Problem 1P

step 1 of 3

The given differential equation is.

$$y' = 3 - 2y$$

Or

$$\frac{dy}{dt} = 3 - 2y$$

In order to draw the direction field for given differential equation we will find the value of $\frac{dy}{dt}$ at different points i.e. for the different values of y.

Y	0	1	3/2	2	3
$\frac{dy}{dt}$	3	1	0	-1	-3

For the equilibrium position $\frac{dy}{dt} = 0$

i.e.

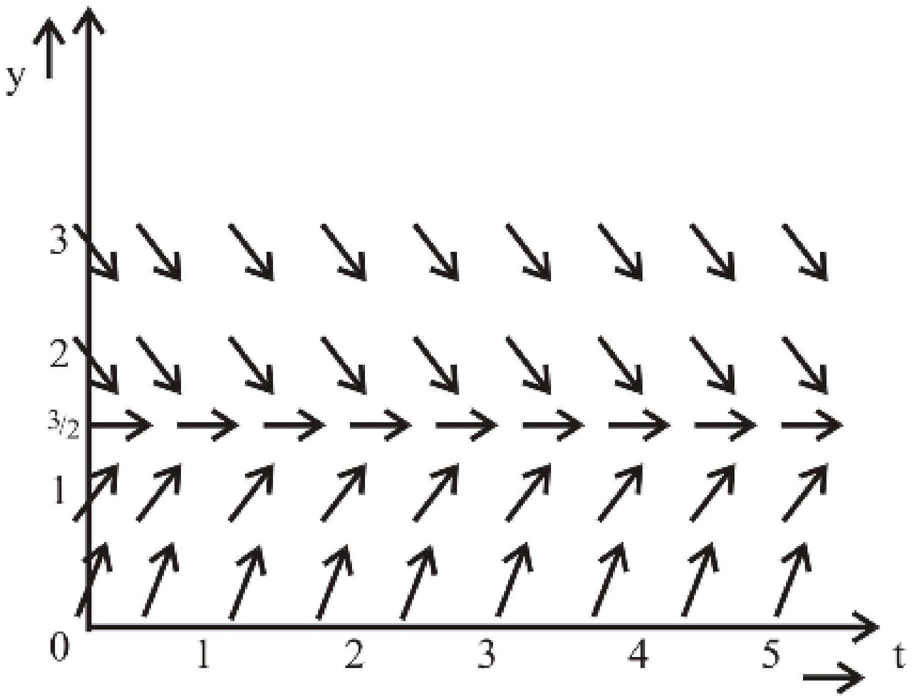
$$3 - 2y = 0$$

i.e.

$$y = \frac{3}{2}$$

step 2 of 3

The direction field is: -



step 3 of 3

In the direction field each line segment is the tangent to the graph of the solution passing through the point.

Now it is clear from the direction field that as $t \to \infty,\ y \to \frac{3}{2}$

Problem 2P

step 1 of 1

The given differential equation is:

$$y' = 2y - 3$$

I.e.

$$\frac{dy}{dt} = 2y - 3$$

In order to draw the direction field for given differential equation, we will find the value of $\frac{dy}{dt}$ for different values of y.

---

y	0	1	3/2	2	3
$\frac{dy}{dt}$	-3	-1	0	1	3

For the equilibrium position $\frac{dy}{dt} = 0$

i.e.

$$2y - 3 = 0$$

i.e.

$$y = \frac{3}{2}$$

Problem 3P

step 1 of 1

The given differential equation is:

$$y' = 3+2y$$

i.e.

$$\frac{dy}{dt} = 3+2y$$

In order to draw the direction field for given differential equation, we will find the value of $\frac{dy}{dt}$ for different values of y.

---

y	0	1	-1	-3/2	-2	-3	-4
$\frac{dy}{dt}$	3	5	1	0	-1	-3	-5

For the equilibrium position $\frac{dy}{dt} = 0$

i.e.

$$3+2y = 0$$

i.e.

$$y = \frac{-3}{2}$$

The direction field is: -

Problem 4P

step 1 of 1

The given differential equation is:

$$y' = -1 - 2y$$

i.e.

$$\frac{dy}{dt} = -1 - 2y$$

In order to draw the direction field for given differential equation, we will find the value of $\frac{dy}{dt}$ for different values of y.

Y	0	1	2	-1/2	-1	-2
$\frac{dy}{dt}$	-1	-3	-5	0	1	3

For the equilibrium position $\frac{dy}{dt} = 0$

i.e.

$$-1 - 2y = 0$$

i.e.

$$y = -1/2$$

The direction field is: -

Problem 5P

step 1 of 1

The given differential equation is:

$$y' = 1 + 2y$$

i.e.

$$\frac{dy}{dt} = 1 + 2y$$

In order to draw the direction field for given differential equation, we will find the value of $\frac{dy}{dt}$ for different values of y.

