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Section 1-1 Linear Equations and Inequalities

Goal: To solve linear equation and linear inequalities**Equality Properties:**

1. If $x = y$ and a is any real number, then $x \pm a = y \pm a$.
2. If $x = y$ and a is any nonzero real number, then $ax = ay$ and $\frac{x}{a} = \frac{y}{a}$.

Inequality Properties:

1. If $x > y$ and a is any real number, then $x \pm a > y \pm a$.
2. If $x > y$ and a is any positive real number, then $ax > ay$ and $\frac{x}{a} > \frac{y}{a}$.
3. If $x > y$ and a is any negative real number, then $ax < ay$ and $\frac{x}{a} < \frac{y}{a}$.

Interval Notation:

A bracket,] or [, is used if the endpoint is included.
A parentheses,) or (, is used if the endpoint is not included.
Infinity, either positive or negative, always uses a parentheses.

In problems 1–3, solve for the variable:

1. $5x + 7 = 9x - 13$
 $5x + 7 - 5x = 9x - 13 - 5x$
 $7 = 4x - 13$
 $7 + 13 = 4x - 13 + 13$
 $20 = 4x$
 $\frac{20}{4} = \frac{4x}{4}$
 $5 = x$

$$\begin{aligned}2. \quad & 7y + 3(6y - 11) = 167 \\ & 7y + 18y - 33 = 167 \\ & 25y - 33 = 167 \\ & 25y - 33 + 33 = 167 + 33 \\ & 25y = 200 \\ & \frac{25y}{25} = \frac{200}{25} \\ & y = 8\end{aligned}$$

$$\begin{aligned}3. \quad & \frac{m}{5} + 4 = \frac{m}{2} + 7 \\ & \frac{10m}{5} + 4(10) = \frac{10m}{2} + 7(10) \\ & 2m + 40 = 5m + 70 \\ & 40 = 3m + 70 \\ & -30 = 3m \\ & -10 = m\end{aligned}$$

In problems 4–6, solve for the variable and place the final answer in interval notation.

$$\begin{aligned}4. \quad & 7x + 5 > 19 \\ & 7x > 14 \\ & x > 2 \\ & (2, \infty)\end{aligned}$$

$$\begin{aligned}5. \quad & -7 < -3x - 1 \leq 17 \\ & -6 < -3x \leq 18 \\ & 2 > x \geq -6 \\ & -6 \leq x < 2 \\ & [-6, 2)\end{aligned}$$

$$\begin{aligned}
 6. \quad & \frac{u}{3} + \frac{3}{4} < \frac{u}{2} - \frac{5}{4} \\
 & \frac{12u}{3} + \frac{3(12)}{4} < \frac{12u}{2} - \frac{5(12)}{4} \\
 & 4u + 9 < 6u - 15 \\
 & 24 < 2u \\
 & 12 < u \\
 & (12, \infty)
 \end{aligned}$$

7. *Break-even Analysis.* A publisher for a promising new novel figures fixed costs (overhead, advances, promotion, copy, editing, typesetting, and so on) at \$67,000 and variable costs (printing, paper, binding, shipping) at \$3.50 for each book produced. If the book is sold to distributors for \$21 each, how many must be produced and sold for the publisher to break even?

Let x = the number of books produced. Since the break even point is the point when cost is the same as the revenue:

$$\begin{aligned}
 21x &= 67,000 + 3.50x \\
 17.50x &= 67,000 \\
 x &= 3828.571429
 \end{aligned}$$

Therefore, the publisher must produce 3829 books to break even.

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Section 1-2 Graphs and Lines

Goal: To find the equations of lines, x -intercepts, and y -intercepts

Slope of a Line: $m = \frac{y_2 - y_1}{x_2 - x_1}$ where $P_1 : (x_1, y_1)$ and $P_2 : (x_2, y_2)$

Slope-Intercept Form of a Line: $y = mx + b$, where m is the slope and $(0, b)$ is the y -intercept.

Equation of a line in standard form: $Ax + By = C$, where A and B are not both zero.

Horizontal Line: $y = b$, slope is zero.

Vertical Line: $x = a$, slope is undefined.

y -intercept: $(0, b)$

x -intercept: $(a, 0)$

In problems 1–12, write the equation of the line in slope-intercept form with the given characteristics:

1. Slope is 6 and y -intercept is $(0, 5)$.

Since the slope and y -intercept are given, $y = 6x + 5$.

2. Slope is -5 and y -intercept is $(0, -6)$.

Since the slope and y -intercept are given, $y = -5x - 6$.

