

12-1.

Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6) \text{ m/s}^2$, where t is in seconds. What is the particle's velocity when $t = 6 \text{ s}$, and what is its position when $t = 11 \text{ s}$?

SOLUTION

$$a = 2t - 6$$

$$dv = a \, dt$$

$$\int_0^v dv = \int_0^t (2t - 6) \, dt$$

$$v = t^2 - 6t$$

$$ds = v \, dt$$

$$\int_0^s ds = \int_0^t (t^2 - 6t) \, dt$$

$$s = \frac{t^3}{3} - 3t^2$$

When $t = 6 \text{ s}$,

$$v = 0$$

Ans.

When $t = 11 \text{ s}$,

$$s = 80.7 \text{ m}$$

Ans.

Ans:

$$s = 80.7 \text{ m}$$

12-2.

If a particle has an initial velocity of $v_0 = 12 \text{ ft/s}$ to the right, at $s_0 = 0$, determine its position when $t = 10 \text{ s}$, if $a = 2 \text{ ft/s}^2$ to the left.

SOLUTION

$$(\rightarrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$= 0 + 12(10) + \frac{1}{2}(-2)(10)^2$$

$$= 20 \text{ ft}$$

Ans.

Ans:

$$s = 20 \text{ ft}$$

12-3.

A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

SOLUTION

$$v = 12 - 3t^2$$

(1)

$$a = \frac{dv}{dt} = -6t|_{t=4} = -24 \text{ m/s}^2$$

Ans.

$$\int_{-10}^s ds = \int_1^t v dt = \int_1^t (12 - 3t^2) dt$$

$$s + 10 = 12t - t^3 - 11$$

$$s = 12t - t^3 - 21$$

$$s|_{t=0} = -21$$

$$s|_{t=10} = -901$$

$$\Delta s = -901 - (-21) = -880 \text{ m}$$

Ans.

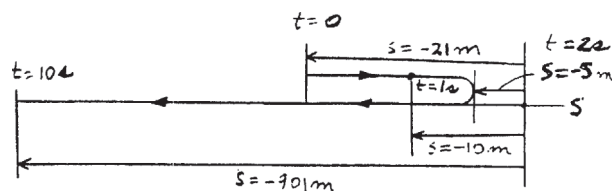
From Eq. (1):

$$v = 0 \text{ when } t = 2 \text{ s}$$

$$s|_{t=2} = 12(2) - (2)^3 - 21 = -5$$

$$s_T = (21 - 5) + (901 - 5) = 912 \text{ m}$$

Ans.



Ans:

$$a = -24 \text{ m/s}^2$$

$$\Delta s = -880 \text{ m}$$

$$s_T = 912 \text{ m}$$

***12–4.**

A particle travels along a straight line with a constant acceleration. When $s = 4$ ft, $v = 3$ ft/s and when $s = 10$ ft, $v = 8$ ft/s. Determine the velocity as a function of position.

SOLUTION

Velocity: To determine the constant acceleration a_c , set $s_0 = 4$ ft, $v_0 = 3$ ft/s, $s = 10$ ft and $v = 8$ ft/s and apply Eq. 12–6.

$$(\pm) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$8^2 = 3^2 + 2a_c(10 - 4)$$

$$a_c = 4.583 \text{ ft/s}^2$$

Using the result $a_c = 4.583 \text{ ft/s}^2$, the velocity function can be obtained by applying Eq. 12–6.

$$(\pm) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v^2 = 3^2 + 2(4.583)(s - 4)$$

$$v = (\sqrt{9.17s - 27.7}) \text{ ft/s}$$

Ans.

Ans:

$$v = (\sqrt{9.17s - 27.7}) \text{ ft/s}$$

12-5.

The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where t is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

SOLUTION

$$v = 6t - 3t^2$$

$$a = \frac{dv}{dt} = 6 - 6t$$

$$\text{At } t = 3 \text{ s}$$

$$a = -12 \text{ m/s}^2$$

$$ds = v dt$$

$$\int_0^s ds = \int_0^t (6t - 3t^2) dt$$

$$s = 3t^2 - t^3$$

$$\text{At } t = 3 \text{ s}$$

$$s = 0$$

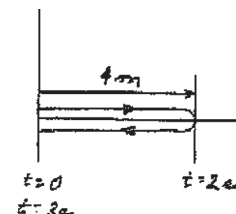
Since $v = 0 = 6t - 3t^2$, when $t = 0$ and $t = 2$ s.

$$\text{when } t = 2 \text{ s, } s = 3(2)^2 - (2)^3 = 4 \text{ m}$$

$$s_T = 4 + 4 = 8 \text{ m}$$

$$(v_{sp})_{\text{avg}} = \frac{s_T}{t} = \frac{8}{3} = 2.67 \text{ m/s}$$

Ans.



Ans.

Ans.

Ans.

Ans:

$$s_T = 8 \text{ m}$$

$$v_{\text{avg}} = 2.67 \text{ m/s}$$

12-6.

The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

SOLUTION

Position: The position of the particle when $t = 6$ s is

$$s|_{t=6s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft} \quad \text{Ans.}$$

Total Distance Traveled: The velocity of the particle can be determined by applying Eq. 12-1.

$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$

The times when the particle stops are

$$4.50t^2 - 27.0t + 22.5 = 0$$

$$t = 1 \text{ s} \quad \text{and} \quad t = 5 \text{ s}$$

The position of the particle at $t = 0$ s, 1 s and 5 s are

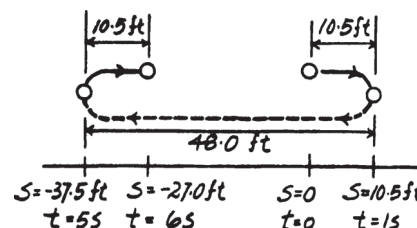
$$s|_{t=0s} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$

$$s|_{t=1s} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ ft}$$

$$s|_{t=5s} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$

From the particle's path, the total distance is

$$s_{\text{tot}} = 10.5 + 48.0 + 10.5 = 69.0 \text{ ft} \quad \text{Ans.}$$



Ans:

$$s|_{t=6s} = -27.0 \text{ ft}$$

$$s_{\text{tot}} = 69.0 \text{ ft}$$

12-7.

A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.

SOLUTION

$$s = t^2 - 6t + 5$$

$$v = \frac{ds}{dt} = 2t - 6$$

$$a = \frac{dv}{dt} = 2$$

$$v = 0 \text{ when } t = 3$$

$$s|_{t=0} = 5$$

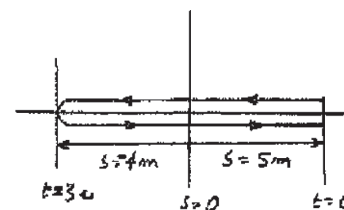
$$s|_{t=3} = -4$$

$$s|_{t=6} = 5$$

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{0}{6} = 0$$

$$(v_{\text{sp}})_{\text{avg}} = \frac{s_T}{\Delta t} = \frac{9 + 9}{6} = 3 \text{ m/s}$$

$$a|_{t=6} = 2 \text{ m/s}^2$$



Ans.

Ans.

Ans.

Ans:

$$v_{\text{avg}} = 0$$

$$(v_{\text{sp}})_{\text{avg}} = 3 \text{ m/s}$$

$$a|_{t=6 \text{ s}} = 2 \text{ m/s}^2$$

***12–8.**

A particle is moving along a straight line such that its position is defined by $s = (10t^2 + 20)$ mm, where t is in seconds. Determine (a) the displacement of the particle during the time interval from $t = 1$ s to $t = 5$ s, (b) the average velocity of the particle during this time interval, and (c) the acceleration when $t = 1$ s.

SOLUTION

$$s = 10t^2 + 20$$

$$(a) \quad s|_{1\text{ s}} = 10(1)^2 + 20 = 30 \text{ mm}$$

$$s|_{5\text{ s}} = 10(5)^2 + 20 = 270 \text{ mm}$$

$$\Delta s = 270 - 30 = 240 \text{ mm}$$

Ans.

$$(b) \quad \Delta t = 5 - 1 = 4 \text{ s}$$

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{240}{4} = 60 \text{ mm/s}$$

Ans.

$$(c) \quad a = \frac{d^2s}{dt^2} = 20 \text{ mm/s}^2 \quad (\text{for all } t)$$

Ans.

Ans:

$$\Delta s = 240 \text{ mm}$$

$$v_{avg} = 60 \text{ mm/s}$$

$$a = 20 \text{ mm/s}^2$$

12-9.

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1) \text{ m/s}^2$, where t is in seconds. If $s = 1 \text{ m}$ and $v = 2 \text{ m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6 \text{ s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$$a = 2t - 1$$

$$dv = a dt$$

$$\int_2^v dv = \int_0^t (2t - 1) dt$$

$$v = t^2 - t + 2$$

$$dx = v dt$$

$$\int_1^s ds = \int_0^t (t^2 - t + 2) dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$$

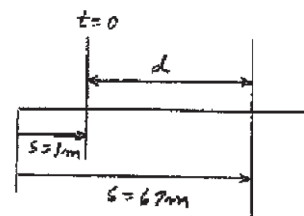
When $t = 6 \text{ s}$

$$v = 32 \text{ m/s}$$

$$s = 67 \text{ m}$$

Since $v \neq 0$ for $0 \leq t \leq 6 \text{ s}$, then

$$d = 67 - 1 = 66 \text{ m}$$



Ans.

Ans.

Ans.

Ans:
 $v = 32 \text{ m/s}$
 $s = 67 \text{ m}$
 $d = 66 \text{ m}$

12–10.

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s², where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use a numerical method to evaluate the integral.

SOLUTION

$$a = \frac{5}{(3s^{1/3} + s^{5/2})}$$

$$a \, ds = v \, dv$$

$$\int_1^2 \frac{5 \, ds}{(3s^{1/3} + s^{5/2})} = \int_0^v v \, dv$$

$$0.8351 = \frac{1}{2} v^2$$

$$v = 1.29 \text{ m/s}$$

Ans.

Ans:
 $v = 1.29 \text{ m/s}$

12–11.

A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8$ m to a position $s_B = +3$ m. Then in another 5 s it moves from s_B to $s_C = -6$ m. Determine the particle's average velocity and average speed during the 9-s time interval.

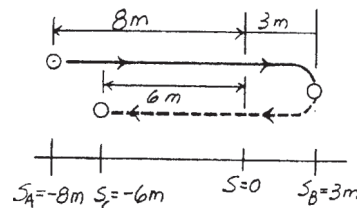
SOLUTION

Average Velocity: The displacement from A to C is $\Delta s = s_C - s_A = -6 - (-8) = 2$ m.

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{2}{4 + 5} = 0.222 \text{ m/s} \quad \text{Ans.}$$

Average Speed: The distances traveled from A to B and B to C are $s_{A \rightarrow B} = 8 + 3 = 11.0$ m and $s_{B \rightarrow C} = 3 + 6 = 9.00$ m, respectively. Then, the total distance traveled is $s_{\text{Tot}} = s_{A \rightarrow B} + s_{B \rightarrow C} = 11.0 + 9.00 = 20.0$ m.

$$(v_{\text{sp}})_{\text{avg}} = \frac{s_{\text{Tot}}}{\Delta t} = \frac{20.0}{4 + 5} = 2.22 \text{ m/s} \quad \text{Ans.}$$



Ans:

$$v_{\text{avg}} = 0.222 \text{ m/s}$$

$$(v_{\text{sp}})_{\text{avg}} = 2.22 \text{ m/s}$$

***12–12.**

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

SOLUTION

$$v = v_1 + a_c t$$

$$120 = 70 + 6000(t)$$

$$t = 8.33(10^{-3}) \text{ hr} = 30 \text{ s}$$

Ans.

$$v^2 = v_1^2 + 2 a_c(s - s_1)$$

$$(120)^2 = 70^2 + 2(6000)(s - 0)$$

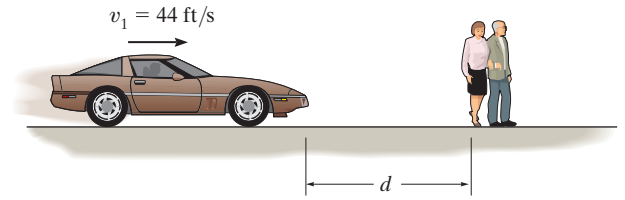
$$s = 0.792 \text{ km} = 792 \text{ m}$$

Ans.

Ans:
 $t = 30 \text{ s}$
 $s = 792 \text{ m}$

12–13.

Tests reveal that a normal driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2 , determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral:* If you must drink, please don't drive!



SOLUTION

Stopping Distance: For normal driver, the car moves a distance of $d' = vt = 44(0.75) = 33.0 \text{ ft}$ before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 33.0 \text{ ft}$ and $v = 0$.

$$\begin{aligned} \left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad v^2 &= v_0^2 + 2a_c(s - s_0) \\ 0^2 &= 44^2 + 2(-2)(d - 33.0) \\ d &= 517 \text{ ft} \end{aligned}$$

Ans.

For a drunk driver, the car moves a distance of $d' = vt = 44(3) = 132 \text{ ft}$ before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 132 \text{ ft}$ and $v = 0$.

$$\begin{aligned} \left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad v^2 &= v_0^2 + 2a_c(s - s_0) \\ 0^2 &= 44^2 + 2(-2)(d - 132) \\ d &= 616 \text{ ft} \end{aligned}$$

Ans.

Ans:
Normal: $d = 517 \text{ ft}$
drunk: $d = 616 \text{ ft}$

12–14.

The position of a particle along a straight-line path is defined by $s = (t^3 - 6t^2 - 15t + 7)$ ft, where t is in seconds. Determine the total distance traveled when $t = 10$ s. What are the particle's average velocity, average speed, and the instantaneous velocity and acceleration at this time?

SOLUTION

$$s = t^3 - 6t^2 - 15t + 7$$

$$v = \frac{ds}{dt} = 3t^2 - 12t - 15$$

When $t = 10$ s,

$$v = 165 \text{ ft/s}$$

$$a = \frac{dv}{dt} = 6t - 12$$

When $t = 10$ s,

$$a = 48 \text{ ft/s}^2$$

When $v = 0$,

$$0 = 3t^2 - 12t - 15$$

The positive root is

$$t = 5 \text{ s}$$

When $t = 0$, $s = 7$ ft

When $t = 5$ s, $s = -93$ ft

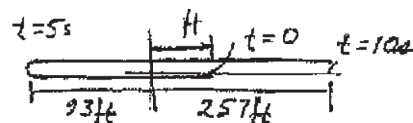
When $t = 10$ s, $s = 257$ ft

Total distance traveled

$$s_T = 7 + 93 + 93 + 257 = 450 \text{ ft}$$

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{257 - 7}{10 - 0} = 25.0 \text{ ft/s}$$

$$(v_{\text{sp}})_{\text{avg}} = \frac{s_T}{\Delta t} = \frac{450}{10} = 45.0 \text{ ft/s}$$



Ans.

Ans.

Ans.

Ans.

Ans.

Ans:

$$v = 165 \text{ ft/s}$$

$$a = 48 \text{ ft/s}^2$$

$$s_T = 450 \text{ ft}$$

$$v_{\text{avg}} = 25.0 \text{ ft/s}$$

$$(v_{\text{sp}})_{\text{avg}} = 45.0 \text{ ft/s}$$

12–15.

A particle is moving with a velocity of v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -kv^3$, where k is a constant, determine its velocity and position as functions of time.

SOLUTION

$$a = \frac{dv}{dt} = -kv^3$$

$$\int_{v_0}^v v^{-3} dv = \int_0^t -k dt$$

$$-\frac{1}{2}(v^{-2} - v_0^{-2}) = -kt$$

$$v = \left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{-\frac{1}{2}}$$

Ans.

$$ds = v dt$$

$$\int_0^s ds = \int_0^t \frac{dt}{\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}}}$$

$$s = \frac{2\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}}}{2k} \Bigg|_0^t$$

$$s = \frac{1}{k} \left[\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}} - \frac{1}{v_0} \right]$$

Ans.

Ans:

$$v = \left(2kt + \frac{1}{v_0^2}\right)^{-1/2}$$

$$s = \frac{1}{k} \left[\left(2kt + \frac{1}{v_0^2}\right)^{1/2} - \frac{1}{v_0} \right]$$

***12–16.**

A particle is moving along a straight line with an initial velocity of 6 m/s when it is subjected to a deceleration of $a = (-1.5v^{1/2}) \text{ m/s}^2$, where v is in m/s. Determine how far it travels before it stops. How much time does this take?

SOLUTION

Distance Traveled: The distance traveled by the particle can be determined by applying Eq. 12–3.

$$\begin{aligned} ds &= \frac{v dv}{a} \\ \int_0^s ds &= \int_{6 \text{ m/s}}^v \frac{v}{-1.5v^{1/2}} dv \\ s &= \int_{6 \text{ m/s}}^v -0.6667 v^{1/2} dv \\ &= \left(-0.4444v^{3/2} + 6.532 \right) \text{ m} \end{aligned}$$

When $v = 0$, $s = -0.4444 \left(0^{3/2} \right) + 6.532 = 6.53 \text{ m}$ **Ans.**

Time: The time required for the particle to stop can be determined by applying Eq. 12–2.

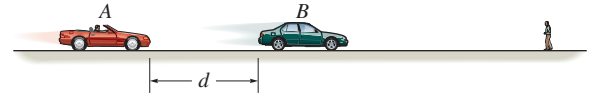
$$\begin{aligned} dt &= \frac{dv}{a} \\ \int_0^t dt &= - \int_{6 \text{ m/s}}^v \frac{dv}{1.5v^{1/2}} \\ t &= -1.333 \left(v^{1/2} \right) \Big|_{6 \text{ m/s}}^v = \left(3.266 - 1.333v^{1/2} \right) \text{ s} \end{aligned}$$

When $v = 0$, $t = 3.266 - 1.333 \left(0^{1/2} \right) = 3.27 \text{ s}$ **Ans.**

Ans:
 $s = 6.53 \text{ m}$
 $t = 3.27 \text{ s}$

12–17.

Car B is traveling a distance d ahead of car A . Both cars are traveling at 60 ft/s when the driver of B suddenly applies the brakes, causing his car to decelerate at 12 ft/s². It takes the driver of car A 0.75 s to react (this is the normal reaction time for drivers). When he applies his brakes, he decelerates at 15 ft/s². Determine the minimum distance d between the cars so as to avoid a collision.



SOLUTION

For B :

$$(\pm) \quad v = v_0 + a_c t$$

$$v_B = 60 - 12 t$$

$$(\pm) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_B = d + 60t - \frac{1}{2} (12) t^2 \quad (1)$$

For A :

$$(\pm) \quad v = v_0 + a_c t$$

$$v_A = 60 - 15(t - 0.75), \quad [t > 0.75]$$

$$(\pm) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 60(0.75) + 60(t - 0.75) - \frac{1}{2} (15) (t - 0.75)^2, \quad [t > 0.75] \quad (2)$$

Require $v_A = v_B$ the moment of closest approach.

$$60 - 12t = 60 - 15(t - 0.75)$$

$$t = 3.75 \text{ s}$$

Worst case without collision would occur when $s_A = s_B$.

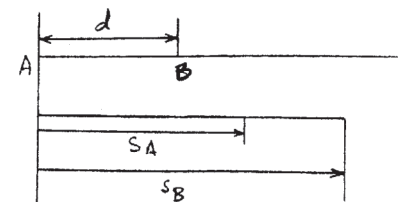
At $t = 3.75$ s, from Eqs. (1) and (2):

$$60(0.75) + 60(3.75 - 0.75) - 7.5(3.75 - 0.75)^2 = d + 60(3.75) - 6(3.75)^2$$

$$157.5 = d + 140.625$$

$$d = 16.9 \text{ ft}$$

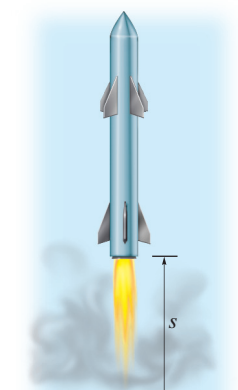
Ans.



Ans:
 $d = 16.9 \text{ ft}$

12–18.

The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s) \text{ m/s}^2$, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100 \text{ m}$. Initially, $v = 0$ and $s = 0$ when $t = 0$.



SOLUTION

$$a \, ds = v \, dv$$

$$\int_0^s (6 + 0.02s) \, ds = \int_0^v v \, dv$$

$$6s + 0.01s^2 = \frac{1}{2}v^2$$

$$v = \sqrt{12s + 0.02s^2}$$

$$ds = v \, dt$$

$$\int_0^{100} \frac{ds}{\sqrt{12s + 0.02s^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{0.02}} \ln \left[\sqrt{12s + 0.02s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}} \right]_0^{100} = t$$

$$t = 5.62 \text{ s}$$

Ans.

Ans:
 $t = 5.62 \text{ s}$

12–19.

A train starts from rest at station *A* and accelerates at 0.5 m/s^2 for 60 s. Afterwards it travels with a constant velocity for 15 min. It then decelerates at 1 m/s^2 until it is brought to rest at station *B*. Determine the distance between the stations.

SOLUTION

Kinematics: For stage (1) motion, $v_0 = 0$, $s_0 = 0$, $t = 60 \text{ s}$, and $a_c = 0.5 \text{ m/s}^2$. Thus,

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(60^2) = 900 \text{ m}$$

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad v = v_0 + a_c t$$

$$v_1 = 0 + 0.5(60) = 30 \text{ m/s}$$

For stage (2) motion, $v_0 = 30 \text{ m/s}$, $s_0 = 900 \text{ m}$, $a_c = 0$ and $t = 15(60) = 900 \text{ s}$. Thus,

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_2 = 900 + 30(900) + 0 = 27\,900 \text{ m}$$

For stage (3) motion, $v_0 = 30 \text{ m/s}$, $v = 0$, $s_0 = 27\,900 \text{ m}$ and $a_c = -1 \text{ m/s}^2$. Thus,

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad v = v_0 + a_c t$$

$$0 = 30 + (-1)t$$

$$t = 30 \text{ s}$$

$$\begin{array}{c} + \\ \rightarrow \end{array} \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_3 = 27\,900 + 30(30) + \frac{1}{2}(-1)(30^2)$$

$$= 28\,350 \text{ m} = 28.4 \text{ km}$$

Ans.

Ans:
 $s = 28.4 \text{ km}$

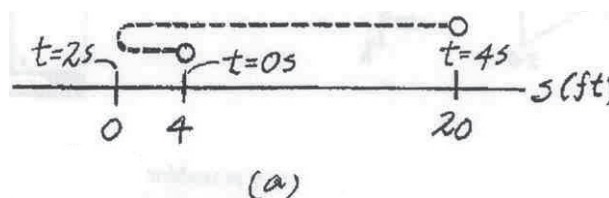
***12–20.**

The velocity of a particle traveling along a straight line is $v = (3t^2 - 6t)$ ft/s, where t is in seconds. If $s = 4$ ft when $t = 0$, determine the position of the particle when $t = 4$ s. What is the total distance traveled during the time interval $t = 0$ to $t = 4$ s? Also, what is the acceleration when $t = 2$ s?

SOLUTION

Position: The position of the particle can be determined by integrating the kinematic equation $ds = v dt$ using the initial condition $s = 4$ ft when $t = 0$ s. Thus,

$$\begin{aligned} \left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad ds &= v dt \\ \int_{4 \text{ ft}}^s ds &= \int_0^t (3t^2 - 6t) dt \\ s \Big|_{4 \text{ ft}}^s &= (t^3 - 3t^2) \Big|_0^t \\ s &= (t^3 - 3t^2 + 4) \text{ ft} \end{aligned}$$



When $t = 4$ s,

$$s|_{4 \text{ s}} = 4^3 - 3(4^2) + 4 = 20 \text{ ft} \quad \text{Ans.}$$

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. Thus,

$$\begin{aligned} v &= 3t^2 - 6t = 0 \\ t(3t - 6) &= 0 \\ t &= 0 \text{ and } t = 2 \text{ s} \end{aligned}$$

The position of the particle at $t = 0$ and 2 s is

$$\begin{aligned} s|_{0 \text{ s}} &= 0 - 3(0^2) + 4 = 4 \text{ ft} \\ s|_{2 \text{ s}} &= 2^3 - 3(2^2) + 4 = 0 \end{aligned}$$

Using the above result, the path of the particle shown in Fig. *a* is plotted. From this figure,

$$s_{\text{Tot}} = 4 + 20 = 24 \text{ ft} \quad \text{Ans.}$$

Acceleration:

$$\begin{aligned} \left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad a &= \frac{dv}{dt} = \frac{d}{dt} (3t^2 - 6t) \\ a &= (6t - 6) \text{ ft/s}^2 \end{aligned}$$

When $t = 2$ s,

$$a|_{t=2 \text{ s}} = 6(2) - 6 = 6 \text{ ft/s}^2 \rightarrow \quad \text{Ans.}$$

Ans:

$$\begin{aligned} s_{\text{Tot}} &= 24 \text{ ft} \\ a|_{t=2 \text{ s}} &= 6 \text{ ft/s}^2 \rightarrow \end{aligned}$$

12–21.

A freight train travels at $v = 60(1 - e^{-t})$ ft/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.



SOLUTION

$$v = 60(1 - e^{-t})$$

$$\int_0^s ds = \int v dt = \int_0^3 60(1 - e^{-t}) dt$$

$$s = 60(t + e^{-t}) \Big|_0^3$$

$$s = 123 \text{ ft}$$

Ans.

$$a = \frac{dv}{dt} = 60(e^{-t})$$

$$\text{At } t = 3 \text{ s}$$

$$a = 60e^{-3} = 2.99 \text{ ft/s}^2$$

Ans.

Ans:
 $s = 123 \text{ ft}$
 $a = 2.99 \text{ ft/s}^2$

12–22.

A sandbag is dropped from a balloon which is ascending vertically at a constant speed of 6 m/s. If the bag is released with the same upward velocity of 6 m/s when $t = 0$ and hits the ground when $t = 8$ s, determine the speed of the bag as it hits the ground and the altitude of the balloon at this instant.

SOLUTION

$$(+\downarrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$\begin{aligned} h &= 0 + (-6)(8) + \frac{1}{2}(9.81)(8)^2 \\ &= 265.92 \text{ m} \end{aligned}$$

During $t = 8$ s, the balloon rises

$$h' = vt = 6(8) = 48 \text{ m}$$

$$\text{Altitude} = h + h' = 265.92 + 48 = 314 \text{ m}$$

Ans.

$$(+\downarrow) \quad v = v_0 + a_c t$$

$$v = -6 + 9.81(8) = 72.5 \text{ m/s}$$

Ans.

Ans:
 $h = 314 \text{ m}$
 $v = 72.5 \text{ m/s}$

12–23.

A particle is moving along a straight line such that its acceleration is defined as $a = (-2v) \text{ m/s}^2$, where v is in meters per second. If $v = 20 \text{ m/s}$ when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

SOLUTION

$$a = -2v$$

$$\frac{dv}{dt} = -2v$$

$$\int_{20}^v \frac{dv}{v} = \int_0^t -2 dt$$

$$\ln \frac{v}{20} = -2t$$

$$v = (20e^{-2t}) \text{ m/s}$$

Ans.

$$a = \frac{dv}{dt} = (-40e^{-2t}) \text{ m/s}^2$$

Ans.

$$\int_0^s ds = v dt = \int_0^t (20e^{-2t}) dt$$

$$s = -10e^{-2t} \Big|_0^t = -10(e^{-2t} - 1)$$

$$s = 10(1 - e^{-2t}) \text{ m}$$

Ans.

Ans:

$$v = (20e^{-2t}) \text{ m/s}$$

$$a = (-40e^{-2t}) \text{ m/s}^2$$

$$s = 10(1 - e^{-2t}) \text{ m}$$

***12–24.**

The acceleration of a particle traveling along a straight line is $a = \frac{1}{4}s^{1/2}$ m/s², where s is in meters. If $v = 0$, $s = 1$ m when $t = 0$, determine the particle's velocity at $s = 2$ m.

SOLUTION

Velocity:

$$\begin{aligned} \left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad v \, dv &= a \, ds \\ \int_0^v v \, dv &= \int_1^s \frac{1}{4}s^{1/2} ds \\ \left. \frac{v^2}{2} \right|_0^v &= \left. \frac{1}{6}s^{3/2} \right|_1^s \\ v &= \frac{1}{\sqrt{3}}(s^{3/2} - 1)^{1/2} \text{ m/s} \end{aligned}$$

When $s = 2$ m, $v = 0.781$ m/s.

Ans.

Ans:
 $v = 0.781$ m/s

12–25.

If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s², where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

SOLUTION

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \quad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81} \left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)} \right]$$

$$9.81t = 50 \ln \left(\frac{1 + 0.01v}{1 - 0.01v} \right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \quad (1)$$

a) When $t = 5$ s, then, from Eq. (1)

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5 \text{ m/s} \quad \text{Ans.}$$

b) If $t \rightarrow \infty$, $\frac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \rightarrow 1$. Then, from Eq. (1)

$$v_{\max} = 100 \text{ m/s} \quad \text{Ans.}$$

Ans:

(a) $v = 45.5 \text{ m/s}$

(b) $v_{\max} = 100 \text{ m/s}$

12-26.

The acceleration of a particle along a straight line is defined by $a = (2t - 9) \text{ m/s}^2$, where t is in seconds. At $t = 0$, $s = 1 \text{ m}$ and $v = 10 \text{ m/s}$. When $t = 9 \text{ s}$, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

SOLUTION

$$a = 2t - 9$$

$$\int_{10}^v dv = \int_0^t (2t - 9) dt$$

$$v - 10 = t^2 - 9t$$

$$v = t^2 - 9t + 10$$

$$\int_1^s ds = \int_0^t (t^2 - 9t + 10) dt$$

$$s - 1 = \frac{1}{3}t^3 - 4.5t^2 + 10t$$

$$s = \frac{1}{3}t^3 - 4.5t^2 + 10t + 1$$

Note when $v = t^2 - 9t + 10 = 0$:

$$t = 1.298 \text{ s and } t = 7.701 \text{ s}$$

$$\text{When } t = 1.298 \text{ s, } s = 7.13 \text{ m}$$

$$\text{When } t = 7.701 \text{ s, } s = -36.63 \text{ m}$$

$$\text{When } t = 9 \text{ s, } s = -30.50 \text{ m}$$

$$(a) \quad s = -30.5 \text{ m}$$

Ans.

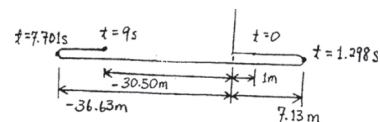
$$(b) \quad s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$$

$$s_{Tot} = 56.0 \text{ m}$$

Ans.

$$(c) \quad v = 10 \text{ m/s}$$

Ans.



Ans:

$$(a) \quad s = -30.5 \text{ m}$$

$$(b) \quad s_{Tot} = 56.0 \text{ m}$$

$$(c) \quad v = 10 \text{ m/s}$$

12–27.

When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f . If this variation of the acceleration can be expressed as $a = (g/v_f^2)(v_f^2 - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

SOLUTION

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)(v_f^2 - v^2)$$

$$\int_0^v \frac{dv}{v_f^2 - v^2} = \frac{g}{v_f^2} \int_0^t dt$$

$$\frac{1}{2v_f} \ln \left(\frac{v_f + v}{v_f - v} \right) \Big|_0^v = \frac{g}{v_f^2} t$$

$$t = \frac{v_f}{2g} \ln \left(\frac{v_f + v}{v_f - v} \right)$$

$$t = \frac{v_f}{2g} \ln \left(\frac{v_f + v_f/2}{v_f - v_f/2} \right)$$

$$t = 0.549 \left(\frac{v_f}{g} \right)$$

Ans.

Ans:

$$t = 0.549 \left(\frac{v_f}{g} \right)$$

***12–28.**

Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3) \text{ ft/s}^2$ and $a_B = (12t^2 - 8) \text{ ft/s}^2$, where t is in seconds. Determine the distance between them when $t = 4 \text{ s}$ and the total distance each has traveled in $t = 4 \text{ s}$.

SOLUTION

Velocity: The velocity of particles A and B can be determined using Eq. 12-2.

$$\begin{aligned} dv_A &= a_A dt \\ \int_0^{v_A} dv_A &= \int_0^t (6t - 3) dt \\ v_A &= 3t^2 - 3t \\ dv_B &= a_B dt \\ \int_0^{v_B} dv_B &= \int_0^t (12t^2 - 8) dt \\ v_B &= 4t^3 - 8t \end{aligned}$$

The times when particle A stops are

$$3t^2 - 3t = 0 \quad t = 0 \text{ s and } t = 1 \text{ s}$$

The times when particle B stops are

$$4t^3 - 8t = 0 \quad t = 0 \text{ s and } t = \sqrt{2} \text{ s}$$

Position: The position of particles A and B can be determined using Eq. 12-1.

$$\begin{aligned} ds_A &= v_A dt \\ \int_0^{s_A} ds_A &= \int_0^t (3t^2 - 3t) dt \\ s_A &= t^3 - \frac{3}{2}t^2 \\ ds_B &= v_B dt \\ \int_0^{s_B} ds_B &= \int_0^t (4t^3 - 8t) dt \\ s_B &= t^4 - 4t^2 \end{aligned}$$

The positions of particle A at $t = 1 \text{ s}$ and 4 s are

$$s_A|_{t=1 \text{ s}} = 1^3 - \frac{3}{2}(1^2) = -0.500 \text{ ft}$$

$$s_A|_{t=4 \text{ s}} = 4^3 - \frac{3}{2}(4^2) = 40.0 \text{ ft}$$

Particle A has traveled

$$d_A = 2(0.5) + 40.0 = 41.0 \text{ ft}$$

Ans.

The positions of particle B at $t = \sqrt{2} \text{ s}$ and 4 s are

$$s_B|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4 \text{ ft}$$

$$s_B|_{t=4} = (4)^4 - 4(4)^2 = 192 \text{ ft}$$

Particle B has traveled

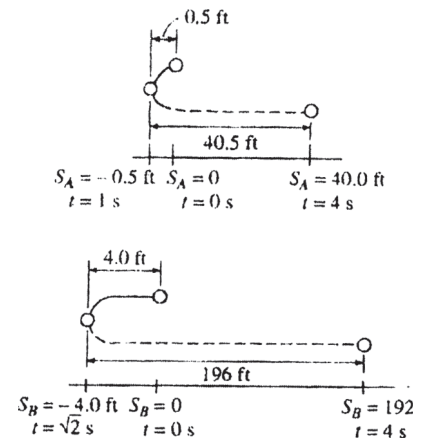
$$d_B = 2(4) + 192 = 200 \text{ ft}$$

Ans.

At $t = 4 \text{ s}$ the distance between A and B is

$$\Delta s_{AB} = 192 - 40 = 152 \text{ ft}$$

Ans.



Ans:

$$d_A = 41.0 \text{ ft}$$

$$d_B = 200 \text{ ft}$$

$$\Delta s_{AB} = 152 \text{ ft}$$

12–29.

A ball *A* is thrown vertically upward from the top of a 30-m-high building with an initial velocity of 5 m/s. At the same instant another ball *B* is thrown upward from the ground with an initial velocity of 20 m/s. Determine the height from the ground and the time at which they pass.

SOLUTION

Origin at roof:

Ball *A*:

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-s = 0 + 5t - \frac{1}{2}(9.81)t^2$$

Ball *B*:

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-s = -30 + 20t - \frac{1}{2}(9.81)t^2$$

Solving,

$$t = 2 \text{ s}$$

Ans.

$$s = 9.62 \text{ m}$$

Distance from ground,

$$d = (30 - 9.62) = 20.4 \text{ m}$$

Ans.

Also, origin at ground,

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 30 + 5t + \frac{1}{2}(-9.81)t^2$$

$$s_B = 0 + 20t + \frac{1}{2}(-9.81)t^2$$

Require

$$s_A = s_B$$

$$30 + 5t + \frac{1}{2}(-9.81)t^2 = 20t + \frac{1}{2}(-9.81)t^2$$

$$t = 2 \text{ s}$$

Ans.

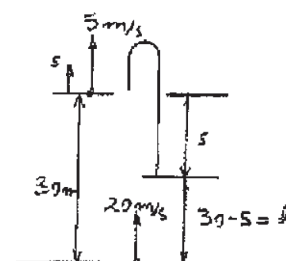
$$s_B = 20.4 \text{ m}$$

Ans.

Ans:

$$h = 20.4 \text{ m}$$

$$t = 2 \text{ s}$$



12–30.

A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t) \text{ m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

SOLUTION

Velocity: $v_0 = 27 \text{ m/s}$ at $t_0 = 0 \text{ s}$. Applying Eq. 12–2, we have

$$\begin{aligned} (+\downarrow) \quad dv &= a dt \\ \int_{27}^v dv &= \int_0^t -6t dt \\ v &= (27 - 3t^2) \text{ m/s} \end{aligned} \quad (1)$$

At $v = 0$, from Eq. (1)

$$0 = 27 - 3t^2 \quad t = 3.00 \text{ s}$$

Distance Traveled: $s_0 = 0 \text{ m}$ at $t_0 = 0 \text{ s}$. Using the result $v = 27 - 3t^2$ and applying Eq. 12–1, we have

$$\begin{aligned} (+\downarrow) \quad ds &= v dt \\ \int_0^s ds &= \int_0^t (27 - 3t^2) dt \\ s &= (27t - t^3) \text{ m} \end{aligned} \quad (2)$$

At $t = 3.00 \text{ s}$, from Eq. (2)

$$s = 27(3.00) - 3.00^3 = 54.0 \text{ m} \quad \textbf{Ans.}$$

Ans:
 $s = 54.0 \text{ m}$

12–31.

The velocity of a particle traveling along a straight line is $v = v_0 - ks$, where k is constant. If $s = 0$ when $t = 0$, determine the position and acceleration of the particle as a function of time.

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SOLUTION

Position:

$$\begin{aligned} (\pm) \quad dt &= \frac{ds}{v} \\ \int_0^t dt &= \int_0^s \frac{ds}{v_0 - ks} \\ t \Big|_0^t &= -\frac{1}{k} \ln(v_0 - ks) \Big|_0^s \\ t &= \frac{1}{k} \ln\left(\frac{v_0}{v_0 - ks}\right) \\ e^{kt} &= \frac{v_0}{v_0 - ks} \\ s &= \frac{v_0}{k} (1 - e^{-kt}) \end{aligned}$$

Ans.

Velocity:

$$\begin{aligned} v &= \frac{ds}{dt} = \frac{d}{dt} \left[\frac{v_0}{k} (1 - e^{-kt}) \right] \\ v &= v_0 e^{-kt} \end{aligned}$$

Acceleration:

$$\begin{aligned} a &= \frac{dv}{dt} = \frac{d}{dt} (v_0 e^{-kt}) \\ a &= -kv_0 e^{-kt} \end{aligned}$$

Ans.

Ans:

$$\begin{aligned} s &= \frac{v_0}{k} (1 - e^{-kt}) \\ a &= -kv_0 e^{-kt} \end{aligned}$$

***12–32.**

Ball A is thrown vertically upwards with a velocity of v_0 . Ball B is thrown upwards from the same point with the same velocity t seconds later. Determine the elapsed time $t < 2v_0/g$ from the instant ball A is thrown to when the balls pass each other, and find the velocity of each ball at this instant.

SOLUTION

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = v_0$, $(s_A)_0 = 0$, $s_A = h$, $t_A = t'$, and $(a_c)_A = -g$.

$$\begin{aligned} (+\uparrow) \quad s_A &= (s_A)_0 + (v_A)_0 t_A + \frac{1}{2}(a_c)_A t_A^2 \\ h &= 0 + v_0 t' + \frac{1}{2}(-g)(t')^2 \\ h &= v_0 t' - \frac{g}{2} t'^2 \end{aligned} \quad (1)$$

$$\begin{aligned} (+\uparrow) \quad v_A &= (v_A)_0 + (a_c)_A t_A \\ v_A &= v_0 + (-g)(t') \\ v_A &= v_0 - gt' \end{aligned} \quad (2)$$

The motion of ball B requires $(v_B)_0 = v_0$, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - t$, and $(a_c)_B = -g$.

$$\begin{aligned} (+\uparrow) \quad s_B &= (s_B)_0 + (v_B)_0 t_B + \frac{1}{2}(a_c)_B t_B^2 \\ h &= 0 + v_0(t' - t) + \frac{1}{2}(-g)(t' - t)^2 \\ h &= v_0(t' - t) - \frac{g}{2}(t' - t)^2 \end{aligned} \quad (3)$$

$$\begin{aligned} (+\uparrow) \quad v_B &= (v_B)_0 + (a_c)_B t_B \\ v_B &= v_0 + (-g)(t' - t) \\ v_B &= v_0 - g(t' - t) \end{aligned} \quad (4)$$

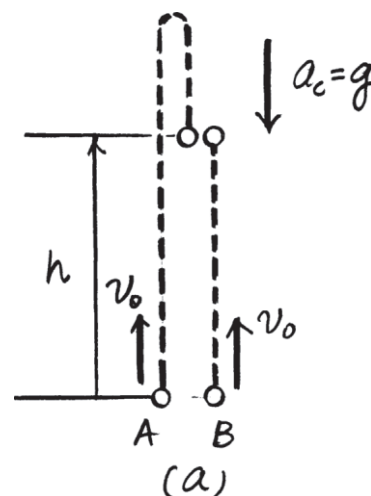
Solving Eqs. (1) and (3),

$$\begin{aligned} v_0 t' - \frac{g}{2} t'^2 &= v_0(t' - t) - \frac{g}{2}(t' - t)^2 \\ t' &= \frac{2v_0 + gt}{2g} \end{aligned} \quad \text{Ans.}$$

Substituting this result into Eqs. (2) and (4),

$$\begin{aligned} v_A &= v_0 - g\left(\frac{2v_0 + gt}{2g}\right) \\ &= -\frac{1}{2}gt = \frac{1}{2}gt \downarrow \end{aligned} \quad \text{Ans.}$$

$$\begin{aligned} v_B &= v_0 - g\left(\frac{2v_0 + gt}{2g} - t\right) \\ &= \frac{1}{2}gt \uparrow \end{aligned} \quad \text{Ans.}$$



$$\begin{aligned} \text{Ans:} \\ t' &= \frac{2v_0 + gt}{2g} \\ v_A &= \frac{1}{2}gt \downarrow \\ v_B &= \frac{1}{2}gt \uparrow \end{aligned}$$

12–33.

As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81 \text{ m/s}^2$ and $R = 6356 \text{ km}$, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

SOLUTION

$$v \, dv = a \, dy$$

$$\int_v^0 v \, dv = -g_0 R^2 \int_0^\infty \frac{dy}{(R + y)^2}$$

$$\frac{v^2}{2} \Big|_v^0 = \frac{g_0 R^2}{R + y} \Big|_0^\infty$$

$$v = \sqrt{2g_0 R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167 \text{ m/s} = 11.2 \text{ km/s}$$

Ans.

Ans:
 $v = 11.2 \text{ km/s}$

12–34.

Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–36), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–36.

SOLUTION

From Prob. 12–36,

$$(+\uparrow) \quad a = -g_0 \frac{R^2}{(R + y)^2}$$

Since $a \, dy = v \, dv$

then

$$-g_0 R^2 \int_{y_0}^y \frac{dy}{(R + y)^2} = \int_0^v v \, dv$$

$$g_0 R^2 \left[\frac{1}{R + y} \right]_{y_0}^y = \frac{v^2}{2}$$

$$g_0 R^2 \left[\frac{1}{R + y} - \frac{1}{R + y_0} \right] = \frac{v^2}{2}$$

Thus

$$v = -R \sqrt{\frac{2g_0(y_0 - y)}{(R + y)(R + y_0)}}$$

Ans.

When $y_0 = 500$ km, $y = 0$,

$$v = -6356(10^3) \sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356 + 500)(10^6)}}$$

$$v = -3016 \text{ m/s} = 3.02 \text{ km/s} \downarrow$$

Ans.

Ans:

$$v = -R \sqrt{\frac{2g_0(y_0 - y)}{(R + y)(R + y_0)}}$$

$$v_{\text{imp}} = 3.02 \text{ km/s}$$

12–35.

A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s^2 . After a time t' it maintains a constant speed so that when $t = 160 \text{ s}$ it has traveled 2000 ft. Determine the time t' and draw the v – t graph for the motion.

SOLUTION

Total Distance Traveled: The distance for part one of the motion can be related to time $t = t'$ by applying Eq. 12–5 with $s_0 = 0$ and $v_0 = 0$.

$$\begin{aligned} (\pm) \quad s &= s_0 + v_0 t + \frac{1}{2} a_c t^2 \\ s_1 &= 0 + 0 + \frac{1}{2} (0.5)(t')^2 = 0.25(t')^2 \end{aligned}$$

The velocity at time t can be obtained by applying Eq. 12–4 with $v_0 = 0$.

$$(\pm) \quad v = v_0 + a_c t = 0 + 0.5t = 0.5t \quad (1)$$

The time for the second stage of motion is $t_2 = 160 - t'$ and the train is traveling at a constant velocity of $v = 0.5t'$ (Eq. (1)). Thus, the distance for this part of motion is

$$(\pm) \quad s_2 = vt_2 = 0.5t'(160 - t') = 80t' - 0.5(t')^2$$

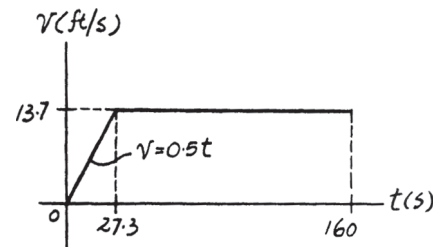
If the total distance traveled is $s_{\text{Tot}} = 2000$, then

$$\begin{aligned} s_{\text{Tot}} &= s_1 + s_2 \\ 2000 &= 0.25(t')^2 + 80t' - 0.5(t')^2 \\ 0.25(t')^2 - 80t' + 2000 &= 0 \end{aligned}$$

Choose a root that is less than 160 s, then

$$t' = 27.34 \text{ s} = 27.3 \text{ s} \quad \text{Ans.}$$

v – t Graph: The equation for the velocity is given by Eq. (1). When $t = t' = 27.34 \text{ s}$, $v = 0.5(27.34) = 13.7 \text{ ft/s}$.



Ans:

$$t' = 27.3 \text{ s.}$$

$$\text{When } t = 27.3 \text{ s, } v = 13.7 \text{ ft/s.}$$

***12–36.**

The s – t graph for a train has been experimentally determined. From the data, construct the v – t and a – t graphs for the motion; $0 \leq t \leq 40$ s. For $0 \leq t \leq 30$ s, the curve is $s = (0.4t^2)$ m, and then it becomes straight for $t \geq 30$ s.

SOLUTION

$0 \leq t \leq 30$:

$$s = 0.4t^2$$

$$v = \frac{ds}{dt} = 0.8t$$

$$a = \frac{dv}{dt} = 0.8$$

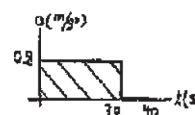
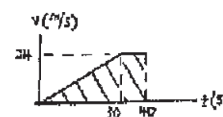
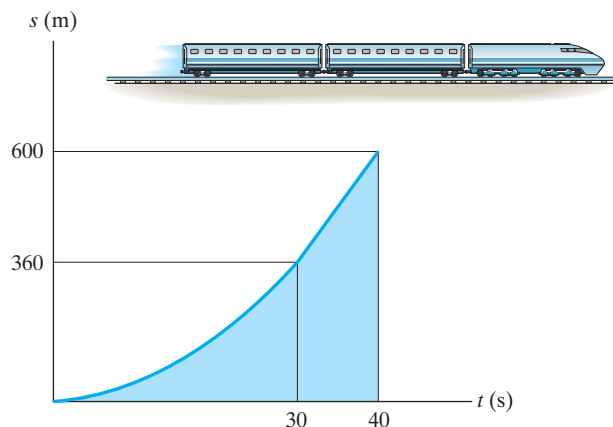
$30 \leq t \leq 40$:

$$s - 360 = \left(\frac{600 - 360}{40 - 30} \right) (t - 30)$$

$$s = 24(t - 30) + 360$$

$$v = \frac{ds}{dt} = 24$$

$$a = \frac{dv}{dt} = 0$$



Ans:

$$s = 0.4t^2$$

$$v = \frac{ds}{dt} = 0.8t$$

$$a = \frac{dv}{dt} = 0.8$$

$$s = 24(t - 30) + 360$$

$$v = \frac{ds}{dt} = 24$$

$$a = \frac{dv}{dt} = 0$$

12-37.

Two rockets start from rest at the same elevation. Rocket *A* accelerates vertically at 20 m/s^2 for 12 s and then maintains a constant speed. Rocket *B* accelerates at 15 m/s^2 until reaching a constant speed of 150 m/s . Construct the a - t , v - t , and s - t graphs for each rocket until $t = 20 \text{ s}$. What is the distance between the rockets when $t = 20 \text{ s}$?

SOLUTION

For rocket *A*

For $t < 12 \text{ s}$

$$+\uparrow v_A = (v_A)_0 + a_A t$$

$$v_A = 0 + 20 t$$

$$v_A = 20 t$$

$$+\uparrow s_A = (s_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(20) t^2$$

$$s_A = 10 t^2$$

When $t = 12 \text{ s}$, $v_A = 240 \text{ m/s}$

$$s_A = 1440 \text{ m}$$

For $t > 12 \text{ s}$

$$v_A = 240 \text{ m/s}$$

$$s_A = 1440 + 240(t - 12)$$

For rocket *B*

For $t < 10 \text{ s}$

$$+\uparrow v_B = (v_B)_0 + a_B t$$

$$v_B = 0 + 15 t$$

$$v_B = 15 t$$

$$+\uparrow s_B = (s_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(15) t^2$$

$$s_B = 7.5 t^2$$

When $t = 10 \text{ s}$, $v_B = 150 \text{ m/s}$

$$s_B = 750 \text{ m}$$

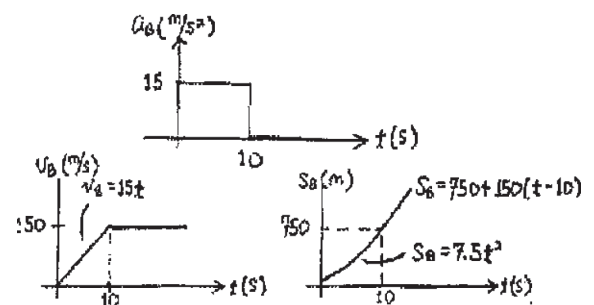
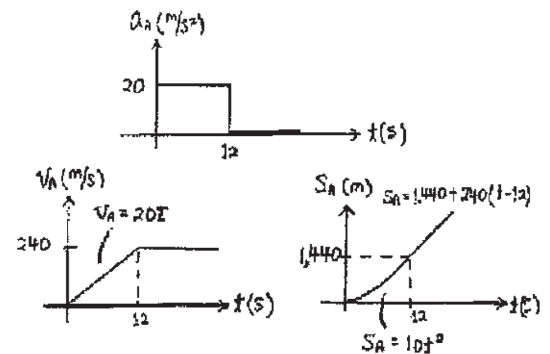
For $t > 10 \text{ s}$

$$v_B = 150 \text{ m/s}$$

$$s_B = 750 + 150(t - 10)$$

When $t = 20 \text{ s}$, $s_A = 3360 \text{ m}$, $s_B = 2250 \text{ m}$

$$\Delta s = 1110 \text{ m} = 1.11 \text{ km}$$



Ans.

Ans:

$$\Delta s = 1.11 \text{ km}$$

12-38.

A particle starts from $s = 0$ and travels along a straight line with a velocity $v = (t^2 - 4t + 3) \text{ m/s}$, where t is in seconds. Construct the $v-t$ and $a-t$ graphs for the time interval $0 \leq t \leq 4 \text{ s}$.

SOLUTION

$a-t$ Graph:

$$a = \frac{dv}{dt} = \frac{d}{dt}(t^2 - 4t + 3)$$

$$a = (2t - 4) \text{ m/s}^2$$

Thus,

$$a|_{t=0} = 2(0) - 4 = -4 \text{ m/s}^2$$

$$a|_{t=2} = 0$$

$$a|_{t=4} = 2(4) - 4 = 4 \text{ m/s}^2$$

The $a-t$ graph is shown in Fig. *a*.

$v-t$ Graph: The slope of the $v-t$ graph is zero when $a = \frac{dv}{dt} = 0$. Thus,

$$a = 2t - 4 = 0 \quad t = 2 \text{ s}$$

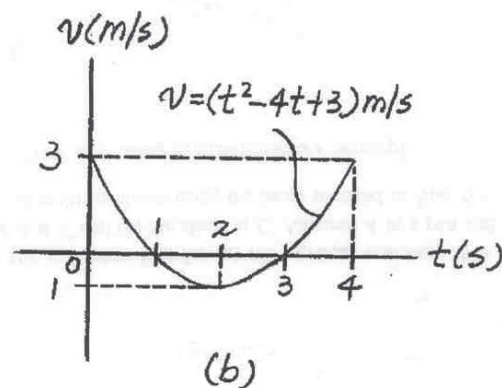
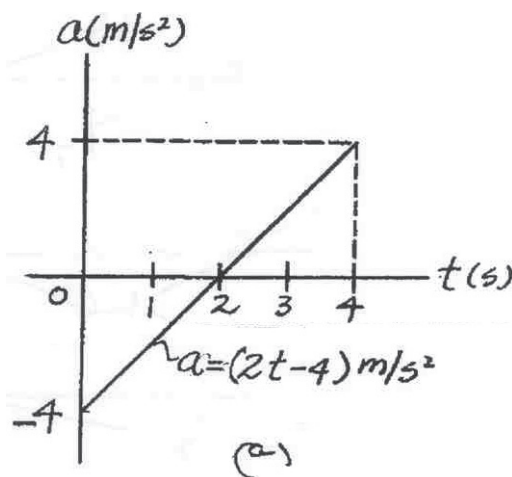
The velocity of the particle at $t = 0 \text{ s}$, 2 s , and 4 s are

$$v|_{t=0} = 0^2 - 4(0) + 3 = 3 \text{ m/s}$$

$$v|_{t=2} = 2^2 - 4(2) + 3 = -1 \text{ m/s}$$

$$v|_{t=4} = 4^2 - 4(4) + 3 = 3 \text{ m/s}$$

The $v-t$ graph is shown in Fig. *b*.



Ans:

$$a|_{t=0} = -4 \text{ m/s}^2$$

$$a|_{t=2} = 0$$

$$a|_{t=4} = 4 \text{ m/s}^2$$

$$v|_{t=0} = 3 \text{ m/s}$$

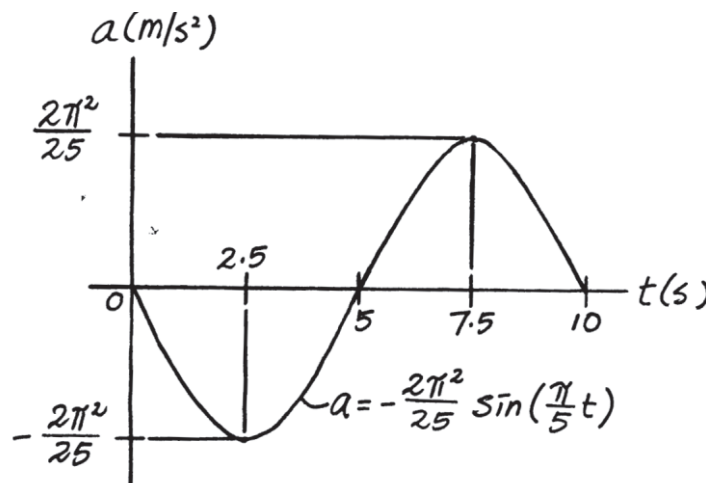
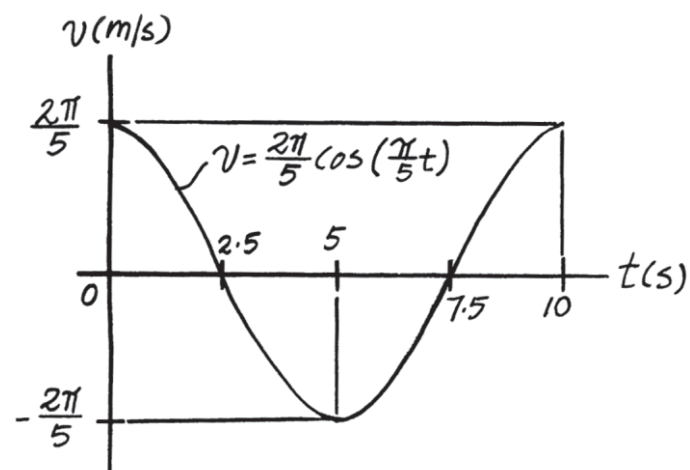
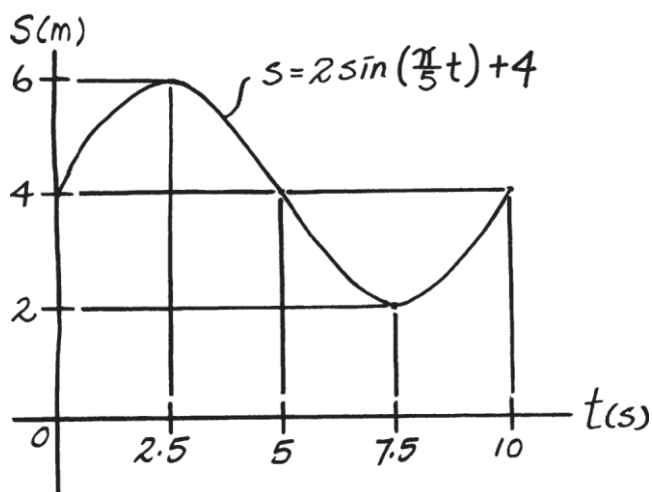
$$v|_{t=2} = -1 \text{ m/s}$$

$$v|_{t=4} = 3 \text{ m/s}$$

12-39.

If the position of a particle is defined by $s = [2 \sin(\pi/5)t + 4]$ m, where t is in seconds, construct the s - t , v - t , and a - t graphs for $0 \leq t \leq 10$ s.

SOLUTION



Ans:

$$s = 2 \sin\left(\frac{\pi}{5}t\right) + 4$$

$$v = \frac{2\pi}{5} \cos\left(\frac{\pi}{5}t\right)$$

$$a = -\frac{2\pi^2}{25} \sin\left(\frac{\pi}{5}t\right)$$

***12–40.**

An airplane starts from rest, travels 5000 ft down a runway, and after uniform acceleration, takes off with a speed of 162 mi/h. It then climbs in a straight line with a uniform acceleration of 3 ft/s^2 until it reaches a constant speed of 220 mi/h. Draw the s - t , v - t , and a - t graphs that describe the motion.

SOLUTION

$$v_1 = 0$$

$$v_2 = 162 \frac{\text{mi}}{\text{h}} \frac{(1 \text{ h}) 5280 \text{ ft}}{(3600 \text{ s})(1 \text{ mi})} = 237.6 \text{ ft/s}$$

$$v_2^2 = v_1^2 + 2 a_c (s_2 - s_1)$$

$$(237.6)^2 = 0^2 + 2(a_c)(5000 - 0)$$

$$a_c = 5.64538 \text{ ft/s}^2$$

$$v_2 = v_1 + a_c t$$

$$237.6 = 0 + 5.64538 t$$

$$t = 42.09 = 42.1 \text{ s}$$

$$v_3 = 220 \frac{\text{mi}}{\text{h}} \frac{(1 \text{ h}) 5280 \text{ ft}}{(3600 \text{ s})(1 \text{ mi})} = 322.67 \text{ ft/s}$$

$$v_3^2 = v_2^2 + 2 a_c (s_3 - s_2)$$

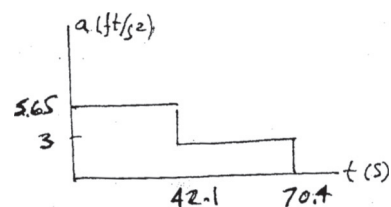
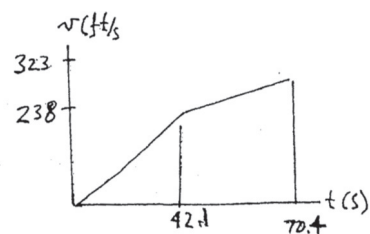
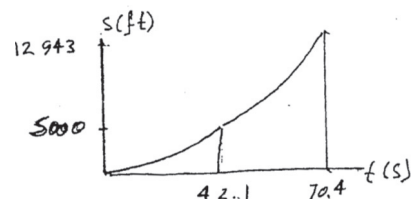
$$(322.67)^2 = (237.6)^2 + 2(3)(s - 5000)$$

$$s = 12\,943.34 \text{ ft}$$

$$v_3 = v_2 + a_c t$$

$$322.67 = 237.6 + 3 t$$

$$t = 28.4 \text{ s}$$



Ans:

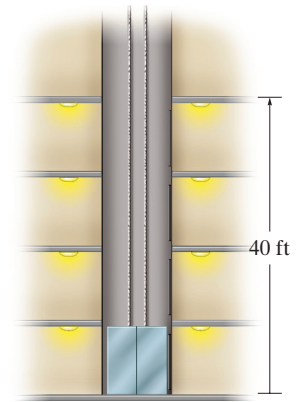
$$s = 12\,943.34 \text{ ft}$$

$$v_3 = v_2 + a_c t$$

$$t = 28.4 \text{ s}$$

12–41.

