For Thought

1. True, since $5(1) = 6 - 1$.

2. True, since $x = 3$ is the solution to both equations.

3. False, -2 is not a solution of the first equation since $\sqrt{-2}$ is not a real number.

4. True, since $x - x = 0$.

5. False, $x = 0$ is the solution. **6.** True

7. False, since $|x| = -8$ has no solution.

8. False, $\dfrac{x}{x-5}$ is undefined at $x = 5$.

9. False, since we should multiply by $-\dfrac{3}{2}$.

10. False, $0 \cdot x + 1 = 0$ has no solution.

1.1 Exercises

1. equation

2. linear

3. equivalent

4. solution set

5. identity

6. inconsistent equation

7. conditional equation

8. extraneous root

9. No, since $2(3) - 4 = 2 \neq 9$. **10.** Yes

11. Yes, since $(-4)^2 = 16$.

12. No, since $\sqrt{16} \neq -4$.

13. Since $3x = 5$, the solution set is $\left\{\dfrac{5}{3}\right\}$.

14. Since $-2x = -3$, the solution set is $\left\{\dfrac{3}{2}\right\}$.

15. Since $-3x = 6$, the solution set is $\{-2\}$.

16. Since $5x = -10$, the solution set is $\{-2\}$.

17. Since $14x = 7$, the solution set is $\left\{\dfrac{1}{2}\right\}$.

18. Since $-2x = 2$, the solution set is $\{-1\}$.

19. Since $7 + 3x = 4x - 4$, the solution set is $\{11\}$.

20. Since $-3x + 15 = 4 - 2x$, the solution set is $\{11\}$.

21. Since $x = -\dfrac{4}{3} \cdot 18$, the solution set is $\{-24\}$.

22. Since $x = \dfrac{3}{2} \cdot (-9)$, the solution set is $\left\{-\dfrac{27}{2}\right\}$.

23. Multiplying by 6 we get

$$\begin{aligned} 3x - 30 &= -72 - 4x \\ 7x &= -42. \end{aligned}$$

The solution set is $\{-6\}$.

24. Multiplying by 4 we obtain

$$\begin{aligned} x - 12 &= 2x + 12 \\ -24 &= x. \end{aligned}$$

The solution set is $\{-24\}$.

25. Multiply both sides of the equation by 12.

$$\begin{aligned} 18x + 4 &= 3x - 2 \\ 15x &= -6 \\ x &= -\frac{2}{5}. \end{aligned}$$

The solution set is $\left\{-\dfrac{2}{5}\right\}$.

26. Multiply both sides of the equation by 30.

$$\begin{aligned} 15x + 6x &= 5x - 10 \\ 16x &= -10 \\ x &= -\frac{5}{8}. \end{aligned}$$

The solution set is $\left\{-\dfrac{5}{8}\right\}$.

27. Note, $3(x - 6) = 3x - 18$ is true by the distributive law. It is an identity and the solution set is R.

28. Subtract $5a$ from both sides of $5a = 6a$ to get $0 = a$. The latter equation is conditional whose solution set is $\{0\}$.

29. Note, $5x = 4x$ is equivalent to $x = 0$. The latter equation is conditional whose solution set is $\{0\}$.

30. Note, $4(y-1) = 4y-4$ is true by the distributive law. The equation is an identity and the solution set is R.

31. Equivalently, we get $2x+6 = 3x-3$ or $9 = x$. The latter equation is conditional whose solution set is $\{9\}$.

32. Equivalently, we obtain $2x+2 = 3x+2$ or $0 = x$. The latter equation is conditional whose solution set is $\{0\}$.

33. Using the distributive property, we find

$$\begin{aligned} 3x-18 &= 3x+18 \\ -18 &= 18. \end{aligned}$$

The equation is inconsistent and the solution set is $\emptyset$.

34. Since $5x = 5x+1$ or $0 = 1$, the equation is inconsistent and the solution set is $\emptyset$.

35. An identity and the solution set is $\{x|x \neq 0\}$.

36. An identity and the solution set is $\{x|x \neq -2\}$.

37. Multiplying by $2(w-1)$, we get

$$\begin{aligned} \frac{1}{w-1} - \frac{1}{2w-2} &= \frac{1}{2w-2} \\ 2-1 &= 1. \end{aligned}$$

An identity and the solution set is $\{w|w \neq 1\}$

38. Multiply by $x(x-3)$.

$$\begin{aligned} (x-3)+x &= 9 \\ 2x &= 12 \end{aligned}$$

A conditional equation with solution set $\{6\}$.

39. Multiply by $6x$.

$$\begin{aligned} 6-2 &= 3+1 \\ 4 &= 4 \end{aligned}$$

An identity with solution set $\{x|x \neq 0\}$.

40. Multiply by $60x$.

$$\begin{aligned} 12-15+20 &= -17x \\ 17 &= -17x \end{aligned}$$

A conditional equation with solution set $\{-1\}$.

41. Multiply by $3(z-3)$.

$$\begin{aligned} 3(z+2) &= -5(z-3) \\ 3z+6 &= -5z+15 \\ 8z &= 9 \end{aligned}$$

A conditional equation with solution set $\left\{\frac{9}{8}\right\}$.

42. Multiply by $(x-4)$.

$$\begin{aligned} 2x-3 &= 5 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

Since division by zero is not allowed, $x = 4$ does not satisfy the original equation. We have an inconsistent equation and so the solution set is $\emptyset$.

43. Multiplying by $(x-3)(x+3)$.

$$\begin{aligned} (x+3)-(x-3) &= 6 \\ 6 &= 6 \end{aligned}$$

An identity with solution set $\{x|x \neq 3, x \neq -3\}$.

44. Multiply by $(x+1)(x-1)$.

$$\begin{aligned} 4(x+1)-9(x-1) &= 3 \\ 4x+4-9x+9 &= 3 \\ -5x &= -10 \end{aligned}$$

A conditional equation with solution set $\{2\}$.

45. Multiply by $(y-3)$.

$$\begin{aligned} 4(y-3)+6 &= 2y \\ 4y-6 &= 2y \\ y &= 3 \end{aligned}$$

Since division by zero is not allowed, $y = 3$ does not satisfy the original equation. We have an inconsistent equation and so the solution set is $\emptyset$.

46. Multiply by $x+6$.

$$\begin{aligned} x-3(x+6) &= (x+6)-6 \\ -2x-18 &= x \\ -6 &= x \end{aligned}$$

Since division by zero is not allowed, $x=-6$ does not satisfy the original equation. We have an inconsistent equation and so the solution set is $\emptyset$.

47. Multiply by $t+3$.

$$\begin{aligned} t+4t+12 &= 2 \\ 5t &= -10 \end{aligned}$$

A conditional equation with solution set $\{-2\}$.

48. Multiply by $x+1$.

$$\begin{aligned} 3x-5(x+1) &= x-11 \\ 3x-5x-5 &= x-11 \\ 6 &= 3x \end{aligned}$$

A conditional equation with solution set $\{2\}$.

49. Since $-4.19 = 0.21x$ and $\dfrac{-4.19}{0.21} \approx -19.952$, the solution set is approximately $\{-19.952\}$.

50. Since $0.92x = 5.9$, we get $x = \dfrac{5.9}{0.92} \approx 6.413$. The solution set is approximately $\{6.413\}$.

51. Divide by 0.06.

$$\begin{aligned} x-3.78 &= \frac{1.95}{0.06} \\ x &= 32.5+3.78 \\ x &= 36.28 \end{aligned}$$

The solution set is $\{36.28\}$.

52. Divide by 0.86.

$$\begin{aligned} 3.7-2.3x &= \frac{4.9}{0.86} \\ -2.3x &= \frac{4.9}{0.86}-3.7 \\ x &= \frac{\frac{4.9}{0.86}-3.7}{-2.3} \\ x &\approx -0.869 \end{aligned}$$

The solution set is approximately $\{-0.869\}$.

53.

$$\begin{aligned} 2a &= -1-\sqrt{17} \\ a &= \frac{-1-\sqrt{17}}{2} \\ a &\approx \frac{-1-4.1231}{2} \\ a &\approx -2.562 \end{aligned}$$

The solution set is approximately $\{-2.562\}$.

54.

$$\begin{aligned} 3c &= \sqrt{38}-4 \\ c &= \frac{\sqrt{38}-4}{3} \\ x &\approx \frac{6.1644-4}{3} \\ x &\approx 0.721 \end{aligned}$$

The solution set is approximately $\{0.721\}$.

55.

$$\begin{aligned} 0.001 &= 3(y-0.333) \\ 0.001 &= 3y-0.999 \\ 1 &= 3y \\ \frac{1}{3} &= y \end{aligned}$$

The solution set is $\left\{\dfrac{1}{3}\right\}$.

56. Multiply by $t-1$.

$$\begin{aligned} (t-1)+0.001 &= 0 \\ t-0.999 &= 0 \end{aligned}$$

The solution set is $\{0.999\}$.

57. Factoring x, we get

$$\begin{aligned} x\left(\frac{1}{0.376}+\frac{1}{0.135}\right) &= 2 \\ x(2.6596+7.4074) &\approx 2 \\ 10.067x &\approx 2 \\ x &\approx 0.199 \end{aligned}$$

The solution set is approximately $\{0.199\}$.

58.

$$\begin{aligned} \frac{1}{x} &= 10.379 - \frac{5}{6.72} \\ 1 &= x\left(10.379 - \frac{5}{6.72}\right) \\ \frac{1}{10.379 - \frac{5}{6.72}} &= x \\ 0.104 &\approx x \end{aligned}$$

The solution set is approximately $\{0.104\}$.

59.

$$\begin{aligned} x^2 + 6.5x + 3.25^2 &= x^2 - 8.2x + 4.1^2 \\ 14.7x &= 4.1^2 - 3.25^2 \\ 14.7x &= 16.81 - 10.5625 \\ 14.7x &= 6.2475 \\ x &= 0.425 \end{aligned}$$

The solution set is $\{0.425\}$.

60.

$$\begin{aligned} 0.25(4x^2 - 6.4x + 2.56) &= x^2 - 1.8x + 0.81 \\ x^2 - 1.6x + 0.64 &= x^2 - 1.8x + 0.81 \\ 0.2x &= 0.17 \\ x &= 0.85 \end{aligned}$$

The solution set is $\{0.85\}$.

61.

$$\begin{aligned} (2.3 \times 10^6)x &= 1.63 \times 10^4 - 8.9 \times 10^5 \\ x &= \frac{1.63 \times 10^4 - 8.9 \times 10^5}{2.3 \times 10^6} \\ x &\approx -0.380 \end{aligned}$$

The solution set is approximately $\{-0.380\}$.

62. Note, $3.45 \times 10^{-8} \approx 0$.

$$\begin{aligned} x &= \frac{1.63 \times 10^4 - 3.45 \times 10^{-8}}{-3.4 \times 10^{-9}} \\ x &\approx -4.794 \times 10^{12} \end{aligned}$$

The solution set is approximately $\{-4.794 \times 10^{12}\}$.

63. Solution set is $\{\pm 8\}$.

64. Solution set is $\{\pm 2.6\}$.

65. Since $x - 4 = \pm 8$, we get $x = 4 \pm 8$.
The solution set is $\{-4, 12\}$.

66. Since $x - 5 = 3.6$ or $x - 5 = -3.6$, we get $x = 8.6$ or $x = 1.4$. The solution set is $\{1.4, 8.6\}$.

67. Since $x - 6 = 0$, we get $x = 6$.
The solution set is $\{6\}$.

68. Since $x - 7 = 0$, we get $x = 7$.
The solution set is $\{7\}$.

69. Since the absolute value of a real number is not a negative number, the equation $|x + 8| = -3$ has no solution. The solution set is $\emptyset$.

70. Since the absolute value of a real number is not a negative number, the equation $|x + 9| = -6$ has no solution. The solution set is $\emptyset$.

71. Since $2x - 3 = 7$ or $2x - 3 = -7$, we get $2x = 10$ or $2x = -4$. The solution set is $\{-2, 5\}$.

72. Since $3x + 4 = 12$ or $3x + 4 = -12$, we find $3x = 8$ or $3x = -16$. The solution set is $\{-16/3, 8/3\}$.

73. Multiplying $\frac{1}{2}|x - 9| = 16$ by 2 we obtain $|x - 9| = 32$. Then $x - 9 = 32$ or $x - 9 = -32$. The solution set is $\{-23, 41\}$.

74. Multiplying $\frac{2}{3}|x + 4| = 8$ by $\frac{3}{2}$ we obtain $|x + 4| = 12$. Then $x + 4 = 12$ or $x + 4 = -12$. The solution set is $\{-16, 8\}$.

75. Since $2|x + 5| = 10$, we find $|x + 5| = 5$. Then $x + 5 = \pm 5$ or $x = \pm 5 - 5$.
The solution set is $\{-10, 0\}$.

76. Since $8 = 4|x + 3|$, we obtain $2 = |x + 3|$. Then $x + 3 = \pm 2$ or $x = \pm 2 - 3$.
The solution set is $\{-5, -1\}$.

77. Dividing $8|3x - 2| = 0$ by 8, we obtain $|3x - 2| = 0$. Then $3x - 2 = 0$ and the solution set is $\{2/3\}$.

78. Dividing $5|6-3x|=0$ by 5, we obtain $|6-3x|=0$. Then $6-3x=0$ and the solution set is $\{2\}$.

79. Subtracting 7, we find $2|x|=-1$ and $|x|=-\dfrac{1}{2}$. Since an absolute value is not equal to a negative number, the solution set is $\emptyset$.

80. Subtracting 5, we obtain $3|x-4|=-5$ and $|x-4|=-\dfrac{5}{3}$. Since an absolute value is not a negative number, the solution set is $\emptyset$.

81. Since $0.95x=190$, the solution set is $\{200\}$.

82. Since $1.1x=121$, the solution set is $\{110\}$.

83.

$$\begin{aligned} 0.1x-0.05x+1 &= 1.2 \\ 0.05x &= 0.2 \end{aligned}$$

The solution set is $\{4\}$.

84.

$$\begin{aligned} 0.03x-0.2 &= 0.2x+0.006 \\ -0.206 &= 0.17x \\ -\frac{0.206}{0.17} &= x \\ -\frac{0.206}{0.17}\cdot\frac{1000}{1000} &= x \\ -\frac{206}{170} &= x \end{aligned}$$

The solution set is $\left\{-\dfrac{103}{85}\right\}$.

85. Simplifying $x^2+4x+4=x^2+4$, we obtain $4x=0$. The solution set is $\{0\}$.

86. Simplifying $x^2-6x+9=x^2-9$, we get $18=6x$. The solution set is $\{3\}$.

87. Since $|2x-3|=|2x+5|$, we get $2x-3=2x+5$ or $2x-3=-2x-5$. Solving for x, we find $-3=5$ (an inconsistent equation) or $4x=-2$. The solution set is $\{-1/2\}$.

88. Squaring the terms, we find $(9x^2-24x+16)+(16x^2+8x+1)=25x^2+20x+4$. Setting the left side to zero, we obtain $0=36x-13$. The solution set is $\{13/36\}$.

89. Multiply by 4.

$$\begin{aligned} 2x+4 &= x-6 \\ x &= -10 \end{aligned}$$

The solution set is $\{-10\}$.

90. Multiply by 12.

$$\begin{aligned} -2(x+3) &= 3(3-x) \\ -2x-6 &= 9-3x \\ x &= 15 \end{aligned}$$

The solution set is $\{15\}$.

91. Multiply by 30.

$$\begin{aligned} 15(y-3)+6y &= 90-5(y+1) \\ 15y-45+6y &= 90-5y-5 \\ 26y &= 130 \end{aligned}$$

The solution set is $\{5\}$.

92. Multiply by 10.

$$\begin{aligned} 2(y-3)-5(y-4) &= 50 \\ 2y-6-5y+20 &= 50 \\ -36 &= 3y \end{aligned}$$

The solution set is $\{-12\}$.

93. Since $7|x+6|=14$, $|x+6|=2$. Then $x+6=2$ or $x+6=-2$. The solution set is $\{-4,-8\}$.

94. From $3=|2x-3|$, it follows that

$$\begin{aligned} 2x-3=3 &\quad \text{or} \quad 2x-3=-3 \\ 2x=6 &\quad \text{or} \quad 2x=0. \end{aligned}$$

The solution set is $\{3,0\}$.

95. Since $-4|2x-3|=0$, we get $|2x-3|=0$. Then $2x-3=0$ and the solution set is $\{3/2\}$.

96. Since $-|3x+1|=|3x+1|$, we get $0=2|3x+1|$. Then $|3x+1|=0$ or $3x+1=0$. The solution set is $\{-1/3\}$.

97. Since $-5|5x+1| = 4$, we find $|5x+1| = -4/5$. Since the absolute value is not a negative number, the solution set is $\emptyset$.

98. Since $|7-3x| = -3$ and the absolute value is not a negative number, the solution set is $\emptyset$.

99. Multiply by $(x-2)(x+2)$.

$$\begin{aligned} 3(x+2)+4(x-2) &= 7x-2 \\ 3x+6+4x-8 &= 7x-2 \\ 7x-2 &= 7x-2 \end{aligned}$$

An identity with solution set $\{x \mid x \neq 2, x \neq -2\}$.

100. Multiply by $(x-1)(x+2)$.

$$\begin{aligned} 2(x+2)-3(x-1) &= 8-x \\ 2x+4-3x+3 &= 8-x \\ 7-x &= 8-x \end{aligned}$$

An inconsistent equation and the solution set is $\emptyset$.

101. Multiply $(x+3)(x-2)$ to both sides of

$$\frac{4}{x+3}+\frac{3}{x-2}=\frac{7x+1}{(x+3)(x-2)}.$$

Then we find

$$\begin{aligned} 4(x-2)+3(x+3) &= 7x+1 \\ 4x-8+3x+9 &= 7x+1 \\ 7x+1 &= 7x+1. \end{aligned}$$

An identity and the solution set is $\{x \mid x \neq 2 \text{ and } x \neq -3\}$.

102. Multiply by $x(x-1)$ to both sides of

$$\frac{3}{x}+\frac{4}{x-1}=\frac{7x-3}{x(x-1)}.$$

Then we get

$$\begin{aligned} 3(x-1)+4x &= 7x-3 \\ 3x-3+4x &= 7x-3 \\ 7x-3 &= 7x-3. \end{aligned}$$

An identity and the solution set is $\{x \mid x \neq 0 \text{ and } x \neq 1\}$.

103. Multiply by $(x-3)(x-4)$.

$$\begin{aligned} (x-4)(x-2) &= (x-3)^2 \\ x^2-6x+8 &= x^2-6x+9 \\ 8 &= 9 \end{aligned}$$

An inconsistent equation and so the solution set is $\emptyset$.

104. Multiply by $(y+4)(y-2)$.

$$\begin{aligned} (y-2)(y-1) &= (y+4)(y+1) \\ y^2-3y+2 &= y^2+5y+4 \\ -2 &= 8y. \end{aligned}$$

A conditional equation and the solution set is $\{-1/4\}$.

105. **a)** About 1995

b) Increasing

c) Let $y = 0.95$. Solving for x, we find

$$\begin{aligned} 0.95 &= 0.0102x+0.644 \\ \frac{0.95-0.644}{0.0102} &= x \\ 30 &= x. \end{aligned}$$

In the year 2020 $(= 1990+30)$, 95% of mothers will be in the labor force.

106. Let $y = 0.644$. Solving for x, we find

$$\begin{aligned} 0.644 &= 0.0102x+0.644 \\ 0 &= 0.0102x \\ 0 &= x. \end{aligned}$$

In the year 1990 $(= 1990+0)$, 64.4% of mothers were in the labor force.

107. Since $B = 21,000 - 0.15B$, we obtain $1.15B = 21,000$ and the bonus is $B = \dfrac{21,000}{1.15} = \$18,260.87$.

108. Since $0.30(200,000) = 60,000$, we find

$$\begin{aligned} S &= 0.06(140,000+0.3S) \\ S &= 8400+0.018S \\ 0.982S &= 8400. \end{aligned}$$

The state tax is $S = \dfrac{8400}{0.982} = \$8553.97$ and the federal tax is $F = 0.30(200,000 - 8553.97) = \$57,433.81$.

109. Rewrite the left-hand side as a sum.

$$\begin{aligned} 10,000 + \frac{500,000,000}{x} &= 12,000 \\ \frac{500,000,000}{x} &= 2,000 \\ 500,000,000 &= 2000x \\ 250,000 &= x \end{aligned}$$

Thus, 250,000 vehicles must be sold.

110. **(a)** The harmonic mean is

$$\frac{5}{\frac{1}{15.1}+\frac{1}{7.3}+\frac{1}{5.9}+\frac{1}{3.6}+\frac{1}{2.8}}$$

which is about \$4.96 trillion.

(b) Let x be the GDP of Brazil, and

$$A = \frac{1}{15.1}+\frac{1}{7.3}+\frac{1}{5.9}+\frac{1}{3.6}+\frac{1}{2.8}.$$

Applying the harmonic mean formula, we get

$$\begin{aligned} \frac{6}{A+\frac{1}{x}} &= 4.26 \\ 6 &= 4.26A + \frac{4.26}{x} \\ 6 - 4.26A &= \frac{4.26}{x} \\ x &= \frac{4.26}{6-4.26A} \\ x &= \$2.49 \text{ trillion} \end{aligned}$$

111. The third side of the triangle is $\sqrt{3}$ by the Pythagorean Theorem. Then draw radial lines from the center of the circle to each of the three sides. Consider the square with side r that is formed with the 90° angle of the triangle. Then the side of length $\sqrt{3}$ is divided into two segments of length r and $\sqrt{3}-r$. Similarly, the side of length 1 is divided into segments of length r and $1-r$.

Note, the center of the circle lies on the bisectors of the angles of the triangle. Using congruent triangles, the hypotenuse consists of line segments of length $\sqrt{3}-r$ and $1-r$. Since the hypotenuse is 2, we have

$$\begin{aligned} (\sqrt{3}-r)+(1-r) &= 2 \\ \sqrt{3}-1 &= 2r. \end{aligned}$$

Thus, the radius is

$$r = \frac{\sqrt{3}-1}{2}.$$

112. The hypotenuse is $\sqrt{2}$ by the Pythagorean Theorem. Then draw radial lines from the center of the circle to each of the three sides. Consider the square with side r that is formed with the 90° angle of the triangle. Then each side of length 1 consists of line segments of length r and $1-r$.

Note, the center of the circle lies on the bisectors of the angles of the triangle. Using congruent triangles, the hypotenuse consists of line segments of length $1-r$ and $1-r$. Since the hypotenuse is $\sqrt{2}$, we have

$$\begin{aligned} (1-r)+(1-r) &= \sqrt{2} \\ 2-\sqrt{2} &= 2r. \end{aligned}$$

Thus, the radius is

$$r = \frac{2-\sqrt{2}}{2}.$$

115. $-3, 0, 7$

116. $20 - 24 = -4$

117. $-5x + 20 - 24 + 12x = 7x - 4$

118. $\dfrac{\frac{1}{4}-\frac{1}{3}}{\frac{1}{5}-\frac{1}{4}} = \dfrac{-\frac{1}{12}}{-\frac{1}{20}} = \dfrac{20}{12} = \dfrac{5}{3}$

119. $\dfrac{\sqrt{5}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{15}}{3}$

120. Since $x \neq 0$, domain is $(-\infty, 0) \cup (0, \infty)$.

121. \$9, \$99, \$999, \$9,999, \$99,999, \$999,999, \$9,999,999 \$99,999,999, \$999,999,999

122. Notice, $5! + 6! + \cdots + 1776!$ is a multiple of $5! = 120$. Then the units digit of $1!+\cdots+1776!$ is the same as the units digit of $1!+2!+3!+4! = 33$. Thus, the units digit of the sum is 3.

1.1 Pop Quiz

1. Since $7x = 6$, we get $x = 6/7$. A conditional equation and the solution set is $\{6/7\}$.

2. Since $\frac{1}{4}x = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$, we get

$$x = 4 \cdot \frac{1}{2} = 2.$$

A conditional equation, the solution set is $\{2\}$.

3. Since $3x - 27 = 3x - 27$ is an identity, the solution set is R.

4. Since $w - 1 = 6$ or $w - 1 = -6$, we get $w = 7$ or $w = -5$. A conditional equation and the solution set is $\{-5, 7\}$.

