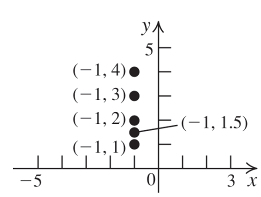
b.



The set of all points of the form $(x, 1)$ is a horizontal line that intersects the y-axis at 1.

12. a. Answers will vary. Sample answer: (0, –2), (0, –1), (0, 0), (0, 1), (0, 2). The x-coordinate is 0.

b.



The set of all points of the form $(-1, y)$ is a vertical line that intersects the x-axis at –1.

In Exercises 13–20, use the distance formula, $d=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$ and the midpoint formula, $(x, y)=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

13. $P(2, 1), Q(3, 5)$

a. $d=\sqrt{(3-2)^2+(5-1)^2}=\sqrt{17}$

b. $M=\left(\frac{2+3}{2}, \frac{1+5}{2}\right)=(2.5, 3)$

14. $P(1, 4), Q(3, 2)$

a. $d=\sqrt{(3-1)^2+(2-4)^2}=\sqrt{8}$

b. $M=\left(\frac{1+3}{2}, \frac{4+2}{2}\right)=(2, 3)$

15. $P(4, 5), Q(1, -2)$

a. $d=\sqrt{(1-4)^2+(-2-5)^2}=\sqrt{(-3)^2+(-7)^2}$
$=\sqrt{9+49}=\sqrt{58}$

b. $M=\left(\frac{4+1}{2}, \frac{5+(-2)}{2}\right)=(2.5, 1.5)$

16. $P(2, 3), Q(-1, 2)$

a. $d=\sqrt{(-1-2)^2+(2-3)^2}=\sqrt{(-3)^2+(-1)^2}$
$=\sqrt{10}$

b. $M=\left(\frac{2+(-1)}{2}, \frac{3+2}{2}\right)=(0.5, 2.5)$

17. $P(-1, -5), Q(2, -3)$

a. $d=\sqrt{(2-(-1))^2+(-3-(-5))^2}$
$=\sqrt{3^2+2^2}=\sqrt{13}$

b. $M=\left(\frac{-1+2}{2}, \frac{-5+(-3)}{2}\right)=(0.5, -4)$

18. $P(-4, 1), Q(-7, -9)$

a. $d=\sqrt{(-7-(-4))^2+(-9-1)^2}$
$=\sqrt{(-3)^2+(-10)^2}=\sqrt{109}$

b. $M=\left(\frac{-4+(-7)}{2}, \frac{1+(-9)}{2}\right)=(-5.5, -4)$

19. $P(x, y), Q(-2, 3)$

a. $d=\sqrt{[(x-(-2))]^2+(y-3)^2}$
$=\sqrt{(x+2)^2+(y-3)^2}$

b. $M=\left(\frac{x+(-2)}{2}, \frac{y+3}{2}\right)=\left(\frac{x-2}{2}, \frac{y+3}{2}\right)$

20. $P(t, k), Q(k, t)$

a. $d=\sqrt{(k-t)^2+(t-k)^2}$
$=\sqrt{\left(k^2-2kt+t^2\right)+\left(t^2-2tk+k^2\right)}$
$=\sqrt{2t^2-4tk+2k^2}=\sqrt{2\left(t^2-2tk+k^2\right)}$
$=\sqrt{2(t-k)^2}=|t-k|\sqrt{2}$

b. $M=\left(\frac{t+k}{2}, \frac{t+k}{2}\right)$

21. $P=(-1,-2), Q=(0,0), R=(1,2)$

$d(P,Q)=\sqrt{(0-(-1))^2+(0-(-2))^2}=\sqrt{5}$
$d(Q,R)=\sqrt{(1-0)^2+(2-0)^2}=\sqrt{5}$
$d(P,R)=\sqrt{(1-(-1))^2+(2-(-2))^2}$
$=\sqrt{2^2+4^2}=\sqrt{20}=2\sqrt{5}$

Because $d(P, Q)+d(Q, R)=d(P, R)$, the points are collinear.

22. $P=(-3,-4), Q=(0,0), R=(3,4)$

$d(P,Q)=\sqrt{(0-(-3))^2+(0-(-4))^2}=\sqrt{25}=5$
$d(Q,R)=\sqrt{(3-0)^2+(4-0)^2}=\sqrt{25}=5$
$d(P,R)=\sqrt{(3-(-3))^2+(4-(-4))^2}$
$=\sqrt{6^2+8^2}=\sqrt{100}=10$

Because $d(P, Q)+d(Q, R)=d(P, R)$, the points are collinear.

23. $P = (9, 6)$, $Q = (0, -3)$, $R = (3, 1)$
It is not possible to arrange the points in such a way so that $d(P, Q) + d(Q, R) = d(P, R)$, so the points are not collinear.

$$d(P,Q) = \sqrt{(9-0)^2 + (6-(-3))^2} = \sqrt{162} = 9\sqrt{2}$$
$$d(Q,R) = \sqrt{(3-0)^2 + (1-(-3))^2} = \sqrt{25} = 5$$
$$d(P,R) = \sqrt{(9-3)^2 + (6-1)^2} = \sqrt{61}$$

The perimeter of triangle PQR is $9\sqrt{2} + 5 + \sqrt{61}.$

24. $P = (-2, 3)$, $Q = (3, 1)$, $R = (2, -1)$
It is not possible to arrange the points in such a way so that $d(P, Q) + d(Q, R) = d(P, R)$, so the points are not collinear.

$$d(P,Q) = \sqrt{(-3-2)^2 + (3-1)^2} = \sqrt{29}$$
$$d(Q,R) = \sqrt{(3-2)^2 + (1-(-1))^2} = \sqrt{5}$$
$$d(P,R) = \sqrt{(-2-2)^2 + (3-(-1))^2} = \sqrt{32} = 4\sqrt{2}$$

The perimeter of triangle PQR is $\sqrt{29} + \sqrt{5} + 4\sqrt{2}.$

25. $$d(P,Q) = \sqrt{(-1-(-5))^2 + (4-5)^2} = \sqrt{17}$$
$$d(Q,R) = \sqrt{(-4-(-1))^2 + (1-4)^2} = 3\sqrt{2}$$
$$d(P,R) = \sqrt{(-4-(-5))^2 + (1-5)^2} = \sqrt{17}$$
The triangle is isosceles.

26. $$d(P,Q) = \sqrt{(6-3)^2 + (6-2)^2} = 5$$
$$d(Q,R) = \sqrt{(-1-6)^2 + (5-6)^2} = 5\sqrt{2}$$
$$d(P,R) = \sqrt{(-1-3)^2 + (5-2)^2} = 5$$
The triangle is isosceles.

27. $$d(P,Q) = \sqrt{(0-(-4))^2 + (7-8)^2} = \sqrt{17}$$
$$d(Q,R) = \sqrt{(-3-0)^2 + (5-7)^2} = \sqrt{13}$$
$$d(P,R) = \sqrt{(-3-(-4))^2 + (5-8)^2} = \sqrt{10}$$
The triangle is scalene.

28. $$d(P,Q) = \sqrt{(-1-6)^2 + (-1-6)^2} = 7\sqrt{2}$$
$$d(Q,R) = \sqrt{(-5-(-1))^2 + (3-(-1))^2} = 4\sqrt{2}$$
$$d(P,R) = \sqrt{(-5-6)^2 + (3-6)^2} = \sqrt{130}$$
The triangle is scalene.

29. $$d(P,Q) = \sqrt{(-1-1)^2 + (1-(-1))^2} = 2\sqrt{2}$$
$$d(Q,R) = \sqrt{\left(-\sqrt{3}-(-1)\right)^2 + \left(-\sqrt{3}-1\right)^2} = \sqrt{\left(3-2\sqrt{3}+1\right) + \left(3+2\sqrt{3}+1\right)} = \sqrt{8} = 2\sqrt{2}$$
$$d(P,R) = \sqrt{\left(-\sqrt{3}-1\right)^2 + \left(-\sqrt{3}-(-1)\right)^2} = \sqrt{\left(3+2\sqrt{3}+1\right) + \left(3-2\sqrt{3}+1\right)} = \sqrt{8} = 2\sqrt{2}$$
The triangle is equilateral.

30. $d(P,Q)=\sqrt{(-1.5-(-0.5))^2+(1-(-1))^2}=\sqrt{5}$

$$d(Q,R)=\sqrt{\left(\left(\sqrt{3}-1\right)-(-1.5)\right)^2+\left(\frac{\sqrt{3}}{2}-1\right)^2}=\sqrt{\left(\left(\sqrt{3}-1\right)^2+3\left(\sqrt{3}-1\right)+2.25\right)+\left(\frac{3}{4}-\sqrt{3}+1\right)}$$
$$=\sqrt{\left(3-2\sqrt{3}+1+3\sqrt{3}-3+2.25\right)+\left(1.75-\sqrt{3}\right)}=\sqrt{5}$$
$$d(P,R)=\sqrt{\left(\left(\sqrt{3}-1\right)-(-0.5)\right)^2+\left(\frac{\sqrt{3}}{2}-(-1)\right)^2}=\sqrt{\left(\left(\sqrt{3}-1\right)^2+\left(\sqrt{3}-1\right)+0.25\right)+\left(\frac{3}{4}+\sqrt{3}+1\right)}$$
$$=\sqrt{\left(3-2\sqrt{3}+1+\sqrt{3}-1+0.25\right)+\left(1.75+\sqrt{3}\right)}=\sqrt{5}$$

The triangle is equilateral.

In exercises 31–36, to determine if a point lies on the graph of the equation, substitute the point's coordinates into the equation to see if the resulting statement is true.

31. $y=x-1$

$-4=-3-1\Rightarrow -4=-4$ ✓

$0=1-1\Rightarrow 0=0$ ✓

$3=4-1\Rightarrow 3=3$ ✓

$3=2-1\Rightarrow 3\neq 1$

On the graph: (–3, –4), (1, 0), (4, 3)
not on the graph: (2, 3)

32. $2y=3x+5$

$2(1)=3(-1)+5\Rightarrow 2=2$ ✓

$2(2)=3(0)+5\Rightarrow 4\neq 5$

$2(0)=3\left(-\frac{5}{3}\right)+5\Rightarrow 0=0$ ✓

$2(4)=3(1)+5\Rightarrow 8=8$ ✓

on the graph: (–1, 1), (1, 4), $\left(-\frac{5}{3},0\right)$

not on the graph: (0, 2)

33. $y=\sqrt{x+1}$

$2=\sqrt{3+1}\Rightarrow 2=2$ ✓

$1=\sqrt{0+1}\Rightarrow 1=1$ ✓

$-3=\sqrt{8+1}\Rightarrow -3\neq 3$

$3=\sqrt{8+1}\Rightarrow 3=3$ ✓

on the graph: (3, 2), (0, 1), (8, 3)
not on the graph: (8, –3)

34. $y=\frac{1}{x}$

$\frac{1}{3}=\frac{1}{3}$ ✓

$1=\frac{1}{1}\Rightarrow 1=1$ ✓

$0\neq\frac{1}{0}$

$\frac{1}{2}=\frac{1}{2}$ ✓

on the graph: (1, 1), $\left(2,\frac{1}{2}\right)$

not on the graph: (0, 0), $\left(-3,\frac{1}{3}\right)$

35. $x^2-y^2=1$

$1^2-0^2=1\Rightarrow 1=1$ ✓

$0^2-(-1)^2=1\Rightarrow 0\neq 1$

$2^2-\sqrt{3}^2=1\Rightarrow 4-3=1\Rightarrow 1=1$ ✓

$2^2-\left(-\sqrt{3}\right)^2=1\Rightarrow 4-3=1\Rightarrow 1=1$ ✓

on the graph: (1, 0), $\left(2,\sqrt{3}\right),\left(2,-\sqrt{3}\right)$

not on the graph: (0, –1)

36. $y^2=x$

$(-1)^2=1\Rightarrow 1=1$ ✓

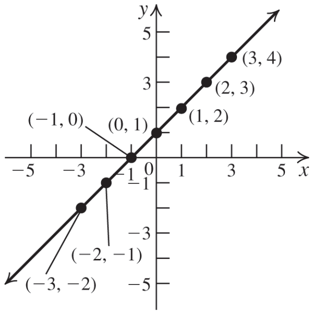
$1^2=1\Rightarrow 1=1$ ✓

$0^2=0\Rightarrow 0=0$ ✓

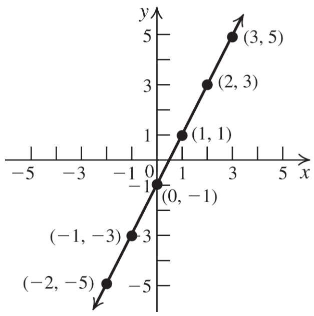
$\left(-\sqrt{2}\right)^2=2\Rightarrow 2=2$ ✓

Each point is on the graph.

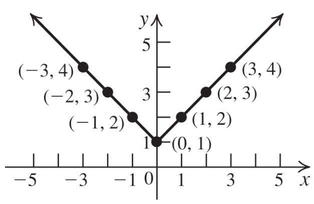
37. $y = x + 1$



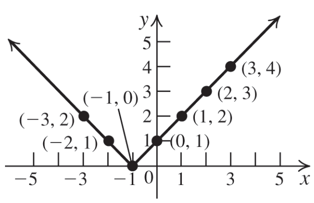
38. $y = 2x - 1$



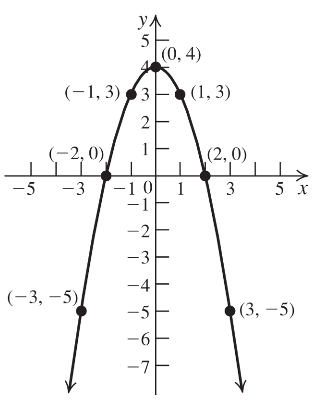
39. $y = |x| + 1$



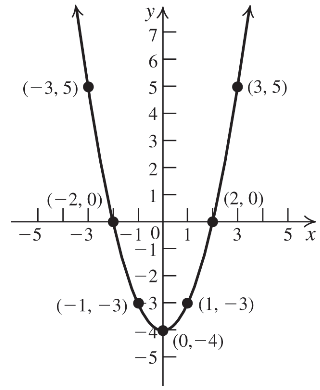
40. $y = |x + 1|$



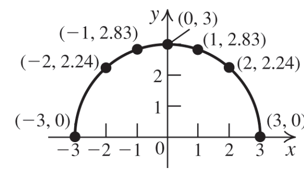
41. $y = 4 - x^2$



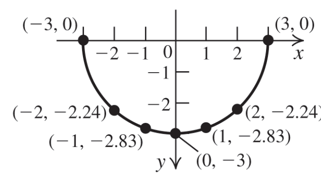
42. $y = x^2 - 4$



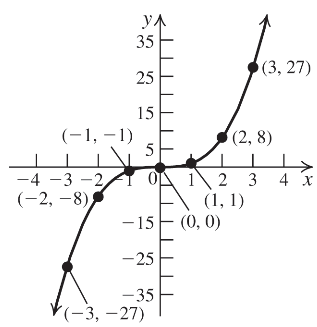
43. $y = \sqrt{9 - x^2}$



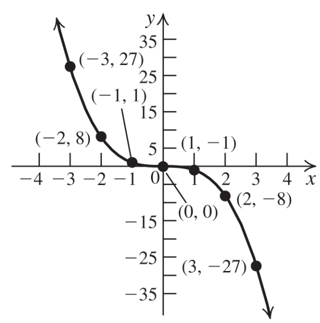
44. $y = -\sqrt{9 - x^2}$



45. $y = x^3$



46. $y = -x^3$



For exercises 47–56, read the answers directly from the given graphs.

47. x-intercepts: –1, 1
y-intercepts: none
symmetries: y-axis

48. x-intercepts: none
y-intercepts: –1, 1
symmetries: x-axis

49. x-intercepts: $-\pi$, 0, π
y-intercepts: 0
symmetries: origin

50. x-intercepts: $-\frac{\pi}{2}, \frac{\pi}{2}$
y-intercepts: 2
symmetries: y-axis

51. x-intercepts: –3, 3
y-intercepts: –2, 2
symmetries: x-axis, y-axis, origin

52. x-intercepts: –2, 2
y-intercepts: –3, 3
symmetries: x-axis, y-axis, origin

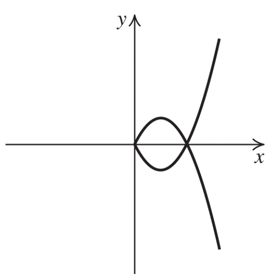
53. x-intercepts: –2, 0, 2
y-intercepts: 0
symmetries: origin

54. x-intercepts: –2, 0, 2
y-intercepts: 0
symmetries: origin

55. x-intercepts: –2, 0, 2
y-intercepts: 0, 3
symmetries: y-axis

56. x-intercepts: 0, 3
y-intercepts: –2, 0, 2
symmetries: x-axis

57.



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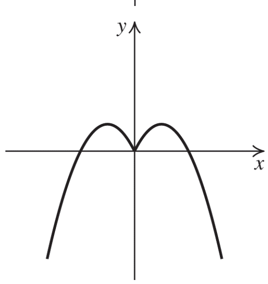

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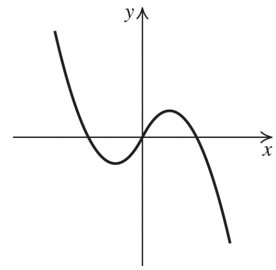
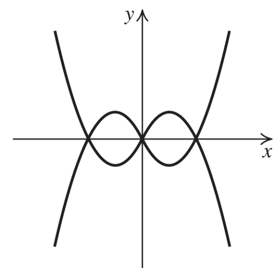


60.



61. To find the x-intercept, let $y = 0$, and solve the equation for x: $3x + 4(0) = 12 \Rightarrow x = 4$. To find the y-intercept, let $x = 0$, and solve the equation for y: $3(0) + 4y = 12 \Rightarrow y = 3$. The x-intercept is 4; the y-intercept is 3.

62. To find the x-intercept, let $y = 0$, and solve the equation for x: $\frac{x}{5} + \frac{0}{3} = 1 \Rightarrow x = 5$. To find the y-intercept, let $x = 0$, and solve the equation for y: $\frac{0}{5} + \frac{y}{3} = 1 \Rightarrow y = 3$. The x-intercept is 5; the y-intercept is 3.

63. To find the x-intercept, let $y = 0$, and solve the equation for x: $2x + 3(0) = 5 \Rightarrow x = \frac{5}{2}$. To find the y-intercept, let $x = 0$, and solve the equation for y: $2(0) + 3y = 5 \Rightarrow y = \frac{5}{3}$. The x-intercept is $5/2$; the y-intercept is $5/3$.

64. To find the x-intercept, let $y = 0$, and solve the equation for x: $\frac{x}{2} - \frac{0}{3} = 1 \Rightarrow x = 2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $\frac{0}{2} - \frac{y}{3} = 1 \Rightarrow y = -3$. The x-intercept is 2; the y-intercept is -3.

65. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = \frac{x+2}{x-1} \Rightarrow x = -2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y = \frac{0+2}{0-1} = -2$. The x-intercept is –2; the y-intercept is –2.

66. To find the x-intercept, let $y = 0$, and solve the equation for x: $x = \frac{0-2}{0+1} \Rightarrow x = -2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $0 = \frac{y-2}{y+1} = 2$. The x-intercept is –2; the y-intercept is 2.

67. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = x^2 - 6x + 8 \Rightarrow x = 4$ or $x = 2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y = 0^2 - 6(0) + 8 \Rightarrow y = 8$. The x-intercepts are 2 and 4; the y-intercept is 8.

68. To find the x-intercept, let $y = 0$, and solve the equation for x: $x = 0^2 - 5(0) + 6 \Rightarrow x = 6$. To find the y-intercept, let $x = 0$, and solve the equation for y: $0 = y^2 - 5y + 6 \Rightarrow y = 2$ or $y = 3$. The x-intercept is 6; the y-intercepts are 2 and 3.

69. To find the x-intercept, let $y = 0$, and solve the equation for x: $x^2 + 0^2 = 4 \Rightarrow x = \pm 2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $0^2 + y^2 = 4 \Rightarrow y = \pm 2$. The x-intercepts are –2 and 2; the y-intercepts are –2 and 2.

70. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = \sqrt{9 - x^2} \Rightarrow x = \pm 3$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y = \sqrt{9 - 0^2} \Rightarrow y = 3$. The x-intercepts are –3 and 3; the y-intercept is 3.

71. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = \sqrt{x^2 - 1} \Rightarrow x = \pm 1$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y = \sqrt{0^2 - 1} \Rightarrow$ no solution. The x-intercepts are –1 and 1; there is no y-intercept.

72. To find the x-intercept, let $y = 0$, and solve the equation for x: $x(0) = 1 \Rightarrow$ no solution. To find the y-intercept, let $x = 0$, and solve the equation for y: $(0)y = 1 \Rightarrow$ no solution. There is no x-intercept; there is no y-intercept.

In exercises 73–80, to test for symmetry with respect to the x-axis, replace y with $-y$ to determine if $(x, -y)$ satisfies the equation. To test for symmetry with respect to the y-axis, replace x with $-x$ to determine if $(-x, y)$ satisfies the equation. To test for symmetry with respect to the origin, replace x with $-x$ and y with $-y$ to determine if $(-x, -y)$ satisfies the equation.

73. $-y = x^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis. $y = (-x)^2 + 1 \Rightarrow y = x^2 + 1$, so the equation is symmetric with respect to the y-axis. $-y = (-x)^2 + 1 \Rightarrow -y = x^2 + 1$, is not the same as the original equation, so the equation is not symmetric with respect to the origin.

74. $x = (-y)^2 + 1 \Rightarrow x = y^2 + 1$, so the equation is symmetric with respect to the x-axis. $-x = y^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis. $-x = (-y)^2 + 1 \Rightarrow -x = y^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

75. $-y = x^3 + x$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis. $y = (-x)^3 - x \Rightarrow y = -x^3 - x \Rightarrow y = -(x^3 + x)$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis. $-y = (-x)^3 - x \Rightarrow -y = -x^3 - x \Rightarrow -y = -(x^3 + x) \Rightarrow y = x^3 + x$, so the equation is symmetric with respect to the origin.

76. $-y = 2x^3 - x$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis. $y = 2(-x)^3 - (-x) \Rightarrow$ $y = -2x^3 + x \Rightarrow y = -2(x^3 - x)$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis.
$-y = 2(-x)^3 - (-x) \Rightarrow -y = -2x^3 + x \Rightarrow$ $-y = -2(x^3 - x) \Rightarrow y = 2x^3 - x$, so the equation is symmetric with respect to the origin.

77. $-y = 5x^4 + 2x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis. $y = 5(-x)^4 + 2(-x)^2 \Rightarrow$ $y = 5x^4 + 2x^2$, so the equation is symmetric with respect to the y-axis.
$-y = 5(-x)^4 + 2(-x) \Rightarrow -y = 5x^4 + 2x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

78. $-y = -3x^6 + 2x^4 + x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = -3(-x)^6 + 2(-x)^4 + (-x)^2 \Rightarrow$
$y = -3x^6 + 2x^4 + x^2$, so the equation is symmetric with respect to the y-axis.
$-y = -3(-x)^6 + 2(-x)^4 + (-x)^2 \Rightarrow$
$-y = -3x^6 + 2x^4 + x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

79. $x^2(-y)^2 + 2x(-y) = 1 \Rightarrow x^2y^2 - 2xy = 1$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$(-x)^2 y^2 + 2(-x)y = 1 \Rightarrow x^2y^2 - 2xy = 1$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis.
$(-x)^2(-y)^2 + 2(-x)(-y) = 1 \Rightarrow$
$x^2y^2 + 2xy = 1.$ so the equation is symmetric with respect to the origin.

80. $x^4 - xy + y^4 = 1$
$x^4 - x(-y) + (-y)^4 = 1 \Rightarrow x^4 + xy + y^4 = 1$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$(-x)^4 - (-x)y + y^4 = 1 \Rightarrow x^4 + xy + y^4 = 1$ is not the same as the original equation, so the equation is symmetric with respect to the y-axis.
$(-x)\ (-x)^4 - (-x)(-y) + (-y)^4 = 1 \Rightarrow$
$x^4 - xy + y^4 = 1$ is the same as the original equation, so the equation is symmetric with respect to the origin.

In exercises 81–84, the equations are given in the standard form of the equation of a circle, $(x-h)^2 + (y-k)^2 = r^2$.

81. $(x+1)^2 + (y-3)^2 = 16$
Center (–1, 3); radius = 4

82. $(x+2)^2 + (y+3)^2 = 11$
Center (–2, –3); radius = $\sqrt{11}$

83. $(x-1)^2 + y^2 = 4$
Center: (1, 0); radius = 2

84. $x^2 + (y+2)^2 = 13$
Center: (0, –2); radius = $\sqrt{13}$

85. $(x-3)^2 + (y-1)^2 = 9$

86. $(x+1)^2 + (y-2)^2 = 4$

87. Find the radius by using the distance formula:
$d = \sqrt{(-1-3)^2 + (5-(-4))^2} = \sqrt{97}.$
The equation of the circle is
$(x-3)^2 + (y+4)^2 = 97.$

88. Find the radius by using the distance formula:
$d = \sqrt{(-1-2)^2 + (1-5)^2} = \sqrt{25} = 5.$
The equation of the circle is
$(x+1)^2 + (y-1)^2 = 25.$

89. The center is 3 units from the y-axis, so the radius is 3. The equation of the circle is
$(x+3)^2 + (y+2)^2 = 9.$

90. The center is 2 units from the x-axis, so the radius is 2. The equation of the circle is $(x-1)^2+(y-2)^2=4$.

91. The length of the diameter is $d=\sqrt{(2-8)^2+(-3-5)^2}=10,$ so the radius is 5. The center is at the midpoint of the diameter, $\left(\frac{2+8}{2}, \frac{-3+5}{2}\right)=(5, 1)$. Thus, the equation of the circle is $(x-5)^2+(y-1)^2=25$.

92. Find the diameter by using the distance formula: $d=\sqrt{(-3-7)^2+(6-4)^2}=\sqrt{104}=2\sqrt{26}$. So the radius is $\sqrt{26}$. Use the midpoint formula to find the center: $M=\left(\frac{7+(-3)}{2},\frac{4+6}{2}\right)=(2,5)$.
The equation of the circle is
$(x-2)^2+(y-5)^2=26$.

93. a. $x^2+y^2+4x-6y+9=0 \Rightarrow$
$x^2+4x+y^2-6y+9=0.$
Now complete the square:
$x^2+4x+4+y^2-6y+9=4 \Rightarrow$
$(x+2)^2+(y-3)^2=4$
This is a circle with center (–2, 3) and radius 2.

b. To find the x-intercepts, let $y = 0$, then solve for x:
$x^2+4x+9=0 \Rightarrow$
$$x=\frac{-4\pm\sqrt{4^2-4(1)(9)}}{2(1)}=\frac{-4\pm\sqrt{-20}}{2} \Rightarrow$$
there are no x-intercepts.
To find the y-intercepts, let $x = 0$, then solve for y:
$y^2-6y+9=0 \Rightarrow (y-3)^2=0 \Rightarrow y=3$
The y-intercept is 3.

94. a. $x^2+y^2-4x+2y+4=0 \Rightarrow$
$x^2-4x+4+y^2+2y=0.$
Now complete the square:
$x^2-4x+4+y^2+2y+1=1. \Rightarrow$
$(x-2)^2+(y+1)^2=1$
This is a circle with center (2, –1) and radius 1.

b. To find the x-intercepts, let $y = 0$, then solve for x:
$x^2-4x+4=0 \Rightarrow (x-2)^2=0 \Rightarrow x=2$
The x-intercept is 2.
To find the y-intercepts, let $x = 0$, then solve for y:
$y^2+2y+4=0 \Rightarrow$
$$y=\frac{-2\pm\sqrt{2^2-4(1)(4)}}{2(1)}=\frac{-2\pm\sqrt{-12}}{2} \Rightarrow$$
there are no y-intercepts.

95. a. $x^2+y^2-2x-2y-4=0 \Rightarrow$
$x^2-2x+y^2-2y=4$
Now complete the square:
$x^2-2x+1+y^2-2y+1=4+1+1 \Rightarrow$
$(x-1)^2+(y-1)^2=6$
This is a circle with center (1, 1) and radius $\sqrt{6}$.

b. To find the x-intercepts, let $y = 0$, then solve for x:
$x^2-2x-4=0 \Rightarrow$
$$x=\frac{2\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)} \Rightarrow$$
$$x=\frac{2\pm\sqrt{20}}{2}=1\pm\sqrt{5}$$
To find the y-intercepts, let $x = 0$, then solve for y: $y^2-2y-4=0 \Rightarrow$
$$y=\frac{2\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)}=\frac{2\pm\sqrt{20}}{2}$$
$$=1\pm\sqrt{5}$$

96. a. $x^2+y^2-4x-2y-15=0 \Rightarrow$
$x^2-4x+y^2-2y=15.$ Now complete the square:
$x^2-4x+4+y^2-2y+1=15+4+1 \Rightarrow$
$(x-2)^2+(y-1)^2=20.$ This is a circle with center (2, 1) and radius $2\sqrt{5}$.

b. To find the x-intercepts, let $y = 0$, then solve for x:

$x^2 - 4x - 15 = 0 \Rightarrow$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-15)}}{2(1)} = 2 \pm \sqrt{19}$$

To find the y-intercepts, let $x = 0$, then solve for y:

$y^2 - 2y - 15 = 0 \Rightarrow (y+3)(y-5) = 0 \Rightarrow$
$y = -3$ or $y = 5$.

97. a. $2x^2 + 2y^2 + 4y = 0 \Rightarrow$

$2(x^2 + y^2 + 2y) = 0 \Rightarrow x^2 + y^2 + 2y = 0$

Now complete the square:

$x^2 + y^2 + 2y + 1 = 0 + 1 \Rightarrow x^2 + (y+1)^2 = 1.$

This is a circle with center (0, –1) and radius 1.

b. To find the x-intercepts, let $y = 0$, then solve for x: $2x^2 = 0 \Rightarrow x = 0$

To find the y-intercepts, let $x = 0$, then solve for y: $2y^2 + 4y = 0 = 0 \Rightarrow 2y(y+2) = 0 \Rightarrow$
$y = -2$ or $y = 0$.

98. a. $3x^2 + 3y^2 + 6x = 0 \Rightarrow$

$3(x^2 + y^2 + 2x) = 0 \Rightarrow x^2 + 2x + y^2 = 0.$

Now complete the square:

$x^2 + 2x + 1 + y^2 = 0 + 1 \Rightarrow (x+1)^2 + y^2 = 1.$

This is a circle with center (–1, 0) and radius 1.

b. To find the x-intercepts, let $y = 0$, then solve for x:

$3x^2 + 6x = 0 \Rightarrow 3x(x+2) = 0 \Rightarrow x = 0$ or $x = -2$. To find the y-intercepts, let $x = 0$, then solve for y: $3y^2 = 0 \Rightarrow y = 0$

99. a. $x^2 + y^2 - 2x + 4y + 5 = 0$

Complete the squares on x and y.

$x^2 + y^2 - 2x + 4y + 5 = 0 \Rightarrow$
$x^2 - 2x + y^2 + 4y = -5 \Rightarrow$
$\left(x^2 - 2x + 1\right) + \left(y^2 + 4y + 4\right) = -5 + 1 + 4 \Rightarrow$
$(x-1)^2 + (y+2)^2 = 0$

b. Since $r^2 = 0 \Rightarrow r = 0$, this is the point $(1, -2)$.

100. a. $x^2 + y^2 + 4x - 6y + 17 = 0$

Complete the squares on x and y.

$x^2 + y^2 + 4x - 6y + 17 = 0 \Rightarrow$
$x^2 + 4x + y^2 - 6y = -17 \Rightarrow$
$\left(x^2 + 4x + 4\right) + \left(y^2 - 6y + 9\right) = -17 + 4 + 9 \Rightarrow$
$(x+2)^2 + (y-3)^2 = -4$

b. Since $r^2 = -4$, this is neither a circle nor a point. There is no solution.

Applying the Concepts

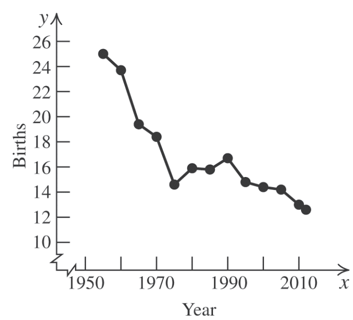
101. Percentage of Android sales in June 2013: 51.5%

102. Percentage of iPhone sales in December 2012: 49.7%

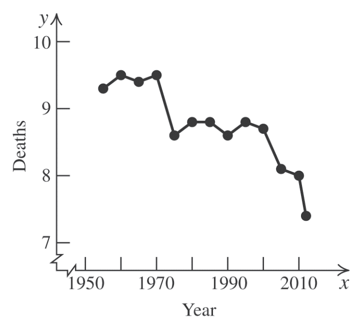
103. Android sales were at a maximum in June 2014.

104. iPhone sales were at a maximum in December 2012.

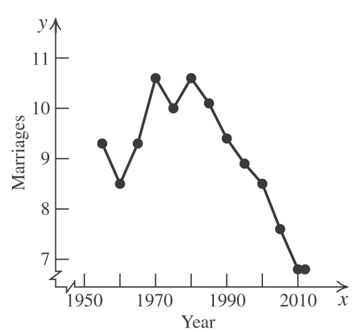
105.



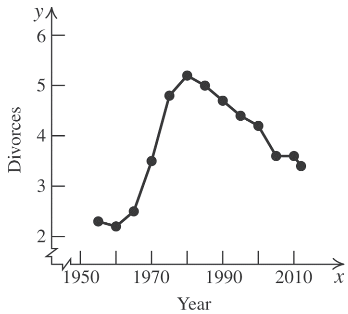
106.



107.



108.



109. 2009 is the midpoint of the initial range, so

$$M_{2009} = \frac{228+320}{2} = 274 .$$

2008 is the midpoint of the range [2007, 2009], so $M_{2008} = \dfrac{228+274}{2} = 251$.

2010 is the midpoint of the range [2009, 2011], so $M_{2010} = \dfrac{274+320}{2} = 297$. So, in 2008, \$251 billion was spent; in 2009, \$274 billion was spent, and \$297 was spent in 2010.

110. 2008 is the midpoint of the initial range, so

$$M_{2008} = \frac{548+925}{2} = 736.5 .$$

2006 is the midpoint of the range [2004, 2008], so

$$M_{2006} = \frac{548+736.5}{2} = 642.25 .$$

2005 is the midpoint of the range [2004, 2006], so

$$M_{2005} = \frac{548+642.25}{2} = 595.125 .$$

2007 is the midpoint of the range [2006, 2008], so

$$M_{2007} = \frac{642.25+736.5}{2} = 689.375$$

Use similar reasoning to find the amounts for 2009, 2010, and 2011. Defense spending was as follows:

Year	Amount spent
2004	\$548 billion
2005	\$595.125 billion
2006	\$642.25 billion
2007	\$689.375 billion
2008	\$736.5 billion
2009	\$783.625 billion
2010	\$830.75 billion
2011	\$877.875 billion
2012	\$925 billion

111. The distance from $P(x, y)$ to the x-axis is $|x|$ while the distance from P to the y-axis is $|y|$. So the equation of the graph is $|x| = |y|$.

112. The distance from $P(x, y)$ to (1, 2) is $\sqrt{(x-1)^2+(y-2)^2}$ while the distance from P to (3, –4) is $\sqrt{(x-3)^2+(y+4)^2}$.
So the equation of the graph is

$$\sqrt{(x-1)^2+(y-2)^2} = \sqrt{(x-3)^2+(y+4)^2} \Rightarrow (x-1)^2+(y-2)^2 = (x-3)^2+(y+4)^2 \Rightarrow x^2-2x+1+y^2-4y+4 = x^2-6x+9+y^2+8y+16 \Rightarrow -2x-4y+5 = -6x+8y+25 \Rightarrow 4x-20 = 12y \Rightarrow$$

$$y = \frac{1}{3}x - \frac{5}{3}.$$

113. The distance from $P(x, y)$ to (2, 0) is $\sqrt{(x-2)^2+y^2}$ while the distance from P to the y-axis is $|x|$. So the equation of the graph is

$$\sqrt{(x-2)^2+y^2} = |x| \Rightarrow (x-2)^2+y^2 = x^2 \Rightarrow . \; x^2-4x+4+y^2 = x^2 \Rightarrow y^2 = 4x-4 \Rightarrow$$

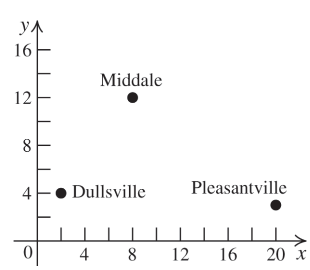
$$\frac{y^2+4}{4} = \frac{y^2}{4}+1 = x$$

114. The distance from P to the point (0, 4) is $\sqrt{x^2+(y-4)^2}$ while the distance from P to the x-axis is $|y|$. So the equation of the graph is

$$\sqrt{x^2+(y-4)^2} = |y| \Rightarrow x^2+(y-4)^2 = y^2 \Rightarrow x^2+y^2-8y+16 = y^2 \Rightarrow x^2 = 8y-16 \Rightarrow$$

$$\frac{x^2+16}{8} = \frac{x^2}{8}+2 = y$$

115. a.



b. $d(D,M) = \sqrt{(800-200)^2 + (1200-400)^2}$
$= 1000$
$d(M, P) = \sqrt{(2000-800)^2 + (300-1200)^2}$
$= 1500$
The distance traveled by the pilot
$= 1000 + 1500 = 2500$ miles.

c. $d(D,P) = \sqrt{(2000-200)^2 + (300-400)^2}$
$= \sqrt{3{,}250{,}000} = 500\sqrt{13}$
≈ 1802.78 miles

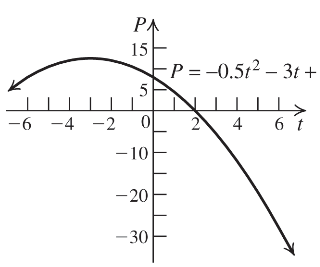
116. First, find the initial length of the rope using the Pythagorean theorem: $c = \sqrt{24^2 + 10^2} = 26$. After t seconds, the length of the rope is $26 - 3t$. Now find the distance from the boat to the dock, x, using the Pythagorean theorem again and solving for x:

$$(26-3t)^2 = x^2 + 10^2$$
$$676 - 156t + 9t^2 = x^2 + 100$$
$$576 - 156t + 9t^2 = x^2$$
$$\sqrt{576 - 156t + 9t^2} = x$$

117. a. Since July 2012 is represented by $t = 0$, March 2012 is represented by $t = -4$. So the monthly profit for March is determined by $P = -0.5(-4)^2 - 3(-4) + 8 = \$12$ million.

b. Since July 2012 is represented by $t = 0$, October 2012 is represented by $t = 3$. So the monthly profit for October is determined by $P = -0.5(3)^2 - 3(3) + 8 = -\$5.5$ million.

c.



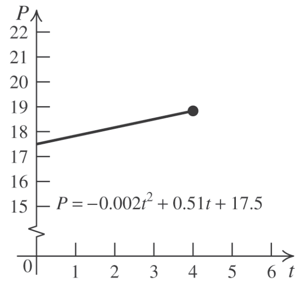
d. To find the t-intercept, set $P = 0$ and solve for t: $0 = -0.5t^2 - 3t + 8 \Rightarrow$

$$t = \frac{3 \pm \sqrt{(-3)^2 - 4(-0.5)(8)}}{2(-0.5)} = \frac{3 \pm \sqrt{25}}{-1}$$
$= 2$ or -8

The t-intercepts represent the months with no profit and no loss. In this case, $t = -8$ makes no sense in terms of the problem, so we disregard this solution. $t = 2$ represents Sept 2012.

e. To find the P-intercept, set $t = 0$ and solve to P: $P = -0.5(0)^2 - 3(0) + 8 \Rightarrow P = 8$.
The P-intercept represents the profit (in millions) in July 2012.

118. a.

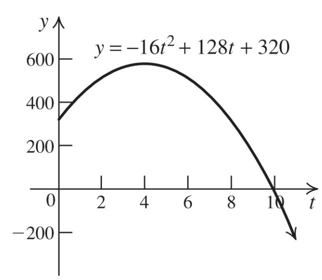


b. To find the P-intercept, set $t = 0$ and solve to P: $P = -0.002(0)^2 + 0.51(0) + 17.5 \Rightarrow$
$P = 17.5$. The P-intercept represents the number of female college students (in millions) in 2005.

119. a.

t	Height = $-16t^2 + 128t + 320$
0	320 feet
1	432 feet
2	512 feet
3	560 feet
4	576 feet
5	560 feet
6	512 feet

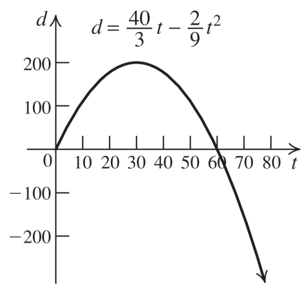
b.



c. $0 \le t \le 10$

d. To find the t-intercept, set $y = 0$ and solve for t: $0 = -16t^2 + 128t + 320 \Rightarrow$
$0 = -16(t^2 - 8t - 20) \Rightarrow 0 = (t - 10)(t + 2) \Rightarrow$
$t = 10$ or $t = -2$. The graph does not apply if $t < 0$, so the t-intercept is 10. This represents the time when the object hits the ground. To find the y-intercept, set $t = 0$ and solve for y:
$y = -16(0)^2 + 128(0) + 320 \Rightarrow y = 320$.
This represents the height of the building.

120. a.

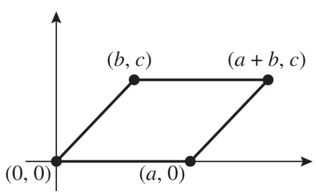


b. $0 \le t \le 60$

c. The total time of the experiment is 60 minutes or 1 hour.

Beyond the Basics

121.



The midpoint of the diagonal connecting (0, 0) and $(a + b, c)$ is $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. The midpoint of the diagonal connecting $(a, 0)$ and (b, c) is also $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. Because the midpoints of the two diagonals are the same, the diagonal bisect each other.

122. a. If AB is one of the diagonals, then DC is the other diagonal, and both diagonals have the same midpoint. The midpoint of AB is $\left(\frac{2+5}{2}, \frac{3+4}{2}\right) = (3.5, 3.5)$. The midpoint of $DC = (3.5, 3.5) = \left(\frac{x+3}{2}, \frac{y+8}{2}\right)$. So we have $3.5 = \frac{x+3}{2} \Rightarrow x = 4$ and $3.5 = \frac{y+8}{2} \Rightarrow y = -1$.
The coordinates of D are (4, –1).

b. If AC is one of the diagonals, then DB is the other diagonal, and both diagonals have the same midpoint. The midpoint of AC is $\left(\frac{2+3}{2}, \frac{3+8}{2}\right) = (2.5, 5.5)$. The midpoint of $DB = (2.5, 5.5) = \left(\frac{x+5}{2}, \frac{y+4}{2}\right)$. So we have $2.5 = \frac{x+5}{2} \Rightarrow x = 0$ and $5.5 = \frac{y+4}{2} \Rightarrow y = 7$.
The coordinates of D are (0, 7).

c. If BC is one of the diagonals, then DA is the other diagonal, and both diagonals have the same midpoint. The midpoint of BC is $\left(\frac{5+3}{2}, \frac{4+8}{2}\right) = (4, 6)$. The midpoint of $DA = (4, 6) = \left(\frac{x+2}{2}, \frac{y+3}{2}\right)$. So we have $4 = \frac{x+2}{2} \Rightarrow x = 6$ and $6 = \frac{y+3}{2} \Rightarrow y = 9$
The coordinates of D are (6, 9).

123. The midpoint of the diagonal connecting (0, 0) and (x, y) is $\left(\frac{x}{2}, \frac{y}{2}\right)$. The midpoint of the diagonal connecting $(a, 0)$ and (b, c) is $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. Because the diagonals bisect each other, the midpoints coincide. So $\frac{x}{2} = \frac{a+b}{2} \Rightarrow x = a + b$, and $\frac{y}{2} = \frac{c}{2} \Rightarrow y = c$.
Therefore, the quadrilateral is a parallelogram.

124. To show that M is the midpoint of the line segment AB, we need to show that the distance between A and M is the same as the distance between M and B and that this distance is half the distance from A to B. Using $A(x_1, y_1)$, $B(x_2, y_2)$, and $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, we have $d(A, B) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ and

$$AM = \sqrt{\left(\frac{x_1+x_2}{2}-x_1\right)^2+\left(\frac{y_1+y_2}{2}-y_2\right)^2} = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2+\left(\frac{y_2-y_1}{2}\right)^2}$$
$$= \sqrt{\frac{(x_2-x_1)^2}{4}+\frac{(y_2-y_1)^2}{4}} = \frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Thus, we have $AM = \frac{1}{2}AB$.

Similarly, we can show that $MB = \frac{1}{2}AB$. Therefore, M is the midpoint of AB.

125. a. Using $A(x_1, y_1)$, $B(x_2, y_2)$, and $C\left(\frac{2x_1+x_2}{3}, \frac{2y_1+y_2}{3}\right)$, we have $d(A, B) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

and

$$d(A,C) = \sqrt{\left(\frac{2x_1+x_2}{3}-x_1\right)^2+\left(\frac{2y_1+y_2}{3}-y_1\right)^2} = \sqrt{\left(\frac{2x_1+x_2-3x_1}{3}\right)^2+\left(\frac{2y_1+y_2-3y_1}{3}\right)^2}$$
$$= \sqrt{\left(\frac{x_2-x_1}{3}\right)^2+\left(\frac{y_2-y_1}{3}\right)^2} = \sqrt{\frac{(x_2-x_1)^2}{9}+\frac{(y_2-y_1)^2}{9}}$$
$$= \frac{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}{3} = \frac{1}{3}d(A,B).$$

b. Using $A(x_1, y_1)$, $B(x_2, y_2)$, and $D\left(\frac{x_1+2x_2}{3}, \frac{y_1+2y_2}{3}\right)$, we have $d(A, B) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

and

$$d(A,D) = \sqrt{\left(\frac{x_1+2x_2}{3}-x_1\right)^2+\left(\frac{y_1+2y_2}{3}-y_1\right)^2} = \sqrt{\left(\frac{x_1+2x_2-3x_1}{3}\right)^2+\left(\frac{y_1+2y_2-3y_1}{3}\right)^2}$$
$$= \sqrt{\left(\frac{2x_2-2x_1}{3}\right)^2+\left(\frac{2y_2-2y_1}{3}\right)^2} = \sqrt{\frac{4(x_2-x_1)^2+4(y_2-y_1)^2}{9}}$$
$$= \frac{2\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}{3} = \frac{2}{3}d(A,B).$$

c.
$$\frac{2x_1+x_2}{3} = \frac{2(-1)+4}{3} = \frac{2}{3}$$
$$\frac{2y_1+y_2}{3} = \frac{2(2)+1}{3} = \frac{5}{3}$$
$$\frac{x_1+2x_2}{3} = \frac{-1+2(4)}{3} = \frac{7}{3}$$
$$\frac{y_1+2y_2}{3} = \frac{2+2(1)}{3} = \frac{4}{3}$$

The points of trisection are $\left(\frac{2}{3}, \frac{5}{3}\right)$ and $\left(\frac{7}{3}, \frac{4}{3}\right)$.

126. D is the midpoint of AB, so its coordinates are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, and

$$CD = \sqrt{\left(x_3 - \frac{x_1+x_2}{2}\right)^2 + \left(y_3 - \frac{y_1+y_2}{2}\right)^2} = \sqrt{\left(\frac{2x_3-(x_1+x_2)}{2}\right)^2 + \left(\frac{2y_3-(y_1+y_2)}{2}\right)^2}$$
$$= \sqrt{\frac{(2x_3-(x_1+x_2))^2 + (2y_3-(y_1+y_2))^2}{4}} = \frac{1}{2}\sqrt{(2x_3-(x_1+x_2))^2 + (2y_3-(y_1+y_2))^2}$$

Because the centroid divides the median CD in a 2:1 ratio, we have $CM = 2MD$ or $CM = \frac{2}{3}d(CD)$.

$$CM = \sqrt{\left(x_3 - \frac{x_1+x_2+x_3}{3}\right)^2 + \left(y_3 - \frac{y_1+y_2+y_3}{3}\right)^2} = \sqrt{\left(\frac{3x_3-(x_1+x_2+x_3)}{3}\right)^2 + \left(\frac{3y_3-(y_1+y_2+y_3)}{3}\right)^2}$$
$$= \sqrt{\frac{(2x_3-(x_1+x_2))^2}{9} + \frac{(2y_3-(y_1+y_2))^2}{9}} = \frac{1}{3}\sqrt{(2x_3-(x_1+x_2))^2 + (2y_3-(y_1+y_2))^2}$$
$$= \frac{2}{3}\cdot\frac{1}{2}\sqrt{(2x_3-(x_1+x_2))^2 + (2y_3-(y_1+y_2))^2} = \frac{2}{3}d(CD)$$

127. the y-axis

128. the x-axis

129. the union of the x- and y-axes

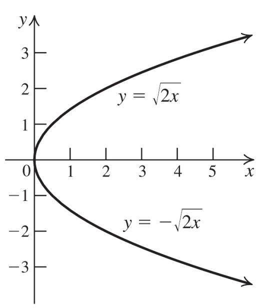
130. the plane without the x- and y-axes

131. Quadrants I and III

132. Quadrants II and IV

Critical Thinking/Discussion/Writing

133. The graph of $y^2 = 2x$ is the union of the graphs of $y = \sqrt{2x}$ and $y = -\sqrt{2x}$.



134. Let (x, y) be a point on the graph. Since the graph is symmetric with regard to the x-axis, then the point $(x, –y)$ is also on the graph. Because the graph is symmetric with regard to the y-axis, the point $(–x, y)$ is also on the graph. Therefore the point $(–x, –y)$ is on the graph, and the graph is symmetric with respect to the origin. The graphs of $y = x$ and $y = x^3$ are examples of a graph that is symmetric with respect to the origin but is not symmetric with respect to the x- and y-axes.

135. a. First find the radius of the circle:
$d(A, B) = \sqrt{(6-0)^2 + (8-1)^2} = \sqrt{85} \Rightarrow$
$r = \frac{\sqrt{85}}{2}$. The center of the circle is $\left(\frac{6+0}{2}, \frac{1+8}{2}\right) = \left(3, \frac{9}{2}\right)$. So the equation of the circle is $(x-3)^2 + \left(y - \frac{9}{2}\right)^2 = \frac{85}{4}$.

To find the x-intercepts, set $y = 0$, and solve for x:

$$(x-3)^2 + \left(0 - \frac{9}{2}\right)^2 = \frac{85}{4} \Rightarrow$$
$$(x-3)^2 + \frac{81}{4} = \frac{85}{4} \Rightarrow x^2 - 6x + 9 = 1 \Rightarrow$$
$$x^2 - 6x + 8 = 0$$

The x-intercepts are the roots of this equation.

b. First find the radius of the circle:
$d(A, B) = \sqrt{(a-0)^2 + (b-1)^2} = \sqrt{a^2 + (b-1)^2} \Rightarrow$
$r = \frac{\sqrt{a^2 + (b-1)^2}}{2}$. The center of the circle is $\left(\frac{a+0}{2}, \frac{b+1}{2}\right) = \left(\frac{a}{2}, \frac{b+1}{2}\right)$ So the equation of the circle is
$$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b+1}{2}\right)^2 = \frac{a^2 + (b-1)^2}{4}.$$

(*continued on next page*)

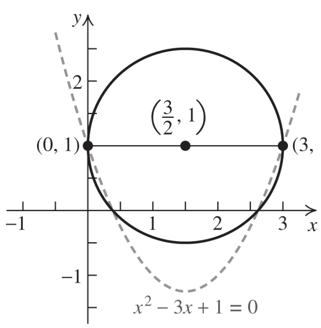
(*continued*)

To find the x-intercepts, set $y = 0$ and solve for x:

$$\left(x-\frac{a}{2}\right)^2+\left(0-\frac{b+1}{2}\right)^2=\frac{a^2+(b-1)^2}{4}$$
$$x^2-a+\frac{a^2}{4}+\frac{(b+1)^2}{4}=\frac{a^2+(b-1)^2}{4}$$
$$4x^2-4ax+a^2+b^2+2b+1=a^2+b^2-2b+1$$
$$4x^2-4ax+4b=0$$
$$x^2-ax+b=0$$

The x-intercepts are the roots of this equation.

c. $a = 3$ and $b = 1$. Approximate the roots of the equation by drawing a circle whose diameter has endpoints $A(0, 1)$ and $B(3, 1)$. The center of the circle is $\left(\frac{3}{2},1\right)$ and the radius is $\frac{3}{2}$.



136. a. First find the radius of the circle:

$d(A,B)=\sqrt{(10-1)^2+(7-0)^2}=\sqrt{130}\Rightarrow$

$r=\frac{\sqrt{130}}{2}$. The center of the circle is $\left(\frac{10+1}{2},\frac{7+0}{2}\right)=\left(\frac{11}{2},\frac{7}{2}\right)$. So the equation of the circle is

$\left(x-\frac{11}{2}\right)^2+\left(y-\frac{7}{2}\right)^2=\left(\frac{\sqrt{130}}{2}\right)^2$. To find the y-intercepts, set $x = 0$, and solve for y:

$\left(0-\frac{11}{2}\right)^2+\left(y-\frac{7}{2}\right)^2=\frac{130}{4}\Rightarrow\left(y-\frac{7}{2}\right)^2=\frac{9}{4}\Rightarrow$

$y^2-7y+\frac{49}{4}=\frac{9}{4}\Rightarrow y^2-7y+\frac{40}{4}=0\Rightarrow$

$y^2-7y+10=0$. The y-intercepts are the roots of this equation.

b. First find the radius of the circle:

$$d(A,B)=\sqrt{(a-1)^2+(b-0)^2}$$
$$=\sqrt{(a-1)^2+b^2}\Rightarrow$$

$r=\frac{\sqrt{(a-1)^2+b^2}}{2}$. The center of the circle is $\left(\frac{a+1}{2},\frac{b+0}{2}\right)=\left(\frac{a+1}{2},\frac{b}{2}\right)$. So the equation of the circle is

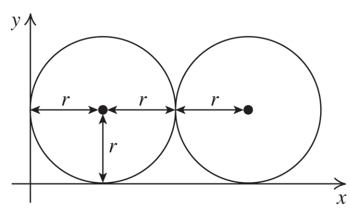
$\left(x-\frac{a+1}{2}\right)^2+\left(y-\frac{b}{2}\right)^2=\frac{(a-1)^2+b^2}{4}$.

