CHAPTER 1
Functions, Graphs, and Models; Linear Functions

Toolbox Exercises

1. $\{1,2,3,4,5,6,7,8\}$ and
$\{x \mid x < 9, x \in N\}$
Remember that $x \in N$ means that x is a natural number.

2. Yes.

3. No. A is not a subset of B. A contains elements 2, 7, and 10 which are not in B.

4. No. $N = \{1,2,3,4,...\}$. Therefore, $\frac{1}{2} \notin N$.

5. Yes. Every integer can be written as a fraction with the denominator equal to 1.

6. Yes. Irrational numbers are by definition numbers that are not rational.

7. Integers. However, note that this set of integers could also be considered as a set of rational numbers. See question 5.

8. Rational numbers

9. Irrational numbers

10. $x > -3$

11. $-3 \le x \le 3$

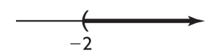
12. $x \le 3$

13. $(-\infty, 7]$

14. $(3, 7]$

15. $(-\infty, 4)$

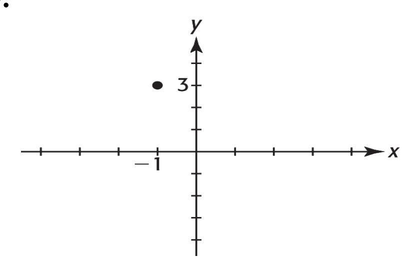
16.



17. Note that $5 > x \ge 2$ implies $2 \le x < 5$, therefore:

18.

19.



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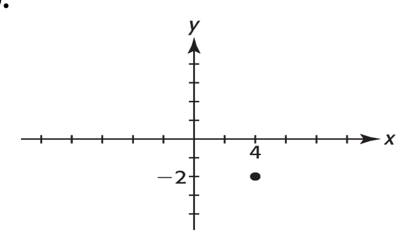

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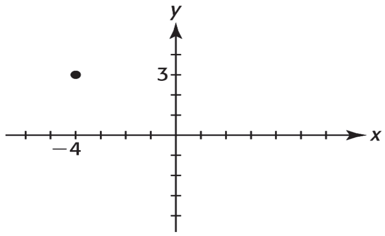

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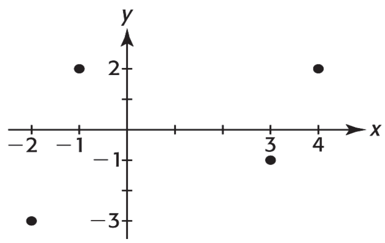

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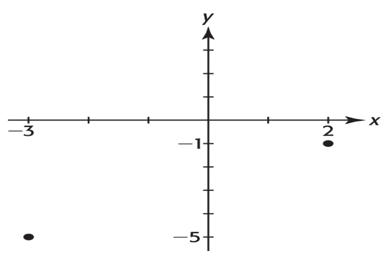


24. Yes, it is a polynomial with degree 4.

25. No, it is not a polynomial since there is a variable in a denominator.

26. No, it is not a polynomial since there is a variable inside a radical.

27. Yes, it is a polynomial with degree 6.

28. The x^2 term has a coefficient of –3. The x term has a coefficient of –4. The constant term is 8.

29. The x^4 term has a coefficient of 5. The x^3 term has a coefficient of 7. The constant term is –3.

30. $\left(z^4-15z^2+20z-6\right)+\left(2z^4+4z^3-12z^2-5\right)$
$=\left(z^4+2z^4\right)+\left(4z^3\right)+\left(-15z^2-12z^2\right)+$
$\left(20z\right)+\left(-6-5\right)$
$=3z^4+4z^3-27z^2+20z-11$

31. $3x+2y^4-2x^3y^4-119-5x-3y^2+5y^4+110$
$=\left(2y^4+5y^4\right)-2x^3y^4-3y^2+$
$\left(3x-5x\right)+\left(-119+110\right)$
$=7y^4-2x^3y^4-3y^2-2x-9$

32. $4\left(p+d\right)=4p+4d$

33. $-2\left(3x-7y\right)=-6x+14y$

34. $-a\left(b+8c\right)$
$=-ab-8ac$

35. $4\left(x-y\right)-\left(3x+2y\right)$
$=4x-4y-3x-2y$
$=1x-6y$
$=x-6y$

36. $4\left(2x-y\right)+4xy-5\left(y-xy\right)-\left(2x-4y\right)$
$=8x-4y+4xy-5y+5xy-2x+4y$
$=\left(8x-2x\right)+\left(4xy+5xy\right)+\left(-4y-5y+4y\right)$
$=6x+9xy-5y$

37. $2x\left(4yz-4\right)-\left(5xyz-3x\right)$
$=8xyz-8x-5xyz+3x$
$=\left(8xyz-5xyz\right)+\left(-8x+3x\right)$
$=3xyz-5x$

38. $\left(4x^2y^3\right)\left(-3a^2x^3\right) = -12a^2x^5y^3$

39. $2xy^3\left(2x^2y + 4xz - 3z^2\right)$
$= 4x^3y^4 + 8x^2y^3z - 6xy^3z^2$

40. $(x-7)(2x+3) = 2x^2 + 3x - 14x - 21$
$= 2x^2 - 11x - 21$

41. $(k-3)^2 = (k-3)(k-3)$
$= k^2 - 3k - 3k + 9$
$= k^2 - 6k + 9$

42. $(4x-7y)(4x+7y) = 16x^2 + 28xy - 28xy - 49y^2$
$= 16x^2 - 49y^2$

43. $\dfrac{12x - 5x^2}{x} = 12 - 5x$

44. $\dfrac{8x^2 + 2x}{2x} = 4x + 1$

Section 1.1 Skills Check

1. Using Table A

a. –5 is an x-value and therefore is an input into the function $f(x)$.

b. $f(-5)$ represents an output from the function.

c. The domain is the set of all inputs. D: $\{-9,-7,-5,6,12,17,20\}$. The range is the set of all outputs. R: $\{4,5,6,7,9,10\}$

d. Each input x into the function f yields exactly one output $y=f(x)$.

2. Using Table B

a. 0 is an x-value and therefore is an input into the function $g(x)$.

b. $g(7)$ represents an output from the function.

c. The domain is the set of all inputs. D: $\{-4,-1,0,1,3,7,12\}$. The range is the set of all outputs. R: $\{3,5,7,8,9,10,15\}$

d. Each input x into the function g yields exactly one output $y=g(x)$.

3. $f(-9)=5$
$f(17)=9$

4. $g(-4)=5$
$g(3)=8$

5. No. In the given table, x is not a function of y. If y is considered the input variable, one input will correspond with more than one output. Specifically, if $y=9$, then $x=12$ or $x=17$.

6. Yes. Each input y produces exactly one output x.

7. **a.** $f(2)=-1$, since $x = 2$ in the table corresponds with $f(x)=-1$.

b.
$$\begin{aligned} f(2) &= 10-3(2)^2 \\ &= 10-3(4) \\ &= 10-12 \\ &= -2 \end{aligned}$$

c. $f(2)=-3$, since $(2,-3)$ is a point on the graph.

8. **a.** $f(-1)=5$, since $(-1,5)$ is a point on the graph.

b. $f(-1)=-8$, since $x = -1$ in the table corresponds with $f(x) = -8$.

c.
$$\begin{aligned} f(-1) &= (-1)^2+3(-1)+8 \\ &= 1-3+8 \\ &= 6 \end{aligned}$$

9.

x	y
0	2
–2	–4

10.

x	y
4	1
–4	–3

11. Recall that $R(x)=5x+8$.

a. $R(-3)=5(-3)+8=-15+8=-7$

b. $R(-1)=5(-1)+8=-5+8=3$

c. $R(2)=5(2)+8=10+8=18$

12. Recall that $C(s)=16-2s^2$.

a. $$\begin{aligned}C(3)&=16-2(3)^2\\&=16-2(9)\\&=16-18\\&=-2\end{aligned}$$

b. $$\begin{aligned}C(-2)&=16-2(-2)^2\\&=16-2(4)\\&=16-8\\&=8\end{aligned}$$

c. $$\begin{aligned}C(-1)&=16-2(-1)^2\\&=16-2(1)\\&=16-2\\&=14\end{aligned}$$

13. Yes. Each input corresponds with exactly one output. The domain is $\{-1,0,1,2,3\}$. The range is $\{-8,-1,2,5,7\}$.

14. No. Each input x does not match with exactly one output y. Specifically, if $x=2$ then $y=-3$ or $y=4$.

15. No. The graph fails the vertical line test. Each input does not match with exactly one output.

16. Yes. The graph passes the vertical line test. Each input matches with exactly one output.

17. Yes. The graph passes the vertical line test. Each input matches with exactly one output.

18. No. The graph fails the vertical line test. Each input does not match with exactly one output.

19. No. If $x=3$, then $y=5$ or $y=7$. One input yields two outputs. The relation is not a function.

20. Yes. Each input x yields exactly one output y.

21. a. Not a function. If $x=4$, then $y=12$ or $y=8$.

b. Yes. Each input yields exactly one output.

22. a. Yes. Each input yields exactly one output.

b. Not a function. If $x=3$, then $y=4$ or $y=6$.

23. a. Not a function. If $x=2$, then $y=3$ or $y=4$.

b. Function. Each input yields exactly one output.

24. a. Function. Each input yields exactly one output.

b. Not a function. If $x=-3$, then $y=3$ or $y=-5$.

25. The domain is the set of all inputs. D: $\{-3,-2,-1,1,3,4\}$. The range is the set of all outputs. R: $\{-8,-4,2,4,6\}$

26. The domain is the set of all inputs. D: $\{-6,-4,-2,0,2,4\}$. The range is the set of all outputs. R: $\{-5,-2,0,1,4,6\}$

27. Considering *y* as a function of *x*, the domain is the set of all inputs, *x*. Therefore the domain is D: $[-10, 8]$. The range is the set of all outputs, *y*. Therefore, the range is R: $[-12, 2]$.

28. Considering *y* as a function of *x*, the domain is the set of all inputs, *x*. Therefore the domain is D: $[-4, 3]$. The range is the set of all outputs, *y*. Therefore, the range is R: $[-1, 4]$.

29. Considering *y* as a function of *x*, the domain is the set of all inputs, *x*. Therefore the domain is D: $(-\infty,\infty)$. The range is the set of all outputs, *y*. Therefore, the range is R: $[-4,\infty)$.

30. Considering *y* as a function of *x*, the domain is the set of all inputs, *x*. Therefore the domain is D: $(-\infty,3]$. The range is the set of all outputs, *y*. Therefore, the range is R: $[0,\infty)$.

31. The input is the number of years after 2000, therefore 2015 is 15 years after 2000 and the input would be $x = 15$. Similarly, 2022 is 22 years after 2000, and the input would be $x = 22$.

32. The input is the number of years after 1990, therefore 1990 is 0 years after 1990 and the input would be $x = 0$. Similarly, 2015 is 25 years after 1990, and the input would be $x = 25$. Therefore, the input would be 0 to 25.

33. No. If $x = 0$, then $(0)^2 + y^2 = 4 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$. So, one input of 0 corresponds with 2 outputs of –2 and 2. Therefore the equation is not a function.

34. Yes. Each input for *x* corresponds with exactly one output for *y*.

35. $F = \frac{9C}{5} + 32,$ where *F* is the Fahrenheit temperature and *C* is the Celsius temperature.

36. $C = 2\pi r$, where *C* is the circumference and *r* is the radius.

37. *C* is found by using the steps; a. subtract 32; b. multiply by 5; c. divide by 9.

38. *D* is found by squaring *E*, multiplying the result by 3, and subtracting 5.

Section 1.1 Exercises

39. **a.** Yes. Each input (*a*, age in years) corresponds with exactly one output (*p*, life insurance premium). The independent variable is *a*, age in years, and the dependent variable is *p*, life insurance premium.

b. No. One input of \$11.81 corresponds with six outputs (*a*, age in years).

40. Yes. Each input (*d*, degree) corresponds with exactly one output (*M*, mean earnings). The independent variable is *d*, highest degree for females, and the dependent variable is *M*, mean earnings in dollars.

41. Yes. Each input (*y*, year) corresponds with exactly one output (*p*, percent). The independent variable is *y*, the year, and the dependent variable is *p*, the percent of Americans who are obese.

42. T, temperature, is a function of m, number of minutes after the power outage, since each value for m corresponds with exactly one value for T. The graph of the equation passes the vertical line test, which implies there is one temperature for each value of m, number of minutes after the power outage.

43. **a.** Yes. Each input (the barcode) corresponds with exactly one output (an item's price).

b. No. Every input (an item's price) could correspond with more than one output (the barcode). Numerous items can have the same price but different barcodes.

44. **a.** Yes. Each input (a child's piano key) corresponds with exactly one output (a musical note). Since the domain is the set of all inputs into the function and there are 12 keys on the child's piano keyboard, there are 12 elements in the domain of the function.

b. Yes. Each input (a note from the child's piano) corresponds with exactly one output (a piano key). Since the range is the set of all outputs from the function and there are 12 keys on the child's piano keyboard, there are 12 elements in the range of the function.

45. Each input (x, years) corresponds with exactly one output (V, value of the property). The graph of the equation passes the vertical line test.

46. Yes. Each input (d, depth) corresponds with exactly one output (p, pressure). The graph of the equation passes the vertical line test.

47. **a.** Yes. Each input (day of the month) corresponds with exactly one output (weight).

b. The domain is the first 14 days of May or D: $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$.

c. The range is $\{171,172,173,174,175,176,177,178\}$.

d. The highest weights were on May 1 and May 3.

e. The lowest weight was on May 14.

f. Three days from May 8 until May 11.

48. **a.** No. One input of 75 matches with two outputs of 70 and 81.

b. Yes. Each input (average score on the final exam) matches with exactly one output (average score on the math placement test).

49. **a.** $P(3) = \$1096.78$. If the car is financed over three years, the payment is \$1096.78.

b. $C(5) = \$42,580.80$.

c. $t = 4$. If the total cost is \$41,014.08, then the car has been financed over four years,

d. Since $C(5)$ = \$42,580.80, and $C(3)$ = \$39,484.08, the savings would be $C(5) - C(3)$ = \$3096.72.

50. **a.** The couple must make payments for 20 years.

$f(103{,}000) = 20$

b. $f(120{,}000) = 30$. It will take the couple 30 years to payoff a \$120,000 mortgage at 7.5%.

c. $f(3 \cdot 40{,}000) = f(120{,}000) = 30$

d. If $A = 40{,}000$ then $f(A) = f(40{,}000) = 5$.

e. $f(3\cdot 40{,}000)=f(120{,}000)=30$
$3\cdot f(40{,}000)=3\cdot 5=15$
The expressions are not equal.

51. **a.** Approximately 22 million

b. $f(1930)=11$. Approximately 11 million women were in the work force in 1930.

c. D: $\{1930,1940,1950,1960,1970,$
$1980,1990,2000,2005,2010,2015\}$

d. Increasing. As the year increases, the number of women in the work force also increases.

52. **a.** When $t=2020$, the ratio is approximately 3 to 1.

b. $f(2005)=4$. For year 2005, the projected ratio of the working-age population to the elderly is 4 to 1.

c. The domain is the set of all possible inputs. In this example, the domain consists of all the years, *t*, represented in the figure. Specifically, the domain is $\{1995,2000,2005,2010,2015,$
$2020,2025,2030\}$.

d. As the years, *t*, increase, the projected ratio of the working-age population to the elderly decreases. Notice that the bars in the figure grow smaller as the time increases.

53. **a.** $f(1890)=26.1$
$f(2015)=29.2$

b. $g(1940)=21.5$
$g(2013)=26.6$

c. If $x=1980$, then $f(x)=24.7$. The median age at first marriage for men in 1980 was 24.7 years.

d. $f(2015)=29.2>22.8=f(1960)$. Therefore, the median age at first marriage for men increased between 1960 and 2015.