---

y	1	0	-1/2	-1	-2
$\frac{dy}{dt}$	3	1	0	-1	-3

For the equilibrium position $\frac{dy}{dt} = 0$

i.e.

$$1 + 2y = 0$$

i.e.

$$y = \frac{-1}{2}$$

The direction field is: -

Problem 6P

step 1 of 1

The given differential equation is:

$$y' = y + 2$$

i.e.

$$\frac{dy}{dt} = y + 2$$

In order to draw the direction field for given differential equation, we will find the value of $\frac{dy}{dt}$ for different values of y.

Problem 7P

step 1 of 1

We need to construct a differential equation of the form $\frac{dy}{dt} = ay + b$.

For which all the solution approach y = 3 as t approaches ∞.

Since all the solutions approach y = 3, therefore the equilibrium solution is y = 3.

When all the solutions approach a particular value, we know that the solutions below the equilibrium solution increase with time and those above it decrease with time and all other solutions approach the equilibrium solution as t becomes large.

Problem 8P

step 1 of 1

We need to construct a differential equation of the form $\frac{dy}{dt} = ay + b$

For which all the solution approach y = 2/3 as t approaches ∞

Since all the solutions approach y = 2/3, therefore the equilibrium solution is y = 2/3.

Now we know when all the solutions approach the equilibrium solution, the solutions below the equilibrium solution increase with time and those above it decrease with time and all other solutions approach the equilibrium solution as t becomes large.

Problem 9P

step 1 of 1

We need to construct a differential equation of the form $\frac{dy}{dt} = ay + b$

For which all other solution diverge from y = 2 as t approaches ∞

Since all the solutions diverge y = 2, therefore the equilibrium solution is y = 2.

*Now we know when all the solutions diverge from the equilibrium solution, the solutions above the equilibrium solution increase with time and those below it decrease with time.

Problem 10P

Step-by-step solution

step 1 of 1

Consider the statement: “All other solutions diverge from $y = \frac{1}{3}$”.

Construct a differential equation of the form, $\frac{dy}{dt} = ay + b$, for which all other solutions diverge from $y = \frac{1}{3}$ as t approaches to ∞

All other solutions diverge from $y = \frac{1}{3}$, means that the equilibrium solution is $y = \frac{1}{3}$.

When the other solutions diverge from equilibrium solution, the solutions above the equilibrium solution increase and the solutions below decrease with time

That means, when $\frac{dy}{dt} < 0$, the solution curves decrease with time.

For this particular sum,

$$3y - 1 < 0$$
$$3y < 1$$
$$y < \frac{1}{3}$$

And, when $\frac{dy}{dt} > 0$, the solution curves increase with time.

$$3y - 1 > 0$$
$$3y > 1$$
$$y > \frac{1}{3}$$

So, at $y = \frac{1}{3}$, the equilibrium solution separates the solution curves which are increasing and decreasing.

Problem 11P

step 1 of 1

The given differential equation is:

$$y' = y(4 - y)$$

i.e. $\frac{dy}{dt} = y(4 - y)$

To find equilibrium solution(s),

Put $\frac{dy}{dt} = 0$

i.e. $y(4 - y) = 0$

i.e. $y = 0, 4$

Thus $y = 0, 4$ are the equilibrium solutions.

Problem 12P

222-1.1-12P

The given differential equation is

$$y' = -y(5-y)$$

i.e. $\frac{dy}{dt} = -y(5-y)$

To find equilibrium solution(s)

