Chapter 1

Functions and Their Graphs

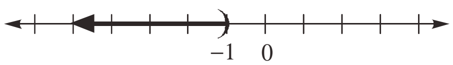
Section 1.1

1. $(-1,3)$

2. $3(-2)^2-5(-2)+\dfrac{1}{(-2)}=3(4)-5(-2)-\dfrac{1}{2}$
$$=12+10-\frac{1}{2}$$
$$=\frac{43}{2} \text{ or } 21\tfrac{1}{2} \text{ or } 21.5$$

3. We must not allow the denominator to be 0.
$x+4\neq 0 \Rightarrow x\neq -4$; Domain: $\{x|x\neq -4\}$.

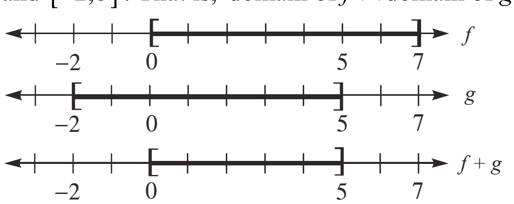
4. $3-2x>5$
$-2x>2$
$x<-1$
Solution set: $\{x\,|\,x<-1\}$ or $(-\infty,-1)$



5. independent; dependent

6. range

7. $[0,5]$
We need the intersection of the intervals $[0,7]$ and $[-2,5]$. That is, domain of $f\cap$ domain of g.



8. $\neq$; f; g

9. $(g-f)(x)$ or $g(x)-f(x)$

10. False; every function is a relation, but not every relation is a function. For example, the relation $x^2+y^2=1$ is not a function.

11. True

12. True

13. False; if the domain is not specified, we assume it is the largest set of real numbers for which the value of f is a real number.

14. False; the domain of $f(x)=\dfrac{x^2-4}{x}$ is $\{x\,|\,x\neq 0\}$.

15. Function
Domain: {Elvis, Colleen, Kaleigh, Marissa}
Range: {Jan. 8, Mar. 15, Sept. 17}

16. Not a function

17. Not a function

18. Function
Domain: {Less than 9th grade, 9th-12th grade, High School Graduate, Some College, College Graduate}
Range: {$18,120, $23,251, $36,055, $45,810, $67,165}

19. Not a function

20. Function
Domain: {–2, –1, 3, 4}
Range: {3, 5, 7, 12}

21. Function
Domain: {1, 2, 3, 4}
Range: {3}

22. Function
Domain: {0, 1, 2, 3}
Range: {–2, 3, 7}

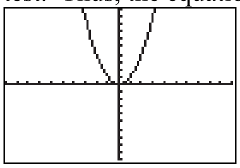
23. Not a function

24. Not a function

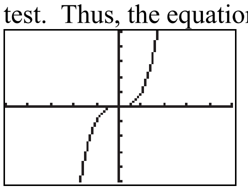
25. Function
Domain: {–2, –1, 0, 1}
Range: {0, 1, 4}

26. Function
Domain: {–2, –1, 0, 1}
Range: {3, 4, 16}

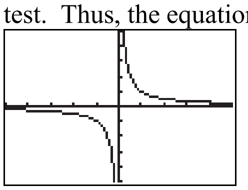
27. Graph $y = x^2$. The graph passes the vertical line test. Thus, the equation represents a function.



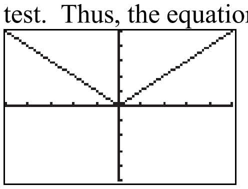
28. Graph $y = x^3$. The graph passes the vertical line test. Thus, the equation represents a function.



29. Graph $y = \frac{1}{x}$. The graph passes the vertical line test. Thus, the equation represents a function.



30. Graph $y = |x|$. The graph passes the vertical line test. Thus, the equation represents a function.



31. $y^2 = 4 - x^2$

Solve for y: $y = \pm\sqrt{4 - x^2}$

For $x = 0, y = \pm 2$. Thus, (0, 2) and (0, –2) are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

32. $y = \pm\sqrt{1 - 2x}$

For $x = 0, y = \pm 1$. Thus, (0, 1) and (0, –1) are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

33. $x = y^2$

Solve for y: $y = \pm\sqrt{x}$

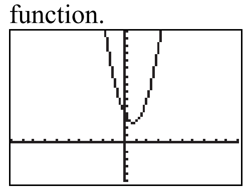
For $x = 1, y = \pm 1$. Thus, (1, 1) and (1, –1) are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

34. $x + y^2 = 1$

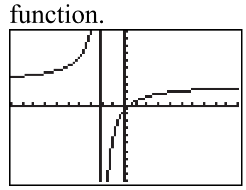
Solve for y: $y = \pm\sqrt{1 - x}$

For $x = 0, y = \pm 1$. Thus, (0, 1) and (0, –1) are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

35. Graph $y = 2x^2 - 3x + 4$. The graph passes the vertical line test. Thus, the equation represents a function.



36. Graph $y = \frac{3x - 1}{x + 2}$. The graph passes the vertical line test. Thus, the equation represents a function.



37. $2x^2 + 3y^2 = 1$

Solve for y:
$$\begin{aligned} 2x^2 + 3y^2 &= 1 \\ 3y^2 &= 1 - 2x^2 \\ y^2 &= \frac{1 - 2x^2}{3} \\ y &= \pm\sqrt{\frac{1 - 2x^2}{3}} \end{aligned}$$

For $x = 0, y = \pm\sqrt{\frac{1}{3}}$. Thus, $\left(0, \sqrt{\frac{1}{3}}\right)$ and $\left(0, -\sqrt{\frac{1}{3}}\right)$ are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

38. $x^2 - 4y^2 = 1$

Solve for y: $x^2 - 4y^2 = 1$

$$4y^2 = x^2 - 1$$

$$y^2 = \frac{x^2 - 1}{4}$$

$$y = \frac{\pm\sqrt{x^2 - 1}}{2}$$

For $x = \sqrt{2}$, $y = \pm\frac{1}{2}$. Thus, $\left(\sqrt{2}, \frac{1}{2}\right)$ and $\left(\sqrt{2}, -\frac{1}{2}\right)$ are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

39. $f(x) = 3x^2 + 2x - 4$

a. $f(0) = 3(0)^2 + 2(0) - 4 = -4$

b. $f(1) = 3(1)^2 + 2(1) - 4 = 3 + 2 - 4 = 1$

c. $f(-1) = 3(-1)^2 + 2(-1) - 4 = 3 - 2 - 4 = -3$

d. $f(-x) = 3(-x)^2 + 2(-x) - 4 = 3x^2 - 2x - 4$

e. $-f(x) = -\left(3x^2 + 2x - 4\right) = -3x^2 - 2x + 4$

f. $f(x+1) = 3(x+1)^2 + 2(x+1) - 4$

$$= 3\left(x^2 + 2x + 1\right) + 2x + 2 - 4$$

$$= 3x^2 + 6x + 3 + 2x + 2 - 4$$

$$= 3x^2 + 8x + 1$$

g. $f(2x) = 3(2x)^2 + 2(2x) - 4 = 12x^2 + 4x - 4$

h. $f(x+h) = 3(x+h)^2 + 2(x+h) - 4$

$$= 3\left(x^2 + 2xh + h^2\right) + 2x + 2h - 4$$

$$= 3x^2 + 6xh + 3h^2 + 2x + 2h - 4$$

40. $f(x) = -2x^2 + x - 1$

a. $f(0) = -2(0)^2 + 0 - 1 = -1$

b. $f(1) = -2(1)^2 + 1 - 1 = -2$

c. $f(-1) = -2(-1)^2 + (-1) - 1 = -4$

d. $f(-x) = -2(-x)^2 + (-x) - 1 = -2x^2 - x - 1$

e. $-f(x) = -\left(-2x^2 + x - 1\right) = 2x^2 - x + 1$

f. $f(x+1) = -2(x+1)^2 + (x+1) - 1$

$$= -2\left(x^2 + 2x + 1\right) + x + 1 - 1$$

$$= -2x^2 - 4x - 2 + x$$

$$= -2x^2 - 3x - 2$$

g. $f(2x) = -2(2x)^2 + (2x) - 1 = -8x^2 + 2x - 1$

h. $f(x+h) = -2(x+h)^2 + (x+h) - 1$

$$= -2\left(x^2 + 2xh + h^2\right) + x + h - 1$$

$$= -2x^2 - 4xh - 2h^2 + x + h - 1$$

41. $f(x) = \dfrac{x}{x^2 + 1}$

a. $f(0) = \dfrac{0}{0^2 + 1} = \dfrac{0}{1} = 0$

b. $f(1) = \dfrac{1}{1^2 + 1} = \dfrac{1}{2}$

c. $f(-1) = \dfrac{-1}{(-1)^2 + 1} = \dfrac{-1}{1+1} = -\dfrac{1}{2}$

d. $f(-x) = \dfrac{-x}{(-x)^2 + 1} = \dfrac{-x}{x^2 + 1}$

e. $-f(x) = -\left(\dfrac{x}{x^2 + 1}\right) = \dfrac{-x}{x^2 + 1}$

f. $f(x+1) = \dfrac{x+1}{(x+1)^2 + 1}$

$$= \frac{x+1}{x^2 + 2x + 1 + 1}$$

$$= \frac{x+1}{x^2 + 2x + 2}$$

g. $f(2x) = \dfrac{2x}{(2x)^2 + 1} = \dfrac{2x}{4x^2 + 1}$

h. $f(x+h) = \dfrac{x+h}{(x+h)^2 + 1} = \dfrac{x+h}{x^2 + 2xh + h^2 + 1}$

42. $f(x)=\dfrac{x^2-1}{x+4}$

a. $f(0)=\dfrac{0^2-1}{0+4}=\dfrac{-1}{4}=-\dfrac{1}{4}$

b. $f(1)=\dfrac{1^2-1}{1+4}=\dfrac{0}{5}=0$

c. $f(-1)=\dfrac{(-1)^2-1}{-1+4}=\dfrac{0}{3}=0$

d. $f(-x)=\dfrac{(-x)^2-1}{-x+4}=\dfrac{x^2-1}{-x+4}$

e. $-f(x)=-\left(\dfrac{x^2-1}{x+4}\right)=\dfrac{-x^2+1}{x+4}$

f. $f(x+1)=\dfrac{(x+1)^2-1}{(x+1)+4}$
$=\dfrac{x^2+2x+1-1}{x+5}=\dfrac{x^2+2x}{x+5}$

g. $f(2x)=\dfrac{(2x)^2-1}{2x+4}=\dfrac{4x^2-1}{2x+4}$

h. $f(x+h)=\dfrac{(x+h)^2-1}{(x+h)+4}=\dfrac{x^2+2xh+h^2-1}{x+h+4}$

43. $f(x)=|x|+4$

a. $f(0)=|0|+4=0+4=4$

b. $f(1)=|1|+4=1+4=5$

c. $f(-1)=|-1|+4=1+4=5$

d. $f(-x)=|-x|+4=|x|+4$

e. $-f(x)=-(|x|+4)=-|x|-4$

f. $f(x+1)=|x+1|+4$

g. $f(2x)=|2x|+4=2|x|+4$

h. $f(x+h)=|x+h|+4$

44. $f(x)=\sqrt{x^2+x}$

a. $f(0)=\sqrt{0^2+0}=\sqrt{0}=0$

b. $f(1)=\sqrt{1^2+1}=\sqrt{2}$

c. $f(-1)=\sqrt{(-1)^2+(-1)}=\sqrt{1-1}=\sqrt{0}=0$

d. $f(-x)=\sqrt{(-x)^2+(-x)}=\sqrt{x^2-x}$

e. $-f(x)=-\left(\sqrt{x^2+x}\right)=-\sqrt{x^2+x}$

f. $f(x+1)=\sqrt{(x+1)^2+(x+1)}$
$=\sqrt{x^2+2x+1+x+1}$
$=\sqrt{x^2+3x+2}$

g. $f(2x)=\sqrt{(2x)^2+2x}=\sqrt{4x^2+2x}$

h. $f(x+h)=\sqrt{(x+h)^2+(x+h)}$
$=\sqrt{x^2+2xh+h^2+x+h}$

45. $f(x)=\dfrac{2x+1}{3x-5}$

a. $f(0)=\dfrac{2(0)+1}{3(0)-5}=\dfrac{0+1}{0-5}=-\dfrac{1}{5}$

b. $f(1)=\dfrac{2(1)+1}{3(1)-5}=\dfrac{2+1}{3-5}=\dfrac{3}{-2}=-\dfrac{3}{2}$

c. $f(-1)=\dfrac{2(-1)+1}{3(-1)-5}=\dfrac{-2+1}{-3-5}=\dfrac{-1}{-8}=\dfrac{1}{8}$

d. $f(-x)=\dfrac{2(-x)+1}{3(-x)-5}=\dfrac{-2x+1}{-3x-5}=\dfrac{2x-1}{3x+5}$

e. $-f(x)=-\left(\dfrac{2x+1}{3x-5}\right)=\dfrac{-2x-1}{3x-5}$

f. $f(x+1)=\dfrac{2(x+1)+1}{3(x+1)-5}=\dfrac{2x+2+1}{3x+3-5}=\dfrac{2x+3}{3x-2}$

g. $f(2x)=\dfrac{2(2x)+1}{3(2x)-5}=\dfrac{4x+1}{6x-5}$

h. $f(x+h)=\dfrac{2(x+h)+1}{3(x+h)-5}=\dfrac{2x+2h+1}{3x+3h-5}$

46. $f(x)=1-\dfrac{1}{(x+2)^2}$

a. $f(0)=1-\dfrac{1}{(0+2)^2}=1-\dfrac{1}{4}=\dfrac{3}{4}$

b. $f(1)=1-\dfrac{1}{(1+2)^2}=1-\dfrac{1}{9}=\dfrac{8}{9}$

c. $f(-1)=1-\dfrac{1}{(-1+2)^2}=1-\dfrac{1}{1}=0$

d. $f(-x)=1-\dfrac{1}{(-x+2)^2}=1-\dfrac{1}{(2-x)^2}$

e. $-f(x)=-\left(1-\dfrac{1}{(x+2)^2}\right)=\dfrac{1}{(x+2)^2}-1$

f. $f(x+1)=1-\dfrac{1}{(x+1+2)^2}=1-\dfrac{1}{(x+3)^2}$

g. $f(2x)=1-\dfrac{1}{(2x+2)^2}=1-\dfrac{1}{4(x+1)^2}$

h. $f(x+h)=1-\dfrac{1}{(x+h+2)^2}$

47. $f(x)=-5x+4$

Domain: $\{x \mid x \text{ is any real number}\}$

48. $f(x)=x^2+2$

Domain: $\{x \mid x \text{ is any real number}\}$

49. $f(x)=\dfrac{x}{x^2+1}$

Domain: $\{x \mid x \text{ is any real number}\}$

50. $f(x)=\dfrac{x^2}{x^2+1}$

Domain: $\{x \mid x \text{ is any real number}\}$

51. $g(x)=\dfrac{x}{x^2-16}$

$x^2-16\neq 0$

$x^2\neq 16 \Rightarrow x\neq \pm 4$

Domain: $\{x \mid x\neq -4,\ x\neq 4\}$

52. $h(x)=\dfrac{2x}{x^2-4}$

$x^2-4\neq 0$

$x^2\neq 4 \Rightarrow x\neq \pm 2$

Domain: $\{x \mid x\neq -2,\ x\neq 2\}$

53. $F(x)=\dfrac{x-2}{x^3+x}$

$x^3+x\neq 0$

$x(x^2+1)\neq 0$

$x\neq 0, \quad x^2\neq -1$

Domain: $\{x \mid x\neq 0\}$

54. $G(x)=\dfrac{x+4}{x^3-4x}$

$x^3-4x\neq 0$

$x(x^2-4)\neq 0$

$x\neq 0, \quad x^2\neq 4$

$x\neq 0, \quad x\neq \pm 2$

Domain: $\{x \mid x\neq -2,\ x\neq 0,\ x\neq 2\}$

55. $h(x)=\sqrt{3x-12}$

$3x-12\geq 0$

$3x\geq 12$

$x\geq 4$

Domain: $\{x \mid x\geq 4\}$

56. $G(x)=\sqrt{1-x}$

$1-x\geq 0$

$-x\geq -1$

$x\leq 1$

Domain: $\{x \mid x\leq 1\}$

57. $f(x)=\dfrac{4}{\sqrt{x-9}}$

$x-9>0$

$x>9$

Domain: $\{x \mid x>9\}$

58. $f(x)=\dfrac{x}{\sqrt{x-4}}$

$x-4>0$

$x>4$

Domain: $\{x\,|\,x>4\}$

59. $p(x)=\sqrt{\dfrac{2}{x-1}}=\dfrac{\sqrt{2}}{\sqrt{x-1}}$

$x-1>0$

$x>1$

Domain: $\{x\,|\,x>1\}$

60. $q(x)=\sqrt{-x-2}$

$-x-2\ge 0$

$-x\ge 2$

$x\le -2$

Domain: $\{x\,|\,x\le -2\}$

61. $P(t)=\dfrac{\sqrt{t-4}}{3t-21}$

$t-4\ge 0$

$t\ge 4$

Also $3t-21\ne 0$

$3t-21\ne 0$

$3t\ne 21$

$t\ne 7$

Domain: $\{t\,|\,t\ge 4, t\ne 7\}$

62. $h(z)=\dfrac{\sqrt{z+3}}{z-2}$

$z+3\ge 0$

$z\ge -3$

Also $z-2\ne 0$

$z\ne 2$

Domain: $\{z\,|\,z\ge -3, z\ne 2\}$

63. $f(x)=3x+4\qquad g(x)=2x-3$

a. $(f+g)(x)=3x+4+2x-3=5x+1$

Domain: $\{x\,|\,x \text{ is any real number}\}$.

b. $(f-g)(x)=(3x+4)-(2x-3)$

$=3x+4-2x+3$

$=x+7$

Domain: $\{x\,|\,x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=(3x+4)(2x-3)$

$=6x^2-9x+8x-12$

$=6x^2-x-12$

Domain: $\{x\,|\,x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{3x+4}{2x-3}$

$2x-3\ne 0\Rightarrow 2x\ne 3\Rightarrow x\ne \dfrac{3}{2}$

Domain: $\left\{x\,\middle|\,x\ne \dfrac{3}{2}\right\}$.

e. $(f+g)(3)=5(3)+1=15+1=16$

f. $(f-g)(4)=4+7=11$

g. $(f\cdot g)(2)=6(2)^2-2-12=24-2-12=10$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{3(1)+4}{2(1)-3}=\dfrac{3+4}{2-3}=\dfrac{7}{-1}=-7$

64. $f(x)=2x+1\qquad g(x)=3x-2$

a. $(f+g)(x)=2x+1+3x-2=5x-1$

Domain: $\{x\,|\,x \text{ is any real number}\}$.

b. $(f-g)(x)=(2x+1)-(3x-2)$

$=2x+1-3x+2$

$=-x+3$

Domain: $\{x\,|\,x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=(2x+1)(3x-2)$

$=6x^2-4x+3x-2$

$=6x^2-x-2$

Domain: $\{x\,|\,x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{2x+1}{3x-2}$

$3x-2\ne 0$

$3x\ne 2\Rightarrow x\ne \dfrac{2}{3}$

Domain: $\left\{x\,\middle|\,x\ne \dfrac{2}{3}\right\}$.

e. $(f+g)(3)=5(3)-1=15-1=14$

f. $(f-g)(4)=-4+3=-1$

g. $(f\cdot g)(2)=6(2)^2-2-2$
$=6(4)-2-2$
$=24-2-2=20$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{2(1)+1}{3(1)-2}=\dfrac{2+1}{3-2}=\dfrac{3}{1}=3$

65. $f(x)=x-1 \qquad g(x)=2x^2$

a. $(f+g)(x)=x-1+2x^2=2x^2+x-1$
Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=(x-1)-(2x^2)$
$=x-1-2x^2$
$=-2x^2+x-1$
Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=(x-1)(2x^2)=2x^3-2x^2$
Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{x-1}{2x^2}$
Domain: $\{x \mid x\neq 0\}$.

e. $(f+g)(3)=2(3)^2+3-1$
$=2(9)+3-1$
$=18+3-1=20$

f. $(f-g)(4)=-2(4)^2+4-1$
$=-2(16)+4-1$
$=-32+4-1=-29$

g. $(f\cdot g)(2)=2(2)^3-2(2)^2$
$=2(8)-2(4)$
$=16-8=8$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{1-1}{2(1)^2}=\dfrac{0}{2(1)}=\dfrac{0}{2}=0$

66. $f(x)=2x^2+3 \qquad g(x)=4x^3+1$

a. $(f+g)(x)=2x^2+3+4x^3+1$
$=4x^3+2x^2+4$
Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=\left(2x^2+3\right)-\left(4x^3+1\right)$
$=2x^2+3-4x^3-1$
$=-4x^3+2x^2+2$
Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=\left(2x^2+3\right)\left(4x^3+1\right)$
$=8x^5+12x^3+2x^2+3$
Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{2x^2+3}{4x^3+1}$
$4x^3+1\neq 0$
$4x^3\neq -1$
$x^3\neq -\dfrac{1}{4}\Rightarrow x\neq \sqrt[3]{-\dfrac{1}{4}}=-\dfrac{\sqrt[3]{2}}{2}$
Domain: $\left\{x \,\middle|\, x\neq -\dfrac{\sqrt[3]{2}}{2}\right\}$.

e. $(f+g)(3)=4(3)^3+2(3)^2+4$
$=4(27)+2(9)+4$
$=108+18+4=130$

f. $(f-g)(4)=-4(4)^3+2(4)^2+2$
$=-4(64)+2(16)+2$
$=-256+32+2=-222$

g. $(f\cdot g)(2)=8(2)^5+12(2)^3+2(2)^2+3$
$=8(32)+12(8)+2(4)+3$
$=256+96+8+3=363$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{2(1)^2+3}{4(1)^3+1}=\dfrac{2(1)+3}{4(1)+1}=\dfrac{2+3}{4+1}=\dfrac{5}{5}=1$

67. $f(x)=\sqrt{x} \qquad g(x)=3x-5$

a. $(f+g)(x)=\sqrt{x}+3x-5$
Domain: $\{x \mid x\geq 0\}$.

b. $(f-g)(x)=\sqrt{x}-(3x-5)=\sqrt{x}-3x+5$
Domain: $\{x \mid x \geq 0\}$.

c. $(f\cdot g)(x)=\sqrt{x}(3x-5)=3x\sqrt{x}-5\sqrt{x}$
Domain: $\{x \mid x \geq 0\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x}}{3x-5}$
$x \geq 0$ and $3x-5 \neq 0$
$3x \neq 5 \Rightarrow x \neq \dfrac{5}{3}$
Domain: $\left\{x \,\middle|\, x \geq 0 \text{ and } x \neq \dfrac{5}{3}\right\}$.

e. $(f+g)(3)=\sqrt{3}+3(3)-5$
$=\sqrt{3}+9-5=\sqrt{3}+4$

f. $(f-g)(4)=\sqrt{4}-3(4)+5$
$=2-12+5=-5$

g. $(f\cdot g)(2)=3(2)\sqrt{2}-5\sqrt{2}$
$=6\sqrt{2}-5\sqrt{2}=\sqrt{2}$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{\sqrt{1}}{3(1)-5}=\dfrac{1}{3-5}=\dfrac{1}{-2}=-\dfrac{1}{2}$

68. $f(x)=|x| \qquad g(x)=x$

a. $(f+g)(x)=|x|+x$
Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=|x|-x$
Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=|x|\cdot x=x|x|$
Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{|x|}{x}$
Domain: $\{x \mid x \neq 0\}$.

e. $(f+g)(3)=|3|+3=3+3=6$

f. $(f-g)(4)=|4|-4=4-4=0$

g. $(f\cdot g)(2)=2|2|=2\cdot 2=4$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{|1|}{1}=\dfrac{1}{1}=1$

69. $f(x)=1+\dfrac{1}{x} \qquad g(x)=\dfrac{1}{x}$

a. $(f+g)(x)=1+\dfrac{1}{x}+\dfrac{1}{x}=1+\dfrac{2}{x}$
Domain: $\{x \mid x \neq 0\}$.

b. $(f-g)(x)=1+\dfrac{1}{x}-\dfrac{1}{x}=1$
Domain: $\{x \mid x \neq 0\}$.

c. $(f\cdot g)(x)=\left(1+\dfrac{1}{x}\right)\dfrac{1}{x}=\dfrac{1}{x}+\dfrac{1}{x^2}$
Domain: $\{x \mid x \neq 0\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{1+\frac{1}{x}}{\frac{1}{x}}=\dfrac{\frac{x+1}{x}}{\frac{1}{x}}=\dfrac{x+1}{x}\cdot\dfrac{x}{1}=x+1$
Domain: $\{x \mid x \neq 0\}$.

e. $(f+g)(3)=1+\dfrac{2}{3}=\dfrac{5}{3}$

f. $(f-g)(4)=1$

g. $(f\cdot g)(2)=\dfrac{1}{2}+\dfrac{1}{(2)^2}=\dfrac{1}{2}+\dfrac{1}{4}=\dfrac{3}{4}$

h. $\left(\dfrac{f}{g}\right)(1)=1+1=2$

70. $f(x)=\sqrt{x-1} \qquad g(x)=\sqrt{4-x}$

a. $(f+g)(x)=\sqrt{x-1}+\sqrt{4-x}$
$x-1 \geq 0$ and $4-x \geq 0$
$x \geq 1$ and $-x \geq -4$
$x \leq 4$
Domain: $\{x \mid 1 \leq x \leq 4\}$.

b. $(f-g)(x)=\sqrt{x-1}-\sqrt{4-x}$
$x-1 \geq 0$ and $4-x \geq 0$
$x \geq 1$ and $-x \geq -4$
$x \leq 4$
Domain: $\{x \mid 1 \leq x \leq 4\}$.

c. $(f \cdot g)(x) = \left(\sqrt{x-1}\right)\left(\sqrt{4-x}\right)$
$= \sqrt{-x^2+5x-4}$
$x-1 \ge 0$ and $4-x \ge 0$
$x \ge 1$ and $-x \ge -4$
$x \le 4$
Domain: $\{x \mid 1 \le x \le 4\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x-1}}{\sqrt{4-x}} = \sqrt{\dfrac{x-1}{4-x}}$
$x-1 \ge 0$ and $4-x > 0$
$x \ge 1$ and $-x > -4$
$x < 4$
Domain: $\{x \mid 1 \le x < 4\}$.

e. $(f+g)(3) = \sqrt{3-1} + \sqrt{4-3}$
$= \sqrt{2} + \sqrt{1} = \sqrt{2} + 1$

f. $(f-g)(4) = \sqrt{4-1} - \sqrt{4-4}$
$= \sqrt{3} - \sqrt{0} = \sqrt{3} - 0 = \sqrt{3}$

g. $(f \cdot g)(2) = \sqrt{-(2)^2+5(2)-4}$
$= \sqrt{-4+10-4} = \sqrt{2}$

h. $\left(\dfrac{f}{g}\right)(1) = \sqrt{\dfrac{1-1}{4-1}} = \sqrt{\dfrac{0}{3}} = \sqrt{0} = 0$

71. $f(x) = \dfrac{2x+3}{3x-2} \qquad g(x) = \dfrac{4x}{3x-2}$

a. $(f+g)(x) = \dfrac{2x+3}{3x-2} + \dfrac{4x}{3x-2}$
$= \dfrac{2x+3+4x}{3x-2} = \dfrac{6x+3}{3x-2}$
$3x-2 \ne 0$
$3x \ne 2 \Rightarrow x \ne \dfrac{2}{3}$
Domain: $\left\{x \,\middle|\, x \ne \dfrac{2}{3}\right\}$.

b. $(f-g)(x) = \dfrac{2x+3}{3x-2} - \dfrac{4x}{3x-2}$
$= \dfrac{2x+3-4x}{3x-2} = \dfrac{-2x+3}{3x-2}$
$3x-2 \ne 0$
$3x \ne 2 \Rightarrow x \ne \dfrac{2}{3}$
Domain: $\left\{x \,\middle|\, x \ne \dfrac{2}{3}\right\}$.

c. $(f \cdot g)(x) = \left(\dfrac{2x+3}{3x-2}\right)\left(\dfrac{4x}{3x-2}\right) = \dfrac{8x^2+12x}{(3x-2)^2}$
$3x-2 \ne 0$
$3x \ne 2 \Rightarrow x \ne \dfrac{2}{3}$
Domain: $\left\{x \,\middle|\, x \ne \dfrac{2}{3}\right\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{\frac{2x+3}{3x-2}}{\frac{4x}{3x-2}} = \dfrac{2x+3}{3x-2} \cdot \dfrac{3x-2}{4x} = \dfrac{2x+3}{4x}$
$3x-2 \ne 0$ and $x \ne 0$
$3x \ne 2$
$x \ne \dfrac{2}{3}$
Domain: $\left\{x \,\middle|\, x \ne \dfrac{2}{3} \text{ and } x \ne 0\right\}$.

e. $(f+g)(3) = \dfrac{6(3)+3}{3(3)-2} = \dfrac{18+3}{9-2} = \dfrac{21}{7} = 3$

f. $(f-g)(4) = \dfrac{-2(4)+3}{3(4)-2} = \dfrac{-8+3}{12-2} = \dfrac{-5}{10} = -\dfrac{1}{2}$

g. $(f \cdot g)(2) = \dfrac{8(2)^2+12(2)}{\left(3(2)-2\right)^2}$
$= \dfrac{8(4)+24}{(6-2)^2} = \dfrac{32+24}{(4)^2} = \dfrac{56}{16} = \dfrac{7}{2}$

h. $\left(\dfrac{f}{g}\right)(1) = \dfrac{2(1)+3}{4(1)} = \dfrac{2+3}{4} = \dfrac{5}{4}$

72. $f(x) = \sqrt{x+1} \qquad g(x) = \dfrac{2}{x}$

a. $(f+g)(x) = \sqrt{x+1} + \dfrac{2}{x}$

$x+1 \ge 0 \quad \text{and} \quad x \ne 0$

$x \ge -1$

Domain: $\{x \mid x \ge -1, \text{ and } x \ne 0\}$.

b. $(f-g)(x) = \sqrt{x+1} - \dfrac{2}{x}$

$x+1 \ge 0 \quad \text{and} \quad x \ne 0$

$x \ge -1$

Domain: $\{x \mid x \ge -1, \text{ and } x \ne 0\}$.

c. $(f \cdot g)(x) = \sqrt{x+1} \cdot \dfrac{2}{x} = \dfrac{2\sqrt{x+1}}{x}$

$x+1 \ge 0 \quad \text{and} \quad x \ne 0$

$x \ge -1$

Domain: $\{x \mid x \ge -1, \text{ and } x \ne 0\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x+1}}{\frac{2}{x}} = \dfrac{x\sqrt{x+1}}{2}$

$x+1 \ge 0 \quad \text{and} \quad x \ne 0$

$x \ge -1$

Domain: $\{x \mid x \ge -1, \text{ and } x \ne 0\}$.

e. $(f+g)(3) = \sqrt{3+1} + \dfrac{2}{3} = \sqrt{4} + \dfrac{2}{3} = 2 + \dfrac{2}{3} = \dfrac{8}{3}$

f. $(f-g)(4) = \sqrt{4+1} - \dfrac{2}{4} = \sqrt{5} - \dfrac{1}{2}$

g. $(f \cdot g)(2) = \dfrac{2\sqrt{2+1}}{2} = \dfrac{2\sqrt{3}}{2} = \sqrt{3}$

h. $\left(\dfrac{f}{g}\right)(1) = \dfrac{1\sqrt{1+1}}{2} = \dfrac{\sqrt{2}}{2}$

73. $f(x) = 3x+1 \qquad (f+g)(x) = 6 - \dfrac{1}{2}x$

$6 - \dfrac{1}{2}x = 3x + 1 + g(x)$

$5 - \dfrac{7}{2}x = g(x)$

$g(x) = 5 - \dfrac{7}{2}x$

74. $f(x) = \dfrac{1}{x} \qquad \left(\dfrac{f}{g}\right)(x) = \dfrac{x+1}{x^2 - x}$

$\dfrac{x+1}{x^2-x} = \dfrac{\frac{1}{x}}{g(x)}$

$g(x) = \dfrac{\frac{1}{x}}{\frac{x+1}{x^2-x}} = \dfrac{1}{x} \cdot \dfrac{x^2-x}{x+1}$

$= \dfrac{1}{x} \cdot \dfrac{x(x-1)}{x+1} = \dfrac{x-1}{x+1}$

75. $f(x) = 4x+3$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{4(x+h)+3-(4x+3)}{h}$

$= \dfrac{4x+4h+3-4x-3}{h}$

$= \dfrac{4h}{h} = 4$

76. $f(x) = -3x+1$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{-3(x+h)+1-(-3x+1)}{h}$

$= \dfrac{-3x-3h+1+3x-1}{h}$

$= \dfrac{-3h}{h} = -3$

77. $f(x) = x^2 - x + 4$

$\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{(x+h)^2 - (x+h) + 4 - (x^2 - x + 4)}{h}$

$= \dfrac{x^2 + 2xh + h^2 - x - h + 4 - x^2 + x - 4}{h}$

$= \dfrac{2xh + h^2 - h}{h}$

$= 2x + h - 1$

78. $f(x) = 3x^2 - 2x + 6$

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{\left[3(x+h)^2 - 2(x+h) + 6\right] - \left[3x^2 - 2x + 6\right]}{h}$$
$$= \frac{3\left(x^2 + 2xh + h^2\right) - 2x - 2h + 6 - 3x^2 + 2x - 6}{h}$$
$$= \frac{3x^2 + 6xh + 3h^2 - 2h - 3x^2}{h} = \frac{6xh + 3h^2 - 2h}{h}$$
$$= 6x + 3h - 2$$

79. $f(x) = \dfrac{1}{x^2}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h}$$
$$= \frac{\dfrac{x^2 - (x+h)^2}{x^2(x+h)^2}}{h}$$
$$= \frac{\dfrac{x - \left(x^2 + 2xh + h^2\right)}{x^2(x+h)^2}}{h}$$
$$= \left(\frac{1}{h}\right)\frac{-2xh - h^2}{x^2(x+h)^2}$$
$$= \left(\frac{1}{h}\right)\frac{h(-2x-h)}{x^2(x+h)^2}$$
$$= \frac{-2x-h}{x^2(x+h)^2} = \frac{-(2x+h)}{x^2(x+h)^2}$$

80. $f(x) = \dfrac{1}{x+3}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\dfrac{1}{x+h+3} - \dfrac{1}{x+3}}{h}$$
$$= \frac{\dfrac{x+3-(x+3+h)}{(x+h+3)(x+3)}}{h}$$
$$= \left(\frac{x+3-x-3-h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)$$
$$= \left(\frac{-h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)$$
$$= \frac{-1}{(x+h+3)(x+3)}$$

81. $f(x) = \sqrt{x}$

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{\sqrt{x+h} - \sqrt{x}}{h}$$
$$= \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$
$$= \frac{x+h-x}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$
$$= \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

82. $f(x) = \sqrt{x+1}$

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$
$$= \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$$
$$= \frac{x+h+1-(x+1)}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)} = \frac{h}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$
$$= \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$$

83. $f(x) = 2x^3 + Ax^2 + 4x - 5$ and $f(2) = 5$

$$f(2) = 2(2)^3 + A(2)^2 + 4(2) - 5$$
$$5 = 16 + 4A + 8 - 5$$
$$5 = 4A + 19$$
$$-14 = 4A$$
$$A = \frac{-14}{4} = -\frac{7}{2}$$

84. $f(x) = 3x^2 - Bx + 4$ and $f(-1) = 12$:

$$f(-1) = 3(-1)^2 - B(-1) + 4$$
$$12 = 3 + B + 4$$
$$B = 5$$

85. $f(x) = \dfrac{3x+8}{2x-A}$ and $f(0) = 2$

$$f(0) = \frac{3(0)+8}{2(0)-A}$$
$$2 = \frac{8}{-A}$$
$$-2A = 8$$
$$A = -4$$

86. $f(x) = \dfrac{2x-B}{3x+4}$ and $f(2) = \dfrac{1}{2}$

$$f(2) = \frac{2(2)-B}{3(2)+4}$$
$$\frac{1}{2} = \frac{4-B}{10}$$
$$5 = 4 - B$$
$$B = -1$$

87. $f(x) = \dfrac{2x-A}{x-3}$ and $f(4) = 0$

$$f(4) = \frac{2(4)-A}{4-3}$$
$$0 = \frac{8-A}{1}$$
$$0 = 8 - A$$
$$A = 8$$

f is undefined when $x = 3$.