5. Since $2x + 12 = 2x + 6$, we obtain $12 = 6$ which is an inconsistent equation. The solution set is $\emptyset$.

6. Since $x^2 + 2x + 1 = x^2 + 1$, we obtain $2x = 0$. This is a conditional equation and the solution set is $\{0\}$.

1.1 Linking Concepts

(a) The power expenditures for runners with masses 60 kg, 65 kg, and 70 kg are
$60(a \cdot 400 - b) \approx 22.9$ kcal/min,
$65(a \cdot 400 - b) \approx 24.8$ kcal/min, and
$70(a \cdot 400 - b) \approx 26.7$ kcal/min, respectively.

(b) Power expenditure increases as the mass of the runner increases (assuming constant velocity).

(c) Since $v = \frac{1}{a}\left(\frac{P}{M} + b\right)$, the velocities are

$$v = \frac{1}{a}\left(\frac{38.7}{80} + b\right) \approx 500 \text{ m/min},$$

$$v = \frac{1}{a}\left(\frac{38.7}{84} + b\right) \approx 477 \text{ m/min},$$

and $v = \frac{1}{a}\left(\frac{38.7}{90} + b\right) \approx 447$ m/min.

(d) The velocity decreases as the mass increases (assuming constant power expenditure).

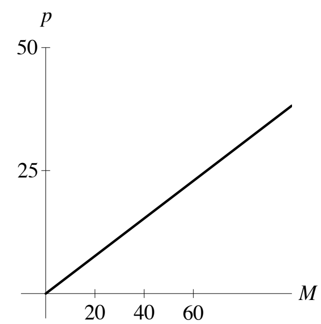
(e) Note, $P = Mva - Mb$. Solving for v, we find

$$\begin{aligned} 52(480)a - 52b &= 50va - 50b \\ 52(480)a - 2b &= 50va \\ \frac{52(480)a - 2b}{50a} &= v \\ 498.2 &\approx v. \end{aligned}$$

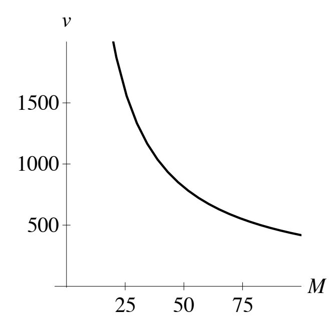
The velocity is approximately 498.2 m/min.

(f) With weights removed and constant power expenditure, a runner's velocity increases.

(g) The first graph shows P versus M (with $v = 400$)



and the second graph shows v versus M (with $P = 40$)



For Thought

1. False, $P(1 + rt) = S$ implies $P = \frac{S}{1 + rt}$.

2. False, since the perimeter is twice the sum of the length and width. **3.** False, since $n + 1$ and $n + 3$ are even integers if n is odd.

4. True **5.** True, since $x + (-3 - x) = -3$.

6. False, since $P = 2S$

7. False, for if the house sells for x dollars then

$$\begin{aligned} x - 0.09x &= 100,000 \\ 0.91x &= 100,000 \\ x &= \$109,890.11. \end{aligned}$$

8. True

9. False, a correct equation is $4(x-2) = 3x - 5$.

10. False, since 9 and $x+9$ differ by x.

1.2 Exercises

1. formula

2. function

3. uniform

4. rate, time

5. $r = \dfrac{I}{Pt}$ **6.** $R = \dfrac{D}{T}$

7. Since $F - 32 = \dfrac{9}{5}C$, $C = \dfrac{5}{9}(F - 32)$.

8. Since $\dfrac{9}{5}C = F - 32$, $\dfrac{9}{5}C + 32 = F$ or

$F = \dfrac{9}{5}C + 32$.

9. Since $2A = bh$, we get $b = \dfrac{2A}{h}$.

10. Since $2A = bh$, we have $h = \dfrac{2A}{b}$.

11. Since $By = C - Ax$, we obtain $y = \dfrac{C - Ax}{B}$.

12. Since $Ax = C - By$, we get $x = \dfrac{C - By}{A}$.

13. Multiplying by $RR_1R_2R_3$, we find

$$R_1R_2R_3 = RR_2R_3 + RR_1R_3 + RR_1R_2$$

$$\begin{aligned} R_1R_2R_3 - RR_1R_3 - RR_1R_2 &= RR_2R_3 \\ R_1(R_2R_3 - RR_3 - RR_2) &= RR_2R_3. \end{aligned}$$

Then $R_1 = \dfrac{RR_2R_3}{R_2R_3 - RR_3 - RR_2}$.

14. Multiplying by $RR_1R_2R_3$, we find

$$R_1R_2R_3 = RR_2R_3 + RR_1R_3 + RR_1R_2$$

$$\begin{aligned} R_1R_2R_3 - RR_2R_3 - RR_1R_2 &= RR_1R_3 \\ R_2(R_1R_3 - RR_3 - RR_1) &= RR_1R_3. \end{aligned}$$

Then $R_2 = \dfrac{RR_1R_3}{R_1R_3 - RR_3 - RR_1}$.

15. Since $a_n - a_1 = (n-1)d$, we obtain

$$\begin{aligned} n - 1 &= \frac{a_n - a_1}{d} \\ n &= \frac{a_n - a_1}{d} + 1 \\ n &= \frac{a_n - a_1 + d}{d}. \end{aligned}$$

16. Multiplying by 2, we get

$$\begin{aligned} 2S_n &= n(a_1 + a_n) \\ 2S_n &= na_1 + na_n \\ 2S_n - na_n &= na_1. \end{aligned}$$

Then $a_1 = \dfrac{2S_n - na_n}{n}$.

17. Since $S = \dfrac{a_1(1 - r^n)}{1 - r}$, we obtain

$$\begin{aligned} a_1(1 - r^n) &= S(1 - r) \\ a_1 &= \frac{S(1 - r)}{1 - r^n}. \end{aligned}$$

18. Since $S = 2LW + H(2L + 2W)$, we obtain

$$\begin{aligned} H(2L + 2W) &= S - 2LW \\ H &= \frac{S - 2LW}{2L + 2W}. \end{aligned}$$

19. Multiplying by 2.37, one finds

$$\begin{aligned} 2.4(2.37) &= L + 2D - F\sqrt{S} \\ 5.688 - L + F\sqrt{S} &= 2D \end{aligned}$$

and $D = \dfrac{5.688 - L + F\sqrt{S}}{2}$.

20. Multiplying by 2.37, one finds

$$\begin{aligned} 2.4(2.37) &= L + 2D - F\sqrt{S} \\ F\sqrt{S} &= L + 2D - 5.688 \\ F &= \frac{L + 2D - 5.688}{\sqrt{S}}. \end{aligned}$$

21. $R = D/T$

22. $T = D/R$

23. Since $LW = A$, we have $W = A/L$.

24. Since $P = 2L + 2W$, we have $2W = P - 2L$ and $W = P/2 - L$.

25. $r = d/2$

26. $d = 2r$

27. By using the formula $I = Prt$, one gets

$$\begin{aligned} 51.30 &= 950r \cdot 1 \\ 0.054 &= r. \end{aligned}$$

The simple interest rate is 5.4%.

28. Note, $I = Prt$ and the interest is \$5.

$$\begin{aligned} 5 &= 100r \cdot \frac{1}{12} \\ 60 &= 100r \\ 0.6 &= r \end{aligned}$$

Simple interest rate is 60%.

29. Since $D = RT$, we find

$$\begin{aligned} 5570 &= 2228 \cdot T \\ 2.5 &= T. \end{aligned}$$

The surveillance takes 2.5 hours.

30. Since $C = 2\pi r$, the radius is $r = \dfrac{72\pi}{2\pi} = 36$ in.

31. Note, $C = \dfrac{5}{9}(F - 32)$. If $F = 23^o F$, then $C = \dfrac{5}{9}(23 - 32) = -5^o\text{C}$.

32. Since $F = \dfrac{9}{5}C + 32$ and $C = 30^o\text{C}$, we get $F = \dfrac{9}{5} \cdot 30 + 32 = 54 + 32 = 86^o\text{F}$.

33. If x is the cost of the car before taxes, then

$$\begin{aligned} 1.08x &= 40,230 \\ x &= \$37,250. \end{aligned}$$

34. If x is the minimum selling price, then

$$\begin{aligned} x - 0.06x - 780 &= 128,000 \\ 0.94x &= 128,780 \\ x &= \$137,000. \end{aligned}$$

35. Let S be the saddle height and let L be the inside measurement.

$$\begin{aligned} S &= 1.09L \\ 37 &= 1.09L \\ \frac{37}{1.09} &= L \\ 33.9 &\approx L \end{aligned}$$

The inside leg measurement is 33.9 inches.

36. Let h, a, and r denote the target heart rate, age, and resting heart rate, respectively. Substituting we obtain

$$\begin{aligned} 144 &= 0.6[220 - (a + r)] + r \\ 144 &= 0.6[220 - (30 + r)] + r \\ 144 &= 0.6[190 - r] + r \\ 144 &= 114 + 0.4r \\ 30 &= 0.4r. \end{aligned}$$

The resting heart rate is $r = \dfrac{30}{0.4} = 75$.

37. Let x be the sale price.

$$\begin{aligned} 1.1x &= 50,600 \\ x &= \frac{50,600}{1.1} \\ x &= \$46,000 \end{aligned}$$

38. Let x be the winning bid.

$$\begin{aligned} 1.05x &= 2.835 \\ x &= \frac{2.835}{1.05} \\ x &= 2.7 \end{aligned}$$

The winning bid is 2.7 million pounds.

39. Let x be the amount of her game-show winnings.

$$\begin{aligned} 0.14\frac{x}{3} + 0.12\frac{x}{6} &= 4000 \\ 6\left(0.14\frac{x}{3} + 0.12\frac{x}{6}\right) &= 24,000 \\ 0.28x + 0.12x &= 24,000 \\ 0.40x &= 24,000 \\ x &= \$60,000 \end{aligned}$$

Her winnings is \$60,000.

40. Let x and $4.7 - x$ be the amounts of the high school and stadium contracts, respectively, in millions of dollars.

$$\begin{aligned} 0.05x + 0.04(4.7 - x) &= 0.223 \\ 0.05x + 0.188 - 0.04x &= 0.223 \\ 0.01x &= 0.035 \\ x &= 3.5 \end{aligned}$$

The high school contract and stadium contract are are \$3.5 million and \$1.2 million, respectively.

41. If x is the length of the shorter piece in feet, then the length of the longer side is $2x + 2$. Then we obtain

$$\begin{aligned} x + x + (2x + 2) &= 30 \\ 4x &= 28 \\ x &= 7. \end{aligned}$$

The length of each shorter piece is 7 ft and the longer piece is $(2 \cdot 7 + 2)$ or 16 ft.

42. If x is the width of the old field, then $(x+3)x$ is the area of the old field. The larger field has an area of $(x + 5)(x + 2)$ and so

$$\begin{aligned} (x + 5)(x + 2) &= (x + 3)x + 46 \\ x^2 + 7x + 10 &= x^2 + 3x + 46 \\ 4x &= 36 \\ x &= 9. \end{aligned}$$

Since $x + 3 = 9 + 3 = 12$, the dimension of the old field is 9 m by 12 m.

43. If x is the length of the side of the larger square lot then $2x$ is the amount of fencing needed to divide the square lot into four smaller lots. The solution to $4x + 2x = 480$ is $x = 80$. The side of the larger square lot is 80 feet and its area is 6400 ft^2.

44. If x is the length of a shorter side, then the longer side is $3x$ and the amount of fencing for the three long sides is $9x$. Since there are four shorter sides and there is 65 feet of fencing, we have $4x + 9x = 65$. The solution to this equation is $x = 5$ and the dimension of each pen is 5 ft by 15 ft.

45. Let d denote the distance from Fairbanks to Coldfoot, which is the same distance from Coldfoot to Deadhorse.

$$\begin{aligned} \frac{d}{50} + \frac{d}{40} &= 11.25 \text{ hr} \\ 90d &= 11.25(2000) \\ d &= 250 \text{ miles} \end{aligned}$$

46. Let x be the distance from Dawson City to Inuvik.

$$\begin{aligned} \frac{x}{23} + 3 &= \frac{x}{20} \\ x &= \frac{-3}{\frac{1}{23} - \frac{1}{20}} \\ x &= 460 \text{ miles} \end{aligned}$$

47. Note, Bobby will complete the remaining 8 laps in $\frac{8}{90}$ of an hour. If Ricky is to finish at the same time as Bobby, then Ricky's average speed s over 10 laps must satisfy $\frac{10}{s} = \frac{8}{90}$. $\left(\text{Note: } time = \frac{distance}{speed}\right)$. The equation is equivalent to $900 = 8s$. Thus, Ricky's average speed must be 112.5 mph.

48. Let d be the distance from the camp to the site of the crab traps. Note, $rate = distance \div time$.

	distance	time	rate
going	d	$1/6$	$6d$
against	d	$1/2$	$2d$

Since the speed going with the tide is increased by 2 mph, his normal speed is $6d-2$. Similarly, since his speed against the tide is decreased by 2 mph, his normal speed is $2d+2$. Then $6d-2 = 2d+2$ for the normal speeds are the same. Solving for d, we get $d = 1$ mile which is the distance between the camp and the site of the crab traps.

49. Let d be the halfway distance between San Antonio and El Paso, and let s be the speed in the last half of the trip. Junior took $\frac{d}{80}$ hours to get to the halfway point and the last half took $\frac{d}{s}$ hours to drive. Since the total distance is $2d$ and $distance = rate \times time$,

$$\begin{aligned} 2d &= 60\left(\frac{d}{80}+\frac{d}{s}\right) \\ 160sd &= 60\,(sd+80d) \\ 160sd &= 60sd+4800d \\ 100sd &= 4800d \\ 100d(s-48) &= 0. \end{aligned}$$

Since $d \neq 0$, the speed for the last half of the trip was $s = 48$ mph.

50. Let A be Parker's average for the remaining games and let n be the number of games in the entire season. The number of points she scored through two-thirds of the season is $18\left(\frac{2n}{3}\right)$ points. If she must have an average of 22 points per game for the entire season, then

$$\begin{aligned} \frac{18\left(\frac{2n}{3}\right)+A\left(\frac{n}{3}\right)}{n} &= 22 \\ \frac{36}{3}+\frac{A}{3} &= 22 \\ 36+A &= 66 \\ A &= 30. \end{aligned}$$

For the remaining games, she must average 30 points per game.

51. If x is the part of the start-up capital invested at 5% and $x+10,000$ is the part invested at 6%, then

$$\begin{aligned} 0.05x+0.06(x+10,000) &= 5880 \\ 0.11x+600 &= 5880 \\ 0.11x &= 5280 \\ x &= 48,000. \end{aligned}$$

Norma invested \$48,000 at 5% and \$58,000 at 6% for a total start-up capital of \$106,000.

52. If x is the amount in cents Bob borrowed at 8% and $\frac{x}{2}$ is the amount borrowed by Betty at 16%, then

$$\begin{aligned} 0.08x+0.16\left(\frac{x}{2}\right) &= 24 \\ 0.08x+0.08x &= 24 \\ 0.16x &= 24 \\ x &= 150 \text{ cents.} \end{aligned}$$

Bob borrowed \$1.50 and Betty borrowed \$0.75.

53. Let x and $1500-x$ be the number of employees from the Northside and Southside, respectively. Then

$$\begin{aligned} (0.05)x+0.80(1500-x) &= 0.5(1500) \\ 0.05x+1200-0.80x &= 750 \\ 450 &= 0.75x \\ 600 &= x. \end{aligned}$$

There were 600 and 900 employees at the Northside and Southside, respectively.

54. Let x be the number of ounces of a 30% solution.

	amt of soln	amt of alcohol
30% soln	x	$0.3x$
80% soln	40	$0.8(40)$
70% soln	$40+x$	$0.7(40+x)$

Adding the amounts of pure alcohol we get

$$\begin{aligned} 0.3x + 0.8(40) &= 0.7(40+x) \\ 0.3x + 32 &= 28 + 0.7x \\ 4 &= 0.4x \\ 10 &= x. \end{aligned}$$

Then 10 ounces of the 30% solution are needed.

55. Let x be the number of hours it takes both combines working together to harvest an entire wheat crop.

	rate
old	1/72
new	1/48
combined	$1/x$

Then $\frac{1}{72} + \frac{1}{48} = \frac{1}{x}$. Multiply both sides by $144x$ and get $2x + 3x = 144$. The solution is $x = 28.8$ hr, which is the time it takes both combines to harvest the entire wheat crop.

56. Let x be the number of hours it takes Rita and Eduardo, working together, to process a batch of claims.

	rate
Rita	1/4
Eduardo	1/2
together	$1/x$

It follows that $\frac{1}{2} + \frac{1}{4} = \frac{1}{x}$. Multiplying both sides by $4x$, we get $2x + x = 4$. The solution is $x = \frac{4}{3}$ hr, which is how long it will take both of them to process a batch of claims.

57. Let t be the number of hours since 8:00 a.m.

	rate	time	work completed
Batman	1/8	$t-2$	$(t-2)/8$
Robin	1/12	t	$t/12$

$$\begin{aligned} \frac{t-2}{8} + \frac{t}{12} &= 1 \\ 24\left(\frac{t-2}{8} + \frac{t}{12}\right) &= 24 \\ 3(t-2) + 2t &= 24 \\ 5t - 6 &= 24 \\ t &= 6 \end{aligned}$$

At 2 p.m., all the crimes have been cleaned up.

58. Let t be the number of hours since noon.

	rate	time	work completed
Della	1/10	$t-3$	$(t-3)/10$
Don	1/15	t	$t/15$

Since the sum of the works completed is 1,

$$\begin{aligned} \frac{t-3}{10} + \frac{t}{15} &= 1 \\ 30\left(\frac{t-3}{10} + \frac{t}{15}\right) &= 30 \\ 3(t-3) + 2t &= 30 \\ 5t - 9 &= 30 \\ t &= 7.8. \end{aligned}$$

They will finish the job in 7.8 hrs or at 7:48 p.m.

59. Since there are 5280 feet to a mile and the circumference of a circle is $C = 2\pi r$, the radius r of the race track is $r = \frac{5280}{2\pi}$. Since the length of a side of the square plot is twice the radius, the area of the plot is

$$\left(2 \cdot \frac{5280}{2\pi}\right)^2 \approx 2,824,672.5 \text{ ft}^2.$$

Dividing this number by $43,560$ results to 64.85 acres which is the acreage of the square lot.

60. Since the volume of a circular cylinder is $V = \pi r^2 h$, we have $12(1.8) = \pi\left(\frac{2.375}{2}\right)^2 \cdot h$. Solving for h, we get $h \approx 4.876$ in., the height of a can of Coke.

61. The area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.

$$\begin{aligned} 90,000 &= \frac{1}{2}h(500+300) \\ 90,000 &= 400h \\ 225 &= h \end{aligned}$$

Thus, the streets are 225 ft apart.

62. The perimeter is the sum of the lengths of the four sides. If w is the width and since there are 3 feet to a yard, then $2w + 2(3 \cdot 120) = 1040$. Solving for w, we obtain $w = 160$ ft.

63. Since the volume of a circular cylinder is $V = \pi r^2 h$, we have $\frac{22,000}{7.5} = \pi 15^2 \cdot h$. Solving for h, we get $h = 4.15$ ft, the depth of water in the pool.

64. The volume of a rectangular box is the product of its length, width, and height. Therefore, $\frac{200,000}{7.5} = 100 \cdot 150 \cdot h$. Solving for h, we obtain $h = 1.78$ ft, the depth of water in the pool.

65. Let r be the radius of the semicircular turns. Since the circumference of a circle is given by $C = 2\pi r$, we have $514 = 2\pi r + 200$. Solving for r, we get

$$r = \frac{157}{\pi} \approx 49.9747 \text{ m}.$$

Note, the width of the rectangular lot is $2r$. Then the dimension of the rectangular lot is 99.9494 m by 199.9494 m; its area is 19,984.82 m^2, which is equivalent to 1.998 hectares.

66. Let r be the radius of the semicircular turns. Since the circumference of a circle is $C = 2\pi r$, we get $514 = 2\pi r + 200$. Solving for r, we have

$$r = \frac{157}{\pi} \approx 49.9747 \text{ m}.$$

By subtracting the area of a circle from the area of a rectangle, we obtain the area outside the track which is

$$2\left[2(49.9747)^2 - \frac{\pi}{2}(49.9747)^2\right] \approx 2,143.85.$$

While the area inside the track is the sum of the areas of a rectangle and a circle, i.e.,

$$100(60) + \pi(30)^2 \approx 8,827.43.$$

The area to be planted with grass is

$$2,143.85 + 8,827.43 \approx 10,971.3 \text{ m}^2.$$

67. Let x be Lorinda's taxable income.

$$\begin{aligned} 17,442 + 0.28(x - 85,650) &= 22,549 \\ 0.28x - 23,982 &= 5,107 \\ 0.28x &= 29,089 \\ x &\approx 103,889. \end{aligned}$$

Lorinda's taxable income is \$103,889.

68. Let x be Glen's taxable income.

$$\begin{aligned} 112,683.50 + 0.35(x - 388,350) &= 285,738 \\ 0.35x - 135,922.50 &= 173,054.50 \\ 0.35x &= 308,977 \\ x &\approx 882,791. \end{aligned}$$

Glen's taxable income is \$882,791.

69. Let x be the amount of water to be added. The volume of the resulting solution is $4 + x$ liters and the amount of pure baneberry in it is 0.05(4) liters. Since the resulting solution is a 3% extract, we have

$$\begin{aligned} 0.03(4+x) &= 0.05(4) \\ 0.12 + 0.03x &= 0.20 \\ x &= \frac{0.08}{0.03} \\ x &= \frac{8}{3}. \end{aligned}$$

The amount of water to be added is $\frac{8}{3}$ liters.

70. Let x be the amount of acetic acid to be added. The volume of the resulting solution is $200 + x$ gallons and the amount of acetic acid is $(200 + x)0.05$ gallons.

$$\begin{aligned} 200(0.04) + x &= (200+x)0.05 \\ 0.95x &= 2 \\ x &= \frac{40}{19}. \end{aligned}$$

The amount of acetic acid to be added is 40/19 gallons.

71. Let x be the number of gallons of gasoline with 12% ethanol that needs to be added. The volume of the resulting solution is $500+x$ gallons and the amount of ethanol is $(500+x)0.1$ gallons.

$$\begin{aligned} 500(0.05)+0.12x &= (500+x)0.1 \\ 0.02x &= 25 \\ x &= 1250. \end{aligned}$$

The amount of gasoline with 12% ethanol to be added is 1250 gallons.

72. Let x be the amount of antifreeze solution (in quarts) to be drained. The amount of pure antifreeze left in the radiator is $0.60(20-x)$. Since pure water will be added, the volume of antifreeze solution will still be 20 quarts, of which 10 quarts is pure antifreeze. (Note, the resulting solution is 50% pure). Then

$$\begin{aligned} 10 &= 0.60(20-x) \\ 0.60x &= 12-10 \\ x &= \frac{10}{3}. \end{aligned}$$

The amount to be drained must be $x = \dfrac{10}{3}$ quarts.

73. The costs of x pounds of dried apples is $(1.20)4x$ and the cost of $(20-x)$ pounds of dried apricots is $4(1.80)(20-x)$. Since the 20 lb-mixture costs \$1.68 per quarter-pound, we obtain

$$\begin{aligned} 4(1.68)(20) &= (1.20)4x+4(1.80)(20-x) \\ 134.4 &= 4.80x+144-7.20x \\ 2.40x &= 9.6 \\ x &= 4. \end{aligned}$$

The mix needs 4 lb of dried apples and 16 lb of dried apricots.

74. Let x be the number of pounds of raisins.

$$\begin{aligned} 3.14(x+12) &= 4.50x+2.80(12) \\ 3.14x+37.68 &= 4.50x+33.6 \\ 4.08 &= 1.36x \\ x &= 3. \end{aligned}$$

The mix requires 3 lb of raisins.

75. Let x and $8-x$ be the number of dimes and nickels, respectively. Since the candy bar costs 55 cents, we have $55 = 10x+5(8-x)$. Solving for x, we find $x=3$. Thus, Dana has 3 dimes and 5 nickels.