Put $\frac{dy}{dt} = 0$

i.e. $-y(5-y) = 0$

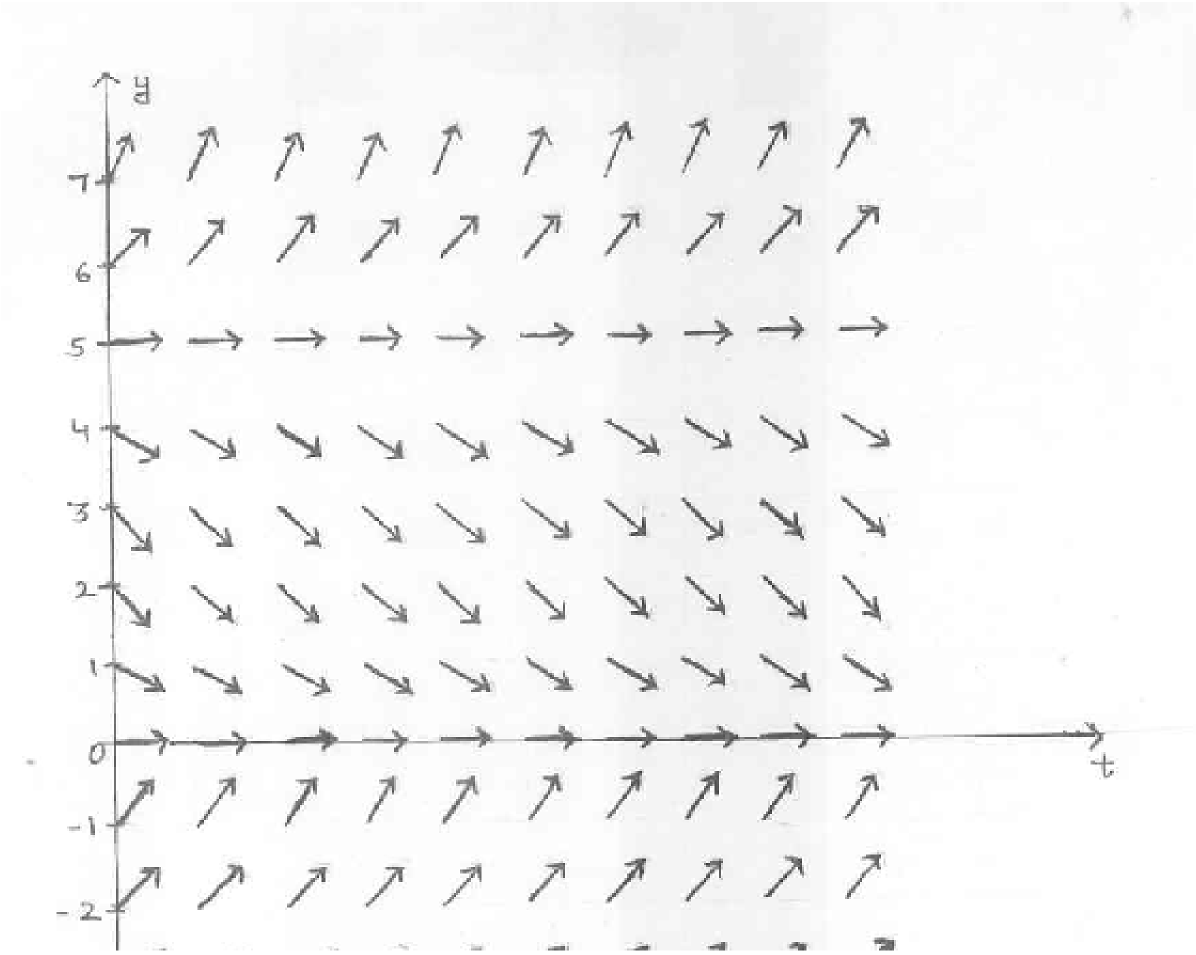
i.e. $y = 0, 5$

Thus the equilibrium solutions are $y = 0, 5$

In order to draw direction field for the given differential equation, we will find the value of $\frac{dy}{dt}$ for different values of y.

y	-2	-1	0	1	2	3	4	5	6	7
$\frac{dy}{dt}$	14	6	0	-4	-6	-6	-4	0	6	14

The direction field is: -



Now clearly from the direction field as t tends to infinity, y diverges from 5 if initial value is greater than 5 and y approaches to zero if initial value is less than 5

Problem 13P

step 1 of 1

The given differential equation is

$$y' = y^2$$

i.e.

$$\frac{dy}{dt} = y^2$$

To find equilibrium position (s)
Put

$$\frac{dy}{dt} = 0$$

i.e.

$$y^2 = 0$$

i.e.

$$y = 0, 0$$

Problem 14P

step 1 of 1

The given differential equation is

$$y' = y(y-2)^2$$

i.e. $\frac{dy}{dt} = y(y-2)^2$

To find equilibrium position (s)

Put $\frac{dy}{dt} = 0$

i.e. $y(y-2)^2 = 0$

i.e.