The elevator starts from rest at the first floor of the building. It can accelerate at 5 ft/s^2 and then decelerate at 2 ft/s^2 . Determine the shortest time it takes to reach a floor 40 ft above the ground. The elevator starts from rest and then stops. Draw the a - t , v - t , and s - t graphs for the motion.



SOLUTION

$$+\uparrow v_2 = v_1 + a_c t_1$$

$$v_{\max} = 0 + 5 t_1$$

$$+\uparrow v_3 = v_2 + a_c t$$

$$0 = v_{\max} - 2 t_2$$

Thus

$$t_1 = 0.4 t_2$$

$$+\uparrow s_2 = s_1 + v_1 t_1 + \frac{1}{2} a_c t_1^2$$

$$h = 0 + 0 + \frac{1}{2}(5)(t_1^2) = 2.5 t_1^2$$

$$+\uparrow 40 - h = 0 + v_{\max} t_2 - \frac{1}{2}(2) t_2^2$$

$$+\uparrow v^2 = v_1^2 + 2 a_c (s - s_1)$$

$$v_{\max}^2 = 0 + 2(5)(h - 0)$$

$$v_{\max}^2 = 10h$$

$$0 = v_{\max}^2 + 2(-2)(40 - h)$$

$$v_{\max}^2 = 160 - 4h$$

Thus,

$$10 h = 160 - 4h$$

$$h = 11.429 \text{ ft}$$

$$v_{\max} = 10.69 \text{ ft/s}$$

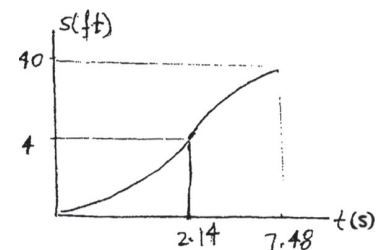
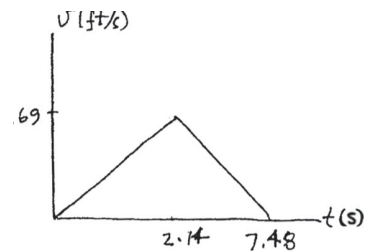
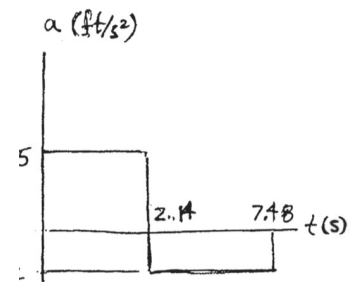
$$t_1 = 2.138 \text{ s}$$

$$t_2 = 5.345 \text{ s}$$

$$t = t_1 + t_2 = 7.48 \text{ s}$$

When $t = 2.145$, $v = v_{\max} = 10.7 \text{ ft/s}$

and $h = 11.4 \text{ ft}$.



Ans.

Ans:

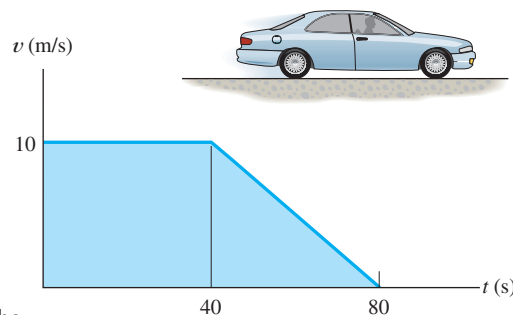
$t = 7.48 \text{ s}$. When $t = 2.14$,

$v = v_{\max} = 10.7 \text{ ft/s}$

$h = 11.4 \text{ ft}$

12-42.

The velocity of a car is plotted as shown. Determine the total distance the car moves until it stops ($t = 80$ s). Construct the $a-t$ graph.



SOLUTION

Distance Traveled: The total distance traveled can be obtained by computing the area under the $v-t$ graph.

$$s = 10(40) + \frac{1}{2}(10)(80 - 40) = 600 \text{ m}$$

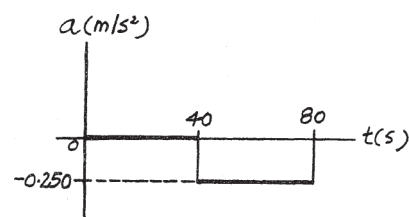
Ans.

$a-t$ Graph: The acceleration in terms of time t can be obtained by applying $a = \frac{dv}{dt}$. For time interval $0 \leq t < 40$ s,

$$a = \frac{dv}{dt} = 0$$

For time interval $40 \text{ s} < t \leq 80 \text{ s}$, $\frac{v - 10}{t - 40} = \frac{0 - 10}{80 - 40}$, $v = \left(-\frac{1}{4}t + 20\right) \text{ m/s}$.

$$a = \frac{dv}{dt} = -\frac{1}{4} = -0.250 \text{ m/s}^2$$



For $0 \leq t < 40$ s, $a = 0$.

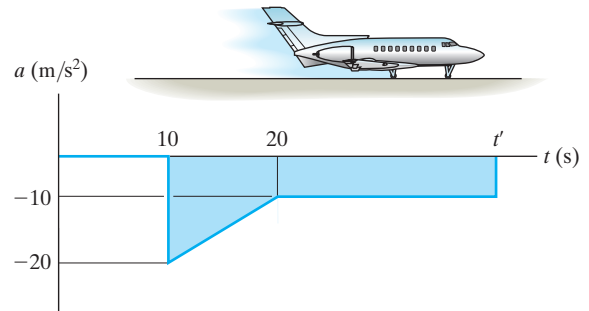
For $40 \text{ s} < t \leq 80$, $a = -0.250 \text{ m/s}^2$.

Ans:

$s = 600 \text{ m}$. For $0 \leq t < 40$ s,
 $a = 0$. For $40 \text{ s} < t \leq 80$ s,
 $a = -0.250 \text{ m/s}^2$

12–43.

The motion of a jet plane just after landing on a runway is described by the a – t graph. Determine the time t' when the jet plane stops. Construct the v – t and s – t graphs for the motion. Here $s = 0$, and $v = 300$ ft/s when $t = 0$.



SOLUTION

v – t Graph. The v – t function can be determined by integrating $dv = a dt$. For $0 \leq t < 10$ s, $a = 0$. Using the initial condition $v = 300$ ft/s at $t = 0$,

$$\int_{300 \text{ ft/s}}^v dv = \int_0^t 0 dt$$

$$v - 300 = 0$$

$$v = 300 \text{ ft/s}$$

Ans.

For $10 \text{ s} < t < 20$ s, $\frac{a - (-20)}{t - 10} = \frac{-10 - (-20)}{20 - 10}$, $a = (t - 30) \text{ ft/s}^2$. Using the initial condition $v = 300$ ft/s at $t = 10$ s,

$$\int_{300 \text{ ft/s}}^v dv = \int_{10 \text{ s}}^t (t - 30) dt$$

$$v - 300 = \left(\frac{1}{2} t^2 - 30t \right) \Big|_{10 \text{ s}}^t$$

$$v = \left\{ \frac{1}{2} t^2 - 30t + 550 \right\} \text{ ft/s}$$

Ans.

At $t = 20$ s,

$$v \Big|_{t=20 \text{ s}} = \frac{1}{2} (20^2) - 30(20) + 550 = 150 \text{ ft/s}$$

For $20 \text{ s} < t < t'$, $a = -10$ ft/s. Using the initial condition $v = 150$ ft/s at $t = 20$ s,

$$\int_{150 \text{ ft/s}}^v dv = \int_{20 \text{ s}}^t -10 dt$$

$$v - 150 = (-10t) \Big|_{20 \text{ s}}^t$$

$$v = (-10t + 350) \text{ ft/s}$$

Ans.

It is required that at $t = t'$, $v = 0$. Thus

$$0 = -10 t' + 350$$

$$t' = 35 \text{ s}$$

Ans.

Using these results, the v – t graph shown in Fig. a can be plotted **s – t Graph.** The s – t function can be determined by integrating $ds = v dt$. For $0 \leq t < 10$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t 300 dt$$

$$s = \{300 t\} \text{ ft}$$

Ans.

At $t = 10$ s,

$$s \Big|_{t=10 \text{ s}} = 300(10) = 3000 \text{ ft}$$

12-43. Continued

For $10 \text{ s} < t < 20 \text{ s}$, the initial condition is $s = 3000 \text{ ft}$ at $t = 10 \text{ s}$.

$$\int_{3000 \text{ ft}}^s ds = \int_{10 \text{ s}}^t \left(\frac{1}{2}t^2 - 30t + 550 \right) dt$$

$$s - 3000 = \left(\frac{1}{6}t^3 - 15t^2 + 550t \right) \Big|_{10 \text{ s}}^t$$

$$s = \left\{ \frac{1}{6}t^3 - 15t^2 + 550t - 1167 \right\} \text{ ft}$$

At $t = 20 \text{ s}$,

$$s = \frac{1}{6}(20^3) - 15(20^2) + 550(20) - 1167 = 5167 \text{ ft}$$

For $20 \text{ s} < t \leq 35 \text{ s}$, the initial condition is $s = 5167 \text{ ft}$ at $t = 20 \text{ s}$.

$$\int_{5167 \text{ ft}}^s ds = \int_{20 \text{ s}}^t (-10t + 350) dt$$

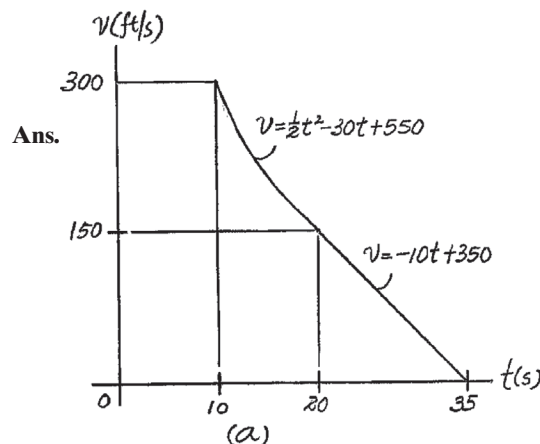
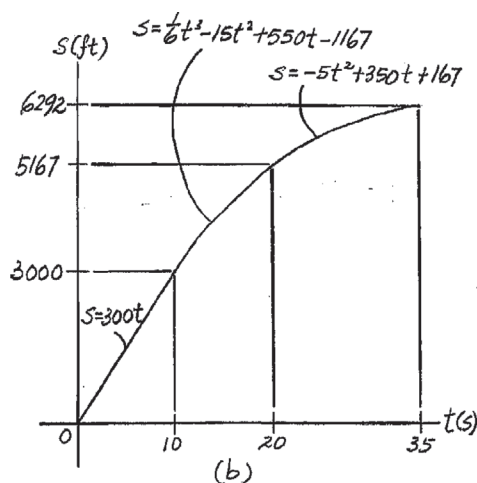
$$s - 5167 = (-5t^2 + 350t) \Big|_{20 \text{ s}}^t$$

$$s = \{-5t^2 + 350t + 167\} \text{ ft}$$

At $t = 35 \text{ s}$,

$$s \Big|_{t=35 \text{ s}} = -5(35^2) + 350(35) + 167 = 6292 \text{ ft}$$

using these results, the s - t graph shown in Fig. b can be plotted.



Ans.

Ans:

$$t' = 35 \text{ s}$$

For $0 \leq t < 10 \text{ s}$,

$$s = \{300t\} \text{ ft}$$

$$v = 300 \text{ ft/s}$$

For $10 \text{ s} < t < 20 \text{ s}$,

$$s = \left\{ \frac{1}{6}t^3 - 15t^2 + 550t - 1167 \right\} \text{ ft}$$

$$v = \left\{ \frac{1}{2}t^2 - 30t + 550 \right\} \text{ ft/s}$$

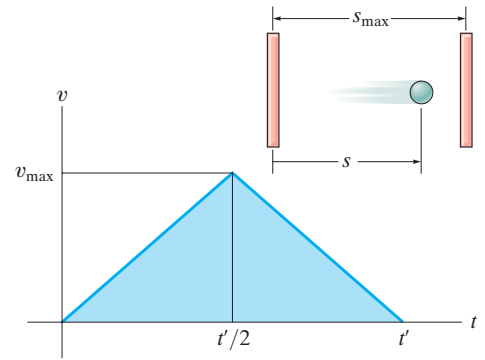
For $20 \text{ s} < t \leq 35 \text{ s}$,

$$s = \{-5t^2 + 350t + 167\} \text{ ft}$$

$$v = (-10t + 350) \text{ ft/s}$$

***12–44.**

The v - t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of 4 m/s^2 . If the plates are spaced 200 mm apart, determine the maximum velocity v_{\max} and the time t' for the particle to travel from one plate to the other. Also draw the s - t graph. When $t = t'/2$ the particle is at $s = 100 \text{ mm}$.



SOLUTION

$$a_c = 4 \text{ m/s}^2$$

$$\frac{s}{2} = 100 \text{ mm} = 0.1 \text{ m}$$

$$v^2 = v_0^2 + 2 a_c (s - s_0)$$

$$v_{\max}^2 = 0 + 2(4)(0.1 - 0)$$

$$v_{\max} = 0.89442 \text{ m/s} = 0.894 \text{ m/s}$$

$$v = v_0 + a_c t'$$

$$0.89442 = 0 + 4\left(\frac{t'}{2}\right)$$

$$t' = 0.44721 \text{ s} = 0.447 \text{ s}$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2} (4)(t)^2$$

$$s = 2 t^2$$

$$\text{When } t = \frac{0.44721}{2} = 0.2236 = 0.224 \text{ s,}$$

$$s = 0.1 \text{ m}$$

$$\int_{0.894}^v ds = - \int_{0.2235}^t 4 dt$$

$$v = -4t + 1.788$$

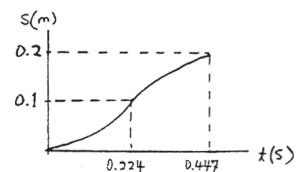
$$\int_{0.1}^s ds = \int_{0.2235}^t (-4t + 1.788) dt$$

$$s = -2t^2 + 1.788t - 0.2$$

$$\text{When } t = 0.447 \text{ s,}$$

$$s = 0.2 \text{ m}$$

Ans.



Ans.

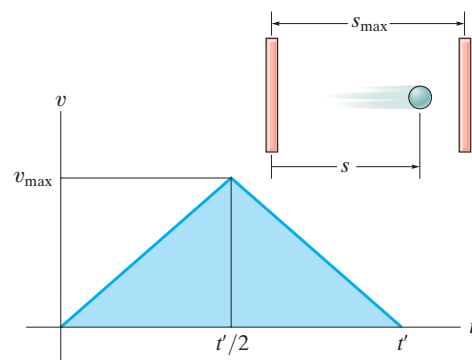
Ans:

$$t' = 0.447 \text{ s}$$

$$s = 0.2 \text{ m}$$

12–45.

The v - t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{\max} = 10$ m/s. Draw the s - t and a - t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



SOLUTION

For $0 < t < 0.1$ s,

$$v = 100 t$$

$$a = \frac{dv}{dt} = 100$$

$$ds = v dt$$

$$\int_0^s ds = \int_0^t 100 t dt$$

$$s = 50 t^2$$

When $t = 0.1$ s,

$$s = 0.5 \text{ m}$$

For $0.1 \text{ s} < t < 0.2$ s,

$$v = -100 t + 20$$

$$a = \frac{dv}{dt} = -100$$

$$ds = v dt$$

$$\int_{0.5}^s ds = \int_{0.1}^t (-100t + 20) dt$$

$$s - 0.5 = (-50 t^2 + 20 t - 1.5)$$

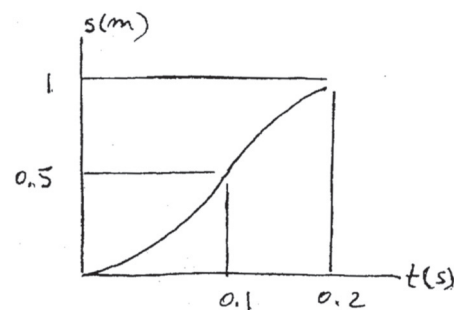
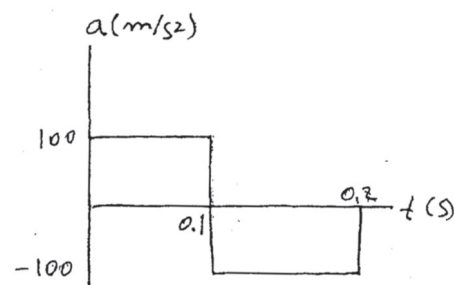
$$s = -50 t^2 + 20 t - 1$$

When $t = 0.2$ s,

$$s = 1 \text{ m}$$

When $t = 0.1$ s, $s = 0.5$ m and a changes from 100 m/s^2

to -100 m/s^2 . When $t = 0.2$ s, $s = 1$ m.

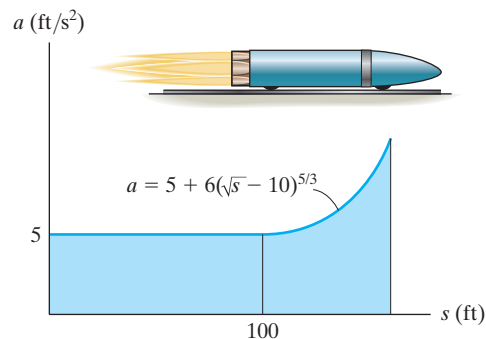


Ans:

When $t = 0.1$ s,
 $s = 0.5$ m and a changes from
 100 m/s^2 to -100 m/s^2 . When $t = 0.2$ s,
 $s = 1$ m.

12–46.

The a - s graph for a rocket moving along a straight track has been experimentally determined. If the rocket starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = 75$ ft, and 125 ft, respectively. Use Simpson's rule with $n = 100$ to evaluate v at $s = 125$ ft.



SOLUTION

$$0 \leq s < 100$$

$$\int_0^v v \, dv = \int_0^s 5 \, ds$$

$$\frac{1}{2} v^2 = 5s$$

$$v = \sqrt{10s}$$

$$\text{At } s = 75 \text{ ft, } v = \sqrt{750} = 27.4 \text{ ft/s}$$

$$\text{At } s = 100 \text{ ft, } v = 31.623$$

$$v \, dv = a \, ds$$

$$\int_{31.623}^v v \, dv = \int_{100}^{125} [5 + 6(\sqrt{s} - 10)^{5/3}] \, ds$$

$$\frac{1}{2} v^2 \Big|_{31.623}^v = 201.0324$$

$$v = 37.4 \text{ ft/s}$$

Ans.

Ans.

Ans:

$$v \Big|_{s=75 \text{ ft}} = 27.4 \text{ ft/s}$$

$$v \Big|_{s=125 \text{ ft}} = 37.4 \text{ ft/s}$$

12-47.

A two-stage rocket is fired vertically from rest at $s = 0$ with the acceleration as shown. After 30 s the first stage, A , burns out and the second stage, B , ignites. Plot the v - t and s - t graphs which describe the motion of the second stage for $0 \leq t \leq 60$ s.

SOLUTION

v - t Graph. The v - t function can be determined by integrating $dv = a \, dt$.

For $0 \leq t < 30$ s, $a = \frac{12}{30}t = \left(\frac{2}{5}t\right)$ m/s². Using the initial condition $v = 0$ at $t = 0$,

$$\int_0^v dv = \int_0^t \frac{2}{5}t \, dt$$

$$v = \left\{ \frac{1}{5}t^2 \right\} \text{ m/s}$$

At $t = 30$ s,

$$v \Big|_{t=30 \text{ s}} = \frac{1}{5}(30^2) = 180 \text{ m/s}$$

For $30 < t \leq 60$ s, $a = 24$ m/s². Using the initial condition $v = 180$ m/s at $t = 30$ s,

$$\int_{180 \text{ m/s}}^v dv = \int_{30 \text{ s}}^t 24 \, dt$$

$$v - 180 = 24t \Big|_{30 \text{ s}}^t$$

$$v = \{24t - 540\} \text{ m/s}$$

At $t = 60$ s,

$$v \Big|_{t=60 \text{ s}} = 24(60) - 540 = 900 \text{ m/s}$$

Using these results, v - t graph shown in Fig. a can be plotted.

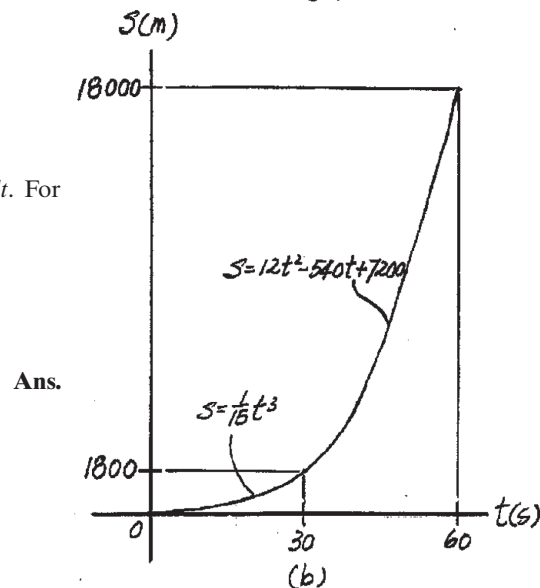
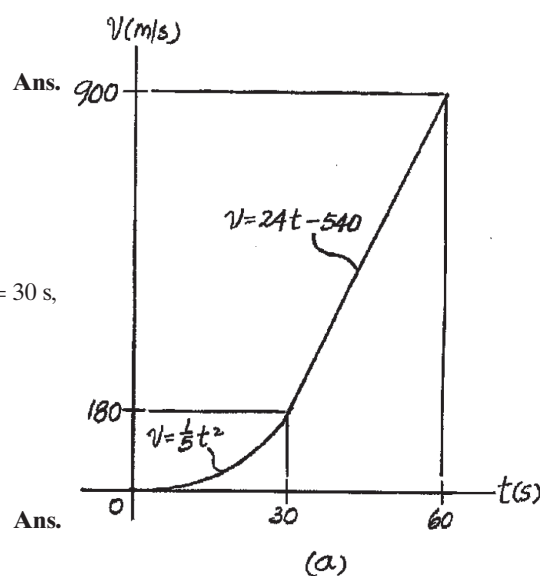
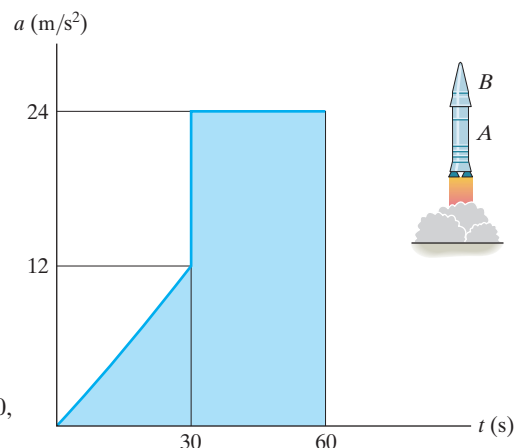
s - t Graph. The s - t function can be determined by integrating $ds = v \, dt$. For $0 \leq t < 30$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t \frac{1}{5}t^2 \, dt$$

$$s = \left\{ \frac{1}{15}t^3 \right\} \text{ m}$$

At $t = 30$ s,

$$s \Big|_{t=30 \text{ s}} = \frac{1}{15}(30^3) = 1800 \text{ m}$$



12–47. Continued

For $30 \text{ s} < t \leq 60 \text{ s}$, the initial condition is $s = 1800 \text{ m}$ at $t = 30 \text{ s}$.

$$\int_{1800 \text{ m}}^s ds = \int_{30 \text{ s}}^t (24t - 540) dt$$

$$s - 1800 = (12t^2 - 540t) \Big|_{30 \text{ s}}^t$$

$$s = \{12t^2 - 540t + 7200\} \text{ m}$$

At $t = 60 \text{ s}$,

$$s \Big|_{t=60 \text{ s}} = 12(60^2) - 540(60) + 7200 = 18000 \text{ m}$$

Using these results, the s – t graph in Fig. b can be plotted.

Ans:

For $0 \leq t < 30 \text{ s}$,

$$v = \left\{ \frac{1}{5} t^2 \right\} \text{ m/s}$$

$$s = \left\{ \frac{1}{15} t^3 \right\} \text{ m}$$

For $30 \leq t \leq 60 \text{ s}$,

$$v = \{24t - 540\} \text{ m/s}$$

$$s = \{12t^2 - 540t + 7200\} \text{ m}$$

***12–48.**

The race car starts from rest and travels along a straight road until it reaches a speed of 26 m/s in 8 s as shown on the v - t graph. The flat part of the graph is caused by shifting gears. Draw the a - t graph and determine the maximum acceleration of the car.

SOLUTION

For $0 \leq t < 4$ s

$$a = \frac{\Delta v}{\Delta t} = \frac{14}{4} = 3.5 \text{ m/s}^2$$

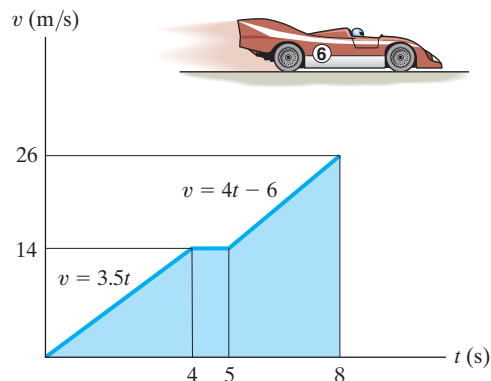
For $4 \text{ s} \leq t < 5$ s

$$a = \frac{\Delta v}{\Delta t} = 0$$

For $5 \text{ s} \leq t < 8$ s

$$a = \frac{\Delta v}{\Delta t} = \frac{26 - 14}{8 - 5} = 4 \text{ m/s}^2$$

$$a_{\max} = 4.00 \text{ m/s}^2$$



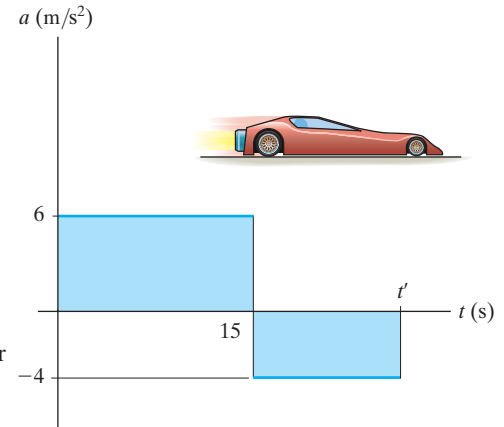
Ans.

Ans:

$$a_{\max} = 4.00 \text{ m/s}^2$$

12–49.

The jet car is originally traveling at a velocity of 10 m/s when it is subjected to the acceleration shown. Determine the car's maximum velocity and the time t' when it stops. When $t = 0, s = 0$.



SOLUTION

v - t Function. The v - t function can be determined by integrating $dv = a dt$. For $0 \leq t < 15$ s, $a = 6$ m/s². Using the initial condition $v = 10$ m/s at $t = 0$,

$$\int_{10 \text{ m/s}}^v dv = \int_0^t 6 dt$$

$$v - 10 = 6t$$

$$v = \{6t + 10\} \text{ m/s}$$

The maximum velocity occurs when $t = 15$ s. Then

$$v_{\max} = 6(15) + 10 = 100 \text{ m/s} \quad \textbf{Ans.}$$

For $15 \text{ s} < t \leq t'$, $a = -4$ m/s². Using the initial condition $v = 100$ m/s at $t = 15$ s,

$$\int_{100 \text{ m/s}}^v dv = \int_{15 \text{ s}}^t -4 dt$$

$$v - 100 = (-4t) \Big|_{15 \text{ s}}^t$$

$$v = \{-4t + 160\} \text{ m/s}$$

It is required that $v = 0$ at $t = t'$. Then

$$0 = -4t' + 160 \quad t' = 40 \text{ s} \quad \textbf{Ans.}$$

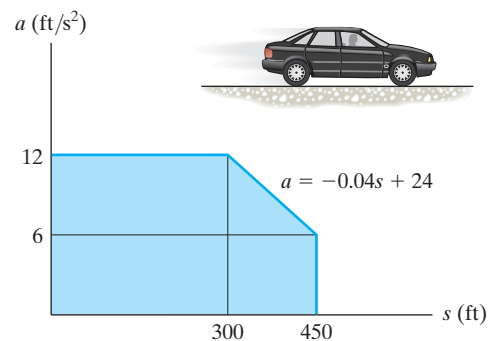
Ans:

$$v_{\max} = 100 \text{ m/s}$$

$$t' = 40 \text{ s}$$

12–50.

The car starts from rest at $s = 0$ and is subjected to an acceleration shown by the a – s graph. Draw the v – s graph and determine the time needed to travel 200 ft.



SOLUTION

For $s < 300$ ft

$$a \, ds = v \, dv$$

$$\int_0^s 12 \, ds = \int_0^v v \, dv$$

$$12s = \frac{1}{2}v^2$$

$$v = 4.90 \, s^{1/2}$$

At $s = 300$ ft, $v = 84.85$ ft/s

For $300 \text{ ft} < s < 450$ ft

$$a \, ds = v \, dv$$

$$\int_{300}^s (24 - 0.04s) \, ds = \int_{84.85}^v v \, dv$$

$$24s - 0.02s^2 - 5400 = 0.5v^2 - 3600$$

$$v = (-0.04s^2 + 48s - 3600)^{1/2}$$

At $s = 450$ ft, $v = 99.5$ ft/s

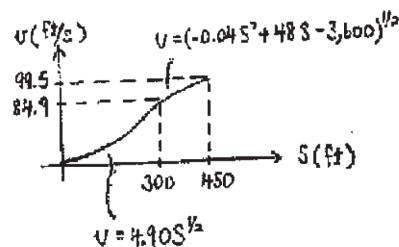
$$v = 4.90 \, s^{1/2}$$

$$\frac{ds}{dt} = 4.90 \, s^{1/2}$$

$$\int_0^{200} s^{-1/2} \, ds = \int_0^t 4.90 \, dt$$

$$2s^{1/2} \Big|_0^{200} = 4.90t$$

$$t = 5.77 \text{ s}$$



Ans.

Ans:

For $0 \leq s < 300$ ft,

$$v = \{4.90 \, s^{1/2}\} \text{ m/s.}$$

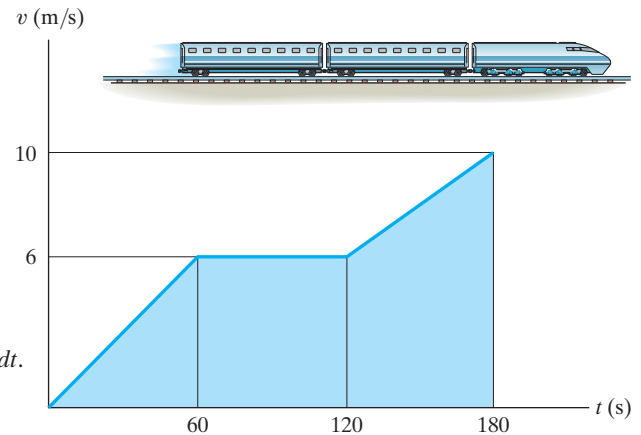
For $300 \text{ ft} < s \leq 450$ ft,

$$v = \{(-0.04s^2 + 48s - 3600)^{1/2}\} \text{ m/s.}$$

$$s = 200 \text{ ft when } t = 5.77 \text{ s.}$$

12-51.

The $v-t$ graph for a train has been experimentally determined. From the data, construct the $s-t$ and $a-t$ graphs for the motion for $0 \leq t \leq 180$ s. When $t = 0, s = 0$.



SOLUTION

$s-t$ Graph. The $s-t$ function can be determined by integrating $ds = v dt$.

For $0 \leq t < 60$ s, $v = \frac{6}{60}t = \left(\frac{1}{10}t\right)$ m/s. Using the initial condition $s = 0$ at $t = 0$,

$$\int_0^s ds = \int_0^t \left(\frac{1}{10}t\right) dt$$

$$s = \left\{ \frac{1}{20}t^2 \right\} \text{ m}$$

Ans.

When $t = 60$ s,

$$s|_{t=60 \text{ s}} = \frac{1}{20}(60^2) = 180 \text{ m}$$

For $60 \text{ s} < t < 120$ s, $v = 6$ m/s. Using the initial condition $s = 180$ m at $t = 60$ s,

$$\int_{180 \text{ m}}^s ds = \int_{60 \text{ s}}^t 6 dt$$

$$s - 180 = 6t \Big|_{60 \text{ s}}^t$$

$$s = \{6t - 180\} \text{ m}$$

Ans.

At $t = 120$ s,

$$s|_{t=120 \text{ s}} = 6(120) - 180 = 540 \text{ m}$$

For $120 \text{ s} < t \leq 180$ s, $\frac{v-6}{t-120} = \frac{10-6}{180-120}$; $v = \left\{ \frac{1}{15}t - 2 \right\}$ m/s. Using the initial

condition $s = 540$ m at $t = 120$ s,

$$\int_{540 \text{ m}}^s ds = \int_{120 \text{ s}}^t \left(\frac{1}{15}t - 2 \right) dt$$

$$s - 540 = \left(\frac{1}{30}t^2 - 2t \right) \Big|_{120 \text{ s}}^t$$

$$s = \left\{ \frac{1}{30}t^2 - 2t + 300 \right\} \text{ m}$$

Ans.

At $t = 180$ s,

$$s|_{t=180 \text{ s}} = \frac{1}{30}(180^2) - 2(180) + 300 = 1020 \text{ m}$$

Using these results, $s-t$ graph shown in Fig. *a* can be plotted.

12-51. Continued

a - t Graph. The a - t function can be determined using $a = \frac{dv}{dt}$.

For $0 \leq t < 60$ s, $a = \frac{d\left(\frac{1}{10}t\right)}{dt} = 0.1 \text{ m/s}^2$

Ans.

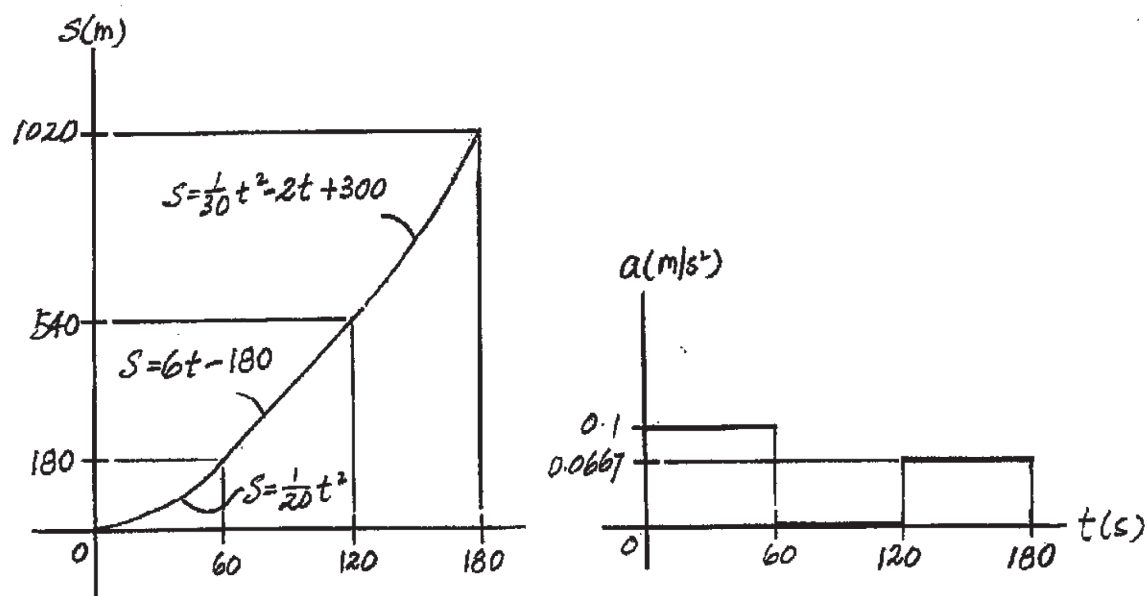
For $60 \text{ s} < t < 120$ s, $a = \frac{d(6)}{dt} = 0$

Ans.

For $120 \text{ s} < t \leq 180$ s, $a = \frac{d\left(\frac{1}{15}t - 2\right)}{dt} = 0.0667 \text{ m/s}^2$

Ans.

Using these results, a - t graph shown in Fig. b can be plotted.



Ans:

For $0 \leq t < 60$ s,

$$s = \left\{ \frac{1}{20}t^2 \right\} \text{ m,}$$

$$a = 0.1 \text{ m/s}^2.$$

For $60 \text{ s} < t < 120$ s,

$$s = \{6t - 180\} \text{ m,}$$

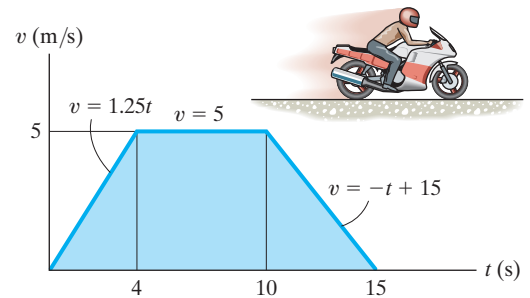
$a = 0$. For $120 \text{ s} < t \leq 180$ s,

$$s = \left\{ \frac{1}{30}t^2 - 2t + 300 \right\} \text{ m,}$$

$$a = 0.0667 \text{ m/s}^2.$$

***12–52.**

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the $v-t$ graph. Determine the total distance the motorcycle travels until it stops when $t = 15$ s. Also plot the $a-t$ and $s-t$ graphs.



SOLUTION

For $t < 4$ s

$$a = \frac{dv}{dt} = 1.25$$

$$\int_0^s ds = \int_0^t 1.25 t dt$$

$$s = 0.625 t^2$$

When $t = 4$ s, $s = 10$ m

For $4 \text{ s} < t < 10 \text{ s}$

$$a = \frac{dv}{dt} = 0$$

$$\int_{10}^s ds = \int_4^t 5 dt$$

$$s = 5t - 10$$

When $t = 10$ s, $s = 40$ m

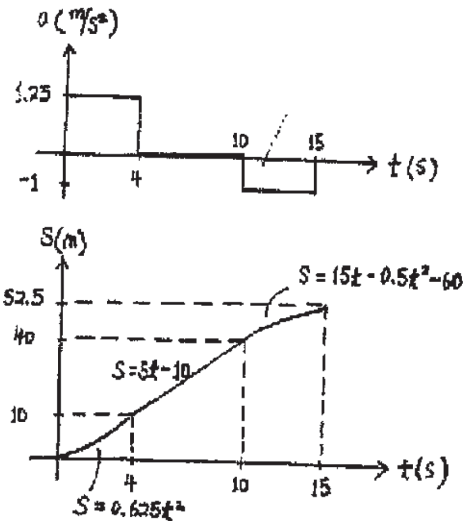
For $10 \text{ s} < t < 15 \text{ s}$

$$a = \frac{dv}{dt} = -1$$

$$\int_{40}^s ds = \int_{10}^t (15 - t) dt$$

$$s = 15t - 0.5t^2 - 60$$

When $t = 15$ s, $s = 52.5$ m



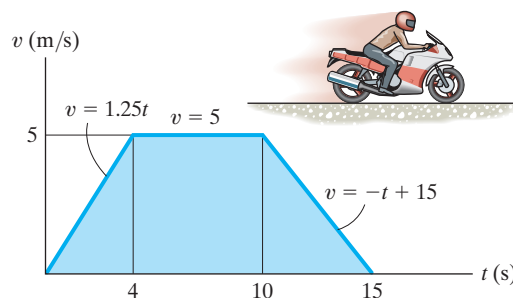
Ans.

Ans:

When $t = 15$ s, $s = 52.5$ m

12–53.

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v – t graph. Determine the motorcycle's acceleration and position when $t = 8$ s and $t = 12$ s.



SOLUTION

At $t = 8$ s

$$a = \frac{dv}{dt} = 0$$

$$\Delta s = \int v \, dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (8 - 4)(5) = 30$$

$$s = 30 \text{ m}$$

At $t = 12$ s

$$a = \frac{dv}{dt} = \frac{-5}{5} = -1 \text{ m/s}^2$$

$$\Delta s = \int v \, dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (10 - 4)(5) + \frac{1}{2}(15 - 10)(5) - \frac{1}{2}\left(\frac{3}{5}\right)(5)\left(\frac{3}{5}\right)(5)$$

$$s = 48 \text{ m}$$

Ans.

Ans.

Ans.

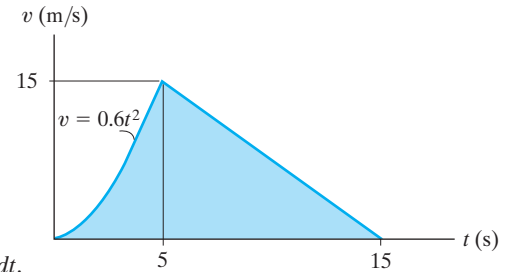
Ans.

Ans:

At $t = 8$ s,
 $a = 0$ and $s = 30$ m.
 At $t = 12$ s,
 $a = -1 \text{ m/s}^2$
 and $s = 48$ m.

12-54.

The $v-t$ graph for the motion of a car as it moves along a straight road is shown. Draw the $s-t$ and $a-t$ graphs. Also determine the average speed and the distance traveled for the 15-s time interval. When $t = 0, s = 0$.



SOLUTION

$s-t$ Graph. The $s-t$ function can be determined by integrating $ds = v dt$. For $0 \leq t < 5$ s, $v = 0.6t^2$. Using the initial condition $s = 0$ at $t = 0$,

$$\int_0^s ds = \int_0^t 0.6t^2 dt$$

$$s = \{0.2t^3\} \text{ m}$$

Ans.

At $t = 5$ s,

$$s|_{t=5\text{ s}} = 0.2(5^3) = 25 \text{ m}$$

For $5 \text{ s} < t \leq 15 \text{ s}$, $\frac{v - 15}{t - 5} = \frac{0 - 15}{15 - 5}$; $v = \frac{1}{2}(45 - 3t)$. Using the initial condition

$s = 25 \text{ m}$ at $t = 5 \text{ s}$,

$$\int_{25\text{ m}}^s ds = \int_{5\text{ s}}^t \frac{1}{2}(45 - 3t) dt$$

$$s - 25 = \frac{45}{2}t - \frac{3}{4}t^2 - 93.75$$

$$s = \left\{ \frac{1}{4}(90t - 3t^2 - 275) \right\} \text{ m}$$

Ans.

At $t = 15 \text{ s}$,

$$s = \frac{1}{4}[90(15) - 3(15^2) - 275] = 100 \text{ m}$$

Ans.

Thus the average speed is

$$v_{\text{avg}} = \frac{s_T}{t} = \frac{100 \text{ m}}{15 \text{ s}} = 6.67 \text{ m/s}$$

Ans.

using these results, the $s-t$ graph shown in Fig. *a* can be plotted.

12-54. Continued

***a-t* Graph.** The *a-t* function can be determined using $a = \frac{dv}{dt}$.

For $0 \leq t < 5$ s, $a = \frac{d(0.6 t^2)}{dt} = \{1.2 t\} \text{ m/s}^2$

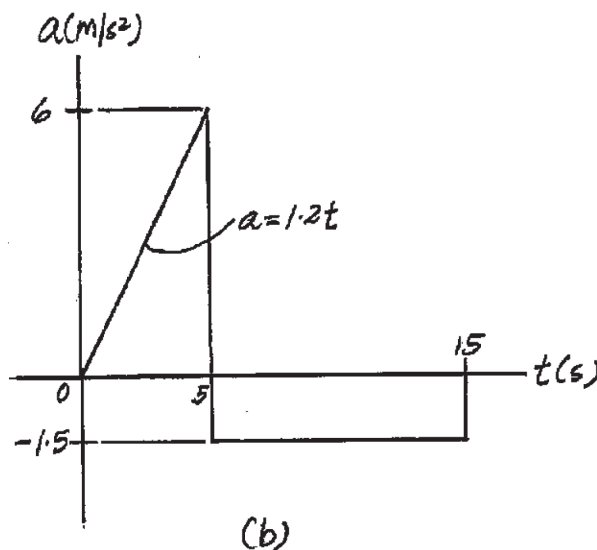
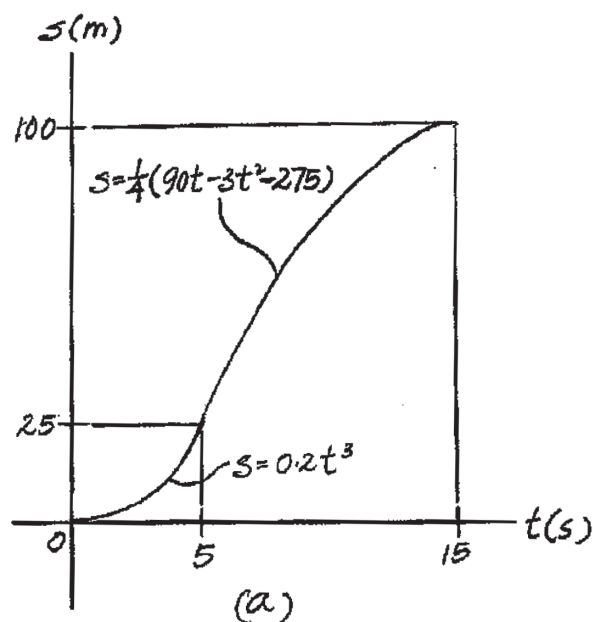
Ans.

At $t = 5$ s, $a = 1.2(5) = 6 \text{ m/s}^2$

Ans.

For $5 \text{ s} < t \leq 15$ s, $a = \frac{d[\frac{1}{2}(45 - 3t)]}{dt} = -1.5 \text{ m/s}^2$

Ans.



Ans:

For $0 \leq t < 5$ s,

$$s = \{0.2t^3\} \text{ m}$$

$$a = \{1.2t\} \text{ m/s}^2$$

For $5 \text{ s} < t \leq 15$ s,

$$s = \left\{ \frac{1}{4}(90t - 3t^2 - 275) \right\} \text{ m}$$

$$a = -1.5 \text{ m/s}^2$$

At $t = 15$ s,

$$s = 100 \text{ m}$$

$$v_{\text{avg}} = 6.67 \text{ m/s}$$

12–55.

An airplane lands on the straight runway, originally traveling at 110 ft/s when $s = 0$. If it is subjected to the decelerations shown, determine the time t' needed to stop the plane and construct the s – t graph for the motion.

SOLUTION

$$v_0 = 110 \text{ ft/s}$$

$$\Delta v = \int a \, dt$$

$$0 - 110 = -3(15 - 5) - 8(20 - 15) - 3(t' - 20)$$

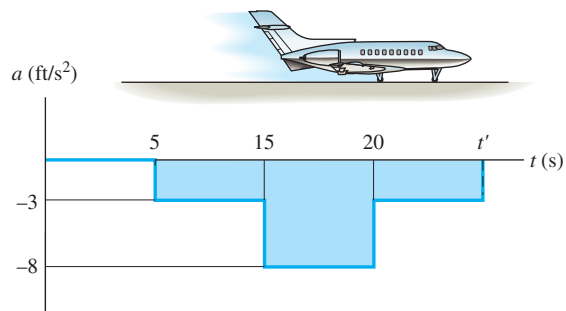
$$t' = 33.3 \text{ s}$$

$$s|_{t=5\text{s}} = 550 \text{ ft}$$

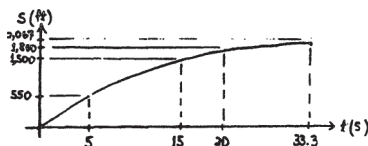
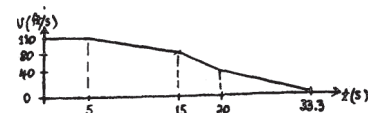
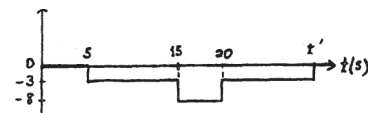
$$s|_{t=15\text{s}} = 1500 \text{ ft}$$

$$s|_{t=20\text{s}} = 1800 \text{ ft}$$

$$s|_{t=33.3\text{s}} = 2067 \text{ ft}$$



Ans.



Ans:

$$t' = 33.3 \text{ s}$$

$$s|_{t=5\text{s}} = 550 \text{ ft}$$

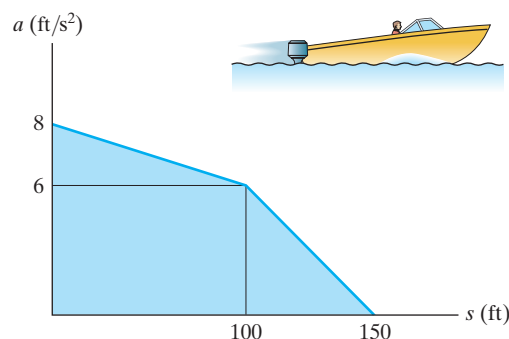
$$s|_{t=15\text{s}} = 1500 \text{ ft}$$

$$s|_{t=20\text{s}} = 1800 \text{ ft}$$

$$s|_{t=33.3\text{s}} = 2067 \text{ ft}$$

***12–56.**

Starting from rest at $s = 0$, a boat travels in a straight line with the acceleration shown by the a – s graph. Determine the boat's speed when $s = 50$ ft, 100 ft, and 150 ft.



SOLUTION

v – s Function. The v – s function can be determined by integrating $v dv = a ds$.

For $0 \leq s < 100$ ft, $\frac{a-8}{s-0} = \frac{6-8}{100-0}$, $a = \left\{ -\frac{1}{50}s + 8 \right\}$ ft/s². Using the initial condition $v=0$ at $s=0$,

$$\int_0^v v dv = \int_0^s \left(-\frac{1}{50}s + 8 \right) ds$$

$$\frac{v^2}{2} \Big|_0^v = \left(-\frac{1}{100}s^2 + 8s \right) \Big|_0^s$$

$$\frac{v^2}{2} = 8s - \frac{1}{100}s^2$$

$$v = \left\{ \sqrt{\frac{1}{50}(800s - s^2)} \right\} \text{ ft/s}$$

At $s = 50$ ft,

$$v|_{s=50 \text{ ft}} = \sqrt{\frac{1}{50}[800(50) - 50^2]} = 27.39 \text{ ft/s} = 27.4 \text{ ft/s} \quad \textbf{Ans.}$$

At $s = 100$ ft,

$$v|_{s=100 \text{ ft}} = \sqrt{\frac{1}{50}[800(100) - 100^2]} = 37.42 \text{ ft/s} = 37.4 \text{ ft/s} \quad \textbf{Ans.}$$

For $100 \text{ ft} < s \leq 150 \text{ ft}$, $\frac{a-0}{s-150} = \frac{6-0}{100-150}$; $a = \left\{ -\frac{3}{25}s + 18 \right\}$ ft/s². Using the initial condition $v = 37.42$ ft/s at $s = 100$ ft,

$$\int_{37.42 \text{ ft/s}}^v v dv = \int_{100 \text{ ft}}^s \left(-\frac{3}{25}s + 18 \right) ds$$

$$\frac{v^2}{2} \Big|_{37.42 \text{ ft/s}}^v = \left(-\frac{3}{50}s^2 + 18s \right) \Big|_{100 \text{ ft}}^s$$

$$v = \left\{ \frac{1}{5} \sqrt{-3s^2 + 900s - 25000} \right\} \text{ ft/s}$$

At $s = 150$ ft

$$v|_{s=150 \text{ ft}} = \frac{1}{5} \sqrt{-3(150^2) + 900(150) - 25000} = 41.23 \text{ ft/s} = 41.2 \text{ ft/s} \quad \textbf{Ans.}$$

Ans:

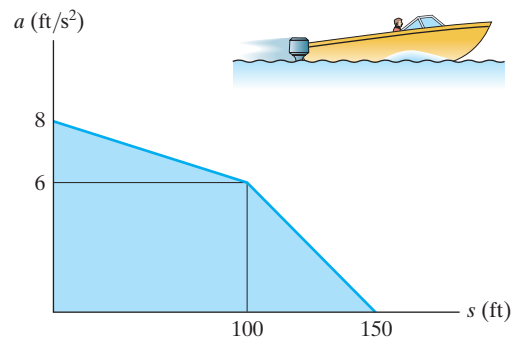
$$v|_{s=50 \text{ ft}} = 27.4 \text{ ft/s}$$

$$v|_{s=100 \text{ ft}} = 37.4 \text{ ft/s}$$

$$v|_{s=150 \text{ ft}} = 41.2 \text{ ft/s}$$

12-57.

Starting from rest at $s = 0$, a boat travels in a straight line with the acceleration shown by the $a-s$ graph. Construct the $v-s$ graph.



SOLUTION

$v-s$ Graph. The $v-s$ function can be determined by integrating $v dv = a ds$. For

$0 \leq s < 100$ ft, $\frac{a-8}{s-0} = \frac{6-8}{100-0}$, $a = \left\{ -\frac{1}{50}s + 8 \right\}$ ft/s² using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v dv = \int_0^s \left(-\frac{1}{50}s + 8 \right) ds$$

$$\frac{v^2}{2} \Big|_0^v = \left(-\frac{1}{100}s^2 + 8s \right) \Big|_0^s$$

$$\frac{v^2}{2} = 8s - \frac{1}{100}s^2$$

$$v = \left\{ \sqrt{\frac{1}{50}(800s - s^2)} \right\} \text{ ft/s}$$

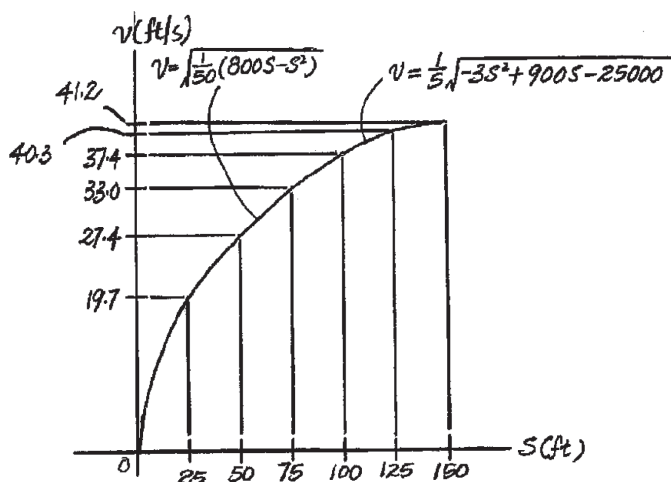
At $s = 25$ ft, 50 ft, 75 ft and 100 ft

$$v|_{s=25 \text{ ft}} = \sqrt{\frac{1}{50}[800(25) - 25^2]} = 19.69 \text{ ft/s}$$

$$v|_{s=50 \text{ ft}} = \sqrt{\frac{1}{50}[800(50) - 50^2]} = 27.39 \text{ ft/s}$$

$$v|_{s=75 \text{ ft}} = \sqrt{\frac{1}{50}[800(75) - 75^2]} = 32.98 \text{ ft/s}$$

$$v|_{s=100 \text{ ft}} = \sqrt{\frac{1}{50}[800(100) - 100^2]} = 37.42 \text{ ft/s}$$



For $100 \text{ ft} < s \leq 150$ ft, $\frac{a-0}{s-150} = \frac{6-0}{100-150}$; $a = \left\{ -\frac{3}{25}s + 18 \right\}$ ft/s² using the

initial condition $v = 37.42$ ft/s at $s = 100$ ft,

$$\int_{37.42 \text{ ft/s}}^v v dv = \int_{100 \text{ ft}}^s \left(-\frac{3}{25}s + 18 \right) ds$$

$$\frac{v^2}{2} \Big|_{37.42 \text{ ft/s}}^v = \left(-\frac{3}{50}s^2 + 18s \right) \Big|_{100 \text{ ft}}^s$$

$$v = \left\{ \frac{1}{5} \sqrt{-3s^2 + 900s - 25000} \right\} \text{ ft/s}$$

At $s = 125$ ft and $s = 150$ ft

$$v|_{s=125 \text{ ft}} = \frac{1}{5} \sqrt{-3(125^2) + 900(125) - 25000} = 40.31 \text{ ft/s}$$

$$v|_{s=150 \text{ ft}} = \frac{1}{5} \sqrt{-3(150^2) + 900(150) - 25000} = 41.23 \text{ ft/s}$$

Ans:

For $0 \leq s < 100$ ft,

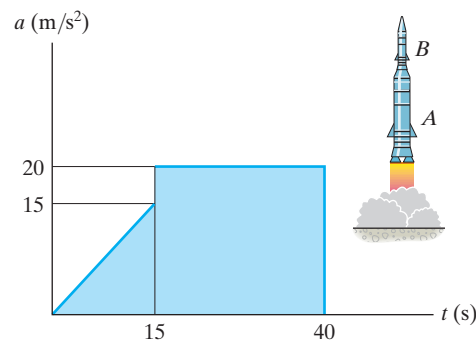
$$v = \left\{ \sqrt{\frac{1}{50}(800s - s^2)} \right\} \text{ ft/s}$$

For $100 \text{ ft} < s \leq 150$ ft,

$$v = \left\{ \frac{1}{5} \sqrt{-3s^2 + 900s - 25000} \right\} \text{ ft/s}$$

12-58.

A two-stage rocket is fired vertically from rest with the acceleration shown. After 15 s the first stage *A* burns out and the second stage *B* ignites. Plot the v - t and s - t graphs which describe the motion of the second stage for $0 \leq t \leq 40$ s.



SOLUTION

For $0 \leq t < 15$

$$a = t$$

$$\int_0^v dv = \int_0^t t \, dt$$

$$v = \frac{1}{2} t^2$$

$$v = 112.5 \text{ when } t = 15 \text{ s}$$

$$\int_0^s ds = \int_0^t \frac{1}{2} t^2 \, dt$$

$$s = \frac{1}{6} t^3$$

$$s = 562.5 \text{ when } t = 15 \text{ s}$$

For $15 < t < 40$

$$a = 20$$

$$\int_{112.5}^v dv = \int_{15}^t 20 \, dt$$

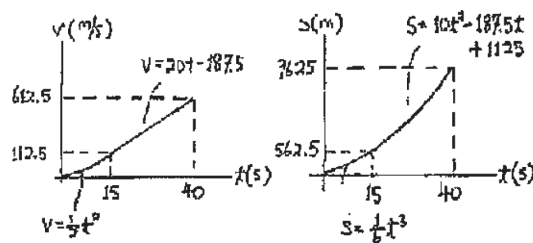
$$v = 20t - 187.5$$

$$v = 612.5 \text{ when } t = 40 \text{ s}$$

$$\int_{562.5}^s ds = \int_{15}^t (20t - 187.5) \, dt$$

$$s = 10t^2 - 187.5t + 1125$$

$$s = 9625 \text{ when } t = 40 \text{ s}$$



Ans:

For $0 \leq t < 15$ s,

$$v = \left\{ \frac{1}{2} t^2 \right\} \text{ m/s}$$

$$s = \left\{ \frac{1}{6} t^3 \right\} \text{ m}$$

For $15 \text{ s} < t \leq 40$ s,

$$v = \{20t - 187.5 \text{ m/s}\}$$

$$s = \{10t^2 - 187.5t + 1125\} \text{ m}$$

12–59.

The speed of a train during the first minute has been recorded as follows:

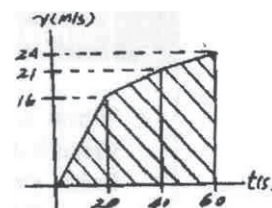
t (s)	0	20	40	60
v (m/s)	0	16	21	24

Plot the v – t graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.

SOLUTION

The total distance traveled is equal to the area under the graph.

$$s_T = \frac{1}{2}(20)(16) + \frac{1}{2}(40 - 20)(16 + 21) + \frac{1}{2}(60 - 40)(21 + 24) = 980 \text{ m} \quad \text{Ans.}$$



Ans:
 $s_T = 980 \text{ m}$

***12–60.**

A man riding upward in a freight elevator accidentally drops a package off the elevator when it is 100 ft from the ground. If the elevator maintains a constant upward speed of 4 ft/s, determine how high the elevator is from the ground the instant the package hits the ground. Draw the v - t curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

SOLUTION

For package:

$$(+\uparrow) \quad v^2 = v_0^2 + 2a_c(s_2 - s_0)$$

$$v^2 = (4)^2 + 2(-32.2)(0 - 100)$$

$$v = 80.35 \text{ ft/s } \downarrow$$

$$(+\uparrow) \quad v = v_0 + a_c t$$

$$-80.35 = 4 + (-32.2)t$$

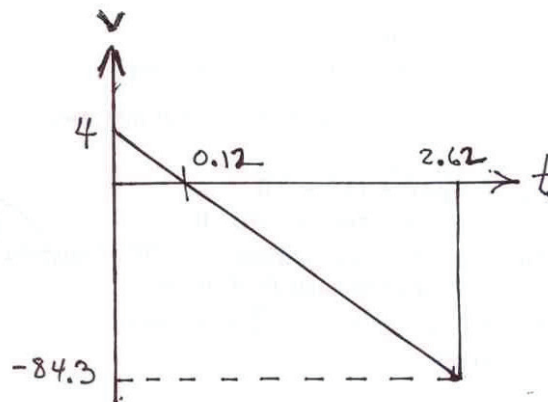
$$t = 2.620 \text{ s}$$

For elevator:

$$(+\uparrow) \quad s_2 = s_0 + vt$$

$$s = 100 + 4(2.620)$$

$$s = 110 \text{ ft}$$



Ans.