54. **a.** $f(2030)=39{,}244$

b. The population of females under the age of 18 is projected to be 39,244,000 in the year 2030.

c. The function is increasing. The number of females under the age of 18 is projected to keep increasing.

55. **a.** In 2020, 5.7 million U.S. citizens age 65 and older are expected to have Alzheimer's disease.

b. $f(2030)=7.7$. In 2030, 7.7 million U.S. citizens age 65 and older are expected to have Alzheimer's disease.

c. 2040. $f(2040)=11$.

d. The function is increasing. Since 2000, the number of U.S citizen's 65 and older that are expected to have Alzheimer's disease in going up.

56. **a.** Yes. Each year, *t*, corresponds with exactly one number of U.S. farms, *N*.

b. $f(1940)=6.3\text{(millions)}$. $f(1940)$ represents the number of U.S. farms in millions in the year 1940.

c. If $f(t)=2.4$, then $t=1990$.

d. $f(2012)=2.1$ implies that in 2012, there were 2.1 million farms in the U.S.

57. a. $f(1995) = 56.0$. In 1995, the birth rate for U.S. girls ages 15 to 19 was 56.0 per 1000 girls.

b. 2005. $f(2005) = 40.5$.

c. The birth rate appears to be at its maximum (59.9) in the year 1990.

d. In the years 2005 to 2009, the birth rate appears to have increased until 2007, then decreased.

58. a. $f(1990) = 3.4$. In 1990 there were 3.4 workers for each retiree.

b. 2030. $f(2030) = 2$.

c. As the years increase, the number of workers available to support retirees decreases. Therefore, funding for social security into the future is problematic. Workers will need to pay a larger portion of their salaries to fund payments to retirees.

59. a. $R(200) = 32(200) = 6400$. The revenue generated from selling 200 golf hats is \$6400.

b. $R(2500) = 32(2500) = \$80{,}000$

60. a. $C(200) = 4000 + 12(200) = 6400$. The cost of producing 200 golf hats is \$6400.

b. $C(2500) = 4000 + 12(2500)$
$= \$34{,}000$

61. a. $f(1000) = 0.857(1000) + 19.35$
$= 857 + 19.35$
$= 876.35$
The monthly charge for using 1000 kilowatt hours is \$876.35.

b. $f(1500) = 0.857(1500) + 19.35$
$= 1285.50 + 19.35$
$= 1304.85$
The monthly charge for using 1500 kilowatt hours is \$1304.85.

62. a. $P(500) = 450(500) - 0.1(500)^2 - 2000$
$= 225{,}000 - 25{,}000 - 2000$
$= 198{,}000$
The profit generated from the production and sale of 500 iPod players is \$198,000.

b. $P(4000)$
$= 450(4000) - 0.1(4000)^2 - 2000$
$= 1{,}800{,}000 - 1{,}600{,}000 - 2000$
$= 198{,}000$
$P(4000) = \$198{,}000$

63. a. $P(100) = 32(100) - 0.1(100)^2 - 1000$
$= 3200 - 1000 - 1000$
$= 1200$
The daily profit from the production and sale of 100 Blue Chief bicycles is \$1200.

b. $P(160) = 32(160) - 0.1(160)^2 - 1000$
$= 5120 - 2560 - 1000 = 1560$
The daily profit from the production and sale of 160 Blue Chief bicycles is \$1560.

64. a. $h(1) = 6 + 96(1) - 16(1)^2$
$= 6 + 96 - 16$
$= 86$
The height of the ball after one second is 86 feet.

b. $h(3)=6+96(3)-16(3)^2$
$=6+288-144$
$=150$
After three seconds the ball is 150 feet high.

c. Test $t=2$.
$h(2)=6+96(2)-16(2)^2$
$=6+192-64$
$=134$

Test $t=4$.
$h(4)=6+96(4)-16(4)^2$
$=6+384-256$
$=134$

Test $t=5$.
$h(5)=6+96(5)-16(5)^2$
$=6+480-400$
$=86$

Since h(1) = 86, and h(2) = 134, and h(3) = 150, and h(4) = 134, and h(5) = 86, it appears that the ball stops climbing after 3 seconds and begins to fall. One might conclude that the ball reaches its maximum height at 3 seconds since at 1 and 5 seconds, and again at 2 and 4 seconds, the respective heights are the same.

65. a. $0.3+0.7n=0$
$0.7n=-0.3$
$\frac{0.7n}{0.7}=\frac{-0.3}{0.7}$
$n=-\frac{3}{7}$
Therefore the domain of $R(n)$ is all real numbers except $-\frac{3}{7}$ or $\left(-\infty,-\frac{3}{7}\right)\text{U}\left(-\frac{3}{7},\infty\right)$.

b. In the context of the problem, n represents the factor for increasing the number of questions on a test. Therefore it makes sense that n is positive ($n>0$).

66. a. Yes, since each value of s produces exactly one value of K_c.

b. Any input into the function must not create a negative number under the radical. Therefore, the radicand, $4s+1$, must be greater than or equal to zero. Isolating s yields:
$4s+1\geq 0$
$4s+1-1\geq 0-1$
$4s\geq -1$
$s\geq -\frac{1}{4}$
Therefore, the domain defined by the equation is all real numbers greater than or equal to $-\frac{1}{4}$ or, in interval notation, $\left[-\frac{1}{4},\infty\right)$.

c. Since s represents wind speed in the given function, and wind speed cannot be less than zero, the domain of the function is restricted based on the physical context of the problem. Even though the domain implied by the function is $\left[-\frac{1}{4},\infty\right)$, the actual domain in the given physical context is $[0,\infty)$.

67. a. Since p is a percentage, $0\leq p\leq 100$. However in the given function, the denominator, $100-p$, cannot equal zero. Therefore, $p\neq 100$. The domain is $0\leq p<100$ or, in interval notation, $[0,100)$.

b. $C(60)=\dfrac{237{,}000(60)}{100-60}=355{,}500$

$C(90)=\dfrac{237{,}000(90)}{100-90}=2{,}133{,}000$

68. a. Any input into the function must not create a negative number under the square root. Therefore, $2p+1\ge 0$. Isolating p yields

$2p+1\ge 0$

$2p\ge -1$

$p\ge -\dfrac{1}{2}$

Since the denominator cannot equal zero, $p\ne -\dfrac{1}{2}$.

Therefore the domain of q is $\left(-\dfrac{1}{2},\infty\right)$.

b. In the context of the problem, p represents the price of a product. Since the price can not be negative, $p\ge 0$. The domain is $[0,\infty)$. Also, since q represents the quantity of the product demanded by consumers, $q\ge 0$. The range is $(0,100]$.

69. a. $V(12)=(12)^2(108-4(12))$

$=144(108-48)$

$=144(60)$

$=8640$

$V(18)=(18)^2(108-4(18))$

$=324(108-72)$

$=324(36)$

$=11{,}664$

b. First, since x represents a side length in the diagram, x must be greater than zero. Second, to satisfy postal restrictions, the length (longest side) plus the girth must be less than or equal to 108 inches. Therefore,

Length + Girth ≤ 108

Length $+4x\le 108$

$4x\le 108-\text{Length}$

$x\le \dfrac{108-\text{Length}}{4}$

$x\le 27-\dfrac{\text{Length}}{4}$

Since x is greatest if the longest side is smallest, let the length equal zero to find the largest value for x.

$x\le 27-\dfrac{0}{4}$

$x\le 27$

Therefore the conditions on x are $0<x\le 27$. If $x=27$, the length would be zero and the package would not exist. Therefore, in the context of the question, $0<x<27$ and the corresponding domain for the function $V(x)$ is $(0,27)$.

c.

x	*Volume*
10	6800
12	8640
14	10192
16	11264
18	**11664**
20	11200
22	9680

The maximum volume occurs when $x=18$. Therefore the dimensions that maximize the volume of the box are 18 inches by 18 inches by 72 inches, a total of 108 inches.

70. a. $S(0)=-4.9(0)^2+98(0)+2=2$

The initial height of the bullet is 2 meters.

b.

t	*Height*
9	487.1
9.5	490.78
10	492
10.5	490.78
11	487.1

$S(9) = 487.1$

$S(10) = 492$

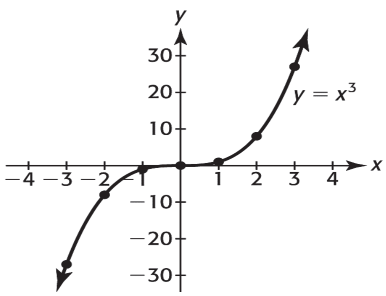
$S(11) = 487.1$

c. The bullet seems to reach a maximum height at 10 seconds and then begins to fall. See the table in part b) for further verification, using 9.5 seconds and 10.5 seconds.

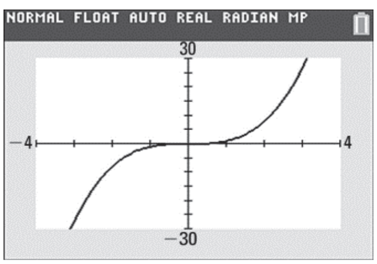
Section 1.2 Skills Check

1. a.

x	$y = x^3$	(x, y)
–3	–27	(–3, –27)
–2	–8	(–2, –8)
–1	–1	(–1, –1)
0	0	(0, 0)
1	1	(1, 1)
2	8	(2, 8)
3	27	(3, 27)



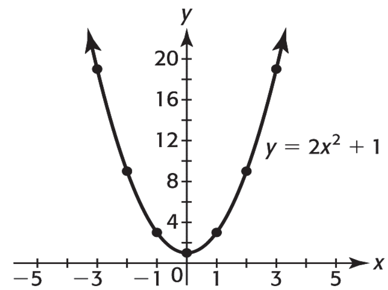
b.



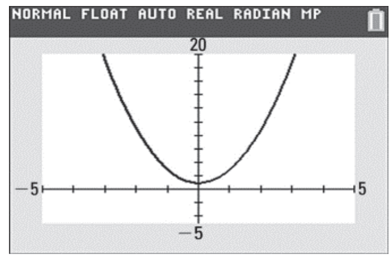
c. Your hand-drawn graph in part a) by plotting points from the table should match the calculator-drawn graph in part b).

2. a.

x	$y = 2x^2 + 1$	(x, y)
–3	19	(–3, 19)
–2	9	(–2, 9)
–1	3	(–1, 3)
0	1	(0, 1)
1	3	(1, 3)
2	9	(2, 9)
3	19	(3, 19)



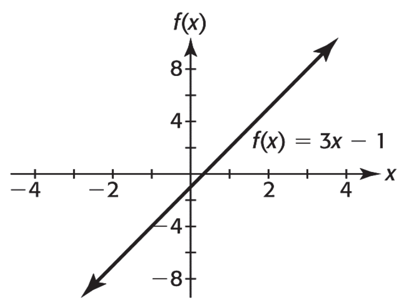
b.



c. Your hand-drawn graph in part a) by plotting points from the table should match the calculator-drawn graph in part b).

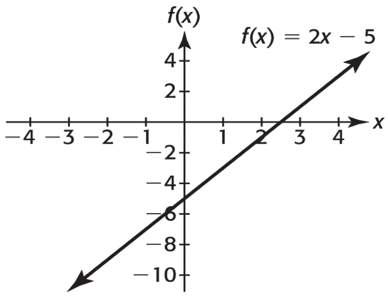
3.

x	–2	–1	0	1	2
y	–7	–4	–1	2	5



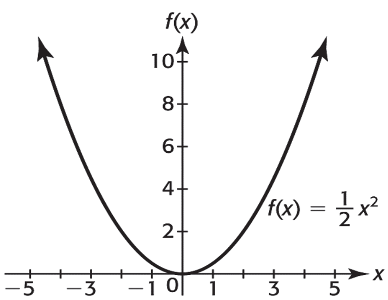
4.

x	-2	-1	0	1	2
y	-9	-7	-5	-3	-1



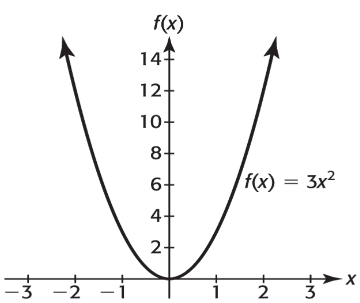
5.

x	-4	-2	0	2	4
y	8	2	0	2	8



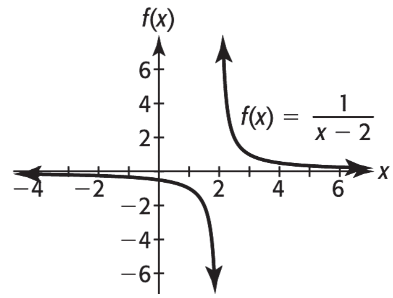
6.

x	-2	-1	0	1	2
y	12	3	0	3	12



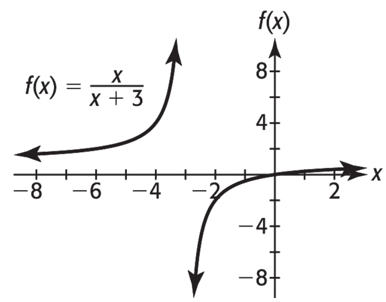
7.

x	0	1	2	3	4
y	$-\frac{1}{2}$	-1	undefined	1	$\frac{1}{2}$

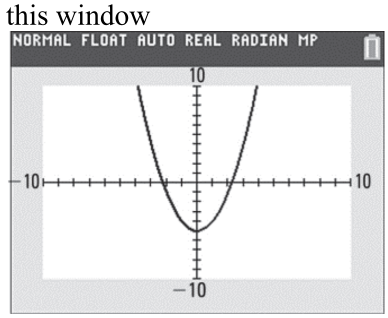


8.

