CHAPTER TWO

PROB #2.1

$I_g = \frac{1}{12}bh^3 = \left(\frac{1}{12}\right)(12)(21)^3 = 9261 \text{ in.}^4$

f_r = modulus of rupture = $7.5\sqrt{f'_c} = 7.5\sqrt{4000}$

$= 474$ psi

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(474)(9261)}{10.5} = 418{,}068 \text{ in. lbs}$$

$= \boxed{34.8 \text{ ft-k}}$

rgcm

PROB #2.2

$I_g = \left(\frac{1}{12}\right)(14)(21)^3 = 10{,}805 \text{ in.}^4$

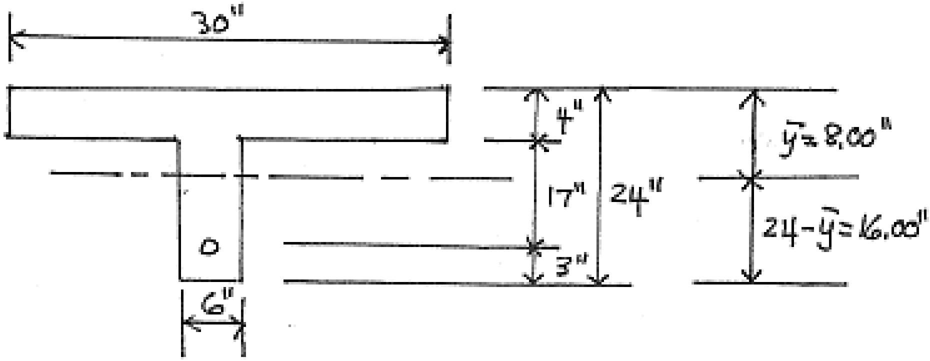
$f_r = 7.5\sqrt{4000} = 474$ psi

$$M_{cr} = \frac{(474)(10805)}{10.5} = 487{,}746 \text{ in. lbs}$$

$= \boxed{40.7 \text{ ft-k}}$

rgcm

PROB # 2.3



$A = (30)(4) + (6)(20) = 240 \text{ in.}^2$

$\bar{y} \text{ from top} = \dfrac{(30)(4)(2) + (6)(20)(14)}{240} = 8.00 \text{ in.}$

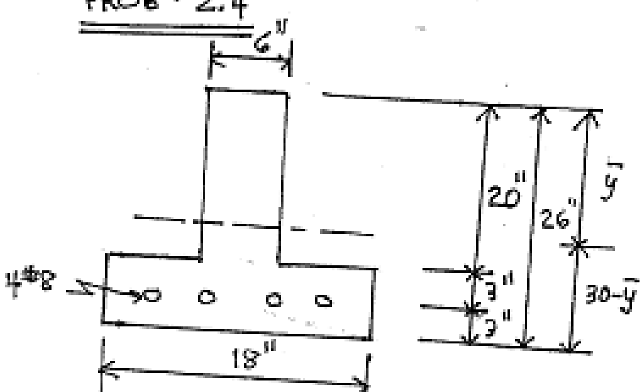
$I_g = \left(\frac{1}{3}\right)(30)(8^3 + 16^3) - \left(\frac{1}{3}\right)(24)(4^3 + 16^3) = 12{,}800 \text{ in.}^3$

$f_r = 7.5\sqrt{4000} = 474 \text{ psi}$

$M_{cr} = \dfrac{(474)(12{,}800)}{16.00} = 379{,}200 \text{ in. lbs.} = \boxed{31.6 \text{ ft-k}}$

✓gcm

PROB # 2.4



$A = (6)(20) + (18)(6)$

$= 228 \text{ in.}^2$

$\bar{y} = \dfrac{6(20)10 + 18(23)6}{228}$

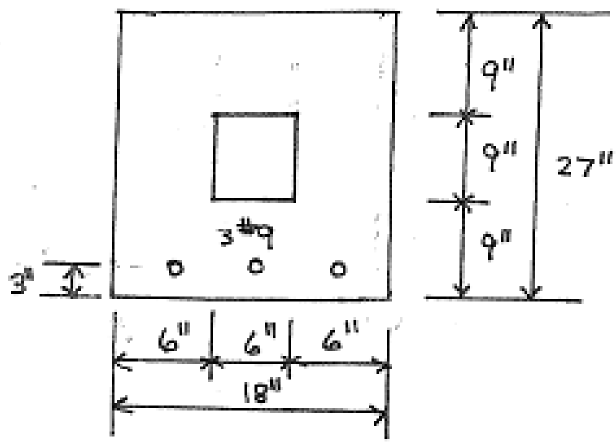
$= 16.16 \text{ in.}$

$I_g = \left(\frac{1}{3}\right)(6)(16.16)^3 + \left(\frac{1}{3}\right)(18)(9.84)^3 - \left(\frac{1}{3}\right)(12)(3.84)^3 = 13{,}930 \text{ in}^4$

$f_r = 7.5\sqrt{4000} = 474 \text{ psi}$

$M_{cr} = \dfrac{(474)(13{,}930)}{9.84} = 671{,}034 \text{ in.-lbs} = \boxed{55.9 \text{ Ft-k}}$ ✓gcm

PROB #2.5



$$I_g = \left(\frac{1}{12}\right)(18)(27)^3 - \left(\frac{1}{12}\right)(6)(9)^3 = 29{,}160 \text{ in.}^4$$

$$f_r = 7.5\sqrt{4000} = 474 \text{ psi}$$

$$M_{cr} = \frac{(474)(29{,}160)}{13.50} = 1{,}023{,}840 \text{ in.-lbs.} = \boxed{85.3 \text{ ft-k}}$$

✓ gcm^c

PROB #2.6

$$I_g = \left(\frac{1}{12}\right)(14)(24)^3 = 16{,}128 \text{ in.}^4$$

$$f_r = 7.5\sqrt{4000} = 474 \text{ psi}$$

$$M_{cr} = \frac{(474)(16{,}128)}{12} = 637{,}056 \text{ in.-lbs}$$

$$= 53.1 \text{ ft-k}$$

$$\frac{w\ell^2}{8} = 53.1$$

$$w_{Total} = \frac{(8)(53.1)}{(28)^2} = 0.542 \text{ k/ft}$$

$$-\text{Beam wt} = -\frac{(16)(24)}{144}(0.150) = -0.400$$

$$w \text{ not including beam wt} = \boxed{0.142 \text{ k/ft}}$$

✓ gcm^c

PROB #2.7

$I_g = \left(\frac{1}{12}\right)(12)(30)^3 - \left(\frac{1}{12}\right)(8)(22)^3 = 19{,}901 \text{ in.}^4$

$f_r = 7.5\sqrt{4000} = 474 \text{ psi}$

$M_{cr} = \frac{(474)(19{,}901)}{15.00} = 628{,}872 \text{ in-lbs} = 52.41 \text{ ft-k}$

$\frac{w_T \ell^2}{8} = 52.41$

$w_T = \frac{(8)(52.41)}{(28)^2} = 0.535 \text{ k/ft}$

$-\text{Bm wt} = \frac{(30)(12)-(8)(22)}{144}(0.150) = -0.192$

w not including beam wt = **0.343 k/ft** ✓ gcm

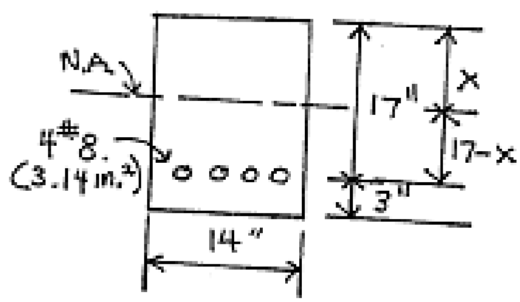
PROB #2.8

Locate N.A.

$(14x)\left(\frac{x}{2}\right) = (8)(3.14)(17-x)$

$7x^2 + 25.12x = 427$

$x = 6.22 \text{ in}$



Moment of Inertia

$I_x = \left(\frac{1}{3}\right)(14)(6.22)^3 + (8)(3.14)(10.78)^2 = 4042 \text{ in.}^4$

Flexural stresses

$f_c = \frac{(12)(60{,}000)(6.22)}{4042} = \boxed{1108 \text{ psi}}$

$f_s = \frac{(8)(12)(60{,}000)(10.78)}{4042} = \boxed{15{,}362 \text{ psi}}$ ✓ gcm

PROB# 2.9

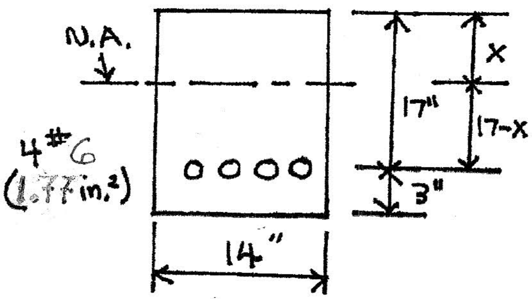
Locate N.A.

$$(14x)\left(\frac{x}{2}\right) = (8)(1.77)(17-x)$$

$$7x^2 = 241 - 14.2x$$

$$7x^2 + 14.2x - 241 = 0$$

$$x = 4.94 \text{ in}$$



Calculate moment of inertia

$$I_x = \left(\frac{1}{3}\right)(14)(4.94)^3 + (8)(1.77)(12.06)^2 = 2622 \text{ in}^4$$