Ans:
 $s = 110 \text{ ft}$

12-61.

Two cars start from rest side by side and travel along a straight road. Car A accelerates at 4 m/s^2 for 10 s and then maintains a constant speed. Car B accelerates at 5 m/s^2 until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a - t , v - t , and s - t graphs for each car until $t = 15 \text{ s}$. What is the distance between the two cars when $t = 15 \text{ s}$?

SOLUTION

Car A :

$$v = v_0 + a_c t$$

$$v_A = 0 + 4t$$

$$\text{At } t = 10 \text{ s, } v_A = 40 \text{ m/s}$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(4)t^2 = 2t^2$$

$$\text{At } t = 10 \text{ s, } s_A = 200 \text{ m}$$

$$t > 10 \text{ s, } ds = v dt$$

$$\int_{200}^{s_A} ds = \int_{10}^t 40 dt$$

$$s_A = 40t - 200$$

$$\text{At } t = 15 \text{ s, } s_A = 400 \text{ m}$$

Car B :

$$v = v_0 + a_c t$$

$$v_B = 0 + 5t$$

$$\text{When } v_B = 25 \text{ m/s, } t = \frac{25}{5} = 5 \text{ s}$$

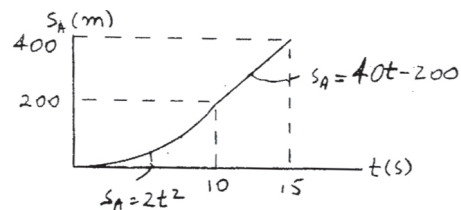
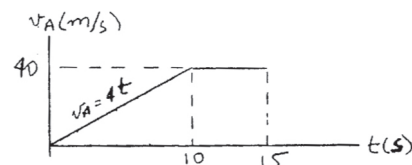
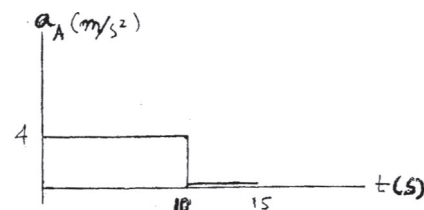
$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(5)t^2 = 2.5t^2$$

When $t = 10 \text{ s}$, $v_A = (v_A)_{\max} = 40 \text{ m/s}$ and $s_A = 200 \text{ m}$.

When $t = 5 \text{ s}$, $s_B = 62.5 \text{ m}$.

When $t = 15 \text{ s}$, $s_A = 400 \text{ m}$ and $s_B = 312.5 \text{ m}$.



12-61. Continued

At $t = 5$ s, $s_B = 62.5$ m

$t > 5$ s, $ds = v dt$

$$\int_{62.5}^{s_B} ds = \int_5^t 25 dt$$

$$s_B - 62.5 = 25t - 125$$

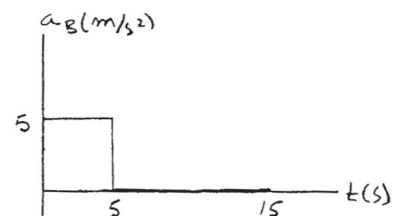
$$s_B = 25t - 62.5$$

When $t = 15$ s, $s_B = 312.5$

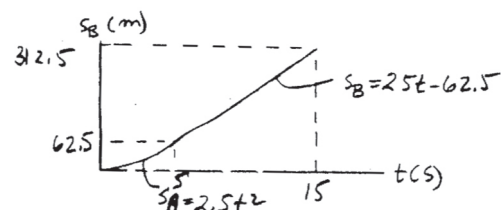
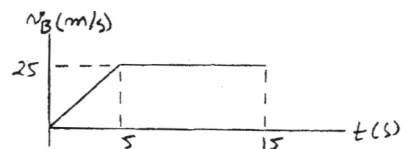
Distance between the cars is

$$\Delta s = s_A - s_B = 400 - 312.5 = 87.5 \text{ m}$$

Car A is ahead of car B .



Ans.



Ans:

When $t = 5$ s,

$$s_B = 62.5 \text{ m.}$$

When $t = 10$ s,

$$v_A = (v_A)_{\max} = 40 \text{ m/s and}$$

$$s_A = 200 \text{ m.}$$

When $t = 15$ s,

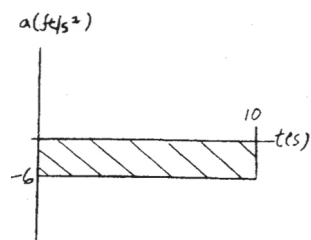
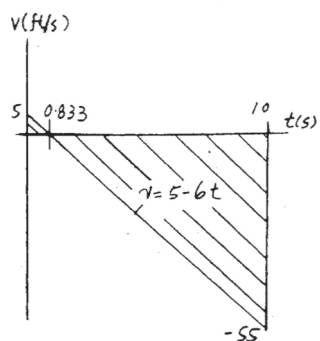
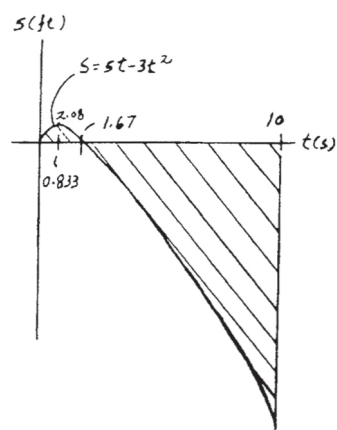
$$s_A = 400 \text{ m and } s_B = 312.5 \text{ m.}$$

$$\Delta s = s_A - s_B = 87.5 \text{ m}$$

12-62.

If the position of a particle is defined as $s = (5t - 3t^2)$ ft, where t is in seconds, construct the s - t , v - t , and a - t graphs for $0 \leq t \leq 10$ s.

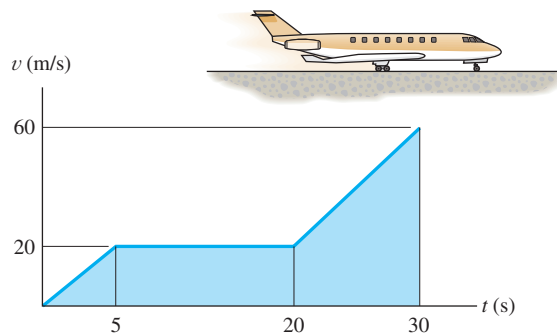
SOLUTION



Ans:
 $v = \{5 - 6t\}$ ft/s
 $a = -6$ ft/s²

12–63.

From experimental data, the motion of a jet plane while traveling along a runway is defined by the $v - t$ graph. Construct the $s - t$ and $a - t$ graphs for the motion. When $t = 0, s = 0$.



SOLUTION

$s - t$ Graph: The position in terms of time t can be obtained by applying

$$v = \frac{ds}{dt}. \text{ For time interval } 0 \leq t < 5 \text{ s, } v = \frac{20}{5}t = (4t) \text{ m/s.}$$

$$ds = v dt$$

$$\int_0^s ds = \int_0^t 4t dt$$

$$s = (2t^2) \text{ m}$$

When $t = 5 \text{ s}$,

$$s = 2(5^2) = 50 \text{ m,}$$

For time interval $5 \text{ s} < t < 20 \text{ s}$,

$$ds = v dt$$

$$\int_{50 \text{ m}}^s ds = \int_5^t 20 dt$$

$$s = (20t - 50) \text{ m}$$

When $t = 20 \text{ s}$,

$$s = 20(20) - 50 = 350 \text{ m}$$

For time interval $20 \text{ s} < t \leq 30 \text{ s}$, $\frac{v - 20}{t - 20} = \frac{60 - 20}{30 - 20}$, $v = (4t - 60) \text{ m/s}$.

$$ds = v dt$$

$$\int_{350 \text{ m}}^s ds = \int_{20}^t (4t - 60) dt$$

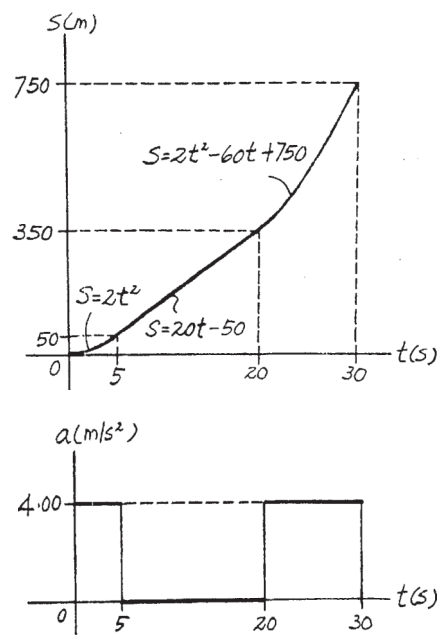
$$s = (2t^2 - 60t + 750) \text{ m}$$

When $t = 30 \text{ s}$,

$$s = 2(30^2) - 60(30) + 750 = 750 \text{ m}$$

$a - t$ Graph: The acceleration function in terms of time t can be obtained by applying $a = \frac{dv}{dt}$. For time interval $0 \leq t < 5 \text{ s}$, $5 \text{ s} < t < 20 \text{ s}$ and

$20 \text{ s} < t \leq 30 \text{ s}$, $a = \frac{dv}{dt} = 4.00 \text{ m/s}^2$, $a = \frac{dv}{dt} = 0$ and $a = \frac{dv}{dt} = 4.00 \text{ m/s}^2$, respectively.



Ans:

For $0 \leq t < 5 \text{ s}$,

$$s = \{2t^2\} \text{ m and } a = 4 \text{ m/s}^2.$$

For $5 \text{ s} < t < 20 \text{ s}$,

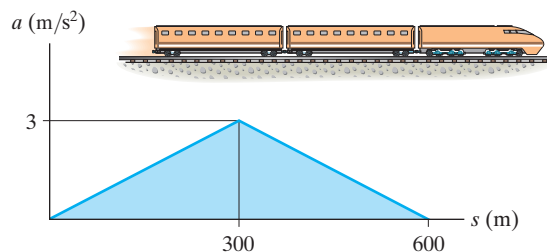
$$s = \{20t - 50\} \text{ m and } a = 0.$$

For $20 \text{ s} < t \leq 30 \text{ s}$,

$$s = \{2t^2 - 60t + 750\} \text{ m and } a = 4 \text{ m/s}^2.$$

***12-64.**

The motion of a train is described by the a - s graph shown. Draw the v - s graph if $v = 0$ at $s = 0$.



SOLUTION

v - s Graph. The v - s function can be determined by integrating $v dv = a ds$.

For $0 \leq s < 300$ m, $a = \left(\frac{3}{300}\right)s = \left(\frac{1}{100}\right)s$ m/s². Using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v dv = \int_0^s \left(\frac{1}{100}\right)s ds$$

$$\frac{v^2}{2} = \frac{1}{200} s^2$$

$$v = \left\{ \frac{1}{10} s \right\} \text{ m/s}$$

At $s = 300$ m,

$$v|_{s=300 \text{ m}} = \frac{1}{10} (300) = 30 \text{ m/s}$$

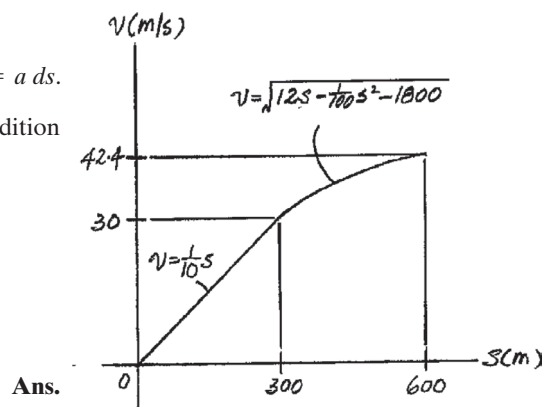
For $300 \text{ m} < s \leq 600$ m, $\frac{a-3}{s-300} = \frac{0-3}{600-300}$; $a = \left\{ -\frac{1}{100}s + 6 \right\}$ m/s², using the initial condition $v = 30$ m/s at $s = 300$ m,

$$\int_{30 \text{ m/s}}^v v dv = \int_{300 \text{ m}}^s \left(-\frac{1}{100}s + 6 \right) ds$$

$$\frac{v^2}{2} \Big|_{30 \text{ m/s}}^v = \left(-\frac{1}{200} s^2 + 6s \right) \Big|_{300 \text{ m}}^s$$

$$\frac{v^2}{2} - 450 = 6s - \frac{1}{200} s^2 - 1350$$

$$v = \left\{ \sqrt{12s - \frac{1}{100} s^2 - 1800} \right\} \text{ m/s}$$



Ans.

At $s = 600$ m,

$$v = \sqrt{12(600) - \frac{1}{100} (600^2) - 1800} = 42.43 \text{ m/s}$$

Using these results, the v - s graph shown in Fig. a can be plotted.

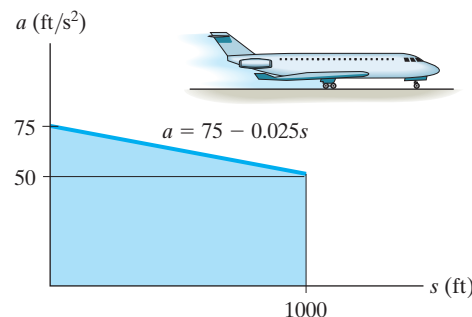
Ans.

Ans:

$$v = \left\{ \frac{1}{10} s \right\} \text{ m/s}$$

12–65.

The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has traveled 1000 ft. Also, how much time is required for it to travel 1000 ft?



SOLUTION

v - s Function. Here, $\frac{a - 75}{s - 0} = \frac{50 - 75}{1000 - 0}$; $a = \{75 - 0.025s\} \text{ ft/s}^2$. The function $v_{(s)}$ can be determined by integrating $v dv = a ds$. Using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v dv = \int_0^s (75 - 0.025s) ds$$

$$\frac{v^2}{2} = 75s - 0.0125s^2$$

$$v = \{\sqrt{150s - 0.025s^2}\} \text{ ft/s}$$

At $s = 1000 \text{ ft}$,

$$\begin{aligned} v &= \sqrt{150(1000) - 0.025(1000^2)} \\ &= 353.55 \text{ ft/s} = 354 \text{ ft/s} \end{aligned}$$

Ans.

Time. t as a function of s can be determined by integrating $dt = \frac{ds}{v}$. Using the initial condition $s = 0$ at $t = 0$;

$$\int_0^t dt = \int_0^s \frac{ds}{\sqrt{150s - 0.025s^2}}$$

$$t = \left[-\frac{1}{\sqrt{0.025}} \sin^{-1}\left(\frac{150 - 0.05s}{150}\right) \right] \Big|_0^s$$

$$t = \frac{1}{\sqrt{0.025}} \left[\frac{\pi}{2} - \sin^{-1}\left(\frac{150 - 0.05s}{150}\right) \right]$$

At $s = 1000 \text{ ft}$,

$$\begin{aligned} t &= \frac{1}{\sqrt{0.025}} \left\{ \frac{\pi}{2} - \sin^{-1}\left[\frac{150 - 0.05(1000)}{150}\right] \right\} \\ &= 5.319 \text{ s} = 5.32 \text{ s} \end{aligned}$$

Ans.

Ans:

$$v = 354 \text{ ft/s}$$

$$t = 5.32 \text{ s}$$

12-66.

The boat travels along a straight line with the speed described by the graph. Construct the s - t and a - s graphs. Also, determine the time required for the boat to travel a distance $s = 400$ m if $s = 0$ when $t = 0$.

SOLUTION

s - t Graph: For $0 \leq s < 100$ m, the initial condition is $s = 0$ when $t = 0$ s.

$$\begin{aligned} (\pm) \quad dt &= \frac{ds}{v} \\ \int_0^t dt &= \int_0^s \frac{ds}{2s^{1/2}} \\ t &= s^{1/2} \\ s &= (t^2) \text{ m} \end{aligned}$$

When $s = 100$ m,

$$100 = t^2 \quad t = 10 \text{ s}$$

For $100 \text{ m} < s \leq 400$ m, the initial condition is $s = 100$ m when $t = 10$ s.

$$\begin{aligned} (\pm) \quad dt &= \frac{ds}{v} \\ \int_{10 \text{ s}}^t dt &= \int_{100 \text{ m}}^s \frac{ds}{0.2s} \\ t - 10 &= 5 \ln \frac{s}{100} \\ \frac{t}{5} - 2 &= \ln \frac{s}{100} \\ e^{t/5-2} &= \frac{s}{100} \\ \frac{e^{t/5}}{e^2} &= \frac{s}{100} \\ s &= (13.53e^{t/5}) \text{ m} \end{aligned}$$

When $s = 400$ m,

$$\begin{aligned} 400 &= 13.53e^{t/5} \\ t &= 16.93 \text{ s} = 16.9 \text{ s} \end{aligned}$$

The s - t graph is shown in Fig. *a*.

a - s Graph: For $0 \text{ m} \leq s < 100$ m,

$$a = v \frac{dv}{ds} = (2s^{1/2})(s^{-1/2}) = 2 \text{ m/s}^2$$

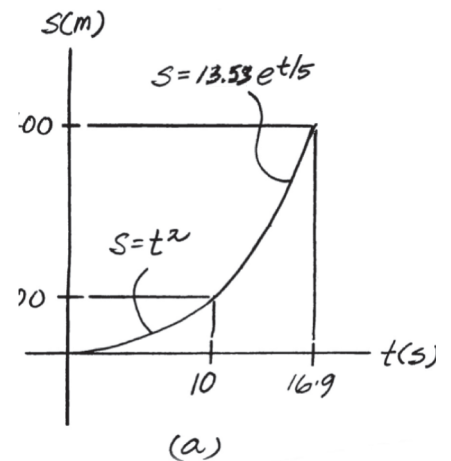
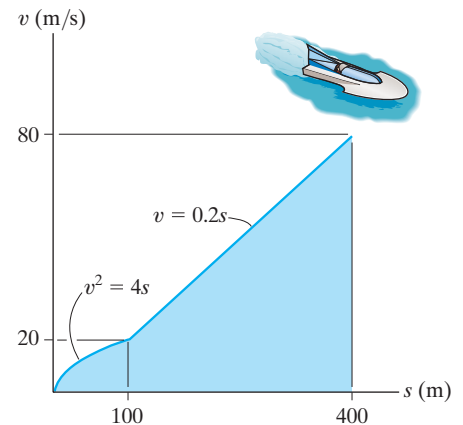
For $100 \text{ m} < s \leq 400$ m,

$$a = v \frac{dv}{ds} = (0.2s)(0.2) = 0.04s$$

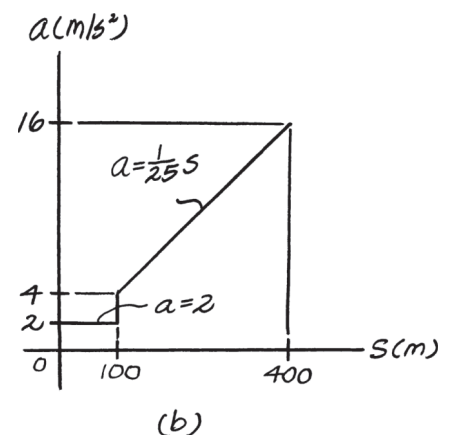
When $s = 100$ m and 400 m,

$$\begin{aligned} a|_{s=100 \text{ m}} &= 0.04(100) = 4 \text{ m/s}^2 \\ a|_{s=400 \text{ m}} &= 0.04(400) = 16 \text{ m/s}^2 \end{aligned}$$

The a - s graph is shown in Fig. *b*.



Ans.



Ans:

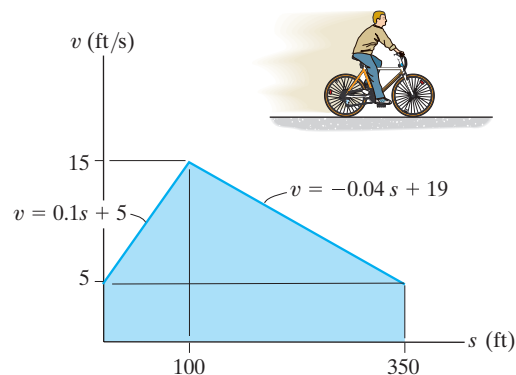
When $s = 100$ m,
 $t = 10$ s.

When $s = 400$ m,
 $t = 16.9$ s.

$a|_{s=100 \text{ m}} = 4 \text{ m/s}^2$
 $a|_{s=400 \text{ m}} = 16 \text{ m/s}^2$

12–67.

The $v-s$ graph of a cyclist traveling along a straight road is shown. Construct the $a-s$ graph.



SOLUTION

$a-s$ Graph: For $0 \leq s < 100$ ft,

$$\left(\frac{+}{\rightarrow} \right) \quad a = v \frac{dv}{ds} = (0.1s + 5)(0.1) = (0.01s + 0.5) \text{ ft/s}^2$$

Thus at $s = 0$ and 100 ft

$$a|_{s=0} = 0.01(0) + 0.5 = 0.5 \text{ ft/s}^2$$

$$a|_{s=100 \text{ ft}} = 0.01(100) + 0.5 = 1.5 \text{ ft/s}^2$$

For $100 \text{ ft} < s \leq 350$ ft,

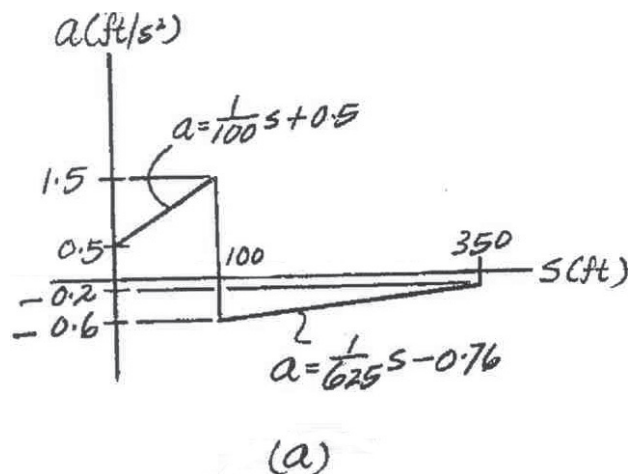
$$\left(\frac{+}{\rightarrow} \right) \quad a = v \frac{dv}{ds} = (-0.04s + 19)(-0.04) = (0.0016s - 0.76) \text{ ft/s}^2$$

Thus at $s = 100$ ft and 350 ft

$$a|_{s=100 \text{ ft}} = 0.0016(100) - 0.76 = -0.6 \text{ ft/s}^2$$

$$a|_{s=350 \text{ ft}} = 0.0016(350) - 0.76 = -0.2 \text{ ft/s}^2$$

The $a-s$ graph is shown in Fig. a .



Thus at $s = 0$ and 100 ft

$$a|_{s=0} = 0.01(0) + 0.5 = 0.5 \text{ ft/s}^2$$

$$a|_{s=100 \text{ ft}} = 0.01(100) + 0.5 = 1.5 \text{ ft/s}^2$$

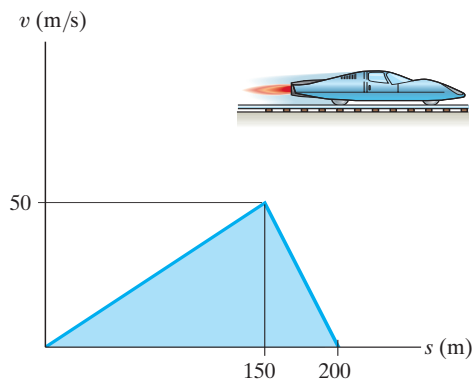
At $s = 100$ ft, a changes from $a_{\max} = 1.5 \text{ ft/s}^2$ to $a_{\min} = -0.6 \text{ ft/s}^2$.

Ans:

At $s = 100$ ft,
 a changes from $a_{\max} = 1.5 \text{ ft/s}^2$
to $a_{\min} = -0.6 \text{ ft/s}^2$.

***12–68.**

The v – s graph for a test vehicle is shown. Determine its acceleration when $s = 100$ m and when $s = 175$ m.



SOLUTION

$$0 \leq s \leq 150\text{m}; \quad v = \frac{1}{3}s,$$

$$dv = \frac{1}{3}ds$$

$$v \, dv = a \, ds$$

$$\frac{1}{3}s \left(\frac{1}{3}ds \right) = a \, ds$$

$$a = \frac{1}{9}s$$

$$\text{At } s = 100 \text{ m}, \quad a = \frac{1}{9}(100) = 11.1 \text{ m/s}^2$$

Ans.

$$150 \leq s \leq 200 \text{ m}; \quad v = 200 - s,$$

$$dv = -ds$$

$$v \, dv = a \, ds$$

$$(200 - s)(-ds) = a \, ds$$

$$a = s - 200$$

$$\text{At } s = 175 \text{ m}, \quad a = 175 - 200 = -25 \text{ m/s}^2$$

Ans.

Ans:

$$\text{At } s = 100 \text{ s}, \quad a = 11.1 \text{ m/s}^2$$

$$\text{At } s = 175 \text{ m}, \quad a = -25 \text{ m/s}^2$$

12–69.

If the velocity of a particle is defined as $\mathbf{v}(t) = \{0.8t^2\mathbf{i} + 12t^{1/2}\mathbf{j} + 5\mathbf{k}\}$ m/s, determine the magnitude and coordinate direction angles α, β, γ of the particle's acceleration when $t = 2$ s.

SOLUTION

$$\mathbf{v}(t) = 0.8t^2\mathbf{i} + 12t^{1/2}\mathbf{j} + 5\mathbf{k}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 1.6\mathbf{i} + 6t^{1/2}\mathbf{j}$$

$$\text{When } t = 2 \text{ s, } \mathbf{a} = 3.2\mathbf{i} + 4.243\mathbf{j}$$

$$a = \sqrt{(3.2)^2 + (4.243)^2} = 5.31 \text{ m/s}^2$$

Ans.

$$u_o = \frac{\mathbf{a}}{a} = 0.6022\mathbf{i} + 0.7984\mathbf{j}$$

$$\alpha = \cos^{-1}(0.6022) = 53.0^\circ$$

Ans.

$$\beta = \cos^{-1}(0.7984) = 37.0^\circ$$

Ans.

$$\gamma = \cos^{-1}(0) = 90.0^\circ$$

Ans.

Ans:
 $a = 5.31 \text{ m/s}^2$
 $\alpha = 53.0^\circ$
 $\beta = 37.0^\circ$
 $\gamma = 90.0^\circ$

12–70.

The velocity of a particle is $\mathbf{v} = \{3\mathbf{i} + (6 - 2t)\mathbf{j}\}$ m/s, where t is in seconds. If $\mathbf{r} = \mathbf{0}$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

SOLUTION

Position: The position \mathbf{r} of the particle can be determined by integrating the kinematic equation $d\mathbf{r} = \mathbf{v}dt$ using the initial condition $\mathbf{r} = \mathbf{0}$ at $t = 0$ as the integration limit. Thus,

$$\begin{aligned} d\mathbf{r} &= \mathbf{v}dt \\ \int_0^{\mathbf{r}} d\mathbf{r} &= \int_0^t [3\mathbf{i} + (6 - 2t)\mathbf{j}] dt \\ \mathbf{r} &= [3t\mathbf{i} + (6t - t^2)\mathbf{j}] \text{ m} \end{aligned}$$

When $t = 1$ s and 3 s,

$$\begin{aligned} \mathbf{r}|_{t=1 \text{ s}} &= 3(1)\mathbf{i} + [6(1) - 1^2]\mathbf{j} = [3\mathbf{i} + 5\mathbf{j}] \text{ m/s} \\ \mathbf{r}|_{t=3 \text{ s}} &= 3(3)\mathbf{i} + [6(3) - 3^2]\mathbf{j} = [9\mathbf{i} + 9\mathbf{j}] \text{ m/s} \end{aligned}$$

Thus, the displacement of the particle is

$$\begin{aligned} \Delta\mathbf{r} &= \mathbf{r}|_{t=3 \text{ s}} - \mathbf{r}|_{t=1 \text{ s}} \\ &= (9\mathbf{i} + 9\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j}) \\ &= \{6\mathbf{i} + 4\mathbf{j}\} \text{ m} \end{aligned}$$

Ans.

Ans:
 $\Delta\mathbf{r} = \{6\mathbf{i} + 4\mathbf{j}\} \text{ m}$

12–71.

A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration of $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}$ ft/s². Determine the particle's position (x, y, z) at $t = 1$ s.

SOLUTION

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\begin{aligned} dv &= \mathbf{a} dt \\ \int_0^v dv &= \int_0^t (6t\mathbf{i} + 12t^2\mathbf{k}) dt \\ v &= \{3t^2\mathbf{i} + 4t^3\mathbf{k}\} \text{ ft/s} \end{aligned}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$\begin{aligned} dr &= \mathbf{v} dt \\ \int_{\mathbf{r}_1}^{\mathbf{r}} d\mathbf{r} &= \int_0^t (3t^2\mathbf{i} + 4t^3\mathbf{k}) dt \\ \mathbf{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) &= t^3\mathbf{i} + t^4\mathbf{k} \\ \mathbf{r} &= \{(t^3 + 3)\mathbf{i} + 2\mathbf{j} + (t^4 + 5)\mathbf{k}\} \text{ ft} \end{aligned}$$

When $t = 1$ s, $\mathbf{r} = (1^3 + 3)\mathbf{i} + 2\mathbf{j} + (1^4 + 5)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}$ ft.

The coordinates of the particle are

$$(4 \text{ ft}, 2 \text{ ft}, 6 \text{ ft})$$

Ans.

Ans:
(4 ft, 2 ft, 6 ft)

***12–72.**

The velocity of a particle is given by $\mathbf{v} = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

SOLUTION

Acceleration: The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\} \text{ m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12(2^2)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\} \text{ m/s}^2$. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2 \text{ m/s}^2 \quad \textbf{Ans.}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \mathbf{v} dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t (16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}) dt$$

$$\mathbf{r} = \left[\frac{16}{3} t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2} t^2 + 2t \right) \mathbf{k} \right] \text{ m}$$

When $t = 2$ s,

$$\mathbf{r} = \frac{16}{3} (2^3)\mathbf{i} + (2^4)\mathbf{j} + \left[\frac{5}{2} (2^2) + 2(2) \right] \mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\} \text{ m.}$$

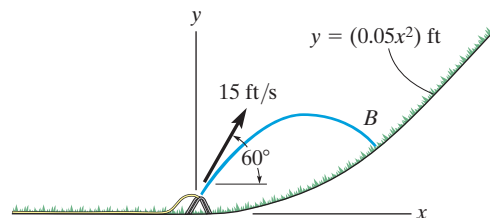
Thus, the coordinate of the particle is

$$(42.7, 16.0, 14.0) \text{ m} \quad \textbf{Ans.}$$

Ans:
(42.7, 16.0, 14.0) m

12-73.

The water sprinkler, positioned at the base of a hill, releases a stream of water with a velocity of 15 ft/s as shown. Determine the point $B(x, y)$ where the water strikes the ground on the hill. Assume that the hill is defined by the equation $y = (0.05x^2)$ ft and neglect the size of the sprinkler.



SOLUTION

$$v_x = 15 \cos 60^\circ = 7.5 \text{ ft/s} \quad v_y = 15 \sin 60^\circ = 12.99 \text{ ft/s}$$

$$\begin{aligned} (\rightarrow) \quad s &= v_0 t \\ x &= 7.5t \end{aligned}$$

$$\begin{aligned} (+\uparrow) \quad s &= s_0 + v_0 t + \frac{1}{2} a_c t^2 \\ y &= 0 + 12.99t + \frac{1}{2} (-32.2)t^2 \\ y &= 1.732x - 0.286x^2 \end{aligned}$$

$$\text{Since } y = 0.05x^2,$$

$$0.05x^2 = 1.732x - 0.286x^2$$

$$x(0.336x - 1.732) = 0$$

$$x = 5.15 \text{ ft}$$

$$y = 0.05(5.15)^2 = 1.33 \text{ ft}$$

Also,

$$(\rightarrow) \quad s = v_0 t$$

$$x = 15 \cos 60^\circ t$$

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$y = 0 + 15 \sin 60^\circ t + \frac{1}{2} (-32.2)t^2$$

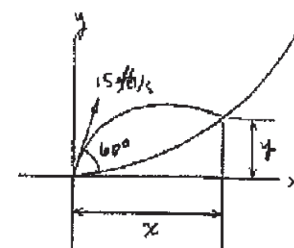
$$\text{Since } y = 0.05x^2$$

$$12.99t - 16.1t^2 = 2.8125t^2 \quad t = 0.6869 \text{ s}$$

So that,

$$x = 15 \cos 60^\circ (0.6869) = 5.15 \text{ ft}$$

$$y = 0.05(5.15)^2 = 1.33 \text{ ft}$$



Ans.

Ans.

Ans.

Ans.

Ans:
(5.15 ft, 1.33 ft)

12–74.

A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}$ ft/s². Determine the particle's position (x, y, z) when $t = 2$ s.

SOLUTION

$$\mathbf{a} = 6t\mathbf{i} + 12t^2\mathbf{k}$$

$$\int_0^v d\mathbf{v} = \int_0^t (6t\mathbf{i} + 12t^2\mathbf{k}) dt$$

$$\mathbf{v} = 3t^2\mathbf{i} + 4t^3\mathbf{k}$$

$$\int_{\mathbf{r}_0}^{\mathbf{r}} d\mathbf{r} = \int_0^t (3t^2\mathbf{i} + 4t^3\mathbf{k}) dt$$

$$\mathbf{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = t^3\mathbf{i} + t^4\mathbf{k}$$

When $t = 2$ s

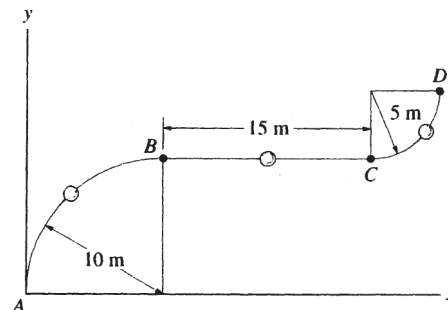
$$\mathbf{r} = \{11\mathbf{i} + 2\mathbf{j} + 21\mathbf{k}\} \text{ ft}$$

Ans.

Ans:
 $\mathbf{r} = \{11\mathbf{i} + 2\mathbf{j} + 21\mathbf{k}\} \text{ ft}$

12–75.

A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D . Determine its average speed when it goes from A to D .



SOLUTION

$$s_T = \frac{1}{4}(2\pi)(10) + 15 + \frac{1}{4}(2\pi)(5) = 38.56$$

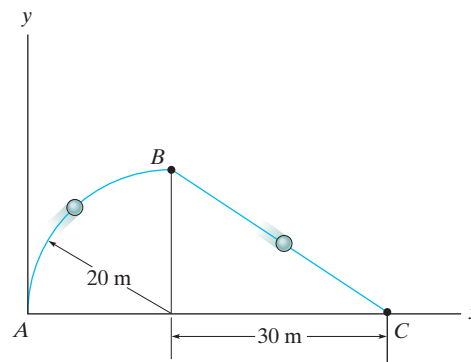
$$v_{sp} = \frac{s_T}{t_t} = \frac{38.56}{2 + 4 + 3} = 4.28 \text{ m/s}$$

Ans.

Ans:
 $(v_{sp})_{\text{avg}} = 4.28 \text{ m/s}$

***12–76.**

A particle travels along the curve from A to B in 5 s. It takes 8 s for it to go from B to C and then 10 s to go from C to A . Determine its average speed when it goes around the closed path.



SOLUTION

The total distance traveled is

$$\begin{aligned} S_{\text{Tot}} &= S_{AB} + S_{BC} + S_{CA} \\ &= 20\left(\frac{\pi}{2}\right) + \sqrt{20^2 + 30^2} + (30 + 20) \\ &= 117.47 \text{ m} \end{aligned}$$

The total time taken is

$$\begin{aligned} t_{\text{Tot}} &= t_{AB} + t_{BC} + t_{CA} \\ &= 5 + 8 + 10 \\ &= 23 \text{ s} \end{aligned}$$

Thus, the average speed is

$$(v_{\text{sp}})_{\text{avg}} = \frac{S_{\text{Tot}}}{t_{\text{Tot}}} = \frac{117.47 \text{ m}}{23 \text{ s}} = 5.107 \text{ m/s} = 5.11 \text{ m/s}$$

Ans.

Ans:
 $(v_{\text{sp}})_{\text{avg}} = 5.11 \text{ m/s}$

12–77.

The position of a crate sliding down a ramp is given by $x = (0.25t^3)$ m, $y = (1.5t^2)$ m, $z = (6 - 0.75t^{5/2})$ m, where t is in seconds. Determine the magnitude of the crate's velocity and acceleration when $t = 2$ s.

SOLUTION

Velocity: By taking the time derivative of x , y , and z , we obtain the x , y , and z components of the crate's velocity.

$$v_x = \dot{x} = \frac{d}{dt}(0.25t^3) = (0.75t^2) \text{ m/s}$$

$$v_y = \dot{y} = \frac{d}{dt}(1.5t^2) = (3t) \text{ m/s}$$

$$v_z = \dot{z} = \frac{d}{dt}(6 - 0.75t^{5/2}) = (-1.875t^{3/2}) \text{ m/s}$$

When $t = 2$ s,

$$v_x = 0.75(2^2) = 3 \text{ m/s} \quad v_y = 3(2) = 6 \text{ m/s} \quad v_z = -1.875(2)^{3/2} = -5.303 \text{ m/s}$$

Thus, the magnitude of the crate's velocity is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{3^2 + 6^2 + (-5.303)^2} = 8.551 \text{ ft/s} = 8.55 \text{ ft} \quad \textbf{Ans.}$$

Acceleration: The x , y , and z components of the crate's acceleration can be obtained by taking the time derivative of the results of v_x , v_y , and v_z , respectively.

$$a_x = \dot{v}_x = \frac{d}{dt}(0.75t^2) = (1.5t) \text{ m/s}^2$$

$$a_y = \dot{v}_y = \frac{d}{dt}(3t) = 3 \text{ m/s}^2$$

$$a_z = \dot{v}_z = \frac{d}{dt}(-1.875t^{3/2}) = (-2.815t^{1/2}) \text{ m/s}^2$$

When $t = 2$ s,

$$a_x = 1.5(2) = 3 \text{ m/s}^2 \quad a_y = 3 \text{ m/s}^2 \quad a_z = -2.8125(2^{1/2}) = -3.977 \text{ m/s}^2$$

Thus, the magnitude of the crate's acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{3^2 + 3^2 + (-3.977)^2} = 5.815 \text{ m/s}^2 = 5.82 \text{ m/s}^2 \quad \textbf{Ans.}$$

Ans:

$$v = 8.55 \text{ ft/s}$$

$$a = 5.82 \text{ m/s}^2$$

12–78.

A rocket is fired from rest at $x = 0$ and travels along a parabolic trajectory described by $y^2 = [120(10^3)x]$ m. If the x component of acceleration is $a_x = \left(\frac{1}{4}t^2\right)$ m/s², where t is in seconds, determine the magnitude of the rocket's velocity and acceleration when $t = 10$ s.

SOLUTION

Position: The parameter equation of x can be determined by integrating a_x twice with respect to t .

$$\begin{aligned}\int dv_x &= \int a_x dt \\ \int_0^{v_x} dv_x &= \int_0^t \frac{1}{4} t^2 dt \\ v_x &= \left(\frac{1}{12} t^3\right) \text{ m/s} \\ \int dx &= \int v_x dt \\ \int_0^x dx &= \int_0^t \frac{1}{12} t^3 dt \\ x &= \left(\frac{1}{48} t^4\right) \text{ m}\end{aligned}$$

Substituting the result of x into the equation of the path,

$$\begin{aligned}y^2 &= 120(10^3)\left(\frac{1}{48} t^4\right) \\ y &= (50t^2) \text{ m}\end{aligned}$$

Velocity:

$$v_y = \dot{y} = \frac{d}{dt}(50t^2) = (100t) \text{ m/s}$$

When $t = 10$ s,

$$v_x = \frac{1}{12}(10^3) = 83.33 \text{ m/s} \qquad v_y = 100(10) = 1000 \text{ m/s}$$

Thus, the magnitude of the rocket's velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{83.33^2 + 1000^2} = 1003 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_y = \dot{v}_y = \frac{d}{dt}(100t) = 100 \text{ m/s}^2$$

When $t = 10$ s,

$$a_x = \frac{1}{4}(10^2) = 25 \text{ m/s}^2$$

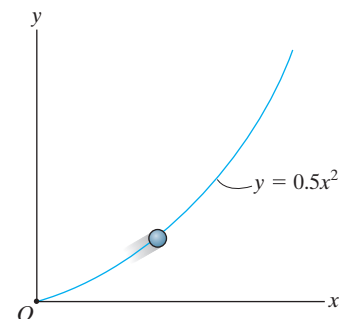
Thus, the magnitude of the rocket's acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{25^2 + 100^2} = 103 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\begin{aligned}\textbf{Ans:} \\ v &= 1003 \text{ m/s} \\ a &= 103 \text{ m/s}^2\end{aligned}$$

12–79.

The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ ft/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.



SOLUTION

Position: The x position of the particle can be obtained by applying the $v_x = \frac{dx}{dt}$.

$$\begin{aligned} dx &= v_x dt \\ \int_0^x dx &= \int_0^t 5t dt \\ x &= (2.50t^2) \text{ ft} \end{aligned}$$

Thus, $y = 0.5(2.50t^2)^2 = (3.125t^4)$ ft. At $t = 1$ s, $x = 2.5(1^2) = 2.50$ ft and $y = 3.125(1^4) = 3.125$ ft. The particle's distance from the origin at this moment is

$$d = \sqrt{(2.50 - 0)^2 + (3.125 - 0)^2} = 4.00 \text{ ft} \quad \textbf{Ans.}$$

Acceleration: Taking the first derivative of the path $y = 0.5x^2$, we have $\dot{y} = x\dot{x}$. The second derivative of the path gives

$$\ddot{y} = \dot{x}^2 + x\ddot{x} \quad (1)$$

However, $\dot{x} = v_x$, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. (1) becomes

$$a_y = v_x^2 + xa_x \quad (2)$$

When $t = 1$ s, $v_x = 5(1) = 5$ ft/s $a_x = \frac{dv_x}{dt} = 5$ ft/s², and $x = 2.50$ ft. Then, from Eq. (2)

$$a_y = 5^2 + 2.50(5) = 37.5 \text{ ft/s}^2$$

Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 37.5^2} = 37.8 \text{ ft/s}^2 \quad \textbf{Ans.}$$

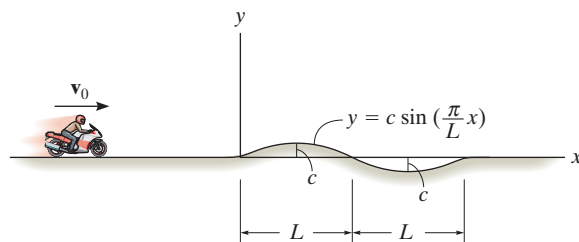
Ans:

$$d = 4.00 \text{ ft}$$

$$a = 37.8 \text{ ft/s}^2$$

***12–80.**

The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



SOLUTION

$$y = c \sin\left(\frac{\pi}{L} x\right)$$

$$\dot{y} = \frac{\pi}{L} c \left(\cos \frac{\pi}{L} x\right) \dot{x}$$

$$v_y = \frac{\pi}{L} c v_x \left(\cos \frac{\pi}{L} x\right)$$

$$v_0^2 = v_y^2 + v_x^2$$

$$v_0^2 = v_x^2 \left[1 + \left(\frac{\pi}{L} c\right)^2 \cos^2\left(\frac{\pi}{L} x\right) \right]$$

$$v_x = v_0 \left[1 + \left(\frac{\pi}{L} c\right)^2 \cos^2\left(\frac{\pi}{L} x\right) \right]^{-\frac{1}{2}}$$

$$v_y = \frac{v_0 \pi c}{L} \left(\cos \frac{\pi}{L} x\right) \left[1 + \left(\frac{\pi}{L} c\right)^2 \cos^2\left(\frac{\pi}{L} x\right) \right]^{-\frac{1}{2}}$$

Ans.

Ans.

Ans:

$$v_x = v_0 \left[1 + \left(\frac{\pi}{L} c\right)^2 \cos^2\left(\frac{\pi}{L} x\right) \right]^{-\frac{1}{2}}$$

$$v_y = \frac{v_0 \pi c}{L} \left(\cos \frac{\pi}{L} x\right) \left[1 + \left(\frac{\pi}{L} c\right)^2 \cos^2\left(\frac{\pi}{L} x\right) \right]^{-\frac{1}{2}}$$

12–81.

A particle travels along the circular path from A to B in 1 s. If it takes 3 s for it to go from A to C , determine its *average velocity* when it goes from B to C .

SOLUTION

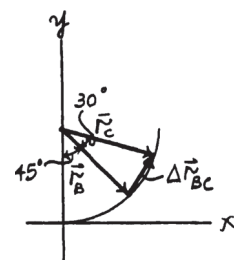
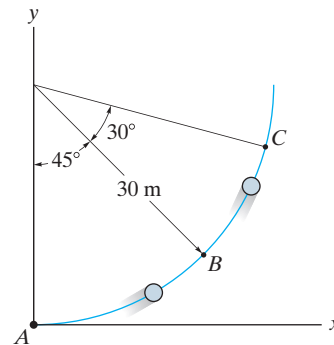
Position: The coordinates for points B and C are $[30 \sin 45^\circ, 30 - 30 \cos 45^\circ]$ and $[30 \sin 75^\circ, 30 - 30 \cos 75^\circ]$. Thus,

$$\begin{aligned}\mathbf{r}_B &= (30 \sin 45^\circ - 0)\mathbf{i} + [(30 - 30 \cos 45^\circ) - 30]\mathbf{j} \\ &= \{21.21\mathbf{i} - 21.21\mathbf{j}\} \text{ m}\end{aligned}$$

$$\begin{aligned}\mathbf{r}_C &= (30 \sin 75^\circ - 0)\mathbf{i} + [(30 - 30 \cos 75^\circ) - 30]\mathbf{j} \\ &= \{28.98\mathbf{i} - 7.765\mathbf{j}\} \text{ m}\end{aligned}$$

Average Velocity: The displacement from point B to C is $\Delta \mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B$
 $= (28.98\mathbf{i} - 7.765\mathbf{j}) - (21.21\mathbf{i} - 21.21\mathbf{j}) = \{7.765\mathbf{i} + 13.45\mathbf{j}\} \text{ m}.$

$$(\mathbf{v}_{BC})_{\text{avg}} = \frac{\Delta \mathbf{r}_{BC}}{\Delta t} = \frac{7.765\mathbf{i} + 13.45\mathbf{j}}{3 - 1} = \{3.88\mathbf{i} + 6.72\mathbf{j}\} \text{ m/s} \quad \textbf{Ans.}$$



Ans:
 $(\mathbf{v}_{BC})_{\text{avg}} = \{3.88\mathbf{i} + 6.72\mathbf{j}\} \text{ m/s}$

12–82.

The roller coaster car travels down the helical path at constant speed such that the parametric equations that define its position are $x = c \sin kt$, $y = c \cos kt$, $z = h - bt$, where c , h , and b are constants. Determine the magnitudes of its velocity and acceleration.

SOLUTION

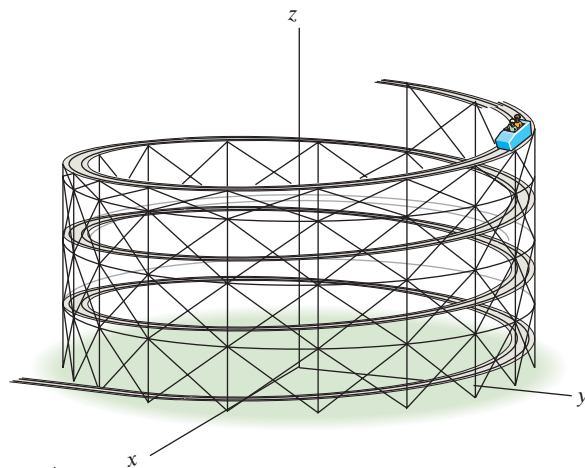
$$x = c \sin kt \quad \dot{x} = ck \cos kt \quad \ddot{x} = -ck^2 \sin kt$$

$$y = c \cos kt \quad \dot{y} = -ck \sin kt \quad \ddot{y} = -ck^2 \cos kt$$

$$z = h - bt \quad \dot{z} = -b \quad \ddot{z} = 0$$

$$v = \sqrt{(ck \cos kt)^2 + (-ck \sin kt)^2 + (-b)^2} = \sqrt{c^2 k^2 + b^2}$$

$$a = \sqrt{(-ck^2 \sin kt)^2 + (-ck^2 \cos kt)^2 + 0} = ck^2$$



Ans.

Ans.

Ans:

$$v = \sqrt{c^2 k^2 + b^2}$$

$$a = ck^2$$

12-83.

Pegs *A* and *B* are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10 m/s, determine the magnitude of the velocity and acceleration of peg *A* when $x = 1$ m.

SOLUTION

Velocity: The x and y components of the peg's velocity can be related by taking the first time derivative of the path's equation.

$$\begin{aligned}\frac{x^2}{4} + y^2 &= 1 \\ \frac{1}{4}(2x\dot{x}) + 2y\dot{y} &= 0 \\ \frac{1}{2}x\dot{x} + 2y\dot{y} &= 0\end{aligned}$$

or

$$\frac{1}{2}xv_x + 2yv_y = 0 \quad (1)$$

At $x = 1$ m,

$$\frac{(1)^2}{4} + y^2 = 1 \quad y = \frac{\sqrt{3}}{2} \text{ m}$$

Here, $v_x = 10$ m/s and $x = 1$. Substituting these values into Eq. (1),

$$\frac{1}{2}(1)(10) + 2\left(\frac{\sqrt{3}}{2}\right)v_y = 0 \quad v_y = -2.887 \text{ m/s} = 2.887 \text{ m/s} \downarrow$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{10^2 + 2.887^2} = 10.4 \text{ m/s} \quad \text{Ans.}$$

Acceleration: The x and y components of the peg's acceleration can be related by taking the second time derivative of the path's equation.

$$\begin{aligned}\frac{1}{2}(\dot{x}\dot{x} + x\ddot{x}) + 2(\dot{y}\dot{y} + y\ddot{y}) &= 0 \\ \frac{1}{2}(\dot{x}^2 + x\ddot{x}) + 2(\dot{y}^2 + y\ddot{y}) &= 0\end{aligned}$$

or

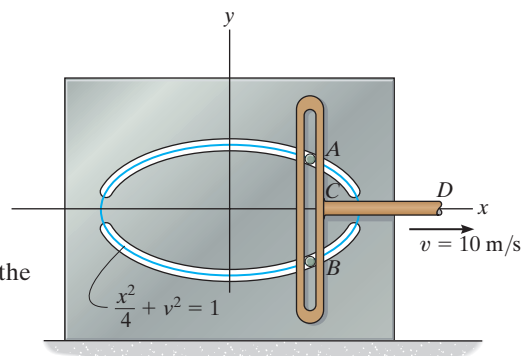
$$\frac{1}{2}(v_x^2 + xa_x) + 2(v_y^2 + ya_y) = 0 \quad (2)$$

Since v_x is constant, $a_x = 0$. When $x = 1$ m, $y = \frac{\sqrt{3}}{2}$ m, $v_x = 10$ m/s, and $v_y = -2.887$ m/s. Substituting these values into Eq. (2),

$$\begin{aligned}\frac{1}{2}(10^2 + 0) + 2\left[(-2.887)^2 + \frac{\sqrt{3}}{2}a_y\right] &= 0 \\ a_y &= -38.49 \text{ m/s}^2 = 38.49 \text{ m/s}^2 \downarrow\end{aligned}$$

Thus, the magnitude of the peg's acceleration is

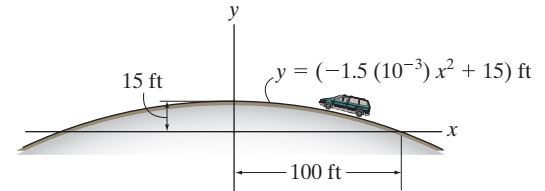
$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + (-38.49)^2} = 38.5 \text{ m/s}^2 \quad \text{Ans.}$$



$$\begin{aligned}\text{Ans:} \\ v &= 10.4 \text{ m/s} \\ a &= 38.5 \text{ m/s}^2\end{aligned}$$

***12–84.**

The van travels over the hill described by $y = (-1.5(10^{-3})x^2 + 15)$ ft. If it has a constant speed of 75 ft/s, determine the x and y components of the van's velocity and acceleration when $x = 50$ ft.



SOLUTION

Velocity: The x and y components of the van's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$y = -1.5(10^{-3})x^2 + 15$$

$$\dot{y} = -3(10^{-3})x\dot{x}$$

or

$$v_y = -3(10^{-3})xv_x$$

When $x = 50$ ft,

$$v_y = -3(10^{-3})(50)v_x = -0.15v_x \quad (1)$$

The magnitude of the van's velocity is

$$v = \sqrt{v_x^2 + v_y^2} \quad (2)$$

Substituting $v = 75$ ft/s and Eq. (1) into Eq. (2),

$$75 = \sqrt{v_x^2 + (-0.15v_x)^2}$$

$$v_x = 74.2 \text{ ft/s} \leftarrow \quad \text{Ans.}$$

Substituting the result of v_x into Eq. (1), we obtain

$$v_y = -0.15(-74.17) = 11.12 \text{ ft/s} = 11.1 \text{ ft/s} \uparrow \quad \text{Ans.}$$

Acceleration: The x and y components of the van's acceleration can be related by taking the second time derivative of the path's equation using the chain rule.

$$\ddot{y} = -3(10^{-3})(\dot{x}\dot{x} + x\ddot{x})$$

or

$$a_y = -3(10^{-3})(v_x^2 + xa_x)$$

When $x = 50$ ft, $v_x = -74.17$ ft/s. Thus,

$$a_y = -3(10^{-3})\left[(-74.17)^2 + 50a_x\right]$$

$$a_y = -(16.504 + 0.15a_x) \quad (3)$$

Since the van travels with a constant speed along the path, its acceleration along the tangent of the path is equal to zero. Here, the angle that the tangent makes with the horizontal at

$$x = 50 \text{ ft is } \theta = \tan^{-1}\left(\frac{dy}{dx}\right)\bigg|_{x=50 \text{ ft}} = \tan^{-1}\left[-3(10^{-3})x\right]\bigg|_{x=50 \text{ ft}} = \tan^{-1}(-0.15) = -8.531^\circ.$$

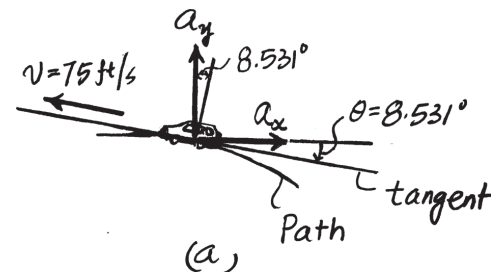
Thus, from the diagram shown in Fig. a ,

$$a_x \cos 8.531^\circ - a_y \sin 8.531^\circ = 0 \quad (4)$$

Solving Eqs. (3) and (4) yields

$$a_x = -2.42 \text{ ft/s} = 2.42 \text{ ft/s}^2 \leftarrow \quad \text{Ans.}$$

$$a_y = -16.1 \text{ ft/s} = 16.1 \text{ ft/s}^2 \downarrow \quad \text{Ans.}$$



Ans:

$$v_x = 74.2 \text{ ft/s} \leftarrow$$

$$v_y = 11.1 \text{ ft/s} \uparrow$$

$$a_x = 2.42 \text{ ft/s}^2 \leftarrow$$

$$a_y = 16.1 \text{ ft/s}^2 \downarrow$$

12–85.

The flight path of the helicopter as it takes off from *A* is defined by the parametric equations $x = (2t^2)$ m and $y = (0.04t^3)$ m, where t is the time in seconds. Determine the distance the helicopter is from point *A* and the magnitudes of its velocity and acceleration when $t = 10$ s.

SOLUTION

$$x = 2t^2 \quad y = 0.04t^3$$

$$\text{At } t = 10 \text{ s,} \quad x = 200 \text{ m} \quad y = 40 \text{ m}$$

$$d = \sqrt{(200)^2 + (40)^2} = 204 \text{ m}$$

$$v_x = \frac{dx}{dt} = 4t$$

$$a_x = \frac{dv_x}{dt} = 4$$

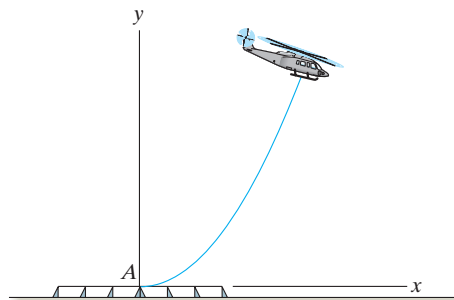
$$v_y = \frac{dy}{dt} = 0.12t^2$$

$$a_y = \frac{dv_y}{dt} = 0.24t$$

$$\text{At } t = 10 \text{ s,}$$

$$v = \sqrt{(40)^2 + (12)^2} = 41.8 \text{ m/s}$$

$$a = \sqrt{(4)^2 + (2.4)^2} = 4.66 \text{ m/s}^2$$



Ans.

Ans.

Ans.

Ans:

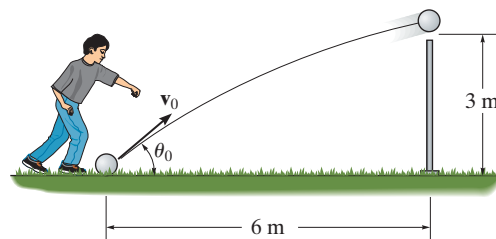
$$d = 204 \text{ m}$$

$$v = 41.8 \text{ m/s}$$

$$a = 4.66 \text{ m/s}^2$$

12-86.

Determine the minimum initial velocity v_0 and the corresponding angle θ_0 at which the ball must be kicked in order for it to just cross over the 3-m high fence.



SOLUTION

Coordinate System: The x - y coordinate system will be set so that its origin coincides with the ball's initial position.

x -Motion: Here, $(v_0)_x = v_0 \cos \theta$, $x_0 = 0$, and $x = 6$ m. Thus,

$$\begin{aligned} \left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad x &= x_0 + (v_0)_x t \\ 6 &= 0 + (v_0 \cos \theta) t \\ t &= \frac{6}{v_0 \cos \theta} \end{aligned} \quad (1)$$

y -Motion: Here, $(v_0)_y = v_0 \sin \theta$, $a_y = -g = -9.81 \text{ m/s}^2$, and $y_0 = 0$. Thus,

$$\begin{aligned} \left(\begin{array}{c} + \\ \uparrow \end{array} \right) \quad y &= y_0 + (v_0)_y t + \frac{1}{2} a_y t^2 \\ 3 &= 0 + v_0 (\sin \theta) t + \frac{1}{2} (-9.81) t^2 \\ 3 &= v_0 (\sin \theta) t - 4.905 t^2 \end{aligned} \quad (2)$$

Substituting Eq. (1) into Eq. (2) yields

$$v_0 = \sqrt{\frac{58.86}{\sin 2\theta - \cos^2 \theta}} \quad (3)$$

From Eq. (3), we notice that v_0 is minimum when $f(\theta) = \sin 2\theta - \cos^2 \theta$ is maximum. This requires $\frac{df(\theta)}{d\theta} = 0$

$$\frac{df(\theta)}{d\theta} = 2 \cos 2\theta + \sin 2\theta = 0$$

$$\tan 2\theta = -2$$

$$2\theta = 116.57^\circ$$

$$\theta = 58.28^\circ = 58.3^\circ$$

Ans.

Substituting the result of θ into Eq. (2), we have

$$(v_0)_{\min} = \sqrt{\frac{58.86}{\sin 116.57^\circ - \cos^2 58.28^\circ}} = 9.76 \text{ m/s} \quad \text{Ans.}$$

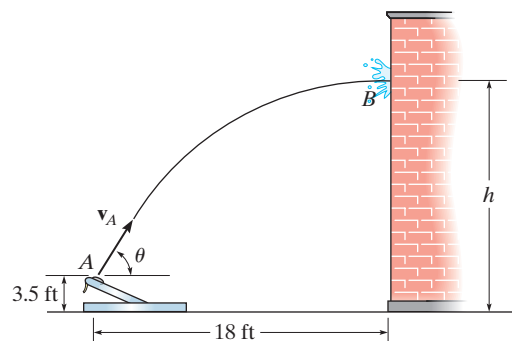
Ans:

$$\theta = 58.3^\circ$$

$$(v_0)_{\min} = 9.76 \text{ m/s}$$

12-87.

The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes 1.5 s to travel from A to B , determine the velocity v_A at which it was launched, the angle of release θ , and the height h .



SOLUTION

$$(\rightarrow) s = v_0 t$$

$$18 = v_A \cos \theta (1.5)$$

$$(+\uparrow) v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = (v_A \sin \theta)^2 + 2(-32.2)(h - 3.5)$$

$$(+\uparrow) v = v_0 + a_c t$$

$$0 = v_A \sin \theta - 32.2(1.5)$$

To solve, first divide Eq. (2) by Eq. (1) to get θ . Then

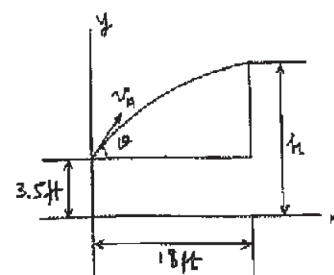
$$\theta = 76.0^\circ$$

$$v_A = 49.8 \text{ ft/s}$$

$$h = 39.7 \text{ ft}$$

(1)

(2)



Ans.