88. $f(x) = \dfrac{x-B}{x-A}, f(2) = 0$ and $f(1)$ is undefined

$$1 - A = 0 \quad \Rightarrow \quad A = 1$$
$$f(2) = \frac{2-B}{2-1}$$
$$0 = \frac{2-B}{1}$$
$$0 = 2 - B$$
$$B = 2$$

89. Let x represent the length of the rectangle.
Then, $\dfrac{x}{2}$ represents the width of the rectangle since the length is twice the width. The function for the area is: $A(x) = x \cdot \dfrac{x}{2} = \dfrac{x^2}{2} = \dfrac{1}{2}x^2$

90. Let x represent the length of one of the two equal sides. The function for the area is:
$$A(x) = \frac{1}{2} \cdot x \cdot x = \frac{1}{2}x^2$$

91. Let x represent the number of hours worked. The function for the gross salary is:
$G(x) = 10x$

92. Let x represent the number of items sold. The function for the gross salary is:
$G(x) = 10x + 100$

93. **a.** P is the dependent variable; a is the independent variable

b. $P(20) = 0.004(20)^2 - 3.792(20) + 317.946$
$= 1.6 - 75.84 + 317.946$
$= 243.706$
In 2011 there are 243.706 million people who are 20 years of age or older.

c. $P(20) = 0.004(0)^2 - 3.792(0) + 317.946$
$= 317.946$
In 2011 there are 317.946 million people.

94. **a.** N is the dependent variable; r is the independent variable

b. $N(3) = -2.08(3)^2 + 22.901(3) - 36.06$
$= -18.72 + 68.703 - 36.06$
$= 13.923$
In 2011, there are 13.923 million housing units with 3 rooms.

95. a. $H(1) = 20 - 4.9(1)^2$
$= 20 - 4.9 = 15.1$ meters
$H(1.1) = 20 - 4.9(1.1)^2$
$= 20 - 4.9(1.21)$
$= 20 - 5.929 = 14.071$ meters
$H(1.2) = 20 - 4.9(1.2)^2$
$= 20 - 4.9(1.44)$
$= 20 - 7.056 = 12.944$ meters
$H(1.3) = 20 - 4.9(1.3)^2$
$= 20 - 4.9(1.69)$
$= 20 - 8.281 = 11.719$ meters

b. $H(x) = 15$:
$15 = 20 - 4.9x^2$
$-5 = -4.9x^2$
$x^2 \approx 1.0204$
$x \approx 1.01$ seconds
$H(x) = 10$:
$10 = 20 - 4.9x^2$
$-10 = -4.9x^2$
$x^2 \approx 2.0408$
$x \approx 1.43$ seconds
$H(x) = 5$:
$5 = 20 - 4.9x^2$
$-15 = -4.9x^2$
$x^2 \approx 3.0612$
$x \approx 1.75$ seconds

c. $H(x) = 0$
$0 = 20 - 4.9x^2$
$-20 = -4.9x^2$
$x^2 \approx 4.0816$
$x \approx 2.02$ seconds

96. a. $H(1) = 20 - 13(1)^2 = 20 - 13 = 7$ meters
$H(1.1) = 20 - 13(1.1)^2 = 20 - 13(1.21)$
$= 20 - 15.73 = 4.27$ meters
$H(1.2) = 20 - 13(1.2)^2 = 20 - 13(1.44)$
$= 20 - 18.72 = 1.28$ meters

b. $H(x) = 15$
$15 = 20 - 13x^2$
$-5 = -13x^2$
$x^2 \approx 0.3846$
$x \approx 0.62$ seconds
$H(x) = 10$
$10 = 20 - 13x^2$
$-10 = -13x^2$
$x^2 \approx 0.7692$
$x \approx 0.88$ seconds
$H(x) = 5$
$5 = 20 - 13x^2$
$-15 = -13x^2$
$x^2 \approx 1.1538$
$x \approx 1.07$ seconds

c. $H(x) = 0$
$0 = 20 - 13x^2$
$-20 = -13x^2$
$x^2 \approx 1.5385$
$x \approx 1.24$ seconds

97. $C(x) = 100 + \dfrac{x}{10} + \dfrac{36,000}{x}$

a. $C(500) = 100 + \dfrac{500}{10} + \dfrac{36,000}{500}$
$= 100 + 50 + 72$
$= \$222$

b. $C(450) = 100 + \dfrac{450}{10} + \dfrac{36,000}{450}$
$= 100 + 45 + 80$
$= \$225$

c. $C(600) = 100 + \dfrac{600}{10} + \dfrac{36,000}{600}$
$= 100 + 60 + 60$
$= \$220$

d. $C(400) = 100 + \dfrac{400}{10} + \dfrac{36,000}{400}$
$= 100 + 40 + 90$
$= \$230$

98. $A(x)=4x\sqrt{1-x^2}$

a. $A\left(\frac{1}{3}\right)=4\cdot\frac{1}{3}\sqrt{1-\left(\frac{1}{3}\right)^2}=\frac{4}{3}\sqrt{\frac{8}{9}}=\frac{4}{3}\cdot\frac{2\sqrt{2}}{3}$

$=\frac{8\sqrt{2}}{9}\approx 1.26\text{ ft}^2$

b. $A\left(\frac{1}{2}\right)=4\cdot\frac{1}{2}\sqrt{1-\left(\frac{1}{2}\right)^2}=2\sqrt{\frac{3}{4}}=2\cdot\frac{\sqrt{3}}{2}$

$=\sqrt{3}\approx 1.73\text{ ft}^2$

c. $A\left(\frac{2}{3}\right)=4\cdot\frac{2}{3}\sqrt{1-\left(\frac{2}{3}\right)^2}=\frac{8}{3}\sqrt{\frac{5}{9}}=\frac{8}{3}\cdot\frac{\sqrt{5}}{3}$

$=\frac{8\sqrt{5}}{9}\approx 1.99\text{ ft}^2$

99. $R(x)=\left(\frac{L}{P}\right)(x)=\frac{L(x)}{P(x)}$

100. $T(x)=(V+P)(x)=V(x)+P(x)$

101. $H(x)=(P\cdot I)(x)=P(x)\cdot I(x)$

102. $N(x)=(I-T)(x)=I(x)-T(x)$

103. a. $P(x)=R(x)-C(x)$

$=\left(-1.2x^2+220x\right)-\left(0.05x^3-2x^2+65x+500\right)$

$=-1.2x^2+220x-0.05x^3+2x^2-65x-500$

$=-0.05x^3+0.8x^2+155x-500$

b. $P(15)=-0.05(15)^3+0.8(15)^2+155(15)-500$

$=-168.75+180+2325-500$

$=\$1836.25$

c. When 15 hundred cell phones are sold, the profit is \$1836.25.

104. a. $P(x)=R(x)-C(x)$

$=30x-\left(0.1x^2+7x+400\right)$

$=30x-0.1x^2-7x-400$

$=-0.1x^2+23x-400$

b. $P(30)=-0.1(30)^2+23(30)-400$

$=-90+690-400$

$=\$200$

c. When 30 clocks are sold, the profit is \$200.

105. a. $R(v)=2.2v;\, B(v)=0.05v^2+0.4\text{v}-15$

$D(v)=R(v)+B(v)$

$=2.2v+0.05v^2+0.4\text{v}-15$

$=0.05v^2+2.6v-15$

b. $D(60)=0.05(60)^2+2.6(60)-15$

$=180+156-15$

$=321$

c. The car will need 321 feet to stop once the impediment is observed.

106. a. $h(x)=2x$

$h(a+b)=2(a+b)=2a+2b$

$=h(a)+h(b)$

$h(x)=2x$ has the property.

b. $g(x)=x^2$

$g(a+b)=(a+b)^2=a^2+2ab+b^2$

Since

$a^2+2ab+b^2\neq a^2+b^2=g(a)+g(b)$,

$g(x)=x^2$ does not have the property.

c. $F(x)=5x-2$

$F(a+b)=5(a+b)-2=5a+5b-2$

Since

$5a+5b-2\neq 5a-2+5b-2=F(a)+F(b)$,

$F(x)=5x-2$ does not have the property.

d. $G(x)=\frac{1}{x}$

$G(a+b)=\frac{1}{a+b}\neq\frac{1}{a}+\frac{1}{b}=G(a)+G(b)$

$G(x)=\frac{1}{x}$ does not have the property.

107. No. The domain of *f* is $\{x\,|\,x \text{ is any real number}\}$, but the domain of *g* is $\{x\,|\,x\neq -1\}$.

108. Answers will vary.

109. $\dfrac{3x-x^3}{(\textit{your age})}$

Section 1.2

1. $x^2 + 4y^2 = 16$
   x-intercepts:
   $x^2 + 4(0)^2 = 16$
   $x^2 = 16$
   $x = \pm 4 \Rightarrow (-4, 0), (4, 0)$

   y-intercepts:
   $(0)^2 + 4y^2 = 16$
   $4y^2 = 16$
   $y^2 = 4$
   $y = \pm 2 \Rightarrow (0, -2), (0, 2)$

2. False; $x = 2y - 2$
   $-2 = 2y - 2$
   $0 = 2y$
   $0 = y$
   The point $(-2, 0)$ is on the graph.

3. vertical

4. $f(5) = -3$

5. $f(x) = ax^2 + 4$
   $a(-1)^2 + 4 = 2 \Rightarrow a = -2$

6. False; it would fail the vertical line test.

7. False; e.g. $y = \frac{1}{x}$.

8. True

9. **a.** $f(0) = 3$ since $(0, 3)$ is on the graph.
   $f(-6) = -3$ since $(-6, -3)$ is on the graph.

   b. $f(6) = 0$ since $(6, 0)$ is on the graph.
   $f(11) = 1$ since $(11, 1)$ is on the graph.

   c. $f(3)$ is positive since $f(3) \approx 3.7$.

   d. $f(-4)$ is negative since $f(-4) \approx -1$.

   e. $f(x) = 0$ when $x = -3$, $x = 6$, and $x = 10$.

   f. $f(x) > 0$ when $-3 < x < 6$, and $10 < x \le 11$.

   g. The domain of f is $\{x \mid -6 \le x \le 11\}$ or $[-6, 11]$.

   h. The range of f is $\{y \mid -3 \le y \le 4\}$ or $[-3, 4]$.

   i. The x-intercepts are -3, 6, and 10.

   j. The y-intercept is 3.

   k. The line $y = \frac{1}{2}$ intersects the graph 3 times.

   l. The line $x = 5$ intersects the graph 1 time.

   m. $f(x) = 3$ when $x = 0$ and $x = 4$.

   n. $f(x) = -2$ when $x = -5$ and $x = 8$.

   o. The zeros are -3, 6, 10.

10. **a.** $f(0) = 0$ since $(0, 0)$ is on the graph.
    $f(6) = 0$ since $(6, 0)$ is on the graph.

    b. $f(2) = -2$ since $(2, -2)$ is on the graph.
    $f(-2) = 1$ since $(-2, 1)$ is on the graph.

    c. $f(3)$ is negative since $f(3) \approx -1$.

    d. $f(-1)$ is positive since $f(-1) \approx 1.0$.

    e. $f(x) = 0$ when $x = 0$, $x = 4$, and $x = 6$.

    f. $f(x) < 0$ when $0 < x < 4$.

    g. The domain of f is $\{x \mid -4 \le x \le 6\}$ or $[-4, 6]$.

    h. The range of f is $\{y \mid -2 \le y \le 3\}$ or $[-2, 3]$.

    i. The x-intercepts are 0, 4, and 6.

    j. The y-intercept is 0.

    k. The line $y = -1$ intersects the graph 2 times.

    l. The line $x = 1$ intersects the graph 1 time.

    m. $f(x) = 3$ when $x = 5$.

    n. $f(x) = -2$ when $x = 2$.

    o. The zeros are 0, 4, 6.

11. Not a function since vertical lines will intersect the graph in more than one point.

12. Function

a. Domain: $\{x | x \text{ is any real number}\}$;

Range: $\{y | y > 0\}$

b. Intercepts: $(0,1)$

c. None

13. Function

a. Domain: $\{x | -\pi \le x \le \pi\}$;

Range: $\{y | -1 \le y \le 1\}$

b. Intercepts: $\left(-\frac{\pi}{2},0\right)$, $\left(\frac{\pi}{2},0\right)$, $(0,1)$

c. Symmetry about *y*-axis.

14. Function

a. Domain: $\{x | -\pi \le x \le \pi\}$;

Range: $\{y | -1 \le y \le 1\}$

b. Intercepts: $(-\pi, 0)$, $(\pi, 0)$, $(0, 0)$

c. Symmetry about the origin.

15. Not a function since vertical lines will intersect the graph in more than one point.

16. Not a function since vertical lines will intersect the graph in more than one point.

17. Function

a. Domain: $\{x | 0 < x < 3\}$;

Range: $\{y | y < 2\}$

b. Intercepts: (1, 0)

c. None

18. Function

a. Domain: $\{x | 0 \le x < 4\}$;

Range: $\{y | 0 \le y < 3\}$

b. Intercepts: (0, 0)

c. None

19. Function

a. Domain: $\{x | x \text{ is any real number}\}$;

Range: $\{y | y \le 2\}$

b. Intercepts: (–3, 0), (3, 0), (0,2)

c. Symmetry about *y*-axis.

20. Function

a. Domain: $\{x | x \ge -3\}$;

Range: $\{y | y \ge 0\}$

b. Intercepts: (–3, 0), (2,0), (0,2)

c. None

21. Function

a. Domain: $\{x | x \text{ is any real number}\}$;

Range: $\{y | y \ge -3\}$

b. Intercepts: (1, 0), (3,0), (0,9)

c. None

22. Function

a. Domain: $\{x | x \text{ is any real number}\}$;

Range: $\{y | y \le 5\}$

b. Intercepts: (–1, 0), (2,0), (0,4)

c. None

23. $f(x) = 2x^2 - x - 1$

a. $f(-1) = 2(-1)^2 - (-1) - 1 = 2$

The point $(-1,2)$ is on the graph of *f*.

b. $f(-2) = 2(-2)^2 - (-2) - 1 = 9$

The point $(-2,9)$ is on the graph of *f*.

c. Solve for x:

$$-1 = 2x^2 - x - 1$$

$$0 = 2x^2 - x$$

$$0 = x(2x-1) \Rightarrow x = 0, x = \tfrac{1}{2}$$

$(0, -1)$ and $\left(\frac{1}{2}, -1\right)$ are on the graph of f.

d. The domain of *f* is $\{x | x \text{ is any real number}\}$.

e. *x*-intercepts:

$$f(x)=0 \Rightarrow 2x^2 - x - 1 = 0$$

$$(2x+1)(x-1) = 0 \Rightarrow x = -\frac{1}{2}, x = 1$$

$\left(-\frac{1}{2},0\right)$ and $(1,0)$

f. y-intercept:
$f(0)=2(0)^2-0-1=-1 \Rightarrow (0,-1)$

g. $(2x+1)(x-4)=0$
$x=-\dfrac{1}{2} \quad x=4$

24. $f(x)=-3x^2+5x$

a. $f(-1)=-3(-1)^2+5(-1)=-8 \neq 2$
The point $(-1,2)$ is not on the graph of *f*.

b. $f(-2)=-3(-2)^2+5(-2)=-22$
The point $(-2,-22)$ is on the graph of *f*.

c. Solve for x:
$-2=-3x^2+5x \Rightarrow 3x^2-5x-2=0$
$(3x+1)(x-2)=0 \Rightarrow x=-\frac{1}{3}, x=2$
(2, –2) and $\left(-\frac{1}{3},-2\right)$ on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. x-intercepts:
$f(x)=0 \Rightarrow -3x^2+5x=0$
$x(-3x+5)=0 \Rightarrow x=0, x=\frac{5}{3}$
$(0,0)$ and $\left(\frac{5}{3},0\right)$

f. y-intercept:
$f(0)=-3(0)^2+5(0)=0 \Rightarrow (0,0)$

g. $-x(3x-5)=0$
$x=0 \quad x=\dfrac{5}{3}$

25. $f(x)=\dfrac{x+2}{x-6}$

a. $f(3)=\dfrac{3+2}{3-6}=-\dfrac{5}{3} \neq 14$
The point $(3,14)$ is not on the graph of *f*.

b. $f(4)=\dfrac{4+2}{4-6}=\dfrac{6}{-2}=-3$
The point $(4,-3)$ is on the graph of *f*.

c. Solve for x:
$$2=\frac{x+2}{x-6}$$
$$2x-12=x+2$$
$$x=14$$
(14, 2) is a point on the graph of f.

d. The domain of *f* is $\{x \mid x \neq 6\}$.

e. x-intercepts:
$f(x)=0 \Rightarrow \dfrac{x+2}{x-6}=0$
$x+2=0 \Rightarrow x=-2 \Rightarrow (-2,0)$

f. y-intercept: $f(0)=\dfrac{0+2}{0-6}=-\dfrac{1}{3} \Rightarrow \left(0,-\dfrac{1}{3}\right)$

g. $x+2=0$
$x=-2$

26. $f(x)=\dfrac{x^2+2}{x+4}$

a. $f(1)=\dfrac{1^2+2}{1+4}=\dfrac{3}{5}$
The point $\left(1,\dfrac{3}{5}\right)$ is on the graph of *f*.

b. $f(0)=\dfrac{0^2+2}{0+4}=\dfrac{2}{4}=\dfrac{1}{2}$
The point $\left(0,\dfrac{1}{2}\right)$ is on the graph of *f*.

c. Solve for x:
$$\frac{1}{2}=\frac{x^2+2}{x+4} \Rightarrow x+4=2x^2+4$$
$$0=2x^2-x$$
$x(2x-1)=0 \Rightarrow x=0 \text{ or } x=\dfrac{1}{2}$
$\left(0,\dfrac{1}{2}\right)$ and $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$ are on the graph of f.

d. The domain of *f* is $\{x \mid x \neq -4\}$.

e. x-intercepts:
$f(x)=0 \Rightarrow \dfrac{x^2+2}{x+4}=0 \Rightarrow x^2+2=0$
This is impossible, so there are no x-intercepts.

f. y-intercept:

$$f(0)=\frac{0^2+2}{0+4}=\frac{2}{4}=\frac{1}{2}\Rightarrow\left(0,\frac{1}{2}\right)$$

g. $x^2+2=0$. There are no real zeros.

27. $f(x)=\dfrac{2x^2}{x^4+1}$

a. $f(-1)=\dfrac{2(-1)^2}{(-1)^4+1}=\dfrac{2}{2}=1$

The point $(-1,1)$ is on the graph of *f*.

b. $f(2)=\dfrac{2(2)^2}{(2)^4+1}=\dfrac{8}{17}$

The point $\left(2,\dfrac{8}{17}\right)$ is on the graph of *f*.

c. Solve for x:

$$1=\frac{2x^2}{x^4+1}$$
$$x^4+1=2x^2$$
$$x^4-2x^2+1=0$$
$$(x^2-1)^2=0$$
$$x^2-1=0\Rightarrow x=\pm1$$

(1,1) and (–1,1) are on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. x-intercept:

$$f(x)=0\Rightarrow\frac{2x^2}{x^4+1}=0$$
$$2x^2=0\Rightarrow x=0\Rightarrow(0,0)$$

f. y-intercept:

$$f(0)=\frac{2(0)^2}{0^4+1}=\frac{0}{0+1}=0\Rightarrow(0,0)$$

g. $2x^2=0$

$x=0$

28. $f(x)=\dfrac{2x}{x-2}$

a. $f\left(\dfrac{1}{2}\right)=\dfrac{2\left(\frac{1}{2}\right)}{\frac{1}{2}-2}=\dfrac{1}{-\frac{3}{2}}=-\dfrac{2}{3}$

The point $\left(\dfrac{1}{2},-\dfrac{2}{3}\right)$ is on the graph of *f*.

b. $f(4)=\dfrac{2(4)}{4-2}=\dfrac{8}{2}=4$

The point $(4, 4)$ is on the graph of *f*.

c. Solve for x:

$$1=\frac{2x}{x-2}\Rightarrow x-2=2x\Rightarrow-2=x$$

(–2,1) is a point on the graph of f.

d. The domain of *f* is $\{x \mid x\neq2\}$.

e. x-intercept:

$$f(x)=0\Rightarrow\frac{2x}{x-2}=0\Rightarrow2x=0$$
$$\Rightarrow x=0\Rightarrow(0,0)$$

f. y-intercept: $f(0)=\dfrac{0}{0-2}=0\Rightarrow(0,0)$

g. $2x=0$

$x=0$

29. $h(x)=-\dfrac{44x^2}{v^2}+x+6$

a.
$$h(8)=-\frac{44(8)^2}{28^2}+(8)+6$$
$$=-\frac{2816}{784}+14\approx10.4\ \text{feet}$$

b.
$$h(12)=-\frac{44(12)^2}{28^2}+(12)+6$$
$$=-\frac{6336}{784}+18\approx9.9\ \text{feet}$$

$h(12)\approx9.9$ represents the height of the ball, in feet, after it has traveled 12 feet in front of the foul line.

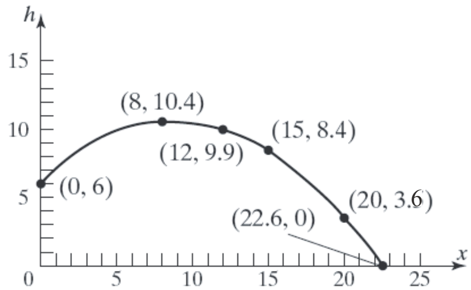
c. From part (a) we know the point $(8,10.4)$ is on the graph and from part (b) we know the point $(12,9.9)$ is on the graph. We could evaluate the function at several more values of x (e.g. $x=0$, $x=15$, and $x=20$) to obtain additional points.

$$h(0) = -\frac{44(0)^2}{28^2} + (0) + 6 = 6$$

$$h(15) = -\frac{44(15)^2}{28^2} + (15) + 6 \approx 8.4$$

$$h(20) = -\frac{44(20)^2}{28^2} + (20) + 6 \approx 3.6$$

Some additional points are $(0,6)$, $(15,8.4)$ and $(20,3.6)$. The complete graph is given below.



d. $h(15) = -\frac{44(15)^2}{28^2} + (15) + 6 \approx 8.4$ feet

No; when the ball is 15 feet in front of the foul line, it will be below the hoop. Therefore it cannot go through the hoop.

In order for the ball to pass through the hoop, we need to have $h(15) = 10$.

$$10 = -\frac{44(15)^2}{v^2} + (15) + 6$$

$$-11 = -\frac{44(15)^2}{v^2}$$

$$v^2 = 4(225)$$

$$v^2 = 900$$

$$v = 30 \text{ ft/sec}$$

The ball must be shot with an initial velocity of 30 feet per second in order to go through the hoop.

30. $h(x) = -\frac{136x^2}{v^2} + 2.7x + 3.5$

a. We want $h(15) = 10$.

$$-\frac{136(15)^2}{v^2} + 2.7(15) + 3.5 = 10$$

$$-\frac{30,600}{v^2} = -34$$

$$v^2 = 900$$

$$v = 30 \text{ ft/sec}$$

The ball needs to be thrown with an initial velocity of 30 feet per second.

b. $h(x) = -\frac{126x^2}{30^2} + 2.7x + 3.5$

which simplifies to

$h(x) = -\frac{34}{225}x^2 + 2.7x + 3.5$

c. Using the velocity from part (b),

$h(9) = -\frac{34}{225}(9)^2 + 2.7(9) + 3.5 = 15.56$ ft

The ball will be 15.56 feet above the floor when it has traveled 9 feet in front of the foul line.

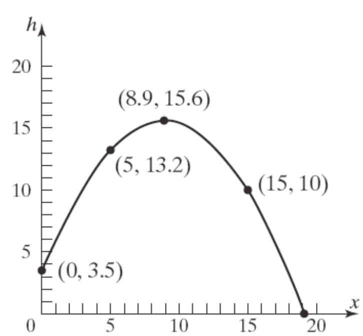
d. Select several values for x and use these to find the corresponding values for h. Use the results to form ordered pairs (x,h). Plot the points and connect with a smooth curve.

$h(0) = -\frac{34}{225}(0)^2 + 2.7(0) + 3.5 = 3.5$ ft

$h(5) = -\frac{34}{225}(5)^2 + 2.7(5) + 3.5 \approx 13.2$ ft

$h(15) = -\frac{24}{225}(15)^2 + 2.7(15) + 3.5 \approx 10$ ft

Thus, some points on the graph are $(0,3.5)$, $(5,13.2)$, and $(15,10)$. The complete graph is given below.



31. $h(x)=\dfrac{-32x^2}{130^2}+x$

a. $h(100)=\dfrac{-32(100)^2}{130^2}+100$

$$=\dfrac{-320,000}{16,900}+100\approx 81.07 \text{ feet}$$

b. $h(300)=\dfrac{-32(300)^2}{130^2}+300$

$$=\dfrac{-2,880,000}{16,900}+300\approx 129.59 \text{ feet}$$

c. $h(500)=\dfrac{-32(500)^2}{130^2}+500$

$$=\dfrac{-8,000,000}{16,900}+500\approx 26.63 \text{ feet}$$

$h(500)\approx 26.63$ feet represents the height of the golf ball, in feet, after it has traveled a horizontal distance of 500 feet.

d. Solving $h(x)=\dfrac{-32x^2}{130^2}+x=0$

$$\frac{-32x^2}{130^2}+x=0$$

$$x\left(\frac{-32x}{130^2}+1\right)=0$$

$$x=0 \quad\text{or}\quad \frac{-32x}{130^2}+1=0$$

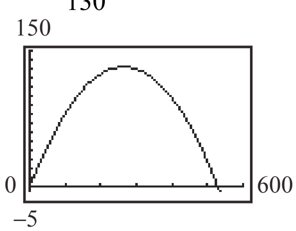
$$1=\frac{32x}{130^2}$$

$$130^2=32x$$

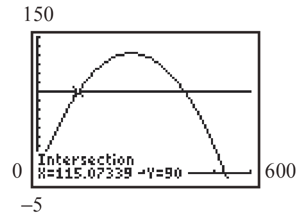
$$x=\frac{130^2}{32}=528.13 \text{ feet}$$

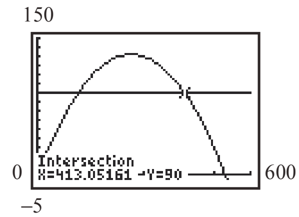
Therefore, the golf ball travels 528.13 feet.

e. $y_1=\dfrac{-32x^2}{130^2}+x$



f. Use INTERSECT on the graphs of $y_1=\dfrac{-32x^2}{130^2}+x$ and $y_2=90$.





The ball reaches a height of 90 feet twice. The first time is when the ball has traveled approximately 115.07 feet, and the second time is when the ball has traveled about 413.05 feet.

g. The ball travels approximately 275 feet before it reaches its maximum height of approximately 131.8 feet.

X	Y_1	
200	124.26	
225	129.14	
250	131.66	
275	131.8	
300	129.59	
325	125	
350	118.05	

X=275

h. The ball travels approximately 264 feet before it reaches its maximum height of approximately 132.03 feet.

X	Y_1	
260	132	
261	132.01	
262	132.02	
263	132.03	
264	132.03	
265	132.03	
266	132.02	

Y_1=132.029112426

X	Y_1	
260	132	
261	132.01	
262	132.02	
263	132.03	
264	132.03	
265	132.03	
266	132.02	

Y_1=132.031242604

X	Y_1	
260	132	
261	132.01	
262	132.02	
263	132.03	
264	132.03	
265	132.03	
266	132.02	

Y_1=132.029585799

32. $A(x)=4x\sqrt{1-x^2}$

a. Domain of $A(x)=4x\sqrt{1-x^2}$; we know that x must be greater than or equal to zero, since x represents a length. We also need $1-x^2\geq 0$, since this expression occurs under a square root. In fact, to avoid Area = 0, we require

$x > 0$ and $1 - x^2 > 0$.

Solve: $1 - x^2 > 0$

$$(1+x)(1-x) > 0$$

Case1: $1 + x > 0$ and $1 - x > 0$

$x > -1$ and $x < 1$

(i.e. $-1 < x < 1$)

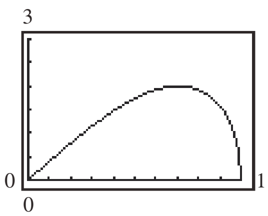
Case2: $1 + x < 0$ and $1 - x < 0$

$x < -1$ and $x > 1$

(which is impossible)

Therefore the domain of A is $\{x \mid 0 < x < 1\}$.

b. Graphing $A(x) = 4x\sqrt{1 - x^2}$



c. When $x = 0.7$ feet, the cross-sectional area is maximized at approximately 1.9996 square feet. Therefore, the length of the base of the beam should be 1.4 feet in order to maximize the cross-sectional area.

X	Y1
.3	1.1447
.4	1.4664
.5	1.7321
.6	1.92
.7	1.9996
.8	1.92
.9	1.5692

X=.7

33. $C(x) = 100 + \dfrac{x}{10} + \dfrac{36000}{x}$

a. $C(480) = 100 + \dfrac{480}{10} + \dfrac{36000}{480}$

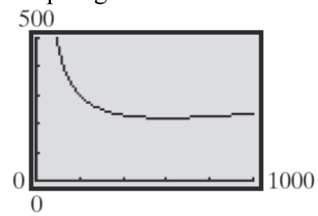
$= 100 + 48 + 75 = \$223.00$

$C(600) = 100 + \dfrac{600}{10} + \dfrac{36000}{600}$

$= 100 + 60 + 60 = \$220.00$

b. The speed must be greater than zero so the domain is: $\{x \mid x > 0\}$

c. Graphing:



d. TblStart = 0; ΔTbl = 50

X	Y1
0	ERROR
50	825
100	470
150	355
200	300
250	269
300	250

Y1=100+X/10+360…

e. The cost per passenger is minimized to about \$220 when the ground speed is roughly 600 miles per hour.

X	Y1
450	225
500	222
550	220.45
600	220
650	220.38
700	221.43
750	223

X=600

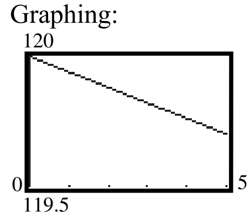
34. $W(h) = m\left(\dfrac{4000}{4000 + h}\right)^2$

a. $h = 14110$ feet ≈ 2.67 miles;

$W(2.67) = 120\left(\dfrac{4000}{4000 + 2.67}\right)^2 \approx 119.84$

On Pike's Peak, Amy will weigh about 119.84 pounds.

b. Graphing:



c. Create a TABLE:

X	Y1
0	120
.5	119.97
1	119.94
1.5	119.91
2	119.88
2.5	119.85
3	119.82

X=0

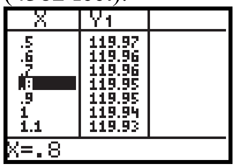
X	Y1
2	119.88
2.5	119.85
3	119.82
3.5	119.79
4	119.76
4.5	119.73
5	119.7

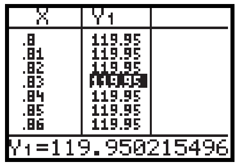
X=5

The weight W will vary from 120 pounds to about 119.7 pounds.

d. By refining the table, Amy will weigh 119.95 lbs at a height of about 0.83 miles

(4382 feet).





e. Yes, 4382 feet is reasonable.

35. a. $(f+g)(2) = f(2)+g(2) = 2+1 = 3$

b. $(f+g)(4) = f(4)+g(4) = 1+(-3) = -2$

c. $(f-g)(6) = f(6)-g(6) = 0-1 = -1$

d. $(g-f)(6) = g(6)-f(6) = 1-0 = 1$

e. $(f \cdot g)(2) = f(2)\cdot g(2) = 2(1) = 2$

f. $\left(\dfrac{f}{g}\right)(4) = \dfrac{f(4)}{g(4)} = \dfrac{1}{-3} = -\dfrac{1}{3}$

36. a. $C(0) = 5000$

This represents the fixed overhead costs. That is, the company will incur costs of \$5000 per day even if no computers are manufactured.

b. $C(10) = 19{,}000$

It costs the company \$19,000 to produce 10 computers in a day.

c. $C(50) = 51{,}000$

It costs the company \$51,000 to produce 50 computers in a day.

d. The domain is $\{q \mid 0 \le q \le 100\}$. This indicates that production capacity is limited to 100 computers in a day.

e. The graph is curved down and rises slowly at first. As production increases, the graph becomes rises more quickly and changes to being curved up.

f. The inflection point is where the graph changes from being curved down to being curved up.

37. a. $C(0) = 80$

This represents the monthly fee. The plan costs \$80 per month even if no minutes are used.

b. $C(1000) = 80$

The monthly charge is \$80 if 1000 minutes are used. Since this is the same as the cost for 0 minutes, all these minutes are included in the base plan.

c. $C(2000) = 210$

The monthly charge is \$210 if 2000 minutes are used.

d. The domain is $\{m \mid 0 \le m \le 14{,}400\}$. The domain implies that there are at most 14,400 anytime minutes in a month.

e. The graph starts off flat (horizontal line), then increases at a constant rate (straight line with positive slope) after $m = 1000$.

38. Answers will vary. From a graph, the domain can be found by visually locating the x-values for which the graph is defined. The range can be found in a similar fashion by visually locating the *y*-values for which the function is defined. If an equation is given, the domain can be found by locating any restricted values and removing them from the set of real numbers. The range can be found by using known properties of the graph of the equation, or estimated by means of a table of values.

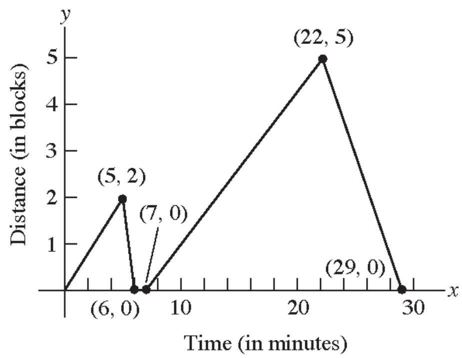
39. The graph of a function can have any number of *x*-intercepts. The graph of a function can have at most one *y*-intercept (otherwise the graph would fail the vertical line test).

40. Yes, the graph of a single point is the graph of a function since it would pass the vertical line test. The equation of such a function would be something like the following: $f(x) = 2$, where $x = 7$.

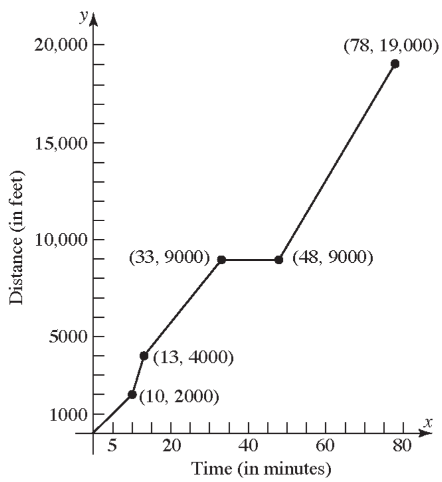
41. (a) III; (b) IV; (c) I; (d) V; (e) II

42. (a) II; (b) V; (c) IV; (d) III; (e) I

43.



44.



45. **a.** 2 hours elapsed; Kevin was between 0 and 3 miles from home.

b. 0.5 hours elapsed; Kevin was 3 miles from home.

c. 0.3 hours elapsed; Kevin was between 0 and 3 miles from home.

d. 0.2 hours elapsed; Kevin was at home.

e. 0.9 hours elapsed; Kevin was between 0 and 2.8 miles from home.

f. 0.3 hours elapsed; Kevin was 2.8 miles from home.

g. 1.1 hours elapsed; Kevin was between 0 and 2.8 miles from home.

h. The farthest distance Kevin is from home is 3 miles.

i. Kevin returned home 2 times.