Flexural stresses

$$f_c = \frac{(12)(60{,}000)(4.94)}{2622} = \boxed{1356 \text{ psi}}$$

$$f_s = \frac{(8)(12)(60{,}000)(12.06)}{2622} = \boxed{26{,}494 \text{ psi}}$$

rgcm©

PROB# 2.10

$d = 22.5$ in

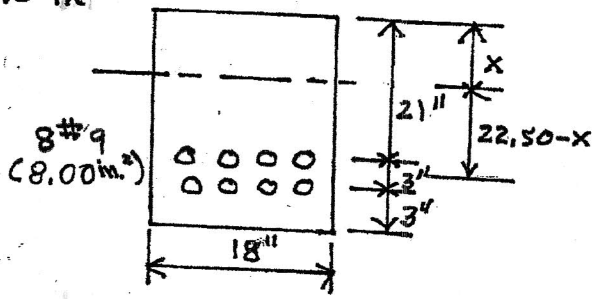
Locate N.A.

$$(18x)\left(\frac{x}{2}\right) = (9)(8.00)(22.50-x)$$

$$9x^2 = 1620 - 72x$$

$$9x^2 + 72x = 1620$$

$$x = 10.0 \text{ in}$$



Moment of inertia

$$I_x = \left(\frac{1}{3}\right)(18)(10.0)^3 + (9)(8.00)(12.5)^2 = 17{,}250 \text{ in}^4$$

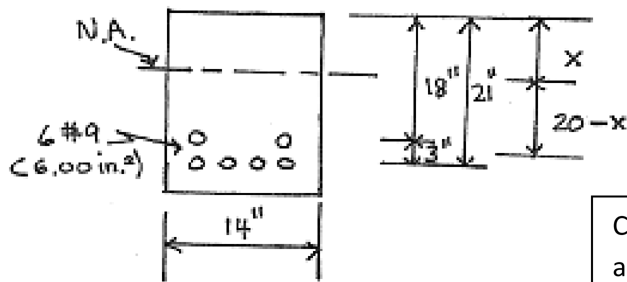
Flexural stresses

$$f_c = \frac{(12)(120{,}000)(10.0)}{17{,}250} = \boxed{835 \text{ psi}}$$

$$f_s = \frac{(9)(12)(120{,}000)(12.5)}{17{,}250} = \boxed{9391 \text{ psi}}$$

rgcm©

PROB #2.11



Centroid of tension steel bars is located at a distance from the bottom of the beam of

$[(4)(1.00\ in^2)(3\ in) + (2)(1.00\ in^2)(6\ in)]/6$ = 4 in., or one inch above the bottom layer. Therefore d = 20 in.

Locate N.A.

$$(14x)\left(\frac{x}{2}\right) = (8)(6.00)(20-x)$$
$$7x^2 = 960 - 48x$$
$$x^2 + 6.857x = 137.143$$
$$(x + 3.428)(x + 3.428) = 137.143 + (3.428)^2 = 148.891$$
$$x + 3.428 = \sqrt{148.891} = 12.202$$
$$x = 8.77 \text{ in.}$$

Moment of Inertia

$$I_x = \left(\frac{1}{3}\right)(14)(8.77)^3 + (8)(6.00)(11.23)^2 = 9201 \text{ in.}^4$$

Flexural stresses

$$f_c = \frac{(12)(110{,}000)(8.77)}{9201} = \boxed{1258 \text{ psi}}$$

$$f_s = \frac{(8)(12)(110{,}000)(11.23)}{9201} = \boxed{12{,}889 \text{ psi}}$$

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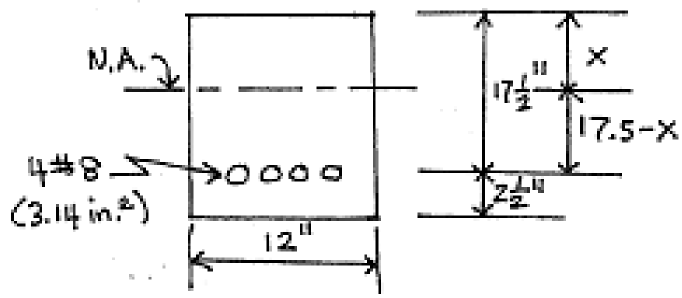
The value of f_s here is at the centroid of the tension steel bars. At the bottom layer of bars, the distance from the neutral axis is one inch larger, so the stress in the bottom layer is calculated using y = 12.33 in. instead of 11.23 in. The resulting value of **f_s = 14,037 psi.**

PROB #2.12



Locate N.A.

$$(12x)\left(\frac{x}{2}\right) = (10)(3.14)(17.5 - x)$$

$$6x^2 = 549.5 - 31.4x$$

$$6x^2 + 31.4x = 549.5$$

$$x = 7.30 \text{ in}^2$$

Moment of Inertia

$$I_x = \left(\frac{1}{3}\right)(12)(7.30)^3 + (10)(3.14)(10.2)^2 = 4823 \text{ in.}^4$$

Moment and Flexural stresses

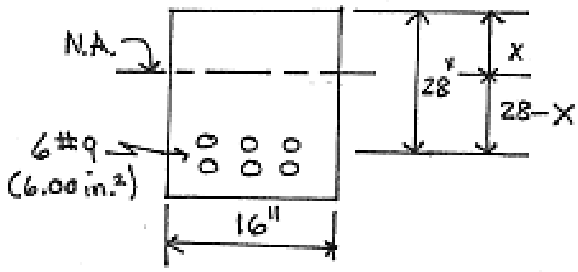
$$M = \frac{wl^2}{8} = \frac{(1.5)(24)^2}{8} = 108 \text{ ft-k}$$

$$f_c = \frac{(12)(108{,}000)(7.30)}{4823} = \boxed{1962 \text{ psi}}$$

$$f_s = \frac{(10)(12)(108{,}000)(10.2)}{4823} = \boxed{27{,}400 \text{ psi}}$$

✓ JCM[c]

PROB# 2-13



Locate N.A.

$$(16x)\left(\frac{x}{2}\right) = (8)(6.00)(28-x)$$

$$8x^2 = 1344 - 48x$$

$$x^2 + 6x = 168$$

$$(x+3)(x+3) = 168 + (3)^2 = 177$$

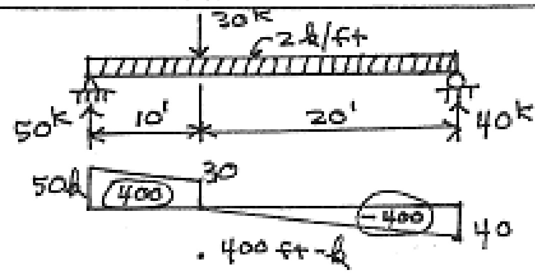
$$x + 3 = \sqrt{177} = 13.30$$

$$x = 10.30 \text{ in.}$$

Moment of Inertia

$$I_x = \left(\frac{1}{3}\right)(16)(10.30)^3 + (8)(6.00)(17.70)^2 = 20{,}866 \text{ in.}^4$$

Moment and Flexural Stresses



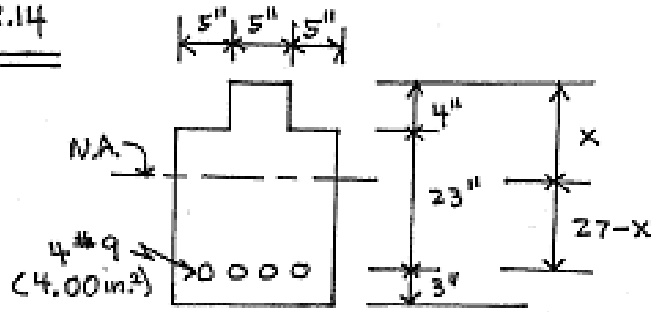
$$f_c = \frac{(12)(400{,}000)(10.30)}{20{,}866} = \boxed{2369 \text{ psi}}$$

gcm

$$f_s = \frac{(8)(12)(400{,}000)(17.70)}{20{,}866} = \boxed{32{,}574 \text{ psi}}$$

The value of f_s here is at the centroid of the tension steel bars. At the bottom layer of bars, the distance from the neutral axis is two inches larger, so the stress in the bottom layer is calculated using y = 19.70 in. instead of 17.70 in. The resulting value of **f_s = 36,255 psi.**

PROB # 2.14



Locate N.A. (assume $x > 4''$)

$$(5x)\left(\frac{x}{2}\right)+(2)(5)(x-4)\left(\frac{x-4}{2}\right)=(9)(4.00)(27-x)$$

$$2.5x^2 + 5x^2 - 40x + 80 = 972 - 36x$$

$$7.5x^2 - 4x = 892$$

$$x = 11.18 \text{ in} > 4 \text{ in. as assumed}$$

Moment of Inertia

$$I_x = \left(\frac{1}{3}\right)(5)(11.18)^3 + (2)\left(\frac{1}{3}\right)(5)(7.18)^3 + (9)(4.00)(15.82)^2$$

$$= 12{,}573 \text{ in.}^4$$

Flexural Stresses

$$f_c = \frac{(12)(70{,}000)(11.18)}{12{,}573} = \boxed{747 \text{ psi}}$$

$$f_s = \frac{(9)(12)(70{,}000)(15.82)}{12{,}573} = \boxed{9512 \text{ psi}}$$

✓ gcm©

PROB # 2.15

From solution of Prob. #2.10

$x = 10.0$ in.