To find the y-intercepts, set $x = 0$ and solve for y:

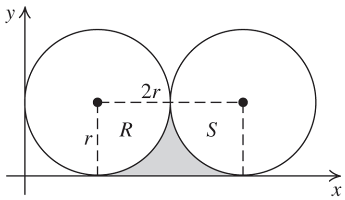
$$\left(0-\frac{a+1}{2}\right)^2+\left(y-\frac{b}{2}\right)^2=\frac{(a-1)^2+b^2}{4}$$
$$\frac{(a+1)^2}{4}+y^2-by+\frac{b^2}{4}=\frac{(a-1)^2+b^2}{4}$$
$$a^2+2a+1+4y^2-4by+b^2$$
$$=a^2-2a+1+b^2$$
$$4y^2-4by+4a=0$$
$$y^2-by+a=0$$

The y-intercepts are the roots of this equation.

137. a. The coordinates of the center of each circle are (r, r) and $(3r, r)$.

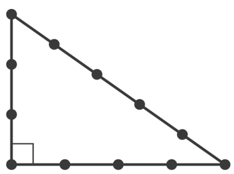


b. To find the area of the shaded region, first find the area of the rectangle shown in the figure below, and then subtract the sum of the areas of the two sectors, *A* and *B*.



$$A_{\text{rect}} = r(2r) = 2r^2$$
$$A_{\text{sector } R} = A_{\text{sector } S} = \frac{1}{4}\pi r^2$$
$$A_{\text{shaded region}} = 2r^2 - \left(\frac{1}{4}\pi r^2 + \frac{1}{4}\pi r^2\right) = 2r^2 - \frac{1}{2}\pi r^2 = \left(2 - \frac{\pi}{2}\right)r^2$$

138.



Getting Ready for the Next Section

139. $\frac{5-3}{6-2} = \frac{2}{4} = \frac{1}{2}$

140. $\frac{1-2}{-2-2} = \frac{-1}{-4} = \frac{1}{4}$

141. $\frac{2-(-3)}{3-13} = \frac{5}{-10} = -\frac{1}{2}$

142. $\frac{3-1}{-2-(-6)} = \frac{2}{4} = \frac{1}{2}$

143. $2x + 3y = 6 \Rightarrow 3y = 6 - 2x \Rightarrow$
$y = \frac{6-2x}{3} = 2 - \frac{2}{3}x$

144. $\frac{x}{2} - \frac{y}{5} = 3 \Rightarrow \frac{x}{2} - 3 = \frac{y}{5} \Rightarrow y = \frac{5}{2}x - 15$

145. $y - 2 - \frac{2}{3}(x+1) = 0 \Rightarrow$
$y = 2 + \frac{2}{3}(x+1) = 2 + \frac{2}{3}x + \frac{2}{3} = \frac{2}{3}x + \frac{8}{3}$

146. $0.1x + 0.2y - 1 = 0 \Rightarrow x + 2y - 10 = 0 \Rightarrow$
$2y = -x + 10 \Rightarrow y = -0.5x + 5$

147. $-\frac{1}{2}$

148. $\frac{1}{3}$

149. $\frac{3}{2}$

150. $-\frac{3}{4}$

1.2 Lines

1.2 Practice Problems

1. $m = \frac{5-(-3)}{-7-6} = -\frac{8}{13}$

A slope of $-\frac{8}{13}$ means that the value of y decreases 8 units for every 13 units increase in x.

2. $P(-2,-3),\ m = -\frac{2}{3}$

$y - (-3) = -\frac{2}{3}[x - (-2)]$

$y + 3 = -\frac{2}{3}(x+2)$ point-slope form

$y + 3 = -\frac{2}{3}x - \frac{4}{3} \Rightarrow y = -\frac{2}{3}x - \frac{13}{3}$

3. $m = \frac{-4-6}{-3-(-1)} = \frac{-10}{-2} = 5$

Use either point to determine the equation of the line. Using $(-3, -4)$, we have

$y - (-4) = 5[x - (-3)] \Rightarrow y + 4 = 5(x+3)$
point-slope form

$y + 4 = 5(x+3) \Rightarrow y + 4 = 5x + 15 \Rightarrow$
$y = 5x + 11$

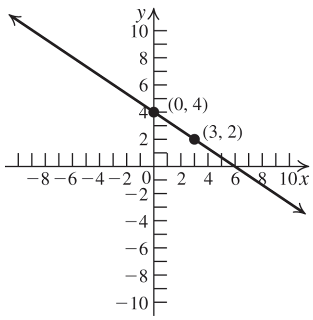
Using $(-1, 6)$, we have

$y - 6 = 5[x - (-1)] \Rightarrow y - 6 = 5(x+1) \Rightarrow$
$y - 6 = 5x + 5 \Rightarrow y = 5x + 11$

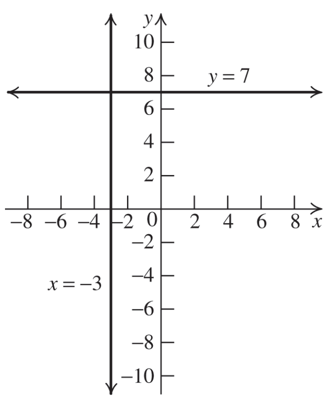
4. $y - y_1 = m(x - x_1) \Rightarrow y - (-3) = 2(x - 0)$
point-slope form

$y - (-3) = 2(x - 0) \Rightarrow$
$y + 3 = 2x \Rightarrow y = 2x - 3$

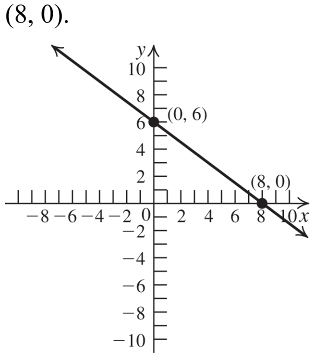
5. The slope is $-\frac{2}{3}$ and the y-intercept is 4. The line goes through (0, 4), so locate a second point by moving two units down and three units right. Thus, the line goes through (3, 2).



6. $x = -3$. The slope is undefined, and there is no y-intercept. The x-intercept is −3.
$y = 7$. The slope is 0, and the y-intercept is 7.



7. First, solve for y to write the equation in slope-intercept form:
$3x + 4y = 24 \Rightarrow 4y = -3x + 24 \Rightarrow$
$y = -\frac{3}{4}x + 6$. The slope is $-\frac{3}{4}$, and the y-intercept is 6. Find the x-intercept by setting $y = 0$ and solving the equation for x:
$0 = -\frac{3}{4}x + 6 \Rightarrow 6 = \frac{3}{4}x \Rightarrow 8 = x$. Thus, the graph passes through the points (0, 6) and (8, 0).



8. Use the equation $H = 2.6x + 65$.
$H_1 = 2.6(43) + 65 = 176.8$
$H_2 = 2.6(44) + 65 = 179.4$
The person is between 176.8 cm and 179.4 cm tall, or between 1.768 m and 1.794 m.

9. **a.** Parallel lines have the same slope, so the slope of the line is $m = \frac{3-7}{2-5} = \frac{-4}{-3} = \frac{4}{3}$.
Using the point-slope form, we have
$y - 5 = \frac{4}{3}[x - (-2)] \Rightarrow 3y - 15 = 4(x + 2) \Rightarrow$
$3y - 15 = 4x + 8 \Rightarrow 4x - 3y + 23 = 0$

b. The slopes of perpendicular lines are negative reciprocals. Write the equation $4x + 5y + 1 = 0$ in slope-intercept form to find its slope: $4x + 5y + 1 = 0 \Rightarrow$
$5y = -4x - 1 \Rightarrow y = -\frac{4}{5}x - \frac{1}{5}$. The slope of a line perpendicular to this lines is $\frac{5}{4}$. Using the point-slope form, we have
$y - (-4) = \frac{5}{4}(x - 3) \Rightarrow 4(y + 4) = 5(x - 3) \Rightarrow$
$4y + 16 = 5x - 15 \Rightarrow 5x - 4y - 31 = 0$

10. Since 2016 is 10 years after 2006, set $x = 10$.
Then $y = 0.44(10) + 6.70 = 11.1$
There were 11.1 million registered motorcycles in the U.S. in 2014.

1.2 Exercises
Concepts and Vocabulary

1. The slope of a horizontal line is <u>0</u>; the slope of a vertical line is <u>undefined</u>.

2. The slope of the line passing through the points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ is given by the formula $m = \underline{\frac{y_2 - y_1}{x_2 - x_1}}$.

3. Every line parallel to the line $y = 3x - 2$ has slope, m, equal to <u>3</u>.

4. Every line perpendicular to the line $y = 3x - 2$ has slope, m, equal to $\underline{-\frac{1}{3}}$.

5. False. The slope of the line $y = -\frac{1}{4}x + 5$ is equal to $-\frac{1}{4}$.

6. False. The y-intercept of the line $y = 2x - 3$ is equal to –3.

7. True

8. True

Building Skills

9. $m = \frac{7-3}{4-1} = \frac{4}{3}$; the graph is rising.

10. $m = \frac{0-4}{2-0} = \frac{-4}{2} = -2$; the graph is falling.

11. $m = \frac{-2-(-2)}{-2-6} = \frac{0}{-8} = 0$; the graph is horizontal.

12. $m = \frac{7-(-4)}{-3-(-3)} = \frac{11}{0} \Rightarrow$ slope is undefined; the graph is vertical.

13. $m = \frac{-3.5-2}{3-0.5} = \frac{-5.5}{2.5} = -2.2$; the graph is falling.

14. $m = \frac{-3-(-2)}{2-3} = \frac{-1}{-1} = 1$; the graph is rising.

15. $m = \frac{5-1}{\left(1+\sqrt{2}\right)-\sqrt{2}} = \frac{4}{1} = 4$; the graph is rising.

16. $m = \frac{3\sqrt{3}-0}{\left(1+\sqrt{3}\right)-\left(1-\sqrt{3}\right)} = \frac{3\sqrt{3}}{2\sqrt{3}} = \frac{3}{2}$; the graph is rising.

17. ℓ_3

18. ℓ_2

19. ℓ_4

20. ℓ_1

21. ℓ_1 passes through the points (2, 3) and (−1, 0). $m_{\ell_1} = \frac{0-3}{-1-2} = 1.$

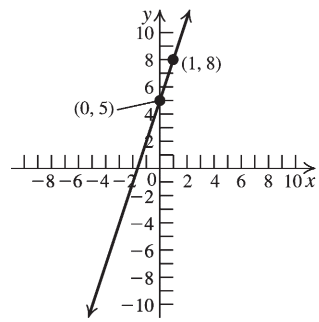
22. ℓ_2 is a horizontal line, so it has slope 0.

23. ℓ_3 passes through the points (2, 3) and (0, −1). $m_{\ell_3} = \frac{-1-3}{0-2} = 2.$

24. ℓ_4 passes through the points (−6, 7) and (0, −1). $m_{\ell_4} = \frac{-1-7}{0-(-6)} = -\frac{4}{3}.$

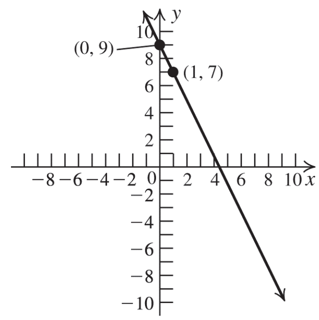
25. $(0, 5)$; $m = 3$

$y = 3x + 5$

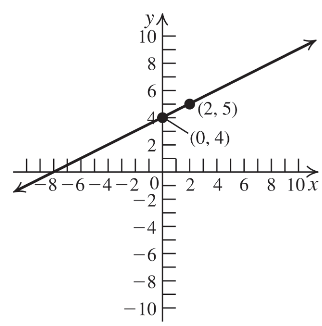


26. $(0, 9)$; $m = -2$

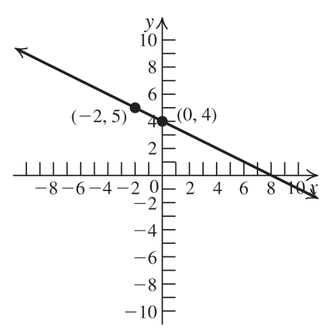
$y = -2x + 9$



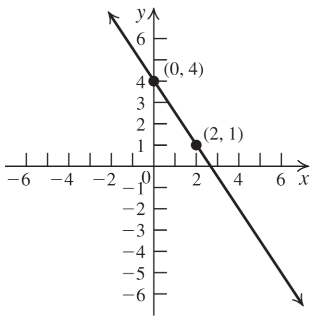
27. $y = \frac{1}{2}x + 4$



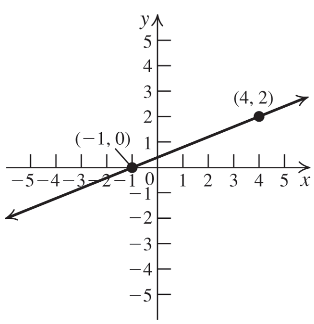
28. $y = -\frac{1}{2}x + 4$



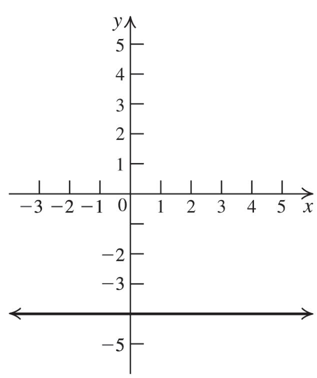
29. $y-1=-\frac{3}{2}(x-2) \Rightarrow y-1=-\frac{3}{2}x+3 \Rightarrow$
$y=-\frac{3}{2}x+4$



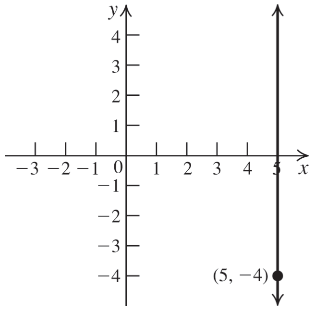
30. $y=\frac{2}{5}(x+1) \Rightarrow y=\frac{2}{5}x+\frac{2}{5}$



31. $y+4=0(x-5) \Rightarrow y+4=0 \Rightarrow y=-4$



32. Because the slope is undefined, the graph is vertical. The equation is $x=5$.



33. $m=\frac{0-1}{1-0}=-1$. The y-intercept is (0, 1), so the equation is $y=-x+1$.

34. $m=\frac{3-1}{1-0}=2$. The y-intercept is (0, 1), so the equation is $y=2x+1$.

35. $m=\frac{3-3}{3-(-1)}=0$ Because the slope = 0, the line is horizontal. Its equation is $y=3$.

36. $m=\frac{7-1}{2-(-5)}=\frac{6}{7}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.
$y-1=\frac{6}{7}(x+5) \Rightarrow y-1=\frac{6}{7}x+\frac{30}{7} \Rightarrow$
$y=\frac{6}{7}x+\frac{37}{7}$

37. $m=\frac{1-(-1)}{1-(-2)}=\frac{2}{3}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.
$y+1=\frac{2}{3}(x+2) \Rightarrow y+1=\frac{2}{3}x+\frac{4}{3} \Rightarrow$
$y=\frac{2}{3}x+\frac{1}{3}$

38. $m=\frac{-9-(-3)}{6-(-1)}=-\frac{6}{7}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.
$y+3=-\frac{6}{7}(x+1) \Rightarrow y+3=-\frac{6}{7}x-\frac{6}{7} \Rightarrow$
$y=-\frac{6}{7}x-\frac{27}{7}$

39. $m=\frac{2-\frac{1}{4}}{0-\frac{1}{2}}=\frac{\frac{7}{4}}{-\frac{1}{2}}=-\frac{7}{2}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.
$y-2=-\frac{7}{2}x \Rightarrow y=-\frac{7}{2}x+2$

40. $m=\frac{3-(-7)}{4-4}=\frac{10}{0} \Rightarrow$ the slope is undefined. So the graph is a vertical line. The equation is $x=4$.

41. $x = 5$ **42.** $y = 1.5$

43. $y = 0$ **44.** $x = 0$

45. $y = 14$ **46.** $y = 2x + 5$

47. $y = -\dfrac{2}{3}x - 4$ **48.** $y = -6x - 3$

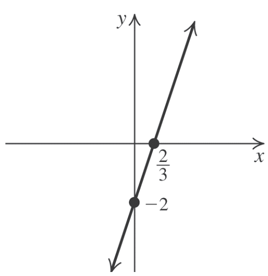
49. $m = \dfrac{4-0}{0-(-3)} = \dfrac{4}{3};\ \ y = \dfrac{4}{3}x + 4$

50. $m = \dfrac{-2-0}{0-(-5)} = -\dfrac{2}{5};\ \ y = -\dfrac{2}{5}x - 2$

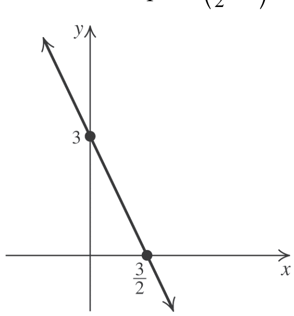
51. $y = 7$ **52.** $x = 4$

53. $y = -5$ **54.** $x = -3$

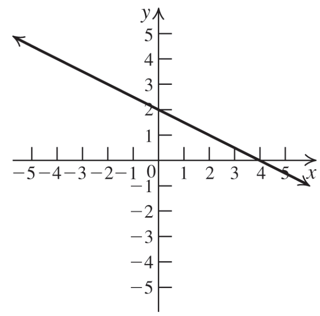
55. $y = 3x - 2$
The slope is 3 and the y-intercept is (0, –2).
$0 = 3x - 2 \Rightarrow 3x = 2 \Rightarrow x = \dfrac{2}{3}$
The x-intercept is $\left(\frac{2}{3}, 0\right)$.



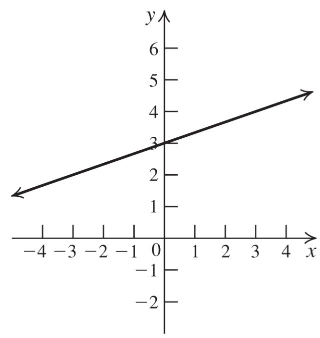
56. $y = -2x + 3$
The slope is –2 and the y-intercept is (0, 3).
$0 = -2x + 3 \Rightarrow 2x = 3 \Rightarrow x = \dfrac{3}{2}$
The x-intercept is $\left(\frac{3}{2}, 0\right)$.



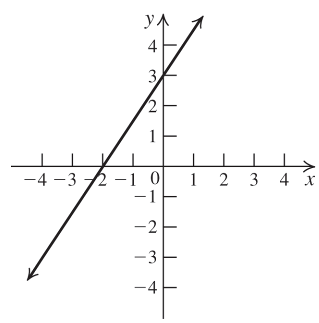
57. $x + 2y - 4 = 0 \Rightarrow 2y = -x + 4 \Rightarrow y = -\dfrac{1}{2}x + 2$.
The slope is $-1/2$, and the y-intercept is (0, 2).
To find the x-intercept, set $y = 0$ and solve for x:
$x + 2(0) - 4 = 0 \Rightarrow x = 4$.



58. $x = 3y - 9 \Rightarrow x + 9 = 3y \Rightarrow y = \dfrac{1}{3}x + 3$. The slope is $1/3$, and the y-intercept is (0, 3). To find the x-intercept, set $y = 0$ and solve for x:
$x = 3(0) - 9 \Rightarrow x = -9$.

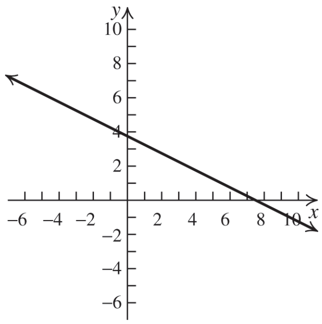


59. $3x - 2y + 6 = 0 \Rightarrow 3x + 6 = 2y \Rightarrow \dfrac{3}{2}x + 3 = y$.
The slope is $3/2$, and the y-intercept is (0, 3).
To find the x-intercept, set $y = 0$ and solve for x:
$3x - 2(0) + 6 = 0 \Rightarrow 3x = -6 \Rightarrow x = -2$.

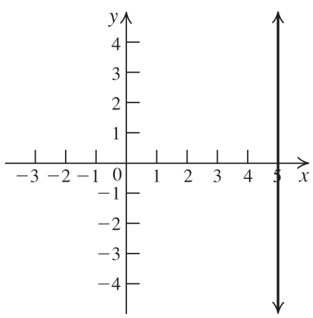


60. $2x = -4y + 15 \Rightarrow 2x - 15 = -4y \Rightarrow$
$-\dfrac{1}{2}x + \dfrac{15}{4} = y$. The slope is $-1/2$, and the y-intercept is $15/4$. To find the x-intercept, set $y = 0$ and solve for x: $2x = -4(0) + 15 \Rightarrow$
$x = 15/2$.

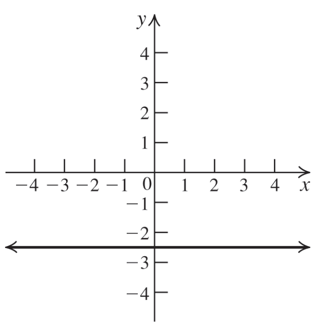
60. $2x = -4y + 15 \Rightarrow 2x - 15 = -4y \Rightarrow$
$-\dfrac{1}{2}x + \dfrac{15}{4} = y$. The slope is $-1/2$, and the y-intercept is $15/4$. To find the x-intercept, set $y = 0$ and solve for x: $2x = -4(0) + 15 \Rightarrow$
$x = 15/2$.



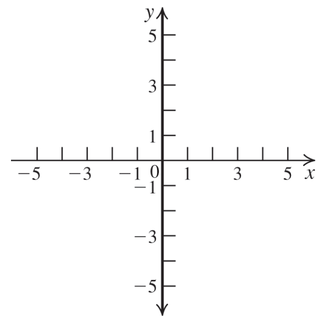
61. $x - 5 = 0 \Rightarrow x = 5$. The slope is undefined, and there is no y-intercept. The x-intercept is 5.



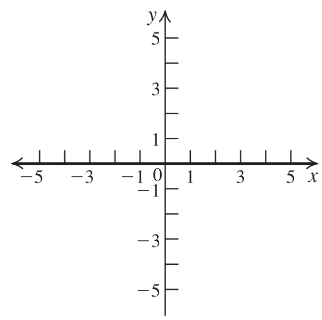
62. $2y + 5 = 0 \Rightarrow y = -\dfrac{5}{2}$. The slope is 0, and the y-intercept is $-5/2$. This is a horizontal line, so there is no x-intercept.



63. $x = 0$. The slope is undefined, and the y-intercepts are the y-axis. This is a vertical line whose x-intercept is 0.



64. $y = 0$. The slope is 0, and the x-intercepts are the x-axis. This is a horizontal line whose y-intercept is 0.



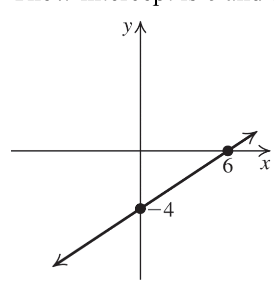
For exercises 65–68, the two-interecpt form of the equation of a line is $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

65. $\dfrac{x}{4} + \dfrac{y}{3} = 1$ **66.** $-\dfrac{x}{3} + \dfrac{y}{2} = 1$

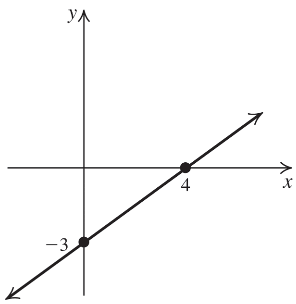
67. $2x + 3y = 6 \Rightarrow \dfrac{2x}{6} + \dfrac{3y}{6} = \dfrac{6}{6} \Rightarrow \dfrac{x}{3} + \dfrac{y}{2} = 1$;
x-intercept = 3; y-intercept = 2

68. $3x - 4y + 12 = 0 \Rightarrow 3x - 4y = -12 \Rightarrow$
$\dfrac{3x}{-12} - \dfrac{4y}{-12} = \dfrac{-12}{-12} \Rightarrow -\dfrac{x}{4} + \dfrac{y}{3} = 1$;
x-intercept = –4; y-intercept = 3

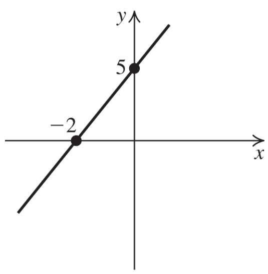
69. $2x - 3y = 12 \Rightarrow \dfrac{2x}{12} - \dfrac{3y}{12} = 1 \Rightarrow \dfrac{x}{6} - \dfrac{y}{4} = 1$
The x-intercept is 6 and the y-intercept is –4.



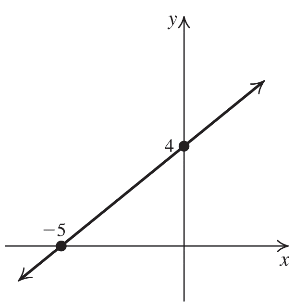
70. $3x-4y=12 \Rightarrow \frac{3x}{12}-\frac{4y}{12}=1 \Rightarrow \frac{x}{4}-\frac{y}{3}=1$
The x-intercept is 4 and the y-intercept is –3.



71. $-5x+2y=10 \Rightarrow -\frac{5x}{10}+\frac{2y}{10}=1 \Rightarrow -\frac{x}{2}+\frac{y}{5}=1$
The x-intercept is –2 and the y-intercept is 5.



72. $-4x+5y=20 \Rightarrow -\frac{4x}{20}+\frac{5y}{20}=1 \Rightarrow -\frac{x}{5}+\frac{y}{4}=1$
The x-intercept is –5 and the y-intercept is 4.



73. $m=\frac{9-4}{7-2}=\frac{5}{5}=1.$ The equation of the line through (2, 4) and (7, 9) is $y-4=1(x-2) \Rightarrow$ $y=x+2$. Check to see if (–1, 1) satisfies the equation by substituting $x=-1$ and $y=1$: $1=-1+2 \Rightarrow 1=1.$ So (–1, 1) lies on the line.

74. $m=\frac{-3-2}{2-7}=\frac{-5}{-5}=1.$ The equation of the line through (7, 2) and (2, –3) is $y-2=1(x-7) \Rightarrow$ $y=x-5.$ Check to see if (5, 1) satisfies the equation by substituting $x=5$ and $y=1$:
$1=5-5 \Rightarrow 1 \neq 0.$
So (5, 1) does not lie on the line.

75. The given line passes through the points (0, 3) and (4, 0), so its slope is $-\frac{3}{4}$. Any line parallel to this line will have the same slope. The line that passes through the origin and is parallel to the given line has equation $y=-\frac{3}{4}x.$

76. From exercise 75, the slope of the given line is $-\frac{3}{4}$. Any line perpendicular to this line will have slope $\frac{4}{3}$. The line that passes through the origin and is perpendicular to the given line has equation $y=\frac{4}{3}x.$

77. The red line passes through the points (–2, 0) and (0, 3), so its slope is $\frac{3}{2}$. The blue line passes through (4, 2) and has the same slope, so its equation is
$y-2=\frac{3}{2}(x-4) \Rightarrow 2y-4=3x-12 \Rightarrow$
$2y=3x-8 \Rightarrow y=\frac{3}{2}x-4$

78. The red line passes through the points (–2, 0) and (0, 3), so its slope is $\frac{3}{2}$. The green line passes through (4, 2) and has slope $-\frac{2}{3}$, so its equation is
$y-2=-\frac{2}{3}(x-4) \Rightarrow 3y-6=-2x+8 \Rightarrow$
$3y=-2x+14 \Rightarrow y=-\frac{2}{3}x+\frac{14}{3}$

79. The slope of $y=3x-1$ is 3. The slope of $y=3x+2$ is also 3. The lines are parallel.

80. The slope of $y=2x+2$ is 2. The slope of $y=-2x+2$ is –2. The lines are neither parallel nor perpendicular.

81. The slope of $y=2x-4$ is 2. The slope of $y=-\frac{1}{2}x+4$ is $-\frac{1}{2}$. The lines are perpendicular.

82. The slope of $y=3x+1$ is 3. The slope of $y=\frac{1}{3}x-1$ is $\frac{1}{3}$.. The lines are neither parallel nor perpendicular.

83. The slope of $3x+8y=7$ is $-3/8$, while the slope of $5x-7y=0$ is $5/7$. The lines are neither parallel nor perpendicular.

84. The slope of $10x+2y=3$ is –5. The slope of $5x+y=-1$ is also –5, so the lines are parallel.

85. The slope of $x = 4y + 8$ is $1/4$. The slope of $y = -4x + 1$ is -4, so the lines are perpendicular.

86. The slope of $y = 3x + 1$ is 3. The slope of $6y + 2x = 0$ is $-1/3$. The lines are perpendicular.

87. Both lines are vertical lines. The lines are parallel.

88. The slope of $2x + 3y = 7$ is $-2/3$, while $y = 2$ is a horizontal line. The lines are neither parallel nor perpendicular.

89. The equation of the line through (2, –3) with slope 3 is $y + 3 = 3(x - 2) \Rightarrow y + 3 = 3x - 6 \Rightarrow$
$y = 3x - 9.$

90. The equation of the line through (–1, 3) with slope –2 is
$y - 3 = -2(x - (-1)) \Rightarrow y - 3 = -2(x + 1) \Rightarrow$
$y - 3 = -2x - 2 \Rightarrow y = -2x + 1.$

91. A line perpendicular to a line with slope $-\frac{1}{2}$ has slope 2. The equation of the line through (–1, 2) with slope 2 is $y - 2 = 2(x - (-1)) \Rightarrow$
$y - 2 = 2(x + 1) \Rightarrow y - 2 = 2x + 2 \Rightarrow$
$y = 2x + 4.$

92. A line perpendicular to a line with slope $\frac{1}{3}$ has slope –3. The equation of the line through (2, –1) with slope –3 is $y - (-1) = -3(x - 2) \Rightarrow$
$y + 1 = -3(x - 2) \Rightarrow y + 1 = -3x + 6 \Rightarrow$
$y = -3x + 5.$

93. The slope of the line joining (1, –2) and (–3, 2) is $\dfrac{2 - (-2)}{-3 - 1} = -1.$ The equation of the line through (–2, –5) with slope –1 is
$y - (-5) = -(x - (-2)) \Rightarrow y + 5 = -(x + 2) \Rightarrow$
$y + 5 = -x - 2 \Rightarrow y = -x - 7.$

94. The slope of the line joining (–2, 1) and (3, 5) is
$\dfrac{5 - 1}{3 - (-2)} = \dfrac{4}{5}.$
The equation of the line through (1, 2) with slope $\frac{4}{5}$ is
$y - 2 = \dfrac{4}{5}(x - 1) \Rightarrow 5(y - 2) = 4(x - 1) \Rightarrow$
$5y - 10 = 4x - 4 \Rightarrow 5y = 4x + 6 \Rightarrow$
$y = \dfrac{4}{5}x + \dfrac{6}{5}.$

95. The slope of the line joining (–3, 2) and (–4, –1) is
$\dfrac{-1 - 2}{-4 - (-3)} = 3.$
A line perpendicular to this line has slope $-\frac{1}{3}$. The equation of the line through (1, –2) with slope $-\frac{1}{3}$ is
$y - (-2) = -\dfrac{1}{3}(x - 1) \Rightarrow 3(y + 2) = -(x - 1) \Rightarrow$
$3y + 6 = -x + 1 \Rightarrow 3y = -x - 5 \Rightarrow$
$y = -\dfrac{1}{3}x - \dfrac{5}{3}.$

96. The slope of the line joining (2, 1) and (4, –1) is
$\dfrac{-1 - 1}{4 - 2} = -1.$
A line perpendicular to this line has slope 1 The equation of the line through (–1, 2) with slope 1 is
$y - 2 = x - (-1) \Rightarrow y - 2 = x + 1 \Rightarrow y = x + 3.$

97. The slope of the line $y = 6x + 5$ is 6. The lines are parallel, so the slope of the new line is also 6. The equation of the line with slope 6 and y-intercept 4 is $y = 6x + 4$.

98. The slope of the line $y = -\frac{1}{2}x + 5$ is $-\frac{1}{2}$. The lines are parallel, so the slope of the new line is also $-\frac{1}{2}$. The equation of the line with slope $-\frac{1}{2}$ and y-intercept 2 is $y = -\frac{1}{2}x + 2.$

99. The slope of the line $y = 6x + 5$ is 6. The lines are perpendicular, so the slope of the new line is $-\frac{1}{6}$. The equation of the line with slope $-\frac{1}{6}$ and y-intercept 4 is $y = -\frac{1}{6}x + 4.$

100. The slope of the line $y = -\frac{1}{2}x + 5$ is $-\frac{1}{2}$. The lines are perpendicular, so the slope of the new line is 2. The equation of the line with slope 2 and y-intercept –4 is $y = 2x - 4.$

101. The slope of $x + y = 1$ is -1. The lines are parallel, so they have the same slope. The equation of the line through (1, 1) with slope -1 is $y-1=-(x-1) \Rightarrow y-1=-x+1 \Rightarrow$ $y=-x+2$.

102. The slope of $-2x+3y=7$ is $2/3$. The lines are parallel, so they have the same slope. The equation of the line through (1, 0) with slope $2/3$ is $y-0=\frac{2}{3}(x-1) \Rightarrow y=\frac{2}{3}x-\frac{2}{3}$.

103. The slope of $3x-9y=18$ is $1/3$. The lines are perpendicular, so the slope of the new line is -3. The equation of the line through (–2, 4) with slope -3 is $y-4=-3(x-(-2)) \Rightarrow$ $y-4=-3x-6 \Rightarrow y=-3x-2$.

104. The slope of $-2x+y=14$ is 2. The lines are perpendicular, so the slope of the new line is $-1/2$. The equation of the line through (0, 2) with slope $-1/2$ is $y=-\frac{1}{2}x+2$.

Applying the Concepts

105. a. The y-intercept represents the initial expenses.

b. The x-intercept represents the point at which the teacher breaks even, i.e., the expenses equal the income.

c. The teacher's profit if there are 16 students in the class is \$640.

d. The slope of the line is
$$\frac{640-(-750)}{16-0}=\frac{1390}{16}=\frac{695}{8}$$
The equation of the line is $P=\frac{695}{8}n-750$.

106. a. The y-intercept represents the initial prepaid amount.

b. The x-intercept represents the total number of minutes the cellphone can be used.

c. The slope of the line is $\frac{0-15}{75-0}=-\frac{15}{75}=-\frac{1}{5}$.

The equation of the line is $P=-\frac{1}{5}t+15$.

d. The cost per minute is $\$\frac{1}{5}=20¢$.

107. slope $=\frac{\text{rise}}{\text{run}} \Rightarrow \frac{4}{40}=\frac{1}{10}$

108. 4 miles = 21,120 feet.

$|\text{slope}|=\frac{\text{rise}}{\text{run}} \Rightarrow \frac{2000}{21{,}120}=\frac{25}{264}$

109. 8 in. in two weeks $\Rightarrow$ the plant grows 4 in. per week. John wants to trim the hedge when it grows 6 in., so he should trim it every $\frac{6}{4}=1.5$ weeks ≈ 10 days.

110. $\frac{2\text{ min.}}{5\text{ in.}}=\frac{x\text{ min.}}{31\text{ in.}} \Rightarrow x=\frac{2\cdot 31}{5}=12.4$ min.

The water will overflow in about 12 min.

111. a. x = the number of weeks; y = the amount of money in the account after x weeks; $y=7x+130$

b. The slope is the amount of money deposited each week; the y-intercept is the initial deposit.

112. a. x = the number of sessions of golf; y = the yearly payment to the club; $y=35x+1000$

b. The slope is the cost per golf session; the y-intercept is the yearly membership fee.

113. a. x = the number of months owed to pay off the refrigerator; y = the amount owed; $y=-15x+600$

b. The slope is the amount that the balance due changes per month; the y-intercept is the initial amount owed.

114. a. x = the number of rupees; y = the number of dollars equal to x rupees.

$y=\frac{1}{50.5}x=.019802x$.

b. The slope is the number of dollars per rupee. The y-intercept is the number of dollars for 0 rupees.

115. a. x = the number of years after 2010; y = the life expectancy of a female born in the year $2010+x$; $y=0.17x+80.8$

b. The slope is the rate of increase in life expectancy; the y-intercept is the current life expectancy.

116. a. $v = -1400(2) + 14,000 = \$11,200$

b. $v = -1400(6) + 14,000 = \$5600$
To find when the tractor will have no value, set $v = 0$ and solve the equation for t:
$0 = -1400t + 14,000 \Rightarrow t = 10$

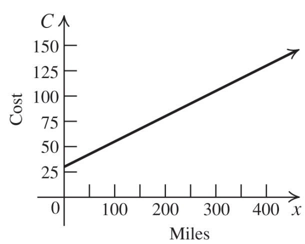
117. There are 30 days in June. For the first 13 days, you used data at a rate of $\frac{435}{13} \approx 33.5$ MB/day.
At the same rate, you will use $33.5(17) = 569.5$ MB for the rest of the month.
435 + 569.5 = 1004.5
So, you don't need to buy extra data. You will have about 20 MB left.

118. For the first three hours, you traveled at $\frac{195}{3} = 65$ mph.
$d = rt \Rightarrow 520 - 195 = 65t \Rightarrow 325 = 65t \Rightarrow$
$t = 5$
You will arrive at your destination five hours after 12 pm or 5 pm.

119. $y = 5x + 40,000$

120. a. $C = 0.25x + 30$

b.



c. $y = 0.25(60) + 30 = \$45$

d. $47.75 = 0.25x + 30 \Rightarrow x = 71$ miles

121. a. The two points are (100, 212) and (0, 32). So the slope is $\frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$.
The equation is
$F - 32 = \frac{9}{5}(C - 0) \Rightarrow F = \frac{9}{5}C + 32$

b. One degree Celsius change in the temperature equals $9/5$ degrees change in degrees Fahrenheit.

c.

C	$F = \frac{9}{5}C + 32$
40ºC	104ºF
25º C	77ºF
–5ºC	23ºF
–10ºC	14ºF

d. $100°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 37.78°\text{C}$

$90°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 32.22°\text{C}$

$75°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 23.89°\text{C}$

$-10°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = -23.33°\text{C}$

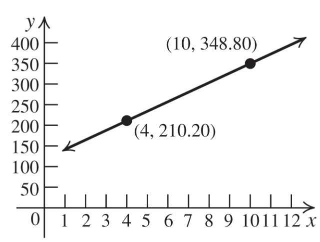
$-20°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = -28.89°\text{C}$

e. $97.6°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 36.44°\text{C}$;

$99.6°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 37.56°\text{C}$

f. Let x = °F = °C. Then $x = \frac{9}{5}x + 32 \Rightarrow$
$-\frac{4}{5}x = 32 \Rightarrow x = -40$. At –40°, °F = °C.

122. a. The two points are (4, 210.20) and (10, 348.80). So the slope is $\frac{348.80 - 210.20}{10-4} = \frac{138.6}{6} = 23.1$. The equation is $y - 348.8 = 23.1(x - 10) \Rightarrow$
$y = 23.1x + 117.8$

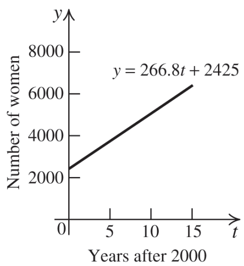


b. The slope represents the cost of producing one modem. The y-intercept represents the fixed cost.

c. $y = 23.1(12) + 117.8 \Rightarrow y = \$395$

123. a. The year 2005 is represented by $t = 0$, and the year 2011 is represented by $t = 6$. The points are (0, 2425) and (6, 4026). So the slope is $\frac{4026-2425}{6} \approx 266.8$ The equation is $y - 2425 = 266.8(t-0) \Rightarrow$ $y = 266.8t + 2425$

b.



c. The year 2008 is represented by $t = 3$. So $y = 266.8(3) + 2425 \Rightarrow y = 3225.4$. Note that there cannot be a fraction of a person, so. there were 3225 women prisoners in 2008.

d. The year 2017 is represented by $t = 12$. So $y = 266.8(12) + 2425 \Rightarrow y = 5626.6$. There will be 5627 women prisoners in 2017.

124. a. The two points are (5, 5.73) and (8, 6.27). The slope is $\frac{6.27-5.73}{8-5} = \frac{0.54}{3} = 0.18.$ The equation is $V - 5.73 = 0.18(x-5) \Rightarrow$ $V = 0.18x + 4.83.$

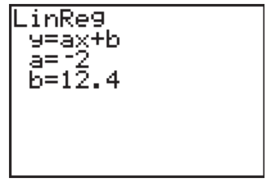
b. The slope represents the monthly change in the number of viewers. The V-intercept represents the number of viewers when the show first started.

c. $V = 0.18(11) + 4.83 \Rightarrow V = 6.81$ million

125. The independent variable t represents the number of years after 2000, with $t = 0$ representing 2000. The two points are (0, 11.7) and (5, 12.7). So the slope is $\frac{12.7-11.7}{5} = 0.2$. The equation is $p - 11.7 = 0.2(t-0) \Rightarrow$ $p = 0.2t + 11.7.$ The year 2010 is represented by $t = 10$. $p = 0.2(10) + 11.7 \Rightarrow p = 13.7\%$.

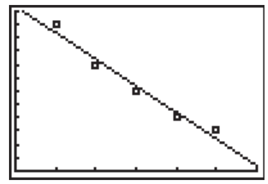
126. The year 2004 is represented by $t = 0$, so the year 2009 is represented by $t = 5$. The two points are (0, 82.7) and (5, 84.2). So the slope is $\frac{84.2-82.7}{5} = \frac{1.5}{5} = 0.3$. The equation is $y = 0.3t + 82.7.$

127. a.



$y = -2x + 12.4$

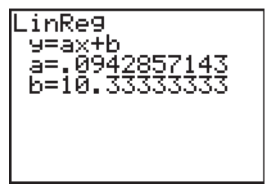
b.



[0, 6, 1] by [0, 12, 1]

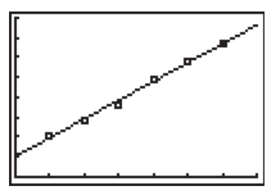
c. The price in the table is given as the number of nickels. 35¢ = 7 nickels, so let $x = 7$. $y = -2(7) + 12.4 = -1.6$ Thus, no newspapers will be sold if the price per copy is 35¢. Note that this is also clear from the graph, which appears to cross the x-axis at approximately $x = 6$.

128. a.



$y \approx 0.09x + 10.3$

b.



[0, 700, 100] by [0, 80, 10]

c. The advertising expenses in the table are given as thousands of dollars, so let $x = 700$. $y \approx 0.09(700) + 10.3 = 73.3$ Sales are given in thousands, so approximately $73.3 \times 1000 = 73{,}300$ computers will be sold.

Beyond the Basics

129. $3 = \frac{c-3}{1-(-2)} \Rightarrow 9 = c - 3 \Rightarrow c = 12$

130. First write the equation in slope-intercept form:

$3x - cy - 2 = 0 \Rightarrow -cy = -3x + 2 \Rightarrow y = \dfrac{3}{c}x - \dfrac{2}{c}$

Now solve for c by setting the y-intercept from the equation equal to –4: $-\dfrac{2}{c} = -4 \Rightarrow c = \dfrac{1}{2}$

131. a. Let $A = (0, 1)$, $B = (1, 3)$, $C = (-1, -1)$.

$m_{AB} = \dfrac{3-1}{1-0} = 2; m_{BC} = \dfrac{-1-3}{-1-1} = \dfrac{-4}{-2} = 2$

$m_{AC} = \dfrac{-1-1}{-1-0} = 2$

The slopes of the three segments are the same, so the points are collinear.

b. $d(A,B) = \sqrt{(1-0)^2 + (3-1)^2} = \sqrt{5}$

$d(B,C) = \sqrt{(-1-1)^2 + (-1-3)^2} = 2\sqrt{5}$

$d(A,C) = \sqrt{(-1-0)^2 + (-1-1)^2} = \sqrt{5}$

Because $d(B, C) = d(A, B) + d(A, C)$, the three points are collinear.

132. a. Let $A = (1, 0.5)$, $B = (2, 0)$, $C = (0.5, 0.75)$.

$m_{AB} = \dfrac{0-0.5}{2-1} = -0.5; m_{BC} = \dfrac{0.75-0}{0.5-2} = -0.5$

$m_{AC} = \dfrac{0.75-0.5}{0.5-1} = -0.5$

The slopes of the three segments are the same, so the points are collinear.

b. $d(A,B) = \sqrt{(1-2)^2 + \left(\dfrac{1}{2}-0\right)^2} = \sqrt{\dfrac{5}{4}} = \dfrac{\sqrt{5}}{2}$

$d(B,C) = \sqrt{\left(\dfrac{1}{2}-2\right)^2 + \left(\dfrac{3}{4}-0\right)^2}$

$= \sqrt{\dfrac{45}{16}} = \dfrac{3\sqrt{5}}{4}$

$d(A,C) = \sqrt{\left(\dfrac{1}{2}-1\right)^2 + \left(\dfrac{3}{4}-\dfrac{1}{2}\right)^2}$

$= \sqrt{\dfrac{5}{16}} = \dfrac{\sqrt{5}}{4}$

Because $d(B, C) = d(A, B) + d(A, C)$, the three points are collinear.

133. a. $m_{AB} = \dfrac{4-1}{-1-1} = -\dfrac{3}{2}; m_{BC} = \dfrac{8-4}{5-(-1)} = \dfrac{2}{3}$.

The product of the slopes = –1, so $AB \perp BC$.

b. $d(A,B) = \sqrt{(-1-1)^2 + (4-1)^2} = \sqrt{13}$

$d(B,C) = \sqrt{(5-(-1))^2 + (8-4)^2} = \sqrt{52}$

$d(A,C) = \sqrt{(5-1)^2 + (8-1)^2} = \sqrt{65}$

$\left(d(A,B)\right)^2 + \left(d(B,C)\right)^2 = \left(d(A,C)\right)^2$, so the triangle is a right triangle.

134. $m_{AB} = \dfrac{2-(-1)}{1-(-4)} = \dfrac{3}{5}; m_{BC} = \dfrac{1-2}{3-1} = -\dfrac{1}{2}$

$m_{CD} = \dfrac{-2-1}{-2-3} = \dfrac{3}{5}; m_{AD} = \dfrac{-2-(-1)}{-2-(-4)} = -\dfrac{1}{2}$

So, $AB \parallel CD$ and $BC \parallel AD$, and $ABCD$ is a parallelogram.

For exercises 135 and 136, refer to Figure 1.34 on page 30 in your text.

135. $\overline{AD}$ and $\overline{BC}$ are parallel because they lie on parallel lines l_1 and l_2. $\overline{AB}$ and $\overline{CD}$ are parallel because they are parallel to the x-axis. Therefore, $ABCD$ is a parallelogram. $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$ because opposite sides of a parallelogram are congruent.

$\triangle ABD \cong \triangle CDB$ by SSS. Then $m_1 = \dfrac{\text{rise}}{\text{run}} = \dfrac{BD}{CD}$ and $m_2 = \dfrac{\text{rise}}{\text{run}} = \dfrac{BD}{AB}$. Since $AB = CD$,

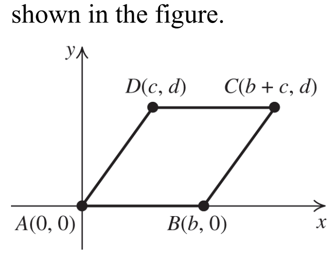
$m_1 = \dfrac{BD}{CD} = \dfrac{BD}{AB} = m_2$.

136. $\triangle OKA \sim \triangle BLO$ because $OL = AK = d$ and $BL = OK = c$. Then, $m_1 = \dfrac{\text{rise}}{\text{run}} = \dfrac{d}{c}$ and

$m_2 = \dfrac{\text{rise}}{\text{run}} = \dfrac{c}{-d} = -\dfrac{c}{d}$.

$m_1 \cdot m_2 = \dfrac{d}{c}\left(-\dfrac{c}{d}\right) = -1$.

137. Let the quadrilateral $ABCD$ be such that $AB \cong CD$ and $AB \parallel CD$. Locate the points as shown in the figure.



Because $AB \parallel CD$, the y-coordinates of C and D are equal.

(*continued on next page*)

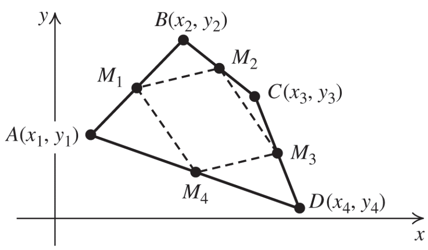
(*continued*)

Because $AB \cong CD$, the x-coordinates of the points are as shown in the figure. The slope of AD is d/c. The slope of BC is $\dfrac{d-0}{b+c-b} = \dfrac{d}{c}$.

So $AD \parallel BC$. $d(A, D) = \sqrt{d^2 + c^2}$.

$d(B, C) = \sqrt{d^2 + ((b+c)-b)^2} = \sqrt{d^2 + c^2}$. So $AD \cong BC$.

138. Let $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ and $D(x_4, y_4)$ be the vertices of the quadrilateral.



Then the midpoint M_1 of AB is $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$; the midpoint M_2 of BC is $\left(\dfrac{x_2+x_3}{2}, \dfrac{y_2+y_3}{2}\right)$; the midpoint M_3 of CD is $\left(\dfrac{x_3+x_4}{2}, \dfrac{y_3+y_4}{2}\right)$; and the midpoint M_4 of AD is $\left(\dfrac{x_1+x_4}{2}, \dfrac{y_1+y_4}{2}\right)$. The slope of M_1M_2 is

$$\frac{\frac{y_1+y_2}{2} - \frac{y_2+y_3}{2}}{\frac{x_1+x_2}{2} - \frac{x_2+x_3}{2}} = \frac{y_1-y_3}{x_1-x_3}.$$

The slope of M_2M_3 is

$$\frac{\frac{y_2+y_3}{2} - \frac{y_3+y_4}{2}}{\frac{x_2+x_3}{2} - \frac{x_3+x_4}{2}} = \frac{y_2-y_4}{x_2-x_4}.$$

The slope of M_3M_4 is

$$\frac{\frac{y_3+y_4}{2} - \frac{y_1+y_4}{2}}{\frac{x_3+x_4}{2} - \frac{x_1+x_4}{2}} = \frac{y_3-y_1}{x_3-x_1} = \frac{y_1-y_3}{x_1-x_3}.$$

The slope of M_1M_4 is

$$\frac{\frac{y_1+y_2}{2} - \frac{y_1+y_4}{2}}{\frac{x_1+x_2}{2} - \frac{x_1+x_4}{2}} = \frac{y_2-y_4}{x_2-x_4}.$$

So $M_1M_2 \parallel M_3M_4$ and $M_2M_3 \parallel M_1M_4$, and $M_1M_2M_3M_4$ is a parallelogram.

139. Let (x, y) be the coordinates of point B. Then

$d(A, B) = 12.5 = \sqrt{(x-2)^2 + (y-2)^2} \Rightarrow$

$(x-2)^2 + (y-2)^2 = 156.25$ and

$m_{AB} = \dfrac{4}{3} = \dfrac{y-2}{x-2} \Rightarrow 4(x-2) = 3(y-2) \Rightarrow$

$y = \dfrac{4}{3}x - \dfrac{2}{3}$. Substitute this into the first equation and solve for x:

$$(x-2)^2 + \left(\left(\frac{4}{3}x - \frac{2}{3}\right) - 2\right)^2 = 156.25$$
$$(x-2)^2 + \left(\frac{4}{3}x - \frac{8}{3}\right)^2 = 156.25$$
$$x^2 - 4x + 4 + \frac{16}{9}x^2 - \frac{64}{9}x + \frac{64}{9} = 156.25$$
$$9x^2 - 36x + 36 + 16x^2 - 64x + 64 = 1406.25$$
$$25x^2 - 100x - 1306.25 = 0$$

Solve this equation using the quadratic formula:

$$x = \frac{100 \pm \sqrt{100^2 - 4(25)(-1306.25)}}{2(25)} = \frac{100 \pm \sqrt{10{,}000 + 130{,}625}}{50} = \frac{100 \pm \sqrt{140{,}625}}{50} = \frac{100 \pm 375}{50} = 9.5 \text{ or } -5.5$$

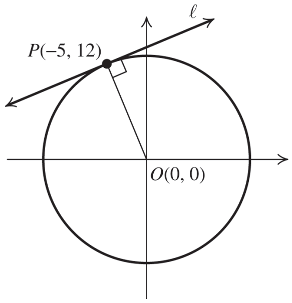
Now find y by substituting the x-values into the slope formula: $\dfrac{4}{3} = \dfrac{y-2}{9.5-2} \Rightarrow y = 12$ or $\dfrac{4}{3} = \dfrac{y-2}{-5.5-2} \Rightarrow y = -8$. So the coordinates of B are (9.5, 12) or (–5.5, –8).

140. Let (x, y) be a point on the circle with (x_1, y_1) and (x_2, y_2) as the endpoints of a diameter. Then the line that passes through (x, y) and (x_1, y_1) is perpendicular to the line that passes through (x, y) and (x_2, y_2), and their slopes are negative reciprocals. So $\dfrac{y-y_1}{x-x_1} = -\dfrac{x-x_2}{y-y_2} \Rightarrow$

$(y-y_1)(y-y_2) = -(x-x_1)(x-x_2) \Rightarrow$

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0.$

141.

