1.1: PROBLEM DEFINITION

No solution provided because student answers will vary.

1.2: PROBLEM DEFINITION

No solution provided because student answers will vary..

1.3: PROBLEM DEFINITION

No solution provided because student answers will vary.

1.4: PROBLEM DEFINITION

Essay question. No solution provided; answers will vary.

1.5: PROBLEM DEFINITION

Situation:

Many engineering students believe that fixing a washing machine is an example of engineering because it involves solving a problem. Write a brief essay in which you address the following questions: Is fixing a washing machine an example of engineering? Why or why not? How do your ideas align or misalign with the definition of engineering given in §1.1?

SOLUTION

Answers will vary. A possible argument is that simply fixing a washing machine is doing the work of a mechanic or electrician. Such work is engineering if new innovation is applied to make the washing machine better for humankind than as originally constructed.

1.6: PROBLEM DEFINITION

No solution provided; answers will vary. Possible answers could be determined by googling "material properties", which would yield answers such as thermal conductivity, electrical conductivity, tensile strength, etc. The next step would be to discuss how each new material property was different for solids, liquids, and gases.

1.7: PROBLEM DEFINITION

Situation:
Based on molecular mechanisms, explain why aluminum melts at 660 °C whereas ice will melt at 0 °C.

SOLUTION

When a solid melts, sufficient energy must be added to overcome the strong intermolecular forces. The intermolecular forces within solid aluminum require more energy to be overcome (to cause melting), than do the intermolecular forces in ice.

1.8: PROBLEM DEFINITION

Situation:

The continuum assumption (select all that apply)

a. applies in a vacuum such as in outer space

b. assumes that fluids are infinitely divisible into smaller and smaller parts

c. is a bad assumption when the length scale of the problem or design is similar to the spacing of the molecules

d. means that density can idealized as a continuous function of position

e. only applies to gases

SOLUTION

The correct answers are b, c, and d.

1.9: PROBLEM DEFINITION

Situation:

A fluid particle

a. is defined as one molecule

b. is small given the scale of the problem being considered

c. is so small that the continuum assumption does not apply

SOLUTION

The correct answer is b.

1.10: PROBLEM DEFINITION

Find: List three common units for each variable:

a. Volume flow rate (Q), mass flow rate ($\dot{m}$), and pressure (p).
b. Force, energy, power.
c. Viscosity, surface tension.

PLAN

Use Table F.1 to find common units

SOLUTION

a. Volume flow rate, mass flow rate, and pressure.

- Volume flow rate, m^3/s, ft^3/s or cfs, cfm or ft^3/m.
- Mass flow rate, kg/s, lbm/s, slug/s.
- Pressure, Pa, bar, psi or $\mathrm{lbf}/\mathrm{in}^2$.

b. Force, energy, power.

- Force, lbf, N, dyne.
- Energy, J, ft·lbf, Btu.
- Power, W, Btu/s, ft·lbf/s.

c. Viscosity.

- Viscosity, Pa·s, kg/(m·s), poise.

1.11: PROBLEM DEFINITION

Situation: The hydrostatic equation has three common forms:

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 = \text{ constant}$$

$$p_z = p_1 + \gamma z_1 = p_2 + \gamma z_2 = \text{ constant}$$

$$\Delta p = -\gamma \Delta z$$

Find: For each variable in these equations, list the name, symbol, and primary dimensions of each variable.

PLAN

Look up variables in Table A.6. Organize results using a table.

SOLUTION

Name	**Symbol**	**Primary dimensions**
pressure	p	M/LT^2
specific weight	γ	M/L^2T^2
elevation	z	L
piezometric pressure	p_z	M/LT^2
change in pressure	Δp	M/LT^2
change in elevation	Δz	L

1.12: PROBLEM DEFINITION

Situation:

Five units are specified.

Find:

Primary dimensions for each given unit: kWh, poise, slug, cfm, cSt.

PLAN

1. Find each primary dimension by using Table F.1.
2. Organize results using a table.

SOLUTION

Unit	**Associated Dimension**	**Associated Primary Dimensions**
kWh	Energy	ML^2/T^2
poise	Viscosity	$M/(L \cdot T)$
slug	Mass	M
cfm	Volume Flow Rate	L^3/T
cSt	Kinematic viscosity	L^2/T

1.13: PROBLEM DEFINITION

Situation:
In the context of measurement, a dimension is:
a. a category for measurement
b. a standard of measurement for size or magnitude
c. an increment for measuring "how much"

SOLUTION

a. a category for measurement

1.14: PROBLEM DEFINITION

Situation:
What is the approximate mass in units of slugs for
a. A 2-liter bottle of water?
b. A typical adult male?
c. A typical automobile?

a)

PLAN

Mass in slugs for: 2-L bottle of water

SOLUTION

$$\left(\frac{2\text{L}}{}\right)\left(\frac{1000\,\text{kg}}{\text{m}^3}\right)\left(\frac{1\,\text{m}^3}{1000\text{L}}\right)\left(\frac{1\,\text{slug}}{14.59\,\text{kg}}\right) = \boxed{0.137\ \text{slug}}$$

b)

PLAN

Answers will vary, but for 180-lb male:

SOLUTION

On earth 1 lbf weighs 1 lbm
To convert to slugs

$$\left(\frac{180\,\text{lb}}{}\right)\left(\frac{1\,\text{slug}}{32.17\,\text{lb}}\right) = \boxed{5.60\ \text{slug}}$$

c)

PLAN

Answers will vary, but for 3000-lb automobile:

SOLUTION

On earth 1 lbf weighs 1 lbm
To convert to slugs

$$\left(\frac{3000\,\text{lb}}{}\right)\left(\frac{1\,\text{slug}}{32.17\,\text{lb}}\right) = \boxed{93.3\ \text{slug}}$$

1.15: PROBLEM DEFINITION

Situation:
In the list below, identify which parameters are dimensions and which paramenters are units: slug, mass, kg, energy/time, meters, horsepower, pressure, and pascals.

SOLUTION

Dimensions: mass, energy/time, pressure
Units: slug, kg, meters, horsepower, pascals

1.16: PROBLEM DEFINITION

Situation:

Of the 3 lists below, which sets of units are consistent? Select all that Apply.

a. pounds-mass, pounds-force, feet, and seconds.

b. slugs, pounds-force, feet, and seconds

c. kilograms, newtons, meters, and seconds.

SOLUTION

Answers (a) and (c) are correct.

Problem 1.17

No solution provided, students are asked to describe the actions for each step of the WWM in their own words.

1.18: PROBLEM DEFINITION

Situation:
Which of these is a correct conversion ratio?