$d - x = 12.5$ in.

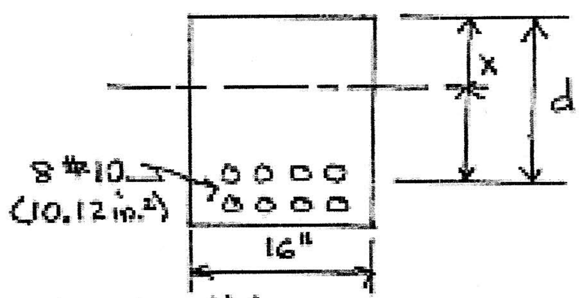
$I_x = 17{,}250 \text{ in}^4$

✓gcm ©

$$M_c = \frac{f_c I_x}{x} = \frac{(1800)(17{,}250)}{10} = 3{,}105{,}000 \text{ in.-lbs} = \boxed{258.8 \text{ ft-k}} \leftarrow$$

$$M_s = \frac{f_s I_x}{n(d-x)} = \frac{(24{,}000)(17{,}250)}{(9)(12.5)} = 3{,}680{,}000 \text{ in.-lbs} = \boxed{306.7 \text{ ft-k}}$$

PROB # 2.16



n = 10

Locate N.A.

$$(16x)\left(\frac{x}{2}\right) = (10)(10.12)(28-x)$$

$$8x^2 = 2834 - 101.2x$$

$$8x^2 + 101.2x = 2834$$

$$x = 13.53 \text{ in}$$

Moment of Inertia

$$I_x = \left(\frac{1}{3}\right)(16)(13.53)^3 + (10)(10.12)(28 - 13.53)^2 = 34{,}400 \text{ in}^4$$

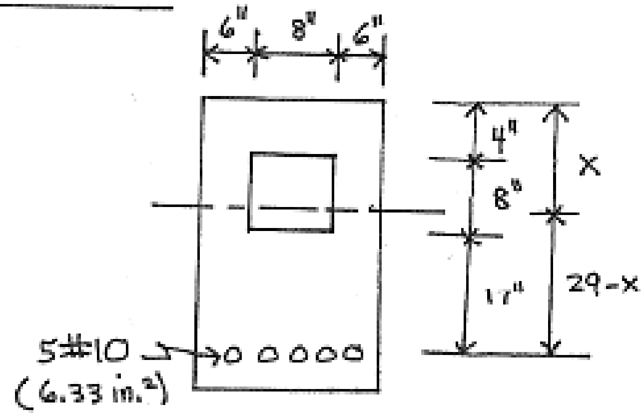
Resisting moment

✓gcm ©

$$M_c = \frac{f_c I_x}{x} = \frac{(1125)(34{,}400)}{13.53} = 2{,}860{,}300 \text{ in.-lbs} = \boxed{238 \text{ ft-k}} \leftarrow$$

$$M_s = \frac{f_s I_x}{n(d-x)} = \frac{(20{,}000)(34{,}400)}{(10)(28 - 13.53)} = 4{,}754{,}600 \text{ in.-lbs} = 396 \text{ ft-k}$$

PROB # 2.17



Assume N.A. passes through hole

$$(12x)\left(\frac{x}{2}\right) + (8)(4)(x-2) = (8)(6.33)(29-x)$$

$$6x^2 + 32x - 64 = 1468.56 - 50.64x$$

$$x^2 + 13.77x = 255.43$$

$$(x + 6.885)(x + 6.885) = 255.43 + (6.885)^2 = 302.83$$

$$x + 6.885 = \sqrt{302.83} = 17.402$$

$$x = 10.52 \text{ in.}$$

Moment of Inertia

$$I_x = \left(\tfrac{1}{3}\right)(20)(10.52)^3 - \left(\tfrac{1}{3}\right)(8)(6.52)^3 + (8)(6.33)(18.48)^2$$

$$= 24{,}317 \text{ in.}^4$$

Resisting Moment

$$M_c = \frac{f_c I_x}{x} = \frac{(1800)(24{,}317)}{10.52} = 4{,}160{,}703 \text{ in.-lbs} = 346.7 \text{ ft-k}$$

$$M_s = \frac{f_s I_x}{m(d-x)} = \frac{(24{,}000)(24{,}317)}{(8)(18.48)} = 3{,}947{,}565 \text{ in.-lbs} = \underline{\underline{329 \text{ ft-k}}}$$

Allowable uniform load

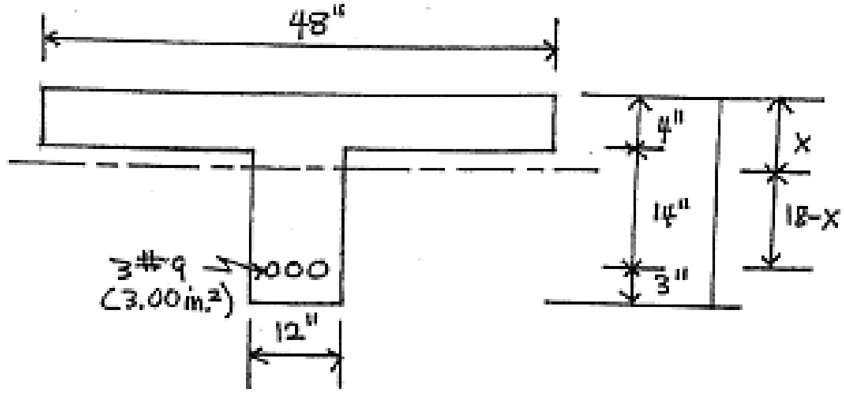
$$\frac{wl^2}{8} = 329$$

$$w = \frac{(8)(329)}{(28)^2} = 3.357 - \frac{(20)(32) - (8)(8)}{144}(0.15) = \boxed{2.757 \text{ k/ft}}$$

JCMc

PROB # 2.18

Assume N.A. is in web



$$(48x)\left(\frac{x}{2}\right)-(36)(x-4)\left(\frac{x-4}{2}\right)=(10)(3.00)(18-x)$$

$$24x^2-18x^2+144x-288=540-30x$$

$$6x^2+174x=828$$

$$x=4.16 \text{ in}$$ (Neutral axis in web)

Moment of Inertia

$$I_x=\left(\frac{1}{3}\right)(48)(4.16)^3-\left(\frac{1}{3}\right)(36)(0.16)^3+10(3.00)(13.84)^2$$

$$=6898 \text{ in.}^4$$

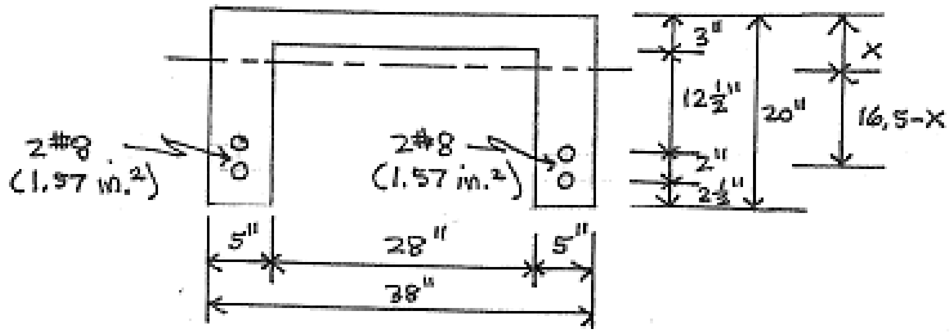
Flexural Stresses

$$f_c=\frac{(12)(100{,}000)(4.16)}{6898}=\boxed{724 \text{ psi}}$$

$$f_s=\frac{(10)(12)(100{,}000)(13.84)}{6898}=\boxed{24{,}077 \text{ psi}}$$

PROB # 2.19

Assume N.A. in stems



$$(38x)\left(\frac{x}{2}\right) - (28)(x-3)\left(\frac{x-3}{2}\right) = (8)(2 \times 1.57)(16.5-x)$$

$$19x^2 - 14x^2 + 84x - 126 = 414.48 - 25.12x$$

$$5x^2 + 109.12x = 540.48$$

$$x^2 + 21.82x = 108.10$$

$$(x + 10.91)(x + 10.91) = 108.10 + (10.91)^2 = 227.13$$

$$x + 10.91 = \sqrt{227.13} = 15.07$$

$$x = 4.16 \text{ in.}$$

Moment of Inertia

$$I_x = \left(\frac{1}{3}\right)(38)(4.16)^3 - \left(\frac{1}{3}\right)(28)(1.16)^3 + (8)(3.14)(12.34)^2$$

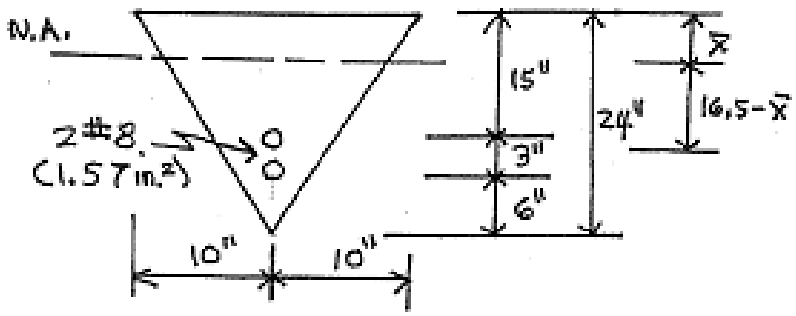
$$= 4722.5 \text{ in.}^4$$

Flexural Stresses

$$f_c = \frac{(12)(130{,}000)(4.16)}{4722.5} = \boxed{1374 \text{ psi}}$$

$$f_s = \frac{(8)(12)(130{,}000)(12.34)}{4722.5} = \boxed{32{,}611 \text{ psi}}$$