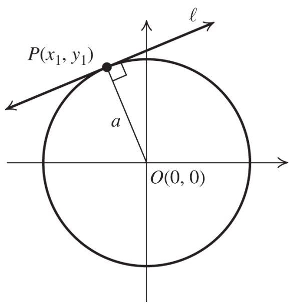


$$m_{\overline{OP}} = \frac{12-0}{-5-0} = -\frac{12}{5}$$

Since the tangent line ℓ is perpendicular to $\overline{OP}$, the slope of ℓ is the negative reciprocal of $-\frac{12}{5}$ or $\frac{5}{12}$. Using the point-slope form, we have

$$y-12 = \frac{5}{12}\left[x-(-5)\right] \Rightarrow y-12 = \frac{5}{12}(x+5) \Rightarrow$$
$$y-12 = \frac{5}{12}x + \frac{25}{12} \Rightarrow y = \frac{5}{12}x + \frac{169}{12}$$

142.



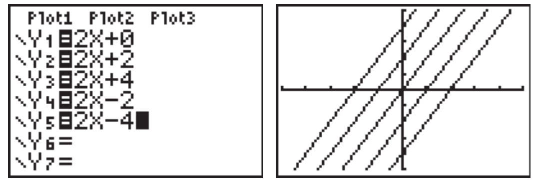
$$m_{\overline{OP}} = \frac{y_1-0}{x_1-0} = \frac{y_1}{x_1}$$

Since the tangent line ℓ is perpendicular to $\overline{OP}$, the slope of ℓ is the negative reciprocal of $\frac{y_1}{x_1}$ or $-\frac{x_1}{y_1}$. Using the point-slope form, we have

$$y-y_1 = -\frac{x_1}{y_1}(x-x_1) \Rightarrow$$
$$yy_1 - y_1^2 = -xx_1 + x_1^2 \Rightarrow xx_1 + yy_1 = x_1^2 + y_1^2$$

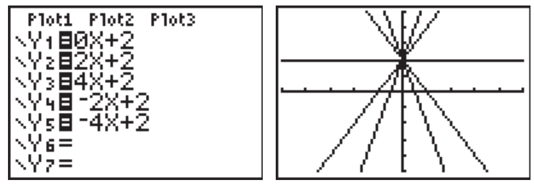
Since the equation of the circle is $x^2 + y^2 = a^2$, we substitute a^2 for $x_1^2 + y_1^2$ to obtain $xx_1 + yy_1 = a^2$.

143.



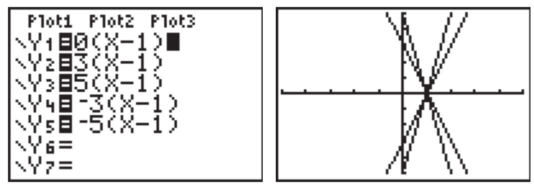
The family of lines has slope 2. The lines have different y-intercepts.

144.



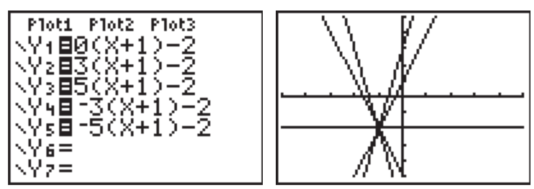
The family of lines has y-intercept 2. The lines have different slopes.

145.



The lines pass through (1, 0). The lines have different slopes.

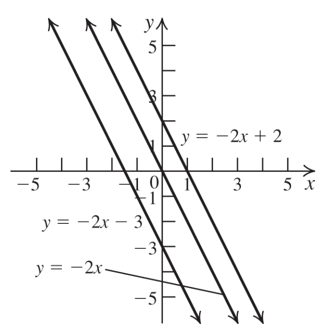
146.



The lines pass through (−1, −2). The lines have different slopes.

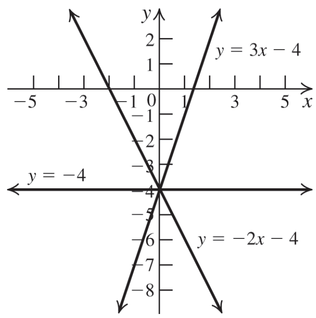
Critical Thinking/Discussion/Writing

147. a.



This is a family of lines parallel to the line $y = -2x$. They all have slope –2.

b.



This is a family of lines that passes through the point (0, –4). Their y-intercept is –4.

148. $\left.\begin{array}{l} y = m_1x + b_1 \\ y = m_2x + b_2 \end{array}\right\} \Rightarrow m_1x + b_1 = m_2x + b_2 \Rightarrow$

$m_1x - m_2x = b_2 - b_1 \Rightarrow x(m_1 - m_2) = b_2 - b_1 \Rightarrow$

$x = \dfrac{b_2 - b_1}{m_1 - m_2}$

a. If $m_1 > m_2 > 0$ and $b_1 > b_2$, then

$x = \dfrac{b_2 - b_1}{m_1 - m_2} = -\dfrac{b_1 - b_2}{m_1 - m_2}.$

b. If $m_1 > m_2 > 0$ and $b_1 < b_2$, then

$x = \dfrac{b_2 - b_1}{m_1 - m_2}.$

c. If $m_1 < m_2 < 0$ and $b_1 > b_2$, then

$x = \dfrac{b_2 - b_1}{m_1 - m_2} = \dfrac{b_1 - b_2}{m_2 - m_1}.$

d. If $m_1 < m_2 < 0$ and $b_1 < b_2$, then

$x = \dfrac{b_2 - b_1}{m_1 - m_2} = -\dfrac{b_2 - b_1}{m_2 - m_1}.$

Getting Ready for the Next Section

149. $x^2 - 4 = 0 \Rightarrow (x+2)(x-2) = 0 \Rightarrow$

$x + 2 = 0 \Rightarrow x = -2$ or

$x - 2 = 0 \Rightarrow x = 2$

Solution: {–2, 2}

150. $1 - x^2 = 0 \Rightarrow (1+x)(1-x) = 0 \Rightarrow$

$1 + x = 0 \Rightarrow x = -1$ or

$1 - x = 0 \Rightarrow x = 1$

Solution: {–1, 1}

151. $x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x + 1 = 0 \Rightarrow x = -1$ or $x - 2 = 0 \Rightarrow x = 2$

Solution: {–1, 2}

152. $x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x + 3 = 0 \Rightarrow x = -3$

$x - 1 = 0 \Rightarrow x = 1$

Solution: {–3, 1}

153. $$\begin{aligned}(3(a+h)+1) - (3a+1) &= (3a+3h+1) - (3a+1) \\ &= 3h\end{aligned}$$

154. $$\begin{aligned}&\left(2(a+h)^2+1\right) - \left(2a^2+1\right) \\ &= \left(2\left(a^2+2ah+h^2\right)+1\right) - \left(2a^2+1\right) \\ &= \left(2a^2+4ah+2h^2+1\right) - \left(2a^2+1\right) \\ &= 4ah + 2h^2\end{aligned}$$

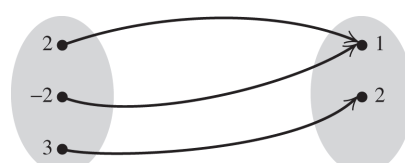
155. $$\begin{aligned}\frac{-(a+h)^2+a^2}{h} &= \frac{-\left(a^2+2ah+h^2\right)+a^2}{h} \\ &= \frac{-2ah-h^2}{h} = \frac{h(-2a-h)}{h} \\ &= -2a-h\end{aligned}$$

156. $$\begin{aligned}\frac{1}{h}\left(\frac{1}{a+h} - \frac{1}{a}\right) &= \frac{1}{h}\left(\frac{a-(a+h)}{a(a+h)}\right) = \frac{-h}{h(a(a+h))} \\ &= -\frac{1}{a(a+h)}\end{aligned}$$

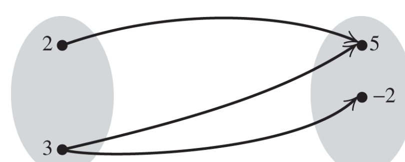
Section 1.3 Functions

1.3 Practice Problems

1. a. The domain of R is {2, −2, 3} and its range is {1, 2}. The relation R is a function because no two ordered pairs in R have the same first component.



b. The domain of S is {2, 3} and its range is {5, −2}. The relation S is not a function because the ordered paired (3, −2) and (3, 5) have the same first component.



2. Solve each equation for y.

a. $2x^2 - y^2 = 1 \Rightarrow 2x^2 - 1 = y^2 \Rightarrow$
$\pm\sqrt{2x^2 - 1} = y$; not a function

b. $x - 2y = 5 \Rightarrow x - 5 = 2y \Rightarrow \frac{1}{2}(x-5) = y$;
a function

3. a. $g(0) = -2(0)^2 + 5(0) = 0$

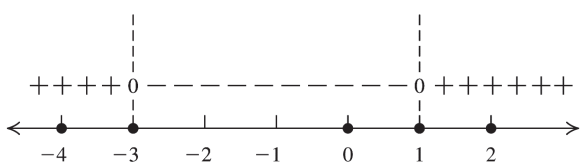
b. $g(-1) = -2(-1)^2 + 5(-1) = -7$

c. $g(x+h) = -2(x+h)^2 + 5(x+h)$
$= -2\left(x^2 + 2xh + h^2\right) + 5x + 5h$
$= -2x^2 - 4xh + 5x - 2h^2 + 5h$

4. $A_{TLMS} = (\text{length})(\text{height}) = (|3-1|)(22)$
$= (2)(22) = 44$ sq. units

5. a. $f(x) = \dfrac{1}{\sqrt{1-x}}$ is not defined when the denominator equals 0 or is less than 0. Since $\sqrt{1-x} \le 0 \Rightarrow x \ge 1$, the domain of f is $(-\infty, 1)$.

b. $g(x) = \sqrt{x^2 + 2x - 3}$ is not defined when
$x^2 + 2x - 3 < 0$.
Use the test point method to see that
$x^2 + 2x - 3 < 0$ on the interval (–3, 1).
Thus, the domain of g is $(-\infty, -3] \cup [1, \infty)$.



$x^2 + 2x - 3 = (x+3)(x-1) \ge 0$ or $(-\infty, -3] \cup [1, \infty)$

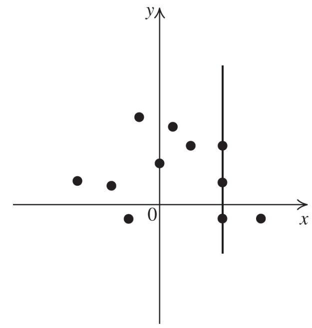
6. $f(x) = x^2$, domain $X = [-3, 3]$

a. $f(x) = 10 \Rightarrow x^2 = 10 \Rightarrow x = \pm\sqrt{10} \approx \pm 3.16$
Since $\sqrt{10} > 3$ and $-\sqrt{10} < -3$, neither solution is in the interval $X = [-3, 3]$. Therefore, 10 is not in the range of f.

b. $f(x) = 4 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$
Since $-3 < -2 < 2 < 3$, 4 is in the range of f.

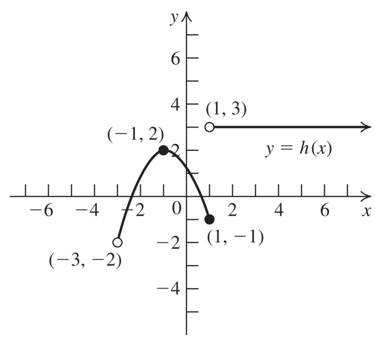
c. The range of f is the interval [0, 9] because for each number y in this interval, the number $x = \sqrt{y}$ is in the interval [−3, 3].

7.



The graph is not a function because a vertical line can be drawn through three points, as shown.

8.



Domain: $(-3, \infty)$; range: $(-2, 2] \cup \{3\}$

9. $y = f(x) = x^2 + 4x - 5$

a. Check whether the ordered pair (2, 7) satisfies the equation:
$7 \stackrel{?}{=} 2^2 + 4(2) - 5$
$7 = 7$ ✓
The point (2, 7) is on the graph.

b. Let $y = -8$, then solve for x:
$-8 = x^2 + 4x - 5 \Rightarrow 0 = x^2 + 4x + 3 \Rightarrow$
$0 = (x+3)(x+1) \Rightarrow x = -3$ or $x = -1$
The points (−3, −8) and (−1, −8) lie on the graph.

c. Let $x = 0$, then solve for y:
$y = 0^2 + 4(0) - 5 = -5$
The y-intercept is −5.

d. Let $y = 0$, then solve for x:
$0 = x^2 + 4x - 5 \Rightarrow 0 = (x+5)(x-1) \Rightarrow$
$x = -5$ or $x = 1$
The x-intercepts are −5 and 1.

10. $f(x)=1-x^2;\ a=2, b=4$
$f(2)=1-2^2=-3;\ f(4)=1-4^2=-15$
$$\frac{f(b)-f(a)}{b-a}=\frac{-15-(-3)}{4-2}=\frac{-12}{2}=-6$$
The average rate of change is −6.

11. $f(x)=1-x^2;\ a=c, b=c+h$
$f(c)=1-c^2$
$$f(c+h)=1-(c+h)^2=1-\left(c^2+2ch+h^2\right)=1-c^2-2ch-h^2$$
$$\frac{f(b)-f(a)}{b-a}=\frac{\left(1-c^2-2ch-h^2\right)-\left(1-c^2\right)}{(c+h)-c}=\frac{-2ch-h^2}{h}=-2c-h$$
The average rate of change is $-2c-h$.

12. $f(x)=-x^2+x-3$
$$f(x+h)=-(x+h)^2+(x+h)-3=-x^2-2xh-h^2+x+h-3$$
$$\frac{f(x+h)-f(x)}{h}=\frac{\left(-x^2-2xh-h^2+x+h-3\right)-\left(-x^2+x-3\right)}{h}=\frac{-2xh-h^2+h}{h}=-2x-h+1$$

13. The domain refers to the number of days, so the domain is $[1,\infty)$. The range is [6, 12).
$C(11)=\frac{1}{2}C(10)+6=\frac{1}{2}(11.988)+6=11.994$.

14. From Example 14 we have $AP=\sqrt{500^2+x^2}$ and $\overline{PD}=1200-x$ feet. If c = the cost on land, the total cost C is given by
$$C=1.3c(PD)+c(AP)=1.3c\sqrt{500^2+x^2}+c(1200-x)$$

1.3 Exercises
Concepts and Vocabulary

1. In the functional notation $y=f(x)$, x is the <u>independent</u> variable.
2. If $f(-2)=7$, then −2 is in the <u>domain</u> of the function f, and 7 is in the <u>range</u> of f.
3. If the point (9, −14) is on the graph of a function f, then $f(9)=\underline{-14}$.
4. The average rate of change of f as x changes from $x=a$ to $x=b$ is $\dfrac{f(b)-f(a)}{b-a}$, $a\neq b$.
5. False. Every function is a relation but not every relation is a function.
6. False. If no **vertical** line intersects the graph of a relation at more than one point, then graph of the relation is the graph of a function.
7. True.
8. True

Building Skills

9. Domain: $\{a, b, c\}$; range: $\{d, e\}$; function
10. Domain: $\{a, b, c\}$; range: $\{d, e, f\}$; function
11. Domain: $\{a, b, c\}$; range: $\{1, 2\}$; function
12. Domain: $\{1, 2, 3\}$; range: $\{a, b, c, d\}$; not a function
13. Domain: $\{-3, -1, 0, 1, 2, 3\}$; range: $\{-8, -3, 0, 1\}$; function
14. Domain: $\{0, 3, 8\}$; range: $\{-3, -2, -1, 1, 2\}$; not a function
15. $x+y=2\Rightarrow y=-x+2$; a function
16. $x=y-1\Rightarrow y=x+1$; a function
17. $y=\dfrac{1}{x}$; a function
18. $xy=-1\Rightarrow y=-\dfrac{1}{x}$; a function
19. $x=|y|\Rightarrow y=x$ or $y=-x$; not a function
20. $x=|y-1|$; not a function
21. $y=\dfrac{1}{\sqrt{2x-5}}$; a function
22. $y=\dfrac{1}{\sqrt{x^2-1}}$; a function
23. $2-y=3x\Rightarrow y=2-3x$; a function
24. $3x-5y=15\Rightarrow y=\dfrac{3}{5}x-3$; a function
25. $x^2+y^2=8\Rightarrow y=\pm\sqrt{-x^2+8}$; not a function
26. $x=y^2\Rightarrow y=\sqrt{x}$ or $y=-\sqrt{x}$; not a function

27. $x^2 + y^3 = 5 \Rightarrow y = \sqrt[3]{5 - x^2}$; a function

28. $x + y^3 = 8 \Rightarrow y = \sqrt[3]{8 - x}$; a function

In exercises 29–32, $f(x) = x^2 - 3x + 1$, $g(x) = \dfrac{2}{\sqrt{x}}$, and $h(x) = \sqrt{2 - x}$.

29. $f(0) = 0^2 - 3(0) + 1 = 1$

$g(0) = \dfrac{2}{\sqrt{0}} \Rightarrow g(0)$ is undefined

$h(0) = \sqrt{2 - 0} = \sqrt{2}$

$f(a) = a^2 - 3a + 1$

$f(-x) = (-x)^2 - 3(-x) + 1 = x^2 + 3x + 1$

30. $f(1) = 1^2 - 3(1) + 1 = -1; g(1) = \dfrac{2}{\sqrt{1}} = 2;$

$h(1) = \sqrt{2 - 1} = 1; g(a) = \dfrac{2}{\sqrt{a}};$

$g(x^2) = \dfrac{2}{\sqrt{x^2}} = \dfrac{2}{|x|}$

31. $f(-1) = (-1)^2 - 3(-1) + 1 = 5;$

$g(-1) = \dfrac{2}{\sqrt{-1}} \Rightarrow g(-1)$ is undefined;

$h(-1) = \sqrt{2 - (-1)} = \sqrt{3};\ h(c) = \sqrt{2 - c}$

$h(-x) = \sqrt{2 - (-x)} = \sqrt{2 + x}$

32. $f(4) = 4^2 - 3(4) + 1 = 5; g(4) = \dfrac{2}{\sqrt{4}} = 1;$

$h(4) = \sqrt{2 - 4} = \sqrt{-2} \Rightarrow h(4)$ is undefined;

$g(2 + k) = \dfrac{2}{\sqrt{2 + k}};$

$$\begin{aligned} f(a + k) &= (a + k)^2 - 3(a + k) + 1 \\ &= a^2 + 2ak - 3a + k^2 - 3k + 1 \end{aligned}$$

33. **a.** $f(0) = \dfrac{2(0)}{\sqrt{4 - 0^2}} = 0$

b. $f(1) = \dfrac{2(1)}{\sqrt{4 - 1^2}} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

c. $f(2) = \dfrac{2(2)}{\sqrt{4 - 2^2}} = \dfrac{4}{0} \Rightarrow f(2)$ is undefined

d. $f(-2) = \dfrac{2(-2)}{\sqrt{4 - (-2)^2}} = \dfrac{-4}{0} \Rightarrow f(-2)$ is undefined

e. $f(-x) = \dfrac{2(-x)}{\sqrt{4 - (-x)^2}} = \dfrac{-2x}{\sqrt{4 - x^2}}$

34. **a.** $g(0) = 2(0) + \sqrt{0^2 - 4} \Rightarrow g(0)$ is undefined

b. $g(1) = 2(1) + \sqrt{1^2 - 4} \Rightarrow g(1)$ is undefined

c. $g(2) = 2(2) + \sqrt{2^2 - 4} = 4$

d. $g(-3) = 2(-3) + \sqrt{(-3)^2 - 4} = -6 + \sqrt{5}$

e. $$\begin{aligned} g(-x) &= 2(-x) + \sqrt{(-x)^2 - 4} \\ &= -2x + \sqrt{x^2 - 4} \end{aligned}$$

35. $$\begin{aligned} A &= (1)(f(0)) + (1)(f(1)) \\ &= 1(2) + (1)(3) = 5 \text{ sq. units} \end{aligned}$$

36. $$\begin{aligned} A &= (1)(f(1)) + (1)(f(2)) \\ &= 1(3) + (1)(6) = 9 \text{ sq. units} \end{aligned}$$

37. $(-\infty, \infty)$

38. $(-\infty, \infty)$

39. The denominator is not defined for $x = 9$. The domain is $(-\infty, 9) \cup (9, \infty)$

40. The denominator is not defined for $x = -9$. The domain is $(-\infty, -9) \cup (-9, \infty)$

41. The denominator is not defined for $x = -1$ or $x = 1$. The numerator is defined for all values of x. The domain is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

42. The denominator is not defined for $x = -2$ or $x = 2$. The domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

43. The denominator is not defined for $x \geq 4$. The domain is $(-\infty, 4)$.

44. The denominator is not defined for $x \leq -2$. The domain is $(-2, \infty)$.

45. The denominator equals 0 if $x = 2$. The numerator is not defined if $x < 1$. The domain is $[1, 2) \cup (2, \infty)$.

46. The denominator equals 0 if $x = 1$. The numerator is not defined if $x \geq 2$. The domain is $(-\infty, 1) \cup (1, 2]$.

47. The denominator equals 0 if $x = -1$ or $x = -2$. The numerator is defined for all values of x. The domain is $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$.

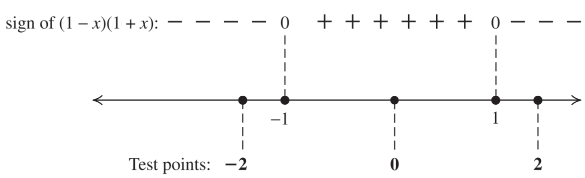
48. The denominator equals 0 if $x = -2$ or $x = -3$. The numerator is defined for all values of x. The domain is $(-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$.

49. The denominator is not defined for $x = 0$. The numerator is defined for all values of x. The domain is $(-\infty, 0) \cup (0, \infty)$.

50. The denominator is defined for all values of x. The domain is $(-\infty, \infty)$.

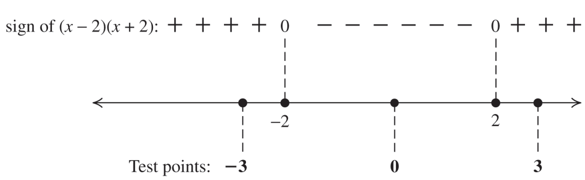
51. $1 - x^2 \geq 0 \Rightarrow (1 + x)(1 - x) \geq 0$

Using the test-point method, we see that this is true on the interval $[-1, 1]$. Thus, the domain is $[-1, 1]$.

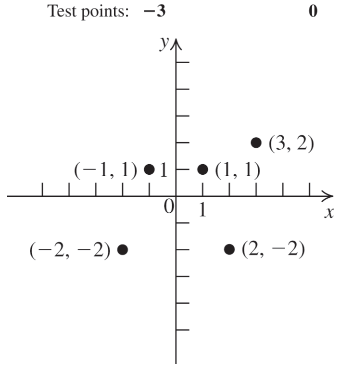


52. $x^2 - 4 \geq 0 \Rightarrow (x + 2)(x - 2) \geq 0$

Using the test-point method, we see that this is true on the intervals $(-\infty, -2]$ and $[2, \infty)$. Thus, the domain is $(-\infty, -2] \cup [2, \infty)$.

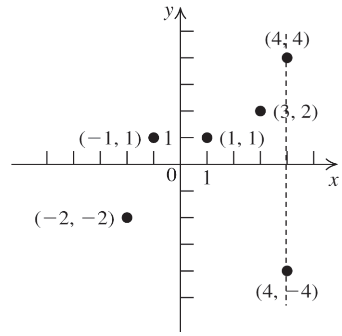


53.



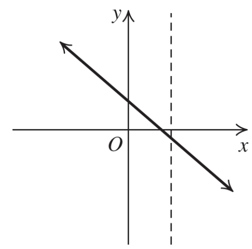
a function

54.



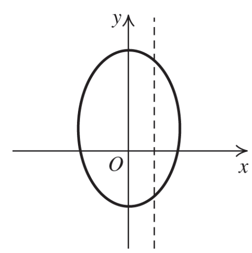
not a function

55.



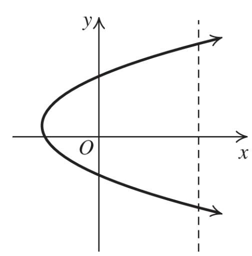
a function

56.



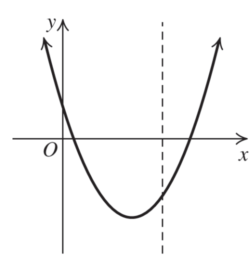
not a function

57.



not a function

58.



a function

59. $f(-4) = -2; f(-1) = 1; f(3) = 5; f(5) = 7$

60. $g(-2) = 5; g(1) = -4; g(3) = 0; g(4) = 5$

61. $h(-2) = -5; h(-1) = 4; h(0) = 3; h(1) = 4$

62. $f(-1)=4; f(0)=0; f(1)=-4$

63. $h(x)=7$, so solve the equation $7=x^2-x+1$.
$x^2-x-6=0 \Rightarrow (x-3)(x+2)=0 \Rightarrow x=-2$ or $x=3$.

64. $H(x)=7$, so solve the equation $7=x^2+x+8$.
$x^2+x+1=0 \Rightarrow x=\dfrac{-1\pm\sqrt{1-4(1)(1)}}{2(1)} \Rightarrow$
$x=\dfrac{-1\pm\sqrt{-3}}{2} \Rightarrow$ there is no real solution.
Therefore, there is no point $(x, 7)$ that lies on the graph of H.

65. **a.** $1=-2(1+1)^2+7 \Rightarrow 1=-1,$ which is false. Therefore, (1, 1) does not lie on the graph of f.

b. $1=-2(x+1)^2+7 \Rightarrow 2(x+1)^2=6 \Rightarrow$
$(x+1)^2=3 \Rightarrow x+1=\pm\sqrt{3} \Rightarrow x=-1\pm\sqrt{3}$
The points $\left(-1-\sqrt{3}, 1\right)$ and $\left(-1+\sqrt{3}, 1\right)$ lie on the graph of f.

c. $y=-2(0+1)^2+7 \Rightarrow y=5$
The y-intercept is (0, 5).

d. $0=-2(x+1)^2+7 \Rightarrow -7=-2(x+1)^2 \Rightarrow$
$\dfrac{7}{2}=(x+1)^2 \Rightarrow \pm\sqrt{\dfrac{7}{2}}=x+1 \Rightarrow$
$x=-1\pm\sqrt{\dfrac{7}{2}}=-1\pm\dfrac{\sqrt{14}}{2}$
The x-intercepts are $\left(-1-\dfrac{\sqrt{14}}{2}, 0\right)$ and $\left(-1+\dfrac{\sqrt{14}}{2}, 0\right)$.

66. **a.** $10=-3(-2)^2-12(-2) \Rightarrow 10=12,$ which is false. Therefore, (−2, 10) does not lie on the graph of f.

b. $g(x)=12$, so solve the equation
$-3x^2-12x=12$.
$-3x^2-12x=12 \Rightarrow x^2+4x=-4 \Rightarrow$
$x^2+4x+4=0 \Rightarrow (x+2)^2=0 \Rightarrow$
$x+2=0 \Rightarrow x=-2$

c. $y=-3(0)^2-12(0) \Rightarrow y=0$
The y-intercept is (0, 0).

d. $0=-3x^2-12x \Rightarrow 0=-3x(x+4) \Rightarrow$
$x=0$ or $x=-4$
The x-intercepts are (0, 0) and (−4, 0).

67. domain: $[-3, 2]$, range: $[-3, 3]$

68. domain: $[-1, 3]$, range: $[-2, 4]$

69. domain: $[-4, 4]$, range: $[-2, 3]$

70. domain: $[-4, 4]$, range: $[-3, 3]$

71. $f(x)=-2x+7;\ a=-1,\ b=3$
$f(3)=-2(3)+7=1;\ f(-1)=-2(-1)+7=9$
$$\text{average rate of change}=\frac{f(3)-f(-1)}{3-(-1)}=\frac{1-9}{4}=-2$$

72. $f(x)=4x-9;\ a=-2,\ b=2$
$f(2)=4(2)-9=-1;\ f(-2)=4(-2)-9=-17$
$$\text{average rate of change}=\frac{f(2)-f(-2)}{2-(-2)}=\frac{-1-(-17)}{4}=4$$

73. $g(x)=2x^2;\ a=0,\ b=5$
$g(5)=2\cdot5^2=50;\ g(0)=2\cdot0^2=0$
$$\text{average rate of change}=\frac{g(5)-g(0)}{5-0}=\frac{50-0}{5}=10$$

74. $g(x)=-4x^2;\ a=-1,\ b=4$
$g(4)=-4\cdot4^2=-64;\ g(-1)=-4(-1)^2=-4$
$$\text{average rate of change}=\frac{g(4)-g(-1)}{4-(-1)}=\frac{-64-(-4)}{5}=-12$$

75. $h(x)=x^2-1;\ a=-2,\ b=0$
$h(0)=0^2-1=-1;\ h(-2)=(-2)^2-1=3$
$$\text{average rate of change}=\frac{h(0)-h(-2)}{0-(-2)}=\frac{-1-3}{2}=-2$$

76. $h(x) = 2 - x^2;\ a = 3,\ b = 4$
$h(4) = 2 - 4^2 = -14;\ h(3) = 2 - 3^2 = -7$
$$\text{average rate of change} = \frac{h(4) - h(3)}{4 - 3} = \frac{-14 - (-7)}{1} = -7$$

77. $f(x) = (3 - x)^2;\ a = 1,\ b = 3$
$f(4) = (3 - 3)^2 = 0;\ f(1) = (3 - 1)^2 = 4$
$$\text{average rate of change} = \frac{f(3) - f(1)}{3 - 1} = \frac{0 - 4}{2} = -2$$

78. $f(x) = (x - 2)^2;\ a = -1,\ b = 5$
$f(5) = (5 - 2)^2 = 9;\ f(-1) = (-1 - 2)^2 = 9$
$$\text{average rate of change} = \frac{f(5) - f(-1)}{5 - (-1)} = \frac{9 - 9}{6} = 0$$

79. $g(x) = x^3;\ a = -1,\ b = 3$
$g(3) = 3^3 = 27;\ g(-1) = (-1)^3 = -1$
$$\text{average rate of change} = \frac{g(3) - g(-1)}{3 - (-1)} = \frac{27 - (-1)}{4} = 7$$

80. $g(x) = -x^3;\ a = -1,\ b = 3$
$g(3) = -3^3 = -27;\ g(-1) = -(-1)^3 = 1$
$$\text{average rate of change} = \frac{g(3) - g(-1)}{3 - (-1)} = \frac{-27 - 1}{4} = -7$$

81. $h(x) = \frac{1}{x};\ a = 2,\ b = 6$
$h(2) = \frac{1}{2};\ h(6) = \frac{1}{6}$
$$\text{average rate of change} = \frac{h(6) - h(2)}{6 - 2} = \frac{\frac{1}{6} - \frac{1}{2}}{4} = -\frac{1}{12}$$

82. $h(x) = \frac{4}{x + 3};\ a = -2,\ b = 4$
$h(4) = \frac{4}{4 + 3} = \frac{4}{7};\ h(-2) = \frac{4}{-2 + 3} = 4$
$$\text{average rate of change} = \frac{h(4) - h(-2)}{4 - (-2)} = \frac{\frac{4}{7} - 4}{6} = -\frac{4}{7}$$

83. $f(x) = 2x,\ \ a = 3$
$$\frac{f(x) - f(a)}{x - a} = \frac{2x - 2(3)}{x - 3} = \frac{2(x - 3)}{x - 3} = 2$$

84. $f(x) = 3x + 2,\ a = 2$
$$\frac{f(x) - f(a)}{x - a} = \frac{(3x + 2) - (3(2) + 2)}{x - 2} = \frac{3x - 6}{x - 2} = \frac{3(x - 2)}{x - 2} = 3$$

85. $f(x) = -x^2,\ a = 1$
$$\frac{f(x) - f(a)}{x - a} = \frac{-x^2 - \left[-(1^2)\right]}{x - 1} = \frac{-x^2 + 1}{x - 1} = \frac{-(x^2 - 1)}{x - 1} = \frac{-(x - 1)(x + 1)}{x - 1} = -(x + 1) = -x - 1$$

86. $f(x) = 2x^2,\ a = -1$
$$\frac{f(x) - f(a)}{x - a} = \frac{2x^2 - 2(-1)^2}{x - (-1)} = \frac{2x^2 - 2}{x + 1} = \frac{2(x - 1)(x + 1)}{x + 1} = 2(x - 1) = 2x - 2$$

87. $f(x) = 3x^2 + x,\ a = 2$
$$\frac{f(x) - f(a)}{x - a} = \frac{(3x^2 + x) - (3(2)^2 + 2)}{x - 2} = \frac{3x^2 + x - 14}{x - 2} = \frac{(3x + 7)(x - 2)}{x - 2} = 3x + 7$$

88. $f(x) = -2x^2 + x,\ a = 3$

$$\frac{f(x)-f(a)}{x-a} = \frac{\left(-2x^2+x\right)-\left(-2(3)^2+3\right)}{x-3} = \frac{-2x^2+x+15}{x-3} = \frac{(-2x-5)(x-3)}{x-3} = -2x-5$$

89. $f(x) = \frac{4}{x},\ a = 1$

$$\frac{f(x)-f(a)}{x-a} = \frac{\frac{4}{x}-\frac{4}{1}}{x-1} = \frac{\frac{4}{x}-4}{x-1} = \frac{\frac{4-4x}{x}}{x-1} = \frac{4-4x}{x(x-1)} = \frac{-4(x-1)}{x(x-1)} = -\frac{4}{x}$$

90. $f(x) = -\frac{4}{x},\ a = 1$

$$\frac{f(x)-f(a)}{x-a} = \frac{-\frac{4}{x}-\left(-\frac{4}{1}\right)}{x-1} = \frac{-\frac{4}{x}+4}{x-1} = \frac{\frac{-4+4x}{x}}{x-1} = \frac{4x-4}{x(x-1)} = \frac{4(x-1)}{x(x-1)} = \frac{4}{x}$$

91. $f(x+h) = -2(x+h)+3 = -2x-2h+3$

$f(x+h)-f(x) = -2x-2h+3-(-2x+3) = -2h$

$$\frac{f(x+h)-f(x)}{h} = \frac{-2h}{h} = -2$$

92. $f(x+h) = 3(x+h)+2 = 3x+3h+2$

$f(x+h)-f(x) = 3x+3h+2-(3x+2) = 3h$

$$\frac{f(x+h)-f(x)}{h} = \frac{3h}{h} = 3$$

93. $f(x+h) = (x+h)^2 = x^2+2xh+h^2$

$f(x+h)-f(x) = x^2+2xh+h^2-x^2 = 2xh+h^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2}{h} = 2x+h$$

94. $f(x+h) = (x+h)^2-(x+h) = x^2+2xh+h^2-x-h$

$f(x+h)-f(x)$
$= \left(x^2+2xh+h^2-x-h\right)-\left(x^2-x\right)$
$= 2xh+h^2-h = h(2x+h-1)$

$$\frac{f(x+h)-f(x)}{h} = \frac{h(2x+h-1)}{h} = 2x+h-1$$

95. $f(x+h) = 3(x+h)^2-2(x+h)+5$
$= 3x^2+6xh+3h^2-2x-2h+5$
$= 3x^2+6xh-2x+3h^2-2h+5$

$f(x+h)-f(x) = 3x^2+6xh-2x+3h^2-2h+5-(3x^2-2x+5)$
$= 6xh+3h^2-2h$

$$\frac{f(x+h)-f(x)}{h} = \frac{6xh+3h^2-2h}{h} = 6x+3h-2$$

96. $f(x+h) = 2(x+h)^2+3(x+h) = 2x^2+4xh+2h^2+3x+3h$

$f(x+h)-f(x)$
$= \left(2x^2+4xh+2h^2+3x+3h\right)-\left(2x^2+3x\right)$
$= 4xh+2h^2+3h = h(4x+2h+3)$

$$\frac{f(x+h)-f(x)}{h} = \frac{h(4x+2h+3)}{h} = 4x+2h+3$$

97. $f(x+h) = 4$

$f(x+h)-f(x) = 4-4 = 0$

$$\frac{f(x+h)-f(x)}{h} = \frac{0}{h} = 0$$

98. $f(x+h) = -3$

$f(x+h)-f(x) = -3-(-3) = 0$

$$\frac{f(x+h)-f(x)}{h} = \frac{0}{h} = 0$$

99. $f(x+h) = \frac{1}{x+h}$

$$f(x+h)-f(x) = \frac{1}{x+h}-\frac{1}{x} = \frac{x}{x(x+h)}-\frac{x+h}{x(x+h)} = -\frac{h}{x(x+h)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-\frac{h}{x(x+h)}}{h} = -\frac{1}{x(x+h)}$$

100. $f(x+h) = -\dfrac{1}{x+h}$

$$f(x+h) - f(x) = -\frac{1}{x+h} - \left(-\frac{1}{x}\right)$$
$$= -\frac{x}{x(x+h)} + \frac{x+h}{x(x+h)}$$
$$= \frac{h}{x(x+h)}$$
$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{h}{x(x+h)}}{h} = \frac{1}{x(x+h)}$$

Applying the Concepts

101. a function, because there is only one high temperature per day.

102. a function because the cost of a first-class stamp on January 1 each year is a single value.

103. not a function because there are several states that begin with N (i.e., New York, New Jersey, New Mexico, Nevada, North Carolina, North Dakota); there are also several states that begin with T and S.

104. not a function because people with the same name may have the same birthday.

105. $A(x) = x^2; A(4) = 16;$ $A(4)$ represents the area of a tile with side 4.

106. $V(x) = x^3; V(3) = 27$ in.^{3}; $V(3)$ represents the volume of a cube with edge 3.

107. It is a function. $S(x) = 6x^2; S(3) = 54$

108. $f(x) = \dfrac{x}{39.37}; f(59) \approx 1.5$ meters

109. a. The domain is [0, 8].

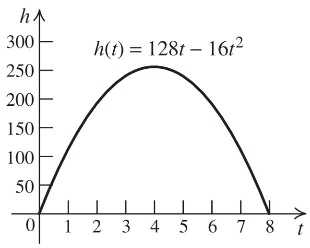
b. $h(2) = 128(2) - 16(2^2) = 192$

$h(4) = 128(4) - 16(4^2) = 256$

$h(6) = 128(6) - 16(6^2) = 192$

c. $0 = 128t - 16t^2 \Rightarrow 0 = 16t(8-t) \Rightarrow$ $t = 0$ or $t = 8$. It will take 8 seconds for the stone to hit the ground.

d.



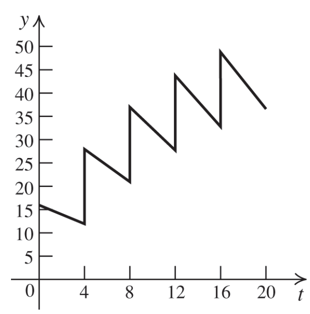
110. a. average rate of change $= \dfrac{f(2004) - f(2000)}{2004 - 2000}$
$= \dfrac{195.2 - 147.3}{4}$
$= 11.975$

The average rate of change of a median-priced home from 2000–2004 was $11,975. This gives 147.3 + 2(11.975) = $171.25 thousand as the price of a home in 2002. This is more than the actual price of $167.6 thousand.

b. average rate of change $= \dfrac{f(2014) - f(2010)}{2014 - 2010}$
$= \dfrac{208.5 - 173.2}{4}$
$= 8.825$

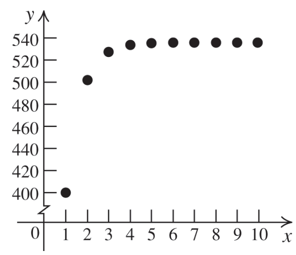
The average rate of change of a median-priced home from 2010–2014 was $8.825 thousand. This gives 208.5 + 8.825 = $217.325 thousand as the price of a home in 2015.

111. After 4 hours, there are (0.75)(16) = 12 ml of the drug. After 8 hours, there are (0.75)(12 + 16) = 21 ml. After 12 hours, there are (0.75)(21 + 16) = 27.75 ml. After 16 hours, there are (0.75)(27.75 + 16) = 32.81 ml. After 20 hours, there are (0.75)(32.81 + 16) = 36.61 ml.



112.

Day	Maximum Concentration
1	(0.8)(500) = 400 mg
2	(0.25)(400) + (0.8)(500) = 500 mg
3	(0.25)(500) + (0.8)(500) = 525 mg
4	(0.25)(525) + (0.8)(500) = 531.25 mg
5	(0.25)(531.25) + (0.8)(500) = 532.81 mg
6	(0.25)(532.81) + (0.8)(500) = 533.20 mg
7	(0.25)(533.20) + (0.8)(500) = 533.30 mg
8	(0.25)(533.33) + (0.8)(500) = 533.33 mg
9	(0.25)(533.33) + (0.8)(500) = 533.33 mg
10	(0.25)(533.33) + (0.8)(500) = 533.33 mg



113. $x + y = 28 \Rightarrow y = 28 - x$

$P = x(28 - x) = 28x - x^2$

114. $P = 60 = 2(x + y) \Rightarrow 30 = x + y \Rightarrow y = 30 - x$

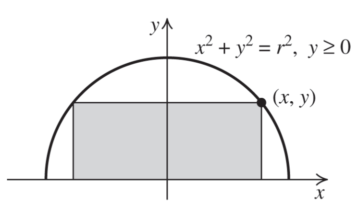
$A = x(30 - x) = 30x - x^2$

115. Note that the length of the base = the width of the base = x.

$V = lwh = x^2h = 64 \Rightarrow h = \dfrac{64}{x^2}$

$$\begin{aligned} S &= 2lw + 2lh + 2wh \\ &= 2x^2 + 2x\left(\frac{64}{x^2}\right) + 2x\left(\frac{64}{x^2}\right) \\ &= 2x^2 + \frac{128}{x} + \frac{128}{x} \\ &= 2x^2 + \frac{256}{x} \end{aligned}$$

116.



a. $x^2 + y^2 = r^2 \Rightarrow y^2 = r^2 - x^2 \Rightarrow y = \sqrt{r^2 - x^2}$

The length of the rectangle is $2x$ and its height is $y = \sqrt{r^2 - x^2}$.

$$\begin{aligned} P &= 2l + 2w = 2(2x) + 2\sqrt{r^2 - x^2} \\ &= 4x + 2\sqrt{r^2 - x^2} \end{aligned}$$

b. $A = lw = 2x\sqrt{r^2 - x^2}$

117. The piece with length x is formed into a circle, so $C = x = 2\pi r \Rightarrow r = \dfrac{x}{2\pi}$. Thus, the area of the circle is $A = \pi r^2 = \pi\left(\dfrac{x}{2\pi}\right)^2 = \dfrac{x^2}{4\pi}$.

The piece with length $20 - x$ is formed into a square, so $P = 20 - x = 4s \Rightarrow s = \dfrac{1}{4}(20 - x)$.

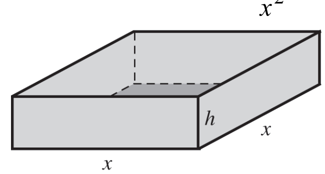
Thus, the area of the square is $s^2 = \left[\dfrac{1}{4}(20 - x)\right]^2 = \dfrac{1}{16}(20 - x)^2$. The sum of the areas is $A = \dfrac{x^2}{4\pi} + \dfrac{1}{16}(20 - x)^2$

118. The volume of the tank is $V = 64 = \pi r^2 h$, so $h = \dfrac{64}{\pi r^2}$. The top is open, so the surface area is given by

$$\begin{aligned} S &= \pi r^2 + 2\pi r h = \pi r^2 + 2\pi r\left(\frac{64}{\pi r^2}\right) \\ &= \pi r^2 + \frac{128}{r} \end{aligned}$$

119. The volume of the pool is

$V = 288 = x^2h \Rightarrow h = \dfrac{288}{x^2}$.



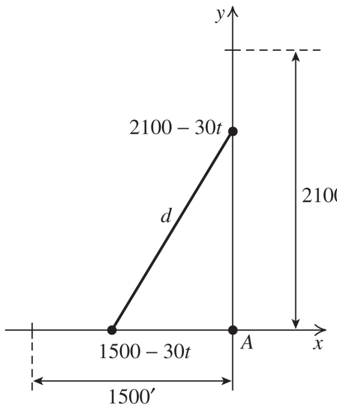
The total area to be tiled is

$4xh = 4\left(\dfrac{288}{x^2}\right) = \dfrac{1152}{x}$

The cost of the tile is $6\left(\dfrac{1152}{x}\right) = \dfrac{6912}{x}$.

The area of the bottom of the pool is x^2, so the cost of the cement is $2x^2$. Therefore, the total cost is $C = 2x^2 + \dfrac{6912}{x}$.

120.

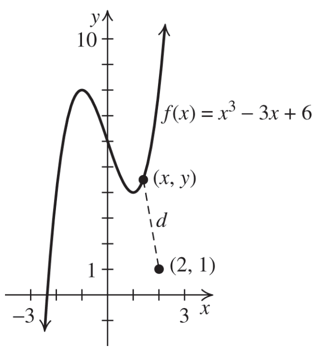


Using the Pythagorean theorem, we have

$d^2 = (1500 - 30t)^2 + (2100 - 30t)^2 \Rightarrow$

$d = \left[(1500 - 30t)^2 + (2100 - 30t)^2\right]^{1/2}$

121.



Using the distance formula we have

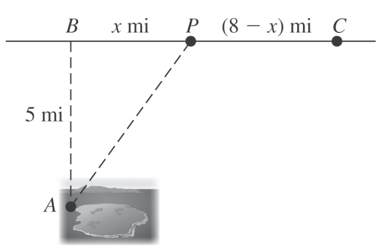
$d = \sqrt{(x-2)^2 + (y-1)^2}$

$= \sqrt{(x-2)^2 + \left[\left(x^2 - 3x + 6\right) - 1\right]^2}$

$= \sqrt{(x-2)^2 + \left(x^2 - 3x + 5\right)^2}$

$= \left[(x-2)^2 + \left(x^2 - 3x + 5\right)^2\right]^{1/2}$

122.



The distance from A to P is $\sqrt{x^2 + 5^2} = \sqrt{x^2 + 25}$ mi. At 4 mi/hr, it will take Julio $\dfrac{\sqrt{x^2 + 25}}{4}$ hr to row that distance.

The distance from P to C is $(8 - x)$ mi, so it will take Julio $\dfrac{8 - x}{5}$ hr to walk that distance. The total time it will take him to travel is

$T = \dfrac{\sqrt{x^2 + 25}}{4} + \dfrac{8 - x}{5}.$

Beyond the Basics

123. $x = \dfrac{y}{y-1} \Rightarrow xy - x = y \Rightarrow -x = y - xy \Rightarrow$

$-x = y(1-x) \Rightarrow -\dfrac{x}{1-x} = y \Rightarrow \dfrac{x}{x-1} = y \Rightarrow$

$f(x) = \dfrac{x}{x-1}$; Domain: $(-\infty, 1) \cup (1, \infty)$.

$f(4) = \dfrac{4}{3}.$

124. $xy = x - y \Rightarrow x = xy + y \Rightarrow x = y(x+1) \Rightarrow$

$y = f(x) = \dfrac{x}{x+1}$

Domain: $(-\infty, -1) \cup (-1, \infty)$

$f(4) = \dfrac{4}{5}.$

125. $x = \dfrac{2}{y-4} \Rightarrow xy - 4x = 2 \Rightarrow xy = 2 + 4x \Rightarrow$

$y = \dfrac{4x+2}{x} \Rightarrow f(x) = \dfrac{4x+2}{x}$;

Domain: $(-\infty, 0) \cup (0, \infty)$. $f(4) = \dfrac{9}{2}.$

126. $xy - 3 = 2y \Rightarrow 2y - xy = -3 \Rightarrow$

$y(2-x) = -3 \Rightarrow y = -\dfrac{3}{2-x} \Rightarrow f(x) = \dfrac{3}{x-2}$

Domain: $(-\infty, 2) \cup (2, \infty)$. $f(4) = \dfrac{3}{2}$

127. $\left(x^2 + 1\right)y + x = 2 \Rightarrow y = \dfrac{2-x}{x^2+1} \Rightarrow$

$f(x) = \dfrac{2-x}{x^2+1}$; Domain: $(-\infty, \infty)$; $f(4) = -\dfrac{2}{17}$

128. $yx^2 - \sqrt{x} = -2y \Rightarrow yx^2 + 2y = \sqrt{x} \Rightarrow$

$y\left(x^2 + 2\right) = \sqrt{x} \Rightarrow y = \dfrac{\sqrt{x}}{x^2+2} \Rightarrow f(x) = \dfrac{\sqrt{x}}{x^2+2}$

Domain: $[0, \infty)$; $f(4) = \dfrac{1}{9}$

129. $f(x) \ne g(x)$ because they have different domains.

130. $f(x) \neq g(x)$ because they have different domains.

131. $f(x) \neq g(x)$ because they have different domains. $g(x)$ is not defined for $x = -1$, while $f(x)$ is defined for all real numbers.

132. $f(x) \neq g(x)$ because they have different domains. $g(x)$ is not defined for $x = 3$, while $f(x)$ is not defined for $x = 3$ or $x = -2$.

133. $f(x) = g(x)$ because $f(3) = 10 = g(3)$ and $f(5) = 26 = g(5)$.

134. $f(x) \neq g(x)$ because $f(2) = 16$ while $g(2) = 13$.

135. $f(2) = 15 = a(2^2) + 2a - 3 \Rightarrow 15 = 6a - 3 \Rightarrow a = 3$.

136. $g(6) = 28 = 6^2 + 6b + b^2 \Rightarrow b^2 + 6b + 8 = 0 \Rightarrow (b+2)(b+4) = 0 \Rightarrow b = -2$ or $b = -4$.

137. $h(6) = 0 = \dfrac{3(6) + 2a}{2(6) - b} \Rightarrow 0 = 18 + 2a \Rightarrow a = -9$

$h(3)$ is undefined $\Rightarrow \dfrac{3(3) + 2(-9)}{2(3) - b}$ has a zero in the denominator. So $6 - b = 0 \Rightarrow b = 6$.

138. $f(x) = 2x - 3 \Rightarrow f\left(x^2\right) = 2x^2 - 3$

$\left(f(x)\right)^2 = (2x-3)^2 = 4x^2 - 12x + 9$

139. $g(x) = x^2 - \dfrac{1}{x^2} \Rightarrow g\left(\dfrac{1}{x}\right) = \dfrac{1}{x^2} - \dfrac{1}{\frac{1}{x^2}} = \dfrac{1}{x^2} - x^2$

$$g(x) + g\left(\frac{1}{x}\right) = \left(x^2 - \frac{1}{x^2}\right) + \left(\frac{1}{x^2} - x^2\right) = 0$$

140. $f(x) = \dfrac{x-1}{x+1} \Rightarrow f\left(\dfrac{x-1}{x+1}\right) = \dfrac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1}$

$$= \frac{\frac{(x-1)-(x+1)}{x+1}}{\frac{(x-1)+(x+1)}{x+1}} = -\frac{2}{2x} = -\frac{1}{x}$$

141. $f(x) = \dfrac{x+3}{4x-5} \Rightarrow$

$$f(t) = \frac{\frac{3+5x}{4x-1} + 3}{4\left(\frac{3+5x}{4x-1}\right) - 5} = \frac{\frac{(3+5x) + 3(4x-1)}{4x-1}}{\frac{(12+20x) - (5(4x-1))}{4x-1}}$$

$$= \frac{(3+5x) + (12x-3)}{(12+20x) - (20x-5)} = \frac{17x}{17} = x$$

Critical Thinking/Discussion/Writing

142. Answers may vary. Sample answers are given.

a. $y = \sqrt{x-2}$ **b.** $y = \dfrac{1}{\sqrt{x-2}}$

c. $y = \sqrt{2-x}$ **d.** $y = \dfrac{1}{\sqrt{2-x}}$

143. a. $ax^2 + bx + c = 0$

b. $y = c$

c. The equation will have no x-intercepts if $b^2 - 4ac < 0$.

d. It is not possible for the equation to have no y-intercepts because $y = f(x)$.

144. a. $f(x) = |x|$ **b.** $f(x) = 0$

c. $f(x) = x$

d. $f(x) = \sqrt{-x^2}$ (Note: the point is the origin.)

e. $f(x) = 1$

f. A vertical line is not a function.

145. a. $\{(a, 1), (b, 1)\}$
$\{(a, 1), (b, 2)\}$
$\{(a, 1), (b, 3)\}$
$\{(a, 2), (b, 1)\}$
$\{(a, 2), (b, 2)\}$
$\{(a, 2), (b, 3)\}$
$\{(a, 3), (b, 1)\}$
$\{(a, 3), (b, 2)\}$
$\{(a, 3), (b, 3)\}$
There are nine functions from X to Y.

b. {(1, a)}, {(2, a)}, {(3, a)}
{(1, a)}, {(2, a)}, {(3, b)}
{(1, a)}, {(2, b}, {(3, a)}
{(1, b)}, {(2, a)}, {(3, a)}
{(1, b)}, {(2, a)}, {(3, a)}
{(1, b)}, {(2, b}, {(3, a)}
{(1, b)}, {(2, a)}, {(3, b)}
{(1, b)}, {(2, b)}, {(3, b)}
There are eight functions from Y to X.

146. If a set X has m elements and a set of Y has n elements, there are n^m functions that can be defined from X to Y. This is true since a function assigns each element of X to an element of Y. There are m possibilities for each element of X, so there are
$\underbrace{n \cdot n \cdot n \cdot \cdots \cdot n}_{m} = n^m$ possible functions.