Ans.

Ans.

Ans:

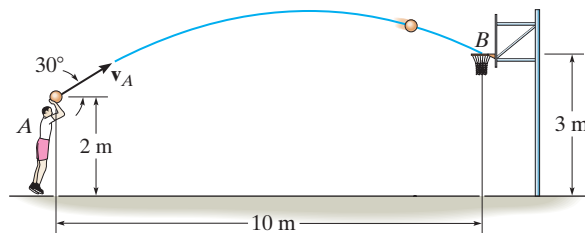
$$\theta = 76.0^\circ$$

$$v_A = 49.8 \text{ ft/s}$$

$$h = 39.7 \text{ ft}$$

***12–88.**

Neglecting the size of the ball, determine the magnitude v_A of the basketball's initial velocity and its velocity when it passes through the basket.



SOLUTION

Coordinate System. The origin of the x - y coordinate system will be set to coincide with point A as shown in Fig. a

Horizontal Motion. Here $(v_A)_x = v_A \cos 30^\circ \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 10 \text{ m} \rightarrow$.

$$\begin{aligned} (+\rightarrow) (s_B)_x &= (s_A)_x + (v_A)_x t \\ 10 &= 0 + v_A \cos 30^\circ t \\ t &= \frac{10}{v_A \cos 30^\circ} \end{aligned}$$

Also,

$$(+\rightarrow) (v_B)_x = (v_A)_x = v_A \cos 30^\circ \quad (2)$$

Vertical Motion. Here, $(v_A)_y = v_A \sin 30^\circ \uparrow$, $(s_A)_y = 0$, $(s_B)_y = 3 - 2 = 1 \text{ m} \uparrow$ and $a_y = 9.81 \text{ m/s}^2 \downarrow$

$$\begin{aligned} (+\uparrow) (s_B)_y &= (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2 \\ 1 &= 0 + v_A \sin 30^\circ t + \frac{1}{2} (-9.81) t^2 \\ 4.905 t^2 - 0.5 v_A t + 1 &= 0 \end{aligned} \quad (3)$$

Also

$$\begin{aligned} (+\uparrow) (v_B)_y &= (v_A)_y + a_y t \\ (v_B)_y &= v_A \sin 30^\circ + (-9.81) t \\ (v_B)_y &= 0.5 v_A - 9.81 t \end{aligned} \quad (4)$$

Solving Eq. (1) and (3)

$$v_A = 11.705 \text{ m/s} = 11.7 \text{ m/s}$$

Ans.

$$t = 0.9865 \text{ s}$$

Substitute these results into Eq. (2) and (4)

$$(v_B)_x = 11.705 \cos 30^\circ = 10.14 \text{ m/s} \rightarrow$$

$$(v_B)_y = 0.5(11.705) - 9.81(0.9865) = -3.825 \text{ m/s} = 3.825 \text{ m/s} \downarrow$$

Thus, the magnitude of \mathbf{v}_B is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{10.14^2 + 3.825^2} = 10.83 \text{ m/s} = 10.8 \text{ m/s} \quad \text{Ans.}$$

And its direction is defined by

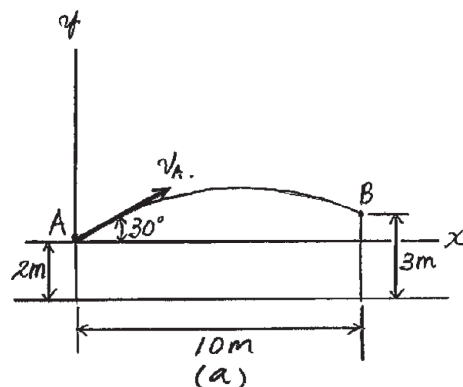
$$\theta_B = \tan^{-1} \left[\frac{(v_B)_y}{(v_B)_x} \right] = \tan^{-1} \left(\frac{3.825}{10.14} \right) = 20.67^\circ = 20.7^\circ \quad \text{Ans.}$$

Ans:

$$v_A = 11.7 \text{ m/s}$$

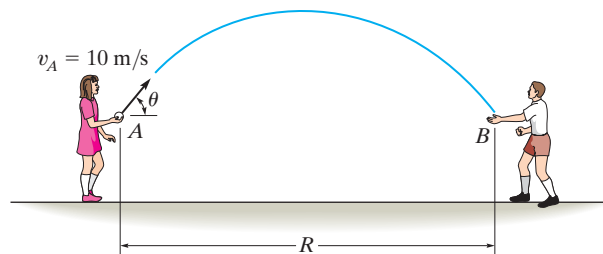
$$v_B = 10.8 \text{ m/s}$$

$$\theta = 20.7^\circ \searrow$$



12-89.

The girl at A can throw a ball at $v_A = 10 \text{ m/s}$. Calculate the maximum possible range $R = R_{\max}$ and the associated angle θ at which it should be thrown. Assume the ball is caught at B at the same elevation from which it is thrown.



SOLUTION

$$(\rightarrow) s = s_0 + v_0 t$$

$$R = 0 + (10 \cos \theta)t$$

$$(+\uparrow) v = v_0 + a_c t$$

$$-10 \sin \theta = 10 \sin \theta - 9.81t$$

$$t = \frac{20}{9.81} \sin \theta$$

$$\text{Thus, } R = \frac{200}{9.81} \sin \theta \cos \theta$$

$$R = \frac{100}{9.81} \sin 2\theta$$

Require,

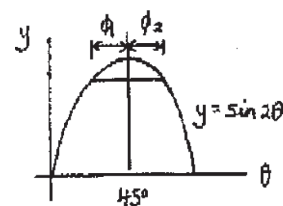
$$\frac{dR}{d\theta} = 0$$

$$\frac{100}{9.81} \cos 2\theta(2) = 0$$

$$\cos 2\theta = 0$$

$$\theta = 45^\circ$$

$$R = \frac{100}{9.81} (\sin 90^\circ) = 10.2 \text{ m}$$



(1)

Ans.

Ans.

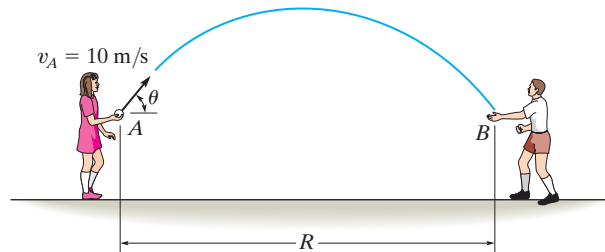
Ans:

$$R_{\max} = 10.2 \text{ m}$$

$$\theta = 45^\circ$$

12–90.

Show that the girl at A can throw the ball to the boy at B by launching it at equal angles measured up or down from a 45° inclination. If $v_A = 10$ m/s, determine the range R if this value is 15° , i.e., $\theta_1 = 45^\circ - 15^\circ = 30^\circ$ and $\theta_2 = 45^\circ + 15^\circ = 60^\circ$. Assume the ball is caught at the same elevation from which it is thrown.



SOLUTION

$$(\rightarrow) s = s_0 + v_0 t$$

$$R = 0 + (10 \cos \theta) t$$

$$(+\uparrow) v = v_0 + a_c t$$

$$-10 \sin \theta = 10 \sin \theta - 9.81 t$$

$$t = \frac{20}{9.81} \sin \theta$$

$$\text{Thus, } R = \frac{200}{9.81} \sin \theta \cos \theta$$

$$R = \frac{100}{9.81} \sin 2\theta \quad (1)$$

Since the function $y = \sin 2\theta$ is symmetric with respect to $\theta = 45^\circ$ as indicated, Eq. (1) will be satisfied if $|\phi_1| = |\phi_2|$

Choosing $\phi = 15^\circ$ or $\theta_1 = 45^\circ - 15^\circ = 30^\circ$ and $\theta_2 = 45^\circ + 15^\circ = 60^\circ$, and substituting into Eq. (1) yields

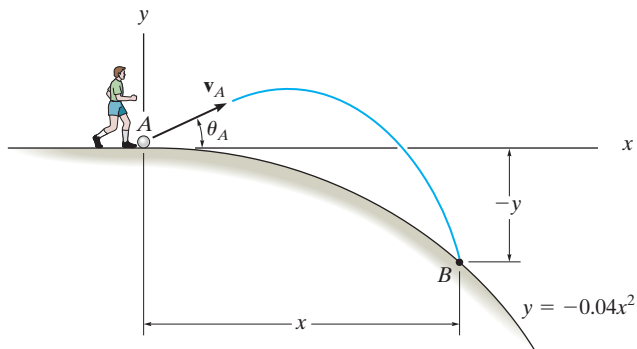
$$R = 8.83 \text{ m}$$

Ans.

Ans:
 $R = 8.83 \text{ m}$

12-91.

The ball at A is kicked with a speed $v_A = 80$ ft/s and at an angle $\theta_A = 30^\circ$. Determine the point $(x, -y)$ where it strikes the ground. Assume the ground has the shape of a parabola as shown.



SOLUTION

$$(v_A)_x = 80 \cos 30^\circ = 69.28 \text{ ft/s}$$

$$(v_A)_y = 80 \sin 30^\circ = 40 \text{ ft/s}$$

$$(\rightarrow) s = s_0 + v_0 t$$

$$x = 0 + 69.28t$$

(1)

$$(+\uparrow) s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-y = 0 + 40t + \frac{1}{2} (-32.2)t^2$$

(2)

$$y = -0.04x^2$$

From Eqs. (1) and (2):

$$-y = 0.5774x - 0.003354x^2$$

$$0.04x^2 = 0.5774x - 0.003354x^2$$

$$0.04335x^2 = 0.5774x$$

$$x = 13.3 \text{ ft}$$

Ans.

Thus

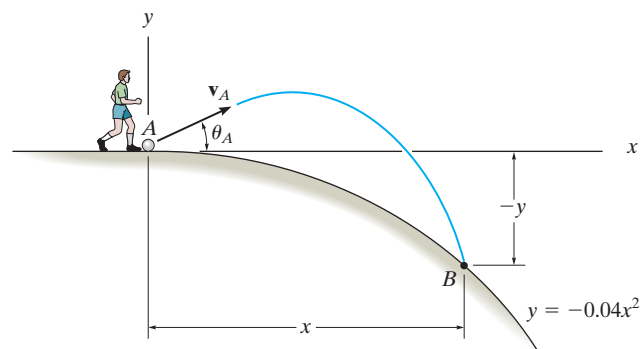
$$y = -0.04 (13.3)^2 = -7.09 \text{ ft}$$

Ans.

Ans:
(13.3 ft, -7.09 ft)

***12–92.**

The ball at A is kicked such that $\theta_A = 30^\circ$. If it strikes the ground at B having coordinates $x = 15$ ft, $y = -9$ ft, determine the speed at which it is kicked and the speed at which it strikes the ground.



SOLUTION

$$(\rightarrow) s = s_0 + v_0 t$$

$$15 = 0 + v_A \cos 30^\circ t$$

$$(+\uparrow) s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-9 = 0 + v_A \sin 30^\circ t + \frac{1}{2} (-32.2) t^2$$

$$v_A = 16.5 \text{ ft/s}$$

$$t = 1.047 \text{ s}$$

$$(\rightarrow) (v_B)_x = 16.54 \cos 30^\circ = 14.32 \text{ ft/s}$$

$$(+\uparrow) v = v_0 + a_c t$$

$$(v_B)_y = 16.54 \sin 30^\circ + (-32.2)(1.047)$$

$$= -25.45 \text{ ft/s}$$

$$v_B = \sqrt{(14.32)^2 + (-25.45)^2} = 29.2 \text{ ft/s}$$

Ans.

Ans.

Ans:

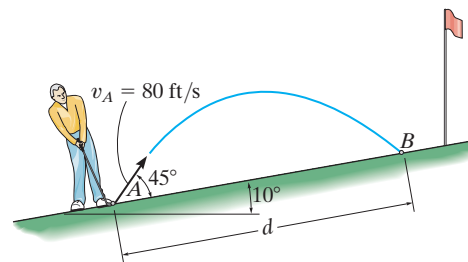
$$v_A = 16.5 \text{ ft/s}$$

$$t = 1.047 \text{ s}$$

$$v_B = 29.2 \text{ ft/s}$$

12–93.

A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.



SOLUTION

$$(\rightarrow) s = s_0 + v_0 t$$

$$d \cos 10^\circ = 0 + 80 \cos 55^\circ t$$

$$(+\uparrow) s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$d \sin 10^\circ = 0 + 80 \sin 55^\circ t - \frac{1}{2} (32.2)(t^2)$$

Solving

$$t = 3.568 \text{ s}$$

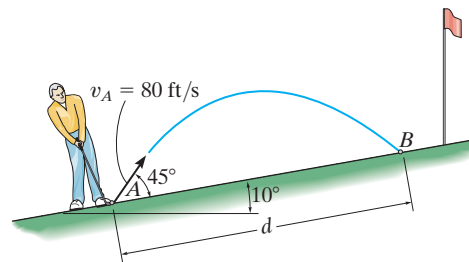
$$d = 166 \text{ ft}$$

Ans.

Ans:
 $d = 166 \text{ ft}$

12–94.

A golf ball is struck with a velocity of 80 ft/s as shown. Determine the speed at which it strikes the ground at *B* and the time of flight from *A* to *B*.



SOLUTION

$$(v_A)_x = 80 \cos 55^\circ = 44.886$$

$$(v_A)_y = 80 \sin 55^\circ = 65.532$$

$$(\rightarrow) s = s_0 + v_0 t$$

$$d \cos 10^\circ = 0 + 45.886t$$

$$(+\uparrow) s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$d \sin 10^\circ = 0 + 65.532(t) + \frac{1}{2}(-32.2)(t^2)$$

$$d = 166 \text{ ft}$$

$$t = 3.568 = 3.57 \text{ s}$$

$$(v_B)_x = (v_A)_x = 45.886$$

$$(+\uparrow) v = v_0 + a_c t$$

$$(v_B)_y = 65.532 - 32.2(3.568)$$

$$(v_B)_y = -49.357$$

$$v_B = \sqrt{(45.886)^2 + (-49.357)^2}$$

$$v_B = 67.4 \text{ ft/s}$$

Ans.

Ans.

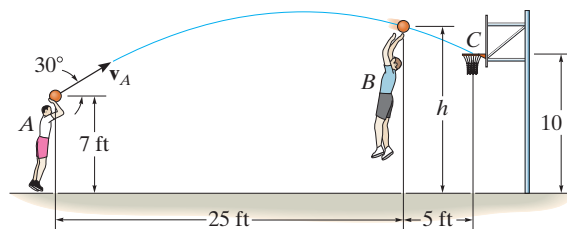
Ans:

$$t = 3.57 \text{ s}$$

$$v_B = 67.4 \text{ ft/s}$$

12–95.

The basketball passed through the hoop even though it barely cleared the hands of the player *B* who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player *B*.



SOLUTION

$$(\rightarrow) \quad s = s_0 + v_0 t$$

$$30 = 0 + v_A \cos 30^\circ t_{AC}$$

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$10 = 7 + v_A \sin 30^\circ t_{AC} - \frac{1}{2} (32.2) (t_{AC}^2)$$

Solving

$$v_A = 36.73 = 36.7 \text{ ft/s}$$

$$t_{AC} = 0.943 \text{ s}$$

$$(\rightarrow) \quad s = s_0 + v_0 t$$

$$25 = 0 + 36.73 \cos 30^\circ t_{AB}$$

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$h = 7 + 36.73 \sin 30^\circ t_{AB} - \frac{1}{2} (32.2) (t_{AB}^2)$$

Solving

$$t_{AB} = 0.786 \text{ s}$$

$$h = 11.5 \text{ ft}$$

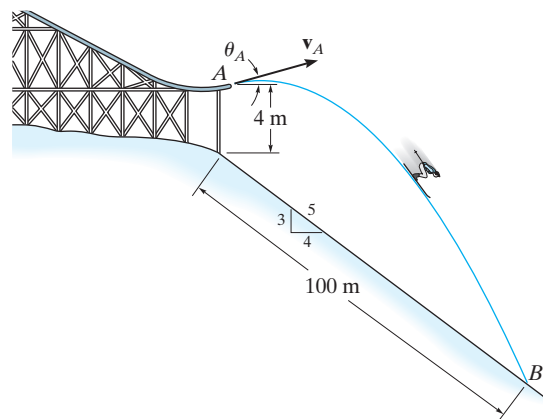
Ans.

Ans.

Ans:
 $v_A = 36.7 \text{ ft/s}$
 $h = 11.5 \text{ ft}$

***12-96.**

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25^\circ$ with the horizontal. If he strikes the ground at B , determine his initial speed v_A and the time of flight t_{AB} .



SOLUTION

$$(\rightarrow) \quad s = v_0 t$$

$$100\left(\frac{4}{5}\right) = v_A \cos 25^\circ t_{AB}$$

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-4 - 100\left(\frac{3}{5}\right) = 0 + v_A \sin 25^\circ t_{AB} + \frac{1}{2}(-9.81)t_{AB}^2$$

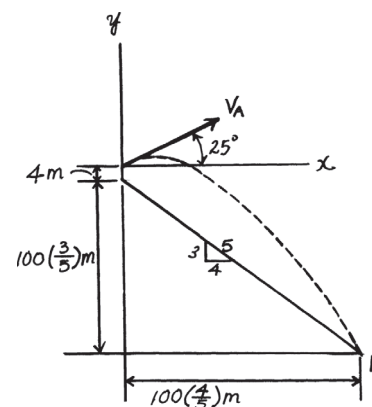
Solving,

$$v_A = 19.4 \text{ m/s}$$

$$t_{AB} = 4.54 \text{ s}$$

Ans.

Ans.



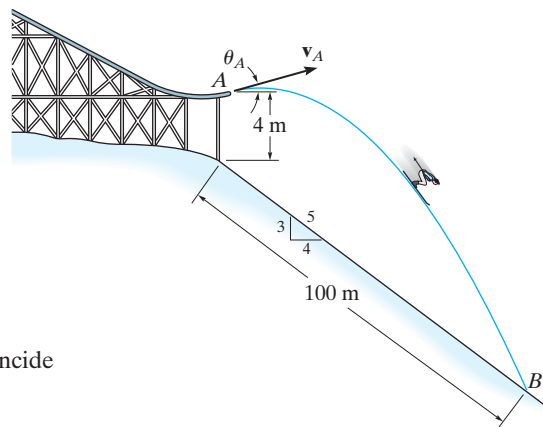
Ans:

$$v_A = 19.4 \text{ m/s}$$

$$t_{AB} = 4.54 \text{ s}$$

12-97.

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25^\circ$ with the horizontal. If he strikes the ground at B , determine his initial speed v_A and the speed at which he strikes the ground.



SOLUTION

Coordinate System: x - y coordinate system will be set with its origin to coincide with point A as shown in Fig. a .

x -motion: Here, $x_A = 0$, $x_B = 100\left(\frac{4}{5}\right) = 80$ m and $(v_A)_x = v_A \cos 25^\circ$.

$$\begin{aligned} (+\rightarrow) \quad x_B &= x_A + (v_A)_x t \\ 80 &= 0 + (v_A \cos 25^\circ) t \\ t &= \frac{80}{v_A \cos 25^\circ} \end{aligned} \quad (1)$$

y -motion: Here, $y_A = 0$, $y_B = -[4 + 100\left(\frac{3}{5}\right)] = -64$ m and $(v_A)_y = v_A \sin 25^\circ$ and $a_y = -g = -9.81$ m/s².

$$\begin{aligned} (+\uparrow) \quad y_B &= y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \\ -64 &= 0 + v_A \sin 25^\circ t + \frac{1}{2} (-9.81) t^2 \\ 4.905 t^2 - v_A \sin 25^\circ t &= 64 \end{aligned} \quad (2)$$

Substitute Eq. (1) into (2) yield

$$4.905 \left(\frac{80}{v_A \cos 25^\circ} \right)^2 = v_A \sin 25^\circ \left(\frac{80}{v_A \cos 25^\circ} \right) = 64$$

$$\left(\frac{80}{v_A \cos 25^\circ} \right)^2 = 20.65$$

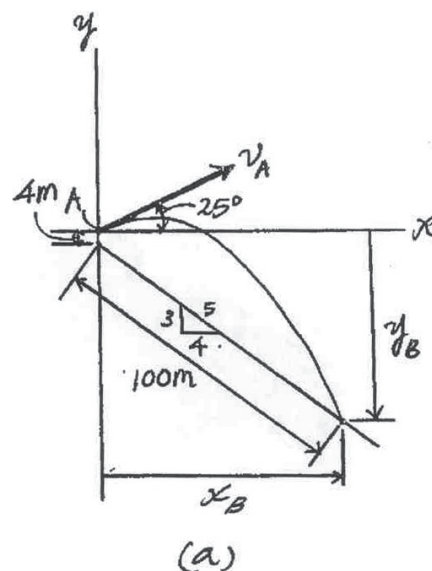
$$\frac{80}{v_A \cos 25^\circ} = 4.545$$

$$v_A = 19.42 \text{ m/s} = 19.4 \text{ m/s}$$

Ans.

Substitute this result into Eq. (1),

$$t = \frac{80}{19.42 \cos 25^\circ} = 4.54465$$



12–97. Continued

Using this result,

$$\begin{aligned} (+\uparrow) \quad (v_B)_y &= (v_A)_y + a_y t \\ &= 19.42 \sin 25^\circ + (-9.81)(4.5446) \\ &= -36.37 \text{ m/s} = 36.37 \text{ m/s} \downarrow \end{aligned}$$

And

$$(\rightarrow) \quad (v_B)_x = (v_A)_x = v_A \cos 25^\circ = 19.42 \cos 25^\circ = 17.60 \text{ m/s} \rightarrow$$

Thus,

$$\begin{aligned} v_B &= \sqrt{(v_B)_x^2 + (v_B)_y^2} \\ &= \sqrt{36.37^2 + 17.60^2} \\ &= 40.4 \text{ m/s} \end{aligned}$$

Ans.

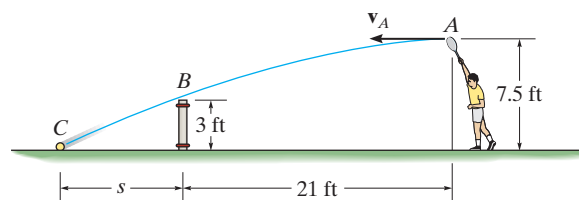
Ans:

$$v_A = 19.4 \text{ m/s}$$

$$v_B = 40.4 \text{ m/s}$$

12–98.

Determine the horizontal velocity v_A of a tennis ball at A so that it just clears the net at B . Also, find the distance s where the ball strikes the ground.



SOLUTION

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 0$. For the ball to travel from A to B , the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 3$ ft, respectively.

$$\begin{aligned} (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2} (a_c)_y t^2 \\ 3 &= 7.5 + 0 + \frac{1}{2} (-32.2) t_1^2 \\ t_1 &= 0.5287 \text{ s} \end{aligned}$$

For the ball to travel from A to C , the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 0$, respectively.

$$\begin{aligned} (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2} (a_c)_y t^2 \\ 0 &= 7.5 + 0 + \frac{1}{2} (-32.2) t_2^2 \\ t_2 &= 0.6825 \text{ s} \end{aligned}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A$. For the ball to travel from A to B , the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 21$ ft, respectively. The time is $t = t_1 = 0.5287$ s.

$$\begin{aligned} (\leftarrow) \quad s_x &= (s_0)_x + (v_0)_x t \\ 21 &= 0 + v_A (0.5287) \\ v_A &= 39.72 \text{ ft/s} = 39.7 \text{ ft/s} \end{aligned} \quad \textbf{Ans.}$$

For the ball to travel from A to C , the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (21 + s)$ ft, respectively. The time is $t = t_2 = 0.6825$ s.

$$\begin{aligned} (\leftarrow) \quad s_x &= (s_0)_x + (v_0)_x t \\ 21 + s &= 0 + 39.72(0.6825) \\ s &= 6.11 \text{ ft} \end{aligned} \quad \textbf{Ans.}$$

Ans:
 $v_A = 39.7 \text{ ft/s}$
 $s = 6.11 \text{ ft}$

12-99.

The missile at A takes off from rest and rises vertically to B , where its fuel runs out in 8 s. If the acceleration varies with time as shown, determine the missile's height h_B and speed v_B . If by internal controls the missile is then suddenly pointed 45° as shown, and allowed to travel in free flight, determine the maximum height attained, h_C , and the range R to where it crashes at D .

SOLUTION

$$a = \frac{40}{8}t = 5t$$

$$dv = a dt$$

$$\int_0^v dv = \int_0^t 5t dt$$

$$v = 2.5t^2$$

$$\text{When } t = 8 \text{ s, } v_B = 2.5(8)^2 = 160 \text{ m/s}$$

$$ds = v dt$$

$$\int_0^s ds = \int_0^t 2.5t^2 dt$$

$$s = \frac{2.5}{3}t^3$$

$$h_B = \frac{2.5}{3}(8)^3 = 426.67 = 427 \text{ m}$$

$$(v_B)_x = 160 \sin 45^\circ = 113.14 \text{ m/s}$$

$$(v_B)_y = 160 \cos 45^\circ = 113.14 \text{ m/s}$$

$$(+\uparrow) v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = (113.14)^2 + 2(-9.81)(s_c - 426.67)$$

$$h_c = 1079.1 \text{ m} = 1.08 \text{ km}$$

$$(\rightarrow) s = s_0 + v_0 t$$

$$R = 0 + 113.14t$$

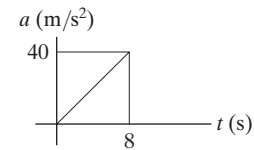
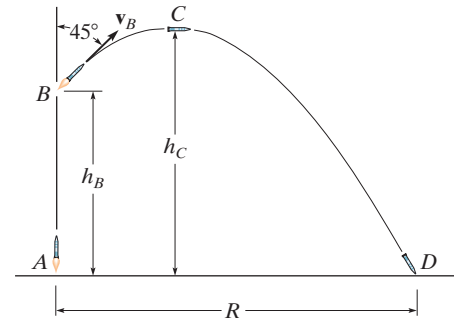
$$(+\uparrow) s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$0 = 426.67 + 113.14t + \frac{1}{2}(-9.81)t^2$$

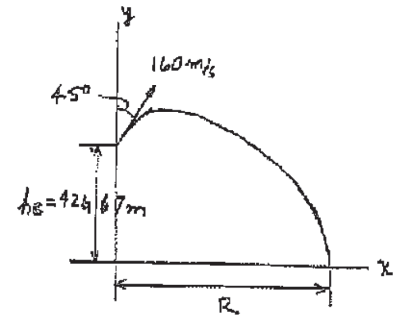
Solving for the positive root, $t = 26.36 \text{ s}$

Then,

$$R = 113.14(26.36) = 2983.0 = 2.98 \text{ km}$$



Ans.



Ans.

Ans.

Ans.

Ans:

$$v_B = 160 \text{ m/s}$$

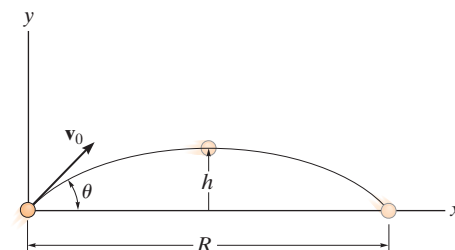
$$h_B = 427 \text{ m}$$

$$h_C = 1.08 \text{ km}$$

$$R = 2.98 \text{ km}$$

***12–100.**

The projectile is launched with a velocity v_0 . Determine the range R , the maximum height h attained, and the time of flight. Express the results in terms of the angle θ and v_0 . The acceleration due to gravity is g .



SOLUTION

$$(\rightarrow) \quad s = s_0 + v_0 t$$

$$R = 0 + (v_0 \cos \theta) t$$

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$0 = 0 + (v_0 \sin \theta) t + \frac{1}{2} (-g) t^2$$

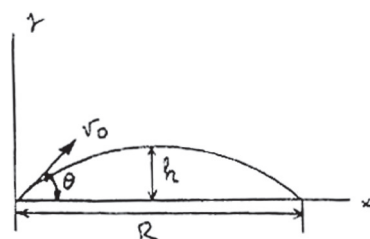
$$0 = v_0 \sin \theta - \frac{1}{2} (g) \left(\frac{R}{v_0 \cos \theta} \right)$$

$$R = \frac{v_0^2}{g} \sin 2\theta$$

$$t = \frac{R}{v_0 \cos \theta} = \frac{v_0^2 (2 \sin \theta \cos \theta)}{v_0 g \cos \theta}$$

$$= \frac{2v_0}{g} \sin \theta$$

Ans.



Ans.

$$(+\uparrow) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = (v_0 \sin \theta)^2 + 2(-g)(h - 0)$$

$$h = \frac{v_0^2}{2g} \sin^2 \theta$$

Ans.

Ans:

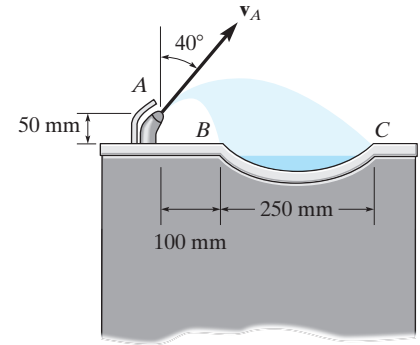
$$R = \frac{v_0^2}{g} \sin 2\theta$$

$$t = \frac{2v_0}{g} \sin \theta$$

$$h = \frac{v_0^2}{2g} \sin^2 \theta$$

12–101.

The drinking fountain is designed such that the nozzle is located from the edge of the basin as shown. Determine the maximum and minimum speed at which water can be ejected from the nozzle so that it does not splash over the sides of the basin at B and C .



SOLUTION

Horizontal Motion:

$$(\rightarrow) \quad s = v_0 t$$

$$R = v_A \sin 40^\circ t \quad t = \frac{R}{v_A \sin 40^\circ} \quad (1)$$

Vertical Motion:

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-0.05 = 0 + v_A \cos 40^\circ t + \frac{1}{2} (-9.81) t^2 \quad (2)$$

Substituting Eq.(1) into (2) yields:

$$-0.05 = v_A \cos 40^\circ \left(\frac{R}{v_A \sin 40^\circ} \right) + \frac{1}{2} (-9.81) \left(\frac{R}{v_A \sin 40^\circ} \right)^2$$

$$v_A = \sqrt{\frac{4.905 R^2}{\sin 40^\circ (R \cos 40^\circ + 0.05 \sin 40^\circ)}}$$

At point B , $R = 0.1$ m.

$$v_{\min} = v_A = \sqrt{\frac{4.905 (0.1)^2}{\sin 40^\circ (0.1 \cos 40^\circ + 0.05 \sin 40^\circ)}} = 0.838 \text{ m/s} \quad \text{Ans.}$$

At point C , $R = 0.35$ m.

$$v_{\max} = v_A = \sqrt{\frac{4.905 (0.35)^2}{\sin 40^\circ (0.35 \cos 40^\circ + 0.05 \sin 40^\circ)}} = 1.76 \text{ m/s} \quad \text{Ans.}$$

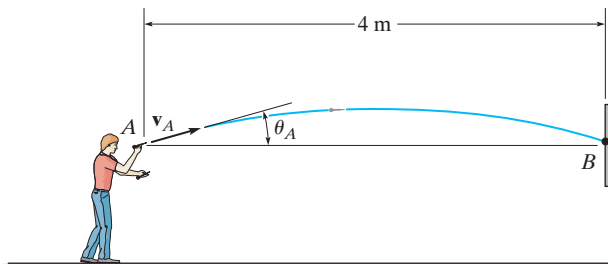
Ans:

$$v_{\min} = 0.838 \text{ m/s}$$

$$v_{\max} = 1.76 \text{ m/s}$$

12-102.

If the dart is thrown with a speed of 10 m/s, determine the shortest possible time before it strikes the target. Also, what is the corresponding angle θ_A at which it should be thrown, and what is the velocity of the dart when it strikes the target?



SOLUTION

Coordinate System. The origin of the x - y coordinate system will be set to coincide with point A as shown in Fig. a .

Horizontal Motion. Here, $(v_A)_x = 10 \cos \theta_A \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 4 \text{ m} \rightarrow$.

$$\begin{aligned} (+\rightarrow) \quad (s_B)_x &= (s_A)_x + (v_A)_x t \\ 4 &= 0 + 10 \cos \theta_A t \\ t &= \frac{4}{10 \cos \theta_A} \end{aligned}$$

Also,

$$(+\rightarrow) \quad (v_B)_x = (v_A)_x = 10 \cos \theta_A \rightarrow$$

Vertical Motion. Here, $(v_A)_y = 10 \sin \theta_A \uparrow$, $(s_A)_y = (s_B)_y = 0$ and $a_y = 9.81 \text{ m/s}^2 \downarrow$

$$\begin{aligned} (+\uparrow) \quad (s_B)_y &= (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2 \\ 0 &= 0 + (10 \sin \theta_A) t + \frac{1}{2} (-9.81) t^2 \\ 4.905 t^2 - (10 \sin \theta_A) t &= 0 \\ t (4.905 t - 10 \sin \theta_A) &= 0 \end{aligned}$$

Since $t \neq 0$, then

$$4.905 t - 10 \sin \theta_A = 0 \quad (3)$$

Also

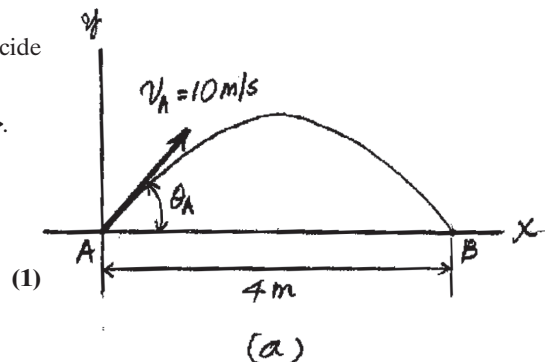
$$\begin{aligned} (+\uparrow) \quad (v_B)_y^2 &= (v_A)_y^2 + 2 a_y [(s_B)_y - (s_A)_y] \\ (v_B)_y^2 &= (10 \sin \theta_A)^2 + 2 (-9.81) (0 - 0) \\ (v_B)_y &= -10 \sin \theta_A = 10 \sin \theta_A \downarrow \end{aligned} \quad (4)$$

Substitute Eq. (1) into (3)

$$\begin{aligned} 4.905 \left(\frac{4}{10 \cos \theta_A} \right) - 10 \sin \theta_A &= 0 \\ 1.962 - 10 \sin \theta_A \cos \theta_A &= 0 \end{aligned}$$

Using the trigonometry identity $\sin 2\theta_A = 2 \sin \theta_A \cos \theta_A$, this equation becomes

$$\begin{aligned} 1.962 - 5 \sin 2\theta_A &= 0 \\ \sin 2\theta_A &= 0.3924 \\ 2\theta_A &= 23.10^\circ \text{ and } 2\theta_A = 156.90^\circ \\ \theta_A &= 11.55^\circ \text{ and } \theta_A = 78.45^\circ \end{aligned}$$



12–102. Continued

Since the shorter time is required, Eq. (1) indicates that smaller θ_A must be chosen.
Thus

$$\theta_A = 11.55^\circ = 11.6^\circ \quad \textbf{Ans.}$$

and

$$t = \frac{4}{10 \cos 11.55^\circ} = 0.4083 \text{ s} = 0.408 \text{ s} \quad \textbf{Ans.}$$

Substitute the result of θ_A into Eq. (2) and (4)

$$(v_B)_x = 10 \cos 11.55^\circ = 9.7974 \text{ m/s} \rightarrow$$

$$(v_B)_y = 10 \sin 11.55^\circ = 2.0026 \text{ m/s} \downarrow$$

Thus, the magnitude of v_B is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{9.7974^2 + 2.0026^2} = 10 \text{ m/s} \quad \textbf{Ans.}$$

And its direction is defined by

$$\theta_B = \tan^{-1} \left[\frac{(v_B)_y}{(v_B)_x} \right] = \tan^{-1} \left(\frac{2.0026}{9.7974} \right) = 11.55^\circ = 11.6^\circ \quad \nwarrow \quad \textbf{Ans.}$$

Ans:

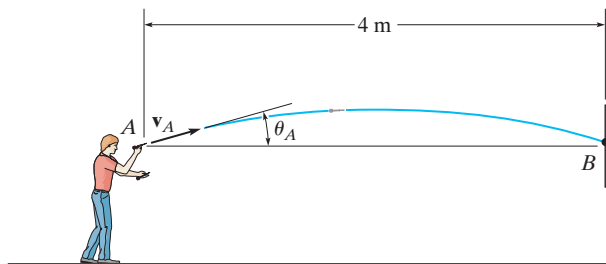
$$\theta_A = 11.6^\circ$$

$$t = 0.408 \text{ s}$$

$$\theta_B = 11.6^\circ \quad \nwarrow$$

12-103.

If the dart is thrown with a speed of 10 m/s, determine the longest possible time when it strikes the target. Also, what is the corresponding angle θ_A at which it should be thrown, and what is the velocity of the dart when it strikes the target?



SOLUTION

Coordinate System. The origin of the x - y coordinate system will be set to coincide with point A as shown in Fig. a .

Horizontal Motion. Here, $(v_A)_x = 10 \cos \theta_A \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 4 \text{ m} \rightarrow$.

$$\begin{aligned} (\rightarrow) \quad (s_B)_x &= (s_A)_x + (v_A)_x t \\ 4 &= 0 + 10 \cos \theta_A t \\ t &= \frac{4}{10 \cos \theta_A} \end{aligned}$$

Also,

$$(\rightarrow) \quad (v_B)_x = (v_A)_x = 10 \cos \theta_A \rightarrow$$

Vertical Motion. Here, $(v_A)_y = 10 \sin \theta_A \uparrow$, $(s_A)_y = (s_B)_y = 0$ and $a_y = -9.81 \text{ m/s}^2 \downarrow$.

$$\begin{aligned} (+\uparrow) \quad (s_B)_y &= (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2 \\ 0 &= 0 + (10 \sin \theta_A) t + \frac{1}{2} (-9.81) t^2 \\ 4.905 t^2 - (10 \sin \theta_A) t &= 0 \\ t (4.905 t - 10 \sin \theta_A) &= 0 \end{aligned}$$

Since $t \neq 0$, then

$$4.905 t - 10 \sin \theta_A = 0 \quad (3)$$

Also,

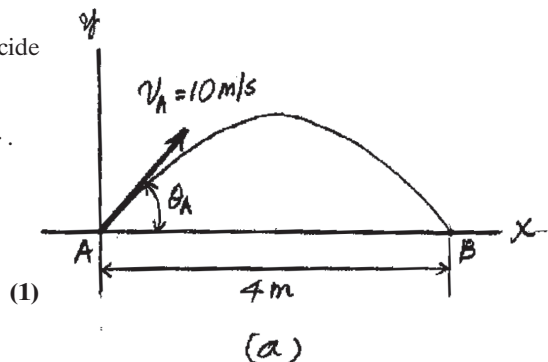
$$\begin{aligned} (v_B)_y^2 &= (v_A)_y^2 + 2 a_y [(s_B)_y - (s_A)_y] \\ (v_B)_y^2 &= (10 \sin \theta_A)^2 + 2 (-9.81) (0 - 0) \\ (v_B)_y &= -10 \sin \theta_A = 10 \sin \theta_A \downarrow \end{aligned} \quad (4)$$

Substitute Eq. (1) into (3)

$$\begin{aligned} 4.905 \left(\frac{4}{10 \cos \theta_A} \right) - 10 \sin \theta_A &= 0 \\ 1.962 - 10 \sin \theta_A \cos \theta_A &= 0 \end{aligned}$$

Using the trigonometry identity $\sin 2\theta_A = 2 \sin \theta_A \cos \theta_A$, this equation becomes

$$\begin{aligned} 1.962 - 5 \sin 2\theta_A &= 0 \\ \sin 2\theta_A &= 0.3924 \\ 2\theta_A &= 23.10^\circ \text{ and } 2\theta_A = 156.90^\circ \\ \theta_A &= 11.55^\circ \text{ and } \theta_A = 78.44^\circ \end{aligned}$$



12–103. Continued

Since the longer time is required, Eq. (1) indicates that larger θ_A must be chosen. Thus,

$$\theta_A = 78.44^\circ = 78.4^\circ \quad \textbf{Ans.}$$

and

$$t = \frac{4}{10 \cos 78.44^\circ} = 1.9974 \text{ s} = 2.00 \text{ s} \quad \textbf{Ans.}$$

Substitute the result of θ_A into Eq. (2) and (4)

$$(v_B)_x = 10 \cos 78.44^\circ = 2.0026 \text{ m/s} \rightarrow$$

$$(v_B)_y = 10 \sin 78.44^\circ = 9.7974 \text{ m/s} \downarrow$$

Thus, the magnitude of v_B is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{2.0026^2 + 9.7974^2} = 10 \text{ m/s} \quad \textbf{Ans.}$$

And its direction is defined by

$$\theta_B = \tan^{-1} \left[\frac{(v_B)_y}{(v_B)_x} \right] = \tan^{-1} \left(\frac{9.7974}{2.0026} \right) = 78.44^\circ = 78.4^\circ \quad \nwarrow \quad \textbf{Ans.}$$

Ans:

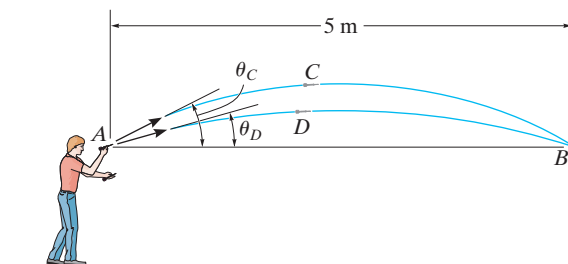
$$\theta_A = 78.4^\circ$$

$$t = 2.00 \text{ s}$$

$$\theta_B = 78.4^\circ \quad \nwarrow$$

***12–104.**

The man at A wishes to throw two darts at the target at B so that they arrive at the *same time*. If each dart is thrown with a speed of 10 m/s, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at θ_C ($>\theta_D$), then the second dart is thrown at θ_D .



SOLUTION

$$(\rightarrow) \quad s = s_0 + v_0 t$$

$$5 = 0 + (10 \cos \theta) t$$

(1)

$$(+\uparrow) \quad v = v_0 + a_c t$$

$$-10 \sin \theta = 10 \sin \theta - 9.81 t$$

$$t = \frac{2(10 \sin \theta)}{9.81} = 2.039 \sin \theta$$

From Eq. (1),

$$5 = 20.39 \sin \theta \cos \theta$$

$$\text{Since } \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\sin 2\theta = 0.4905$$

$$\text{The two roots are } \theta_D = 14.7^\circ$$

Ans.

$$\theta_C = 75.3^\circ$$

Ans.

$$\text{From Eq. (1): } t_D = 0.517 \text{ s}$$

$$t_C = 1.97 \text{ s}$$

$$\text{So that } \Delta t = t_C - t_D = 1.45 \text{ s}$$

Ans.

Ans:

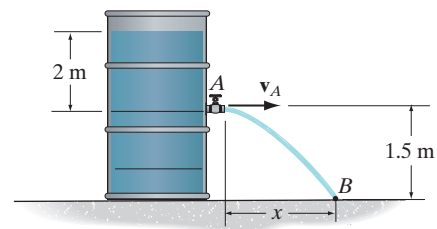
$$\theta_D = 14.7^\circ$$

$$\theta_C = 75.3^\circ$$

$$\Delta t = t_C - t_D = 1.45 \text{ s}$$

12–105.

The velocity of the water jet discharging from the orifice can be obtained from $v = \sqrt{2gh}$, where $h = 2$ m is the depth of the orifice from the free water surface. Determine the time for a particle of water leaving the orifice to reach point B and the horizontal distance x where it hits the surface.



SOLUTION

Coordinate System: The x – y coordinate system will be set so that its origin coincides with point A . The speed of the water that the jet discharges from A is

$$v_A = \sqrt{2(9.81)(2)} = 6.264 \text{ m/s}$$

x -Motion: Here, $(v_A)_x = v_A = 6.264$ m/s, $x_A = 0$, $x_B = x$, and $t = t_A$. Thus,

$$\begin{aligned} (\rightarrow) \quad x_B &= x_A + (v_A)_x t \\ x &= 0 + 6.264 t_A \end{aligned} \quad (1)$$

y -Motion: Here, $(v_A)_y = 0$, $a_y = -g = -9.81$ m/s², $y_A = 0$ m, $y_B = -1.5$ m, and $t = t_A$. Thus,

$$\begin{aligned} (+\uparrow) \quad y_B &= y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \\ -1.5 &= 0 + 0 + \frac{1}{2} (-9.81) t_A^2 \\ t_A &= 0.553 \text{ s} \end{aligned} \quad \text{Ans.}$$

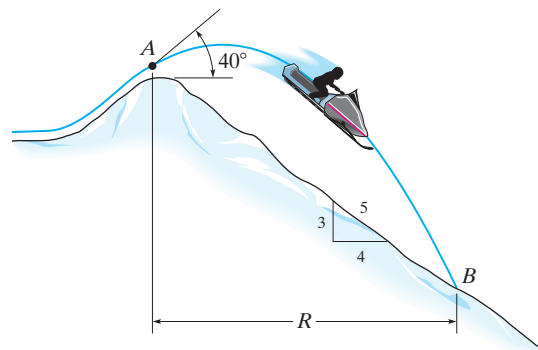
Thus,

$$x = 0 + 6.264(0.553) = 3.46 \text{ m} \quad \text{Ans.}$$

Ans:
 $t_A = 0.553 \text{ s}$
 $x = 3.46 \text{ m}$

12–106.

The snowmobile is traveling at 10 m/s when it leaves the embankment at *A*. Determine the time of flight from *A* to *B* and the range *R* of the trajectory.



SOLUTION

$$(\rightarrow) \quad s_B = s_A + v_A t$$

$$R = 0 + 10 \cos 40^\circ t$$

$$(+\uparrow) \quad s_B = s_A + v_A t + \frac{1}{2} a_c t^2$$

$$-R\left(\frac{3}{4}\right) = 0 + 10 \sin 40^\circ t - \frac{1}{2} (9.81) t^2$$

Solving:

$$R = 19.0 \text{ m}$$

$$t = 2.48 \text{ s}$$

Ans.

Ans.

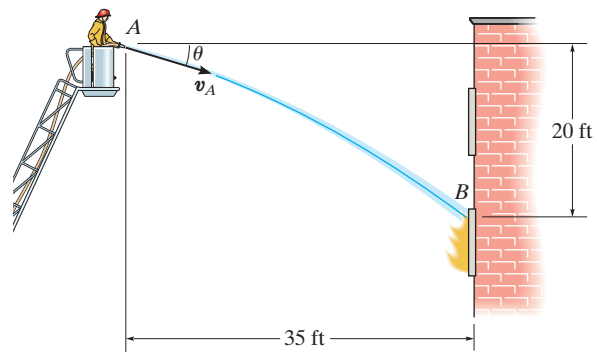
Ans:

$$R = 19.0 \text{ m}$$

$$t = 2.48 \text{ s}$$

12-107.

The fireman wishes to direct the flow of water from his hose to the fire at B . Determine two possible angles θ_1 and θ_2 at which this can be done. Water flows from the hose at $v_A = 80 \text{ ft/s}$.



SOLUTION

$$(\rightarrow) \quad s = s_0 + v_0 t$$

$$35 = 0 + (80)(\cos \theta)t$$

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-20 = 0 - 80 (\sin \theta)t + \frac{1}{2} (-32.2)t^2$$

Thus,

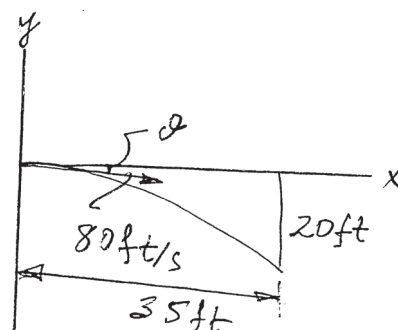
$$20 = 80 \sin \theta \frac{0.4375}{\cos \theta} t + 16.1 \left(\frac{0.1914}{\cos^2 \theta} \right)$$

$$20 \cos^2 \theta = 17.5 \sin 2\theta + 3.0816$$

Solving,

$$\theta_1 = 24.9^\circ \quad (\text{below the horizontal})$$

$$\theta_2 = 85.2^\circ \quad (\text{above the horizontal})$$



Ans.

Ans.

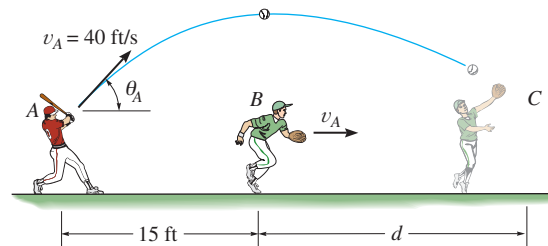
Ans:

$$\theta_1 = 24.9^\circ \searrow$$

$$\theta_2 = 85.2^\circ \nearrow$$

***12–108.**

The baseball player *A* hits the baseball at $v_A = 40$ ft/s and $\theta_A = 60^\circ$ from the horizontal. When the ball is directly overhead of player *B* he begins to run under it. Determine the constant speed at which *B* must run and the distance d in order to make the catch at the same elevation at which the ball was hit.



SOLUTION

Vertical Motion: The vertical component of initial velocity for the football is $(v_0)_y = 40 \sin 60^\circ = 34.64$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 0$, respectively.

$$(+\uparrow) \quad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2} (a_c)_y t^2$$

$$0 = 0 + 34.64t + \frac{1}{2} (-32.2) t^2$$

$$t = 2.152 \text{ s}$$

Horizontal Motion: The horizontal component of velocity for the baseball is $(v_0)_x = 40 \cos 60^\circ = 20.0$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = R$, respectively.

$$(\rightarrow) \quad s_x = (s_0)_x + (v_0)_x t$$

$$R = 0 + 20.0(2.152) = 43.03 \text{ ft}$$

The distance for which player *B* must travel in order to catch the baseball is

$$d = R - 15 = 43.03 - 15 = 28.0 \text{ ft}$$

Ans.

Player *B* is required to run at a same speed as the horizontal component of velocity of the baseball in order to catch it.

$$v_B = 40 \cos 60^\circ = 20.0 \text{ ft/s}$$

Ans.

Ans:

$$d = 28.0 \text{ ft}$$

$$v_B = 20.0 \text{ ft/s}$$

12–109.

The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes 1.5 s to travel from A to B , determine the velocity v_A at which it was launched, the angle of release θ , and the height h .

SOLUTION

$$(\rightarrow) \quad s = v_0 t$$

$$18 = v_A \cos \theta (1.5)$$

$$(+\uparrow) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = (v_A \sin \theta)^2 + 2(-32.2)(h - 3.5)$$

$$(+\uparrow) \quad v = v_0 + a_c t$$

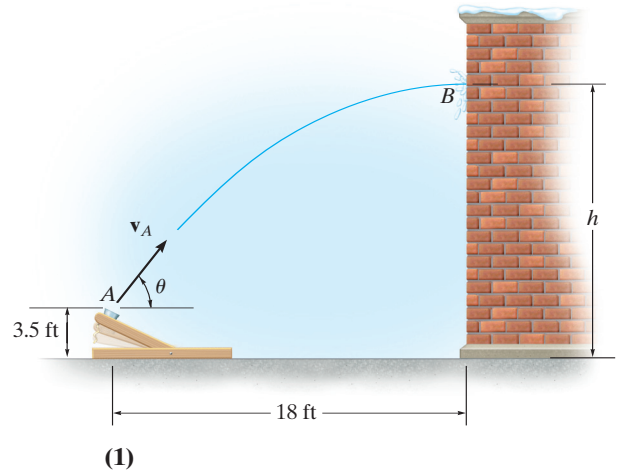
$$0 = v_A \sin \theta - 32.2(1.5)$$

To solve, first divide Eq. (2) by Eq. (1), to get θ . Then

$$\theta = 76.0^\circ$$

$$v_A = 49.8 \text{ ft/s}$$

$$h = 39.7 \text{ ft}$$

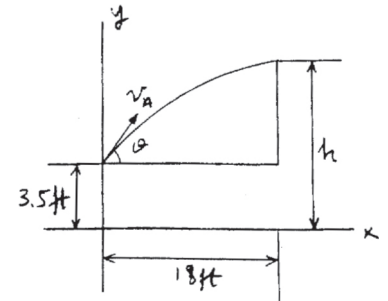


(2)

Ans.

Ans.

Ans.



Ans:

$$\theta = 76.0^\circ$$

$$v_A = 49.8 \text{ ft/s}$$

$$h = 39.7 \text{ ft}$$

12–110.

An automobile is traveling on a curve having a radius of 800 ft. If the acceleration of the automobile is 5 ft/s^2 , determine the constant speed at which the automobile is traveling.

SOLUTION

Acceleration: Since the automobile is traveling at a constant speed, $a_t = 0$.

Thus, $a_n = a = 5 \text{ ft/s}^2$. Applying Eq. 12–20, $a_n = \frac{v^2}{\rho}$, we have

$$v = \sqrt{\rho a_n} = \sqrt{800(5)} = 63.2 \text{ ft/s} \qquad \mathbf{Ans.}$$

Ans:
 $v = 63.2 \text{ ft/s}$

12–111.

Determine the maximum constant speed a race car can have if the acceleration of the car cannot exceed 7.5 m/s^2 while rounding a track having a radius of curvature of 200 m.

SOLUTION

Acceleration: Since the speed of the race car is constant, its tangential component of acceleration is zero, i.e., $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho}$$

$$7.5 = \frac{v^2}{200}$$

$$v = 38.7 \text{ m/s}$$

Ans.

Ans:
 $v = 38.7 \text{ m/s}$

***12–112.**

A boat has an initial speed of 16 ft/s. If it then increases its speed along a circular path of radius $\rho = 80$ ft at the rate of $\dot{v} = (1.5s)$ ft/s, where s is in feet, determine the time needed for the boat to travel $s = 50$ ft.

SOLUTION

$$a_t = 1.5s$$

$$\int_0^s 1.5s \, ds = \int_{16}^v v \, dv$$

$$0.75 s^2 = 0.5 v^2 - 128$$

$$v = \frac{ds}{dt} = \sqrt{256 + 1.5 s^2}$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 170.7}} = \int_0^t 1.225 \, dt$$

$$\ln \left(s + \sqrt{s^2 + 170.7} \right) \Big|_0^s = 1.225t$$

$$\ln \left(s + \sqrt{s^2 + 170.7} \right) - 2.570 = 1.225t$$

At $s = 50$ ft,

$$t = 1.68 \text{ s}$$

Ans.

Ans:
 $t = 1.68 \text{ s}$

12–113.

The position of a particle is defined by $\mathbf{r} = \{4(t - \sin t)\mathbf{i} + (2t^2 - 3)\mathbf{j}\}$ m, where t is in seconds and the argument for the sine is in radians. Determine the speed of the particle and its normal and tangential components of acceleration when $t = 1$ s.

SOLUTION

$$\mathbf{r} = 4(t - \sin t)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 4(1 - \cos t)\mathbf{i} + (4t)\mathbf{j}$$

$$\mathbf{v}|_{t=1} = 1.83879\mathbf{i} + 4\mathbf{j}$$

$$v = \sqrt{(1.83879)^2 + (4)^2} = 4.40 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{4}{1.83879}\right) = 65.312^\circ \angle \theta$$

$$\mathbf{a} = 4 \sin t \mathbf{i} + 4\mathbf{j}$$

$$\mathbf{a}|_{t=1} = 3.3659\mathbf{i} + 4\mathbf{j}$$

$$a = \sqrt{(3.3659)^2 + (4)^2} = 5.22773 \text{ m/s}^2$$

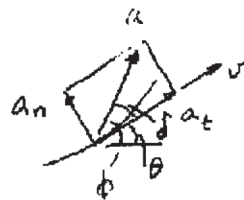
$$\phi = \tan^{-1}\left(\frac{4}{3.3659}\right) = 49.920^\circ \angle \phi$$

$$\delta = \theta - \phi = 15.392^\circ$$

$$a_t = 5.22773 \cos 15.392^\circ = 5.04 \text{ m/s}^2$$

$$a_n = 5.22773 \sin 15.392^\circ = 1.39 \text{ m/s}^2$$

Ans.



Ans.

Ans.

Ans:

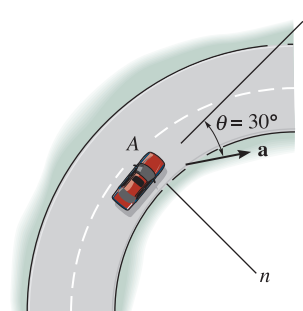
$$v = 4.40 \text{ m/s}$$

$$a_t = 5.04 \text{ m/s}^2$$

$$a_n = 1.39 \text{ m/s}^2$$

12–114.

The automobile has a speed of 80 ft/s at point *A* and an acceleration **a** having a magnitude of 10 ft/s², acting in the direction shown. Determine the radius of curvature of the path at point *A* and the tangential component of acceleration.



SOLUTION

Acceleration: The tangential acceleration is

$$a_t = a \cos 30^\circ = 10 \cos 30^\circ = 8.66 \text{ ft/s}^2 \quad \textbf{Ans.}$$

and the normal acceleration is $a_n = a \sin 30^\circ = 10 \sin 30^\circ = 5.00 \text{ ft/s}^2$. Applying

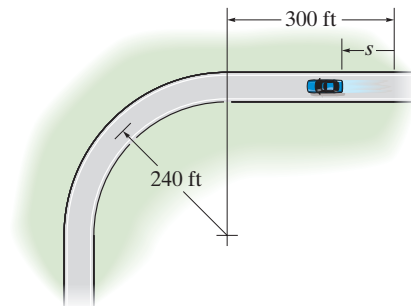
Eq. 12–20, $a_n = \frac{v^2}{\rho}$, we have

$$\rho = \frac{v^2}{a_n} = \frac{80^2}{5.00} = 1280 \text{ ft} \quad \textbf{Ans.}$$

Ans:
 $a_t = 8.66 \text{ ft/s}^2$
 $\rho = 1280 \text{ ft}$

12–115.

The automobile is originally at rest at $s = 0$. If its speed is increased by $\dot{v} = (0.05t^2) \text{ ft/s}^2$, where t is in seconds, determine the magnitudes of its velocity and acceleration when $t = 18 \text{ s}$.



SOLUTION

$$a_t = 0.05t^2$$

$$\int_0^v dv = \int_0^t 0.05 t^2 dt$$

$$v = 0.0167 t^3$$

$$\int_0^s ds = \int_0^t 0.0167 t^3 dt$$

$$s = 4.167(10^{-3}) t^4$$

When $t = 18 \text{ s}$, $s = 437.4 \text{ ft}$

Therefore the car is on a curved path.

$$v = 0.0167(18^3) = 97.2 \text{ ft/s}$$

Ans.

$$a_n = \frac{(97.2)^2}{240} = 39.37 \text{ ft/s}^2$$

$$a_t = 0.05(18^2) = 16.2 \text{ ft/s}^2$$

$$a = \sqrt{(39.37)^2 + (16.2)^2}$$

$$a = 42.6 \text{ ft/s}^2$$

Ans.

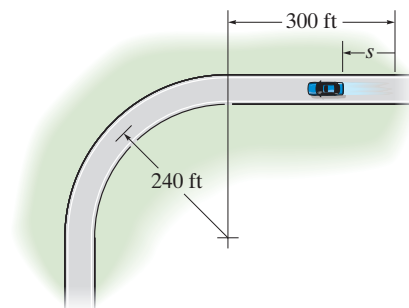
Ans:

$$v = 97.2 \text{ ft/s}$$

$$a = 42.6 \text{ ft/s}^2$$

***12–116.**

The automobile is originally at rest $s = 0$. If it then starts to increase its speed at $\dot{v} = (0.05t^2) \text{ ft/s}^2$, where t is in seconds, determine the magnitudes of its velocity and acceleration at $s = 550 \text{ ft}$.



SOLUTION

The car is on the curved path.

$$a_t = 0.05 t^2$$

$$\int_0^v dv = \int_0^t 0.05 t^2 dt$$

$$v = 0.0167 t^3$$

$$\int_0^s ds = \int_0^t 0.0167 t^3 dt$$

$$s = 4.167(10^{-3}) t^4$$

$$550 = 4.167(10^{-3}) t^4$$

$$t = 19.06 \text{ s}$$

So that

$$v = 0.0167(19.06)^3 = 115.4$$

$$v = 115 \text{ ft/s}$$

Ans.

$$a_n = \frac{(115.4)^2}{240} = 55.51 \text{ ft/s}^2$$

$$a_t = 0.05(19.06)^2 = 18.17 \text{ ft/s}^2$$

$$a = \sqrt{(55.51)^2 + (18.17)^2} = 58.4 \text{ ft/s}^2$$

Ans.

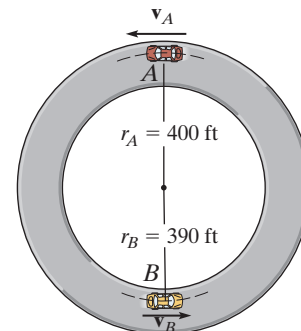
Ans:

$$v = 115 \text{ ft/s}$$

$$a = 58.4 \text{ ft/s}^2$$

12-117.