46. **a.** Michael travels fastest between 7 and 7.4 minutes. That is, $(7, 7.4)$.

b. Michael's speed is zero between 4.2 and 6 minutes. That is, $(4.2, 6)$.

c. Between 0 and 2 minutes, Michael's speed increased from 0 to 30 miles/hour.

d. Between 4.2 and 6 minutes, Michael was stopped (i.e, his speed was 0 miles/hour).

e. Between 7 and 7.4 minutes, Michael was traveling at a steady rate of 50 miles/hour.

f. Michael's speed is constant between 2 and 4 minutes, between 4.2 and 6 minutes, between 7 and 7.4 minutes, and between 7.6 and 8 minutes. That is, on the intervals (2, 4), (4.2, 6), (7, 7.4), and (7.6, 8).

47. Answers (graphs) will vary. Points of the form $(5, y)$ and of the form $(x, 0)$ cannot be on the graph of the function.

48. The only such function is $f(x) = 0$ because it is the only function for which $f(x) = -f(x)$. Any other such graph would fail the vertical line test.

49. Answers may vary. Two points that fall on the same vertical line have the same first element which violates the definition of a function.

Section 1.3

1. $2 < x < 5$

2. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{8-3}{3-(-2)} = \dfrac{5}{5} = 1$

3. x-axis: $y \to -y$

$$(-y) = 5x^2 - 1$$
$$-y = 5x^2 - 1$$
$$y = -5x^2 + 1 \quad \text{different}$$

y-axis: $x \to -x$

$$y = 5(-x)^2 - 1$$
$$y = 5x^2 - 1 \quad \text{same}$$

origin: $x \to -x$ and $y \to -y$

$$(-y) = 5(-x)^2 - 1$$
$$-y = 5x^2 - 1$$
$$y = -5x^2 + 1 \quad \text{different}$$

The equation has symmetry with respect to the y-axis only.

4. $y - y_1 = m(x - x_1)$
$y - (-2) = 5(x - 3)$
$y + 2 = 5(x - 3)$

5. $y = x^2 - 9$
x-intercepts:
$0 = x^2 - 9$
$x^2 = 9 \to x = \pm 3$
y-intercept:
$y = (0)^2 - 9 = -9$
The intercepts are $(-3,0)$, $(3,0)$, and $(0,-9)$.

6. increasing

7. even; odd

8. True

9. True

10. False; odd functions are symmetric with respect to the origin. Even functions are symmetric with respect to the y-axis.

11. Yes

12. No, it is increasing.

13. No, it only increases on (5, 10).

14. Yes

15. f is increasing on the intervals $(-8,-2)$, $(0,2)$, $(5,\infty)$.

16. f is decreasing on the intervals: $(-\infty,-8)$, $(-2,0)$, $(2,5)$.

17. Yes. The local maximum at $x = 2$ is 10.

18. No. There is a local minimum at $x = 5$; the local minimum is 0.

19. f has local maxima at $x = -2$ and $x = 2$. The local maxima are 6 and 10, respectively.

20. f has local minima at $x = -8$, $x = 0$ and $x = 5$. The local minima are –4, 0, and 0, respectively.

21. **a.** Intercepts: (–2, 0), (2, 0), and (0, 3).

b. Domain: $\{x \mid -4 \le x \le 4\}$ or $[-4, 4]$;
Range: $\{y \mid 0 \le y \le 3\}$ or $[0, 3]$.

c. Increasing: (–2, 0) and (2, 4);
Decreasing: (–4, –2) and (0, 2).

d. Since the graph is symmetric with respect to the y-axis, the function is <u>even</u>.

22. **a.** Intercepts: (–1, 0), (1, 0), and (0, 2).

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;
Range: $\{y \mid 0 \le y \le 3\}$ or $[0, 3]$.

c. Increasing: (–1, 0) and (1, 3);
Decreasing: (–3, –1) and (0, 1).

d. Since the graph is symmetric with respect to the y-axis, the function is <u>even</u>.

23. **a.** Intercepts: (0, 1).

b. Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y > 0\}$ or $(0, \infty)$.

c. Increasing: $(-\infty, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is <u>neither</u> even nor odd.

24. **a.** Intercepts: (1, 0).

b. Domain: $\{x \mid x > 0\}$ or $(0, \infty)$;
Range: $\{y \mid y \text{ is any real number}\}$.

c. Increasing: $(0, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is <u>neither</u> even nor odd.

25. **a.** Intercepts: $(-\pi, 0)$, $(\pi, 0)$, and $(0, 0)$.

b. Domain: $\{x \mid -\pi \le x \le \pi\}$ or $[-\pi, \pi]$;
Range: $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

c. Increasing: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$;
Decreasing: $\left(-\pi, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \pi\right)$.

d. Since the graph is symmetric with respect to the origin, the function is <u>odd</u>.

26. a. Intercepts: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right)$, and $(0, 1)$.

b. Domain: $\{x \mid -\pi \le x \le \pi\}$ or $[-\pi, \pi]$;
Range: $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

c. Increasing: $(-\pi, 0)$; Decreasing: $(0, \pi)$.

d. Since the graph is symmetric with respect to the y-axis, the function is even.

27. a. Intercepts: $\left(\frac{1}{3}, 0\right), \left(\frac{5}{2}, 0\right)$, and $\left(0, \frac{1}{2}\right)$.

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;
Range: $\{y \mid -1 \le y \le 2\}$ or $[-1, 2]$.

c. Increasing: $(2, 3)$; Decreasing: $(-1, 1)$;
Constant: $(-3, -1)$ and $(1, 2)$

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is neither even nor odd.

28. a. Intercepts: $(-2.3, 0), (3, 0)$, and $(0, 1)$.

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;
Range: $\{y \mid -2 \le y \le 2\}$ or $[-2, 2]$.

c. Increasing: $(-3, -2)$ and $(0, 2)$;
Decreasing: $(2, 3)$; Constant: $(-2, 0)$.

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is neither even nor odd.

29. a. f has a local maximum of 3 at $x = 0$.

b. f has a local minimum of 0 at both $x = -2$ and $x = 2$.

30. a. f has a local maximum of 2 at $x = 0$.

b. f has a local minimum of 0 at both $x = -1$ and $x = 1$.

31. a. f has a local maximum of 1 at $x = \frac{\pi}{2}$.

b. f has a local minimum of -1 at $x = -\frac{\pi}{2}$.

32. a. f has a local maximum of 1 at $x = 0$.

b. f has a local minimum of -1 both at $x = -\pi$ and $x = \pi$.

33. $f(x) = 4x^3$
$f(-x) = 4(-x)^3 = -4x^3 = -f(x)$
Therefore, f is odd.

34. $f(x) = 2x^4 - x^2$
$f(-x) = 2(-x)^4 - (-x)^2 = 2x^4 - x^2 = f(x)$
Therefore, f is even.

35. $g(x) = -3x^2 - 5$
$g(-x) = -3(-x)^2 - 5 = -3x^2 - 5 = g(x)$
Therefore, g is even.

36. $h(x) = 3x^3 + 5$
$h(-x) = 3(-x)^3 + 5 = -3x^3 + 5$
h is neither even nor odd.

37. $F(x) = \sqrt[3]{x}$
$F(-x) = \sqrt[3]{-x} = -\sqrt[3]{x} = -F(x)$
Therefore, F is odd.

38. $G(x) = \sqrt{x}$
$G(-x) = \sqrt{-x}$
G is neither even nor odd.

39. $f(x) = x + |x|$
$f(-x) = -x + |-x| = -x + |x|$
f is neither even nor odd.

40. $f(x) = \sqrt[3]{2x^2 + 1}$
$f(-x) = \sqrt[3]{2(-x)^2 + 1} = \sqrt[3]{2x^2 + 1} = f(x)$
Therefore, f is even.

41. $g(x) = \dfrac{x^2 + 3}{x^2 - 1}$

$$g(-x) = \frac{(-x)^2 + 3}{(-x)^2 - 1} = \frac{x^2 + 3}{x^2 - 1} = g(x)$$

Therefore, g is even.

42. $h(x)=\dfrac{x}{x^2-1}$

$h(-x)=\dfrac{-x}{(-x)^2-1}=\dfrac{-x}{x^2-1}=-h(x)$

Therefore, h is odd.

43. $h(x)=\dfrac{-x^3}{3x^2-9}$

$h(-x)=\dfrac{-(-x)^3}{3(-x)^2-9}=\dfrac{x^3}{3x^2-9}=-h(x)$

Therefore, h is odd.

44. $F(x)=\dfrac{2x}{|x|}$

$F(-x)=\dfrac{2(-x)}{|-x|}=\dfrac{-2x}{|x|}=-F(x)$

Therefore, F is odd.

45. Absolute maximum of 4 at $x=1$.
Absolute minimum of 1 at $x=5$.
Local maximum of 3 at $x=3$.
Local minimum of 2 at $x=2$.

46. Absolute maximum of 4 at $x=4$.
Absolute minimum of 0 at $x=5$.
Local maximum of 4 at $x=4$.
Local minimum of 1 at $x=1$.

47. Absolute minimum of 1 at $x=1$.
Absolute maximum of 4 at x = 3.
Local maximum of 3 at $x=4$.
Local minimum of 1 at $x=1$.

48. Absolute minimum of 1 at $x=0$.
No absolute maximum.
No local maximum.
No local minimum.

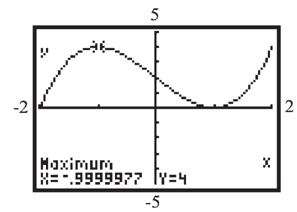
49. Absolute minimum of 0 at $x=0$.
No absolute maximum.
Local maximum of 3 at $x=2$.
Local minimum of 0 at $x=0$ and 2 at $x=3$.

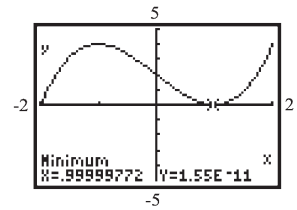
50. Absolute maximum of 4 at $x=2$.
No absolute minimum.
Local maximum of 4 at $x=2$.
Local minimum of 2 at $x=0$.

51. No absolute maximum or minimum.
No local maximum or minimum.

52. No absolute maximum or minimum.
No local maximum or minimum.

53. $f(x)=x^3-3x+2$ on the interval $(-2,2)$
Use MAXIMUM and MINIMUM on the graph of $y_1=x^3-3x+2$.





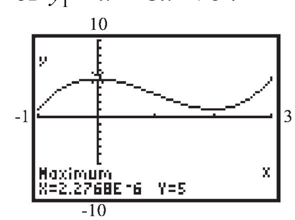
local maximum at: $(-1,4)$;

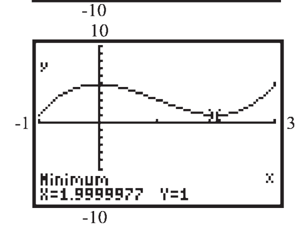
local minimum at: $(1,0)$

f is increasing on: $(-2,-1)$ and $(1,2)$;

f is decreasing on: $(-1,1)$

54. $f(x)=x^3-3x^2+5$ on the interval $(-1,3)$
Use MAXIMUM and MINIMUM on the graph of $y_1=x^3-3x^2+5$.





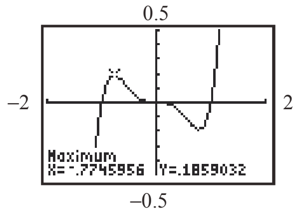
local maximum at: $(0,5)$;

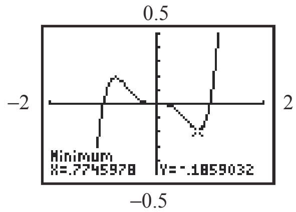
local minimum at: $(2,1)$

f is increasing on: $(-1,0)$ and $(2,3)$;

f is decreasing on: $(0,2)$

55. $f(x)=x^5-x^3$ on the interval $(-2,2)$
Use MAXIMUM and MINIMUM on the graph of $y_1=x^5-x^3$.





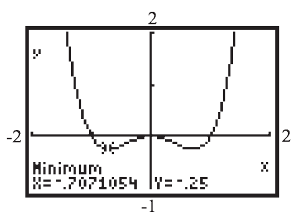
local maximum at: $(-0.77, 0.19)$;

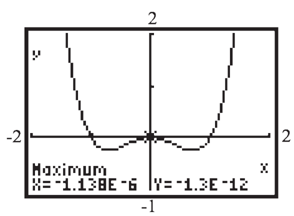
local minimum at: $(0.77, -0.19)$;

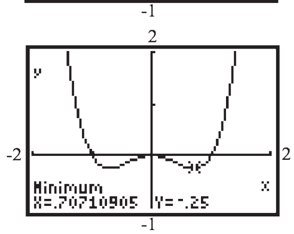
f is increasing on: $(-2, -0.77)$ and $(0.77, 2)$;

f is decreasing on: $(-0.77, 0.77)$

56. $f(x) = x^4 - x^2$ on the interval $(-2, 2)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = x^4 - x^2$.







local maximum at: $(0, 0)$;

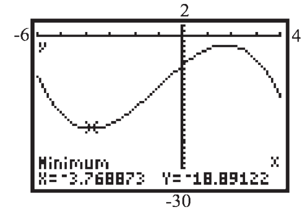
local minimum at: $(-0.71, -0.25)$, $(0.71, -0.25)$

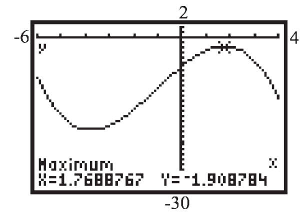
f is increasing on: $(-0.71, 0)$ and $(0.71, 2)$;

f is decreasing on: $(-2, -0.71)$ and $(0, 0.71)$

57. $f(x) = -0.2x^3 - 0.6x^2 + 4x - 6$ on the interval $(-6, 4)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.2x^3 - 0.6x^2 + 4x - 6$.





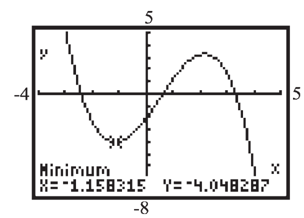
local maximum at: $(1.77, -1.91)$;

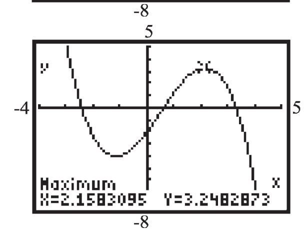
local minimum at: $(-3.77, -18.89)$

f is increasing on: $(-3.77, 1.77)$;

f is decreasing on: $(-6, -3.77)$ and $(1.77, 4)$

58. $f(x) = -0.4x^3 + 0.6x^2 + 3x - 2$ on the interval $(-4, 5)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^3 + 0.6x^2 + 3x - 2$.





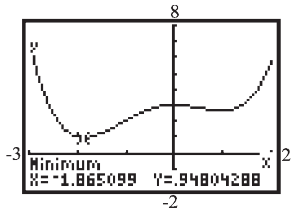
local maximum at: $(2.16, 3.25)$;

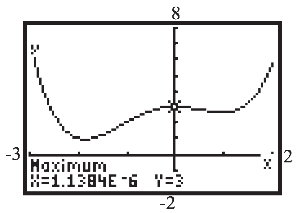
local minimum at: $(-1.16, -4.05)$

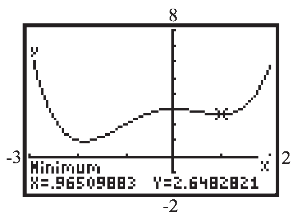
f is increasing on: $(-1.16, 2.16)$;

f is decreasing on: $(-4, -1.16)$ and $(2.16, 5)$

59. $f(x) = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$ on the interval $(-3, 2)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$.







local maximum at: $(0,3)$;

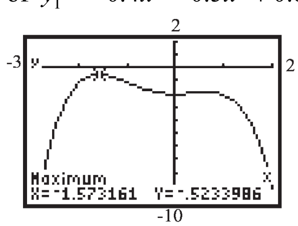
local minimum at: $(-1.87,0.95)$, $(0.97,2.65)$

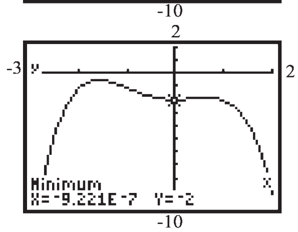
f is increasing on: $(-1.87,0)$ and $(0.97,2)$;

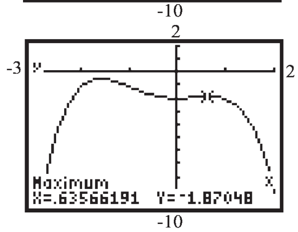
f is decreasing on: $(-3,-1.87)$ and $(0,0.97)$

60. $f(x)=-0.4x^4-0.5x^3+0.8x^2-2$ on the interval $(-3,2)$

Use MAXIMUM and MINIMUM on the graph of $y_1=-0.4x^4-0.5x^3+0.8x^2-2$.







local maxima at: $(-1.57,-0.52)$, $(0.64,-1.87)$;

local minimum at: $(0,-2)$

f is increasing on: $(-3,-1.57)$ and $(0,0.64)$;

f is decreasing on: $(-1.57,0)$ and $(0.64,2)$

61. $f(x)=-2x^2+4$

a. Average rate of change of *f* from $x=0$ to $x=2$

$$\frac{f(2)-f(0)}{2-0}=\frac{\left(-2(2)^2+4\right)-\left(-2(0)^2+4\right)}{2}$$
$$=\frac{(-4)-(4)}{2}=\frac{-8}{2}=-4$$

b. Average rate of change of *f* from $x=1$ to $x=3$:

$$\frac{f(3)-f(1)}{3-1}=\frac{\left(-2(3)^2+4\right)-\left(-2(1)^2+4\right)}{2}$$
$$=\frac{(-14)-(2)}{2}=\frac{-16}{2}=-8$$

c. Average rate of change of *f* from $x=1$ to $x=4$:

$$\frac{f(4)-f(1)}{4-1}=\frac{\left(-2(4)^2+4\right)-\left(-2(1)^2+4\right)}{3}$$
$$=\frac{(-28)-(2)}{3}=\frac{-30}{3}=-10$$

62. $f(x)=-x^3+1$

a. Average rate of change of *f* from $x=0$ to $x=2$:

$$\frac{f(2)-f(0)}{2-0}=\frac{\left(-(2)^3+1\right)-\left(-(0)^3+1\right)}{2}$$
$$=\frac{-7-1}{2}=\frac{-8}{2}=-4$$

b. Average rate of change of *f* from $x=1$ to $x=3$:

$$\frac{f(3)-f(1)}{3-1}=\frac{\left(-(3)^3+1\right)-\left(-(1)^3+1\right)}{2}$$
$$=\frac{-26-(0)}{2}=\frac{-26}{2}=-13$$

c. Average rate of change of *f* from $x = -1$ to $x = 1$:

$$\frac{f(1)-f(-1)}{1-(-1)} = \frac{\left(-(1)^3+1\right)-\left(-(-1)^3+1\right)}{2}$$
$$= \frac{0-2}{2} = \frac{-2}{2} = -1$$

63. $g(x) = x^3 - 2x + 1$

a. Average rate of change of *g* from $x = -3$ to $x = -2$:

$$\frac{g(-2)-g(-3)}{-2-(-3)}$$
$$= \frac{\left[(-2)^3-2(-2)+1\right]-\left[(-3)^3-2(-3)+1\right]}{1}$$
$$= \frac{(-3)-(-20)}{1} = \frac{17}{1} = 17$$

b. Average rate of change of *g* from $x = -1$ to $x = 1$:

$$\frac{g(1)-g(-1)}{1-(-1)}$$
$$= \frac{\left[(1)^3-2(1)+1\right]-\left[(-1)^3-2(-1)+1\right]}{2}$$
$$= \frac{(0)-(2)}{2} = \frac{-2}{2} = -1$$

c. Average rate of change of *g* from $x = 1$ to $x = 3$:

$$\frac{g(3)-g(1)}{3-1}$$
$$= \frac{\left[(3)^3-2(3)+1\right]-\left[(1)^3-2(1)+1\right]}{2}$$
$$= \frac{(22)-(0)}{2} = \frac{22}{2} = 11$$

64. $h(x) = x^2 - 2x + 3$

a. Average rate of change of *h* from $x = -1$ to $x = 1$:

$$\frac{h(1)-h(-1)}{1-(-1)}$$
$$= \frac{\left[(1)^2-2(1)+3\right]-\left[(-1)^2-2(-1)+3\right]}{2}$$
$$= \frac{(2)-(6)}{2} = \frac{-4}{2} = -2$$

b. Average rate of change of *h* from $x = 0$ to $x = 2$:

$$\frac{h(2)-h(0)}{2-0}$$
$$= \frac{\left[(2)^2-2(2)+3\right]-\left[(0)^2-2(0)+3\right]}{2}$$
$$= \frac{(3)-(3)}{2} = \frac{0}{2} = 0$$

c. Average rate of change of *h* from $x = 2$ to $x = 5$:

$$\frac{h(5)-h(2)}{5-2}$$
$$= \frac{\left[(5)^2-2(5)+3\right]-\left[(2)^2-2(2)+3\right]}{3}$$
$$= \frac{(18)-(3)}{3} = \frac{15}{3} = 5$$

65. $f(x) = 5x - 2$

a. Average rate of change of *f* from 1 to 3:

$$\frac{\Delta y}{\Delta x} = \frac{f(3)-f(1)}{3-1} = \frac{13-3}{3-1} = \frac{10}{2} = 5$$

Thus, the average rate of change of *f* from 1 to 3 is 5.

b. From (a), the slope of the secant line joining $(1, f(1))$ and $(3, f(3))$ is 5. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 3 = 5(x-1)$$
$$y - 3 = 5x - 5$$
$$y = 5x - 2$$

66. $f(x) = -4x + 1$

a. Average rate of change of *f* from 2 to 5:

$$\frac{\Delta y}{\Delta x} = \frac{f(5)-f(2)}{5-2} = \frac{-19-(-7)}{5-2}$$
$$= \frac{-12}{3} = -4$$

Therefore, the average rate of change of *f* from 2 to 5 is -4.

b. From (a), the slope of the secant line joining $(2, f(2))$ and $(5, f(5))$ is -4. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\sec}(x - x_1)$$
$$y - (-7) = -4(x - 2)$$
$$y + 7 = -4x + 8$$
$$y = -4x + 1$$

67. $g(x) = x^2 - 2$

a. Average rate of change of g from -2 to 1:

$$\frac{\Delta y}{\Delta x} = \frac{g(1) - g(-2)}{1 - (-2)} = \frac{-1 - 2}{1 - (-2)} = \frac{-3}{3} = -1$$

Therefore, the average rate of change of g from -2 to 1 is -1.

b. From (a), the slope of the secant line joining $(-2, g(-2))$ and $(1, g(1))$ is -1. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\sec}(x - x_1)$$
$$y - 2 = -1(x - (-2))$$
$$y - 2 = -x - 2$$
$$y = -x$$

68. $g(x) = x^2 + 1$

a. Average rate of change of g from -1 to 2:

$$\frac{\Delta y}{\Delta x} = \frac{g(2) - g(-1)}{2 - (-1)} = \frac{5 - 2}{2 - (-1)} = \frac{3}{3} = 1$$

Therefore, the average rate of change of g from -1 to 2 is 1.

b. From (a), the slope of the secant line joining $(-1, g(-1))$ and $(2, g(2))$ is 1. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\sec}(x - x_1)$$
$$y - 2 = 1(x - (-1))$$
$$y - 2 = x + 1$$
$$y = x + 3$$

69. $h(x) = x^2 - 2x$

a. Average rate of change of h from 2 to 4:

$$\frac{\Delta y}{\Delta x} = \frac{h(4) - h(2)}{4 - 2} = \frac{8 - 0}{4 - 2} = \frac{8}{2} = 4$$

Therefore, the average rate of change of h from 2 to 4 is 4.

b. From (a), the slope of the secant line joining $(2, h(2))$ and $(4, h(4))$ is 4. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\sec}(x - x_1)$$
$$y - 0 = 4(x - 2)$$
$$y = 4x - 8$$

70. $h(x) = -2x^2 + x$

a. Average rate of change from 0 to 3:

$$\frac{\Delta y}{\Delta x} = \frac{h(3) - h(0)}{3 - 0} = \frac{-15 - 0}{3 - 0} = \frac{-15}{3} = -5$$

Therefore, the average rate of change of h from 0 to 3 is -5.

b. From (a), the slope of the secant line joining $(0, h(0))$ and $(3, h(3))$ is -5. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\sec}(x - x_1)$$
$$y - 0 = -5(x - 0)$$
$$y = -5x$$

71. a. $g(x) = x^3 - 27x$

$$g(-x) = (-x)^3 - 27(-x)$$
$$= -x^3 + 27x$$
$$= -(x^3 - 27x)$$
$$= -g(x)$$

Since $g(-x) = -g(x)$, the function is odd.

b. Since $g(x)$ is odd then it is symmetric about the origin so there exist a local maximum at $x = -3$.

$$g(-3) = (-3)^3 - 27(-3) = -27 + 81 = 54$$

So there is a local maximum of 54 at $x = -3$.

72. $f(x) = -x^3 + 12x$

a. $f(-x) = -(-x)^3 + 12(-x)$

$$= x^3 - 12x$$
$$= -(-x^3 + 12x)$$
$$= -f(x)$$

Since $f(-x) = -f(x)$, the function is odd.

b. Since $f(x)$ is odd then it is symmetric about the origin so there exist a local maximum at $x=-3$.

$f(-2)=-(-2)^3+12(-2)=8-24=-16$

So there is a local maximum of -16 at $x=-2$.

73. $F(x)=-x^4+8x^2+8$

a.
$$\begin{aligned} F(-x) &= -(-x)^4+8(-x)^2+8 \\ &= -x^4+8x+8 \\ &= F(x) \end{aligned}$$

Since $F(-x)=F(x)$, the function is even.

b. Since the function is even, its graph has y-axis symmetry. The second local maximum is in quadrant II and is 24 and occurs at $x=-2$.

c. Because the graph has y-axis symmetry, the area under the graph between $x=0$ and $x=3$ bounded below by the x-axis is the same as the area under the graph between $x=-3$ and $x=0$ bounded below the x-axis. Thus, the area is 47.4 square units.

74. $G(x)=-x^4+32x^2+144$

a.
$$\begin{aligned} G(-x) &= -(-x)^4+32(-x)^2+144 \\ &= -x^4+32x^2+144 \\ &= G(x) \end{aligned}$$

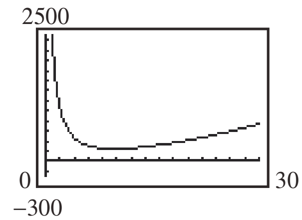
Since $G(-x)=G(x)$, the function is even.

b. Since the function is even, its graph has y-axis symmetry. The second local maximum is in quadrant II and is 400 and occurs at $x=-4$.

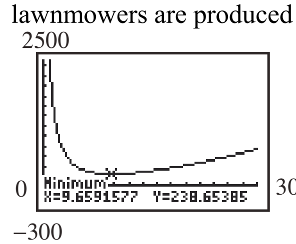
c. Because the graph has y-axis symmetry, the area under the graph between $x=0$ and $x=6$ bounded below by the x-axis is the same as the area under the graph between $x=-6$ and $x=0$ bounded below the x-axis. Thus, the area is 1612.8 square units.

75. $\overline{C}(x)=0.3x^2+21x-251+\dfrac{2500}{x}$

a. $y_1=0.3x^2+21x-251+\dfrac{2500}{x}$



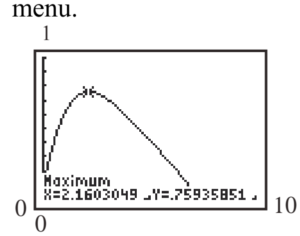
b. Use MINIMUM. Rounding to the nearest whole number, the average cost is minimized when approximately 10 lawnmowers are produced per hour.



c. The minimum average cost is approximately \$239 per mower.

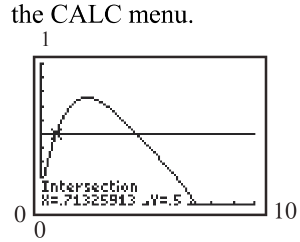
76. a. $C(t)=-.002t^4+.039t^3-.285t^2+.766t+.085$

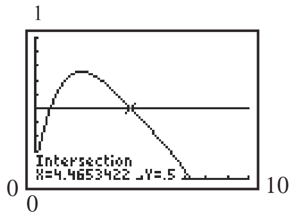
Graph the function on a graphing utility and use the Maximum option from the CALC menu.



The concentration will be highest after about 2.16 hours.

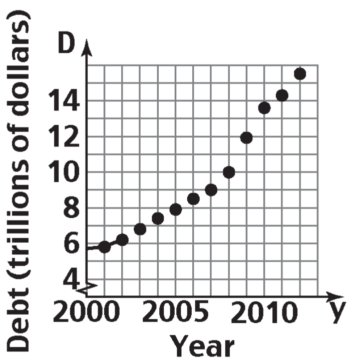
b. Enter the function in Y1 and 0.5 in Y2. Graph the two equations in the same window and use the Intersect option from the CALC menu.





After taking the medication, the woman can feed her child within the first 0.71 hours (about 42 minutes) or after 4.47 hours (about 4hours 28 minutes) have elapsed.

77. **a.**



b. The slope represents the average rate of change of the debt from 2001 to 2006.

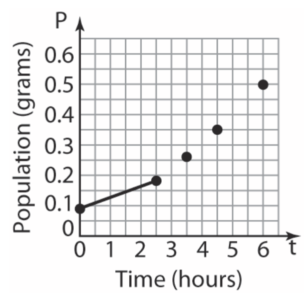
c. $$\text{avg. rate of change} = \frac{P(2004)-P(2002)}{2004-2002} = \frac{7379-6228}{2} = \frac{1151}{2} = \$\ 575.5 \text{ billion/yr}$$

d. $$\text{avg. rate of change} = \frac{P(2008)-P(2006)}{2008-2006} = \frac{10025-8507}{2} = \frac{1518}{2} = \$\ 759 \text{ billion/yr}$$

e. $$\text{avg. rate of change} = \frac{P(2012)-P(2010)}{2012-2010} = \frac{16066-13562}{2} = \frac{2504}{2} = \$\ 1252 \text{ billion}$$

c. The average rate of change is increasing as time passes.

78. **a.**



b. The slope represents the average rate of change of the population from 0 to 2.5 hours.

c. $$\text{avg. rate of change} = \frac{P(2.5)-P(0)}{2.5-0} = \frac{0.18-0.09}{2.5-0} = \frac{0.09}{2.5} = 0.036 \text{ gram per hour}$$

c. $$\text{avg. rate of change} = \frac{P(6)-P(4.5)}{6-4.5} = \frac{0.50-0.35}{6-4.5} = \frac{0.15}{1.5} = 0.1 \text{ gram per hour}$$

d. The average rate of change is increasing as time passes.

79. $f(x)=x^2$

a. Average rate of change of *f* from $x=0$ to $x=1$:

$$\frac{f(1)-f(0)}{1-0} = \frac{1^2-0^2}{1} = \frac{1}{1} = 1$$

b. Average rate of change of *f* from $x=0$ to $x=0.5$:

$$\frac{f(0.5)-f(0)}{0.5-0}=\frac{(0.5)^2-0^2}{0.5}=\frac{0.25}{0.5}=0.5$$

c. Average rate of change of *f* from $x=0$ to $x=0.1$:

$$\frac{f(0.1)-f(0)}{0.1-0}=\frac{(0.1)^2-0^2}{0.1}=\frac{0.01}{0.1}=0.1$$

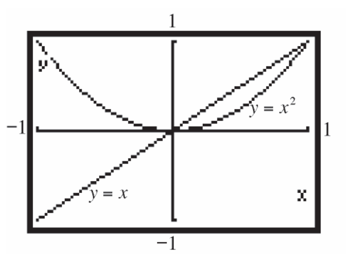
d. Average rate of change of *f* from $x=0$ to $x=0.01$:

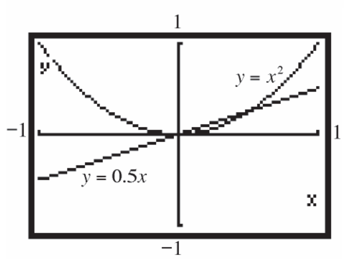
$$\frac{f(0.01)-f(0)}{0.01-0}=\frac{(0.01)^2-0^2}{0.01}$$
$$=\frac{0.0001}{0.01}=0.01$$

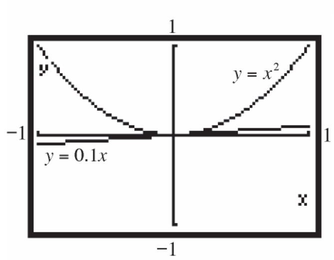
e. Average rate of change of *f* from $x=0$ to $x=0.001$:

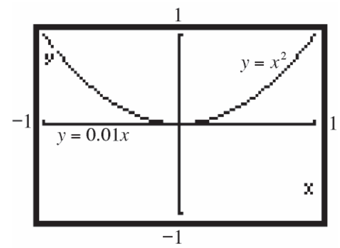
$$\frac{f(0.001)-f(0)}{0.001-0}=\frac{(0.001)^2-0^2}{0.001}$$
$$=\frac{0.000001}{0.001}=0.001$$

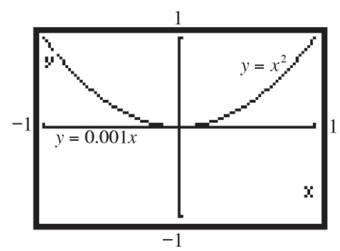
f. Graphing the secant lines:











g. The secant lines are beginning to look more and more like the tangent line to the graph of *f* at the point where $x=0$.

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number zero.

80. $f(x)=x^2$

a. Average rate of change of *f* from $x=1$ to $x=2$:

$$\frac{f(2)-f(1)}{2-1}=\frac{2^2-1^2}{1}=\frac{3}{1}=3$$

b. Average rate of change of *f* from $x=1$ to $x=1.5$:

$$\frac{f(1.5)-f(1)}{1.5-1}=\frac{(1.5)^2-1^2}{0.5}=\frac{1.25}{0.5}=2.5$$

c. Average rate of change of *f* from $x=1$ to $x=1.1$:

$$\frac{f(1.1)-f(1)}{1.1-1}=\frac{(1.1)^2-1^2}{0.1}=\frac{0.21}{0.1}=2.1$$

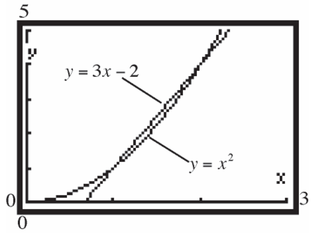
d. Average rate of change of *f* from $x=1$ to $x=1.01$:

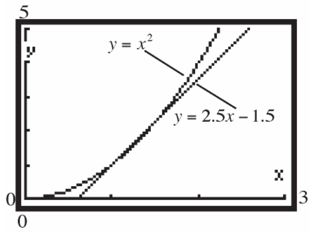
$$\frac{f(1.01)-f(1)}{1.01-1}=\frac{(1.01)^2-1^2}{0.01}=\frac{0.0201}{0.01}=2.01$$

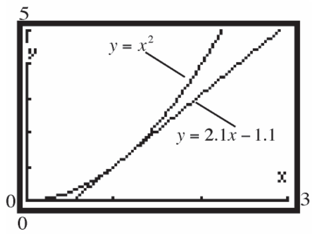
e. Average rate of change of *f* from $x=1$ to $x=1.001$:

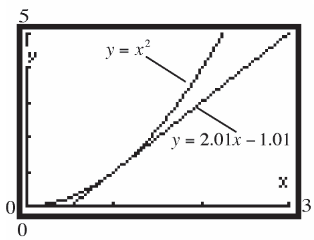
$$\frac{f(1.001)-f(1)}{1.001-1}=\frac{(1.001)^2-1^2}{0.001}$$
$$=\frac{0.002001}{0.001}=2.001$$

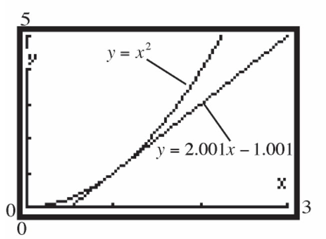
f. Graphing the secant lines:











g. The secant lines are beginning to look more and more like the tangent line to the graph of *f* at the point where $x = 1$.