76. Let x and $x+1$ be the number of dimes and nickels, respectively. The number of quarters is

$$8-[x+(x+1)] = 7-2x.$$

Since the newspaper costs 75 cents, we have $10x+5(x+1)+25(7-2x) = 75$. The solution is $x=3$. Thus, he used 3 dimes, 4 nickels, and 1 quarter.

77. Let x be the amount of water needed. The volume of the resulting solution is $200+x$ ml and the amount of active ingredient in it is $0.4(200)$ or 80 ml. Since the resulting solution is a 25% extract, we have

$$\begin{aligned} 0.25(200+x) &= 80 \\ 200+x &= 320 \\ x &= 120. \end{aligned}$$

The amount of water needed is 120 ml.

78. Let x be the number of pounds of cashews. Then the cost of the cashews and peanuts is $30+5x$, and the mixture weighs $(12+x)$ lb. Since the mixture costs \$4 per pound, we obtain

$$\begin{aligned} 4(12+x) &= 30+5x \\ 48+4x &= 30+5x \\ 18 &= x. \end{aligned}$$

The mix needs 18 lb of cashews.

79. Let x be the number of gallons of the stronger solution. The amount of salt in the new solution is $5(0.2)+x(0.5)$ or $(1+0.5x)$ lb, and the volume of new solution is $(5+x)$ gallons. Since the new solution contains 0.3 lb of salt per gallon, we obtain

$$\begin{aligned} 0.30(5+x) &= 1+0.5x \\ 1.5+0.3x &= 1+0.5x \\ 0.5 &= 0.2x. \end{aligned}$$

Then $x = 2.5$ gallons, the required amount of the stronger solution.

80. Let x be the amount in gallons to be removed. The amount of salt in the new solution is $0.2(5 - x) + (0.5)x$ or $(1 + 0.3x)$ lb, and the volume of new solution is 5 gallons. Since the new solution contains 0.3 lb of salt per gallon, we find

$$\begin{aligned} 1 + 0.3x &= 5(0.3) \\ 0.3x &= 0.5 \end{aligned}$$

Then $x = 5/3$ gallons, the amount to be removed.

81. Let x be the number of hours it takes both pumps to drain the pool simultaneously.

	The part drained in 1 hr
Together	$1/x$
Large pump	$1/5$
Small pump	$1/8$

It follows that $\frac{1}{5} + \frac{1}{8} = \frac{1}{x}$. Multiplying both sides by $40x$, we find $8x + 5x = 40$. Then $x = 40/13$ hr, or about 3 hr and 5 min.

82. Let t be the number of hours that the small pump was used. Since $1/8$ is the part drained by the small pump in 1 hour, the part drained by the small pump in t hours is $t/8$.

Similarly, the part drained by the large pump in $(6 - t)$ hours is $(6 - t)/5$.

Since the sum of the parts drained is 1, we obtain

$$\frac{t}{8} + \frac{6 - t}{5} = 1.$$

Multiplying both sides by 40, we find $5t + (48 - 8t) = 40$ or $8 = 3t$. Then $x = 8/3$ hr, i.e., the small was used for 2 hr, 40 min. The time for the large pump was $(6 - x)$, or 3 hr, 20 min.

83. Let x and $20 - x$ be the number of gallons of the needed 15% and 10% alcohol solutions, respectively. Since the resulting mixture is 12% alcohol, we find

$$\begin{aligned} 0.15x + 0.10(20 - x) &= 20(0.12) \\ 0.05x &= 0.4 \\ x &= 8. \end{aligned}$$

Then 8 gallons of the 15% alcohol solution and 12 gallons of the 10% alcohol solution are needed.

84. Let x and $100 - x$ be the number of quarts needed of the 20% and pure alcohol solutions, respectively. Since the resulting mixture is 30% alcohol, we find

$$\begin{aligned} 0.2x + (100 - x) &= 100(0.3) \\ 70 &= 0.8x \\ 87.5 &= x \end{aligned}$$

Then 87.5 quarts of the 20% alcohol solution and 12.5 quarts pure alcohol are needed.

85. The amount of salt is

$$10 \text{ kg} \times 1\% = 0.1 \text{ kg}.$$

Let x be the amount of salt water after the evaporation. Since 2% salt is the new concentration, we obtain $\frac{0.1}{x} = 0.02$. Then $x = 5$ kg salt water after the evaporation.

86. Let x be the number of men who were laid off. The number of women is $15,000(0.03) = 450$. Since 5% of the remaining workforce are women,

$$\begin{aligned} (15,000 - x)0.05 &= 450 \\ 15,000 - x &= 9000. \end{aligned}$$

The solution is $x = 6000$ men laid off.

87.
a) Decreasing
b) If $M = 40$, then $40 = -1.64n + 74.48$. Solving for n, we find $n \approx 21$. In the year 2021 $(= 2000 + n)$, the mortality rate is 40 per 1000 live births.

88.
a) Increasing
b) If $L = 4$, then $4 = 0.044n + 2.776$. Solving for n, we find $n = 1.224/0.044 \approx 28$. In year 2028 $(= 2000 + n)$, it is predicted that there will be $L = 4$ billion workers.

89. If h is the number of hours it will take two hikers to pick a gallon of wild berries, then

$$\begin{aligned} \frac{1}{2}+\frac{1}{2} &= \frac{1}{h} \\ 1 &= \frac{1}{h} \\ 1 &= h. \end{aligned}$$

Two hikers can pick a gallon of wild berries in 1 hr.

If m is the number of minutes it will take two mechanics to change the oil of a Saturn, then

$$\begin{aligned} \frac{1}{6}+\frac{1}{6} &= \frac{1}{m} \\ \frac{1}{3} &= \frac{1}{m} \\ m &= 3. \end{aligned}$$

Two mechanics can change the oil in 3 minutes.

If w is the number of minutes it will take 60 mechanics to change the oil, then

$$\begin{aligned} 60\cdot\frac{1}{6} &= \frac{1}{w} \\ w &= \frac{1}{10}\text{ min} \\ w &= 6\text{ sec.} \end{aligned}$$

So, 60 mechanics working together can change the oil in 6 sec (an unreasonable situation and answer).

90. a. The average speed is

$$\frac{70(3)+60(1)}{3+1} = 67.5\text{ mph.}$$

If the times T_1 and T_2 over the two time intervals are the same, i.e., $T_1 = T_2$, then the average speed over the two time intervals is the average of the two speeds. To prove this, let V_1 and V_2 be the average speeds in the first and second time intervals respectively. Then the average velocity over the two time intervals (as shown in the left side of the equation) is equal to the average of the two speeds (as shown in the right side of the equation).

$$\begin{aligned} \frac{V_1T_1+V_2T_2}{T_1+T_2} &= \frac{V_1T_1+V_2T_1}{T_1+T_1} \\ &= \frac{T_1(V_1+V_2)}{2T_1} \\ &= \frac{(V_1+V_2)}{2} \end{aligned}$$

b. The average speed is

$$\frac{180+160}{\dfrac{180}{60}+\dfrac{160}{40}} \approx 48.57\text{ mph.}$$

Let D_1 and D_2 be the distances, and let R_1 and R_2 be the speeds in the first and second distance intervals, respectively. If

$$\frac{D_1}{R_1} = \frac{D_2}{R_2}$$

then the average speed over the two distance intervals is equal to the average of the two speeds. As shown below, the average speed over the two distance intervals is the left side of the equation while the average of the two speeds is the right side.

$$\begin{aligned} \frac{D_1+D_2}{\dfrac{D_1}{R_1}+\dfrac{D_2}{R_2}} &= \frac{D_1+D_2}{2\dfrac{D_1}{R_1}} \text{ since } \frac{D_1}{R_1} = \frac{D_2}{R_2} \\ &= \frac{(D_1+D_2)\dfrac{R_1}{D_1}}{2} \\ &= \frac{R_1+\dfrac{D_2R_1}{D_1}}{2} \\ \frac{D_1+D_2}{\dfrac{D_1}{R_1}+\dfrac{D_2}{R_2}} &= \frac{R_1+R_2}{2} \text{ since } R_2 = \frac{D_2R_1}{D_1} \end{aligned}$$

91.

$$\begin{aligned} \frac{x}{2}-\frac{x}{9} &= \frac{1}{6}-\frac{1}{3} \\ \frac{7x}{18} &= -\frac{3}{18} \end{aligned}$$

$$\begin{aligned} 7x &= -3 \\ x &= -\frac{3}{7} \end{aligned}$$

The solution set is $\{-3/7\}$.

92.

$$\begin{aligned} 2x-3 &= \pm 8 \\ 2x &= 3 \pm 8 \\ 2x &= -5, 11 \\ x &= -\frac{5}{2}, \frac{11}{2} \end{aligned}$$

The solution set is $\{-5/2, 11/2\}$.

93.

$$\begin{aligned} 0.999x &= 9990 \\ x &= 10,000 \end{aligned}$$

The solution set is $\{10,000\}$.

94. $24a^4b$

95. $\left((-27)^{1/3}\right)^{-5} = (-3)^{-5} = \dfrac{-1}{243}$

96. **a)** $x(x^4-1) = x(x^2-1)(x^2+1) = x(x-1)(x+1)(x^2+1)$

b) $(w+3)(4w+1)$

97. Let x be the rate of the current in the river, and let $2x$ be the rate that Milo and Bernard can paddle. Since the rate going upstream is x and it takes 21 hours going upstream, the distance going upstream is $21x$.

Note, the rate going downstream is $3x$. Then the time going downstream is

$$\text{time} = \frac{\text{distance}}{\text{rate}} = \frac{21x}{3x} = 7 \text{ hours.}$$

Thus, in order to meet Vince at 5pm, Milo and Bernard must start their return trip at 10am.

98. Let x be the number of minutes it would take the hare to pass the tortoise for the first time.

$$\begin{aligned} \frac{x}{6} &= \frac{x}{10} + 1 \\ 10x &= 6x + 60 \\ x &= 15 \text{ min.} \end{aligned}$$

1.2 Pop Quiz

1. Since $dy = -dx + w$, we get $y = -x + \dfrac{w}{d}$.

2. If w is the width, then the length is $w+3$. Since the perimeter is 62 feet, we obtain

$$\begin{aligned} 2w + 2(w+3) &= 62 \\ 4w + 6 &= 62 \\ 4w &= 56 \\ w &= 14 \end{aligned}$$

The width is $w = 14$ feet.

3. Let x be the amount of water to be added. The volume of the resulting solution is $x+3$ liters, and the amount of alcohol in the solution is $0.7(3)$ or 2.1 liters. Since the resulting solution is 50% alcohol, we obtain

$$\begin{aligned} 0.5(x+3) &= 2.1 \\ x+3 &= 4.2 \\ x &= 1.2 \end{aligned}$$

The amount of water that should be added is 1.2 liters.

1.2 Linking Concepts

(a) 1965

(b) If 150 million more tons were generated than recovered, then

$$\begin{aligned} (3.14n + 87.1) - (0.576n + 3.78) &= 150 \\ 2.564n + 83.32 &= 150 \\ 2.564n &= 66.68 \\ n &\approx 26. \end{aligned}$$

In 1986 ($= 1960 + 26$), 150 million more tons were generated than recovered.

(c) Let $p = 0.13$.

$$\begin{aligned} \frac{0.576n + 3.78}{3.14n + 87.1} &= p \\ 0.576n + 3.78 &= 3.14np + 87.1p \end{aligned}$$

$$\begin{aligned} n(0.576 - 3.14p) &= 87.1p - 3.78 \\ n &= \frac{87.1p - 3.78}{0.576 - 3.14p} \\ n &= \frac{87.1(0.13) - 3.78}{0.576 - 3.14(0.13)} \\ n &\approx 45. \end{aligned}$$

In 2005, 13% of solid waste will be recovered.

(d) Using $p = 0.14$ in part (c), one finds

$$\begin{aligned} n &= \frac{87.1p - 3.78}{0.576 - 3.14p} \\ n &= \frac{87.1(0.14) - 3.78}{0.576 - 3.14(0.14)} \\ n &\approx 61.7. \end{aligned}$$

In 2022 ($= 1960 + 62$), 14% of solid waste will be recovered.

Similarly, when $p = 0.15$ and $p = 0.16$, we obtain $n \approx 88.4$ and $n \approx 138.0$, respectively. In 2048 ($= 1960+88$) and 2098 ($= 1960+138$), the percentages of recovered solid waste are 15% and 16%, respectively.

(e) If we substitute $p = 0.25$ into the formula $n = \dfrac{87.1p - 3.78}{0.576 - 3.14p}$, we obtain $n \approx -86.1$.

No, the recovery rate will never be 25%.

(f) If n is a large number, then

$$p = 100 \cdot \frac{0.576n + 3.78}{3.14n + 87.1} \approx 100 \cdot \frac{0.576}{3.14} \approx 18.3\%.$$

The maximum recovery percentage is 18.3%.

For Thought

1. False, the point $(2, -3)$ is in Quadrant IV.

2. False, the point $(4, 0)$ does not belong to any quadrant.

3. False, since the distance is $\sqrt{(a-c)^2 + (b-d)^2}$.

4. False, since $Ax + By = C$ is a linear equation.

5. True, since the x-intercept can be obtained by replacing y by 0.

6. False, since $\sqrt{7^2 + 9^2} = \sqrt{130} \approx 11.4$

7. True

8. True

9. True

10. False, it is a circle of radius $\sqrt{5}$.

1.3 Exercises

1. ordered

2. abcissa, ordinate

3. Cartesian

4. origin

5. circle

6. completing the square

7. linear equation

8. y-intercept

9. $(4, 1)$, Quadrant I

10. $(-3, 2)$, Quadrant II

11. $(1, 0)$, x-axis

12. $(-1, -5)$, Quadrant III

13. $(5, -1)$, Quadrant IV

14. $(0, -3)$, y-axis

15. $(-4, -2)$, Quadrant III

16. $(-2, 0)$, x-axis

17. $(-2, 4)$, Quadrant II

18. $(1, 5)$, Quadrant I

19. Distance is $\sqrt{(4-1)^2 + (7-3)^2} = \sqrt{9+16} = \sqrt{25} = 5$, midpoint is $(2.5, 5)$

20. Distance is $\sqrt{144 + 25} = 13$, midpoint is $(3, 0.5)$

21. Distance is $\sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$, midpoint is $(0, -1)$

22. Distance is $\sqrt{4+4} = 2\sqrt{2}$, midpoint is $(0, 1)$

23. Distance is $\sqrt{(12-5)^2 + (-11-13)^2} = \sqrt{49+576} = \sqrt{625} = 25$, and the midpoint is $\left(\dfrac{12+5}{2}, \dfrac{-11+13}{2}\right) = \left(\dfrac{17}{2}, 1\right)$

24. Distance is $\sqrt{(-4-4)^2 + (-7-8)^2} = \sqrt{64+225} = \sqrt{289} = 17$, and the midpoint is $\left(\dfrac{-4+4}{2}, \dfrac{-7+8}{2}\right) = \left(0, \dfrac{1}{2}\right)$

25. Distance is $\sqrt{(-1+3\sqrt{3}-(-1))^2 + (4-1)^2} = \sqrt{27+9} = 6$, midpoint is $\left(\dfrac{-2+3\sqrt{3}}{2}, \dfrac{5}{2}\right)$

26. Distance is $\sqrt{8+16} = 2\sqrt{6}$, midpoint is $(1, 0)$

27. Distance is $\sqrt{(1.2+3.8)^2 + (4.8+2.2)^2} = \sqrt{25+49} = \sqrt{74}$, midpoint is $(-1.3, 1.3)$

28. Distance is $\sqrt{49+81} = \sqrt{130}$, midpoint is $(1.2, -3)$

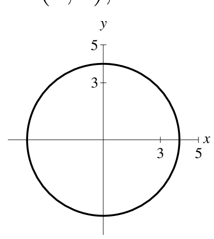
29. Distance is $\sqrt{(a-b)^2 + 0} = |a-b|$, midpoint is $\left(\dfrac{a+b}{2}, 0\right)$

30. Distance is $\dfrac{|a-b|}{2}$, midpoint is $\left(\dfrac{3a+b}{4}, 0\right)$

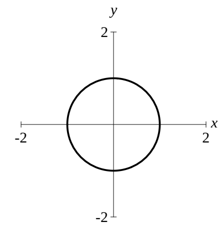
31. Distance is $\dfrac{\sqrt{\pi^2+4}}{2}$, midpoint is $\left(\dfrac{3\pi}{4}, \dfrac{1}{2}\right)$

32. Distance is $\dfrac{\sqrt{\pi^2+4}}{2}$, midpoint is $\left(\dfrac{\pi}{4}, \dfrac{1}{2}\right)$

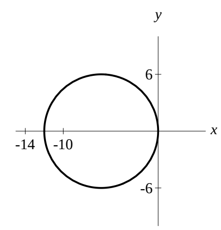
33. Center$(0, 0)$, radius 4



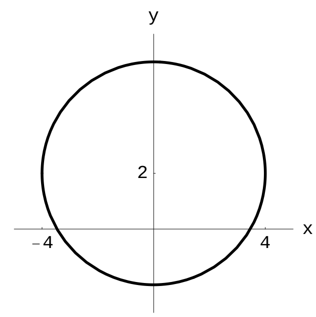
34. Center $(0, 0)$, radius 1



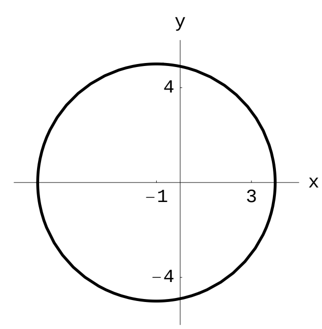
35. Center $(-6, 0)$, radius 6



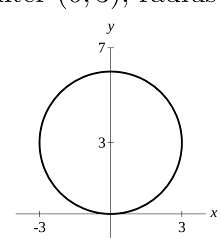
36. Center $(0, 2)$, radius 4



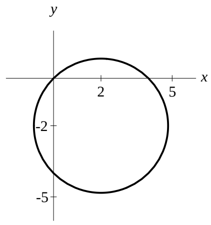
37. Center $(-1, 0)$, radius 5



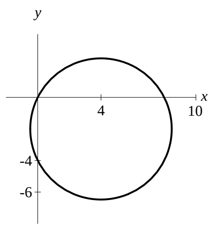
38. Center $(0, 3)$, radius 3



39. Center $(2, -2)$, radius $2\sqrt{2}$



40. Center $(4, -2)$, radius $2\sqrt{5}$



41. $x^2 + y^2 = 49$

42. $x^2 + y^2 = 25$

43. $(x+2)^2 + (y-5)^2 = 1/4$

44. $(x+1)^2 + (y+6)^2 = 1/9$

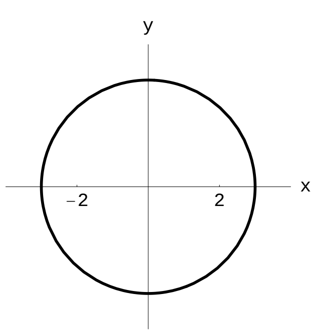
45. The distance between $(3, 5)$ and the origin is $\sqrt{34}$ which is the radius. The standard equation is $(x-3)^2 + (y-5)^2 = 34$.

46. The distance between $(-3, 9)$ and the origin is $\sqrt{90}$ which is the radius. The standard equation is $(x+3)^2 + (y-9)^2 = 90$.

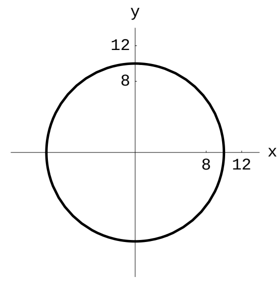
47. The distance between $(5, -1)$ and $(1, 3)$ is $\sqrt{32}$ which is the radius. The standard equation is $(x-5)^2 + (y+1)^2 = 32$.

48. The distance between $(-2, -3)$ and $(2, 5)$ is $\sqrt{80}$ which is the radius. The standard equation is $(x+2)^2 + (y+3)^2 = 80$.

49. Center $(0, 0)$, radius 3



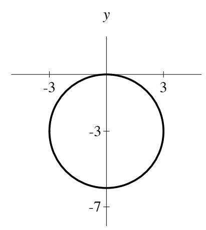
50. Center $(0, 0)$, radius 10



51. Completing the square, we have

$$\begin{aligned} x^2 + (y^2 + 6y + 9) &= 0 + 9 \\ x^2 + (y+3)^2 &= 9. \end{aligned}$$

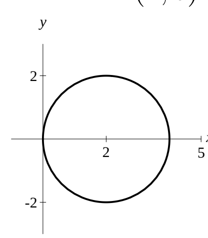
The center is $(0, -3)$ and the radius is 3.



52. Completing the square, we obtain

$$\begin{aligned} (x^2 - 4x + 4) + y^2 &= 4 \\ (x-2)^2 + y^2 &= 4. \end{aligned}$$

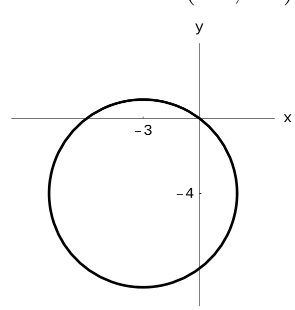
The center is $(2, 0)$ and the radius is 2.



53. Completing the square, we obtain

$$\begin{aligned} (x^2 + 6x + 9) + (y^2 + 8y + 16) &= 9 + 16 \\ (x+3)^2 + (y+4)^2 &= 25. \end{aligned}$$

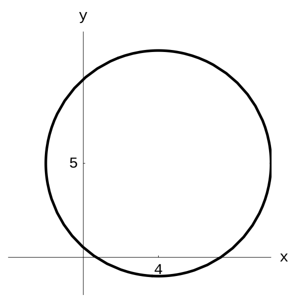
The center is $(-3, -4)$ and the radius is 5.



54. Completing the square, we find

$$\begin{aligned}(x^2-8x+16)+(y^2-10y+25) &= -5+16+25\\ (x-4)^2+(y-5)^2 &= 36.\end{aligned}$$

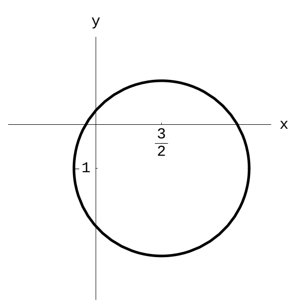
The center is $(4,5)$ and the radius is 6.



55. Completing the square, we find

$$\begin{aligned}\left(x^2-3x+\frac{9}{4}\right)+\left(y^2+2y+1\right) &= \frac{3}{4}+\frac{9}{4}+1\\ \left(x-\frac{3}{2}\right)^2+(y+1)^2 &= 4.\end{aligned}$$

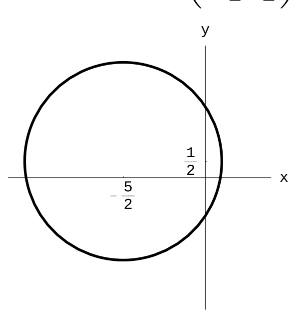
The center is $\left(\frac{3}{2},-1\right)$ and the radius is 2.



56. Completing the square, we find

$$\begin{aligned}\left(x^2+5x+\frac{25}{4}\right)+\left(y^2-y+\frac{1}{4}\right) &= \frac{5}{2}+\frac{25}{4}+\frac{1}{4}\\ \left(x+\frac{5}{2}\right)^2+\left(y-\frac{1}{2}\right)^2 &= 9.\end{aligned}$$

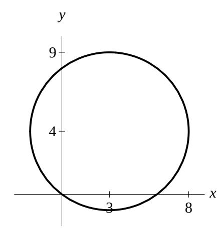
The center is $\left(-\frac{5}{2},\frac{1}{2}\right)$ and the radius is 3.



57. Completing the square, we obtain

$$\begin{aligned}(x^2-6x+9)+(y^2-8y+16) &= 9+16\\ (x-3)^2+(y-4)^2 &= 25.\end{aligned}$$

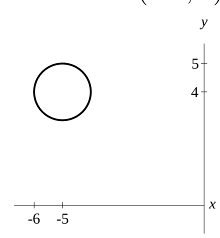
The center is $(3,4)$ and the radius is 5.