The two cars A and B travel along the circular path at constant speeds $v_A = 80 \text{ ft/s}$ and $v_B = 100 \text{ ft/s}$, respectively. If they are at the positions shown when $t = 0$, determine the time when the cars are side by side, and the time when they are 90° apart.



SOLUTION

- a) Referring to Fig. a , when cars A and B are side by side, the relation between their angular displacements is

$$\theta_B = \theta_A + \pi \quad (1)$$

Here, $s_A = v_A t = 80 t$ and $s_B = v_B t = 100 t$. Apply the formula $s = r\theta$ or $\theta = \frac{s}{r}$. Then

$$\theta_B = \frac{s_B}{r_B} = \frac{100 t}{390} = \frac{10}{39} t$$

$$\theta_A = \frac{s_A}{r_A} = \frac{80 t}{400} = \frac{1}{5} t$$

Substitute these results into Eq. (1)

$$\frac{10}{39} t = \frac{1}{5} t + \pi$$

$$t = 55.69 \text{ s} = 55.7 \text{ s}$$

Ans.

- (b) Referring to Fig. a , when cars A and B are 90° apart, the relation between their angular displacements is

$$\theta_B + \frac{\pi}{2} = \theta_A + \pi$$

$$\theta_B = \theta_A + \frac{\pi}{2} \quad (2)$$

Here, $s_A = v_A t = 80 t$ and $s_B = v_B t = 100 t$. Applying the formula $s = r\theta$ or $\theta = \frac{s}{r}$.

Then

$$\theta_B = \frac{s_B}{r_B} = \frac{100 t}{390} = \frac{10}{39} t$$

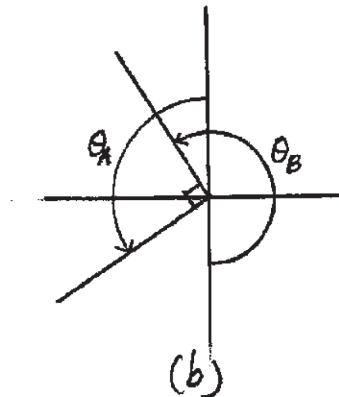
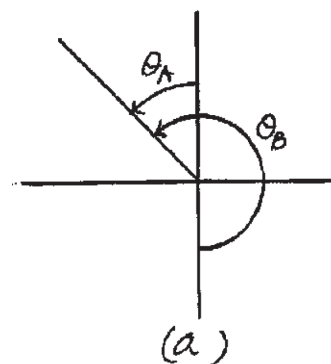
$$\theta_A = \frac{s_A}{r_A} = \frac{80 t}{400} = \frac{1}{5} t$$

Substitute these results into Eq. (2)

$$\frac{10}{39} t = \frac{1}{5} t + \frac{\pi}{2}$$

$$t = 27.84 \text{ s} = 27.8 \text{ s}$$

Ans.



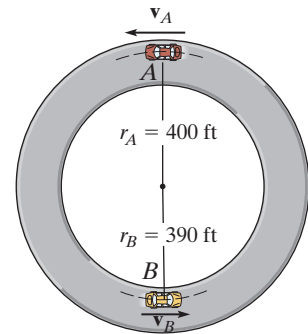
Ans:

When cars A and B are side by side, $t = 55.7 \text{ s}$.

When cars A and B are 90° apart, $t = 27.8 \text{ s}$.

12-118.

Cars A and B are traveling around the circular race track. At the instant shown, A has a speed of 60 ft/s and is increasing its speed at the rate of 15 ft/s^2 until it travels for a distance of $100\pi \text{ ft}$, after which it maintains a constant speed. Car B has a speed of 120 ft/s and is decreasing its speed at 15 ft/s^2 until it travels a distance of $65\pi \text{ ft}$, after which it maintains a constant speed. Determine the time when they come side by side.



SOLUTION

Referring to Fig. a , when cars A and B are side by side, the relation between their angular displacements is

$$\theta_A = \theta_B + \pi \quad (1)$$

The constant speed achieved by cars A and B can be determined from

$$(v_A)_c^2 = (v_A)_0^2 + 2(a_A)_t[s_A - (s_0)_A]$$

$$(v_A)_c^2 = 60^2 + 2(15)(100\pi - 0)$$

$$(v_A)_c = 114.13 \text{ ft/s}$$

$$(v_B)_c^2 = (v_B)_0^2 + 2(a_B)_t[s_B - (s_0)_B]$$

$$(v_B)_c^2 = 120^2 + 2(-15)(65\pi - 0)$$

$$(v_B)_c = 90.96 \text{ ft/s}$$

The time taken to achieve these constant speeds can be determined from

$$(v_A)_c = (v_A)_0 + (a_A)_t(t_A)_1$$

$$114.13 = 60 + 15(t_A)_1$$

$$(t_A)_1 = 3.6084 \text{ s}$$

$$(v_B)_c = (v_B)_0 + (a_B)_t(t_B)_1$$

$$90.96 = 120 + (-15)(t_B)_1$$

$$(t_B)_1 = 1.9359 \text{ s}$$

Let t be the time taken for cars A and B to be side by side. Then, the times at which cars A and B travel with constant speed are $(t_A)_2 = t - (t_A)_1 = t - 3.6084$ and $(t_B)_2 = t - (t_B)_1 = t - 1.9359$. Here, $(s_A)_1 = 100\pi$, $(s_A)_2 = (v_A)_c(t_A)_2 = 114.13(t - 3.6084)$, $(s_B)_1 = 65\pi$ and $(s_B)_2 = (v_B)_c(t_B)_2 = 90.96(t - 1.9359)$.

Using the formula $s = r\theta$ or $\theta = \frac{s}{r}$,

$$\begin{aligned} \theta_A = (\theta_A)_1 + (\theta_A)_2 &= \frac{(s_A)_1}{r_A} + \frac{(s_A)_2}{r_A} = \frac{100\pi}{400} + \frac{114.13(t - 3.6084)}{400} \\ &= 0.2853t - 0.24414 \end{aligned}$$

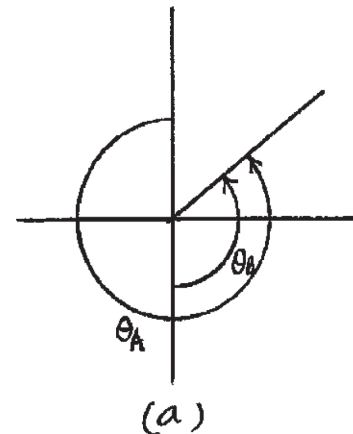
$$\begin{aligned} \theta_B = (\theta_B)_1 + (\theta_B)_2 &= \frac{(s_B)_1}{r_B} + \frac{(s_B)_2}{r_B} = \frac{65\pi}{390} + \frac{90.96(t - 1.9359)}{390} \\ &= 0.2332t + 0.07207 \end{aligned}$$

Substitute these results into Eq. (1),

$$0.2853t - 0.24414 = 0.2332t + 0.07207 + \pi$$

$$t = 66.39 \text{ s} = 66.4 \text{ s}$$

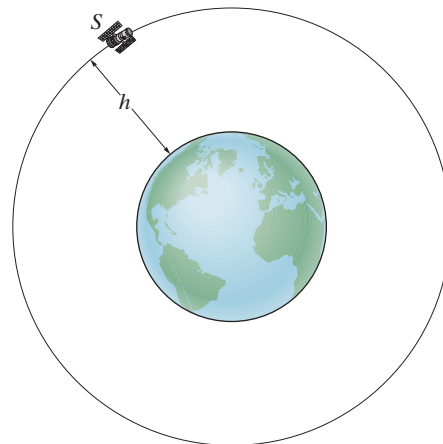
Ans.



Ans:
 $t = 66.4 \text{ s}$

12–119.

The satellite S travels around the earth in a circular path with a constant speed of 20 Mm/h. If the acceleration is 2.5 m/s^2 , determine the altitude h . Assume the earth's diameter to be 12 713 km.



SOLUTION

$$v = 20 \text{ Mm/h} = \frac{20(10^6)}{3600} = 5.56(10^3) \text{ m/s}$$

Since $a_t = \frac{dv}{dt} = 0$, then,

$$a = a_n = 2.5 = \frac{v^2}{\rho}$$

$$\rho = \frac{(5.56(10^3))^2}{2.5} = 12.35(10^6) \text{ m}$$

The radius of the earth is

$$\frac{12\,713(10^3)}{2} = 6.36(10^6) \text{ m}$$

Hence,

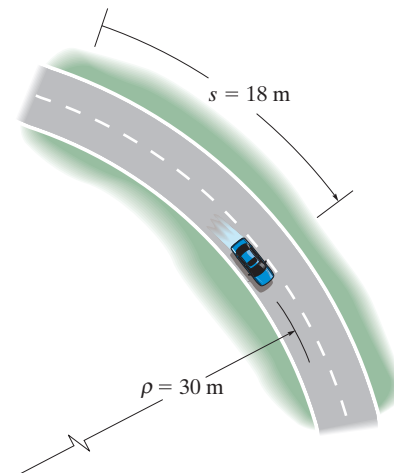
$$h = 12.35(10^6) - 6.36(10^6) = 5.99(10^6) \text{ m} = 5.99 \text{ Mm}$$

Ans.

Ans:
 $h = 5.99 \text{ Mm}$

***12–120.**

The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t) \text{ m/s}^2$, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18 \text{ m}$ starting from rest. Neglect the size of the car.



SOLUTION

$$\begin{aligned}\int_0^v dv &= \int_0^t 0.5e^t dt \\ v &= 0.5(e^t - 1) \\ \int_0^{18} ds &= 0.5 \int_0^t (e^t - 1) dt \\ 18 &= 0.5(e^t - t - 1)\end{aligned}$$

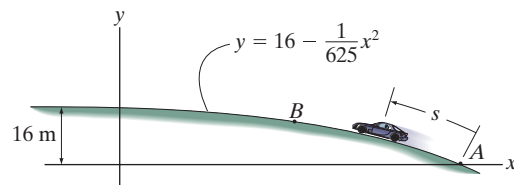
Solving,

$$\begin{aligned}t &= 3.7064 \text{ s} \\ v &= 0.5(e^{3.7064} - 1) = 19.85 \text{ m/s} = 19.9 \text{ m/s} && \text{Ans.} \\ a_t &= \dot{v} = 0.5e^t|_{t=3.7064 \text{ s}} = 20.35 \text{ m/s}^2 \\ a_n &= \frac{v^2}{\rho} = \frac{19.85^2}{30} = 13.14 \text{ m/s}^2 \\ a &= \sqrt{a_t^2 + a_n^2} = \sqrt{20.35^2 + 13.14^2} = 24.2 \text{ m/s}^2 && \text{Ans.}\end{aligned}$$

$$\begin{aligned}\text{Ans:} \\ v &= 19.9 \text{ m/s} \\ a &= 24.2 \text{ m/s}^2\end{aligned}$$

12–121.

The car passes point A with a speed of 25 m/s after which its speed is defined by $v = (25 - 0.15s)$ m/s. Determine the magnitude of the car's acceleration when it reaches point B , where $s = 51.5$ m and $x = 50$ m.



SOLUTION

Velocity: The speed of the car at B is

$$v_B = [25 - 0.15(51.5)] = 17.28 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2(10^{-3})x$$

$$\frac{d^2y}{dx^2} = -3.2(10^{-3})$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-3.2(10^{-3})x\right)^2\right]^{3/2}}{\left|-3.2(10^{-3})\right|} \bigg|_{x=50 \text{ m}} = 324.58 \text{ m}$$

Acceleration:

$$a_n = \frac{v_B^2}{\rho} = \frac{17.28^2}{324.58} = 0.9194 \text{ m/s}^2$$

$$a_t = v \frac{dv}{ds} = (25 - 0.15s)(-0.15) = (0.225s - 3.75) \text{ m/s}^2$$

When the car is at B ($s = 51.5$ m)

$$a_t = [0.225(51.5) - 3.75] = -2.591 \text{ m/s}^2$$

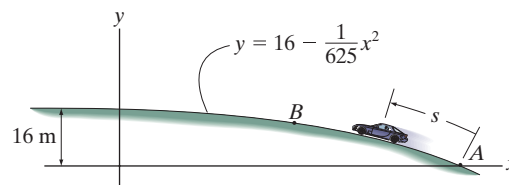
Thus, the magnitude of the car's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.591)^2 + 0.9194^2} = 2.75 \text{ m/s}^2 \quad \text{Ans.}$$

Ans:
 $a = 2.75 \text{ m/s}^2$

12–122.

If the car passes point *A* with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5 \text{ m/s}^2$, determine the magnitude of the car's acceleration when $s = 101.68 \text{ m}$ and $x = 0$.



SOLUTION

Velocity: The speed of the car at *C* is

$$v_C^2 = v_A^2 + 2a_t(s_C - s_A)$$

$$v_C^2 = 20^2 + 2(0.5)(100 - 0)$$

$$v_C = 22.361 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2(10^{-3})x$$

$$\frac{d^2y}{dx^2} = -3.2(10^{-3})$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-3.2(10^{-3})x\right)^2\right]^{3/2}}{|-3.2(10^{-3})|} \bigg|_{x=0} = 312.5 \text{ m}$$

Acceleration:

$$a_t = \dot{v} = 0.5 \text{ m/s}^2$$

$$a_n = \frac{v_C^2}{\rho} = \frac{22.361^2}{312.5} = 1.60 \text{ m/s}^2$$

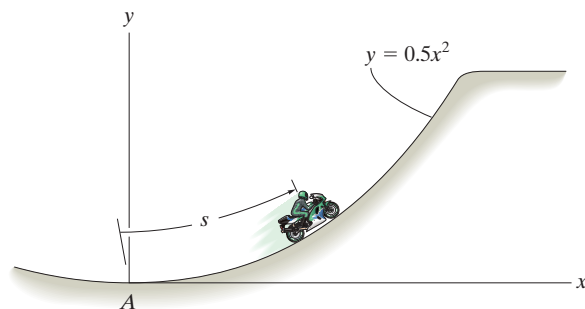
The magnitude of the car's acceleration at *C* is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 1.60^2} = 1.68 \text{ m/s}^2 \quad \textbf{Ans.}$$

Ans:
 $a = 1.68 \text{ m/s}^2$

12–123.

The motorcycle is traveling at 1 m/s when it is at A . If the speed is then increased at $\dot{v} = 0.1 \text{ m/s}^2$, determine its speed and acceleration at the instant $t = 5 \text{ s}$.



SOLUTION

$$a_t = \dot{v} = 0.1$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 1(5) + \frac{1}{2}(0.1)(5)^2 = 6.25 \text{ m}$$

$$\int_0^{6.25} ds = \int_0^x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$y = 0.5x^2$$

$$\frac{dy}{dx} = x$$

$$\frac{d^2y}{dx^2} = 1$$

$$6.25 = \int_0^x \sqrt{1 + x^2} dx$$

$$6.25 = \frac{1}{2} \left[x\sqrt{1 + x^2} + \ln \left(x + \sqrt{1 + x^2} \right) \right]_0^x$$

$$x\sqrt{1 + x^2} + \ln \left(x + \sqrt{1 + x^2} \right) = 12.5$$

Solving,

$$x = 3.184 \text{ m}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}}{\left| \frac{d^2y}{dx^2} \right|} = \frac{[1 + x^2]^{\frac{3}{2}}}{|1|} \bigg|_{x=3.184} = 37.17 \text{ m}$$

$$v = v_0 + a_c t$$

$$= 1 + 0.1(5) = 1.5 \text{ m/s}$$

Ans.

$$a_n = \frac{v^2}{\rho} = \frac{(1.5)^2}{37.17} = 0.0605 \text{ m/s}^2$$

$$a = \sqrt{(0.1)^2 + (0.0605)^2} = 0.117 \text{ m/s}^2$$

Ans.

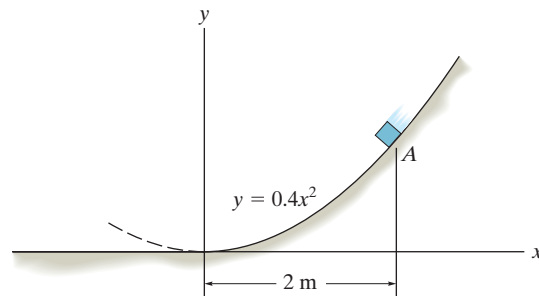
Ans:

$$v = 1.5 \text{ m/s}$$

$$a = 0.117 \text{ m/s}^2$$

***12–124.**

The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at A ($x_A = 2$ m, $y_A = 1.6$ m), the speed is $v = 8$ m/s and the increase in speed is $dv/dt = 4$ m/s². Determine the magnitude of the acceleration of the box at this instant.



SOLUTION

$$y = 0.4x^2$$

$$\left. \frac{dy}{dx} \right|_{x=2 \text{ m}} = 0.8x \Big|_{x=2 \text{ m}} = 1.6$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=2 \text{ m}} = 0.8$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\left| \frac{d^2y}{dx^2} \right|} \Big|_{x=2 \text{ m}} = \frac{\left[1 + (1.6)^2 \right]^{3/2}}{|0.8|} = 8.396 \text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{8^2}{8.396} = 7.622 \text{ m/s}^2$$

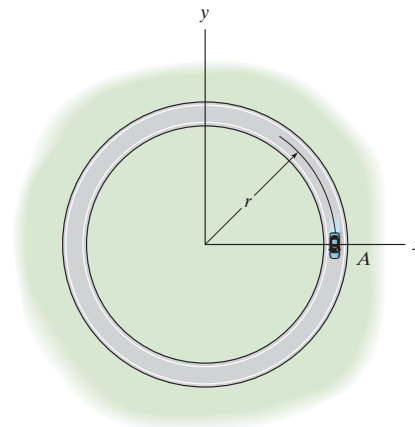
$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4)^2 + (7.622)^2} = 8.61 \text{ m/s}^2$$

Ans.

Ans:
 $a = 8.61 \text{ m/s}^2$

12–125.

The car travels around the circular track having a radius of $r = 300$ m such that when it is at point A it has a velocity of 5 m/s, which is increasing at the rate of $\dot{v} = (0.06t)$ m/s², where t is in seconds. Determine the magnitudes of its velocity and acceleration when it has traveled one-third the way around the track.



SOLUTION

$$a_t = v = 0.06t$$

$$dv = a_t dt$$

$$\int_s^v dv = \int_0^t 0.06t dt$$

$$v = 0.03t^2 + 5$$

$$ds = v dt$$

$$\int_0^s ds = \int_0^t (0.03t^2 + 5) dt$$

$$s = 0.01t^3 + 5t$$

$$s = \frac{1}{3} (2\pi(300)) = 628.3185$$

$$0.01t^3 + 5t - 628.3185 = 0$$

Solve for the positive root,

$$t = 35.58 \text{ s}$$

$$v = 0.03(35.58)^2 + 5 = 42.978 \text{ m/s} = 43.0 \text{ m/s}$$

Ans.

$$a_n = \frac{v^2}{\rho} = \frac{(42.978)^2}{300} = 6.157 \text{ m/s}^2$$

$$a_t = 0.06(35.58) = 2.135 \text{ m/s}^2$$

$$a = \sqrt{(6.157)^2 + (2.135)^2} = 6.52 \text{ m/s}^2$$

Ans.

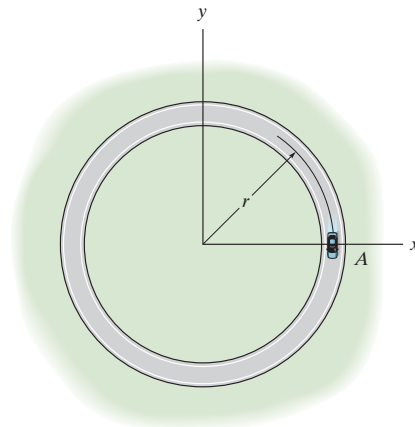
Ans:

$$v = 43.0 \text{ m/s}$$

$$a = 6.52 \text{ m/s}^2$$

12–126.

The car travels around the portion of a circular track having a radius of $r = 500$ ft such that when it is at point A it has a velocity of 2 ft/s, which is increasing at the rate of $\dot{v} = (0.002t)$ ft/s², where t is in seconds. Determine the magnitudes of its velocity and acceleration when it has traveled three-fourths the way around the track.



SOLUTION

$$a_t = 0.002 \text{ s}$$

$$a_t ds = v dv$$

$$\int_0^s 0.002s \, ds = \int_2^v v \, dv$$

$$0.001s^2 = \frac{1}{2}v^2 - \frac{1}{2}(2)^2$$

$$v^2 = 0.002s^2 + 4$$

$$s = \frac{3}{4}[2\pi(500)] = 2356.194 \text{ ft}$$

$$v^2 = 0.002(2356.194)^2 + 4$$

$$v = 105.39 \text{ ft/s} = 105 \text{ ft/s}$$

Ans.

$$a_n = \frac{v^2}{\rho} = \frac{(105.39)^2}{500} = 22.21 \text{ ft/s}^2$$

$$a_t = 0.002(2356.194) = 4.712 \text{ ft/s}^2$$

$$a = \sqrt{(22.21)^2 + (4.712)^2} = 22.7 \text{ ft/s}^2$$

Ans.

Ans:

$$v = 105 \text{ ft/s}$$

$$a = 22.7 \text{ ft/s}^2$$

12–127.

At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.

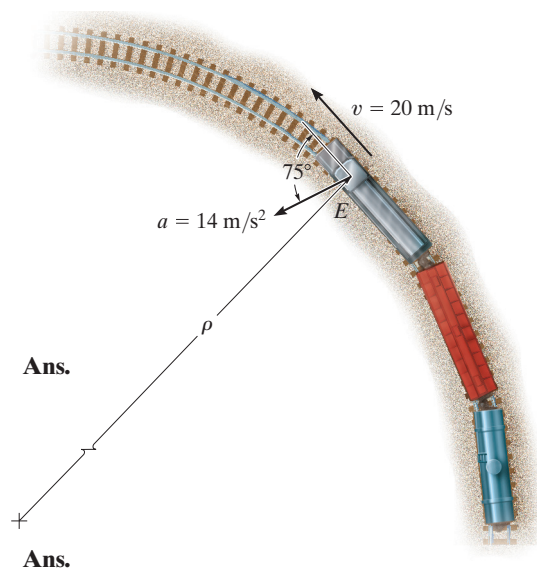
SOLUTION

$$a_t = 14 \cos 75^\circ = 3.62 \text{ m/s}^2$$

$$a_n = 14 \sin 75^\circ$$

$$a_n = \frac{(20)^2}{\rho}$$

$$\rho = 29.6 \text{ m}$$



Ans:
 $a_t = 3.62 \text{ m/s}^2$
 $\rho = 29.6 \text{ m}$

***12–128.**

The car has an initial speed $v_0 = 20$ m/s. If it increases its speed along the circular track at $s = 0$, $a_t = (0.8s)$ m/s², where s is in meters, determine the time needed for the car to travel $s = 25$ m.

SOLUTION

The distance traveled by the car along the circular track can be determined by integrating $v dv = a_t ds$. Using the initial condition $v = 20$ m/s at $s = 0$,

$$\int_{20 \text{ m/s}}^v v dv = \int_0^s 0.8 s ds$$

$$\frac{v^2}{2} \Big|_{20 \text{ m/s}}^v = 0.4 s^2$$

$$v = \left\{ \sqrt{0.8 (s^2 + 500)} \right\} \text{ m/s}$$

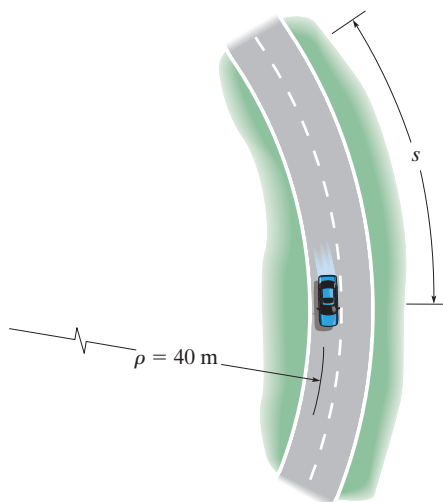
The time can be determined by integrating $dt = \frac{ds}{v}$ with the initial condition $s = 0$ at $t = 0$.

$$\int_0^t dt = \int_0^{25 \text{ m}} \frac{ds}{\sqrt{0.8(s^2 + 500)}}$$

$$t = \frac{1}{\sqrt{0.8}} \left[\ln(s + \sqrt{s^2 + 500}) \right] \Big|_0^{25 \text{ m}}$$

$$= 1.076 \text{ s} = 1.08 \text{ s}$$

Ans.



Ans:
 $t = 1.08 \text{ s}$

12–129.

The car starts from rest at $s = 0$ and increases its speed at $a_t = 4 \text{ m/s}^2$. Determine the time when the magnitude of acceleration becomes 20 m/s^2 . At what position s does this occur?

SOLUTION

Acceleration. The normal component of the acceleration can be determined from

$$a_r = \frac{v^2}{\rho}; \quad a_r = \frac{v^2}{40}$$

From the magnitude of the acceleration

$$a = \sqrt{a_t^2 + a_n^2}; \quad 20 = \sqrt{4^2 + \left(\frac{v^2}{40}\right)^2} \quad v = 28.00 \text{ m/s}$$

Velocity. Since the car has a constant tangential acceleration of $a_t = 4 \text{ m/s}^2$,

$$v = v_0 + a_t t; \quad 28.00 = 0 + 4t$$

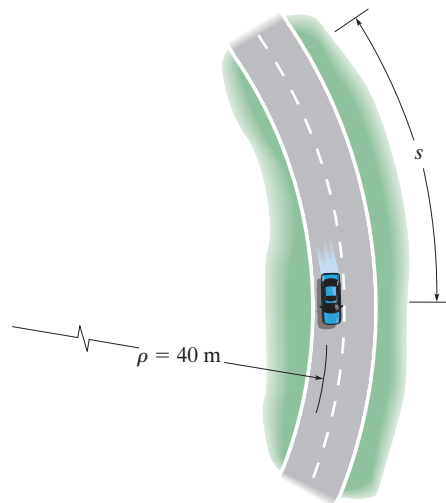
$$t = 6.999 \text{ s} = 7.00 \text{ s}$$

Ans.

$$v^2 = v_0^2 + 2a_t s; \quad 28.00^2 = 0^2 + 2(4) s$$

$$s = 97.98 \text{ m} = 98.0 \text{ m}$$

Ans.



Ans:
 $t = 7.00 \text{ s}$
 $s = 98.0 \text{ m}$

12–130.

A boat is traveling along a circular curve having a radius of 100 ft. If its speed at $t = 0$ is 15 ft/s and is increasing at $\dot{v} = (0.8t)$ ft/s², determine the magnitude of its acceleration at the instant $t = 5$ s.

SOLUTION

$$\int_{15}^v dv = \int_0^5 0.8t dt$$

$$v = 25 \text{ ft/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{25^2}{100} = 6.25 \text{ ft/s}^2$$

At $t = 5$ s,

$$a_t = \dot{v} = 0.8(5) = 4 \text{ ft/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4^2 + 6.25^2} = 7.42 \text{ ft/s}^2$$

Ans.

Ans:

$$a = 7.42 \text{ ft/s}^2$$

12–131.

A boat is traveling along a circular path having a radius of 20 m. Determine the magnitude of the boat's acceleration when the speed is $v = 5$ m/s and the rate of increase in the speed is $\dot{v} = 2$ m/s².

SOLUTION

$$a_t = 2 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{5^2}{20} = 1.25 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 1.25^2} = 2.36 \text{ m/s}^2$$

Ans.

Ans:

$$a = 2.36 \text{ m/s}^2$$

***12–132.**

Starting from rest, a bicyclist travels around a horizontal circular path, $\rho = 10$ m, at a speed of $v = (0.09t^2 + 0.1t)$ m/s, where t is in seconds. Determine the magnitudes of his velocity and acceleration when he has traveled $s = 3$ m.

SOLUTION

$$\int_0^s ds = \int_0^t (0.09t^2 + 0.1t) dt$$

$$s = 0.03t^3 + 0.05t^2$$

When $s = 3$ m, $3 = 0.03t^3 + 0.05t^2$

Solving,

$$t = 4.147 \text{ s}$$

$$v = \frac{ds}{dt} = 0.09t^2 + 0.1t$$

$$v = 0.09(4.147)^2 + 0.1(4.147) = 1.96 \text{ m/s}$$

Ans.

$$a_t = \frac{dv}{dt} = 0.18t + 0.1 \bigg|_{t=4.147 \text{ s}} = 0.8465 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{1.96^2}{10} = 0.3852 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8465)^2 + (0.3852)^2} = 0.930 \text{ m/s}^2$$

Ans.

Ans:

$$v = 1.96 \text{ m/s}$$

$$a = 0.930 \text{ m/s}^2$$

12–133.

A particle travels around a circular path having a radius of 50 m. If it is initially traveling with a speed of 10 m/s and its speed then increases at a rate of $\dot{v} = (0.05 v) \text{ m/s}^2$, determine the magnitude of the particle's acceleration four seconds later.

SOLUTION

Velocity: Using the initial condition $v = 10 \text{ m/s}$ at $t = 0 \text{ s}$,

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{10 \text{ m/s}}^v \frac{dv}{0.05v}$$

$$t = 20 \ln \frac{v}{10}$$

$$v = (10e^{t/20}) \text{ m/s}$$

When $t = 4 \text{ s}$,

$$v = 10e^{4/20} = 12.214 \text{ m/s}$$

Acceleration: When $v = 12.214 \text{ m/s}$ ($t = 4 \text{ s}$),

$$a_t = 0.05(12.214) = 0.6107 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(12.214)^2}{50} = 2.984 \text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.6107^2 + 2.984^2} = 3.05 \text{ m/s}^2$$

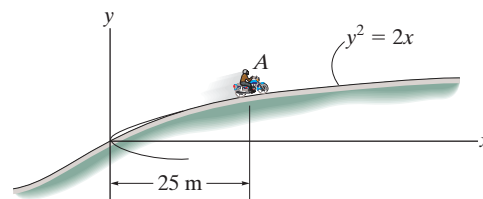
Ans.

Ans:

$$a = 3.05 \text{ m/s}^2$$

12–134.

The motorcycle is traveling at a constant speed of 60 km/h. Determine the magnitude of its acceleration when it is at point *A*.



SOLUTION

Radius of Curvature:

$$y = \sqrt{2}x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{2}x^{-1/2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}\sqrt{2}x^{-3/2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(\frac{1}{2}\sqrt{2}x^{-1/2}\right)^2\right]^{3/2}}{\left|-\frac{1}{4}\sqrt{2}x^{-3/2}\right|} \bigg|_{x=25 \text{ m}} = 364.21 \text{ m}$$

Acceleration: The speed of the motorcycle at *a* is

$$v = \left(60 \frac{\text{km}}{\text{h}}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 16.67 \text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{364.21} = 0.7627 \text{ m/s}^2$$

Since the motorcycle travels with a constant speed, $a_t = 0$. Thus, the magnitude of the motorcycle's acceleration at *A* is

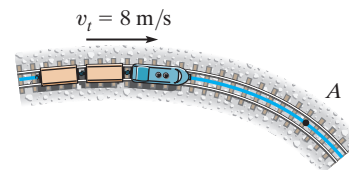
$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0^2 + 0.7627^2} = 0.763 \text{ m/s}^2 \quad \textbf{Ans.}$$

Ans:

$$a = 0.763 \text{ m/s}^2$$

12–135.

When $t = 0$, the train has a speed of 8 m/s , which is increasing at 0.5 m/s^2 . Determine the magnitude of the acceleration of the engine when it reaches point A , at $t = 20 \text{ s}$. Here the radius of curvature of the tracks is $\rho_A = 400 \text{ m}$.



SOLUTION

Velocity. The velocity of the train along the track can be determined by integrating $dv = a_t dt$ with initial condition $v = 8 \text{ m/s}$ at $t = 0$.

$$\int_{8 \text{ m/s}}^v dv = \int_0^t 0.5 dt$$

$$v - 8 = 0.5 t$$

$$v = \{0.5 t + 8\} \text{ m/s}$$

At $t = 20 \text{ s}$,

$$v|_{t=20 \text{ s}} = 0.5(20) + 8 = 18 \text{ m/s}$$

Acceleration. Here, the tangential component is $a_t = 0.5 \text{ m/s}^2$. The normal component can be determined from

$$a_n = \frac{v^2}{\rho} = \frac{18^2}{400} = 0.81 \text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$\begin{aligned} a &= \sqrt{a_t^2 + a_n^2} \\ &= \sqrt{0.5^2 + 0.81^2} \\ &= 0.9519 \text{ m/s}^2 = 0.952 \text{ m/s}^2 \end{aligned}$$

Ans.

Ans:

$$a = 0.952 \text{ m/s}^2$$

***12–136.**

At a given instant the jet plane has a speed of 550 m/s and an acceleration of 50 m/s² acting in the direction shown. Determine the rate of increase in the plane's speed, and also the radius of curvature ρ of the path.

SOLUTION

Acceleration. With respect to the n - t coordinate established as shown in Fig. *a*, the tangential and normal components of the acceleration are

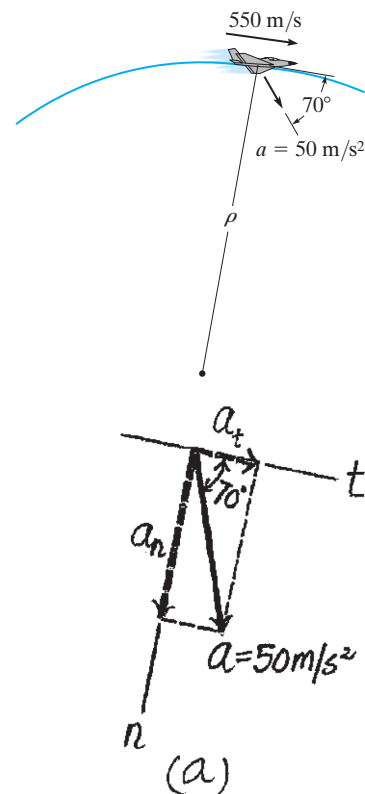
$$a_t = 50 \cos 70^\circ = 17.10 \text{ m/s}^2 = 17.1 \text{ m/s}^2 \quad \text{Ans.}$$

$$a_n = 50 \sin 70^\circ = 46.98 \text{ m/s}^2$$

However,

$$a_n = \frac{v^2}{\rho}; \quad 46.98 = \frac{550^2}{\rho}$$

$$\rho = 6438.28 \text{ m} = 6.44 \text{ km} \quad \text{Ans.}$$



Ans:

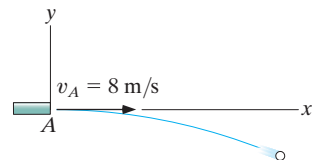
$$a_t = 17.1 \text{ m/s}^2$$

$$a_n = 46.98 \text{ m/s}^2$$

$$\rho = 6.44 \text{ km}$$

12–137.

The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.



SOLUTION

$$v_x = 8 \text{ m/s}$$

$$(\rightarrow) \quad s = v_0 t$$

$$x = 8t$$

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$y = 0 + 0 + \frac{1}{2} (-9.81) t^2$$

$$y = -4.905 t^2$$

$$y = -4.905 \left(\frac{x}{8} \right)^2$$

$$y = -0.0766 x^2 \quad (\text{Parabola})$$

$$v = v_0 + a_c t$$

$$v_y = 0 - 9.81 t$$

When $t = 0.25$ s,

$$v_y = -2.4525 \text{ m/s}$$

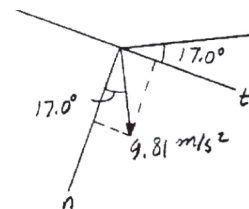
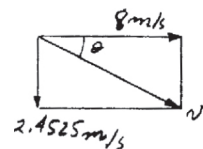
$$v = \sqrt{(8)^2 + (2.4525)^2} = 8.37 \text{ m/s}$$

$$\theta = \tan^{-1} \left(\frac{2.4525}{8} \right) = 17.04^\circ$$

$$a_x = 0 \quad a_y = 9.81 \text{ m/s}^2$$

$$a_n = 9.81 \cos 17.04^\circ = 9.38 \text{ m/s}^2$$

$$a_t = 9.81 \sin 17.04^\circ = 2.88 \text{ m/s}^2$$



Ans.

Ans.

Ans.

Ans.

Ans:

$$y = -0.0766 x^2$$

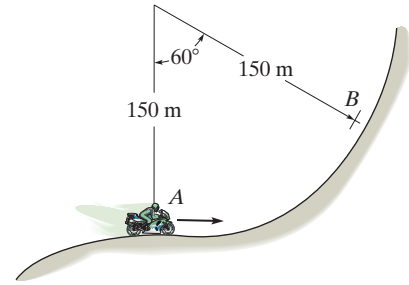
$$v = 8.37 \text{ m/s}$$

$$a_n = 9.38 \text{ m/s}^2$$

$$a_t = 2.88 \text{ m/s}^2$$

12–138.

The motorcycle is traveling at 40 m/s when it is at *A*. If the speed is then decreased at $\dot{v} = -(0.05 \text{ s}) \text{ m/s}^2$, where *s* is in meters measured from *A*, determine its speed and acceleration when it reaches *B*.



SOLUTION

Velocity. The velocity of the motorcycle along the circular track can be determined by integrating $v dv = a_t ds$ with the initial condition $v = 40 \text{ m/s}$ at $s = 0$. Here, $a_t = -0.05s$.

$$\int_{40 \text{ m/s}}^v v dv = \int_0^s -0.05 s ds$$

$$\frac{v^2}{2} \Big|_{40 \text{ m/s}}^v = -0.025 s^2 \Big|_0^s$$

$$v = \{ \sqrt{1600 - 0.05 s^2} \} \text{ m/s}$$

At *B*, $s = r\theta = 150 \left(\frac{\pi}{3} \right) = 50\pi \text{ m}$. Thus

$$v_B = v|_{s=50\pi \text{ m}} = \sqrt{1600 - 0.05(50\pi)^2} = 19.14 \text{ m/s} = 19.1 \text{ m/s}$$

Ans.

Acceleration. At *B*, the tangential and normal components are

$$a_t = 0.05(50\pi) = 2.5\pi \text{ m/s}^2$$

$$a_n = \frac{v_B^2}{\rho} = \frac{19.14^2}{150} = 2.4420 \text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(2.5\pi)^2 + 2.4420^2} = 8.2249 \text{ m/s}^2 = 8.22 \text{ m/s}^2$$

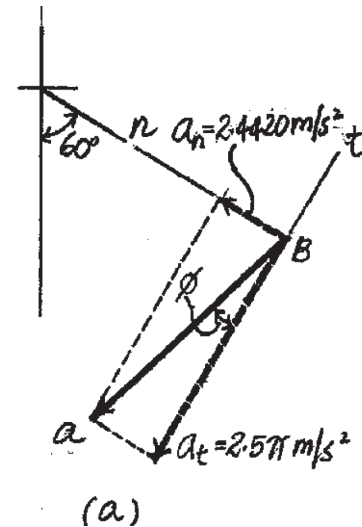
Ans.

And its direction is defined by angle ϕ measured from the negative *t*-axis, Fig. *a*.

$$\phi = \tan^{-1} \left(\frac{a_n}{a_t} \right) = \tan^{-1} \left(\frac{2.4420}{2.5\pi} \right)$$

$$= 17.27^\circ = 17.3^\circ$$

Ans.



Ans:

$$v_B = 19.1 \text{ m/s}$$

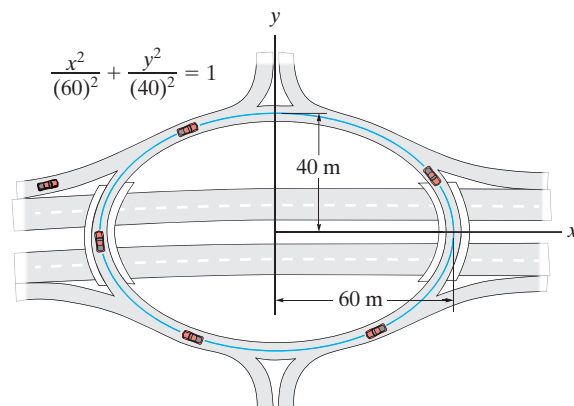
$$a = 8.22 \text{ m/s}^2$$

$$\phi = 17.3^\circ$$

up from negative-*t* axis

12–139.

Cars move around the “traffic circle” which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the minimum acceleration experienced by the passengers.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = -\frac{b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)^2$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{b^4}{a^2y^2}\right)\left(\frac{x^2}{a^2}\right)$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \frac{b^4}{a^2y^2}\left(1 - \frac{y^2}{b^2}\right)$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \frac{b^4}{a^2y^2} + \frac{b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{-b^2x}{a^2y}\right)^2\right]^{3/2}}{\left|\frac{-b^4}{a^2y^3}\right|}$$

At $x = 0, y = b$,

$$\rho = \frac{a^2}{b}$$

12–139. Continued

Thus

$$a_t = 0$$

$$a_{\min} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{a^2}{b}} = \frac{v^2 b}{a^2}$$

Set $a = 60$ m, $b = 40$ m,

$$v = \frac{60(10)^3}{3600} = 16.67 \text{ m/s}$$

$$a_{\min} = \frac{(16.67)^2(40)}{(60)^2} = 3.09 \text{ m/s}^2$$

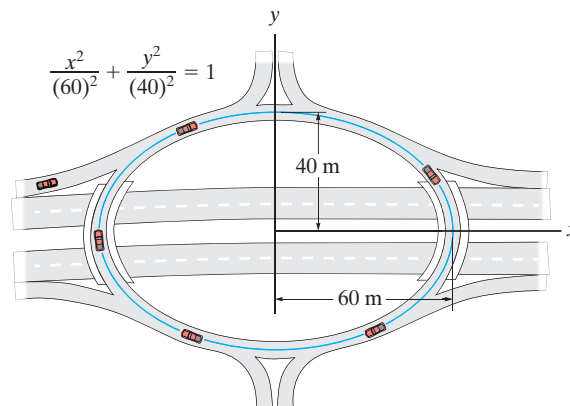
Ans.

Ans:

$$a_{\min} = 3.09 \text{ m/s}^2$$

*12–140.

Cars move around the “traffic circle” which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the maximum acceleration experienced by the passengers.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = -\frac{b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)^2$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{-b^2x}{a^2y}\right)^2\right]^{3/2}}{\left|\frac{-b^4}{a^2y^3}\right|}$$

At $x = a, y = 0$,

$$\rho = \frac{b^2}{a}$$

Then

$$a_t = 0$$

$$a_{\max} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{b^2}{a}} = \frac{v^2 a}{b^2}$$

$$\text{Set } a = 60 \text{ m, } b = 40 \text{ m, } v = \frac{60(10^3)}{3600} = 16.67 \text{ m/s}$$

$$a_{\max} = \frac{(16.67)^2(60)}{(40)^2} = 10.4 \text{ m/s}^2$$

Ans.

Ans:

$$a_{\max} = 10.4 \text{ m/s}^2$$

12-141.

A package is dropped from the plane which is flying with a constant horizontal velocity of $v_A = 150$ ft/s. Determine the normal and tangential components of acceleration and the radius of curvature of the path of motion (a) at the moment the package is released at A, where it has a horizontal velocity of $v_A = 150$ ft/s, and (b) *just before* it strikes the ground at B.

SOLUTION

Initially (Point A):

$$(a_n)_A = g = 32.2 \text{ ft/s}^2$$

$$(a_t)_A = 0$$

$$(a_n)_A = \frac{v_A^2}{\rho_A}; \quad 32.2 = \frac{(150)^2}{\rho_A}$$

$$\rho_A = 698.8 \text{ ft}$$

$$(v_B)_x = (v_A)_x = 150 \text{ ft/s}$$

$$(+\downarrow) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$(v_B)_y^2 = 0 + 2(32.2)(1500 - 0)$$

$$(v_B)_y = 310.8 \text{ ft/s}$$

$$v_B = \sqrt{(150)^2 + (310.8)^2} = 345.1 \text{ ft/s}$$

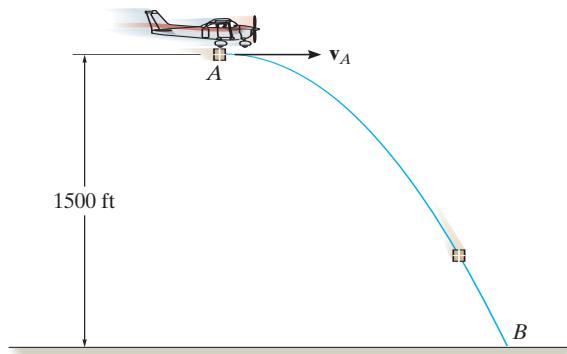
$$\theta = \tan^{-1}\left(\frac{v_{B_y}}{v_{B_x}}\right) = \tan^{-1}\left(\frac{310.8}{150}\right) = 64.23^\circ$$

$$(a_n)_B = g \cos \theta = 32.2 \cos 64.24^\circ = 14.0 \text{ ft/s}^2$$

$$(a_t)_B = g \sin \theta = 32.2 \sin 64.24^\circ = 29.0 \text{ ft/s}^2$$

$$(a_n)_B = \frac{v_B^2}{\rho_B}; \quad 14.0 = \frac{(345.1)^2}{\rho_B}$$

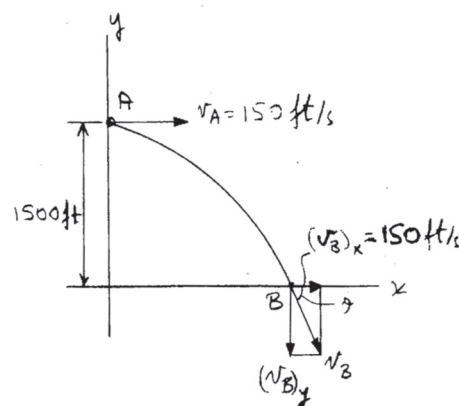
$$\rho_B = 8509.8 \text{ ft} = 8.51(10^3) \text{ ft}$$



Ans.

Ans.

Ans.



Ans.

Ans.

Ans.

Ans:

$$(a_n)_A = g = 32.2 \text{ ft/s}^2$$

$$(a_t)_A = 0$$

$$\rho_A = 699 \text{ ft}$$

$$(a_n)_B = 14.0 \text{ ft/s}^2$$

$$(a_t)_B = 29.0 \text{ ft/s}^2$$

$$\rho_B = 8.51(10^3) \text{ ft}$$

12–142.

The race car has an initial speed $v_A = 15 \text{ m/s}$ at A . If it increases its speed along the circular track at the rate $a_t = (0.4s) \text{ m/s}^2$, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150 \text{ m}$.

SOLUTION

$$a_t = 0.4s = \frac{v \, dv}{ds}$$

$$a \, ds = v \, dv$$

$$\int_0^s 0.4s \, ds = \int_{15}^v v \, dv$$

$$\left. \frac{0.4s^2}{2} \right|_0^s = \left. \frac{v^2}{2} \right|_{15}^v$$

$$\frac{0.4s^2}{2} = \frac{v^2}{2} - \frac{225}{2}$$

$$v^2 = 0.4s^2 + 225$$

$$v = \frac{ds}{dt} = \sqrt{0.4s^2 + 225}$$

$$\int_0^s \frac{ds}{\sqrt{0.4s^2 + 225}} = \int_0^t dt$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 562.5}} = 0.632 \, 456t$$

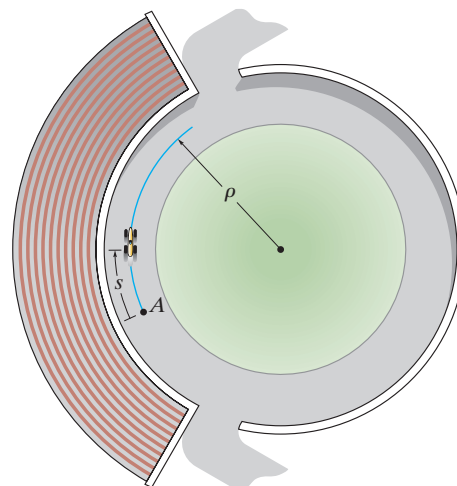
$$\left. \ln(s + \sqrt{s^2 + 562.5}) \right|_0^s = 0.632 \, 456t$$

$$\ln(s + \sqrt{s^2 + 562.5}) - 3.166 \, 196 = 0.632 \, 456t$$

At $s = 20 \text{ m}$,

$$t = 1.21 \text{ s}$$

Ans.

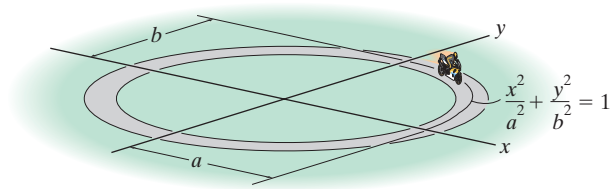


Ans:

$$t = 1.21 \text{ s}$$

12–143.

The motorcycle travels along the elliptical track at a constant speed v . Determine its greatest acceleration if $a > b$.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{b^2x}{a^2y}\right)^2\right]^{3/2}}{\frac{-b^4}{a^2y^3}}$$

At $x = a, y = 0$,

$$\rho = \frac{b^2}{a}$$

Then

$$a_t = 0$$

$$a_{\max} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{b^2}{a}} = \frac{v^2a}{b^2}$$

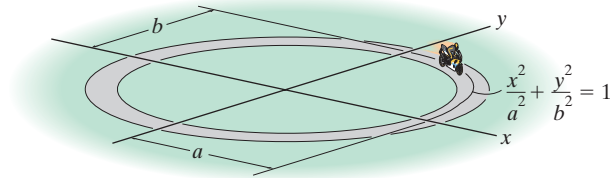
Ans.

Ans:

$$a_{\max} = \frac{v^2a}{b^2}$$

***12–144.**

The motorcycle travels along the elliptical track at a constant speed v . Determine its smallest acceleration if $a > b$.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = -\frac{b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{b^2x}{a^2y}\right)^2\right]^{3/2}}{\frac{-b^4}{a^2y^3}}$$

At $x = 0, y = b$,

$$|\rho| = \frac{a^2}{b}$$

Thus

$$a_t = 0$$

$$a_{\min} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{a^2}{b}} = \frac{v^2b}{a^2}$$

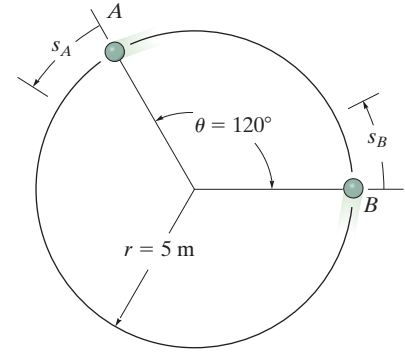
Ans.

Ans:

$$a_{\min} = \frac{v^2b}{a^2}$$

12–145.

Particles A and B are traveling counter-clockwise around a circular track at a constant speed of 8 m/s . If at the instant shown the speed of A begins to increase by $(a_t)_A = (0.4s_A) \text{ m/s}^2$, where s_A is in meters, determine the distance measured counterclockwise along the track from B to A when $t = 1 \text{ s}$. What is the magnitude of the acceleration of each particle at this instant?



SOLUTION

Distance Traveled: Initially the distance between the particles is

$$d_0 = \rho d\theta = 5 \left(\frac{120^\circ}{180^\circ} \right) \pi = 10.47 \text{ m}$$

When $t = 1 \text{ s}$, B travels a distance of

$$d_B = 8(1) = 8 \text{ m}$$

The distance traveled by particle A is determined as follows:

$$\begin{aligned} v dv &= a ds \\ \int_{8 \text{ m/s}}^v v dv &= \int_0^s 0.4 s ds \\ v &= 0.6325 \sqrt{s^2 + 160} \\ dt &= \frac{ds}{v} \\ \int_0^t dt &= \int_0^s \frac{ds}{0.6325 \sqrt{s^2 + 160}} \\ 1 &= \frac{1}{0.6325} \left(\ln \left[\frac{\sqrt{s^2 + 160} + s}{\sqrt{160}} \right] \right) \\ s &= 8.544 \text{ m} \end{aligned} \quad (1)$$

Thus the distance between the two cyclists after $t = 1 \text{ s}$ is

$$d = 10.47 + 8.544 - 8 = 11.0 \text{ m} \quad \text{Ans.}$$

Acceleration:

For A , when $t = 1 \text{ s}$,

$$\begin{aligned} (a_t)_A &= \dot{v}_A = 0.4(8.544) = 3.4176 \text{ m/s}^2 \\ v_A &= 0.6325 \sqrt{8.544^2 + 160} = 9.655 \text{ m/s} \\ (a_n)_A &= \frac{v_A^2}{\rho} = \frac{9.655^2}{5} = 18.64 \text{ m/s}^2 \end{aligned}$$

The magnitude of the A 's acceleration is

$$a_A = \sqrt{3.4176^2 + 18.64^2} = 19.0 \text{ m/s}^2 \quad \text{Ans.}$$

For B , when $t = 1 \text{ s}$,

$$\begin{aligned} (a_t)_B &= \dot{v}_B = 0 \\ (a_n)_B &= \frac{v_B^2}{\rho} = \frac{8^2}{5} = 12.80 \text{ m/s}^2 \end{aligned}$$

The magnitude of the B 's acceleration is

$$a_B = \sqrt{0^2 + 12.80^2} = 12.8 \text{ m/s}^2 \quad \text{Ans.}$$

Ans:

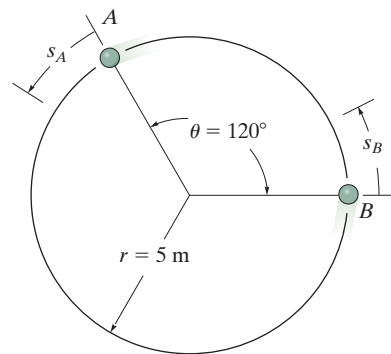
$$d = 11.0 \text{ m}$$

$$a_A = 19.0 \text{ m/s}^2$$

$$a_B = 12.8 \text{ m/s}^2$$

12–146.

Particles A and B are traveling around a circular track at a speed of 8 m/s at the instant shown. If the speed of B is increasing by $(a_t)_B = 4 \text{ m/s}^2$, and at the same instant A has an increase in speed of $(a_t)_A = 0.8t \text{ m/s}^2$, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?



SOLUTION

Distance Traveled: Initially the distance between the two particles is $d_0 = \rho\theta$
 $= 5\left(\frac{120^\circ}{180^\circ}\pi\right) = 10.47 \text{ m}$. Since particle B travels with a constant acceleration, distance can be obtained by applying equation

$$s_B = (s_0)_B + (v_0)_B t + \frac{1}{2} a_c t^2$$

$$s_B = 0 + 8t + \frac{1}{2}(4)t^2 = (8t + 2t^2) \text{ m} \quad [1]$$

The distance traveled by particle A can be obtained as follows.

$$\begin{aligned} dv_A &= a_A dt \\ \int_{8 \text{ m/s}}^{v_A} dv_A &= \int_0^t 0.8t dt \\ v_A &= (0.4t^2 + 8) \text{ m/s} \end{aligned} \quad [2]$$

$$\begin{aligned} ds_A &= v_A dt \\ \int_0^{s_A} ds_A &= \int_0^t (0.4t^2 + 8) dt \\ s_A &= 0.1333t^3 + 8t \end{aligned}$$

In order for the collision to occur

$$\begin{aligned} s_A + d_0 &= s_B \\ 0.1333t^3 + 8t + 10.47 &= 8t + 2t^2 \end{aligned}$$

Solving by trial and error $t = 2.5074 \text{ s} = 2.51 \text{ s}$ **Ans.**

Note: If particle A strikes B then, $s_A = 5\left(\frac{240^\circ}{180^\circ}\pi\right) + s_B$. This equation will result in $t = 14.6 \text{ s} > 2.51 \text{ s}$.

Acceleration: The tangential acceleration for particle A and B when $t = 2.5074$ are $(a_t)_A = 0.8t = 0.8(2.5074) = 2.006 \text{ m/s}^2$ and $(a_t)_B = 4 \text{ m/s}^2$, respectively. When $t = 2.5074 \text{ s}$, from Eq. [1], $v_A = 0.4(2.5074^2) + 8 = 10.51 \text{ m/s}$ and $v_B = (v_0)_B + a_c t = 8 + 4(2.5074) = 18.03 \text{ m/s}$. To determine the normal acceleration, apply Eq. 12–20.

$$\begin{aligned} (a_n)_A &= \frac{v_A^2}{\rho} = \frac{10.51^2}{5} = 22.11 \text{ m/s}^2 \\ (a_n)_B &= \frac{v_B^2}{\rho} = \frac{18.03^2}{5} = 65.01 \text{ m/s}^2 \end{aligned}$$

The magnitude of the acceleration for particles A and B just before collision are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{2.006^2 + 22.11^2} = 22.2 \text{ m/s}^2 \quad \textbf{Ans.}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{4^2 + 65.01^2} = 65.1 \text{ m/s}^2 \quad \textbf{Ans.}$$

Ans:

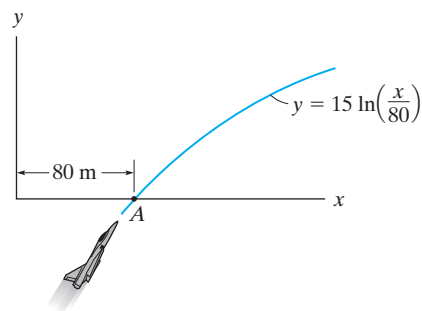
$$t = 2.51 \text{ s}$$

$$a_A = 22.2 \text{ m/s}^2$$

$$a_B = 65.1 \text{ m/s}^2$$

12–147.

The jet plane is traveling with a speed of 120 m/s which is decreasing at 40 m/s² when it reaches point A. Determine the magnitude of its acceleration when it is at this point. Also, specify the direction of flight, measured from the x axis.



SOLUTION

$$y = 15 \ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x} \bigg|_{x=80 \text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2} \bigg|_{x=80 \text{ m}} = -0.002344$$

$$\rho \bigg|_{x=80 \text{ m}} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} \bigg|_{x=80 \text{ m}}$$

$$= \frac{[1 + (0.1875)^2]^{3/2}}{|-0.002344|} = 449.4 \text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(120)^2}{449.4} = 32.04 \text{ m/s}^2$$

$$a_t = -40 \text{ m/s}^2$$

$$a = \sqrt{(-40)^2 + (32.04)^2} = 51.3 \text{ m/s}^2$$

Ans.

Since

$$\frac{dy}{dx} = \tan \theta = 0.1875$$

$$\theta = 10.6^\circ$$

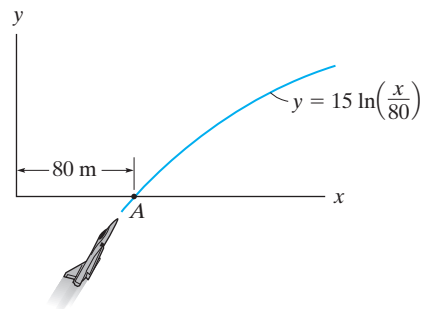
Ans.

Ans:

$$\theta = 10.6^\circ$$

***12–148.**

The jet plane is traveling with a constant speed of 110 m/s along the curved path. Determine the magnitude of the acceleration of the plane at the instant it reaches point A ($y = 0$).



SOLUTION

$$y = 15 \ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x} \bigg|_{x=80 \text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2} \bigg|_{x=80 \text{ m}} = -0.002344$$

$$\begin{aligned} \rho \bigg|_{x=80 \text{ m}} &= \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} \bigg|_{x=80 \text{ m}} \\ &= \frac{[1 + (0.1875)^2]^{3/2}}{|-0.002344|} = 449.4 \text{ m} \end{aligned}$$

$$a_n = \frac{v^2}{\rho} = \frac{(110)^2}{449.4} = 26.9 \text{ m/s}^2$$

Since the plane travels with a constant speed, $a_t = 0$. Hence

$$a = a_n = 26.9 \text{ m/s}^2$$

Ans.

Ans:

$$a = 26.9 \text{ m/s}^2$$

12–149.

The train passes point B with a speed of 20 m/s which is decreasing at $a_t = -0.5 \text{ m/s}^2$. Determine the magnitude of acceleration of the train at this point.

SOLUTION

Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 200\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2e^{\frac{x}{1000}}$$

$$\frac{d^2y}{dx^2} = 0.2\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2(10^{-3})e^{\frac{x}{1000}}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(0.2e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2(10^{-3})e^{\frac{x}{1000}}\right|} \bigg|_{x=400 \text{ m}} = 3808.96 \text{ m}$$

Acceleration:

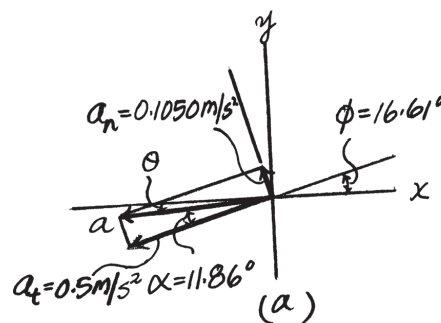
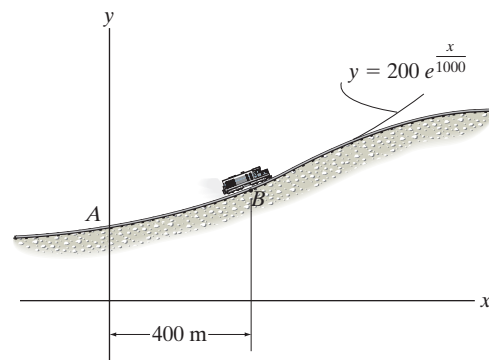
$$a_t = \dot{v} = -0.5 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{20^2}{3808.96} = 0.1050 \text{ m/s}^2$$

The magnitude of the train's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.5)^2 + 0.1050^2} = 0.511 \text{ m/s}^2$$

Ans.