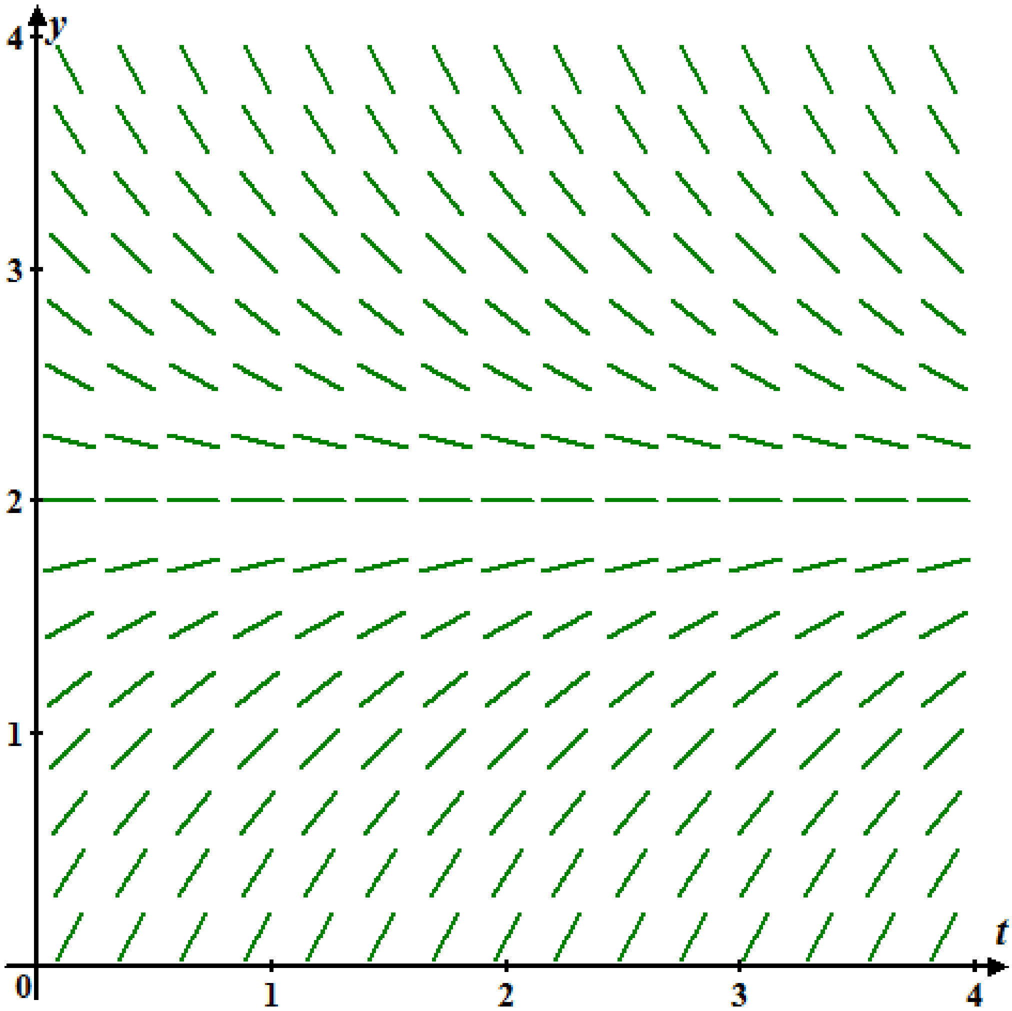
$$y = 0, 2, 2$$

Problem 15P

Step-by-step solution

step 1 of 1

Consider the following direction field:



Problem 16P

step 1 of 1

The direction field of fig 1.1.6 indicates that it has an equilibrium solution for $y = 2$ and y diverges to 2 as $t \to \infty$. Therefore the differential equation $(c)\, y' = y - 2$ corresponds to the given direction field.

Problem 17P

step 1 of 1

The direction field of fig 1.1.7 indicates that it has an equilibrium solution for $y = -2$ and y converges to -2 as $t \to \infty$. Therefore the differential equation $(g)\, y' = -2 - y$ corresponds to the given direction field.

Problem 18P

step 1 of 1

The direction field of fig 1.1.8 indicates that it has an equilibrium solution for y = -2 and y diverges from -2 as $t \to \infty$. Therefore the differential equation $(b)\, y' = 2 + y$ corresponds to the given direction field.

Problem 19P

step 1 of 1

The direction field of fig 1.1.9 indicates that it has two equilibrium solutions for y = 0 and y = 3. Also it indicates that $y \to 3$ if initial value is positive and y diverges from zero if initial value is negative. Therefore the differential equation $(h)\, y' = y(3 - y)$ corresponds to the given direction field.

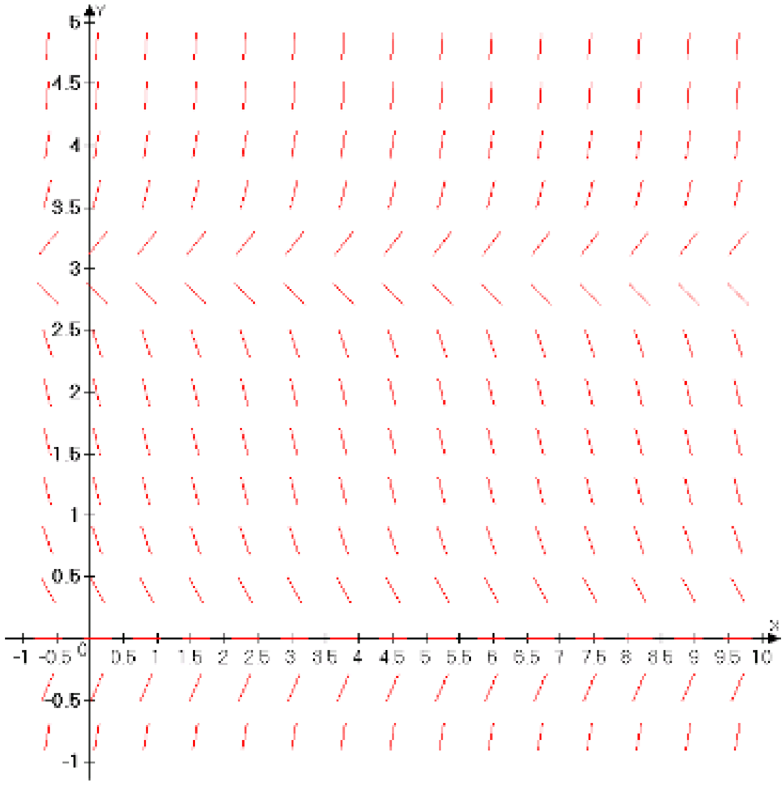
Problem 20P

step 1 of 2

The direction field of fig 1.1.10 indicates that it has two equilibrium solutions for y = 0 and y = 3. Also it indicates that y approaches zero if initial value is less than 3 and diverges from 3 if initial value is greater than 3. Therefore the differential equation $(e)\, y' = y(y - 3)$ corresponds to the given direction field.