58. Completing the square, we find

$$\begin{aligned}(x^2+10x+25)+(y^2-8y+16) &= -40+25+16\\ (x+5)^2+(y-4)^2 &= 1.\end{aligned}$$

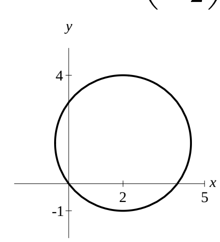
The center is $(-5,4)$ and the radius is 1.



59. Completing the square, we obtain

$$\begin{aligned}(x^2-4x+4)+\left(y^2-3y+\frac{9}{4}\right) &= 4+\frac{9}{4}\\ (x-2)^2+\left(y-\frac{3}{2}\right)^2 &= \frac{25}{4}.\end{aligned}$$

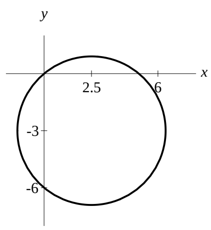
The center is $\left(2,\frac{3}{2}\right)$ and the radius is $\frac{5}{2}$.



60. Completing the square, we obtain

$$\begin{aligned}\left(x^2-5x+\frac{25}{4}\right)+\left(y^2+6y+9\right) &= 0+\frac{25}{4}+9\\ \left(x-\frac{5}{2}\right)^2+(y+3)^2 &= \frac{61}{4}.\end{aligned}$$

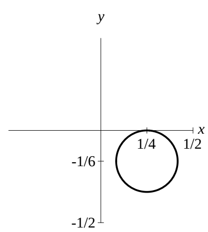
The center is $\left(\frac{5}{2}, -3\right)$ and the radius is $\frac{\sqrt{61}}{2}$.



61. Completing the square, we obtain

$$\begin{aligned}\left(x^2 - \frac{1}{2}x + \frac{1}{16}\right) + \left(y^2 + \frac{1}{3}y + \frac{1}{36}\right) &= \frac{1}{36} \\ \left(x - \frac{1}{4}\right)^2 + \left(y + \frac{1}{6}\right)^2 &= \frac{1}{36}.\end{aligned}$$

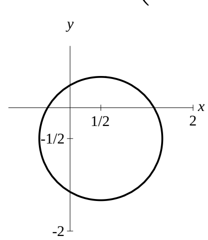
The center is $\left(\frac{1}{4}, -\frac{1}{6}\right)$ and the radius is $\frac{1}{6}$.



62. Completing the square, we obtain

$$\begin{aligned}\left(x^2 - x + \frac{1}{4}\right) + \left(y^2 + y + \frac{1}{4}\right) &= \frac{1}{2} + \frac{1}{4} + \frac{1}{4} \\ \left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 &= 1.\end{aligned}$$

The center is $\left(\frac{1}{2}, -\frac{1}{2}\right)$ and the radius is 1.



63. a. Since the center is $(0, 0)$ and the radius is 7, the standard equation is $x^2 + y^2 = 49$.

b. The radius, which is the distance between $(1, 0)$ and $(3, 4)$, is given by

$$\sqrt{(3-1)^2 + (4-0)^2} = \sqrt{20}.$$

Together with the center $(1, 0)$, it follows that the standard equation is

$$(x-1)^2 + y^2 = 20.$$

c. Using the midpoint formula, the center is

$$\left(\frac{3-1}{2}, \frac{5-1}{2}\right) = (1, 2).$$

The diameter is

$$\sqrt{(3-(-1))^2 + (5-(-1))^2} = \sqrt{52}.$$

Since the square of the radius is

$$\left(\frac{1}{2}\sqrt{52}\right)^2 = 13,$$

the standard equation is

$$(x-1)^2 + (y-2)^2 = 13.$$

64. a. The center is $(0, -1)$, the radius is 4, and the standard equation is

$$x^2 + (y+1)^2 = 16.$$

b. The radius which is the distance between $(1, 3)$ and $(0, 0)$ is given by

$$\sqrt{(1-0)^2 + (3-0)^2} = \sqrt{10}.$$

Together with the center $(1, 3)$, it follows that the standard equation is

$$(x-1)^2 + (y-3)^2 = 10.$$

c. The center by using the midpoint formula is

$$\left(\frac{-2+3}{2}, \frac{2+(-3)}{2}\right) = \left(\frac{1}{2}, -\frac{1}{2}\right).$$

The diameter is

$$\sqrt{(-2-3)^2 + (2-(-3))^2} = \sqrt{50}.$$

Since the square of the radius is $\left(\frac{1}{2}\sqrt{50}\right)^2 = \frac{25}{2}$, the standard equation is $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{25}{2}$.

65. **a.** Since the center is $(2, -3)$ and the radius is 2, the standard equation is

$$(x-2)^2 + (y+3)^2 = 4.$$

b. The center is $(-2, 1)$, the radius is 1, and the standard equation is

$$(x+2)^2 + (y-1)^2 = 1.$$

c. The center is $(3, -1)$, the radius is 3, and the standard equation is

$$(x-3)^2 + (y+1)^2 = 9.$$

d. The center is $(0, 0)$, the radius is 1, and the standard equation is

$$x^2 + y^2 = 1.$$

66. **a.** Since the center is $(-2, -4)$ and the radius is 3, the standard equation is

$$(x+2)^2 + (y+4)^2 = 9.$$

b. The center is $(2, 3)$, the radius is 4, and the standard equation is

$$(x-2)^2 + (y-3)^2 = 16.$$

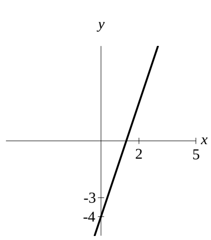
c. The center is $(0, -1)$, the radius is 3, and the standard equation is

$$x^2 + (y+1)^2 = 9.$$

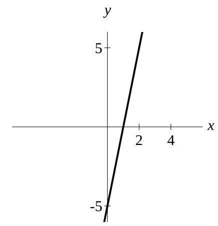
d. The center is $(0, 0)$, the radius is 5, and the standard equation is

$$x^2 + y^2 = 25.$$

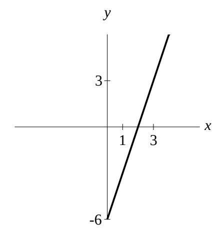
67. $y = 3x - 4$ goes through $(0, -4)$, $\left(\dfrac{4}{3}, 0\right)$.



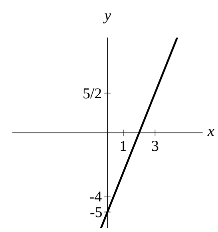
68. $y = 5x - 5$ goes through $(0, -5), (1, 0)$.



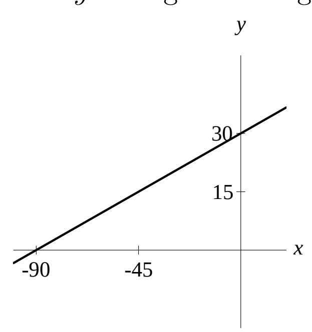
69. $3x - y = 6$ goes through $(0, -6)$, $(2, 0)$.



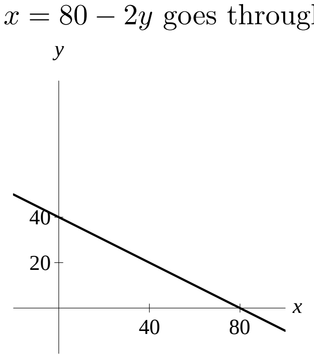
70. $5x - 2y = 10$ goes through $(0, -5)$, $(2, 0)$.



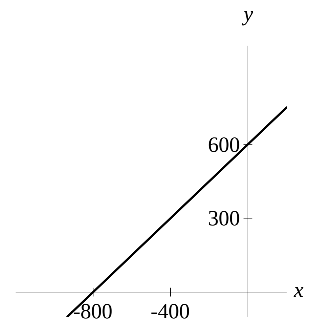
71. $x = 3y - 90$ goes through $(0, 30), (-90, 0)$.



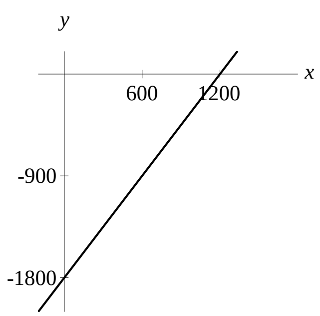
72. $x = 80 - 2y$ goes through $(0, 40), (80, 0)$.



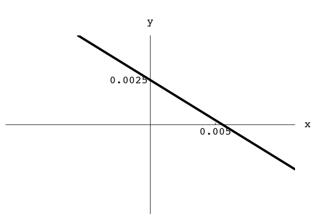
73. $\frac{2}{3}y - \frac{1}{2}x = 400$ goes through $(0, 600), (-800, 0)$.



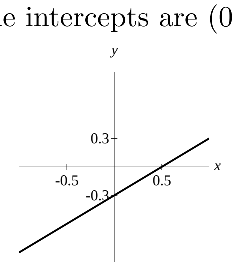
74. $\frac{1}{2}x - \frac{1}{3}y = 600$ goes through $(0, -1800), (1200, 0)$.



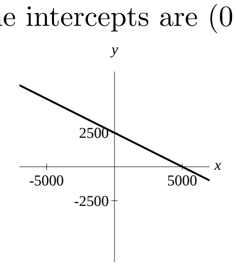
75. The intercepts are $(0, 0.0025), (0.005, 0)$.



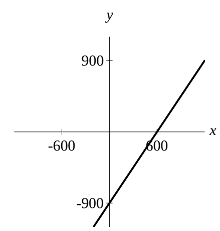
76. The intercepts are $(0, -0.3), (0.5, 0)$.



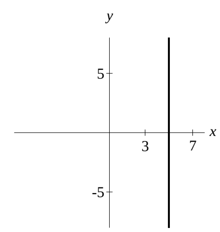
77. The intercepts are $(0, 2500), (5000, 0)$.



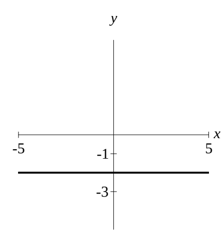
78. The intercepts are $(0, -900), (600, 0)$.



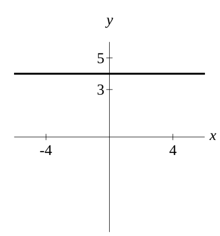
79. $x = 5$



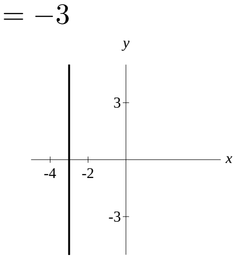
80. $y = -2$



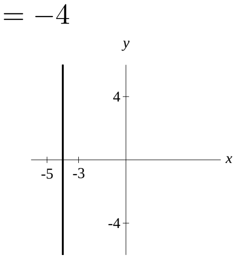
81. $y = 4$



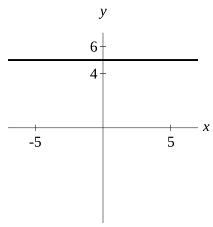
82. $x = -3$



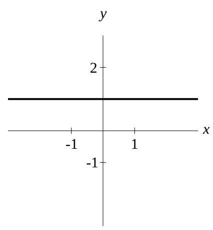
83. $x = -4$



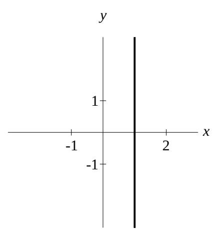
84. $y = 5$



85. Solving for y, we have $y = 1$.



86. Solving for x, we get $x = 1$.



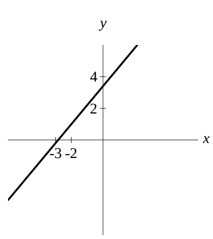
87. Since the x-intercept of $y = 2.4x - 8.64$ is $(3.6, 0)$, the solution set of $2.4x - 8.64 = 0$ is $\{3.6\}$.

88. Since the x-intercept of $y = 8.84 - 1.3x$ is $(6.8, 0)$, the solution set of $8.84 - 1.3x = 0$ is $\{6.8\}$.

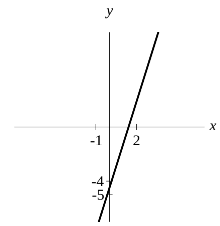
89. Since the x-intercept of $y = -\dfrac{3}{7}x + 6$ is $(14, 0)$, the solution set of $-\dfrac{3}{7}x + 6 = 0$ is $\{14\}$.

90. Since the x-intercept of $y = \dfrac{5}{6}x + 30$ is $(-36, 0)$, the solution set of $\dfrac{5}{6}x + 6 = 0$ is $\{-36\}$.

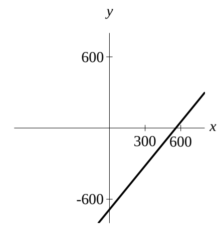
91. The solution is $x = -\dfrac{3.4}{12} \approx -2.83$.



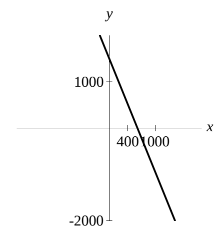
92. The solution is $\dfrac{4.5}{3.2} \approx 1.41$



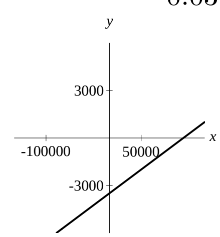
93. The solution is $\dfrac{687}{1.23} \approx 558.54$



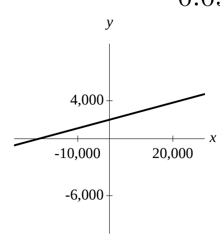
94. The solution is $\dfrac{1500}{2.46} \approx 609.76$



95. The solution is $\dfrac{3497}{0.03} \approx 116,566.67$



96. The solution is $\dfrac{-2000}{0.09} \approx -22,222.22$

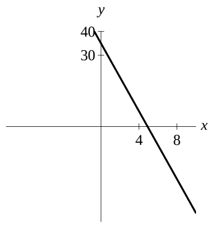


97. Note,

$$\begin{aligned} 4.3 - 3.1(2.3x) + 3.1(9.9) &= 0 \\ 4.3 - 7.13x + 30.69 &= 0 \\ 34.99 - 7.13x &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{34.99}{7.13} \\ x &\approx 4.91. \end{aligned}$$

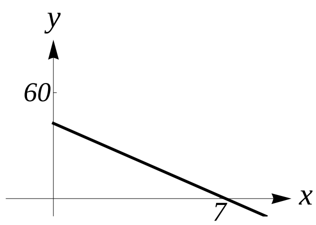
The solution set is $\{4.91\}$.



98. Simplifying, we obtain

$$\begin{aligned} 9.4x - (15.295x - 42.6512) &= 0 \\ -5.895x + 42.6512 &= 0 \\ x &= \frac{42.6512}{5.895} \\ x &\approx 7.24. \end{aligned}$$

The solution set is $\{7.24\}$.



99. **a)** Let $0 < r < 2$ be the radius of the smallest circle centered at $(2-r, 0)$. Apply the Pythagorean theorem to the right triangle with vertices at $(2-r,0)$, $(0,0)$, and $(0,-1)$. Then

$$\begin{aligned} 1 + (2-r)^2 &= (r+1)^2 \\ 1 + (4 - 4r + r^2) &= r^2 + 2r + 1 \\ 4 &= 6r. \end{aligned}$$

Then $r = 2/3$, and the diameter of each smallest circle is $4/3$ cm.

b) Since $r = 2/3$, the centers of the smallest circles are at $(2-r, 0)$ and $(-2+r, 0)$. Equivalently, the centers are at $(\pm 4/3, 0)$. Thus, the equations of the smallest circles are

$$(x - 4/3)^2 + y^2 = 4/9$$

and

$$(x + 4/3)^2 + y^2 = 4/9.$$

100. Since the two tangent lines are perpendicular, each of the tangent lines passes through the center of the other circle. The line segment joining the centers of the circle is the hypotenuse. Since the radii are 5 and 12, the hypotenuse is 13 by the Pythagorean theorem. Then the distance between the centers is 13 units.

101. **a)** Substitute $h = 0$ into $h = 0.229n + 2.913$. Solving for n, we get

$$n = -\frac{2.913}{0.229} \approx -12.72.$$

Then the n-intercept is near $(-12.72, 0)$. There were no unmarried-couple households in 1977 (i.e., 13 years before 1990). The answer does not make sense.

b) Let $n = 0$. Solving for h, one finds $h = 0.229(0) + 2.913 = 2.913$.
The h-intercept is $(0, 2.913)$.
In 1990 (i.e., $n = 0$), there were 2,913,000 unmarried-couple households.

c) If $n = 25$, then

$$h = 0.229(25) + 2.913 \approx 8.6.$$

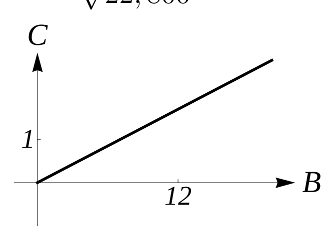
In 2015 (i.e., $n = 25$), it is predicted that there will be 8.6 million unmarried-couple households.

102. The midpoint is

$$\left(\frac{0+42}{2}, \frac{20.8+26.1}{2} \right) = (21, 23.5).$$

The median age at first marriage in 1991 $(= 1970 + 21)$ was 23.5 years.

103. Given $D = 22,800$ lbs, the graph of $C = \dfrac{4B}{\sqrt[3]{22,800}}$ is given below.



For Island Packet 40, $C = \dfrac{4(12 + \frac{11}{12})}{\sqrt[3]{22,800}} \approx 1.8.$

104. The maximum allowable beam B satisfies

$$\begin{aligned} \frac{4B}{\sqrt[3]{22,800}} &= 2 \\ B &= \frac{\sqrt[3]{22,800}}{2} \\ B &\approx 14.178 \\ B &\approx 14 \text{ ft}, 2 \text{ in.} \end{aligned}$$

From the graph in Exercise 103, for a fixed displacement, a boat is more likely to capsize as its beam gets larger.

105. By the distance formula, we find

$$AB = \sqrt{(1+4)^2 + (1+5)^2} = \sqrt{25+36} = \sqrt{61}.$$

Similarly, we obtain

$$BC = \sqrt{61}$$

and

$$AC = \sqrt{244} = 2\sqrt{61}.$$

Since

$$AB + BC = AC$$

we conclude that A, B, and C are collinear.

106. One can assume the vertices of the right triangle are $C(0,0)$, $A(a,0)$, and $B(0,b)$.

The midpoint of the hypotenuse is $\left(\frac{a}{2}, \frac{b}{2}\right)$.

The distance between the midpoint and C is $\frac{\sqrt{a^2+b^2}}{2}$, which is half the distance between A and B. Thus, the midpoint is equidistant from all vertices.

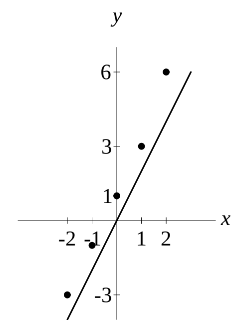
107. The distance between $(10,0)$ and $(0,0)$ is 10. The distance between $(1,3)$ and the origin is $\sqrt{10}$.

If two points have integer coordinates, then the distance between them is of the form $\sqrt{s^2+t^2}$ where s^2, t^2 lies in the set

$$\{0, 1, 2^2, 3^2, 4^2, ...\} = \{0, 1, 4, 9, 16, ...\}.$$

Note, there exists no pair s^2 and t^2 in $\{0, 1, 4, 9, 16, ...\}$ satisfying $s^2 + t^2 = 19$. Thus, one cannot find two points with integer coordinates whose distance between them is $\sqrt{19}$.

108. Five points satisfying $y > 2x$ are $(-2,-3)$, $(-1,-1)$, $(0,1)$, $(1,3)$, and $(2,6)$.



The solution set to $y > 2x$ consists of all points in the xy-plane that lie above the line $y = 2x$.

110. The distance between $(0,0)$ and $(2m, m^2-1)$ is $\sqrt{(2m)^2 + (m^2-1)^2} = \sqrt{4m^2 + m^4 - 2m^2 + 1} = \sqrt{m^4 + 2m^2 + 1} = \sqrt{(m^2+1)^2} = m^2 + 1$

The distance between $(0,0)$ and $(2mn, m^2 - n^2)$ is $\sqrt{(2mn)^2 + (m^2-n^2)^2} = \sqrt{4m^2n^2 + m^4 - 2m^2n^2 + n^4} = \sqrt{m^4 + 2m^2n^2 + n^4} = \sqrt{(m^2+n^2)^2} = m^2 + n^2$.

111.

a) Conditional equation, solution is $x = \frac{1}{2}$.

b) Identity, both equivalent to $2x + 4$.

c) Inconsistent equation, i.e., no solution.

112. Multiply both sides by $x^2 - 9$.

$$\begin{aligned} 4(x+3) + (x-3) &= x \\ 5x + 9 &= x \\ 4x &= -9 \\ x &= -\frac{9}{4} \end{aligned}$$

The solution set is $\{-9/4\}$.

113. Cross-multiply to obtain

$$\begin{aligned} (x-2)(x+9) &= (x+3)(x+4) \\ x^2 + 7x - 18 &= x^2 + 7x + 12 \\ -18 &= 12 \end{aligned}$$

Inconsistent, the solution set is $\emptyset$.

114. If x is the selling price, then

$$\begin{aligned} 0.92x &= 180,780 \\ x &= \$196,500 \end{aligned}$$

115.

$$\begin{aligned} ax + b &= cx + d \\ ax - cx &= d - b \\ x(a - c) &= d - b \\ x &= \frac{d - b}{a - c} \end{aligned}$$

116. $\left(3a^9b^5\right)^3 = 27a^{27}b^{15}$

117. Let x be the uniform width of the swath. When Eugene is half done, we find

$$(300 - 2x)(400 - 2x) = \frac{300(400)}{2}.$$

We could rewrite the equation as

$$\begin{aligned} x^2 - 350x + 15,000 &= 0 \\ (x - 50)(x - 300) &= 0. \end{aligned}$$

Then $x = 50$ or $x = 300$. Since $x = 300$ is not possible, the width of the swath is $x = 50$ feet.

118. Since $10^{40} = 2^{40}5^{40}$, the number of positive factors is $41 \times 41 = 1681$.

1.3 Pop Quiz

1. The distance is $\sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$.

2. Center $(3, -5)$, radius 9

3. Completing the square, we find

$$\begin{aligned} (x^2 + 4x + 4) + (y^2 - 10y + 25) &= -28 + 4 + 25 \\ (x + 2)^2 + (y - 5)^2 &= 1. \end{aligned}$$

The center is $(-2, 5)$ and the radius is 1.

4. The distance between $(3, 4)$ and the origin is 5, which is the radius. The circle is given by $(x - 3)^2 + (y - 4)^2 = 25$.

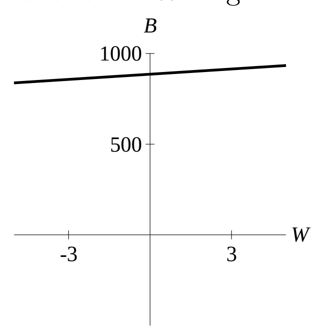
5. By setting $x = 0$ and $y = 0$ in $2x - 3y = 12$ we find $-3y = 12$ and $2x = 12$, respectively. Since $y = -4$ and $x = 6$ are the solutions of the two equations, the intercepts are $(0, -4)$ and $(6, 0)$.

6. $(5, -1)$

1.3 Linking Concepts

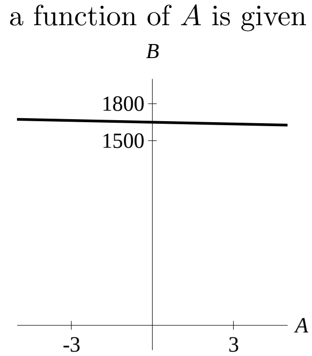
a) For a 21 year old, 5′10", 160 pound male, one finds the values $W = 160(0.4356) = 69.696$ kg, $H = \left(5 + \frac{10}{12}\right)(30.48) = 177.8$ cm, and $A = 21$. With these values, one finds $B = 655.096 + 9.563W + 1.85H - 4.676A$ $\approx 1,552.3$ calories.

b) If one fixes $H = 177.8$ cm. and $A = 21$, then $B = 655.096 + 9.563W + 328.93 - 98.196$ or $B = 9.563W + 885.83$. A graph of B as a function of W is given below.