Ans:

$$a = 0.511 \text{ m/s}^2$$

12–150.

The train passes point A with a speed of 30 m/s and begins to decrease its speed at a constant rate of $a_t = -0.25 \text{ m/s}^2$. Determine the magnitude of the acceleration of the train when it reaches point B , where $s_{AB} = 412 \text{ m}$.

SOLUTION

Velocity: The speed of the train at B can be determined from

$$v_B^2 = v_A^2 + 2a_t(s_B - s_A)$$

$$v_B^2 = 30^2 + 2(-0.25)(412 - 0)$$

$$v_B = 26.34 \text{ m/s}$$

Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 0.2e^{\frac{x}{1000}}$$

$$\frac{d^2y}{dx^2} = 0.2(10^{-3})e^{\frac{x}{1000}}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(0.2e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2(10^{-3})e^{\frac{x}{1000}}\right|} \bigg|_{x=400 \text{ m}} = 3808.96 \text{ m}$$

Acceleration:

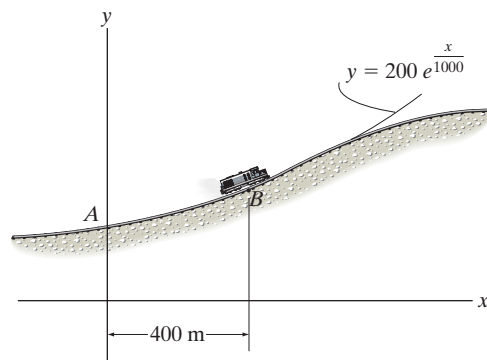
$$a_t = \dot{v} = -0.25 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{26.34^2}{3808.96} = 0.1822 \text{ m/s}^2$$

The magnitude of the train's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.25)^2 + 0.1822^2} = 0.309 \text{ m/s}^2$$

Ans.



Ans:

$$a = 0.309 \text{ m/s}^2$$

12-151.

The particle travels with a constant speed of 300 mm/s along the curve. Determine the particle's acceleration when it is located at point (200 mm, 100 mm) and sketch this vector on the curve.

SOLUTION

$$v = 300 \text{ mm/s}$$

$$a_t = \frac{dv}{dt} = 0$$

$$y = \frac{20(10^3)}{x}$$

$$\left. \frac{dy}{dx} \right|_{x=200} = -\frac{20(10^3)}{x^2} = -0.5$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=200} = \frac{40(10^3)}{x^3} = 5(10^{-3})$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (-0.5)^2\right]^{\frac{3}{2}}}{\left|5(10^{-3})\right|} = 279.5 \text{ mm}$$

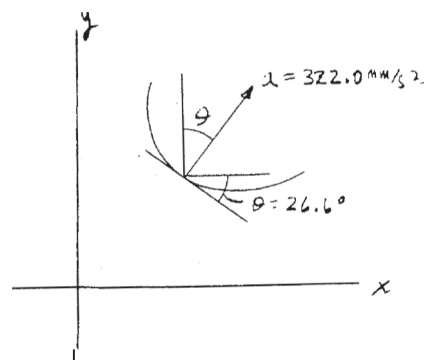
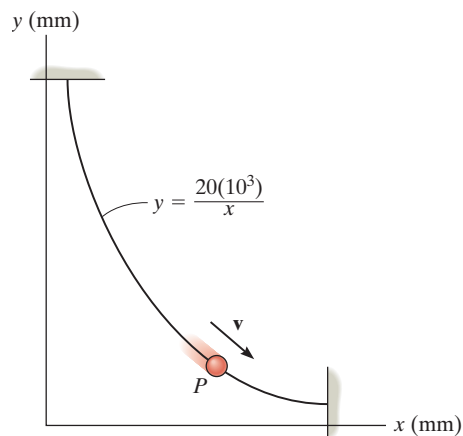
$$a_n = \frac{v^2}{\rho} = \frac{(300)^2}{279.5} = 322 \text{ mm/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2}$$

$$= \sqrt{(0)^2 + (322)^2} = 322 \text{ mm/s}^2$$

$$\text{Since } \frac{dy}{dx} = -0.5,$$

$$\theta = \tan^{-1}(-0.5) = 26.6^\circ \nearrow$$



Ans.

Ans.

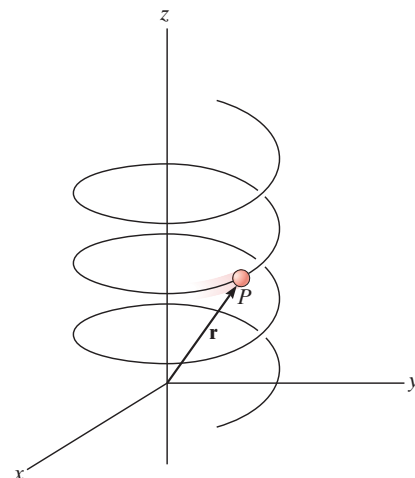
Ans:

$$a = 322 \text{ mm/s}^2$$

$$\theta = 26.6^\circ \nearrow$$

***12–152.**

A particle P travels along an elliptical spiral path such that its position vector \mathbf{r} is defined by $\mathbf{r} = \{2 \cos(0.1t)\mathbf{i} + 1.5 \sin(0.1t)\mathbf{j} + (2t)\mathbf{k}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. When $t = 8$ s, determine the coordinate direction angles α , β , and γ , which the binormal axis to the osculating plane makes with the x , y , and z axes. *Hint:* Solve for the velocity \mathbf{v}_P and acceleration \mathbf{a}_P of the particle in terms of their \mathbf{i} , \mathbf{j} , \mathbf{k} components. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?



SOLUTION

$$\mathbf{r}_P = 2 \cos(0.1t)\mathbf{i} + 1.5 \sin(0.1t)\mathbf{j} + 2t\mathbf{k}$$

$$\mathbf{v}_P = \dot{\mathbf{r}} = -0.2 \sin(0.1t)\mathbf{i} + 0.15 \cos(0.1t)\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = \ddot{\mathbf{r}} = -0.02 \cos(0.1t)\mathbf{i} - 0.015 \sin(0.1t)\mathbf{j}$$

When $t = 8$ s,

$$\mathbf{v}_P = -0.2 \sin(0.8 \text{ rad})\mathbf{i} + 0.15 \cos(0.8 \text{ rad})\mathbf{j} + 2\mathbf{k} = -0.14347\mathbf{i} + 0.10451\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = -0.02 \cos(0.8 \text{ rad})\mathbf{i} - 0.015 \sin(0.8 \text{ rad})\mathbf{j} = -0.013934\mathbf{i} - 0.01076\mathbf{j}$$

Since the binormal vector is perpendicular to the plane containing the n - t axis, and \mathbf{a}_P and \mathbf{v}_P are in this plane, then by the definition of the cross product,

$$\mathbf{b} = \mathbf{v}_P \times \mathbf{a}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.14347 & 0.10451 & 2 \\ -0.013934 & -0.01076 & 0 \end{vmatrix} = 0.02152\mathbf{i} - 0.027868\mathbf{j} + 0.003\mathbf{k}$$

$$b = \sqrt{(0.02152)^2 + (-0.027868)^2 + (0.003)^2} = 0.035338$$

$$\mathbf{u}_b = 0.60899\mathbf{i} - 0.78862\mathbf{j} + 0.085\mathbf{k}$$

$$\alpha = \cos^{-1}(0.60899) = 52.5^\circ \quad \text{Ans.}$$

$$\beta = \cos^{-1}(-0.78862) = 142^\circ \quad \text{Ans.}$$

$$\gamma = \cos^{-1}(0.085) = 85.1^\circ \quad \text{Ans.}$$

Note: The direction of the binormal axis may also be specified by the unit vector $\mathbf{u}_b' = -\mathbf{u}_b$, which is obtained from $\mathbf{b}' = \mathbf{a}_P \times \mathbf{v}_P$.

$$\text{For this case, } \alpha = 128^\circ, \beta = 37.9^\circ, \gamma = 94.9^\circ \quad \text{Ans.}$$

Ans:

$$\alpha = 52.5^\circ$$

$$\beta = 142^\circ$$

$$\gamma = 85.1^\circ$$

$$\alpha = 128^\circ, \beta = 37.9^\circ, \gamma = 94.9^\circ$$

12-153.

The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m, where t is in seconds. Determine the normal and tangential components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

Velocity: Here, $\mathbf{r} = \{(2t + t^2)\mathbf{i} + t^2\mathbf{j}\}$ m. To determine the velocity \mathbf{v} , apply Eq. 12-7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{(2 + 2t)\mathbf{i} + 2t\mathbf{j}\} \text{ m/s}$$

When $t = 2$ s, $\mathbf{v} = [2 + 2(2)]\mathbf{i} + 2(2)\mathbf{j} = \{6\mathbf{i} + 4\mathbf{j}\}$ m/s. Then $v = \sqrt{6^2 + 4^2} = 7.21$ m/s. Since the velocity is always directed tangent to the path,

$$v_n = 0 \quad \text{and} \quad v_t = 7.21 \text{ m/s} \quad \text{Ans.}$$

The velocity \mathbf{v} makes an angle $\theta = \tan^{-1} \frac{4}{6} = 33.69^\circ$ with the x axis.

Acceleration: To determine the acceleration \mathbf{a} , apply Eq. 12-9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{2\mathbf{i} + 2\mathbf{j}\} \text{ m/s}^2$$

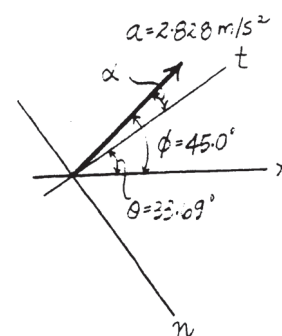
Then

$$a = \sqrt{2^2 + 2^2} = 2.828 \text{ m/s}^2$$

The acceleration \mathbf{a} makes an angle $\phi = \tan^{-1} \frac{2}{2} = 45.0^\circ$ with the x axis. From the figure, $\alpha = 45^\circ - 33.69 = 11.31^\circ$. Therefore,

$$a_n = a \sin \alpha = 2.828 \sin 11.31^\circ = 0.555 \text{ m/s}^2 \quad \text{Ans.}$$

$$a_t = a \cos \alpha = 2.828 \cos 11.31^\circ = 2.77 \text{ m/s}^2 \quad \text{Ans.}$$



Ans:

$$v_n = 0$$

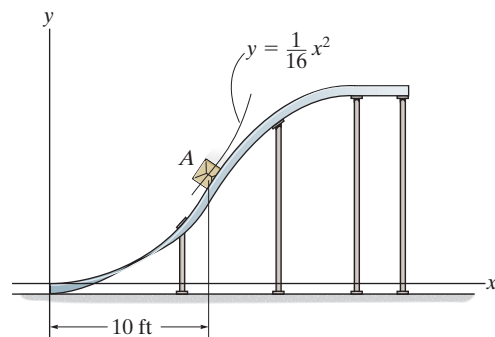
$$v_t = 7.21 \text{ m/s}$$

$$a_n = 0.555 \text{ m/s}^2$$

$$a_t = 2.77 \text{ m/s}^2$$

12–154.

If the speed of the crate at A is 15 ft/s, which is increasing at a rate $\dot{v} = 3 \text{ ft/s}^2$, determine the magnitude of the acceleration of the crate at this instant.



SOLUTION

Radius of Curvature:

$$y = \frac{1}{16}x^2$$

$$\frac{dy}{dx} = \frac{1}{8}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{8}$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(\frac{1}{8}x\right)^2\right]^{3/2}}{\left|\frac{1}{8}\right|} \bigg|_{x=10 \text{ ft}} = 32.82 \text{ ft}$$

Acceleration:

$$a_t = \dot{v} = 3 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{15^2}{32.82} = 6.856 \text{ ft/s}^2$$

The magnitude of the crate's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 6.856^2} = 7.48 \text{ ft/s}^2$$

Ans.

Ans:

$$a = 7.48 \text{ ft/s}^2$$

12–155.

A particle is moving along a circular path having a radius of 4 in. such that its position as a function of time is given by $\theta = \cos 2t$, where θ is in radians and t is in seconds. Determine the magnitude of the acceleration of the particle when $\theta = 30^\circ$.

SOLUTION

When $\theta = \frac{\pi}{6}$ rad, $\frac{\pi}{6} = \cos 2t \quad t = 0.5099 \text{ s}$

$$\dot{\theta} = \frac{d\theta}{dt} = -2 \sin 2t \bigg|_{t=0.5099 \text{ s}} = -1.7039 \text{ rad/s}$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = -4 \cos 2t \bigg|_{t=0.5099 \text{ s}} = -2.0944 \text{ rad/s}^2$$

$$r = 4 \quad \dot{r} = 0 \quad \ddot{r} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4(-1.7039)^2 = -11.6135 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(-2.0944) + 0 = -8.3776 \text{ in./s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-11.6135)^2 + (-8.3776)^2} = 14.3 \text{ in./s}^2$$

Ans.

Ans:

$$a = 14.3 \text{ in./s}^2$$

***12–156.**

For a short time a rocket travels up and to the right at a constant speed of 800 m/s along the parabolic path $y = 600 - 35x^2$. Determine the radial and transverse components of velocity of the rocket at the instant $\theta = 60^\circ$, where θ is measured counterclockwise from the x axis.

SOLUTION

$$y = 600 - 35x^2$$

$$\dot{y} = -70x\dot{x}$$

$$\frac{dy}{dx} = -70x$$

$$\tan 60^\circ = \frac{y}{x}$$

$$y = 1.732051x$$

$$1.732051x = 600 - 35x^2$$

$$x^2 + 0.049487x - 17.142857 = 0$$

Solving for the positive root,

$$x = 4.1157 \text{ m}$$

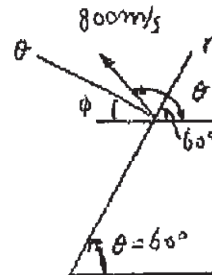
$$\tan \theta' = \frac{dy}{dx} = -288.1$$

$$\theta' = 89.8011^\circ$$

$$\phi = 180^\circ - 89.8011^\circ - 60^\circ = 30.1989^\circ$$

$$v_r = 800 \cos 30.1989^\circ = 691 \text{ m/s}$$

$$v_\theta = 800 \sin 30.1989^\circ = 402 \text{ m/s}$$



Ans.

Ans.

Ans:

$$v_r = 691 \text{ m/s}$$

$$v_\theta = 402 \text{ m/s}$$

12–157.

A particle moves along a path defined by polar coordinates $r = (2e^t)$ ft and $\theta = (8t^2)$ rad, where t is in seconds. Determine the components of its velocity and acceleration when $t = 1$ s.

SOLUTION

When $t = 1$ s,

$$r = 2e^t = 5.4366$$

$$\dot{r} = 2e^t = 5.4366$$

$$\ddot{r} = 2e^t = 5.4366$$

$$\theta = 8t^2$$

$$\dot{\theta} = 16t = 16$$

$$\ddot{\theta} = 16$$

$$v_r = \dot{r} = 5.44 \text{ ft/s}$$

Ans.

$$v_\theta = r\dot{\theta} = 5.4366(16) = 87.0 \text{ ft/s}$$

Ans.

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 5.4366 - 5.4366(16)^2 = -1386 \text{ ft/s}^2$$

Ans.

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5.4366(16) + 2(5.4366)(16) = 261 \text{ ft/s}^2$$

Ans.

Ans:

$$v_r = 5.44 \text{ ft/s}$$

$$v_\theta = 87.0 \text{ ft/s}$$

$$a_r = -1386 \text{ ft/s}^2$$

$$a_\theta = 261 \text{ ft/s}^2$$

12–158.

An airplane is flying in a straight line with a velocity of 200 mi/h and an acceleration of 3 mi/h². If the propeller has a diameter of 6 ft and is rotating at an angular rate of 120 rad/s, determine the magnitudes of velocity and acceleration of a particle located on the tip of the propeller.

SOLUTION

$$v_{Pl} = \left(\frac{200 \text{ mi}}{\text{h}} \right) \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = 293.3 \text{ ft/s}$$

$$a_{Pl} = \left(\frac{3 \text{ mi}}{\text{h}^2} \right) \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right)^2 = 0.00122 \text{ ft/s}^2$$

$$v_{Pr} = 120(3) = 360 \text{ ft/s}$$

$$v = \sqrt{v_{Pl}^2 + v_{Pr}^2} = \sqrt{(293.3)^2 + (360)^2} = 464 \text{ ft/s} \quad \textbf{Ans.}$$

$$a_{Pr} = \frac{v_{Pr}^2}{\rho} = \frac{(360)^2}{3} = 43200 \text{ ft/s}^2$$

$$a = \sqrt{a_{Pl}^2 + a_{Pr}^2} = \sqrt{(0.00122)^2 + (43200)^2} = 43.2(10^3) \text{ ft/s}^2 \quad \textbf{Ans.}$$

Ans:

$$v = 464 \text{ ft/s}$$

$$a = 43.2(10^3) \text{ ft/s}^2$$

12-159.

The small washer is sliding down the cord OA . When it is at the midpoint, its speed is 28 m/s and its acceleration is 7 m/s^2 . Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.

SOLUTION

The position of the washer can be defined using the cylindrical coordinate system (r, θ and z) as shown in Fig. a . Since θ is constant, there will be no transverse component for \mathbf{v} and \mathbf{a} . The velocity and acceleration expressed as Cartesian vectors are

$$\mathbf{v} = v \left(\frac{\mathbf{r}_{AO}}{r_{AO}} \right) = 28 \left[\frac{(0-2)\mathbf{i} + (0-3)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-2)^2 + (0-3)^2 + (0-6)^2}} \right] = \{-8\mathbf{i} - 12\mathbf{j} - 24\mathbf{k}\} \text{ m/s}$$

$$\mathbf{a} = a \left(\frac{\mathbf{r}_{AO}}{r_{AO}} \right) = 7 \left[\frac{(0-2)\mathbf{i} + (0-3)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-2)^2 + (0-3)^2 + (0-6)^2}} \right] = \{-2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{u}_r = \frac{\mathbf{r}_{OB}}{r_{OB}} = \frac{2\mathbf{i} + 3\mathbf{j}}{\sqrt{2^2 + 3^2}} = \frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}$$

$$\mathbf{u}_z = \mathbf{k}$$

Using vector dot product

$$v_r = \mathbf{v} \cdot \mathbf{u}_r = (-8\mathbf{i} - 12\mathbf{j} - 24\mathbf{k}) \cdot \left(\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j} \right) = -8 \left(\frac{2}{\sqrt{13}} \right) + \left[-12 \left(\frac{3}{\sqrt{13}} \right) \right] = -14.42 \text{ m/s}$$

$$v_z = \mathbf{v} \cdot \mathbf{u}_z = (-8\mathbf{i} - 12\mathbf{j} - 24\mathbf{k}) \cdot (\mathbf{k}) = -24.0 \text{ m/s}$$

$$a_r = \mathbf{a} \cdot \mathbf{u}_r = (-2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}) \cdot \left(\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j} \right) = -2 \left(\frac{2}{\sqrt{13}} \right) + \left[-3 \left(\frac{3}{\sqrt{13}} \right) \right] = -3.606 \text{ m/s}^2$$

$$a_z = \mathbf{a} \cdot \mathbf{u}_z = (-2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}) \cdot \mathbf{k} = -6.00 \text{ m/s}^2$$

Thus, in vector form

$$\mathbf{v} = \{-14.2\mathbf{u}_r - 24.0\mathbf{u}_z\} \text{ m/s}$$

Ans.

$$\mathbf{a} = \{-3.61\mathbf{u}_r - 6.00\mathbf{u}_z\} \text{ m/s}^2$$

Ans.

These components can also be determined using trigonometry by first obtain angle ϕ shown in Fig. a .

$$OA = \sqrt{2^2 + 3^2 + 6^2} = 7 \text{ m} \quad OB = \sqrt{2^2 + 3^2} = \sqrt{13}$$

Thus,

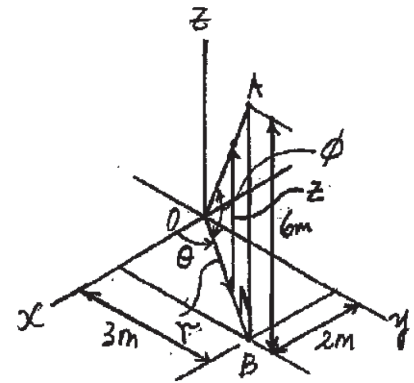
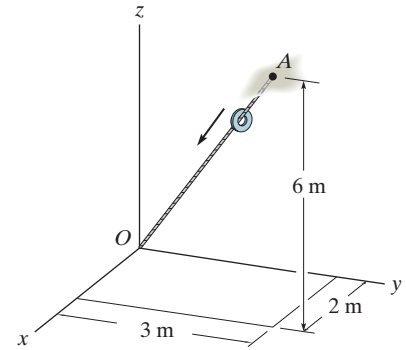
$$\sin \phi = \frac{6}{7} \text{ and } \cos \phi = \frac{\sqrt{13}}{7}. \text{ Then}$$

$$v_r = -v \cos \phi = -28 \left(\frac{\sqrt{13}}{7} \right) = -14.42 \text{ m/s}$$

$$v_z = -v \sin \phi = -28 \left(\frac{6}{7} \right) = -24.0 \text{ m/s}$$

$$a_r = -a \cos \phi = -7 \left(\frac{\sqrt{13}}{7} \right) = -3.606 \text{ m/s}^2$$

$$a_z = -a \sin \phi = -7 \left(\frac{6}{7} \right) = -6.00 \text{ m/s}^2$$



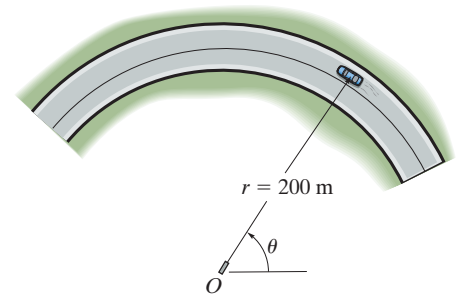
Ans:

$$\mathbf{v} = \{-14.2\mathbf{u}_r - 24.0\mathbf{u}_z\} \text{ m/s}$$

$$\mathbf{a} = \{-3.61\mathbf{u}_r - 6.00\mathbf{u}_z\} \text{ m/s}^2$$

***12–160.**

A radar gun at O rotates with the angular velocity of $\dot{\theta} = 0.1 \text{ rad/s}$ and angular acceleration of $\ddot{\theta} = 0.025 \text{ rad/s}^2$, at the instant $\theta = 45^\circ$, as it follows the motion of the car traveling along the circular road having a radius of $r = 200 \text{ m}$. Determine the magnitudes of velocity and acceleration of the car at this instant.



SOLUTION

Time Derivatives: Since r is constant,

$$\dot{r} = \ddot{r} = 0$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 200(0.1) = 20 \text{ m/s}$$

Thus, the magnitude of the car's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 20^2} = 20 \text{ m/s} \quad \textbf{Ans.}$$

Acceleration:

$$a_r = \dot{r} - r\dot{\theta}^2 = 0 - 200(0.1^2) = -2 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 200(0.025) + 0 = 5 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-2)^2 + 5^2} = 5.39 \text{ m/s}^2 \quad \textbf{Ans.}$$

Ans:

$$v = 20 \text{ m/s}$$

$$a = 5.39 \text{ m/s}^2$$

12–161.

If a particle moves along a path such that $r = (2 \cos t)$ ft and $\theta = (t/2)$ rad, where t is in seconds, plot the path $r = f(\theta)$ and determine the particle's radial and transverse components of velocity and acceleration.

SOLUTION

$$r = 2 \cos t \quad \dot{r} = -2 \sin t \quad \ddot{r} = -2 \cos t$$

$$\theta = \frac{t}{2} \quad \dot{\theta} = \frac{1}{2} \quad \ddot{\theta} = 0$$

$$v_r = \dot{r} = -2 \sin t$$

Ans.

$$v_\theta = r\dot{\theta} = (2 \cos t)\left(\frac{1}{2}\right) = \cos t$$

Ans.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2 \cos t - (2 \cos t)\left(\frac{1}{2}\right)^2 = -\frac{5}{2} \cos t$$

Ans.

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2 \cos t(0) + 2(-2 \sin t)\left(\frac{1}{2}\right) = -2 \sin t$$

Ans.

Ans:

$$v_r = -2 \sin t$$

$$v_\theta = \cos t$$

$$a_r = -\frac{5}{2} \cos t$$

$$a_\theta = -2 \sin t$$

12–162.

If a particle moves along a path such that $r = (e^{at})$ m and $\theta = t$, where t is in seconds, plot the path $r = f(\theta)$, and determine the particle's radial and transverse components of velocity and acceleration.

SOLUTION

$$r = e^{at} \quad \dot{r} = ae^{at} \quad \ddot{r} = a^2e^{at}$$

$$\theta = t \quad \dot{\theta} = 1 \quad \ddot{\theta} = 0$$

$$v_r = \dot{r} = ae^{at}$$

$$v_\theta = r\dot{\theta} = e^{at}(1) = e^{at}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = a^2e^{at} - e^{at}(1)^2 = e^{at}(a^2 - 1)$$

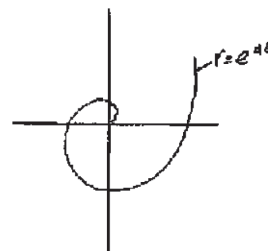
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = e^{at}(0) + 2(ae^{at})(1) = 2ae^{at}$$

Ans.

Ans.

Ans.

Ans.



Ans:

$$v_r = ae^{at}$$

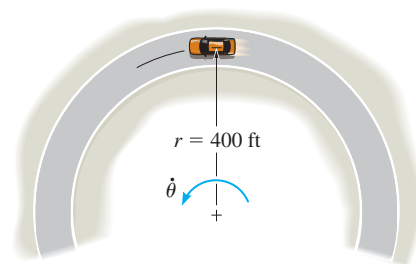
$$v_\theta = e^{at}$$

$$a_r = e^{at}(a^2 - 1)$$

$$a_\theta = 2ae^{at}$$

12-163.

The car travels along the circular curve having a radius $r = 400$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.025$ rad/s, which is decreasing at the rate $\ddot{\theta} = -0.008$ rad/s². Determine the radial and transverse components of the car's velocity and acceleration at this instant and sketch these components on the curve.



SOLUTION

$$r = 400 \quad \dot{r} = 0 \quad \ddot{r} = 0$$

$$\dot{\theta} = 0.025 \quad \ddot{\theta} = -0.008$$

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 400(0.025) = 10 \text{ ft/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.025)^2 = -0.25 \text{ ft/s}^2$$

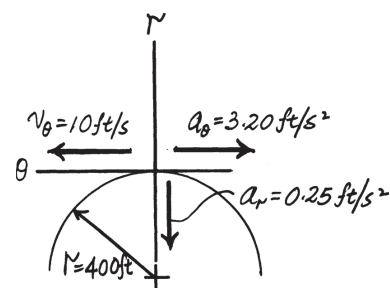
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(-0.008) + 0 = -3.20 \text{ ft/s}^2$$

Ans.

Ans.

Ans.

Ans.



Ans:

$$v_r = 0$$

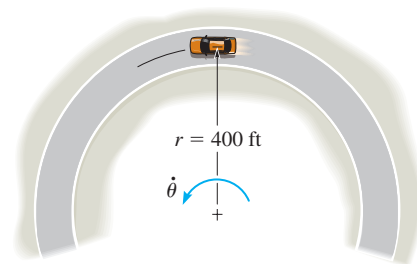
$$v_\theta = 10 \text{ ft/s}$$

$$a_r = -0.25 \text{ ft/s}^2$$

$$a_\theta = -3.20 \text{ ft/s}^2$$

***12–164.**

The car travels along the circular curve of radius $r = 400$ ft with a constant speed of $v = 30$ ft/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the car's acceleration.



SOLUTION

$$r = 400 \text{ ft} \quad \dot{r} = 0 \quad \ddot{r} = 0$$

$$v_r = \dot{r} = 0 \quad v_\theta = r\dot{\theta} = 400(\dot{\theta})$$

$$v = \sqrt{(0)^2 + (400\dot{\theta})^2} = 30$$

$$\dot{\theta} = 0.075 \text{ rad/s}$$

Ans.

$$\ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.075)^2 = -2.25 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(0) + 2(0)(0.075) = 0$$

$$a = \sqrt{(-2.25)^2 + (0)^2} = 2.25 \text{ ft/s}^2$$

Ans.

Ans:

$$\dot{\theta} = 0.075 \text{ rad/s}$$

$$a = 2.25 \text{ ft/s}^2$$

12–165.

The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, $\dot{\mathbf{a}}$, in terms of its cylindrical components, using Eq. 12–32.

SOLUTION

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta + \ddot{z}\mathbf{u}_z$$

$$\dot{\mathbf{a}} = (\dddot{r} - \dot{r}\dot{\theta}^2 - 2r\ddot{\theta}\dot{\theta})\mathbf{u}_r + (\ddot{r} - r\dot{\theta}^2)\dot{\mathbf{u}}_r + (\dot{r}\ddot{\theta} + r\ddot{\theta} + 2\dot{r}\dot{\theta} + 2\dot{r}\ddot{\theta})\mathbf{u}_\theta + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\dot{\mathbf{u}}_\theta + \dddot{z}\mathbf{u}_z + \ddot{z}\dot{\mathbf{u}}_z$$

$$\text{But, } \mathbf{u}_r = \dot{\theta}\mathbf{u}_\theta \quad \dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r \quad \dot{\mathbf{u}}_z = 0$$

Substituting and combining terms yields

$$\dot{\mathbf{a}} = (\dddot{r} - 3r\dot{\theta}^2 - 3r\ddot{\theta}\dot{\theta})\mathbf{u}_r + (3\dot{r}\ddot{\theta} + r\ddot{\theta} + 3\dot{r}\dot{\theta} - r\dot{\theta}^3)\mathbf{u}_\theta + (\ddot{z})\mathbf{u}_z \quad \text{Ans.}$$

Ans:

$$\dot{\mathbf{a}} = (\dddot{r} - 3r\dot{\theta}^2 - 3r\ddot{\theta}\dot{\theta})\mathbf{u}_r + (3\dot{r}\ddot{\theta} + r\ddot{\theta} + 3\dot{r}\dot{\theta} - r\dot{\theta}^3)\mathbf{u}_\theta + (\ddot{z})\mathbf{u}_z$$

12-166.

A particle is moving along a circular path having a radius of 6 in. such that its position as a function of time is given by $\theta = \sin 3t$, where θ is in radians, the argument for the sine are in radians, and t is in seconds. Determine the acceleration of the particle at $\theta = 30^\circ$. The particle starts from rest at $\theta = 0^\circ$.

SOLUTION

$$r = 6 \text{ in.}, \quad \dot{r} = 0, \quad \ddot{r} = 0$$

$$\theta = \sin 3t$$

$$\dot{\theta} = 3 \cos 3t$$

$$\ddot{\theta} = -9 \sin 3t$$

$$\text{At } \theta = 30^\circ,$$

$$\frac{30^\circ}{180^\circ} \pi = \sin 3t$$

$$t = 10.525 \text{ s}$$

Thus,

$$\dot{\theta} = 2.5559 \text{ rad/s}$$

$$\ddot{\theta} = -4.7124 \text{ rad/s}^2$$

$$a_r = \dot{r} - r\dot{\theta}^2 = 0 - 6(2.5559)^2 = -39.196$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 6(-4.7124) + 0 = -28.274$$

$$a = \sqrt{(-39.196)^2 + (-28.274)^2} = 48.3 \text{ in./s}^2$$

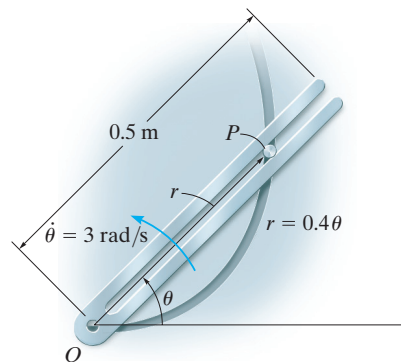
Ans.

Ans:

$$a = 48.3 \text{ in./s}^2$$

12–167.

The slotted link is pinned at O , and as a result of the constant angular velocity $\dot{\theta} = 3 \text{ rad/s}$ it drives the peg P for a short distance along the spiral guide $r = (0.4 \theta) \text{ m}$, where θ is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \pi/3 \text{ rad}$.



SOLUTION

$$\dot{\theta} = 3 \text{ rad/s} \quad r = 0.4 \theta$$

$$\dot{r} = 0.4 \dot{\theta}$$

$$\ddot{r} = 0.4 \ddot{\theta}$$

$$\text{At } \theta = \frac{\pi}{3}, \quad r = 0.4189$$

$$\dot{r} = 0.4(3) = 1.20$$

$$\ddot{r} = 0.4(0) = 0$$

$$v = \dot{r} = 1.20 \text{ m/s}$$

Ans.

$$v_{\theta} = r \dot{\theta} = 0.4189(3) = 1.26 \text{ m/s}$$

Ans.

$$a_r = \ddot{r} - r \dot{\theta}^2 = 0 - 0.4189(3)^2 = -3.77 \text{ m/s}^2$$

Ans.

$$a_{\theta} = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 0 + 2(1.20)(3) = 7.20 \text{ m/s}^2$$

Ans.

Ans:

$$v_r = 1.20 \text{ m/s}$$

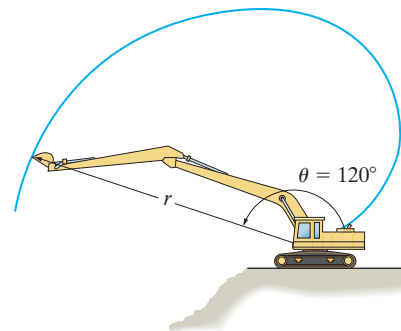
$$v_{\theta} = 1.26 \text{ m/s}$$

$$a_r = -3.77 \text{ m/s}^2$$

$$a_{\theta} = 7.20 \text{ m/s}^2$$

***12–168.**

For a short time the bucket of the backhoe traces the path of the cardioid $r = 25(1 - \cos \theta)$ ft. Determine the magnitudes of the velocity and acceleration of the bucket when $\theta = 120^\circ$ if the boom is rotating with an angular velocity of $\dot{\theta} = 2$ rad/s and an angular acceleration of $\ddot{\theta} = 0.2$ rad/s² at the instant shown.



SOLUTION

$$r = 25(1 - \cos \theta) = 25(1 - \cos 120^\circ) = 37.5 \text{ ft}$$

$$\dot{r} = 25 \sin \theta \dot{\theta} = 25 \sin 120^\circ (2) = 43.30 \text{ ft/s}$$

$$\ddot{r} = 25[\cos \theta \ddot{\theta} + \sin \theta \dot{\theta}^2] = 25[\cos 120^\circ (0.2) + \sin 120^\circ (2)^2] = -45.67 \text{ ft/s}^2$$

$$v_r = \dot{r} = 43.30 \text{ ft/s}$$

$$v_\theta = r\dot{\theta} = 37.5(2) = 75 \text{ ft/s}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{43.30^2 + 75^2} = 86.6 \text{ ft/s}$$

Ans.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -45.67 - 37.5(2)^2 = -195.67 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 37.5(0.2) + 2(43.30)(2) = 180.71 \text{ ft/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-195.67)^2 + 180.71^2} = 266 \text{ ft/s}^2$$

Ans.

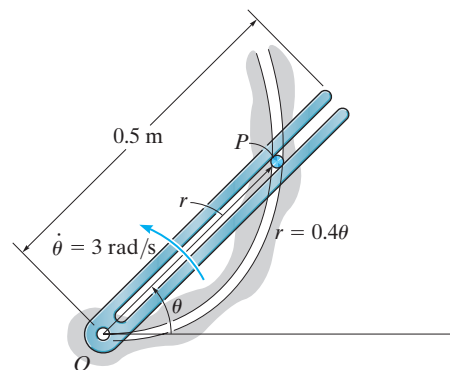
Ans:

$$v = 86.6 \text{ ft/s}$$

$$a = 266 \text{ ft/s}^2$$

12–169.

The slotted link is pinned at O , and as a result of the constant angular velocity $\dot{\theta} = 3 \text{ rad/s}$ it drives the peg P for a short distance along the spiral guide $r = (0.4 \theta) \text{ m}$, where θ is in radians. Determine the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = 0.5 \text{ m}$.



SOLUTION

$$r = 0.4 \theta$$

$$\dot{r} = 0.4 \dot{\theta}$$

$$\ddot{r} = 0.4 \ddot{\theta}$$

$$\dot{\theta} = 3$$

$$\ddot{\theta} = 0$$

At $r = 0.5 \text{ m}$,

$$\theta = \frac{0.5}{0.4} = 1.25 \text{ rad}$$

$$\dot{r} = 1.20$$

$$\ddot{r} = 0$$

$$v_r = \dot{r} = 1.20 \text{ m/s}$$

$$v_\theta = r \dot{\theta} = 0.5(3) = 1.50 \text{ m/s}$$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 0.5(3)^2 = -4.50 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20 \text{ m/s}^2$$

Ans.

Ans.

Ans.

Ans.

Ans:

$$v_r = 1.20 \text{ m/s}$$

$$v_\theta = 1.50 \text{ m/s}$$

$$a_r = -4.50 \text{ m/s}^2$$

$$a_\theta = 7.20 \text{ m/s}^2$$

12–170.

A particle moves in the x - y plane such that its position is defined by $\mathbf{r} = \{2t\mathbf{i} + 4t^2\mathbf{j}\}$ ft, where t is in seconds. Determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

$$\mathbf{r} = 2t\mathbf{i} + 4t^2\mathbf{j}|_{t=2} = 4\mathbf{i} + 16\mathbf{j}$$

$$\mathbf{v} = 2\mathbf{i} + 8t\mathbf{j}|_{t=2} = 2\mathbf{i} + 16\mathbf{j}$$

$$\mathbf{a} = 8\mathbf{j}$$

$$\theta = \tan^{-1}\left(\frac{16}{4}\right) = 75.964^\circ$$

$$v = \sqrt{(2)^2 + (16)^2} = 16.1245 \text{ ft/s}$$

$$\phi = \tan^{-1}\left(\frac{16}{2}\right) = 82.875^\circ$$

$$a = 8 \text{ ft/s}^2$$

$$\phi - \theta = 6.9112^\circ$$

$$v_r = 16.1245 \cos 6.9112^\circ = 16.0 \text{ ft/s}$$

Ans.

$$v_\theta = 16.1245 \sin 6.9112^\circ = 1.94 \text{ ft/s}$$

Ans.

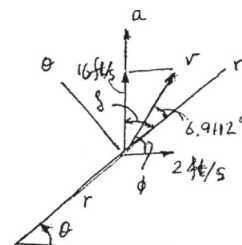
$$\delta = 90^\circ - \theta = 14.036^\circ$$

$$a_r = 8 \cos 14.036^\circ = 7.76 \text{ ft/s}^2$$

Ans.

$$a_\theta = 8 \sin 14.036^\circ = 1.94 \text{ ft/s}^2$$

Ans.



Ans:

$$v_r = 16.0 \text{ ft/s}$$

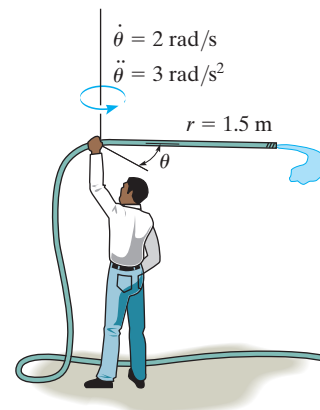
$$v_\theta = 1.94 \text{ ft/s}$$

$$a_r = 7.76 \text{ ft/s}^2$$

$$a_\theta = 1.94 \text{ ft/s}^2$$

12-171.

At the instant shown, the man is twirling a hose over his head with an angular velocity $\dot{\theta} = 2 \text{ rad/s}$ and an angular acceleration $\ddot{\theta} = 3 \text{ rad/s}^2$. If it is assumed that the hose lies in a horizontal plane, and water is flowing through it at a constant rate of 3 m/s , determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, $r = 1.5 \text{ m}$.



SOLUTION

$$r = 1.5$$

$$\dot{r} = 3$$

$$\ddot{r} = 0$$

$$\dot{\theta} = 2$$

$$\ddot{\theta} = 3$$

$$v_r = \dot{r} = 3$$

$$v_\theta = r\dot{\theta} = 1.5(2) = 3$$

$$v = \sqrt{(3)^2 + (3)^2} = 4.24 \text{ m/s}$$

Ans.

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(2)^2 = 6$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.5(3) + 2(3)(2) = 16.5$$

$$a = \sqrt{(6)^2 + (16.5)^2} = 17.6 \text{ m/s}^2$$

Ans.

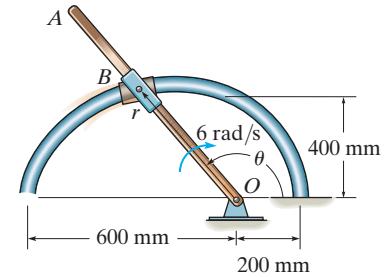
Ans:

$$v = 4.24 \text{ m/s}$$

$$a = 17.6 \text{ m/s}^2$$

***12–172.**

The rod OA rotates clockwise with a constant angular velocity of 6 rad/s . Two pin-connected slider blocks, located at B , move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 200(2 - \cos \theta) \text{ mm}$. Determine the speed of the slider blocks at the instant $\theta = 150^\circ$.



SOLUTION

Velocity. Using the chain rule, the first and second time derivatives of r can be determined.

$$r = 200(2 - \cos \theta)$$

$$\dot{r} = 200 (\sin \theta) \dot{\theta} = \{200 (\sin \theta) \dot{\theta}\} \text{ mm/s}$$

$$\ddot{r} = \{200[(\cos \theta)\dot{\theta}^2 + (\sin \theta)\ddot{\theta}]\} \text{ mm/s}^2$$

The radial and transverse components of the velocity are

$$v_r = \dot{r} = \{200 (\sin \theta) \dot{\theta}\} \text{ mm/s}$$

$$v_\theta = r\dot{\theta} = \{200(2 - \cos \theta)\dot{\theta}\} \text{ mm/s}$$

Since $\dot{\theta}$ is in the opposite sense to that of positive θ , $\dot{\theta} = -6 \text{ rad/s}$. Thus, at $\theta = 150^\circ$,

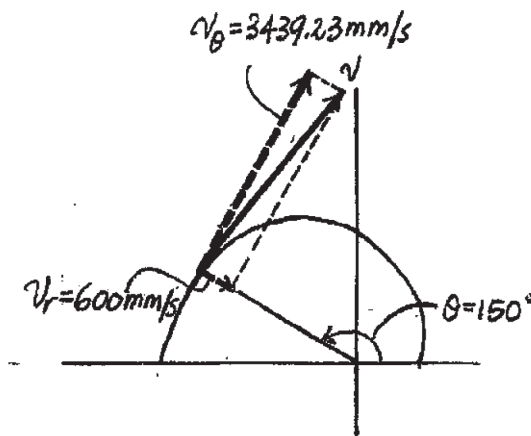
$$v_r = 200(\sin 150^\circ)(-6) = -600 \text{ mm/s}$$

$$v_\theta = 200(2 - \cos 150^\circ)(-6) = -3439.23 \text{ mm/s}$$

Thus, the magnitude of the velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-600)^2 + (-3439.23)^2} = 3491 \text{ mm/s} = 3.49 \text{ m/s} \quad \text{Ans.}$$

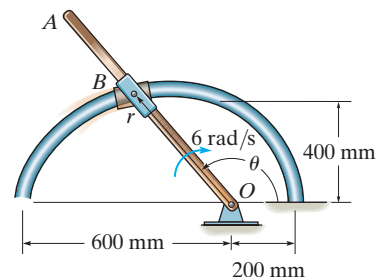
These components are shown in Fig. *a*



Ans:
 $v = 3.49 \text{ m/s}$

12-173.

Determine the magnitude of the acceleration of the slider blocks in Prob. 12-172 when $\theta = 150^\circ$.



SOLUTION

Acceleration. Using the chain rule, the first and second time derivatives of r can be determined

$$r = 200(2 - \cos \theta)$$

$$\dot{r} = 200 (\sin \theta) \dot{\theta} = \{ 200 (\sin \theta) \dot{\theta} \} \text{ mm/s}$$

$$\ddot{r} = \{ 200[(\cos \theta) \ddot{\theta} + (\sin \theta) \dot{\theta}^2] \} \text{ mm/s}^2$$

Here, since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$. Since $\dot{\theta}$ is in the opposite sense to that of positive θ , $\dot{\theta} = -6 \text{ rad/s}$. Thus, at $\theta = 150^\circ$

$$r = 200(2 - \cos 150^\circ) = 573.21 \text{ mm}$$

$$\dot{r} = 200(\sin 150^\circ)(-6) = -600 \text{ mm/s}$$

$$\ddot{r} = 200[(\cos 150^\circ)(-6)^2 + \sin 150^\circ(0)] = -6235.38 \text{ mm/s}^2$$

The radial and transverse components of the acceleration are

$$a_r = \ddot{r} - r\dot{\theta}^2 = -6235.38 - 573.21(-6)^2 = -26870.77 \text{ mm/s}^2 = -26.87 \text{ m/s}^2$$

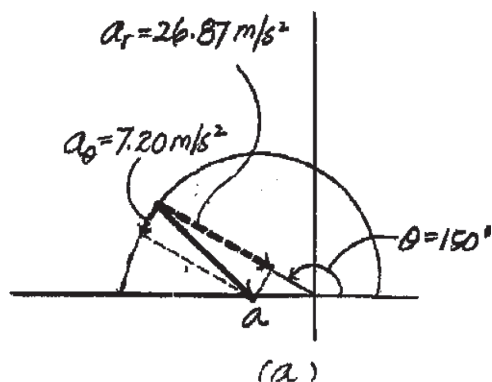
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 573.21(0) + 2(-600)(-6) = 7200 \text{ mm/s}^2 = 7.20 \text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-26.87)^2 + 7.20^2} = 27.82 \text{ m/s}^2 = 27.8 \text{ m/s}^2$$

Ans.

These components are shown in Fig. a .

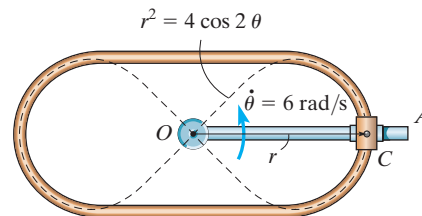


Ans:

$$a = 27.8 \text{ m/s}^2$$

12-174.

A double collar C is pin connected together such that one collar slides over a fixed rod and the other slides over a rotating rod. If the geometry of the fixed rod for a short distance can be defined by a lemniscate, $r^2 = (4 \cos 2\theta) \text{ ft}^2$, determine the collar's radial and transverse components of velocity and acceleration at the instant $\theta = 0^\circ$ as shown. Rod OA is rotating at a constant rate of $\dot{\theta} = 6 \text{ rad/s}$.



SOLUTION

$$r^2 = 4 \cos 2\theta$$

$$r\dot{r} = -4 \sin 2\theta \dot{\theta}$$

$$r\ddot{r} = \dot{r}^2 = -4 \sin 2\theta \ddot{\theta} - 8 \cos 2\theta \dot{\theta}^2$$

$$\text{when } \theta = 0, \dot{\theta} = 6, \ddot{\theta} = 0$$

$$r = 2, \dot{r} = 0, \ddot{r} = -144$$

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 2(6) = 12 \text{ ft/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -144 - 2(6)^2 = -216 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(0) + 2(0)(6) = 0$$

Ans.

Ans.

Ans.

Ans.

Ans:

$$v_r = 0$$

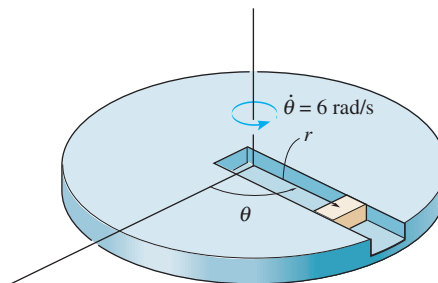
$$v_\theta = 12 \text{ ft/s}$$

$$a_r = -216 \text{ ft/s}^2$$

$$a_\theta = 0$$

12–175.

A block moves outward along the slot in the platform with a speed of $\dot{r} = (4t)$ m/s, where t is in seconds. The platform rotates at a constant rate of 6 rad/s. If the block starts from rest at the center, determine the magnitudes of its velocity and acceleration when $t = 1$ s.



SOLUTION

$$\dot{r} = 4t|_{t=1} = 4 \quad \ddot{r} = 4$$

$$\dot{\theta} = 6 \quad \ddot{\theta} = 0$$

$$\int_0^1 dr = \int_0^1 4t \, dt$$

$$r = 2t^2|_0^1 = 2 \text{ m}$$

$$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} = \sqrt{(4)^2 + [2(6)]^2} = 12.6 \text{ m/s}$$

Ans.

$$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (\ddot{\theta}r + 2\dot{r}\dot{\theta})^2} = \sqrt{[4 - 2(6)^2]^2 + [0 + 2(4)(6)]^2}$$

$$= 83.2 \text{ m/s}^2$$

Ans.

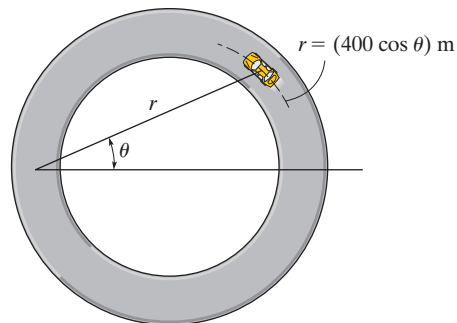
Ans:

$$v = 12.6 \text{ m/s}$$

$$a = 83.2 \text{ m/s}^2$$

***12–176.**

The car travels around the circular track with a constant speed of 20 m/s. Determine the car's radial and transverse components of velocity and acceleration at the instant $\theta = \pi/4$ rad.



SOLUTION

$$v = 20 \text{ m/s}$$

$$\theta = \frac{\pi}{4} = 45^\circ$$

$$r = 400 \cos \theta$$

$$\dot{r} = -400 \sin \theta \dot{\theta}$$

$$\ddot{r} = -400(\cos \theta (\dot{\theta})^2 + \sin \theta \ddot{\theta})$$

$$v^2 = (\dot{r})^2 + (r\dot{\theta})^2$$

$$0 = \ddot{r} + r\dot{\theta}(\ddot{\theta} + \dot{\theta})$$

Thus

$$r = 282.84$$

$$(20)^2 = [-400 \sin 45^\circ \dot{\theta}]^2 + [282.84 \dot{\theta}]^2$$

$$\dot{\theta} = 0.05$$

$$\dot{r} = -14.14$$

$$0 = -14.14[-400(\cos 45^\circ)(0.05)^2 + \sin 45^\circ \ddot{\theta}] + 282.84(0.05)[-14.14(0.05) + 282.84\ddot{\theta}]$$

$$\ddot{\theta} = 0$$

$$\ddot{r} = -0.707$$

$$v_r = \dot{r} = -14.1 \text{ m/s}$$

Ans.

$$v_\theta = r\dot{\theta} = 282.84(0.05) = 14.1 \text{ m/s}$$

Ans.

$$a_r = \ddot{r} - r(\dot{\theta})^2 = -0.707 - 282.84(0.05)^2 = -1.41 \text{ m/s}^2$$

Ans.

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \ddot{\theta} + 2(-14.14)(0.05) = -1.41 \text{ m/s}^2$$

Ans.

Ans:

$$v_r = -14.1 \text{ m/s}$$

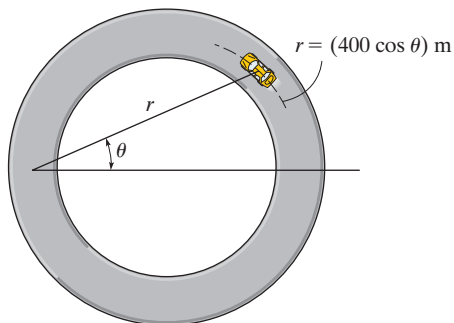
$$v_\theta = 14.1 \text{ m/s}$$

$$a_r = -1.41 \text{ m/s}^2$$

$$a_\theta = -1.41 \text{ m/s}^2$$

12-177.

The car travels around the circular track such that its transverse component is $\theta = (0.006t^2)$ rad, where t is in seconds. Determine the car's radial and transverse components of velocity and acceleration at the instant $t = 4$ s.



SOLUTION

$$\theta = 0.006t^2|_{t=4} = 0.096 \text{ rad} = 5.50^\circ$$

$$\dot{\theta} = 0.012t|_{t=4} = 0.048 \text{ rad/s}$$

$$\ddot{\theta} = 0.012 \text{ rad/s}^2$$

$$r = 400 \cos \theta$$

$$\dot{r} = -400 \sin \theta \dot{\theta}$$

$$\ddot{r} = -400(\cos \theta (\dot{\theta})^2 + \sin \theta \ddot{\theta})$$

$$\text{At } \theta = 0.096 \text{ rad}$$

$$r = 398.158 \text{ m}$$

$$\dot{r} = -1.84037 \text{ m/s}$$

$$\ddot{r} = -1.377449 \text{ m/s}^2$$

$$v_r = \dot{r} = -1.84 \text{ m/s}$$

Ans.

$$v_\theta = r \dot{\theta} = 398.158(0.048) = 19.1 \text{ m/s}$$

Ans.

$$a_r = \ddot{r} - r (\dot{\theta})^2 = -1.377449 - 398.158(0.048)^2 = -2.29 \text{ m/s}^2$$

Ans.

$$a_\theta = r \ddot{\theta} = 2\dot{r} \dot{\theta} = 398.158 (0.012) + 2(-1.84037)(0.048) = 4.60 \text{ m/s}^2$$

Ans.

Ans:

$$v_r = -1.84 \text{ m/s}$$

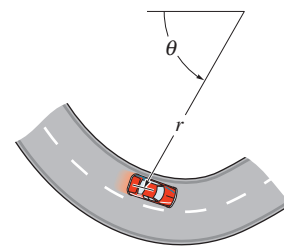
$$v_\theta = 19.1 \text{ m/s}$$

$$a_r = -2.29 \text{ m/s}^2$$

$$a_\theta = 4.60 \text{ m/s}^2$$

12–178.

The car travels along a road which for a short distance is defined by $r = (200/\theta)$ ft, where θ is in radians. If it maintains a constant speed of $v = 35$ ft/s, determine the radial and transverse components of its velocity when $\theta = \pi/3$ rad.



SOLUTION

$$r = \frac{200}{\theta} \bigg|_{\theta=\pi/3 \text{ rad}} = \frac{600}{\pi} \text{ ft}$$

$$\dot{r} = -\frac{200}{\theta^2} \dot{\theta} \bigg|_{\theta=\pi/3 \text{ rad}} = -\frac{1800}{\pi^2} \dot{\theta}$$

$$v_r = \dot{r} = -\frac{1800}{\pi^2} \dot{\theta} \quad v_\theta = r\dot{\theta} = \frac{600}{\pi} \dot{\theta}$$

$$v^2 = v_r^2 + v_\theta^2$$

$$35^2 = \left(-\frac{1800}{\pi^2} \dot{\theta} \right)^2 + \left(\frac{600}{\pi} \dot{\theta} \right)^2$$

$$\dot{\theta} = 0.1325 \text{ rad/s}$$

$$v_r = -\frac{1800}{\pi^2} (0.1325) = -24.2 \text{ ft/s}$$

Ans.

$$v_\theta = \frac{600}{\pi} (0.1325) = 25.3 \text{ ft/s}$$

Ans.

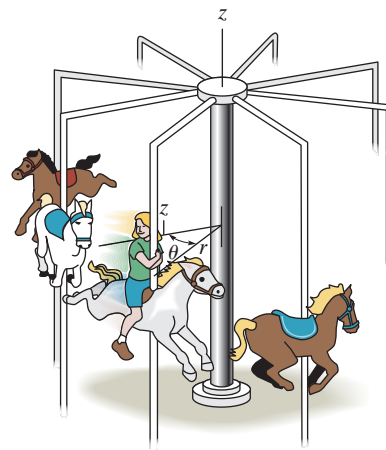
Ans:

$$v_r = -24.2 \text{ ft/s}$$

$$v_\theta = 25.3 \text{ ft/s}$$

12–179.

A horse on the merry-go-round moves according to the equations $r = 8$ ft, $\theta = (0.6t)$ rad, and $z = (1.5 \sin \theta)$ ft, where t is in seconds. Determine the cylindrical components of the velocity and acceleration of the horse when $t = 4$ s.



SOLUTION

$$r = 8 \quad \theta = 0.6t$$

$$\dot{r} = 0 \quad \dot{\theta} = 0.6$$

$$\ddot{r} = 0 \quad \ddot{\theta} = 0$$

$$z = 1.5 \sin \theta$$

$$\dot{z} = 1.5 \cos \theta \dot{\theta}$$

$$\ddot{z} = -1.5 \sin \theta (\dot{\theta})^2 + 1.5 \cos \theta \ddot{\theta}$$

$$\text{At } t = 4 \text{ s}$$

$$\theta = 2.4$$

$$\dot{z} = -0.6637$$

$$\ddot{z} = -0.3648$$

$$v_r = 0$$

$$v_\theta = 4.80 \text{ ft/s}$$

$$v_z = -0.664 \text{ ft/s}$$

$$a_r = 0 - 8(0.6)^2 = -2.88 \text{ ft/s}^2$$

$$a_\theta = 0 + 0 = 0$$

$$a_z = -0.365 \text{ ft/s}^2$$

Ans.

Ans.

Ans.

Ans.

Ans.

Ans.

Ans:

$$v_r = 0$$

$$v_\theta = 4.80 \text{ ft/s}$$

$$v_z = -0.664 \text{ ft/s}$$

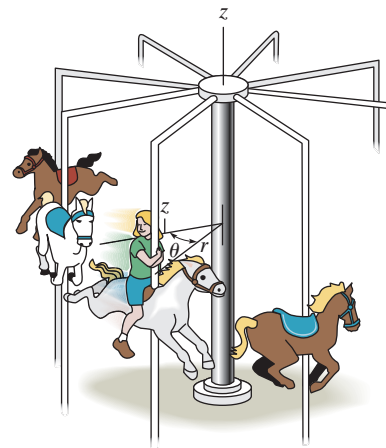
$$a_r = -2.88 \text{ ft/s}^2$$

$$a_\theta = 0$$

$$a_z = -0.365 \text{ ft/s}^2$$

***12–180.**

A horse on the merry-go-round moves according to the equations $r = 8$ ft, $\dot{\theta} = 2$ rad/s and $z = (1.5 \sin \theta)$ ft, where t is in seconds. Determine the maximum and minimum magnitudes of the velocity and acceleration of the horse during the motion.



SOLUTION

$$r = 8$$

$$\dot{r} = 0 \quad \dot{\theta} = 2$$

$$\ddot{r} = 0 \quad \ddot{\theta} = 0$$

$$z = 1.5 \sin \theta$$

$$\dot{z} = 1.5 \cos \theta \dot{\theta}$$

$$\ddot{z} = -1.5 \sin \theta (\dot{\theta})^2 + 1.5 \cos \theta \ddot{\theta}$$

$$v_r = \dot{r} = 0$$

$$v_\theta = r \dot{\theta} = 8(2) = 16 \text{ ft/s}$$

$$(v_z)_{\max} = \dot{z} = 1.5(\cos 0^\circ)(2) = 3 \text{ ft/s}$$

$$(v_z)_{\min} = \dot{z} = 1.5(\cos 90^\circ)(2) = 0$$

$$v_{\max} = \sqrt{(16)^2 + (3)^2} = 16.3 \text{ ft/s}$$

$$v_{\min} = \sqrt{(16)^2 + (0)^2} = 16 \text{ ft/s}$$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 8(2)^2 = -32 \text{ ft/s}^2$$

$$a_\theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 0 + 0 = 0$$

$$(a_z)_{\max} = \ddot{z} = -1.5(\sin 90^\circ)(2)^2 = -6$$

$$(a_z)_{\min} = \ddot{z} = -1.5(\sin 0^\circ)(2)^2 = 0$$

$$a_{\max} = \sqrt{(-32)^2 + (0)^2 + (-6)^2} = 32.6 \text{ ft/s}^2$$

$$a_{\min} = \sqrt{(-32)^2 + (0)^2 + (0)^2} = 32 \text{ ft/s}^2$$

Ans.

Ans.

Ans.

Ans.