Getting Ready for the Next Section

147. $m = \dfrac{-2-0}{2-0} = -1$
$y = -x$

148. $m = \dfrac{-2-3}{4-(-1)} = -1$
$y - 3 = -(x-(-1)) \Rightarrow y - 3 = -x - 1 \Rightarrow$
$y = -x + 2$

149. $m = \dfrac{4-2}{1-(-3)} = \dfrac{1}{2}$
$y - 2 = \dfrac{1}{2}(x-(-3)) \Rightarrow 2y - 4 = x + 3 \Rightarrow$
$2y = x + 7 \Rightarrow y = \dfrac{1}{2}x + \dfrac{7}{2}$

150. $m = \dfrac{-3-(-5)}{8-3} = \dfrac{2}{5}$
$y - (-5) = \dfrac{2}{5}(x-3) \Rightarrow 5y + 25 = 2x - 6 \Rightarrow$
$5y = 2x - 31 \Rightarrow y = \dfrac{2}{5}x - \dfrac{31}{5}$

151. a. $f(-x) = 2(-x)^2 - 3(-x) = 2x^2 + 3x$

b. $-f(x) = -(2x^2 - 3x) = -2x^2 + 3x$

152. a. $f(-x) = 3(-x)^4 - 7(-x)^2 + 5$
$= 3x^4 - 7x^2 + 5$

b. $-f(x) = -(3x^4 - 7x^2 + 5)$
$= -3x^4 + 7x^2 - 5$

153. a. $f(-x) = (-x)^3 - 2(-x) = -x^3 + 2x$

b. $-f(x) = -(x^3 - 2x) = -x^3 + 2x$

154. a. $f(-x) = 2(-x)^3 - 5(-x)^2 + (-x)$
$= -2x^3 - 5x^2 - x$

b. $-f(x) = -(2x^3 - 5x^2 + x)$
$= -2x^3 + 5x^2 - x$

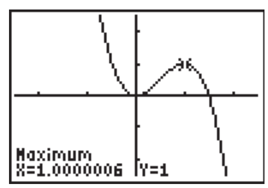
1.4 A Library of Functions

1.4 Practice Problems

1. Using the formula
Shark length = (0.96)(tooth height) − 0.22,
gives:
Shark length = (0.96)(16.4) − 0.22 = 15.524 m

2. g is decreasing on $(0, 3)$, $(12, 13)$, and $(15, 24)$; increasing on $(3, 12)$ and $(13, 15)$

3. relative maxima of 3640 at $x = 12$ and 4070 at $x = 15$; relative minima of 40 at $x = 3$ and 3490 at $x = 13$.

4.


Relative minimum: (0, 0)
Relative maximum: (1, 1)

5. a. $g(-x) = 3(-x)^4 - 5(-x)^2$
$= 3x^4 - 5x^2 = f(x) \Rightarrow$
$g(x)$ is even.

b. $f(-x) = 4(-x)^5 + 2(-x)^3 = -4x^5 - 2x^3$
$= -(4x^5 + 2x^3) = -f(x) \Rightarrow$
$f(x)$ is odd.

c. $h(-x) = 2(-x) + 1 = -2x + 1$
$\neq h(x)$
$\neq h(-x) \Rightarrow h$ is neither even nor odd.

6. $f(x) = \begin{cases} x^2 & \text{if } x \le -1 \\ 2x & \text{if } x > -1 \end{cases}$

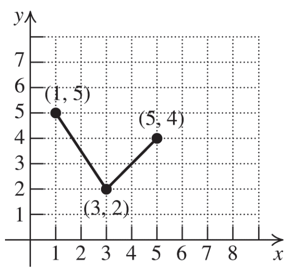
$f(-2) = (-2)^2 = 4;\ \ f(3) = 2(3) = 6$

7. a. $f(x)=\begin{cases}50+4(x-55) & 56\le x<75\\ 200+5(x-75) & x\ge 75\end{cases}$

b. The fine for driving 60 mph is $50+4(60-55)=\$70.$

c. The fine for driving 90 mph is $200+5(90-75)=\$275.$

8. The graph of f is made up of two parts: a line segment passing through (1, 5) and (3, 2) on the interval [1, 3], and a line segment passing through (3, 2) and (5, 4) on the interval [3, 5].



For the first line segment:

$m=\dfrac{2-5}{3-1}=-\dfrac{3}{2}$

$y-5=-\dfrac{3}{2}(x-1)\Rightarrow 2y-10=-3(x-1)\Rightarrow$

$2y-10=-3x+3\Rightarrow 2y=-3x+13\Rightarrow$

$y=-\dfrac{3}{2}x+\dfrac{13}{2}$

For the second line segment:

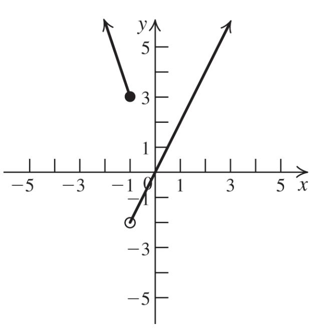
$m=\dfrac{4-2}{5-3}=1$

$y-4=x-5\Rightarrow y=x-1$

The piecewise function is

$$g(x)=\begin{cases}-\dfrac{3}{2}x+\dfrac{13}{2} & \text{if } 1\le x\le 3\\ x-1 & \text{if } 3<x\le 5\end{cases}$$

9. $f(x)=\begin{cases}-3x & \text{if } x\le -1\\ 2x & \text{if } x>-1\end{cases}$



Graph $f(x)=-3x$ on the interval $(-\infty,-1]$, and graph $f(x)=2x$ on the interval $(-1,\infty)$.

10. $f(x)=[\![x]\!]$

$f(-3.4)=-4;\ f(4.7)=4$

1.4 Exercises
Concepts and Vocabulary

1. A function f is decreasing if $x_1<x_2$ implies that $\underline{f(x_1)>f(x_2)}$.

2. A function f has a relative maximum at $x=a$ if there is an interval (x_1, x_2) containing a such that $\underline{f(a)\ge f(x)}$ for every x in the interval (x_1, x_2).

3. A function is even if for all x in the domain of f we have $\underline{f(-x)=f(x)}$.

4. A function that uses different rules for assigning output values on different parts of the domain is called a <u>piecewise function</u>.

5. True

6. True

7. False. At a point on a graph where a function changes direction from increasing to decreasing, the function has a relative *maximum*.

8. False. The graph of an odd function is symmetric about the origin.

Building Skills

9. Increasing on $(-\infty, \infty)$

10. Decreasing on $(-\infty, \infty)$

11. Increasing on $(-\infty, 2)$, decreasing on $(2, \infty)$

12. Decreasing on $(-\infty, 3)$, increasing on $(3, \infty)$

13. Increasing on $(-\infty, -2)$, constant on $(-2, 2)$, increasing on $(2, \infty)$

14. Decreasing on $(-\infty, -1)$, constant on $(-1, 4)$, decreasing on $(4, \infty)$

15. Increasing on $(-\infty, -3)$ and $\left(-\frac{1}{2}, 2\right)$, decreasing on $\left(-3, -\frac{1}{2}\right)$ and $(2, \infty)$

16. Increasing on $(-3, -1)$, $(0, 1)$, and $(2, \infty)$. Decreasing on $(-\infty, -3)$, $(-1, 0)$, and $(1, 2)$.

17. No relative extrema

18. No relative extrema

19. (2, 10) is a relative maximum point and a turning point.

20. (3, 2) is a relative minimum point and a turning point.

21. Any point on (x, 2) is a relative maximum and a relative minimum point on the interval (–2, 2). Relative maximum at (−2, 2); relative minimum at (2, 2). None of these points are turning points.

22. Any point on (x, 3) is a relative maximum and a relative minimum point on the interval (–1, 4). Relative maximum at (4, 3); relative minimum at (−1, 3). None of these points are turning points.

23. (–3, 4) and (2, 5) are relative maxima points and turning points. $\left(-\frac{1}{2}, -2\right)$ is a relative minimum and a turning point.

24. (–3, –2), (0, 0), and (2, –3) are relative minimum points and turning points. (–1, 1) and (1, 2) are relative maximum points and turning points.

For exercises 25−34, recall that the graph of an even function is symmetric about the y-axis, and the graph of an odd function is symmetric about the origin.

25. The graph is symmetric with respect to the origin. The function is odd.

26. The graph is symmetric with respect to the origin. The function is odd.

27. The graph has no symmetries, so the function is neither odd nor even.

28. The graph has no symmetries, so the function is neither odd nor even.

29. The graph is symmetric with respect to the origin. The function is odd.

30. The graph is symmetric with respect to the origin. The function is odd.

31. The graph is symmetric with respect to the y-axis. The function is even.

32. The graph is symmetric with respect to the y-axis. The function is even.

33. The graph is symmetric with respect to the origin. The function is odd.

34. The graph is symmetric with respect to the origin. The function is odd.

For exercises 35−48, $f(-x) = f(x) \Rightarrow f(x)$ is even and $f(-x) = -f(x) \Rightarrow f(x)$ is odd.

35. $f(-x) = 2(-x)^4 + 4 = 2x^4 + 4 = f(x) \Rightarrow$ $f(x)$ is even.

36. $g(-x) = 3(-x)^4 - 5 = 3x^4 - 5 = g(x) \Rightarrow$ $g(x)$ is even.

37. $f(-x) = 5(-x)^3 - 3(-x) = -5x^3 + 3x$
$= -(5x^3 - 3x) = -f(x) \Rightarrow$
$f(x)$ is odd.

38. $g(-x) = 2(-x)^3 + 4(-x)$
$= -2x^3 - 4x = -g(x) \Rightarrow$
$g(x)$ is odd.

39. $f(-x) = 2(-x) + 4 = -2x + 4$
$\neq -f(x) \neq f(x) \Rightarrow$
$f(x)$ is neither even nor odd.

40. $g(-x) = 3(-x) + 7 = -3x + 7$
$\neq -g(x) \neq g(x) \Rightarrow$
$g(x)$ is neither even nor odd.

41. $f(-x) = \dfrac{1}{(-x)^2 + 4} = \dfrac{1}{x^2 + 4} = f(x) \Rightarrow$
$f(x)$ is even.

42. $g(-x) = \dfrac{(-x)^2 + 2}{(-x)^4 + 1} = \dfrac{x^2 + 2}{x^4 + 1} = g(x) \Rightarrow$
$g(x)$ is even.

43. $f(-x) = \dfrac{(-x)^3}{(-x)^2 + 1} = -\dfrac{x^3}{x^2 + 1} = -f(x) \Rightarrow$
$f(x)$ is odd.

44. $g(-x) = \dfrac{(-x)^4 + 3}{2(-x)^3 - 3(-x)} = \dfrac{x^4 + 3}{-2x^3 + 3x}$
$= -\dfrac{x^4 + 3}{2x^3 - 3x} = -f(x) \Rightarrow f(x)$ is odd.

45. $f(-x) = \dfrac{-x}{(-x)^5 - 3(-x)^3} = \dfrac{-x}{-x^5 + 3x^3}$
$= \dfrac{(-1)(-x)}{(-1)\left(-x^5 + 3x^3\right)} = \dfrac{x}{x^5 - 3x^3} = f(x)$

Thus, $f(x)$ is even.

46. $g(-x)=\dfrac{(-x)^3+2(-x)}{2(-x)^5-3(-x)}=\dfrac{-x^3-2x}{-2x^5+3x}$
$=\dfrac{(-1)\left(-x^3-2x\right)}{(-1)\left(-2x^5+3x\right)}=\dfrac{x^3+2x}{2x^5-3x}=f(x)$

Thus, $f(x)$ is even.

47. $f(-x)=\dfrac{(-x)^2-2(-x)}{5(-x)^4+7}=\dfrac{x^2+2x}{5x^4+7}$
$\neq -f(x)\neq f(x)$

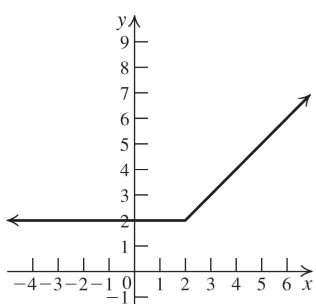
Thus, $f(x)$ is neither even nor odd.

48. $g(-x)=\dfrac{3(-x)^2+7}{(-x)-3}=\dfrac{3x^2+7}{-x-3}\neq -g(x)\neq g(x)$

Thus, $g(x)$ is neither even nor odd.

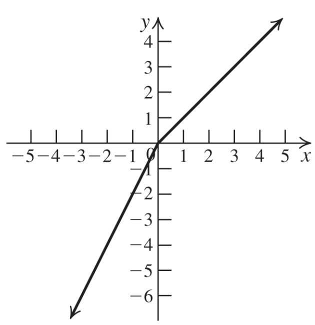
49. a. $f(1)=2; f(2)=2; f(3)=3$

b.



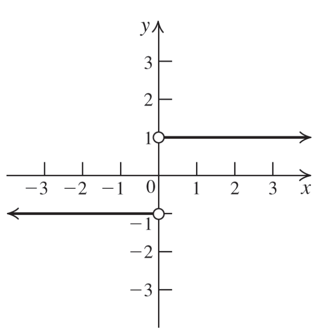
50. a. $g(-1)=-2; g(0)=0; g(1)=1$

b.



51. a. $f(-15)=-1; f(12)=1$

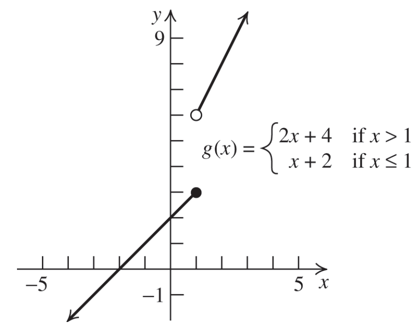
b.



c. Domain: $(-\infty,0)\cup(0,\infty)$; range: $\{-1,1\}$

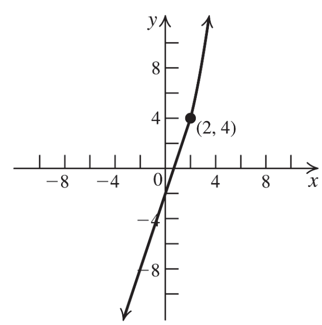
52. a. $g(-3)=-3+2=-1;\ g(1)=1+2=3;$
$g(3)=2(3)+4=10$

b.



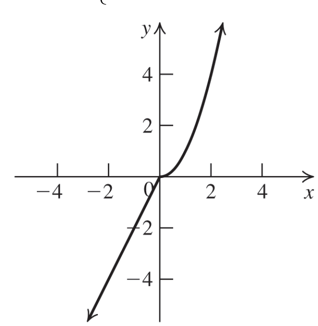
c. Domain: $(-\infty,\infty)$; range: $(-\infty,3]\cup(6,\infty]$

53. $f(x)=\begin{cases}x^2 & \text{if } x\ge 2\\ 3x-2 & \text{if } x<2\end{cases}$



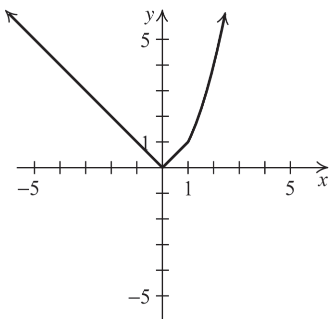
Range: $(-\infty,\infty)$

54. $f(x)=\begin{cases}2x & \text{if } x<0\\ x^2 & \text{if } x\ge 0\end{cases}$



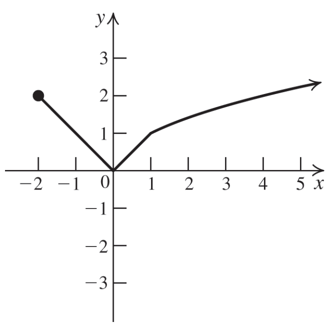
Range: $(-\infty,\infty)$

55. $f(x)=\begin{cases} |x| & \text{if } x<1 \\ x^2 & \text{if } x\ge 1 \end{cases}$



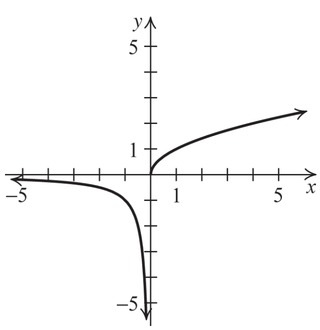
Range: $[0, \infty)$

56. $f(x)=\begin{cases} |x| & \text{if } -2\le x<1 \\ \sqrt{x} & \text{if } x\ge 1 \end{cases}$



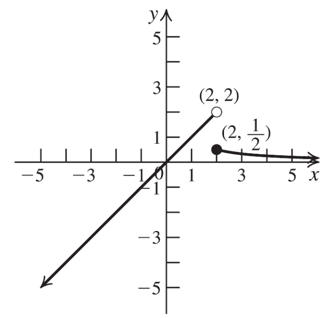
Range: $[0, \infty)$

57. $g(x)=\begin{cases} \dfrac{1}{x} & \text{if } x<0 \\ \sqrt{x} & \text{if } x\ge 0 \end{cases}$



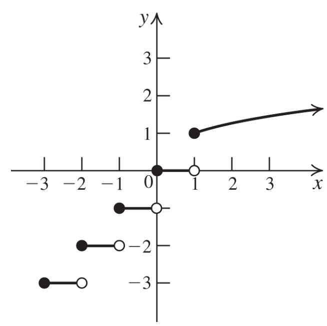
Range: $(-\infty, \infty)$

58. $h(x)=\begin{cases} \dfrac{1}{x} & \text{if } x\ge 2 \\ x & \text{if } x<2 \end{cases}$



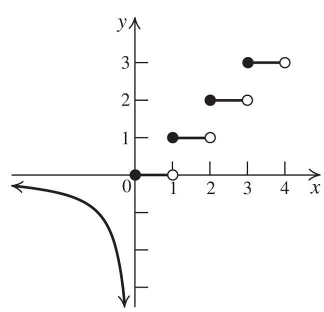
Range: $(-\infty, 2)$

59. $f(x)=\begin{cases} [\![x]\!] & \text{if } x<1 \\ \sqrt[3]{x} & \text{if } x\ge 1 \end{cases}$



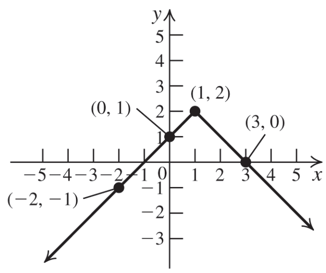
Range: $\{\ldots, -3, -2, -1, 0\}\cup[1, \infty)$

60. $f(x)=\begin{cases} \dfrac{1}{x} & \text{if } x<0 \\ [\![x]\!] & \text{if } x\ge 0 \end{cases}$



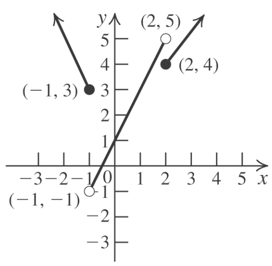
Range: $(-\infty, 0]\cup\{1, 2, 3, \ldots\}$

61. $f(x)=\begin{cases}2x+3 & \text{if } x<-2\\ x+1 & -2\le x<1\\ -x+3 & \text{if } x\ge 1\end{cases}$



Range: $(-\infty, 2]$

62. $f(x)=\begin{cases}-2x+1 & \text{if } x\le -1\\ 2x+1 & -1<x<2\\ x+2 & \text{if } x\ge 2\end{cases}$



Range: $(-1, \infty)$

63. The graph of g is made up of two parts: a line segment passing through (1, 2) and (2, 5) on the interval [1, 2], and a line segment passing through (2, 5) and (5, 1) on the interval [2, 5]. For the first line segment:

$m=\dfrac{5-2}{2-1}=3$

$y-5=3(x-2)\Rightarrow y-5=3x-6\Rightarrow$

$y=3x-1$

For the second line segment:

$m=\dfrac{1-5}{5-2}=-\dfrac{4}{3}$

$y-1=-\dfrac{4}{3}(x-5)\Rightarrow 3(y-1)=-4(x-5)\Rightarrow$

$3y-3=-4x+20\Rightarrow 3y=-4x+23\Rightarrow$

$y=-\dfrac{4}{3}x+\dfrac{23}{3}$

The piecewise function is

$$g(x)=\begin{cases}3x-1 & \text{if } 1\le x\le 2\\ -\dfrac{4}{3}x+\dfrac{23}{3} & \text{if } 2<x\le 5\end{cases}$$

64. The graph of g is made up of two parts: a line segment passing through (1, 5) and (2, 2) on the interval [1, 2], and a line segment passing through (2, 2) and (5, 3) on the interval [2, 5]. For the first line segment:

$m=\dfrac{2-5}{2-1}=-3$

$y-2=-3(x-2)\Rightarrow y-2=-3x+6\Rightarrow$

$y=-3x+8$

For the second line segment:

$m=\dfrac{3-2}{5-2}=\dfrac{1}{3}$

$y-3=\dfrac{1}{3}(x-5)\Rightarrow 3(y-3)=x-5\Rightarrow$

$3y-9=x-5\Rightarrow 3y=x+4\Rightarrow$

$y=\dfrac{1}{3}x+\dfrac{4}{3}$

The piecewise function is

$$g(x)=\begin{cases}-3x+8 & \text{if } 1\le x\le 2\\ \dfrac{1}{3}x+\dfrac{4}{3} & \text{if } 2<x\le 5\end{cases}$$

65. The graph of g is made up of two parts: a line segment passing through (1, 1) and (4, 3) on the interval [1, 4], and a line segment passing through (4, 3) and (6, 5) on the interval [4, 6]. For the first line segment:

$m=\dfrac{3-1}{4-1}=\dfrac{2}{3}$

$y-1=\dfrac{2}{3}(x-1)\Rightarrow 3(y-1)=2(x-1)\Rightarrow$

$3y-3=2x-2\Rightarrow 3y=2x+1\Rightarrow y=\dfrac{2}{3}x+\dfrac{1}{3}$

For the second line segment:

$m=\dfrac{5-3}{6-4}=1$

$y-5=x-6\Rightarrow y=x-1$

The piecewise function is

$$g(x)=\begin{cases}\dfrac{2}{3}x+\dfrac{1}{3} & \text{if } 1\le x\le 4\\ x-1 & \text{if } 4<x\le 6\end{cases}$$

66. The graph of g is made up of two parts: a line segment passing through (1, 5) and (4, 4) on the interval [1, 4], and a line segment passing through (4, 4) and (6, 1) on the interval [4, 6]. For the first line segment:

$m = \frac{4-5}{4-1} = -\frac{1}{3}$

$y - 5 = -\frac{1}{3}(x-1) \Rightarrow 3(y-5) = -(x-1) \Rightarrow$
$3y - 15 = -x + 1 \Rightarrow 3y = -x + 16 \Rightarrow$
$y = -\frac{1}{3}x + \frac{16}{3}$

For the second line segment:

$m = \frac{1-4}{6-4} = -\frac{3}{2}$

$y - 1 = -\frac{3}{2}(x-6) \Rightarrow 2(y-1) = -3(x-6) \Rightarrow$
$2y - 2 = -3x + 18 \Rightarrow 2y = -3x + 20 \Rightarrow$
$y = -\frac{3}{2}x + 10$

The piecewise function is

$$g(x) = \begin{cases} -\frac{1}{3}x + \frac{16}{3} & \text{if } 1 \le x \le 4 \\ -\frac{3}{2}x + 10 & \text{if } 4 < x \le 6 \end{cases}$$

67. The graph of g is made up of two parts: a line segment passing through (1, 1) and (2, 5) on the interval [1, 2], and a line segment passing through (2, 5) and (6, 6) on the interval [2, 6]. For the first line segment:

$m = \frac{5-1}{2-1} = 4$

$y - 1 = 4(x-1) \Rightarrow y - 1 = 4x - 4 \Rightarrow$
$y = 4x - 3$

For the second line segment:

$m = \frac{6-5}{6-2} = \frac{1}{4}$

$y - 5 = \frac{1}{4}(x-2) \Rightarrow 4(y-5) = x - 2 \Rightarrow$
$4y - 20 = x - 2 \Rightarrow 4y = x + 18 \Rightarrow$
$y = \frac{1}{4}x + \frac{9}{2}$

The piecewise function is

$$g(x) = \begin{cases} 4x - 3 & \text{if } 1 \le x \le 2 \\ \frac{1}{4}x + \frac{9}{2} & \text{if } 2 < x \le 6 \end{cases}$$

68. The graph of g is made up of two parts: a line segment passing through (1, 1) and (4, 2) on the interval [1, 4], and a line segment passing through (4, 2) and (6, 4) on the interval [4, 6]. For the first line segment:

$m = \frac{2-1}{4-1} = \frac{1}{3}$

$y - 1 = \frac{1}{3}(x-1) \Rightarrow 3(y-1) = x - 1 \Rightarrow$
$3y - 3 = x - 1 \Rightarrow 3y = x + 2 \Rightarrow y = \frac{1}{3}x + \frac{2}{3}$

For the second line segment:

$m = \frac{4-2}{6-4} = 1$

$y - 4 = x - 6 \Rightarrow y = x - 2$

The piecewise function is

$$g(x) = \begin{cases} \frac{1}{3}x + \frac{2}{3} & \text{if } 1 \le x \le 4 \\ x - 2 & \text{if } 4 < x \le 6 \end{cases}$$

69. The graph of g is made up of two parts: a line segment passing through (1, 6) and (3, 2) on the interval [1, 3], and a line segment passing through (3, 2) and (6, 1) on the interval [3, 6]. For the first line segment:

$m = \frac{2-6}{3-1} = -2$

$y - 2 = -2(x-3) \Rightarrow y - 2 = -2x + 6 \Rightarrow$
$y = -2x + 8$

For the second line segment:

$m = \frac{1-2}{6-3} = -\frac{1}{3}$

$y - 1 = -\frac{1}{3}(x-6) \Rightarrow y - 1 = -\frac{1}{3}x + 2 \Rightarrow$
$y = -\frac{1}{3}x + 3$

The piecewise function is

$$g(x) = \begin{cases} -2x + 8 & \text{if } 1 \le x \le 3 \\ -\frac{1}{3}x + 3 & \text{if } 3 < x \le 6 \end{cases}$$

70. The graph of g is made up of two parts: a line segment passing through (1, 6) and (4, 5) on the interval [1, 4], and a line segment passing through (4, 5) and (6, 3) on the interval [4, 6]. For the first line segment:

$m = \frac{5-6}{4-1} = -\frac{1}{3}$

$y - 6 = -\frac{1}{3}(x-1) \Rightarrow 3(y-6) = -(x-1) \Rightarrow$
$3y - 18 = -x + 1 \Rightarrow 3y = -x + 19 \Rightarrow$
$y = -\frac{1}{3}x + \frac{19}{3}$

(*continued on next page*)

(*continued*)

For the second line segment:

$$m = \frac{3-5}{6-4} = -1$$

$$y - 3 = -(x-6) \Rightarrow y - 3 = -x + 6 \Rightarrow$$

$$y = -x + 9$$

The piecewise function is

$$g(x) = \begin{cases} -\frac{1}{3}x + \frac{19}{3} & \text{if } 1 \le x \le 4 \\ -x + 9 & \text{if } 4 < x \le 6 \end{cases}$$

Applying the Concepts

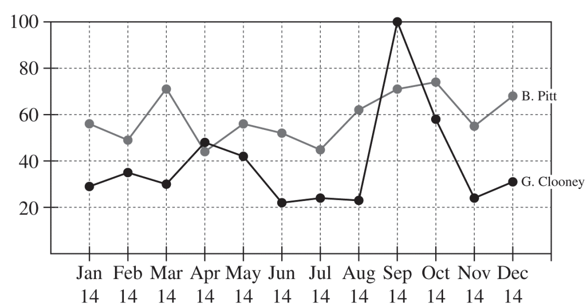
71. a. Increasing: (2006, 2009), (2011, 2012), (2013, 2014)
Decreasing: (2009, 2011), (2012, 2013)

b. Relative maxima: 251.1 at x = 2009, 293.2 at x = 2012
Relative minima: 21.5 at x = 2011, 187.0 at x = 2013

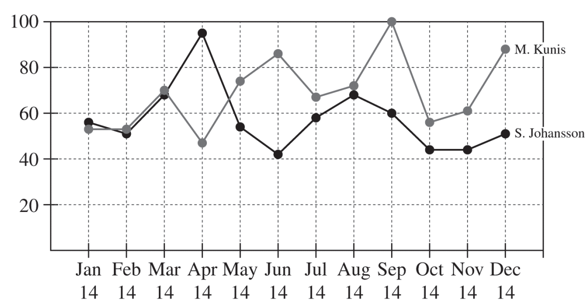
72. a. Increasing: (Jan., June), (July, Sept.)
Decreasing: (June, July), (Sept., Dec.)

b. Relative maxima: 185 in June, 185 in Sept.
Relative minima: 132 in July

73.



74.



75. a. $f(x) = \frac{x}{33.81}$; domain: $[0, \infty)$; range: $[0, \infty)$.

b. $f(3) = \frac{3}{33.81} \approx 0.0887$.
This means that 3 oz = 0.0887 liter.

c. $f(12) = \frac{12}{33.81} \approx 0.3549$ liter.

76. a. $B(0) = -1.8(0) + 212 = 212$. The y-intercept is 212. This means that water boils at 212°F at sea level. $0 = -1.8h + 212 \Rightarrow h \approx 117.78$. The h-intercept is approximately 117.78. This means that water boils at 0°F at approximately 117,780 feet above sea level.

b. Domain: closed interval from 0 to the end of the atmosphere, in thousands of feet.

c. $98.6 = -1.8h + 212 \Rightarrow h = 63$. Water boils at 98.6°F at 63,000 feet. It is dangerous because 98.6°F is the temperature of human blood.

77. a. $P(0) = \frac{1}{33}(0) + 1 = 1$.
The y-intercept is 1. This means that the pressure at sea level ($d = 0$) is 1 atm. $0 = \frac{1}{33}d + 1 \Rightarrow d = -33$.
d can't be negative, so there is no d-intercept.

b. $P(0) = 1; P(10) = \frac{1}{33}(10) + 1 \approx 1.3;$
$P(33) = \frac{1}{33}(33) + 1 = 2;$
$P(100) = \frac{1}{33}(100) + 1 \approx 4.03.$

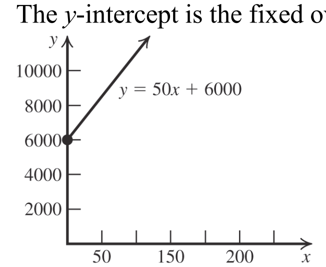
c. $5 = \frac{1}{33}d + 1 \Rightarrow d = 132$ feet
The pressure is 5 atm at 132 feet.

78. a. $V(90) = 1055 + 1.1(90) = 1154$ ft/sec
The speed of sound at 90ºF is 1154 feet per second.

b. $1100 = 1055 + 1.1T \Rightarrow T \approx 40.91$°F
The speed of sound is 1100 feet per second at approximately 40.91ºF.

79. a. $C(x) = 50x + 6000$

b. The y-intercept is the fixed overhead cost.



c. $11,500 = 50x + 6000 \Rightarrow 110$
110 printers were manufactures on a day when the total cost was $11,500.

80. a. The rate of change (slope) is 100. Find the y-intercept by using the point (10, 750): $750 = 100(10) + b \Rightarrow b = -250.$ The equation is $f(p) = 100p - 250.$

b. $f(15) = 100(15) - 250 = 1250$
When the price is $15 per unit, there are 1250 units.

c. $1750 = 100p - 250 \Rightarrow p = \$20.$
1750 units can be supplied at $20 per unit.

81. a. $R = 900 - 30x$

b. $R(6) = 900 - 30(6) = 720$
If you move in 6 days after the first of the month, the rent is $720.

c. $600 = 900 - 30x \Rightarrow x = 10$
You moved in ten days after first of the month.

82. a. Let $t = 0$ represent the year 2010. The rate of change (slope) is $\frac{1150 - 1120}{2} = 15.$
The y-intercept is 1120, so the equation is $f(t) = 15t + 1120.$

b. $f(8) = 15(8) + 1120 = 1240$
The average SAT score will be 1240 in 2018.

c. $1300 = 15t + 1120 \Rightarrow t = 12.$
2012 + 12 = 2022.
The average SAT score will be 1300 in 2022.

83. The rate of change (slope) is $\frac{100 - 40}{20 - 80} = -1$.
Use the point (20, 100) to find the equation of the line: $100 = -20 + b \Rightarrow b = 120.$ The equation of the line is $y = -x + 120.$ Now solve $50 = -x + 120 \Rightarrow x = 70.$ Age 70 corresponds to 50% capacity.

84. a. $y = \frac{2}{25}(5)(60) = 24$
The dosage for a five-year-old child is 24 mg.

b. $60 = \frac{2}{25}(60)a \Rightarrow a = 12.5$
A child would have to be 12.5 years old to be prescribed an adult dosage.

85. a. The rate of change (slope) is $\frac{50 - 30}{420 - 150} = \frac{2}{27}.$
The equation of the line is

$$y - 30 = \frac{2}{27}(x - 150)$$
$$y = \frac{2}{27}(x - 150) + 30$$

b. $y = \frac{2}{27}(350 - 150) + 30$
$y = \frac{1210}{27} \approx 44.8$
There can't be a fractional number of deaths, so round up. There will be about 45 deaths when $x = 350$ milligrams per cubic meter.

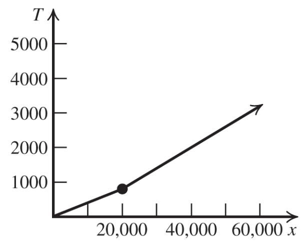
c. $45 = \frac{2}{27}(x - 150) + 30 \Rightarrow x = 352.5$
If the number of deaths per month is 45, the concentration of sulfur dioxide in the air is 352.5 mg/m^3.

86. a. The rate of change is $\frac{1}{3}$. The y-intercept is $\frac{47}{12}$, so the equation is
$y = L(S) = \frac{1}{3}S + \frac{47}{12}.$

b. $L(4) = \frac{1}{3}(4) + \frac{47}{12} = 5.25$
A child's size 4 shoe has insole length 5.25 inches.

c. $\frac{61}{10} = \frac{1}{3}x + \frac{47}{12} \Rightarrow x = 6.55 \approx 6.5$
A child whose insole length is 6.1 inches wears a size 6.5 shoe.

87. a.



b.(i) $T(12,000) = 0.04(12,000) = \$480$

(ii) $T(20,000) = 800 + 0.06(20,000 - 20,000)$
$= \$800$

(iii) $T(50,000) = 800 + 0.06(50,000 - 20,000)$
$= \$2600$

c. (i) $600 = 0.04x \Rightarrow x = \$15,000$

(ii) From the answers in part **b**, a \$1400 tax liability lies between the tax liability for incomes of \$20,000 to \$50,000, so use the function $T(x) = 800 + 0.06(x - 20,000)$.

$$1400 = 800 + 0.06(x - 20,000)$$
$$600 = 0.06(x - 20,000)$$
$$10,000 = x - 20,000 \Rightarrow x = \$30,000$$

(iii)
$$2300 = 800 + 0.06(x - 20,000)$$
$$1500 = 0.06(x - 20,000)$$
$$25,000 = x - 20,000 \Rightarrow x = \$45,000$$

88. a.
$$f(x) = \begin{cases} 0.1x & \text{if } 0 < x \le 9225 \\ 922.50 + 0.15(x - 9225) & \text{if } 9225 < x \le 37,450 \\ 5156.25 + 0.25(x - 37,450) & \text{if } 37,450 < x \le 90,750, \\ 18,481.25 + 0.28(x - 90,750) & \text{if } 90,750 < x \le 189,300 \\ 46,075.25 + 0.33(x - 189,300) & \text{if } 189,300 < x \le 411,500 \\ 119,401.25 + 0.35(x - 411,500) & \text{if } 411,500 < x \le 413,200 \\ 119,996.25 + 0.396(x - 413,200) & \text{if } 413,200 < x \end{cases}$$

b. (i) $f(35,000) = 922.50 + 0.15(35,000 - 9225) = \$4788.75 \approx \$4789$

(ii) $f(100,000) = 18,481.25 + 0.28(100,000 - 90,750) = \$21,071.25 \approx \$21,071$

(iii) $f(500,000) = 119,996.25 + 0.396(500,000 - 413,200) = \$154,369.05 \approx \$154,369$

c. (i) $3500 = 922.50 + 0.15(x - 9225) \Rightarrow 2577.5 = 0.15(x - 9225) \Rightarrow 17183.33 = x - 9225 \Rightarrow x \approx \$26,408$

(ii) $12,700 = 5156.25 + 0.25(x - 37,450) \Rightarrow 7543.75 = 0.25(x - 37,450) \Rightarrow 30,175 = x - 37,450 \Rightarrow$
$x = \$67,625$

(iii) $35,000 = 18,481.25 + 0.28(x - 90,750) \Rightarrow 16518.75 = 0.28(x - 90,750) \Rightarrow$
$58995.54 = x - 90,750 \Rightarrow x \approx \$149,746$

Beyond the Basics

89. $2(3) - 1 = a - 3(3) \Rightarrow 5 = a - 9 \Rightarrow a = 14$

90. $1 - 3 = 3a + 3 \Rightarrow -2 = 3a + 3 \Rightarrow -5 = 3a \Rightarrow$
$a = -\dfrac{5}{3}$

91. a. Domain: $(-\infty, \infty)$; range: $[0,1)$

b. The function is increasing on $(n, n + 1)$ for every integer n.

c. $f(-x) = -x - [\![-x]\!] \ne -f(x) \ne f(x)$, so the function is neither even nor odd.

92. a. Domain: $(-\infty, 0) \cup [1, \infty)$

range: $\left\{\dfrac{1}{n} : n \ne 0, n \text{ an integer}\right\}$

b. The function is constant on $(n, n + 1)$ for every nonzero integer n.

c. $f(-x) = \dfrac{1}{[\![-x]\!]} \ne -f(x) \ne f(x)$, so the function is neither even nor odd.

93. a. (i) $WCI(2) = 40$

(ii) $WCI(16)$
$$= 91.4 + (91.4 - 40) \cdot \left(0.0203(16) - 0.304\sqrt{16} - 0.474\right) \approx 21$$

(iii) $WCI(50) = 1.6(40) - 55 = 9$

b. (i)
$$-58 = 91.4 + (91.4 - T)\bullet (0.0203(36) - 0.304\sqrt{36} - 0.474$$
$$-58 = 91.4 + (91.4 - T)(-1.5672)$$
$$-58 = 91.4 - 143.24 + 1.5672T$$
$$-58 = -51.84 + 1.5672T \Rightarrow T \approx -4°\text{F}$$

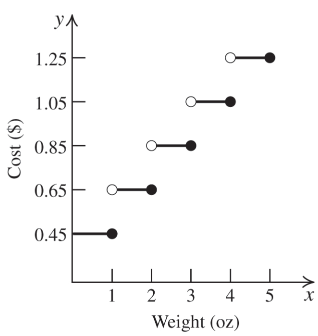
(ii) $-10 = 1.6T - 55 \Rightarrow T \approx 28°\text{F}$

94. a.
$$C(x) = 20\left(f(x) - 1\right) + 45 = 20\left(-[\![-x]\!] - 1\right) + 45$$

b.
$$C(2.3) = 20\left(-[\![-2.3]\!] - 1\right) + 45 = 20\left[-(-3) - 1\right] + 45 = 20(2) + 45 = 85$$

It will cost 85¢ to mail a first-class letter weighing 2.3 oz.

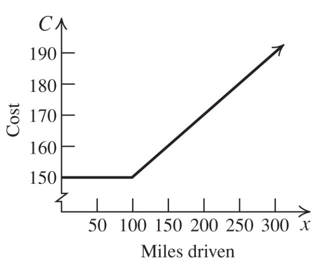
c.



95. $C(x) = 2[\![x]\!] + 4$

96. a.
$$C(x) = \begin{cases} 150 & \text{if } x \le 100 \\ 0.2[\![x - 100]\!] + 150 & \text{if } x > 100 \end{cases}$$

b.



c.
$$190 = 0.2[\![x - 99]\!] + 150$$
$$40 = 0.2[\![x - 99]\!] \Rightarrow 200 = [\![x - 99]\!] \Rightarrow$$
$$x = 300 \text{ miles}$$

Critical Thinking/Discussion/Writing

97. D

98. C

99. a. If f is even, then f is increasing on $(-\infty, -1)$ and decreasing on $(-1, 0)$.

b. If f is odd, then f is decreasing on $(-\infty, -1)$ and increasing on $(-1, 0)$.

100. a. If f is even, then f has a relative maximum at $x = -1$ and a relative minimum at $x = -3$.

b. If f is odd, then f has a relative minimum at $x = -1$ and a relative maximum at $x = -3$.

101. $f(x) = \dfrac{x + |x|}{2}$

102. $f(x) = \dfrac{x - |x|}{2}$

103. $f(x) = [\![x + 0.5]\!]$

104. $f(x) = [\![x]\!] + [\![-x]\!] + 1$

Getting Ready for the Next Section

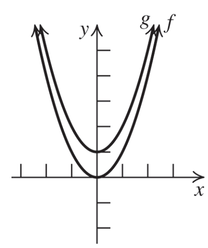
105. If we add 3 to each y-coordinate of the graph of f, we will obtain the graph of $y = \underline{f(x) + 3}.$

106. If we subtract 2 from each x-coordinate of the graph of f, we will obtain the graph of $y = \underline{f(x + 2)}.$

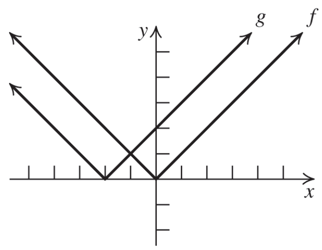
107. If we replace each x-coordinate with its opposite in the graph of f, we will obtain the graph of $y = \underline{f(-x)}.$

108. If we replace each y-coordinate with its opposite in the graph of f, we will obtain the graph of $y = \underline{-f(x)}.$

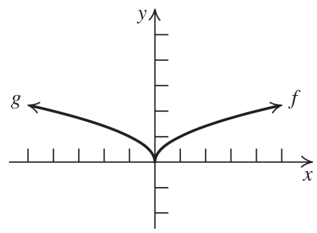
109.



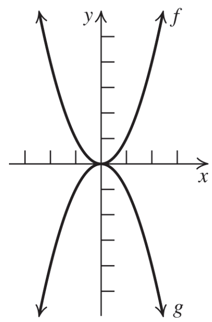
110.



111.



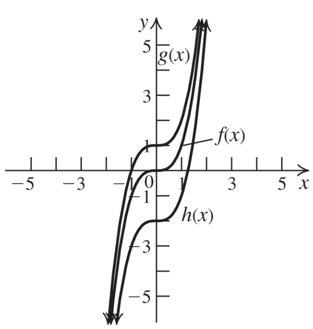
112.



1.5 Transformations of Functions

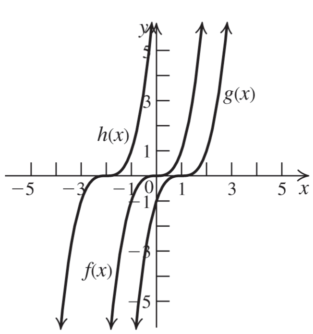
1.5 Practice Problems

1.



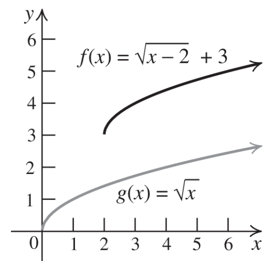
The graph of g is the graph of f shifted one unit up. The graph of h is the graph of f shifted two units down.

2.



The graph of g is the graph of f shifted one unit to the right. The graph of h is the graph of f shifted two units to the left.

3. The graph of $f(x) = \sqrt{x-2} + 3$ is the graph of $g(x) = \sqrt{x}$ shifted two units to the right and three units up.

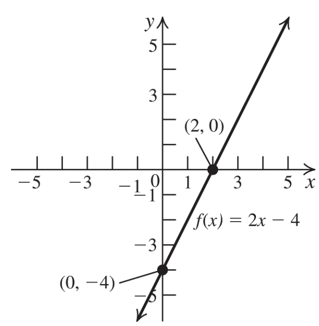


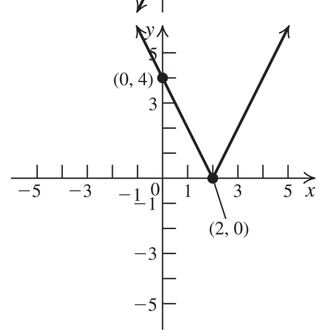
4. The graph of $y = -(x-1)^2 + 2$ can be obtained from the graph of $y = x^2$ by first shifting the graph of $y = x^2$ one unit to the right. Reflect the resulting graph about the x-axis, and then shift the graph two units up.

5. The graph of $y = 2x - 4$ is obtained from the graph of $y = 2x$ by shifting it down by four units. We know that

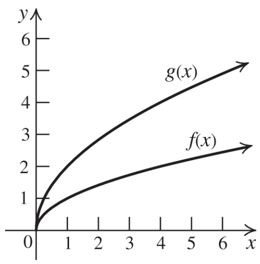
$$|y| = \begin{cases} y & \text{if } y \ge 0 \\ -y & \text{if } y < 0. \end{cases}$$

This means that the portion of the graph on or above the x-axis $(y \ge 0)$ is unchanged while the portion of the graph below the x-axis $(y < 0)$ is reflected above the x-axis.



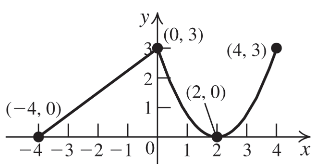


6.

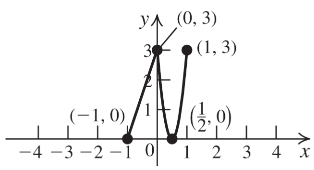


The graph of g is the graph of f stretched vertically by multiplying each of its y-coordinates by 2.

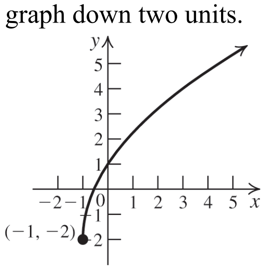
7. a.



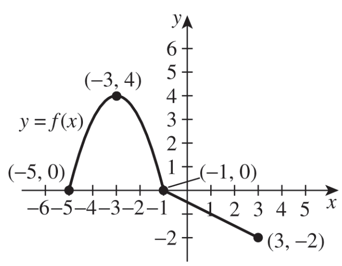
b.



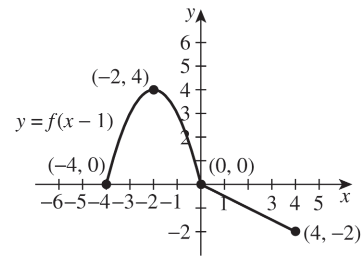
8. Start with the graph of $y = \sqrt{x}$. Shift the graph one unit to the left, then stretch the graph vertically by a factor of three. Shift the resulting graph down two units.



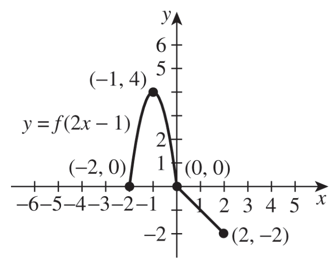
9.



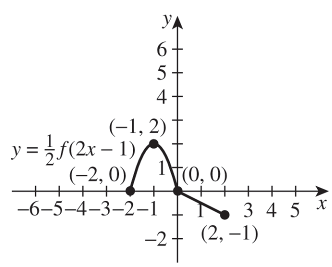
Shift the graph one unit right to graph $y = f(x-1)$.



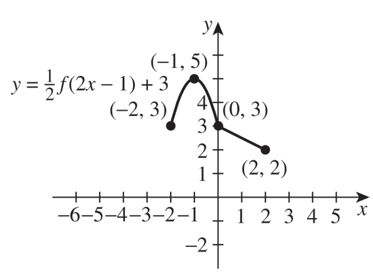
Compress horizontally by a factor of 2. Multiply each x-coordinate by $\frac{1}{2}$ to graph $y = f(2x-1)$.



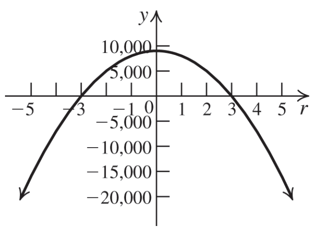
Compress vertically by a factor of $\frac{1}{2}$. Multiply each y-coordinate by $\frac{1}{2}$ to graph $y = \frac{1}{2} f(2x-1)$.



Shift the graph up three units to graph $y = \frac{1}{2} f(2x-1) + 3$.



10. The equation is
$v(r) = 10^3 \left(3^2 - r^2\right) = 9000 - 1000r^2.$
This is obtained by stretching the graph of $y = r^2$ vertically by a factor of 1000, reflecting the resulting graph in the x-axis, and then shifting the graph 9000 units up.



1.5 Exercises
Concepts and Vocabulary

1. The graph of $y = f(x) - 3$ is found by vertically shifting the graph of $y = f(x)$ three units down.
2. The graph of $y = f(x+5)$ is found by horizontally shifting the graph of $y = f(x)$ five units to the left.
3. The graph of $y = f(-x)$ is found by reflecting the graph of $y = f(x)$ about the y-axis.
4. The graph of $y = -f(x)$ is found by reflecting the graph of $y = f(x)$ about the x-axis.
5. False. The graph of $y = f(bx)$ is a horizontal compression of the graph of $y = f(x)$ if $b > 1$.
6. True
7. False. The graphs are the same if the function is an even function.
8. True

Building Skills

9. a. The graph of g is the graph of f shifted two units up.

b. The graph of h is the graph of f shifted one unit down.

10. a. The graph of g is the graph of f shifted one unit up.

b. The graph of h is the graph of f shifted two units down.

11. a. The graph of g is the graph of f shifted one unit to the left.

b. The graph of h is the graph of f shifted two units to the right.

12.a. The graph of g is the graph of f shifted two units to the left.

b. The graph of h is the graph of f shifted three units to the right.

13. a. The graph of g is the graph of f shifted one unit left and two units down.

b. The graph of h is the graph of f shifted one unit right and three units up.

14. a. The graph of g is the graph of f reflected about the x-axis.

b. The graph of h is the graph of f reflected about the y-axis.

15. a. The graph of g is the graph of f reflected about the x-axis.

b. The graph of h is the graph of f reflected about the y-axis.

16. a. The graph of g is the graph of f stretched vertically by a factor of 2.

b. The graph of h is the graph of f compressed horizontally by a factor of 2.

17. a. The graph of g is the graph of f vertically stretched by a factor of 2.

b. The graph of h is the graph of f horizontally compressed by a factor of 2.

18. a. The graph of g is the graph of f shifted two units to the right and one unit up.

b. The graph of h is the graph of f shifted one unit to the left, reflected about the x-axis, and then shifted two units up.

19. a. The graph of g is the graph of f reflected about the x-axis and then shifted one unit up.

b. The graph of h is the graph of f reflected about the y-axis and then shifted one unit up.

20. a. The graph of g is the graph of f shifted one unit to the right and then shifted two units up.

b. The graph of h is the graph of f stretched vertically by a factor of three and then shifted one unit down.

21. a. The graph of g is the graph of f shifted one unit up.

b. The graph of h is the graph of f shifted one unit to the left.

22. **a.** The graph of g is the graph of f shifted one unit left, vertically stretched by a factor of 2, reflected about the y-axis, and then shifted 4 units up.

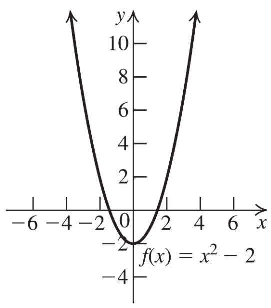
b. The graph of h is the graph of f shifted one unit to the right, reflected about the x-axis, and then shifted three units up.

23. e **24.** c **25.** g **26.** h

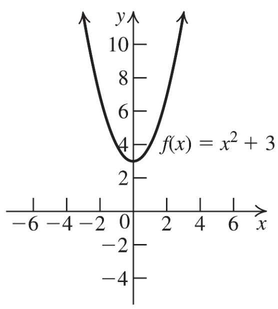
27. i **28.** a **29.** b **30.** k

31. l **32.** f **33.** d **34.** j

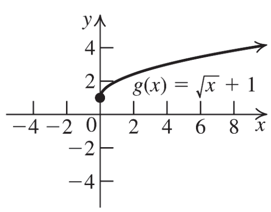
35.



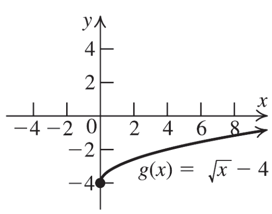
36.



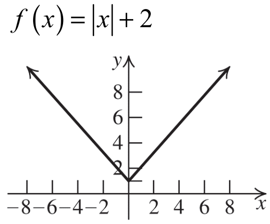
37.



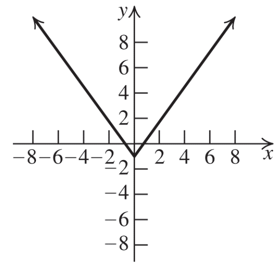
38.



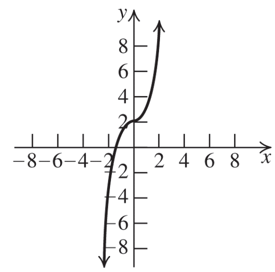
39. $f(x) = |x| + 2$



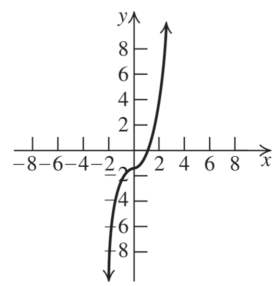
40. $f(x) = |x| - 1$



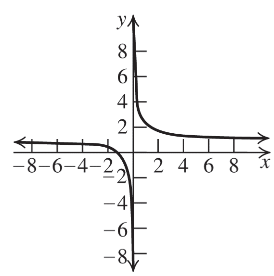
41. $f(x) = x^3 + 2$



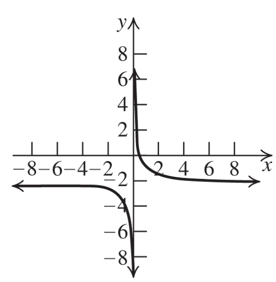
42. $f(x) = x^3 - 1$



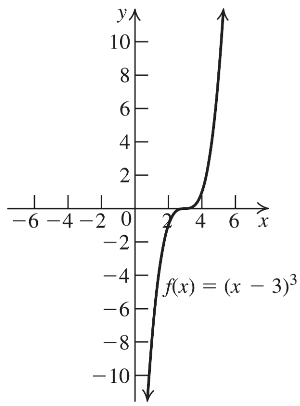
43. $f(x) = \dfrac{1}{x} + 1$



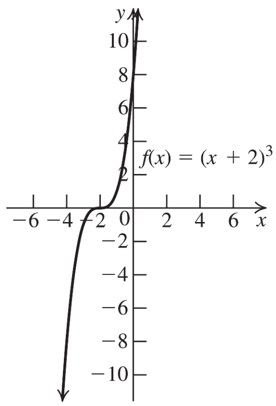
44. $f(x) = \dfrac{1}{x} - 2$



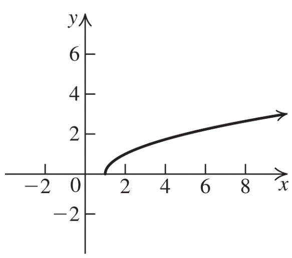
45.