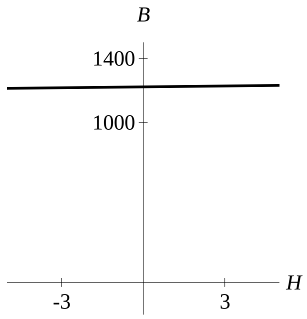
c) If H and A are fixed, then B increases as W increases.

d) If one fixes $H = 177.8$ cm and $W = 69.696$ kg, then $B = 655.096 + 666.503 + 328.93 - 4.676A$ or $B = -4.676A + 1650.529$. A graph of B as a function of A is given in the next column.



e) If H and W are fixed, then the basic energy requirement B decreases as A increases.

f) If one fixes $A = 21$ cm and $W = 69.696$ kg, then $B = 655.096 + 666.503 + 1.85H - 98.196$ or $B = 1.85H + 1223.403$. A graph of B as a function of H is given below.



g) If A and W are fixed, then the basic energy requirement B increases as H increases

h) The equations in parts (b), (d), and (f) are of the form $y = mx+b$. If $m > 0$, then y increases as x increases . If $m < 0$, then y decreases as x increases.

For Thought

1. False, the slope is $\dfrac{3-2}{3-2} = 1$.

2. False, the slope is $\dfrac{5-1}{-3-(-3)} = \dfrac{4}{0}$ which is undefined.

3. False, slopes of vertical lines are undefined.

4. False, it is a vertical line. **5.** True

6. False, $x = 1$ cannot be written in the slope-intercept form.

7. False, the slope is -2.

8. True **9.** False **10.** True

1.4 Exercises

1. rise

2. run

3. slope

4. point-slope

5. slope-intercept

6. parallel

7. perpendicular

8. is a function of

9. $\dfrac{5-3}{4+2} = \dfrac{1}{3}$ **10.** $\dfrac{6-2}{3+1} = 1$

11. $\dfrac{3+5}{1-3} = -4$ **12.** $\dfrac{-1+3}{2-5} = -\dfrac{2}{3}$

13. $\dfrac{2-2}{5+3} = 0$ **14.** $\dfrac{0-0}{0-5} = 0$

15. $\dfrac{1/2 - 1/4}{1/4 - 1/8} = \dfrac{1/4}{1/8} = 2$

16. $\dfrac{1/3 - 1/2}{1/6 - (-1/3)} = \dfrac{-1/6}{3/6} = -\dfrac{1}{3}$

17. $\dfrac{3-(-1)}{5-5} = \dfrac{4}{0}$, no slope

18. $\dfrac{-6-2}{-7-(-7)} = \dfrac{-8}{0}$, no slope

19. The slope is $m = \dfrac{4-(-1)}{3-(-1)} = \dfrac{5}{4}$. Since $y+1 = \dfrac{5}{4}(x+1)$, we get $y = \dfrac{5}{4}x + \dfrac{5}{4} - 1$ or $y = \dfrac{5}{4}x + \dfrac{1}{4}$.

20. The slope is $m = \dfrac{5-1}{3-(-2)} = \dfrac{4}{5}$. Since $y-1 = \dfrac{4}{5}(x+2)$, we get $y = \dfrac{4}{5}x + \dfrac{8}{5} + 1$ or $y = \dfrac{4}{5}x + \dfrac{13}{5}$.

21. The slope is $m = \dfrac{-1-6}{4-(-2)} = -\dfrac{7}{6}$. Since $y + 1 = -\dfrac{7}{6}(x-4)$, we obtain $y = -\dfrac{7}{6}x + \dfrac{14}{3} - 1$ or $y = -\dfrac{7}{6}x + \dfrac{11}{3}$.

22. The slope is $m = \dfrac{1-5}{2-(-3)} = -\dfrac{4}{5}$. Since $y - 1 = -\dfrac{4}{5}(x-2)$, we obtain $y = -\dfrac{4}{5}x + \dfrac{8}{5} + 1$ or $y = -\dfrac{4}{5}x + \dfrac{13}{5}$.

23. The slope is $m = \dfrac{5-5}{-3-3} = 0$. Since $y - 5 = 0(x-3)$, we get $y = 5$.

24. The slope is $m = \dfrac{4-4}{2-(-6)} = 0$. Since $y - 4 = 0(x-2)$, we get $y = 4$.

25. Since $m = \dfrac{12-(-3)}{4-4} = \dfrac{15}{0}$ is undefined, the equation of the vertical line is $x = 4$.

26. Since $m = \dfrac{4-6}{-5-(-5)} = \dfrac{-2}{0}$ is undefined, the equation of the vertical line is $x = -5$.

27. The slope of the line through $(0,-1)$ and $(3,1)$ is $m = \dfrac{2}{3}$. Since the y-intercept is $(0,-1)$, the line is given by $y = \dfrac{2}{3}x - 1$.

28. The slope of the line through $(0,2)$ and $(3,-1)$ is $m = -1$. Since the y-intercept is $(0,2)$, the line is given by $y = -x + 2$.

29. The slope of the line through $(1,4)$ and $(-1,-1)$ is $m = \dfrac{5}{2}$. Solving for y in $y + 1 = \dfrac{5}{2}(x+1)$, we get $y = \dfrac{5}{2}x + \dfrac{3}{2}$.

30. The slope of the line through $(-2,4)$ and $(1,0)$ is $m = -\dfrac{4}{3}$. Solving for y in $y - 0 = -\dfrac{4}{3}(x-1)$, we get $y = -\dfrac{4}{3}x + \dfrac{4}{3}$.

31. The slope of the line through $(0,4)$ and $(2,0)$ is $m = -2$. Since the y-intercept is $(0,4)$, the line is given by $y = -2x + 4$.

32. The slope of the line through $(0,0)$ and $(4,1)$ is $m = \dfrac{1}{4}$. Since the y-intercept is $(0,0)$, the line is given by $y = \dfrac{1}{4}x$.

33. The slope of the line through $(1,4)$ and $(-3,-2)$ is $m = \dfrac{3}{2}$. Solving for y in $y - 4 = \dfrac{3}{2}(x-1)$, we get $y = \dfrac{3}{2}x + \dfrac{5}{2}$.

34. The slope of the line through $(4,2)$ and $(-1,-1)$ is $m = \dfrac{3}{5}$. Solving for y in $y + 1 = \dfrac{3}{5}(x+1)$, we get $y = \dfrac{3}{5}x - \dfrac{2}{5}$.

35. $y = \dfrac{3}{5}x - 2$, slope is $\dfrac{3}{5}$, y-intercept is $(0,-2)$

36. $y = x - \dfrac{1}{2}$, slope is 1, y-intercept is $(0,-1/2)$

37. Since $y - 3 = 2x - 8$, $y = 2x - 5$.
The slope is 2 and y-intercept is $(0,-5)$.

38. Since $y + 5 = -3x - 3$, $y = -3x - 8$.
The slope is -3 and y-intercept is $(0,-8)$.

39. Since $y + 1 = \dfrac{1}{2}x + \dfrac{3}{2}$, $y = \dfrac{1}{2}x + \dfrac{1}{2}$.
The slope is $\dfrac{1}{2}$ and y-intercept is $\left(0, \dfrac{1}{2}\right)$.

40. Since $y - 2 = -\dfrac{3}{2}x - \dfrac{15}{2}$, $y = -\dfrac{3}{2}x - \dfrac{11}{2}$.
The slope is $-\dfrac{3}{2}$ and y-intercept is $\left(0, -\dfrac{11}{2}\right)$.

41. Since $y = 4$, the slope is $m = 0$ and the y-intercept is $(0,4)$.

42. Since $y = 5$, the slope is $m = 0$ and the y-intercept is $(0,5)$.

43.

$$\begin{aligned} y - 5 &= \frac{1}{4}(x+8) \\ y - 5 &= \frac{1}{4}x + 2 \\ y &= \frac{1}{4}x + 7 \end{aligned}$$

44.

$$\begin{aligned} y - 9 &= -\frac{1}{3}(x-6) \\ y - 9 &= -\frac{1}{3}x + 2 \\ y &= -\frac{1}{3}x + 11 \end{aligned}$$

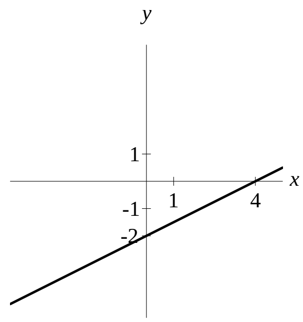
45.

$$\begin{aligned} y + 2 &= -\frac{1}{2}(x+3) \\ y + 2 &= -\frac{1}{2}x - \frac{3}{2} \\ y &= -\frac{1}{2}x - \frac{7}{2} \end{aligned}$$

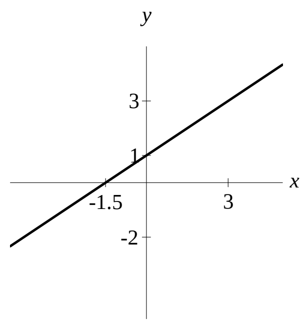
46.

$$\begin{aligned} y - 3 &= \frac{2}{3}(x-4) \\ y - 3 &= \frac{2}{3}x - \frac{8}{3} \\ y &= \frac{2}{3}x + \frac{1}{3} \end{aligned}$$

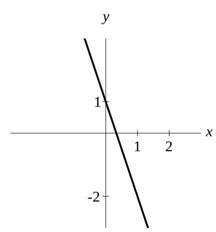
47. $y = \frac{1}{2}x - 2$ goes through the points $(0, -2), (2, -1)$, and $(4, 0)$.



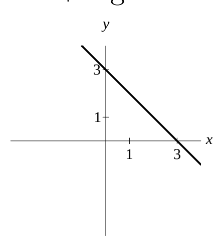
48. $y = \frac{2}{3}x + 1$ goes through $(0, 1), (-1.5, 0)$



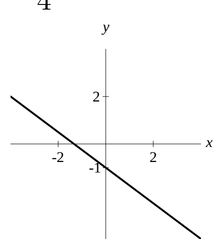
49. $y = -3x + 1$ goes through $(0, 1), (1, -2)$



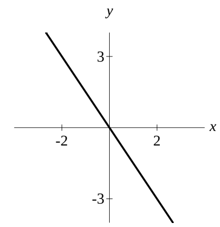
50. $y = -x + 3$ goes through $(0, 3), (1, 2)$



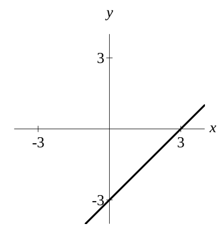
51. $y = -\frac{3}{4}x - 1$ goes through $(0, -1), (-4/3, 0)$



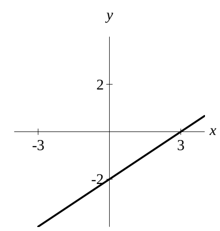
52. $y = -\frac{3}{2}x$ goes through $(0, 0), (2, -3)$



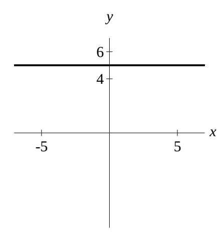
53. $x - y = 3$ goes through $(0, -3), (3, 0)$



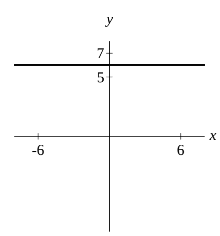
54. $2x - 3y = 6$ goes through $(0, -2), (3, 0)$



55. $y = 5$ is a horizontal line



56. $y = 6$ is a horizontal line



57. Since $m = \frac{4}{3}$ and $y - 0 = \frac{4}{3}(x - 3)$, we have $4x - 3y = 12$.

58. Since $m = \frac{3}{2}$ and $y - 0 = \frac{3}{2}(x + 2)$, we have $3x - 2y = -6$.

59. Since $m = \frac{4}{5}$ and $y - 3 = \frac{4}{5}(x-2)$, we obtain $5y - 15 = 4x - 8$ and $4x - 5y = -7$.

60. Since $m = \frac{5}{6}$ and $y + 1 = \frac{5}{6}(x-4)$, we obtain $6y + 6 = 5x - 20$ and $5x - 6y = 26$.

61. $x = -4$ is a vertical line.

62. $y = 6$ is a horizontal line.

63. Note, the slope is

$$m = \frac{\frac{2}{3}+2}{2+\frac{1}{2}} = \frac{8/3}{5/2} = \frac{16}{15}.$$

Using the point-slope form, we obtain a standard equation of the line using only integers.

$$\begin{aligned} y - \frac{2}{3} &= \frac{16}{15}(x-2) \\ 15y - 10 &= 16(x-2) \\ 15y - 10 &= 16x - 32 \\ -16x + 15y &= -22 \\ 16x - 15y &= 22 \end{aligned}$$

64. Note, the slope is

$$m = \frac{\frac{1}{8}+3}{-5-\frac{3}{4}} = \frac{25/8}{-23/4} = -\frac{25}{46}.$$

Using the point-slope form, we get a standard equation of the line using only integers.

$$\begin{aligned} y + 3 &= -\frac{25}{46}\left(x - \frac{3}{4}\right) \\ 46y + 138 &= -25\left(x - \frac{3}{4}\right) \\ 184y + 552 &= -25(4x-3) \\ 184y + 552 &= -100x + 75 \\ 100x + 184y &= -477 \end{aligned}$$

65. The slope is

$$m = \frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{2}+\frac{1}{3}} = \frac{1/20}{5/6} = \frac{3}{50}.$$

Using the point-slope form, we get a standard equation of the line using only integers.

$$\begin{aligned} y - \frac{1}{4} &= \frac{3}{50}\left(x - \frac{1}{2}\right) \\ 100y - 25 &= 6\left(x - \frac{1}{2}\right) \\ 100y - 25 &= 6x - 3 \\ -22 &= 6x - 100y \\ 3x - 50y &= -11 \end{aligned}$$

66. The slope is

$$m = \frac{\frac{1}{4}+\frac{1}{6}}{-\frac{3}{8}-\frac{1}{2}} = \frac{5/12}{-7/8} = -\frac{10}{21}.$$

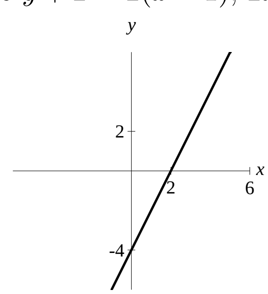
Using the point-slope form, we obtain a standard equation of the line using only integers.

$$\begin{aligned} y - \frac{1}{4} &= -\frac{10}{21}\left(x + \frac{3}{8}\right) \\ 84y - 21 &= -40\left(x + \frac{3}{8}\right) \\ 84y - 21 &= -40x - 15 \\ 40x + 84y &= 6 \\ 20x + 42y &= 3 \end{aligned}$$

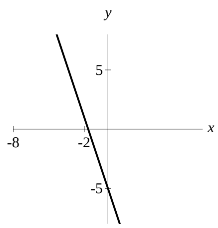
67. 0.5 **68.** 1/3 **69.** -1

70. $-3/2$ **71.** 0 **72.** 0

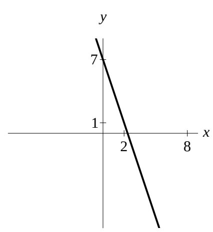
73. Since $y + 2 = 2(x-1)$, $2x - y = 4$



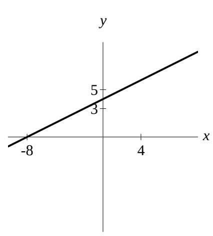
74. Since $y - 4 = -3(x + 3)$, $3x + y = -5$



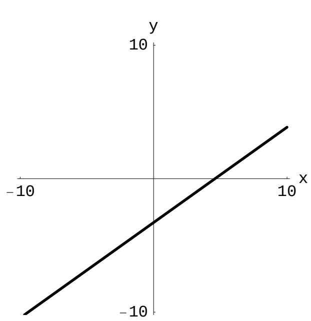
75. Since slope of $y = -3x$ is -3 and $y - 4 = -3(x - 1)$, we obtain $3x + y = 7$.



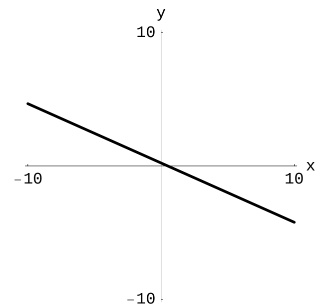
76. Since slope of $y = \frac{1}{2}x + 6$ is $\frac{1}{2}$ and $y - 3 = \frac{1}{2}(x+2)$, we get $2y - 6 = x + 2$ and $x - 2y = -8$



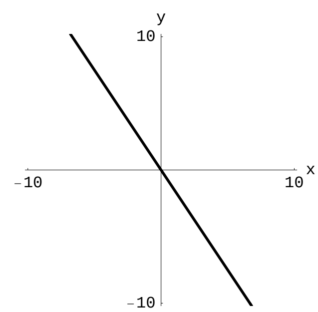
77. Since the slope of $5x - 7y = 35$ is $\frac{5}{7}$, we obtain $y - 1 = \frac{5}{7}(x - 6)$. Multiplying by 7, we get $7y - 7 = 5x - 30$ or equivalently $5x - 7y = 23$.



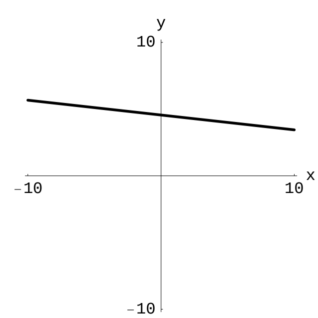
78. Since the slope of $4x + 9y = 5$ is $-\frac{4}{9}$, we obtain $y - 2 = -\frac{4}{9}(x + 4)$. Multiplying by 9, we get $9y - 18 = -4x - 16$ or equivalently $4x + 9y = 2$.



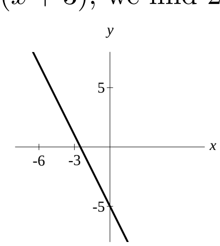
79. Since the slope of $y = \frac{2}{3}x + 5$ is $\frac{2}{3}$, we obtain $y + 3 = -\frac{3}{2}(x - 2)$. Multiplying by 2, we find $2y + 6 = -3x + 6$ or equivalently $3x + 2y = 0$.



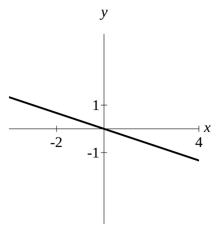
80. Since the slope of $y = 9x + 5$ is 9, we obtain $y - 4 = -\frac{1}{9}(x - 5)$. Multiplying by 9, we find $9y - 36 = -x + 5$ or equivalently $x + 9y = 41$.



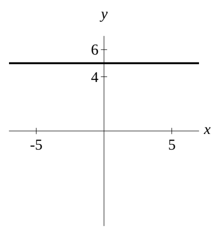
81. Since slope of $y = \frac{1}{2}x - \frac{3}{2}$ is $\frac{1}{2}$ and $y - 1 = -2(x + 3)$, we find $2x + y = -5$.



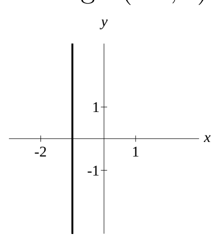
82. Since slope of $y = 3x - 9$ is 3 and $y - 0 = -\frac{1}{3}(x - 0)$, we have $x + 3y = 0$



83. Since $x = 4$ is a vertical line, the horizontal line through $(2, 5)$ is $y = 5$.



84. Since $y = 9$ is a horizontal line, the vertical line through $(-1, 3)$ is $x = -1$.



85. Since $\dfrac{5-3}{8+2} = \dfrac{1}{5} = -\dfrac{1}{a}$, we find $a = -5$.

86. Since $\dfrac{a-4}{7-3} = \dfrac{a-4}{4} = \dfrac{2}{3}$, we get $a = \dfrac{20}{3}$.

87. Since $\dfrac{a-3}{-2-a} = -\dfrac{1}{2}$, we obtain $2a - 6 = 2 + a$ and $a = 8$.

88. Since $\dfrac{a+4}{-1-3} = \dfrac{a+4}{-4} = a$, we obtain $a + 4 = -4a$ and $a = -\dfrac{4}{5}$.

89. Plot the points $A(-1, 2)$, $B(2, -1)$, $C(3, 3)$, and $D(-2, -2)$, respectively. The slopes of the opposite sides are $m_{AC} = m_{BD} = 1/4$ and $m_{AD} = m_{BC} = 4$. Since the opposite sides are parallel, it is a parallelogram.

90. Plot the points $A(-1, 1)$, $B(-2, -5)$, $C(2, -4)$, and $D(3, 2)$, respectively. The slopes of the opposite sides are $m_{AB} = m_{CD} = 6$ and $m_{AD} = m_{BC} = 1/4$. Since the opposite sides are parallel, it is a parallelogram.

91. Plot the points $A(-5, -1)$, $B(-3, -4)$, $C(3, 0)$, and $D(1, 3)$, respectively. The slopes of the opposite sides are

$$m_{AB} = m_{CD} = -3/2$$

and

$$m_{AD} = m_{BC} = 2/3.$$

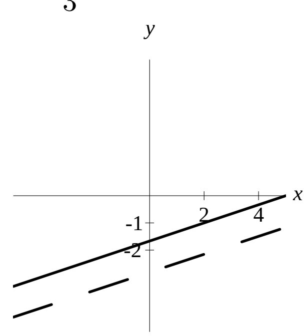
Since the adjacent sides are perpendicular, it is a rectangle.

92. Plot the points $A(-5, -1)$, B$(1, -4)$, $C(4, 2)$, and $D(-1, 5)$. The lengths of the opposite sides AB and CD are $AB = \sqrt{45}$ and $CD = \sqrt{34}$, respectively. Since $CD \neq AB$, it is not a square.

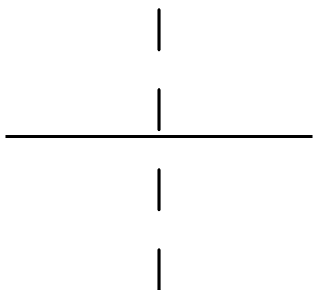
93. Plot the points $A(-5, 1)$, $B(-2, -3)$, and $C(4, 2)$, respectively. The slopes of the sides are $m_{AB} = -4/3$, $m_{BC} = 5/6$ and $m_{AC} = 1/9$. It is not a right triangle since no two sides are perpendicular.

94. Plot the points $A(-4, -3)$, $B(1, -2)$, $C(2, 3)$, and $D(-3, 2)$. Since all sides have equal length and $AB = CD = AD = BC = \sqrt{26}$, it is a rhombus.

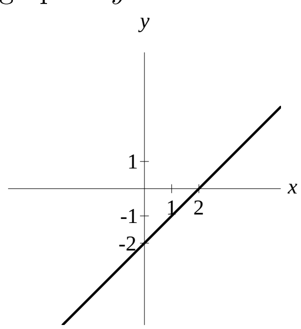
95. Yes, they appear to be parallel. However, they are not parallel since their slopes are not equal, i.e., $\dfrac{1}{3} \neq 0.33$.



96. Yes, they are perpendicular since the product of their slopes is -1, i.e., $(99)\dfrac{-1}{99} = -1$.



97. Since $x^3 - 8 = (x-2)(x^2+2x+4)$, we obtain $\dfrac{x^3-8}{x^2+2x+4} = x - 2$. A linear function for the graph is $y = x - 2$.



98. Since $\dfrac{x^3+2x^2-5x-6}{x^2+x-6} = x+1$, a linear function for the graph is $y = x+1$ where $x \neq 2, -3$. A factorization is given below.

$$\begin{aligned} x^3+2x^2-5x-6 &= (x+1)(x^2+x-6) \\ &= (x+1)(x+3)(x-2) \end{aligned}$$

99. The slope is $\dfrac{212-32}{100-0} = \dfrac{9}{5}$.

Since $F - 32 = \dfrac{9}{5}(C-0)$, $F = \dfrac{9}{5}C + 32$.

When $C = 150$, $F = \dfrac{9}{5}(150) + 32 = 302^o$F.