SOLUTION

Answers (a) and (b) are correct

1.19: PROBLEM DEFINITION

Situation:
If the local atmospheric pressure is 93 kPa, use the grid method to find the pressure in units of
a. psia
b. psf
c. bar
d. atmospheres
e. feet of water
f. inches of mercury

PLAN

Follow the process given in the text. Look up conversion ratios in Table F.1 (EFM 10e).

a)

SOLUTION

$$\left(\frac{93\,\mathrm{kPa}}{}\right)\left(\frac{1000\,\mathrm{Pa}}{1\,\mathrm{kPa}}\right)\left(\frac{1.450\times10^{-4}\ \mathrm{psi}}{\mathrm{Pa}}\right)$$

$$\boxed{93\,\mathrm{kPa} = 13.485\ \mathrm{psia}}$$

b)

SOLUTION

$$\left(\frac{93\,\mathrm{kPa}}{}\right)\left(\frac{1000\,\mathrm{Pa}}{1\,\mathrm{kPa}}\right)\left(\frac{1.450\times10^{-4}\ \mathrm{psi}}{\mathrm{Pa}}\right)\left(\frac{144\,\mathrm{in}^2}{1\,\mathrm{ft}^2}\right)$$

$$\boxed{93\,\mathrm{kPa} = 1941.8\ \mathrm{psf}}$$

c)

SOLUTION

$$\left(\frac{93\,\mathrm{kPa}}{}\right)\left(\frac{1000\,\mathrm{Pa}}{1\,\mathrm{kPa}}\right)\left(\frac{1\,\mathrm{bar}}{100000\,\mathrm{Pa}}\right)$$

$$\boxed{93\,\mathrm{kPa} = 0.93\,\mathrm{bar}}$$

d)

SOLUTION

$$\left(\frac{93\,\text{kPa}}{}\right)\left(\frac{1000\,\text{Pa}}{1\,\text{kPa}}\right)\left(\frac{1.450\times10^{-4}\ \text{psi}}{\text{Pa}}\right)\left(\frac{1\,\text{atm}}{14.7\ \text{psi}}\right)$$

$$\boxed{93\,\text{kPa} = 0.917\,\text{atm}}$$

e)

SOLUTION

$$\left(\frac{93\,\text{kPa}}{}\right)\left(\frac{1000\,\text{Pa}}{1\,\text{kPa}}\right)\left(\frac{0.004019\ \text{in-H}_2\text{0}}{\text{Pa}}\right)\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)$$

$$\boxed{93\,\text{kPa} = 31.15\ \text{ft-H}_2\text{O}}$$

f)

SOLUTION

$$\left(\frac{93\,\text{kPa}}{}\right)\left(\frac{1000\,\text{Pa}}{1\,\text{kPa}}\right)\left(\frac{1\,\text{in}\cdot\text{HG}}{3386.39\,\text{Pa}}\right)$$

$$\boxed{93\,\text{kPa} = 27.46\ \text{in-HG}}$$

1.20: PROBLEM DEFINITION

Apply the grid method.

Situation:

Density of ideal gas is given by:

$$\rho = \frac{p}{RT}$$

$p = 60$ psi, $R = 1716\,\text{ft} \cdot \text{lbf/slug} \cdot {}^\circ\text{R}$.
$T = 180\,^\circ\text{F} = 640\ {}^\circ\text{R}$.

Find:

Calculate density (in lbm/ft^3).

PLAN

Use the definition of density.
Follow the process for the grid method given in the text.
Look up conversion formulas in Table F.1 (EFM 10e).

SOLUTION

(Note, cancellation of units not shown below, but student should show cancellations on handworked problems.)

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \left(\frac{60\,\text{lbf}}{\text{in}^2}\right)\left(\frac{12\,\text{in}}{\text{ft}}\right)^2\left(\frac{\text{slug}\cdot{}^\text{o}\,\text{R}}{1716\,\text{ft}\cdot\text{lbf}}\right)\left(\frac{1.0}{640\ {}^\text{o}\text{R}}\right)\left(\frac{32.17\ \text{lbm}}{1.0\,\text{slug}}\right)\end{aligned}$$

$$\boxed{\rho = 0.253\ \ \text{lbm/ft}^3}$$

1.21: PROBLEM DEFINITION

Apply the grid method.

Situation:

Wind is hitting a window of building.

$\Delta p = \frac{\rho V^2}{2}$.

$\rho = 1.2\,\mathrm{kg/m^3}$, $V = 60$ mph.

Find:

a. Express the answer in pascals.

b. Express the answer in pounds force per square inch (psi).

c. Express the answer in inches of water column (inch $H_2 0$).

PLAN

Follow the process for the grid method given in the text. Look up conversion ratios in Table F.1.

SOLUTION

a)

Pascals.

$$\begin{aligned}\Delta p &= \frac{\rho V^2}{2} \\ &= \frac{1}{2}\left(\frac{1.2\,\mathrm{kg}}{\mathrm{m^3}}\right)\left(\frac{60\text{ mph}}{1.0}\right)^2\left(\frac{1.0\,\mathrm{m/s}}{2.237\text{ mph}}\right)^2\left(\frac{\mathrm{Pa\cdot m\cdot s^2}}{\mathrm{kg}}\right)\end{aligned}$$

$$\boxed{\Delta p = 432\,\mathrm{Pa}}$$

b)

Pounds per square inch.

$$\Delta p = 432\,\mathrm{Pa}\left(\frac{1.450\times 10^{-4}\text{ psi}}{\mathrm{Pa}}\right)$$

$$\boxed{\Delta p = 0.062\,6\text{ psi}}$$

c)

Inches of water column

$$\Delta p = 432\,\mathrm{Pa}\left(\frac{0.004019\text{ in-H}_2\text{0}}{\mathrm{Pa}}\right)$$

$$\boxed{\Delta p = 1.74\text{ in-H}_2\text{0}}$$

1.22: PROBLEM DEFINITION

Apply the grid method.

Situation:

Force is given by $F = ma$.

a) $m = 10\,\text{kg}$, $a = 10\,\text{m/s}^2$.

b) $m = 10\,\text{lbm}$, $a = 10\,\text{ft/s}^2$.

c) $m = 10\,\text{slug}$, $a = 10\,\text{ft/s}^2$.

Find:

Calculate force.

PLAN

Follow the process for the grid method given in the text. Look up conversion ratios in Table F.1.

SOLUTION

a)

Force in newtons for $m = 10\,\text{kg}$ and $a = 10\,\text{m/s}^2$.

$$\begin{aligned} F &= ma \\ &= (10\,\text{kg}) \left(10\frac{\text{m}}{\text{s}^2}\right) \left(\frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}\right) \end{aligned}$$

$$\boxed{F = 100\,\text{N}}$$

b)

Force in lbf for $m = 10$ lbm and $a = 10\,\text{ft/s}^2$.

$$\begin{aligned} F &= ma \\ &= (10\ \text{lbm}) \left(10\frac{\text{ft}}{\text{s}^2}\right) \left(\frac{\text{lbf} \cdot \text{s}^2}{32.2\ \text{lbm} \cdot \text{ft}}\right) \end{aligned}$$

$$\boxed{F = 3.11\,\text{lbf}}$$

c)

Force in newtons for $m = 10$ slug and acceleration is $a = 10\,\text{ft/s}^2$.

$$\begin{aligned} F &= ma \\ &= (10\,\text{slug}) \left(10\frac{\text{ft}}{\text{s}^2}\right) \left(\frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}}\right) \left(\frac{4.448\,\text{N}}{\text{lbf}}\right) \end{aligned}$$

$$\boxed{F = 445\,\text{N}}$$

1.23: PROBLEM DEFINITION

Apply the grid method.