Ans:

$$v_{\max} = 16.3 \text{ ft/s}$$

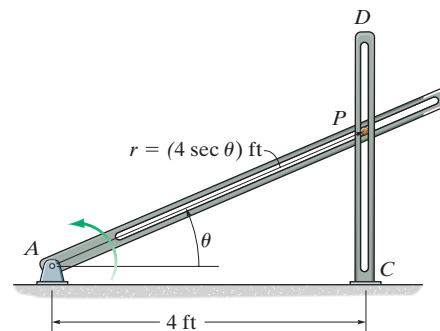
$$v_{\min} = 16 \text{ ft/s}$$

$$a_{\max} = 32.6 \text{ ft/s}^2$$

$$a_{\min} = 32 \text{ ft/s}^2$$

12–181.

If the slotted arm AB rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 2 \text{ rad/s}$, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30^\circ$. The peg is constrained to move in the slots of the fixed bar CD and rotating bar AB .



SOLUTION

Time Derivatives:

$$r = 4 \sec \theta$$

$$\dot{r} = (4 \sec \theta (\tan \theta) \dot{\theta}) \text{ ft/s} \quad \dot{\theta} = 2 \text{ rad/s}$$

$$\ddot{r} = 4 [\sec \theta (\tan \theta) \ddot{\theta} + \dot{\theta} (\sec \theta (\sec^2 \theta) \dot{\theta} + \tan \theta \sec \theta (\tan \theta) \dot{\theta})] \quad \ddot{\theta} = 0$$

$$= 4 [\sec \theta (\tan \theta) \ddot{\theta} + \dot{\theta}^2 (\sec^3 \theta + \tan^2 \theta \sec \theta)] \text{ ft/s}^2$$

When $\theta = 30^\circ$,

$$r|_{\theta=30^\circ} = 4 \sec 30^\circ = 4.619 \text{ ft}$$

$$\dot{r}|_{\theta=30^\circ} = (4 \sec 30^\circ \tan 30^\circ)(2) = 5.333 \text{ ft/s}$$

$$\ddot{r}|_{\theta=30^\circ} = 4[0 + 2^2(\sec^3 30^\circ + \tan^2 30^\circ \sec 30^\circ)] = 30.79 \text{ ft/s}^2$$

Velocity:

$$v_r = \dot{r} = 5.333 \text{ ft/s} \quad v_\theta = r\dot{\theta} = 4.619(2) = 9.238 \text{ ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{5.333^2 + 9.238^2} = 10.7 \text{ ft/s} \quad \text{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 30.79 - 4.619(2^2) = 12.32 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(5.333)(2) = 21.23 \text{ ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{12.32^2 + 21.23^2} = 24.6 \text{ ft/s}^2 \quad \text{Ans.}$$

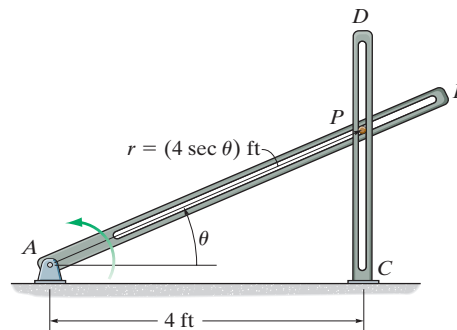
Ans:

$$v = 10.7 \text{ ft/s}$$

$$a = 24.6 \text{ ft/s}^2$$

12-182.

The peg is constrained to move in the slots of the fixed bar CD and rotating bar AB . When $\theta = 30^\circ$, the angular velocity and angular acceleration of arm AB are $\dot{\theta} = 2 \text{ rad/s}$ and $\ddot{\theta} = 3 \text{ rad/s}^2$, respectively. Determine the magnitudes of the velocity and acceleration of the peg P at this instant.



SOLUTION

Time Derivatives:

$$r = 4 \sec \theta$$

$$\dot{r} = (4 \sec \theta (\tan \theta) \dot{\theta}) \text{ ft/s} \quad \dot{\theta} = 2 \text{ rad/s}$$

$$\ddot{r} = 4[\sec \theta (\tan \theta) \ddot{\theta} + \dot{\theta} (\sec \theta \sec^2 \theta \dot{\theta} + \tan \theta \sec \theta (\tan \theta) \dot{\theta})] \quad \ddot{\theta} = 3 \text{ rad/s}^2$$

$$= 4[\sec \theta (\tan \theta) \ddot{\theta} + \dot{\theta}^2 (\sec^3 \theta + \tan^2 \theta \sec \theta)] \text{ ft/s}^2$$

When $\theta = 30^\circ$,

$$r|_{\theta=30^\circ} = 4 \sec 30^\circ = 4.619 \text{ ft}$$

$$\dot{r}|_{\theta=30^\circ} = (4 \sec 30^\circ \tan 30^\circ)(2) = 5.333 \text{ ft/s}$$

$$\ddot{r}|_{\theta=30^\circ} = 4[(\sec 30^\circ \tan 30^\circ)(3) + 2^2(\sec^3 30^\circ + \tan^2 30^\circ \sec 30^\circ)] = 38.79 \text{ ft/s}^2$$

Velocity:

$$v_r = \dot{r} = 5.333 \text{ ft/s}$$

$$v_\theta = r\dot{\theta} = 4.619(2) = 9.238 \text{ ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{5.333^2 + 9.238^2} = 10.7 \text{ ft/s} \quad \text{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 38.79 - 4.619(2^2) = 20.32 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.619(3) + 2(5.333)(2) = 35.19 \text{ ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{20.32^2 + 35.19^2} = 40.6 \text{ ft/s}^2 \quad \text{Ans.}$$

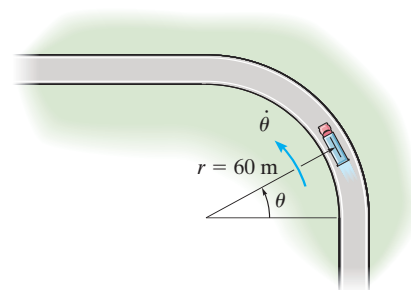
Ans:

$$v = 10.7 \text{ ft/s}$$

$$a = 40.6 \text{ ft/s}^2$$

12–183.

A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a constant speed $v = 20$ m/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the truck's acceleration.



SOLUTION

$$r = 60$$

$$\dot{r} = 0$$

$$\ddot{r} = 0$$

$$v = 20$$

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 60\dot{\theta}$$

$$v = \sqrt{(v_r)^2 + (v_\theta)^2}$$

$$20 = 60\dot{\theta}$$

$$\dot{\theta} = 0.333 \text{ rad/s}$$

Ans.

$$\begin{aligned} a_r &= \ddot{r} - r(\dot{\theta})^2 \\ &= 0 - 60(0.333)^2 \\ &= -6.67 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} \\ &= 60\ddot{\theta} \end{aligned}$$

Since

$$v = r\dot{\theta}$$

$$\dot{v} = \dot{r}\dot{\theta} + r\ddot{\theta}$$

$$0 = 0 + 60\ddot{\theta}$$

$$\ddot{\theta} = 0$$

Thus,

$$a_\theta = 0$$

$$a = |a_r| = 6.67 \text{ m/s}^2$$

Ans.

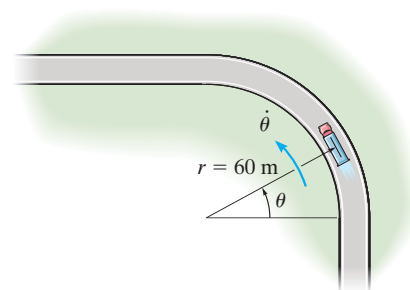
Ans:

$$\dot{\theta} = 0.333 \text{ rad/s}$$

$$a = 6.67 \text{ m/s}^2$$

***12–184.**

A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a speed of 20 m/s which is increasing at 3 m/s^2 . Determine the truck's radial and transverse components of acceleration.



SOLUTION

$$r = 60$$

$$a_t = 3 \text{ m/s}^2$$

$$a_n = \frac{v^2}{r} = \frac{(20)^2}{60} = 6.67 \text{ m/s}^2$$

$$a_r = -a_n = -6.67 \text{ m/s}^2$$

$$a_\theta = a_t = 3 \text{ m/s}^2$$

Ans.

Ans.

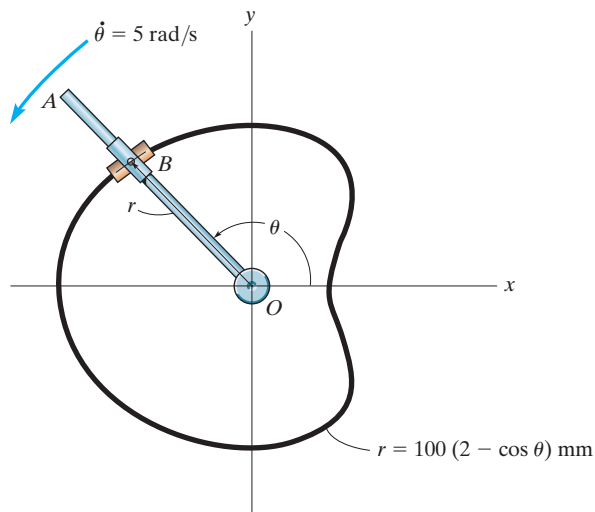
Ans:

$$a_r = -6.67 \text{ m/s}^2$$

$$a_\theta = 3 \text{ m/s}^2$$

12–185.

The rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5 \text{ rad/s}$. Two pin-connected slider blocks, located at B , move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 100(2 - \cos \theta) \text{ mm}$. Determine the speed of the slider blocks at the instant $\theta = 120^\circ$.



SOLUTION

$$\dot{\theta} = 5$$

$$r = 100(2 - \cos \theta)$$

$$\dot{r} = 100 \sin \theta \dot{\theta} = 500 \sin \theta$$

$$\ddot{r} = 500 \cos \theta \dot{\theta} = 2500 \cos \theta$$

$$\text{At } \theta = 120^\circ,$$

$$v_r = \dot{r} = 500 \sin 120^\circ = 433.013$$

$$v_\theta = r\dot{\theta} = 100(2 - \cos 120^\circ)(5) = 1250$$

$$v = \sqrt{(433.013)^2 + (1250)^2} = 1322.9 \text{ mm/s} = 1.32 \text{ m/s}$$

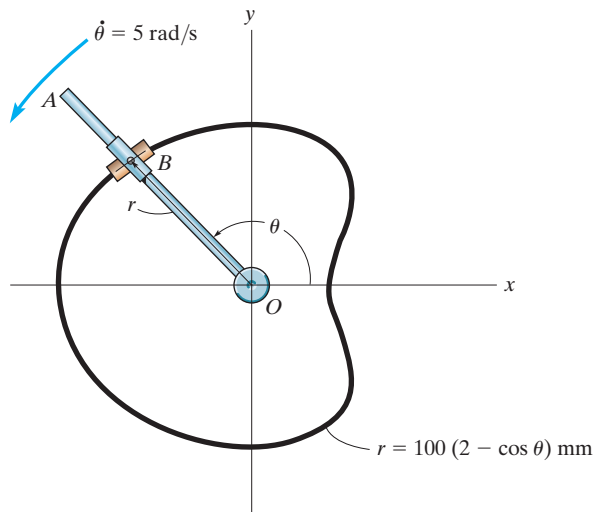
Ans.

Ans:

$$v = 1.32 \text{ m/s}$$

12–186.

Determine the magnitude of the acceleration of the slider blocks in Prob. 12–185 when $\theta = 120^\circ$.



SOLUTION

$$\dot{\theta} = 5$$

$$\ddot{\theta} = 0$$

$$r = 100(2 - \cos \theta)$$

$$\dot{r} = 100 \sin \theta \dot{\theta} = 500 \sin \theta$$

$$\ddot{r} = 500 \cos \theta \dot{\theta} = 2500 \cos \theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 2500 \cos \theta - 100(2 - \cos \theta)(5)^2 = 5000(\cos 120^\circ - 1) = -7500 \text{ mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(500 \sin \theta)(5) = 5000 \sin 120^\circ = 4330.1 \text{ mm/s}^2$$

$$a = \sqrt{(-7500)^2 + (4330.1)^2} = 8660.3 \text{ mm/s}^2 = 8.66 \text{ m/s}^2$$

Ans.

Ans:

$$a = 8.66 \text{ m/s}^2$$

12–187.

The searchlight on the boat anchored 2000 ft from shore is turned on the automobile, which is traveling along the straight road at a constant speed of 80 ft/s. Determine the angular rate of rotation of the light when the automobile is $r = 3000$ ft from the boat.

SOLUTION

$$r = 2000 \csc \theta$$

$$\dot{r} = -2000 \csc \theta \cot \theta \dot{\theta}$$

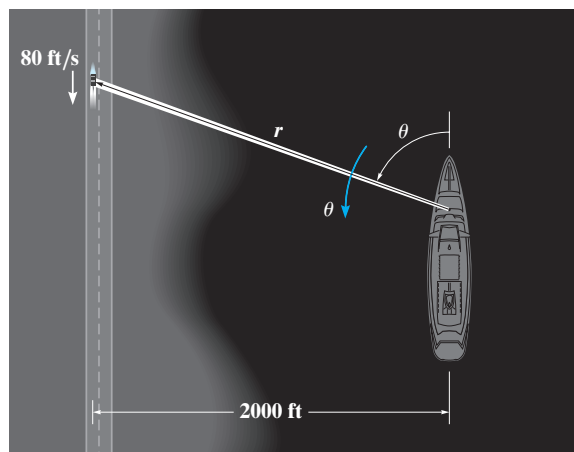
$$\text{At } r = 3000 \text{ ft, } \theta = 41.8103^\circ$$

$$\dot{r} = -3354.102 \dot{\theta}$$

$$v = \sqrt{(\dot{r})^2 + (r \dot{\theta})^2}$$

$$(80)^2 = [(-3354.102)^2 + (3000)^2](\dot{\theta})^2$$

$$\dot{\theta} = 0.0177778 = 0.0178 \text{ rad/s}$$



Ans.

Ans:

$$\dot{\theta} = 0.0178 \text{ rad/s}$$

***12–188.**

If the car in Prob. 12–187 is accelerating at 15 ft/s^2 and has a velocity of 80 ft/s at the instant $r = 3000 \text{ ft}$, determine the required angular acceleration $\ddot{\theta}$ of the light at this instant.

SOLUTION

$$r = 2000 \csc \theta$$

$$\dot{r} = -2000 \csc \theta \cot \theta \dot{\theta}$$

$$\text{At } r = 3000 \text{ ft}, \theta = 41.8103^\circ$$

$$\dot{r} = -3354.102 \dot{\theta}$$

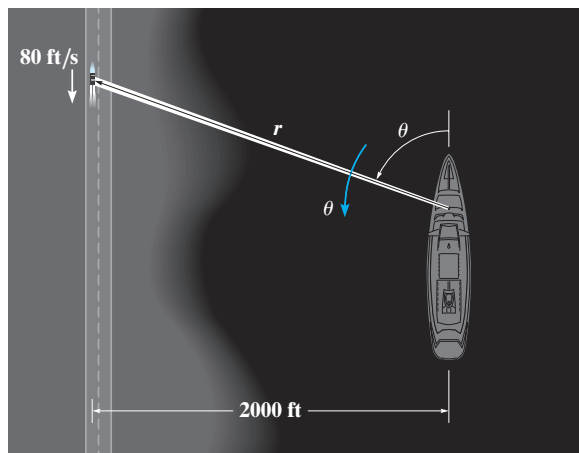
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$a_\theta = 3000\ddot{\theta} + 2(-3354.102)(0.0177778)^2$$

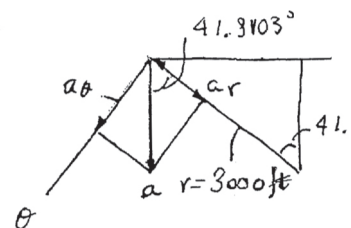
$$\text{Since } a_\theta = 15 \sin 41.8103^\circ = 10 \text{ m/s}$$

Then,

$$\ddot{\theta} = 0.00404 \text{ rad/s}^2$$



Ans.

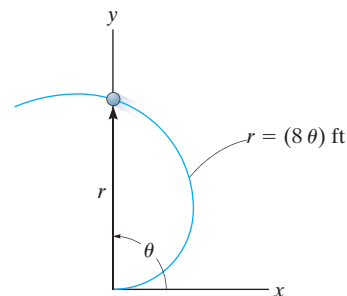


Ans:

$$\ddot{\theta} = 0.00404 \text{ rad/s}^2$$

12–189.

A particle moves along an Archimedean spiral $r = (8\theta)$ ft, where θ is given in radians. If $\dot{\theta} = 4$ rad/s (constant), determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = \pi/2$ rad. Sketch the curve and show the components on the curve.



SOLUTION

Time Derivatives: Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi \text{ ft} \quad \dot{r} = 8\dot{\theta} = 8(4) = 32.0 \text{ ft/s} \quad \ddot{r} = 8\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 32.0 \text{ ft/s}$$

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3 \text{ ft/s}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4\pi(4^2) = -201 \text{ ft/s}^2$$

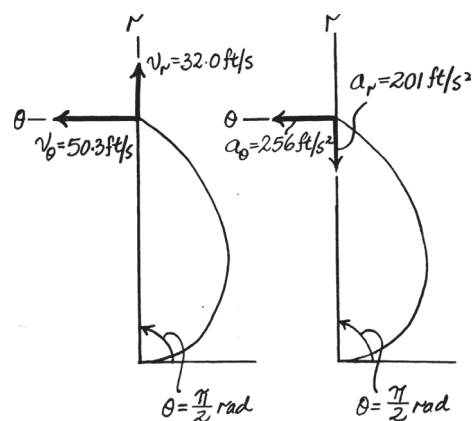
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(32.0)(4) = 256 \text{ ft/s}^2$$

Ans.

Ans.

Ans.

Ans.



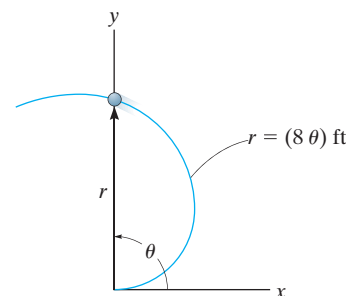
Ans:

$$a_r = -201 \text{ ft/s}^2$$

$$a_\theta = 256 \text{ ft/s}^2$$

12–190.

Solve Prob. 12–189 if the particle has an angular acceleration $\ddot{\theta} = 5 \text{ rad/s}^2$ when $\dot{\theta} = 4 \text{ rad/s}$ at $\theta = \pi/2 \text{ rad}$.



SOLUTION

Time Derivatives: Here,

$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi \text{ ft} \quad \dot{r} = 8\dot{\theta} = 8(4) = 32.0 \text{ ft/s}$$

$$\ddot{r} = 8\ddot{\theta} = 8(5) = 40 \text{ ft/s}^2$$

Velocity: Applying Eq. 12–25, we have

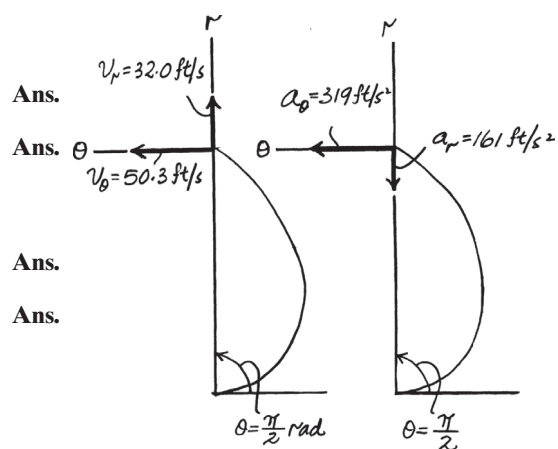
$$v_r = \dot{r} = 32.0 \text{ ft/s}$$

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3 \text{ ft/s}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 40 - 4\pi(4^2) = -161 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4\pi(5) + 2(32.0)(4) = 319 \text{ ft/s}^2$$



Ans:

$$v_r = 32.0 \text{ ft/s}$$

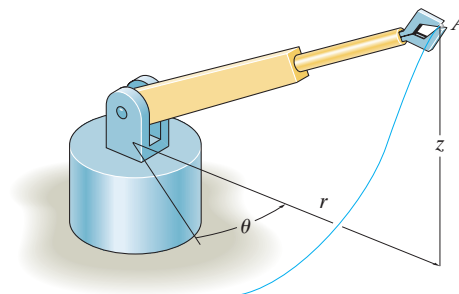
$$v_\theta = 50.3 \text{ ft/s}$$

$$a_r = -161 \text{ ft/s}^2$$

$$a_\theta = 319 \text{ ft/s}^2$$

12–191.

The arm of the robot moves so that $r = 3$ ft is constant, and its grip A moves along the path $z = (3 \sin 4\theta)$ ft, where θ is in radians. If $\theta = (0.5t)$ rad, where t is in seconds, determine the magnitudes of the grip's velocity and acceleration when $t = 3$ s.



SOLUTION

$$\theta = 0.5t \quad r = 3 \quad z = 3 \sin 2t$$

$$\dot{\theta} = 0.5 \quad \dot{r} = 0 \quad \dot{z} = 6 \cos 2t$$

$$\ddot{\theta} = 0 \quad \ddot{r} = 0 \quad \ddot{z} = -12 \sin 2t$$

At $t = 3$ s,

$$z = -0.8382$$

$$\dot{z} = 5.761$$

$$\ddot{z} = 3.353$$

$$v_r = 0$$

$$v_\theta = 3(0.5) = 1.5$$

$$v_z = 5.761$$

$$v = \sqrt{(0)^2 + (1.5)^2 + (5.761)^2} = 5.95 \text{ ft/s}$$

Ans.

$$a_r = 0 - 3(0.5)^2 = -0.75$$

$$a_\theta = 0 + 0 = 0$$

$$a_z = 3.353$$

$$a = \sqrt{(-0.75)^2 + (0)^2 + (3.353)^2} = 3.44 \text{ ft/s}^2$$

Ans.

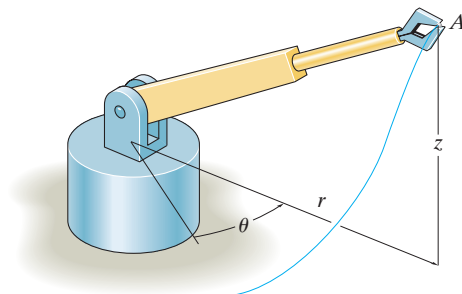
Ans:

$$v = 5.95 \text{ ft/s}$$

$$a = 3.44 \text{ ft/s}^2$$

***12–192.**

For a short time the arm of the robot is extending at a constant rate such that $\dot{r} = 1.5 \text{ ft/s}$ when $r = 3 \text{ ft}$, $z = (4t^2) \text{ ft}$, and $\theta = 0.5t \text{ rad}$, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the grip A when $t = 3 \text{ s}$.



SOLUTION

$$\begin{aligned}\theta &= 0.5t \text{ rad} & r &= 3 \text{ ft} & z &= 4t^2 \text{ ft} \\ \dot{\theta} &= 0.5 \text{ rad/s} & \dot{r} &= 1.5 \text{ ft/s} & \dot{z} &= 8t \text{ ft/s} \\ \ddot{\theta} &= 0 & \ddot{r} &= 0 & \ddot{z} &= 8 \text{ ft/s}^2\end{aligned}$$

At $t = 3 \text{ s}$,

$$\begin{aligned}\theta &= 1.5 & r &= 3 & z &= 36 \\ \dot{\theta} &= 0.5 & \dot{r} &= 1.5 & \dot{z} &= 24 \\ \ddot{\theta} &= 0 & \ddot{r} &= 0 & \ddot{z} &= 8\end{aligned}$$

$$v_r = 1.5$$

$$v_\theta = 3(0.5) = 1.5$$

$$v_z = 24$$

$$v = \sqrt{(1.5)^2 + (1.5)^2 + (24)^2} = 24.1 \text{ ft/s}$$

Ans.

$$a_r = 0 - 3(0.5)^2 = -0.75$$

$$a_\theta = 0 + 2(1.5)(0.5) = 1.5$$

$$a_z = 8$$

$$a = \sqrt{(-0.75)^2 + (1.5)^2 + (8)^2} = 8.17 \text{ ft/s}^2$$

Ans.

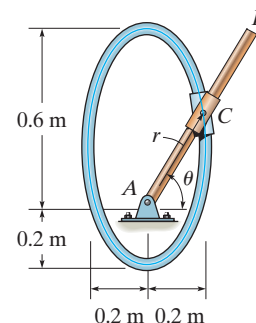
Ans:

$$v = 24.1 \text{ ft/s}$$

$$a = 8.17 \text{ ft/s}^2$$

12–193.

The double collar C is pin connected together such that one collar slides over the fixed rod and the other slides over the rotating rod AB . If the angular velocity of AB is given as $\dot{\theta} = (e^{0.5t^2})$ rad/s, where t is in seconds, and the path defined by the fixed rod is $r = |(0.4 \sin \theta + 0.2)|$ m, determine the radial and transverse components of the collar's velocity and acceleration when $t = 1$ s. When $t = 0$, $\theta = 0$. Use Simpson's rule with $n = 50$ to determine θ at $t = 1$ s.



SOLUTION

$$\dot{\theta} = e^{0.5t^2} \Big|_{t=1} = 1.649 \text{ rad/s}$$

$$\ddot{\theta} = e^{0.5t^2} t \Big|_{t=1} = 1.649 \text{ rad/s}^2$$

$$\theta = \int_0^1 e^{0.5t^2} dt = 1.195 \text{ rad} = 68.47^\circ$$

$$r = 0.4 \sin \theta + 0.2$$

$$\dot{r} = 0.4 \cos \theta \dot{\theta}$$

$$\ddot{r} = -0.4 \sin \theta \dot{\theta}^2 + 0.4 \cos \theta \ddot{\theta}$$

At $t = 1$ s,

$$r = 0.5721$$

$$\dot{r} = 0.2421$$

$$\ddot{r} = -0.7697$$

$$v_r = \dot{r} = 0.242 \text{ m/s}$$

Ans.

$$v_\theta = r \dot{\theta} = 0.5721(1.649) = 0.943 \text{ m/s}$$

Ans.

$$a_r = \ddot{r} - r \dot{\theta}^2 = -0.7697 - 0.5721(1.649)^2$$

$$a_r = -2.33 \text{ m/s}^2$$

Ans.

$$a_\theta = r \ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$= 0.5721(1.649) + 2(0.2421)(1.649)$$

$$a_\theta = 1.74 \text{ m/s}^2$$

Ans.

Ans:

$$v_r = 0.242 \text{ m/s}$$

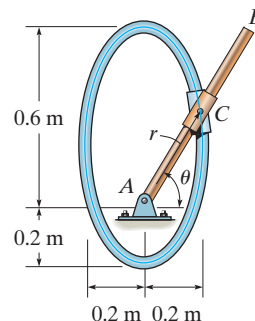
$$v_\theta = 0.943 \text{ m/s}$$

$$a_r = -2.33 \text{ m/s}^2$$

$$a_\theta = 1.74 \text{ m/s}^2$$

12–194.

The double collar C is pin connected together such that one collar slides over the fixed rod and the other slides over the rotating rod AB . If the mechanism is to be designed so that the largest speed given to the collar is 6 m/s, determine the required constant angular velocity $\dot{\theta}$ of rod AB . The path defined by the fixed rod is $r = (0.4 \sin \theta + 0.2)$ m.



SOLUTION

$$r = 0.4 \sin \theta + 0.2$$

$$\dot{r} = 0.4 \cos \theta \dot{\theta}$$

$$v_r = \dot{r} = 0.4 \cos \theta \dot{\theta}$$

$$v_\theta = r \dot{\theta} = (0.4 \sin \theta + 0.2) \dot{\theta}$$

$$v^2 = v_r^2 + v_\theta^2$$

$$(6)^2 = [(0.4 \cos \theta)^2 + (0.4 \sin \theta + 0.2)^2](\dot{\theta})^2$$

$$36 = [0.2 + 0.16 \sin \theta](\dot{\theta})^2$$

The greatest speed occurs when $\theta = 90^\circ$.

$$\dot{\theta} = 10.0 \text{ rad/s}$$

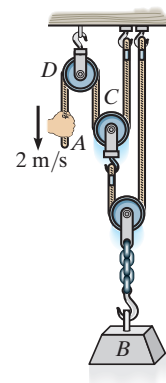
Ans.

Ans:

$$\dot{\theta} = 10.0 \text{ rad/s}$$

12–195.

If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



SOLUTION

Position-Coordinate Equation: Datum is established at fixed pulley D . The position of point A , block B and pulley C with respect to datum are s_A , s_B , and s_C respectively. Since the system consists of two cords, two position-coordinate equations can be derived.

$$(s_A - s_C) + (s_B - s_C) + s_B = l_1 \quad (1)$$

$$s_B + s_C = l_2 \quad (2)$$

Eliminating s_C from Eqs. (1) and (2) yields

$$s_A + 4s_B = l_1 = 2l_2$$

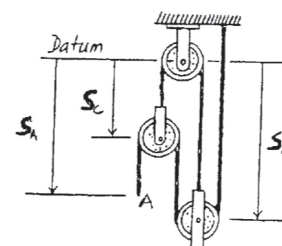
Time Derivative: Taking the time derivative of the above equation yields

$$v_A + 4v_B = 0 \quad (3)$$

Since $v_A = 2$ m/s, from Eq. (3)

$$(+\downarrow) \quad 2 + 4v_B = 0$$

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \quad \text{Ans.}$$

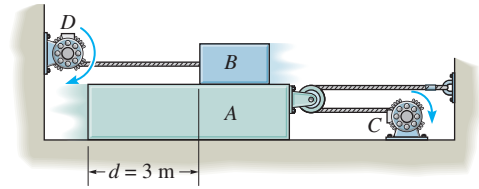


Ans:

$$v_B = 0.5 \text{ m/s}$$

***12–196.**

The motor at C pulls in the cable with an acceleration $a_C = (3t^2) \text{ m/s}^2$, where t is in seconds. The motor at D draws in its cable at $a_D = 5 \text{ m/s}^2$. If both motors start at the same instant from rest when $d = 3 \text{ m}$, determine (a) the time needed for $d = 0$, and (b) the velocities of blocks A and B when this occurs.



SOLUTION

For A :

$$s_A + (s_A - s_C) = l$$

$$2v_A = v_C$$

$$2a_A = a_C = -3t^2$$

$$a_A = -1.5t^2 = 1.5t^2 \rightarrow$$

$$v_A = 0.5t^3 \rightarrow$$

$$s_A = 0.125t^4 \rightarrow$$

For B :

$$a_B = 5 \text{ m/s}^2 \leftarrow$$

$$v_B = 5t \leftarrow$$

$$s_B = 2.5t^2 \leftarrow$$

Require $s_A + s_B = d$

$$0.125t^4 + 2.5t^2 = 3$$

$$\text{Set } u = t^2 \quad 0.125u^2 + 2.5u = 3$$

The positive root is $u = 1.1355$. Thus,

$$t = 1.0656 = 1.07 \text{ s}$$

Ans.

$$v_A = 0.5(1.0656)^3 = 0.6050$$

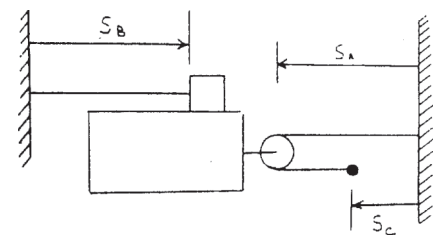
$$v_B = 5(1.0656) = 5.3281 \text{ m/s}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$0.6050\mathbf{i} = -5.3281\mathbf{i} + v_{A/B}\mathbf{i}$$

$$v_{A/B} = 5.93 \text{ m/s} \rightarrow$$

Ans.



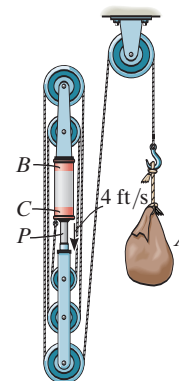
Ans:

$$t = 1.07 \text{ s}$$

$$v_{A/B} = 5.93 \text{ m/s} \rightarrow$$

12–197.

The pulley arrangement shown is designed for hoisting materials. If *BC remains fixed* while the plunger *P* is pushed downward with a speed of 4 ft/s, determine the speed of the load at *A*.



SOLUTION

$$5 s_B + (s_B - s_A) = l$$

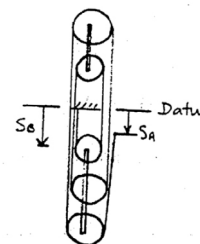
$$6 s_B - s_A = l$$

$$6 v_B - v_A = 0$$

$$6(4) = v_A$$

$$v_A = 24 \text{ ft/s}$$

Ans.

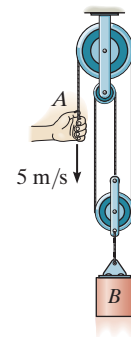


Ans:

$$v = 24 \text{ ft/s}$$

12–198.

If the end of the cable at A is pulled down with a speed of 5 m/s, determine the speed at which block B rises.



SOLUTION

Position Coordinate. The positions of pulley B and point A are specified by position coordinates s_B and s_A , respectively, as shown in Fig. a . This is a single-cord pulley system. Thus,

$$\begin{aligned} s_B + 2(s_B - a) + s_A &= l \\ 3s_B + s_A &= l + 2a \end{aligned} \quad (1)$$

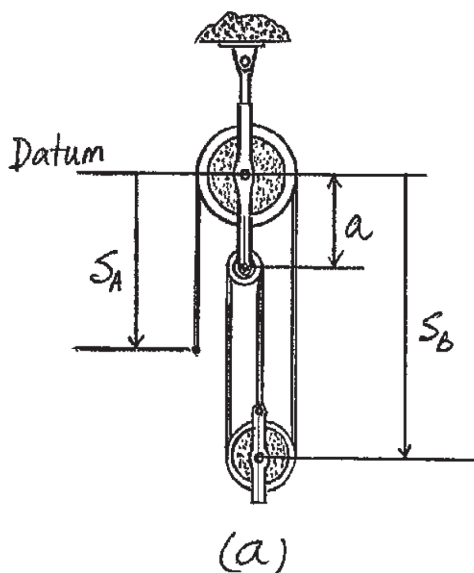
Time Derivative. Taking the time derivative of Eq. (1),

$$3v_B + v_A = 0 \quad (2)$$

Here $v_A = +5$ m/s, since it is directed toward the positive sense of s_A . Thus,

$$3v_B + 5 = 0 \quad v_B = -1.667 \text{ m/s} = 1.67 \text{ m/s} \uparrow \quad \text{Ans.}$$

The negative sign indicates that v_B is directed toward the negative sense of s_B .

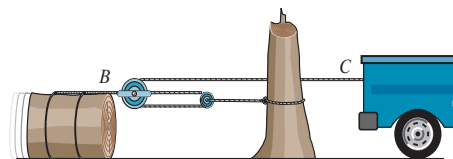


Ans:

$$v_B = 1.67 \text{ m/s}$$

12–199.

Determine the displacement of the log if the truck at C pulls the cable 4 ft to the right.



SOLUTION

$$2s_B + (s_B - s_C) = l$$

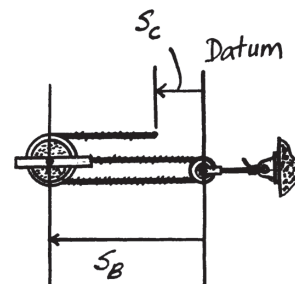
$$3s_B - s_C = l$$

$$3\Delta s_B - \Delta s_C = 0$$

Since $\Delta s_C = -4$, then

$$3\Delta s_B = -4$$

$$\Delta s_B = -1.33 \text{ ft} = 1.33 \text{ ft} \rightarrow$$



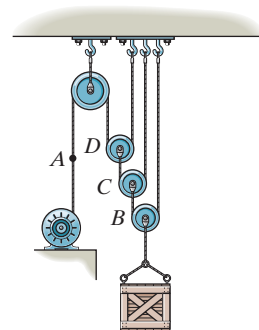
Ans.

Ans:

$$\Delta s_B = 1.33 \text{ ft} \rightarrow$$

***12–200.**

Determine the constant speed at which the cable at A must be drawn in by the motor in order to hoist the load 6 m in 1.5 s.



SOLUTION

$$v_B = \frac{6}{1.5} = 4 \text{ m/s} \uparrow$$

$$s_B + (s_B - s_C) = l_1$$

$$s_C + (s_C - s_D) = l_2$$

$$s_A + 2s_D = l_3$$

Thus,

$$2s_B - s_C = l_1$$

$$2s_C - s_D = l_2$$

$$s_A + 2s_D = l_3$$

$$2v_A = v_C$$

$$2v_C = v_D$$

$$v_A = -2v_D$$

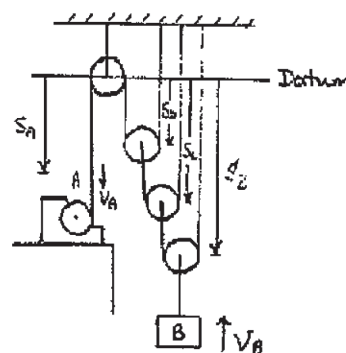
$$2(2v_B) = v_D$$

$$v_A = -2(4v_B)$$

$$v_A = -8v_B$$

$$v_A = -8(-4) = 32 \text{ m/s} \downarrow$$

Ans.

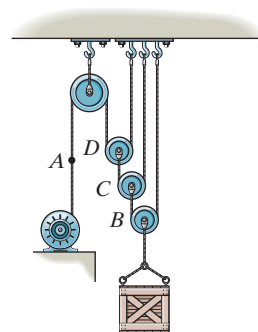


Ans:

$$v_A = 32 \text{ m/s} \downarrow$$

12–201.

Starting from rest, the cable can be wound onto the drum of the motor at a rate of $v_A = (3t^2)$ m/s, where t is in seconds. Determine the time needed to lift the load 7 m.



SOLUTION

$$v_B = \frac{6}{1.5} = 4 \text{ m/s } \uparrow$$

$$s_B + (s_B - s_C) = l_1$$

$$s_C + (s_C - s_D) = l_2$$

$$s_A + 2s_D = l_3$$

Thus,

$$2s_B - s_C = l_1$$

$$2s_C - s_D = l_2$$

$$s_A + 2s_D = l_3$$

$$2v_B = v_C$$

$$2v_C = v_D$$

$$v_A = -2v_D$$

$$v_A = -8v_B$$

$$3t^2 = -8v_B$$

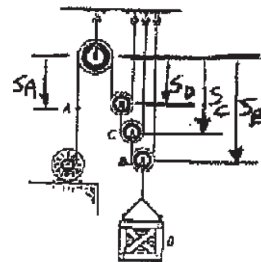
$$v_B = \frac{-3}{8}t^2$$

$$s_B = \int_0^t \frac{-3}{8}t^2 dt$$

$$s_B = \frac{-1}{8}t^3$$

$$-7 = \frac{-1}{8}t^3$$

$$t = 3.83 \text{ s}$$



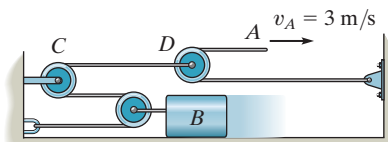
Ans.

Ans:

$$t = 3.83 \text{ s}$$

12-202.

If the end A of the cable is moving at $v_A = 3 \text{ m/s}$, determine the speed of block B .



SOLUTION

Position Coordinates. The positions of pulley B , D and point A are specified by position coordinates s_B , s_D and s_A respectively as shown in Fig. a . The pulley system consists of two cords which give

$$2s_B + s_D = l_1 \quad (1)$$

and

$$\begin{aligned} (s_A - s_D) + (b - s_D) &= l_2 \\ s_A - 2s_D &= l_2 - b \end{aligned} \quad (2)$$

Time Derivative. Taking the time derivatives of Eqs. (1) and (2), we get

$$2v_B + v_D = 0 \quad (3)$$

$$v_A - 2v_D = 0 \quad (4)$$

Eliminate v_D from Eqs. (3) and (4),

$$v_A + 4v_B = 0 \quad (5)$$

Here $v_A = +3 \text{ m/s}$ since it is directed toward the positive sense of s_A .

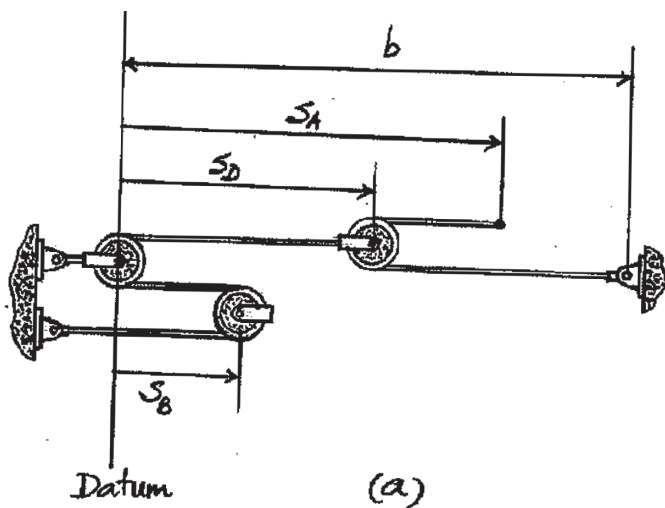
Thus

$$3 + 4v_B = 0$$

$$v_B = -0.75 \text{ m/s} = 0.75 \text{ m/s} \leftarrow$$

Ans.

The negative sign indicates that v_D is directed toward the negative sense of s_B .

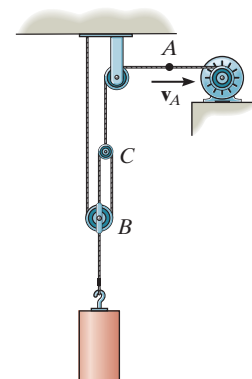


Ans:

$$v_B = 0.75 \text{ m/s}$$

12-203.

Determine the time needed for the load at B to attain a speed of 10 m/s , starting from rest, if the cable is drawn into the motor with an acceleration of 3 m/s^2 .



SOLUTION

Position Coordinates. The position of pulleys B , C and point A are specified by position coordinates s_B , s_C and s_A respectively as shown in Fig. a . The pulley system consists of two cords which gives

$$\begin{aligned} s_B + 2(s_B - s_C) &= l_1 \\ 3s_B - 2s_C &= l_1 \end{aligned} \quad (1)$$

And

$$s_C + s_A = l_2 \quad (2)$$

Time Derivative. Taking the time derivative twice of Eqs. (1) and (2),

$$3a_B - 2a_C = 0 \quad (3)$$

And

$$a_C + a_A = 0 \quad (4)$$

Eliminate a_C from Eqs. (3) and (4)

$$3a_B + 2a_A = 0$$

Here, $a_A = +3 \text{ m/s}^2$ since it is directed toward the positive sense of s_A . Thus,

$$3a_B + 2(3) = 0 \quad a_B = -2 \text{ m/s}^2 = 2 \text{ m/s}^2 \uparrow$$

The negative sign indicates that a_B is directed toward the negative sense of s_B . Applying kinematic equation of constant acceleration,

$$+\uparrow \quad v_B = (v_B)_0 + a_B t$$

$$10 = 0 + 2 t$$

$$t = 5.00 \text{ s}$$

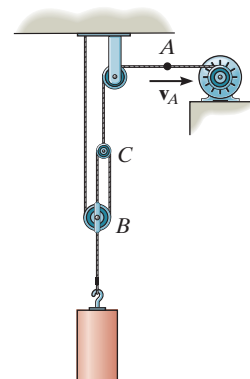
Ans.

Ans:

$$t = 5.00 \text{ s}$$

***12-204.**

The cable at A is being drawn toward the motor at $v_A = 8 \text{ m/s}$. Determine the velocity of the block.



SOLUTION

Position Coordinates. The position of pulleys B , C and point A are specified by position coordinates s_B , s_C and s_A respectively as shown in Fig. a . The pulley system consists of two cords which give

$$\begin{aligned} s_B + 2(s_B - s_C) &= l_1 \\ 3s_B - 2s_C &= l_1 \end{aligned} \quad (1)$$

And

$$s_C + s_A = l_2 \quad (2)$$

Time Derivative. Taking the time derivatives of Eqs. (1) and (2), we get

$$3v_B - 2v_C = 0 \quad (3)$$

And

$$v_C + v_A = 0 \quad (4)$$

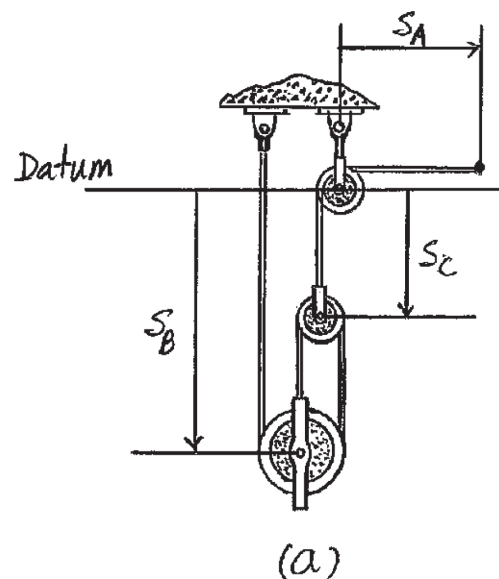
Eliminate v_C from Eqs. (3) and (4),

$$3v_B + 2v_A = 0$$

Here $v_A = +8 \text{ m/s}$ since it is directed toward the positive sense of s_A . Thus,

$$3v_B + 2(8) = 0 \quad v_B = -5.33 \text{ m/s} = 5.33 \text{ m/s} \uparrow \quad \text{Ans.}$$

The negative sign indicates that v_B is directed toward the negative sense of s_B .

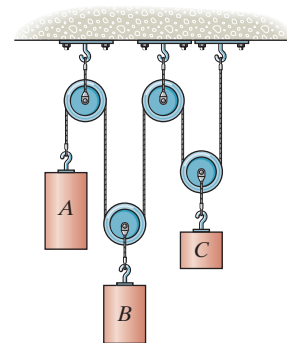


Ans:

$$v_B = 5.33 \text{ m/s} \uparrow$$

12–205.

If block *A* of the pulley system is moving downward at 6 ft/s while block *C* is moving down at 18 ft/s, determine the relative velocity of block *B* with respect to *C*.



SOLUTION

$$s_A + 2s_B + 2s_C = l$$

$$v_A + 2v_B + 2v_C = 0$$

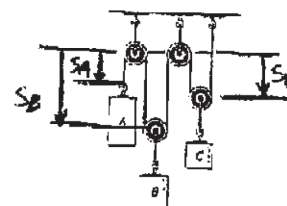
$$6 + 2v_B + 2(18) = 0$$

$$v_B = -21 \text{ ft/s} = 21 \text{ ft/s} \uparrow$$

$$+ \downarrow v_B = v_C + v_{B/C}$$

$$-21 = 18 + v_{B/C}$$

$$v_{B/C} = -39 \text{ ft/s} = 39 \text{ ft/s} \uparrow$$



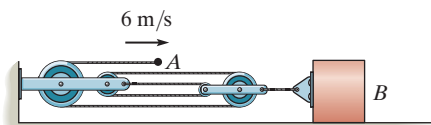
Ans.

Ans:

$$v_{B/C} = 39 \text{ ft/s} \uparrow$$

12-206.

Determine the speed of the block at B .



SOLUTION

Position Coordinate. The positions of pulley B and point A are specified by position coordinates s_B and s_A respectively as shown in Fig. a . This is a single cord pulley system. Thus,

$$\begin{aligned} s_B + 2(s_B - a - b) + (s_B - a) + s_A &= l \\ 4s_B + s_A &= l + 3a + 2b \end{aligned} \quad (1)$$

Time Derivative. Taking the time derivative of Eq. (1),

$$4v_B + v_A = 0 \quad (2)$$

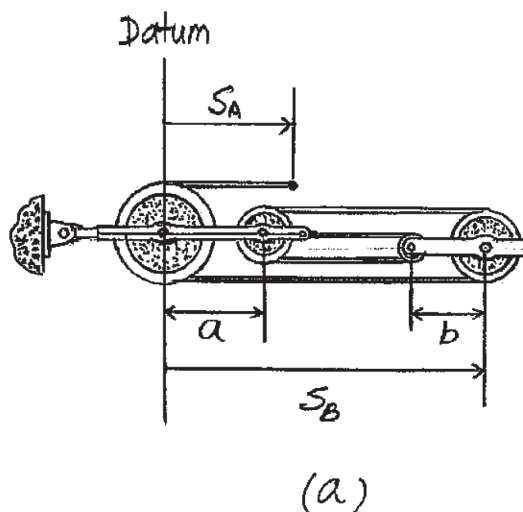
Here, $v_A = +6 \text{ m/s}$ since it is directed toward the positive sense of s_A . Thus,

$$4v_B + 6 = 0$$

$$v_B = -1.50 \text{ m/s} = 1.50 \text{ m/s} \leftarrow$$

Ans.

The negative sign indicates that v_B is directed towards negative sense of s_B .



Ans:

$$v_B = 1.50 \text{ m/s}$$

12–207.

Determine the speed of block *A* if the end of the rope is pulled down with a speed of 4 m/s.

SOLUTION

Position Coordinates: By referring to Fig. *a*, the length of the cord written in terms of the position coordinates s_A and s_B is

$$s_B + s_A + 2(s_A - a) = l$$

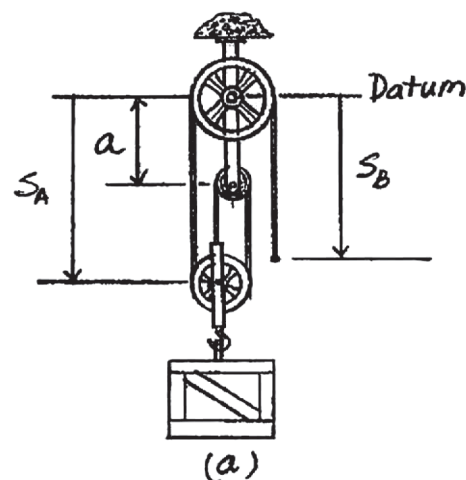
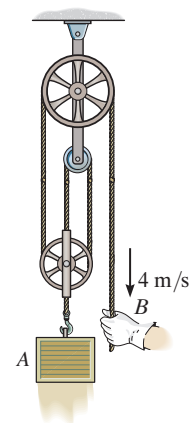
$$s_B + 3s_A = l + 2a$$

Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \quad v_B + 3v_A = 0$$

Here, $v_B = 4$ m/s. Thus,

$$4 + 3v_A = 0 \quad v_A = -1.33 \text{ m/s} = 1.33 \text{ m/s} \uparrow \quad \text{Ans.}$$

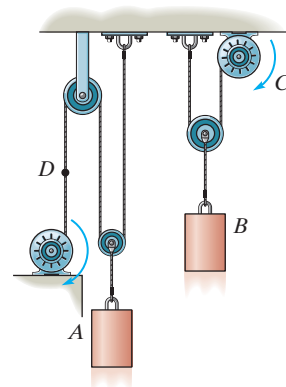


Ans:

$$v_A = 1.33 \text{ m/s}$$

***12–208.**

The motor draws in the cable at C with a constant velocity of $v_C = 4 \text{ m/s}$. The motor draws in the cable at D with a constant acceleration of $a_D = 8 \text{ m/s}^2$. If $v_D = 0$ when $t = 0$, determine (a) the time needed for block A to rise 3 m, and (b) the relative velocity of block A with respect to block B when this occurs.



SOLUTION

(a) $a_D = 8 \text{ m/s}^2$

$$v_D = 8t$$

$$s_D = 4t^2$$

$$s_D + 2s_A = l$$

$$\Delta s_D = -2\Delta s_A$$

$$\Delta s_A = -2t^2$$

$$-3 = -2t^2$$

$$t = 1.2247 = 1.22 \text{ s}$$

(b) $v_A = s_A = -4t = -4(1.2247) = -4.90 \text{ m/s} = 4.90 \text{ m/s} \uparrow$

$$s_B + (s_B - s_C) = l$$

$$2v_B = v_C = -4$$

$$v_B = -2 \text{ m/s} = 2 \text{ m/s} \uparrow$$

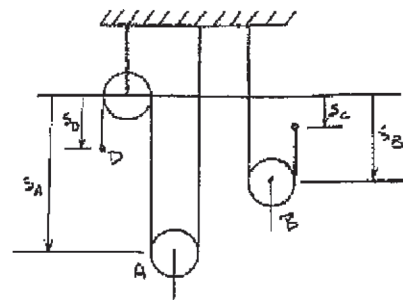
(+↓) $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$$-4.90 = -2 + v_{A/B}$$

$$v_{A/B} = -2.90 \text{ m/s} = 2.90 \text{ m/s} \uparrow$$

(1)

Ans.



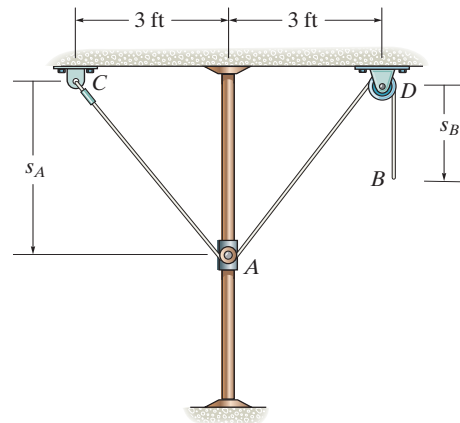
Ans.

Ans:

$$v_{A/B} = 2.90 \text{ m/s} \uparrow$$

12–209.

The cord is attached to the pin at C and passes over the two pulleys at A and D . The pulley at A is attached to the smooth collar that travels along the vertical rod. Determine the velocity and acceleration of the end of the cord at B if at the instant $s_A = 4$ ft the collar is moving upwards at 5 ft/s, which is decreasing at 2 ft/s².



SOLUTION

$$2\sqrt{s_A^2 + 3^2} + s_B = l$$

$$2\left(\frac{1}{2}\right)(s_A^2 + 9)^{-\frac{1}{2}}(2s_A \dot{s}_A) + \dot{s}_B = 0$$

$$\dot{s}_B = -\frac{2s_A \dot{s}_A}{(s_A^2 + 9)^{\frac{1}{2}}}$$

$$\ddot{s}_B = -2\dot{s}_A^2(s_A^2 + 9)^{-\frac{1}{2}} - (2s_A \ddot{s}_A)(s_A^2 + 9)^{-\frac{1}{2}} - (2s_A \dot{s}_A)\left[\left(-\frac{1}{2}\right)(s_A^2 + 9)^{-\frac{3}{2}}(2s_A \dot{s}_A)\right]$$

$$\ddot{s}_B = -\frac{2(\dot{s}_A + s_A \ddot{s}_A)}{(s_A^2 + 9)^{\frac{1}{2}}} + \frac{2(s_A \dot{s}_A)^2}{(s_A^2 + 9)^{\frac{3}{2}}}$$

At $s_A = 4$ ft,

$$v_B = \dot{s}_B = -\frac{2(4)(-5)}{(4^2 + 9)^{\frac{1}{2}}} = 8 \text{ ft/s} \downarrow \quad \text{Ans.}$$

$$a_B = \ddot{s}_B = -\frac{2[(-5)^2 + (4)(2)]}{(4^2 + 9)^{\frac{1}{2}}} + \frac{2[(4)(-5)]^2}{(4^2 + 9)^{\frac{3}{2}}} = -6.80 \text{ ft/s}^2 = 6.80 \text{ ft/s}^2 \uparrow \quad \text{Ans.}$$

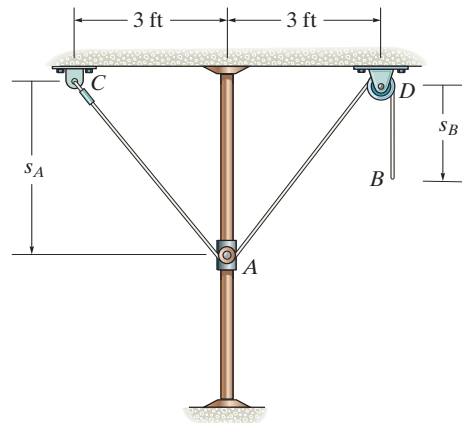
Ans:

$$v_B = 8 \text{ ft/s} \downarrow$$

$$a_B = 6.80 \text{ ft/s}^2 \uparrow$$

12–210.

The 16-ft-long cord is attached to the pin at C and passes over the two pulleys at A and D . The pulley at A is attached to the smooth collar that travels along the vertical rod. When $s_B = 6$ ft, the end of the cord at B is pulled downwards with a velocity of 4 ft/s and is given an acceleration of 3 ft/s². Determine the velocity and acceleration of the collar at this instant.



SOLUTION

$$2\sqrt{s_A^2 + 3^2} + s_B = l$$

$$2\left(\frac{1}{2}\right)(s_A^2 + 9)^{-\frac{1}{2}}(2s_A\dot{s}_A) + \dot{s}_B = 0$$

$$\dot{s}_B = -\frac{2s_A\dot{s}_A}{(s_A^2 + 9)^{\frac{1}{2}}}$$

$$\ddot{s}_B = -2\dot{s}_A^2(s_A^2 + 9)^{-\frac{1}{2}} - (2s_A\ddot{s}_A)(s_A^2 + 9)^{-\frac{1}{2}} - \left(2s_A\dot{s}_A\right)\left[\left(-\frac{1}{2}\right)(s_A^2 + 9)^{-\frac{3}{2}}(2s_A\dot{s}_A)\right]$$

$$\ddot{s}_B = -\frac{2(\dot{s}_A^2 + s_A\ddot{s}_A)}{(s_A^2 + 9)^{\frac{1}{2}}} + \frac{2(s_A\dot{s}_A)^2}{(s_A^2 + 9)^{\frac{3}{2}}}$$

$$\text{At } s_B = 6 \text{ ft, } \dot{s}_B = 4 \text{ ft/s, } \ddot{s}_B = 3 \text{ ft/s}^2$$

$$2\sqrt{s_A^2 + 3^2} + 6 = 16$$

$$s_A = 4 \text{ ft}$$

$$4 = -\frac{2(4)(\dot{s}_A)}{(4^2 + 9)^{\frac{1}{2}}}$$

$$v_A = \dot{s}_A = -2.5 \text{ ft/s} = 2.5 \text{ ft/s} \uparrow$$

Ans.

$$3 = -\frac{2[(-2.5)^2 + 4(\dot{s}_A)]}{(4^2 + 9)^{\frac{1}{2}}} + \frac{2[4(-2.5)]^2}{(4^2 + 9)^{\frac{3}{2}}}$$

$$a_A = \ddot{s}_A = -2.4375 = 2.44 \text{ ft/s}^2 \uparrow$$

Ans.

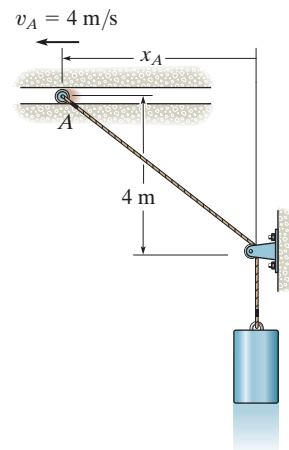
Ans:

$$v_A = 2.5 \text{ ft/s} \uparrow$$

$$a_A = 2.44 \text{ ft/s}^2 \uparrow$$

12-211.

The roller at A is moving with a velocity of $v_A = 4 \text{ m/s}$ and has an acceleration of $a_A = 2 \text{ m/s}^2$ when $x_A = 3 \text{ m}$. Determine the velocity and acceleration of block B at this instant.



SOLUTION

Position Coordinates. The position of roller A and block B are specified by position coordinates x_A and y_B respectively as shown in Fig. a . We can relate these two position coordinates by considering the length of the cable, which is constant

$$\sqrt{x_A^2 + 4^2} + y_B = l$$

$$y_B = l - \sqrt{x_A^2 + 16} \quad (1)$$

Velocity. Taking the time derivative of Eq. (1) using the chain rule,

$$\frac{dy_B}{dt} = 0 - \frac{1}{2} (x_A^2 + 16)^{-\frac{1}{2}} (2x_A) \frac{dx_A}{dt}$$

$$\frac{dy_B}{dt} = - \frac{x_A}{\sqrt{x_A^2 + 16}} \frac{dx_A}{dt}$$

However, $\frac{dy_B}{dt} = v_B$ and $\frac{dx_A}{dt} = v_A$. Then

$$v_B = - \frac{x_A}{\sqrt{x_A^2 + 16}} v_A \quad (2)$$

At $x_A = 3 \text{ m}$, $v_A = +4 \text{ m/s}$ since \mathbf{v}_A is directed toward the positive sense of x_A . Then Eq. (2) give

$$v_B = - \frac{3}{\sqrt{3^2 + 16}} (4) = -2.40 \text{ m/s} = 2.40 \text{ m/s} \uparrow \quad \text{Ans.}$$

The negative sign indicates that \mathbf{v}_B is directed toward the negative sense of y_B .