100. The slope is $\dfrac{35-23}{500-200} = 0.04$.

Since $C - 23 = 0.04(n-200)$, $C = 0.04n + 15$. Then 700 cards costs $C = 0.04(700) + 15 =$ \$43.

101. In 10 years, the new 18-wheeler depreciated by \$110,000. Then the slope of the linear function is $-\$11,000$/year. Thus, the linear value V is

$$V = 130,000 - 11,000n$$

If $n = 7$ years old, the value of truck is

$$130,000 - 11,000(7) = \$53,000.$$

102. The slope is

$$\frac{17,300 - 10,500}{18-10} = 850.$$

The linear function satisfies

$$10,500 = 850(10) + b.$$

Then $b = 2000$. The linear cost is

$$C = 850n + 2000.$$

If $n = 16$, the cost is

$$850(16) + 2000 = \$15,600$$

103. The linear function through $(1, 49)$ and $(2, 48)$ is $c = 50 - n$. With $n = 40$ people in a tour, she charges \$10 each and make \$400.

104. To express n as a function of p we write $n = 8000 - (p-10)500$ or $n = -500p + 13,000$.

If $p = \$20$, then the number of tickets expected to be sold is $n = 13,000 - 500(20) =$ 3,000 tickets; and the money to be taken in is \$20(3000) = \$60,000.

105. The slope is $\dfrac{75-95}{4000} = -0.005$. Since $S - 95 = -0.005(D-0)$, we obtain $S = -0.005D + 95$.

106. If $D = 1100$, then his air speed is $S = -0.005(1100) + 95 = 89.5$ mph.

107. Let c and p be the number of computers and printers, respectively. Since $60,000 = 2000c + 1500p$, we have

$$\begin{aligned} 2000c &= -1500p + 60,000 \\ c &= -\frac{3}{4}p + 30. \end{aligned}$$

The slope is $-\dfrac{3}{4}$, i.e., if 4 more printers are purchased then 3 fewer computers must be purchased.

108. Let c and h be the bonus in dollars of each carpenter and helper, respectively. Since $2400 = 9c + 3h$, we obtain

$$\begin{aligned} 3h &= -9c + 2400 \\ h &= -3c + 800. \end{aligned}$$

The slope is -3, i.e., if each carpenter gets an extra dollar, then each helper will receive \$3 less in bonus.

109. Using the equation of the line given by $y = -\frac{3x}{5} + \frac{43}{5}$, the y-values are integers exactly for $x = -4, 1, 6, 11, 16$ in $[-9, 21]$. The points with integral coordinates are $(-4, 11)$, $(1, 8)$, $(6, 5)$, $(11, 2)$, and $(16, -1)$.

110. The opposite sides of a parallelogram are parallel. The "rise" and "run" from $(2, -3)$ to $(4, 1)$ are 4 and 2, respectively.

From the point $(-1, 2)$, add the "rise=4" and "run=2" to the y- and x-coordinates; we get $(1, 6)$.

Next, subtract the "rise=4" and "run=2" from the coordinates of $(-1, 2)$; we get $(-3, -2)$.

Likewise, the "rise" and "run" from $(-1, 2)$ to $(4, 1)$ are -1 and 5, respectively.

From the coordinates of $(2, -3)$, add the "rise $= -1$" and "run=5" to the y- and x-coordinates; we get $(7, -4)$.

There are only three possible vertices, namely, $(1, 6)$, $(-3, -2)$, and $(7, -4)$.

111.

$$\begin{aligned} d &= \frac{|5(3) - 12(-6) - 2|}{\sqrt{5^2 + (-12)^2}} \\ d &= \frac{85}{\sqrt{169}} \\ d &= \frac{85}{13} \end{aligned}$$

112.

$$\begin{aligned} d &= \frac{|3(-4) + 4(8) - 9|}{\sqrt{3^2 + 4^2}} \\ d &= \frac{11}{\sqrt{25}} \\ d &= \frac{11}{5} \end{aligned}$$

113.

$$\begin{aligned} d &= \frac{|(-5)(1) + (1)(3) + 4|}{\sqrt{(-5)^2 + 1^2}} \\ d &= \frac{2}{\sqrt{26}} \\ d &= \frac{2\sqrt{26}}{26} \\ d &= \frac{\sqrt{26}}{13} \end{aligned}$$

114.

$$\begin{aligned} d &= \frac{|(4)(-2) + (1)(5) - 1|}{\sqrt{4^2 + 1^2}} \\ d &= \frac{4}{\sqrt{17}} \\ d &= \frac{4\sqrt{17}}{17} \end{aligned}$$

115. Let $b_1 \neq b_2$. If $y = mx + b_1$ and $y = mx + b_2$ have a point (s, t) in common, then $ms + b_1 = ms + b_2$. After subtracting ms from both sides, we get $b_1 = b_2$; a contradiction. Thus, $y = mx + b_1$ and $y = mx + b_2$ have no points in common if $b_1 \neq b_2$.

116. Suppose $m_1 \neq m_2$. Solving for x in $m_1x + b_1 = m_2x + b_2$ one obtains $x(m_1 - m_2) = b_2 - b_1$. Rewriting, one obtains $x = \dfrac{b_2 - b_1}{m_1 - m_2}$ which is well-defined since $m_1 \neq m_2$.

Thus, if $m_1 \neq m_2$, then the lines $y = m_1x + b_1$ and $y = m_2x + b_2$ have a point of intersection. Exercise 115 shows that if two distinct non-vertical lines have equal slopes then they have no points in common.

117.

$$\begin{aligned} 3 &= 5|x - 4| \\ \frac{3}{5} &= |x - 4| \\ \pm\frac{3}{5} &= x - 4 \\ 4 \pm \frac{3}{5} &= x \end{aligned}$$

The solution set is $\{17/5, 23/5\}$.

118. Let x be Shanna's rate.

$$\begin{aligned} \frac{1}{24}+\frac{1}{x} &= \frac{1}{18} \\ \frac{1}{x} &= \frac{1}{18}-\frac{1}{24} \\ x &= 72 \end{aligned}$$

Shanna can do the job alone in 72 minutes.

119. The midpoint or center is $((1+3)/2, (3+9)/2)$ or $(2,6)$. The radius is $\sqrt{(2-1)^2+(6-3)^2} = \sqrt{10}$. The circle is given by

$$(x-2)^2+(x-6)^2 = 10.$$

120.

a) -10
b) $-(3)^{-3} = -(1/27) = -\frac{1}{27}$
c) 19
d) $(-2)^2 = 4$

121.

$$\begin{aligned} (x^2+wx)-(2w+2x) &= \\ x(x+w)-2(w+x) &= \\ (x+w)(x-2) & \end{aligned}$$

122. Rewrite x^3+2x-5 where $x-3$ is a multiple.

$$\begin{aligned} x^3+2x-5 &= \\ x^2(x-3)+3x(x-3)+11(x-3)+28 & \end{aligned}$$

Then the remainder is 28.

123. Let x be the number of ants. Then

$$x = 10a+6 = 7b+2 = 11c+2 = 13d+2$$

for some positive integers a, b, c, d.

The smallest positive x satisfying the system of equations is $x = 4006$ ants.

124. Consider an isosceles right triangle with vertices $S(0,0)$, $T(1,2)$, and $U(3,1)$. Then the angle $\angle TUS = 45°$.

Let V be the point $(0,1)$. Note, $\angle SUV = A$ and $\angle TUV = B$. Then $A+B = 45°$.

Since C is an angle of the isosceles right triangle with vertices at $(2,0)$, $(3,0)$, and $(3,1)$, then $C = 45°$. Thus, $C = A+B$.

1.4 Pop Quiz

1. $\dfrac{9-6}{-4-5} = \dfrac{3}{-9} = -\dfrac{1}{3}$

2. The slope is $m = \dfrac{8-4}{6-3} = \dfrac{4}{3}$. Since $y-4 = \dfrac{4}{3}(x-3)$, we obtain $y-4 = \dfrac{4}{3}x-4$ or $y = \dfrac{4}{3}x$.

3. Since $y = \dfrac{2}{7}x-\dfrac{1}{7}$, the slope is $m = \dfrac{2}{7}$.

4. Since slope of $y = 3x-1$ is 3 and $y-7 = 3(x-0)$, we obtain $y = 3x+7$.

5. Since slope of $y = \dfrac{1}{2}x+4$ is $\dfrac{1}{2}$ and $y-8 = -2(x-0)$, we find $y = -2x+8$.

1.4 Linking Concepts

(a) The slope is $\dfrac{22,000-10,000}{24,000-6,000} = \dfrac{2}{3}$. Since $D - 10,000 = \dfrac{2}{3}(E-6,000)$, we get $D = \dfrac{2}{3}E+6000$.

(b) If $E = \$60,000$, then

$$D = \frac{2}{3}(60,000)+6000 = \$46,000$$

(c) Solving $\dfrac{2}{3}x+6000 = x$, we obtain the break-even point $x = \$18,000$.

(d) Taxes are paid if $D < E$. In this case, the percentage, P, of the earned income that is paid in taxes is

$$\begin{aligned} P &= \left(\frac{E-D}{E}\right)100 \\ &= \left(1-\frac{D}{E}\right)100 \\ &= \left(1-\frac{\frac{2}{3}E+6000}{E}\right)100 \\ &= \left(1-\left(\frac{2}{3}+\frac{6000}{E}\right)\right)100 \\ &= \left(\frac{1}{3}-\frac{6000}{E}\right)100. \end{aligned}$$

For the following earned incomes, we calculate and tabulate the corresponding values of P.

E	\$25,000	\$100,000	\$2,000,000
P	9.3%	27.3%	33%

(e) The maximum of

$$P = \left(\frac{1}{3} - \frac{6000}{E}\right) 100$$

is $\frac{100}{3}$% or approximately 33.33%.

For Thought

1. False, since $x = 1$ is a solution of the first equation and not of the second equation.

2. False, since $x^2 + 1 = 0$ cannot be factored with real coefficients.

3. False, $\left(x + \frac{2}{3}\right)^2 = x^2 + \frac{4}{3}x + \frac{9}{4}$.

4. False, the solutions to $(x-3)(2x+5) = 0$ are $x = 3$ and $x = -\frac{5}{2}$.

5. False, $x^2 = 0$ has only $x = 0$ as its solution.

6. True, since $a = 1, b = -3$, and $c = 1$, then by the quadratic formula we obtain
$x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$.

7. False, the quadratic formula can be used to solve any quadratic equation.

8. False, $x^2 + 1 = 0$ has only imaginary zeros.

9. True, for $b^2 - 4ac = 12^2 - 4(4)(9) = 0$.

10. True, $x^2 - 6x + 9 = (x-3)^2 = 0$ has only one real solution, namely, $x = 3$.

1.5 Exercises

1. quadratic

2. quadratic

3. discriminant

4. square root property

5. Since $(x-5)(x+4) = 0$, the solution set is $\{5, -4\}$.

6. Since $(x-2)(x+4) = 0$, the solution set is $\{2, -4\}$.

7. Since $a^2 + 3a + 2 = (a+2)(a+1) = 0$, the solution set is $\{-2, -1\}$.

8. Since $b^2 - 4b - 12 = (b-6)(b+2) = 0$, the solution set is $\{6, -2\}$.

9. Since $(2x+1)(x-3) = 0$, the solution set is $\left\{-\frac{1}{2}, 3\right\}$.

10. Since $(2x-1)(x-2) = 0$, the solution set is $\left\{\frac{1}{2}, 2\right\}$.

11. Since $(2x-1)(3x-2) = 0$, the solution set is $\left\{\frac{1}{2}, \frac{2}{3}\right\}$.

12. Since $(4x-3)(3x-2) = 0$, the solution set is $\left\{\frac{3}{4}, \frac{2}{3}\right\}$.

13. Note, $y^2 + y - 12 = 30$. Subtracting 30 from both sides, one obtains $y^2 + y - 42 = 0$ or $(y+7)(y-6) = 0$. The solution set is $\{-7, 6\}$.

14. Note, $w^2 - 3w + 2 = 6$. Subtracting 6 from both sides, one gets $w^2 - 3w - 4 = 0$ or $(w+1)(w-4) = 0$. The solution set is $\{-1, 4\}$.

15. Since $x^2 = 5$, the solution set is $\{\pm\sqrt{5}\}$.

16. Since $\pm\sqrt{8} = \pm 2\sqrt{2}$, the solution set is $\{\pm 2\sqrt{2}\}$.

17. Since $x^2 = -\frac{2}{3}$, we find $x = \pm i\frac{\sqrt{2}}{\sqrt{3}}$.
The solution set is $\left\{\pm i\frac{\sqrt{6}}{3}\right\}$.

18. Since $x^2 = -8$, we find $x = \pm i\sqrt{8}$.
The solution set is $\left\{\pm 2i\sqrt{2}\right\}$.

19. Since $x - 3 = \pm 3$, we get $x = 3 \pm 3$.
The solution set is $\{0, 6\}$.

20. Since $x+1=\pm\dfrac{3}{2}$ then $x=-1\pm\dfrac{3}{2}=\dfrac{-2\pm 3}{2}$.

The solution set is $\left\{\dfrac{1}{2},-\dfrac{5}{2}\right\}$.

21. By the square root property, we get $3x-1=\pm 0=0$. Solving for x, we obtain $x=\dfrac{1}{3}$. The solution set is $\left\{\dfrac{1}{3}\right\}$.

22. Using the square root property, we find $5x+2=\pm 0=0$. Solving for x, we get $x=-\dfrac{2}{5}$.

The solution set is $\left\{-\dfrac{2}{5}\right\}$.

23. Since $x-\dfrac{1}{2}=\pm\dfrac{5}{2}$, it follows that $x=\dfrac{1}{2}\pm\dfrac{5}{2}$. The solution set is $\{-2,3\}$.

24. Since $3x-1=\pm\dfrac{1}{2}$, we get $3x=1\pm\dfrac{1}{2}$. Thus,

$$x=\frac{1\pm\frac{1}{2}}{3}=\frac{1\pm\frac{1}{2}}{3}\cdot\frac{2}{2}=\frac{2\pm 1}{6}.$$

The solution set is $\left\{\dfrac{1}{6},\dfrac{1}{2}\right\}$.

25. Since $x+2=\pm 2i$, the solution set is $\{-2\pm 2i\}$.

26. Since $x-3=\pm i\sqrt{20}$, the solution set is $\left\{3\pm 2i\sqrt{5}\right\}$.

27. Since $x-\dfrac{2}{3}=\pm\dfrac{2}{3}$, we get

$$x=\frac{2}{3}\pm\frac{2}{3}=\frac{4}{3},0.$$

The solution set is $\left\{0,\dfrac{4}{3}\right\}$.

28. Since $x+\dfrac{3}{2}=\pm\dfrac{\sqrt{2}}{2}$, we obtain

$$x=-\frac{3}{2}\pm\frac{\sqrt{2}}{2}.$$

The solution set is $\left\{\dfrac{-3\pm\sqrt{2}}{2}\right\}$.

29. $x^2-12x+\left(\dfrac{12}{2}\right)^2=x^2-12x+6^2=x^2-12x+36$

30. $y^2+20y+\left(\dfrac{20}{2}\right)^2=y^2+20y+10^2=y^2+20y+100$

31. $r^2+3r+\left(\dfrac{3}{2}\right)^2=r^2+3r+\dfrac{9}{4}$

32. $t^2-7t+\left(\dfrac{7}{2}\right)^2=t^2-7t+\dfrac{49}{4}$

33. $w^2+\dfrac{1}{2}w+\left(\dfrac{1}{4}\right)^2=w^2+\dfrac{1}{2}w+\dfrac{1}{16}$

34. $p^2-\dfrac{2}{3}p+\left(\dfrac{1}{3}\right)^2=p^2-\dfrac{2}{3}p+\dfrac{1}{9}$

35. By completing the square, we derive

$$\begin{aligned}x^2+6x &= -1\\ x^2+6x+9 &= -1+9\\ (x+3)^2 &= 8\\ x+3 &= \pm 2\sqrt{2}.\end{aligned}$$

The solution set is $\{-3\pm 2\sqrt{2}\}$.

36.

$$\begin{aligned}x^2-10x &= -5\\ x^2-10x+25 &= -5+25\\ (x-5)^2 &= 20\\ x-5 &= \pm\sqrt{20}\\ x &= 5\pm 2\sqrt{5}\end{aligned}$$

The solution set is $\{5\pm 2\sqrt{5}\}$.

37. By completing the square, we find

$$\begin{aligned}n^2-2n &= 1\\ n^2-2n+1 &= 1+1\\ (n-1)^2 &= 2\\ n-1 &= \pm\sqrt{2}.\end{aligned}$$

The solution set is $\{1\pm\sqrt{2}\}$.

38.

$$\begin{aligned} m^2 - 12m &= -33 \\ m^2 - 12m + 36 &= -33 + 36 \\ (m-6)^2 &= 3 \\ m - 6 &= \pm\sqrt{3} \end{aligned}$$

The solution set is $\{6 \pm \sqrt{3}\}$.

39.

$$\begin{aligned} h^2 + 3h &= 1 \\ h^2 + 3h + \frac{9}{4} &= 1 + \frac{9}{4} \\ (h + \frac{3}{2})^2 &= \frac{13}{4} \\ h + \frac{3}{2} &= \pm\frac{\sqrt{13}}{2} \end{aligned}$$

The solution set is $\left\{\dfrac{-3 \pm \sqrt{13}}{2}\right\}$.

40.

$$\begin{aligned} t^2 - 5t &= -2 \\ t^2 - 5t + \frac{25}{4} &= -2 + \frac{25}{4} \\ (t - \frac{5}{2})^2 &= \frac{17}{4} \\ t - \frac{5}{2} &= \pm\frac{\sqrt{17}}{2} \end{aligned}$$

The solution set is $\left\{\dfrac{5 \pm \sqrt{17}}{2}\right\}$.

41.

$$\begin{aligned} x^2 + \frac{5}{2}x &= 6 \\ x^2 + \frac{5}{2}x + \frac{25}{16} &= 6 + \frac{25}{16} \\ \left(x + \frac{5}{4}\right)^2 &= \frac{121}{16} \\ x &= -\frac{5}{4} \pm \frac{11}{4} \end{aligned}$$

The solution set is $\left\{-4, \dfrac{3}{2}\right\}$.

42.

$$\begin{aligned} x^2 + \frac{1}{3}x &= \frac{2}{3} \\ x^2 + \frac{1}{3}x + \frac{1}{36} &= \frac{24}{36} + \frac{1}{36} \\ \left(x + \frac{1}{6}\right)^2 &= \frac{25}{36} \\ x &= -\frac{1}{6} \pm \frac{5}{6} \end{aligned}$$

The solution set is $\left\{-1, \dfrac{2}{3}\right\}$.

43.

$$\begin{aligned} x^2 + \frac{2}{3}x &= -\frac{1}{3} \\ x^2 + \frac{2}{3}x + \frac{1}{9} &= -\frac{3}{9} + \frac{1}{9} \\ \left(x + \frac{1}{3}\right)^2 &= -\frac{2}{9} \\ x &= -\frac{1}{3} \pm i\frac{\sqrt{2}}{3} \end{aligned}$$

The solution set is $\left\{\dfrac{-1 \pm i\sqrt{2}}{3}\right\}$.

44.

$$\begin{aligned} x^2 + \frac{4}{5}x &= -\frac{3}{5} \\ x^2 + \frac{4}{5}x + \frac{4}{25} &= -\frac{15}{25} + \frac{4}{25} \\ \left(x + \frac{2}{5}\right)^2 &= -\frac{11}{25} \\ x &= -\frac{2}{5} \pm i\frac{\sqrt{11}}{5} \end{aligned}$$

The solution set is $\left\{\dfrac{-2 \pm i\sqrt{11}}{5}\right\}$.

45. Since $a = 1, b = 3, c = -4$ and $x = \dfrac{-3 \pm \sqrt{3^2 - 4(1)(-4)}}{2(1)} = \dfrac{-3 \pm \sqrt{25}}{2} = \dfrac{-3 \pm 5}{2}$, the solution set is $\{-4, 1\}$.

46. Since $a = 1, b = 8, c = 12$ and $x = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(12)}}{2(1)} = \dfrac{-8 \pm \sqrt{16}}{2} = \dfrac{-8 \pm 4}{2}$, the solution set is $\{-6, -2\}$.

47. Since $a = 2, b = -5, c = -3$ and
$x = \dfrac{5 \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)} = \dfrac{5 \pm \sqrt{49}}{4} =$
$\dfrac{5 \pm 7}{4}$, the solution set is $\left\{-\dfrac{1}{2}, 3\right\}$.

48. Since $a = 2, b = 3, c = -2$ and
$x = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{2(2)} = \dfrac{-3 \pm \sqrt{25}}{4} =$
$\dfrac{-3 \pm 5}{4}$, the solution set is $\left\{-2, \dfrac{1}{2}\right\}$.

49. Since $a = 9, b = 6, c = 1$ and
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(9)(1)}}{2(9)} = \dfrac{-6 \pm 0}{18}$,
the solution set is $\left\{-\dfrac{1}{3}\right\}$.

50. Since $a = 16, b = -24, c = 9$ and
$x = \dfrac{24 \pm \sqrt{(-24)^2 - 4(16)(9)}}{2(16)} = \dfrac{24 \pm 0}{32}$,
the solution set is $\left\{\dfrac{3}{4}\right\}$.

51. Since $a = 2, b = 0, c = -3$ and
$x = \dfrac{0 \pm \sqrt{0^2 - 4(2)(-3)}}{2(2)} = \dfrac{\pm\sqrt{24}}{4} =$
$\dfrac{\pm 2\sqrt{6}}{4}$, the solution set is $\left\{\pm\dfrac{\sqrt{6}}{2}\right\}$.

52. Since $a = -2, b = 0, c = 5$ and
$x = \dfrac{0 \pm \sqrt{0^2 - 4(-2)(5)}}{2(-2)} = \dfrac{\pm\sqrt{40}}{-4} =$
$\dfrac{\pm 2\sqrt{10}}{-4}$, the solution set is $\left\{\pm\dfrac{\sqrt{10}}{2}\right\}$.

53. In $x^2 - 4x + 5 = 0$, $a = 1, b = -4, c = 5$.
Then $x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} =$
$\dfrac{4 \pm \sqrt{-4}}{2} = \dfrac{4 \pm 2i}{2}$.
The solution set is $\{2 \pm i\}$.

54. In $x^2 - 6x + 13 = 0$, $a = 1, b = -6, c = 13$.
Then $x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)} =$
$\dfrac{6 \pm \sqrt{-16}}{2} = \dfrac{6 \pm 4i}{2}$.
The solution set is $\{3 \pm 2i\}$.

55. Note, $a = 1, b = -2$, and $c = 4$.
Then $x = \dfrac{2 \pm \sqrt{4 - 16}}{2} = \dfrac{2 \pm 2i\sqrt{3}}{2}$.
The solution set is $\left\{1 \pm i\sqrt{3}\right\}$.

56. Note, $a = 1, b = -4$, and $c = 9$.
Then $x = \dfrac{4 \pm \sqrt{16 - 36}}{2} = \dfrac{4 \pm 2i\sqrt{5}}{2}$.
The solution set is $\left\{2 \pm i\sqrt{5}\right\}$.

57. Since $2x^2 - 2x + 5 = 0$, we find $a = 2, b = -2$,
and $c = 5$. Then $x = \dfrac{2 \pm \sqrt{4 - 40}}{4} = \dfrac{2 \pm 6i}{4}$.
The solution set is $\left\{\dfrac{1}{2} \pm \dfrac{3}{2}i\right\}$.

58. Since $9x^2 - 12x + 5 = 0$, we find $a = 9, b = -12$,
and $c = 5$. Then $x = \dfrac{12 \pm \sqrt{144 - 180}}{18} =$
$\dfrac{12 \pm 6i}{18}$. The solution set is $\left\{\dfrac{2}{3} \pm \dfrac{1}{3}i\right\}$.

59. Since $a = 4, b = -8, c = 7$ and
$x = \dfrac{8 \pm \sqrt{64 - 112}}{8} = \dfrac{8 \pm \sqrt{-48}}{8} =$
$\dfrac{8 \pm 4i\sqrt{3}}{8}$, the solution set is $\left\{1 \pm \dfrac{\sqrt{3}}{2}i\right\}$.