step 2 of 2

Figure 1.1.10 - (e)



Problem 21P

step 1 of 1

Assume that the amount of chemical is donated by q. Then the rate of change of chemical in the pond is $\frac{dq}{dt}$, which is equal to the rate of chemical flowing inside the pond minus the rate of chemical flowing out of the pond i.e. the rate of change of chemical

$$\frac{dq}{dt} = \text{Rate in} - \text{rate out.}$$

The rate which the chemical flowing in is 300×0.01 g/hr
And the rate which chemical leaves the pond is $(300/1000000) \times q$ g/hr.

Problem 22P

Step-by-step solution

step 1 of 1

Consider the data: A spherical raindrop evaporates at a rate proportional to its surface area.

Here write a differential equation for the volume of the raindrop as a function of time.

Assume that V denotes the volume of the spherical raindrop and r be its radius. Then the surface area of the raindrop is given by $4\pi r^2$.

And its volume is $V=\frac{4}{3}\pi r^3$

$$r^3=\frac{3}{4\pi}V$$

$$r=\left(\frac{3}{4\pi}\right)^{\frac{1}{3}}V^{\frac{1}{3}}$$

Problem 23P

step 1 of 1

Assume that the temperature of the body denoted by u.

Then the rate of change of temperature is given by $\frac{du}{dt}$

It is given that by Newton's law of cooling the rate of change of temperature is proportional to the difference between temperature of body and temperature of its surroundings. Let it be T i.e.

$$\frac{du}{dt}\alpha\ u-T$$

Or $$\frac{du}{dt}=-k(u-T)$$

where k is the constant of proportionality, and it is taken negative because the body is cooling down and so its temperature is decreasing

Problem 24P

step 1 of 1

Assume that q is the amount of drug present in the bloodstream at any time.

Then rate of change of drug in the body is $\frac{dq}{dt}$

Now $\frac{dq}{dt}$ = amount of drug entering in body – amount of drug leaving (which is not absorbed).

Now amount of drug entering body is

$$5mg/cm^3\times 100cm^3/hr$$

Amount of drug leaving = 0.4 × q mg/hr

Problem 25P

Step-by-step solution

step 1 of 1

Assume that v is the velocity of the mass m, which is slowly falling.

Then according to Newton' s second law, the force exerted on the body is the product mass and acceleration.

That means

$$F = ma$$
$$= m\frac{dv}{dt} \quad \left(\text{Since acceleration, } a = \frac{dv}{dt}\right)$$

This force is equal to the sum of weight of the body $(w = mg)$ and the drag force which is due to air resistance. But the drag force always acts in the opposite direction of the motion, so it is taken negative. Now it is given that the drag force is proportional to square of velocity. Then drag force is $-kv^2$, where k a constant of proportionality called drag coefficient is.

Therefore, the force is sum of weight of the body and the drag force.

That is $m\frac{dv}{dt} = mg - kv^2$, here mg is the weight of the body

Problem 26P

Step-by-step solution

step 1 of 1

Consider the differential equation.

$$y' = -2 + t - y$$

Rewrite the given differential equation as follows:

$$\frac{dy}{dx} = -2 + t - y$$

To sketch the direction field of the given differential equation, take some coordinate points (t, y) on the ty-plane to draw some tangent line in the direction field.