Acceleration. Taking the time derivative of Eq. (2),

$$\frac{dv_B}{dt} = - \left[x_A \left(-\frac{1}{2} \right) (x_A^2 + 16)^{-3/2} (2x_A) \frac{dx_A}{dt} + (x_A^2 + 16)^{-1/2} \frac{dx_A}{dt} \right]$$

$$v_A - x_A (x_A^2 + 16)^{-1/2} \frac{dv_A}{dt}$$

However, $\frac{dv_B}{dt} = a_B$, $\frac{dv_A}{dt} = a_A$ and $\frac{dx_A}{dt} = v_A$. Then

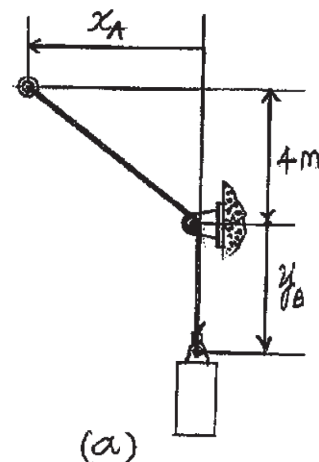
$$a_B = \frac{x_A^2 v_A^2}{(x_A^2 + 16)^{3/2}} - \frac{v_A^2}{(x_A^2 + 16)^{1/2}} - \frac{x_A a_A}{(x_A^2 + 16)^{1/2}}$$

$$a_B = - \frac{16 v_A^2 + a_A x_A (x_A^2 + 16)}{(x_A^2 + 16)^{3/2}}$$

At $x_A = 3 \text{ m}$, $v_A = +4 \text{ m/s}$, $a_A = +2 \text{ m/s}^2$ since \mathbf{v}_A and \mathbf{a}_A are directed toward the positive sense of x_A .

$$a_B = - \frac{16(4^2) + 2(3)(3^2 + 16)}{(3^2 + 16)^{3/2}} = -3.248 \text{ m/s}^2 = 3.25 \text{ m/s}^2 \uparrow \quad \text{Ans.}$$

The negative sign indicates that \mathbf{a}_B is directed toward the negative sense of y_B .



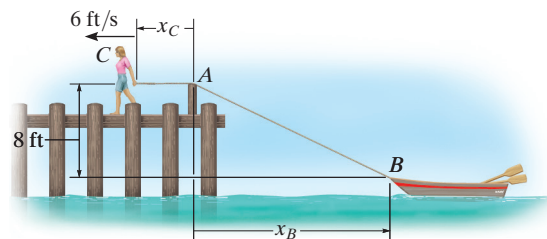
Ans:

$$v_B = 2.40 \text{ m/s} \uparrow$$

$$a_B = 3.25 \text{ m/s}^2 \uparrow$$

***12–212.**

The girl at C stands near the edge of the pier and pulls in the rope *horizontally* at a constant speed of 6 ft/s. Determine how fast the boat approaches the pier at the instant the rope length AB is 50 ft.



SOLUTION

The length l of cord is

$$\sqrt{(8)^2 + x_B^2} + x_C = l$$

Taking the time derivative:

$$\frac{1}{2}[(8)^2 + x_B^2]^{-1/2} 2x_B \dot{x}_B + \dot{x}_C = 0 \quad (1)$$

$$\dot{x}_C = 6 \text{ ft/s}$$

When $AB = 50$ ft,

$$x_B = \sqrt{(50)^2 - (8)^2} = 49.356 \text{ ft}$$

From Eq. (1)

$$\frac{1}{2}[(8)^2 + (49.356)^2]^{-1/2} 2(49.356)(\dot{x}_B) + 6 = 0$$

$$\dot{x}_B = -6.0783 = 6.08 \text{ ft/s} \leftarrow$$

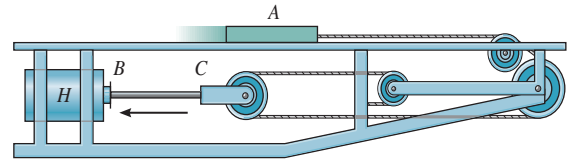
Ans.

Ans:

$$\dot{x}_B = 6.08 \text{ ft/s} \leftarrow$$

12–213.

If the hydraulic cylinder H draws in rod BC at 2 ft/s, determine the speed of slider A .



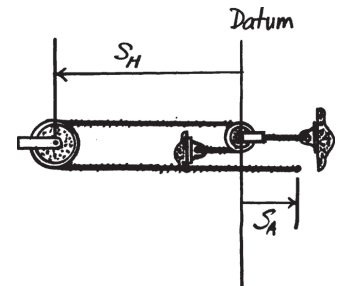
SOLUTION

$$2s_H + s_A = l$$

$$2v_H = -v_A$$

$$2(2) = -v_A$$

$$v_A = -4 \text{ ft/s} = 4 \text{ ft/s} \leftarrow$$



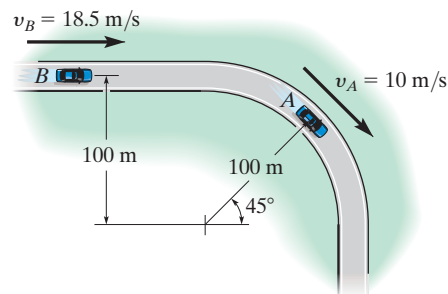
Ans.

Ans:

$$v_A = 4 \text{ ft/s}$$

12-214.

At the instant shown, the car at A is traveling at 10 m/s around the curve while increasing its speed at 5 m/s². The car at B is traveling at 18.5 m/s along the straightaway and increasing its speed at 2 m/s². Determine the relative velocity and relative acceleration of A with respect to B at this instant.



SOLUTION

$$v_A = 10 \cos 45^\circ \mathbf{i} - 10 \sin 45^\circ \mathbf{j} = \{7.071\mathbf{i} - 7.071\mathbf{j}\} \text{ m/s}$$

$$v_B = \{18.5\mathbf{i}\} \text{ m/s}$$

$$v_{A/B} = v_A - v_B = (7.071\mathbf{i} - 7.071\mathbf{j}) - 18.5\mathbf{i} = \{-11.429\mathbf{i} - 7.071\mathbf{j}\} \text{ m/s}$$

$$v_{A/B} = \sqrt{(-11.429)^2 + (-7.071)^2} = 13.4 \text{ m/s}$$

Ans.

$$\theta = \tan^{-1} \frac{7.071}{11.429} = 31.7^\circ \nearrow$$

Ans.

$$(a_A)_n = \frac{v_A^2}{\rho} = \frac{10^2}{100} = 1 \text{ m/s}^2 \quad (a_A)_t = 5 \text{ m/s}^2$$

$$\mathbf{a}_A = (5 \cos 45^\circ - 1 \cos 45^\circ)\mathbf{i} + (-1 \sin 45^\circ - 5 \sin 45^\circ)\mathbf{j} = \{2.828\mathbf{i} - 4.243\mathbf{j}\} \text{ m/s}^2$$

$$\mathbf{a}_B = \{2\mathbf{i}\} \text{ m/s}^2$$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B = (2.828\mathbf{i} - 4.243\mathbf{j}) - 2\mathbf{i} = \{0.828\mathbf{i} - 4.24\mathbf{j}\} \text{ m/s}^2$$

$$a_{A/B} = \sqrt{0.828^2 + (-4.243)^2} = 4.32 \text{ m/s}^2$$

Ans.

$$\theta = \tan^{-1} \frac{4.243}{0.828} = 79.0^\circ \searrow$$

Ans.

Ans:

$$v_{A/B} = 13.4 \text{ m/s}$$

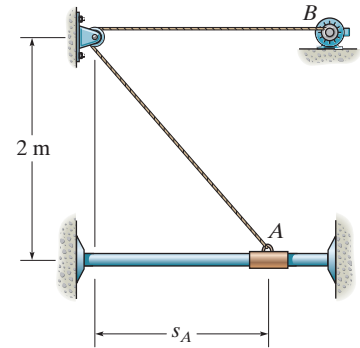
$$\theta_v = 31.7^\circ \nearrow$$

$$a_{A/B} = 4.32 \text{ m/s}^2$$

$$\theta_a = 79.0^\circ \searrow$$

12-215.

The motor draws in the cord at B with an acceleration of $a_B = 2 \text{ m/s}^2$. When $s_A = 1.5 \text{ m}$, $v_B = 6 \text{ m/s}$. Determine the velocity and acceleration of the collar at this instant.



SOLUTION

Position Coordinates. The position of collar A and point B are specified by s_A and s_B respectively as shown in Fig. a . We can relate these two position coordinates by considering the length of the cable, which is constant.

$$\begin{aligned} s_B + \sqrt{s_A^2 + 2^2} &= l \\ s_B &= l - \sqrt{s_A^2 + 4} \end{aligned} \quad (1)$$

Velocity. Taking the time derivative of Eq. (1),

$$\begin{aligned} \frac{ds_B}{dt} &= 0 - \frac{1}{2} (s_A^2 + 4)^{-1/2} \left(2s_A \frac{ds_A}{dt} \right) \\ \frac{ds_B}{dt} &= - \frac{s_A}{\sqrt{s_A^2 + 4}} \frac{ds_A}{dt} \end{aligned}$$

However, $\frac{ds_B}{dt} = v_B$ and $\frac{ds_A}{dt} = v_A$. Then this equation becomes

$$v_B = - \frac{s_A}{\sqrt{s_A^2 + 4}} v_A \quad (2)$$

At the instant $s_A = 1.5 \text{ m}$, $v_B = +6 \text{ m/s}$. v_B is positive since it is directed toward the positive sense of s_B .

$$\begin{aligned} 6 &= - \frac{1.5}{\sqrt{1.5^2 + 4}} v_A \\ v_A &= -10.0 \text{ m/s} = 10.0 \text{ m/s} \leftarrow \end{aligned}$$

Ans.

The negative sign indicates that v_A is directed toward the negative sense of s_A .

Acceleration. Taking the time derivative of Eq. (2),

$$\begin{aligned} \frac{dv_B}{dt} &= - \left[s_A \left(-\frac{1}{2} \right) (s_A^2 + 4)^{-3/2} \left(2s_A \frac{ds_A}{dt} \right) + (s_A^2 + 4)^{-1/2} \frac{ds_A}{dt} \right] \\ v_A - s_A (s_A^2 + 4)^{-1/2} \frac{dv_A}{dt} \end{aligned}$$

However, $\frac{dv_B}{dt} = a_B$, $\frac{dv_A}{dt} = a_A$ and $\frac{ds_A}{dt} = v_A$. Then

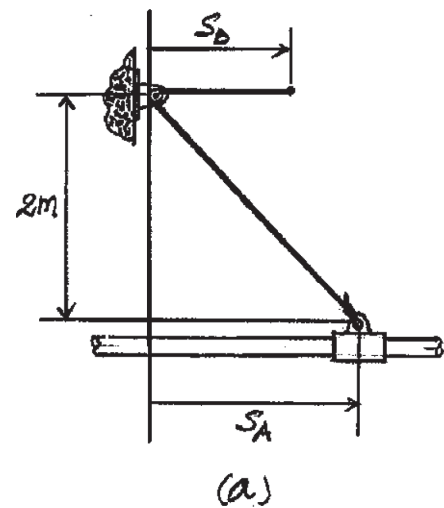
$$\begin{aligned} a_B &= \frac{s_A^2 v_A^2}{(s_A^2 + 4)^{3/2}} - \frac{v_A^2}{(s_A^2 + 4)^{1/2}} - \frac{a_A s_A}{(s_A^2 + 4)^{1/2}} \\ a_B &= - \frac{4v_A^2 + a_A s_A (s_A^2 + 4)}{(s_A^2 + 4)^{3/2}} \end{aligned}$$

At the instant $s_A = 1.5 \text{ m}$, $a_B = +2 \text{ m/s}^2$. a_B is positive since it is directed toward the positive sense of s_B . Also, $v_A = -10.0 \text{ m/s}$. Then

$$\begin{aligned} 2 &= - \left[\frac{4(-10.0)^2 + a_A(1.5)(1.5^2 + 4)}{(1.5^2 + 4)^{3/2}} \right] \\ a_A &= -46.0 \text{ m/s}^2 = 46.0 \text{ m/s}^2 \leftarrow \end{aligned}$$

Ans.

The negative sign indicates that a_A is directed toward the negative sense of s_A .

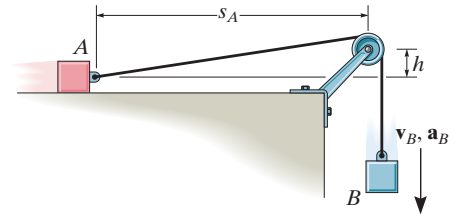


Ans:

$$\begin{aligned} v_A &= 10.0 \text{ m/s} \leftarrow \\ a_A &= 46.0 \text{ m/s}^2 \leftarrow \end{aligned}$$

***12–216.**

If block B is moving down with a velocity v_B and has an acceleration a_B , determine the velocity and acceleration of block A in terms of the parameters shown.



SOLUTION

$$l = s_B + \sqrt{s_A^2 + h^2}$$

$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2} 2s_A \dot{s}_A$$

$$v_A = \dot{s}_A = \frac{-\dot{s}_B(s_A^2 + h^2)^{1/2}}{s_A}$$

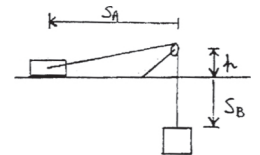
$$v_A = -v_B \left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2}$$

Ans.

$$a_A = \dot{v}_A = -\dot{v}_B \left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2} - v_B \left(\frac{1}{2}\right) \left(1 + \left(\frac{h}{s_A}\right)^2\right)^{-1/2} (h^2) (-2)(s_A)^{-3} \dot{s}_A$$

$$a_A = -a_B \left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2} + \frac{v_A v_B h^2}{s_A^3} \left(1 + \left(\frac{h}{s_A}\right)^2\right)^{-1/2}$$

Ans.



Ans:

$$v_A = -v_B \left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2}$$

$$a_A = -a_B \left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2} + \frac{v_A v_B h^2}{s_A^3} \left(1 + \left(\frac{h}{s_A}\right)^2\right)^{-1/2}$$

12-217.

The crate C is being lifted by moving the roller at A downward with a constant speed of $v_A = 2 \text{ m/s}$ along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1 \text{ m}$. When the roller is at B , the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.

SOLUTION

$$x_C + \sqrt{x_A^2 + (4)^2} = l$$

$$\dot{x}_C + \frac{1}{2}(x_A^2 + 16)^{-1/2}(2x_A)(\dot{x}_A) = 0$$

$$\ddot{x}_C - \frac{1}{2}(x_A^2 + 16)^{-3/2}(2x_A)(\dot{x}_A)^2 + (x_A^2 + 16)^{-1/2}(\dot{x}_A)^2 + (x_A^2 + 16)^{-1/2}(x_A)(\ddot{x}_A) = 0$$

$l = 8 \text{ m}$, and when $s = 1 \text{ m}$,

$$x_C = 3 \text{ m}$$

$$x_A = 3 \text{ m}$$

$$v_A = \dot{x}_A = 2 \text{ m/s}$$

$$a_A = \ddot{x}_A = 0$$

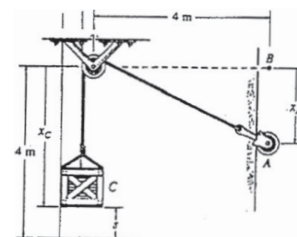
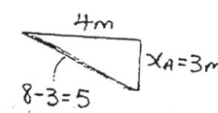
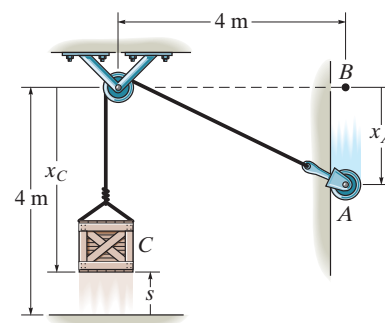
Thus,

$$v_C + [(3)^2 + 16]^{-1/2}(3)(2) = 0$$

$$v_C = -1.2 \text{ m/s} = 1.2 \text{ m/s} \uparrow$$

$$a_C - [(3)^2 + 16]^{-3/2}(3)^2(2)^2 + [(3)^2 + 16]^{-1/2}(2)^2 + 0 = 0$$

$$a_C = -0.512 \text{ m/s}^2 = 0.512 \text{ m/s}^2 \uparrow$$



Ans.

Ans.

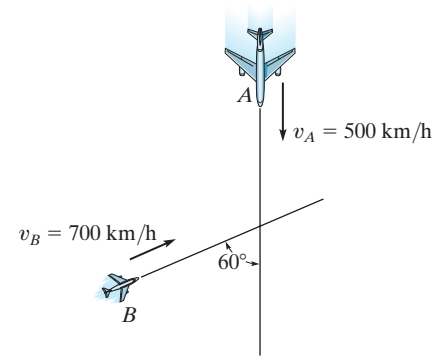
Ans:

$$v_C = 1.2 \text{ m/s} \uparrow$$

$$a_C = 0.512 \text{ m/s}^2 \uparrow$$

12-218.

Two planes, A and B , are flying at the same altitude. If their velocities are $v_A = 500$ km/h and $v_B = 700$ km/h such that the angle between their straight-line courses is $\theta = 60^\circ$, determine the velocity of plane B with respect to plane A .



SOLUTION

Relative Velocity. Express \mathbf{v}_A and \mathbf{v}_B in Cartesian vector form,

$$\mathbf{v}_A = \{-500\mathbf{j}\} \text{ km/h}$$

$$\mathbf{v}_B = \{700 \sin 60^\circ \mathbf{i} + 700 \cos 60^\circ \mathbf{j}\} \text{ km/h} = \{350\sqrt{3}\mathbf{i} + 350\mathbf{j}\} \text{ km/h}$$

Applying the relative velocity equation.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$350\sqrt{3}\mathbf{i} + 350\mathbf{j} = -500\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{350\sqrt{3}\mathbf{i} + 850\mathbf{j}\} \text{ km/h}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is

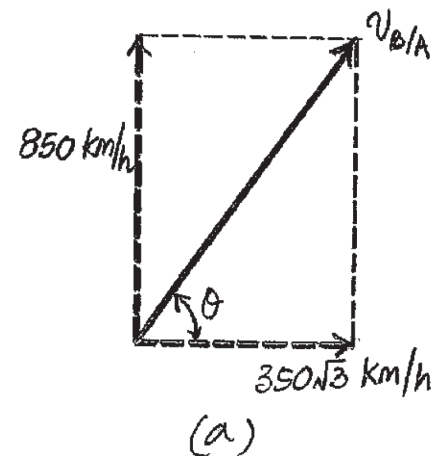
$$v_{B/A} = \sqrt{(350\sqrt{3})^2 + 850^2} = 1044.03 \text{ km/h} = 1044 \text{ km/h}$$

Ans.

And its direction is defined by angle θ , Fig. a .

$$\theta = \tan^{-1}\left(\frac{850}{350\sqrt{3}}\right) = 54.50^\circ = 54.5^\circ \swarrow$$

Ans.



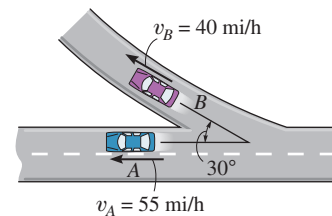
Ans:

$$v_{B/A} = 1044 \text{ km/h}$$

$$\theta = 54.5^\circ \swarrow$$

12-219.

At the instant shown, cars A and B are traveling at speeds of 55 mi/h and 40 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2 , while A maintains a constant speed, determine the velocity and acceleration of B with respect to A . Car B moves along a curve having a radius of curvature of 0.5 mi.



SOLUTION

$$v_B = -40 \cos 30^\circ \mathbf{i} + 40 \sin 30^\circ \mathbf{j} = \{-34.64\mathbf{i} + 20\mathbf{j}\} \text{ mi/h}$$

$$v_A = \{-55\mathbf{i}\} \text{ mi/h}$$

$$v_{B/A} = v_B - v_A$$

$$= (-34.64\mathbf{i} + 20\mathbf{j}) - (-55\mathbf{i}) = \{20.36\mathbf{i} + 20\mathbf{j}\} \text{ mi/h}$$

$$v_{B/A} = \sqrt{20.36^2 + 20^2} = 28.5 \text{ mi/h}$$

Ans.

$$\theta = \tan^{-1} \frac{20}{20.36} = 44.5^\circ \swarrow$$

Ans.

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{40^2}{0.5} = 3200 \text{ mi/h}^2 \quad (a_B)_t = 1200 \text{ mi/h}^2$$

$$\begin{aligned} \mathbf{a}_B &= (3200 \cos 60^\circ - 1200 \cos 30^\circ)\mathbf{i} + (3200 \sin 60^\circ + 1200 \sin 30^\circ)\mathbf{j} \\ &= \{560.77\mathbf{i} + 3371.28\mathbf{j}\} \text{ mi/h}^2 \end{aligned}$$

$$\mathbf{a}_A = 0$$

$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$$

$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\} - 0 = \{560.77\mathbf{i} + 3371.28\mathbf{j}\} \text{ mi/h}^2$$

$$a_{B/A} = \sqrt{(560.77)^2 + (3371.28)^2} = 3418 \text{ mi/h}^2$$

Ans.

$$\theta = \tan^{-1} \frac{3371.28}{560.77} = 80.6^\circ \swarrow$$

Ans.

Ans:

$$v_{B/A} = 28.5 \text{ mi/h}$$

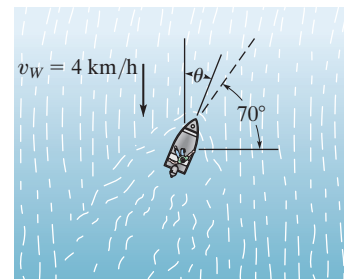
$$\theta_v = 44.5^\circ \swarrow$$

$$a_{B/A} = 3418 \text{ mi/h}^2$$

$$\theta_a = 80.6^\circ \swarrow$$

***12–220.**

The boat can travel with a speed of 16 km/h in still water. The point of destination is located along the dashed line. If the water is moving at 4 km/h, determine the bearing angle θ at which the boat must travel to stay on course.



SOLUTION

$$\mathbf{v}_B = \mathbf{v}_W + \mathbf{v}_{B/W}$$

$$v_B \cos 70^\circ \mathbf{i} + v_B \sin 70^\circ \mathbf{j} = -4\mathbf{j} + 16 \sin \theta \mathbf{i} + 16 \cos \theta \mathbf{j}$$

$$(\rightarrow) \quad v_B \cos 70^\circ = 0 + 16 \sin \theta$$

$$(+\uparrow) \quad v_B \sin 70^\circ = -4 + 16 \cos \theta$$

$$2.748 \sin \theta - \cos \theta + 0.25 = 0$$

Solving,

$$\theta = 15.1^\circ$$

Ans.

Ans:

$$\theta = 15.1^\circ$$

12–221.

Two boats leave the pier P at the same time and travel in the directions shown. If $v_A = 40$ ft/s and $v_B = 30$ ft/s, determine the velocity of boat A relative to boat B . How long after leaving the pier will the boats be 1500 ft apart?

SOLUTION

Relative Velocity:

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$40 \sin 30^\circ \mathbf{i} + 40 \cos 30^\circ \mathbf{j} = 30 \cos 45^\circ \mathbf{i} + 30 \sin 45^\circ \mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-1.213 \mathbf{i} + 13.43 \mathbf{j}\} \text{ ft/s}$$

Thus, the magnitude of the relative velocity $\mathbf{v}_{A/B}$ is

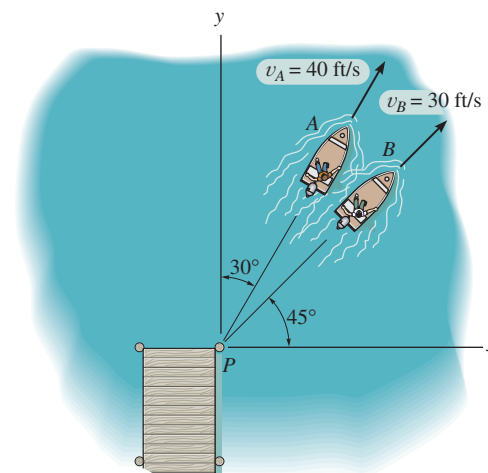
$$v_{A/B} = \sqrt{(-1.213)^2 + 13.43^2} = 13.48 \text{ ft/s} = 13.5 \text{ ft/s} \quad \text{Ans.}$$

And its direction is

$$\theta = \tan^{-1} \frac{13.43}{1.213} = 84.8^\circ \quad \text{Ans.}$$

One can obtain the time t required for boats A and B to be 1500 ft apart by noting that boat B is at rest and boat A travels at the relative speed $v_{A/B} = 13.48$ ft/s for a distance of 1500 ft. Thus

$$t = \frac{1500}{v_{A/B}} = \frac{1500}{13.48} = 111.26 \text{ s} = 1.85 \text{ min} \quad \text{Ans.}$$



Ans:

$$v_B = 13.5 \text{ ft/s}$$

$$\theta = 84.8^\circ$$

$$t = 1.85 \text{ min}$$

12–222.

A car is traveling north along a straight road at 50 km/h. An instrument in the car indicates that the wind is coming from the east. If the car's speed is 80 km/h, the instrument indicates that the wind is coming from the northeast. Determine the speed and direction of the wind.

SOLUTION

Solution I

Vector Analysis: For the first case, the velocity of the car and the velocity of the wind relative to the car expressed in Cartesian vector form are $\mathbf{v}_c = [50\mathbf{j}]$ km/h and $\mathbf{v}_{w/c} = (v_{w/c})_1 \mathbf{i}$. Applying the relative velocity equation, we have

$$\begin{aligned}\mathbf{v}_w &= \mathbf{v}_c + \mathbf{v}_{w/c} \\ \mathbf{v}_w &= 50\mathbf{j} + (v_{w/c})_1 \mathbf{i} \\ \mathbf{v}_w &= (v_{w/c})_1 \mathbf{i} + 50\mathbf{j} \quad (1)\end{aligned}$$

For the second case, $v_c = [80\mathbf{j}]$ km/h and $\mathbf{v}_{w/c} = (v_{w/c})_2 \cos 45^\circ \mathbf{i} + (v_{w/c})_2 \sin 45^\circ \mathbf{j}$. Applying the relative velocity equation, we have

$$\begin{aligned}\mathbf{v}_w &= \mathbf{v}_c + \mathbf{v}_{w/c} \\ \mathbf{v}_w &= 80\mathbf{j} + (v_{w/c})_2 \cos 45^\circ \mathbf{i} + (v_{w/c})_2 \sin 45^\circ \mathbf{j} \\ \mathbf{v}_w &= (v_{w/c})_2 \cos 45^\circ \mathbf{i} + [80 + (v_{w/c})_2 \sin 45^\circ] \mathbf{j} \quad (2)\end{aligned}$$

Equating Eqs. (1) and (2) and then the \mathbf{i} and \mathbf{j} components,

$$(v_{w/c})_1 = (v_{w/c})_2 \cos 45^\circ \quad (3)$$

$$50 = 80 + (v_{w/c})_2 \sin 45^\circ \quad (4)$$

Solving Eqs. (3) and (4) yields

$$(v_{w/c})_2 = -42.43 \text{ km/h} \quad (v_{w/c})_1 = -30 \text{ km/h}$$

Substituting the result of $(v_{w/c})_1$ into Eq. (1),

$$\mathbf{v}_w = [-30\mathbf{i} + 50\mathbf{j}] \text{ km/h}$$

Thus, the magnitude of \mathbf{v}_w is

$$v_w = \sqrt{(-30)^2 + 50^2} = 58.3 \text{ km/h} \quad \text{Ans.}$$

and the directional angle θ that \mathbf{v}_w makes with the x axis is

$$\theta = \tan^{-1} \left(\frac{50}{-30} \right) = 59.0^\circ \nwarrow \quad \text{Ans.}$$

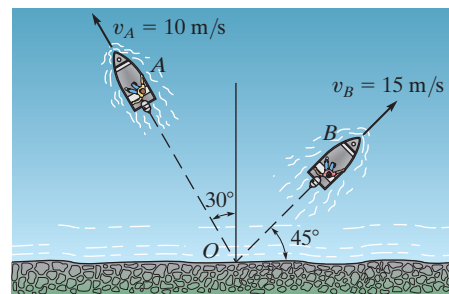
Ans:

$$v_w = 58.3 \text{ km/h}$$

$$\theta = 59.0^\circ \nwarrow$$

12-223.

Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 10 \text{ m/s}$ and $v_B = 15 \text{ m/s}$, determine the velocity of boat A with respect to boat B . How long after leaving the shore will the boats be 600 m apart?



SOLUTION

Relative Velocity. The velocity triangle shown in Fig. *a* is drawn based on the relative velocity equation $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$. Using the cosine law,

$$v_{A/B} = \sqrt{10^2 + 15^2 - 2(10)(15) \cos 75^\circ} = 15.73 \text{ m/s} = 15.7 \text{ m/s} \quad \text{Ans.}$$

Then, the sine law gives

$$\frac{\sin \phi}{10} = \frac{\sin 75^\circ}{15.73} \quad \phi = 37.89^\circ$$

The direction of $\mathbf{v}_{A/B}$ is defined by

$$\theta = 45^\circ - \phi = 45^\circ - 37.89^\circ = 7.11^\circ \quad \swarrow$$

Alternatively, we can express \mathbf{v}_A and \mathbf{v}_B in Cartesian vector form

$$\mathbf{v}_A = \{-10 \sin 30^\circ \mathbf{i} + 10 \cos 30^\circ \mathbf{j}\} \text{ m/s} = \{-5.00 \mathbf{i} + 5\sqrt{3} \mathbf{j}\} \text{ m/s}$$

$$\mathbf{v}_B = \{15 \cos 45^\circ \mathbf{i} + 15 \sin 45^\circ \mathbf{j}\} \text{ m/s} = \{7.5\sqrt{2} \mathbf{i} + 7.5\sqrt{2} \mathbf{j}\} \text{ m/s}.$$

Applying the relative velocity equation

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-5.00 \mathbf{i} + 5\sqrt{3} \mathbf{j} = 7.5\sqrt{2} \mathbf{i} + 7.5\sqrt{2} \mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-15.61 \mathbf{i} - 1.946 \mathbf{j}\} \text{ m/s}$$

Thus the magnitude of $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-15.61)^2 + (-1.946)^2} = 15.73 \text{ m/s} = 15.7 \text{ m/s}$$

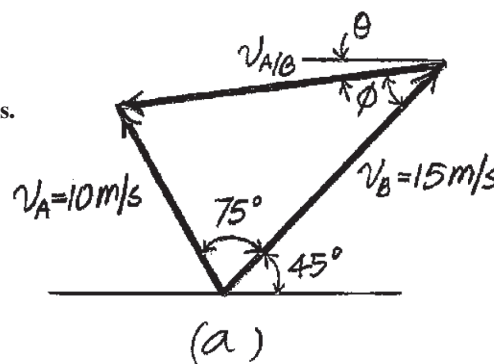
And its direction is defined by angle θ , Fig. *b*,

$$\theta = \tan^{-1}\left(\frac{1.946}{15.61}\right) = 7.1088^\circ = 7.11^\circ \quad \swarrow$$

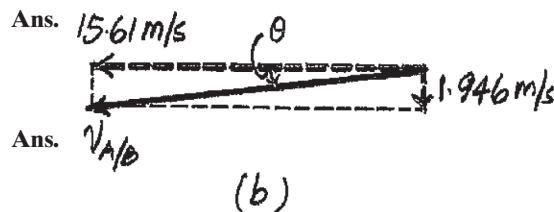
Here $s_{A/B} = 600 \text{ m}$. Thus

$$t = \frac{s_{A/B}}{v_{A/B}} = \frac{600}{15.73} = 38.15 \text{ s} = 38.1 \text{ s}$$

Ans.



Ans.



Ans.

Ans:

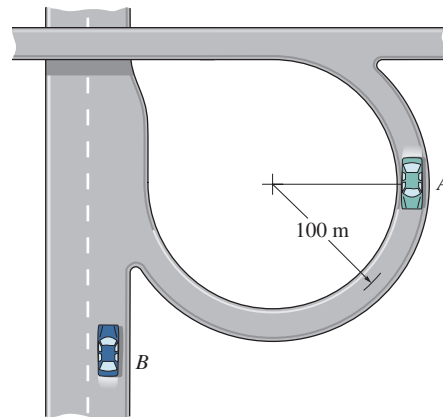
$$v_{A/B} = 15.7 \text{ m/s}$$

$$\theta = 7.11^\circ \quad \swarrow$$

$$t = 38.1 \text{ s}$$

***12–224.**

At the instant shown, car A has a speed of 20 km/h, which is being increased at the rate of 300 km/h^2 as the car enters an expressway. At the same instant, car B is decelerating at 250 km/h^2 while traveling forward at 100 km/h. Determine the velocity and acceleration of A with respect to B .



SOLUTION

$$\mathbf{v}_A = \{-20\mathbf{j}\} \text{ km/h} \quad \mathbf{v}_B = \{100\mathbf{j}\} \text{ km/h}$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= (-20\mathbf{j} - 100\mathbf{j}) = \{-120\mathbf{j}\} \text{ km/h}$$

$$v_{A/B} = 120 \text{ km/h} \downarrow$$

Ans.

$$(a_A)_n = \frac{v_A^2}{\rho} = \frac{20^2}{0.1} = 4000 \text{ km/h}^2 \quad (a_A)_t = 300 \text{ km/h}^2$$

$$\mathbf{a}_A = -4000\mathbf{i} + (-300\mathbf{j})$$

$$= \{-4000\mathbf{i} - 300\mathbf{j}\} \text{ km/h}^2$$

$$\mathbf{a}_B = \{-250\mathbf{j}\} \text{ km/h}^2$$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B$$

$$= (-4000\mathbf{i} - 300\mathbf{j}) - (-250\mathbf{j}) = \{-4000\mathbf{i} - 50\mathbf{j}\} \text{ km/h}^2$$

$$a_{A/B} = \sqrt{(-4000)^2 + (-50)^2} = 4000 \text{ km/h}^2$$

Ans.

$$\theta = \tan^{-1} \frac{50}{4000} = 0.716^\circ \nearrow$$

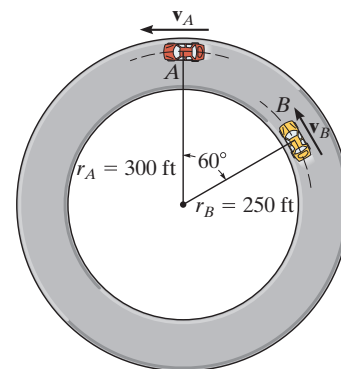
Ans.

Ans:

$$\begin{aligned} v_{A/B} &= 120 \text{ km/h} \downarrow \\ a_{A/B} &= 4000 \text{ km/h}^2 \\ \theta &= 0.716^\circ \nearrow \end{aligned}$$

12-225.

Cars *A* and *B* are traveling around the circular race track. At the instant shown, *A* has a speed of 90 ft/s and is increasing its speed at the rate of 15 ft/s², whereas *B* has a speed of 105 ft/s and is decreasing its speed at 25 ft/s². Determine the relative velocity and relative acceleration of car *A* with respect to car *B* at this instant.



SOLUTION

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-90\mathbf{i} = -105 \sin 30^\circ \mathbf{i} + 105 \cos 30^\circ \mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-37.5\mathbf{i} - 90.93\mathbf{j}\} \text{ ft/s}$$

$$v_{A/B} = \sqrt{(-37.5)^2 + (-90.93)^2} = 98.4 \text{ ft/s}$$

Ans.

$$\theta = \tan^{-1}\left(\frac{90.93}{37.5}\right) = 67.6^\circ \swarrow$$

Ans.

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$-15\mathbf{i} - \frac{(90)^2}{300}\mathbf{j} = 25 \cos 60^\circ \mathbf{i} - 25 \sin 60^\circ \mathbf{j} - 44.1 \sin 60^\circ \mathbf{i} - 44.1 \cos 60^\circ \mathbf{j} + \mathbf{a}_{A/B}$$

$$\mathbf{a}_{A/B} = \{10.69\mathbf{i} + 16.70\mathbf{j}\} \text{ ft/s}^2$$

$$a_{A/B} = \sqrt{(10.69)^2 + (16.70)^2} = 19.8 \text{ ft/s}^2$$

Ans.

$$\theta = \tan^{-1}\left(\frac{16.70}{10.69}\right) = 57.4^\circ \nearrow$$

Ans.

Ans:

$$v_{A/B} = 98.4 \text{ ft/s}$$

$$\theta_v = 67.6^\circ \swarrow$$

$$a_{A/B} = 19.8 \text{ ft/s}^2$$

$$\theta_a = 57.4^\circ \nearrow$$

12–226.

A man walks at 5 km/h in the direction of a 20-km/h wind. If raindrops fall vertically at 7 km/h in *still air*, determine the direction in which the drops appear to fall with respect to the man.

SOLUTION

Relative Velocity: The velocity of the rain must be determined first. Applying Eq. 12–34 gives

$$\mathbf{v}_r = \mathbf{v}_w + \mathbf{v}_{r/w} = 20\mathbf{i} + (-7\mathbf{j}) = \{20\mathbf{i} - 7\mathbf{j}\} \text{ km/h}$$

Thus, the relative velocity of the rain with respect to the man is

$$\mathbf{v}_r = \mathbf{v}_m + \mathbf{v}_{r/m}$$

$$20\mathbf{i} - 7\mathbf{j} = 5\mathbf{i} + \mathbf{v}_{r/m}$$

$$\mathbf{v}_{r/m} = \{15\mathbf{i} - 7\mathbf{j}\} \text{ km/h}$$

The magnitude of the relative velocity $\mathbf{v}_{r/m}$ is given by

$$v_{r/m} = \sqrt{15^2 + (-7)^2} = 16.6 \text{ km/h}$$

Ans.

And its direction is given by

$$\theta = \tan^{-1} \frac{7}{15} = 25.0^\circ \swarrow$$

Ans.



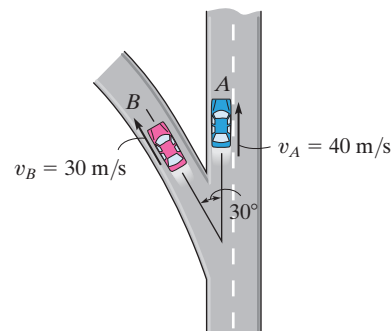
Ans:

$$v_{r/m} = 16.6 \text{ km/h}$$

$$\theta = 25.0^\circ \swarrow$$

12-227.

At the instant shown, cars A and B are traveling at velocities of 40 m/s and 30 m/s , respectively. If B is increasing its velocity by 2 m/s^2 , while A maintains a constant velocity, determine the velocity and acceleration of B with respect to A . The radius of curvature at B is $\rho_B = 200 \text{ m}$.



SOLUTION

Relative velocity. Express \mathbf{v}_A and \mathbf{v}_B as Cartesian vectors.

$$\mathbf{v}_A = \{40\mathbf{j}\} \text{ m/s} \quad \mathbf{v}_B = \{-30 \sin 30^\circ \mathbf{i} + 30 \cos 30^\circ \mathbf{j}\} \text{ m/s} = \{-15\mathbf{i} + 15\sqrt{3}\mathbf{j}\} \text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$-15\mathbf{i} + 15\sqrt{3}\mathbf{j} = 40\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{-15\mathbf{i} - 14.02\mathbf{j}\} \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{(-15)^2 + (-14.02)^2} = 20.53 \text{ m/s} = 20.5 \text{ m/s}$$

And its direction is defined by angle θ , Fig. a

$$\theta = \tan^{-1}\left(\frac{14.02}{15}\right) = 43.06^\circ = 43.1^\circ \checkmark$$

Relative Acceleration. Here, $(a_B)_t = 2 \text{ m/s}^2$ and $(a_B)_n = \frac{v_B^2}{\rho} = \frac{30^2}{200} = 4.50 \text{ m/s}^2$ and their directions are shown in Fig. b . Then, express \mathbf{a}_B as a Cartesian vector,

$$\begin{aligned} \mathbf{a}_B &= (-2 \sin 30^\circ - 4.50 \cos 30^\circ)\mathbf{i} + (2 \cos 30^\circ - 4.50 \sin 30^\circ)\mathbf{j} \\ &= \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\} \text{ m/s}^2 \end{aligned}$$

Applying the relative acceleration equation with $\mathbf{a}_A = \mathbf{0}$,

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$-4.8971\mathbf{i} - 0.5179\mathbf{j} = \mathbf{0} + \mathbf{a}_{B/A}$$

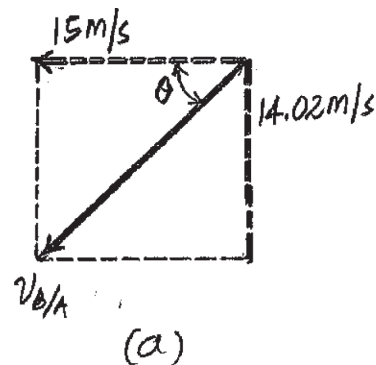
$$\mathbf{a}_{B/A} = \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\} \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/A}$ is

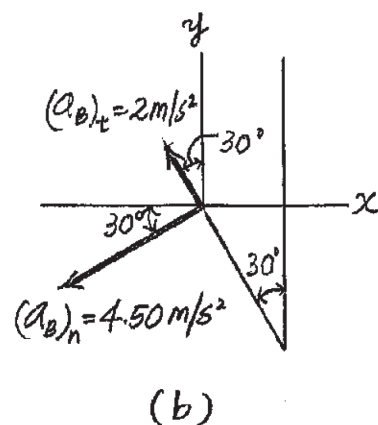
$$a_{B/A} = \sqrt{(-4.8971)^2 + (-0.5179)^2} = 4.9244 \text{ m/s}^2 = 4.92 \text{ m/s}^2$$

And its direction is defined by angle θ' , Fig. c ,

$$\theta' = \tan^{-1}\left(\frac{0.5179}{4.8971}\right) = 6.038^\circ = 6.04^\circ \checkmark$$

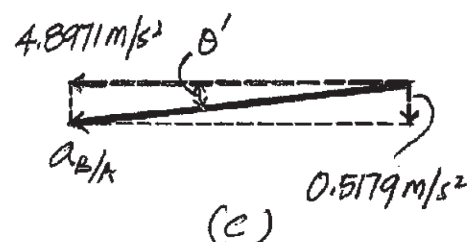


Ans.



(b)

Ans.



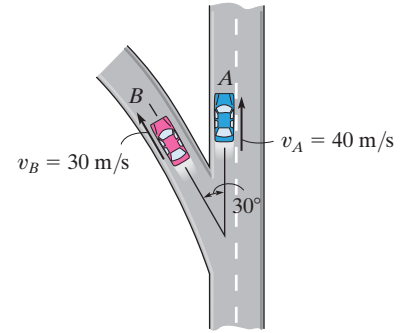
Ans.

Ans:

$$\begin{aligned} v_{B/A} &= 20.5 \text{ m/s} \\ \theta_v &= 43.1^\circ \checkmark \\ a_{B/A} &= 4.92 \text{ m/s}^2 \\ \theta_a &= 6.04^\circ \checkmark \end{aligned}$$

***12–228.**

At the instant shown, cars A and B are traveling at velocities of 40 m/s and 30 m/s , respectively. If A is increasing its speed at 4 m/s^2 , whereas the speed of B is decreasing at 3 m/s^2 , determine the velocity and acceleration of B with respect to A . The radius of curvature at B is $\rho_B = 200 \text{ m}$.



SOLUTION

Relative velocity. Express v_A and v_B as Cartesian vector.

$$\mathbf{v}_A = \{40\mathbf{j}\} \text{ m/s} \quad \mathbf{v}_B = \{-30 \sin 30^\circ \mathbf{i} + 30 \cos 30^\circ \mathbf{j}\} \text{ m/s} = \{-15\mathbf{i} + 15\sqrt{3}\mathbf{j}\} \text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$-15\mathbf{i} + 15\sqrt{3}\mathbf{j} = 40\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{-15\mathbf{i} - 14.02\mathbf{j}\} \text{ m/s}$$

Thus the magnitude of $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{(-15)^2 + (-14.02)^2} = 20.53 \text{ m/s} = 20.5 \text{ m/s} \quad \text{Ans.}$$

And its direction is defined by angle θ , Fig. a .

$$\theta = \tan^{-1}\left(\frac{14.02}{15}\right) = 43.06^\circ = 43.1^\circ \quad \text{Ans.}$$

Relative Acceleration. Here $(a_B)_t = 3 \text{ m/s}^2$ and $(a_B)_n = \frac{v_B^2}{\rho} = \frac{30^2}{200} = 4.5 \text{ m/s}^2$ and their directions are shown in Fig. b . Then express \mathbf{a}_B as a Cartesian vector,

$$\begin{aligned} \mathbf{a}_B &= (3 \sin 30^\circ - 4.50 \cos 30^\circ)\mathbf{i} + (-3 \cos 30^\circ - 4.50 \sin 30^\circ)\mathbf{j} \\ &= \{-2.3971\mathbf{i} - 4.8481\mathbf{j}\} \text{ m/s}^2 \end{aligned}$$

Applying the relative acceleration equation with $\mathbf{a}_A = \{4\mathbf{j}\} \text{ m/s}^2$,

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$-2.3971\mathbf{i} - 4.8481\mathbf{j} = 4\mathbf{j} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{-2.3971\mathbf{i} - 8.8481\mathbf{j}\} \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/A}$ is

$$a_{B/A} = \sqrt{(-2.3971)^2 + (-8.8481)^2} = 9.167 \text{ m/s}^2 = 9.17 \text{ m/s}^2 \quad \text{Ans.}$$

And its direction is defined by angle θ' , Fig. c

$$\theta' = \tan^{-1}\left(\frac{8.8481}{2.3971}\right) = 74.84^\circ = 74.8^\circ \quad \text{Ans.}$$

Ans:

$$v_{B/A} = 20.5 \text{ m/s}$$

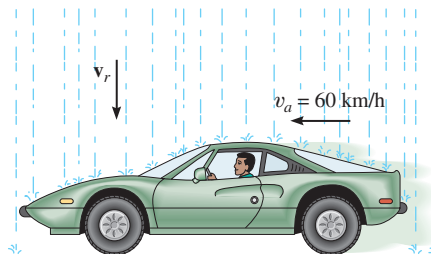
$$\theta = 43.1^\circ \quad \text{Ans.}$$

$$a_{B/A} = 9.17 \text{ m/s}^2$$

$$\theta' = 74.8^\circ \quad \text{Ans.}$$

12–229.

A passenger in an automobile observes that raindrops make an angle of 30° with the horizontal as the auto travels forward with a speed of 60 km/h. Compute the terminal (constant) velocity \mathbf{v}_r of the rain if it is assumed to fall vertically.



SOLUTION

$$\mathbf{v}_r = \mathbf{v}_a + \mathbf{v}_{r/a}$$

$$-v_r \mathbf{j} = -60 \mathbf{i} + v_{r/a} \cos 30^\circ \mathbf{i} - v_{r/a} \sin 30^\circ \mathbf{j}$$

$$(\rightarrow) \quad 0 = -60 + v_{r/a} \cos 30^\circ$$

$$(+\uparrow) \quad -v_r = 0 - v_{r/a} \sin 30^\circ$$

$$v_{r/a} = 69.3 \text{ km/h}$$

$$v_r = 34.6 \text{ km/h}$$

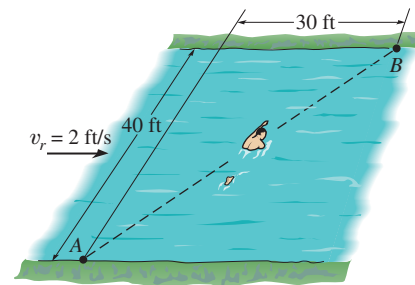
Ans.

Ans:

$$v_r = 34.6 \text{ km/h} \downarrow$$

12-230.

A man can swim at 4 ft/s in still water. He wishes to cross the 40-ft-wide river to point B , 30 ft downstream. If the river flows with a velocity of 2 ft/s, determine the speed of the man and the time needed to make the crossing. *Note:* While in the water he must not direct himself toward point B to reach this point. Why?



SOLUTION

Relative Velocity:

$$v_m = v_r + v_{m/r}$$

$$\frac{3}{5}v_m \mathbf{i} + \frac{4}{5}v_m \mathbf{j} = 2\mathbf{i} + 4 \sin \theta \mathbf{i} + 4 \cos \theta \mathbf{j}$$

Equating the i and j components, we have

$$\frac{3}{5}v_m = 2 + 4 \sin \theta \quad (1)$$

$$\frac{4}{5}v_m = 4 \cos \theta \quad (2)$$

Solving Eqs. (1) and (2) yields

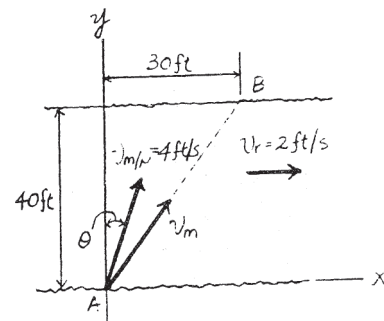
$$\theta = 13.29^\circ$$

$$v_m = 4.866 \text{ ft/s} = 4.87 \text{ ft/s} \quad \text{Ans.}$$

Thus, the time t required by the boat to travel from points A to B is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{40^2 + 30^2}}{4.866} = 10.3 \text{ s} \quad \text{Ans.}$$

In order for the man to reach point B , the man has to direct himself at an angle $\theta = 13.3^\circ$ with y axis.



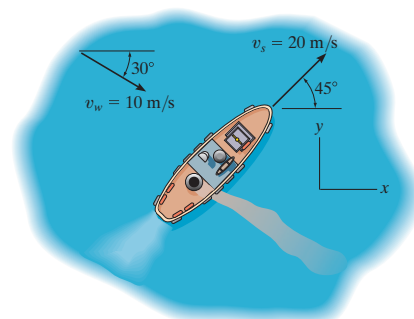
Ans:

$$v_m = 4.87 \text{ ft/s}$$

$$t = 10.3 \text{ s}$$

12-231.

The ship travels at a constant speed of $v_s = 20$ m/s and the wind is blowing at a speed of $v_w = 10$ m/s, as shown. Determine the magnitude and direction of the horizontal component of velocity of the smoke coming from the smoke stack as it appears to a passenger on the ship.



SOLUTION

Solution I

Vector Analysis: The velocity of the smoke as observed from the ship is equal to the velocity of the wind relative to the ship. Here, the velocity of the ship and wind expressed in Cartesian vector form are $\mathbf{v}_s = [20 \cos 45^\circ \mathbf{i} + 20 \sin 45^\circ \mathbf{j}]$ m/s $= [14.14\mathbf{i} + 14.14\mathbf{j}]$ m/s and $\mathbf{v}_w = [10 \cos 30^\circ \mathbf{i} - 10 \sin 30^\circ \mathbf{j}] = [8.660\mathbf{i} - 5\mathbf{j}]$ m/s. Applying the relative velocity equation,

$$\mathbf{v}_w = \mathbf{v}_s + \mathbf{v}_{w/s}$$

$$8.660\mathbf{i} - 5\mathbf{j} = 14.14\mathbf{i} + 14.14\mathbf{j} + \mathbf{v}_{w/s}$$

$$\mathbf{v}_{w/s} = [-5.482\mathbf{i} - 19.14\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{w/s}$ is given by

$$v_w = \sqrt{(-5.482)^2 + (-19.14)^2} = 19.9 \text{ m/s} \quad \text{Ans.}$$

and the direction angle θ that $\mathbf{v}_{w/s}$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{19.14}{5.482}\right) = 74.0^\circ \quad \text{Ans.}$$

Solution II

Scalar Analysis: Applying the law of cosines by referring to the velocity diagram shown in Fig. *a*,

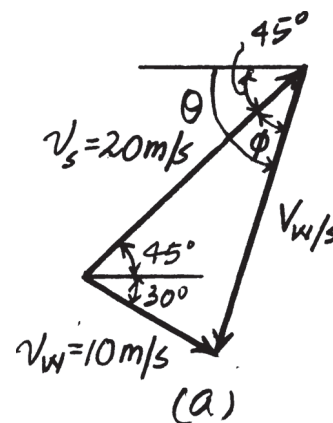
$$\begin{aligned} v_{w/s} &= \sqrt{20^2 + 10^2 - 2(20)(10) \cos 75^\circ} \\ &= 19.91 \text{ m/s} = 19.9 \text{ m/s} \end{aligned} \quad \text{Ans.}$$

Using the result of $v_{w/s}$ and applying the law of sines,

$$\frac{\sin \phi}{10} = \frac{\sin 75^\circ}{19.91} \quad \phi = 29.02^\circ$$

Thus,

$$\theta = 45^\circ + \phi = 74.0^\circ \quad \text{Ans.}$$

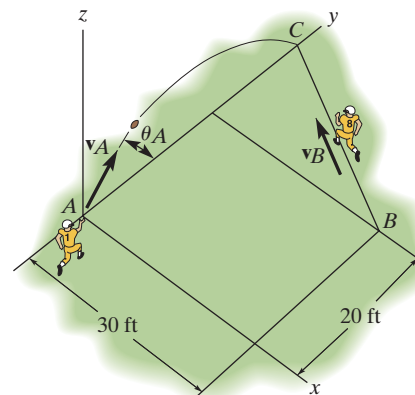


Ans:

$$\begin{aligned} v_{w/s} &= 19.9 \text{ m/s} \\ \theta &= 74.0^\circ \end{aligned}$$

***12–232.**

The football player at A throws the ball in the y – z plane at a speed $v_A = 50$ ft/s and an angle $\theta_A = 60^\circ$ with the horizontal. At the instant the ball is thrown, the player is at B and is running with constant speed along the line BC in order to catch it. Determine this speed, v_B , so that he makes the catch at the same elevation from which the ball was thrown.



SOLUTION

$$(\rightarrow) s = s_0 + v_0 t$$

$$d_{AC} = 0 + (50 \cos 60^\circ) t$$

$$(+\uparrow) v = v_0 + a_c t$$

$$-50 \sin 60^\circ = 50 \sin 60^\circ - 32.2 t$$

$$t = 2.690 \text{ s}$$

$$d_{AC} = 67.24 \text{ ft}$$

$$d_{BC} = \sqrt{(30)^2 + (67.24 - 20)^2} = 55.96 \text{ ft}$$

$$v_B = \frac{55.96}{2.690} = 20.8 \text{ ft/s}$$

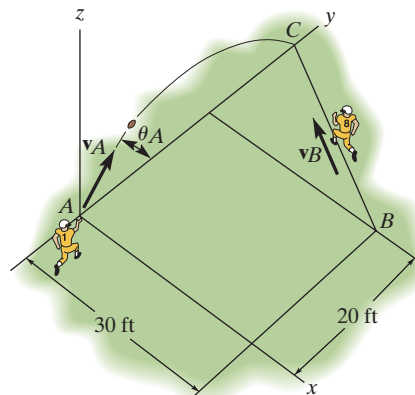
Ans.

Ans:

$$v_B = 20.8 \text{ ft/s}$$

12–233.

The football player at A throws the ball in the y – z plane with a speed $v_A = 50$ ft/s and an angle $\theta_A = 60^\circ$ with the horizontal. At the instant the ball is thrown, the player is at B and is running at a constant speed of $v_B = 23$ ft/s along the line BC . Determine if he can reach point C , which has the same elevation as A , before the ball gets there.



SOLUTION

$$(\rightarrow) s = s_0 + v_0 t$$

$$d_{AC} = 0 + (50 \cos 60^\circ) t$$

$$(+\uparrow) v = v_0 + a_c t$$

$$-50 \sin 60^\circ = 50 \sin 60^\circ - 32.2 t$$

$$t = 2.690 \text{ s}$$

$$d_{AC} = 67.24 \text{ ft}$$

$$d_{BC} = \sqrt{(30)^2 + (67.24 - 20)^2} = 55.96 \text{ ft}$$

$$v_B = \frac{d_{BC}}{t} = \frac{55.96}{(2.690)} = 20.8 \text{ ft/s}$$

$$\text{Since } v_B = 20.8 \text{ ft/s} < (v_B)_{\max} = 23 \text{ ft/s}$$

Yes, he can catch the ball.

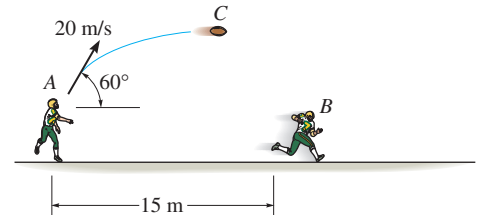
Ans.

Ans:

Yes, he can catch the ball.

12-234.

At a given instant the football player at A throws a football C with a velocity of 20 m/s in the direction shown. Determine the constant speed at which the player at B must run so that he can catch the football at the same elevation at which it was thrown. Also calculate the relative velocity and relative acceleration of the football with respect to B at the instant the catch is made. Player B is 15 m away from A when A starts to throw the football.



SOLUTION

Ball:

$$(\rightarrow) s = s_0 + v_0 t$$

$$s_C = 0 + 20 \cos 60^\circ t$$

$$(+\uparrow) v = v_0 + a_c t$$

$$-20 \sin 60^\circ = 20 \sin 60^\circ - 9.81 t$$

$$t = 3.53 \text{ s}$$

$$s_C = 35.31 \text{ m}$$

Player B :

$$(\rightarrow) s_B = s_0 + v_B t$$

Require,

$$35.31 = 15 + v_B(3.53)$$

$$v_B = 5.75 \text{ m/s}$$

Ans.

At the time of the catch

$$(v_C)_x = 20 \cos 60^\circ = 10 \text{ m/s} \rightarrow$$

$$(v_C)_y = 20 \sin 60^\circ = 17.32 \text{ m/s} \downarrow$$

$$v_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$10\mathbf{i} - 17.32\mathbf{j} = 5.75\mathbf{i} + (v_{C/B})_x\mathbf{i} + (v_{C/B})_y\mathbf{j}$$

$$(\rightarrow) 10 = 5.75 + (v_{C/B})_x$$

$$(+\uparrow) -17.32 = (v_{C/B})_y$$

$$(v_{C/B})_x = 4.25 \text{ m/s} \rightarrow$$

$$(v_{C/B})_y = 17.32 \text{ m/s} \downarrow$$

$$v_{C/B} = \sqrt{(4.25)^2 + (17.32)^2} = 17.8 \text{ m/s}$$

Ans.

$$\theta = \tan^{-1}\left(\frac{17.32}{4.25}\right) = 76.2^\circ \searrow$$

Ans.

$$a_C = \mathbf{a}_B + \mathbf{a}_{C/B}$$

$$-9.81 \mathbf{j} = 0 + \mathbf{a}_{C/B}$$

$$a_{C/B} = 9.81 \text{ m/s}^2 \downarrow$$

Ans.

Ans:

$$v_B = 5.75 \text{ m/s}$$

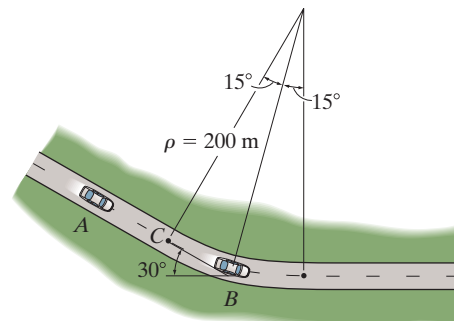
$$v_{C/B} = 17.8 \text{ m/s}$$

$$\theta = 76.2^\circ \searrow$$

$$a_{C/B} = 9.81 \text{ m/s}^2 \downarrow$$

12-235.

At the instant shown, car A travels along the straight portion of the road with a speed of 25 m/s. At this same instant car B travels along the circular portion of the road with a speed of 15 m/s. Determine the velocity of car B relative to car A .



SOLUTION

Velocity: Referring to Fig. a , the velocity of cars A and B expressed in Cartesian vector form are

$$\mathbf{v}_A = [25 \cos 30^\circ \mathbf{i} - 25 \sin 30^\circ \mathbf{j}] \text{ m/s} = [21.65\mathbf{i} - 12.5\mathbf{j}] \text{ m/s}$$

$$\mathbf{v}_B = [15 \cos 15^\circ \mathbf{i} - 15 \sin 15^\circ \mathbf{j}] \text{ m/s} = [14.49\mathbf{i} - 3.882\mathbf{j}] \text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$14.49\mathbf{i} - 3.882\mathbf{j} = 21.65\mathbf{i} - 12.5\mathbf{j} + \mathbf{v}_{B/A}$$

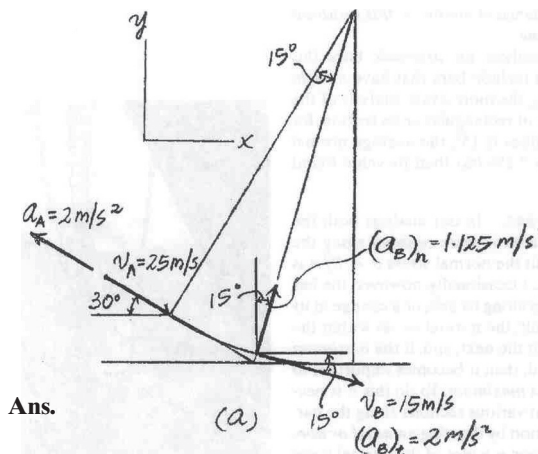
$$\mathbf{v}_{B/A} = [-7.162\mathbf{i} + 8.618\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is given by

$$v_{B/A} = \sqrt{(-7.162)^2 + 8.618^2} = 11.2 \text{ m/s}$$

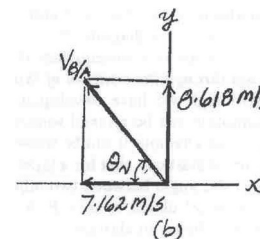
The direction angle θ_v of $\mathbf{v}_{B/A}$ measured down from the negative x axis, Fig. b is

$$\theta_v = \tan^{-1}\left(\frac{8.618}{7.162}\right) = 50.3^\circ \quad \swarrow$$



Ans.

Ans.



Ans:

$$v_{B/A} = 11.2 \text{ m/s}$$

$$\theta = 50.3^\circ$$