PROB #2.20



Locate N.A.

$$(20x)\left(\frac{x}{2}\right)-\left(\frac{1}{2}\right)(x)\left(\frac{10}{24}x\right)(2)\left(\frac{1}{3}x\right)=(9)(1.57)(16.5-x)$$

$$10x^2 - 0.139x^3 = 233.5 - 14.13x$$

$$x^2 - 0.0139x^3 + 1.413x = 23.35$$

$x = 4.29$" from equation solution

Moment of Inertia

$$I_x = \left(\frac{1}{3}\right)(20)(4.29)^3 - \frac{(0.417 \times 4.29)(2)(4.29)^3}{12}$$

$$+ 9(1.57)(12.21)^2 = 2636 \text{ in.}^4$$

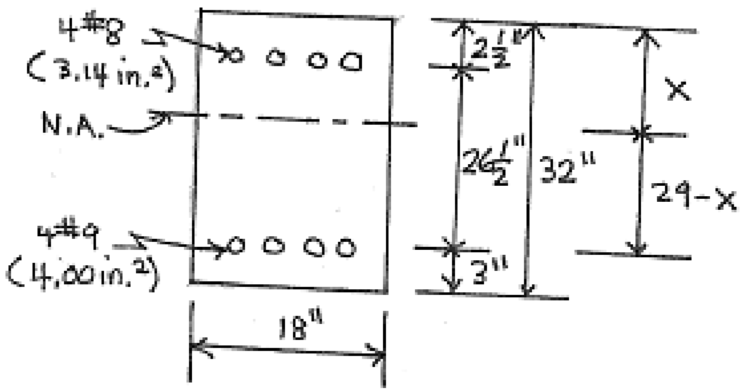
Flexural Stresses

$$f_c = \frac{(12)(90,000)(4.29)}{2636} = \boxed{1758 \text{ psi}}$$

$$f_s = \frac{(9)(12)(90,000)(12.21)}{2636} = \boxed{45,023 \text{ psi}}$$

✓ JCM

PROB #2.21



Locate N.A.

$$(18x)\left(\frac{x}{2}\right)+(17)(3.14)(x-2.5)=(9)(4.00)(29-x)$$

$$9x^2+53.38x-133.45=1044-36.00x$$

$$9x^2+89.38x=1177.45$$

$$x^2+9.93x=130.83$$

$$(x+4.965)(x+4.965)=130.83+(4.965)^2=155.655$$

$$x+4.965=\sqrt{155.655}=12.476$$

$$x=7.51\text{ in.}$$

Moment of Inertia

$$I_x=\left(\frac{1}{3}\right)(18)(7.51)^3+(17)(3.14)(5.01)^2+(9)(4.00)(21.49)^2$$

$$=20{,}507\text{ in.}^4$$

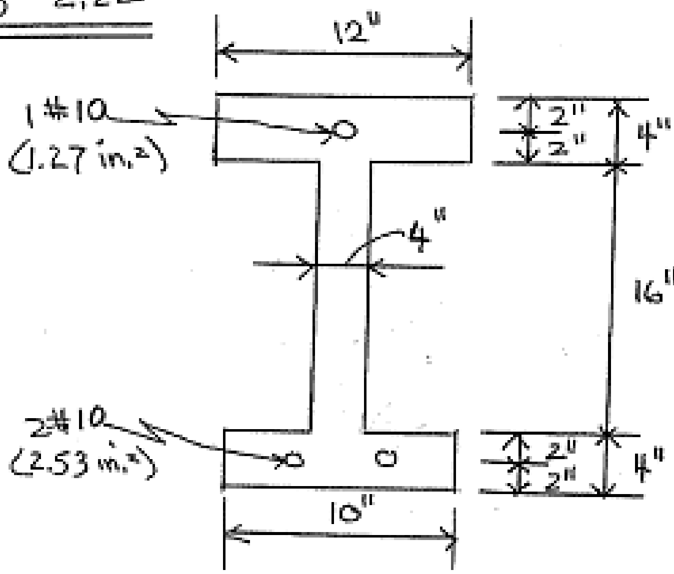
Flexural Stresses

$$f_c=\frac{(12)(320{,}000)(7.51)}{20{,}507}=\boxed{1406\text{ psi}}$$ ✓ JCMc

$$f_s'=\frac{(18)(12)(320{,}000)(5.01)}{20{,}507}=\boxed{16{,}886\text{ psi}}$$

$$f_s=\frac{(9)(12)(320{,}000)(21.49)}{20{,}507}=\boxed{36{,}217\text{ psi}}$$

PROB # 2.22



Assume N.A. is in web

$$(12-4)(4)(x-2)+(4x)\left(\frac{x}{2}\right)+(15)(1.27)(x-2)=(8)(2.53)(22-x)$$ ← 2n-1

$$32x-64+2x^2+19x-38=445-20.24x$$

$$2x^2+71.24x=547$$

$$x=6.49 \text{ in.}$$

∴ N.A. in web as assumed

Moment of Inertia

$$I_x=\left(\tfrac{1}{3}\right)(12)(6.49)^3-\left(\tfrac{1}{3}\right)(8)(2.49)^3+(15)(1.27)(4.49)^2+(8)(2.53)(15.51)^2$$

$$=6305 \text{ in.}^4$$

Allowable resisting moment

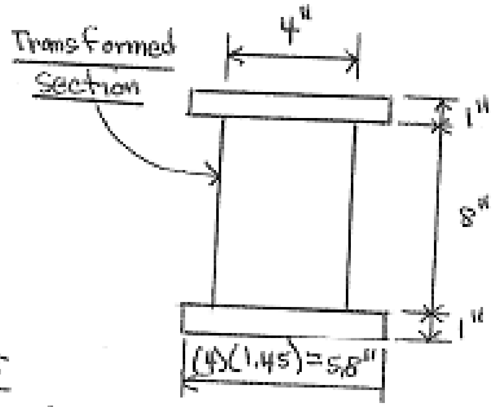
$$M_c=\frac{(1800)(6305)}{6.49}=1{,}748{,}690 \text{ in.-lbs}=145.7 \text{ ft-k}$$

$$M_s'=\frac{(24{,}000)(6305)}{(2)(8)(4.49)}=2{,}106{,}347 \text{ in.-lbs}=175.5 \text{ ft-k}$$

$$M_s=\frac{(24{,}000)(6305)}{(8)(15.51)}=1{,}219{,}536 \text{ in.-lbs}=\boxed{101.6 \text{ ft-k}}$$

PROB # 2.23

$n = \frac{29 \times 10^6}{20 \times 10^6} = 1.45$



Moment of Inertia

$I_x = \left(\frac{1}{12}\right)(5.8)(10)^3 - \left(\frac{1}{12}\right)(1.8)(8)^3$

$= 406.53 \text{ in.}^4$

Allowable resisting moment

M_{res} for 29×10^6 psi steel

$= \frac{(29000)(406.53)}{(1.45)(5.00)} = 1{,}682{,}193 \text{ in.-lbs} = 140.18 \text{ ft lb} \leftarrow$

M_{res} for 20×10^6 psi steel $= \frac{(29000)(406.53)}{4.00}$

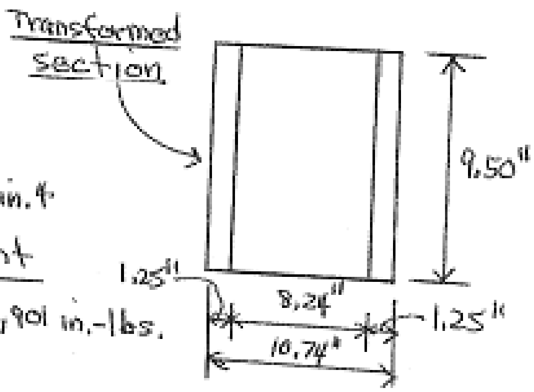
$= 2{,}032{,}650 \text{ in. lbs} = 169.39 \text{ ft-k}$

$M_{res} =$ **140.18 ft-k**

✓ gcm©

PROB # 2.24

$n = \frac{29 \times 10^6}{1.76 \times 10^6} = 16.48$



Moment of inertia

$I_x = \left(\frac{1}{12}\right)(10.74)(9.50)^3 = 767.35 \text{ in.}^4$

Allowable resisting moment

$M_{wood} = \frac{(1875)(767.35)}{4.75} = 302{,}901 \text{ in.-lbs.}$

$= 25.24 \text{ ft-k}$

$M_{steel} = \frac{(24{,}000)(767.35)}{(16.48)(4.75)} = 235{,}263 \text{ in.-lbs}$