60. Since $a = 9, b = -6, c = 4$ and
$x = \dfrac{6 \pm \sqrt{36 - 144}}{18} = \dfrac{6 \pm \sqrt{-108}}{18} =$
$\dfrac{6 \pm 6i\sqrt{3}}{18}$, the solution set is $\left\{\dfrac{1}{3} \pm \dfrac{\sqrt{3}}{3}i\right\}$.

61. Since $a = 3.2, b = 7.6$, and $c = -9$,
$x = \dfrac{-7.6 \pm \sqrt{(7.6)^2 - 4(3.2)(-9)}}{2(3.2)} \approx$
$\dfrac{-7.6 \pm \sqrt{172.96}}{6.4} \approx \dfrac{-7.6 \pm 13.151}{6.4}$.
The solution set is $\{-3.24, 0.87\}$.

62. Since $1.5x^2 - 6.3x - 10.1 = 0$, $a = 1.5, b = -6.3, c = -10.1$ and
$$x = \frac{6.3 \pm \sqrt{(-6.3)^2 - 4(1.5)(-10.1)}}{2(1.5)} \approx$$
$$\frac{6.3 \pm \sqrt{100.29}}{3} \approx \frac{6.3 \pm 10.014}{3}.$$
The solution set is $\{-1.24, 5.44\}$.

63. Note, $a = 3.25, b = -4.6$, and $c = -22$. Then $x = \dfrac{4.6 \pm \sqrt{(-4.6)^2 - 4(3.25)(-22)}}{2(3.25)}$ $= \dfrac{4.6 \pm \sqrt{307.16}}{6.5}$. The solution set is $\{-1.99, 3.40\}$.

64. Note, $a = 4.76, b = 6.12$, and $c = -55.3$. Then $x = \dfrac{-6.12 \pm \sqrt{(6.12)^2 - 4(4.76)(-55.3)}}{2(4.76)}$ $= \dfrac{-6.12 \pm \sqrt{1090.3664}}{9.52}$. The solution set is $\{-4.11, 2.83\}$.

65. The discriminant is
$$(-30)^2 - 4(9)(25) = 900 - 900 = 0.$$
Only one solution and it is real.

66. The discriminant is $(28)^2 - 4(4)(49) = 784 - 784 = 0$. Only one solution and it is real.

67. The discriminant is
$$(-6)^2 - 4(5)(2) = 36 - 40 = -4.$$
There are no real solutions.

68. The discriminant is $5^2 - 4(3)(5) = 25 - 60 = -35$. There are no real solutions.

69. The discriminant is
$$12^2 - 4(7)(-1) = 144 + 28 = 172.$$
There are two distinct real solutions.

70. The discriminant is $(-7)^2 - 4(3)(3) = 49 - 36 = 13$. There are two distinct real solutions.

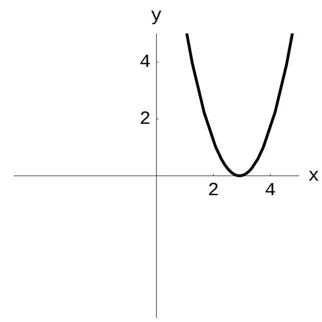
71. Note, x-intercepts are $\left(-\dfrac{2}{3}, 0\right)$ and $\left(\dfrac{1}{2}, 0\right)$. The solution set is $\left\{-\dfrac{2}{3}, \dfrac{1}{2}\right\}$.

72. Note, x-intercepts are $(-3, 0)$ and $(2, 0)$. The solution set is $\{-3, 2\}$.

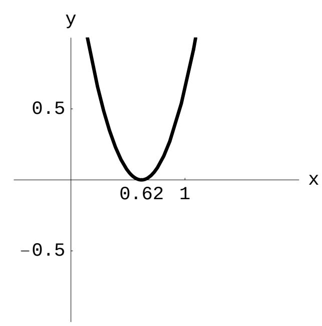
73. Since the x-intercepts are $(-3, 0)$ and $(5, 0)$, the solution set is $\{-3, 5\}$.

74. From the x-intercepts $(1, 0)$ and $(4, 0)$, we conclude that the solution set is $\{1, 4\}$.

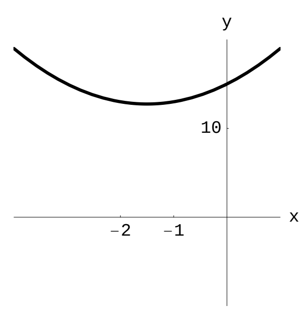
75. Note, the graph of $y = 1.44x^2 - 8.4x + 12.25$ has exactly one x-intercept.



76. Note, the graph of $y = 4.41x^2 - 5.46x + 1.69$ has exactly one x-intercept.

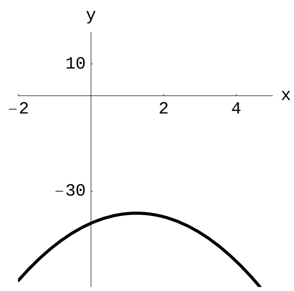


77. Note, the graph of $y = x^2 + 3x + 15$ has no x-intercept.



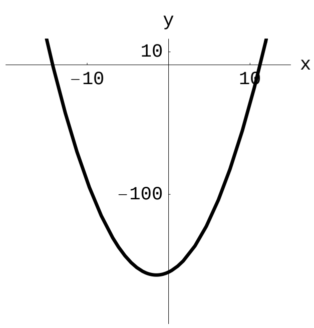
Thus, $x^2 + 3x + 15 = 0$ has no real solution.

78. The graph of $y = -2x^2 + 5x - 40$ has no x-intercept.



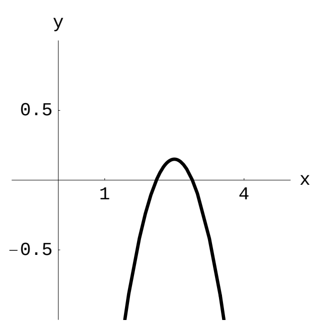
Then $-2x^2 + 5x - 40 = 0$ has no real solution.

79. The graph of $y = x^2 + 3x - 160$ has two x-intercepts.



Then $x^2 + 3x - 160 = 0$ has two real solutions.

80. The graph of $y = -x^2 + 5x - 6.1$ has two x-intercepts.



Thus, $-x^2 + 5x - 6.1 = 0$ has two real solutions.

81. Set the right-hand side to 0.

$$\begin{aligned} x^2 - \frac{4}{3}x - \frac{5}{9} &= 0 \\ 9x^2 - 12x - 5 &= 0 \\ (3x+1)(3x-5) &= 0 \end{aligned}$$

The solution set is $\left\{-\dfrac{1}{3}, \dfrac{5}{3}\right\}$.

82. Set the right-hand side to 0.

$$\begin{aligned} x^2 - \frac{2}{7}x - \frac{2}{49} &= 0 \\ 49x^2 - 14x - 2 &= 0 \end{aligned}$$

By the quadratic formula,

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(49)(-2)}}{2(49)} = \frac{14 \pm 14\sqrt{3}}{98}$$

and the solution set is $\left\{\dfrac{1 \pm \sqrt{3}}{7}\right\}$.

83. Since $x^2 = \sqrt{2}$, $x = \pm\sqrt{\sqrt{2}} = \pm\sqrt[4]{2}$. The solution set is $\left\{\pm\sqrt[4]{2}\right\}$.

84. By taking the square roots in $x^2 = \dfrac{1}{\sqrt{2}}$, we get

$$x = \pm\frac{1}{\sqrt{\sqrt{2}}} = \pm\frac{1}{\sqrt[4]{2}} = \pm\frac{1}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{8}}{\sqrt[4]{8}} = \pm\frac{\sqrt[4]{8}}{\sqrt[4]{16}}.$$

The solution set is $\left\{\pm\dfrac{\sqrt[4]{8}}{2}\right\}$.

85. By the quadratic formula,

$$x = \frac{-\sqrt{6} \pm \sqrt{(-\sqrt{6})^2 - 4(12)(-1)}}{2(12)} =$$

$$\frac{-\sqrt{6} \pm \sqrt{54}}{24} = \frac{-\sqrt{6} \pm 3\sqrt{6}}{24} = \frac{2\sqrt{6}}{24}, \frac{-4\sqrt{6}}{24}.$$

The solution set is $\left\{-\dfrac{\sqrt{6}}{6}, \dfrac{\sqrt{6}}{12}\right\}$.

86. By the quadratic formula,

$$x = \frac{\sqrt{5} \pm \sqrt{(-\sqrt{5})^2 - 4(-10)(1)}}{2(-10)} =$$

$$\frac{\sqrt{5} \pm \sqrt{45}}{-20} = \frac{\sqrt{5} \pm 3\sqrt{5}}{-20} = \frac{4\sqrt{5}}{-20}, \frac{-2\sqrt{5}}{-20}.$$

The solution set is $\left\{\dfrac{-\sqrt{5}}{5}, \dfrac{\sqrt{5}}{10}\right\}$.

87. Since $x^2 + 6x - 72 = (x+12)(x-6) = 0$, the solution set is $\{-12, 6\}$.

88. Since $x(x+4) = 96$, $x^2 + 4x - 96 = 0$, $(x+12)(x-8) = 0$. The solution set is $\{-12, 8\}$.

89. Multiply by x to get $x^2 = x + 1$.
So $x^2 - x - 1 = 0$ and by the quadratic formula,
$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}.$$
The solution set is $\left\{\frac{1 \pm \sqrt{5}}{2}\right\}$.

90. Multiply by x to get $x^2 = 1$.
The solution set is $\{\pm 1\}$.

91. Multiply by x^2 to get $28x - 7 = 7x^2$.
Applying the quadratic formula to
$7x^2 - 28x + 7 = 0$, we obtain
$$x = \frac{28 \pm \sqrt{588}}{14} = \frac{28 \pm 14\sqrt{3}}{14}.$$
The solution set is $\left\{2 \pm \sqrt{3}\right\}$.

92. Multiply by $x^2/2$ to get $10x - 23 = x^2$.
Applying the quadratic formula to
$x^2 - 10x + 23 = 0$, we get
$$x = \frac{10 \pm \sqrt{8}}{2} = \frac{10 \pm 2\sqrt{2}}{2}.$$
The solution set is $\left\{5 \pm \sqrt{2}\right\}$.

93. Multiplying by $(3 - x)(x + 7)$, we get
$$\begin{aligned}(x - 12)(x + 7) &= (x + 4)(3 - x)\\ x^2 - 5x - 84 &= -x^2 - x + 12\\ 2x^2 - 4x - 96 &= 0\\ 2(x - 8)(x + 6) &= 0.\end{aligned}$$
The solution set is $\{8, -6\}$.

94. Multiplying by $(x - 2)(x + 1)$, we get
$$\begin{aligned}(x - 9)(x + 1) &= -(x + 3)(x - 2)\\ x^2 - 8x - 9 &= -(x^2 + x - 6)\\ 2x^2 - 7x - 15 &= 0\\ (2x + 3)(x - 5) &= 0.\end{aligned}$$
The solution set is $\left\{-\frac{3}{2}, 5\right\}$.

95. Multiplying by $(x + 2)(x + 3)$, we find
$$\begin{aligned}(x - 8)(x + 3) &= (x + 2)(2x - 1)\\ x^2 - 5x - 24 &= 2x^2 + 3x - 2\\ 0 &= x^2 + 8x + 22\\ -22 + 16 &= x^2 + 8x + 16\\ -6 &= (x + 4)^2.\end{aligned}$$
Since the right side is not a negative number, the solution set is the empty set $\emptyset$.

96. Multiplying by $x(x + 1)$, we find
$$\begin{aligned}x^2 - 1 &= 3x^2\\ -1 &= 2x^2.\end{aligned}$$
Since the left side is not a negative number, the solution set is the empty set $\emptyset$.

97. Multiplying by $(2x + 1)(2x + 3)$, we find
$$\begin{aligned}(2x + 3)^2 &= 8(2x + 1)\\ 4x^2 + 12x + 9 &= 16x + 8\\ 4x^2 - 4x + 1 &= 0\\ (2x - 1)^2 &= 0.\end{aligned}$$
Thus, the solution set is $\left\{\frac{1}{2}\right\}$.

98. Multiplying by $(6x + 5)(2x + 3)$, we obtain
$$\begin{aligned}(2x + 3)^2 &= 2(6x + 5)\\ 4x^2 + 12x + 9 &= 12x + 10\\ 4x^2 &= 1\\ x^2 &= \frac{1}{4}.\end{aligned}$$
Thus, the solution set is $\left\{\pm\frac{1}{2}\right\}$.

99. Since $r^2 = \frac{A}{\pi}$, $r = \pm\sqrt{\frac{A}{\pi}}$.

100. Apply the quadratic formula to $(2\pi)r^2 + (2\pi h)r - S = 0$ where $a = 2\pi$, $b = 2\pi h$, and $c = -S$.
$$\begin{aligned}r &= \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-S)}}{2(2\pi)}\\ r &= \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8S\pi}}{4\pi}\end{aligned}$$

$$\begin{aligned} r &= \frac{-2\pi h \pm \sqrt{4\left(\pi^2 h^2 + 2S\pi\right)}}{4\pi} \\ r &= \frac{-2\pi h \pm 2\sqrt{\pi^2 h^2 + 2S\pi}}{4\pi} \\ r &= \frac{-\pi h \pm \sqrt{\pi^2 h^2 + 2\pi S}}{2\pi}. \end{aligned}$$

101. We use the quadratic formula to solve

$$x^2 + (2k)x + 3 = 0.$$

Since $a = 1, b = 2k$, and $c = 3$, we obtain

$$\begin{aligned} x &= \frac{-2k \pm \sqrt{(2k)^2 - 4(1)(3)}}{2(1)} \\ x &= \frac{-2k \pm \sqrt{4k^2 - 12}}{2} \\ x &= \frac{-2k \pm \sqrt{4\left(k^2 - 3\right)}}{2} \\ x &= \frac{-2k \pm 2\sqrt{k^2 - 3}}{2} \\ x &= -k \pm \sqrt{k^2 - 3}. \end{aligned}$$

102. Apply the quadratic formula to $hy^2 - ky - p = 0$ where $a = h, b = -k$, and $c = -p$. Then

$$\begin{aligned} y &= \frac{k \pm \sqrt{(-k)^2 - 4(h)(-p)}}{2(h)} \\ &= \frac{k \pm \sqrt{k^2 + 4hp}}{2h}. \end{aligned}$$

103. We use the quadratic formula to solve

$$2y^2 + (4x)y - x^2 = 0.$$

Since $a = 2, b = 4x$, and $c = -x^2$, we obtain

$$y = \frac{-4x \pm \sqrt{(4x)^2 - 4(2)(-x^2)}}{2(2)} =$$

$$\begin{aligned} \frac{-4x \pm \sqrt{16x^2 + 8x^2}}{4} &= \frac{-4x \pm \sqrt{24x^2}}{4} \\ \frac{-4x \pm 2|x|\sqrt{6}}{4} &= x\left(-1 \pm \frac{\sqrt{6}}{2}\right). \end{aligned}$$

Note that we used $|x| = \sqrt{x^2}$.

104. Multiplying $\frac{1}{x+h} - \frac{1}{x} = h$ by $x(x+h)$, we obtain $x - (x+h) = hx^2 + h^2x$. Rewriting and applying the quadratic formula to $0 = hx^2 + h^2x + h$, we get

$$\begin{aligned} x &= \frac{-h^2 \pm \sqrt{h^4 - 4h^2}}{2h} \\ &= \frac{-h^2 \pm \sqrt{h^2(h^2-4)}}{2h} = \frac{-h^2 \pm |h|\sqrt{h^2-4}}{2h} \\ &= \frac{-h^2 \pm h\sqrt{h^2-4}}{2h} = \frac{-h \pm \sqrt{h^2-4}}{2}. \end{aligned}$$

105. From the revenue function,

$$\begin{aligned} x(40 - 0.001x) &= 175,000 \\ 40x - 0.001x^2 &= 175,000 \end{aligned}$$

By applying the quadratic formula to $0.001x^2 - 40x + 175,000 = 0$, we get

$$\begin{aligned} x &= \frac{40 \pm \sqrt{(-40)^2 - 4(0.001)(175,000)}}{0.002} \\ x &= \frac{40 \pm \sqrt{900}}{0.002} = \frac{40 \pm 30}{0.002} \\ x &= 5000 \text{ or } 35,000. \end{aligned}$$

Then 5000 units or 35,000 units must be produced weekly.

106. Let x be the number of dollars that a price of a CD increases. If the revenue is \$120,000, then

$$\begin{aligned} (x+16)(10,000 - 1000x) &= 120,000 \\ -1000x^2 - 6000x + 160,000 &= 120,000 \\ 1000x^2 + 6000x - 40,000 &= 0 \\ x^2 + 6x - 40 &= 0 \\ (x+10)(x-4) &= 0 \\ x &= -10, 4. \end{aligned}$$

Then the price of a CD is $\$16 + x = \$6$ or \$20.

107. The height S (in feet) of the ball from the ground t seconds after it was tossed is given by $S = -16t^2 + 40t + 4$. When the height is 4 feet,

$$\begin{aligned} -16t^2 + 40t + 4 &= 4 \\ -16t^2 + 40t &= 0 \\ -8t(2t-5) &= 0 \\ t &= 0, \frac{5}{2}. \end{aligned}$$

The ball returns to a height of 4 ft in 2.5 sec.

108. Assume the velocity is zero when the sky diver steps out of the plane. The height S (in feet) of the sky diver from the ground t seconds after stepping out of the plane is given by $S = -16t^2 + 5000$. When the sky diver reaches a height of 4000 feet, we obtain

$$\begin{aligned} -16t^2 + 5000 &= 4000 \\ 1000 &= 16t^2 \\ t &= \pm 2.5\sqrt{10} \approx \pm 7.9. \end{aligned}$$

The sky diver drops to 4000 ft in 7.9 secs.

109. Let d be the diagonal distance across the field from one goal to the other. By the Pythagorean Theorem, we obtain $d = \sqrt{300^2 + 160^2} = 340$ ft.

110. If l is the length of the flag, then its width is $w = \dfrac{34 - 2l}{2} = 17 - l$. By the Pythagorean Theorem, we find

$$\begin{aligned} l^2 + (17 - l)^2 &= 13^2 \\ l^2 + (289 - 34l + l^2) &= 169 \\ 2l^2 - 34l + 120 &= 0 \\ l^2 - 17l + 60 &= 0 \\ (l - 12)(l - 5) &= 0 \\ l &= 12, 5. \end{aligned}$$

Since $w = 17 - l$, then $w = 5$ when $l = 12$; and $w = 12$ when $l = 5$. These are the same dimensions. Then the length is 12 in and the width is 5 in.

111. Let w and $2w + 2$ be the width and length. From the given area of the court, we obtain

$$\begin{aligned} (2w + 2)w &= 312 \\ 2w^2 + 2w - 312 &= 0 \\ w^2 + w - 156 &= 0 \\ (w - 12)(w + 13) &= 0. \end{aligned}$$

Then $w = 12$ yd and the length is 26 yd. The distance between two opposite corners (by the Pythagorean Theorem) is $\sqrt{12^2 + 26^2} = 2\sqrt{205} \approx 28.6$ yd.

112. Since the dimensions of the bottom of the box are $14 - 2x$ and $11 - 2x$, then from its area we find

$$\begin{aligned} (14 - 2x)(11 - 2x) &= 80 \\ 4x^2 - 50x + 154 &= 80 \\ 4x^2 - 50x + 74 &= 0 \\ 2x^2 - 25x + 37 &= 0. \end{aligned}$$

By the quadratic formula, $x = \dfrac{25 \pm \sqrt{329}}{4} \approx 10.78, 1.72$. Since x should be less than 5.5 inches, then the size of the square that must be cut is approximately 1.72 in. by 1.72 in.

113. Substituting the values of S and A, we find that the displacement is

$$\begin{aligned} \frac{1}{2^{12}} d^2 (18.8)^3 - 822^3 &= 0 \\ \frac{1}{2^{12}} d^2 (18.8)^3 &= 822^3 \\ d &= \sqrt{\frac{822^3 (2^{12})}{18.8^3}} \\ d &\approx 18,503.4 \text{ lbs.} \end{aligned}$$

114. Let r be the distance, in miles, from Charleston. Since the area of a circle is πr^2,

$$\begin{aligned} \pi r^2 &= 1,500,000 \\ r &= \sqrt{\frac{1,500,000}{\pi}} \approx 700. \end{aligned}$$

The earthquake was felt as far away as 700 miles from Charleston.

115. By choosing an appropriate coordinate system, we can assume the circle is given by $(x+r)^2+(y-r)^2 = r^2$ where $r > 0$ is the radius of the circle and $(-5, 1)$ is the common point between the block and the circle. Note, the radius is less than 5 feet. Substitute $x = -5$ and $y = 1$. Then we obtain

$$\begin{aligned} (-5 + r)^2 + (1 - r)^2 &= r^2 \\ r^2 - 10r + 25 + 1 - 2r + r^2 &= r^2 \\ r^2 - 12r + 26 &= 0. \end{aligned}$$

The solutions of the last quadratic equation are $r = 6 \pm \sqrt{10}$. Since $r < 5$, the radius of the circle is $r = 6 - \sqrt{10}$ ft.

116. With an appropriate coordinate system, we can assume the circle is given by $(x+r)^2 + (y-r)^2 = r^2$ where $r > 0$ is the radius of the circle. From the problem, we find that $(-10, 5)$ is a point on the circle. Substituting $x = -10$ and $y = 5$, we obtain

$$\begin{aligned}
(-10+r)^2 + (5-r)^2 &= r^2 \\
100 - 20r + r^2 + 25 - 10r + r^2 &= r^2 \\
r^2 - 30r + 125 &= 0 \\
r^2 - 30r &= -125 \\
(r-15)^2 &= -125 + 225 \\
(r-15)^2 &= 100 \\
r &= 15 \pm 10 \\
r &= 5, 25.
\end{aligned}$$

Hence, the possible radii are 5 in. and 25 in.

117. Let x be the normal speed of the tortoise in ft/hr.

	distance	rate	time
hwy	24	$x+2$	$24/(x+2)$
off hwy	24	x	$24/x$

Since 24 minutes is 2/5 of an hour, we get

$$\begin{aligned}
\frac{2}{5} + \frac{24}{x+2} &= \frac{24}{x} \\
2x(x+2) + 24(5)x &= 24(5)(x+2) \\
2x^2 + 4x + 120x &= 120x + 240 \\
x^2 + 2x - 120 &= 0 \\
(x+12)(x-10) &= 0 \\
x &= -12, 10.
\end{aligned}$$

The normal speed of the tortoise is 10 ft/hr.

118. Let x be the average speed at night.

	distance	rate	time
day	600	$x+20$	$600/(x+20)$
night	400	x	$400/x$

Since the race took 35 hours, we have

$$\begin{aligned}
\frac{600}{x+20} + \frac{400}{x} &= 35 \\
600x + 400(x+20) &= 35x(x+20) \\
0 &= 35x^2 - 300x - 8000 \\
0 &= 7x^2 - 60x - 1600.
\end{aligned}$$

By the quadratic formula, $x = 20, -\dfrac{80}{7}$. Since $x = 20$, the daytime average speed is 40 mph.

119. Using $v_1^2 = v_0^2 + 2gS$ with $S = 1.07$ and $v_1 = 0$, we find that

$$\begin{aligned}
v_0^2 + 2(-9.8)(1.07) &= 0 \\
v_0^2 - 20.972 &= 0 \\
v_0 = \pm\sqrt{20.972} &\approx \pm 4.58.
\end{aligned}$$

His initial upward velocity is 4.58 m/sec.

Using $S = \dfrac{1}{2}gt^2 + v_0 t$ with $S = 0$ and $v_o = 4.58$, we find that his time t in the air satisfies

$$\begin{aligned}
\frac{1}{2}(-9.8)t^2 + 4.58t &= 0 \\
t(4.58 - 4.9t) &= 0 \\
t &\approx 0, 0.93.
\end{aligned}$$

Carter is in the air for 0.93 seconds.