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number 2.

81. $f(x) = 2x + 5$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{2(x+h) + 5 - 2x - 5}{h} = \dfrac{2h}{h} = 2$

b. When $x = 1$:

$h = 0.5 \Rightarrow m_{\text{sec}} = 2$

$h = 0.1 \Rightarrow m_{\text{sec}} = 2$

$h = 0.01 \Rightarrow m_{\text{sec}} = 2$

as $h \to 0$, $m_{\text{sec}} \to 2$

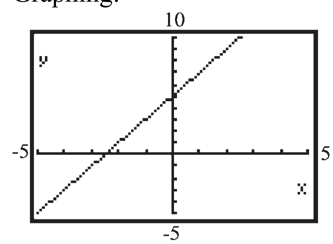
c. Using the point $(1, f(1)) = (1, 7)$ and slope, $m = 2$, we get the secant line:

$y - 7 = 2(x - 1)$

$y - 7 = 2x - 2$

$y = 2x + 5$

d. Graphing:



The graph and the secant line coincide.

82. $f(x) = -3x + 2$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{-3(x+h) + 2 - (-3x+2)}{h} = \dfrac{-3h}{h} = -3$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = -3$

$h = 0.1 \Rightarrow m_{\text{sec}} = -3$

$h = 0.01 \Rightarrow m_{\text{sec}} = -3$

as $h \to 0$, $m_{\text{sec}} \to -3$

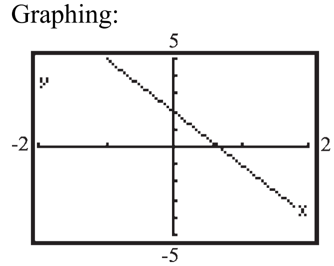
c. Using point $(1, f(1)) = (1, -1)$ and slope $= -3$, we get the secant line:

$y - (-1) = -3(x - 1)$

$y + 1 = -3x + 3$

$y = -3x + 2$

d. Graphing:



The graph and the secant line coincide.

83. $f(x) = x^2 + 2x$

a. $m_{\sec} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$

$= \dfrac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$

$= \dfrac{2xh + h^2 + 2h}{h}$

$= 2x + h + 2$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\sec} = 2\cdot 1 + 0.5 + 2 = 4.5$

$h = 0.1 \Rightarrow m_{\sec} = 2\cdot 1 + 0.1 + 2 = 4.1$

$h = 0.01 \Rightarrow m_{\sec} = 2\cdot 1 + 0.01 + 2 = 4.01$

as $h \to 0$, $m_{\sec} \to 2\cdot 1 + 0 + 2 = 4$

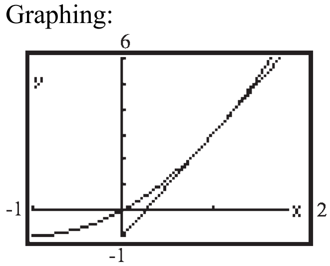
c. Using point $(1, f(1)) = (1,3)$ and slope = 4.01, we get the secant line:

$y - 3 = 4.01(x-1)$

$y - 3 = 4.01x - 4.01$

$y = 4.01x - 1.01$

d. Graphing:



84. $f(x) = 2x^2 + x$

a. $m_{\sec} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{2(x+h)^2 + (x+h) - (2x^2 + x)}{h}$

$= \dfrac{2(x^2 + 2xh + h^2) + x + h - 2x^2 - x}{h}$

$= \dfrac{2x^2 + 4xh + 2h^2 + x + h - 2x^2 - x}{h}$

$= \dfrac{4xh + 2h^2 + h}{h}$

$= 4x + 2h + 1$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\sec} = 4\cdot 1 + 2(0.5) + 1 = 6$

$h = 0.1 \Rightarrow m_{\sec} = 4\cdot 1 + 2(0.1) + 1 = 5.2$

$h = 0.01 \Rightarrow m_{\sec} = 4\cdot 1 + 2(0.01) + 1 = 5.02$

as $h \to 0$, $m_{\sec} \to 4\cdot 1 + 2(0) + 1 = 5$

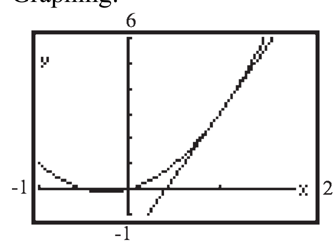
c. Using point $(1, f(1)) = (1,3)$ and slope = 5.02, we get the secant line:

$y - 3 = 5.02(x-1)$

$y - 3 = 5.02x - 5.02$

$y = 5.02x - 2.02$

d. Graphing:



85. $f(x) = 2x^2 - 3x + 1$

a. $m_{\sec} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h}$

$= \dfrac{2(x^2 + 2xh + h^2) - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$

$= \dfrac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$

$= \dfrac{4xh + 2h^2 - 3h}{h}$

$= 4x + 2h - 3$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\sec} = 4\cdot 1 + 2(0.5) - 3 = 2$

$h = 0.1 \Rightarrow m_{\sec} = 4\cdot 1 + 2(0.1) - 3 = 1.2$

$h = 0.01 \Rightarrow m_{\sec} = 4\cdot 1 + 2(0.01) - 3 = 1.02$

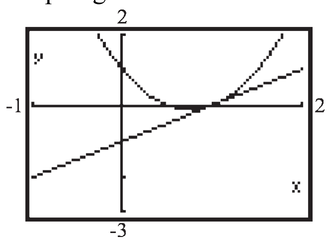
as $h \to 0$, $m_{\sec} \to 4\cdot 1 + 2(0) - 3 = 1$

c. Using point $(1, f(1)) = (1,0)$ and slope = 1.02, we get the secant line:

$y - 0 = 1.02(x-1)$

$y = 1.02x - 1.02$

d. Graphing:



86. $f(x) = -x^2 + 3x - 2$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{-(x+h)^2 + 3(x+h) - 2 - (-x^2 + 3x - 2)}{h}$

$= \dfrac{-(x^2 + 2xh + h^2) + 3x + 3h - 2 + x^2 - 3x + 2}{h}$

$= \dfrac{-x^2 - 2xh - h^2 + 3x + 3h - 2 + x^2 - 3x + 2}{h}$

$= \dfrac{-2xh - h^2 + 3h}{h}$

$= -2x - h + 3$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.5 + 3 = 0.5$

$h = 0.1 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.1 + 3 = 0.9$

$h = 0.01 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.01 + 3 = 0.99$

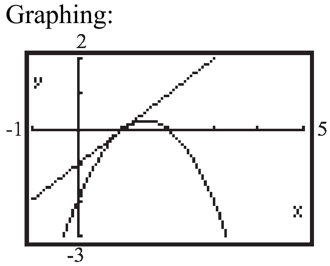
as $h \to 0$, $m_{\text{sec}} \to -2 \cdot 1 - 0 + 3 = 1$

c. Using point $(1, f(1)) = (1, 0)$ and slope = 0.99, we get the secant line:

$y - 0 = 0.99(x - 1)$

$y = 0.99x - 0.99$

d. Graphing:



87. $f(x) = \dfrac{1}{x}$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{\left(\dfrac{1}{x+h} - \dfrac{1}{x}\right)}{h} = \dfrac{\left(\dfrac{x - (x+h)}{(x+h)x}\right)}{h}$

$= \left(\dfrac{x - x - h}{(x+h)x}\right)\left(\dfrac{1}{h}\right) = \left(\dfrac{-h}{(x+h)x}\right)\left(\dfrac{1}{h}\right)$

$= -\dfrac{1}{(x+h)x}$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = -\dfrac{1}{(1+0.5)(1)}$

$= -\dfrac{1}{1.5} = -\dfrac{2}{3} \approx -0.667$

$h = 0.1 \Rightarrow m_{\text{sec}} = -\dfrac{1}{(1+0.1)(1)}$

$= -\dfrac{1}{1.1} = -\dfrac{10}{11} \approx -0.909$

$h = 0.01 \Rightarrow m_{\text{sec}} = -\dfrac{1}{(1+0.01)(1)}$

$= -\dfrac{1}{1.01} = -\dfrac{100}{101} \approx -0.990$

as $h \to 0$, $m_{\text{sec}} \to -\dfrac{1}{(1+0)(1)} = -\dfrac{1}{1} = -1$

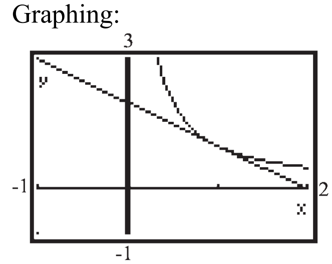
c. Using point $(1, f(1)) = (1, 1)$ and slope = $-\dfrac{100}{101}$, we get the secant line:

$y - 1 = -\dfrac{100}{101}(x - 1)$

$y - 1 = -\dfrac{100}{101}x + \dfrac{100}{101}$

$y = -\dfrac{100}{101}x + \dfrac{201}{101}$

d. Graphing:



88. $f(x)=\dfrac{1}{x^2}$

a. $$m_{\text{sec}}=\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right)}{h}$$
$$=\frac{\left(\dfrac{x^2-(x+h)^2}{(x+h)^2x^2}\right)}{h}$$
$$=\left(\frac{x^2-\left(x^2+2xh+h^2\right)}{(x+h)^2x^2}\right)\left(\frac{1}{h}\right)$$
$$=\left(\frac{-2xh-h^2}{(x+h)^2x^2}\right)\left(\frac{1}{h}\right)$$
$$=\frac{-2x-h}{(x+h)^2x^2}=\frac{-2x-h}{\left(x^2+2xh+h^2\right)x^2}$$

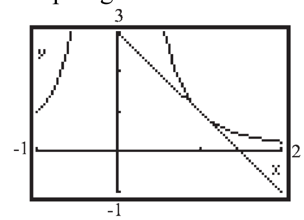
b. When $x = 1$,

$$h=0.5\Rightarrow m_{\text{sec}}=\frac{-2\cdot 1-0.5}{(1+0.5)^2 1^2}=-\frac{10}{9}\approx -1.1111$$
$$h=0.1\Rightarrow m_{\text{sec}}=\frac{-2\cdot 1-0.1}{(1+0.1)^2 1^2}=-\frac{210}{121}\approx -1.7355$$
$$h=0.01\Rightarrow m_{\text{sec}}=\frac{-2\cdot 1-0.01}{(1+0.01)^2 1^2}$$
$$=-\frac{20{,}100}{10{,}201}\approx -1.9704$$
$$\text{as } h\to 0,\ m_{\text{sec}}\to\frac{-2\cdot 1-0}{(1+0)^2 1^2}=-2$$

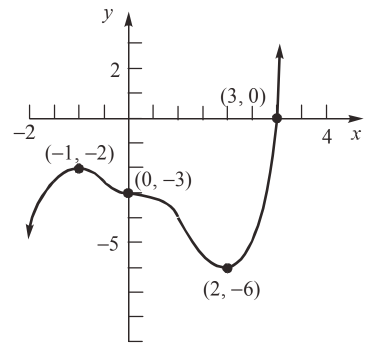
c. Using point $(1,f(1))=(1,1)$ and slope $= -1.9704$, we get the secant line:

$$y-1=-1.9704(x-1)$$
$$y-1=-1.9704x+1.9704$$
$$y=-1.9704x+2.9704$$

d. Graphing:



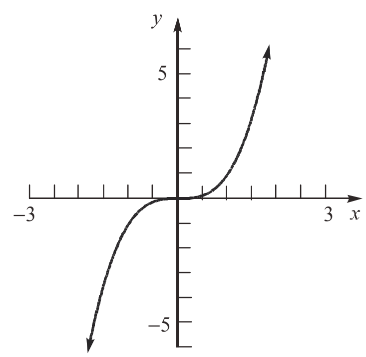
89. Answers will vary. One possibility follows:



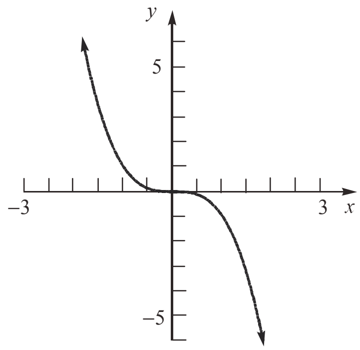
90. Answers will vary. See solution to Problem 89 for one possibility.

91. A function that is increasing on an interval can have at most one x-intercept on the interval. The graph of f could not "turn" and cross it again or it would start to decrease.

92. An increasing function is a function whose graph goes up as you read from left to right.

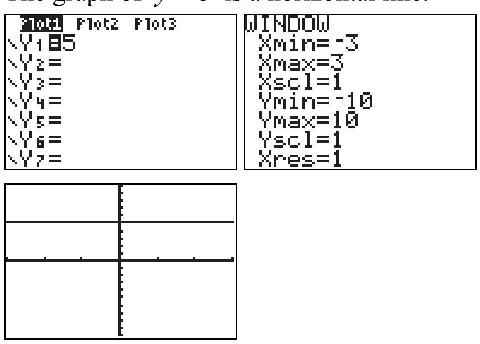


A decreasing function is a function whose graph goes down as you read from left to right.



93. To be an even function we need $f(-x) = f(x)$ and to be an odd function we need $f(-x) = -f(x)$. In order for a function be both even and odd, we would need $f(x) = -f(x)$. This is only possible if $f(x) = 0$.

94. The graph of $y = 5$ is a horizontal line.



The local maximum is $y = 5$ and it occurs at each *x*-value in the interval.

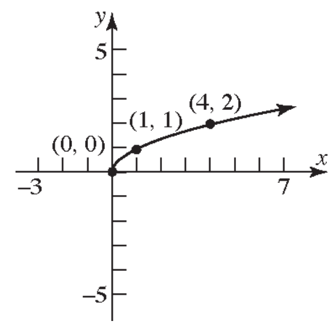
95. Not necessarily. It just means $f(5) > f(2)$. The function could have both increasing and decreasing intervals.

96. $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{b - b}{x_2 - x_1} = 0$

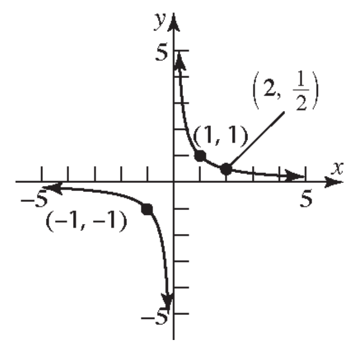
$\dfrac{f(2) - f(-2)}{2 - (-2)} = \dfrac{0 - 0}{4} = 0$

Section 1.4

1. $y = \sqrt{x}$



2. $y = \dfrac{1}{x}$



3. $y = x^3 - 8$

<u>*y*-intercept:</u>

Let $x = 0$, then $y = (0)^3 - 8 = -8$.

<u>*x*-intercept:</u>

Let $y = 0$, then

$$\begin{aligned} 0 &= x^3 - 8 \\ x^3 &= 8 \\ x &= 2 \end{aligned}$$

The intercepts are $(0, -8)$ and $(2, 0)$.

4. $(-\infty, 0)$

5. piecewise-defined

6. True

7. False; the cube root function is odd and increasing on the interval $(-\infty, \infty)$.

8. False; the domain and range of the reciprocal function are both the set of real numbers except for 0.

9. C

10. A

11. E

12. G

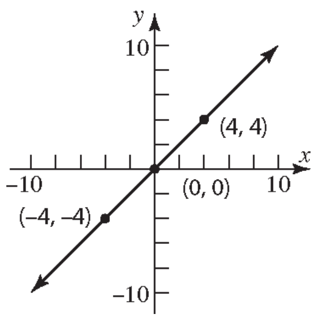
13. B

14. D

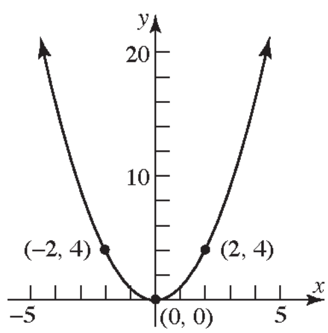
15. F

16. H

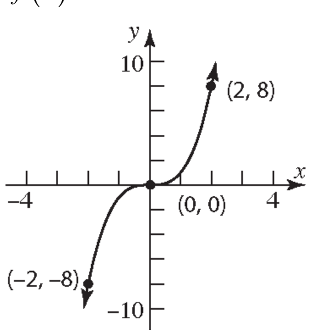
17. $f(x) = x$



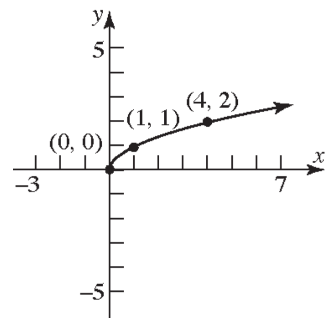
18. $f(x) = x^2$



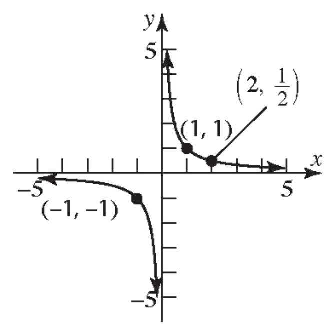
19. $f(x) = x^3$



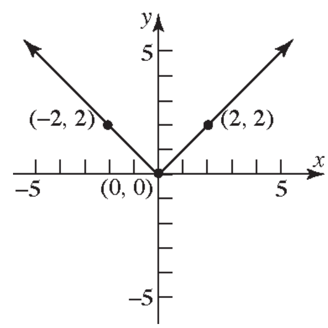
20. $f(x) = \sqrt{x}$



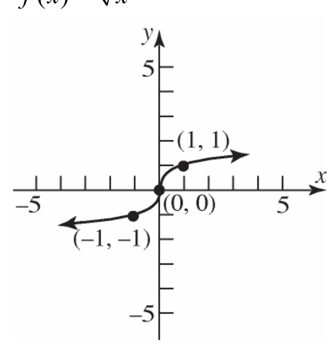
21. $f(x) = \dfrac{1}{x}$



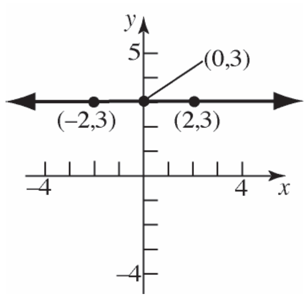
22. $f(x) = |x|$



23. $f(x) = \sqrt[3]{x}$



24. $f(x) = 3$



25. **a.** $f(-2) = (-2)^2 = 4$

b. $f(0) = 2$

c. $f(2) = 2(2) + 1 = 5$

26. **a.** $f(-2) = -3(-2) = 6$

b. $f(-1) = 0$

c. $f(0) = 2(0)^2 + 1 = 1$

27. **a.** $f(0) = 2(0) - 4 = -4$

b. $f(1) = 2(1) - 4 = -2$

c. $f(2) = 2(2) - 4 = 0$

d. $f(3) = (3)^3 - 2 = 25$

28. **a.** $f(-1) = (-1)^3 = -1$

b. $f(0) = (0)^3 = 0$

c. $f(1) = 3(1) + 2 = 5$

d. $f(3) = 3(3) + 2 = 11$

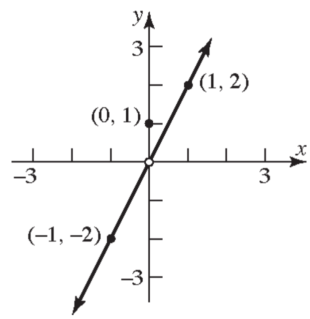
29. $f(x) = \begin{cases} 2x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: none
y-intercept:
$f(0) = 1$

The only intercept is $(0,1)$.

c. Graph:



d. Range: $\{y \mid y \neq 0\}$; $(-\infty, 0) \cup (0, \infty)$

e. The graph is not continuous. There is a jump at $x = 0$.

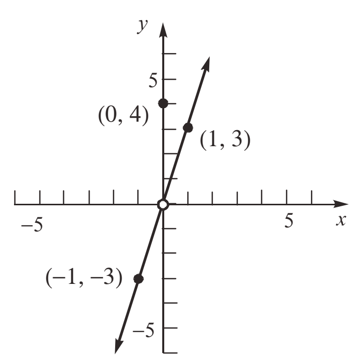
30. $f(x) = \begin{cases} 3x & \text{if } x \neq 0 \\ 4 & \text{if } x = 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: none
y-intercept: $f(0) = 4$

The only intercept is $(0,4)$.

c. Graph:



d. Range: $\{y \mid y \neq 0\}$; $(-\infty, 0) \cup (0, \infty)$

e. The graph is not continuous. There is a jump at $x = 0$.

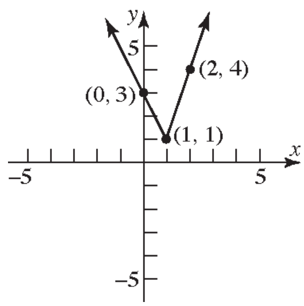
31. $f(x) = \begin{cases} -2x + 3 & \text{if } x < 1 \\ 3x - 2 & \text{if } x \geq 1 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: none
y-intercept: $f(0) = -2(0) + 3 = 3$

The only intercept is $(0,3)$.

c. Graph:



d. Range: $\{y \mid y \geq 1\}$; $[1, \infty)$

e. The graph is continuous. There are no holes or gaps.

32. $f(x) = \begin{cases} x+3 & \text{if } x < -2 \\ -2x-3 & \text{if } x \geq -2 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

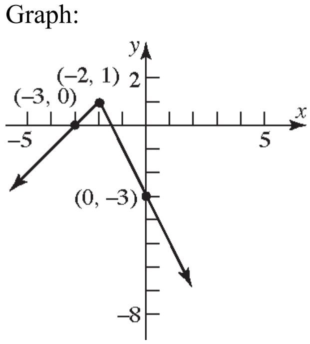
b. $x+3=0$ $\quad$ $-2x-3=0$
$x=-3$ $\quad$ $-2x=3$
$x=-\dfrac{3}{2}$

x-intercepts: $-3, -\dfrac{3}{2}$

y-intercept: $f(0) = -2(0) - 3 = -3$

The intercepts are $(-3,0)$, $\left(-\dfrac{3}{2},0\right)$, and $(0,-3)$.

c. Graph:



d. Range: $\{y \mid y \leq 1\}$; $(-\infty, 1]$

e. The graph is continuous. There are no holes or gaps.

33. $f(x) = \begin{cases} x+3 & \text{if } -2 \leq x < 1 \\ 5 & \text{if } x = 1 \\ -x+2 & \text{if } x > 1 \end{cases}$

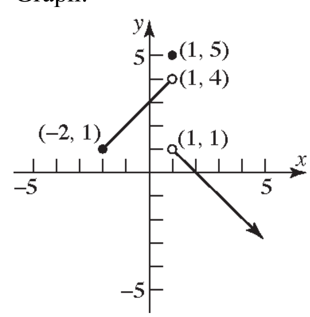
a. Domain: $\{x \mid x \geq -2\}$; $[-2, \infty)$

b. $x+3=0$ $\quad$ $-x+2=0$
$x=-3$ $\quad$ $-x=-2$
(not in domain) $\quad$ $x=2$
x-intercept: 2

y-intercept: $f(0) = 0+3 = 3$

The intercepts are $(2,0)$ and $(0,3)$.

c. Graph:



d. Range: $\{y \mid y < 4,\ y = 5\}$; $(-\infty, 4) \cup \{5\}$

e. The graph is not continuous. There is a jump at $x = 1$.

34. $f(x) = \begin{cases} 2x+5 & \text{if } -3 \leq x < 0 \\ -3 & \text{if } x = 0 \\ -5x & \text{if } x > 0 \end{cases}$

a. Domain: $\{x \mid x \geq -3\}$; $[-3, \infty)$

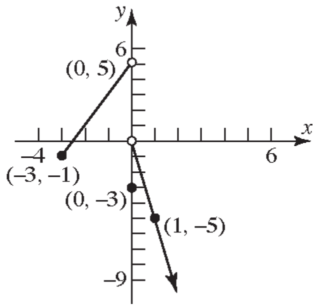
b. $2x+5=0$ $\quad$ $-5x=0$
$2x=-5$ $\quad$ $x=0$
$x=-\dfrac{5}{2}$ $\quad$ (not in domain of piece)

x-intercept: $-\dfrac{5}{2}$

y-intercept: $f(0) = -3$

The intercepts are $\left(-\dfrac{5}{2},0\right)$ and $(0,-3)$.

c. Graph:



d. Range: $\{y \mid y < 5\}$; $(-\infty, 5)$

e. The graph is not continuous. There is a jump at $x = 0$.

35. $f(x) = \begin{cases} 1+x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. $1 + x = 0 \qquad x^2 = 0$

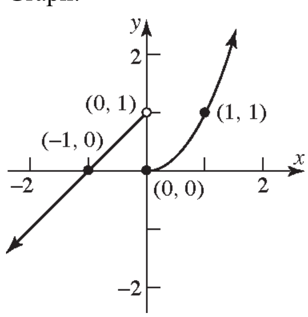
$x = -1 \qquad x = 0$

x-intercepts: $-1, 0$

y-intercept: $f(0) = 0^2 = 0$

The intercepts are $(-1, 0)$ and $(0, 0)$.

c. Graph:



d. Range: $\{y \mid y \text{ is any real number}\}$

e. The graph is not continuous. There is a jump at $x = 0$.

36. $f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ \sqrt[3]{x} & \text{if } x \geq 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. $\dfrac{1}{x} = 0 \qquad \sqrt[3]{x} = 0$

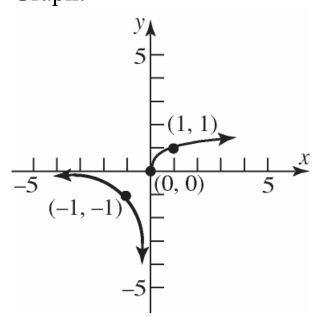
(no solution) $\qquad x = 0$

x-intercept: 0

y-intercept: $f(0) = \sqrt[3]{0} = 0$

The only intercept is $(0, 0)$.

c. Graph:



d. Range: $\{y \mid y \text{ is any real number}\}$

e. The graph is not continuous. There is a break at $x = 0$.

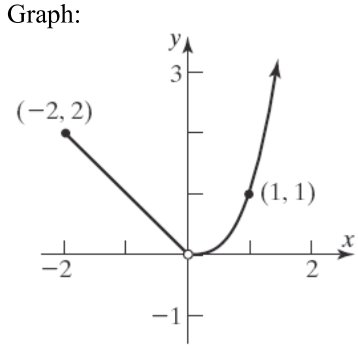
37. $f(x) = \begin{cases} |x| & \text{if } -2 \leq x < 0 \\ x^3 & \text{if } x > 0 \end{cases}$

a. Domain: $\{x \mid -2 \leq x < 0 \text{ and } x > 0\}$ or $\{x \mid x \geq -2, x \neq 0\}$; $[-2, 0) \cup (0, \infty)$.

b. x-intercept: none
There are no x-intercepts since there are no values for x such that $f(x) = 0$.

y-intercept:
There is no y-intercept since $x = 0$ is not in the domain.

c. Graph:



d. Range: $\{y \mid y > 0\}$; $(0, \infty)$

e. The graph is not continuous. There is a hole at $x = 0$.

38. $f(x) = \begin{cases} 2-x & \text{if } -3 \le x < 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$

a. Domain: $\{x \mid -3 \le x < 1 \text{ and } x > 1\}$ or $\{x \mid x \ge -3, x \ne 1\}$; $[-3,1) \cup (1,\infty)$.

b. $2 - x = 0$ $\quad$ $\sqrt{x} = 0$

$x = 2$ $\quad$ $x = 0$

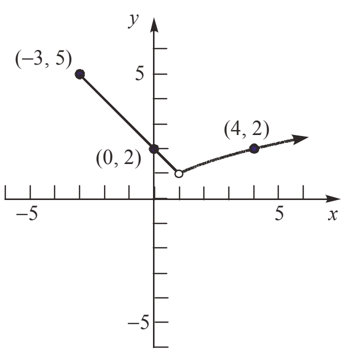
(not in domain of piece)

no x-intercepts

y-intercept: $f(0) = 2 - 0 = 2$

The intercept is $(0,2)$.

c. Graph:



d. Range: $\{y \mid y > 1\}$; $(1,\infty)$

e. The graph is not continuous. There is a hole at $x = 1$.

39. $f(x) = 2\,\text{int}(x)$

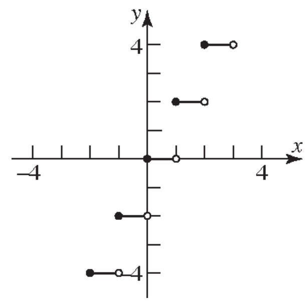
a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercepts:
All values for x such that $0 \le x < 1$.

y-intercept: $f(0) = 2\,\text{int}(0) = 0$

The intercepts are all ordered pairs $(x, 0)$ when $0 \le x < 1$.

c. Graph:



d. Range: $\{y \mid y \text{ is an even integer}\}$

e. The graph is not continuous. There is a jump at each integer value of x.

40. $f(x) = \text{int}(2x)$

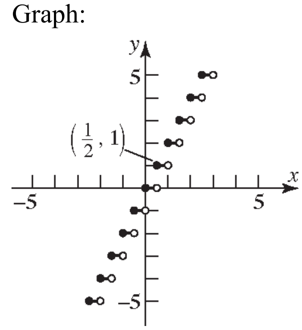
a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercepts:
All values for x such that $0 \le x < \frac{1}{2}$.

y-intercept: $f(0) = \text{int}(2(0)) = \text{int}(0) = 0$

The intercepts are all ordered pairs $(x, 0)$ when $0 \le x < \frac{1}{2}$.

c. Graph:



d. Range: $\{y \mid y \text{ is an integer}\}$

e. The graph is not continuous. There is a jump at each $x = \dfrac{k}{2}$, where k is an integer.

41. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} -x & \text{if } -1 \le x \le 0 \\ \dfrac{1}{2}x & \text{if } 0 < x \le 2 \end{cases}$$

42. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} x & \text{if } -1 \le x \le 0 \\ 1 & \text{if } 0 < x \le 2 \end{cases}$$

43. Answers may vary. One possibility follows:
$$f(x)=\begin{cases}-x & \text{if } x \le 0\\ -x+2 & \text{if } 0 < x \le 2\end{cases}$$

44. Answers may vary. One possibility follows:
$$f(x)=\begin{cases}2x+2 & \text{if } -1 \le x \le 0\\ x & \text{if } x > 0\end{cases}$$

45. **a.** $f(1.2)=\text{int}\left(2(1.2)\right)=\text{int}(2.4)=2$

b. $f(1.6)=\text{int}\left(2(1.6)\right)=\text{int}(3.2)=3$

c. $f(-1.8)=\text{int}\left(2(-1.8)\right)=\text{int}(-3.6)=-4$

46. **a.** $f(1.2)=\text{int}\left(\dfrac{1.2}{2}\right)=\text{int}(0.6)=0$

b. $f(1.6)=\text{int}\left(\dfrac{1.6}{2}\right)=\text{int}(0.8)=0$

c. $f(-1.8)=\text{int}\left(\dfrac{-1.8}{2}\right)=\text{int}(-0.9)=-1$

47. $C=\begin{cases}39.99 & \text{if } 0 < x \le 450\\ 0.45x-162.51 & \text{if } x > 450\end{cases}$

a. $C(200)=\$39.99$

b. $C(465)=0.45(465)-162.51=\$46.74$

c. $C(451)=0.45(451)-162.51=\$40.44$

48. $F(x)=\begin{cases}2 & \text{if } 0 < x \le 1\\ 4 & \text{if } 1 < x \le 3\\ 10 & \text{if } 3 < x \le 4\\ 5\,\text{int}(x+1)+2 & \text{if } 4 < x < 9\\ 51 & \text{if } 9 \le x \le 24\end{cases}$

a. $F(2)=4$
Parking for 2 hours costs \$4.

b. $F(7)=5\,\text{int}(7+1)+2=42$
Parking for 7 hours costs \$42.

c. $F(15)=51$
Parking for 15 hours costs \$51.

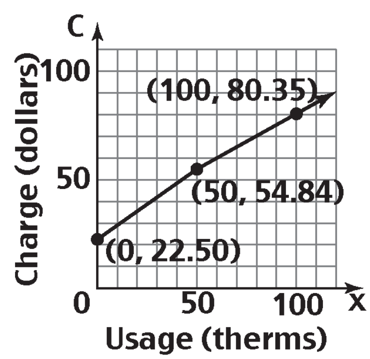
d. $24 \text{ min}\cdot\dfrac{1 \text{ hr}}{60 \text{ min}}=0.4 \text{ hr}$
$F(8.4)=5\,\text{int}(8.4+1)+2=5(9)+2=47$
Parking for 8 hours and 24 minutes costs \$47.