3. Slope is $\frac{2}{5}$ and passes through the point $(-10, 6)$.

$$y - y_1 = m(x - x_1)$$

$$y - 6 = \frac{2}{5}(x - (-10))$$

$$y - 6 = \frac{2}{5}x + 4$$

$$y = \frac{2}{5}x + 10$$

4. Slope is $\frac{-4}{5}$ and passes through the point $(2, -3)$.

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = \frac{-4}{5}(x - 2)$$

$$y + 3 = \frac{-4}{5}x + \frac{8}{5}$$

$$y = \frac{-4}{5}x - \frac{7}{5}$$

5. Passes through the points $(3, 6)$ and $(6, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 6}{6 - 3} = \frac{-3}{3} = -1$$

$$y - y_1 = m(x - x_1)$$

$$y - 6 = -1(x - 3)$$

$$y - 6 = -x + 3$$

$$y = -x + 9$$

6. Passes through the points $(-1, 4)$ and $(2, -2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{2 - (-1)} = \frac{-6}{3} = -2$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -2(x - (-1))$$

$$y - 4 = -2x - 2$$

$$y = -2x + 2$$

7. Passes through the points (0, 6) and (5, 0).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 6}{5 - 0} = \frac{-6}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-6}{5}(x - 5)$$

$$y = \frac{-6}{5}x + 6$$

8. Passes through the points (0, 4) and (-1, 0).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{-1 - 0} = \frac{-4}{-1} = 4$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 4(x - (-1))$$

$$y = 4x + 4$$

9. A horizontal line that passes through the point (-1, 7).

A horizontal line is parallel to the x -axis and in the form $y = b$, therefore the equation is $y = 7$.

10. A horizontal line that passes through the point (2, -5).

A horizontal line is parallel to the x -axis and in the form $y = b$, therefore the equation is $y = -5$.

11. A vertical line that passes through the point (-4, 5).

A vertical line is parallel to the y -axis and in the form $x = a$, therefore the equation is $x = -4$.

12. A vertical line that passes through the point (2, -8).

A vertical line is parallel to the y -axis and in the form $x = a$, therefore the equation is $x = 2$.

In problems 13–17, find the x -intercept and the y -intercept.

Solutions for the x -intercept will be found by setting $y = 0$, and the y -intercepts will be found by setting $x = 0$.

13. $y = -3x + 3$

$$y = -3(0) + 3$$

$$y = 3$$

$(0, 3)$ y -intercept

$$0 = -3x + 3$$

$$3x = 3$$

$$x = 1$$

$(1, 0)$ x -intercept

14. $y = -2x - 2$

$$y = -2(0) - 2$$

$$y = -2$$

$(0, -2)$ y -intercept

$$0 = -2x - 2$$

$$2x = -2$$

$$x = -1$$

$(-1, 0)$ x -intercept

15. $y = \frac{1}{2}x - 1$

$$y = \frac{1}{2}(0) - 1$$

$$y = -1$$

$(0, -1)$ y -intercept

$$0 = \frac{1}{2}x - 1$$

$$-\frac{1}{2}x = -1$$

$$x = 2$$

$(2, 0)$ x -intercept

16. $y = \frac{4}{3}x - 4$

$$y = \frac{4}{3}(0) - 4$$

$$y = -4$$

$(0, -4)$ y -intercept

$$0 = \frac{4}{3}x - 4$$

$$-\frac{4}{3}x = -4$$

$$x = 3$$

$(3, 0)$ x -intercept

17. $x - 3y = -3$

$$0 - 3y = -3$$

$$y = 1$$

$(0, 1)$ y -intercept

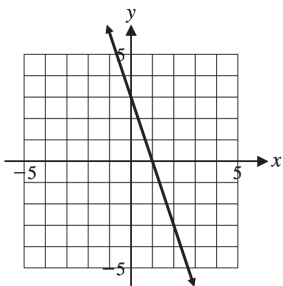
$$x - 3(0) = -3$$

$$x = -3$$

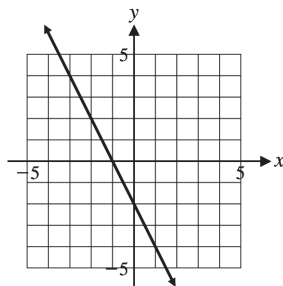
$(-3, 0)$ x -intercept

18. Graph each line in problems 13–17.

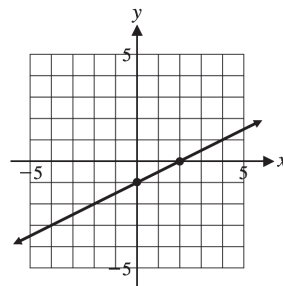
Graph for 13



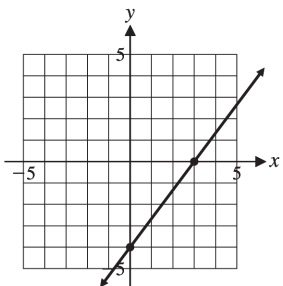
Graph for 14



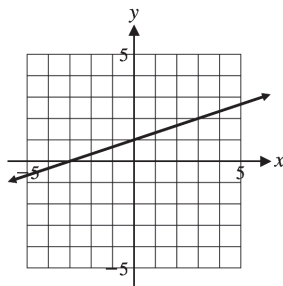
Graph for 15



Graph for 16



Graph for 17



19. A piece of equipment used in a landfill has an original value of \$100,000. After two years of use, the piece of equipment is valued at \$75,000.
- a) If the depreciation of the equipment is assumed to be linear, find an equation to relate the value (V) of the equipment over time (t).
- b) What would the value of the piece of equipment be after 5 years?
- c) In how many years would the value of the piece of equipment be \$0?