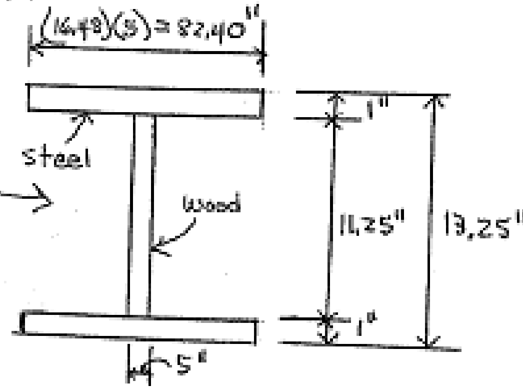
$=$ **19.61 ft-k** $\leftarrow$

✓ gcm©

PROB # 2.25

$$m = \frac{29 \times 10^6}{1.76 \times 10^6} = 16.48$$

Transformed section



Moment of Inertia

$$I_x = \left(\tfrac{1}{12}\right)(82.40)(13.25)^3 - \left(\tfrac{1}{12}\right)(77.40)(11.25)^3$$

$$= 6789.6 \text{ in.}^4$$

Allowable resisting moment

$$M_{wood} = \frac{(1800)(6789.6)}{5.625} = 2{,}172{,}662 \text{ in.-lbs}$$

$$= 181.06 \text{ ft-k}$$

$$M_{steel} = \frac{(24{,}000)(6789.6)}{(16.48)(6.625)}$$

$$= 1{,}492{,}493 \text{ in.-lbs} = \boxed{124.4 \text{ ft-k}} \leftarrow$$

✓ JCM©

PROB # 2.26

Using 3 # 8 bars (2.35 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(2.35)(60)}{(0.85)(4)(16)} = 2.96 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (2.35)(60)\left(21 - \frac{2.96}{2}\right)$$

$$= 2752 \text{ in.-k} = \boxed{229.3 \text{ ft-k}}$$

rjcm©

PROB # 2.27

Using 6 # 9 bars (6.00 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(6.00)(60)}{(0.85)(4)(16)} = 6.62 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (6.00)(60)\left(26.25 - \frac{6.62}{2}\right)$$

$$= 8258.4 \text{ in.-k} = \boxed{688.2 \text{ ft-k}}$$

rjcm©

PROB # 2.28

Using 4 # 10 bars (5.06 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(5.06)(60)}{(0.85)(4)(16)} = 5.58 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (5.06)(60)\left(25 - \frac{5.58}{2}\right)$$

$$= 6743 \text{ in.-k} = \boxed{561.9 \text{ ft-k}}$$

rjcm©

PROB #2.29

Using 6 #10 bars (7.59 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(7.59)(60)}{(0.85)(4)(18)} = 7.44 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (7.59)(60)\left(26 - \frac{7.44}{2}\right)$$

$$= 10{,}146.3 \text{ in.-k} = \boxed{845.5 \text{ ft-k}}$$ ✓ gcm©

PROB #2.30

Using 3 #9 bars (3.00 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3.00)(60)}{(0.85)(4)(14)} = 3.78 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3.00)(60)\left(21 - \frac{3.78}{2}\right)$$

$$= 3440 \text{ in.-k} = \boxed{286.6 \text{ ft-k}}$$ ✓ gcm©

PROB #2.31

Using 8 #9 bars (8.00 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(8.00)(60)}{(0.85)(4)(16)} = 8.82 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (8.00)(60)\left(27 - \frac{8.82}{2}\right)$$

$$= 10{,}843.2 \text{ in.-k} = \boxed{903.6 \text{ ft-k}}$$ ✓ gcm©

PROB # 2.32

Using 4 #10 bars (5.06 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(5.06)(60)}{(0.85)(5)(14)} = 5.10 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (5.06)(60)\left(20.5 - \frac{5.10}{2}\right)$$

$$= 5450 \text{ ft-k} = \boxed{454.1 \text{ ft-k}}$$ ✓gcm©

PROB # 2.33

Using 4 #10 bars (5.06 in.²)

$$a = \frac{A_s f_y}{0.8 f'_c b} = \frac{(5.06)(75)}{(0.85)(5)(21)} = 4.25 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (5.06)(75)\left(28 - \frac{4.25}{2}\right)$$

$$= 9819.6 \text{ in.-k} = \boxed{818.3 \text{ ft-k}}$$ ✓gcm©

PROB # 2.34

Using 6 #11 bars (9.37 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(9.37)(60)}{(0.85)(3)(22)} = 10.02 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (9.37)(60)\left(36 - \frac{10.02}{2}\right)$$

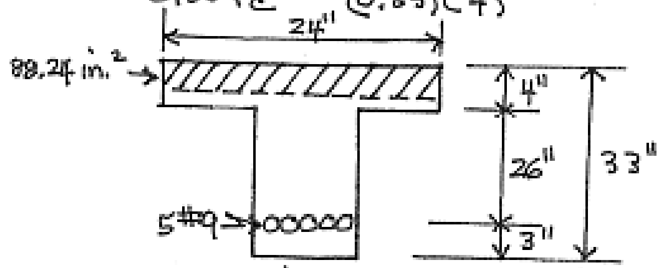
$$= 17{,}422 \text{ in.-k} = \boxed{1451.8 \text{ ft-k}}$$ ✓gcm©

PROB # 2.35

Using 5 #9 bars ($5.00\ in.^2$)

$$0.85 f'_c A_c = A_s f_y$$

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(5.00)(60)}{(0.85)(4)} = 88.24\ in.^2$$



Noting that $88.24\ in.^2 < (4)(24) = 96\ in.^2$

∴ N.A. is in flange

$$a = \frac{88.24}{24} = 3.68\ in.$$

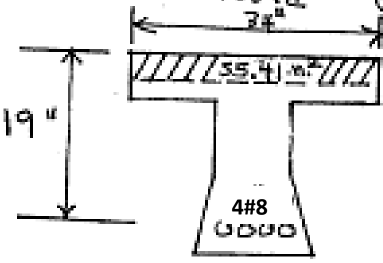
$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (5.0)(60)\left(30 - \frac{3.68}{2}\right)$$

$$= 8448\ in.\text{-}k = \boxed{704\ ft\text{-}k}$$ ✓ JCMc

PROB # 2.36

Using 4 #8 bars ($3.14\ in.^2$)

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(3.14)(60)}{(0.85)(4)} = 55.41\ in.^2$$



$$a = \frac{55.41}{34} = 1.63\ in.$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3.14)(60)\left(19 - \frac{1.63}{2}\right)$$

$$= 3426\ in.\text{-}k = \boxed{285.5\ ft\text{-}k}$$ ✓ JCMc

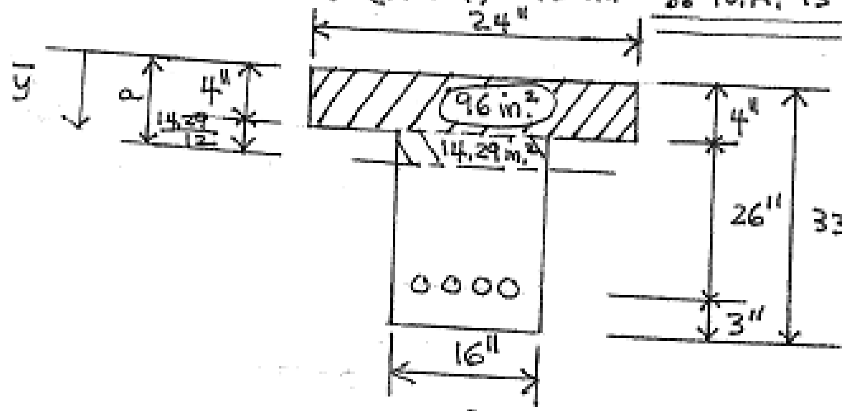
PROB #2.37

Using 4 #11 bars (6.25 in.²)

$0.85 f'_c A_c = A_s f_y$

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(6.25)(60)}{(0.85)(4)} = 110.29 \text{ in.}^2$$

$> (4)(24) = 96 \text{ in.}^2$ ∴ N.A. is in web



$$a = 4 + \frac{14.29}{1} = 4 + 0.898 = 4.893 \text{ in.}$$

$$\bar{y} \text{ from top} = \frac{(96)(2) + (14.29)\left(4 + \frac{0.893}{2}\right)}{110.29} = 2.32 \text{ in.}$$

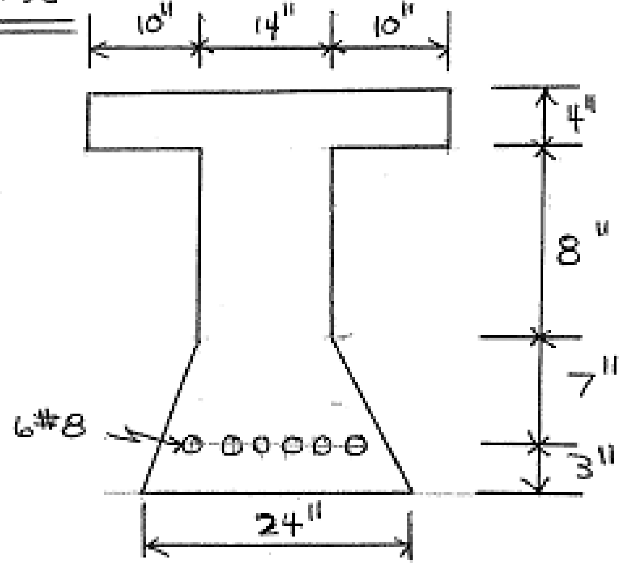
$z = 30 - 2.32 = 27.68$ in.