Situation:

A cyclist is travelling along a road.
$P = FV$.
$V = 24\,\mathrm{mi/h}$, $F = 5\,\mathrm{lbf}$.

Find:

a) Find power in watts.
b) Find the energy in food calories to ride for 1 hour.

PLAN

Follow the process for the grid method given in the text. Look up conversion ratios in Table F.1.

SOLUTION

a)

Power

$$\begin{aligned} P &= FV \\ &= (5\,\mathrm{lbf})\left(\frac{4.448\,\mathrm{N}}{\mathrm{lbf}}\right)(24\ \mathrm{mph})\left(\frac{1.0\,\mathrm{m/s}}{2.237\ \mathrm{mph}}\right)\left(\frac{\mathrm{W}\cdot\mathrm{s}}{\mathrm{N}\cdot\mathrm{m}}\right) \end{aligned}$$

$$\boxed{P = 239\,\mathrm{W}}$$

b)

Energy

$$\begin{aligned} \Delta E &= P\Delta t \\ &= \left(\frac{239\,\mathrm{J}}{\mathrm{s}}\right)(1\,\mathrm{h})\left(\frac{3600\,\mathrm{s}}{\mathrm{h}}\right)\left(\frac{1.0\ \text{calorie (nutritional)}}{4187\,\mathrm{J}}\right) \end{aligned}$$

$$\boxed{\Delta E = 205\ \text{calories}}$$

1.24: PROBLEM DEFINITION

Apply the grid method.

Situation:

A pump operates for one year.
$P = 20\,\text{hp}$.
The pump operates for 20 hours/day.
Electricity costs \$0.10/kWh.

Find:

The cost (U.S. dollars) of operating the pump for one year.

PLAN

1. Find energy consumed using $E = Pt$, where P is power and t is time.
2. Find cost using $C = E \times (\$0.1/\text{kWh})$.

SOLUTION

1. Energy Consumed

$$
\begin{aligned}
E &= Pt \\
&= (20\,\text{hp})\left(\frac{\text{W}}{1.341 \times 10^{-3}\,\text{hp}}\right)\left(\frac{20\,\text{h}}{\text{d}}\right)\left(\frac{365\,\text{d}}{\text{year}}\right) \\
&= 1.09 \times 10^{8}\,\text{W}\cdot\text{h}\left(\frac{\text{kWh}}{1000\,\text{W}\cdot\text{h}}\right)\text{ per year}
\end{aligned}
$$

$$\boxed{E = 1.09 \times 10^{5}\text{ kWh per year}}$$

2. Cost

$$
\begin{aligned}
C &= E(\$0.1/\text{kWh}) \\
&= \left(1.09 \times 10^{5}\text{ kWh}\right)\left(\frac{\$0.10}{\text{kWh}}\right)
\end{aligned}
$$

$$\boxed{C = \$10,900}$$

1.25: PROBLEM DEFINITION

Situation:
Start with the Ideal Gas Law and prove that
a. Boyle's law is true.
b. Charles' law is true.

PLAN

Start with Ideal Gas Law

$$p\forall = nR_uT$$

SOLUTION

a) If temperature is held constant, then

$$p\forall = nR_u \times \text{ constant}$$

for a given # of molecules of a given gas,

$$\begin{aligned} p\forall &= \text{constant} \\ &\Rightarrow \text{Boyle's Law is True} \end{aligned}$$

b) Starting again with

$$p\forall = nR_uT$$

If the pressure is held constant, for a given number of molecules (n), of a given gas,

$$\begin{aligned} \frac{\forall}{T} &= \text{constant} \\ &\Rightarrow \text{Charles' Law is True} \end{aligned}$$

1.26: PROBLEM DEFINITION

Situation:

Calculate the number of moles in:
a) One cubic cm of water at room conditions
b) One cubic cm of air at room conditions

a)

PLAN

1. The density of water at room conditions is known (Table A.5 EFM10e), and the volume is given, so:

$$m = \rho \cancel{V}$$

2. From the Internet, water has a molar mass of 18 g/mol, use this to determine the number of moles in this sample.
3. Avogadro's number says that there are 6×10^{23} molecules/mol

SOLUTION

1.

$$m = \rho_{\text{water}} \cancel{V}$$

Assume conditions are atmospheric with $T = 20°C$ and $\rho = 998 \frac{\text{kg}}{\text{m}^3}$

$$\begin{aligned} m_{\text{water}} &= \left(\frac{998\,\text{kg}}{\text{m}^3}\right)\left(\frac{1\,\text{m}^3}{100^3\,\text{cm}^3}\right)\left(1\,\text{cm}^3\right) \\ &= \boxed{0.001\ \text{kg}} \end{aligned}$$

2. To determine the number of moles:

$$\begin{aligned} & (0.0010\,\text{kg})\left(\frac{1\,\text{mol}}{18\,\text{g}}\right)\left(\frac{1000\,\text{g}}{1\,\text{kg}}\right) \\ = & \boxed{0.055\ \text{mol}} \end{aligned}$$

3. Using Avogadro's number

$$\begin{aligned} & (0.055\,\text{mol})\left(\frac{6 \times 10^{23}\ \text{molecules}}{\text{mol}}\right) \\ = & \boxed{3.3 \times 10^{22}\ \text{molecules}} \end{aligned}$$

b)

PLAN

1. The density of air at room conditions is known (Table A.3 EFM10e), and the volume is given, so:

$$m = \rho \cancel{V}$$

2. From the Internet, dry air has a molar mass of 28.97 g/mol, use this to determine the number of moles in this sample.
3. Avogadro's number says that there are 6×10^{23} molecules/mol

SOLUTION

1.

$$m = \rho_{\text{air}} \cancel{V}$$

Assume conditions are atmospheric with $T = 20°C$ and $\rho = 1.20 \frac{\text{kg}}{\text{m}^3}$

$$\begin{aligned} m_{\text{air}} &= \left(\frac{1.20\,\text{kg}}{\text{m}^3}\right)\left(\frac{1\,\text{m}^3}{100^3\,\text{cm}^3}\right)\left(1\,\text{cm}^3\right) \\ &= \boxed{1.2 \times 10^{-6}\,\text{kg}} \end{aligned}$$

2. To determine the number of moles:

$$\begin{aligned} & \left(1.2 \times 10^{-6}\,\text{kg}\right)\left(\frac{1\,\text{mol}}{28.97\,\text{g}}\right)\left(\frac{1000\,\text{g}}{1\,\text{kg}}\right) \\ = & \boxed{4.14 \times 10^{-5}\,\text{mol}} \end{aligned}$$

3. Using Avogadro's number

$$\begin{aligned} & \left(4.14 \times 10^{-5}\,\text{mol}\right)\left(\frac{6 \times 10^{23}\ \text{molecules}}{\text{mol}}\right) \\ = & \boxed{2.5 \times 10^{19}\ \text{molecules}} \end{aligned}$$

REVIEW

There are more moles in one cm^{3}of water than one cm^3 of dry air. This makes sense, because the molecules in a liquid are held together by weak inter-molecular bonding, and in gases they are not; see Table 1.1 in Section 1.2 (EFM 10e).