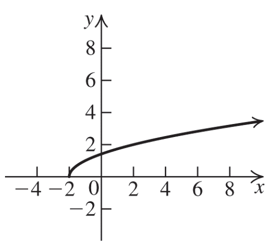
46.



47. $f(x) = \sqrt{x-1}$



48. $f(x) = \sqrt{x+2}$



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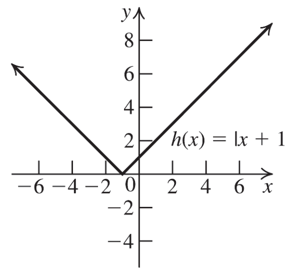
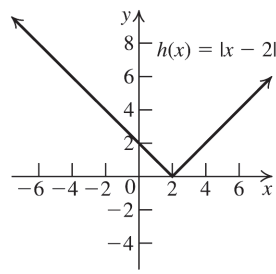
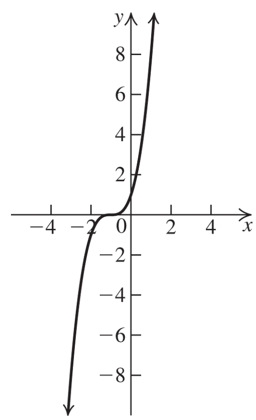


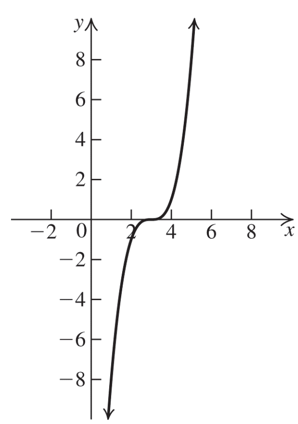
50.



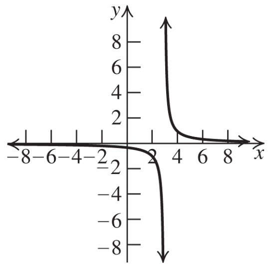
51. $f(x) = (x+1)^3$



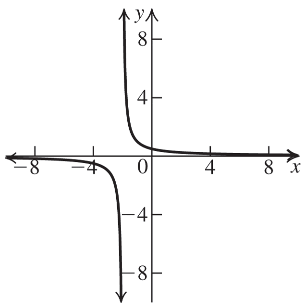
52. $f(x) = (x-3)^3$



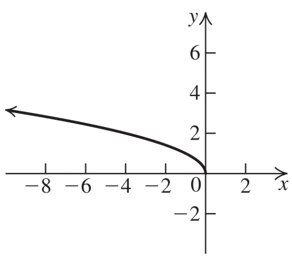
53. $f(x) = \dfrac{1}{x-3}$



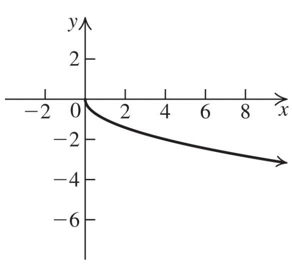
54. $f(x) = \dfrac{1}{x+2}$



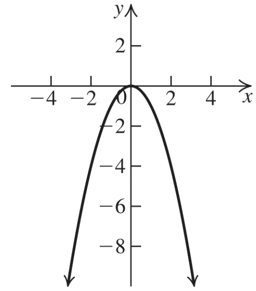
55. $f(x) = \sqrt{-x}$



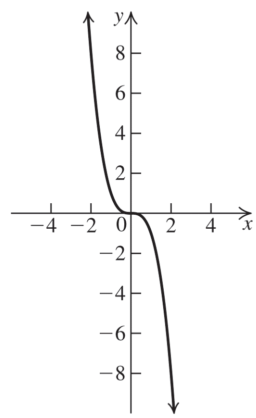
56. $f(x) = -\sqrt{x}$



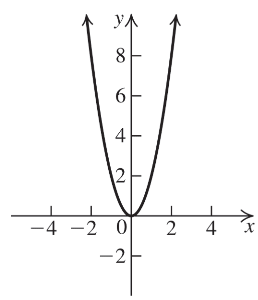
57. $f(x) = -x^2$



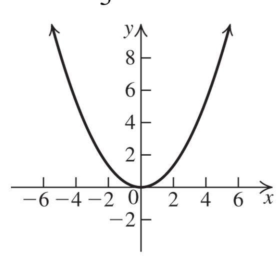
58. $f(x) = -x^3$



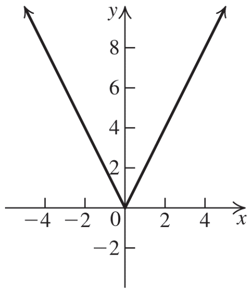
59. $f(x) = 2x^2$



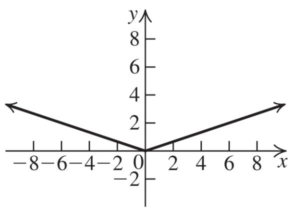
60. $f(x) = \dfrac{1}{3}x^2$



61. $f(x) = 2|x|$

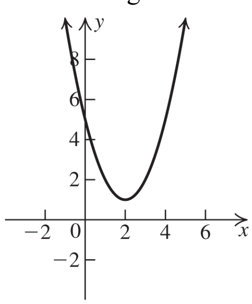


62. $f(x) = \frac{1}{3}|x|$



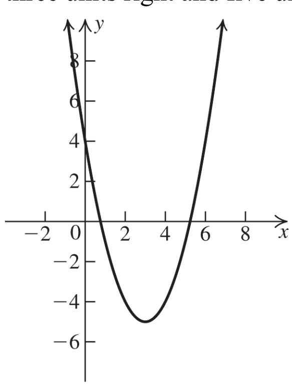
63. $f(x) = (x-2)^2 + 1$

Start with the graph of $f(x) = x^2$, then shift it two units right and one unit up.



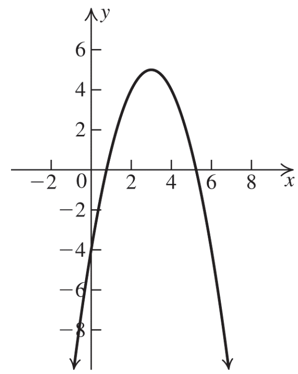
64. $f(x) = (x-3)^2 - 5$

Start with the graph of $f(x) = x^2$, then shift it three units right and five units down.



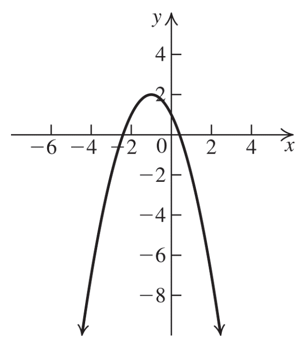
65. $f(x) = 5 - (x-3)^2$

Start with the graph of $f(x) = x^2$, then shift it three units right. Reflect the graph across the x-axis. Shift it five units up.



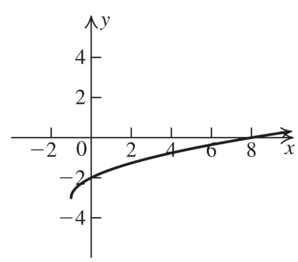
66. $f(x) = 2 - (x+1)^2$

Start with the graph of $f(x) = x^2$, then shift it one unit left. Reflect the graph across the x-axis. Shift it two units up.



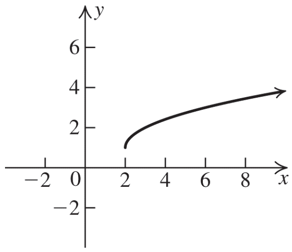
67. $f(x) = \sqrt{x+1} - 3$

Start with the graph of $f(x) = \sqrt{x}$, then shift it one unit left and three units down.



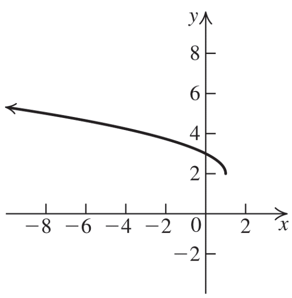
68. $f(x) = \sqrt{x-2} + 1$

Start with the graph of $f(x) = \sqrt{x}$, then shift it two units right and one unit up



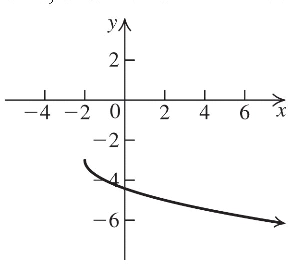
69. $f(x) = \sqrt{1-x} + 2$

Start with the graph of $f(x) = \sqrt{x}$, then shift it one unit left. Reflect the graph across the y-axis, and then shift it two units up.



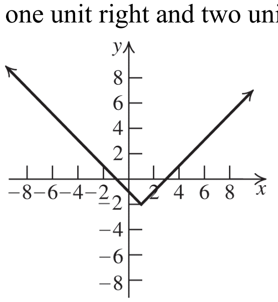
70. $f(x) = -\sqrt{x+2} - 3$

Start with the graph of $f(x) = \sqrt{x}$, then shift it two units left. Reflect the graph across the x-axis, and then shift it three units down.



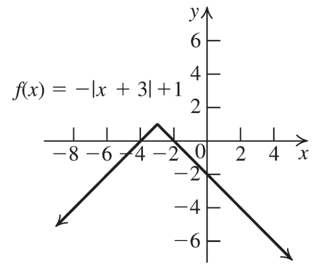
71. $f(x) = |x-1| - 2$

Start with the graph of $f(x) = |x|$, then shift it one unit right and two units down.



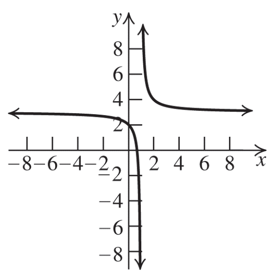
72. $f(x) = -|x+3| + 1$

Start with the graph of $f(x) = |x|$, then shift it three units left. Reflect the graph across the x-axis, and then shift it one unit up.



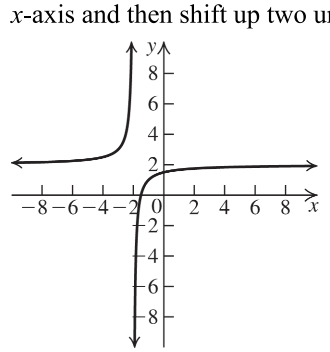
73. $f(x) = \dfrac{1}{x-1} + 3$

Start with the graph of $f(x) = \frac{1}{x}$, then shift it one unit right and three units up.



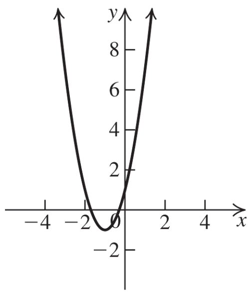
74. $f(x) = 2 - \dfrac{1}{x+2}$

Start with the graph of $f(x) = \frac{1}{x}$, then shift it two units left. Reflect the graph across the x-axis and then shift up two units up.



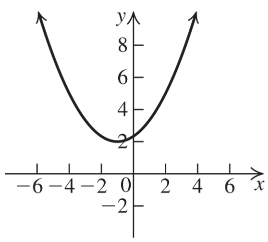
75. $f(x) = 2(x+1)^2 - 1$

Start with the graph of $f(x) = x^2$, then shift it one unit left. Stretch the graph vertically by a factor of 2, then shift it one unit down.



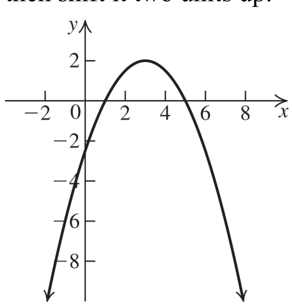
76. $f(x) = \frac{1}{3}(x+1)^2 + 2$

Start with the graph of $f(x) = x^2$, then shift it one unit left. Compress the graph vertically by a factor of 1/3, then shift it two units up.



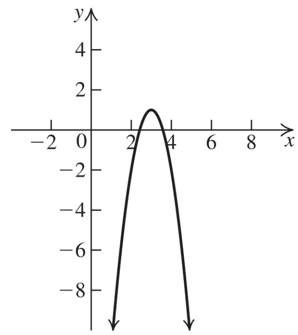
77. $f(x) = 2 - \frac{1}{2}(x-3)^2$

Start with the graph of $f(x) = x^2$, then shift it three units right. Compress the graph vertically by a factor of 1/2, reflect it across the x-axis, then shift it two units up.



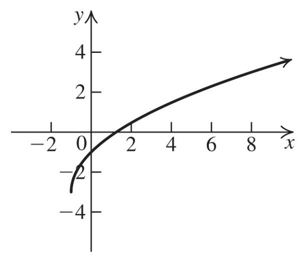
78. $f(x) = 1 - 3(x-3)^2$

Start with the graph of $f(x) = x^2$, then shift it three units right. Stretch the graph vertically by a factor of 3, reflect it across the x-axis, then shift it one unit up.



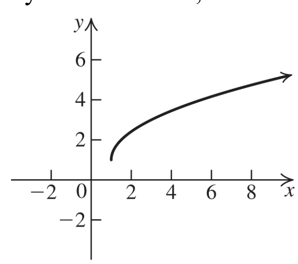
79. $f(x) = 2\sqrt{x+1} - 3$

Start with the graph of $f(x) = \sqrt{x}$, then shift it one unit left. Stretch the graph vertically by a factor of 2, and then shift it three units down.



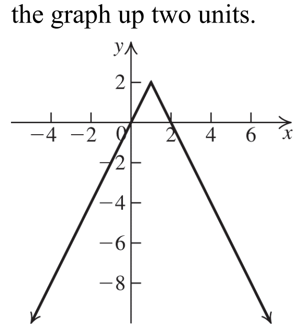
80. $f(x) = \sqrt{2x-2} + 1$

Start with the graph of $f(x) = \sqrt{x}$, then shift it two units right. Compress the graph horizontally by a factor of 1/2, and then shift it one unit up.



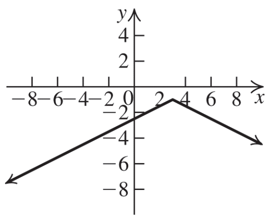
81. $f(x) = -2|x-1| + 2$

Start with the graph of $f(x) = |x|$, then shift it one unit right. Stretch the graph vertically by a factor of 2, then reflect it across the x-axis. Shift the graph up two units.



82. $f(x) = -\frac{1}{2}|3-x|-1$

Start with the graph of $f(x) = |x|$, then shift it three units right. Compress the graph vertically by a factor of 1/2, then reflect it across the y-axis. Reflect the graph across the x-axis, and then shift the graph down one unit.



83. $y = x^3 + 2$

84. $y = \sqrt{x+3}$

85. $y = -|x|$

86. $y = \sqrt{-x}$

87. $y = (x-3)^2 + 2$

88. $y = -(x+2)^2$

89. $y = -\sqrt{x+3} - 2$

90. $y = -\frac{1}{2}\left(\sqrt{x} - 2\right)$

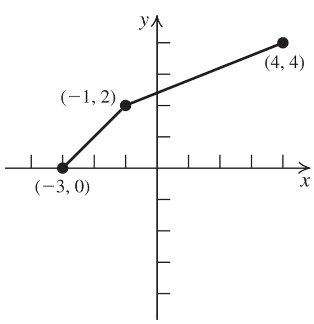
91. $y = 3(-x+4)^3 + 2$

92. $y = -(-x+1)^3 + 1$

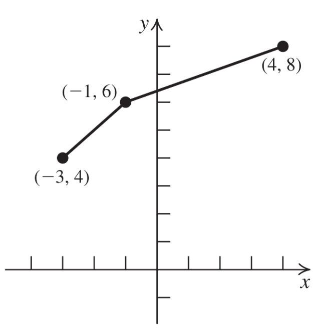
93. $y = -2|x-4| - 3$

94. $y = \frac{1}{2}|-x-2| - 3$

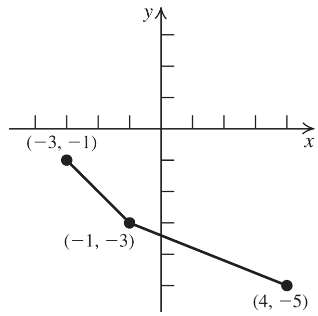
95.



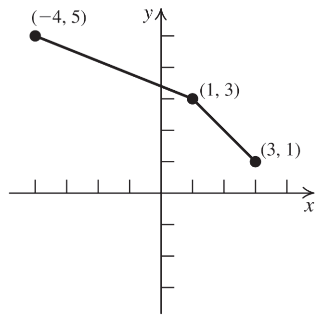
96.



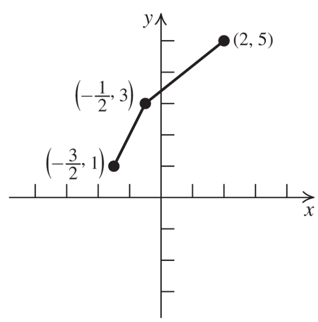
97.



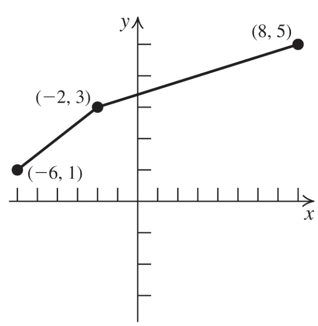
98.



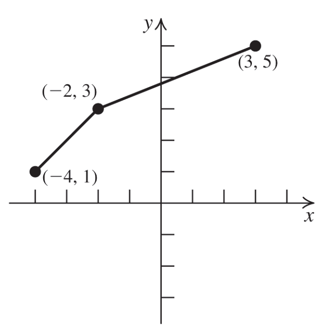
99.



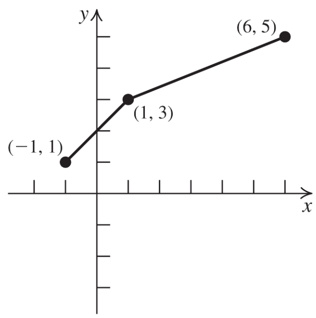
100.



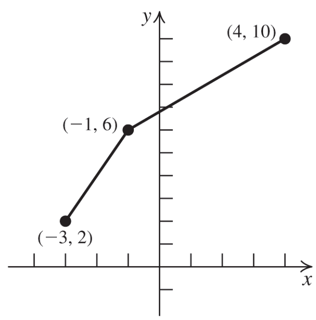
101.



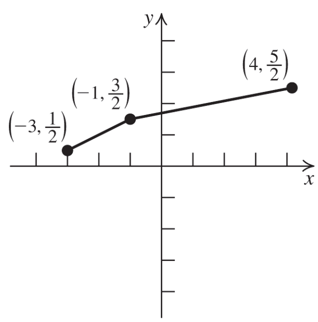
102.



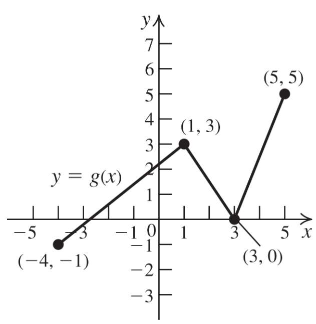
103.



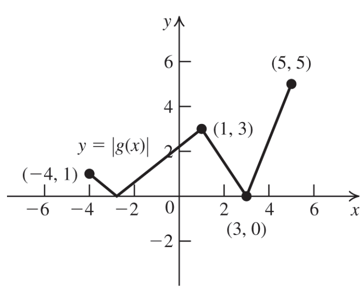
104.



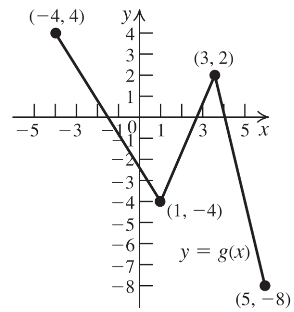
105. a.



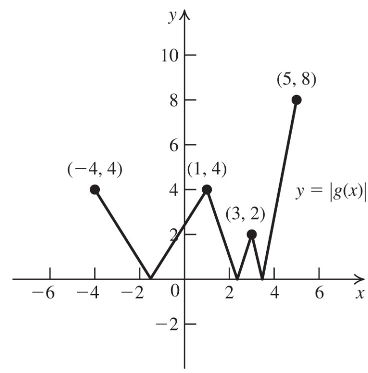
b.



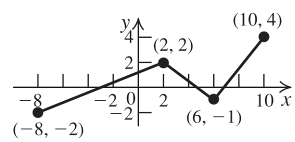
106. a.



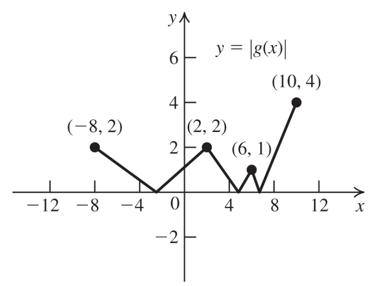
b.



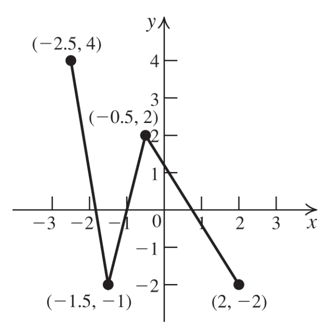
107. a.



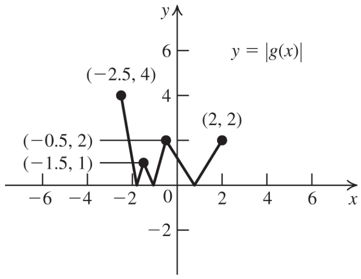
b.



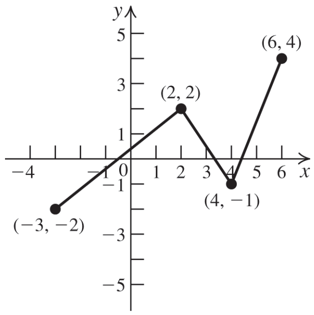
108. a.



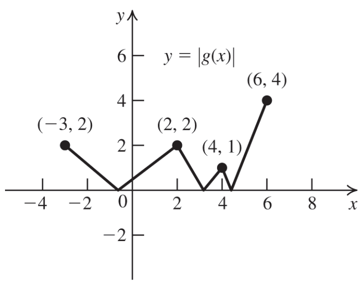
b.



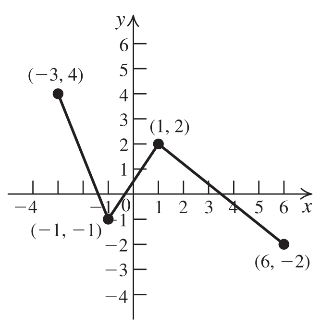
109. a.



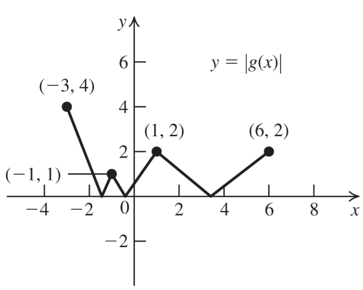
b.



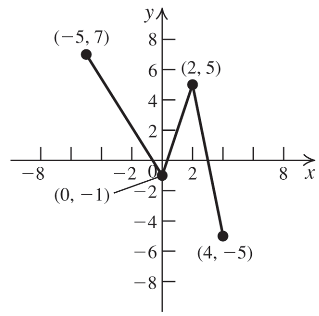
110. a.



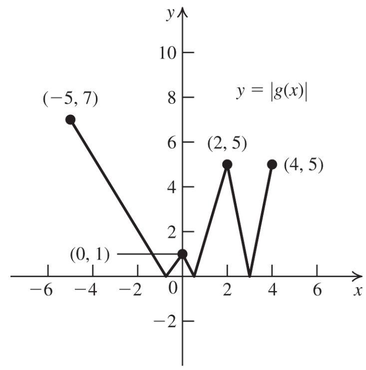
b.



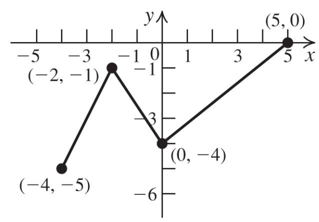
111. a.



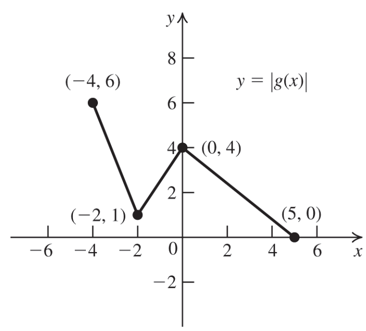
b.



112. a.



b.



Applying the Concepts

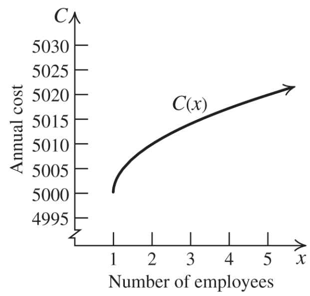
113. $g(x) = f(x) + 800$

114. $h(x) = 1.05f(x)$

115. $p(x) = 1.02\left(f\left(x\right) + 500\right)$

116. $g(x) = \begin{cases} 1.1f\left(x\right) & \text{if } f\left(x\right) < 30,000 \\ 1.02f\left(x\right) & \text{if } f\left(x\right) \geq 30,000 \end{cases}$

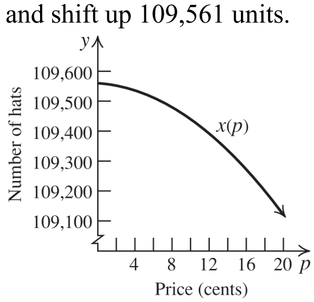
117. a. Shift one unit right, stretch vertically by a factor of 10, and shift 5000 units up.



b. $C(400) = 5000 + 10\sqrt{400 - 1} = \$5199.75$

118. $1.1C(x) = 1.1\left(5000 + 10\sqrt{x-1}\right)$
$= 5500 + 11\sqrt{x-1}$

119. a. Shift one unit left, reflect across the x-axis, and shift up 109,561 units.



b. $69{,}160 = 109{,}561 - (p+1)^2$
$40{,}401 = (p+1)^2$
$201 = p + 1 \Rightarrow p = 200¢ = \$2.00$

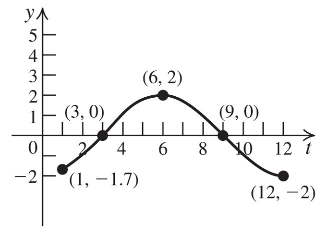
c. $0 = 109{,}561 - (p+1)^2$
$109{,}561 = (p+1)^2$
$331 = p + 1 \Rightarrow p = 330¢ = \$3.30$

120. Write $R(p)$ in the form $-3(p-h)^2 + k$:

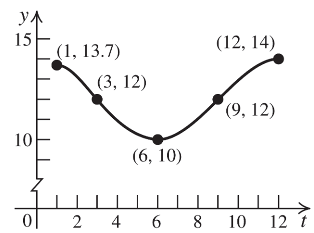
$R(p) = -3p^2 + 600p = -3(p^2 - 200p)$
Complete the square
$= -3(p^2 - 200p + 10{,}000) + 30{,}000$
$= -3(p-100)^2 + 30{,}000$

To graph this, shift $R(p)$ 100 units to the right, stretch by a factor of 3, reflect about the x-axis, and shift by 30,000 units up.

121. The first coordinate gives the month; the second coordinate gives the hours of daylight. From March to September, there is daylight more than half of the day each day. From September to March, more than half of the day is dark each day.



122. The graph shows the number of hours of darkness.

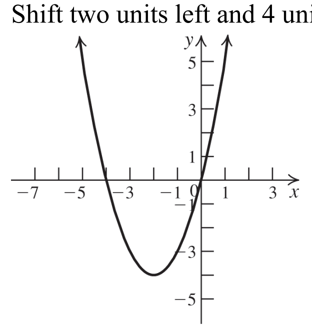


Beyond the Basics

123. The graph is shifted one unit right then reflected about the x-axis, and finally reflected about the y-axis. The equation is $g(x) = -f(1-x)$.

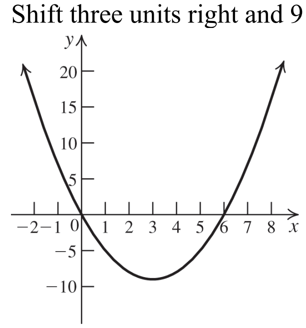
124. The graph is shifted two units left and then reflected about the y-axis.
The equation is $g(x) = f(-2-x)$.

125. Shift two units left and 4 units down.



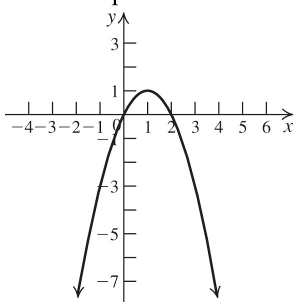
126. $f(x) = x^2 - 6x = \left(x^2 - 6x + 9\right) - 9$
$= (x-3)^2 - 9$

Shift three units right and 9 units down.



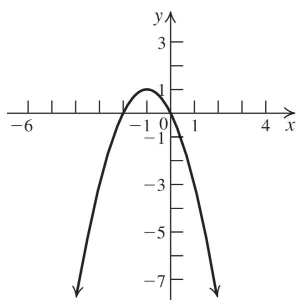
127. $f(x) = -x^2 + 2x = -\left(x^2 - 2x + 1\right) + 1$
$= -(x-1)^2 + 1$

Shift one unit right, reflect in the x-axis, shift one unit up.



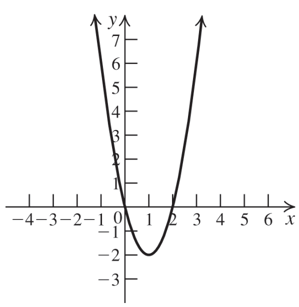
128. $f(x) = -x^2 - 2x = -\left(x^2 + 2x + 1\right) + 1$
$= -(x+1)^2 + 1$

Shift one unit left, reflect across the x-axis, shift one unit up.



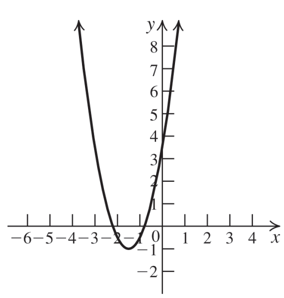
129. $f(x) = 2x^2 - 4x = 2\left(x^2 - 2x + 1\right) - 2$
$= 2(x-1)^2 - 2$

Shift one unit right, stretch vertically by a factor of 2, shift two units down.



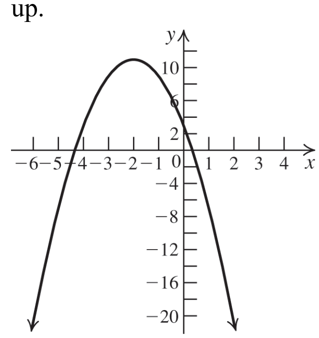
130. $f(x) = 2x^2 + 6x + 3.5$
$= 2\left(x^2 + 3x + 1.75 + 0.5\right) - 1$
$= 2(x+1.5)^2 - 1$

Shift 1.5 units left, stretch vertically by a factor of 2, shift one unit down.



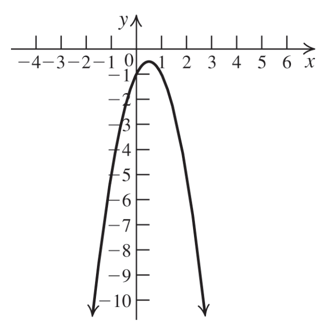
131. $f(x) = -2x^2 - 8x + 3 = -2(x^2 + 4x - 1.5)$
$= -2\left(x^2 + 4x - 1.5 + 5.5\right) + 11$
$= -2(x+2)^2 + 11$

Shift two units left, stretch vertically by a factor of 2, reflect across the x-axis, shift eleven units up.

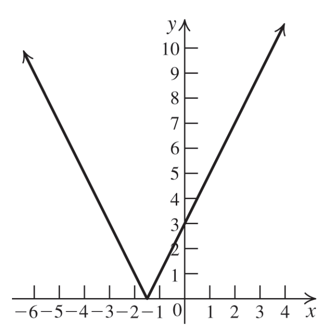


132. $f(x) = -2x^2 + 2x - 1 = -2(x^2 - x + 0.5)$
$= -2\left(x^2 - x + 0.5 - 0.25\right) - 0.5$
$= -2\left(x^2 - x + 0.25\right) - 0.5$
$= -2(x-0.5)^2 - 0.5$

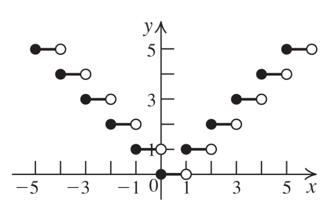
Shift 0.5 unit right, stretch vertically by a factor of 2, reflect across the x-axis, shift 0.5 unit down.



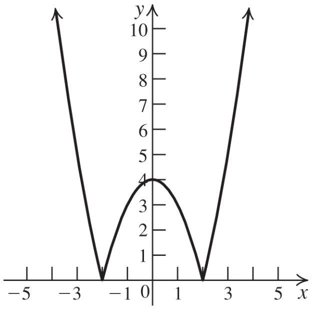
133.



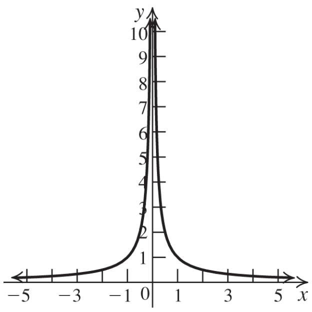
134.



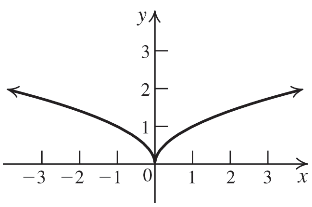
135.



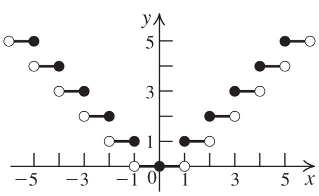
136.



137.



138.



Critical Thinking/Discussion/Writing

139. a. $y = f(x+2)$ is the graph of $y = f(x)$ shifted two units left. So the x-intercepts are $-1 - 2 = -3$, $0 - 2 = -2$, and $2 - 2 = 0$.

b. $y = f(x-2)$ is the graph of $y = f(x)$ shifted two units right. So the x-intercepts are $-1 + 2 = 1$, $0 + 2 = 2$, and $2 + 2 = 4$.

c. $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis. The x-intercepts are the same, -1, 0, 2.

d. $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis. The x-intercepts are the opposites, 1, 0, -2.

e. $y = f(2x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of 1/2. The x-intercepts are $-\frac{1}{2}$, 0, 1.

f. $y = f\left(\frac{1}{2}x\right)$ is the graph of $y = f(x)$ stretched horizontally by a factor of 2. The x-intercepts are -2, 0, 4.

140. a. $y = f(x) + 2$ is the graph of $y = f(x)$ shifted two units up. The y-intercept is $2 + 2 = 4$.

b. $y = f(x) - 2$ is the graph of $y = f(x)$ shifted two units down. The y-intercept is $2 - 2 = 0$.

c. $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis. The y-intercept is the opposite, -2.

d. $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis. The y-intercept is the same, 2.

e. $y = 2f(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 2. The y-intercept is 4.

f. $y = \frac{1}{2}f(x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of 1/2. The y-intercept is 1.

141. a. $y = f(x+2)$ is the graph of $y = f(x)$ shifted two units left. The domain is $[-1-2, 3-2] = [-3, 1]$. The range is the same, $[-2, 1]$.

b. $y = f(x) - 2$ is the graph of $y = f(x)$ shifted two units down. The domain is the same, $[-1, 3]$. The range is $[-2-2, 1-2] = [-4, -1]$.

c. $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis. The domain is the same, $[-1, 3]$. The range is the opposite, $[-1, 2]$.

d. $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis. The domain is the opposite, $[-3, 1]$. The range is the same, $[-2, 1]$.

e. $y = 2f(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 2. The domain is the same, $[-1, 3]$. The range is $[2(-2), 2(1)] = [-4, 2]$.

f. $y = \frac{1}{2}f(x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of 1/2. The domain is the same, $[-1, 3]$. The range is $\left[\frac{1}{2}(-2), \frac{1}{2}(1)\right] = \left[-1, \frac{1}{2}\right]$.

142. a. $y = f(x+2)$ is the graph of $y = f(x)$ shifted two units left. So the relative maximum is at $x = 1 - 2 = -1$, and the relative minimum is at $x = 2 - 2 = 0$.

b. $y = f(x) - 2$ is the graph of $y = f(x)$ shifted two units down. The locations of the relative maximum and minimum do not change. Relative maximum at $x = 1$, relative minimum at $x = 2$.

c. $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis. The relative maximum and relative minimum switch. The relative maximum occurs at $x = 2$, and the relative minimum occurs at $x = 1$.

d. $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis. The relative maximum and relative minimum occur at their opposites. The relative maximum occurs at $x = -1$, and the relative minimum occurs at $x = -2$.

e. $y = 2f(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 2. The locations of the relative maximum and minimum do not change. Relative maximum at $x = 1$, relative minimum at $x = 2$.

f. $y = \frac{1}{2}f(x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of 1/2. The locations of the relative maximum and minimum do not change. Relative maximum at $x = 1$, relative minimum at $x = 2$.

Getting Ready for the Next Section

143. $\left(x^2 + 2x\right) + \left(6x^3 - 2x + 5\right) = 6x^3 + x^2 + 5$

144. $\left(x^3 + 2\right) - \left(2x^3 + 5x - 3\right) = -x^3 - 5x + 5$

145. $(x-2)\left(x^2 + 2x + 4\right)$
$$= x^3 + 2x^2 + 4x - 2x^2 - 4x - 8$$
$$= x^3 - 8$$

146. $\left(x^2 + x + 1\right)\left(x^2 - x + 1\right)$
$$= x^4 - x^3 + x^2 + x^3 - x^2 + x + x^2 - x + 1$$
$$= x^4 + x^2 + 1$$

147. $f(x) = \dfrac{2x-3}{x^2 - 5x + 6}$

The function is not defined when the denominator is zero.

$x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x = 2, 3$

The domain is $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$.

148. $f(x) = \dfrac{x-2}{x^2 - 4}$

The function is not defined when the denominator is zero.

$x^2 - 4 = 0 \Rightarrow (x+2)(x-2) = 0 \Rightarrow x = -2, 2$

The domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

149. $f(x) = \sqrt{2x-3}$

The function is defined only if $2x - 3 \ge 0$.

$2x - 3 \ge 0 \Rightarrow 2x \ge 3 \Rightarrow x \ge \dfrac{3}{2}$

The domain is $\left[\dfrac{3}{2}, \infty\right)$.

150. $f(x) = \dfrac{1}{\sqrt{5-2x}}$

The function is defined only if $5 - 2x > 0$.

$5 - 2x > 0 \Rightarrow -2x > -5 \Rightarrow x < \dfrac{5}{2}$

The domain is $\left(-\infty, \dfrac{5}{2}\right)$.

151. $f(x) = x^2 - 3$
$$f(2x) = (2x)^2 - 3 = 4x^2 - 3$$
$$f(x+1) = (x+1)^2 - 3 = x^2 + 2x + 1 - 3 = x^2 + 2x - 2$$
$$f(2x-1) = (2x-1)^2 - 3 = 4x^2 - 4x + 1 - 3 = 4x^2 - 4x - 2$$

152. $f(x) = x^2 - 2x$
$$f(2x) = (2x)^2 - 2(2x) = 4x^2 - 4x$$
$$f(x+1) = (x+1)^2 - 2(x+1) = x^2 + 2x + 1 - 2x - 2 = x^2 - 1$$
$$f(2x-1) = (2x-1)^2 - 2(2x-1) = 4x^2 - 4x + 1 - (4x - 2) = 4x^2 - 8x + 3$$

153. $f(x) = \sqrt{4-x}$
$$f(2x) = \sqrt{4-2x}$$
$$f(x+1) = \sqrt{4-(x+1)} = \sqrt{3-x}$$
$$f(2x-1) = \sqrt{4-(2x-1)} = \sqrt{5-2x}$$

154. $f(x) = \dfrac{x-2}{x+1}$
$$f(2x) = \frac{2x-2}{2x+1}$$
$$f(x+1) = \frac{(x+1)-2}{(x+1)+1} = \frac{x-1}{x+2}$$
$$f(2x-1) = \frac{(2x-1)-2}{(2x-1)+1} = \frac{2x-3}{2x}$$

1.6 Combining Functions; Composite Functions

1.6 Practice Problems

1. $f(x) = 3x - 1,\ g(x) = x^2 + 2$
$$(f+g)(x) = f(x) + g(x) = 3x - 1 + x^2 + 2 = x^2 + 3x + 1$$
$$(f-g)(x) = f(x) - g(x) = (3x-1) - (x^2 + 2) = -x^2 + 3x - 3$$
$$(fg)(x) = f(x) \cdot g(x) = (3x-1)(x^2+2) = 3x^3 - x^2 + 6x - 2$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x-1}{x^2+2}$$

2. $f(x) = \sqrt{x-1},\ g(x) = \sqrt{3-x}$

The domain of f is $[1, \infty)$ and the domain of g is $(-\infty, 3]$. The intersection of D_f and D_g, $D_f \cap D_g = [1, 3]$.

The domain of fg is $[1, 3]$.

The domain of $\dfrac{f}{g}$ is $[1, 3)$.

The domain of $\dfrac{g}{f}$ is $(1, 3]$.

3. $f(x) = -5x,\ g(x) = x^2 + 1$

a. $(f \circ g)(0) = f(g(0)) = f(0^2 + 1) = f(1) = -5$

b. $(g \circ f)(0) = g(f(0)) = g(-5 \cdot 0) = g(0) = 1$

4. $f(x) = 2 - x,\ g(x) = 2x^2 + 1$

a.
$$(g \circ f)(x) = g(f(x)) = g(2-x) = 2(2-x)^2 + 1 = 2(4 - 4x + x^2) + 1 = 8 - 8x + 2x^2 + 1 = 2x^2 - 8x + 9$$

b.
$$(f \circ g)(x) = f(g(x)) = f(2x^2 + 1) = 2 - (2x^2 + 1) = 1 - 2x^2$$

c.
$$(g \circ g)(x) = g(g(x)) = g(2x^2 + 1) = 2(2x^2 + 1)^2 + 1 = 2(4x^4 + 4x^2 + 1) + 1 = 8x^4 + 8x^2 + 3$$

5. $f(x) = \sqrt{x+1},\ g(x) = \dfrac{2}{x-3}$

Let $A = \{x \mid g(x) \text{ is defined}\}$.

$g(x)$ is not defined if $x = 3$, so $A = (-\infty, 3) \cup (3, \infty)$.

Let $B = \{x \mid f(g(x)) \text{ is defined}\}$.
$$f(g(x)) = \sqrt{\frac{2}{x-3} + 1} = \sqrt{\frac{2 + x - 3}{x-3}} = \sqrt{\frac{x-1}{x-3}}$$

(*continued on next page*)

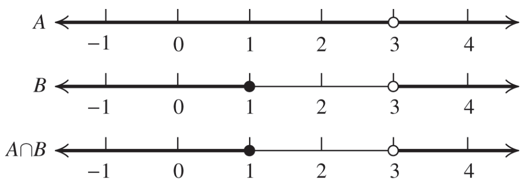
(*continued*)

$f(g(x))$ is not defined if $x = 3$ or if $\frac{x-1}{x-3} < 0$.

$x - 1 = 0 \Rightarrow x = 1$

Interval	Test point	Value of $\frac{x-1}{x-3}$	Result
$(-\infty, 1]$	0	$\frac{1}{3}$	+
$[1, 3)$	2	-1	−
$(3, \infty)$	4	3	+

$f(g(x))$ is not defined for $[1, 3)$, so $B = (-\infty, 1] \cup (3, \infty)$.



The domain of $f \circ g$ is $A \cap B = (-\infty, 1] \cup (3, \infty)$.

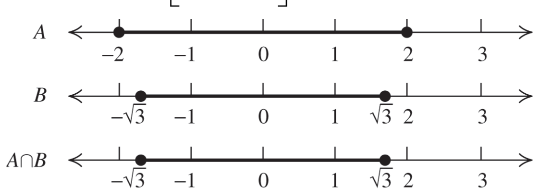
6. $f(x) = \sqrt{x-1}$, $g(x) = \sqrt{4-x^2}$

a. $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{4-x^2}\right)$
$= \sqrt{\sqrt{4-x^2} - 1}$

The function $g(x) = \sqrt{4-x^2}$ is defined for $4 - x^2 \ge 0 \Rightarrow x^2 \le 4 \Rightarrow -2 \le x \le 2$. So, $A = [-2, 2]$.

The function $f(g(x))$ is defined for
$\sqrt{4-x^2} - 1 \ge 0 \Rightarrow \sqrt{4-x^2} \ge 1 \Rightarrow$
$4 - x^2 \ge 1 \Rightarrow -x^2 \ge -3 \Rightarrow x^2 \le 3 \Rightarrow$
$-\sqrt{3} \le x \le \sqrt{3}$

So, $B = \left[-\sqrt{3}, \sqrt{3}\right]$.

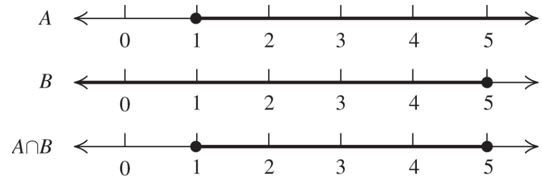


The domain of $f \circ g$ is $A \cap B = \left[-\sqrt{3}, \sqrt{3}\right]$.

b. $(g \circ f)(x) = g(f(x)) = \sqrt{4 - \left(\sqrt{x-1}\right)^2}$
$= \sqrt{4 - (x-1)} = \sqrt{5-x}$

The function $f(x) = \sqrt{x-1}$ is defined for $x - 1 \ge 0 \Rightarrow x \ge 1$. So, $A = [1, \infty)$.

The function $g(f(x))$ is defined for $5 - x \ge 0 \Rightarrow 5 \ge x$, or $x \le 5$. So, $B = (-\infty, 5]$.



The domain of $g \circ f$ is $A \cap B = [1, 5]$.

7. $H(x) = \frac{1}{\sqrt{2x^2+1}}$, $g(x) = \sqrt{2x^2+1}$

If $f(x) = \frac{1}{x}$, then

$H(x) = (f \circ g)(x) = f\left(\sqrt{2x^2+1}\right) = \frac{1}{\sqrt{2x^2+1}}$.

8. $f(x) = 2 - x^2$; $g(x) = 3x - 1$

First compute the average rate of change of g from $x = -1$ to $x = 2$.

$$\text{ARC}[g]_{-1}^{2} = \frac{g(2) - g(-1)}{2 - (-1)} = \frac{[3(2)-1] - [3(-1)-1]}{3} = \frac{5 - (-4)}{3} = \frac{9}{3} = 3$$

To compute the average rate of change of f, modify the range from $g(-1) = -4$ to $g(2) = 5$.

$$\text{ARC}[f]_{-4}^{5} = \frac{f(5) - f(-4)}{5 - (-4)} = \frac{\left[2 - 5^2\right] - \left[2 - (-4)^2\right]}{9} = \frac{-23 - (-14)}{9} = \frac{-9}{9} = -1$$

Finally,

$$\text{ARC}[f \circ g]_{-1}^{2} = \text{ARC}[f]_{-4}^{5} \cdot \text{ARC}[g]_{-1}^{2} = -1 \cdot 3 = -3$$

9. a. $\text{CBR} = 0.32(11.55 + 144.84g)$

Since g is constant over the whole time period, the woman will burn the calories at the same rate given by the equation.

$\text{CBR} = 0.32(11.55 + 144.84 \cdot 0.03) \approx 5.09$

The exercise lasts 1 hour, so she will burn $60 \cdot 5.09 \approx 305$ calories.

b. Express g as a piecewise function:

$$g(t) = \begin{cases} 0.02 & \text{if } 0 \le t \le 10 \\ 0.05 & \text{if } 10 < t \le 40 \\ 0.04 & \text{if } 40 < t \le 60 \end{cases}$$

On each of the intervals, the woman will burn calories at a different rate.

$$\text{CBR}(t) = \begin{cases} 0.32(11.55 + 144.84 \cdot 0.02) \approx 4.62 \\ \quad \text{if } 0 \le t \le 10 \\ 0.32(11.55 + 144.84 \cdot 0.05) \approx 6.01 \\ \quad \text{if } 10 < t \le 40 \\ 0.32(11.55 + 144.84 \cdot 0.04) \approx 5.55 \\ \quad \text{if } 40 < t \le 60 \end{cases}$$

Taking into account the length of the intervals, estimate the total number of burned calories:

$4.62(10) + 6.01(30) + 5.55(20) \approx 338$ cal.

10. a. $r(x) = x - 4500$

b. $d(x) = x - 0.06x = 0.94x$

c. i. $(r \circ d)(x) = r(0.94x) = 0.94x - 4500$

ii. $(d \circ r)(x) = d(x - 4500)$
$= 0.94(x - 4500)$
$= 0.94x - 4230$

d. $(d \circ r)(x) - (r \circ d)(x)$
$= (0.94x - 4230) - (0.94x - 4500)$
$= 270$

1.6 Exercises
Concepts and Vocabulary

1. $(f \cdot g)(x) = \underline{f(x) \cdot g(x)}$.

2. The domain of the function $f + g$ consists of those values of x that are <u>common</u> to the domains of f and g.

3. The composition of the function f with the function g is written as $f \circ g$ and is defined by $f \circ g(x) = \underline{f(g(x))}$.

4. The domain of the composite function $f \circ g$ consists of those values of x in the domain of g for which $g(x)$ <u>is in the domain of f</u>.

5. False. For example, if $f(x) = 2x$ and $g(x) = x^2$, then $(f \circ g)(x) = f(g(x)) = f(x^2) = 2x^2$, while $(g \circ f)(x) = g(f(x)) = g(2x) = 4x^2$.