49. **a.** Charge for 50 therms:
$$C=22.25+0.3922(50)+0.25963(50)$$
$$=\$54.84$$

b. Charge for 500 therms:
$$C=22.25+0.3922(500)+0.11806(450)$$
$$+0.25963(50)$$
$$=\$284.46$$

c. For $0 \le x \le 50$:
$$C=22.25+0.25963x+0.3922x$$
$$=0.65183x+22.25$$
For $x > 50$:
$$C=22.25+0.25963(50)+0.11806(x-50)$$
$$+0.3922x$$
$$=22.25+12.9815+0.11806x-5.903$$
$$+0.3922x$$
$$=0.51026x+29.3285$$
The monthly charge function:
$$C=\begin{cases}0.65183x+22.5 & \text{for } 0 \le x \le 50\\ 0.51026x+29.3285 & \text{for } x > 50\end{cases}$$

d. Graph:



50. **a.** Charge for 20 therms:
$$C=19.50+0.65403(20)+0.53668(20)$$
$$=\$43.31$$

b. Charge for 150 therms:

$$C = 19.50 + 0.65403(30)$$
$$+0.04235(120) + 0.53668(150)$$
$$= \$124.70$$

c. For $0 \le x \le 30$:

$$C = 19.50 + 0.65403x + 0.53668x$$
$$= 1.19071x + 19.50$$

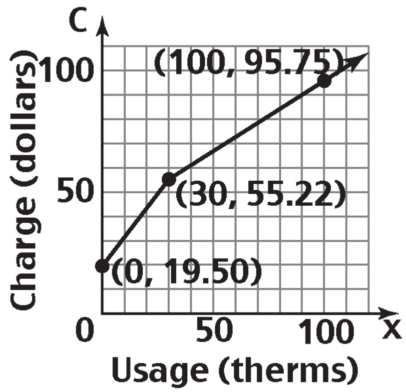
For $x > 30$:

$$C = 19.50 + 0.65403(30)$$
$$+ 0.04235(x - 30) + 0.53668x$$
$$= 19.50 + 19.6209 + 0.04235x - 1.2705$$
$$+ 0.53668x$$
$$= 0.57903x + 37.8504$$

The monthly charge function:

$$C = \begin{matrix} 1.19071x + 19.50 & \text{for } 0 \le x \le 30 \\ 0.57903x + 37.8504 & \text{for } x > 50 \end{matrix}$$

d. Graph:



51. For schedule X:

$$f(x) = \begin{matrix} 0.10x & \text{if } \ 0 < x \le 8925 \\ 892.50 + 0.15(x - 8925) & \text{if } \ 8500 < x \le 36,250 \\ 4991.25 + 0.25(x - 36,250) & \text{if } \ 36,250 < x \le 87,850 \\ 17,891.25 + 0.28(x - 87,850) & \text{if } \ 87,850 < x \le 183,250 \\ 44,603.25 + 0.33(x - 183,250) & \text{if } \ 183,250 < x \le 398,350 \\ 115,586.25 + 0.35(x - 398,350) & \text{if } \ 398,350 < x \le 400,000 \\ 116,163.75 + 0.396(x - 400,000) & \text{if } \ x > 400,000 \end{matrix}$$

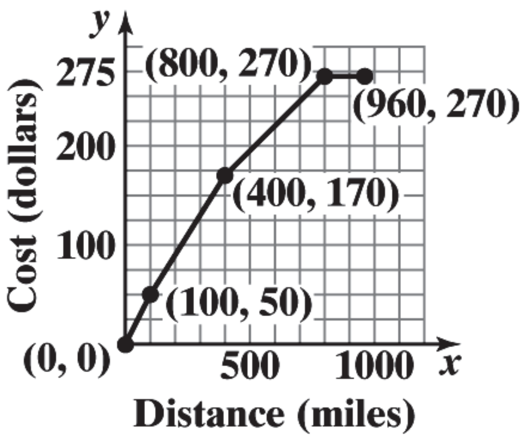
52. For Schedule $\mathrm{Y} - 1$:

$$f(x) = \begin{matrix} 0.10x & \text{if } \ 0 < x \le 17,850 \\ 1785.00 + 0.15(x - 17,850) & \text{if } \ 17,850 < x \le 72,500 \\ 9982.50 + 0.25(x - 72,500) & \text{if } \ 72,500 < x \le 146,400 \\ 28,457.50 + 0.28(x - 146,400) & \text{if } \ 146,400 < x \le 223,050 \\ 49,919.50 + 0.33(x - 223,050) & \text{if } \ 223,050 < x \le 398,350 \\ 107,768.50 + 0.35(x - 398,350) & \text{if } \ 398,350 < x \le 450,000 \\ 125,846 + 0.396(x - 450,000) & \text{if } \ x > 450,000 \end{matrix}$$

53. a. Let x represent the number of miles and C be the cost of transportation.

$$C(x) = \begin{cases} 0.50x & \text{if } 0 \le x \le 100 \\ 0.50(100) + 0.40(x - 100) & \text{if } 100 < x \le 400 \\ 0.50(100) + 0.40(300) + 0.25(x - 400) & \text{if } 400 < x \le 800 \\ 0.50(100) + 0.40(300) + 0.25(400) + 0(x - 800) & \text{if } 800 < x \le 960 \end{cases}$$

$$C(x)=\begin{cases}0.50x & \text{if } 0\le x\le 100\\ 10+0.40x & \text{if } 100<x\le 400\\ 70+0.25x & \text{if } 400<x\le 800\\ 270 & \text{if } 800<x\le 960\end{cases}$$

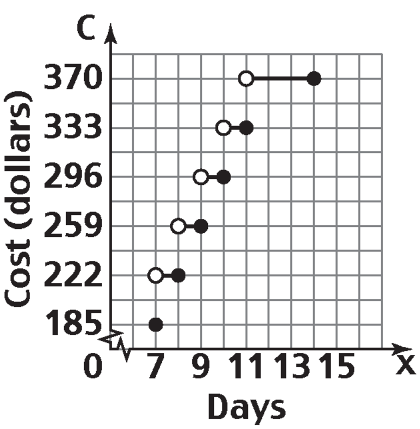


b. For hauls between 100 and 400 miles the cost is: $C(x)=10+0.40x$.

c. For hauls between 400 and 800 miles the cost is: $C(x)=70+0.25x$.

54. Let x = number of days car is used. The cost of renting is given by

$$C(x)=\begin{cases}185 & \text{if } x=7\\ 222 & \text{if } 7<x\le 8\\ 259 & \text{if } 8<x\le 9\\ 296 & \text{if } 9<x\le 10\\ 333 & \text{if } 10<x\le 11\\ 370 & \text{if } 11<x\le 14\end{cases}$$



55. a. Let s = the credit score of an individual who wishes to borrow \$300,000 with an 80% LTV ratio. The adverse market delivery charge is given by

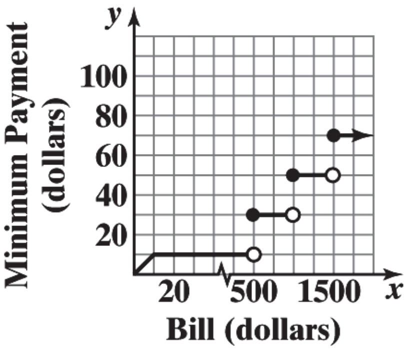
$$C(s)=\begin{cases}9000 & \text{if } s\le 659\\ 7500 & \text{if } 660\le s\le 679\\ 5250 & \text{if } 680\le s\le 699\\ 3000 & \text{if } 700\le s\le 719\\ 1500 & \text{if } 720\le s\le 739\\ 750 & \text{if } s\ge 740\end{cases}$$

b. 725 is between 720 and 739 so the charge would be \$1500.

c. 670 is between 660 and 679 so the charge would be \$7500.

56. Let x = the amount of the bill in dollars. The minimum payment due is given by

$$f(x)=\begin{cases}x & \text{if } 0\le x<10\\ 10 & \text{if } 10\le x<500\\ 30 & \text{if } 500\le x<1000\\ 50 & \text{if } 1000\le x<1500\\ 70 & \text{if } x\ge 1500\end{cases}$$



57. **a.** $W = 10°C$

b. $W = 33 - \dfrac{(10.45 + 10\sqrt{5} - 5)(33-10)}{22.04} \approx 4°C$

c. $W = 33 - \dfrac{(10.45 + 10\sqrt{15} - 15)(33-10)}{22.04} \approx -3°C$

d. $W = 33 - 1.5958(33-10) = -4°C$

e. When $0 \le v < 1.79$, the wind speed is so small that there is no effect on the temperature.

f. When the wind speed exceeds 20, the wind chill depends only on the air temperature.

58. **a.** $W = -10°C$

b. $W = 33 - \dfrac{\left(10.45 + 10\sqrt{5} - 5\right)\left(33-(-10)\right)}{22.04}$

$\approx -21°C$

c. $W = 33 - \dfrac{\left(10.45 + 10\sqrt{15} - 15\right)\left(33-(-10)\right)}{22.04}$

$\approx -34°C$

d. $W = 33 - 1.5958\left(33-(-10)\right) = -36°C$

59. Let x = the number of ounces and $C(x)$ = the postage due.

For $0 < x \le 1$: $C(x) = \$0.92$

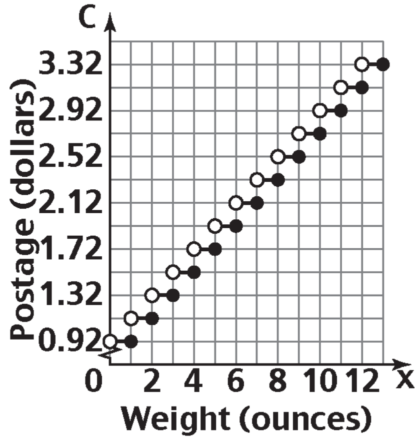
For $1 < x \le 2$: $C(x) = 0.92 + 0.20 = \$1.12$

For $2 < x \le 3$: $C(x) = 0.92 + 2(0.20) = \$1.32$

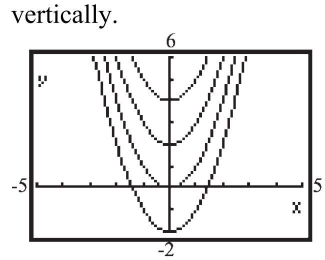
For $3 < x \le 4$: $C(x) = 0.92 + 3(0.20) = \$1.52$

$\vdots$

For $12 < x \le 13$: $C(x) = 0.92 + 12(0.20) = \$3.32$

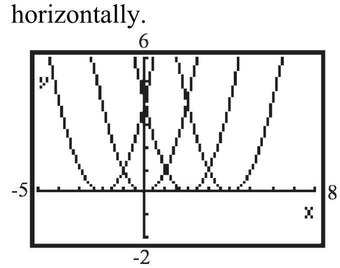


60. Each graph is that of $y = x^2$, but shifted vertically.



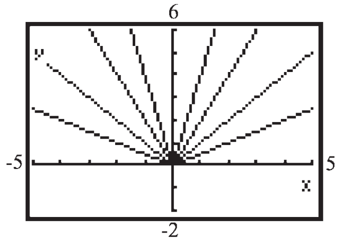
If $y = x^2 + k,\ k > 0$, the shift is up k units; if $y = x^2 - k,\ k > 0$, the shift is down k units. The graph of $y = x^2 - 4$ is the same as the graph of $y = x^2$, but shifted down 4 units. The graph of $y = x^2 + 5$ is the graph of $y = x^2$, but shifted up 5 units.

61. Each graph is that of $y = x^2$, but shifted horizontally.



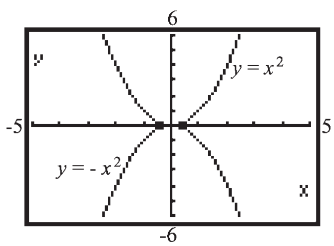
If $y = (x-k)^2,\ k > 0$, the shift is to the right k units; if $y = (x+k)^2,\ k > 0$, the shift is to the left k units. The graph of $y = (x+4)^2$ is the same as the graph of $y = x^2$, but shifted to the left 4 units. The graph of $y=(x-5)^2$ is the graph of $y=x^2$, but shifted to the right 5 units.

62. Each graph is that of $y=|x|$, but either compressed or stretched vertically.

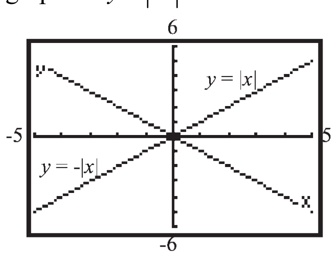


If $y=k|x|$ and $k>1$, the graph is stretched vertically; if $y=k|x|$ and $0<k<1$, the graph is compressed vertically. The graph of $y=\frac{1}{4}|x|$ is the same as the graph of $y=|x|$, but compressed vertically. The graph of $y=5|x|$ is the same as the graph of $y=|x|$, but stretched vertically.

63. The graph of $y=-x^2$ is the reflection of the graph of $y=x^2$ about the x-axis.

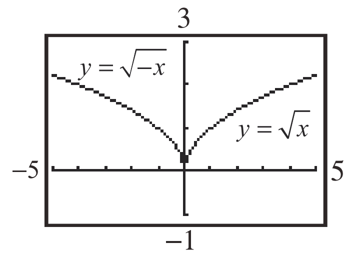


The graph of $y=-|x|$ is the reflection of the graph of $y=|x|$ about the x-axis.

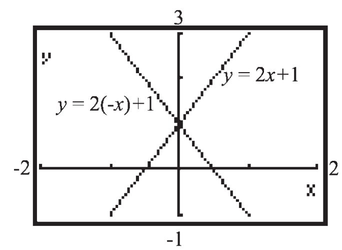


Multiplying a function by –1 causes the graph to be a reflection about the x-axis of the original function's graph.

64. The graph of $y=\sqrt{-x}$ is the reflection about the y-axis of the graph of $y=\sqrt{x}$.

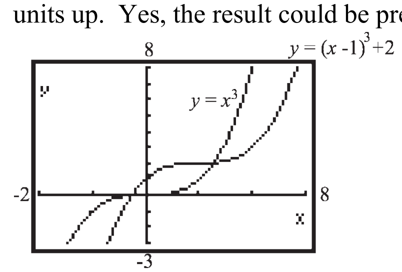


The same type of reflection occurs when graphing $y=2x+1$ and $y=2(-x)+1$.

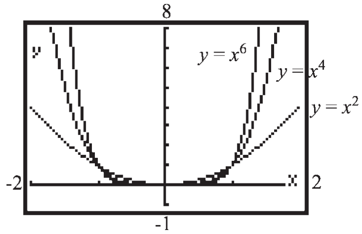


The graph of $y=f(-x)$ is the reflection about the y-axis of the graph of $y=f(x)$.

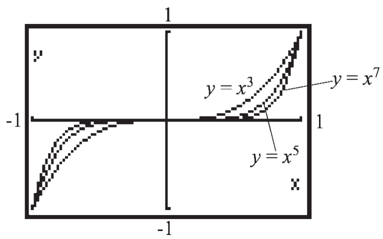
65. The graph of $y=(x-1)^3+2$ is a shifting of the graph of $y=x^3$ one unit to the right and two units up. Yes, the result could be predicted.



66. The graphs of $y=x^n$, n a positive even integer, are all U-shaped and open upward. All go through the points $(-1,1)$, $(0,0)$, and $(1,1)$. As n increases, the graph of the function is narrower for $|x|>1$ and flatter for $|x|<1$.



67. The graphs of $y=x^n$, n a positive odd integer, all have the same general shape. All go through the points $(-1,-1)$, $(0,0)$, and $(1,1)$. As n increases, the graph of the function increases at a greater rate for $|x|>1$ and is flatter around 0 for $|x|<1$.



68. $f(x)=\begin{cases}1 & \text{if } x \text{ is rational}\\ 0 & \text{if } x \text{ is irrational}\end{cases}$

Yes, it is a function.

Domain = $\{x \mid x \text{ is any real number}\}$ or $(-\infty,\infty)$

Range = $\{0, 1\}$ or $\{y \mid y=0 \text{ or } y=1\}$

y-intercept: $x=0 \Rightarrow x$ is rational $\Rightarrow y=1$
So the y-intercept is $y=1$.

x-intercept: $y=0 \Rightarrow x$ is irrational
So the graph has infinitely many x-intercepts, namely, there is an x-intercept at each irrational value of x.

$f(-x)=1=f(x)$ when x is rational;
$f(-x)=0=f(x)$ when x is irrational.
Thus, f is even.

The graph of f consists of 2 infinite clusters of distinct points, extending horizontally in both directions. One cluster is located 1 unit above the x-axis, and the other is located along the x-axis.

69. For $0<x<1$, the graph of $y=x^r$, r rational and $r>0$, flattens down toward the x-axis as r gets bigger. For $x>1$, the graph of $y=x^r$ increases at a greater rate as r gets bigger.

Section 1.5

1. horizontal; right

2. y

3. vertical; up

4. True; the graph of $y=-f(x)$ is the reflection about the x-axis of the graph of $y=f(x)$.

5. False; to obtain the graph of $y=f(x+2)-3$ you shift the graph of $y=f(x)$ to the *left* 2 units and down 3 units.

6. True

7. B

8. E

9. H

10. D

11. I

12. A

13. L

14. C

15. F

16. J

17. G

18. K

19. $y=(x-4)^3$

20. $y=(x+4)^3$

21. $y=x^3+4$

22. $y=x^3-4$

23. $y=(-x)^3=-x^3$

24. $y=-x^3$

25. $y=4x^3$

26. $y=\left(\frac{1}{4}x\right)^3=\frac{1}{64}x^3$

27. (1) $y=\sqrt{x}+2$
(2) $y=-\left(\sqrt{x}+2\right)$
(3) $y=-\left(\sqrt{-x}+2\right)=-\sqrt{-x}-2$

28. (1) $y=-\sqrt{x}$
(2) $y=-\sqrt{x-3}$
(3) $y=-\sqrt{x-3}-2$

29. (1) $y=-\sqrt{x}$
(2) $y=-\sqrt{x}+2$
(3) $y=-\sqrt{x+3}+2$

30. (1) $y=\sqrt{x}+2$
(2) $y=\sqrt{-x}+2$
(3) $y=\sqrt{-(x+3)}+2=\sqrt{-x-3}+2$

31. (c); To go from $y=f(x)$ to $y=-f(x)$ we reflect about the *x*-axis. This means we change the sign of the *y*-coordinate for each point on the graph of $y=f(x)$. Thus, the point $(3, 6)$ would become $(3,-6)$.

32. (d); To go from $y=f(x)$ to $y=f(-x)$, we reflect each point on the graph of $y=f(x)$ about the *y*-axis. This means we change the sign of the *x*-coordinate for each point on the graph of $y=f(x)$. Thus, the point $(3,6)$ would become $(-3,6)$.

33. (c); To go from $y=f(x)$ to $y=2f(x)$, we stretch vertically by a factor of 2. Multiply the *y*-coordinate of each point on the graph of $y=f(x)$ by 2. Thus, the point $(1,3)$ would become $(1,6)$.

34. (c); To go from $y=f(x)$ to $y=f(2x)$, we compress horizontally by a factor of 2. Divide the *x*-coordinate of each point on the graph of $y=f(x)$ by 2. Thus, the point $(4,2)$ would become $(2,2)$.

35. **a.** The graph of $y=f(x+2)$ is the same as the graph of $y=f(x)$, but shifted 2 units to the left. Therefore, the *x*-intercepts are -7 and 1.

b. The graph of $y=f(x-2)$ is the same as the graph of $y=f(x)$, but shifted 2 units to the right. Therefore, the *x*-intercepts are -3 and 5.

c. The graph of $y=4f(x)$ is the same as the graph of $y=f(x)$, but stretched vertically by a factor of 4. Therefore, the *x*-intercepts are still -5 and 3 since the *y*-coordinate of each is 0.

d. The graph of $y=f(-x)$ is the same as the graph of $y=f(x)$, but reflected about the y-axis. Therefore, the *x*-intercepts are 5 and -3.

36. **a.** The graph of $y=f(x+4)$ is the same as the graph of $y=f(x)$, but shifted 4 units to the left. Therefore, the *x*-intercepts are -12 and -3.

b. The graph of $y=f(x-3)$ is the same as the graph of $y=f(x)$, but shifted 3 units to the right. Therefore, the *x*-intercepts are -5 and 4.

c. The graph of $y=2f(x)$ is the same as the graph of $y=f(x)$, but stretched vertically by a factor of 2. Therefore, the *x*-intercepts are still -8 and 1 since the *y*-coordinate of each is 0.

d. The graph of $y=f(-x)$ is the same as the graph of $y=f(x)$, but reflected about the *y*-axis. Therefore, the *x*-intercepts are 8 and -1.

37. **a.** The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is increasing on the interval $(-3,3)$.

b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is increasing on the interval $(4,10)$.

c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x-axis. Therefore, we can say that the graph of $y = -f(x)$ must be *decreasing* on the interval $(-1,5)$.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, we can say that the graph of $y = f(-x)$ must be *decreasing* on the interval $(-5,1)$.

38. **a.** The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is decreasing on the interval $(-4,5)$.

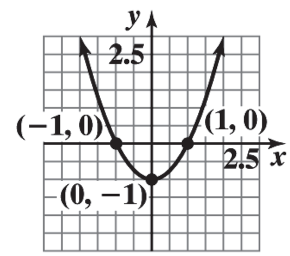
b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is decreasing on the interval $(3,12)$.

c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x-axis. Therefore, we can say that the graph of $y = -f(x)$ must be *increasing* on the interval $(-2,7)$.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, we can say that the graph of $y = f(-x)$ must be *increasing* on the interval $(-7,2)$.

39. $f(x) = x^2 - 1$

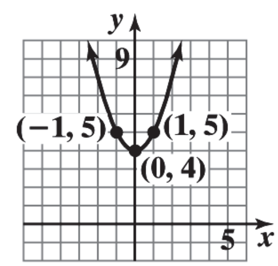
Using the graph of $y = x^2$, vertically shift downward 1 unit.



The domain is $(-\infty,\infty)$ and the range is $[-1,\infty)$.

40. $f(x) = x^2 + 4$

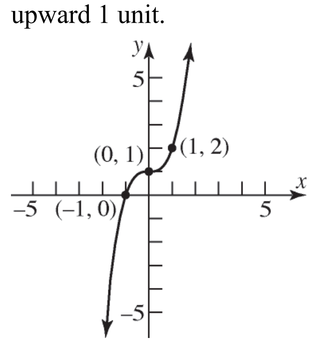
Using the graph of $y = x^2$, vertically shift upward 4 units.



The domain is $(-\infty,\infty)$ and the range is $[4,\infty)$.

41. $g(x) = x^3 + 1$

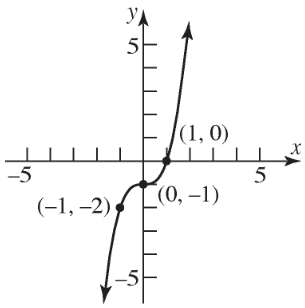
Using the graph of $y = x^3$, vertically shift upward 1 unit.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

42. $g(x) = x^3 - 1$

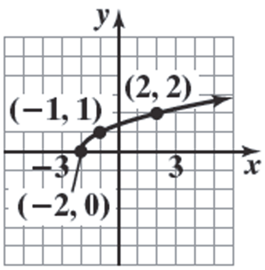
Using the graph of $y = x^3$, vertically shift downward 1 unit.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

43. $h(x) = \sqrt{x+2}$

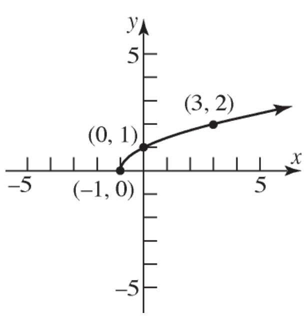
Using the graph of $y = \sqrt{x}$, horizontally shift to the left 2 units.



The domain is $-2, \infty)$ and the range is $[0, \infty)$.

44. $h(x) = \sqrt{x+1}$

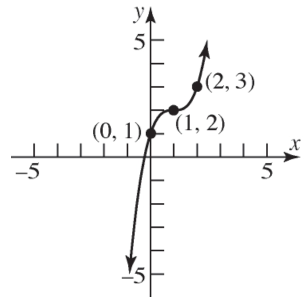
Using the graph of $y = \sqrt{x}$, horizontally shift to the left 1 unit.



The domain is $[-1, \infty)$ and the range is $[0, \infty)$.

45. $f(x) = (x-1)^3 + 2$

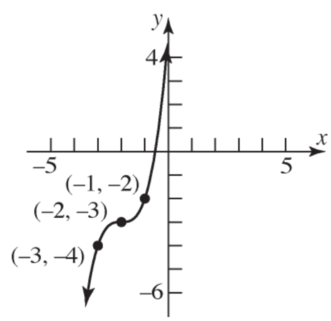
Using the graph of $y = x^3$, horizontally shift to the right 1 unit $\left[y = (x-1)^3\right]$, then vertically shift up 2 units $\left[y = (x-1)^3 + 2\right]$.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

46. $f(x) = (x+2)^3 - 3$

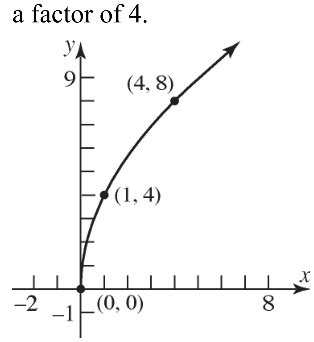
Using the graph of $y = x^3$, horizontally shift to the left 2 units $\left[y = (x+2)^3\right]$, then vertically shift down 3 units $\left[y = (x+2)^3 - 3\right]$.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

47. $g(x) = 4\sqrt{x}$

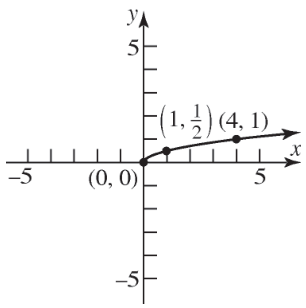
Using the graph of $y = \sqrt{x}$, vertically stretch by a factor of 4.



The domain is $[0, \infty)$ and the range is $[0, \infty)$.

48. $g(x)=\frac{1}{2}\sqrt{x}$

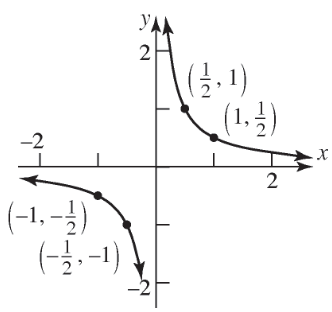
Using the graph of $y=\sqrt{x}$, vertically compress by a factor of $\frac{1}{2}$.



The domain is $\left[0,\infty\right)$ and the range is $\left[0,\infty\right)$.

49. $h(x)=\frac{1}{2x}=\left(\frac{1}{2}\right)\left(\frac{1}{x}\right)$

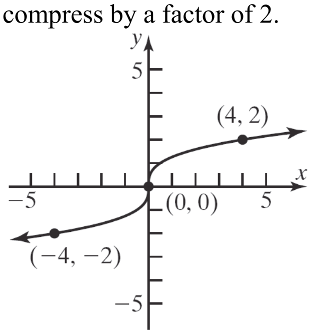
Using the graph of $y=\frac{1}{x}$, vertically compress by a factor of $\frac{1}{2}$.



The domain is $\left(-\infty,0\right)\cup\left(0,\infty\right)$ and the range is $\left(-\infty,0\right)\cup\left(0,\infty\right)$.

50. $h(x)=\sqrt[3]{2x}$

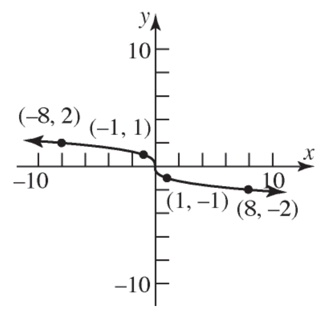
Using the graph of $y=\sqrt[3]{x}$, horizontally compress by a factor of 2.



The domain is $\left(-\infty,\infty\right)$ and the range is $\left(-\infty,\infty\right)$.

51. $f(x)=-\sqrt[3]{x}$

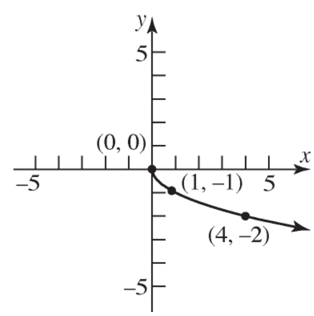
Using the graph of $y=\sqrt[3]{x}$, reflect the graph about the x-axis.



The domain is $\left(-\infty,\infty\right)$ and the range is $\left(-\infty,\infty\right)$.

52. $f(x)=-\sqrt{x}$

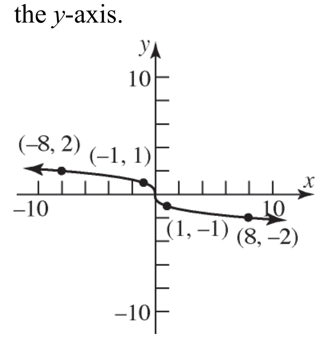
Using the graph of $y=\sqrt{x}$, reflect the graph about the x-axis.



The domain is $\left[0,\infty\right)$ and the range is $\left(-\infty,0\right]$.

53. $g(x)=\sqrt[3]{-x}$

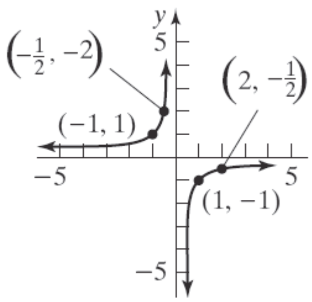
Using the graph of $y=\sqrt[3]{x}$, reflect the graph about the y-axis.



The domain is $\left(-\infty,\infty\right)$ and the range is $\left(-\infty,\infty\right)$.

54. $g(x) = \dfrac{1}{-x}$

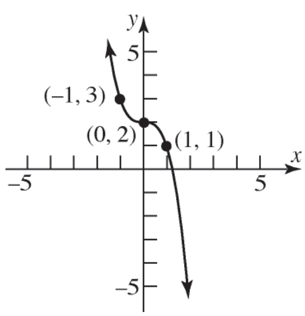
Using the graph of $y = \dfrac{1}{x}$, reflect the graph about the y-axis.



The domain is $(-\infty, 0) \cup (0, \infty)$ and the range is $(-\infty, 0) \cup (0, \infty)$.

55. $h(x) = -x^3 + 2$

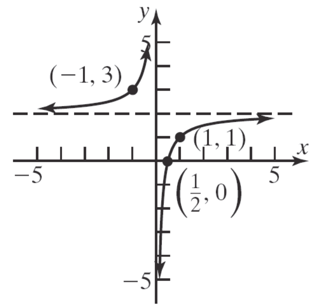
Using the graph of $y = x^3$, reflect the graph about the x-axis $\left[y = -x^3\right]$, then shift vertically upward 2 units $\left[y = -x^3 + 2\right]$.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

56. $h(x) = \dfrac{1}{-x} + 2$

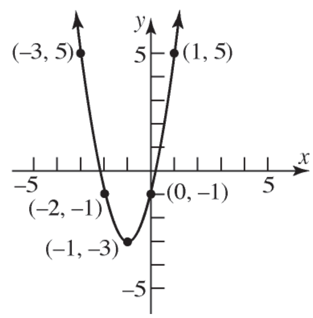
Using the graph of $y = \dfrac{1}{x}$, reflect the graph about the y-axis $\left[y = \dfrac{1}{-x}\right]$, then shift vertically upward 2 units $\left[y = \dfrac{1}{-x} + 2\right]$.



The domain is $(-\infty, 0) \cup (0, \infty)$ and the range is $(-\infty, 2) \cup (2, \infty)$.

57. $f(x) = 2(x+1)^2 - 3$

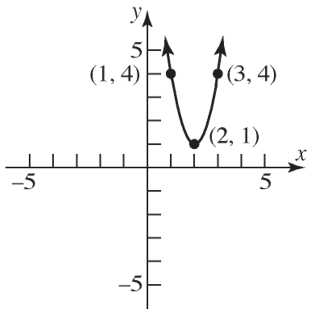
Using the graph of $y = x^2$, horizontally shift to the left 1 unit $\left[y = (x+1)^2\right]$, vertically stretch by a factor of 2 $\left[y = 2(x+1)^2\right]$, and then vertically shift downward 3 units $\left[y = 2(x+1)^2 - 3\right]$.



The domain is $(-\infty, \infty)$ and the range is $[-3, \infty)$.

58. $f(x) = 3(x-2)^2 + 1$

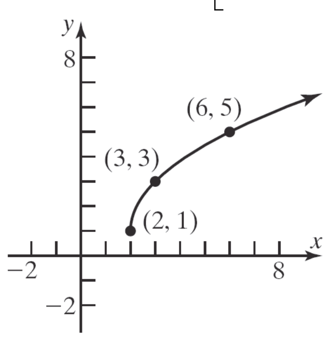
Using the graph of $y = x^2$, horizontally shift to the right 2 units $\left[y = (x-2)^2\right]$, vertically stretch by a factor of 3 $\left[y = 3(x-2)^2\right]$, and then vertically shift upward 1 unit $\left[y = 3(x-2)^2 + 1\right]$.



The domain is $(-\infty, \infty)$ and the range is $[1, \infty)$.

59. $g(x) = 2\sqrt{x-2} + 1$

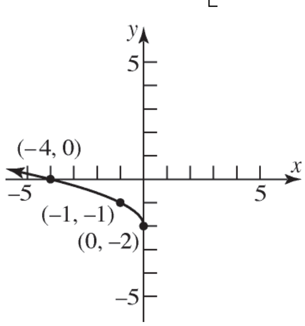
Using the graph of $y = \sqrt{x}$, horizontally shift to the right 2 units $\left[y = \sqrt{x-2}\right]$, vertically stretch by a factor of 2 $\left[y = 2\sqrt{x-2}\right]$, and vertically shift upward 1 unit $\left[y = 2\sqrt{x-2} + 1\right]$.



The domain is $[2, \infty)$ and the range is $[1, \infty)$.

61. $h(x) = \sqrt{-x} - 2$

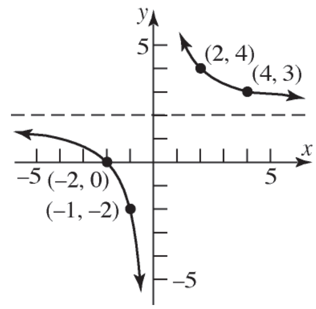
Using the graph of $y = \sqrt{x}$, reflect the graph about the y-axis $\left[y = \sqrt{-x}\right]$ and vertically shift downward 2 units $\left[y = \sqrt{-x} - 2\right]$.



The domain is $(-\infty, 0]$ and the range is $[-2, \infty)$.

62. $h(x) = \dfrac{4}{x} + 2 = 4\left(\dfrac{1}{x}\right) + 2$

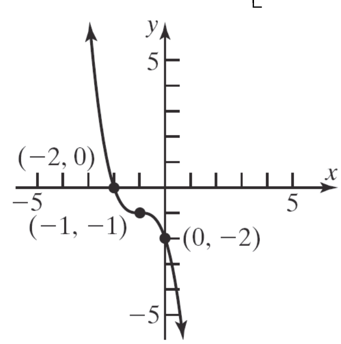
Stretch the graph of $y = \dfrac{1}{x}$ vertically by a factor of 4 $\left[y = 4 \cdot \dfrac{1}{x} = \dfrac{4}{x}\right]$ and vertically shift upward 2 units $\left[y = \dfrac{4}{x} + 2\right]$.



The domain is $(-\infty, 0) \cup (0, \infty)$ and the range is $(-\infty, 2) \cup (2, \infty)$.

63. $f(x) = -(x+1)^3 - 1$

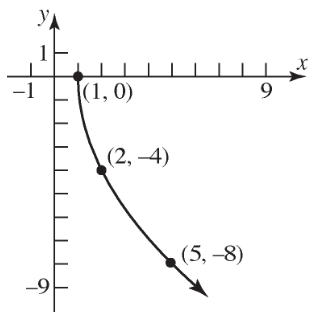
Using the graph of $y = x^3$, horizontally shift to the left 1 unit $\left[y = (x+1)^3\right]$, reflect the graph about the x-axis $\left[y = -(x+1)^3\right]$, and vertically shift downward 1 unit $\left[y = -(x+1)^3 - 1\right]$.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

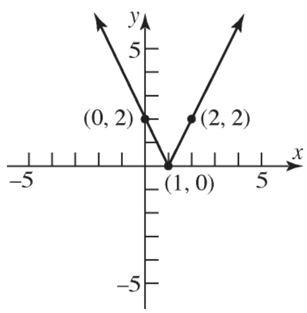
64. $f(x) = -4\sqrt{x-1}$

Using the graph of $y = \sqrt{x}$, horizontally shift to the right 1 unit $\left[y = \sqrt{x-1}\right]$, reflect the graph about the x-axis $\left[y = -\sqrt{x-1} \right]$, and stretch vertically by a factor of 4 $\left[y = -4\sqrt{x-1} \right]$.



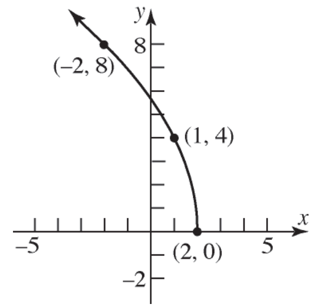
The domain is $\left[1, \infty\right)$ and the range is $\left(-\infty, 0\right]$.

65. $g(x) = 2|1-x| = 2|-(-1+x)| = 2|x-1|$
Using the graph of $y = |x|$, horizontally shift to the right 1 unit $\left[y = |x-1| \right]$, and vertically stretch by a factor or 2 $\left[y = 2|x-1| \right]$.



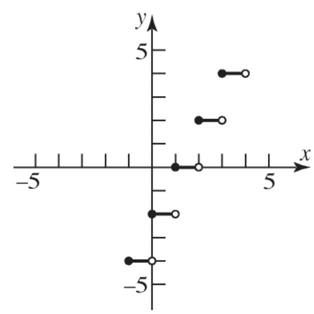
The domain is $\left(-\infty, \infty\right)$ and the range is $\left[0, \infty\right)$.

66. $g(x) = 4\sqrt{2-x} = 4\sqrt{-(x-2)}$
Using the graph of $y = \sqrt{x}$, reflect the graph about the y-axis $\left[y = \sqrt{-x} \right]$, horizontally shift to the right 2 units $\left[y = \sqrt{-(x-2)} \right]$, and vertically stretch by a factor of 4 $\left[y = 4\sqrt{-(x-2)} \right]$.