1.27: PROBLEM DEFINITION

Situation:
Start with the molar form of the Ideal Gas Law, and show the steps to prove that the mass form is correct.

SOLUTION

The molar form is:

$$p\forall = nR_uT$$

Where n = number of moles of gas, and the Universal Gas Constant = R_u = $8.314\,\mathrm{J/\,mol\cdot K}$.
Specific gas constants are given by

$$\begin{aligned} R_{\text{specific}} &= R = \frac{R_u}{\text{molar mass of a gas}} \\ &= \left(\frac{8.314\,\mathrm{J}}{\mathrm{mol\cdot K}}\right)\left(\frac{\text{X moles}}{\mathrm{g}}\right) \\ &= 8.314\ \text{X}\ \frac{\mathrm{J}}{\mathrm{g\cdot K}} \end{aligned}$$

Indeed, we see that the units for gas constants, R, in table A.2 (EFM10e), are

$$\boxed{\frac{\mathrm{J}}{\mathrm{g\cdot K}}}$$

So

$$\begin{aligned} p\forall &= (R_{\text{specific}})\,(m)\,(T) \quad \text{and } \rho = \frac{m}{\forall} \\ &\boxed{\mathrm{p} = \rho \mathrm{RT}} \end{aligned}$$

$\boxed{\text{Thus the mass form is correct.}}$

1.28: PROBLEM DEFINITION

Situation:
Start with the universal gas constant and show that $R_{N_2} = 297\frac{\text{J}}{\text{kg}\cdot\text{K}}$.

SOLUTION

Start with universal gas constant:

$$R_u = \frac{8.314\,\text{J}}{\text{mol}\cdot\text{K}}$$

The molar mass of nitrogen, N_2, is 28.02 g/mol.

$$\begin{aligned} R_{N_2} &= \frac{R_u}{\text{molar mass}} = \left(\frac{8.314\,\text{J}}{\text{mol}\cdot\text{K}}\right)\left(\frac{1\,\text{mol}}{28.02\,\text{g}}\right)\left(\frac{1000\,\text{g}}{1\,\text{kg}}\right) \\ &= \boxed{296.7\frac{\text{J}}{\text{kg}\cdot\text{K}}} \end{aligned}$$

1.29: PROBLEM DEFINITION

Situation:
Spherical tank of CO_2, does $p_2 = 4p_1$?

Case 1:
$p = 3$ atm
$T = 20°C$
Volume is constant inside the tank

Case 2:
$p = ?$
$T = 80°C$
Volume for case 2 is equivalent to that in case 1

PLAN

1. Volume inside the tank is constant, as is the mass.
   Mass is related to volume by density.
2. Use the Ideal Gas Law to find P_2

SOLUTION

1. Mass in terms of density

$$\begin{aligned} m &= \rho \cancel{V} \\ \text{For both case 1 and 2, } \rho_1 &= \frac{m}{\cancel{V}} = \rho_2, \text{ because mass is contained by the tank.} \end{aligned}$$

2. Ideal Gas Law for constant volume

$$\begin{aligned} \rho &= \frac{p}{RT} \\ \rho_{1,2} &= \frac{p_1}{RT_1} = \frac{p_2}{RT_2} \\ \frac{p_1}{T_1} &= \frac{p_2}{T_2} \end{aligned}$$

Therefore, if $T_2 = 4T_1$, Then $p_2 = 4p_1$; however, the Ideal Gas Law applies ONLY if the temperature is absolute, which for this system means Kelvin. In the problem statement, the temperatures were given in Centigrade. We need to convert the given temperatures to Kelvin in order to relate them to the pressures. We see that the ratio of temperatures in K is not 1:4. Rather,
$20°C = 293.15\,K$, and
$80°C = 353.15\,K$

Therefore, $\frac{T_2}{T_1} = \frac{353.15\,K}{293.15\,K} = \frac{p_2}{p_1} = 1.2$

$\boxed{\Rightarrow \text{No, } p_2 \text{ does not equal } 4p_1\text{. Instead, } p_2 = 1.2\ p_1}$

REVIEW

Always convert T to Rankine (traditional) or Kelvin (SI) when working with Ideal Gas Law.

1.30: PROBLEM DEFINITION

Situation:

An engineer needs to know the local density for an experiment with a glider.
$z = 2500\,\mathrm{ft}$.
Local temperature $= 74.3\,^\circ\mathrm{F} = 296.7\,\mathrm{K}$.
Local pressure $= 27.3$ in.-Hg $= 92.45\,\mathrm{kPa}$.

Find:

Calculate density of air using local conditions.
Compare calculated density with the value from Table A.2, and make a recommendation.

Properties:

From Table A.2, $R_{\mathrm{air}} = 287\ \frac{\mathrm{J}}{\mathrm{kg\cdot K}} = 287\ \frac{\mathrm{N\cdot m}}{\mathrm{kg\cdot K}}$, $\rho = 1.22\,\mathrm{kg/\,m^3}$.

PLAN

Calculate density by applying the ideal gas law for local conditions.

SOLUTION

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{92,450\,\mathrm{N/\,m^2}}{\left(287\frac{\mathrm{N\cdot m}}{\mathrm{kg\cdot K}}\right)(296.7\,\mathrm{K})} \\ &= 1.086\ \mathrm{kg/m^3}\end{aligned}$$

$$\boxed{\rho = 1.09\ \mathrm{kg/m^3}\ \text{(local conditions)}}$$

Table value. From Table A.2

$$\boxed{\rho = 1.22\ \mathrm{kg/m^3}\ \text{(table value)}}$$

The density difference (local conditions versus table value) is about 12%. Most of this difference is due to the effect of elevation on atmospheric pressure.

$\boxed{\text{Recommendation–use the local value of density because the effects of elevation are significant}}$.

REVIEW

Note: Use absolute pressure when working with the ideal gas law.

1.31: PROBLEM DEFINITION

Situation:
Carbon dioxide.

Find:
Density and specific weight of CO_2.

Properties:
From Table A.2, $R_{CO_2} = 189$ J/kg·K.
$p = 300\,\mathrm{kPa}$, $T = 60\,^\circ\mathrm{C}$.

PLAN

1. First, apply the ideal gas law to find density.
2. Then, calculate specific weight using $\gamma = \rho g$.

SOLUTION

1. Ideal gas law

$$\begin{aligned}
\rho_{CO_2} &= \frac{P}{RT} \\
&= \frac{300,000\,\mathrm{kPa}}{(189\,\mathrm{J/\,kg\,K})\,(60+273)\,\mathrm{K}} \\
&\boxed{\rho_{CO_2} = 4.767\ \mathrm{kg/m^3}}
\end{aligned}$$

2. Specific weight

$$\gamma = \rho g$$

Thus

$$\begin{aligned}
\gamma_{CO_2} &= \rho_{CO_2} \times g \\
&= 4.767\,\mathrm{kg/\,m^3} \times 9.81\,\mathrm{m/\,s^2} \\
&\boxed{\gamma_{CO_2} = 46.764\ \mathrm{N/m^3}}
\end{aligned}$$

REVIEW

Always use absolute pressure when working with the ideal gas law.