$M_n = A_s f_y z = (6.25)(60)(27.68)$

$= 10{,}380 \text{ in.-k} =$ **865 ft-k**

✓ J C McC

PROB #2.38



using 6#8 bars ($A_s = 4.71 \text{ in.}^2$)

$$0.85 f'_c A_c = A_s f_y$$

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(4.71)(60)}{(0.85)(4)} = 83.12 \text{ in.}^2$$

$$< (4)(34) = 136 \text{ in.}^2$$ ∴ N.A. is in flange

$$a = \frac{83.12}{34} = 2.44 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (4.71)(60)\left(19 - \frac{2.44}{2}\right)$$

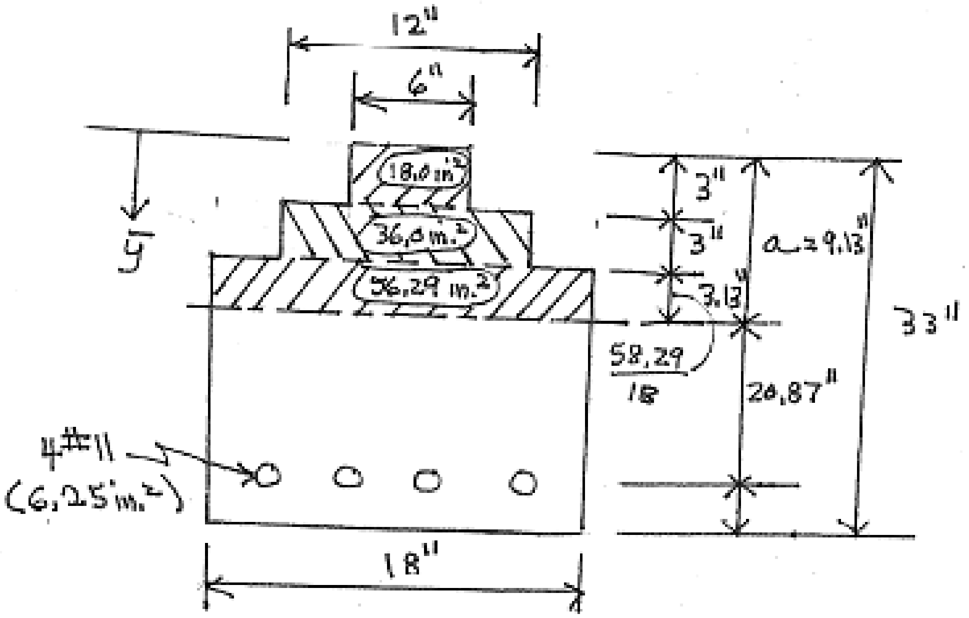
$$= 5024 \text{ in.-k} = \boxed{418.7 \text{ ft-k}}$$

✓ CMC

PROB #2.39

$$0.85 f'_c A_c = A_s f_y$$

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(6.25)(60)}{(0.85)(4)} = 110.29 \text{ in.}^2$$



$$\bar{y} \text{ from top} = \frac{(18)(1.50) + (36)(4.5) + (56.29)(6 + \frac{3.13}{2})}{110.29}$$

$$= 5.575 \text{ in.}$$

$$z = 30 - 5.575 = 24.425 \text{ in.}$$

$$M_n = A_s f_y z = (6.25)(60)(24.425)$$

$$= 9159.4 \text{ in.-k} = \boxed{763.3 \text{ ft-k}}$$

~gcm

PROB # 2.40

Using 3 #9 bars (3.00 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3.00)(60)}{(0.85)(4)(14)} = 3.78 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3.00)(60)\left(23 - \frac{3.78}{2}\right)$$

$$= 3800 \text{ in.-k} = 316.6 \text{ ft-k}$$

$$\frac{w_n L^2}{8} = M_n = 316.6$$

$$w_n = \frac{(8)(316.6)}{(18)^2} = \boxed{7.82 \text{ k/ft}}$$

vJCMc

PROB # 2.41

Using 4 #10 bars (5.06 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(5.06)(60)}{(0.85)(3)(16)} = 7.44 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (5.06)(60)\left(23 - \frac{7.44}{2}\right)$$

$$= 5853.4 \text{ in.-k} = 487.8 \text{ ft-k}$$

$$\frac{w_n L^2}{8} = M_n = 487.8$$

$$w_n = \frac{(8)(487.8)}{(24)^2} = \boxed{6.77 \text{ k/ft}}$$

vJCMc

PROB # 2.42

$$I_g = \left(\frac{1}{12}\right)(350)(600)^3 = 6.3 \times 10^9 \ mm^4$$

$$f_r = 0.7\sqrt{f'_c} = 0.7\sqrt{28} = 3.704 \ MPa$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(3.704)(6.3 \times 10^9)}{300}$$

$$= 7.78 \times 10^7 \ N\cdot mm = \boxed{77.8 \ kN\cdot m}$$ rgcm✓

PROB # 2.43

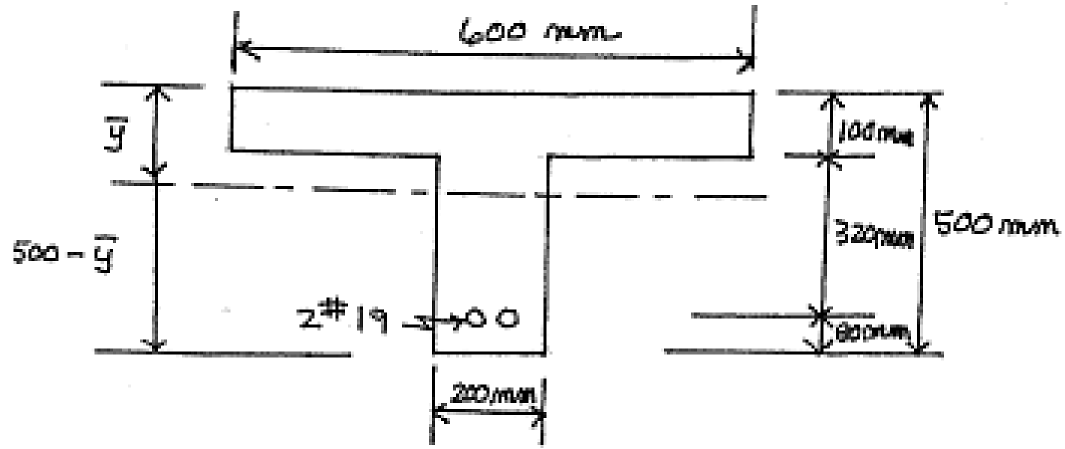
$$I_g = \left(\frac{1}{12}\right)(300)(500)^3 = 3.125 \times 10^9 \ mm^4$$

$$f_r = 0.7\sqrt{f'_c} = 0.7\sqrt{28} = 3.704 \ MPa$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(3.704)(3.125 \times 10^9)}{250}$$

$$= 4.630 \times 10^7 \ N\cdot mm = \boxed{46.30 \ kN\cdot m}$$ ugcm✓

PROB #2.44



$$A = (600)(100) + (200)(400) = 140\,000 \text{ mm}^2$$

$$\bar{y} = \frac{(60\,000)(50) + (80\,000)(300)}{140\,000} = 192{,}86 \text{ mm}$$

$$I_x = \left(\frac{1}{3}\right)(200)(192{,}86^3 + 307{,}14^3) + \left(\frac{1}{12}\right)(400)(100)^3 + (400)(100)(142{,}86)^2 = 3{,}26 \times 10^9 \text{ mm}^4$$

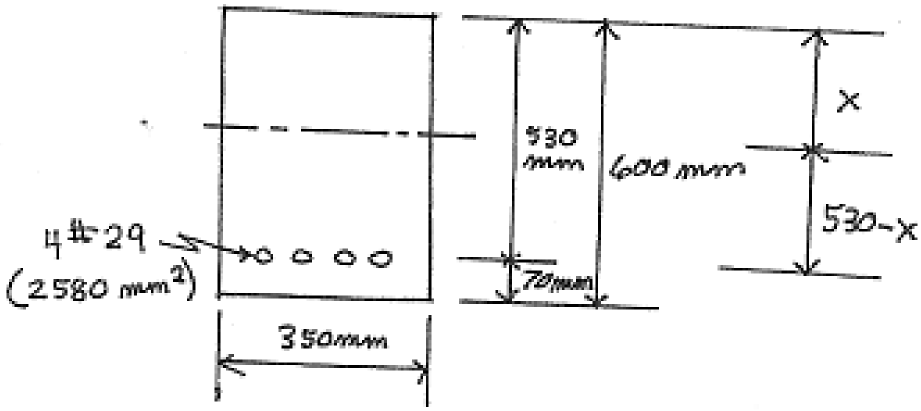
$$f_r = 0{,}7\sqrt{f'_c} = 0{,}7\sqrt{28} = 3.704 \text{ MPa}$$

$$M_{cr} = \frac{f_r I_x}{d - \bar{y}} = \frac{(3{,}704)(3{,}26 \times 10^9)}{307{,}14}$$

$$= 3{,}931 \times 10^7 \text{ N}\cdot\text{mm} = \boxed{39.31 \text{ kN}\cdot\text{m}}$$

✓ gcm≥

PROB# 2.45



Locate N.A.