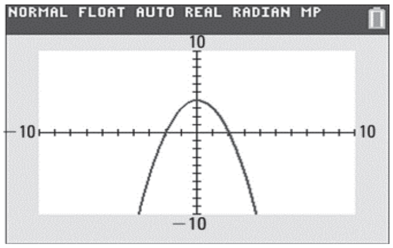
x	-5	-4	-3	-2	-1
y	$\frac{5}{2}$	4	undefined	-2	$-\frac{1}{2}$



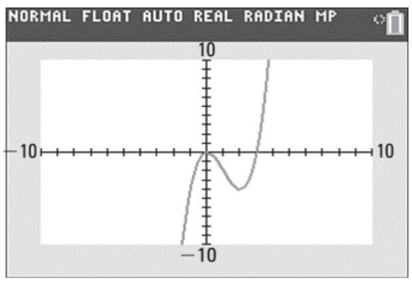
9. $y = x^2 - 5$, yes there is a turning point in this window



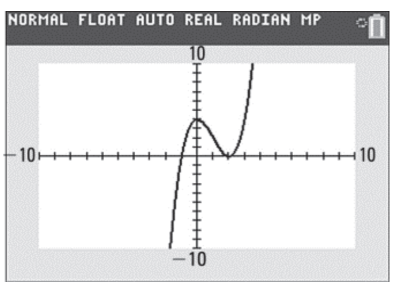
10. $y = 4 - x^2$, yes there is a turning point in this window



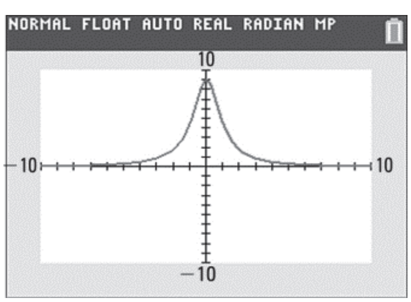
11. $y = x^3 - 3x^2$, yes there are 2 turning points in this window



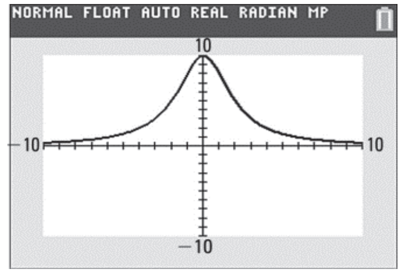
12. $y = x^3 - 3x^2 + 4$, yes there are 2 turning points in this window



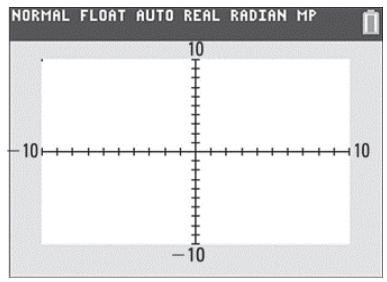
13. $y = 9/\left(x^2 + 1\right)$, yes there is a turning point in this window



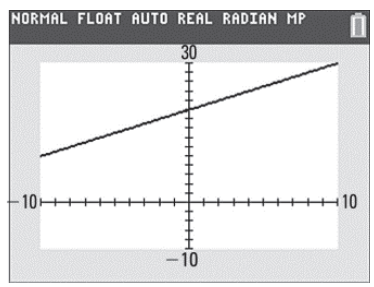
14. $y = 40/\left(x^2 + 4\right)$, yes there is a turning point in this window



15. a. $y = x + 20$

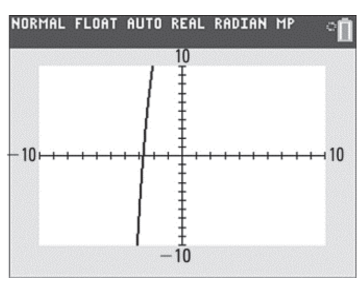


b. $y = x + 20$

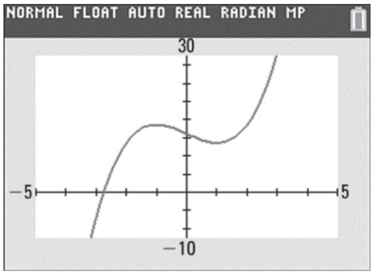


View b) is better.

16. a. $y = x^3 - 3x + 13$

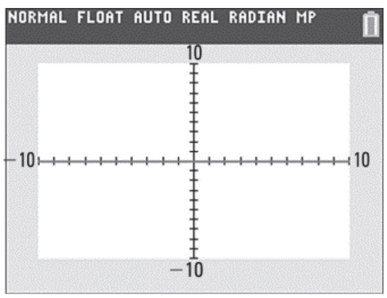


b. $y = x^3 - 3x + 13$

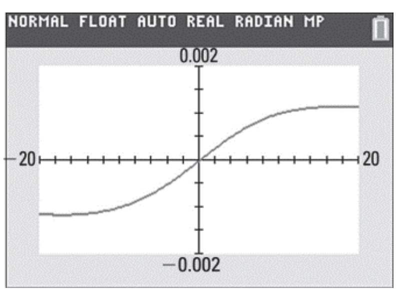


View b) is better.

17. a. $y = \dfrac{0.04(x - 0.1)}{x^2 + 300}$

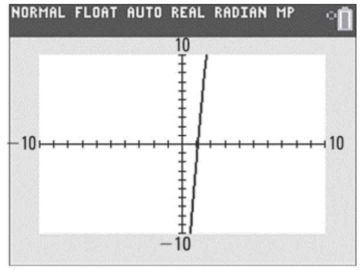


b. $y = \dfrac{0.04(x - 0.1)}{x^2 + 300}$

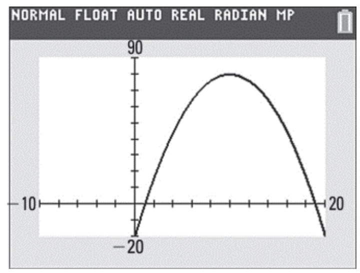


View b) is better.

18. a. $y = -x^2 + 20x - 20$



b. $y = -x^2 + 20x - 20$

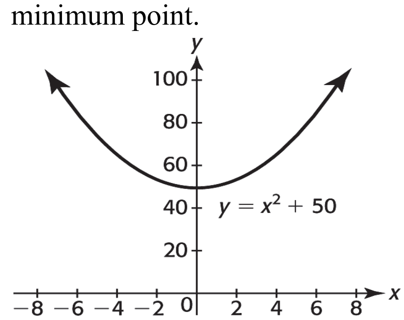


View b) is better.

19. When $x = -8$ or $x = 8$, $y = 114$. When $x = 0$, $y = 50$. Letting y vary from -5 to 100 gives one view.

$y = x^2 + 50$

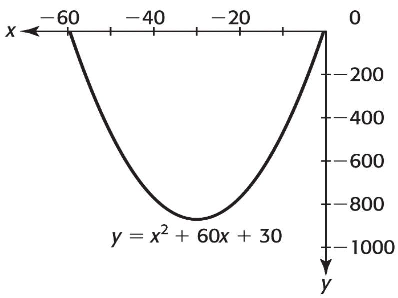
The turning point is at $(0, 50)$ and is a minimum point.



20. When $x = -60, y = 30$. When $x = 0, y = 30$. When $x = -30, y = -870$. Letting y vary from -1000 to 0 gives one view.

$$y = x^2 + 60x + 30$$

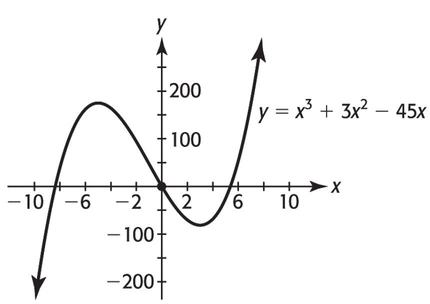
The turning point is at $(-30, -870)$ and is a minimum point.



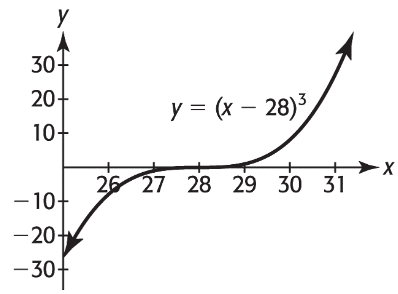
21. When $x = -10, y = -250$. When $x = 10, y = 850$. When $x = 0, y = 0$. Letting y vary from -200 to 300 gives one view.

$$y = x^3 + 3x^2 - 45x$$

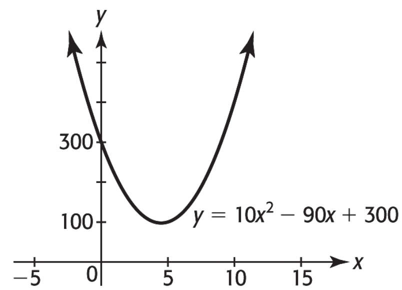
The turning points are at $(-5, 175)$, a maximum, and at $(3, -81)$, a minimum.



22. When $x = 28,\ y = 0$. When $x = 25,\ y = -27$. When $x = 31,\ y = 27$. Letting y vary from -30 to 30 gives one view. There is no turning point.

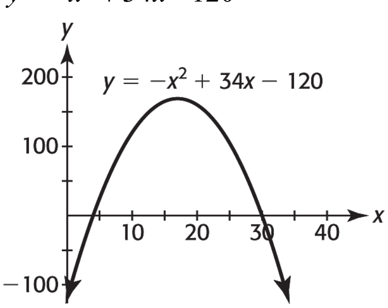


23.



Note that answers for the window may vary.

24. $y = -x^2 + 34x - 120$



Note that answers for the window may vary.

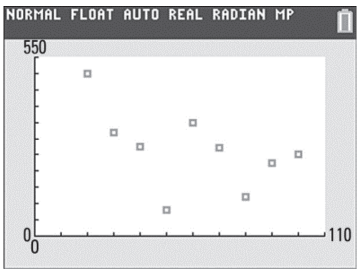
25.

t	$S(t) = 5.2t - 10.5$
12	51.9
16	72.7
28	135.1
43	213.1

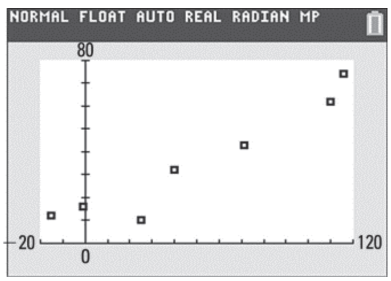
26.

q	$f(q)=3q^2-5q+8$
–8	240
–5	108
24	1616
43	5340

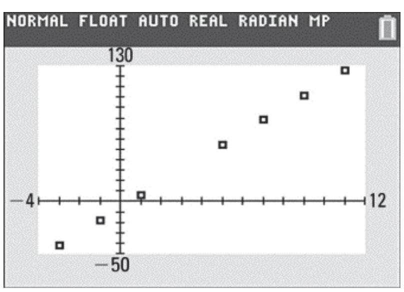
27.



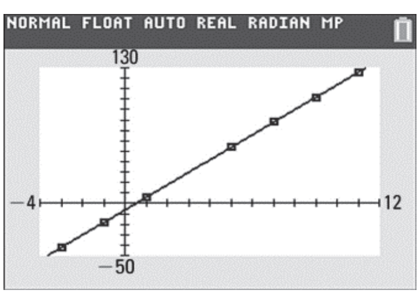
28.



29. a.

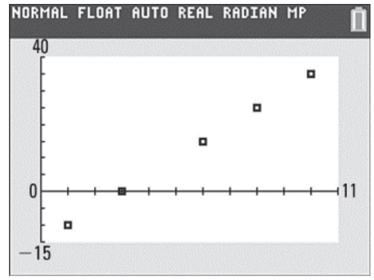


b.

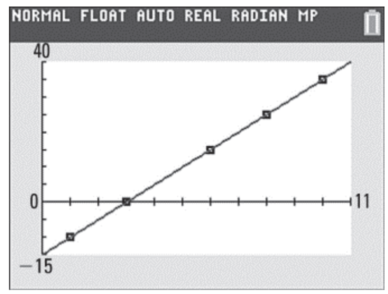


c. Yes. Yes.

30. a.



b.



c. Yes. Yes.

31. a.
$$\begin{aligned} f(20) &= (20)^2 - 5(20) \\ &= 400 - 100 \\ &= 300 \end{aligned}$$

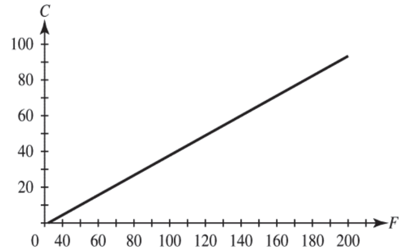
b. $x = 20$ implies 20 years after 2000. Therefore the answer to part a) yields the millions of dollars earned in 2020.

32. a.
$$\begin{aligned} f(10) &= 100(10)^2 - 5(10) \\ &= 10{,}000 - 50 \\ &= 9950 \end{aligned}$$

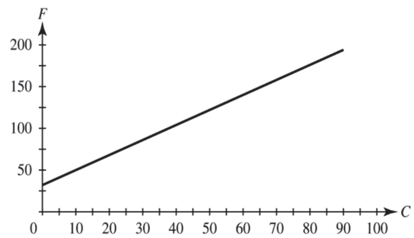
b. In 2010, $x = 10$. Therefore, 9950 thousands of units or 9,950,000 units are produced in 2010.

Section 1.2 Exercises

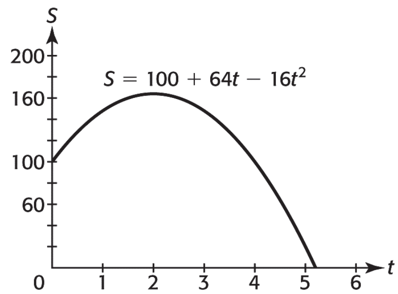
33.



34.



35. a.



b.

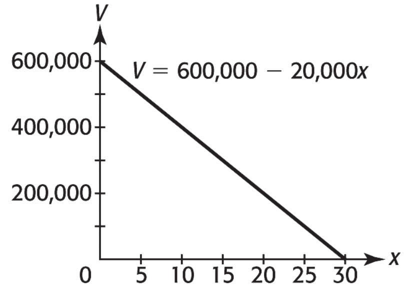
X	Y1
0	100
1	148
2	164
3	148
4	100
5	20
6	-92

X=0

Considering the table, $S = 148$ feet when x is 1 or when x is 3. The height is the same for two different times because the height of the ball increases, reaches a maximum height, and then decreases.

c. From the table in part b), it appears the maximum height is 164 feet, occurring 2 seconds into the flight of the ball.

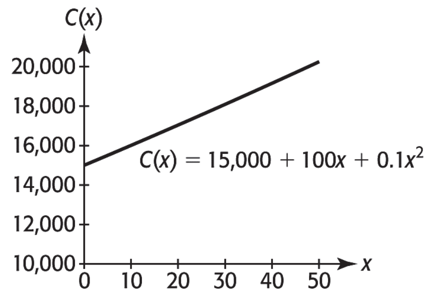
36. a.



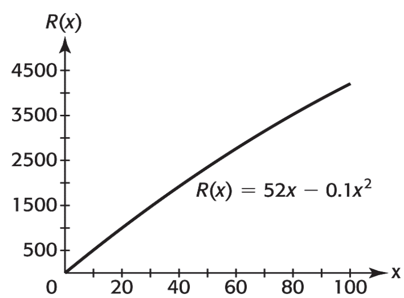
b. When $x = 10$,

$$V = 600{,}000 - 20{,}000(10)$$
$$= 600{,}000 - 200{,}000$$
$$= \$400{,}000$$

37.



38.



39. a. $P = 0.79x + 20.86$

$$= 0.79(70) + 20.86$$
$$= 55.3 + 20.86$$
$$= 76.16$$

The model predicts that there will be 76.16 million women in the workforce in 2020.

b. 80; 2030 – 1950 = 80

$$P = 0.79x + 20.86$$
$$= 0.79(80) + 20.86$$
$$= 63.2 + 20.86$$
$$= 84.06$$

The model predicts that there will be 84.06 million (or 84,060,000) women in the workforce in 2030.

40. a. $x = \text{Year} - 2000$

For 2005, $x = 2005 - 2000 = 5$

For 2016, $x = 2016 - 2000 = 16$

b. For $x = 10$.

$$y = -0.002(10)^4 + 0.051(10)^3 - 0.371(10)^2 + 1.055(10) + 4.234$$
$$= 8.684$$

The unemployment rate in the U.S. for 2010 was 8.684%.

c. For 2015, $x = 15$. Therefore,

$$y = -0.002(15)^4 + 0.051(15)^3 - 0.371(15)^2 + 1.055(15) + 4.234$$
$$= 7.459$$

The unemployment rate in the U.S. for 2015 was 7.459%.

41. a. $t = \text{Year} - 2005$

For 2010, $t = 2010 - 2005 = 5$

For 2016, $t = 2016 - 2005 = 11$

b. $P = f(10)$ represents the value of P in 2015 $(2005 + 10 = 2015)$.

$$f(10) = 1.84(10) + 60.89 = 79.29$$

79.29 represents the percent of households with Internet access in 2015.

c. $x_{\min} = 2005 - 2005 = 0$

$x_{\max} = 2020 - 2005 = 15$

42. a. $t = \text{Year} - 2000$

For 2012, $t = 2012 - 2000 = 12$

For 2016, $t = 2016 - 2000 = 16$

For 2020, $t = 2020 - 2000 = 20$

b. $S = f(18)$ represents the value of S in 2018 $(2000 + 18 = 2018)$.

$$f(18) = 2.4807(18)^3 - 52.25(18)^2 + 528.68(18) + 5192.6$$
$$= 12{,}246.9$$

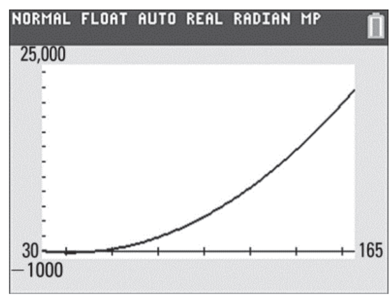
Approximately 12,247 would be the Federal Tax per Capita in 2018.