6. True

7. False. The domain of $f \cdot g$ may include $g(x) = 0$, but the domain of $\frac{f}{g}$ cannot include $g(x) = 0$.

8. True

Building Skills

9. $(f + g)(-2) = f(-2) + g(-2) = 1 + 2 = 3$

10. $(f + g)(2) = f(2) + g(2) = -2 + (-1) = -3$

11. $(f - g)(4) = f(4) - g(4) = -2 - 1 = -3$

12. $(f - g)(-1) = f(-1) - g(-1) = 1 - (-4) = 5$

13. $(f \cdot g)(-1) = f(-1) \cdot g(-1) = 1 \cdot (-4) = -4$

14. $(f \cdot g)(2) = f(2) \cdot g(2) = -2 \cdot (-1) = 2$

15. $\left(\frac{f}{g}\right)(-2) = \frac{f(-2)}{g(-2)} = \frac{1}{2}$

16. $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{-2}{-1} = 2$

17. $(f \circ g)(1) = f(g(1)) = f(-2) = 1$

18. $(g \circ f)(1) = g(f(1)) = g(-2) = 2$

19. $(f \circ g)(-3) = f(g(-3)) = f(0) = 0$

20. $(g \circ f)(-3) = g(f(-3)) = g(1) = -2$

21. a. $(f + g)(-1) = f(-1) + g(-1)$
$= 2(-1) + -(-1) = -2 + 1 = -1$

b. $(f - g)(0) = f(0) - g(0) = 2(0) - (-0) = 0$

c. $(f \cdot g)(2) = f(2) \cdot g(2) = 2(2) \cdot (-2) = -8$

d. $\left(\dfrac{f}{g}\right)(1)=\dfrac{f(1)}{g(1)}=\dfrac{2(1)}{-1}=-2$

22. a. $(f+g)(-1)=f(-1)+g(-1)$
$=\left(1-(-1)^2\right)+(-1+1)=0$

b. $(f-g)(0)=f(0)-g(0)$
$=(1-0^2)-(0+1)=0$

c. $(f\cdot g)(2)=f(2)\cdot g(2)$
$=(1-2^2)\cdot(2+1)=-9$

d. $\left(\dfrac{f}{g}\right)(1)=\dfrac{f(1)}{g(1)}=\dfrac{1-1^2}{1+1}=0$

23. a. $(f+g)(-1)=f(-1)+g(-1)$
$=\dfrac{1}{\sqrt{-1+2}}+\left(2(-1)+1\right)=0$

b. $(f-g)(0)=f(0)-g(0)$
$=\dfrac{1}{\sqrt{0+2}}-\left(2(0)+1\right)=\dfrac{\sqrt{2}}{2}-1$

c. $(f\cdot g)(2)=f(2)\cdot g(2)$
$=\dfrac{1}{\sqrt{2+2}}\cdot\left(2(2)+1\right)=\dfrac{5}{2}$

d. $\left(\dfrac{f}{g}\right)(1)=\dfrac{f(1)}{g(1)}=\dfrac{\frac{1}{\sqrt{1+2}}}{2(1)+1}=\dfrac{1}{3\sqrt{3}}=\dfrac{\sqrt{3}}{9}$

24. a. $(f+g)(-1)=f(-1)+g(-1)$
$=\dfrac{-1}{(-1)^2-6(-1)+8}+\left(3-(-1)\right)$
$=-\dfrac{1}{15}+4=\dfrac{59}{15}$

b. $(f-g)(0)=f(0)-g(0)$
$=\dfrac{0}{0^2-6(0)+8}-(3-0)=-3$

c. $(f\cdot g)(2)=f(2)\cdot g(2)$
$=\dfrac{2}{2^2-6(2)+8}\cdot(3-2)=\dfrac{2}{0}\cdot 1\Rightarrow$
the product does not exist.

d. $\left(\dfrac{f}{g}\right)(1)=\dfrac{f(1)}{g(1)}=\dfrac{\frac{1}{1^2-6(1)+8}}{3-1}=\dfrac{\frac{1}{3}}{2}=\dfrac{1}{6}$

25. a. $f+g=x^2+x-3;$ domain: $(-\infty,\infty)$

b. $f-g=x-3-x^2=-x^2+x-3;$
domain: $(-\infty,\infty)$

c. $f\cdot g=(x-3)x^2=x^3-3x^2;$
domain: $(-\infty,\infty)$

d. $\dfrac{f}{g}=\dfrac{x-3}{x^2};$ domain: $(-\infty,0)\cup(0,\infty)$

e. $\dfrac{g}{f}=\dfrac{x^2}{x-3};$ domain: $(-\infty,3)\cup(3,\infty)$

26. a. $f+g=x^2+2x-1;$ domain: $(-\infty,\infty)$

b. $f-g=2x-1-x^2=-x^2+2x-1;$
domain: $(-\infty,\infty)$

c. $f\cdot g=(2x-1)x^2=2x^3-x^2;$
domain: $(-\infty,\infty)$

d. $\dfrac{f}{g}=\dfrac{2x-1}{x^2};$ domain: $(-\infty,0)\cup(0,\infty)$

e. $\dfrac{g}{f}=\dfrac{x^2}{2x-1};$ domain: $\left(-\infty,\dfrac{1}{2}\right)\cup\left(\dfrac{1}{2},\infty\right)$

27. a. $f+g=\left(x^3-1\right)+\left(2x^2+5\right)=x^3+2x^2+4;$
domain: $(-\infty,\infty)$

b. $f-g=\left(x^3-1\right)-\left(2x^2+5\right)=x^3-2x^2-6;$
domain: $(-\infty,\infty)$

c. $f\cdot g=\left(x^3-1\right)\left(2x^2+5\right)$
$=2x^5+5x^3-2x^2-5;$
domain: $(-\infty,\infty)$

d. $\dfrac{f}{g}=\dfrac{x^3-1}{2x^2+5};$ domain: $(-\infty,\infty)$

e. $\dfrac{g}{f}=\dfrac{2x^2+5}{x^3-1};$ domain: $(-\infty,1)\cup(1,\infty)$

28. a. $f+g=\left(x^2-4\right)+\left(x^2-6x+8\right)$
$=2x^2-6x+4;$ domain: $(-\infty,\infty)$

b. $f-g=\left(x^2-4\right)-\left(x^2-6x+8\right)=6x-12;$
domain: $(-\infty,\infty)$

c. $f\cdot g=\left(x^2-4\right)\left(x^2-6x+8\right)$
$=x^4-6x^3+4x^2+24x-32;$
domain: $(-\infty,\infty)$

d. $\frac{f}{g} = \frac{x^2 - 4}{x^2 - 6x + 8} = \frac{(x+2)(x-2)}{(x-2)(x-4)} = \frac{x+2}{x-4}$

The denominator of the original fraction = 0 if $x = 2$ or $x = 4$, so the domain is $(-\infty, 2) \cup (2, 4) \cup (4, \infty)$.

e. $\frac{g}{f} = \frac{x^2 - 6x + 8}{x^2 - 4} = \frac{(x-2)(x-4)}{(x-2)(x+2)} = \frac{x-4}{x+2}$

The denominator of the original fraction = 0 if $x = 2$ or $x = 4$, so the domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

29. a. $f + g = 2x + \sqrt{x} - 1$; domain: $[0, \infty)$

b. $f - g = 2x - \sqrt{x} - 1$; domain: $[0, \infty)$

c. $f \cdot g = (2x - 1)\sqrt{x} = 2x\sqrt{x} - \sqrt{x}$; domain: $[0, \infty)$

d. $\frac{f}{g} = \frac{2x-1}{\sqrt{x}}$; domain: $(0, \infty)$

e. $\frac{g}{f} = \frac{\sqrt{x}}{2x-1}$; the numerator is defined only for $x \ge 0$, while the denominator = 0 when $x = \frac{1}{2}$, so the domain is $\left[0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

30. a. $f + g = x - 1 + \sqrt{x}$; domain: $[0, \infty)$

b. $f - g = x - 1 - \sqrt{x}$; domain: $[0, \infty)$

c. $f \cdot g = (x - 1)\sqrt{x}$; domain: $[0, \infty)$

d. $\frac{f}{g} = \frac{x-1}{\sqrt{x}}$; domain: $(0, \infty)$

e. $\frac{g}{f} = \frac{\sqrt{x}}{x-1}$; the numerator is defined only for $x \ge 0$, while the denominator = 0 when $x = 1$, so the domain is $[0, 1) \cup (1, \infty)$.

31. a. $f + g = x - 6 + \sqrt{x-3}$; domain: $[3, \infty)$

b. $f - g = x - 6 - \sqrt{x-3}$; domain: $[3, \infty)$

c. $f \cdot g = (x - 6)\sqrt{x-3}$; domain: $[3, \infty)$

d. $\frac{f}{g} = \frac{x-6}{\sqrt{x-3}}$; domain: $(3, \infty)$

e. $\frac{g}{f} = \frac{\sqrt{x-3}}{x-6}$; the numerator is defined only for $x \ge 3$, while the denominator = 0 when $x = 6$, so the domain is $[3, 6) \cup (6, \infty)$.

32. a. $f + g = x + 2 + \sqrt{1-x}$; domain: $(-\infty, 1]$

b. $f - g = x + 2 - \sqrt{1-x}$; domain: $(-\infty, 1]$

c. $f \cdot g = (x + 2)\sqrt{1-x}$; domain: $(-\infty, 1]$

d. $\frac{f}{g} = \frac{x+2}{\sqrt{1-x}}$; domain: $(-\infty, 1]$

e. $\frac{g}{f} = \frac{\sqrt{1-x}}{x+2}$; the numerator is defined only for $x \le 1$, while the denominator = 0 when $x = -2$, so the domain is $(-\infty. -2) \cup (-2, 1]$.

33. a. $f + g = 1 - \frac{2}{x+1} + \frac{1}{x}$

$= \frac{x(x+1) - 2x + (x+1)}{x(x+1)}$

$= \frac{x^2 + x - 2x + x + 1}{x(x+1)} = \frac{x^2 + 1}{x(x+1)}$

domain: $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

b. $f - g = 1 - \frac{2}{x+1} - \frac{1}{x}$

$= \frac{x(x+1) - 2x - (x+1)}{x(x+1)}$

$= \frac{x^2 + x - 2x - x - 1}{x(x+1)} = \frac{x^2 - 2x - 1}{x(x+1)}$

domain: $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

c. $f \cdot g = \left(1 - \frac{2}{x+1}\right)\frac{1}{x} = \left(\frac{x+1-2}{x+1}\right)\frac{1}{x}$

$= \left(\frac{x-1}{x+1}\right)\frac{1}{x} = \frac{x-1}{x(x+1)}$

domain: $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

d. $\frac{f}{g} = \frac{1 - \frac{2}{x+1}}{\frac{1}{x}} = \left(1 - \frac{2}{x+1}\right)(x)$

$= \left(\frac{x+1-2}{x+1}\right)x = \frac{x(x-1)}{x+1}$

domain: $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

e. $\dfrac{g}{f}=\dfrac{\frac{1}{x}}{1-\frac{2}{x+1}}=\dfrac{\frac{1}{x}(x+1)}{\left(1-\frac{2}{x+1}\right)(x+1)}=\dfrac{\frac{x+1}{x}}{x+1-2}$

$=\dfrac{\frac{x+1}{x}}{x-1}=\dfrac{x+1}{x(x-1)}$

The denominator equals zero when $x = 0$ or $x = 1$, so the domain is $(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(0,\infty)$.

34. a. $f+g=\left(1-\dfrac{1}{x}\right)+\dfrac{1}{x}=1$

Neither f nor g is defined for $x = 0$, so the domain is $(-\infty,0)\cup(0,\infty)$.

b. $f-g=\left(1-\dfrac{1}{x}\right)-\dfrac{1}{x}=1-\dfrac{2}{x}$;
domain: $(-\infty,0)\cup(0,\infty)$.

c. $f\cdot g=\left(1-\dfrac{1}{x}\right)\left(\dfrac{1}{x}\right)=\dfrac{1}{x}-\dfrac{1}{x^2}=\dfrac{x-1}{x^2}$;
domain: $(-\infty,0)\cup(0,\infty)$.

d. $\dfrac{f}{g}=\dfrac{1-\frac{1}{x}}{\frac{1}{x}}=\dfrac{\frac{x-1}{x}}{\frac{1}{x}}=x-1$

Neither f nor g is defined for $x = 0$, so the domain is $(-\infty,0)\cup(0,\infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{1}{x}}{1-\frac{1}{x}}=\dfrac{\frac{1}{x}}{\frac{x-1}{x}}=\dfrac{1}{x-1}$

Neither f nor g is defined for $x = 0$, and g/f is not defined for $x = 1$, so the domain is $(-\infty,0)\cup(0,1)\cup(1,\infty)$.

35. a. $f+g=\dfrac{2}{x+1}+\dfrac{x}{x+1}=\dfrac{2+x}{x+1}$

Neither f nor g is defined for $x = -1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

b. $f-g=\dfrac{2}{x+1}-\dfrac{x}{x+1}=\dfrac{2-x}{x+1}$.
domain: $(-\infty,-1)\cup(-1,\infty)$.

c. $f\cdot g=\left(\dfrac{2}{x+1}\right)\left(\dfrac{x}{x+1}\right)=\dfrac{2x}{(x+1)^2}$.
domain: $(-\infty,-1)\cup(-1,\infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{2}{x+1}}{\frac{x}{x+1}}=\dfrac{2}{x}$. Neither f nor g is defined for $x = -1$, and f/g is not defined for $x = 0$, so the domain is $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$.

e. $\dfrac{f}{g}=\dfrac{\frac{x}{x+1}}{\frac{2}{x+1}}=\dfrac{x}{2}$. Neither f nor g is defined for $x = -1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

36. $f(x)=\dfrac{5x-1}{x+1}$; $g(x)=\dfrac{4x+10}{x+1}$

a. $f+g=\dfrac{5x-1}{x+1}+\dfrac{4x+10}{x+1}=\dfrac{9x+9}{x+1}$

$=\dfrac{9(x+1)}{x+1}=9$

Neither f nor g is defined for $x = -1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

b. $f-g=\dfrac{5x-1}{x+1}-\dfrac{4x+10}{x+1}=\dfrac{x-11}{x+1}$

Neither f nor g is defined for $x = -1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

c. $f\cdot g=\dfrac{5x-1}{x+1}\cdot\dfrac{4x+10}{x+1}=\dfrac{20x^2+46x-10}{x^2+2x+1}$

Neither f nor g is defined for $x = -1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{5x-1}{x+1}}{\frac{4x+10}{x+1}}=\dfrac{5x-1}{4x+10}$

Neither f nor g is defined for $x = -1$ and f/g is not defined for $x=-5/2$, so the domain is $\left(-\infty,-\dfrac{5}{2}\right)\cup\left(-\dfrac{5}{2},-1\right)\cup(-1,\infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{4x+10}{x+1}}{\frac{5x-1}{x+1}}=\dfrac{4x+10}{5x-1}$

Neither f nor g is defined for $x = -1$ and g/f is not defined for $x=1/5$, so the domain is $(-\infty,-1)\cup\left(-1,\dfrac{1}{5}\right)\cup\left(\dfrac{1}{5},\infty\right)$.

37. $f(x)=\dfrac{x^2}{x+1}$; $g(x)=\dfrac{2x}{x^2-1}$

a. $f+g=\dfrac{x^2}{x+1}+\dfrac{2x}{x^2-1}=\dfrac{x^2(x-1)}{x^2-1}+\dfrac{2x}{x^2-1}$
$=\dfrac{x^3-x^2+2x}{x^2-1}$

f is not defined for $x=-1$, *g* is not defined for $x=\pm 1$, and $f+g$ is not defined for either -1 or 1, so the domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

b. $f-g=\dfrac{x^2}{x+1}-\dfrac{2x}{x^2-1}=\dfrac{x^2(x-1)}{x^2-1}-\dfrac{2x}{x^2-1}$
$=\dfrac{x^3-x^2-2x}{x^2-1}=\dfrac{x\left(x^2-x-2\right)}{x^2-1}$
$=\dfrac{x(x-2)(x+1)}{(x-1)(x+1)}=\dfrac{x^2-2x}{x-1}$

f is not defined for $x=-1$, *g* is not defined for $x=\pm 1$, and $f-g$ is not defined for 1, so the domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

c. $f\cdot g=\dfrac{x^2}{x+1}\cdot\dfrac{2x}{x^2-1}=\dfrac{2x^3}{x^3+x^2-x-1}$

f is not defined for $x=-1$, *g* is not defined for $x=\pm 1$, and *fg* is not defined for either -1 or 1, so the domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{x^2}{x+1}}{\frac{2x}{x^2-1}}=\dfrac{x^2}{x+1}\cdot\dfrac{x^2-1}{2x}=\dfrac{x(x-1)}{2}$

f is not defined for $x=-1$, *g* is not defined for $x=\pm 1$, and f/g is not defined for either -1, 0, or 1, so the domain is $(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{2x}{x^2-1}}{\frac{x^2}{x+1}}=\dfrac{2x}{x^2-1}\cdot\dfrac{x+1}{x^2}=\dfrac{2}{x(x-1)}$

Neither *f* nor *g* is defined for $x=-1$ and g/f is not defined for $x=0$ or $x=1$, so the domain is $(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$.

38. $f(x)=\dfrac{x-3}{x^2-25}$; $g(x)=\dfrac{x-3}{x^2+9x+20}$

a. $f+g=\dfrac{x-3}{x^2-25}+\dfrac{x-3}{x^2+9x+20}$
$=\dfrac{x-3}{(x-5)(x+5)}+\dfrac{x-3}{(x+4)(x+5)}$
$=\dfrac{(x-3)(x+4)+(x-3)(x-5)}{(x-5)(x+5)(x+4)}$
$=\dfrac{x^2+x-12+x^2-8x+15}{x^3+4x^2-25x-100}$
$=\dfrac{2x^2-7x+3}{x^3+4x^2-25x-100}$

f is not defined for $x=-5$ and $x=5$, *g* is not defined for $x=-4$ and $x=-5$, and $f+g$ is not defined for -5, 5 or -4, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,5)\cup(5,\infty)$.

b. $f-g=\dfrac{x-3}{x^2-25}-\dfrac{x-3}{x^2+9x+20}$
$=\dfrac{x-3}{(x-5)(x+5)}-\dfrac{x-3}{(x+4)(x+5)}$
$=\dfrac{(x-3)(x+4)-(x-3)(x-5)}{(x-5)(x+5)(x+4)}$
$=\dfrac{x^2+x-12-\left(x^2-8x+15\right)}{x^3+4x^2-25x-100}$
$=\dfrac{9x-27}{x^3+4x^2-25x-100}$

f is not defined for $x=-5$ and $x=5$, *g* is not defined for $x=-4$ and $x=-5$, and $f+g$ is not defined for -5, 5, or -4, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,5)\cup(5,\infty)$.

c. $f\cdot g=\dfrac{x-3}{x^2-25}\cdot\dfrac{x-3}{x^2+9x+20}$
$=\dfrac{(x-3)^2}{\left(x^2-25\right)\left(x^2+9x+20\right)}$
$=\dfrac{x^2-6x+9}{x^4+9x^3-5x^2-225x-500}$

f is not defined for $x=-5$ and $x=5$, *g* is not defined for $x=-4$ and $x=-5$, and *fg* is not defined for -5, 5, or -4, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,5)\cup(5,\infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{x-3}{x^2-25}}{\frac{x-3}{x^2+9x+20}}$
$=\dfrac{x-3}{(x-5)(x+5)}\cdot\dfrac{(x+5)(x+4)}{x-3}=\dfrac{x+4}{x-5}$

f is not defined for $x=-5$ and $x=5$, g is not defined for $x=-4$ and $x=-5$, and f/g is not defined for $x=5$, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,3)\cup(3,5)\cup(5,\infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{x-3}{x^2+9x+20}}{\frac{x-3}{x^2-25}}$
$=\dfrac{x-3}{(x+5)(x+4)}\cdot\dfrac{(x-5)(x+5)}{x-3}=\dfrac{x-5}{x+4}$

f is not defined for $x=-5$ and $x=5$, g is not defined for $x=-4$ and $x=-5$, and g/f is not defined for $x=-4$, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,3)\cup(3,5)\cup(5,\infty)$.

39. $f(x)=\sqrt{x-1};\ g(x)=\sqrt{5-x}$

a. $f\cdot g=\sqrt{x-1}\cdot\sqrt{5-x}$
f is not defined for $x<1$, g is not defined for $x>5$. The domain is [1, 5].

b. $\dfrac{f}{g}=\dfrac{\sqrt{x-1}}{\sqrt{5-x}}$
f is not defined for $x<1$, g is not defined for $x>5$. The denominator is zero when $x=5$. The domain is [1, 5).

40. $f(x)=\sqrt{x-2};\ g(x)=\sqrt{x+2}$

a. $f\cdot g=\sqrt{x-2}\cdot\sqrt{x+2}=\sqrt{x^2-4}$
f is not defined for $x<2$, g is not defined for $x<-2$. The domain is $[2,\infty)$.

b. $\dfrac{f}{g}=\dfrac{\sqrt{x-2}}{\sqrt{x+2}}$
f is not defined for $x<2$, g is not defined for $x<-2$. The denominator is zero when $x=-2$. The domain is $[2,\infty)$.

41. $f(x)=\sqrt{x+2};\ g(x)=\sqrt{9-x^2}$

a. $f\cdot g=\sqrt{x+2}\cdot\sqrt{9-x^2}$
f is not defined for $x<-2$, g is defined for [–3, 3]. The domain is $[-2,3]$.

b. $\dfrac{f}{g}=\dfrac{\sqrt{x+2}}{\sqrt{9-x^2}}$
f is not defined for $x<-2$, g is defined for [–3, 3]. The denominator is zero when $x=-3$ or $x=3$. The domain is $[-2,3)$.

42. $f(x)=\sqrt{x^2-4};\ g(x)=\sqrt{25-x^2}$

a. $f\cdot g=\sqrt{x^2-4}\cdot\sqrt{25-x^2}$
f is defined for $x\le -2$ or $x\ge 2$, g is defined for [–5, 5]. The domain is $[-5,-2]\cup[2,5]$.

b. $\dfrac{f}{g}=\dfrac{\sqrt{x^2-4}}{\sqrt{25-x^2}}$
f is defined for $x\le -2$ or $x\ge 2$, g is defined for [–5, 5]. The denominator is zero when $x=-5$ or $x=5$. The domain is $(-5,-2]\cup[2,5)$.

43. $(g\circ f)(x)=2\left(x^2-1\right)+3=2x^2+1;$
$(g\circ f)(2)=2\left(2^2-1\right)+3=9;$
$(g\circ f)(-3)=2\left((-3)^2-1\right)+3=19$

44. $(g\circ f)(x)=3|x+1|^2-1=3\left|x^2+2x+1\right|-1;$
$(g\circ f)(2)=3|2+1|^2-1=26;$
$(g\circ f)(-3)=3|(-3)+1|^2-1=11$

45. $(f\circ g)(2)=2\left(2(2^2)-3\right)+1=11$

46. $(g\circ f)(2)=2(2(2)+1)^2-3=47$

47. $(f\circ g)(-3)=2\left(2(-3)^2-3\right)+1=31$

48. $(g\circ f)(-5)=2(2(-5)+1)^2-3=159$

49. $(f\circ g)(0)=2\left(2(0^2)-3\right)+1=-5$

50. $(g\circ f)\left(\frac{1}{2}\right)=2\left(2\left(\frac{1}{2}\right)+1\right)^2-3=5$

51. $(f\circ g)(-c)=2\left(2(-c)^2-3\right)+1=4c^2-5$

52. $(f\circ g)(c)=2\left(2c^2-3\right)+1=4c^2-5$

53. $(g \circ f)(a) = 2(2a+1)^2 - 3$
$= 2(4a^2 + 4a + 1) - 3$
$= 8a^2 + 8a - 1$

54. $(g \circ f)(-a) = 2(2(-a)+1)^2 - 3$
$= 2(4a^2 - 4a + 1) - 3$
$= 8a^2 - 8a - 1$

55. $(f \circ f)(1) = 2(2(1)+1) + 1 = 7$

56. $(g \circ g)(-1) = 2\left(2(-1)^2 - 3\right)^2 - 3 = -1$

57. $(f \circ g)(x) = \dfrac{2}{\frac{1}{x}+1} = \dfrac{2}{\frac{x+1}{x}} = \dfrac{2x}{x+1}$

The domain of g is $(-\infty, 0) \cup (0, \infty)$. Since -1 is not in the domain of f, we must exclude those values of x that make $g(x) = -1$.

$\dfrac{1}{x} = -1 \Rightarrow x = -1$

Thus, the domain of $f \circ g$ is $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$.

58. $(f \circ g)(x) = \dfrac{1}{\frac{2}{x+3} - 1} = \dfrac{1}{\frac{2-(x+3)}{x+3}}$
$= \dfrac{x+3}{-x-1} = -\dfrac{x+3}{x+1}.$

The domain of g is $(-\infty, -3) \cup (-3, \infty)$. Since 1 is not in the domain of f, we must exclude those values of x that make $g(x) = 1$.

$\dfrac{2}{x+3} = 1 \Rightarrow 2 = x + 3 \Rightarrow x = -1$

Thus, the domain of $f \circ g$ is $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$.

59. $(f \circ g)(x) = \sqrt{(2-3x) - 3} = \sqrt{-1-3x}.$

The domain of g is $(-\infty, \infty)$. Since f is not defined for $(-\infty, 3)$, we must exclude those values of x that make $g(x) < 3$.

$2 - 3x < 3 \Rightarrow -3x < 1 \Rightarrow x > -\dfrac{1}{3}$

Thus, the domain of $f \circ g$ is $\left(-\infty, -\dfrac{1}{3}\right].$

60. $(f \circ g)(x) = \dfrac{2+5x}{(2+5x)-1} = \dfrac{2+5x}{1+5x}$

The domain of g is $(-\infty, \infty)$. Since f is not defined for $x = 1$ we must exclude those values of x that make $g(x) = 1$.

$2 + 5x = 1 \Rightarrow 5x = -1 \Rightarrow x = -\dfrac{1}{5}$

Thus, the domain of $f \circ g$ is $\left(-\infty, -\dfrac{1}{5}\right) \cup \left(-\dfrac{1}{5}, \infty\right).$

61. $(f \circ g)(x) = \left|x^2 - 1\right|$; domain: $(-\infty, \infty)$

62. $(f \circ g)(x) = 3|x-1| - 2$; domain: $(-\infty, \infty)$

63. **a.** $(f \circ g)(x) = 2(x+4) - 3 = 2x + 5$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = (2x-3) + 4 = 2x + 1$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 2(2x-3) - 3 = 4x - 9$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = (x+4) + 4 = x + 8$; domain: $(-\infty, \infty)$

64. **a.** $(f \circ g)(x) = (3x-5) - 3 = 3x - 8$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 3(x-3) - 5 = 3x - 14$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = (x-3) - 3 = x - 6$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 3(3x-5) - 5 = 9x - 20$; domain: $(-\infty, \infty)$

65. **a.** $(f \circ g)(x) = 1 - 2(1+x^2) = -2x^2 - 1$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 1 + (1-2x)^2 = 4x^2 - 4x + 2$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 1 - 2(1-2x) = 4x - 1$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 1 + (1+x^2)^2 = x^4 + 2x^2 + 2$; domain: $(-\infty, \infty)$

66. **a.** $(f \circ g)(x) = 2(2x^2) - 3 = 4x^2 - 3$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 2(2x-3)^2 = 8x^2 - 24x + 18;$
domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 2(2x-3) - 3 = 4x - 9;$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 2\left(2x^2\right)^2 = 8x^4;$
domain: $(-\infty, \infty)$

67. a. $$\begin{aligned}(f \circ g)(x) &= 2(2x-1)^2 + 3(2x-1)\\ &= 2(4x^2 - 4x + 1) + 6x - 3\\ &= 8x^2 - 2x - 1; \text{domain: } (-\infty, \infty)\end{aligned}$$

b. $(g \circ f)(x) = 2(2x^2 + 3x) - 1 = 4x^2 + 6x - 1;$
domain: $(-\infty, \infty)$

c. $$\begin{aligned}(f \circ f)(x) &= 2\left(2x^2 + 3x\right)^2 + 3\left(2x^2 + 3x\right)\\ &= 2\left(4x^4 + 12x^3 + 9x^2\right) + 6x^2 + 9x\\ &= 8x^4 + 24x^3 + 24x^2 + 9x;\end{aligned}$$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 2(2x-1) - 1 = 4x - 3;$
domain: $(-\infty, \infty)$

68. a. $(f \circ g)(x) = (2x)^2 + 3(2x) = 4x^2 + 6x;$
domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 2(x^2 + 3x) = 2x^2 + 6x;$
domain: $(-\infty, \infty)$

c. $$\begin{aligned}(f \circ f)(x) &= \left(x^2 + 3x\right)^2 + 3\left(x^2 + 3x\right)\\ &= x^4 + 6x^3 + 9x^2 + 3x^2 + 9x\\ &= x^4 + 6x^3 + 12x^2 + 9x;\end{aligned}$$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 2(2x) = 4x;$ domain: $(-\infty, \infty)$

69. a. $(f \circ g)(x) = \left(\sqrt{x}\right)^2 = x;$ domain: $[0, \infty)$

b. $(g \circ f)(x) = \sqrt{x^2} = |x|;$ domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = \left(x^2\right)^2 = x^4;$ domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \sqrt{\sqrt{x}} = \sqrt[4]{x};$ domain: $[0, \infty)$

70. a. $$\begin{aligned}(f \circ g)(x) &= \left(\sqrt{x+2}\right)^2 + 2\sqrt{x+2}\\ &= x + 2 + 2\sqrt{x+2}; \text{domain: } [-2, \infty)\end{aligned}$$

b. $(g \circ f)(x) = \sqrt{x^2 + 2x + 2};$ domain: $(-\infty, \infty)$

c. $$\begin{aligned}(f \circ f)(x) &= (x^2 + 2x)^2 + 2(x^2 + 2x)\\ &= x^4 + 4x^3 + 4x^2 + 2x^2 + 4x\\ &= x^4 + 4x^3 + 6x^2 + 4x;\end{aligned}$$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \sqrt{\sqrt{x+2} + 2};$ domain: $[-2, \infty)$

71. a. $$\begin{aligned}(f \circ g)(x) &= \frac{1}{2\left(\frac{1}{x^2}\right) - 1} = \frac{1}{\frac{2 - x^2}{x^2}}\\ &= \frac{x^2}{2 - x^2} = -\frac{x^2}{x^2 - 2}.\end{aligned}$$
The domain of g is $(-\infty, 0) \cup (0, \infty)$. Since $\frac{1}{2}$ is not in the domain of f, we must find those values of x that make $g(x) = \frac{1}{2}$.
$\frac{1}{x^2} = \frac{1}{2} \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$
Thus, the domain of $f \circ g$ is
$\left(-\infty, -\sqrt{2}\right) \cup \left(-\sqrt{2}, 0\right) \cup \left(0, \sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$.

b. $(g \circ f) = \dfrac{1}{\left(\frac{1}{2x-1}\right)^2} = (2x-1)^2$

The domain of f is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$. Since 0 is not in the domain of g, we must find those values of x that make $f(x) = 0$. However, there are no such values, so the domain of $g \circ f$ is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

c. $$\begin{aligned}(f \circ f)(x) &= \frac{1}{2\left(\frac{1}{2x-1}\right) - 1} = \frac{1}{\frac{2 - 2x + 1}{2x - 1}}\\ &= \frac{2x-1}{3-2x} = -\frac{2x-1}{2x-3}\end{aligned}$$
The domain of f is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.
$-\frac{2x-1}{2x-3}$ is defined for $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$,
so the domain of $f \circ f$ is
$\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$.

d. $(g \circ g)(x) = \dfrac{1}{\left(\dfrac{1}{x^2}\right)} = x^4.$

The domain of g is $(-\infty, 0) \cup (0, \infty)$, while $g \circ g = x^4$ is defined for all real numbers. Thus, the domain of $g \circ g$ is $(-\infty, 0) \cup (0, \infty)$.

72. a. $(f \circ g)(x) = \dfrac{x}{x+1} - 1 = \dfrac{x-(x+1)}{x+1} = -\dfrac{1}{x+1}$

The domain of g is $(-\infty, -1) \cup (-1, \infty)$. Since f is defined for all real numbers, there are no values that must be excluded. Thus, the domain of $f \circ g$ is $(-\infty, -1) \cup (-1, \infty)$.

b. $(g \circ f)(x) = \dfrac{x-1}{(x-1)+1} = \dfrac{x-1}{x}$

The domain of f is all real numbers. Since g is not defined for $x = -1$, we must exclude those values of x that make $f(x) = -1$.

$x - 1 = -1 \Rightarrow x = 0$

Thus, the domain of $g \circ f$ is $(-\infty, 0) \cup (0, \infty)$.

c. $(f \circ f)(x) = (x-1) - 1 = x - 2$;
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \dfrac{\dfrac{x}{x+1}}{\dfrac{x}{x+1}+1} = \dfrac{\dfrac{x}{x+1}}{\dfrac{x+x+1}{x+1}} = \dfrac{x}{2x+1}$

The domain of g is $(-\infty, -1) \cup (-1, \infty)$, while $\dfrac{x}{2x+1}$ is defined for $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.
The domain of $g \circ g$ is
$\left(-\infty, -1\right) \cup \left(-1. -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

73. a. $(f \circ g)(x) = \sqrt{\sqrt{4-x} - 1}$; domain: $(-\infty, 3]$

b. $(g \circ f)(x) = \sqrt{4 - \sqrt{x-1}}$; domain: $[1, 17]$

c. $(f \circ f)(x) = \sqrt{\sqrt{x-1} - 1}$; domain: $[2, \infty)$

d. $(g \circ g)(x) = \sqrt{4 - \sqrt{4-x}}$; domain: $[-12, 4]$

74. a. $(f \circ g)(x) = \left(\sqrt{4-x^2}\right)^2 - 4 = -x^2$
domain: $[-2, 2]$

b. $(g \circ f)(x) = \sqrt{4 - \left(x^2 - 4\right)^2}$
domain: $\left[-\sqrt{6}, -\sqrt{2}\right] \cup \left[\sqrt{2}, \sqrt{6}\right]$

c. $(f \circ f)(x) = \left(x^2 - 4\right)^2 - 4$
$= \left(x^4 - 8x^2 + 16\right) - 4$
$= x^4 - 8x^2 + 12$
domain: $(-\infty, \infty)$

d. $\sqrt{4 - \left(\sqrt{4-x^2}\right)^2} = \sqrt{4 - \left(4 - x^2\right)}$
$= \sqrt{4 - 4 + x^2} = \sqrt{x^2} = |x|$
domain: $[-2, 2]$

75. a. $(f \circ g)(x) = \dfrac{1 - \frac{x+3}{x-4}}{\frac{x+3}{x-4} + 2} = \dfrac{\left(1 - \frac{x+3}{x-4}\right)(x-4)}{\left(\frac{x+3}{x-4} + 2\right)(x-4)}$
$= \dfrac{(x-4) - (x+3)}{(x+3) + 2(x-4)} = -\dfrac{7}{3x-5}$

The domain of g is $(-\infty, 4) \cup (4, \infty)$. The denominator of $f \circ g$ is 0 when $x = \frac{5}{3}$, so the domain of $f \circ g$ is
$\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, 4\right) \cup (4, \infty)$.

b. $(g \circ f)(x) = \dfrac{\frac{1-x}{x+2} + 3}{\frac{1-x}{x+2} - 4} = \dfrac{\left(\frac{1-x}{x+2} + 3\right)(x+2)}{\left(\frac{1-x}{x+2} - 4\right)(x+2)}$
$= \dfrac{(1-x) + 3(x+2)}{(1-x) - 4(x+2)} = \dfrac{2x+7}{-5x-7}$
$= -\dfrac{2x+7}{5x+7}$

The domain of f is $(-\infty, -2) \cup (-2, \infty)$. The denominator of $g \circ f$ is 0 when $x = -\frac{7}{5}$, so, the domain of $g \circ f$ is
$(-\infty, -2) \cup \left(-2, -\frac{7}{5}\right) \cup \left(-\frac{7}{5}, \infty\right)$.

c. $(f \circ f)(x) = \dfrac{1 - \frac{1-x}{x+2}}{\frac{1-x}{x+2} + 2} = \dfrac{\left(1 - \frac{1-x}{x+2}\right)(x+2)}{\left(\frac{1-x}{x+2} + 2\right)(x+2)}$
$= \dfrac{(x+2) - (1-x)}{(1-x) + 2(x+2)} = \dfrac{2x+1}{x+5}$

The domain of f is $(-\infty, -2) \cup (-2, \infty)$. The denominator of $f \circ f$ is 0 when $x = -5$, so, the domain of $f \circ f$ is
$(-\infty, -5) \cup (-5, -2) \cup (-2, \infty)$.

d. $(g\circ g)(x)=\dfrac{\frac{x+3}{x-4}+3}{\frac{x+3}{x-4}-4}=\dfrac{\left(\frac{x+3}{x-4}+3\right)(x-4)}{\left(\frac{x+3}{x-4}-4\right)(x-4)}$
$=\dfrac{(x+3)+3(x-4)}{(x+3)-4(x-4)}=\dfrac{4x-9}{-3x+19}$
$=-\dfrac{4x-9}{3x-19}$

The domain of g is $(-\infty,4)\cup(4,\infty)$. The denominator of $g\circ g$ is 0 when $x=\frac{19}{3}$, so the domain of $g\circ g$ is $(-\infty,4)\cup\left(4,\frac{19}{3}\right)\cup\left(\frac{19}{3},\infty\right)$.

76. a. $(f\circ g)(x)=\dfrac{\frac{x+1}{x-1}+2}{\frac{x+1}{x-1}-3}=\dfrac{\left(\frac{x+1}{x-1}+2\right)(x-1)}{\left(\frac{x+1}{x-1}-3\right)(x-1)}$
$=\dfrac{(x+1)+2(x-1)}{(x+1)-3(x-1)}=\dfrac{3x-1}{-2x+4}$
$=-\dfrac{3x-1}{2x-4}$

The domain of g is $(-\infty,1)\cup(1,\infty)$. The denominator of $f\circ g$ is 0 when $x=2$, so the domain of $f\circ g$ is $(-\infty,1)\cup(1,2)\cup(2,\infty)$.

b. $(g\circ f)(x)=\dfrac{\frac{x+2}{x-3}+1}{\frac{x+2}{x-3}-1}=\dfrac{\left(\frac{x+2}{x-3}+1\right)(x-3)}{\left(\frac{x+2}{x-3}-1\right)(x-3)}$
$=\dfrac{(x+2)+(x-3)}{(x+2)-(x-3)}=\dfrac{2x-1}{5}$

The domain of f is $(-\infty,3)\cup(3,\infty)$. The denominator of $g\circ f$ is never 0, so, the domain of $g\circ f$ is $(-\infty,3)\cup(3,\infty)$.

c. $(f\circ f)(x)=\dfrac{\frac{x+2}{x-3}+2}{\frac{x+2}{x-3}-3}=\dfrac{\left(\frac{x+2}{x-3}+2\right)(x-3)}{\left(\frac{x+2}{x-3}-3\right)(x-3)}$
$=\dfrac{(x+2)+2(x-3)}{(x+2)-3(x-3)}$
$=\dfrac{3x-4}{-2x+11}=-\dfrac{3x-4}{2x-11}$

The domain of f is $(-\infty,3)\cup(3,\infty)$. The denominator of $f\circ f$ is 0 when $x=\frac{11}{2}$, so, the domain of $f\circ f$ is $(-\infty,3)\cup\left(3,\frac{11}{2}\right)\cup\left(\frac{11}{2},\infty\right)$.

d. $(g\circ g)(x)=\dfrac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}=\dfrac{\left(\frac{x+1}{x-1}+1\right)(x-1)}{\left(\frac{x+1}{x-1}-1\right)(x-1)}$
$=\dfrac{(x+1)+(x-1)}{(x+1)-(x-1)}=\dfrac{2x}{2}=x$

The domain of g is $(-\infty,1)\cup(1,\infty)$. The denominator of $g\circ g$ is never 0 so the domain of $g\circ g$ is $(-\infty,1)\cup(1,\infty)$.

77. a. $(f\circ g)(x)=1+\dfrac{1}{\frac{1+x}{1-x}}=1+\dfrac{1-x}{1+x}$
$=\dfrac{1+x+1-x}{1+x}=\dfrac{2}{1+x}.$

The domain of g is $(-\infty,1)\cup(1,\infty)$. Since 0 is not in the domain of f, we must find those values of x that make $g(x)=0$.

$\dfrac{1+x}{1-x}=0\Rightarrow 1+x=0\Rightarrow x=-1$

Thus, the domain of $f\circ g$ is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

b. $(g\circ f)(x)=\dfrac{1+\left(1+\frac{1}{x}\right)}{1-\left(1+\frac{1}{x}\right)}=\dfrac{2+\frac{1}{x}}{-\frac{1}{x}}\cdot\dfrac{x}{x}$
$=\dfrac{2x+1}{-1}=-2x-1$

The domain of f is $(-\infty,0)\cup(0,\infty)$. Since 1 is not in the domain of g, we must find those values of x that make $f(x)=1$.

$1+\dfrac{1}{x}=1\Rightarrow\dfrac{1}{x}=0$

There are no values of x that make this true, so there are no additional values to be excluded from the domain of $g\circ f$. Thus, the domain of $g\circ f$ is $(-\infty,0)\cup(0,\infty)$.

c. $(f\circ f)(x)=1+\dfrac{1}{1+\frac{1}{x}}=1+\dfrac{1}{\frac{x+1}{x}}=1+\dfrac{x}{x+1}$
$=\dfrac{2x+1}{x+1}$

The domain of f is $(-\infty,0)\cup(0,\infty)$. $\dfrac{2x+1}{x+1}$ is undefined for $(-\infty,-1)\cup(-1,\infty)$, so the domain of $f\circ f$ is $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$.

d. $(g \circ g)(x) = \dfrac{1+\dfrac{1+x}{1-x}}{1-\dfrac{1+x}{1-x}} = \dfrac{\dfrac{2}{1-x}}{\dfrac{-2x}{1-x}} = -\dfrac{1}{x}.$
The domain of g is $(-\infty, 1) \cup (1, \infty)$, while $-\dfrac{1}{x}$ is defined for $(-\infty, 0) \cup (0, \infty)$. The domain of $g \circ g$ is $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$.

78. a. $(f \circ g)(x) = \sqrt[3]{(x^3+1)+1} = \sqrt[3]{x^3+2}$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = \left(\sqrt[3]{x+1}\right)^3 + 1 = x + 2$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = \sqrt[3]{\sqrt[3]{x+1}+1}$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = (x^3+1)^3 + 1$; domain: $(-\infty, \infty)$

79. The domain of f is $(-\infty, 3) \cup (3, \infty)$. Because the domain of g is $[0, 6]$, the domain of $f \circ g$ cannot extend beyond those values. The value of g is 3 for $(1, 3]$, so those values are not in the domain of $f \circ g$. Therefore, the domain of $f \circ g$ is $[0, 1] \cup (3, 6]$.

80. The domain of f is $(-\infty, 2) \cup (2, \infty)$. Because the domain of g is $[0, 6]$, the domain of $f \circ g$ cannot extend beyond those values. The value of g is 2 for $[0, 1]$, so those values are not in the domain of $f \circ g$. Therefore, the domain of $f \circ g$ is $(1, 6]$.

81. The domain of f is $(-\infty, 3]$. Because the domain of g is $[0, 6]$, the domain of $f \circ g$ cannot extend beyond those values. $g(x) > 3$ for $x > 4$, so the domain of $f \circ g$ is $[0, 4]$.

82. The domain of f is $[2, \infty)$. Because the domain of g is $[0, 6]$, the domain of $f \circ g$ cannot extend beyond those values. $g(x) < 3$ for $3 < x \le 4$, so the domain of $f \circ g$ is $[0, 3] \cup (4, 6]$.

83. $(f \circ g) = \begin{cases} 2^2 - 2 = 2 & \text{if } 0 \le x \le 1 \\ 4^2 - 2 = 14 & \text{if } 1 < x \le 3 \\ 3^2 - 2 = 7 & \text{if } 3 < x \le 6 \end{cases}$

84. $(f \circ g) = \begin{cases} 1 - 2^2 = -3 & \text{if } 0 \le x \le 1 \\ 1 - 4^2 = -15 & \text{if } 1 < x \le 3 \\ 1 - 3^2 = -8 & \text{if } 3 < x \le 6 \end{cases}$

85. $(f \circ g) = \begin{cases} \dfrac{1}{2+2} = \dfrac{1}{4} & \text{if } 0 \le x \le 1 \\ \dfrac{1}{2+4} = \dfrac{1}{6} & \text{if } 1 < x \le 3 \\ \dfrac{1}{2+3} = \dfrac{1}{5} & \text{if } 3 < x \le 6 \end{cases}$

86. $(f \circ g) = \begin{cases} \dfrac{1}{1-2} = -1 & \text{if } 0 \le x \le 1 \\ \dfrac{1}{1-4} = -\dfrac{1}{3} & \text{if } 1 < x \le 3 \\ \dfrac{1}{1-3} = -\dfrac{1}{2} & \text{if } 3 < x \le 6 \end{cases}$

In exercises 87−96, sample answers are given. Other answers are possible.

87. $H(x) = \sqrt{x+2} \Rightarrow f(x) = \sqrt{x}, g(x) = x + 2$

88. $H(x) = |3x+2| \Rightarrow f(x) = |x|, g(x) = 3x + 2$

89. $H(x) = \left(x^2 - 3\right)^{10} \Rightarrow f(x) = x^{10}, g(x) = x^2 - 3$

90. $H(x) = \sqrt{3x^2} + 5 \Rightarrow f(x) = \sqrt{x} + 5, g(x) = 3x^2$

91. $H(x) = \dfrac{1}{3x-5} \Rightarrow f(x) = \dfrac{1}{x}, g(x) = 3x - 5$

92. $H(x) = \dfrac{5}{2x+3} \Rightarrow f(x) = \dfrac{5}{x}, g(x) = 2x + 3$

93. $H(x) = \sqrt[3]{x^2 - 7} \Rightarrow f(x) = \sqrt[3]{x}, g(x) = x^2 - 7$

94. $H(x) = \sqrt[4]{x^2 + x + 1} \Rightarrow f(x) = \sqrt[4]{x}$, $g(x) = x^2 + x + 1$

95. $H(x) = \dfrac{1}{|x^3 - 1|} \Rightarrow f(x) = \dfrac{1}{|x|}, g(x) = x^3 - 1$

96. $H(x) = \sqrt[3]{1 + \sqrt{x}} \Rightarrow f(x) = \sqrt[3]{x}, g(x) = 1 + \sqrt{x}$

97. $f(x) = -x^2 + 2;\ g(x) = 1 - 2x$

First compute the average rate of change of g from $x = 1$ to $x = 2$.

$$\begin{aligned}\text{ARC}[g]_1^2 &= \frac{g(2) - g(1)}{2-1}\\ &= \frac{[1-2(2)] - [1-2(1)]}{1}\\ &= -3 - (-1) = -2\end{aligned}$$

To compute the average rate of change of f, modify the range from $g(1) = -1$ to $g(2) = -3$.

$$\begin{aligned}\text{ARC}[f]_{-3}^{-1} &= \frac{f(-1) - f(-3)}{-1-(-3)}\\ &= \frac{\left[(-1)^2 + 2\right] - \left[(-3)^2 + 2\right]}{2}\\ &= \frac{3-11}{2} = \frac{-8}{2} = -4\end{aligned}$$

Finally,

$$\begin{aligned}\text{ARC}[f \circ g]_1^2 &= \text{ARC}[f]_{-3}^{-1} \cdot \text{ARC}[g]_1^2\\ &= -4(-2) = 8\end{aligned}$$

98. $f(x) = 1 - x^2;\ g(x) = 1 + 3x$

First compute the average rate of change of g from $x = -1$ to $x = 1$.

$$\begin{aligned}\text{ARC}[g]_{-1}^1 &= \frac{g(1) - g(-1)}{1-(-1)}\\ &= \frac{[1+3(1)] - [1+3(-1)]}{2}\\ &= \frac{4-(-2)}{2} = 3\end{aligned}$$

To compute the average rate of change of f, modify the range from $g(-1) = -2$ to $g(1) = 4$.

$$\begin{aligned}\text{ARC}[f]_{-2}^4 &= \frac{f(4) - f(-2)}{4-(-2)}\\ &= \frac{\left[1-(4)^2\right] - \left[1-(-2)^2\right]}{6}\\ &= \frac{-15-(-3)}{6} = -2\end{aligned}$$

Finally,

$$\begin{aligned}\text{ARC}[f \circ g]_{-1}^1 &= \text{ARC}[f]_{-2}^4 \cdot \text{ARC}[g]_{-1}^1\\ &= -2(3) = -6\end{aligned}$$

99. $f(x) = x^3 + 2;\ g(x) = 1 - x^2$

First compute the average rate of change of g from $x = 1$ to $x = 2$.

$$\begin{aligned}\text{ARC}[g]_1^2 &= \frac{g(2) - g(1)}{2-1} = \frac{\left[1-2^2\right] - \left[1-1^2\right]}{1}\\ &= -3\end{aligned}$$

To compute the average rate of change of f, modify the range from $g(1) = 0$ to $g(2) = -3$.

$$\begin{aligned}\text{ARC}[f]_{-3}^0 &= \frac{f(0) - f(-3)}{0-(-3)}\\ &= \frac{\left[(0)^3 + 2\right] - \left[(-3)^3 + 2\right]}{3}\\ &= \frac{2-(-25)}{3} = \frac{27}{3} = 9\end{aligned}$$

Finally,

$$\begin{aligned}\text{ARC}[f \circ g]_1^2 &= \text{ARC}[f]_{-3}^0 \cdot \text{ARC}[g]_1^2\\ &= 9(-3) = -27\end{aligned}$$

100. $f(x) = 1 - x^3;\ g(x) = x^2 + 1$

First compute the average rate of change of g from $x = -1$ to $x = 0$.

$$\begin{aligned}\text{ARC}[g]_{-1}^0 &= \frac{g(0) - g(-1)}{0-(-1)}\\ &= \frac{\left[0^2 + 1\right] - \left[(-1)^2 + 1\right]}{1} = -1\end{aligned}$$

To compute the average rate of change of f, modify the range from $g(-1) = 2$ to $g(0) = 1$.

$$\begin{aligned}\text{ARC}[f]_1^2 &= \frac{f(2) - f(1)}{1-2}\\ &= \frac{\left[1-(2)^3\right] - \left[1-(1)^3\right]}{-1} = -7\end{aligned}$$

Finally,

$$\begin{aligned}\text{ARC}[f \circ g]_{-1}^0 &= \text{ARC}[f]_1^2 \cdot \text{ARC}[g]_{-1}^0\\ &= -7(-1) = 7\end{aligned}$$

101. $f(x)=\dfrac{1}{4+x};\ g(x)=x^2-1$

First compute the average rate of change of g from $x = 1$ to $x = 2$.

$$\text{ARC}[g]_1^2=\frac{g(2)-g(1)}{2-1}=\frac{\left[2^2-1\right]-\left[1^2-1\right]}{1}=3$$

To compute the average rate of change of f, modify the range from $g(1)=0$ to $g(2)=3$.

$$\text{ARC}[f]_0^3=\frac{f(3)-f(0)}{3-0}=\frac{\frac{1}{4+3}-\frac{1}{4+0}}{3}=\frac{\frac{1}{7}-\frac{1}{4}}{3}=\frac{-\frac{3}{28}}{3}=-\frac{1}{28}$$

Finally,

$$\text{ARC}[f\circ g]_1^2=\text{ARC}[f]_0^3\cdot\text{ARC}[g]_1^2=-\frac{1}{28}(3)=-\frac{3}{28}$$

102. $f(x)=\dfrac{1}{2+x};\ g(x)=x^2+1$

First compute the average rate of change of g from $x = 0$ to $x = 2$.

$$\text{ARC}[g]_0^2=\frac{g(2)-g(0)}{2-0}=\frac{\left[2^2+1\right]-\left[0^2+1\right]}{2}=2$$

To compute the average rate of change of f, modify the range from $g(0)=1$ to $g(2)=5$.

$$\text{ARC}[f]_1^5=\frac{f(5)-f(1)}{5-1}=\frac{\frac{1}{2+5}-\frac{1}{2+1}}{4}=\frac{\frac{1}{7}-\frac{1}{3}}{4}=\frac{-\frac{4}{21}}{4}=-\frac{1}{21}$$

Finally,

$$\text{ARC}[f\circ g]_0^2=\text{ARC}[f]_1^5\cdot\text{ARC}[g]_0^2=-\frac{1}{21}(2)=-\frac{2}{21}$$

Applying the Concepts

103. a. $f(x)$ is the cost function.

b. $g(x)$ is the revenue function.

c. $h(x)$ is the selling price of x shirts including sales tax.

d. $P(x)$ is the profit function.

104. a.
$$\begin{aligned}C(p)&=C(5000-5p)\\&=4(5000-5p)+12{,}000\\&=20{,}000-20p+12{,}000\\&=32{,}000-20p\end{aligned}$$

b. $R(p)=px=p(5000-5p)=5000p-5p^2$

c.
$$\begin{aligned}P(p)&=R(p)-C(p)\\&=5000p-5p^2-(32{,}000-20p)\\&=-5p^2+5020p-32{,}000\end{aligned}$$

105. a.
$$\begin{aligned}P(x)&=R(x)-C(x)=25x-(350+5x)\\&=20x-350\end{aligned}$$

b. $P(20)=20(20)-350=50.$ This represents the profit when 20 radios are sold.

c. $P(x)=20x-350; 500=20x-350\Rightarrow x=43$

d. $C=350+5x\Rightarrow x=\dfrac{C-350}{5}=x(C).$

$$(R\circ x)(C)=25\left(\frac{C-350}{5}\right)=5C-1750.$$

This function represents the revenue in terms of the cost C.

106. a. $g(x)=0.04x$

b. $h(x)$ is the after tax selling price of merchandise worth x dollars.

c. $f(x)=0.02h(x)+3$

d. $T(x)$ represents the total price of merchandise worth x dollars, including the shipping and handling fee.

107. a. $f(x)=0.7x$

b. $g(x)=x-5$

c. $(g\circ f)(x)=0.7x-5$

d. $(f\circ g)(x)=0.7(x-5)$

e.
$$\begin{aligned}(f\circ g)-(g\circ f)&=0.7(x-5)-(0.7x-5)\\&=0.7x-3.5-0.7x+5\\&=\$1.50\end{aligned}$$

108. a. $f(x)=0.8x$

b. $g(x)=0.9x$

c. $(g\circ f)(x)=0.9(0.8x)=0.72x$

d. $(f \circ g)(x) = 0.8(0.9x) = 0.72x$

e. They are the same.

109. a. $f(x) = 1.1x; g(x) = x + 8$

b. $(f \circ g)(x) = 1.1(x+8) = 1.1x + 8.8$
This represents a final test score computed by first adding 8 points to the original score and then increasing the total by 10%.

c. $(g \circ f)(x) = 1.1x + 8.$ This represents a final test score computed by first increasing the original score by 10% and then adding 8 points.

d. $(f \circ g)(70) = 1.1(70+8) = 85.8;$
$(g \circ f)(70) = 1.1(70) + 8 = 85.0;$

e. $(f \circ g)(x) \neq (g \circ f)(x)$

f. (i) $(f \circ g)(x) = 1.1x + 8.8 \geq 90 \Rightarrow x \geq 73.82$

(ii) $(g \circ f)(x) = 1.1x + 8 \geq 90 \Rightarrow x \geq 74.55$

110. a. $f(x)$ is a function that models 3% of an amount x.

b. $g(x)$ represents the amount of money that qualifies for a 3% bonus.

c. Her bonus is represented by $(f \circ g)(x)$.

d. $200 + 0.03(17,500 - 8000) = \$485$

e. $521 = 200 + 0.03(x - 8000) \Rightarrow x = \$18,700$

111. a. $f(x) = \pi x^2$

b. $g(x) = \pi(x+30)^2$

c. $g(x) - f(x)$ represents the area between the fountain and the fence.

d. The circumference of the fence is $2\pi(x+30)$.
$10.5(2\pi(x+30)) = 4200 \Rightarrow$
$\pi(x+30) = 200 \Rightarrow$
$\pi x + 30\pi = 200 \Rightarrow \pi x = 200 - 30\pi.$

$$\begin{aligned} g(x) - f(x) &= \pi(x+30)^2 - \pi x^2 \\ &= \pi(x^2 + 60x + 900) - \pi x^2 \\ &= 60\pi x + 900\pi. \end{aligned}$$

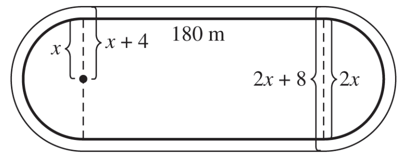
Now substitute $200 - 30\pi$ for πx to compute the estimate:

$$\begin{aligned} &1.75[60(200 - 30\pi) + 900\pi] \\ &= 1.75(12,000 - 900\pi) \approx \$16,052. \end{aligned}$$

112. a. $$\begin{aligned} f(x) &= 180(2x+8) + \pi(x+4)^2 \\ &= 1440 + 360x + \pi(x+4)^2 \end{aligned}$$

b. $g(x) = 2x(180) + \pi x^2 = 360x + \pi x^2$

c. $f(x) - g(x)$ represents the area of the track.

d.