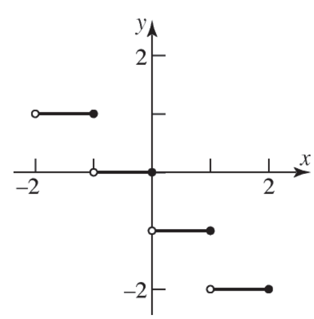
The domain is $\left(-\infty, 2\right]$ and the range is $\left[0, \infty\right)$.

67. $h(x) = 2\,\text{int}(x-1)$
Using the graph of $y = \text{int}(x)$, horizontally shift to the right 1 unit $\left[y = \text{int}(x-1) \right]$, and vertically stretch by a factor of 2 $\left[y = 2\,\text{int}(x-1) \right]$.



The domain is $\left(-\infty, \infty\right)$ and the range is $\left\{ y \mid y \text{ is an even integer} \right\}$.

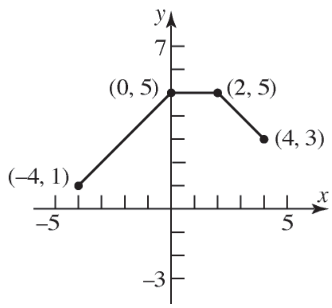
68. $h(x) = \text{int}(-x)$
Reflect the graph of $y = \text{int}(x)$ about the y-axis.



The domain is $\left(-\infty, \infty\right)$ and the range is $\left\{ y \mid y \text{ is an integer} \right\}$.

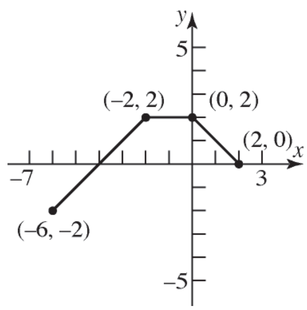
69. a. $F(x) = f(x) + 3$

Shift up 3 units.



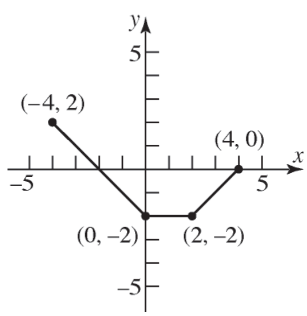
b. $G(x) = f(x+2)$

Shift left 2 units.



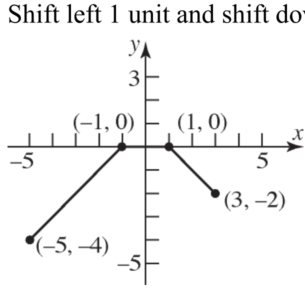
c. $P(x) = -f(x)$

Reflect about the x-axis.



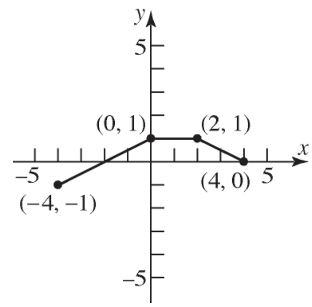
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



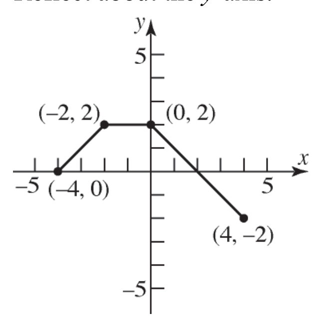
e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



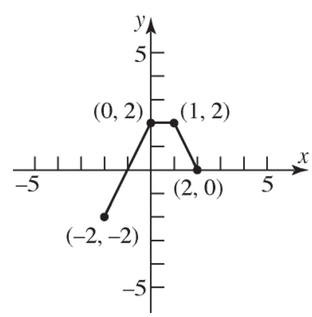
f. $g(x) = f(-x)$

Reflect about the y-axis.



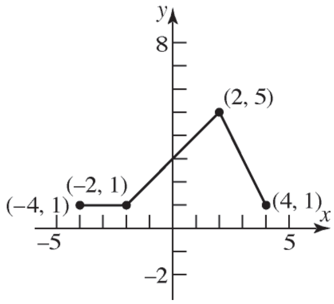
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



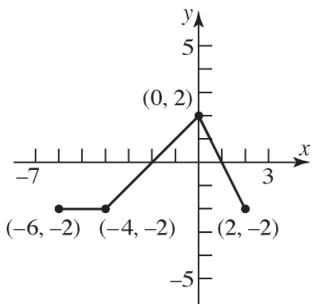
70. **a.** $F(x) = f(x) + 3$

Shift up 3 units.



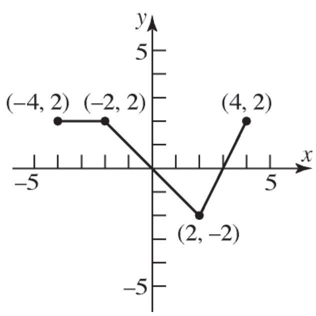
b. $G(x) = f(x+2)$

Shift left 2 units.



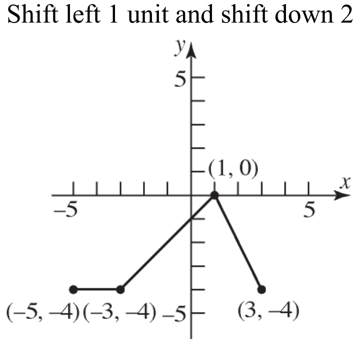
c. $P(x) = -f(x)$

Reflect about the x-axis.



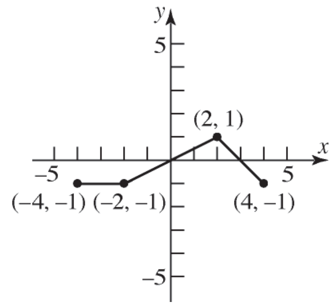
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



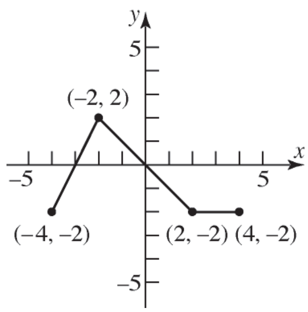
e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



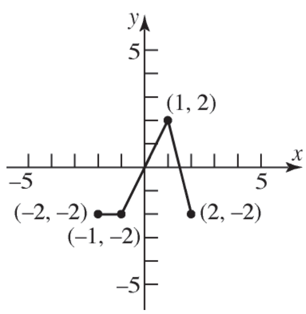
f. $g(x) = f(-x)$

Reflect about the y-axis.



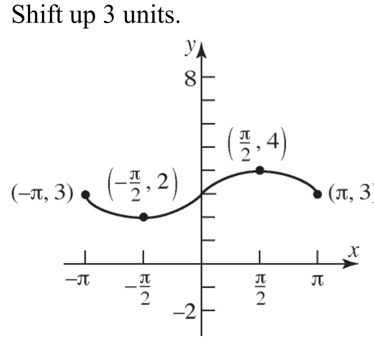
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.

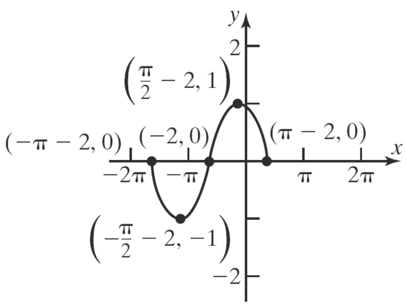


71. **a.** $F(x) = f(x) + 3$

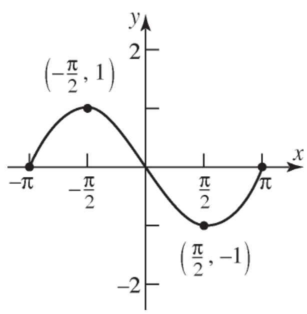
Shift up 3 units.



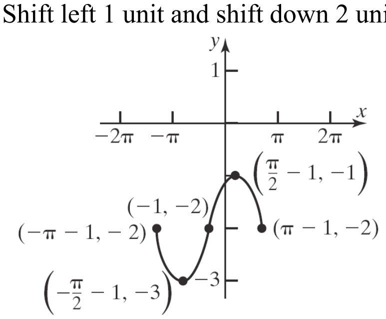
b. $G(x) = f(x+2)$
Shift left 2 units.



c. $P(x) = -f(x)$
Reflect about the x-axis.

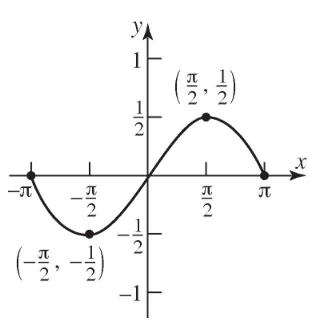


d. $H(x) = f(x+1) - 2$
Shift left 1 unit and shift down 2 units.

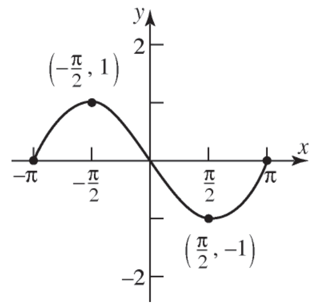


e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.

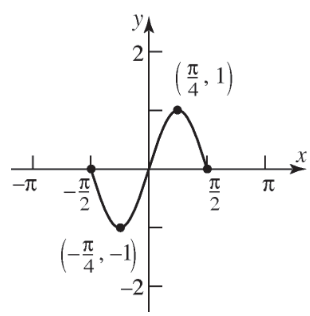


f. $g(x) = f(-x)$
Reflect about the y-axis.

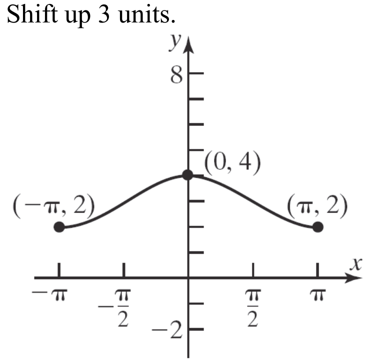


g. $h(x) = f(2x)$

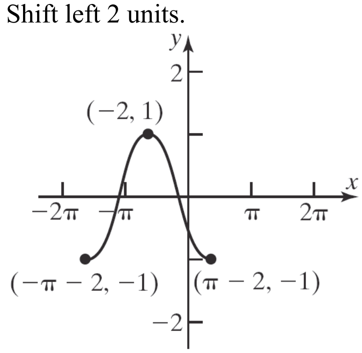
Compress horizontally by a factor of $\frac{1}{2}$.



72. a. $F(x) = f(x) + 3$
Shift up 3 units.

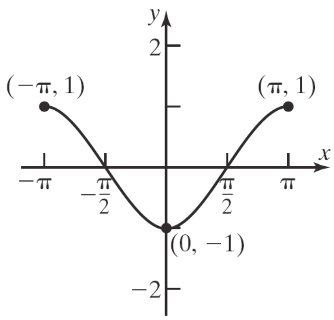


b. $G(x) = f(x+2)$
Shift left 2 units.



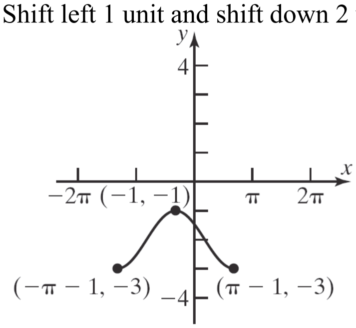
c. $P(x) = -f(x)$

Reflect about the x-axis.



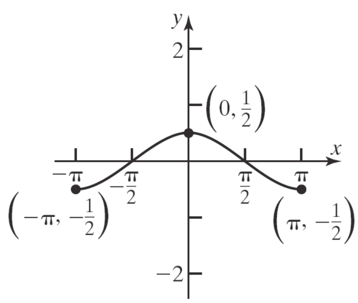
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



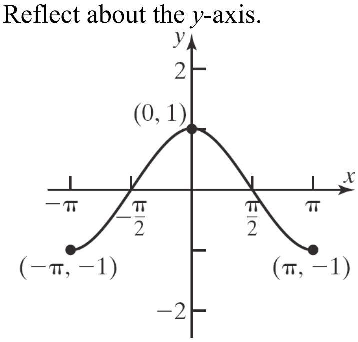
e. $Q(x) = \dfrac{1}{2}f(x)$

Compress vertically by a factor of $\dfrac{1}{2}$.



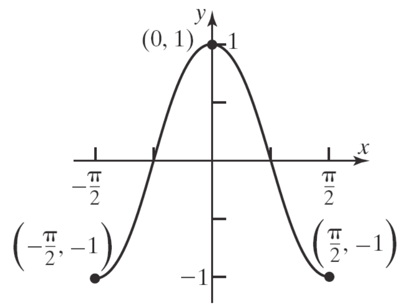
f. $g(x) = f(-x)$

Reflect about the y-axis.



g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.

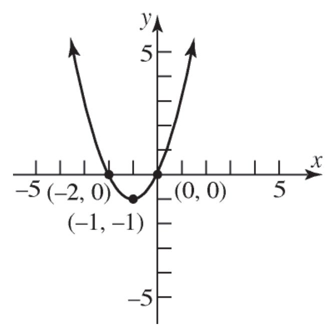


73. $f(x) = x^2 + 2x$

$f(x) = (x^2 + 2x + 1) - 1$

$f(x) = (x+1)^2 - 1$

Using $f(x) = x^2$, shift left 1 unit and shift down 1 unit.

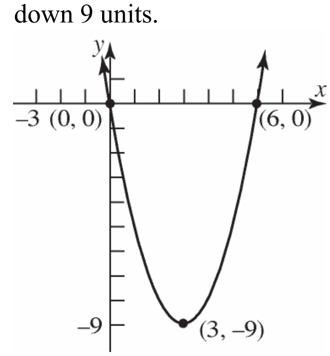


74. $f(x) = x^2 - 6x$

$f(x) = (x^2 - 6x + 9) - 9$

$f(x) = (x-3)^2 - 9$

Using $f(x) = x^2$, shift right 3 units and shift down 9 units.

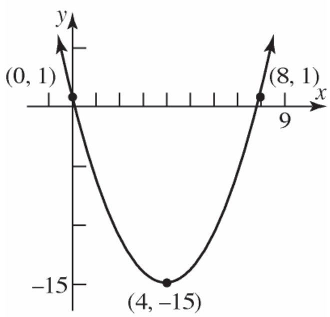


75. $f(x)=x^2-8x+1$

$f(x)=\left(x^2-8x+16\right)+1-16$

$f(x)=\left(x-4\right)^2-15$

Using $f(x)=x^2$, shift right 4 units and shift down 15 units.

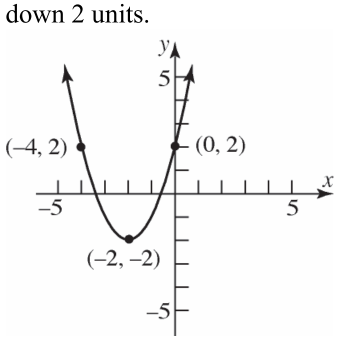


76. $f(x)=x^2+4x+2$

$f(x)=\left(x^2+4x+4\right)+2-4$

$f(x)=\left(x+2\right)^2-2$

Using $f(x)=x^2$, shift left 2 units and shift down 2 units.

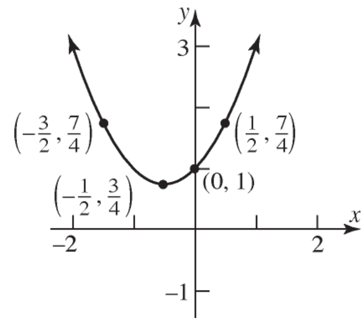


77. $f(x)=x^2+x+1$

$f(x)=\left(x^2+x+\frac{1}{4}\right)+1-\frac{1}{4}$

$f(x)=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$

Using $f(x)=x^2$, shift left $\frac{1}{2}$ unit and shift up $\frac{3}{4}$ unit.

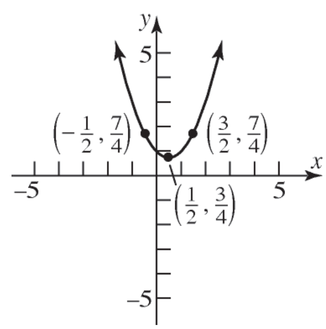


78. $f(x)=x^2-x+1$

$f(x)=\left(x^2-x+\frac{1}{4}\right)+1-\frac{1}{4}$

$f(x)=\left(x-\frac{1}{2}\right)^2+\frac{3}{4}$

Using $f(x)=x^2$, shift right $\frac{1}{2}$ unit and shift up $\frac{3}{4}$ unit.



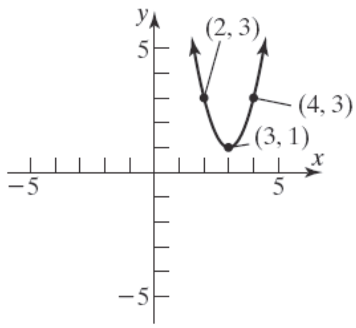
79. $f\left(x\right)=2x^2-12x+19$

$=2\left(x^2-6x\right)+19$

$=2\left(x^2-6x+9\right)+19-18$

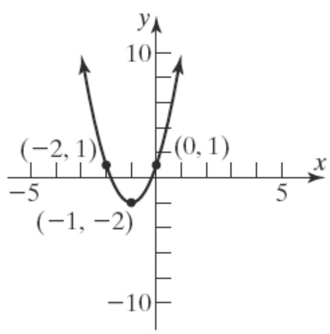
$=2\left(x-3\right)^2+1$

Using $f\left(x\right)=x^2$, shift right 3 units, vertically stretch by a factor of 2, and then shift up 1 unit.



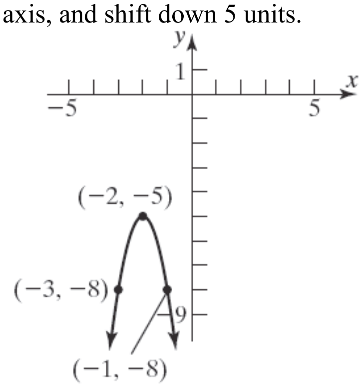
80. $f(x) = 3x^2 + 6x + 1$
$= 3(x^2 + 2x) + 1$
$= 3(x^2 + 2x + 1) + 1 - 3$
$= 3(x+1)^2 - 2$

Using $f(x) = x^2$, shift left 1 unit, vertically stretch by a factor of 3, and shift down 2 units.



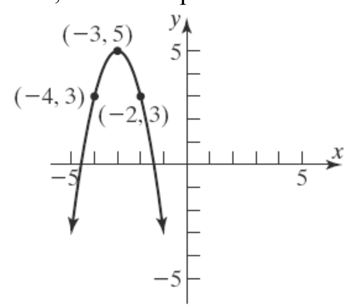
81. $f(x) = -3x^2 - 12x - 17$
$= -3(x^2 + 4x) - 17$
$= -3(x^2 + 4x + 4) - 17 + 12$
$= -3(x+2)^2 - 5$

Using $f(x) = x^2$, shift left 2 units, stretch vertically by a factor of 3, reflect about the x-axis, and shift down 5 units.



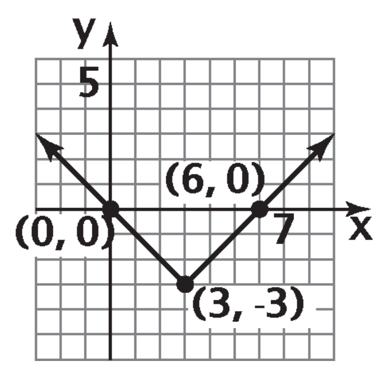
82. $f(x) = -2x^2 - 12x - 13$
$= -2(x^2 + 6x) - 13$
$= -2(x^2 + 6x + 9) - 13 + 18$
$= -2(x+3)^2 + 5$

Using $f(x) = x^2$, shift left 3 units, stretch vertically by a factor of 2, reflect about the x-axis, and shift up 5 units.



83. **a.** $f(x) = |x-3| - 3$

Using the graph of $y = |x|$, horizontally shift to the right 3 units $y = |x-3|$ and vertically shift downward 3 units $y = |x-3| - 3$.

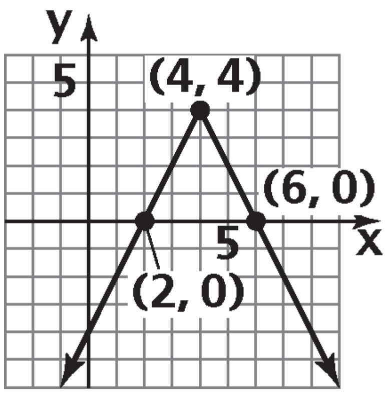


b. $A = \frac{1}{2}bh$
$= \frac{1}{2}(6)(3) = 9$

The area is 9 square units.

84. **a.** $f(x) = -2|x-4| + 4$

Using the graph of $y = |x|$, horizontally shift to the right 4 units $y = |x-4|$, vertically stretch by a factor of 2 and flip on the x-axis $y = -2|x-4|$, and vertically shift upward 4 units $y = -2|x-4|+4$.



b. $A = \frac{1}{2}bh$

$= \frac{1}{2}(4)(4) = 8$

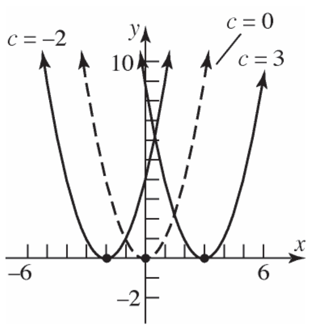
The area is 8 square units.

85. $y = (x-c)^2$

If $c = 0$, $y = x^2$.

If $c = 3$, $y = (x-3)^2$; shift right 3 units.

If $c = -2$, $y = (x+2)^2$; shift left 2 units.

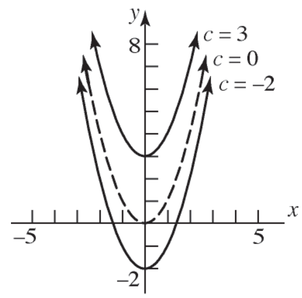


86. $y = x^2 + c$

If $c = 0$, $y = x^2$.

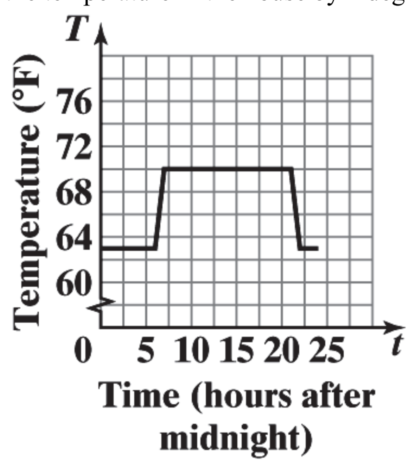
If $c = 3$, $y = x^2 + 3$; shift up 3 units.

If $c = -2$, $y = x^2 - 2$; shift down 2 units.

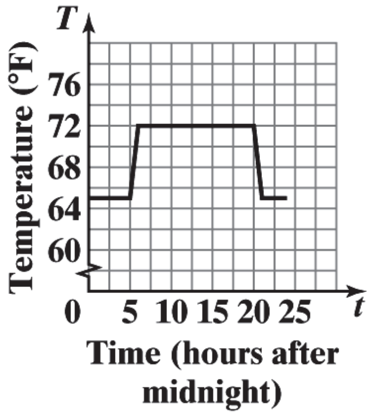


87. a. From the graph, the thermostat is set at 72°F during the daytime hours. The thermostat appears to be set at 65°F overnight.

b. To graph $y = T(t) - 2$, the graph of $T(t)$ is shifted down 2 units. This change will lower the temperature in the house by 2 degrees.



c. To graph $y = T(t+1)$, the graph of $T(t)$ should be shifted left one unit. This change will cause the program to switch between the daytime temperature and overnight temperature one hour sooner. The home will begin warming up at 5am instead of 6am and will begin cooling down at 8pm instead of 9pm.



88. a. $R(0) = 28.6(0)^2 + 300(0) + 4843 = 4843$

The estimated worldwide music revenue for 2012 is \$4843 million.

$R(3) = 28.6(3)^2 + 300(3) + 4843$
$= 6000.4$

The estimated worldwide music revenue for 2015 is \$6000.4 million.

$R(5) = 28.6(5)^2 + 300(5) + 4843$
$= 7058$

The estimated worldwide music revenue for 2017 is \$7058 million.

b. $r(x) = R(x-2)$

$$\begin{aligned} &= 28.6(x-2)^2 + 300(x-2) + 4843 \\ &= 28.6(x^2 - 4x + 4) + 300(x-2) \\ &\quad + 4843 \\ &= 28.6x^2 - 114.4x + 114.4 + 300x \\ &\quad - 600 + 4843 \\ &= 28.6x^2 + 185.6x + 4357.4 \end{aligned}$$

c. The graph of $r(x)$ is the graph of $R(x)$ shifted 2 units to the left. Thus, $r(x)$ represents the estimated worldwide music revenue, x years after 2010.

$r(2) = 28.6(2)^2 + 185.6(2) + 4357.4 = 4843$

The estimated worldwide music revenue for 2012 is \$4843 million.

$r(5) = 28.6(5)^2 + 185.6(5) + 4357.4$
$= 6000.4$

The estimated worldwide music revenue for 2015 is \$6000.4 million.

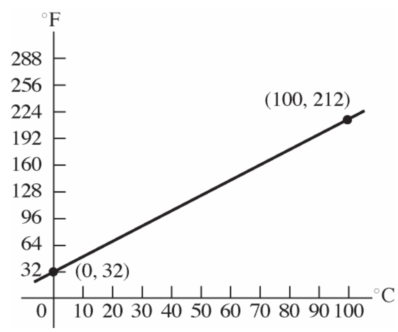
$r(7) = 28.6(7)^2 + 185.6(7) + 4357.4$
$= 7058$

The estimated worldwide music revenue for 2017 is \$7058 million.

d. In $r(x)$, x represents the number of years after 2000 (see the previous part).

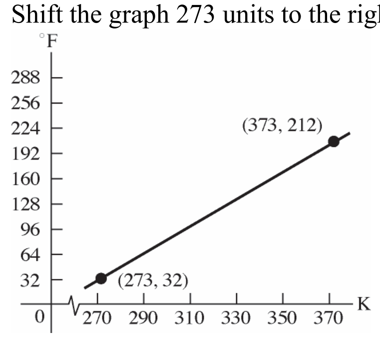
e. Answers will vary. One advantage might be that it is easier to determine what value should be substituted for x when using $r(x)$ instead of $R(x)$ to estimate worldwide music revenue.

89. $F = \frac{9}{5}C + 32$

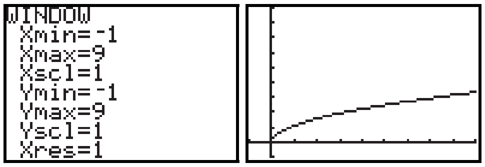


$F = \frac{9}{5}(K - 273) + 32$

Shift the graph 273 units to the right.

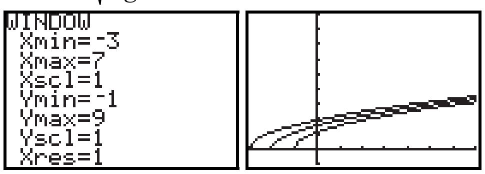


90. **a.** $T = 2\pi\sqrt{\dfrac{l}{g}}$



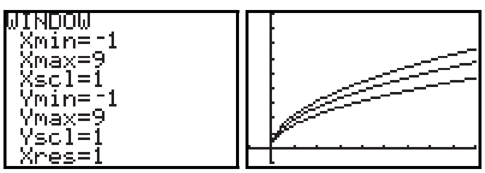
b. $T_1 = 2\pi\sqrt{\dfrac{l+1}{g}}$; $T_2 = 2\pi\sqrt{\dfrac{l+2}{g}}$;

$T_3 = 2\pi\sqrt{\dfrac{l+3}{g}}$



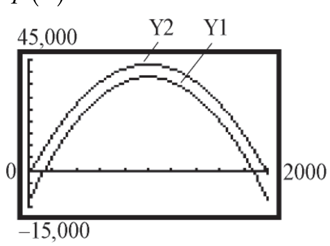
c. As the length of the pendulum increases, the period increases.

d. $T_1 = 2\pi\sqrt{\dfrac{2l}{g}}$; $T_2 = 2\pi\sqrt{\dfrac{3l}{g}}$; $T_3 = 2\pi\sqrt{\dfrac{4l}{g}}$



e. If the length of the pendulum is multiplied by k, the period is multiplied by $\sqrt{k}$.

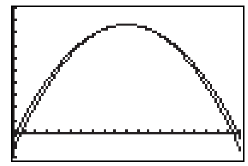
91. **a.** $p(x) = -0.05x^2 + 100x - 2000$



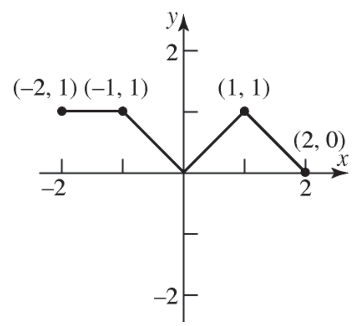
b. Select the 10% tax since the profits are higher.

c. The graph of Y1 is obtained by shifting the graph of $p(x)$ vertically down 10,000 units. The graph of Y2 is obtained by multiplying the *y*-coordinate of the graph of $p(x)$ by 0.9. Thus, Y2 is the graph of $p(x)$ vertically compressed by a factor of 0.9.

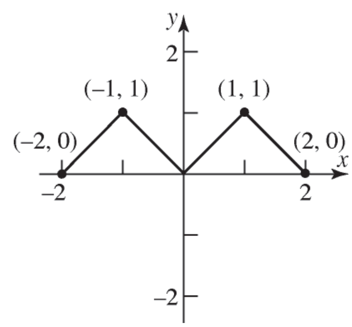
d. Select the 10% tax since the graph of $Y1 = 0.9p(x) \geq Y2 = -0.05x^2 + 100x - 6800$ for all x in the domain.



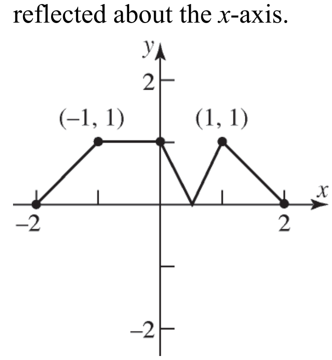
92. **a.** $y = |f(x)|$



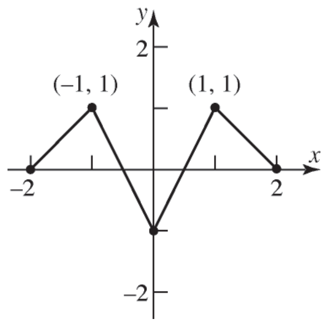
b. $y = f(|x|)$



93. **a.** To graph $y = |f(x)|$, the part of the graph for *f* that lies in quadrants III or IV is reflected about the *x*-axis.



b. To graph $y = f(|x|)$, the part of the graph for *f* that lies in quadrants II or III is replaced by the reflection of the part in quadrants I and IV reflected about the *y*-axis.



94. a. The graph of $y = f(x+3)-5$ is the graph of $y = f(x)$ but shifted left 3 units and down 5 units. Thus, the point $(1,3)$ becomes the point $(-2,-2)$.

b. The graph of $y = -2f(x-2)+1$ is the graph of $y = f(x)$ but shifted right 2 units, stretched vertically by a factor of 2, reflected about the *x*-axis, and shifted up 1 unit. Thus, the point $(1,3)$ becomes the point $(3,-5)$.

c. The graph of $y = f(2x+3)$ is the graph of $y = f(x)$ but shifted left 3 units and horizontally compressed by a factor of 2. Thus, the point $(1,3)$ becomes the point $(-1,3)$.

95. a. The graph of $y = g(x+1)-3$ is the graph of $y = g(x)$ but shifted left 1 unit and down 3 units. Thus, the point $(-3,5)$ becomes the point $(-4,2)$.

b. The graph of $y = -3g(x-4)+3$ is the graph of $y = g(x)$ but shifted right 4 units, stretched vertically by a factor of 3, reflected about the *x*-axis, and shifted up 3 units. Thus, the point $(-3,5)$ becomes the point $(1,-12)$.

c. The graph of $y = g(3x+9)$ is the graph of $y = f(x)$ but shifted left 9 units and horizontally compressed by a factor of 3. Thus, the point $(-3,5)$ becomes the point $(-4,5)$.

96. The graph of $y = 4f(x)$ is a vertical stretch of the graph of *f* by a factor of 4, while the graph of $y = f(4x)$ is a horizontal compression of the graph of *f* by a factor of $\frac{1}{4}$.

97. The graph of $y = f(x)-2$ will shift the graph of $y = f(x)$ down by 2 units. The graph of $y = f(x-2)$ will shift the graph of $y = f(x)$ to the right by 2 units.

98. The graph of $y = \sqrt{-x}$ is the graph of $y = \sqrt{x}$ but reflected about the *y*-axis. Therefore, our region is simply rotated about the *y*-axis and does not change shape. Instead of the region being bounded on the right by $x = 4$, it is bounded on the left by $x = -4$. Thus, the area of the second region would also be $\dfrac{16}{3}$ square units.

99. The range of $f(x) = x^2$ is $0,\infty)$. The graph of $g(x) = f(x)+k$ is the graph of f shifted up k units if $k > 0$ and down $|k|$ units if $k < 0$, so the range of g is $k,\infty)$.

100. The domain of $g(x) = \sqrt{x}$ is $0,\infty)$. The graph of $g(x-k)$ is the graph of g shifted k units to the right, so the domain of $g(x-k)$ is $k,\infty)$.

Section 1.6

1. a. The distance d from P to the origin is $d=\sqrt{x^2+y^2}$. Since P is a point on the graph of $y=x^2-8$, we have:

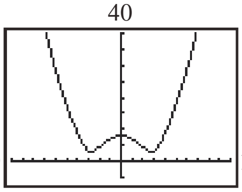
$$d(x)=\sqrt{x^2+(x^2-8)^2}=\sqrt{x^4-15x^2+64}$$

b. $d(0)=\sqrt{0^4-15(0)^2+64}=\sqrt{64}=8$

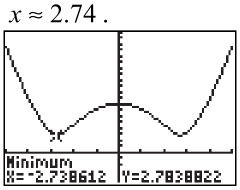
c. $d(1)=\sqrt{(1)^4-15(1)^2+64}$

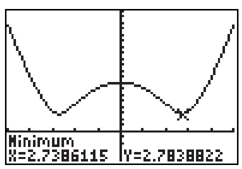
$=\sqrt{1-15+64}=\sqrt{50}=5\sqrt{2}\approx 7.07$

d.



e. d is smallest when $x\approx -2.74$ or when $x\approx 2.74$.





2. a. The distance d from P to $(0,-1)$ is $d=\sqrt{x^2+(y+1)^2}$. Since P is a point on the graph of $y=x^2-8$, we have:

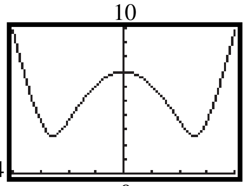
$$d(x)=\sqrt{x^2+(x^2-8+1)^2}$$

$$=\sqrt{x^2+\left(x^2-7\right)^2}=\sqrt{x^4-13x^2+49}$$

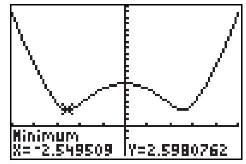
b. $d(0)=\sqrt{0^4-13(0)^2+49}=\sqrt{49}=7$

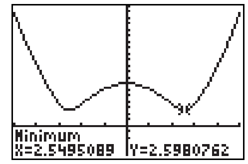
c. $d(-1)=\sqrt{(-1)^4-13(-1)^2+49}=\sqrt{37}\approx 6.08$

d.



e. d is smallest when $x\approx -2.55$ or when $x\approx 2.55$.