.

c. $x_{\min} = 2005 - 2000 = 0$

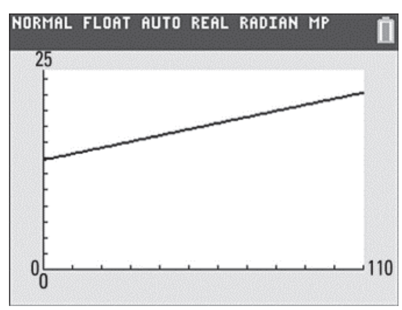
$x_{\max} = 2020 - 2000 = 20$

43. a.



b. Use the Trace feature, and when x = 130, P = 10,984,200.

44. a.



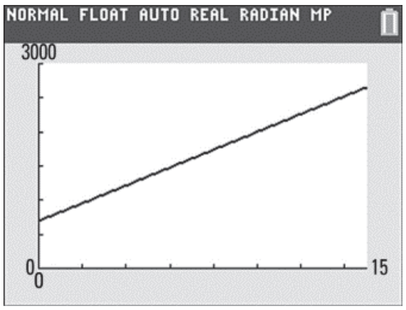
b. $\text{Year} = x + 1950$

For $x = 0$, $\text{Year} = 0 + 1950 = 1950$

For $x = 110$, $\text{Year} = 110 + 1950 = 2060$

c. Use the Trace feature, and when $x = 88$, $y = 20.6$ years.

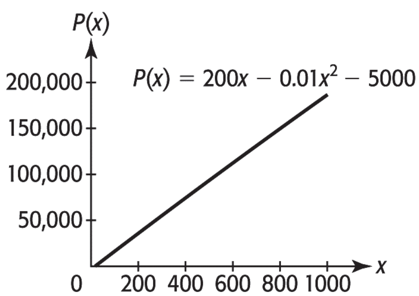
45. a.



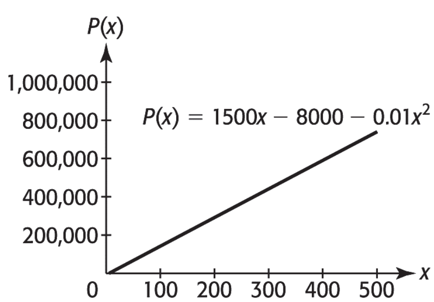
b. $y = 130.7(13) + 699.7 = 2398.8$
In 2023, the balance of federal direct student loans will be 2398.8 billion dollars.

c. $y = 130.7(19) + 699.7 = 3183$
In 2029, the balance of federal direct student loans will be 3183 billion dollars.

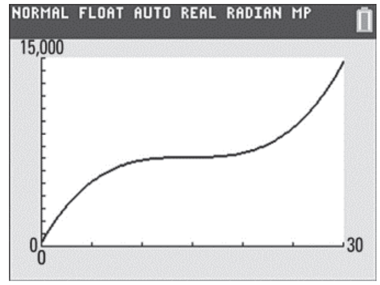
46.



47.



48. a. $B(x) = 2.11x^3 - 93.0x^2 + 1370x + 394$

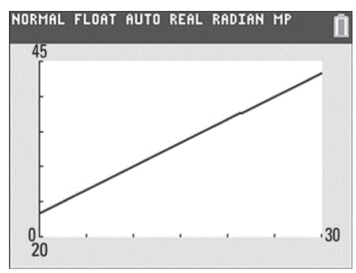


b. The tax burden increased. Reading the graph from left to right, as x increases $B(x)$ also increases

49. a. $x = \text{Year} - 2000$
For 2000, $x = 2000 - 2000 = 0$
For 2030, $x = 2030 - 2000 = 30$

b. For $x = 0$.
$y = 0.665(0) + 23.4$
$= 23.4$
For $x = 30$.
$y = 0.665(30) + 23.4$
$= 43.35$

c.

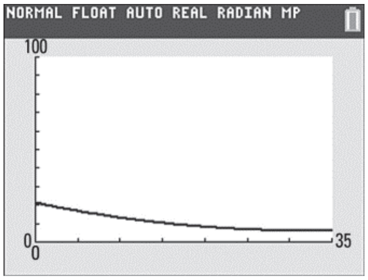


50. $y = 0.0152x^2 - 0.9514x + 21.5818$

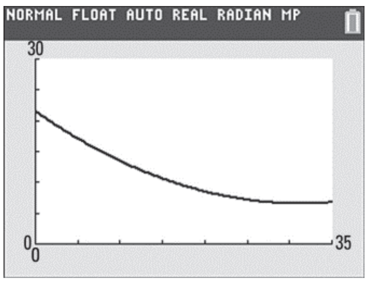
a. Since the base year is 1975, 1975-2010 correspond to values of x between 0 and 35.

b. Since percentages are between 0 and 100, y must correspond to values between 0 and 100.

c.



d.



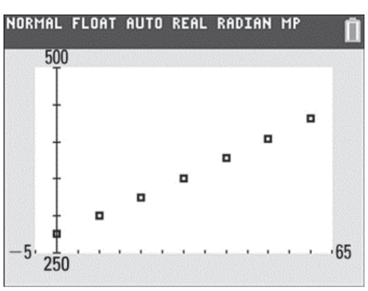
e. 2013 corresponds to $x = 2013 - 1975 = 38$. When $x = 38$, $y = 7.377$. Thus in 2013, approximately 7.38% of high school seniors will have used cocaine.

51. a. 299.9 million, or 299,900,000

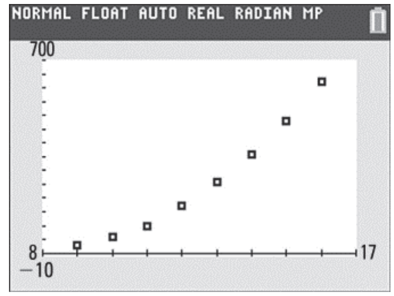
b.

Years after 2000	Population (millions)
0	275.3
10	299.9
20	324.9
30	351.1
40	377.4
50	403.7
60	432

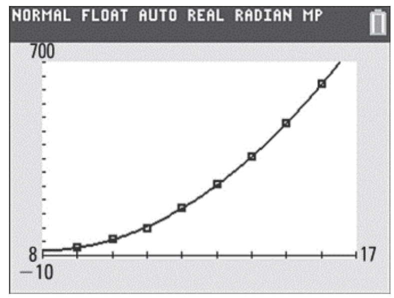
c.



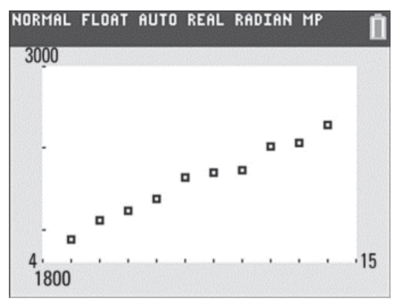
52. a.



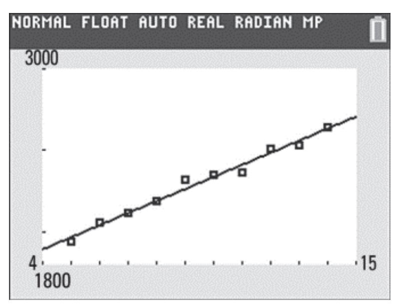
b.



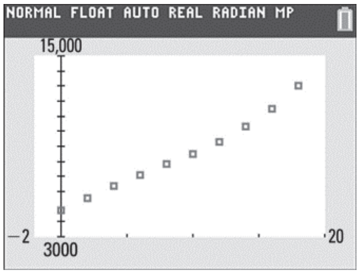
53. a.



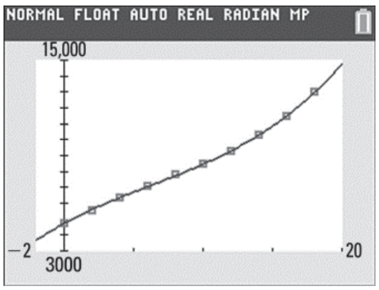
b.



54. a.



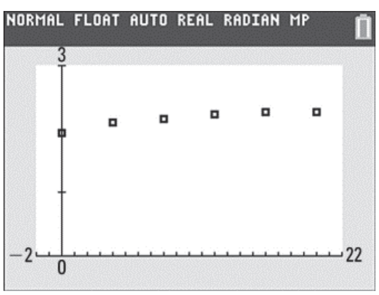
b.



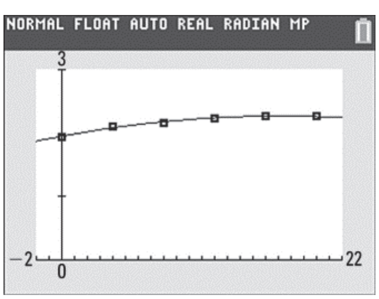
c. Yes, the function is a good visual fit to the data.

55. a. In 2026 the crude oil production will be 2.26 billion barrels.

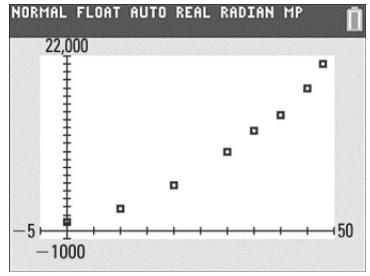
b.



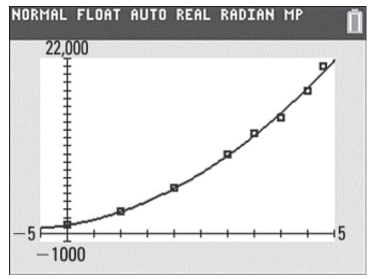
c.



56. a.



b.



c. For $x = 47$.

$$y = 6.20(47)^2 + 104(47) + 1110$$
$$= 19{,}693.8$$

In the year 2017, the GDP will be 19,693.8 billion dollars.

Section 1.3 Skills Check

1. Recall that linear functions must be in the form $f(x) = ax + b$.

   a. Not linear. The equation has a 2nd degree (squared) term.

   b. Linear.

   c. Not linear. The x-term is in the denominator of a fraction.

2. No. A vertical line is not a function.

3. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-6-6}{28-4} = \dfrac{-12}{24} = -\dfrac{1}{2}$

4. $$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-(-10)}{8-8} \\ &= \frac{14}{0} \\ &= \text{undefined} \end{aligned}$$
   Zero in the denominator creates an undefined expression.

5. The given line passes through $(-2,0)$ and $(0,4)$. Therefore the slope is
   $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-0}{0-(-2)} = \dfrac{4}{2} = 2$

6. Since the line is horizontal, the slope of the line is zero. $m = 0$.

7. a. x-intercept: Let $y = 0$ and solve for x.

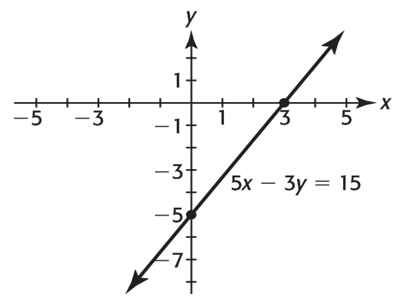
   $$\begin{aligned} 5x - 3(0) &= 15 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

   y-intercept: Let $x = 0$ and solve for y.

   $$\begin{aligned} 5(0) - 3y &= 15 \\ -3y &= 15 \\ y &= -5 \end{aligned}$$

   x-intercept: (3, 0), y-intercept: (0, –5)

   b.



8. a. x-intercept: Let $y = 0$ and solve for x.

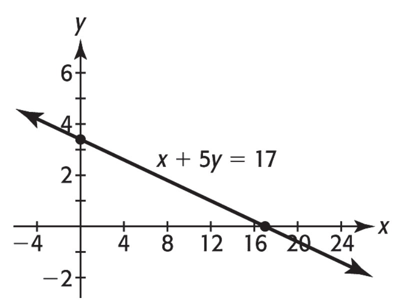
   $$\begin{aligned} x + 5(0) &= 17 \\ x &= 17 \end{aligned}$$

   y-intercept: Let $x = 0$ and solve for y.

   $$\begin{aligned} 0 + 5y &= 17 \\ 5y &= 17 \\ y &= \frac{17}{5} \\ y &= 3.4 \end{aligned}$$

   x-intercept: (17, 0), y-intercept: (0, 3.4)

   b.



9. a. x-intercept: Let $y = 0$ and solve for x.

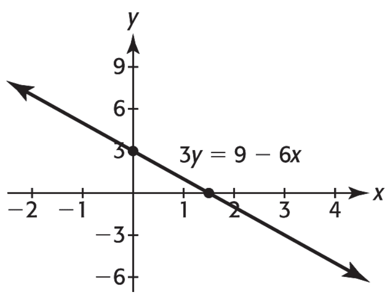
$$3(0) = 9 - 6x$$
$$0 = 9 - 6x$$
$$0 + 6x = 9 - 6x + 6x$$
$$6x = 9$$
$$x = \frac{9}{6}$$
$$x = \frac{3}{2} = 1.5$$

y-intercept: Let $x = 0$ and solve for y.

$$3y = 9 - 6(0)$$
$$3y = 9$$
$$y = 3$$

x-intercept: (1.5, 0), y-intercept: (0, 3)

b.



10. a. x-intercept: Let $y = 0$ and solve for x.

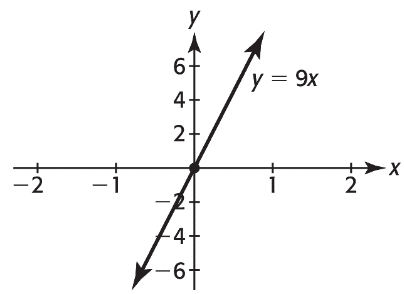
$$0 = 9x$$
$$x = 0$$

y-intercept: Let $x = 0$ and solve for y.

$$y = 9(0)$$
$$y = 0$$

x-intercept: (0, 0), y-intercept: (0, 0). Note that the origin, (0, 0), is both an x- and y-intercept. To graph, use the slope, $m = 9 = \frac{9}{1}$, or find another point from the equation, like $(1, 9)$ or $(-1, -9)$.

b.



11. Horizontal lines have a slope of **zero**. Vertical lines have an **undefined** slope.

12. Since the slope is undefined, the line is vertical.

13. a. Positive. The graph is rising.

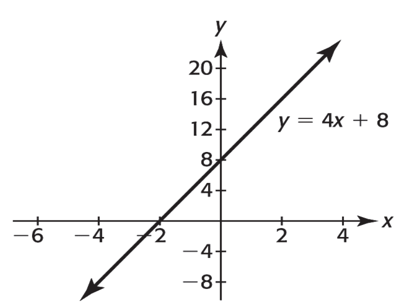
b. Undefined. The line is vertical.

14. a. Negative. The graph is falling.

b. Zero. The line is horizontal.

15. a. $m = 4, b = 8$

b.



16. a. $3x+2y=7$

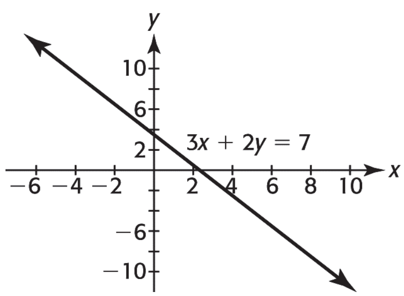
$$\frac{2y}{2}=\frac{-3x+7}{2}$$

$$y=\frac{-3x+7}{2}$$

$$y=-\frac{3}{2}x+\frac{7}{2}$$

$$m=-\frac{3}{2},\ b=\frac{7}{2}$$

b.