(i) First find the radius of the inner track:
$900 = 2\pi x + 360 \Rightarrow \dfrac{270}{\pi} = x$. Use this value to compute $f(x) - g(x)$.

$$\begin{aligned} &f\left(\frac{270}{\pi}\right) - g\left(\frac{270}{\pi}\right) \\ &= \left(1440 + 360\left(\frac{270}{\pi}\right) + \pi\left(\frac{270}{\pi} + 4\right)^2\right) \\ &\quad - \left(360\left(\frac{270}{\pi}\right) + \pi\left(\frac{270}{\pi}\right)^2\right) \\ &= 1440 + 360\left(\frac{270}{\pi}\right) + \frac{270^2}{\pi} + 2160 \\ &\quad + 16\pi - 360\left(\frac{270}{\pi}\right) - \frac{270^2}{\pi} \\ &= 3600 + 16\pi \approx 3650.27 \text{ square meters} \end{aligned}$$

(ii) The outer perimeter
$$= 360 + 2\pi\left(\frac{270}{\pi} + 4\right) \approx 925.13 \text{ meters}$$

113. a. The area of the oil slick is a function of its radius r, $A = f(r) = \pi r^2$. The radius is a function of time because it increases at a rate of 2 mile per hour. Therefore, $r = g(t) = 2t$. So, the composite function is
$A = f(g(t)) = f(2t) = \pi(2t)^2 = 4\pi t^2.$

b. $A(6) = 4\pi(6)^2 = 144\pi \approx 452$ square miles

114. a. $(f \circ g)(t) = \dfrac{4}{3}\pi(2t)^3 = \dfrac{32}{3}\pi t^3$

b. $V(t) = \dfrac{4}{3}\pi(2t)^3 = \dfrac{32}{3}\pi t^3$

c. They are the same.

Beyond the Basics

115. a. When you are looking for the domain of the sum of two functions that are given as sets, you are looking for the intersection of their domains. Since the x-values that f and g have in common are −2, 1, and 3, the domain of $f + g$ is $\{-2, 1, 3\}$. Now add the y-values.

$(f+g)(-2) = 3 + 0 = 3$

$(f+g)(1) = 2 + (-2) = 0$

$(f+g)(3) = 0 + 2 = 2$

Thus, $f + g = \{(-2, 3), (1, 0), (3, 2)\}$.

b. When you are looking for the domain of the product of two functions that are given as sets, you are looking for the intersection of their domains. Since the x-values that f and g have in common are −2, 1, and 3, the domain of $f + g$ is $\{-2, 1, 3\}$. Now multiply the y-values.

$(fg)(-2) = 3 \cdot 0 = 0$

$(fg)(1) = 2 \cdot (-2) = -4$

$(fg)(3) = 0 \cdot 2 = 0$

Thus, $fg = \{(-2, 0), (1, -4), (3, 0)\}$.

c. When you are looking for the domain of the quotient of two functions that are given as sets, you are looking for the intersection of their domains and values of x that do not cause the denominator to equal zero. The x-values that f and g have in common are −2, 1, and 3; however, $g(-2) = 0$, so the domain is $\{1, 3\}$. Now divide the y-values.

$$\left(\frac{f}{g}\right)(1) = \frac{2}{-2} = -1$$

$$\left(\frac{f}{g}\right)(3) = \frac{0}{2} = 0$$

Thus, $\frac{f}{g} = \{(1, -1), (3, 0)\}$.

d. When you are looking for the domain of the composition of two functions that are given as sets, you are looking for values that come from the domain of the inside function and when you plug those values of x into the inside function, the output is in the domain of the outside function.

$f(g(-2)) = f(0)$, which is undefined

$f(g(0)) = f(2) = 1,$

$f(g(1)) = f(-2) = 3,$

$f(g(3)) = f(2) = 1$

Thus, $f \circ g = \{(0, 1), (1, 3), (3, 1)\}$.

116. When you are looking for the domain of the sum of two functions, you are looking for the intersection of their domains. The domain of f is $[-2, 3]$, while the domain of g is $[-3, 3]$. The intersection of the two domains is $[-2, 3]$, so the domain of $f + g$ is $[-2, 3]$.
For the interval $[-2, 1]$,
$f + g = 2x + (x + 1) = 3x + 1$.
For the interval $(1, 2)$
$f + g = (x + 1) + (x + 1) = 2x + 2$.
For the interval $[2, 3]$,
$f + g = (x + 1) + (2x - 1) = 3x$.
Thus,

$$(f+g)(x) = \begin{cases} 3x+1 & \text{if } -2 \le x \le 1 \\ 2x+2 & \text{if } 1 < x < 2 \\ 3x & \text{if } 2 \le x \le 3. \end{cases}$$

117. $(f \circ g)(x) = f(g(x)) = f(2x+1) = x^2 - 2$

$2x + 1 = 5 \Rightarrow x = 2$

$f(5) = 2^2 - 2 = 2$

118. $(f \circ g)(x) = f(g(x)) = f(1-3x) = \sqrt{5+x}$

$1 - 3x = 4 \Rightarrow x = -1$

$f(4) = \sqrt{5-1} = 2$

119. a. $f(-x) = h(-x) + h(-(-x)) = h(-x) + h(x)$
$= f(x) \Rightarrow f(x)$ is an even function.

b. $g(-x) = h(-x) - h(-(-x)) = h(-x) - h(x)$
$= -g(x) \Rightarrow g(x)$ is an odd function.

c. $\begin{cases} f(x) = h(x) + h(-x) \\ g(x) = h(x) - h(-x) \Rightarrow \end{cases}$

$f(x) + g(x) = 2h(x) \Rightarrow$

$$h(x) = \frac{f(x)+g(x)}{2} = \frac{f(x)}{2} + \frac{g(x)}{2} \Rightarrow$$

$h(x)$ is the sum of an even function and an odd function.

120. a. $h(x) = x^2 - 2x + 3 \Rightarrow f(x) = x^2$ (even),
$g(x) = -2x + 3$ (odd) or $f(x) = x^2 + 3$ (even),
$g(x) = -2x$ (odd)

b. $h(x) = [\![x]\!] + x \Rightarrow f(x) = \dfrac{[\![x]\!] + [\![-x]\!]}{2}$ (even),

$g(x) = x + \dfrac{[\![x]\!] - [\![-x]\!]}{2}$ (odd)

1.6 Critical Thinking/Discussion/Writing

121. **a.** The domain of $f(x)$ is $(-\infty, 0) \cup [1, \infty)$.

b. The domain of $g(x)$ is [0, 2].

c. The domain of $f(x)+g(x)$ is [1, 2].

d. The domain of $\dfrac{f(x)}{g(x)}$ is [1, 2).

122. **a.** The domain of *f* is $(-\infty, 0)$. The domain of $f \circ f$ is $\varnothing$ because $f \circ f = \dfrac{1}{\sqrt{-\frac{1}{\sqrt{-x}}}}$ and the denominator is the square root of a negative number.

b. The domain of *f* is $(-\infty, 1)$. The domain of $f \circ f$ is $(-\infty, 0)$ because $f \circ f = \dfrac{1}{\sqrt{1-\frac{1}{\sqrt{1-x}}}}$ and the denominator must be greater than 0. If $x = 0$, then the denominator = 0.

123. **a.** The sum of two even functions is an even function.
$f(x) = f(-x)$ and $g(x) = g(-x) \Rightarrow$
$(f+g)(x) = f(x) + g(x) = f(-x) + g(-x)$
$= (f+g)(-x)$.

b. The sum of two odd functions is an odd function.
$f(-x) = -f(x)$ and $g(-x) = -g(x) \Rightarrow$
$(f+g)(-x) = f(-x) + g(-x) = -f(x) - g(x)$
$= -(f+g)(x)$.

c. The sum of an even function and an odd function is neither even nor odd.
$f(x)$ even $\Rightarrow f(x) = f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x) = -g(x) \Rightarrow f(-x) + g(-x) =$
$f(x) + (-g(x))$, which is neither even nor odd.

d. The product of two even functions is an even function.
$f(x) = f(-x)$ and $g(x) = g(-x) \Rightarrow$
$(f \cdot g)(x) = f(x) \cdot g(x) = f(-x) \cdot (g(-x))$
$= (f \cdot g)(-x)$.

e. The product of two odd functions is an even function.
$f(-x) = -f(x)$ and $g(-x) = -g(x) \Rightarrow$
$(f \cdot g)(-x) = f(-x) \cdot g(-x) = -f(x) \cdot (-g(x))$
$= (f \cdot g)(x)$.

f. The product of an even function and an odd function is an odd function.
$f(x)$ even $\Rightarrow f(x) = f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x) = -g(x) \Rightarrow$
$f(-x) \cdot g(-x) = f(x) \cdot (-g(x)) = -(f \cdot g)(x)$

124. **a.** $f(-x) = -f(x)$ and $g(-x) = -g(x) \Rightarrow$
$(f \circ g)(-x) = f(g(-x)) = f(-g(x)) =$
$-f(g(x)) \Rightarrow (f \circ g)(x)$ is odd.

b. $f(x) = f(-x)$ and $g(x) = g(-x) \Rightarrow$
$(f \circ g)(-x) = f(g(-x)) = f(g(x)) \Rightarrow$
$(f \circ g)(x)$ is even.

c. $f(x)$ odd $\Rightarrow f(-x) = -f(x)$ and
$g(x)$ even $\Rightarrow g(x) = g(-x) \Rightarrow (f \circ g)(-x)$
$f(g(x)) = f(g(-x)) \Rightarrow (f \circ g)(x)$ is even.

d. $f(x)$ even $\Rightarrow f(x) = f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x) = -g(x) \Rightarrow (f \circ g)(-x) = f(-g(x))$
$= f(g(x)) = (f \circ g)(-x) \Rightarrow (f \circ g)(x)$ is even.

Getting Ready for the Next Section

125. $x = 2y + 3 \Rightarrow x - 3 = 2y \Rightarrow y = \dfrac{x-3}{2}$

126. $x = y^2 + 1,\ y \ge 0 \Rightarrow x - 1 = y^2 \Rightarrow y = \sqrt{x-1}$

127. $x^2 + y^2 = 4,\ x \le 0 \Rightarrow x^2 = 4 - y^2 \Rightarrow$
$x = -\sqrt{4 - y^2}$

128. $2x - \dfrac{1}{y} = 3 \Rightarrow -\dfrac{1}{y} = 3 - 2x \Rightarrow \dfrac{1}{y} = -3 + 2x \Rightarrow$
$y = \dfrac{1}{2x-3}$

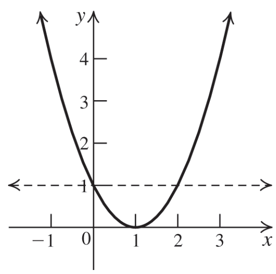
129. $\dfrac{1+\frac{1}{1+x}}{2-\frac{1}{1+x}} = \dfrac{\left(1+\frac{1}{1+x}\right)(1+x)}{\left(2-\frac{1}{1+x}\right)(1+x)} = \dfrac{(1+x)+1}{2(1+x)-1}$
$= \dfrac{x+2}{2x+1}$

130. $\dfrac{2+\dfrac{x}{1-x}}{1-\dfrac{x}{1-x}}=\dfrac{\left(2+\dfrac{x}{1-x}\right)(1-x)}{\left(1-\dfrac{x}{1-x}\right)(1-x)}=\dfrac{2(1-x)+x}{(1-x)-x}$
$=\dfrac{2-x}{1-2x}=\dfrac{x-2}{2x-1}$

Section 1.7 Inverse Functions

1.7 Practice Problems

1. $f(x)=(x-1)^2$ is not one-to-one because the horizontal line $y = 1$ intersects the graph at two different points.

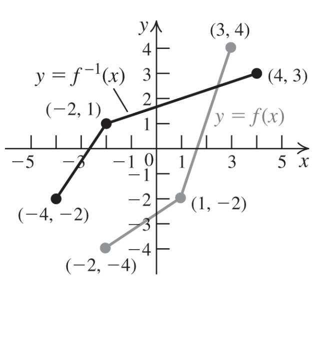


2. a. $f^{-1}(12)=-3$

b. $f(9)=4$

3. $f(x)=3x-1,\ g(x)=\dfrac{x+1}{3}$
$(f\circ g)(x)=f\left(\dfrac{x+1}{3}\right)=3\left(\dfrac{x+1}{3}\right)-1=x$
$(g\circ f)(x)=g(3x-1)=\dfrac{3x-1+1}{3}=x$
Since $f(g(x))=g(f(x))=x$, the two functions are inverses.

4. The graph of f^{-1} is the reflection of the graph of *f* in the line $y = x$.



5. $f(x)=-2x+3$ is a one-to-one function, so the function has an inverse. Interchange the variables and solve for *y*:
$f(x)=y=-2x+3\Rightarrow x=-2y+3\Rightarrow$
$\dfrac{x-3}{-2}=y\Rightarrow y=f^{-1}(x)=\dfrac{3-x}{2}.$

6. Interchange the variables and solve for *y*:
$f(x)=y=\dfrac{x}{x+3},x\neq 3$
$x=\dfrac{y}{y+3}\Rightarrow xy+3x=y\Rightarrow 3x=y-xy\Rightarrow$
$3x=y(1-x)\Rightarrow\dfrac{3x}{1-x}=y\Rightarrow$
$f^{-1}(x)=\dfrac{3x}{1-x},\ x\neq 1$

7. $f(x)=\dfrac{x}{x+3}$
The function is not defined if the denominator is zero, so the domain is $(-\infty,-3)\cup(-3,\infty)$. The range of the function is the same as the domain of the inverse (see practice problem 6), thus the range is $(-\infty,1)\cup(1,\infty)$.

8. *G* is one-to-one since the domain is restricted, so an inverse exists.
$G(x)=y=x^2-1,x\leq 0.$ Interchange the variables and solve for y:
$x=y^2-1,\ y\leq 0\Rightarrow y=G^{-1}=-\sqrt{x+1}.$

9. $f(x)=2-(3x-1)^3$
$\text{ARC}[f]_0^1=\dfrac{f(1)-f(0)}{1-0}=\dfrac{-6-3}{1}=-9$
$\text{ARC}\left[f^{-1}\right]_{f(0)=3}^{f(1)=-6}=\dfrac{1}{\text{ARC}[f]_0^1}=-\dfrac{1}{9}$

10. From the text, we have $d=\dfrac{11p}{5}-33$.
$d=\dfrac{11\cdot 1650}{5}-33=3597$
The bell was 3597 feet below the surface when the gauge failed.

1.7 Exercises Concepts and Vocabulary

1. A function *f* is one-to-one if for any two different numbers x_1 and x_2, $x_1\neq x_2$, in the domain of *f*, we have $\underline{f(x_1)\neq f(x_2)}$.

2. A function f is one-to-one if every horizontal line intersects the graph of f at no more than one point.

3. $f^{-1} \circ f(x) = \underline{x}$

4. The graph of f^{-1} is a reflection of the graph of f about the line $y = x$.

5. True **6.** False

7. False **8.** True.

Building Skills

9. one-to-one **10.** not one-to-one

11. not one-to-one **12.** one-to-one

13. not one-to-one **14.** not one-to-one

15. one-to-one **16.** not one-to-one

17. $f(2) = 7 \Rightarrow f^{-1}(7) = 2$

18. $f^{-1}(4) = -7 \Rightarrow f(-7) = 4$

19. $f(-1) = 2 \Rightarrow f^{-1}(2) = -1$

20. $f^{-1}(-3) = 5 \Rightarrow f(5) = -3$

21. a. $f(3) = 2(3) - 3 = 3$

b. Using the result from part (a), $f^{-1}(3) = 3$.

c. $\left(f \circ f^{-1}\right)(19) = f\left(f^{-1}(19)\right) = 19$

d. $\left(f \circ f^{-1}\right)(5) = f\left(f^{-1}(5)\right) = 5$

22. a. $f(2) = 2^3 = 8$

b. Using the result from part (a), $f^{-1}(8) = 2$.

c. $\left(f \circ f^{-1}\right)(15) = f\left(f^{-1}(15)\right) = 15$

d. $\left(f \circ f^{-1}\right)(27) = f\left(f^{-1}(27)\right) = 27$

23. a. $f(1) = 1^3 + 1 = 2$

b. Using the result from part (a), $f^{-1}(2) = 1$.

c. $\left(f \circ f^{-1}\right)(269) = f\left(f^{-1}(269)\right) = 269$

24. a. $g(1) = \sqrt[3]{2\left(1^3\right) - 1} = \sqrt[3]{1} = 1$

b. Using the result from part (a), $g^{-1}(1) = 1$.

c. $\left(g^{-1} \circ g\right)(135) = g^{-1}\left(g(135)\right) = 135$

25. a. not one-to-one

b. not one-to-one

c. one-to-one

26. a. one-to-one

b. not one-to-one

c. not one-to-one

27. First: Put the milk back in the fridge.
Second: Close the fridge door.

28. First: Take shoes off.
Second: Take socks off.

29. First: Remove makeup
Second: Go to sleep

30. First: Remove a tree.
Second: Fill in a hole

31. $f(x) = 2x + 3$
First: Subtract 3.
Second: Divide by 2.
$f^{-1}(x) = \frac{x-3}{2}$

32. $f(x) = 3(x-2)$
First: Divide by 3.
Second: Add 2.
$f^{-1}(x) = \frac{x}{3} + 2$

33. $f(x) = x^3 + 2$
First: Subtract 2.
Second: Take the cube root.
$f^{-1}(x) = \sqrt[3]{x-2}$

34. $f(x) = (x-3)^3$
First: Take the cube root.
Second: Add 3.
$f^{-1}(x) = \sqrt[3]{x} + 3$

35. $f\left(g(x)\right) = 3\left(\frac{x-1}{3}\right) + 1 = x - 1 + 1 = x$

$g\left(f(x)\right) = \frac{(3x+1)-1}{3} = \frac{3x}{3} = x$

36. $f(g(x)) = 2-3\left(\dfrac{2-x}{3}\right) = 2-2+x = x$

$g(f(x)) = \dfrac{2-(2-3x)}{3} = \dfrac{3x}{3} = x$

37. $f(g(x)) = \left(\sqrt[3]{x}\right)^3 = x$

$g(f(x)) = \sqrt[3]{x^3} = x$

38. $f(g(x)) = g(f(x)) = \dfrac{1}{\frac{1}{x}} = x$

39. $f(g(x)) = 2\left(\sqrt[5]{\dfrac{x-1}{2}}\right)^5 + 1 = 2\left(\dfrac{x-1}{2}\right)+1$

$= x-1+1 = x$

$g(f(x)) = \sqrt[5]{\dfrac{(2x^5+1)-1}{2}} = \sqrt[5]{\dfrac{2x^5}{2}} = \sqrt[5]{x^5} = x$

40. $f(g(x)) = \left(1-3\left(\dfrac{1-\sqrt[3]{x}}{3}\right)\right)^3$

$= \left(1-1+\sqrt[3]{x}\right)^3 = x$

$g(f(x)) = \dfrac{1-\sqrt[3]{(1-3x)^3}}{3} = \dfrac{1-(1-3x)}{3}$

$= \dfrac{3x}{3} = x$

41. $f(g(x)) = \dfrac{\frac{1+2x}{1-x}-1}{\frac{1+2x}{1-x}+2} = \dfrac{\frac{1+2x-(1-x)}{1-x}}{\frac{1+2x+2(1-x)}{1-x}} = \dfrac{3x}{3} = x$

$g(f(x)) = \dfrac{1+2\left(\frac{x-1}{x+2}\right)}{1-\frac{x-1}{x+2}} = \dfrac{1+\frac{2x-2}{x+2}}{1-\frac{x-1}{x+2}}$

$= \dfrac{\frac{x+2+2x-2}{x+2}}{\frac{x+2-(x-1)}{x+2}} = \dfrac{3x}{3} = x$

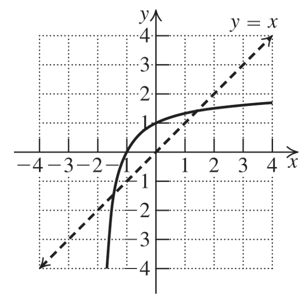
42. $f(g(x)) = \dfrac{3\left(\frac{x+2}{x-3}\right)+2}{\frac{x+2}{x-3}-1} = \dfrac{\frac{3x+6}{x-3}+\frac{2(x-3)}{x-3}}{\frac{x+2}{x-3}-\frac{1(x-3)}{x-3}}$

$= \dfrac{\frac{3x+6+2x-6}{x-3}}{\frac{x+2-x+3}{x-3}} = \dfrac{5x}{5} = x$

$g(f(x)) = \dfrac{\frac{3x+2}{x-1}+2}{\frac{3x+2}{x-1}-3} = \dfrac{\frac{3x+2}{x-1}+\frac{2(x-1)}{x-1}}{\frac{3x+2}{x-1}-\frac{3(x-1)}{x-1}}$

$= \dfrac{\frac{3x+2+2x-2}{x-1}}{\frac{3x+2-3x+3}{x-1}} = \dfrac{5x}{5} = x$

43.



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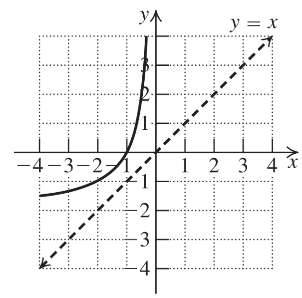

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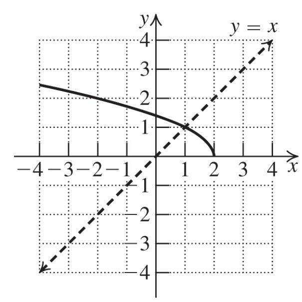

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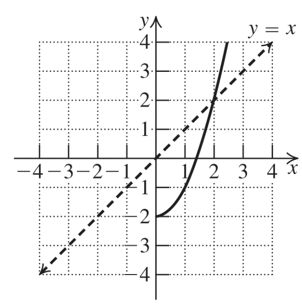

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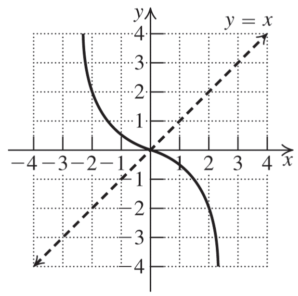

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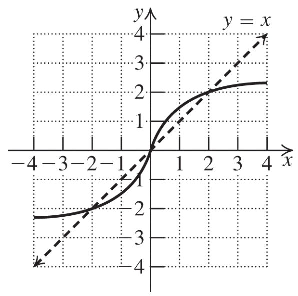
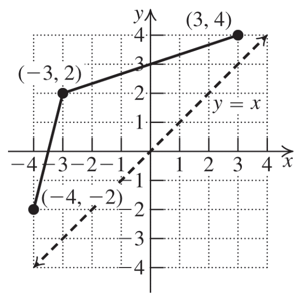
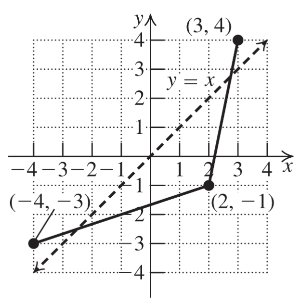


49.



50.



51. $f(x) = 3x - 1$

Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

Replace $f(x)$ with y, interchange x and y, and then solve for y.

$y = 3x - 1 \Rightarrow x = 3y - 1 \Rightarrow x + 1 = 3y \Rightarrow$

$y = f^{-1}(x) = \dfrac{x+1}{3}$

52. $f(x) = 2x + 3$

Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

Replace $f(x)$ with y, interchange x and y, and then solve for y.

$y = 2x + 3 \Rightarrow x = 2y + 3 \Rightarrow x - 3 = 2y \Rightarrow$

$y = f^{-1}(x) = \dfrac{x-3}{2}$

53. $f(x) = \sqrt[3]{\dfrac{x+1}{3}} + 2$

Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

Replace $f(x)$ with y, interchange x and y, and then solve for y.

$y = \sqrt[3]{\dfrac{x+1}{3}} + 2 \Rightarrow x = \sqrt[3]{\dfrac{y+1}{3}} + 2 \Rightarrow$

$x - 2 = \sqrt[3]{\dfrac{y+1}{3}} \Rightarrow (x-2)^3 = \dfrac{y+1}{3} \Rightarrow$

$3(x-2)^3 = y + 1 \Rightarrow$

$y = f^{-1}(x) = 3(x-2)^3 - 1$

54. $f(x) = \sqrt[3]{\dfrac{x-2}{3}} - 1$

Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

Replace $f(x)$ with y, interchange x and y, and then solve for y.

$y = \sqrt[3]{\dfrac{x-2}{3}} - 1 \Rightarrow x = \sqrt[3]{\dfrac{y-2}{3}} - 1 \Rightarrow$

$x + 1 = \sqrt[3]{\dfrac{y-2}{3}} \Rightarrow (x+1)^3 = \dfrac{y-2}{3} \Rightarrow$

$3(x+1)^3 = y - 2 \Rightarrow$

$y = f^{-1}(x) = 3(x+1)^3 + 2$

55. $f(x) = (3x-1)^3 + 2$

Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

Replace $f(x)$ with y, interchange x and y, and then solve for y.

$y = (3x-1)^3 + 2 \Rightarrow x = (3y-1)^3 + 2 \Rightarrow$

$x - 2 = (3y-1)^3 \Rightarrow \sqrt[3]{x-2} = 3y - 1 \Rightarrow$

$\sqrt[3]{x-2} + 1 = 3y \Rightarrow$

$y = f^{-1}(x) = \dfrac{\sqrt[3]{x-2} + 1}{3}$

56. $f(x) = (2x+1)^3 - 3$

Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

Replace $f(x)$ with y, interchange x and y, and then solve for y.

$y = (2x+1)^3 - 3 \Rightarrow x = (2y+1)^3 - 3 \Rightarrow$
$x + 3 = (2y+1)^3 \Rightarrow \sqrt[3]{x+3} = 2y+1 \Rightarrow$
$\sqrt[3]{x+3} - 1 = 2y \Rightarrow$
$y = f^{-1}(x) = \dfrac{\sqrt[3]{x+3}-1}{2}$

57. $f(x) = \dfrac{2}{1+x}$

Domain: $(-\infty, -1) \cup (-1, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

Replace $f(x)$ with y, interchange x and y, and then solve for y.

$y = \dfrac{2}{1+x} \Rightarrow x = \dfrac{2}{1+y} \Rightarrow x(1+y) = 2 \Rightarrow$
$1 + y = \dfrac{2}{x} \Rightarrow y = f^{-1}(x) = \dfrac{2}{x} - 1$

58. $f(x) = 1 - \dfrac{1}{x+1}$

Domain: $(-\infty, -1) \cup (-1, \infty)$

Range: $(-\infty, 1) \cup (1, \infty)$

Replace $f(x)$ with y, interchange x and y, and then solve for y.

$y = 1 - \dfrac{1}{x+1} \Rightarrow x = 1 - \dfrac{1}{y+1} \Rightarrow$
$x - 1 = -\dfrac{1}{y+1} \Rightarrow (x-1)(y+1) = -1 \Rightarrow$
$y + 1 = -\dfrac{1}{x-1} \Rightarrow y = f^{-1}(x) = -\dfrac{1}{x-1} - 1$

59. $f(x) = \dfrac{x+1}{x-2}$

Domain: $(-\infty, 2) \cup (2, \infty)$

Range: $(-\infty, 1) \cup (1, \infty)$

Replace $f(x)$ with y, interchange x and y, and then solve for y.

$y = \dfrac{x+1}{x-2} \Rightarrow x = \dfrac{y+1}{y-2} \Rightarrow x(y-2) = y+1 \Rightarrow$
$xy - 2x = y + 1 \Rightarrow xy - y = 2x + 1 \Rightarrow$
$y(x-1) = 2x+1 \Rightarrow y = f^{-1}(x) = \dfrac{2x+1}{x-1}$

60. $f(x) = \dfrac{1-2x}{1+x}$

Domain: $(-\infty, -1) \cup (-1, \infty)$

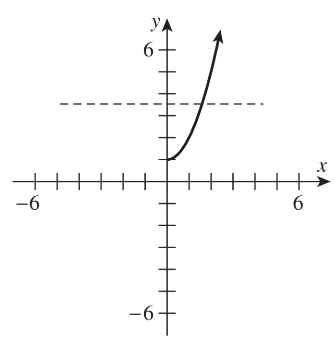
Range: $(-\infty, -2) \cup (-2, \infty)$

Replace $f(x)$ with y, interchange x and y, and then solve for y.

$y = \dfrac{1-2x}{1+x} \Rightarrow x = \dfrac{1-2y}{1+y} \Rightarrow x(1+y) = 1 - 2y \Rightarrow$
$x + xy = 1 - 2y \Rightarrow xy + 2y = 1 - x \Rightarrow$
$y(x+2) = 1 - x \Rightarrow$
$y = f^{-1}(x) = \dfrac{1-x}{x+2} = -\dfrac{x-1}{x+2}$

In exercises 61–64, note that the domain of f is the range of f^{-1}, and the range of f is the domain of f^{-1}.

61. $f(x) = x^2 + 1,\ x \geq 0$



The horizontal line test confirms that f is one-to-one.

Domain of f: $[0, \infty)$

Range of f: $[1, \infty)$

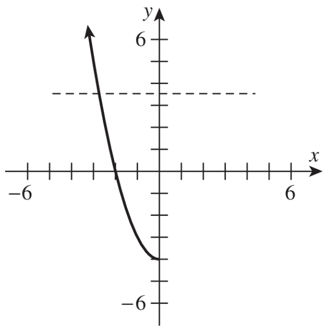
Find $f^{-1}(x)$ by interchanging the variables and solving for y.

$y = x^2 + 1 \Rightarrow x = y^2 + 1 \Rightarrow x - 1 = y^2 \Rightarrow$
$y = f^{-1}(x) = \sqrt{x-1}$

Domain of f^{-1}: $[1, \infty)$

Range of f^{-1}: $[0, \infty)$

62. $f(x) = x^2 - 4,\ x \le 0$



The horizontal line test confirms that *f* is one-to-one.

Domain of *f*: $(-\infty, 0]$

Range of *f*: $[-4, \infty)$

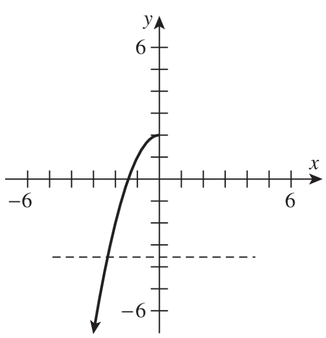
Find $f^{-1}(x)$ by interchanging the variables and solving for *y*.

$y = x^2 - 4 \Rightarrow x = y^2 - 4 \Rightarrow x + 4 = y^2 \Rightarrow$
$y = f^{-1}(x) = -\sqrt{x+4}$

Domain of $f^{-1}: [-4, \infty)$

Range of $f^{-1}: (-\infty, 0]$

63. $f(x) = -x^2 + 2,\ x \le 0$



The horizontal line test confirms that *f* is one-to-one.

Domain of *f*: $(-\infty, 0]$; range of *f*: $(-\infty, 2]$

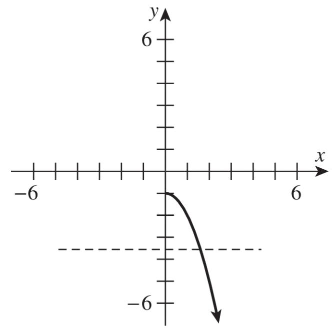
Find $f^{-1}(x)$ by interchanging the variables and solving for *y*.

$y = -x^2 + 2 \Rightarrow x = -y^2 + 2 \Rightarrow x - 2 = -y^2 \Rightarrow$
$y^2 = 2 - x \Rightarrow y = f^{-1}(x) = -\sqrt{2-x}$

Domain of $f^{-1}: (-\infty, 2]$

Range of $f^{-1}: (-\infty, 0]$

64. $f(x) = -x^2 - 1,\ x \ge 0$



The horizontal line test confirms that *f* is one-to-one.

Domain of *f*: $[0, \infty)$

Range of *f*: $(-\infty, -1]$

Find $f^{-1}(x)$ by interchanging the variables and solving for *y*.

$y = -x^2 - 1 \Rightarrow x = -y^2 - 1 \Rightarrow x + 1 = -y^2 \Rightarrow$
$y^2 = -x - 1 \Rightarrow y = f^{-1}(x) = \sqrt{-x-1}$

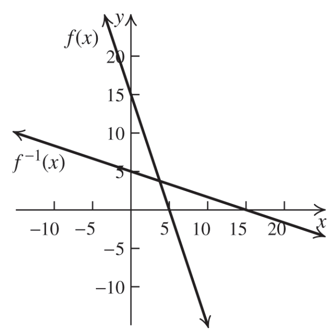
Domain of $f^{-1}: (-\infty, -1]$

Range of $f^{-1}: [0, \infty)$

For exercises 65–76, we include the equation of the inverse for reference. However, you should graph f^{-1} by reflecting the graph of *f* with respect to the line $y = x$. See Example 4 in the text.

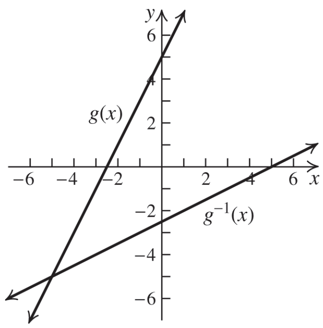
65. $f(x) = 15 - 3x$

$f^{-1}(x) = \dfrac{15 - x}{3} = 5 - \dfrac{1}{3}x$



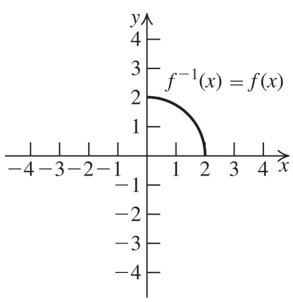
66. $g(x) = 2x + 5$

$g^{-1}(x) = \frac{x-5}{2} = \frac{1}{2}x - \frac{5}{2}$



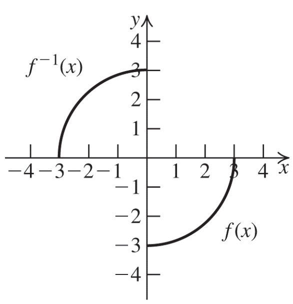
67. $f(x) = \sqrt{4 - x^2},\ x \geq 0$

$f^{-1}(x) = \sqrt{4 - x^2},\ x \geq 0.$



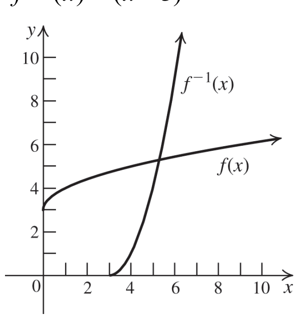
68. $f(x) = y = -\sqrt{9 - x^2},\ x \geq 0$

$f^{-1}(x) = \sqrt{9 - x^2},\ x \leq 0$



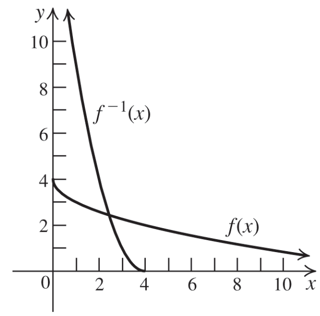
69. $f(x) = y = \sqrt{x} + 3$

$f^{-1}(x) = (x-3)^2$



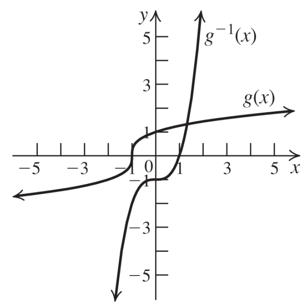
70. $f(x) = y = 4 - \sqrt{x}$

$f^{-1}(x) = (x-4)^2$



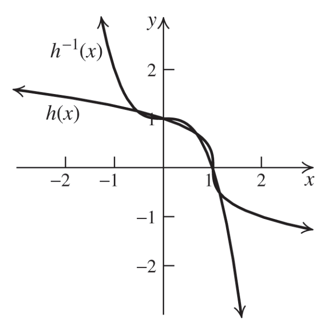
71. $g(x) = y = \sqrt[3]{x+1}$

$g^{-1}(x) = x^3 - 1$



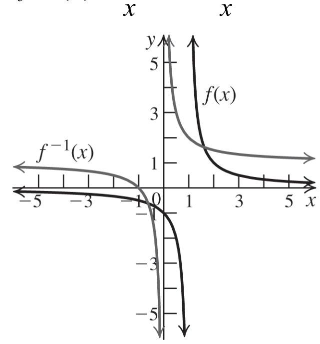
72. $h(x) = y = \sqrt[3]{1-x}$

$g^{-1}(x) = 1 - x^3$



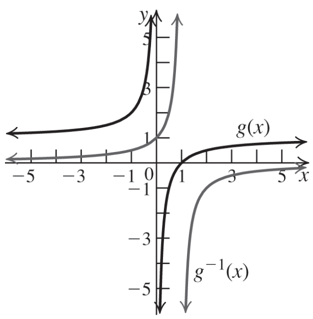
73. $f(x) = y = \frac{1}{x-1}$

$f^{-1}(x) = \frac{1}{x} + 1 = \frac{1+x}{x}$



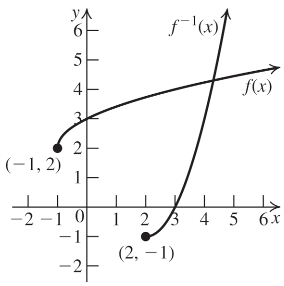
74. $g(x) = y = 1 - \dfrac{1}{x}$

$g^{-1}(x) = -\dfrac{1}{x-1} = \dfrac{1}{1-x}$



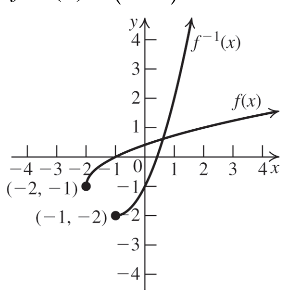
75. $f(x) = y = 2 + \sqrt{x+1}$

$f^{-1}(x) = (x-2)^2 - 1 = x^2 - 4x + 3$



76. $f(x) = y = -1 + \sqrt{x+2}$

$f^{-1}(x) = (x+1)^2 - 2 = x^2 + 2x - 1$



77. $f(x) = y = \dfrac{x+1}{x-2}$. Interchange the variables and solve for $y: x = \dfrac{y+1}{y-2} \Rightarrow xy - 2x = y + 1 \Rightarrow$ $xy - y = 2x + 1 \Rightarrow y(x-1) = 2x + 1 \Rightarrow$ $y = f^{-1}(x) = \dfrac{2x+1}{x-1}$.

Domain of f: $(-\infty, 2) \cup (2, \infty)$; range of f: $(-\infty, 1) \cup (1, \infty)$.

78. $g(x) = y = \dfrac{x+2}{x+1}$. Interchange the variables and solve for $y: x = \dfrac{y+2}{y+1} \Rightarrow xy + x = y + 2 \Rightarrow$ $xy - y = -x + 2 \Rightarrow y(x-1) = -x + 2 \Rightarrow$ $y = g^{-1}(x) = \dfrac{-x+2}{x-1} = \dfrac{x-2}{1-x}$.

Domain of g: $(-\infty, -1) \cup (-1, \infty)$; range of g: $(-\infty, 1) \cup (1, \infty)$.

79. $f(x) = y = \dfrac{1-2x}{1+x}$. Interchange the variables and solve for $y: x = \dfrac{1-2y}{1+y} \Rightarrow$ $x + xy = 1 - 2y \Rightarrow xy + 2y = 1 - x \Rightarrow$ $y(x+2) = 1 - x \Rightarrow y = f^{-1}(x) = \dfrac{1-x}{x+2}$.

Domain of f: $(-\infty, -1) \cup (-1, \infty)$; range of f: $(-\infty, -2) \cup (-2, \infty)$.

80. $h(x) = y = \dfrac{x-1}{x-3}$. Interchange the variables and solve for $y: x = \dfrac{y-1}{y-3} \Rightarrow xy - 3x = y - 1 \Rightarrow$ $xy - y = 3x - 1 \Rightarrow y(x-1) = 3x - 1 \Rightarrow$ $y = h^{-1}(x) = \dfrac{3x-1}{x-1}$.

Domain of h: $(-\infty, 3) \cup (3, \infty)$; range of h: $(-\infty, 1) \cup (1, \infty)$.

81. $f(x) = (3x-1)^3 + 2$

$$\text{ARC}[f]_0^1 = \frac{f(1) - f(0)}{1-0} = \frac{10-1}{1} = 9$$

$$\text{ARC}\left[f^{-1}\right]_{f(0)=1}^{f(1)=10} = \frac{1}{\text{ARC}[f]_0^1} = \frac{1}{9}$$

82. $f(x) = 1 - (2x-1)^3$

$$\text{ARC}[f]_0^1 = \frac{f(1) - f(0)}{1-0} = \frac{0-2}{1} = -2$$

$$\text{ARC}\left[f^{-1}\right]_{f(0)=2}^{f(1)=0} = \frac{1}{\text{ARC}[f]_0^1} = -\frac{1}{2}$$

83. $f(x) = \dfrac{3}{x-1}$

$$\text{ARC}[f]_4^9 = \frac{f(9) - f(4)}{9-4} = \frac{\frac{3}{8} - 1}{5} = -\frac{1}{8}$$

$$\text{ARC}\left[f^{-1}\right]_{f(4)=1}^{f(9)=3/8} = \frac{1}{\text{ARC}[f]_4^9} = -8$$

84. $f(x)=1-\dfrac{2}{x}$

$$\text{ARC}[f]_1^3=\frac{f(3)-f(1)}{3-1}=\frac{\frac{1}{3}-(-1)}{2}=\frac{2}{3}$$
$$\text{ARC}\left[f^{-1}\right]_{f(1)=-1}^{f(3)=1/3}=\frac{1}{\text{ARC}[f]_2^3}=\frac{3}{2}$$

85. $f(x)=\dfrac{2x+3}{x-2}$

$$\text{ARC}[f]_3^5=\frac{f(5)-f(3)}{5-3}=\frac{\frac{13}{3}-9}{2}=-\frac{7}{3}$$
$$\text{ARC}\left[f^{-1}\right]_{f(3)=9}^{f(5)=13/3}=\frac{1}{\text{ARC}[f]_1^3}=-\frac{3}{7}$$

86. $f(x)=\dfrac{3x+1}{x+1}$

$$\text{ARC}[f]_0^2=\frac{f(2)-f(0)}{2-0}=\frac{\frac{7}{3}-1}{2}=\frac{2}{3}$$
$$\text{ARC}\left[f^{-1}\right]_{f(0)=1}^{f(2)=7/3}=\frac{1}{\text{ARC}[f]_0^2}=\frac{3}{2}$$

Applying the Concepts

87. a. $K(C)=C+273\Rightarrow$
$C(K)=K-273=K^{-1}(C)$.
This represents the Celsius temperature corresponding to a given Kelvin temperature.

b. $C(300)=300-273=27°\text{C}$

c. $K(22)=22+273=295\,\text{K}$

88. a. The two points are (212, 373) and (32, 273).
The rate of change is $\dfrac{373-273}{212-32}=\dfrac{100}{180}=\dfrac{5}{9}$.

$$273=\frac{5}{9}(32)+b\Rightarrow b=\frac{2297}{9}\Rightarrow$$
$$K(F)=\frac{5}{9}F+\frac{2297}{9}.$$

b. $K=\dfrac{5}{9}F+\dfrac{2297}{9}\Rightarrow K-\dfrac{2297}{9}=\dfrac{5}{9}F\Rightarrow$
$$9K-2297=5F\Rightarrow F(K)=\frac{9}{5}K-\frac{2297}{5}$$
This represents the Fahrenheit temperature corresponding to a given Kelvin temperature.

c. $K(98.6)=\dfrac{5}{9}(98.6)+\dfrac{2297}{9}=310\text{ K}$

89. a.
$$\begin{aligned}F(K(C))&=\frac{9}{5}(C+273)-\frac{2297}{5}\\&=\frac{9}{5}C+\frac{9(273)}{5}-\frac{2297}{5}\\&=\frac{9}{5}C+\frac{160}{5}=\frac{9}{5}C+32\end{aligned}$$

b.
$$\begin{aligned}C(K(F))&=\frac{5}{9}F+\frac{2297}{9}-273\\&=\frac{5}{9}F+\frac{2297-2457}{9}\\&=\frac{5}{9}F-\frac{160}{9}\end{aligned}$$

90.
$$\begin{aligned}F(C(x))&=\frac{9}{5}\left(\frac{5}{9}x-\frac{160}{9}\right)+32\\&=x-32+32=x\\C(F(x))&=\frac{5}{9}\left(\frac{9}{5}x+32\right)-\frac{160}{9}\\&=x+\frac{160}{9}-\frac{160}{9}=x\end{aligned}$$
Therefore, F and C are inverses of each other.

91. a. $E(x)=0.75x$ where x represents the number of dollars; $D(x)=1.25x$ where x represents the number of euros.

b. $E(D(x))=0.75(1.25x)=0.9375x\neq x$.
Therefore, the two functions are not inverses.

c. She loses money either way.

92. a. $w=4+0.05x\Rightarrow w-4=0.05x\Rightarrow$
$x=20w-80$.
This represents the food sales in terms of his hourly wage.

b. $x=20(12)-80=\$160$

93. a. $7=4+0.05x\Rightarrow x=\$60$. This means that if food sales $\le \$60$, he will receive the minimum hourly wage. If food sales $> \$60$, his wages will be based on food sales.
$$w=\begin{cases}4+0.05x & \text{if } x>60\\ 7 & \text{if } x\le 60\end{cases}$$

b. The function does not have an inverse because it is constant on (0, 60), and it is not one-to-one.

c. If the domain is restricted to $[60,\infty)$, the function has an inverse.

94. a. $T = 1.11\sqrt{l} \Rightarrow l = \left(\frac{T}{1.11}\right)^2$. This shows the length as the function of the period.

b. $l = \left(\frac{2}{1.11}\right)^2 \approx 3.2$ ft

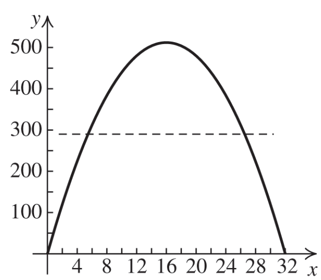
c. $T = 1.11\sqrt{70} \approx 9.3$ sec

95. a. $V = 8\sqrt{x} \Rightarrow \frac{V}{8} = \sqrt{x} \Rightarrow \frac{1}{64}V^2 = x = V^{-1}(x)$
This represents the height of the water in terms of the velocity.

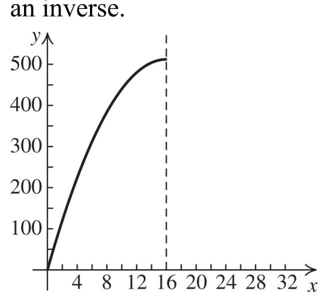
b. (i) $x = \frac{1}{64}\left(30^2\right) = 14.0625$ ft

(ii) $x = \frac{1}{64}\left(20^2\right) = 6.25$ ft

96. a. $y = 64x - 2x^2$ has no inverse because it is not one-to-one across its domain, [0, 32]. (It fails the horizontal line test.)



However, if the domain is restricted to [0, 16], the function is one-to-one, and it has an inverse.



$y = 64x - 2x^2 \Rightarrow 2x^2 - 64x + y = 0 \Rightarrow$

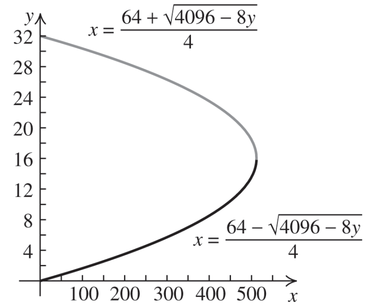
$x = \frac{64 \pm \sqrt{64^2 - 8y}}{4} \Rightarrow$

$x = \frac{64 \pm \sqrt{4096 - 8y}}{4} = \frac{64 \pm 2\sqrt{1024 - 2y}}{4}$

$= \frac{32 \pm \sqrt{1024 - 2y}}{2}$

$1024 - 2y \ge 0 \Rightarrow 0 \le y \le 512.$

(Because y is a number of feet, it cannot be negative.) This is the range of the original function. The domain of the original function is [0, 16], which is the range of the inverse. The range of $x = \frac{32 + \sqrt{1024 - 2y}}{2}$ is [16, 32], so this is not the inverse. The range of $x = \frac{32 - \sqrt{1024 - 2y}}{2}$, $0 \le y \le 512$, is [0, 16], so this is the inverse.



Note that the bottom half of the graph is the inverse.

b. (i) $x = \frac{64 - \sqrt{4096 - 8(32)}}{4} \approx 0.51$ ft

(ii) $x = \frac{64 - \sqrt{4096 - 8(256)}}{4} \approx 4.69$ ft

(iii) $x = \frac{64 - \sqrt{4096 - 8(512)}}{4} \approx 16$ ft

97. a. The function represents the amount she still owes after x months.

b. $y = 36,000 - 600x$. Interchange the variables and solve for y: $x = 36,000 - 600y \Rightarrow$

$600y = 36,000 - x \Rightarrow y = 60 - \frac{x}{600} \Rightarrow$

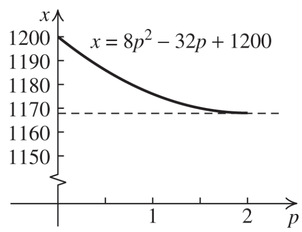
$f^{-1}(x) = 60 - \frac{1}{600}x$. This represents the number of months that have passed from the first payment until the balance due is \$$x$.

c. $y = 60 - \frac{1}{600}(22,000) = 23.33 \approx 24$ months
There are 24 months remaining.

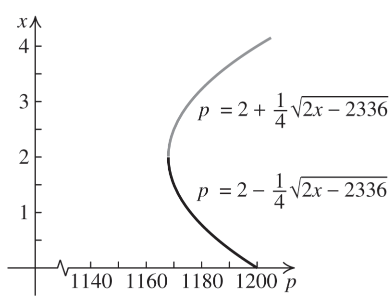
98. a. To find the inverse, solve
$x = 8p^2 - 32p + 1200$ for p:

$$8p^2 - 32p + 1200 - x = 0 \Rightarrow$$
$$p = \frac{32 \pm \sqrt{(-32)^2 - 4(8)(1200 - x)}}{2(8)}$$
$$= \frac{32 \pm \sqrt{1024 - 38,400 + 32x}}{16}$$
$$= \frac{32 \pm \sqrt{32x - 37376}}{16} = \frac{32 \pm 4\sqrt{2x - 2336}}{16}$$
$$= 2 \pm \frac{1}{4}\sqrt{2x - 2336}$$

Because the domain of the original function is (0, 2], its range is (1168, 1200].



So the domain of the inverse is (1168, 1200], and its range is (0, 2]. The range of $p = 2 + \frac{1}{4}\sqrt{2x - 2336}$ is (2, 4], so it is not the inverse. The range of $p = 2 - \frac{1}{4}\sqrt{2x - 2336}, 1168 \le x < 1200,$ is (0, 2], so it is the inverse. This gives the price of computer chips in terms of the demand x.



Note that the bottom half of the graph is the inverse.

b. $p = 2 - \frac{1}{4}\sqrt{2(1180.5) - 2336} = \$0.75$

99. a. $F(r) = GMm\frac{1}{r^2}$ is a one-to-one function because it is decreasing on $(0, \infty)$.

b. $F = GMm\frac{1}{r^2} \Rightarrow \frac{F}{GMm} = \frac{1}{r^2} \Rightarrow$
$r^2 = \frac{GMm}{F} \Rightarrow r = \sqrt{\frac{GMm}{F}}$

c. $y = GMm\frac{1}{x^2} \Rightarrow r = GMm\frac{1}{y^2} \Rightarrow$
$\frac{x}{GMm} = \frac{1}{y^2} \Rightarrow y^2 = \frac{GMm}{x} \Rightarrow$
$y = f^{-1}(x) = \sqrt{\frac{GMm}{x}}$

100. a. $V(r) = \frac{4}{3}\pi r^3$ is a one-to-one function because it is increasing on $(0, \infty)$.

b. $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{3V}{4\pi} = r^3 \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$

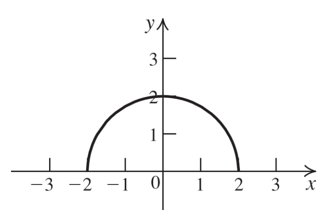
c. $V(x) = y = \frac{4}{3}\pi x^3 \Rightarrow x = \frac{4}{3}\pi y^3 \Rightarrow$
$y^3 = \frac{3x}{4\pi} \Rightarrow y = V^{-1}(x) = \sqrt[3]{\frac{3x}{4\pi}}$

Beyond the Basics

101. $f(g(3)) = f(1) = 3, f(g(5)) = f(3) = 5,$ and $f(g(2)) = f(4) = 2 \Rightarrow f(g(x)) = x$ for each x.
$g(f(1)) = g(3) = 1, g(f(3)) = g(5) = 3,$ and $g(f(4)) = g(2) = 4 \Rightarrow g(f(x)) = x$ for each x.
So, f and g are inverses.

102. $f(g(-2)) = f(1) = -2, f(g(0)) = f(2) = 0,$ $f(g(-3)) = f(3) = -3,$ and $f(g(-2)) = f(1) = -2 \Rightarrow f(g(x)) = x$ for each x.
$g(f(1)) = g(-2) = 1, g(f(2)) = g(0) = 2,$ $g(f(3)) = g(-3) = 3,$ and $g(f(4)) = g(1) = 4$ $\Rightarrow g(f(x)) = x$ for each x.
So f and g are inverses.

103. a.



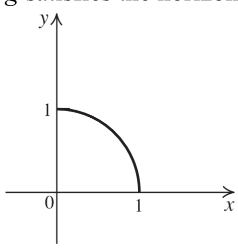
b. f is not one-to-one

c. Domain: [–2, 2]; range: [0, 2]

104. a. Domain: $(-\infty, 2)\cup[3, \infty)$. Note that the domain is not $(-\infty, 2)\cup(2, \infty)$ because $[\![x]\!] = 2$ for $2 \le x < 3$.

b. The function is not one-to-one. The function is constant on each interval $[n, n + 1)$, n an integer.