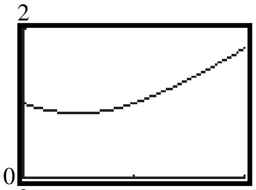


3. a. The distance d from P to the point $(1, 0)$ is $d=\sqrt{(x-1)^2+y^2}$. Since P is a point on the graph of $y=\sqrt{x}$, we have:

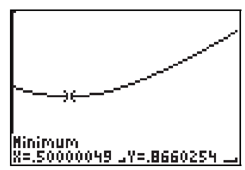
$$d(x)=\sqrt{(x-1)^2+\left(\sqrt{x}\right)^2}=\sqrt{x^2-x+1}$$

where $x\geq 0$.

b.



c. d is smallest when $x=\frac{1}{2}$.

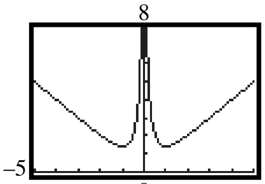


4. a. The distance d from P to the origin is $d=\sqrt{x^2+y^2}$. Since P is a point on the graph of $y=\dfrac{1}{x}$, we have:

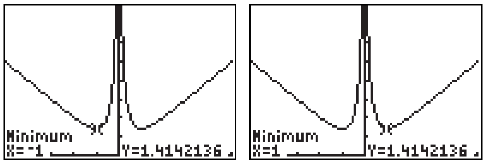
$$d(x)=\sqrt{x^2+\left(\frac{1}{x}\right)^2}=\sqrt{x^2+\frac{1}{x^2}}=\sqrt{\frac{x^4+1}{x^2}}$$

$$=\frac{\sqrt{x^2+1}}{|x|}$$

b.



c. d is smallest when $x = -1$ or $x = 1$.



5. By definition, a triangle has area $A = \frac{1}{2}b\,h$, b = base, h = height. From the figure, we know that $b = x$ and $h = y$. Expressing the area of the triangle as a function of x, we have:
$A(x) = \frac{1}{2}xy = \frac{1}{2}x\left(x^3\right) = \frac{1}{2}x^4$.

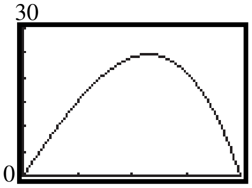
6. By definition, a triangle has area $A = \frac{1}{2}b\,h$, b=base, h = height. Because one vertex of the triangle is at the origin and the other is on the x-axis, we know that $b = x$ and $h = y$. Expressing the area of the triangle as a function of x, we have:
$A(x) = \frac{1}{2}xy = \frac{1}{2}x\left(9 - x^2\right) = \frac{9}{2}x - \frac{1}{2}x^3$.

7. a. $A(x) = xy = x\left(16 - x^2\right) = -x^3 + 16x$

b. Domain: $\{x \mid 0 < x < 4\}$

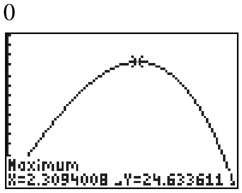
c. The area is largest when $x \approx 2.31$.

30



0 4

0

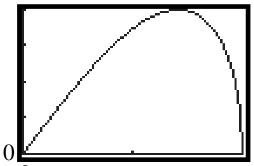


8. a. $A(x) = 2xy = 2x\sqrt{4 - x^2}$

b. $p(x) = 2(2x) + 2(y) = 4x + 2\sqrt{4 - x^2}$

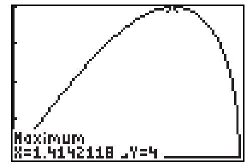
c. Graphing the area equation:

4



0 2

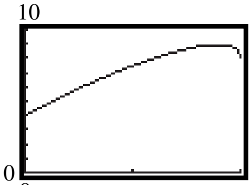
0



The area is largest when $x \approx 1.41$.

d. Graphing the perimeter equation:

10



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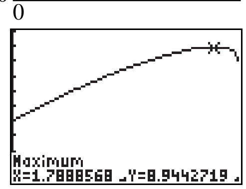


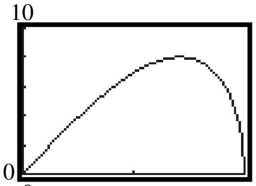
The perimeter is largest when $x \approx 1.79$.

9. a. In Quadrant I, $x^2 + y^2 = 4 \rightarrow y = \sqrt{4 - x^2}$
$A(x) = (2x)(2y) = 4x\sqrt{4 - x^2}$

b. $p(x) = 2(2x) + 2(2y) = 4x + 4\sqrt{4 - x^2}$

c. Graphing the area equation:

10



0 2

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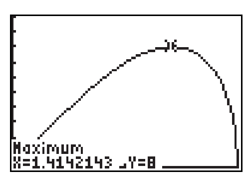
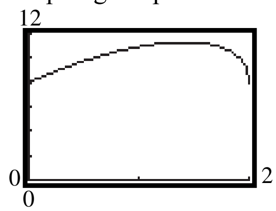
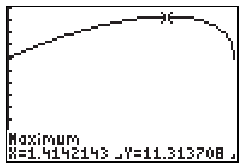


The area is largest when $x \approx 1.41$.

d. Graphing the perimeter equation:





The perimeter is largest when $x \approx 1.41$.

10. a. $A(r) = (2r)(2r) = 4r^2$

b. $p(r) = 4(2r) = 8r$

11. a. $C =$ circumference, $A =$ total area,
$r =$ radius, $x =$ side of square

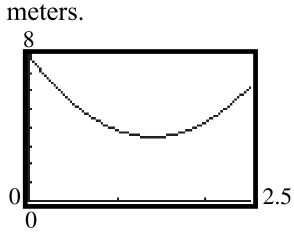
$C = 2\pi r = 10 - 4x \quad \Rightarrow \quad r = \frac{5-2x}{\pi}$

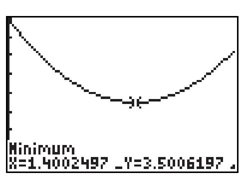
Total Area = $\text{area}_{\text{square}} + \text{area}_{\text{circle}} = x^2 + \pi r^2$

$$A(x) = x^2 + \pi\left(\frac{5-2x}{\pi}\right)^2 = x^2 + \frac{25 - 20x + 4x^2}{\pi}$$

b. Since the lengths must be positive, we have:
$10 - 4x > 0$ and $x > 0$
$-4x > -10$ and $x > 0$
$x < 2.5$ and $x > 0$
Domain: $\{x \mid 0 < x < 2.5\}$

c. The total area is smallest when $x \approx 1.40$ meters.





12. a. $C =$ circumference, $A =$ total area,
$r =$ radius, $x =$ side of equilateral triangle

$C = 2\pi r = 10 - 3x \Rightarrow r = \frac{10 - 3x}{2\pi}$

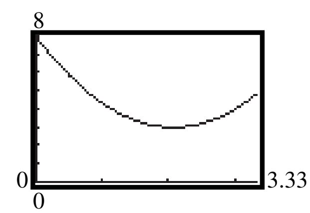
The height of the equilateral triangle is $\frac{\sqrt{3}}{2}x$.

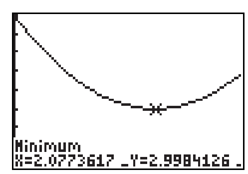
$$\begin{aligned} \text{Total Area} &= \text{area}_{\text{triangle}} + \text{area}_{\text{circle}} \\ &= \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right) + \pi r^2 \end{aligned}$$

$$\begin{aligned} A(x) &= \frac{\sqrt{3}}{4}x^2 + \pi\left(\frac{10-3x}{2\pi}\right)^2 \\ &= \frac{\sqrt{3}}{4}x^2 + \frac{100 - 60x + 9x^2}{4\pi} \end{aligned}$$

b. Since the lengths must be positive, we have:
$10 - 3x > 0$ and $x > 0$
$-3x > -10$ and $x > 0$
$x < \frac{10}{3}$ and $x > 0$
Domain: $\left\{x \,\middle|\, 0 < x < \frac{10}{3}\right\}$

c. The area is smallest when $x \approx 2.08$ meters.





13. a. Since the wire of length x is bent into a circle, the circumference is x. Therefore, $C(x) = x$.

b. Since $C = x = 2\pi r$, $r = \frac{x}{2\pi}$.

$$A(x) = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}.$$

14. a. Since the wire of length x is bent into a square, the perimeter is x. Therefore, $p(x) = x$.

b. Since $P = x = 4s$, $s = \frac{1}{4}x$, we have

$$A(x) = s^2 = \left(\frac{1}{4}x\right)^2 = \frac{1}{16}x^2.$$

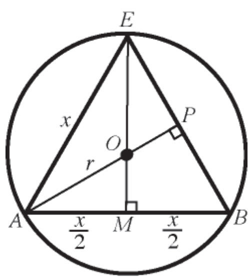
15. a. A = area, r = radius; diameter = $2r$

$$A(r) = (2r)(r) = 2r^2$$

b. p = perimeter

$$p(r) = 2(2r) + 2r = 6r$$

16. C = circumference, r = radius;
x = length of a side of the triangle



Since ΔABC is equilateral, $EM = \frac{\sqrt{3}x}{2}$.

Therefore, $OM = \frac{\sqrt{3}x}{2} - OE = \frac{\sqrt{3}x}{2} - r$

In ΔOAM, $r^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2} - r\right)^2$

$$r^2 = \frac{x^2}{4} + \frac{3}{4}x^2 - \sqrt{3}\,rx + r^2$$

$$\sqrt{3}\,rx = x^2$$

$$r = \frac{x}{\sqrt{3}}$$

Therefore, the circumference of the circle is

$$C(x) = 2\pi r = 2\pi\left(\frac{x}{\sqrt{3}}\right) = \frac{2\pi\sqrt{3}}{3}x$$

17. Area of the equilateral triangle

$$A = \frac{1}{2}x \cdot \frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2$$

From problem 16, we have $r^2 = \frac{x^2}{3}$.

Area inside the circle, but outside the triangle:

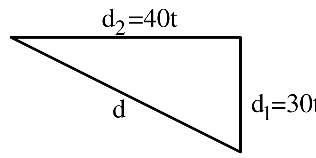
$$A(x) = \pi r^2 - \frac{\sqrt{3}}{4}x^2$$

$$= \pi\frac{x^2}{3} - \frac{\sqrt{3}}{4}x^2 = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)x^2$$

18. $d^2 = d_1^{\,2} + d_2^{\,2}$

$$d^2 = (30t)^2 + (40t)^2$$

$$d(t) = \sqrt{900t^2 + 1600t^2} = \sqrt{2500t^2} = 50t$$



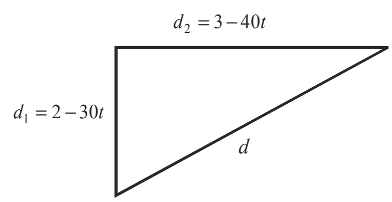
19. a. $d^2 = d_1^{\,2} + d_2^{\,2}$

$$d^2 = (2 - 30t)^2 + (3 - 40t)^2$$

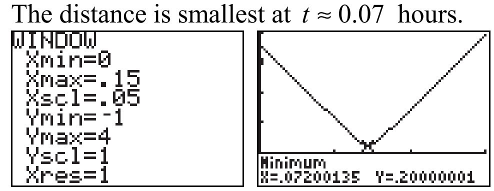
$$d(t) = \sqrt{(2 - 30t)^2 + (3 - 40t)^2}$$

$$= \sqrt{4 - 120t + 900t^2 + 9 - 240t + 1600t^2}$$

$$= \sqrt{2500t^2 - 360t + 13}$$



b. The distance is smallest at $t \approx 0.07$ hours.



20. r = radius of cylinder, h = height of cylinder,
V = volume of cylinder

$$r^2 + \left(\frac{h}{2}\right)^2 = R^2 \Rightarrow r^2 + \frac{h^2}{4} = R^2 \Rightarrow r^2 = R^2 - \frac{h^2}{4}$$

$$V = \pi r^2 h$$

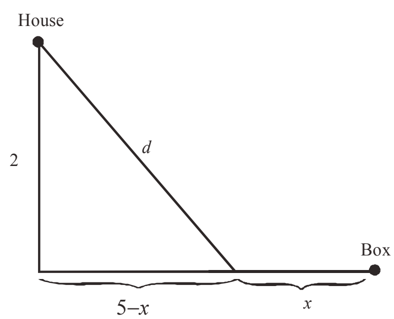
$$V(h) = \pi\left(R^2 - \frac{h^2}{4}\right)h = \pi h\left(R^2 - \frac{h^2}{4}\right)$$

21. r = radius of cylinder, h = height of cylinder, V = volume of cylinder

By similar triangles: $\frac{H}{R} = \frac{H-h}{r}$

$$Hr = R(H-h)$$
$$Hr = RH - Rh$$
$$Rh = RH - Hr$$
$$h = \frac{RH - Hr}{R} = \frac{H(R-r)}{R}$$
$$V = \pi r^2 h = \pi r^2 \left(\frac{H(R-r)}{R}\right) = \frac{\pi H (R-r) r^2}{R}$$

22. a. The total cost of installing the cable along the road is $500x$. If cable is installed x miles along the road, there are $5-x$ miles between the road to the house and where the cable ends along the road.



$$d = \sqrt{(5-x)^2 + 2^2}$$
$$= \sqrt{25 - 10x + x^2 + 4} = \sqrt{x^2 - 10x + 29}$$

The total cost of installing the cable is:

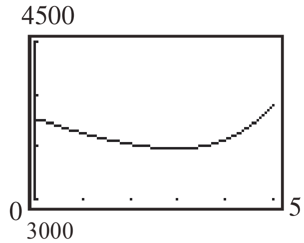
$$C(x) = 500x + 700\sqrt{x^2 - 10x + 29}$$

Domain: $\{ x \mid 0 \le x \le 5 \}$

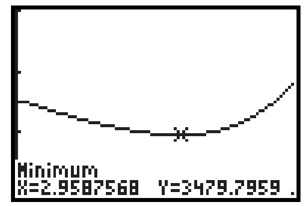
b. $C(1) = 500(1) + 700\sqrt{1^2 - 10(1) + 29}$
$= 500 + 700\sqrt{20} = \$3630.50$

c. $C(3) = 500(3) + 700\sqrt{3^2 - 10(3) + 29}$
$= 1500 + 700\sqrt{8} = \$3479.90$

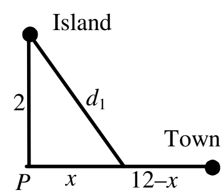
d.



e. Using MINIMUM, the graph indicates that $x \approx 2.96$ miles results in the least cost.



23. a. The time on the boat is given by $\frac{d_1}{3}$. The time on land is given by $\frac{12-x}{5}$.



$$d_1 = \sqrt{x^2 + 2^2} = \sqrt{x^2 + 4}$$

The total time for the trip is:

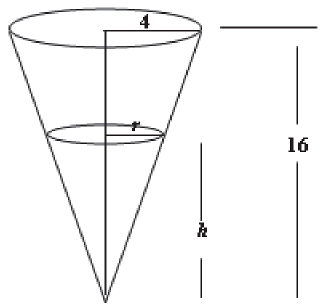
$$T(x) = \frac{12-x}{5} + \frac{d_1}{3} = \frac{12-x}{5} + \frac{\sqrt{x^2+4}}{3}$$

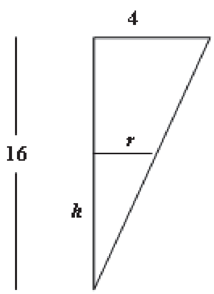
b. Domain: $\{ x \mid 0 \le x \le 12 \}$

c. $T(4) = \frac{12-4}{5} + \frac{\sqrt{4^2+4}}{3}$
$= \frac{8}{5} + \frac{\sqrt{20}}{3} \approx 3.09$ hours

d. $T(8) = \frac{12-8}{5} + \frac{\sqrt{8^2+4}}{3}$
$= \frac{4}{5} + \frac{\sqrt{68}}{3} \approx 3.55$ hours

24. Consider the diagrams shown below.





There is a pair of similar triangles in the diagram. Since the smaller triangle is similar to the larger triangle, we have the proportion

$$\frac{r}{h} = \frac{4}{16} \Rightarrow \frac{r}{h} = \frac{1}{4} \Rightarrow r = \frac{1}{4}h$$

Substituting into the volume formula for the conical portion of water gives

$$V(h) = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{4}h\right)^2 h = \frac{\pi}{48}h^3.$$

25. a. length $= 24-2x$; width $= 24-2x$; height $= x$

$V(x) = x(24-2x)(24-2x) = x(24-2x)^2$

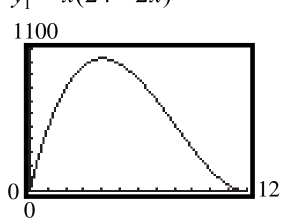
b. $V(3) = 3(24-2(3))^2 = 3(18)^2$

$= 3(324) = 972 \text{ in}^3.$

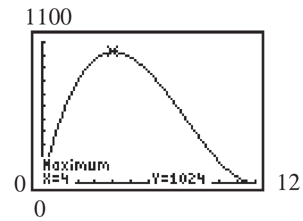
c. $V(10) = 10(24-2(10))^2 = 10(4)^2$

$= 10(16) = 160 \text{ in}^3.$

d. $y_1 = x(24-2x)^2$



Use MAXIMUM.



The volume is largest when $x = 4$ inches.

26. a. Let $A =$ amount of material, $x =$ length of the base, $h =$ height, and $V =$ volume.

$$V = x^2 h = 10 \Rightarrow h = \frac{10}{x^2}$$

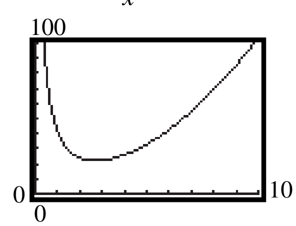
$$\begin{aligned}\text{Total Area } A &= (\text{Area}_{\text{base}}) + (4)(\text{Area}_{\text{side}}) \\ &= x^2 + 4xh \\ &= x^2 + 4x\left(\frac{10}{x^2}\right) \\ &= x^2 + \frac{40}{x}\end{aligned}$$

$$A(x) = x^2 + \frac{40}{x}$$

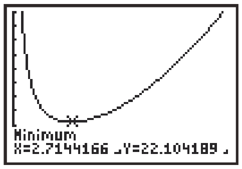
b. $A(1) = 1^2 + \frac{40}{1} = 1 + 40 = 41 \text{ ft}^2$

c. $A(2) = 2^2 + \frac{40}{2} = 4 + 20 = 24 \text{ ft}^2$

d. $y_1 = x^2 + \frac{40}{x}$



100



0 10

0

The amount of material is least when $x = 2.71$ ft.

Section 1.7

1. $y = kx$

2. False. If *y* varies directly with *x*, then $y = kx$, where *k* is a constant.

3. $y = kx$
$2 = 10k$
$k = \frac{2}{10} = \frac{1}{5}$
$y = \frac{1}{5}x$

4. $v = kt$
$16 = 2k$
$8 = k$
$v = 8t$

5. $A = kx^2$
$4\pi = k(2)^2$
$4\pi = 4k$
$\pi = k$
$A = \pi x^2$

6. $V = kx^3$
$36\pi = k(3)^3$
$36\pi = 27k$
$k = \frac{36\pi}{27} = \frac{4}{3}\pi$
$V = \frac{4}{3}\pi x^3$

7. $F = \frac{k}{d^2}$
$10 = \frac{k}{5^2}$
$10 = \frac{k}{25}$
$k = 250$
$F = \frac{250}{d^2}$

8. $y = \frac{k}{\sqrt{x}}$
$4 = \frac{k}{\sqrt{9}}$
$4 = \frac{k}{3}$
$k = 12$
$y = \frac{12}{\sqrt{x}}$

9. $z = k\left(x^2 + y^2\right)$
$5 = k\left(3^2 + 4^2\right)$
$5 = k(25)$
$k = \frac{5}{25} = \frac{1}{5}$
$z = \frac{1}{5}\left(x^2 + y^2\right)$

10. $T = k\left(\sqrt[3]{x}\right)\left(d^2\right)$
$18 = k\left(\sqrt[3]{8}\right)\left(3^2\right)$
$18 = k\left(18\right)$
$1 = k$
$T = d^2\sqrt[3]{x}$

11. $M = \dfrac{kd^2}{\sqrt{x}}$

$24 = \dfrac{k\left(4^2\right)}{\sqrt{9}}$

$24 = \dfrac{16k}{3}$

$k = 24\left(\dfrac{3}{16}\right) = \dfrac{9}{2}$

$M = \dfrac{9d^2}{2\sqrt{x}}$

12. $z = k\left(x^3 + y^2\right)$

$1 = k\left(2^3 + 3^2\right)$

$1 = k(17)$

$k = \dfrac{1}{17}$

$z = \dfrac{1}{17}\left(x^3 + y^2\right)$

13. $T^2 = \dfrac{ka^3}{d^2}$

$2^2 = \dfrac{k\left(2^3\right)}{4^2}$

$4 = \dfrac{k(8)}{16}$

$4 = \dfrac{k}{2}$

$k = 8$

$T^2 = \dfrac{8a^3}{d^2}$

14. $z^3 = k\ \left(x^2 + y^2\right)$

$2^3 = k\ \left(9^2 + 4^2\right)$

$8 = k(97)$

$k = \dfrac{8}{97}$

$z^3 = \dfrac{8}{97}\left(x^2 + y^2\right)$

15. $V = \dfrac{4\pi}{3}r^3$

16. $c^2 = a^2 + b^2$

17. $A = \dfrac{1}{2}bh$

18. $p = 2(l + w)$

19. $F = \left(6.67\times 10^{-11}\right)\left(\dfrac{mM}{d^2}\right)$

20. $T = \dfrac{2\pi}{\sqrt{32}}\sqrt{l}$

21. $p = kB$

$6.49 = k(1000)$

$0.00649 = k$

Therefore we have the linear equation $p = 0.00649B$.

If $B = 145000$, then $p = 0.00649(145000) = \$941.05$.

22. $p = kB$

$8.99 = k(1000)$

$0.00899 = k$

Therefore we have the linear equation $p = 0.00899B$.

If $B = 175000$, then $p = 0.00899(175000) = \$1573.25$.

23. $s = kt^2$

$16 = k(1)^2$

$k = 16$

Therefore, we have equation $s = 16t^2$.

If t = 3 seconds, then $s = 16(3)^2 = 144$ feet.

If s = 64 feet, then

$64 = 16t^2$

$t^2 = 4$

$t = \pm 2$

Time must be positive, so we disregard $t = -2$.

It takes 2 seconds to fall 64 feet.

24. $v = kt$

$64 = k(2)$

$k = 32$

Therefore, we have the linear equation $v = 32t$.

If $t = 3$ seconds, then $v = 32(3) = 96$ ft/sec.

25. $E = kW$

$3 = k(20)$

$k = \frac{3}{20}$

Therefore, we have the linear equation $E = \frac{3}{20}W$.

If $W = 15$, then $E = \frac{3}{20}(15) = 2.25$.

26. $R = \frac{k}{l}$

$256 = \frac{k}{48}$

$k = 12{,}288$

Therefore, we have the equation $R = \frac{12{,}288}{l}$.

If $R = 576$, then

$576 = \frac{12{,}288}{l}$

$576l = 12{,}288$

$l = \frac{12{,}288}{576} = \frac{64}{3}$ inches

27. $R = kg$

$47.40 = k(12)$

$3.95 = k$

Therefore, we have the linear equation $R = 3.95g$.

If $g = 10.5$, then $R = (3.95)(10.5) \approx \$41.48$.

28. $C = kA$

$23.75 = k(5)$

$4.75 = k$

Therefore, we have the linear equation $C = 4.75A$.

If $A = 3.5$, then $C = (4.75)(3.5) = \$16.63$.

29. $D = \frac{k}{p}$

a. $D = 156$, $p = 2.75$;

$156 = \frac{k}{2.75}$

$k = 429$

So, $D = \frac{429}{p}$.

b. $D = \frac{429}{3} = 143$ bags of candy

30. $t = \frac{k}{s}$

a. $t = 40$, $s = 30$;

$40 = \frac{k}{30}$

$k = 1200$

So, we have the equation $t = \frac{1200}{s}$.

b. $t = \frac{1200}{40} = 30$ minutes

31. $V = \frac{k}{P}$

$V = 600, P = 150$;

$600 = \frac{k}{150}$

$k = 90{,}000$

So, we have the equation $V = \frac{90{,}000}{P}$

If $P = 200$, then $V = \frac{90{,}000}{200} = 450 \text{ cm}^3$.

32. $i = \frac{k}{R}$

If $i = 30, R = 8$, then $30 = \frac{k}{8}$ and $k = 240$.

So, we have the equation $i = \frac{240}{R}$.

If $R = 10$, then $i = \frac{240}{10} = 24$ amperes.

33. $W = \frac{k}{d^2}$

If $W = 125, d = 3960$ then

$125 = \frac{k}{3960^2}$ and $k = 1{,}960{,}200{,}000$

So, we have the equation $W = \dfrac{1{,}960{,}200{,}000}{d}$.

At the top of Mt. McKinley, we have $d = 3960 + 3.8 = 3963.8$, so

$$W = \frac{1{,}960{,}200{,}000}{(3963.8)^2} \approx 124.76 \text{ pounds.}$$

34. $I = \dfrac{k}{d^2}$

If $I = 0.075, d = 2$, then

$0.075 = \dfrac{k}{2^2}$ and $k = 0.3$.

So, we have the equation $I = \dfrac{0.3}{d^2}$.

If $d = 5$, then $I = \dfrac{0.3}{5^2} = 0.012$ foot-candle.

35. $V = \pi r^2 h$

36. $V = \dfrac{\pi}{3} r^2 h$

37. $W = \dfrac{k}{d^2}$

$$55 = \frac{k}{3960^2}$$
$$k = 862{,}488{,}000$$

So, we have the equation $W = \dfrac{862{,}488{,}000}{d^2}$.

If d=3965, then

$$W = \frac{862{,}488{,}000}{3965^2} \approx 54.86 \text{ pounds.}$$

38. $F = kAv^2$

$$11 = k(20)(22)^2$$
$$11 = 9860k$$
$$k = \frac{11}{9680} = \frac{1}{880}$$

So, we have the equation $F = \dfrac{1}{880} Av^2$.

If $A = 47.125$ and $v = 36.5$, then

$$F = \frac{1}{880}(47.125)(36.5)^2 \approx 71.34 \text{ pounds.}$$

39. $h = ksd^3$

$$36 = k(75)(2)^3$$
$$36 = 600k$$
$$0.06 = k$$

So, we have the equation $h = 0.06sd^3$.

If $h = 45$ and $s = 125$, then

$$45 = (0.06)(125)d^3$$
$$45 = 7.5d^3$$
$$6 = d^3$$
$$d = \sqrt[3]{6} \approx 1.82 \text{ inches}$$

40. $V = \dfrac{kT}{P}$

$$100 = \frac{k(300)}{15}$$
$$100 = 20k$$
$$5 = k$$

So, we have the equation $V = \dfrac{5T}{P}$.

If $V = 80$ and $T = 310$, then

$$80 = \frac{5(310)}{P}$$
$$80P = 1550$$
$$P = \frac{1550}{80} = 19.375 \text{ atmospheres}$$

41. $K = kmv^2$

$$1250 = k(25)(10)^2$$
$$1250 = 2500k$$
$$k = 0.5$$

So, we have the equation $K = 0.5mv^2$.

If $m = 25$ and $v = 15$, then
$K = 0.5(25)(15)^2 = 2812.5$ Joules

42. $R = \dfrac{kl}{d^2}$

$1.24 = \dfrac{k(432)}{(4)^2}$

$1.24 = 27k$

$k = \dfrac{1.24}{27}$

So, we have the equation $R = \dfrac{1.24l}{27d^2}$.

If $R = 1.44$ and $d = 3$, then

$1.44 = \dfrac{1.24l}{27(3)^2}$

$1.44 = \dfrac{1.24l}{243}$

$349.92 = 1.24l$

$l = \dfrac{349.92}{1.24} \approx 282.2$ feet

43. $S = \dfrac{kpd}{t}$

$100 = \dfrac{k(25)(5)}{0.75}$

$75 = 125k$

$0.6 = k$

So, we have the equation $S = \dfrac{0.6pd}{t}$.

If $p = 40$, $d = 8$, and $t = 0.50$, then

$S = \dfrac{0.6(40)(8)}{0.50} = 384$ psi.

44. $S = \dfrac{kwt^2}{l}$

$750 = \dfrac{k(4)(2)^2}{8}$

$750 = 2k$

$375 = k$

So, we have the equation $S = \dfrac{375wt^2}{l}$.

If $l = 10$, $w = 6$, and $t = 2$, then

$S = \dfrac{375(6)(2)^2}{10} = 900$ pounds.

45 – 48. Answers will vary.

Chapter 1 Review Exercises

1. This relation represents a function.
Domain = {–1, 2, 4}; Range = {0, 3}.

2. This relation does not represent a function, since 4 is paired with two different values.

3. $f(x) = \dfrac{3x}{x^2 - 1}$

a. $f(2) = \dfrac{3(2)}{(2)^2 - 1} = \dfrac{6}{4-1} = \dfrac{6}{3} = 2$

b. $f(-2) = \dfrac{3(-2)}{(-2)^2 - 1} = \dfrac{-6}{4-1} = \dfrac{-6}{3} = -2$

c. $f(-x) = \dfrac{3(-x)}{(-x)^2 - 1} = \dfrac{-3x}{x^2 - 1}$

d. $-f(x) = -\left(\dfrac{3x}{x^2 - 1}\right) = \dfrac{-3x}{x^2 - 1}$

e. $f(x-2) = \dfrac{3(x-2)}{(x-2)^2 - 1}$
$= \dfrac{3x - 6}{x^2 - 4x + 4 - 1} = \dfrac{3(x-2)}{x^2 - 4x + 3}$

f. $f(2x) = \dfrac{3(2x)}{(2x)^2 - 1} = \dfrac{6x}{4x^2 - 1}$

4. $f(x) = \sqrt{x^2 - 4}$

a. $f(2) = \sqrt{2^2 - 4} = \sqrt{4-4} = \sqrt{0} = 0$

b. $f(-2) = \sqrt{(-2)^2 - 4} = \sqrt{4-4} = \sqrt{0} = 0$

c. $f(-x) = \sqrt{(-x)^2 - 4} = \sqrt{x^2 - 4}$

d. $-f(x) = -\sqrt{x^2 - 4}$

e. $f(x-2) = \sqrt{(x-2)^2 - 4}$
$= \sqrt{x^2 - 4x + 4 - 4}$
$= \sqrt{x^2 - 4x}$

f. $f(2x)=\sqrt{(2x)^2-4}=\sqrt{4x^2-4}$
$=\sqrt{4\left(x^2-1\right)}=2\sqrt{x^2-1}$

5. $f(x)=\dfrac{x^2-4}{x^2}$

a. $f(2)=\dfrac{2^2-4}{2^2}=\dfrac{4-4}{4}=\dfrac{0}{4}=0$

b. $f(-2)=\dfrac{(-2)^2-4}{(-2)^2}=\dfrac{4-4}{4}=\dfrac{0}{4}=0$

c. $f(-x)=\dfrac{(-x)^2-4}{(-x)^2}=\dfrac{x^2-4}{x^2}$

d. $-f(x)=-\left(\dfrac{x^2-4}{x^2}\right)=\dfrac{4-x^2}{x^2}=-\dfrac{x^2-4}{x^2}$

e. $f(x-2)=\dfrac{(x-2)^2-4}{(x-2)^2}=\dfrac{x^2-4x+4-4}{(x-2)^2}$
$=\dfrac{x^2-4x}{(x-2)^2}=\dfrac{x(x-4)}{(x-2)^2}$

f. $f(2x)=\dfrac{(2x)^2-4}{(2x)^2}=\dfrac{4x^2-4}{4x^2}$
$=\dfrac{4\left(x^2-1\right)}{4x^2}=\dfrac{x^2-1}{x^2}$

6. $f(x)=\dfrac{x}{x^2-9}$
The denominator cannot be zero:
$x^2-9\neq 0$
$(x+3)(x-3)\neq 0$
$x\neq -3$ or 3
Domain: $\{x \mid x\neq -3, x\neq 3\}$

7. $f(x)=\sqrt{2-x}$
The radicand must be non-negative:
$2-x\geq 0$
$x\leq 2$
Domain: $\{x \mid x\leq 2\}$ or $(-\infty,2]$

8. $g(x)=\dfrac{|x|}{x}$
The denominator cannot be zero:
$x\neq 0$
Domain: $\{x \mid x\neq 0\}$

9. $f(x)=\dfrac{x}{x^2+2x-3}$
The denominator cannot be zero:
$x^2+2x-3\neq 0$
$(x+3)(x-1)\neq 0$
$x\neq -3$ or 1
Domain: $\{x \mid x\neq -3,\ x\neq 1\}$

10. $f(x)=\dfrac{\sqrt{x+1}}{x^2-4}$
The denominator cannot be zero:
$x^2-4\neq 0$
$x^2\neq 4$
$x\neq \pm 2$

The radicand in the numerator must be non-negative:
$x+1\geq 0$
$x\geq -1$

Domain: $\{x \mid x\geq -1, x\neq 2\}$

11. $f(x)=\dfrac{x}{\sqrt{x+8}}$
The denominator cannot be zero and the radicand must be non-negative:
$x+8>0$
$x>-8$

Domain: $\{x \mid x>-8\}$

12. $f(x)=2-x \quad g(x)=3x+1$
$(f+g)(x)=f(x)+g(x)$
$=2-x+3x+1=2x+3$
Domain: $\{x \mid x$ is any real number$\}$

$(f-g)(x)=f(x)-g(x)$
$=2-x-(3x+1)$
$=2-x-3x-1$
$=-4x+1$
Domain: $\{x \mid x$ is any real number$\}$

$$\begin{aligned}(f\cdot g)(x) &= f(x)\cdot g(x)\\ &= (2-x)(3x+1)\\ &= 6x+2-3x^2-x\\ &= -3x^2+5x+2\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{2-x}{3x+1}$$

$3x+1\neq 0$

$3x\neq -1 \Rightarrow x\neq -\frac{1}{3}$

Domain: $\left\{x \,\middle|\, x\neq -\frac{1}{3}\right\}$

13. $f(x)=3x^2+x+1 \qquad g(x)=3x$

$$\begin{aligned}(f+g)(x) &= f(x)+g(x)\\ &= 3x^2+x+1+3x\\ &= 3x^2+4x+1\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$\begin{aligned}(f-g)(x) &= f(x)-g(x)\\ &= 3x^2+x+1-3x\\ &= 3x^2-2x+1\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$\begin{aligned}(f\cdot g)(x) &= f(x)\cdot g(x)\\ &= (3x^2+x+1)(3x)\\ &= 9x^3+3x^2+3x\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{3x^2+x+1}{3x}$$

$3x\neq 0 \Rightarrow x\neq 0$

Domain: $\{x \mid x\neq 0\}$

14. $f(x)=\dfrac{x+1}{x-1} \qquad g(x)=\dfrac{1}{x}$

$$\begin{aligned}(f+g)(x) &= f(x)+g(x)\\ &= \frac{x+1}{x-1}+\frac{1}{x}=\frac{x(x+1)+1(x-1)}{x(x-1)}\\ &= \frac{x^2+x+x-1}{x(x-1)}=\frac{x^2+2x-1}{x(x-1)}\end{aligned}$$

Domain: $\{x \mid x\neq 0, x\neq 1\}$

$$\begin{aligned}(f-g)(x) &= f(x)-g(x)\\ &= \frac{x+1}{x-1}-\frac{1}{x}=\frac{x(x+1)-1(x-1)}{x(x-1)}\\ &= \frac{x^2+x-x+1}{x(x-1)}=\frac{x^2+1}{x(x-1)}\end{aligned}$$

Domain: $\{x \mid x\neq 0, x\neq 1\}$

$$(f\cdot g)(x)=f(x)\cdot g(x)=\left(\frac{x+1}{x-1}\right)\left(\frac{1}{x}\right)=\frac{x+1}{x(x-1)}$$

Domain: $\{x \mid x\neq 0, x\neq 1\}$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{\frac{x+1}{x-1}}{\frac{1}{x}}=\left(\frac{x+1}{x-1}\right)\left(\frac{x}{1}\right)=\frac{x(x+1)}{x-1}$$

Domain: $\{x \mid x\neq 0, x\neq 1\}$

15. $f(x)=-2x^2+x+1$

$$\begin{aligned}&\frac{f(x+h)-f(x)}{h}\\ &= \frac{-2(x+h)^2+(x+h)+1-(-2x^2+x+1)}{h}\\ &= \frac{-2(x^2+2xh+h^2)+x+h+1+2x^2-x-1}{h}\\ &= \frac{-2x^2-4xh-2h^2+x+h+1+2x^2-x-1}{h}\\ &= \frac{-4xh-2h^2+h}{h}=\frac{h(-4x-2h+1)}{h}\\ &= -4x-2h+1\end{aligned}$$

16. **a.** Domain: $\{x \mid -4\leq x\leq 3\}$; $[-4, 3]$

Range: $\{y \mid -3\leq y\leq 3\}$; $[-3, 3]$

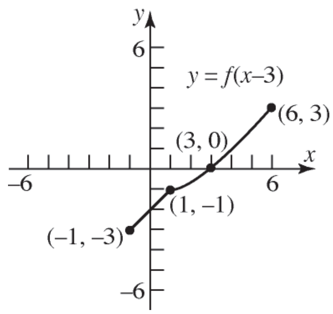
b. Intercept: $(0,0)$

c. $f(-2)=-1$

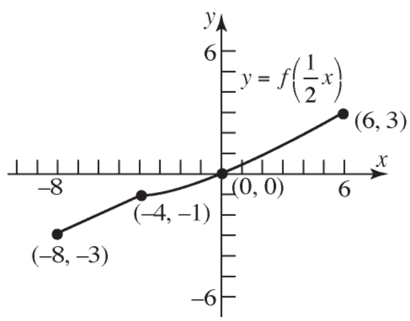
d. $f(x)=-3$ when $x=-4$

e. $f(x)>0$ when $0<x\le 3$
$\{x\,|\,0<x\le 3\}$

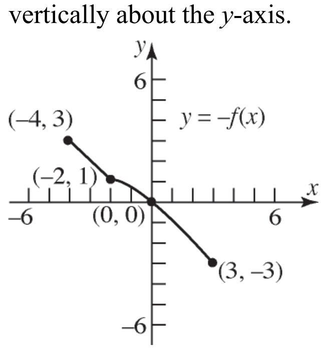
f. To graph $y=f(x-3)$, shift the graph of f horizontally 3 units to the right.



g. To graph $y=f\left(\frac{1}{2}x\right)$, stretch the graph of f horizontally by a factor of 2.



h. To graph $y=-f(x)$, reflect the graph of f vertically about the y-axis.