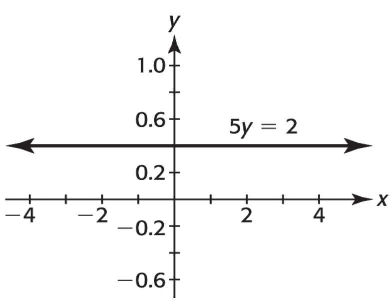


17. a. $5y=2$

$$y=\frac{2}{5}, \text{horizontal line}$$

$$m=0,\ b=\frac{2}{5}$$

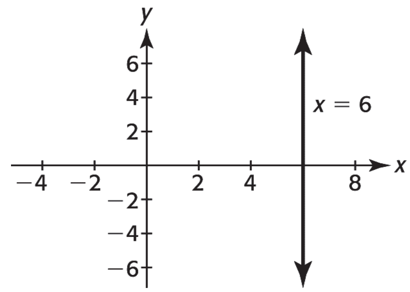
b.



18. a. $x=6$, vertical line

undefined slope, no y-intercept

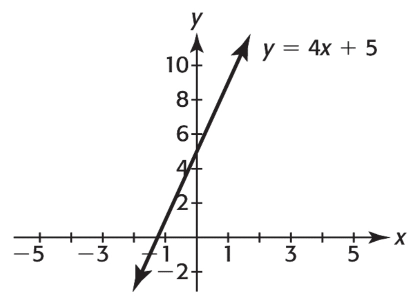
b.



19. a. $m=4,\ b=5$

b. Rising. The slope is positive

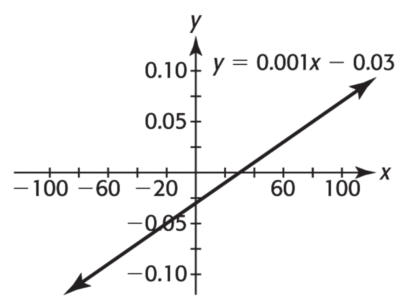
c.



20. a. $m=0.001,\ b=-0.03$

b. Rising. The slope is positive.

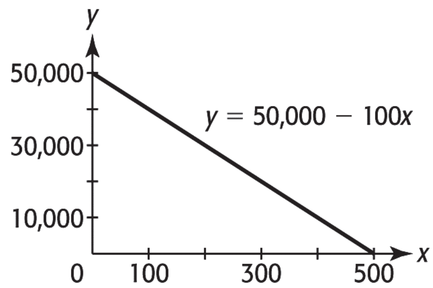
c.



21. a. $m=-100,\ b=50{,}000$

b. Falling. The slope is negative.

c.



22. Steepness refers to the amount of vertical change compared to the amount of horizontal change between two points on a line, regardless of the direction of the line. The steeper line would have the greater absolute value of its slope. Since the slope in exercise 19 is 4, and in exercise 20 it is 0.001, and in exercise 21 it is -100, exercise 20 (m = .001) is the least steep, followed by exercise 19 (m = 4). Exercise 21 displays the greatest steepness since m = -100 which gives $|-100| = 100$.

23. For a linear function, the rate of change is equal to the slope. $m = 4$.

24. For a linear function, the rate of change is equal to the slope. $m = \frac{1}{3}$.

25. For a linear function, the rate of change is equal to the slope. $m = -15$.

26. For a linear function, the rate of change is equal to the slope. $m = 300$.

27. For a linear function, the rate of change is equal to the slope.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-3}{4-(-1)} = \frac{-10}{5} = -2.$$

28. For a linear function, the rate of change is equal to the slope.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{6-2} = \frac{2}{4} = \frac{1}{2}.$$

29. a. The identity function is $y = x$. Graph *ii* represents the identity function.

b. The constant function is $y = k$, where k is a real number. In this case, $k = 3$. Graph *i* represents a constant function.

30. The slope of the identity function is one ($m = 1$).

31. a. The slope of a constant function is zero ($m = 0$).

b. The rate of change of a constant function equals the slope, which is zero.

32. The rate of change of the identity function equals the slope, which is one.

Section 1.3 Exercises

33. Yes, the function is linear since it is written in the form $y = mx + b$. The independent variable is x, the number of years after 1990.

34. No it is not linear since the function does not fit the form $y = ax + b$.

35. **a.** The function is linear since it is written in the form $y = ax + b$.

b. The slope is 0.077. The life expectancy is projected to increase by approximately 0.1 year per year.

36. **a.** The slope is 130.7.

b. The balance of federal direct loans is projected to increase at a rate of $130.7 billion per year.

37. **a.** x-intercept: Let $y = 0$ and solve for x.

$$132x + 1000y = 9570$$
$$132x + 1000(0) = 9570$$
$$132x = 9570$$
$$x = 72.5$$

The x-intercept is $(72.5, 0)$.

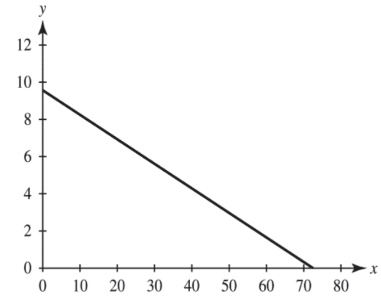
b. y-intercept: Let $x = 0$ and solve for y.

$$132x + 1000y = 9570$$
$$132(0) + 1000y = 9570$$
$$1000y = 9570$$
$$y = 9.57$$

The y-intercept is $(0, 9.57)$.

In 1980, the marriage rate for unmarried women was about 10 per 1000.

c. Integer values of $x \geq 0$ on the graph represent years 1980 and after.



38. **a.** x-intercept: Let $p = 0$ and solve for x.

$$25p + 21x = 1215$$
$$25(0) + 21x = 1215$$
$$21x = 1215$$
$$x = \frac{405}{7}$$

The x-intercept is $\left(\frac{405}{7}, 0\right)$.

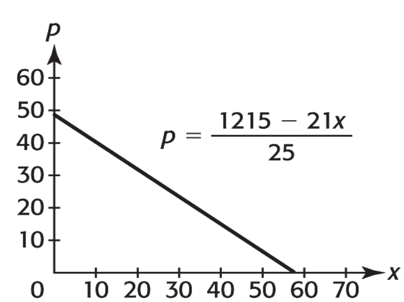
b. p-intercept: Let $x = 0$ and solve for p.

$$25p + 21x = 1215$$
$$25p + 21(0) = 1215$$
$$25p = 1215$$
$$p = 48.6$$

The y-intercept is $(0, 48.6)$.

In 2000, the percentage of high school students who had ever used marijuana was 48.6%.

c. Integer values of $x \geq 0$ on the graph represent years 2000 and after



39. x-intercept: Let $R = 0$ and solve for x.

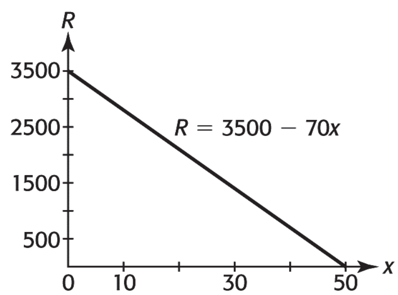
$$R = 3500 - 70x$$
$$0 = 3500 - 70x$$
$$70x = 3500$$
$$x = \frac{3500}{70} = 50$$

The x-intercept is $(50,0)$.

R-intercept: Let $x = 0$ and solve for R.

$$R = 3500 - 70x$$
$$R = 3500 - 70(0)$$
$$R = 3500$$

The R-intercept is $(0,3500)$.



40. a. p-intercept: Let $t = 0$ and solve for p.

$$p = 43.3 - 0.50t$$
$$p = 43.3 - 0.50(0)$$
$$p = 43.3$$

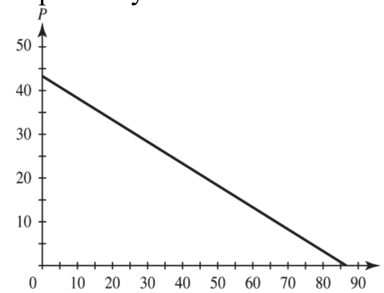
The x-intercept is $(43.3,0)$.

t-intercept: Let $p = 0$ and solve for t.

$$p = 43.3 - 0.50t$$
$$0 = 43.3 - 0.50t$$
$$0.50t = 43.3$$
$$t = 86.6$$

The t-intercept is $(0,86.6)$.

b. Integer values of $t \ge 0$ on the graph represent years 1960 and after



41. a. The data can be modeled by a constant function. Every input x yields the same output y.

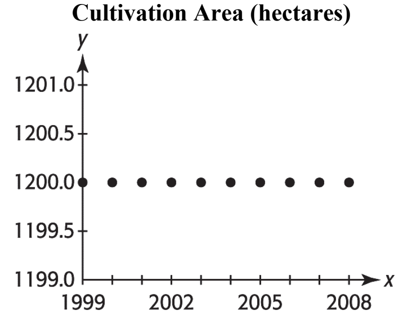
b. $y = 11.81$

c. A constant function has a slope equal to zero.

d. For a linear function the rate of change is equal to the slope. $m = 0$.

42. a.

Cultivation Area (hectares)

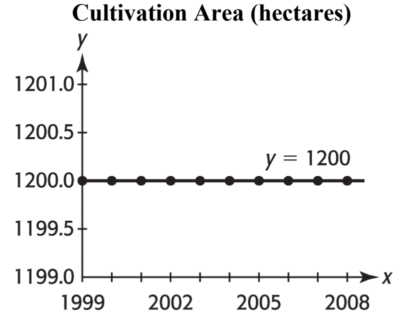


b. The data can be modeled by a constant function.

c. $y = 1200$

d.

Cultivation Area (hectares)



43. a. The slope is 0.328.

b. From 2010 through 2040, the disposable income is projected to increase by \$328 million per year.

44. The average rate of growth over this period of time is 0.465 percentage points per year.

$$\frac{34.3-15.7}{2050-2010}=0.465$$

45. a. The $P-$intercept is 20.86. Approximately 21 million women were in the workforce in 1950.

b. The slope is 0.79.

c. Rate of change of the number of women in the workforce is 790 thousand per year.

46. a. The slope is 0.063.

b. The world population is projected to grow by 63 million per year during this period.

47. a. $m=0.057$

b. From 1990 to 2050, the percent of the U.S. population that is black increased by 0.057 percentage points per year.

48. a. $33p-18d=496$

Solving for p:

$33p=18d+496$

$$p=\frac{18d+496}{33}$$

$$p=\frac{18}{33}d+\frac{496}{33}$$

$$p=\frac{6}{11}d+\frac{496}{33}$$

Therefore, $m=\dfrac{6}{11}$

b. For every one unit increase in depth, there is a corresponding $\frac{6}{11}$ pound per square inch increase in pressure

49. a. For a linear function, the rate of change is equal to the slope. $m=\frac{12}{7}$. The slope is positive.

b. For each one degree increase in temperature, there is a $\frac{12}{7}$ increase in the number of cricket chirps per minute. More generally, as the temperature increases, the number of chirps increases.

50. a. $m=11.23$
S-intercept: $b=6.205$

b. The *S*-intercept represents the total amount spent for wireless communications in 1995. Therefore in 1995, the amount spent on wireless communication in the U.S. was 6.205 billion dollars.

c. The slope represents the annual change in the amount spent on wireless communications. Therefore, the amount spent on wireless communications in the U.S. increased by 11.23 billion each year.

51. a. The rate of change of revenue for call centers in the Philippines from 2006 to 2010, was 0.975 billion dollars per year.

b. 2010 corresponds to $x=2010-2000=10$. When x = 10,
$R=0.975(10)-3.45$
$R=9.75-3.45=6.30$

Thus in 2010, the revenue for call centers in the Philippines was 6.3 billion dollars.

c. No, it would not be valid since the result would be a negative number of dollars.

52. a. $m = 1.36$
y-intercept: $b = 68.8$

b. After 2000, the percent of the U.S. population with Internet access increased by 1.36 percentage points per year.

53. a.
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{700{,}000 - 1{,}310{,}000}{20 - 10} \\ &= \frac{-610{,}000}{10} \\ &= -61{,}000 \end{aligned}$$

b. Based on the calculation in part a), the property value decreases by \$61,000 each year. The annual rate of change is –\$61,000 per year.

54. a.
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{100.3 - 43.8}{2050 - 1950} \\ &= \frac{56.5}{100} \\ &= 0.565 \end{aligned}$$

b. Based on the calculation in part a), the number of men in the workforce increased by 0.565 million (or 565,000) each year from 1950 to 2050.

55. Marginal profit is the rate of change of the profit function.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{9000 - 4650}{375 - 300} \\ &= \frac{4350}{75} \\ &= 58 \end{aligned}$$

The marginal profit is \$58 per unit.

56. Marginal cost is the rate of change of the cost function.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3530 - 2690}{500 - 200} \\ &= \frac{840}{300} \\ &= 2.8 \end{aligned}$$

The marginal cost is \$2.80 per unit.

57. a. $m = 0.56$

b. The marginal cost is \$0.56 per unit.

c. Manufacturing one additional golf ball each month increases the cost by \$0.56 or 56 cents.

58. a. $m = 98$

b. The marginal cost is \$98 per unit.

c. Manufacturing one additional television each month increases the cost by \$98.

59. a. $m = 1.60$

b. The marginal revenue is \$1.60 per unit.

c. Selling one additional golf ball each month increases total revenue by \$1.60.

60. **a.** $m = 198$

b. The marginal revenue is \$198 per unit

c. Selling one additional television each month increases total revenue by \$198.

61. The marginal profit is \$19 per unit. Note that $m = 19$.

62. The marginal profit is \$939 per unit. Note that $m = 939$.

Section 1.4 Skills Check

1. $m = 4,\ b = \frac{1}{2}$. The equation is $y = 4x + \frac{1}{2}$.

2. $m = 5,\ b = \frac{1}{3}$. The equation is $y = 5x + \frac{1}{3}$.

3. $m = \frac{1}{3},\ b = 3$. The equation is $y = \frac{1}{3}x + 3$.

4. $m = -\frac{1}{2},\ b = -8$. The equation is $y = -\frac{1}{2}x - 8$.

5.
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-6) &= -\frac{3}{4}(x - 4) \\ y + 6 &= -\frac{3}{4}x + 3 \\ y &= -\frac{3}{4}x - 3 \end{aligned}$$

6.
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= -\frac{1}{2}(x - (-4)) \\ y - 3 &= -\frac{1}{2}(x + 4) \\ y - 3 &= -\frac{1}{2}x - 2 \\ y &= -\frac{1}{2}x + 1 \end{aligned}$$

7. $x = 9$

8. $y = -10$

9. Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 1}{4 - (-2)} = \frac{6}{6} = 1$

Equation:
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 7 &= 1(x - 4) \\ y - 7 &= x - 4 \\ y &= x + 3 \end{aligned}$$

10. Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 3}{2 - (-1)} = \frac{3}{3} = 1$

Equation:
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 6 &= 1(x - 2) \\ y - 6 &= x - 2 \\ y &= x + 4 \end{aligned}$$

11. Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{5 - (-3)} = \frac{0}{8} = 0$

The line is horizontal. The equation of the line is $y = 2$.

12. Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{9 - 9} = \frac{3}{0} =$ undefined

The line is vertical. The equation of the line is $x = 9$.

13. With the given intercepts, the line passes through the points (–5, 0) and (0, 4). The slope of the line is
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{0 - (-5)} = \frac{4}{5}$.