$$(350x)\left(\frac{x}{2}\right) = (9)(2580)(530-x)$$

$$175x^2 = 1.2306 \times 10^7 - 23220x$$

$$x^2 + 132.69x = 70\,323$$

$$(x + 66.34)(x + 66.34) = 70\,323 + (66.34)^2 = 74\,724$$

$$x + 66.34 = \sqrt{74\,124} = 273.36$$

$$x = 207.02 \text{ mm}$$

Moment of Inertia

$$I_x = \left(\frac{1}{3}\right)(350)(207.02)^3 + (9)(2580)(322.98)^2$$

$$= 3.457 \times 10^9 \text{ mm}^4$$

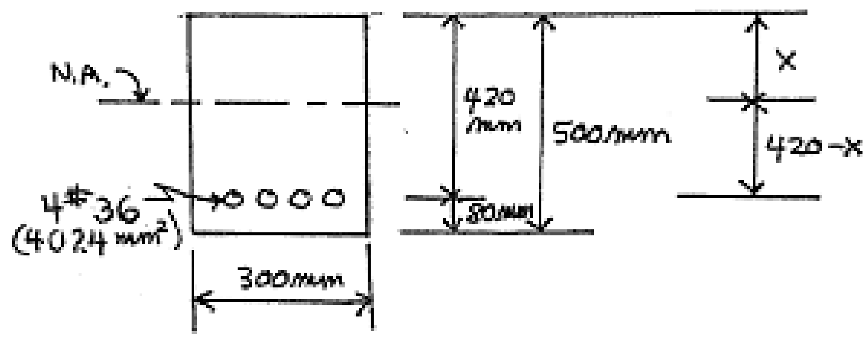
Flexural Stresses

$$f_c = \frac{(130)(10)^6(207.02)}{3.457 \times 10^9} = \boxed{7.785 \text{ MPa}}$$

$$f_s = \frac{(9)(130)(10)^6(322.98)}{3.457 \times 10^9} = \boxed{109.311 \text{ MPa}}$$

rgcm©

PROB # 2.46



Locate N.A.

$$(300x)\left(\frac{x}{2}\right) = (9)(4024)(420-x)$$

$$150x^2 = 15.21 \times 10^6 - 36{,}216x$$

$$150x^2 + 36{,}216x = 15.21 \times 10^6$$

$$x = 219.8 \text{ mm}$$

Moment of Inertia

$$I_x = \left(\frac{1}{3}\right)(300)(219.8)^3 + (9)(4024)(200.2)^2 = 2.51 \times 10^9 \text{ mm}^4$$

Flexural Stresses

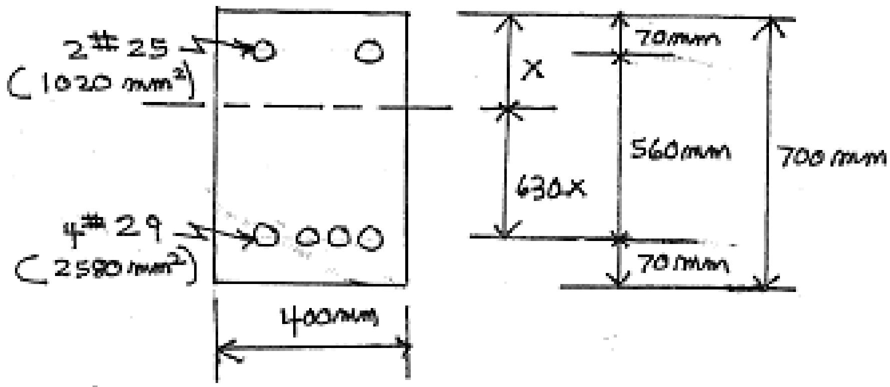
$$M = \frac{(20)(8)^2}{8} = 160 \text{ kN.m}$$

$$f_c = \frac{(160)(10)^6(219.8)}{2.51 \times 10^9} = \boxed{14.01 \text{ MPa}}$$

$$f_s = \frac{(9)(160)(10)^6(200.2)}{2.51 \times 10^9} = \boxed{114.9 \text{ MPa}}$$

✓ JCM

PROB #2.47



Locate N.A.

$$(400x)\left(\frac{x}{2}\right)+(16-1)(1020)(x-70)=(8)(2580)(630-x)$$

$$200x^2+15\ 300x-1\ 071\ 000=13\ 003\ 200-20\ 640x$$

$$200x^2+35\ 940x=14\ 074\ 200$$

$$x^2+179.7x=70\ 371$$

$$(x+89.85)(x+89.85)=70\ 371+(89.85)^2=78\ 444$$

$$x+89.85=\sqrt{78\ 444}=280.08$$

$$x=190.23\text{ mm}$$

Moment of Inertia

$$I_x=\left(\frac{1}{3}\right)(400)(190.23)^3+(16-1)(1020)(120.23)^2+(8)(2580)(439.77)^2=5.131\times10^9\text{ mm}^4$$

Flexural Stresses

$$f_c=\frac{(275)(10)^6(190.23)}{5.131\times10^9}=\boxed{10.20\text{ MPa}}$$

$$f_s'=\frac{(16)(275)(10)^6(120.23)}{5.131\times10^9}=\boxed{103.10\text{ MPa}}$$

$$f_s=\frac{(8)(275)(10)^6(439.77)}{5.131\times10^9}=\boxed{188.56\text{ MPa}}$$

JCMc

PROB #2.48

Using 3 #36 bars (3018 mm²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3018)(350)}{(0.85)(35)(300)} = 118.4 \text{ mm}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3018)(350)\left(600 - \frac{118.4}{2}\right)$$

$$= 5.71 \times 10^8 \text{ N·mm} = \boxed{571 \text{ kN.m}}$$

✓ gcm©

PROB #2.49

Using 3 #36 bars (3018 mm²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3018)(350)}{(0.85)(28)(320)} = 138.69 \text{ mm}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3018)(350)\left(600 - \frac{138.69}{2}\right)$$

$$= 5.605 \times 10^8 \text{ N·mm} = \boxed{560.5 \text{ kN.m}}$$

✓ gcm©

PROB #2.50

Using 3 #25 bars (1530 mm²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(1530)(420)}{(0.85)(24)(350)} = 90.0 \text{ mm}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (1530)(420)\left(530 - \frac{90}{2}\right)$$

$$= 3.117 \times 10^8 \text{ N·mm} = \boxed{311.7 \text{ kN.m}}$$

✓ gcm©

PROB #2.51

Using 3 #25 bars (1530 mm²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(1530)(420)}{(0.85)(24)(370)} = 85.14 \text{ mm}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (1530)(420)\left(530 - \frac{85.14}{2}\right)$$

$$= 3.132 \times 10^8 \text{ N·mm} = \boxed{313.2 \text{ kN.m}}$$

✓ gcm©

PROB # 2.52

Using 6 #25 bars (3060 mm²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3060)(420)}{(0.85)(24)(350)} = 180 \text{ mm}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3060)(420)\left(495 - \frac{180}{2}\right)$$

$$= 5.205 \times 10^8 \text{ N}\cdot\text{mm} = \boxed{520.5 \text{ kN}\cdot\text{m}}$$ ✓ gcm

PROB # 2.53

Using 4 #36 bars (4024 mm²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(4024)(350)}{(0.85)(35)(300)} = 157.8 \text{ mm}$$

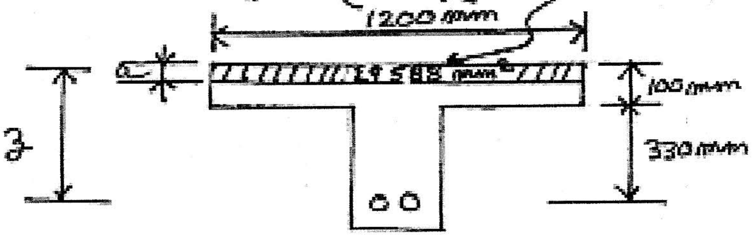
$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (4024)(350)\left(600 - \frac{157.8}{2}\right)$$

$$= 7.34 \times 10^8 \text{ N}\cdot\text{mm} = \boxed{734 \text{ kN}\cdot\text{m}}$$ ✓ gcm

PROB # 2.54

Using 2 #36 bars (2012 mm²)

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(2012)(350)}{(0.85)(28)} = 29\,588 \text{ mm}^2$$



$$T = A_s f_y = (2012)(350) = 704\,200 \text{ N}$$

$$a = \frac{29{,}588}{1200} = 24.66 \text{ mm}$$

$$z = 430 - \frac{24.66}{2} = 417.7 \text{ mm}$$

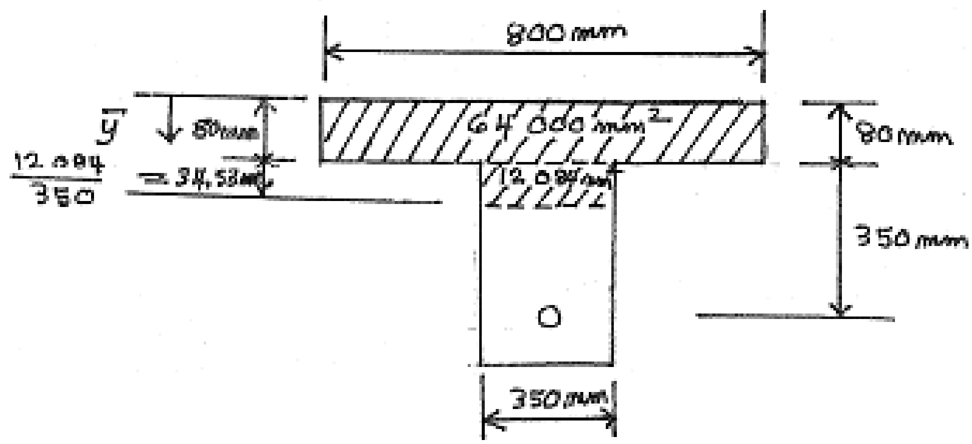
$$M_n = Tz = (704\,200)(417.7) = 2.941 \times 10^8 \text{ N}\cdot\text{mm}$$

$$= \boxed{294 \text{ kN}\cdot\text{m}}$$ ✓ gcm

PROB #2.55

Using 6 #36 bars ($6036\ mm^2$)

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(6036)(300)}{(0.85)(28)} = 76\ 084\ mm^2$$



$$\bar{y} = \frac{(64\ 000)(40) + (12\ 084)(80 + \frac{34.53}{2})}{76\ 084} = 49.10\ mm$$

$$z = 430 - 49.10 = 380.90\ mm$$

$$T = A_s f_y = (6036)(300) = 1\ 810\ 800\ N$$

$$M_n = Tz = (1\ 810\ 800)(380.90)$$

$$= 6.897 \times 10^8\ N \cdot mm$$

$$= \boxed{689.7\ kN \cdot m}$$

✓ gcm

Problem 2.56 – Repeat Prob. 2.27 using the spreadsheet for Chapter 2

Open the Chap. 2 spreadsheet, and select the Rectangular Beam Moment Strength Worksheet. Input the values for cells colored yellow.