1.32: PROBLEM DEFINITION

Situation:
Methane gas.

Find:
Density (kg/m^3).

Properties:
From Table A.2, $R_{\text{Methane}} = 518\ \frac{\text{J}}{\text{kg}\cdot\text{K}}$
$p = 300\,\text{kPa}$, $T = 60\,^\circ\text{C}$.

PLAN

1. Apply the ideal gas law to find density.

SOLUTION

1. Ideal gas law

$$\begin{aligned}\rho_{\text{Methane}} &= \frac{p}{RT} \\ &= \frac{300,000\frac{\text{N}}{\text{m}^2}}{518\frac{\text{J}}{\text{kg}\cdot\text{K}}(60+273\,\text{K})}\end{aligned}$$

$$\boxed{\rho_{\text{Methane}} = 1.74\ \text{kg/m}^3}$$

REVIEW

Always use absolute pressure when working with the ideal gas law.

1.33: PROBLEM DEFINITION

Situation:
Find D for 10 moles of methane gas.
$p = 2\,\text{bar} = 29\frac{\text{lbf}}{\text{in}^2} = 4176\frac{\text{lbf}}{\text{ft}^2}$
$T = 70^\circ\,\text{F} = 529.7^\circ$ R

Properties:
$R_{\text{methane}} = 3098\frac{\text{ft}\cdot\text{lbf}}{\text{slug}\cdot\,^\circ\text{R}}$

PLAN

1. Find volume to get diameter.
2. Moles of methane can be related to mass by molecular weight.
3. Mass and volume are related by density.
4. Ideal Gas Law for constant volume.

$$\rho = \frac{p}{RT}$$

SOLUTION

1.

$$\begin{aligned} \forall_{\text{sphere}} &= \frac{4}{3}\pi r^3 = \frac{1}{6}\pi D^3 \\ \implies D &= \sqrt[3]{\frac{6\forall}{\pi}} \end{aligned}$$

2. Methane, CH_4, has a molecular weight of $\frac{16\,\text{g}}{\text{mol}}$.
Thus, 10 moles of methane weighs 160 g.
3.

$$\rho = \frac{m}{\forall} = \frac{\text{Known}}{\text{Unknown}}$$

4.

$$\rho = \frac{P}{RT} = \frac{\text{Known}}{\text{Known}} = \frac{4176\,\text{lbf}/\,\text{ft}^2}{3098\,\text{ft}\,\text{lbf}/\,\text{slug}^\circ R}$$

5. Solve for density, then go back and solve for volume, yielding $\forall = 4.31\,\text{ft}^3$.
6. Use volume to solve for diameter

$$\boxed{D = 2.02\ \text{ft}}$$

REVIEW

Always convert Temperature to Rankine (traditional) or Kelvin (SI) when working with Ideal Gas Law.

1.34: PROBLEM DEFINITION

Natural gas is stored in a spherical tank.

Find:

Ratio of final mass to initial mass in the tank.

Properties:

$p_{atm} = 100\,\text{kPa}$, $p_1 = 100\,\text{kPa-gage}$.
$p_2 = 200\,\text{kPa-gage}$, $T = 10\,^\circ\text{C}$.

PLAN

Use the ideal gas law to develop a formula for the ratio of final mass to initial mass.

SOLUTION

1. Mass in terms of density

$$M = \rho \forall \tag{1}$$

2. Ideal gas law

$$\rho = \frac{p}{RT} \tag{2}$$

3. Combine Eqs. (1) and (2)

$$\begin{aligned} M &= \rho \forall \\ &= (p/RT)\forall \end{aligned}$$

4. Volume and gas temperature are constant, so

$$\frac{M_2}{M_1} = \frac{p_2}{p_1}$$

and

$$\begin{aligned} \frac{M_2}{M_1} &= \frac{300\text{ kPa}}{200\text{ kPa}} \\ & \boxed{\tfrac{M_2}{M_1} = 1.5} \end{aligned}$$

1.35: PROBLEM DEFINITION

Situation:
Wind and water at 100 °C and 5 atm.

Find:
Ratio of density of water to density of air.

Properties:
Air, Table A.2: $R_{\text{air}} = 287$ J/kg·K.
Water (100°C), Table A.5: $\rho_{\text{water}} = 958$ kg/m^3.

PLAN

Apply the ideal gas law to air.

SOLUTION

Ideal gas law

$$\begin{aligned}\rho_{\text{air}} &= \frac{p}{RT} \\ &= \frac{506,600\,\text{Pa}}{(287\,\text{J/kg K})\,(100+273)\,\text{K}} \\ &= 4.73\ \text{kg/m}^3\end{aligned}$$

For water

$$\rho_{\text{water}} = 958\ \text{kg/m}^3$$

Ratio

$$\frac{\rho_{\text{water}}}{\rho_{\text{air}}} = \frac{958\,\text{kg/m}^3}{4.73\,\text{kg/m}^3}$$

$$\boxed{\frac{\rho_{\text{water}}}{\rho_{\text{air}}} = 203}$$

REVIEW

Always use absolute pressures when working with the ideal gas law.

1.36: PROBLEM DEFINITION

Situation:
Oxygen fills a tank.
$\not{V}_{\text{tank}} = 6\,\text{ft}^3$, $W_{\text{tank}} = 90\,\text{lbf}$.

Find:
Weight (tank plus oxygen).

Properties:
From Table A.2, $R_{O_2} = 1555$ ft·lbf/(slug $\cdot^{o}$ R).
$p = 400$ psia, $T = 70\,°\text{F}$.

PLAN

1. Apply the ideal gas law to find density of oxygen.
2. Find the weight of the oxygen using specific weight (γ) and add this to the weight of the tank.

SOLUTION

1. Ideal gas law

$$\begin{aligned}
p_{\text{abs.}} &= 400 \text{ psia} \times 144 \text{ psf/psi} = 57,600 \text{ psf} \\
T &= 460 + 70 = 530° R \\
\rho &= \frac{p}{RT} \\
&= \frac{57,600 \text{ psf}}{(1555\,\text{ft lbf/ slug}^{o} R)\,(530^{o} R)} \\
\rho &= 0.070 \text{ slugs/ft}^3
\end{aligned}$$

2. Specific weight

$$\begin{aligned}
\gamma &= \rho g \\
&= 0.070 \frac{\text{slug}}{\text{ft}^3} \times 32.2 \frac{\text{ft}}{\text{s}^2} \\
\gamma &= 2.25 \text{ lbf/ft}^3
\end{aligned}$$

3. Weight of filled tank

$$\begin{aligned}
W_{\text{oxygen}} &= 2.25 \text{ lbf/ft}^3 \times 6 \text{ ft}^3 \\
&= 13.50 \text{ lbf} \\
W_{\text{total}} &= W_{\text{oxygen}} + W_{\text{tank}} \\
&= 13.5\,\text{lbf} + 90\,\text{lbf} \\
&\boxed{W_{\text{total}} = 103.5 \text{ lbf}}
\end{aligned}$$

REVIEW

1. For compressed gas in a tank, pressures are often very high and the ideal gas assumption is invalid. For this problem the pressure is about 34 atmospheres–it is a good idea to check a thermodynamics reference to analyze whether or not real gas effects are significant.
2. Always use absolute pressure when working with the ideal gas law.