Solution:

- a) Since the value started at \$100,000 and after two (2) years it was worth \$75,000, the equipment depreciated as follows:

$$m = \frac{75,000 - 100,000}{2} = \frac{-25,000}{2} = -12,500$$

Since the slope is $-12,500$ and the equipment had a starting value of \$100,000, the equation is:

$$V = -12,500t + 100,000$$

- b) Substitute 5 in for t :

$$V = -12,500t + 100,000$$

$$V = -12,500(5) + 100,000$$

$$V = -62,500 + 100,000$$

$$V = 37,500$$

Therefore, the equipment will be worth \$37,500 after 5 years.

- c) Find the value of t when $V=0$.

$$V = -12,500t + 100,000$$

$$0 = -12,500t + 100,000$$

$$12,500t = 100,000$$

$$t = 8$$

Therefore, the value will be \$0 after 8 years.

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Section 1-3 Linear Regression

Goal: To interpret slopes and find linear regression equations

Solving Real-World Problems

1. Construct a mathematical model.
2. Solve the mathematical model.
3. Interpret the solution.

Linear Regression on a Graphing Calculator

1. Enter the data in columns L_1 and L_2 .
2. In the “STAT” mode, find the “LinReg” function.
3. Read the display to find the values of the slope and the y -intercept.

In problems 1–3, use the given information to answer the questions.

1. *Depreciation.* A new car worth \$36,000 is depreciating in value by \$4000 per year.
 - a) Find the linear model for the current value of the car, v , and the number of years, y , after it was purchased.
 - b) Interpret the slope of the model.
 - c) If the car is 5 years old, what does the model predict for its value?
 - d) After how many years will the car be worth nothing?

SOLUTION:

- a) $v = -4000y + 36,000$
- b) The value of the car decreases \$4000 for every year after it was purchased.
- c) $v = -4000y + 36,000$
 $v = -4000(5) + 36,000$
 $v = -20,000 + 36,000$
 $v = 16,000$

- The car is worth \$16,000 after it is 5 years old.
- d) $v = -4000y + 36,000$
 $0 = -4000y + 36,000$
 $4000y = 36,000$
 $y = 9$

The car will be worth nothing after 9 years.

2. *Health Club Membership.* A health club offers membership for a fee of \$99 plus a monthly fee of \$10 per month.
- a) Find the linear model for the membership fee, f , and the number of months, m , since you have been a member.
- b) Interpret the slope of the model.
- c) If you have been a member for 36 months, what does the model predict for the fee you have paid so far?
- d) After how many months will you have paid the health club \$279?

SOLUTION:

- a) $f = 10m + 99$
- b) The fee will increase by \$10 for every additional month of membership.
- c) $f = 10m + 99$
 $f = 10(36) + 99$
 $f = 360 + 99$
 $f = 459$
The fee after 36 months will be \$459.
- d) $f = 10m + 99$
 $279 = 10m + 99$
 $180 = 10m$
 $18 = m$
After 18 months, you will have paid \$279 for your membership.

3. *Stress.* The table below shows the relationship between a stress test score and the diastolic blood pressure for 8 patients. A linear regression model for this data is:

$$y = 0.56x + 41.71$$

where x represents the stress test score and y represents the blood pressure.

Stress Test Score, x	55	62	58	78	92	88	75	80
Blood Pressure, y	70	85	72	85	96	90	82	85

- a) Interpret the slope of the model.
- b) Use the model to predict the blood pressure for a person with a stress test score of 75
- c) Use the model to estimate the stress test score for if the diastolic blood pressure was 90.

SOLUTION:

- a) For every 1 point increase in the stress test score, the diastolic blood pressure will increase by 0.56 points.
- b)
$$y = 0.56x + 41.71$$
$$y = 0.56(75) + 41.71$$
$$y = 42 + 41.71$$
$$y = 83.71$$

A person with a stress test score of 75 will have an approximate diastolic blood pressure of 84.
- c)
$$y = 0.56x + 41.71$$
$$90 = 0.56x + 41.71$$
$$48.29 = 0.56x$$
$$86.23 = x$$

A person with a diastolic blood pressure of 90 will have a stress test score of approximately 86.