17. a. Domain: $\{x\,|\,x \text{ is any real number}\}$
Range: $\{y\,|\,y \text{ is any real number}\}$

b. Increasing: $(-\infty,-2)$ and $(2,\infty)$;
Decreasing: $(-2,2)$

c. Local minimum is -1 at $x=2$;
Local maximum is 1 at $x=-2$

d. Absolute minimum is -3 at $x=-4$;
Absolute maximum is 3 at $x=4$

e. The graph is symmetric with respect to the origin.

f. The function is odd.

g. x-intercepts: $-3, 0, 3$;
y-intercept: 0

18. $f(x)=x^3-4x$

$$f(-x)=(-x)^3-4(-x)=-x^3+4x$$
$$=-\left(x^3-4x\right)=-f(x)$$

f is odd.

19. $g(x)=\dfrac{4+x^2}{1+x^4}$

$$g(-x)=\frac{4+(-x)^2}{1+(-x)^4}=\frac{4+x^2}{1+x^4}=g(x)$$

g is even.

20. $G(x)=1-x+x^3$

$$G(-x)=1-(-x)+(-x)^3$$
$$=1+x-x^3\ne -G(x) \text{ or } G(x)$$

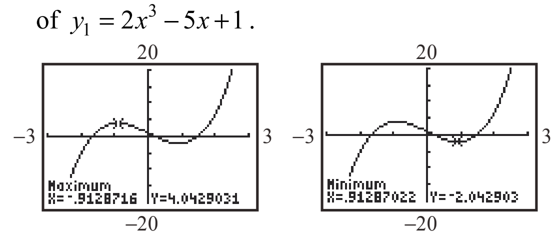
G is neither even nor odd.

21. $f(x)=\dfrac{x}{1+x^2}$

$$f(-x)=\frac{-x}{1+(-x)^2}=\frac{-x}{1+x^2}=-f(x)$$

f is odd.

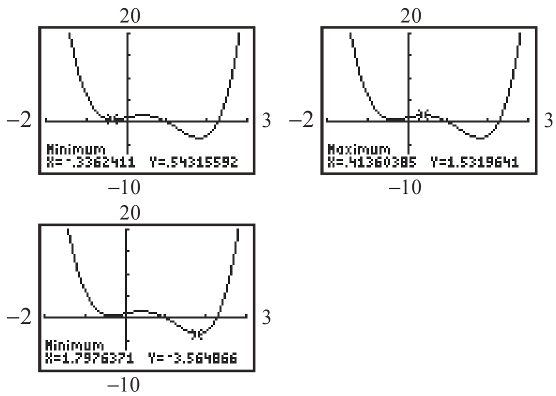
22. $f(x)=2x^3-5x+1$ on the interval $(-3,3)$
Use MAXIMUM and MINIMUM on the graph of $y_1=2x^3-5x+1$.



local maximum: 4.04 when $x\approx -0.91$

local minimum: -2.04 when $x = 0.91$
f is increasing on: $(-3, -0.91)$ and $(0.91, 3)$;
f is decreasing on: $(-0.91, 0.91)$.

23. $f(x) = 2x^4 - 5x^3 + 2x + 1$ on the interval $(-2, 3)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = 2x^4 - 5x^3 + 2x + 1$.



local maximum: 1.53 when $x = 0.41$
local minima: 0.54 when $x = -0.34$, -3.56 when $x = 1.80$
f is increasing on: $(-0.34, 0.41)$ and $(1.80, 3)$;
f is decreasing on: $(-2, -0.34)$ and $(0.41, 1.80)$.

24. $f(x) = 8x^2 - x$

a. $$\frac{f(2)-f(1)}{2-1} = \frac{8(2)^2 - 2 - [8(1)^2 - 1]}{1} = 32 - 2 - (7) = 23$$

b. $$\frac{f(1)-f(0)}{1-0} = \frac{8(1)^2 - 1 - [8(0)^2 - 0]}{1} = 8 - 1 - (0) = 7$$

c. $$\frac{f(4)-f(2)}{4-2} = \frac{8(4)^2 - 4 - [8(2)^2 - 2]}{2} = \frac{128 - 4 - (30)}{2} = \frac{94}{2} = 47$$

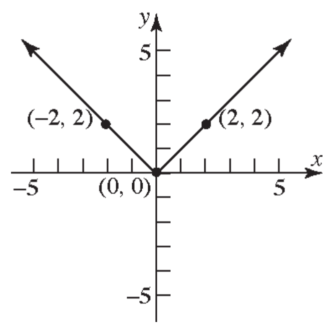
25. $f(x) = 2 - 5x$
$$\frac{f(3)-f(2)}{3-2} = \frac{[2-5(3)] - [2-5(2)]}{3-2} = \frac{(2-15)-(2-10)}{1} = -13 - (-8) = -5$$

26. $f(x) = 3x - 4x^2$
$$\frac{f(3)-f(2)}{3-2} = \frac{\left[3(3)-4(3)^2\right] - \left[3(2)-4(2)^2\right]}{3-2} = \frac{(9-36)-(6-16)}{1} = -27 + 10 = -17$$

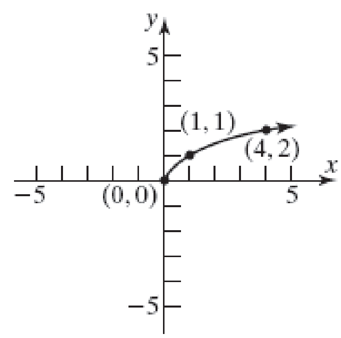
27. The graph does not pass the Vertical Line Test and is therefore not a function.

28. The graph passes the Vertical Line Test and is therefore a function.

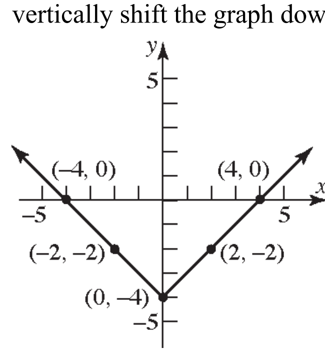
29. $f(x) = |x|$



30. $f(x) = \sqrt{x}$

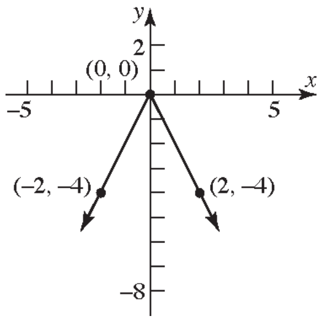


31. $F(x) = |x| - 4$. Using the graph of $y = |x|$, vertically shift the graph downward 4 units.



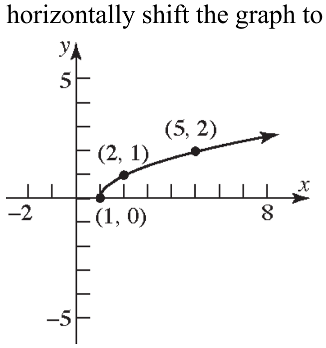
Intercepts: (–4,0), (4,0), (0,–4)
Domain: $\{x \mid x \text{ is any real number}\}$
Range: $\{y \mid y \geq -4\}$ or $[-4, \infty)$

32. $g(x) = -2|x|$. Reflect the graph of $y = |x|$ about the x-axis and vertically stretch the graph by a factor of 2.



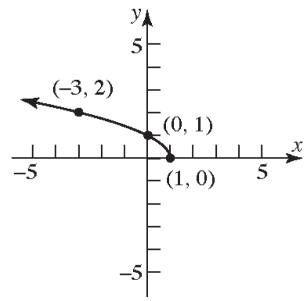
Intercepts: (0, 0)
Domain: $\{x \mid x \text{ is any real number}\}$
Range: $\{y \mid y \leq 0\}$ or $(-\infty, 0]$

33. $h(x) = \sqrt{x-1}$. Using the graph of $y = \sqrt{x}$, horizontally shift the graph to the right 1 unit.



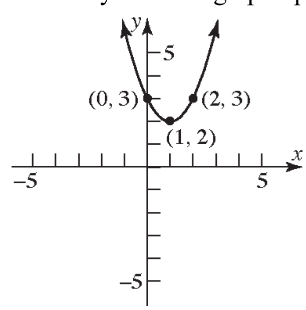
Intercept: (1, 0)
Domain: $\{x \mid x \geq 1\}$ or $[1, \infty)$
Range: $\{y \mid y \geq 0\}$ or $[0, \infty)$

34. $f(x) = \sqrt{1-x} = \sqrt{-(x-1)}$. Reflect the graph of $y = \sqrt{x}$ about the y-axis and horizontally shift the graph to the right 1 unit.



Intercepts: (1, 0), (0, 1)
Domain: $\{x \mid x \leq 1\}$ or $(-\infty, 1]$
Range: $\{y \mid y \geq 0\}$ or $[0, \infty)$

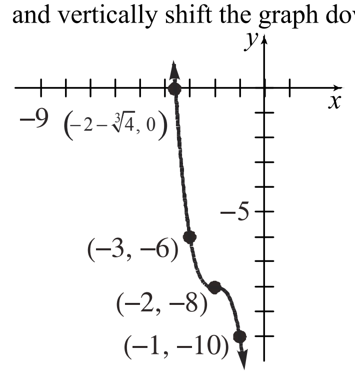
35. $h(x) = (x-1)^2 + 2$. Using the graph of $y = x^2$, horizontally shift the graph to the right 1 unit and vertically shift the graph up 2 units.



Intercepts: (0, 3)
Domain: $\{x \mid x \text{ is any real number}\}$
Range: $\{y \mid y \geq 2\}$ or $[2, \infty)$

36. $g(x) = -2(x+2)^3 - 8$

Using the graph of $y = x^3$, horizontally shift the graph to the left 2 units, vertically stretch the graph by a factor of 2, reflect about the x-axis, and vertically shift the graph down 8 units.



Intercepts: (0,–24), $\left(-2-\sqrt[3]{4}, 0\right) \approx (-3.6, 0)$

Domain: $\{x \mid x \text{ is any real number}\}$

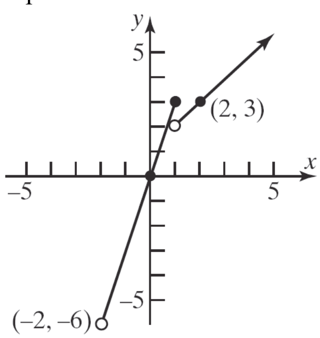
Range: $\{y \mid y \text{ is any real number}\}$

37. $f(x)=\begin{cases} 3x & \text{if } -2<x\le 1 \\ x+1 & \text{if } x>1 \end{cases}$

a. Domain: $\{x \mid x>-2\}$ or $(-2,\infty)$

b. Intercept: $(0,0)$

c. Graph:



d. Range: $\{y \mid y>-6\}$ or $(-6,\infty)$

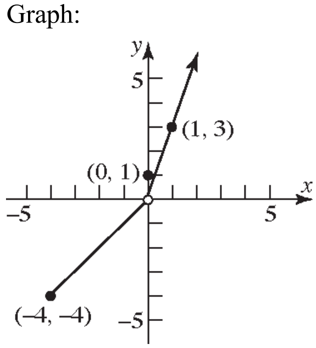
e. There is a jump in the graph at $x=1$. Therefore, the function is not continuous.

38. $f(x)=\begin{cases} x & \text{if } -4\le x<0 \\ 1 & \text{if } x=0 \\ 3x & \text{if } x>0 \end{cases}$

a. Domain: $\{x \mid x\ge -4\}$ or $[-4,\infty)$

b. Intercept: (0, 1)

c. Graph:



d. Range: $\{y \mid y\ge -4,\ y\ne 0\}$

e. There is a jump at $x=0$. Therefore, the function is not continuous.

39. $f(x)=\dfrac{Ax+5}{6x-2}$ and $f(1)=4$

$$\frac{A(1)+5}{6(1)-2}=4$$
$$\frac{A+5}{4}=4$$
$$A+5=16$$
$$A=11$$

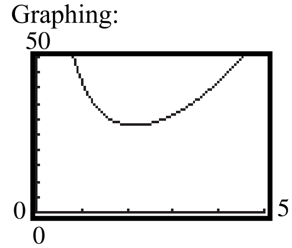
40. a. $x^2h=10 \;\Rightarrow\; h=\dfrac{10}{x^2}$

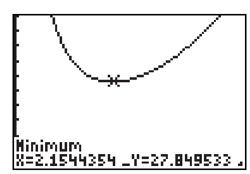
$$A(x)=2x^2+4xh$$
$$=2x^2+4x\left(\frac{10}{x^2}\right)$$
$$=2x^2+\frac{40}{x}$$

b. $A(1)=2\cdot 1^2+\dfrac{40}{1}=2+40=42\text{ ft}^2$

c. $A(2)=2\cdot 2^2+\dfrac{40}{2}=8+20=28\text{ ft}^2$

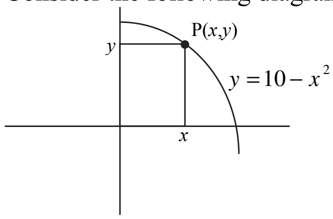
d. Graphing:





The area is smallest when $x\approx 2.15$ feet.

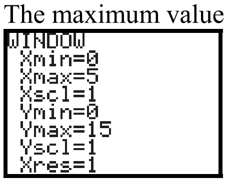
41. a. Consider the following diagram:

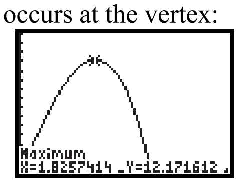


The area of the rectangle is $A = xy$. Thus, the area function for the rectangle is:

$A(x) = x(10 - x^2) = -x^3 + 10x$

b. The maximum value occurs at the vertex:





The maximum area is roughly:

$$A(1.83) = -(1.83)^3 + 10(1.83)$$
$$\approx 12.17 \text{ square units}$$

42. $p = kB$

$854 = k(130{,}000)$

$$k = \frac{854}{130{,}000} = \frac{427}{65{,}000}$$

Therefore, we have the equation $p = \frac{427}{65{,}000}B$.

If $B = 165{,}000$, then

$$p = \frac{427}{65{,}000}(165{,}000) = \$1083.92.$$

43. $w = \frac{k}{d^2}$

$$200 = \frac{k}{3960^2}$$

$$k = (200)\left(3960^2\right) = 3{,}136{,}320{,}000$$

Therefore, we have the equation

$$w = \frac{3{,}136{,}320{,}000}{d^2}.$$

If $d = 3960 + 1 = 3961$ miles, then

$$w = \frac{3{,}136{,}320{,}000}{3961^2} \approx 199.9 \text{ pounds.}$$

44. $H = ksd$

$135 = k(7.5)(40)$

$135 = 300k$

$k = 0.45$

So, we have the equation $H = 0.45sd$.

If $s = 12$ and $d = 35$, then

$H = 0.45(12)(35) = 189$ BTU

Chapter 1 Test

1. a. $\{(2,5),(4,6),(6,7),(8,8)\}$

This relation is a function because there are no ordered pairs that have the same first element and different second elements.

Domain: $\{2,4,6,8\}$

Range: $\{5,6,7,8\}$

b. $\{(1,3),(4,-2),(-3,5),(1,7)\}$

This relation is not a function because there are two ordered pairs that have the same first element but different second elements.

c. This relation is not a function because the graph fails the vertical line test.

d. This relation is a function because it passes the vertical line test.

Domain: $\{x \mid x \text{ is any real number}\}$

Range: $\{y \mid y \ge 2\}$ or $[2, \infty)$

2. $f(x) = \sqrt{4 - 5x}$

The function tells us to take the square root of $4 - 5x$. Only nonnegative numbers have real square roots so we need $4 - 5x \ge 0$.

$$4 - 5x \ge 0$$
$$4 - 5x - 4 \ge 0 - 4$$
$$-5x \ge -4$$
$$\frac{-5x}{-5} \le \frac{-4}{-5}$$
$$x \le \frac{4}{5}$$

Domain: $\left\{x \middle| x \le \frac{4}{5}\right\}$ or $\left(-\infty, \frac{4}{5}\right]$

$$f(-1) = \sqrt{4 - 5(-1)} = \sqrt{4 + 5} = \sqrt{9} = 3$$

3. $g(x)=\dfrac{x+2}{|x+2|}$

The function tells us to divide $x+2$ by $|x+2|$. Division by 0 is undefined, so the denominator can never equal 0. This means that $x \neq -2$.

Domain: $\{x \mid x \neq -2\}$

$$g(-1)=\frac{(-1)+2}{|(-1)+2|}=\frac{1}{|1|}=1$$

4. $h(x)=\dfrac{x-4}{x^2+5x-36}$

The function tells us to divide $x-4$ by $x^2+5x-36$. Since division by 0 is not defined, we need to exclude any values which make the denominator 0.

$$x^2+5x-36=0$$
$$(x+9)(x-4)=0$$
$$x=-9 \text{ or } x=4$$

Domain: $\{x \mid x \neq -9, x \neq 4\}$

(note: there is a common factor of $x-4$ but we must determine the domain prior to simplifying)

$$h(-1)=\frac{(-1)-4}{(-1)^2+5(-1)-36}=\frac{-5}{-40}=\frac{1}{8}$$

5. a. To find the domain, note that all the points on the graph will have an x-coordinate between -5 and 5, inclusive. To find the range, note that all the points on the graph will have a y-coordinate between -3 and 3, inclusive.

Domain: $\{x \mid -5 \le x \le 5\}$ or $[-5, 5]$

Range: $\{y \mid -3 \le y \le 3\}$ or $[-3, 3]$

b. The intercepts are $(0,2)$, $(-2,0)$, and $(2,0)$.

x-intercepts: -2, 2

y-intercept: 2

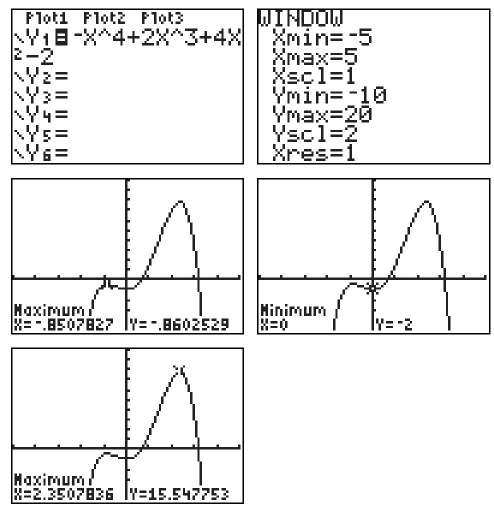
c. $f(1)$ is the value of the function when $x=1$. According to the graph, $f(1)=3$.

d. Since $(-5,-3)$ and $(3,-3)$ are the only points on the graph for which $y=f(x)=-3$, we have $f(x)=-3$ when $x=-5$ and $x=3$.

e. To solve $f(x)<0$, we want to find x-values such that the graph is below the x-axis. The graph is below the x-axis for values in the domain that are less than -2 and greater than 2. Therefore, the solution set is $\{x \mid -5 \le x < -2 \text{ or } 2 < x \le 5\}$. In interval notation we would write the solution set as $[-5,-2) \cup (2,5]$.

6. $f(x)=-x^4+2x^3+4x^2-2$

We set Xmin = -5 and Xmax = 5. The standard Ymin and Ymax will not be good enough to see the whole picture so some adjustment must be made.



We see that the graph has a local maximum of -0.86 (rounded to two places) when $x=-0.85$ and another local maximum of 15.55 when $x=2.35$. There is a local minimum of -2 when $x=0$. Thus, we have

Local maxima: $f(-0.85) \approx -0.86$

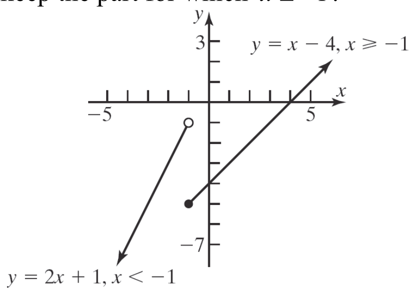
$f(2.35) \approx 15.55$

Local minima: $f(0)=-2$

The function is increasing on the intervals $(-5,-0.85)$ and $(0,2.35)$ and decreasing on the intervals $(-0.85,0)$ and $(2.35,5)$.

7. a. $f(x)=\begin{cases} 2x+1 & x<-1 \\ x-4 & x \ge -1 \end{cases}$

To graph the function, we graph each "piece". First we graph the line $y=2x+1$ but only keep the part for which $x<-1$. Then we plot the line $y=x-4$ but only keep the part for which $x \ge -1$.



b. To find the intercepts, notice that the only piece that hits either axis is $y = x - 4$.

$y = x - 4 \qquad y = x - 4$

$y = 0 - 4 \qquad 0 = x - 4$

$y = -4 \qquad 4 = x$

The intercepts are $(0,-4)$ and $(4,0)$.

c. To find $g(-5)$ we first note that $x = -5$ so we must use the first "piece" because $-5 < -1$.

$g(-5) = 2(-5) + 1 = -10 + 1 = -9$

d. To find $g(2)$ we first note that $x = 2$ so we must use the second "piece" because $2 \ge -1$.

$g(2) = 2 - 4 = -2$

8. The average rate of change from 3 to 4 is given by

$$\frac{\Delta y}{\Delta x} = \frac{f(4) - f(3)}{4 - 3}$$

$$= \frac{\left(3(4)^2 - 2(4) + 4\right) - \left(3(3)^2 - 2(3) + 4\right)}{4 - 3}$$

$$= \frac{44 - 25}{4 - 3} = \frac{19}{1} = 19$$

9. **a.** $f - g = \left(2x^2 + 1\right) - (3x - 2)$

$= 2x^2 + 1 - 3x + 2 = 2x^2 - 3x + 3$

b. $f \cdot g = \left(2x^2 + 1\right)(3x - 2) = 6x^3 - 4x^2 + 3x - 2$

c. $f(x + h) - f(x)$

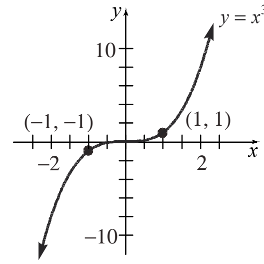
$= \left(2(x + h)^2 + 1\right) - \left(2x^2 + 1\right)$

$= \left(2\left(x^2 + 2xh + h^2\right) + 1\right) - \left(2x^2 + 1\right)$

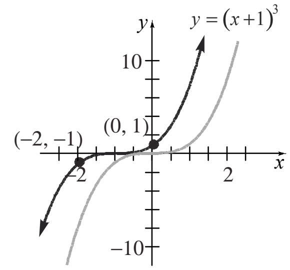
$= 2x^2 + 4xh + 2h^2 + 1 - 2x^2 - 1$

$= 4xh + 2h^2$

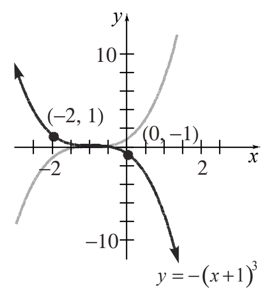
10. **a.** The basic function is $y = x^3$ so we start with the graph of this function.



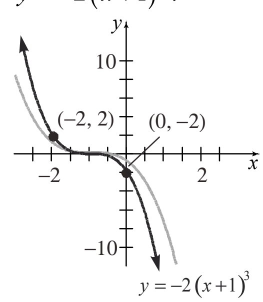
Next we shift this graph 1 unit to the left to obtain the graph of $y = (x+1)^3$.



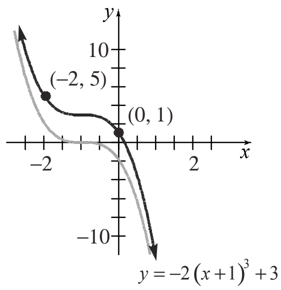
Next we reflect this graph about the x-axis to obtain the graph of $y = -(x+1)^3$.



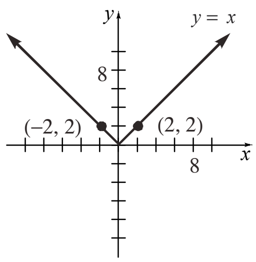
Next we stretch this graph vertically by a factor of 2 to obtain the graph of $y = -2(x+1)^3$.



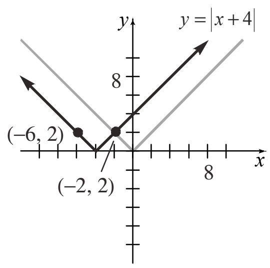
The last step is to shift this graph up 3 units to obtain the graph of $y = -2(x+1)^3 + 3$.



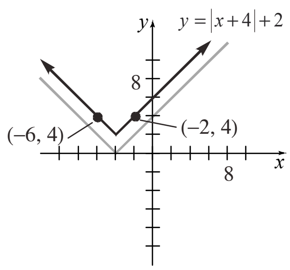
b. The basic function is $y = |x|$ so we start with the graph of this function.



Next we shift this graph 4 units to the left to obtain the graph of $y = |x+4|$.

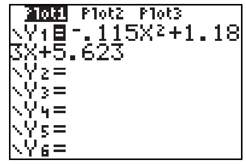


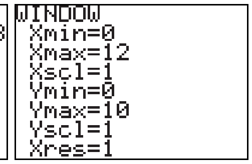
Next we shift this graph up 2 units to obtain the graph of $y = |x+4| + 2$.

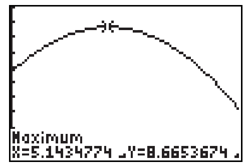


11. a. $r(x) = -0.115x^2 + 1.183x + 5.623$

For the years 1992 to 2004, we have values of *x* between 0 and 12. Therefore, we can let Xmin = 0 and Xmax = 12. Since *r* is the interest rate as a percent, we can try letting Ymin = 0 and Ymax = 10.



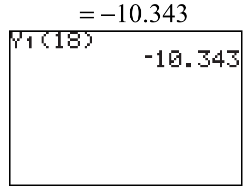




The highest rate during this period appears to be 8.67%, occurring in 1997 ($x \approx 5$).

b. For 2010, we have $x = 2010 - 1992 = 18$.

$$\begin{aligned} r(18) &= -0.115(18)^2 + 1.183(18) + 5.623 \\ &= -10.343 \end{aligned}$$



The model predicts that the interest rate will be -10.343%. This is not a reasonable value since it implies that the bank would be paying interest to the borrower.

12. a. Let x = width of the rink in feet. Then the length of the rectangular portion is given by $2x - 20$. The radius of the semicircular portions is half the width, or $r = \frac{x}{2}$.

To find the volume, we first find the area of the surface and multiply by the thickness of the ice. The two semicircles can be combined to form a complete circle, so the area is given by

$$\begin{aligned} A &= l \cdot w + \pi r^2 \\ &= (2x-20)(x) + \pi\left(\frac{x}{2}\right)^2 \\ &= 2x^2 - 20x + \frac{\pi x^2}{4} \end{aligned}$$

We have expressed our measures in feet so we need to convert the thickness to feet as well.

$$0.75 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{0.75}{12} \text{ ft} = \frac{1}{16} \text{ ft}$$

Now we multiply this by the area to obtain the volume. That is,

$$V(x) = \frac{1}{16}\left(2x^2 - 20x + \frac{\pi x^2}{4}\right)$$

$$V(x) = \frac{x^2}{8} - \frac{5x}{4} + \frac{\pi x^2}{64}$$

b. If the rink is 90 feet wide, then we have $x = 90$.

$$V(90) = \frac{90^2}{8} - \frac{5(90)}{4} + \frac{\pi(90)^2}{64} \approx 1297.61$$

The volume of ice is roughly 1297.61 ft^3.

13.

$$R = \frac{kl}{r^2}$$

$$10 = \frac{k(50)}{(6\times10^{-3})^2}$$

$$10(6\times10^{-3})^2 = 50k$$

$$k = \frac{10(6\times10^{-3})^2}{50} = 7.2\times10^{-6}$$

So, we have the equation $R = \dfrac{(7.2\times10^{-6})l}{r^2}$.

If $l = 100$ and $r = 7\times10^{-3}$, then

$$R = \frac{(7.2\times10^{-6})(100)}{(7\times10^{-3})^2} \approx 14.69 \text{ ohms}$$

Chapter 1 Projects

Project I – Internet Based Project – Answers will vary

Project II

1. Silver: $C(x) = 20 + 0.16(x - 200) = 0.16x - 12$

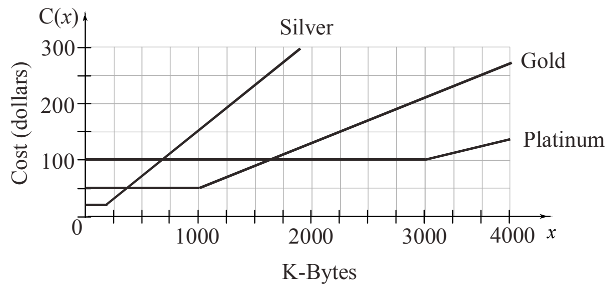
$$C(x) = \begin{cases} 20 & 0 \le x \le 200 \\ 0.16x - 12 & x > 200 \end{cases}$$

Gold: $C(x) = 50 + 0.08(x - 1000) = 0.08x - 30$

$$C(x) = \begin{cases} 50.00 & 0 \le x \le 1000 \\ 0.08x - 30 & x > 1000 \end{cases}$$

Platinum: $C(x) = 100 + 0.04(x - 3000)$
$= 0.04x - 20$

$$C(x) = \begin{cases} 100.00 & 0 \le x \le 3000 \\ 0.04x - 20 & x > 3000 \end{cases}$$



3. Let y = #K-bytes of service over the plan minimum.

Silver: $20 + 0.16y \le 50$
$0.16y \le 30$
$y \le 187.5$

Silver is the best up to $187.5 + 200 = 387.5$ K-bytes of service.

Gold: $50 + 0.08y \le 100$
$0.08y \le 50$
$y \le 625$

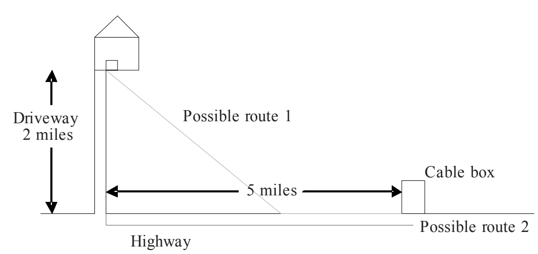
Gold is the best from 387.5 K-bytes to $625 + 1000 = 1625$ K-bytes of service.

Platinum: Platinum will be the best if more than 1625 K-bytes is needed.

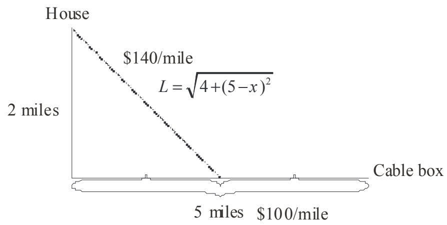
4. Answers will vary.

Project III

1.



2.



$C(x)=100x+140L$

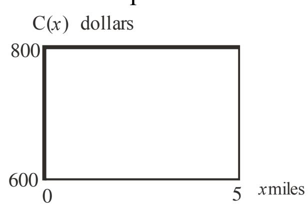
$C(x)=100x+140\sqrt{4+(5-x)^2}$

3.

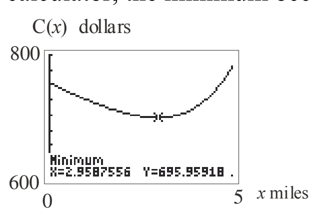
x	$C(x)$
0	$100(0)+140\sqrt{4+25}\approx \$753.92$
1	$100(1)+140\sqrt{4+16}\approx \$726.10$
2	$100(2)+140\sqrt{4+9}\approx \$704.78$
3	$100(3)+140\sqrt{4+4}\approx \$695.98$
4	$100(4)+140\sqrt{4+1}\approx \$713.05$
5	$100(5)+140\sqrt{4+0}=\$780.00$

The choice where the cable goes 3 miles down the road then cutting up to the house seems to yield the lowest cost.

4. Since all of the costs are less than $800, there would be a profit made with any of the plans.



Using the MINIMUM function on a graphing calculator, the minimum occurs at $x\approx 2.96$.



The minimum cost occurs when the cable runs for 2.96 mile along the road.

6. $C(4.5) = 100(4.5) + 140\sqrt{4+(5-4.5)^2}$

$\approx \$738.62$

The cost for the Steven's cable would be $738.62.

7. 5000(738.62) = $3,693,100 State legislated
5000(695.96) = $3,479,800 cheapest cost
It will cost the company $213,300 more.

Project IV

1. $A=\pi r^2$
2. $r=2.2t$
3. $r=2.2(2)=4.4$ ft
   $r=2.2(2.5)=5.5$ ft
4. $A=\pi(4.4)^2=60.82\text{ ft}^2$
   $A=\pi(5.5)^2=95.03\text{ ft}^2$
5. $A=\pi(2.2t)^2=4.84\pi t^2$
6. $A=4.84\pi(2)^2=60.82\text{ ft}^2$
   $A=4.84\pi(2.5)^2=95.03\text{ ft}^2$
7. $\dfrac{A(2.5)-A(2)}{2.5-2}=\dfrac{95.03-60.82}{0.5}=68.42\text{ ft/hr}$
8. $\dfrac{A(3.5)-A(3)}{3.5-3}=\dfrac{186.27-136.85}{0.5}=98.84\text{ ft/hr}$
9. The average rate of change is increasing.
10. 150 yds = 450 ft
    $r=2.2t$
    $t=\dfrac{450}{2.2}=204.5$ hours
11. 6 miles = 31680 ft
    Therefore, we need a radius of 15,840 ft.
    $t=\dfrac{15{,}840}{2.2}=7200$ hours