1.37: PROBLEM DEFINITION

Situation:

Oxygen is released from a tank through a valve.
$\cancel{V} = 4\,\mathrm{m}^3$.

Find:

Mass of oxygen that has been released.

Properties:

$R_{O_2} = 260\frac{\mathrm{J}}{\mathrm{kg\cdot K}}$.
$p_1 = 700\,\mathrm{kPa}$, $T_1 = 20\,^\circ\mathrm{C}$.
$p_2 = 500\,\mathrm{kPa}$, $T_2 = 20\,^\circ\mathrm{C}$.

PLAN

1. Use ideal gas law, expressed in terms of density and the gas-specific (not universal) gas constant.
2. Find the density for the case before the gas is released; and then mass from density, given the tank volume.
3. Find the density for the case after the gas is released, and the corresponding mass.
4. Calculate the mass difference, which is the mass released.

SOLUTION

1. Ideal gas law

$$\rho = \frac{p}{RT}$$

2. Density and mass for case 1

$$\begin{aligned}\rho_1 &= \frac{700,000\frac{\mathrm{N}}{\mathrm{m}^2}}{(260\frac{\mathrm{N\cdot m}}{\mathrm{kg\cdot K}})(293\,\mathrm{K})}\\ \rho_1 &= 9.19\frac{\mathrm{kg}}{\mathrm{m}^3}\end{aligned}$$

$$\begin{aligned}M_1 &= \rho_1\cancel{V}\\ &= 9.19\frac{\mathrm{kg}}{\mathrm{m}^3}\times 4\,\mathrm{m}^3\\ M_1 &= 36.8\ \mathrm{kg}\end{aligned}$$

3. Density and mass for case 2

$$\begin{aligned}\rho_2 &= \frac{500,000\frac{\mathrm{N}}{\mathrm{m}^2}}{(260\frac{\mathrm{N\cdot m}}{\mathrm{kg\cdot K}})(293\,\mathrm{K})}\\ \rho_2 &= 6.56\frac{\mathrm{kg}}{\mathrm{m}^3}\end{aligned}$$

$$
\begin{aligned}
M_2 &= \rho_1 \cancel{V} \\
&= 6.56 \frac{\text{kg}}{\text{m}^3} \times 4\,\text{m}^3 \\
M_2 &= 26.3 \text{ kg}
\end{aligned}
$$

4. Mass released from tank

$$
\begin{aligned}
M_1 - M_2 &= 36.8 - 26.3 \\
& \boxed{M_1 - M_2 = 10.5 \text{ kg}}
\end{aligned}
$$

1.38: PROBLEM DEFINITION

Situation:

Properties of air.

Find:

Specific weight ($\mathrm{N/m^3}$).
Density ($\mathrm{kg/m^3}$).

Properties:

From Table A.2, $R = 287\ \frac{\mathrm{J}}{\mathrm{kg\cdot K}}$.
$p = 600\,\mathrm{kPa}$, $T = 50\,^\circ\mathrm{C}$.

PLAN

First, apply the ideal gas law to find density. Then, calculate specific weight using $\gamma = \rho g$.

SOLUTION

1. Ideal gas law

$$\begin{aligned}\rho_{\text{air}} &= \frac{P}{RT} \\ &= \frac{600,000\,\mathrm{Pa}}{(287\,\mathrm{J/\,kg\,K})\,(50+273)\,\mathrm{K}} \\ &\boxed{\rho_{\text{air}} = 6.47\ \mathrm{kg/m^3}}\end{aligned}$$

2. Specific weight

$$\begin{aligned}\gamma_{\text{air}} &= \rho_{\text{air}} \times g \\ &= 6.47\,\mathrm{kg/\,m^3} \times 9.81\,\mathrm{m/\,s^2} \\ &\boxed{\gamma_{\text{air}} = 63.5\ \mathrm{N/\,m^3}}\end{aligned}$$

REVIEW

Always use absolute pressure when working with the ideal gas law.

1.39: PROBLEM DEFINITION

Situation:
Consider a mass of air in the atmosphere.
$\forall = 1\,\text{mi}^3$.

Find:
Mass of air using units of slugs and kg.

Properties:
From Table A.2, $\rho_{\text{air}} = 0.00237$ slugs/ft^3.

Assumptions:
The density of air is the value at sea level for standard conditions.

SOLUTION

Units of slugs

$$M = \rho \forall$$
$$M = 0.00237 \tfrac{\text{slug}}{\text{ft}^3} \times (5280)^3\,\text{ft}^3$$
$$\boxed{M = 3.49 \times 10^8 \ \ \text{slugs}}$$

Units of kg

$$M = \left(3.49 \times 10^8\,\text{slug}\right) \times \left(14.59 \frac{\text{kg}}{\text{slug}}\right)$$
$$\boxed{M = 5.09 \times 10^9 \ \ \text{kg}}$$

REVIEW

The mass will probably be somewhat less than this because density decreases with altitude.

1.40: PROBLEM DEFINITION

Situation:

For a cyclist, temperature changes affect air density, thereby affecting both aerodynamic drag and tire pressure.

Find:

a.) Plot air density versus temperature for a range of -10°C to 50°C.
b.) Plot tire pressure versus temperature for the same temperature range.

Properties:

From Table A.2, $R_{\text{air}} = 287$ J/kg/K.
Initial conditions for part b: $p = 450\,\text{kPa}$, $T = 20\,°\text{C}$.

Assumptions:

For part b, assume that the bike tire volume does not change.

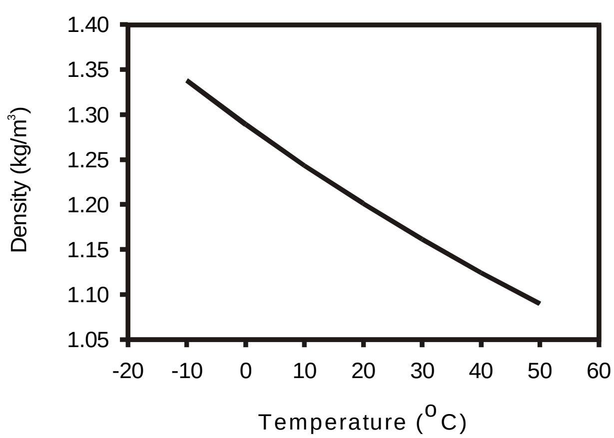
PLAN

Apply the ideal gas law.