Equation:
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= \frac{4}{5}(x - (-5)) \\ y &= \frac{4}{5}(x + 5) \\ y &= \frac{4}{5}x + 4 \end{aligned}$$

14. With the given intercepts, the line passes through the points (4, 0) and (0, –5). The slope of the line is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-5-0}{0-(4)}=\frac{-5}{-4}=\frac{5}{4}.$$

Equation:
$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-0&=\frac{5}{4}(x-4)\\ y&=\frac{5}{4}(x-4)\\ y&=\frac{5}{4}x-5 \end{aligned}$$

15.
$$\begin{aligned} 3x+y&=4\\ y&=-3x+4\\ m&=-3 \end{aligned}$$

Since the new line is parallel with the given line, the slopes of both lines are the same.

Equation:
$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-(-6)&=-3(x-4)\\ y+6&=-3x+12\\ y&=-3x+6 \end{aligned}$$

16.
$$\begin{aligned} 2x+y&=-3\\ y&=-2x-3\\ m&=-2 \end{aligned}$$

Since the new line is parallel with the given line, the slopes of both lines are the same.

Equation:
$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-(-3)&=-2(x-5)\\ y+3&=-2x+10\\ y&=-2x+7 \end{aligned}$$

17.
$$\begin{aligned} 2x+3y&=7\\ 3y&=-2x+7\\ \frac{3y}{3}&=\frac{-2x+7}{3}\\ y&=-\frac{2}{3}x+\frac{7}{3}\\ m&=-\frac{2}{3} \end{aligned}$$

Since the new line is perpendicular with the given line, the slope of the new line is $m_\perp=-\frac{1}{m}$, where m is the slope of the given line. $m_\perp=-\frac{1}{m}=-\frac{1}{\left(-\frac{2}{3}\right)}=\frac{3}{2}$.

Equation:
$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-7&=\frac{3}{2}(x-(-3))\\ y-7&=\frac{3}{2}x+\frac{9}{2}\\ y-7+7&=\frac{3}{2}x+\frac{9}{2}+7\\ y&=\frac{3}{2}x+\frac{23}{2} \end{aligned}$$

18.
$$\begin{aligned} 3x+2y&=-8\\ 2y&=-3x-8\\ \frac{2y}{2}&=\frac{-3x-8}{2}\\ y&=-\frac{3}{2}x-4\\ m&=-\frac{3}{2} \end{aligned}$$

Since the new line is perpendicular with the given line, the slope of the new line is $m_\perp=-\frac{1}{m}$, where m is the slope of the given line. $m_\perp=-\frac{1}{m}=-\frac{1}{\left(-\frac{3}{2}\right)}=\frac{2}{3}$.

Equation: $y - y_1 = m(x - x_1)$

$$y - 5 = \frac{2}{3}\left(x - (-4)\right)$$
$$y - 5 = \frac{2}{3}x + \frac{8}{3}$$
$$y - 5 + 5 = \frac{2}{3}x + \frac{8}{3} + 5$$
$$y = \frac{2}{3}x + \frac{23}{3}$$

19. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{13 - (-5)}{4 - (-2)} = \dfrac{18}{6} = 3$

Equation: $y - y_1 = m(x - x_1)$

$$y - 13 = 3(x - 4)$$
$$y - 13 = 3x - 12$$
$$y = 3x + 1$$

20. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-11 - 7}{2 - (-4)} = \dfrac{-18}{6} = -3$

Equation: $y - y_1 = m(x - x_1)$

$$y - 7 = -3(x - (-4))$$
$$y - 7 = -3x - 12$$
$$y = -3x - 5$$

21. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{(-5) - (-5)}{1 - 0} = \dfrac{0}{1} = 0$

Equation: $y - y_1 = m(x - x_1)$

$$y - (-5) = 0(x - 1)$$
$$y + 5 = 0$$
$$y = -5$$

22. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - 0}{3 - 3} = \dfrac{1}{0} = \text{undefined}$

Lines with undefined slopes are vertical lines. Every point on the vertical line through (3,0) has an x – coordinate of 3. Thus the equation of the line is $x = 3$.

23. For a linear function, the rate of change is equal to the slope. Therefore, $m = -15$. The equation is

$$y - y_1 = m(x - x_1)$$
$$y - 12 = -15(x - 0)$$
$$y - 12 = -15x$$
$$y = -15x + 12.$$

24. For a linear function, the rate of change is equal to the slope. Therefore, $m = -8$. The equation is

$$y - y_1 = m(x - x_1)$$
$$y - (-7) = -8(x - 0)$$
$$y + 7 = -8x$$
$$y = -8x - 7.$$

25. For a linear function, the rate of change is equal to the slope. Therefore, $m = \frac{2}{3}$. The equation is

$$y - y_1 = m(x - x_1)$$
$$y - 9 = \tfrac{2}{3}(x - 3)$$
$$y - 9 = \tfrac{2}{3}x - 2$$
$$y = \tfrac{2}{3}x + 7.$$

26. For a linear function, the rate of change is equal to the slope. Therefore, $m = -\frac{1}{5}$. The equation is

$$y - y_1 = m(x - x_1)$$
$$y - 12 = -\tfrac{1}{5}(x - (-2))$$
$$y - 12 = -\tfrac{1}{5}x - \tfrac{2}{5}$$
$$y = -\tfrac{1}{5}x + \tfrac{58}{5}.$$

27. $$\frac{f(b)-f(a)}{b-a}=\frac{f(2)-f(-1)}{2-(-1)}=\frac{(2)^2-(-1)^2}{3}=\frac{4-1}{3}=\frac{3}{3}=1$$

The average rate of change between the two points is 1.

28. $$\frac{f(b)-f(a)}{b-a}=\frac{f(2)-f(-1)}{2-(-1)}=\frac{(2)^3-(-1)^3}{3}=\frac{8+1}{3}=\frac{9}{3}=3$$

The average rate of change between the two points is 3.

29. $$\frac{f(b)-f(a)}{b-a}=\frac{f(1)-f(-2)}{1-(-2)}=\frac{-2-7}{3}=\frac{-9}{3}=-3$$

30. $$\frac{f(b)-f(a)}{b-a}=\frac{f(2)-f(-1)}{2-(-1)}=\frac{-4-2}{3}=\frac{-6}{3}=-2$$

31. $$\begin{aligned}y-y_1&=m(x-x_1)\\y-(-10)&=-3(x-1)\\y+10&=-3x+3\\y&=-3x-7.\end{aligned}$$

The y–intercept is $b=-7.$

32. $$\begin{aligned}y-y_1&=m(x-x_1)\\y-8&=-\tfrac{3}{2}(x-(-2))\\y-8&=-\tfrac{3}{2}x-3\\y&=-\tfrac{3}{2}x+5.\end{aligned}$$

The y–intercept is $b=5.$

33. $$\begin{aligned}f(x+h)&=45-15(x+h)\\&=45-15x-15h\end{aligned}$$

$$\begin{aligned}&f(x+h)-f(x)\\&=45-15x-15h-[45-15x]\\&=45-15x-15h-45+15x\\&=-15h\end{aligned}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{-15h}{h}=-15$$

34. $$\begin{aligned}f(x+h)&=32(x+h)+12\\&=32x+32h+12\end{aligned}$$

$$\begin{aligned}&f(x+h)-f(x)\\&=32x+32h+12-[32x+12]\\&=32x+32h+12-32x-12\\&=32h\end{aligned}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{32h}{h}=32$$

35. $f(x+h) = 2(x+h)^2 + 4$
$= 2\left(x^2 + 2xh + h^2\right) + 4$
$= 2x^2 + 4xh + 2h^2 + 4$

$f(x+h) - f(x)$
$= 2x^2 + 4xh + 2h^2 + 4 - \left[2x^2 + 4\right]$
$= 2x^2 + 4xh + 2h^2 + 4 - 2x^2 - 4$
$= 4xh + 2h^2$

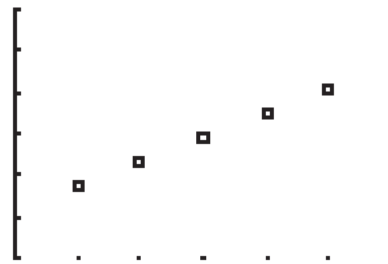
$$\frac{f(x+h)-f(x)}{h} = \frac{4xh+2h^2}{h} = \frac{h(4x+2h)}{h} = 4x + 2h$$

36. $f(x+h) = 3(x+h)^2 + 1$
$= 3\left(x^2 + 2xh + h^2\right) + 1$
$= 3x^2 + 6xh + 3h^2 + 1$

$f(x+h) - f(x)$
$= 3x^2 + 6xh + 3h^2 + 1 - \left[3x^2 + 1\right]$
$= 3x^2 + 6xh + 3h^2 + 1 - 3x^2 - 1$
$= 6xh + 3h^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{6xh+3h^2}{h} = \frac{h(6x+3h)}{h} = 6x + 3h$$

37. a. The difference in the y-coordinates is consistently 30, while the difference in the x-coordinates is consistently 10. Considering the scatter plot below, a line fits the data exactly.

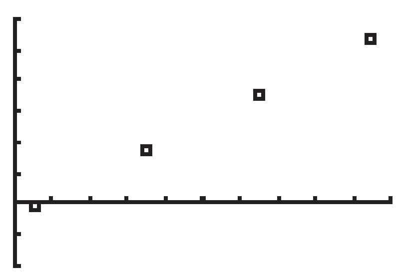


[0, 60] by [500, 800]

b. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{615-585}{20-10} = \dfrac{30}{10} = 3$

Equation: $y - y_1 = m(x - x_1)$
$y - 585 = 3(x - 10)$
$y - 585 = 3x - 30$
$y = 3x + 555$

38. a. The difference in the y-coordinates is consistently 9, while the difference in the x-coordinates is consistently 6. Considering the scatter plot below, a line fits the data exactly.



[0, 20] by [–10, 30]

b. Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{26.5 - 17.5}{19 - 13}$$
$$= \frac{9}{6}$$
$$= \frac{3}{2}$$

Equation:

$$y - y_1 = m(x - x_1)$$
$$y - 26.5 = \frac{3}{2}(x - 19)$$
$$y - 26.5 = \frac{3}{2}x - \frac{57}{2}$$
$$y = \frac{3}{2}x - 28.5 + 26.5$$
$$y = \frac{3}{2}x - 2$$

Section 1.4 Exercises

39. Let x = kWh used and let y = monthly charge in dollars. Then the equation is $y = 0.1034x + 12.00$.

40. Let x = minutes used and let y = monthly charge in dollars. Then the equation is $y = 0.10x + 2.99$.

41. Let t = number of years, and let y = value of the machinery after t years. Then the equation is $y = 36{,}000 - 3{,}600t$.

42. a. Sleep needed $= 8 + \frac{1}{4}(18 - 10)$

$$= 8 + \frac{1}{4}(8)$$
$$= 10 \text{ hours}$$

b. Sleep needed $= 8 + \frac{1}{4}(18 - 14)$

$$= 8 + \frac{1}{4}(4)$$
$$= 9 \text{ hours}$$

c. Let x = age in years, and let y = hours of sleep. Using the results of parts a) and b), the two ordered pairs are (10, 10) and (14, 9).

The resulting slope is $\frac{9 - 10}{14 - 10} = -.25$

Then using the point-slope form of a linear equation and one of the ordered pairs, (14, 9):

$$y - 9 = -0.25(x - 14)$$
$$y - 9 = -0.25x + 3.5$$
$$y = -0.25x + 12.5$$

d. Let $x = 18$.

$y = -0.25(18) + 12.5 = -4.5 + 12.5 = 8$

43. a. Let t = age in years, and let H = heart rate. Using the information given:

The resulting slope is $\frac{6.5}{-10} = -0.65$

Then using the point-slope form of a linear equation and the ordered pair (60, 104):

$$H - 104 = -0.65(t - 60)$$
$$H - 104 = -0.65t + 39$$
$$H = -0.65t + 143$$

b. Let $t = 40$.

$H = -0.65(40) + 143 = -26 + 143 = 117.$

The desired heart rate for a 40 year old person is 117 beats per minute.

44. **a.** Let t = age in years, and let h = heart rate. Using the information given:

The resulting slope is $\frac{8}{-10} = -0.8$ Then using the point-slope form of a linear equation and the ordered pair (50, 136):

$$h - 136 = -0.8(t - 50)$$
$$h - 136 = -0.8t + 40$$
$$H = -0.8t + 176$$

b. Let $t = 75$.

$h = -0.8(75) + 176 = -60 + 176 = 116.$

The desired heart rate for a 75 year old person is 116 beats per minute.

45. **a.** From year 0 to year 5, the automobile depreciates from a value of \$26,000 to a value of \$1,000. Therefore, the total depreciation is 26,000–1000 or \$25,000.

b. Since the automobile depreciates for 5 years in a straight-line (linear) fashion, each year the value declines by $\frac{25{,}000}{5} = \$5{,}000$.

c. Let t = the number of years, and let s = the value of the automobile at the end of t years. Then, based on parts a) and b) the linear equation modeling the value is $s = -5000t + 26{,}000$.

46. $P = 2.5\%(75{,}000)y$

$= 1875y$

where y = the number of years of service, and P = the annual pension in dollars.

47. Notice that the x and y values always match. The number of deputies always equals the number of patrol cars. Therefore the equation is $y = x$, where x represents the number of deputies, and y represents the number of patrol cars.

48. **a.** Notice that the y values are always the same, regardless of the x value. The market share (%) is constant. Therefore the equation is $y = 66$, where x represents the number of months after April 2010, and y represents Google's market share (%).

b. For this time period, the rate of change of Google's market share (%) is zero.

49.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9000 - 4650}{375 - 300} = \frac{4350}{75} = 58$$

Equation:

$$P - p_1 = m(x - x_1)$$
$$P - 4650 = 58(x - 300)$$
$$P - 4650 = 58x - 17{,}400$$
$$P = 58x - 12{,}750$$

50.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3580 - 2680}{500 - 200} = \frac{900}{300} = 3$$

Equation:

$$y - y_1 = m(x - x_1)$$
$$y - 2680 = 3(x - 200)$$
$$y - 2680 = 3x - 600$$
$$y = 3x - 600 + 2680$$
$$y = 3x + 2080$$

51. $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{700,000 - 1,310,000}{20 - 10}$

$= \frac{-610,000}{10}$

$= -61,000$

Equation:

$y - y_1 = m(x - x_1)$

$y - 700,000 = -61,000(x - 20)$

$y - 700,000 = -61,000x + 1,220,000$

$y = -61,000x + 1,920,000$

$V = -61,000x + 1,920,000$

52. a. At $t = 0$, $y = 860,000$.

b. $(0,860,000),(25,0)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0 - 860,000}{25 - 0}$

$= \frac{-860,000}{25}$

$= -34,400$

Equation:

$y - y_1 = m(x - x_1)$

$y - 0 = -34,400(x - 25)$

$y = -34,400x + 860,000$

$y = 860,000 - 34,400t$

where t = the number of years, and y = the property value.

53. a. The slope of the equation will be $m = 160$ and the point used will be (66, 2000):

Equation:

$y - y_1 = m(x - x_1)$

$y - 2000 = 160(x - 66)$

$y - 2000 = 160x - 10560$

$y = 160x - 8560$

where x = the age of the person between 66 and 70 inclusive, and y = the monthly benefit received.

b. Let $x = 70$.

$y = 160(70) - 8560$

$= 11200 - 8560$

$= 2640$

The monthly benefit for a recipient starting their benefits at age 70 would be \$2640.

54. a. Let x = the year, and y = U.S. CPI. The goal is to write $y = f(x)$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{108.6 - 100}{2016 - 2012} = \frac{8.6}{4} = 2.15$

Equation:

$y - y_1 = m(x - x_1)$

$y - 100 = 2.15(x - 2012)$

$y - 100 = 2.15x - 4325.8$

$y = 2.15x - 4225.8$

b. Let $x = 2020$.