Nominal Moment Strength Singly Reinforced Rectangular Beam

		Units
f'_c =	4000	psi
b =	16	in.
d =	26.25	in.
A_s =	6	in^2
f_y =	60	ksi
a =	6.62	in.
M_n =	8258.8	in.-k
	688.2	kip-ft

ϕM_n =	619.4	kip-ft

Problem 2.57 – Repeat Prob. 2.28 using the spreadsheet for Chapter 2

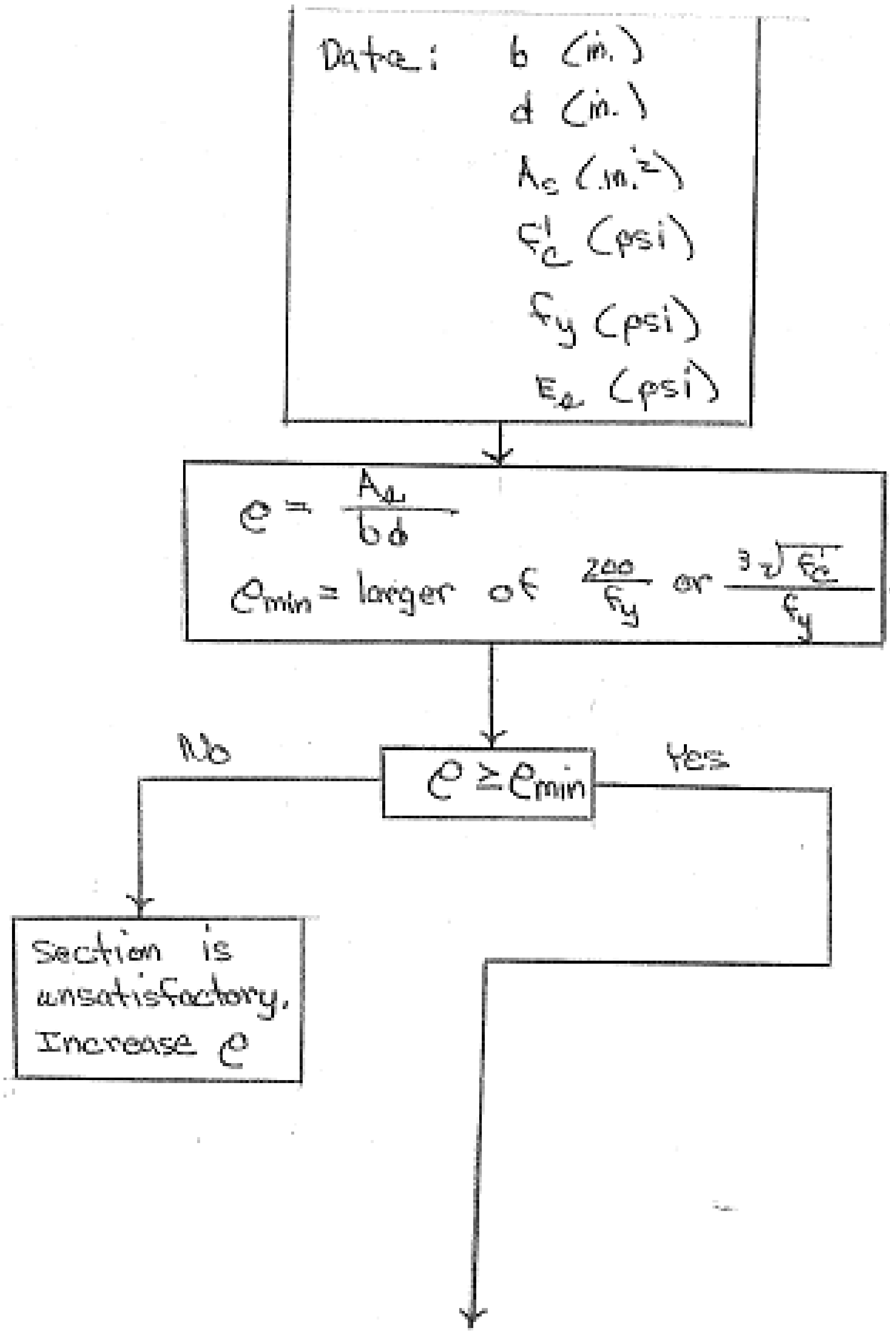
Open the Chap. 2 spreadsheet, and select the Rectangular Beam Moment Strength Worksheet. Input the values for cells colored yellow.

Nominal Moment Strength Singly Reinforced Rectangular Beam

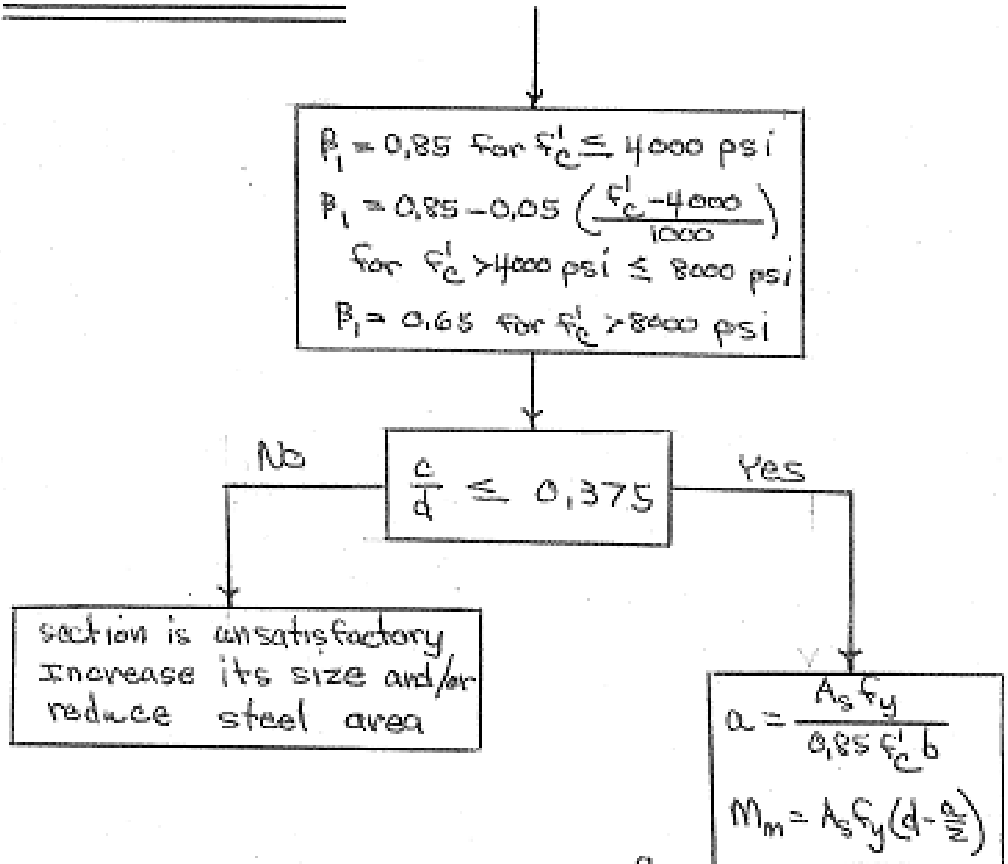
		Units
f'_c =	4000	psi
b =	16	in.
d =	25	in.
A_s =	5.06	in^2
f_y =	60	ksi
a =	5.58	in.
M_n =	6742.8	in.-k
	561.9	kip-ft
ϕM_n =	505.7	kip-ft

PROB# 2.58 (1)

Flexural analysis of singly reinforced rectangular beams



PROB # 2,58 (2)



$\beta_1 = 0{,}85$ for $f'_c \le 4000$ psi
$\beta_1 = 0{,}85 - 0{,}05\left(\frac{f'_c - 4000}{1000}\right)$
for $f'_c > 4000$ psi ≤ 8000 psi
$\beta_1 = 0{,}65$ for $f'_c > 8000$ psi
$\frac{c}{d} \le 0{,}375$
No
Yes
section is unsatisfactory Increase its size and/or reduce steel area
$a = \frac{A_s f_y}{0{,}85 f'_c b}$
$M_n = A_s f_y \left(d - \frac{a}{2}\right)$