SOLUTION

a.) Ideal gas law

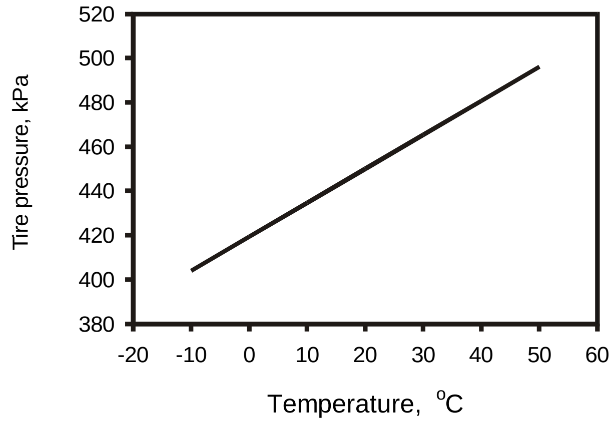
$$\rho = \frac{p}{RT} = \frac{101000\,\text{Pa}}{(287\,\text{J/kg K})\,(273+T)}$$



b.) If the volume is constant, since mass can't change, then density must be constant. Thus

$$\frac{p}{T} = \frac{p_o}{T_o}$$

$$p = 450\,\text{kPa}\left(\frac{T}{20\,°\text{C}}\right)$$



Tire pressure, kPa
520
500
480
460
440
420
400
380
-20
-10
0
10
20
30
40
50
60
Temperature, °C

1.41: PROBLEM DEFINITION

Situation:

Design of a CO_2 cartridge to inflate a rubber raft.

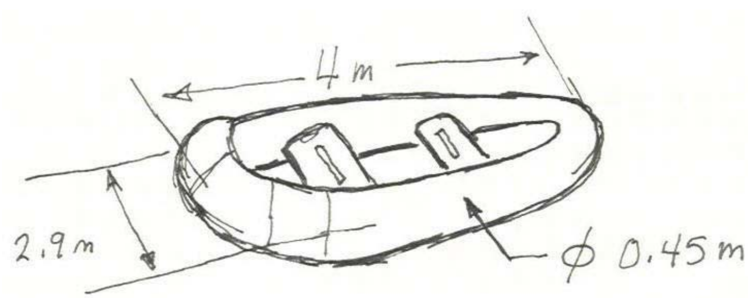
Inflation pressure = 3 psi above p_{atm} = 17.7 psia = 122 kPa abs.

Find:

Estimate the volume of the raft.

Calculate the mass of CO_2 (in grams) to inflate the raft.

Sketch:



Assumptions:

CO_2 in the raft is at 62 °F = 290 K.

Volume of the raft ≈ Volume of a cylinder with $D = 0.45\,\text{m}$ & $L = 16\,\text{m}$ (8 meters for the length of the sides and 8 meters for the lengths of the ends plus center tubes).

Properties:

CO_2, Table A.2, $R = 189$ J/kg·K.

PLAN

Since mass is related to volume by $m = \rho V$, the steps are:
1. Find volume using the formula for a cylinder.
2. Find density using the ideal gas law (IGL).
3. Calculate mass.

SOLUTION

1. Volume

$$\begin{aligned} V &= \frac{\pi D^2}{4} \times L \\ &= \left(\frac{\pi \times 0.45^2}{4} \times 16 \right) \text{m}^3 \\ & \boxed{V = 2.54 \ \text{m}^3} \end{aligned}$$

2. Ideal gas law

$$
\begin{aligned}
\rho &= \frac{p}{RT} \\
&= \frac{122,000\,\mathrm{N/\,m^2}}{(189\,\mathrm{J/\,kg\cdot\;K})\,(290\,\mathrm{K})} \\
&= 2.226\ \mathrm{kg/m^3}
\end{aligned}
$$

3. Mass of CO_2

$$
\begin{aligned}
m &= \rho \forall \\
&= \left(2.226\ \mathrm{kg/m^3}\right)\left(2.54\,\mathrm{m^3}\right) \\
&\boxed{m = 5660\,\mathrm{g}}
\end{aligned}
$$

REVIEW

The final mass (5.66 kg = 12.5 lbm) is large. This would require a large and potentially expensive CO_2 tank. Thus, this design idea may be impractical for a product that is driven by cost.

1.42: PROBLEM DEFINITION

Situation:

A helium filled balloon is being designed.

$r = 1.3\,\text{m}$, $z = 80,000\,\text{ft}$.

Find:

Weight of helium inside balloon.

Properties:

From Table A.2, $R_{He} = 2077$ J/kg·K.

$p = 0.89\,\text{bar} = 89\,\text{kPa}$, $T = 22\,°\text{C} = 295.2\,\text{K}$.

PLAN

Weight is given by $W = mg$. Mass is related to volume by $M = \rho * \forall$. Density can be found using the ideal gas law.

SOLUTION

Volume in a sphere

$$\begin{aligned} \forall &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi\,(1.3\,\text{m})^3 \\ &= 9.203\,\text{m}^3 \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{89,000\,\text{N/m}^2}{(2077\,\text{J/kg}\cdot\text{K})\,(295.2\,\text{K})} \\ &= 0.145\ \text{kg/m}^3 \end{aligned}$$

Weight of helium

$$\begin{aligned} W &= \rho \times \forall \times g \\ &= \left(0.145\ \text{kg/m}^3\right) \times \left(9.203\,\text{m}^3\right) \times \left(9.81\,\text{m/s}^2\right) \\ &= 13.10\,\text{N} \end{aligned}$$

$$\boxed{\text{Weight} = 13.1\ \text{N}}$$

1.43: PROBLEM DEFINITION

*Note: solutions for this problem will vary, but should include the steps indicated in **bold**. The steps below are outlined in detail in Example 1.2 in §1.7 (EFM 10e). With our students, we place particular emphasis on the "Define the Situation" step.*

Problem Statement

Apply the WWM and Grid Method to find the acceleraton for a force of 2 N acting on an object of 7 ounces.

Define the situation *(summarize the physics, check for inconsistent units)*

A force acting on a body is causing it to accelerate.

The physics of this situation are described by Newton's 2nd Law of motion, $F = ma$

The units are inconsistent

State the Goal

a <== the acceleration of the object

Generate Ideas and Make a Plan

1. Apply Grid Method
2. Apply Newton's 2nd Law of motion, $F = ma$.
3. Do calculations, and conversions to SI units.
4. Answer should be in m/s^2

Take Action (Execute the Plan)

$$\begin{aligned} F &= ma \\ \frac{2\,\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2} &= \left(\frac{7\ \mathrm{oz}}{}\right)\left(\frac{1\,\mathrm{lb}}{16\ \mathrm{oz}}\right)\left(\frac{1\,\mathrm{kg}}{2.2\,\mathrm{lb}}\right)\left(\frac{a\,\mathrm{m}}{\mathrm{s}^2}\right) \\ & \boxed{\mathrm{a} = 10.06\ \frac{\mathrm{m}}{\mathrm{s}^2}} \end{aligned}$$

Review the Solution to the Problem

(typical student reflective comment)

This is a straightforword $F = ma$ problem, but in the real world you should always check whether the units are from different systems, and do the appropriate conversions if they are.

1.44: PROBLEM DEFINITION

Situation:
From Example 1.2 in §1.7, state the 3 steps that an engineer takes to "State the Goal".

SOLUTION

1. List the variable(s) to be solved for.
2. List the units on these variables.
3. Describe each variable(s) with a short statement.