$y = 2.15(2020) - 4225.8$

$= 4343 - 4225.8$

$= 117.2$

The U.S. CPI in the year 2020 is projected to be 117.2.

55. a. Notice that the change in the x-values is consistently 1 while the change in the y-values is consistently 0.045 percentage points per drink. Therefore the table represents a linear function. The rate of change is the slope of the linear function.

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{0.045}{1} = 0.045$$

b. Let x = the number of drinks, and let y = the blood alcohol percent. Using the slope 0.045 and one of points (5, 0.225), the equation is:

$$y - y_1 = m(x - x_1)$$
$$y - 0.225 = 0.045(x - 5)$$
$$y - 0.225 = 0.045x - 0.225$$
$$y = 0.045x$$

56. a. Notice that the change in the x-values is consistently 1 while the change in the y-values is consistently 0.017 percentage points per drink. Therefore the table represents a linear function. The rate of change is the slope of the linear function.

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{0.017}{1} = 0.017$$

b. Let x = the number of drinks, and let y = the blood alcohol percent. Using the slope 0.017 and one of the points (0, 0), the equation is

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 0.017(x - 0)$$
$$y = 0.017x$$

57. a. Let t = the year at the beginning of the decade, and let g = average number of men in the workforce during the decade. Using points (1950, 43.8) and (2050, 100.3) to calculate the slope yields:

$$m = \frac{g_2 - g_1}{t_2 - t_1}$$
$$= \frac{100.3 - 43.8}{2050 - 1950}$$
$$= \frac{56.5}{100} = 0.565$$

Equation:

$$g - g_1 = m(t - t_1)$$
$$g - 43.8 = 0.565(t - 1950)$$
$$g - 43.8 = 0.565t - 1101.75$$
$$g = 0.565t - 1057.95$$

b. Yes. Consider the following table of values based on the equation in comparison to the actual data points.

t	g = Equation Values	Actual Values
1950	43.8	43.8
1960	49.45	46.4
1970	55.1	51.2
1980	60.75	61.5
1990	66.4	69
2000	72.05	75.2
2010	77.7	82.2
2015	80.525	84.2
2020	83.35	85.4
2030	89	88.5
2040	94.65	94
2050	100.3	100.3

c. They are the same since the points $(1950, 43.8)$ and $(2050, 100.3)$ were used to calculate the slope of the linear model.

58. a. Let t = the year, and let w = the total population (in billions). Using points (1950, 2.556) and (2050, 9.346) to calculate the slope yields:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{9.346 - 2.556}{2050 - 1950} \\ &= \frac{6.79}{100} \\ &= 0.0679 \end{aligned}$$

Equation:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ w - 2.556 &= 0.0679(t - 1950) \\ w - 2.556 &= 0.0679t - 132.405 \\ w &= 0.0679t - 129.849 \end{aligned}$$

b. Yes, the line appears to be a reasonable fit to the data.

c. Each year, between 1950 and 2050, the world population is projected to increase by 67.9 million each year.

d. Let $x = 2025$.

$$\begin{aligned} w &= 0.0679(2025) - 129.849 \\ &= 137.4975 - 129.849 \\ &= 7.6485 \end{aligned}$$

The world population in the year 2025 is projected to be approximately 7.649 billion people.

59. a. Let x = the year, and let p = the percent of Americans considered obese. Using points (2000, 22.1) and (2030, 42.2) to calculate the slope yields:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{42.2 - 22.1}{2030 - 2000} \\ &= \frac{20.1}{30} \\ &= 0.67 \end{aligned}$$

Equation:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ p - 22.1 &= 0.67(x - 2000) \\ p - 22.1 &= 0.67x - 1340 \\ p &= 0.67x - 1317.9 \end{aligned}$$

b. Let $x = 2020$.

$$\begin{aligned} p &= 0.67(2020) - 1317.9 \\ &= 1353.4 - 1317.9 \\ &= 35.5 \end{aligned}$$

The percent of Americans that will be considered obese in the year 2020 is projected to be 35.5%.

60. a. The line would be increasing

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{100.9 - 99}{2011 - 2010} \\ &= \frac{1.9}{1} \\ &= 1.9 \end{aligned}$$

b. Let x = the year, and let y = the number of U.S. households with pay TV. Using points (2012, 100.8) and (2017, 94.6) to calculate the slope yields:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{94.6 - 100.8}{2017 - 2012} \\ &= \frac{-6.2}{5} \\ &= -1.24 \end{aligned}$$

Equation:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 100.8 &= -1.24(x - 2012) \\ y - 100.8 &= -1.24x + 2494.88 \\ y &= -1.24x + 2595.68 \end{aligned}$$

.

c. Decreasing because the slope is a negative value.

d. Let $x = 2020$.
$$y = -1.24(2020) + 2595.68$$
$$= -2504.8 + 2595.68$$
$$= 90.88$$
The number of U.S. households with pay TV in the year 2020 is projected to be 90.88 million.

61. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
$$= \frac{81.6 - 18.4}{80 - 0}$$
$$= \frac{63.2}{80} = 0.79$$

b. It is the same as part a). The average rate of change in the number of women in the workforce between 1950 and 2030 is increasing at a rate of 0.79 million per year.

c. $y - y_1 = m(x - x_1)$
$$y - 18.4 = 0.79(x - 0)$$
$$y - 18.4 = 0.79x$$
$$y = 0.79x + 18.4$$

62. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
$$= \frac{26.3 - 96.3}{63 - 7}$$
$$= \frac{-70}{56} = -1.25$$

b. The average rate of change of the birth rate between 1957 and 2013 was -1.25 births per 1000 girls.

c. The birth rate has decreased over this period by 1.25% per year.

d. Let $x =$ the number of years after 1950, and let $y =$ the birth rate for U.S. girls.
$$y - y_1 = m(x - x_1)$$
$$y - 26.3 = -1.25(x - 63)$$
$$y - 26.3 = -1.25x + 78.75$$
$$y = -1.25x + 105.05$$

63. a. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{f(2012) - f(1960)}{2009 - 1960}$
$$= \frac{1,570,397 - 212,953}{2012 - 1960}$$
$$= \frac{1,357,444}{52}$$
$$= 26,104.69231$$
$$\approx 26,105 \text{ / year}$$

b. The slope of the line connecting the two points is the same as the average rate of change between the two points. Based on part a), $m = 26,105$.

c. The equation of the secant line, using the rounded slope from part a), is given by:
$$y - y_1 = m(x - x_1)$$
$$y - 1,570,397 = 26,105(x - 2012)$$
$$y - 1,570,397 = 26,105x - 52,523,260$$
$$y = 26,105x - 50,952,863$$

d. No. The points on the scatter plot do not approximate a linear pattern.

64. a. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{f(5) - f(1)}{5 - 1}$
$$= \frac{1492 - 1083}{4}$$
$$= \frac{409}{4}$$
$$= 102.25$$

b. On average from year 1 to year 5, the worth of the investment increases by \$102.25 per year.

c. The slope is the same as the average rate of change, 102.25.

d.
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1083 &= 102.25(x - 1) \\ y - 1083 &= 102.25x - 102.25 \\ y &= 102.25x + 980.75 \end{aligned}$$

65. a. No.

b. Yes. The points seem to follow a straight line pattern for years between 2010 and 2030.

c.
$$\begin{aligned} \frac{f(b) - f(a)}{b - a} &= \frac{f(2030) - f(2010)}{2030 - 2010} \\ &= \frac{2.2 - 3.9}{2030 - 2010} \\ &= \frac{-1.7}{20} \\ &= -0.085 \end{aligned}$$

The average annual rate of change of the data over this period of time is -0.085 points per year.

d.
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3.9 &= -0.085(x - 2010) \\ y - 3.9 &= -0.085x + 170.85 \\ y &= -0.085x + 174.75 \end{aligned}$$

66. a. No. The points in the scatter plot do not lie approximately on a line.

b.
$$\begin{aligned} \frac{f(b) - f(a)}{b - a} &= \frac{f(1950) - f(1930)}{1950 - 1930} \\ &= \frac{16.443 - 10.519}{20} \\ &= \frac{5.924}{20} \\ &= 0.2962 \end{aligned}$$

The average rate of change is 0.2962 million (296,200) women per year.

c.
$$\begin{aligned} &\frac{f(b) - f(a)}{b - a} \\ &= \frac{f(2010) - f(1950)}{1990 - 1950} \\ &= \frac{75.500 - 16.443}{60} \\ &= \frac{59.057}{60} \\ &= 0.98428\overline{3} \approx 0.984 \end{aligned}$$

The average rate of change is approximately 0.984 million (984,000) women per year.

d. Yes. Since the graph curves, the average rate of change is not constant. The points do not lie exactly along a line.

67. a. Let x = the year, and let y = the number of White non-Hispanic individuals in the U.S. civilian non-institutional population 16 years and older. Then, the average rate of change between 2000 and 2050 is given by:

$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= \frac{169.4 - 153.1}{2050 - 2000} \\ &= \frac{16.3}{50} \\ &= 0.326 \end{aligned}$$

The annual average increase in the number of White non-Hispanic individuals in the U.S. civilian non-institutional population 16 years and older is 0.326 million per year.

b.
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 153.1 &= 0.326(x - 2000) \\ y - 153.1 &= 0.326x - 652 \\ y &= 0.326x - 498.9 \end{aligned}$$

c. 2020 corresponds to $x = 2020$.

$$y = 0.326x - 498.9$$
$$y = 0.326(2020) - 498.9$$
$$y = 658.52 - 498.9$$
$$y = 159.62$$

The number of White non-Hispanic individuals in the U.S. civilian non-institutional population 16 years and older in 2020 is projected to be 159.62 million people.

68. a. Let x represent the number of clients in Group 1, and y represent the number of clients in Group 2. Then,

Group 1 Expense +

Group 2 Expense = Total Expense

$$300x + 200y = 100,000$$

b.
$$300x + 200y = 100,000$$
$$200y = -300x + 100,000$$
$$y = \frac{-300x + 100,000}{200}$$
$$y = \frac{-300}{200}x + \frac{100,000}{200}$$
$$y = -1.5x + 500$$

The y-intercept is 500. If no clients from the first group are served, then 500 clients from the second group can be served. The slope is –1.5. For each one person increase in the number of clients served from the first group there is a corresponding decrease of 1.5 clients served from the second group.

c. $10(-1.5) = -15$

Fifteen fewer clients can be served from the second group.

69.
$$\frac{f(b) - f(a)}{b - a} = \frac{f(4) - f(2)}{4 - 2} = \frac{256 - 192}{4 - 2} = \frac{64}{2} = 32$$

The average velocity over this period of time is 32 ft/sec.

70.
$$\frac{f(b) - f(a)}{b - a} = \frac{f(3) - f(2)}{3 - 2} = \frac{-48 - 0}{3 - 2} = \frac{-48}{1} = -48$$

The average velocity over this period of time is –48 ft/sec.

71. First calculate $f(2+h)$.

$$\begin{aligned} f(2+h) &= -16(2+h)^2 + 128(2+h) \\ &= -16(h^2 + 4h + 4) + 128(2+h) \\ &= -16h^2 - 64h - 64 + 256 + 128h \\ &= -16h^2 + 64h + 192 \end{aligned}$$

Now Substitute into the difference quotient:

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{f(2+h) - f(2)}{h} \\ &= \frac{\left(-16h^2 + 64h + 192\right) - \left(-16(2)^2 + 128(2)\right)}{h} \\ &= \frac{-16h^2 + 64h + 192 + 64 - 256}{h} \\ &= \frac{-16h^2 + 64h}{h} \\ &= -16h + 64 \end{aligned}$$

72. First calculate $f(3+h)$.

$$\begin{aligned} f(3+h) &= 112(3+h) - 16(3+h)^2 \\ &= 112(3+h) - 16(h^2+6h+9) \\ &= 336 + 112h - 16h^2 - 96h - 144 \\ &= -16h^2 + 16h + 192 \end{aligned}$$

Now Substitute into the difference quotient:

$$\frac{f(x+h)-f(x)}{h} = \frac{f(3+h)-f(3)}{h}$$

$$= \frac{\left(-16h^2+16h+192\right)-\left(112(3)-16(3)^2\right)}{h}$$

$$= \frac{-16h^2+16h+192-336+144}{h}$$

$$= \frac{-16h^2+16h}{h}$$

$$= -16h+16$$

Chapter 1 Skills Check

1. The table represents a function because each x matches with exactly one y.

2. Domain = $\{-3,-1,1,3,5,7,9,11,13\}$

Range = $\{9,6,3,0,-3,-6,-9,-12,-15\}$

3. $f(3)=0$

4. Yes. The rate of change between any two pairs of values is constant.
The slope is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{6-9}{-1-(-3)}=\frac{-3}{2}=-\frac{3}{2}.$$

Calculating the equation:

$$y-y_1=m(x-x_1)$$
$$y-9=-\frac{3}{2}(x-(-3))$$
$$y-9=-\frac{3}{2}x-\frac{9}{2}$$
$$y=-\frac{3}{2}x-\frac{9}{2}+9$$
$$y=-\frac{3}{2}x-\frac{9}{2}+\frac{18}{2}$$
$$y=-\frac{3}{2}x+\frac{9}{2}$$

5. **a.** $C(3)=16-2(3)^2$
$=16-2(9)=16-18=-2$

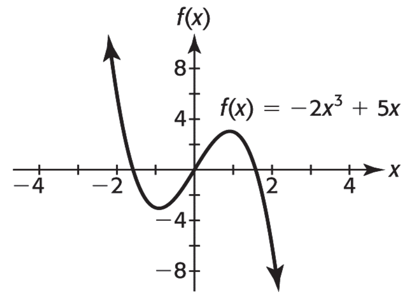
b. $C(-2)=16-2(-2)^2$
$=16-2(4)=16-8=8$

c. $C(-1)=16-2(-1)^2$
$=16-2(1)=16-2=14$

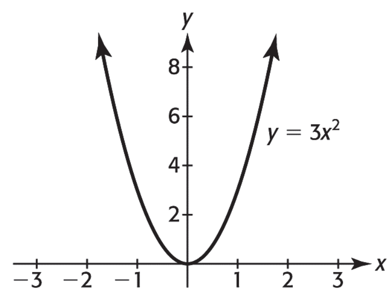
6. **a.** $f(-3)=1$

b. $f(-3)=-10$

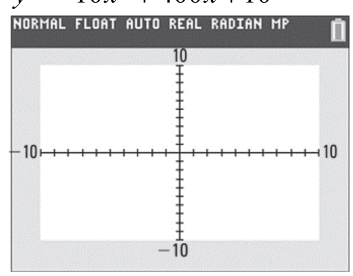
7.

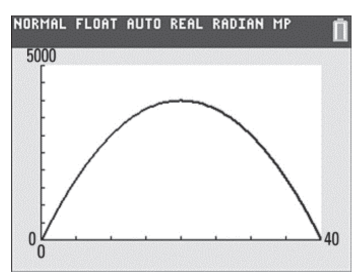


8.



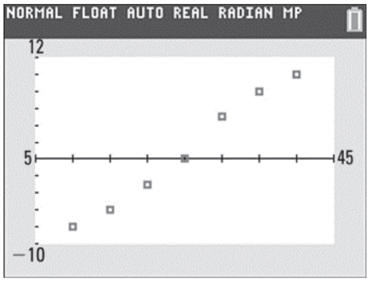
9. $y=-10x^2+400x+10$





The second view is better.

10.