Problem 27P

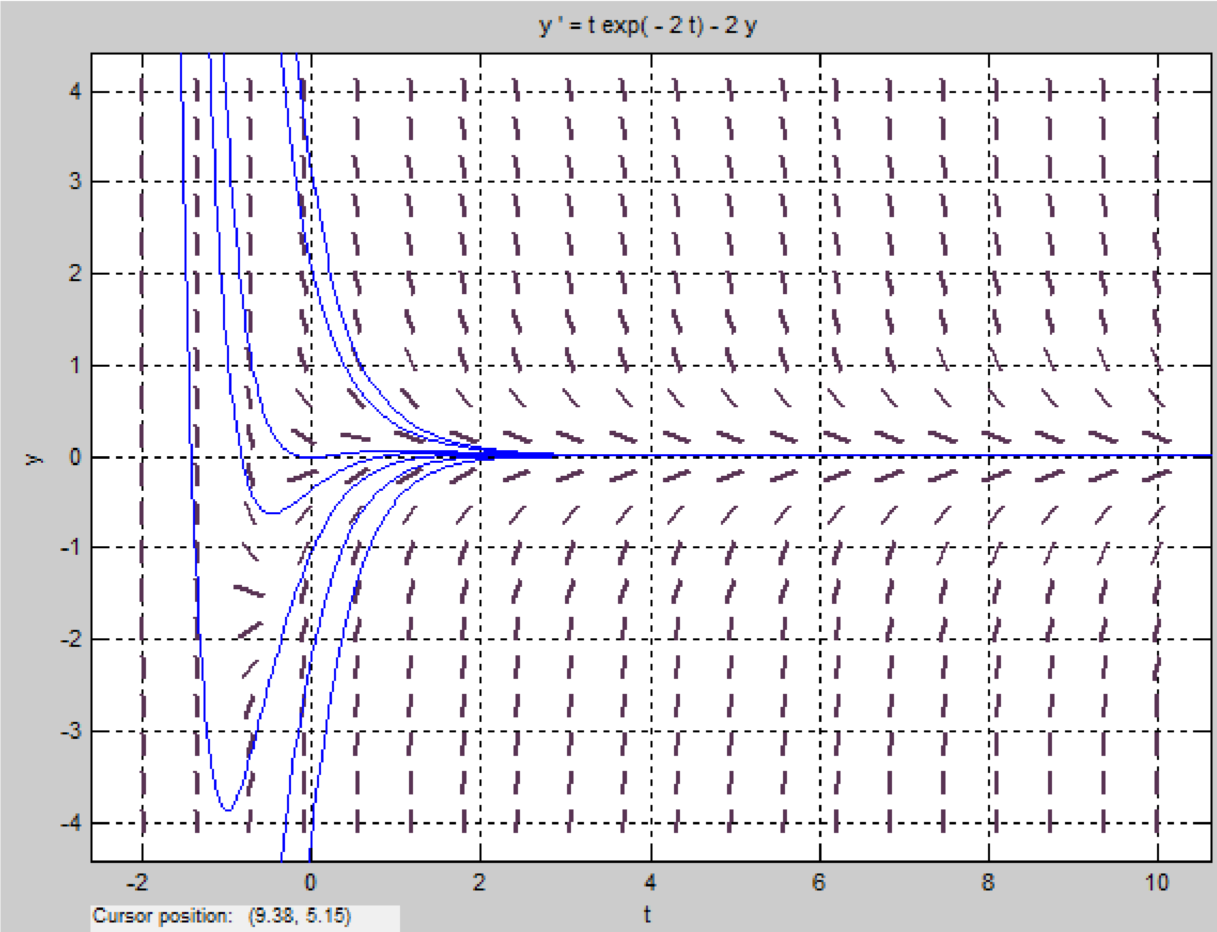
Step-by-step solution

step 1 of 1

Consider the differential equation

$$\frac{dy}{dt} = te^{-2t} - 2y$$

Sketch the differential equation using mat lab.



From the graph, the curves represent the solutions of this differential equation for different values to the arbitrary constants in the solution. From the direction field it is observed that as $t \to \infty$ the solutions $y(t)$ of the differential equation tends to zero.

From the graph at $t = 0$ the initial value of y can take any value so the behavior of y does not depend on this initial value of t.

Problem 28P

Step-by-step solution

step 1 of 1

Consider the following differential equation:

$$y' = e^{-t} + y$$

The objective is to draw a direction field for the given differential equation and determine the behavior of y as $t \to \infty$.

Rewrite the given differential equation as follows:

$$\frac{dy}{dt} = e^{-t} + y$$

To sketch the direction field of the given differential equation, take some coordinate points (t, y) on the ty-plane to draw some tangent line in the direction field.

Problem 29P

Step-by-step solution

step 1 of 1

Consider the differential equation,

$$y' = t + 2y.$$

The object is to sketch the direction field and determine the behaviour of y as $t \to \infty$.

Use maple software commands to sketch the direction field of the differential equation.

Maple Input Commands: with(DEtools);

Maple output:

> *with(DEtools)*;

Maple input command: ode:=diff(y(t),t=t+2*y(t));

Maple output:

> $ode := diff(y(t), t) = t + 2*y$

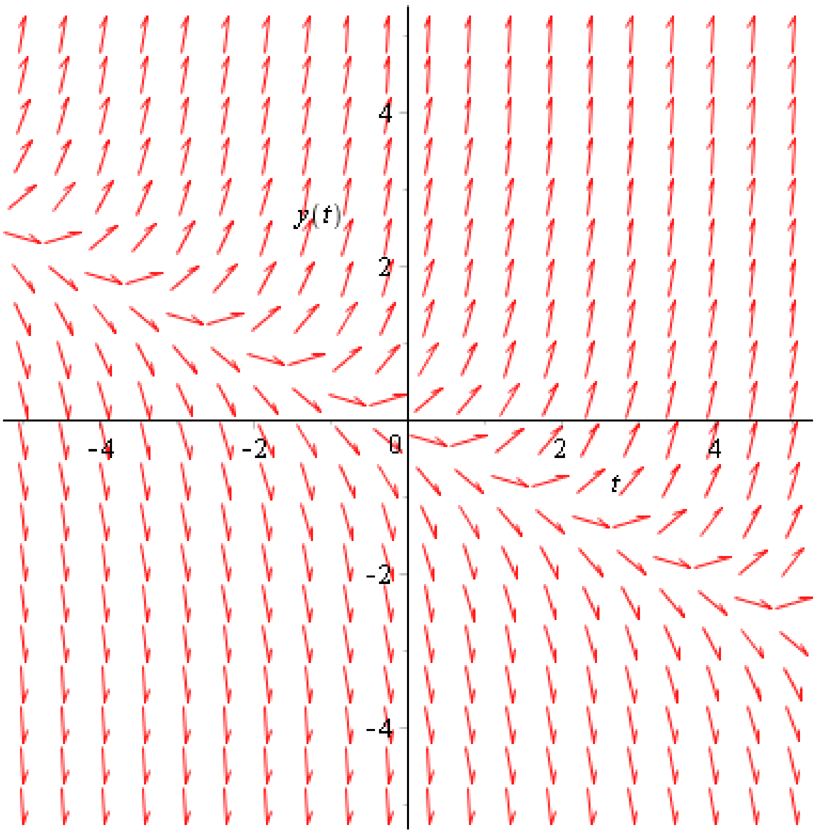
$$\frac{d}{dt} y(t) = t + 2y$$

Maple input command: DEplot(ode,y(t),t=-5..5,y=-5..5)

Maple output:

> $DEplot(ode, y(t), t = -5..5, y = -5..5)$

The direction filed plot of the differential equation is shown below:



Problem 30P

Step-by-step solution

step 1 of 1

Consider the following differential equation:

$$y' = 3\sin t + 1 + y$$

Rewrite the given differential equation as follows:

$$\frac{dy}{dt} = 3\sin t + 1 + y$$

Sketch the direction field of the given differential equation; take some coordinate points (t, y) on the ty-plane to draw some tangent line in the direction field.

Problem 31P

Step-by-step solution

step 1 of 1

Consider the differential equation.

$$y' = 2t - 1 - y^2$$

Rewrite the given differential equation as follows:

$$\frac{dy}{dx} = 2t - 1 - y^2$$

To sketch the direction field of the given differential equation, take some coordinate points (t, y) on the ty-plane to draw some tangent line in the direction field.

Problem 32P

Step-by-step solution

step 1 of 1

10923-1.1-32P RID: 1411| 14/01/2016

Consider the differential equation,

$$y' = \frac{-(2t + y)}{2y}.$$

Need to draw a direction field by using maple software.

Use maple software to draw the direction field as follows:

Maple input command: with(DEtools)

Maple output:

> *with(DEtools)*

Maple input command: ode:=diff(y(t),t)=-(2*t+y)/(2*y)

Maple output:

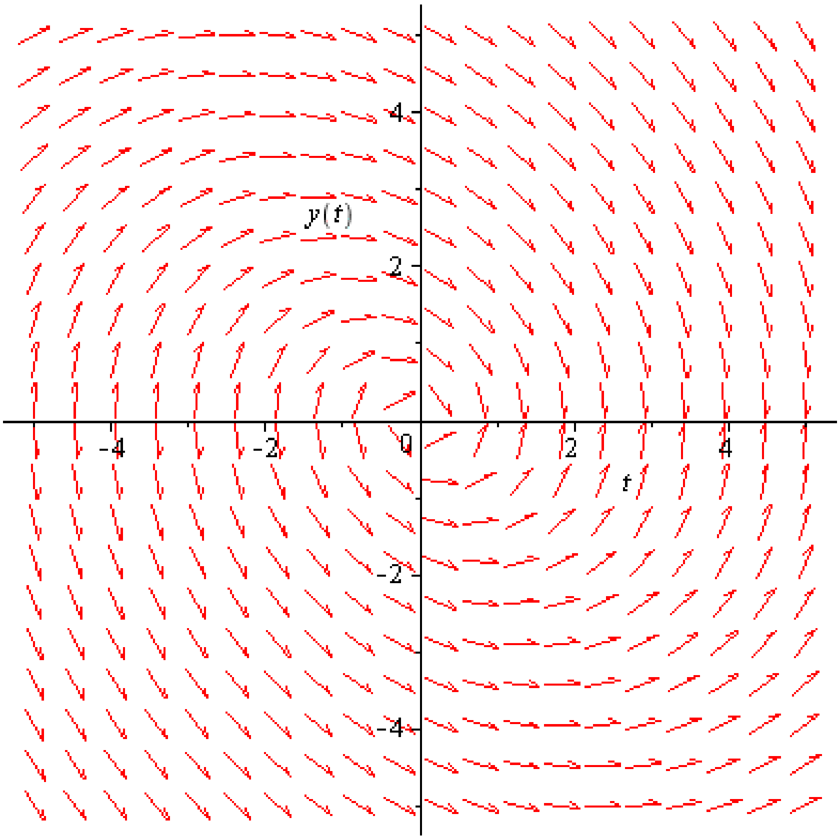
> *ode := diff(y(t), t) =-(2*t + y)/(2*y)*

$$ode := \frac{d}{dt} y(t) = -\frac{1}{2} \frac{2t + y}{y}$$

Maple input command: DEplot(ode,y(t),t=-5..5,y=-5..5);

Maple output:

> *DEplot(ode, y(t), t =-5 ..5, y =-5 ..5);*



Use this direction field to analyse how the solution $y(t)$ of the DE $y' = \frac{-(2t+y)}{2y}$ will behave as $t \to \infty$, depending on the initial condition $y(t_0) = y_0$. If $t_0 \geq 0$, then as $t \to \infty$, the solution $y(t) \to 0$. If $t_0 = 0$, then $y(t) \to 0$, and if $t_0 < 0$, then $y(t) \to 0$.