1.45: PROBLEM DEFINITION

Situation:

For Problem 1.37 (10e), complete the "Define the Situation", "State the Goal", and "Generate Ideas and Make a Plan" operations of the WWM.

Answers will vary. A representative solution is provided here.

Define the Situation

Oxygen is released from a tank through a valve.

The volume of the tank is $\forall = 4\,\text{m}^3$.

$\text{R}_{O_2} = 260\frac{\text{J}}{\text{kg}\cdot\text{K}}$.

$p_1 = 700\,\text{kPa}$, $T_1 = 20\,°\text{C}$.

$p_2 = 500\,\text{kPa}$, $T_2 = 20\,°\text{C}$.

State the Goal

Find the mass of oxygen that has been released.

Generate Ideas and Make a Plan

Recognize that density, which is $\frac{M}{\forall}$, is related to p and $\forall$ via the ideal gas law.

Specific steps are as follows:

1. Use ideal gas law, expressed in terms of density and the gas-specific (not universal) gas constant.
2. Find the density for the case before the gas is released; and then mass from density, given the tank volume.
3. Find the density for the case after the gas is released, and the corresponding mass.
4. Calculate the mass difference, which is the mass released.

Take Action

1. Ideal gas law

$$\rho = \frac{p}{RT}$$

2. Density and mass for case 1

$$\begin{aligned}
\rho_1 &= \frac{700,000\frac{\text{N}}{\text{m}^2}}{(260\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}})(293\,\text{K})} \\
\rho_1 &= 9.19\frac{\text{kg}}{\text{m}^3}
\end{aligned}$$

$$\begin{aligned}
M_1 &= \rho_1\forall \\
&= 9.19\frac{\text{kg}}{\text{m}^3} \times 4\,\text{m}^3 \\
M_1 &= 36.8\text{ kg}
\end{aligned}$$

3. Density and mass for case 2

$$\begin{aligned}
\rho_2 &= \frac{500,000\frac{\text{N}}{\text{m}^2}}{(260\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}})(293\,\text{K})} \\
\rho_2 &= 6.56\frac{\text{kg}}{\text{m}^3}
\end{aligned}$$

$$
\begin{aligned}
M_2 &= \rho_1 \cancel{V} \\
&= 6.56\frac{\mathrm{kg}}{\mathrm{m}^3} \times 4\,\mathrm{m}^3 \\
M_2 &= 26.3 \text{ kg}
\end{aligned}
$$

4. Mass released from tank

$$
\begin{aligned}
M_1 - M_2 &= 36.8 - 26.25 \\
&\boxed{M_1 - M_2 = 10.5 \text{ kg}}
\end{aligned}
$$

Review the Solution and the Process

(typical student reflections could include...)

The important concept in this problem is that density, which is $\frac{M}{\cancel{V}}$, is related to p and $\cancel{V}$ via the ideal gas law.

Also, always remember that when you use the ideal gas law, you must convert the T to absolute T.

1.46: PROBLEM DEFINITION

Situation:

The hydrostatic equation is

$$\frac{p}{\gamma} + z = C$$

p is pressure, γ is specific weight, z is elevation and C is a constant.

Find:

Prove that the hydrostatic equation is dimensionally homogeneous.

PLAN

Show that each term has the same primary dimensions. Thus, show that the primary dimensions of p/γ equal the primary dimensions of z. Find primary dimensions using Table F.1.

SOLUTION

1. Primary dimensions of p/γ:

$$\left[\frac{p}{\gamma}\right] = \frac{[p]}{[\gamma]} = \left(\frac{M}{LT^2}\right)\left(\frac{L^2T^2}{M}\right) = L$$

2. Primary dimensions of z :

$$[z] = L$$

3. Dimensional homogeneity. Since the primary dimensions of each term is length, the equation is dimensionally homogeneous. Note that the constant C in the equation will also have the same primary dimension.

1.47: PROBLEM DEFINITION

Situation:
Four terms are given in the problem statement.

Find: Primary dimensions of each term.
a) $\rho V^2/\sigma$ (kinetic pressure).
b) T (torque).
c) P (power).
d) $\rho V^2 L/\sigma$ (Weber number).

SOLUTION

a. Kinetic pressure:

$$\left[\frac{\rho V^2}{2}\right] = [\rho]\,[V]^2 = \left(\frac{M}{L^3}\right)\left(\frac{L}{T}\right)^2 = \frac{M}{L \cdot T^2}$$

b. Torque.

$$[\text{Torque}] = [\text{Force}]\,[\text{Distance}] = \left(\frac{ML}{T^2}\right)(L) = \frac{M \cdot L^2}{T^2}$$

c. Power (from Table F.1).

$$[P] = \frac{M \cdot L^2}{T^3}$$

d. Weber Number:

$$\left[\frac{\rho V^2 L}{\sigma}\right] = \frac{[\rho]\,[V]^2\,[L]}{[\sigma]} = \frac{(M/L^3)\,(L/T)^2\,(L)}{(M/T^2)} = []$$

Thus, this is a dimensionless group

1.48: PROBLEM DEFINITION

Situation:

The power provided by a centrifugal pump is given by:

$$P = \dot{m}gh$$

Find:

Prove that the above equation is dimensionally homogenous.

PLAN

1. Look up primary dimensions of P and $\dot{m}$ using Table F.1.
2. Show that the primary dimensions of P are the same as the primary dimensions of $\dot{m}gh$.

SOLUTION

1. Primary dimensions:

$$\begin{aligned} [P] &= \frac{M \cdot L^2}{T^3} \\ [\dot{m}] &= \frac{M}{T} \\ [g] &= \frac{L}{T^2} \\ [h] &= L \end{aligned}$$

2. Primary dimensions of $\dot{m}gh$:

$$[\dot{m}gh] = [\dot{m}]\,[g]\,[h] = \left(\frac{M}{T}\right)\left(\frac{L}{T^2}\right)(L) = \frac{M \cdot L^2}{T^3}$$

Since $[\dot{m}gh] = [P]$, $\boxed{\text{The power equation is dimensionally homogenous.}}$

1.49: PROBLEM DEFINITION

Situation:

Two terms are specified.

a. $\int \rho V^2 dA$.

b. $\frac{d}{dt}\int_{\forall} \rho V d\forall$.

Find:

Primary dimensions for each term.

PLAN

1. To find primary dimensions for term a, use the idea that an integral is defined using a sum.
2. To find primary dimensions for term b, use the idea that a derivative is defined using a ratio.

SOLUTION

Term a:

$$\left[\int \rho V^2 dA\right] = [\rho]\left[V^2\right][A] = \left(\frac{M}{L^3}\right)\left(\frac{L}{T}\right)^2 \left(L^2\right) = \boxed{\frac{ML}{T^2}}$$

Term b:

$$\left[\frac{d}{dt}\int_{\forall} \rho V d\forall\right] = \frac{\left[\int \rho V d\forall\right]}{[t]} = \frac{[\rho]\,[V]\,[\forall]}{[t]} = \frac{\left(\frac{M}{L^3}\right)\left(\frac{L}{T}\right)\left(L^3\right)}{T} = \boxed{\frac{ML}{T^2}}$$