11. a. Since $y=\sqrt{2x-8}$ will not be a real number if $2x-8<0$, the only values of x that yield real outputs to the function are values that satisfy $2x-8\geq 0$. Isolating x yields:

$$2x-8+8\geq 0+8$$
$$\frac{2x}{2}\geq\frac{8}{2}$$
$$x\geq 4$$

Therefore the domain is D: $[4,\infty)$.

b. The denominator of the function will be zero if $x-6=0$ or $x=6$. This implies $x\neq 6$. The domain is all real numbers except 6 or in interval notation D: $(-\infty,6)\text{U}(6,\infty)$.

12. The slope of the line through the two given points is: $m=\frac{y_2-y_1}{x_2-x_1}=\frac{38-16}{-1-(-12)}=\frac{22}{11}=2$

The slope of the line from the given equation is $-1/2$. Since the slopes are negative reciprocals of each other, the lines are perpendicular.

13. The slope of the given line is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-3-4}{5-(-1)}=\frac{-7}{6}=-\frac{7}{6}.$$

Since two parallel lines have the same slope, the slope of any line parallel to this one is also $m=-\frac{7}{6}$.

Since the slopes of perpendicular lines are negative reciprocals of one another,

$$m_\perp=-\frac{1}{m}=-\frac{1}{\left(-\frac{7}{6}\right)}=-\left(-\frac{6}{7}\right)=\frac{6}{7}$$

14. $m=\frac{y_2-y_1}{x_2-x_1}=\frac{-16-6}{8-(-4)}=\frac{-22}{12}=-\frac{11}{6}$

15. a. x-intercept: Let $y = 0$ and solve for x.

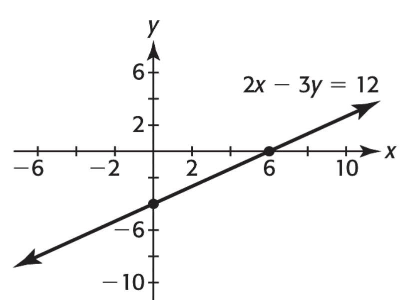
$$2x-3(0)=12$$
$$2x=12$$
$$x=6$$

y-intercept: Let $x = 0$ and solve for y.

$$2(0)-3y=12$$
$$-3y=12$$
$$y=-4$$

x-intercept: (6, 0), y-intercept: (0, –4)

b.



16. Solving for y:

$$2x-3y=12$$
$$-3y=-2x+12$$
$$\frac{-3y}{-3}=\frac{-2x+12}{-3}$$
$$y=\frac{2}{3}x-4$$

Since the model is linear, the rate of change is equal to the slope of the equation. The slope, m, is $\frac{2}{3}$.

17. The slope is $m=-6$ The y-intercept is $(0,3)$.

18. Since the function is linear, the rate of change is the slope. $m=-6$.

19. $y=mx+b$

$$y=\frac{1}{3}x+3$$

20.

$$y-y_1=m(x-x_1)$$
$$y-(-6)=-\frac{3}{4}(x-4)$$
$$y+6=-\frac{3}{4}x+3$$
$$y=-\frac{3}{4}x-3$$

21. The slope is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{6-3}{2-(-1)}=\frac{3}{3}=1.$$

Solving for the equation:

$$y-y_1=m(x-x_1)$$
$$y-6=1(x-2)$$
$$y-6=x-2$$
$$y=x+4$$

22. $y=x^2$

$x=0$: $\quad y=(0)^2=0 \qquad (0,0)$

$x=3$: $\quad y=(3)^2=9 \qquad (3,9)$

The average rate of change is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{9-0}{3-0}=\frac{9}{3}=3.$$

23. **a.** $f(x+h)=5-4(x+h)$
$=5-4x-4h$

b. $f(x+h)-f(x)$
$=[5-4(x+h)]-[5-4x]$
$=5-4x-4h-5+4x$
$=-4h$

c. $\dfrac{f(x+h)-f(x)}{h}$
$=\dfrac{-4h}{h}$
$=-4$

24. **a.** $f(x+h)$
$=10(x+h)-50$
$=10x+10h-50$

b. $f(x+h)-f(x)$
$=[10x+10h-50]-[10x-50]$
$=10x+10h-50-10x+50$
$=10h$

c. $\dfrac{f(x+h)-f(x)}{h}$
$=\dfrac{10h}{h}$
$=10$

Chapter 1 Review Exercises

25. a. Yes. If only the Democratic percentages are considered, each year matches with exactly one black voter percentage.

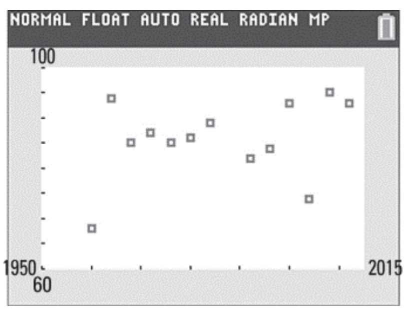
b. $f(1992)=82$. The table indicates that in 1992, 82% of black voters supported a Democratic candidate for president.

c. When $f(y)=94, y=1964$. The table indicates that in 1964, 94% of black voters supported a Democratic candidate for president.

26. a. The domain is $\{1960,\ 1964,\ 1968,\ 1972,\ 1976,\ 1980,\ 1984, 1992, 1996, 2000, 2004, 2008, 2012\}$

b. No. 1982 was not a presidential election year.

27.



28. a. For 1972 to 2012:

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{93-87}{2012-1972}=\frac{6}{40}=0.15$$

b. The average annual rate of change is the same as the slope in part a), therefore = 0.15 percentage points per year.

c. For 1972 to 1984:

$$\frac{f(b)-f(a)}{b-a}=\frac{89-87}{1984-1972}=\frac{2}{12}\approx 0.1667$$

No, the rates of change are not equal.

29. a. Each amount borrowed matches with exactly one monthly payment. The change in y is 89.62 for each change in x of 5000.

b. $f(25{,}000)=448.11$. Therefore, borrowing \$25,000 to buy a car from the dealership results in a monthly payment of \$448.11.

c. If $f(A)=\$358.49$, then $A=\$20{,}000$.

30. a. Domain = $\{10{,}000,\ 15{,}000,\ 20{,}000,\ 25{,}000,\ 30{,}000\}$

Range = $\{179.25,\ 268.87,\ 358.49,\ 448.11,\ 537.73\}$

b. No. \$12,000 is not in the domain of the function.

31. a.

$$\begin{aligned} f(28{,}000) &= 0.017924(28{,}000)+0.01 \\ &= 501.872+0.01 \\ &= 501.882 \end{aligned}$$

The predicted monthly payment on a car loan of \$28,000 is \$501.88.

b. Any positive input could be used for A. Borrowing a negative amount of money does not make sense in the context of the problem.

32. a. $f(1960) = 15.9$. A 65-year old woman in 1960 is expected to live 15.9 more years. Her overall life expectancy is 80.9 years.

b. $f(2010) = 19.4$. A 65-year old woman in 2010 has a life expectancy of 84.4 years.

c. Since $f(1990) = 19$, the average woman is expected to live 19 years past age 65 in 1990.

33. a. $g(2020) = 16.9$. A 65-year old man in 2020 is expected to live 16.9 more years. His overall life expectancy is 81.9 years.

b. Since $g(1950) = 12.8$, a 65-year old man in 1950 has a life expectancy of 77.8 years.

c. Since $g(1990) = 15$, the average man is expected to live 15 years past age 65 (or to age 80) in 1990. $g(1990) = 15$.

34. No; The function cannot be written in linear form, $y = mx + b$.

35. a. Let t = age, and let h = heart rate in beats per minute. Using points (20, 160) and (90, 104) to calculate the slope yields:

$$m = \frac{h_2 - h_1}{t_2 - t_1} = \frac{104 - 160}{90 - 20} = \frac{-56}{70} = -0.8$$

Equation:

$$\begin{aligned} h - h_1 &= m(t - t_1) \\ h - 160 &= -0.8(t - 20) \\ h - 160 &= -0.8t + 16 \\ h &= -0.8t + 176 \end{aligned}$$

b. Yes. Consider the following table of values based on the equation in comparison to the actual data points.

t	h = Equation Values	Actual Values
20	160	160
30	152	152
40	144	144
50	136	136
60	128	128
70	120	120
80	112	112
90	104	104

c. They are the same since the points $(20,160)$ and $(90,104)$ were used to calculate the slope of the linear model.

36. a. Using points (2010, 1.94) and (2030, 2.26) to calculate the slope yields:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.26 - 1.94}{2030 - 2010} = \frac{0.32}{20} = 0.016$$

b. Let x = the year, and let y = crude oil production in billions of barrels. The equation will be:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1.94 &= 0.016(x - 2010) \\ y - 1.94 &= 0.016x - 32.16 \\ y &= 0.016x - 30.22 \end{aligned}$$

37. $f(x) = 4500$

38. a. Let x = the number of months past May 2016, and $f(x)$ = the average weekly hours worked. The function is $f(x) = 33.8$.

b. This is a constant function since the rate of change is zero.

39. a. $R(120) = 564(120) = 67,680$. The revenue when 120 units are produced is \$67,680.

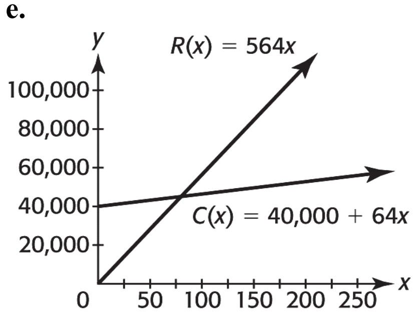
b. $C(120) = 40,000 + 64(120) = 47,680$. The cost when 120 units are produced is \$47,680.

c. Marginal Cost $= MC = 64 = \$64$. Note that MC is the slope of the cost function.

Marginal Revenue $= MR = 564 = \$564$. Note that MR is the slope of the revenue function.

d. $m = 64$

e.



40. a. $$\begin{aligned} P(x) &= 564x - (40,000 + 64x) \\ &= 564x - 40,000 - 64x \\ &= 500x - 40,000 \end{aligned}$$

b. $$\begin{aligned} P(120) &= 500(120) - 40,000 \\ &= 60,000 - 40,000 \\ &= \$20,000 \end{aligned}$$

c. Break-even occurs when $R(x) = C(x)$ or alternately $P(x) = R(x) - C(x) = 0$.

$$\begin{aligned} 500x - 40,000 &= 0 \\ 500x &= 40,000 \\ x &= \frac{40,000}{500} \\ x &= 80 \end{aligned}$$

Eighty units represent break-even for the company.

d. $MP =$ the slope of $P(x) = 500$

e. $MP = MR - MC$

41. a. Let $x = 0$, and solve for y.

$$\begin{aligned} y + 3000(0) &= 300,000 \\ y &= 300,000 \end{aligned}$$

The initial value of the property is \$300,000.

b. Let $y = 0$, and solve for x.

$$\begin{aligned} 0 + 3000x &= 300,000 \\ 3000x &= 300,000 \\ x &= 100 \end{aligned}$$

The value of the property after 100 years is zero dollars.

42. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{895 - 455}{250 - 150} = \dfrac{440}{100} = 4.4$.

The average rate of change is \$4.40 per unit.

b. For a linear function, the slope is the average rate of change. Referring to part a), the slope is 4.4.

c. $$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 455 &= 4.4(x - 150) \\ y - 455 &= 4.4x - 660 \\ y &= 4.4x - 205 \\ P(x) &= 4.4x - 205 \end{aligned}$$

d. $MP =$ the slope of $P(x) = 4.4$ or \$4.40 per unit.

e. Break-even occurs when $R(x) = C(x)$ or alternately $P(x) = R(x) - C(x) = 0$.

$$4.4x - 205 = 0$$
$$4.4x = 205$$
$$x = \frac{205}{4.4}$$
$$x = 46.59\overline{09} \approx 47$$

The company will break even selling approximately 47 units.

Group Activities/Extended Applications

1. Body Mass Index

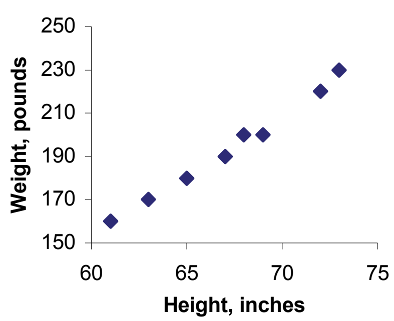
1. A person uses the table to determine his or her BMI by locating the entry in the table that corresponds to his or her height and weight. The entry in the table is the person's BMI.

2. If a person's BMI is 30 or higher, the person is considered obese and at risk for health problems.

3. **a.** Determine the heights and weights that produce a BMI of exactly 30 based on the table.

Height (inches)	Weight (pounds)
61	160
63	170
65	180
67	190
68	200
69	200
72	220
73	230

b.

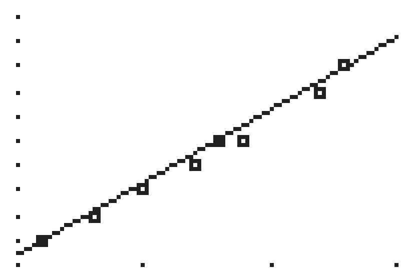


A linear model is reasonable, but not exact.

c.
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{230-160}{73-61} \\ &= \frac{70}{12} \\ &= 5.8\overline{3} \end{aligned}$$

$$y - y_1 = m(x - x_1)$$
$$y - 160 = \frac{70}{12}(x - 61)$$
$$y - 160 = \frac{70}{12}x - \frac{4270}{12}$$
$$y = \frac{70}{12}x - \frac{4270}{12} + \frac{1920}{12}$$
$$y = \frac{70}{12}x - \frac{2350}{12}$$
$$y = 5.8\overline{3}x - 195.8\overline{3}$$

d.



[60, 75] by [150, 250]

The line fits the data points well, but not perfectly.

e. Any data point that lies exactly along the line generated from the model will yield a BMI of 30. If a height is substituted into the model, the output weight would generate a BMI of 30. That weight or any higher weight for the given height would place a person at risk for health problems.

2. Total Revenue, Total Cost, and Profit

Let x represent the number of units produced and sold.

1. The revenue function is $R(x) = 98x$.

2. Marginal Revenue $= MR = 98 = \$98$.
Note that MR is the slope of the revenue function.

3. $C(x) = 23x + 262{,}500$

4. Marginal Cost $= MC = 23 = \$23$. Note that MC is the slope of the cost function.

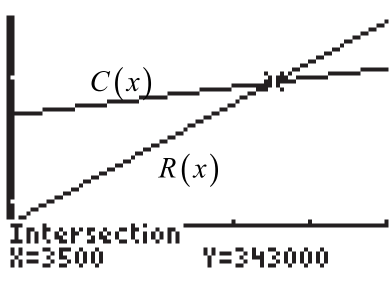
Neither. $MC \cdot x$ is the variable cost.

5.
$$P(x) = R(x) - C(x)$$
$$= [98x] - [23x - 262{,}500]$$
$$= 75x - 262{,}500$$

$MP =$ the slope of $P(x) = 75$

6. If $x = 0$, then
$R(0) = 98(0) = 0$
$C(0) = 23(0) + 262{,}500 = 262{,}500$
$P(0) = 75(0) - 262{,}500 = -262{,}500$

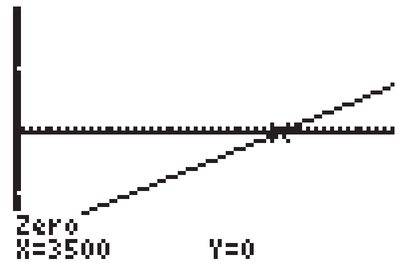
7.



[0, 5000] by [0, 500,000]

The intersection point is approximately $x = 3500$ units.

8.



[0, 5000] by [–300,000, 300,000]

$x = 3500$ is the intersection point

9. The intersection points in questions 7 and 8 represent the value of x where $R(x) = C(x)$. Therefore, the points represent the break-even production level for the MP3 players. If the company produces and sells 3500 MP3 players, it will break even.