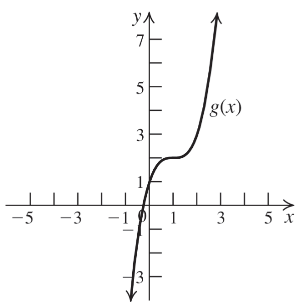
105. a. g satisfies the horizontal line test.



b. $y = \sqrt{1-x^2}$. Interchange the variables and solve for $y : x = \sqrt{1-y^2} \Rightarrow$
$x^2 = 1-y^2 \Rightarrow y^2 = 1-x^2 \Rightarrow$
$y = g^{-1}(x) = \sqrt{1-x^2}$

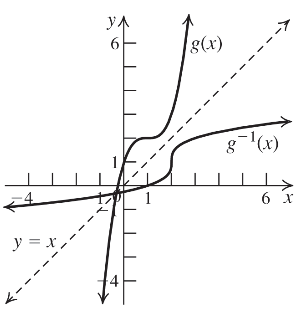
c. Domain of f = range of f: [0, 1]

106. a. The graph of g is the graph of f shifted one unit to the right and two units up.



b. $g(x) = y = (x-1)^3 + 2$. Interchange the variables and solve for $y : x = (y-1)^3 + 2 \Rightarrow$
$y = g^{-1}(x) = \sqrt[3]{x-2} + 1$.

c.



107. We can show that the sum of increasing functions is also an increasing function as follows: If $f(x)$ and $g(x)$ are increasing functions, then if $x_1 < x_2$, $f(x_1) < f(x_2)$ and $g(x_1) < g(x_2)$. Adding the two inequalities gives $f(x_1) + g(x_1) < f(x_2) + g(x_2)$. Thus, the sum is also increasing, and, therefore, the sum must be a one-to-one function. We know that f^{-1} exists because f is a one-to-one function and an inverse exists.

108. a. $f(x) = x^3 + x + 1$

$$\text{ARC}[f]_1^2 = \frac{f(2)-f(1)}{2-1} = \frac{11-3}{1} = 8$$
$$\text{ARC}\left[f^{-1}\right]_{f(1)=3}^{f(2)=11} = \frac{1}{\text{ARC}[f]_1^2} = \frac{1}{8}$$

b. It is difficult to compute the average rate of change directly because it is impossible to represent f^{-1} as a concise algebraic formula.

109. a. $M = \left(\frac{3+7}{2}, \frac{7+3}{2}\right) = (5, 5)$.

Since the coordinates of M satisfy the equation $y = x$, it lies on the line.

b. The slope of $y = x$ is 1, while the slope of $\overline{PQ}$ is $\frac{3-7}{7-3} = -1$. So, $y = x$ is perpendicular to $\overline{PQ}$.

110. $M = \left(\frac{a+b}{2}, \frac{b+a}{2}\right)$.

Since the coordinates of M satisfy the equation $y = x$, it lies on the line. The slope of the line segment between the two points is $\frac{b-a}{a-b} = -1$, while the slope of $y = x$ is 1. So the two lines are perpendicular, and the points (a, b) and (b, a) are symmetric about the line $y = x$.

111. a. (i) $f(x) = y = 2x - 1$
Interchange the variables and solve for y:
$x = 2y - 1 \Rightarrow y = f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$

(ii) $g(x) = y = 3x + 4$
Interchange the variables and solve for y:
$x = 3y + 4 \Rightarrow y = g^{-1}(x) = \frac{1}{3}x - \frac{4}{3}$

(iii) $(f \circ g)(x) = 2(3x + 4) - 1 = 6x + 7$

(iv) $(g \circ f)(x) = 3(2x - 1) + 4 = 6x + 1$

(v) $(f \circ g)(x) = y = 6x + 7$
Interchange the variables and solve for y:
$x = 6y + 7 \Rightarrow (f \circ g)^{-1}(x) = \frac{1}{6}x - \frac{7}{6}$

(vi) $(g \circ f)(x) = y = 6x + 1$
Interchange the variables and solve for y:
$x = 6y + 1 \Rightarrow (f \circ g)^{-1}(x) = \frac{1}{6}x - \frac{1}{6}$

(vii) $(f^{-1} \circ g^{-1})(x) = \frac{1}{2}\left(\frac{1}{3}x - \frac{4}{3}\right) + \frac{1}{2}$
$= \frac{1}{6}x - \frac{2}{3} + \frac{1}{2} = \frac{1}{6}x - \frac{1}{6}$

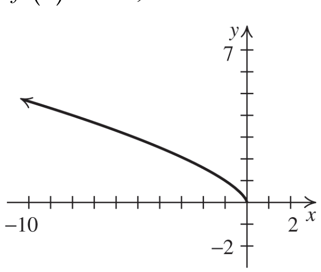
(viii) $(g^{-1} \circ f^{-1})(x) = \frac{1}{3}\left(\frac{1}{2}x + \frac{1}{2}\right) - \frac{4}{3}$
$= \frac{1}{6}x + \frac{1}{6} - \frac{4}{3} = \frac{1}{6}x - \frac{7}{6}$

b. **(i)** $(f \circ g)^{-1}(x) = \frac{1}{6}x - \frac{7}{6}$
$= (g^{-1} \circ f^{-1})(x)$

(ii) $(g \circ f)^{-1}(x) = \frac{1}{6}x - \frac{1}{6}$
$= (f^{-1} \circ g^{-1})(x)$

For $f(x) = 2x + 3,\ g(x) = x^3 - 1$

112. $f(x) = x^{2/3},\ x \le 0$



It is clear from the graph that the function satisfies the horizontal line test. Thus, the function is one-to-one. To find the inverse, interchange the variables and solve for y.
$f(x) = y = x^{2/3} \Rightarrow x = y^{2/3} \Rightarrow x^3 = y^2$
Now we must take the square root of each side. We choose the negative square root because x is restricted to those values less than or equal to zero.
$x^3 = y^2 \Rightarrow -\sqrt{x^3} = y \Rightarrow y = f^{-1}(x) = -x^{3/2}$.

Critical Thinking/Discussion/Writing

113. No. For example, $f(x) = x^3 - x$ is odd, but it does not have an inverse, because $f(0) = f(1)$, so it is not one-to-one.

114. Yes. The function $f = \{(0,1)\}$ is even, and it has an inverse: $f^{-1} = \{(1,0)\}$.

115. Yes, because increasing and decreasing functions are one-to-one.

116. **a.** $R = \{(-1,1), (0,0), (1,1)\}$

b. $R = \{(-1,1), (0,0), (1,2)\}$

Getting Ready for the Next Section

117. $x^2 - x - 12 = (x + 3)(x - 4)$

118. $x^2 - 5x + 6 = (x - 2)(x - 3)$

119. $(x^2 + 2x - 8) = (x + 4)(x - 2)$

120. $x^2 + 7x + 10 = (x + 2)(x + 5)$

121. $x^2 - 7x + 12 = 0$
$(x - 3)(x - 4) = 0$
$x - 3 = 0 \mid x - 4 = 0$
$x = 3 \mid x = 4$
Solution: $\{3, 4\}$

122. $x^2 - x - 6 = 0$
$(x + 2)(x - 3) = 0$
$x + 2 = 0 \mid x - 3 = 0$
$x = -2 \mid x = 3$
Solution: $\{-2, 3\}$

123. $3x^2 + 7x + 2 = 0$

$(3x+1)(x+2) = 0$

$3x + 1 = 0 \Rightarrow x = -\frac{1}{3}$ | $x + 2 = 0 \Rightarrow x = -2$

Solution: $\left\{-2, -\frac{1}{3}\right\}$

124. $x^2 - 4x + 1 = 0$

Use the quadratic formula.

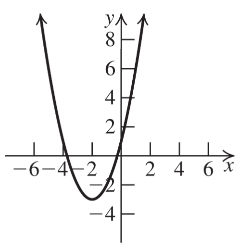
$a = 1, b = -4, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

Solution: $\left\{2 - \sqrt{3}, 2 + \sqrt{3}\right\}$

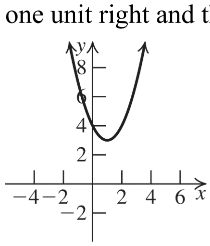
125. $y = (x+2)^2 - 3$

Start with the graph of $f(x) = x^2$, then shift it two units left and three units down.



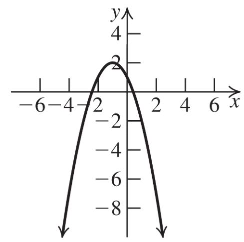
126. $y = (x-1)^2 + 3$

Start with the graph of $f(x) = x^2$, then shift it one unit right and three units up.



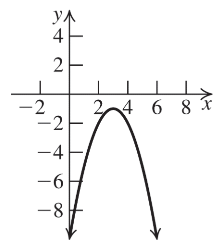
127. $y = -(x+1)^2 + 2$

Start with the graph of $f(x) = x^2$, then shift it one unit left. Reflect the graph across the x-axis and then shift it two units up



128. $y = -(x-3)^2 - 1$

Start with the graph of $f(x) = x^2$, then shift it three units right. Reflect the graph across the x-axis and then shift it one unit down.



Chapter 1 Review Exercises

1. False. The midpoint is $\left(\frac{-3+3}{2}, \frac{1+11}{2}\right) = (0, 6)$.
2. False. The equation is a circle with center $(-2, -3)$ and radius $\sqrt{5}$.
3. True
4. False. A graph that is symmetric with respect to the origin is the graph of an odd function. A graph that is symmetric with respect to the y-axis is the graph of an even function.
5. False. The slope is 4/3 and the y-intercept is 3.
6. False. The slope of a line that is perpendicular to a line with slope 2 is –1/2.
7. True
8. False. There is no graph because the radius cannot be negative.
9. **a.** $d(P,Q) = \sqrt{(-1-3)^2 + (3-5)^2} = 2\sqrt{5}$

   b. $M = \left(\frac{3+(-1)}{2}, \frac{5+3}{2}\right) = (1, 4)$

   c. $m = \frac{3-5}{-1-3} = \frac{1}{2}$

10. a. $d(P,Q)=\sqrt{(3-(-3))^2+(-1-5)^2}=6\sqrt{2}$

b. $M=\left(\frac{-3+3}{2},\frac{5+(-1)}{2}\right)=(0,2)$

c. $m=\frac{-1-5}{3-(-3)}=-1$

11. a. $d(P,Q)=\sqrt{(9-4)^2+(-8-(-3))^2}=5\sqrt{2}$

b. $M=\left(\frac{4+9}{2},\frac{-3+(-8)}{2}\right)=\left(\frac{13}{2},-\frac{11}{2}\right)$

c. $m=\frac{-8-(-3)}{9-4}=-1$

12. a. $d(P,Q)=\sqrt{(-7-2)^2+(-8-3)^2}=\sqrt{202}$

b. $M=\left(\frac{2+(-7)}{2},\frac{3+(-8)}{2}\right)=\left(-\frac{5}{2},-\frac{5}{2}\right)$

c. $m=\frac{-8-3}{-7-2}=\frac{11}{9}$

13. a. $D(P,Q)=\sqrt{(5-2)^2+(-2-(-7))^2}=\sqrt{34}$

b. $M=\left(\frac{2+5}{2},\frac{-7+(-2)}{2}\right)=\left(\frac{7}{2},-\frac{9}{2}\right)$

c. $m=\frac{-2-(-7)}{5-2}=\frac{5}{3}$

14. a. $d(P,Q)=\sqrt{(10-(-5))^2+(-3-4)^2}=\sqrt{274}$

b. $M=\left(\frac{-5+10}{2},\frac{4+(-3)}{2}\right)=\left(\frac{5}{2},\frac{1}{2}\right)$

c. $m=\frac{-3-4}{10-(-5)}=-\frac{7}{15}$

15. $m_{AC}=\frac{0-5}{3-0}=-\frac{5}{3}; m_{CB}=\frac{0-(-3)}{3-(-2)}=\frac{3}{5}$

$m_{AC}\cdot m_{CB}=-1\Rightarrow AC\perp CB$, so $\triangle ABC$ is a right triangle.

16. $d(A,B)=\sqrt{(4-1)^2+(8-2)^2}=3\sqrt{5}$

$d(C,D)=\sqrt{(10-7)^2+(5-(-1))^2}=3\sqrt{5}$

$d(A,C)=\sqrt{(7-1)^2+(-1-2)^2}=3\sqrt{5}$

$d(B,D)=\sqrt{(10-4)^2+(5-8)^2}=3\sqrt{5}$

The four sides are equal, so the quadrilateral is a rhombus.

17. $A=(-6,3), B=(4,5)$

$d(A,O)=\sqrt{(-6-0)^2+(3-0)^2}=\sqrt{45}$

$d(B,O)=\sqrt{(4-0)^2+(5-0)^2}=\sqrt{41}$

(4, 5) is closer to the origin.

18. $A=(-6,4), B=(5,10), C=(2,3)$

$d(A,C)=\sqrt{(2-(-6))^2+(3-4)^2}=\sqrt{65}$

$d(B,C)=\sqrt{(2-5)^2+(3-10)^2}=\sqrt{58}$

(5, 10) is closer to (2, 3).

19. $A=(-5,3), B=(4,7), C=(x,0)$

$$\begin{aligned}d(A,C)&=\sqrt{(x-(-5))^2+(0-3)^2}\\&=\sqrt{(x+5)^2+9}\\d(B,C)&=\sqrt{(x-4)^2+(0-7)^2}\\&=\sqrt{(x-4)^2+49}\end{aligned}$$

$d(A,C)=d(B,C)\Rightarrow$

$\sqrt{(x+5)^2+9}=\sqrt{(x-4)^2+49}$

$(x+5)^2+9=(x-4)^2+49$

$x^2+10x+34=x^2-8x+65$

$x=\frac{31}{18}\Rightarrow$ The point is $\left(\frac{31}{18},0\right)$.

20. $A=(-3,-2), B(2,-1), C(0,y)$

$$\begin{aligned}d(A,C)&=\sqrt{(0-(-3))^2+(y-(-2))^2}\\&=\sqrt{(y+2)^2+9}\\d(B,C)&=\sqrt{(0-(2))^2+(y-(-1))^2}\\&=\sqrt{(y+1)^2+4}\end{aligned}$$

$d(A,C)=d(B,C)\Rightarrow$

$\sqrt{(y+2)^2+9}=\sqrt{(y+1)^2+4}$

$(y+2)^2+9=(y+1)^2+4$

$y^2+4y+13=y^2+2y+5$

$y=-4\Rightarrow$ The point is $(0,-4)$.

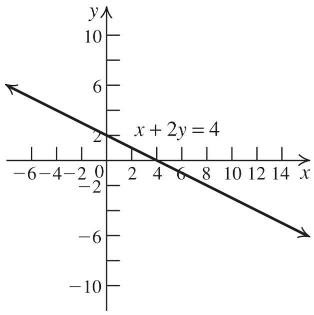
21. Not symmetric with respect to the *x*-axis; symmetric with respect to the *y*-axis; not symmetric with respect to the origin.

22. Not symmetric with respect to the *x*-axis; not symmetric with respect to the *y*-axis; symmetric with respect to the origin.

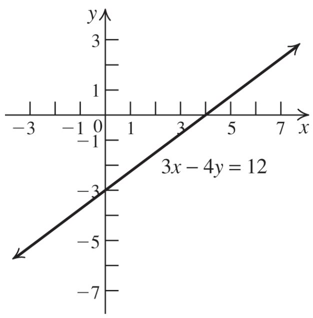
23. Symmetric with respect to the *x*-axis; not symmetric with respect to the *y*-axis; not symmetric with respect to the origin.

24. Symmetric with respect to the *x*-axis; symmetric with respect to the *y*-axis; symmetric with respect to the origin.

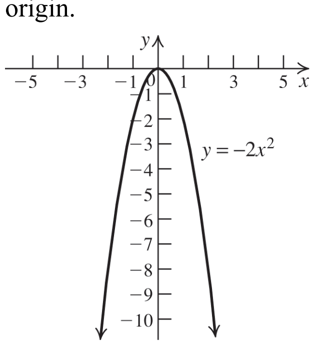
25. x-intercept: 4; y-intercept: 2; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



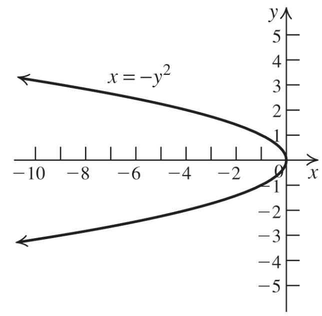
26. x-intercept: 4; y-intercept: –3; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



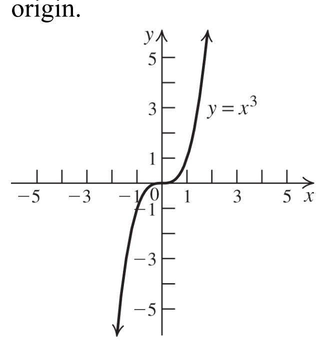
27. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.



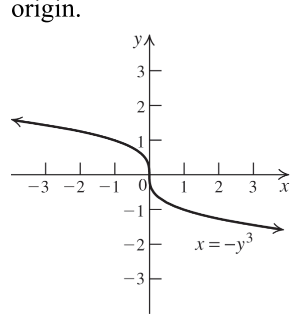
28. x-intercept: 0; y-intercept: 0; symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



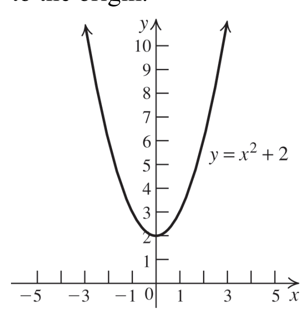
29. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.



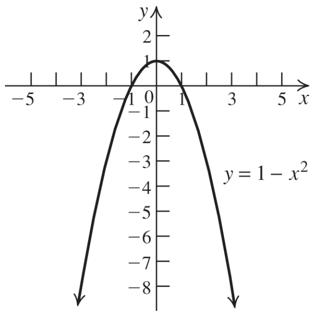
30. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.



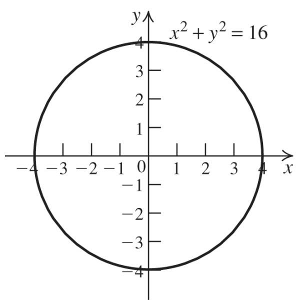
31. No x-intercept; y-intercept: 2; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.



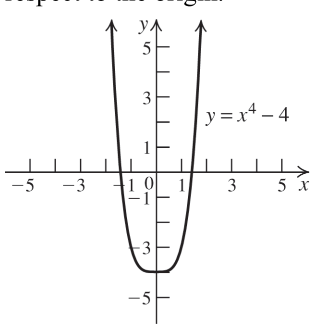
32. x-intercepts: –1, 1; y-intercept: 1; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.



33. x-intercepts: –4, 4; y-intercepts: –4, 4; symmetric with respect to the x-axis; symmetric with respect to the y-axis; symmetric with respect to the origin.



34. x-intercepts: $-\sqrt{2}, \sqrt{2}$; y-intercept: –4; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.



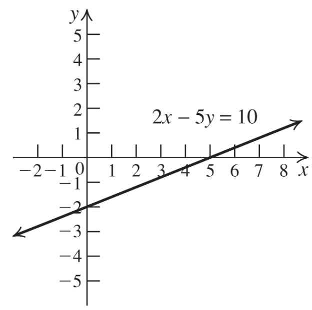
35. $(x-2)^2 + (y+3)^2 = 25$

36. The center of the circle is the midpoint of the diameter. $M = \left(\dfrac{5+(-5)}{2} + \dfrac{2+4}{2}\right) = (0, 3)$. The length of the radius is the distance from the center to one of the endpoints of the diameter = $\sqrt{(5-0)^2 + (2-3)^2} = \sqrt{26}$. The equation of the circle is $x^2 + (y-3)^2 = 26$.

37. The radius is 2, so the equation of the circle is $(x+2)^2 + (y+5)^2 = 4$.

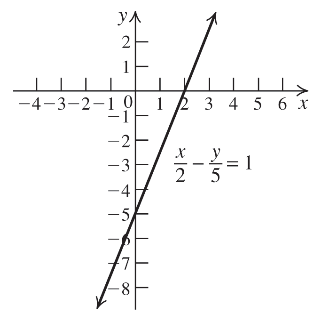
38. $2x - 5y = 10 \Rightarrow y = \dfrac{2}{5}x - 2$.

Line with slope $\dfrac{2}{5}$, y-intercept –2, and x-intercept 5.

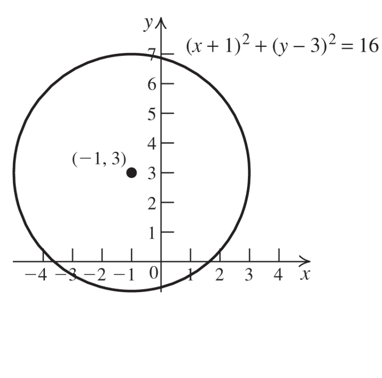


39. $\dfrac{x}{2} - \dfrac{y}{5} = 1 \Rightarrow 5x - 2y = 10 \Rightarrow y = \dfrac{5}{2}x - 5$.

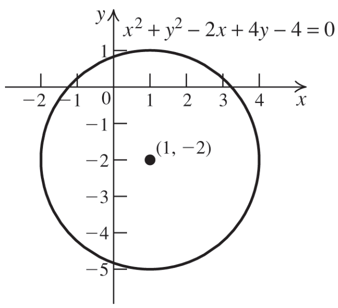
Line with slope $\dfrac{5}{2}$, y-intercept –5, and x-intercept 2.



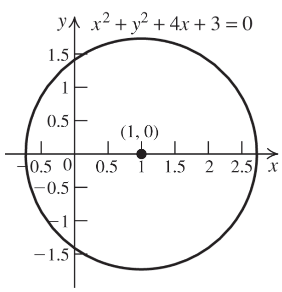
40. Circle with center (–1, 3) and radius 4.
intercepts:
$(x+1)^2 + (-3)^2 = 16 \Rightarrow x = -1 \pm \sqrt{7}$
$(1)^2 + (y-3)^2 = 16 \Rightarrow y = 3 \pm \sqrt{15}$



41. $x^2 + y^2 - 2x + 4y - 4 = 0 \Rightarrow$
$x^2 - 2x + 1 + y^2 + 4y + 4 = 4 + 1 + 4 \Rightarrow$
$(x-1)^2 + (y+2)^2 = 9.$
Circle with center (1, –2) and radius 3.
intercepts:
$x^2 - 2x - 4 = 0 \Rightarrow x = 1 \pm \sqrt{5}$
$y^2 + 4y - 4 = 0 \Rightarrow y = -2 \pm 2\sqrt{2}$



42. $3x^2 + 3y^2 - 6x - 6 = 0 \Rightarrow x^2 - 2x + y^2 = 2 \Rightarrow$
$x^2 - 2x + 1 + y^2 = 2 + 1 \Rightarrow (x-1)^2 + y^2 = 3.$
Circle with center (1, 0) and radius $\sqrt{3}$.
intercepts:
$3x^2 - 6x - 6 = 0 = 9 \Rightarrow x = 1 \pm \sqrt{3}$
$y^2 + 4x + 3 = 0 \Rightarrow y = -2 \pm 2\sqrt{2}$



43. $y - 2 = -2(x-1) \Rightarrow y = -2x + 4$

44. $m = \dfrac{5-0}{0-2} = -\dfrac{5}{2}; y = -\dfrac{5}{2}x + 5$

45. $m = \dfrac{7-3}{-1-1} = -2; 3 = -2(1) + b \Rightarrow 5 = b \Rightarrow$
$y = -2x + 5$

46. $2x + 3y = 7 \Rightarrow 3y = -2x + 7 \Rightarrow y = -\dfrac{2}{3}x + \dfrac{7}{3}$

The slope of the line is $m = -\dfrac{2}{3}$, so the slope of the parallel line is $m = -\dfrac{2}{3}$.
Using the point-slope formula, we have
$y - 2 = -\dfrac{2}{3}(x-1) \Rightarrow y - 2 = -\dfrac{2}{3}x + \dfrac{2}{3} \Rightarrow$
$y = -\dfrac{2}{3}x + \dfrac{8}{3}.$

47. $3x - 4y = 12 \Rightarrow -4y = -3x + 12 \Rightarrow y = \dfrac{3}{4}x - 3$

The slope of the line is $m = \dfrac{3}{4}$, so the slope of the perpendicular line is $m = -\dfrac{4}{3}$.
Using the point-slope formula, we have
$y - 3 = -\dfrac{4}{3}(x-1) \Rightarrow y - 3 = -\dfrac{4}{3}x + \dfrac{4}{3} \Rightarrow$
$y = -\dfrac{4}{3}x + \dfrac{13}{3}.$

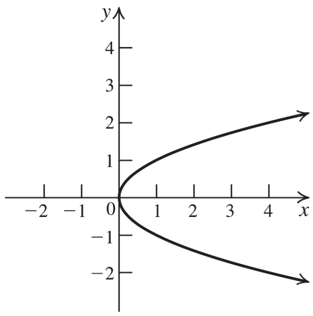
48. **a.** $y = 3x - 2 \Rightarrow m = 3; y = 3x + 2 \Rightarrow m = 3.$
The slopes are equal, so the lines are parallel.

b. $3x - 5y + 7 \Rightarrow m = 3/5;$
$5x - 3y + 2 = 0 \Rightarrow m = 5/3.$
The slopes are neither equal nor negative reciprocals, so the lines are neither parallel nor perpendicular.

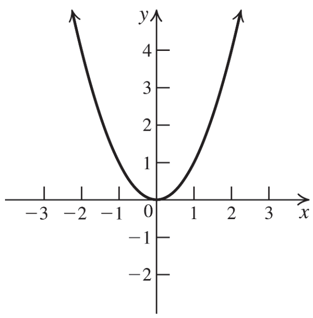
c. $ax + by + c = 0 \Rightarrow m = -a/b;$
$bx - ay + d = 0 \Rightarrow m = b/a.$
The slopes are negative reciprocals, so the lines are perpendicular.

d. $y + 2 = \dfrac{1}{3}(x-3) \Rightarrow m = \dfrac{1}{3};$
$y - 5 = 3(x-3) \Rightarrow m = 3.$
The slopes are neither equal nor negative reciprocals, so the lines are neither parallel nor perpendicular.

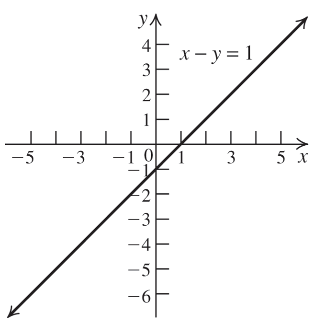
49. not a function



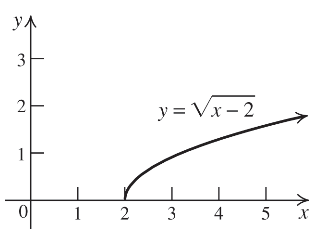
50. function



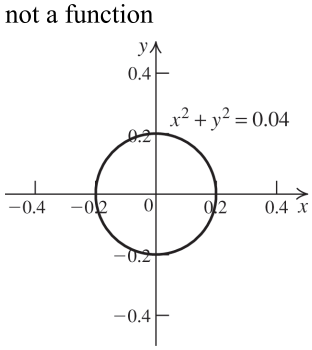
51. function



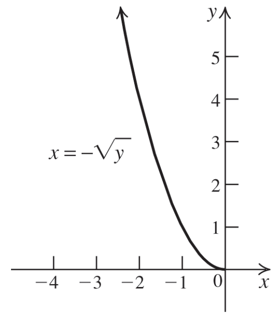
52. function



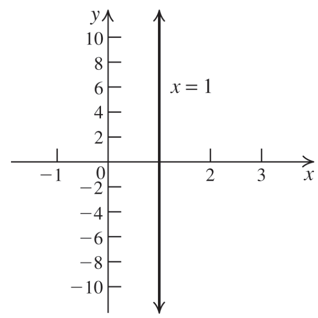
53. not a function



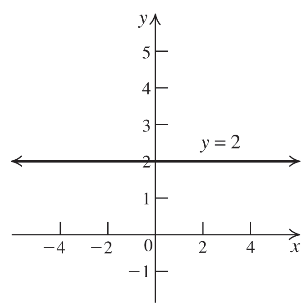
54. function



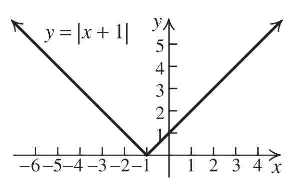
55. not a function



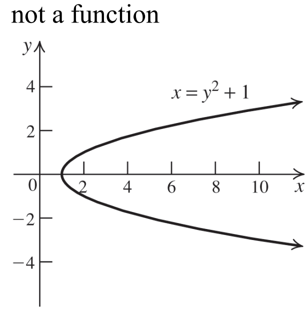
56. function



57. function



58. not a function



59. $f(-2) = 3(-2) + 1 = -5$

60. $g(-2) = (-2)^2 - 2 = 2$

61. $f(x) = 4 \Rightarrow 3x + 1 = 4 \Rightarrow x = 1$

62. $g(x) = 2 \Rightarrow x^2 - 2 = 2 \Rightarrow x = \pm 2$

63. $(f+g)(1) = f(1) + g(1)$
$= (3(1)+1) + (1^2 - 2) = 3$

64. $(f-g)(-1) = f(-1) - g(-1)$
$= (3(-1)+1) - ((-1)^2 - 2) = -1$

65. $(f \cdot g)(-2) = f(-2) \cdot g(-2)$
$= (3(-2)+1) \cdot ((-2)^2 - 2) = -10$

66. $(g \cdot f)(0) = g(0) \cdot f(0)$
$= (0^2 - 2) \cdot (3(0)+1) = -2$

67. $(f \circ g)(3) = 3(3^2 - 2) + 1 = 22$

68. $(g \circ f)(-2) = (3(-2)+1)^2 - 2 = 23$

69. $(f \circ g)(x) = 3(x^2 - 2) + 1 = 3x^2 - 5$

70. $(g \circ g)(x) = (x^2 - 2)^2 - 2 = x^4 - 4x^2 + 2$

71. $f(a+h) = 3(a+h) + 1 = 3a + 3h + 1$

72. $g(a-h) = (a-h)^2 - 2 = a^2 - 2ah + h^2 - 2$

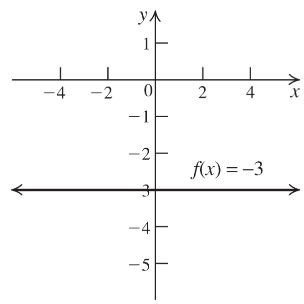
73. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{(3(x+h)+1) - (3x+1)}{h}$
$= \dfrac{3x + 3h + 1 - 3x - 1}{h} = \dfrac{3h}{h} = 3$

74. $\dfrac{g(x+h) - g(x)}{h} = \dfrac{((x+h)^2 - 2) - (x^2 - 2)}{h}$
$= \dfrac{x^2 + 2xh + h^2 - 2 - x^2 + 2}{h}$
$= \dfrac{2xh + h^2}{h} = 2x + h$

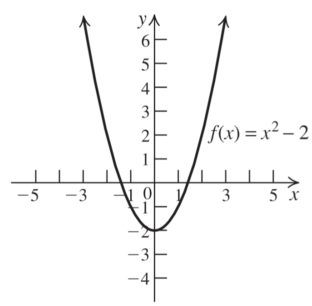
75. $(f \circ g)(-1) = f(g(-1)) = f(0) = 1$
$(f \circ g)(1) = f(g(1)) = f(2) = 5$
$(f \circ g)(2) = f(g(2)) = f(3)$, which is undefined. Thus,
$f \circ g = \{(-1, 1), (1, 5)\}$.

76. $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right) = \sqrt{\dfrac{1}{x} + 1}$
$= \sqrt{\dfrac{1+x}{x}}$
The domain of g is $(-\infty, 0) \cup (0, \infty)$, while the domain of f is $[-1, \infty)$. Thus, the domain of $f \circ g$ is $(-\infty, -1] \cup (0, \infty)$.

77. Domain: $(-\infty, \infty)$; range: $\{-3\}$.
Constant on $(-\infty, \infty)$.

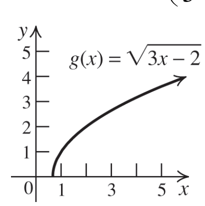


78. Domain: $(-\infty, \infty)$; range: $[-2, \infty)$.
Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.

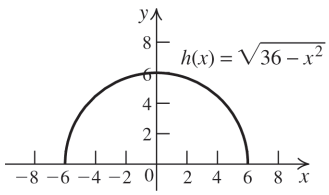


79. Domain: $\left[\dfrac{2}{3}, \infty\right)$; range: $[0, \infty)$.

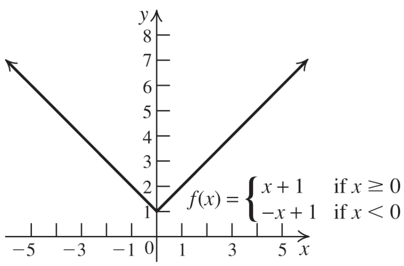
Increasing on $\left(\dfrac{2}{3}, \infty\right)$.



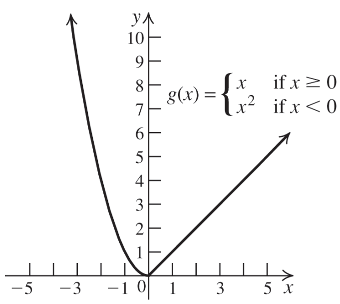
80. Domain: $[-6, 6]$; range: $[0,6]$.
Increasing on $(-6, 0)$; decreasing on $(0, 6)$.



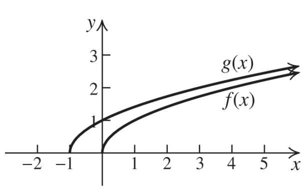
81. Domain: $(-\infty, \infty)$; range: $[1, \infty)$. Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.



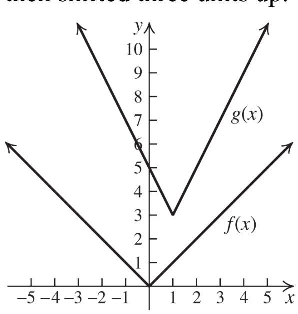
82. Domain: $(-\infty, \infty)$; range: $[0,\infty)$.
Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.



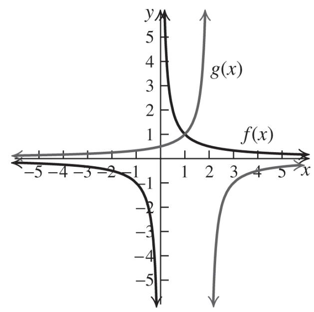
83. The graph of g is the graph of f shifted one unit left.



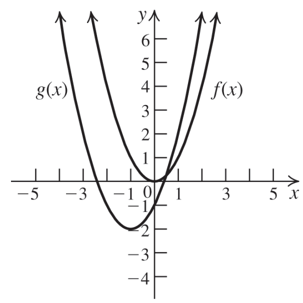
84. The graph of g is the graph of f shifted one unit right, stretched vertically by a factor of 2, and then shifted three units up.



85. The graph of g is the graph of f shifted two units right and then reflected across the x-axis.



86. The graph of g is the graph of f shifted one unit left and two units down.



87. $f(-x) = (-x)^2 + (-x)^4 = x^2 + x^4 = f(x) \Rightarrow$
$f(x)$ is even. Not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

88. $f(-x) = (-x)^3 + (-x) = -x^3 - x = -f(x) \Rightarrow$
$f(x)$ is odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.

89. $f(-x) = |-x| + 3 = |x| + 3 = f(x) \Rightarrow$
$f(x)$ is even. Not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

90. $f(-x) = -3x + 5 \neq f(x)$ or $f(-x) \Rightarrow f(x)$ is neither even nor odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.

91. $f(-x) = \sqrt{-x} \neq f(x)$ or $f(-x) \Rightarrow f(x)$ is neither even nor odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.

92. $f(-x) = -\dfrac{2}{x} = -f(x) \Rightarrow f(x)$ is odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.

Answers may vary in exercises 93−96.

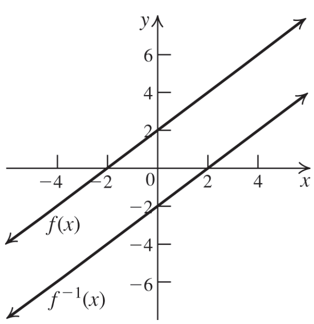
93. $f(x) = \sqrt{x^2 - 4} \Rightarrow f(x) = (g \circ h)(x)$ where $g(x) = \sqrt{x}$ and $h(x) = x^2 - 4$.

94. $g(x) = \left(x^2 - x + 2\right)^{50} \Rightarrow g(x) = (f \circ h)(x)$ where $f(x) = x^{50}$ and $h(x) = x^2 - x + 2$.

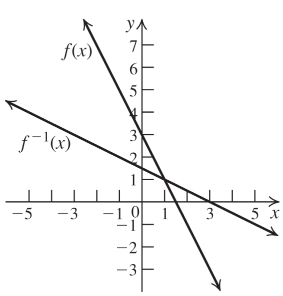
95. $h(x) = \sqrt{\dfrac{x-3}{2x+5}} \Rightarrow h(x) = (f \circ g)(x)$ where $f(x) = \sqrt{x}$ and $g(x) = \dfrac{x-3}{2x+5}$.

96. $H(x) = (2x-1)^3 + 5 \Rightarrow H(x) = (f \circ g)(x)$ where $f(x) = x^3 + 5$ and $g(x) = 2x - 1$.

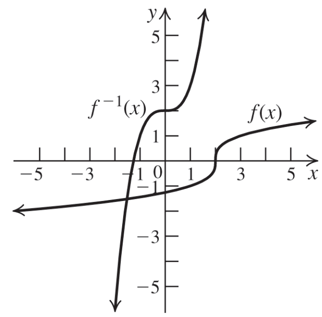
97. $f(x)$ is one-to-one. $f(x) = y = x + 2$.
Interchange the variables and solve for y:
$x = y + 2 \Rightarrow y = f^{-1}(x) = x - 2$.



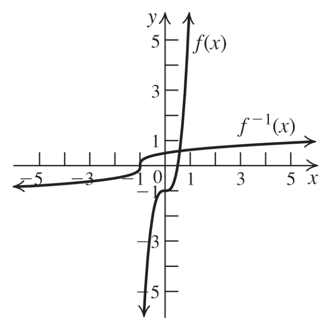
98. $f(x)$ is one-to-one. $f(x) = y = -2x + 3$.
Interchange the variables and solve for y:
$x = -2y + 3 \Rightarrow y = f^{-1}(x) = -\dfrac{1}{2}x + \dfrac{3}{2}$.



99. $f(x)$ is one-to-one. $f(x) = y = \sqrt[3]{x-2}$.
Interchange the variables and solve for y:
$x = \sqrt[3]{y-2} \Rightarrow y = f^{-1}(x) = x^3 + 2$.



100. $f(x)$ is one-to-one. $f(x) = y = 8x^3 - 1$.
Interchange the variables and solve for y:
$x = 8y^3 - 1 \Rightarrow y = \sqrt[3]{\dfrac{x+1}{8}} \Rightarrow$
$y = f^{-1}(x) = \dfrac{1}{2}\sqrt[3]{x+1}$.



101. $f(x) = y = 4 + \sqrt{x-1}$. Interchange the variables and solve for y: $x = 4 + \sqrt{y-1} \Rightarrow$
$x - 4 = \sqrt{y-1} \Rightarrow (x-4)^2 = y - 1 \Rightarrow$
$f^{-1} = y = x^2 - 8x + 17$
Domain of f: $[1, \infty)$; range of f: $[4, \infty)$.
Note that the inverse function is defined for $x \geq 4$.

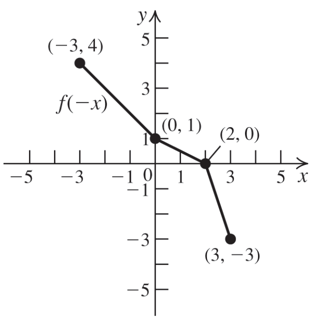
102. $f(x) = y = -3 + \sqrt{x+2}$. Interchange the variables and solve for y: $x = -3 + \sqrt{y+2} \Rightarrow$
$x + 3 = \sqrt{y+2} \Rightarrow (x+3)^2 = y + 2 \Rightarrow$
$f^{-1} = y = x^2 + 6x + 7$
Domain of f: $[-2, \infty)$; range of f: $[-3, \infty)$.
Note that the inverse function is defined for $x \geq -3$.

103. a. $A=(-3,-3), B=(-2,0), C=(0,1), D=(3,4)$.
Find the equation of each segment:
$$m_{AB}=\frac{0-(-3)}{-2-(-3)}=3.\ 0=3(-2)+b\Rightarrow b=6.$$
The equation of AB is $y=3x+6$.
$$m_{BC}=\frac{1-0}{0-(-2)}=\frac{1}{2}; b=1.$$ The equation of BC is $y=\frac{1}{2}x+1$.
$$m_{CD}=\frac{4-1}{3-0}=1; b=1.$$ The equation of CD is $y=x+1$.
$$\text{So, } f(x)=\begin{cases}3x+6 & \text{if } -3\le x\le -2\\ \frac{1}{2}x+1 & \text{if } -2<x<0\\ x+1 & \text{if } 0\le x\le 3\end{cases}$$

b. Domain: [–3,3]; range: [–3,4]

c. x-intercept: –2; y-intercept: 1

d.



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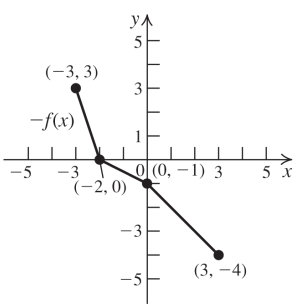

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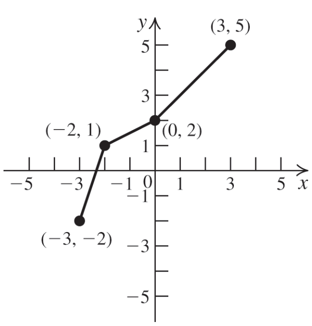

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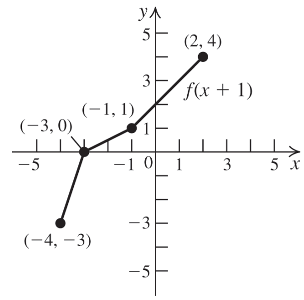

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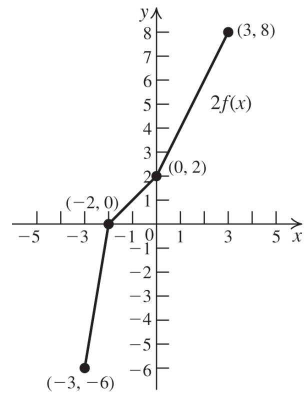

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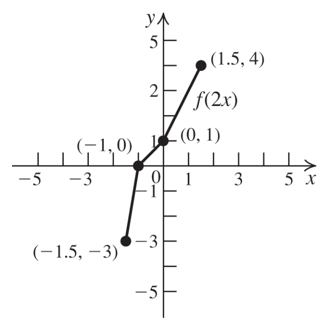

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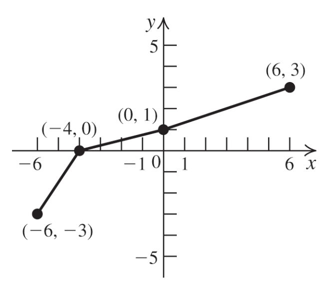
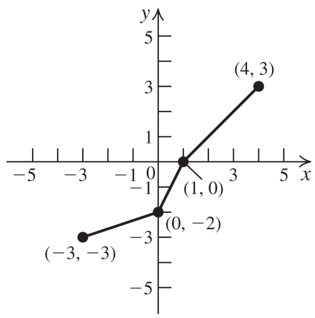


k. f is one-to-one because it satisfies the horizontal line test.

l.



Applying the Concepts

104. a. rate of change (slope)
$= \dfrac{25.95-19.2}{25-10} = 0.45.$
$19.2 = 0.45(10) + b \Rightarrow b = 14.7.$ The equation is $P = 0.45d + 14.7.$

b. The slope represents the amount of increase in pressure (in pounds per square inch) as the diver descends one foot deeper. The y-intercept represents the pressure at the surface of the sea.

c. $P = 0.45(160) + 14.7 = 86.7$ lb/in.^{2}

d. $104.7 = 0.45d + 14.7 \Rightarrow 200$ feet

105. a. rate of change (slope) $= \dfrac{173{,}000 - 54{,}000}{223{,}000 - 87{,}000}$
$= 0.875.$
$54{,}000 = 0.875(87{,}000) + b \Rightarrow$
$b = -22{,}125.$ The equation is
$C = 0.875w - 22{,}125.$

b. The slope represents the cost to dispose of one pound of waste. The x-intercept represents the amount of waste that can be disposed with no cost. The y-intercept represents the fixed cost.

c. $C = 0.875(609{,}000) - 22{,}125 = \$510{,}750$

d. $1{,}000{,}000 = 0.875w - 22{,}125 \Rightarrow$
$w = 1{,}168{,}142.86$ pounds

106. a. At 60 mph = 1 mile per minute, so if the speedometer is correct, the number of minutes elapsed is equal to the number of miles driven.

b. The odometer is based on the speedometer, so if the speedometer is incorrect, so is the odometer.

107. a. $f(2) = 100 + 55(2) - 3(2)^2 = \$198.$ She started with \$100, so she won \$98.

b. She was winning at a rate of \$49/hour.

c. $0 = 100 + 55t - 3t^2 \Rightarrow (-t+20)(3t+5) \Rightarrow$ $t = 20$ or $t = -5/3.$ Since t represents the amount of time, we reject $t = -5/3.$ Chloe will lose all her money after playing for 20 hours.

d. \$100/20 = \$5/hour.

108. If $100 < x \le 500$, then the sales price per case is \$4 – 0.2(4) = \$3.20. The first 100 cases cost \$400.

$$f(x) = \begin{cases} 4x & \text{if } 0 \le x \le 100 \\ 3.2x + 80 & \text{if } 100 < x \le 500 \\ 3x + 180 & \text{if } x > 500 \end{cases}$$

109. a. $(L \circ x)(t) = 0.5\sqrt{\left(1 + 0.002t^2\right)^2 + 4}$
$= 0.5\sqrt{0.000004t^4 + 0.004t^2 + 5}$

b. $(L \circ x)(5) = 0.5\sqrt{\left(1 + 0.002(5^2)\right)^2 + 4}$
$= 0.5\sqrt{(1.05)^2 + 4} = 0.5\sqrt{5.1025}$
≈ 1.13

110. a. Revenue = number of units × price per unit:
$x \cdot p = (5000 + 50t + 10t^2)(10 + 0.5t)$
$= 5t^3 + 125t^2 + 3000t + 50{,}000$

b. $p = 10 + 0.5t \Rightarrow t = 2p - 20.$
$x(t) = x(2p - 20)$
$= 5000 + 50(2p - 20) + 10(2p - 20)^2$
$= 40p^2 - 700p + 8000,$ which is the number of toys made at price p. The revenue is $p\left(40p^2 - 700p + 8000\right) =$
$40p^3 - 700p^2 + 8000p.$

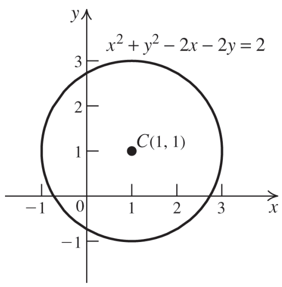
Chapter 1 Practice Test A

1. To test if the graph is symmetric with respect to the y-axis, replace x with $-x$:
$3(-x) + 2(-x)y^2 = 1 \Rightarrow -3x - 2xy^2 = 1,$ which is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. To test if the graph is symmetric with respect to the x-axis, replace y with $-y$:

$3x+2x(-y)^2=1 \Rightarrow 3x+2xy^2=1,$ which is the same as the original equation, so the graph is symmetric with respect to the x-axis.
To test if the graph is symmetric with respect to the origin, replace x with $-x$ and y with $-y$:
$3(-x)+2(-x)(-y)^2=1 \Rightarrow -3x-2xy^2=1,$
which is not the same as the original equation, so the graph is not symmetric with respect to the origin.

2. $0=x^2(x-3)(x+1) \Rightarrow x=0$ or $x=3$ or $x=-1$
$y=0^2(0-3)(0+1) \Rightarrow y=0.$ The x-intercepts are 0, 3, and –1; the y-intercept is 0.

3.



Intercepts:
$y^2-2y=2 \Rightarrow y=1\pm\sqrt{3}$
$x^2-2x=2 \Rightarrow x=1\pm\sqrt{3}$

4. $7=-1(2)+b \Rightarrow 9=b.$
The equation is $y=-x+9.$

5. $8x-2y=7 \Rightarrow y=4x-\dfrac{7}{2} \Rightarrow$ the slope of the line is 4. $-1=4(2)+b \Rightarrow b=-9.$
So the equation is $y=4x-9.$

6. $(fg)(2)=f(2)\cdot g(2)=(-2(2)+1)\left(2^2+3(2)+2\right)$
$=(-3)(12)=-36$

7. $g(f(2))=g(2(2)-3)=g(1)=1-2(1)^2=-1$

8. $(f\circ f)(x)=(x^2-2x)^2-2(x^2-2x)$
$=x^4-4x^3+4x^2-2x^2+4x$
$=x^4-4x^3+2x^2+4x$

9. a. $f(-1)=(-1)^3-2=-3$

b. $f(0)=0^3-2=-2$

c. $f(1)=1-2(1)^2=-1$

10. $1-x>0 \Rightarrow x<1; x$ must also be greater than or equal to 0, so the domain is [0, 1).

11. $x^2+x-6\ge 0 \Rightarrow (x+3)(x-2)\ge 0.$ Test the intervals $(-\infty,-3),(-3,2),$ and $(2,\infty).$ The inequality is true for $(-\infty,-3]$ and $[2,\infty),$ so the domain is $(-\infty,-3]\cup[2,\infty).$

12. $\dfrac{f(4)-f(1)}{4-1}=\dfrac{(2(4)+7)-(2(1)+7)}{3}=2$

13. $f(-x)=2(-x)^4-\dfrac{3}{(-x)^2}=2x^4-\dfrac{3}{x^2}=f(x)\Rightarrow$
$f(x)$ is even.

14. Increasing on $(-\infty,0)$ and $(2,\infty);$ decreasing on (0, 2).

15. Shift the graph of f three units to the right.

16. $25=25-(2t-5)^2 \Rightarrow 0=-(2t-5)^2 \Rightarrow$
$0=2t-5 \Rightarrow t=\dfrac{5}{2}=2.5$ seconds

17. 2

18. $f(x)=y=1+\dfrac{1}{x}.$ Interchange the variables and solve for y: $x=1+\dfrac{1}{y} \Rightarrow xy=y+1 \Rightarrow$
$xy-y=1 \Rightarrow y(x-1)=1 \Rightarrow$
$y=f^{-1}(x)=\dfrac{1}{x-1}$

19. $A(x)=100x+1000$

20. a. $C(230)=0.25(230)+30=\$87.50$

b. $57.50=0.25m+30 \Rightarrow m=110$ miles

Chapter 1 Practice Test B

1. To test if the graph is symmetric with respect to the y-axis, replace x with $-x$:
$|-x|+2|y|=2 \Rightarrow |x|+2|y|=2,$ which is the same as the original equation, so the graph is symmetric with respect to the y-axis. To test if the graph is symmetric with respect to the x-axis, replace y with $-y$:
$|x|+2|-y|=2 \Rightarrow |x|+2|y|=2,$ which is the same as the original equation, so the graph is symmetric with respect to the x-axis. To test if the graph is symmetric with respect to the origin, replace x with $-x,$ and y with $-y$:
$|-x|+2|-y|=2 \Rightarrow |x|+2|y|=2,$ which is the same as the original equation, so the graph is symmetric with respect to the origin. The answer is D.

2. $0 = x^2 - 9 \Rightarrow x = \pm 3; y = 0^2 + 9 \Rightarrow y = 9.$ The x-intercepts are ±3; the y-intercept is 9. The answer is B.

3. D **4.** D **5.** C

6. Suppose the coordinates of the second point are (a, b). Then $-\frac{1}{2} = \frac{b-2}{a-3}$. Substitute each of the points given into this equation to see which makes it true. The answer is B.

7. Find the slope of the original line: $6x - 3y = 5 \Rightarrow y = 2x - \frac{5}{3}$. The slope is 2. The equation of the line with slope 2, passing through (–1, 2) is $y - 2 = 2(x+1)$. The answer is D.

8. $(f \circ g)(x) = 3\left(2 - x^2\right) - 5 = 1 - 3x^2.$
The answer is B.

9. $(f \circ f)(x) = 2\left(2x^2 - x\right)^2 - \left(2x^2 - x\right)$
$= 8x^4 - 8x^3 + x.$ The answer is A.

10. $g(a-1) = \frac{1-(a-1)}{1+(a-1)} = \frac{2-a}{a}$. The answer is C.

11. $1 - x \geq 0 \Rightarrow x \leq 1;$ x must also be greater than or equal to 0, so the domain is [0, 1]. The answer is A.

12. $x^2 + 6x - 7 \geq 0 \Rightarrow (x+7)(x-1) \geq 0.$ Test the intervals $(-\infty, -7), (-7, 1),$ and $(1, \infty).$ The inequality is true for $(-\infty, -7]$ and $[1, \infty),$ so the answer is B.

13. A **14.** A **15.** B

16. D **17.** C

18. $f(x) = y = \frac{1-3x}{5+2x}$. Interchange the variables and solve for y: $x = \frac{1-3y}{5+2y} \Rightarrow$
$5x + 2xy = 1 - 3y \Rightarrow 2xy + 3y = 1 - 5x$
$y(2x+3) = 1 - 5x \Rightarrow y = f^{-1}(x) = \frac{1-5x}{2x+3}.$
The answer is C.

19. $w = 5x - 190; w = 5(70) - 190 = 160.$ The answer is B.

20. $50 = 0.2m + 25 \Rightarrow m = 125.$ The answer is A.