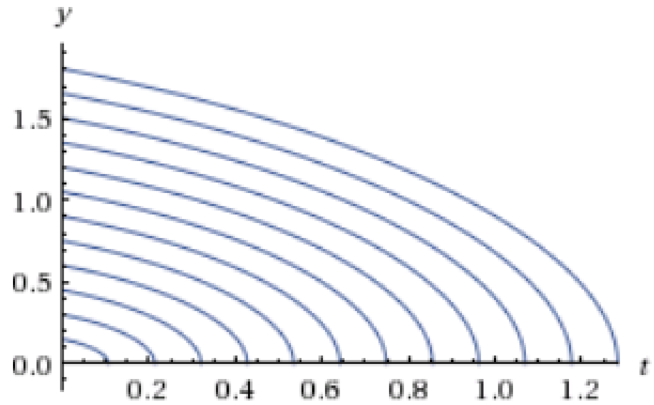
As $t \to \infty$ then the behaviour of $y(t)$ is tends to zero.

Vectors in the direction field point upward if $\frac{-(2t+y)}{2y} > 0$, which occurs in two cases when $2t + y < 0$ and $y > 0$ or when $2t + y > 0$ and $y < 0$.

Since the vectors point downward if $\frac{-(2t+y)}{2y}<0$, which occurs when both $2t+y>0$ and $y>0$.

The solution curves are looks like a spiral centred at the origin.

The solution curve of the differential equation is shown below:



Problem 33P

Step-by-step solution

step 1 of 1

Consider the differential equation:

$$y'=\frac{y^3}{6}-y-\frac{t^2}{3}.$$

Use MAPLE commands to sketch the direction field of the differential equation.

Maple Input Commands:

```
> with(DEtools):
> ode:=diff(y(t),t) = (y^3/6)-y-(t^2/3);
>DEplot(ode,y(t),t=-3..6,y=-5..5);
```

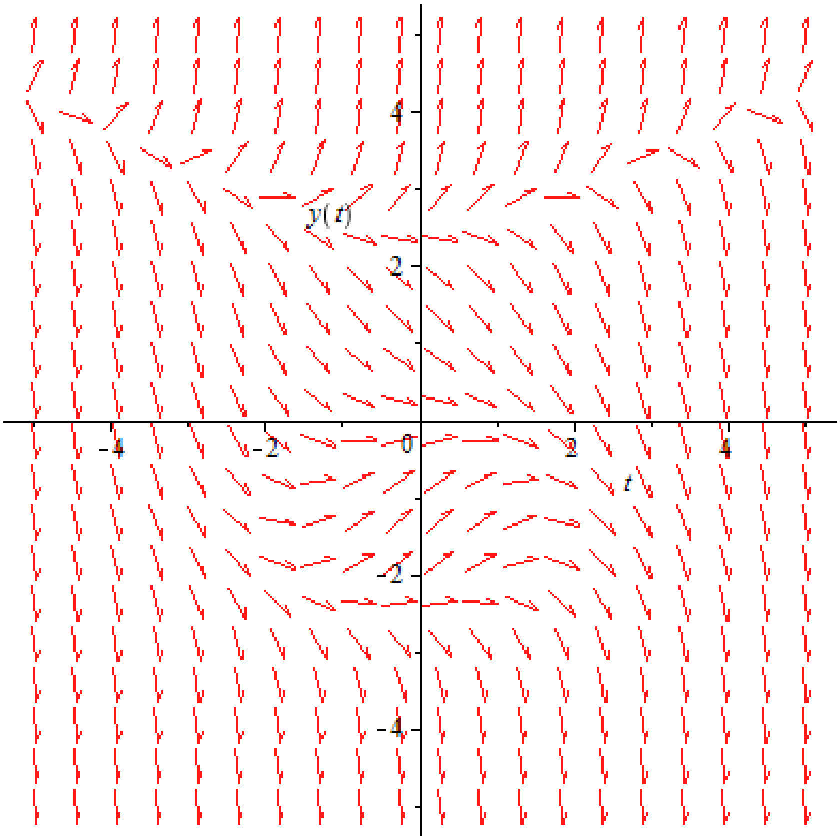
Maple output:

> $with(DEtools):$

> ode := diff(y(t),t) = (y^3/6)-y-(t^2/3);

$$ode := \frac{d}{dt}y(t)=\frac{1}{6}y^3-y-\frac{1}{3}t^2$$

> $DEplot(ode, y(t), t=-5..5, y=-5..5);$


y(t)
t
4
2
-2
-4
-4
-2
0
2
4