120. **a)** If one substitutes $h = 18,000$ into the equation

$$a = 3.89 \times 10^{-10}h^2 - 3.48 \times 10^{-5}h + 1,$$

one finds $a \approx 0.5$ atm.

b) If $a = 0.52$, then

$$0.52 = 3.89 \times 10^{-10}h^2 - 3.48 \times 10^{-5}h + 1.$$

Set the left side to 0 to obtain

$$0 = 3.89 \times 10^{-10}h^2 - 3.48 \times 10^{-5}h + 0.48.$$

By the quadratic formula, one finds

$$h \approx 17,000 \text{ or } h \approx 72,422.$$

Since $h = 72,422$ ft is a very high altitude, the altitude of highest human settlements is about 17,000 ft.

c) By using the points $(28000, 0.33)$, $(20000, 0.46)$, $(10000, 0.69)$, and $(0, 1)$, and the quadratic regression feature of a calculator, we obtain

$$a = \left(3.89 \times 10^{-10}\right) h^2 - \left(3.48 \times 10^{-5}\right) h + 1.$$

121. Let x and $x - 2$ be the number of days it takes to design a direct-mail package using traditional methods and a computer, respectively.

	rate
together	$2/7$
computer	$1/(x-2)$
traditional	$1/x$

$$\begin{aligned}
\frac{1}{x-2} + \frac{1}{x} &= \frac{2}{7} \\
7x + (7x - 14) &= 2(x^2 - 2x) \\
0 &= 2x^2 - 18x + 14 \\
0 &= x^2 - 9x + 7 \\
x &= \frac{9 \pm \sqrt{81 - 28}}{2} \\
x &= \frac{9 \pm \sqrt{53}}{2} \\
x &\approx 8.14, 0.86
\end{aligned}$$

Curt, using traditional methods, can do the job in 8.14 days.

Note, $x \approx 0.86$ days has to be excluded since $x - 2$ is negative when $x \approx 0.86$.

122. Let x be the number of hours it takes Stephanie to sew all the sequins.

	rate
together	$1/17$
Stephanie	$1/x$
Maria	$1/(x-10)$

Adding the rates,

$$\begin{aligned}
\frac{1}{x-10} + \frac{1}{x} &= \frac{1}{17} \\
17x + 17(x - 10) &= x^2 - 10x \\
0 &= x^2 - 44x + 170 \\
x &= \frac{44 \pm \sqrt{44^2 - 4(1)(170)}}{2} \\
x &= \frac{44 \pm \sqrt{1256}}{2} \\
x &= \frac{44 \pm 2\sqrt{314}}{2} \\
x &= 22 \pm \sqrt{314} \approx 39.7, 4.28
\end{aligned}$$

Stephanie can sew on all sequins in 39.7 hrs.

If $x \approx 4.28$ hrs., then $x - 10 \approx -5.72$ hrs. which is impossible.

123. Let x and $x - 10$ be the number of pounds of white meat chicken in a Party Size bucket and a Big Family Size bucket, respectively. From the ratios, we obtain

$$\begin{aligned}
\frac{8}{x} &= \frac{3}{x-10} + 0.10 \\
8(x - 10) &= 3x + 0.10x(x - 10) \\
0 &= 0.10x^2 - 6x + 80 \\
0 &= x^2 - 60x + 800 \\
0 &= (x - 40)(x - 20) \\
x &= 40, 20
\end{aligned}$$

A Party Size bucket weighs 20 or 40 lbs.

124. Let x be the amount of water, in quarts, that was in the radiator originally. After adding 2 quarts of antifreeze, the ratio of antifreeze to water becomes $\frac{2}{x+2}$. Adding a quart of antifreeze and a quart of water changes the ratio to $\frac{3}{x+4}$. Then

$$\begin{aligned}
0.03 + \frac{2}{x+2} &= \frac{3}{x+4} \\
0.03(x + 2)(x + 4) + 2(x + 4) &= 3(x + 2) \\
0.03(x^2 + 6x + 8) + 2x + 8 &= 3x + 6 \\
0.03x^2 - 0.82x + 2.24 &= 0
\end{aligned}$$

$$\begin{aligned}
x &= \frac{0.82 \pm \sqrt{(-0.82)^2 - 4(0.03)(2.24)}}{2(0.03)} \\
x &\approx 24.255, 3.078.
\end{aligned}$$

Steve's radiator originally had 24.255 or 3.078 quarts of water.

125. **a)** Let x be the number of years since 1980. With the aid of a graphing calculator, the quadratic regression curve is approximately

$$y = -0.054x^2 + 0.996x + 51.78.$$

b) Using the regression curve in part a) and the quadratic formula, we find that the positive solution to

$$0 = ax^2 + bx + c$$

is

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \approx 42.$$

In the year 2022 ($= 1980 + 42$), the extrapolated birth rate will be zero.

126. **a)** Let x be the number of years since 1980. With the aid of a graphing calculator, the regression line is approximately

$$y = -0.625x + 58.532.$$

b) Using the regression line in part a), we find that the solution to

$$0 = -0.625x + 58.532$$

is

$$x = \frac{58.532}{0.625} \approx 94.$$

In the year 2074 ($= 1980 + 94$), the extrapolated linear birth rate will be zero.

127. Since the lines are parallel, we find C such that the point $(-2, 6)$ satisfies

$$4x - 5y = C$$

Then $-8 - 30 = C$ or $C = -38$. The standard form is $4x - 5y = -38$.

128. The domain does not include the solutions of

$$x^2 + 4x - 5 = (x + 5)(x - 1) = 0$$

which are $x = -5, 1$. Then the domain is

$$\{x| \ x \neq -5, x \neq 1\}.$$

129. Let x be the amount she invested in a CD.

$$\begin{aligned} 0.05x + (x + 4000)0.06 &= 1230 \\ 0.11x + 240 &= 1230 \\ 0.11x &= 990 \\ x &= \$9000 \end{aligned}$$

130. Apply the method of completing the square.

$$\begin{aligned} x^2 + \left(y^2 - 3y + \frac{9}{4}\right) &= \frac{9}{4} \\ x^2 + \left(y - \frac{3}{2}\right)^2 &= \frac{9}{4}. \end{aligned}$$

The radius is $\frac{3}{2}$.

131. $-\sqrt{2}$

132. $y = 0.8x + 3.2$

133. a) Let $m = l + 1$ and $n = w + 1$ where l and w are relatively prime. The number of streets that the crow passes as it flies from $(1, 1)$ to (m, n) is l. When the crow crosses an avenue, it will cross over two blocks bounded by the same streets (and not over an intersection) since l and w are relatively prime. Since the crow will cross over $w-1$ avenues between the first block and the last block, the crow flies over $w - 1$ additional blocks. Thus, the total number of blocks that the crow flies over is

$$l + w - 1 = m + n - 3.$$

b) Let $m = l + 1$ and $n = w + 1$ where d is the greatest common divisor of l and w. Write $l = dl_1$ and $w = dw_1$ where l_1 and w_1 are relatively prime. Then we can break the flight of the crow into d segments described as follows:

(a) $(1, 1)$ to $(l_1 + 1, w_1 + 1)$

(b) $(l_1 + 1, w_1 + 1)$ to $(2l_1 + 1, 2w_1 + 1)$, and so on, and finally from

(c) $((d - 1)l_1 + 1, (d - 1)w_1 + 1)$ to $(dl_1 + 1, dw_1 + 1)$.

By part a), the number of blocks that the crow flies over in each segment is

$$l_1 + w_1 - 1.$$

Thus, the total number of blocks that the crow flies over is

$$d(l_1 + w_1 - 1) = l + w - d = m + n - 2 - d.$$

134. a) Let x be the volume of the bucket in quarts. Initially, the radiator is filled entirely with 20 quarts of pure antifreeze. If x quarts is taken from the radiator, and the radiator is filled with water, then $20 - x$ quarts of pure antifreeze is left.

If x quarts again is taken, and the radiator is filled with water, then the amount of pure antifreeze left is

$$\begin{aligned}(20 - x) - \left(\frac{20 - x}{20}\right) x &= \\ (20 - x)\left(1 - \frac{x}{20}\right) &= \\ \frac{(20 - x)^2}{20}.\end{aligned}$$

If the current content of the radiator is 50% pure antifreeze, then

$$\begin{aligned}\frac{(20 - x)^2}{20} &= 10 \\ (20 - x)^2 &= 2(10)^2 \\ 20 - x &= 10\sqrt{2} \\ x &= 20 - 10\sqrt{2}.\end{aligned}$$

b) If it takes three times to get a mix that is 50% antifreeze, then

$$\begin{aligned}\frac{(20 - x)^2}{20} - \left(\frac{\frac{(20-x)^2}{20}}{20}\right) x &= 10 \\ \frac{(20 - x)^2}{20}\left(1 - \frac{x}{20}\right) &= 10 \\ \frac{(20 - x)^3}{20^2} &= 10 \\ 20 - \sqrt[3]{10(20)^2} &= x \\ x &\approx 4.126 \text{ qt.}\end{aligned}$$

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1.5 Pop Quiz

1. $\{\pm\sqrt{2}\}$

2. Since $x^2 - 2x - 48 = (x - 8)(x + 6) = 0$, the solution set is $\{-6, 8\}$.

3. Completing the square, we find

$$\begin{aligned}x^2 - 4x + 4 &= 1 + 4 \\ (x - 2)^2 &= 5 \\ x &= 2 \pm \sqrt{5}.\end{aligned}$$

The solution set is $\{2 \pm \sqrt{5}\}$.

4. In $2x^2 - 4x - 3 = 0$, we have $a = 2, b = -4$, and $c = -3$. By the quadratic formuula, we find $x = \dfrac{4 \pm \sqrt{16 - (-24)}}{4} = \dfrac{4 \pm \sqrt{40}}{4} = \dfrac{4 \pm 2\sqrt{10}}{4}$. The solution set is $\left\{\dfrac{2 \pm \sqrt{10}}{2}\right\}$.

5. Since the discriminant is negative, i.e.,

$$b^2 - 4ac = 81 - 4(5)(5) < 0$$

the equation $5x^2 - 9x + 5 = 0$ has no real solution.

1.5 Linking Concepts

a) Atlanta's GB is

$$GB = \frac{(38 - 35) + (29 - 24)}{2} = \frac{8}{2} = 4.$$

Philadelphia's GB is

$$GB = \frac{(38 - 34) + (31 - 24)}{2} = \frac{11}{2} = 5.5.$$

b) Chicago's GB is given by

$$\frac{(18 - 22) + (16 - 13)}{2} = -0.5.$$

c) In terms of GB statistics, Chicago is not behind Pittsburgh.

d) If D_a is the deficit for Atlanta, then

$$\begin{aligned}
\frac{38}{62+D_a} &= \frac{35+D_a}{64+D_a} \\
38(64+D_a) &= (35+D_a)(62+D_a) \\
2432+38D_a &= D_a^2+97D_a+2170 \\
0 &= D_a^2+59D_a-262 \\
D_a &= \frac{-59\pm\sqrt{59^2-4(1)(-262)}}{2} \\
D_a &\approx 4.15, -63.1.
\end{aligned}$$

Atlanta's deficit is $D_a \approx 4.1$.

If D_p is the deficit for Philadelphia, then

$$\begin{aligned}
\frac{38}{62+D_p} &= \frac{34+D_p}{65+D_p} \\
38(65+D_p) &= (34+D_p)(62+D_p) \\
2470+38D_p &= D_p^2+96D_p+2108 \\
0 &= D_p^2+58D_p-362 \\
D_p &= \frac{-58\pm\sqrt{58^2-4(1)(-362)}}{2} \\
D_p &\approx 5.7, -63.7.
\end{aligned}$$

Philadelphia's deficit is $D_p \approx 5.7$.

If D_c is the deficit for Chicago, then

$$\begin{aligned}
\frac{18}{31+D_c} &= \frac{22+D_c}{38+D_c} \\
18(38+D_c) &= (22+D_c)(31+D_c) \\
684+18D_c &= D_c^2+53D_c+682 \\
D_c^2+35D_c-2 &= 0 \\
D_c &= \frac{-35\pm\sqrt{35^2-4(1)(-2)}}{2} \\
D_c &\approx 0.057, -35.06.
\end{aligned}$$

Chicago's deficit is $D_c \approx 0.06$.

e) As seen in part b), the GB can be negative as it is for Chicago.

If a better team has a win-loss record of (a, b) and a second team has a win-loss record (c, d) where $\frac{a}{a+b} > \frac{c}{c+d}$, then the deficit D of the second team is always positive. To see this, observe that from

$$\frac{a}{a+b+D} = \frac{c+D}{c+d+D}$$

one can obtain $D^2 + (b+c)D + bc - ad = 0$. Using the quadratic formula, we find

$$D = \frac{-(b+c)\pm\sqrt{(b+c)^2+4(ad-bc)}}{2}.$$

Since $\frac{a}{a+b} > \frac{c}{c+d}$, it follows that $ad - bc > 0$. Thus, $\sqrt{(b+c)^2+4(ad-bc)} > (b+c)$ and the values of D cannot be negative.

f) From the tabulated values,

Team	GB	D
Atlanta	4.0	4.15
Philadelphia	5.5	5.7
Chicago	−0.5	0.06

the deficit D is a better measure of how far a team is from first place since a team with a negative GB may mislead us to believe that it is in first place.

For Thought

1. False, since

$$(\sqrt{x-1}+\sqrt{x})^2 = (x-1) + 2\sqrt{x(x-1)} + x.$$

2. False, since -1 is a solution of the first and not of the second equation.

3. False, since -27 is a solution of the first equation but not of the second.

4. False, rather let $u = x^{1/4}$ and $u^2 = x^{1/2}$.

5. True, since $x - 1 = \pm 4^{-3/2}$.

6. False, $\left(-\frac{1}{32}\right)^{-2/5} = (-32)^{2/5} = (-2)^2 = 4$.

7. False, $x = -2$ is not a solution.

8. True

9. True

10. False, since $(x^3)^2 = x^6$.

1.6 Exercises

1. Factor: $x^2(x+3)-4(x+3)=0$
$(x^2-4)(x+3)=(x-2)(x+2)(x+3)=0$
The solution set is $\{\pm 2,-3\}$.

2. Factor: $x^2(x-1)-5(x-1)=0$
$(x^2-5)(x-1)=(x-\sqrt{5})(x+\sqrt{5})(x-1)=0$
The solution set is $\{\pm\sqrt{5},1\}$.

3. Factor: $2x^2(x+500)-(x+500)=0$
$(2x^2-1)(x+500)=0$
The solution set is $\left\{\pm\dfrac{\sqrt{2}}{2},-500\right\}$.

4. Factor the left-hand side.

$$\begin{aligned} 3x^2(x-400)-2(x-400) &= 0 \\ (3x^2-2)(x-400) &= 0 \\ x^2=\frac{2}{3} \quad &\text{or} \quad x=400 \end{aligned}$$

The solution set is $\left\{\pm\dfrac{\sqrt{6}}{3},400\right\}$.

5. Set the right-hand side to 0 and factor.

$$\begin{aligned} a(a^2-15a+5) &= 0 \\ a=\frac{15\pm\sqrt{(-15)^2-4(1)(5)}}{2} \quad &\text{or} \quad a=0 \\ a=\frac{15\pm\sqrt{205}}{2} \quad &\text{or} \quad a=0 \end{aligned}$$

The solution set is $\left\{\dfrac{15\pm\sqrt{205}}{2},0\right\}$.

6. Factor: $b(b^2-9b+20)=b(b-5)(b-4)=0$
The solution set is $\{0,4,5\}$.

7. Factor: $3y^2(y^2-4)=3y^2(y-2)(y+2)=0$
The solution set is $\{0,\pm 2\}$.

8. Factor: $5m^2(m^2-2m+1)=5m^2(m-1)^2=0$
The solution set is $\{0,1\}$.

9. Factor: $(a^2-4)(a^2+4)=$
$(a-2)(a+2)(a-2i)(a+2i)=0$.
The solution set is $\{\pm 2,\pm 2i\}$.

10. Factoring, we obtain

$$w(w^3+2^3)=w(w+2)(w^2-2w+4)=0.$$

Then

$$\begin{aligned} w=0,-2 \quad &\text{or} \quad w=\frac{2\pm\sqrt{4-4(1)(4)}}{2} \\ w=0,-2 \quad &\text{or} \quad w=\frac{2\pm\sqrt{-12}}{2} \\ w=0,-2 \quad &\text{or} \quad w=\frac{2\pm 2i\sqrt{3}}{2} \end{aligned}$$

and the solution set is $\{0,-2,1\pm i\sqrt{3}\}$.

11. Squaring each side, we get

$$\begin{aligned} x+1 &= x^2-10x+25 \\ 0 &= x^2-11x+24=(x-8)(x-3). \end{aligned}$$

Checking $x=3$, we get $2\neq -2$. Then $x=3$ is an extraneous root. The solution set is $\{8\}$.

12. Squaring each side,

$$\begin{aligned} x-1 &= x^2-14x+49 \\ 0 &= x^2-15x+50=(x-5)(x-10). \end{aligned}$$

Checking $x=5$, we get $2\neq -2$. Then $x=5$ is an extraneous root. The solution set is $\{10\}$.

13. Isolate the radical and then square each side.

$$\begin{aligned} x &= x^2-40x+400 \\ 0 &= x^2-41x+400=(x-25)(x-16) \end{aligned}$$

Checking $x=16$, we get $2\neq -6$. Then $x=16$ is an extraneous root. The solution set is $\{25\}$.

14. Isolate the radical and then square each side.

$$\begin{aligned} x &= x^2-4x+4 \\ 0 &= x^2-5x+4=(x-4)(x-1) \end{aligned}$$

Checking $x=1$, we get $4\neq 2$. Then $x=1$ is an extraneous root. The solution set is $\{4\}$.

15. Isolate the radical and then square each side.

$$\begin{aligned} 2w &= \sqrt{1-3w} \\ 4w^2 &= 1-3w \\ 4w^2+3w-1 &= (4w-1)(w+1)=0 \\ w &= \frac{1}{4},-1 \end{aligned}$$

Checking $w = -1$, we get $-1 \neq 1$.
Then $w = -1$ is an extraneous root.

The solution set is $\left\{\frac{1}{4}\right\}$.

16. Isolate the radical and then square each side.

$$\begin{aligned} 3t &= \sqrt{2-3t} \\ 9t^2 &= 2-3t \\ 9t^2+3t-2 &= (3t-1)(3t+2) = 0 \\ t &= \frac{1}{3}, -\frac{2}{3} \end{aligned}$$

Checking $t = -\frac{2}{3}$, we get $-\frac{2}{3} \neq \frac{2}{3}$.

Then $t = -\frac{2}{3}$ is an extraneous root.

The solution set is $\left\{\frac{1}{3}\right\}$.

17. Multiply both sides by $z\sqrt{4z+1}$ and square each side.

$$\begin{aligned} \sqrt{4z+1} &= 3z \\ 4z+1 &= 9z^2 \\ 0 &= 9z^2-4z-1 \end{aligned}$$

By the quadratic formula,

$$\begin{aligned} z &= \frac{4 \pm \sqrt{16-4(9)(-1)}}{18} \\ z &= \frac{4 \pm \sqrt{52}}{18} = \frac{4 \pm 2\sqrt{13}}{18} = \frac{2 \pm \sqrt{13}}{9}. \end{aligned}$$

Since $z = \frac{2-\sqrt{13}}{9} < 0$ and the right-hand side of the original equation is nonnegative, $z = \frac{2-\sqrt{13}}{9}$ is an extraneous root.

The solution set is $\left\{\frac{2+\sqrt{13}}{9}\right\}$.

18. Multiply both sides by $p\sqrt{9p+1}$.

$$\begin{aligned} \sqrt{9p+1}-2p &= 0 \\ \sqrt{9p+1} &= 2p \\ 9p+1 &= 4p^2 \\ 4p^2-9p-1 &= 0 \end{aligned}$$

By the quadratic formula,

$$p = \frac{9 \pm \sqrt{81-4(4)(-1)}}{8} = \frac{9 \pm \sqrt{97}}{8}.$$

If $p = \frac{9-\sqrt{97}}{8}$ then the left-hand side of the original equation becomes negative and so $p = \frac{9-\sqrt{97}}{8}$ is an extraneous root.

The solution set is $\left\{\frac{9+\sqrt{97}}{8}\right\}$.

19. Squaring each side, one obtains

$$\begin{aligned} x^2-2x-15 &= 9 \\ x^2-2x-24 &= (x-6)(x+4) = 0. \end{aligned}$$

The solution set is $\{-4, 6\}$.

20. Squaring each side, one obtains

$$\begin{aligned} 3x^2+5x-3 &= x^2 \\ 2x^2+5x-3 &= (2x-1)(x+3) = 0 \\ x &= -3, \frac{1}{2}. \end{aligned}$$

Checking $x = -3$, notice that the left-hand side and the right hand-side of the original equation have opposite signs. Then $x = -3$ is an extraneous root. The solution set is $\left\{\frac{1}{2}\right\}$.

21. Isolate a radical and square each side.

$$\begin{aligned} \sqrt{x+40} &= \sqrt{x}+4 \\ x+40 &= x+8\sqrt{x}+16 \\ 24 &= 8\sqrt{x} \\ 3 &= \sqrt{x} \\ 9 &= x \end{aligned}$$

The solution set is $\{9\}$.

22. Isolate a radical and square each side.

$$\begin{aligned} \sqrt{x-36} &= 2-\sqrt{x} \\ x-36 &= 4-4\sqrt{x}+x \\ -40 &= -4\sqrt{x} \\ 10 &= \sqrt{x} \\ 100 &= x \end{aligned}$$

Checking $x = 100$ we get $18 = 2$, a contradiction. There is no solution and the solution set is the empty set $\emptyset$.

23. Isolate a radical and square each side.

$$\begin{aligned} \sqrt{n+4} &= 5-\sqrt{n-1} \\ n+4 &= 25-10\sqrt{n-1}+(n-1) \\ -20 &= -10\sqrt{n-1} \\ 2 &= \sqrt{n-1} \\ 4 &= n-1 \end{aligned}$$

The solution set is $\{5\}$.

24. Isolate a radical and square each side.

$$\begin{aligned} \sqrt{y+10} &= 2+\sqrt{y-2} \\ y+10 &= 4+4\sqrt{y-2}+(y-2) \\ 8 &= 4\sqrt{y-2} \\ 2 &= \sqrt{y-2} \\ 4 &= y-2 \end{aligned}$$

The solution set is $\{6\}$.

25. Isolate a radical and square each side.

$$\begin{aligned} \sqrt{2x+5} &= 9-\sqrt{x+6} \\ 2x+5 &= 81-18\sqrt{x+6}+(x+6) \\ x-82 &= -18\sqrt{x+6} \\ x^2-164x+6724 &= 324(x+6) \\ x^2-488x+4780 &= 0 \\ (x-10)(x-478) &= 0 \end{aligned}$$

Checking $x=478$ we get $53 \neq 9$ and $x=478$ is an extraneous root. The solution set is $\{10\}$.

26. Isolate a radical and square each side.

$$\begin{aligned} \sqrt{3x-2} &= 2+\sqrt{x-2} \\ 3x-2 &= 4+4\sqrt{x-2}+(x-2) \\ 2x-4 &= 4\sqrt{x-2} \\ x-2 &= 2\sqrt{x-2} \\ x^2-4x+4 &= 4(x-2) \\ x^2-8x+12 &= 0 \\ (x-6)(x-2) &= 0 \\ x &= 6, 2 \end{aligned}$$

The solution set is $\{2, 6\}$.

27. Raise each side to the power $3/2$.
Then $x = \pm 2^{3/2} = \pm 8^{1/2} = \pm 2\sqrt{2}$.
The solution set is $\left\{\pm 2\sqrt{2}\right\}$.

28. Raise each side to the power $3/2$.

Then $x = \pm\left(\dfrac{1}{2}\right)^{3/2} = \pm\left(\dfrac{1}{8}\right)^{1/2} =$

$\pm\dfrac{1}{2\sqrt{2}} = \pm\dfrac{\sqrt{2}}{4}$. The solution set is $\left\{\pm\dfrac{\sqrt{2}}{4}\right\}$.

29. Raise each side to the power $-3/4$.

Thus, $w = \pm(16)^{-3/4} = \pm(2)^{-3} = \pm\dfrac{1}{8}$.

The solution set is $\left\{\pm\dfrac{1}{8}\right\}$.

30. Raise each side to the power $-2/3$.
Thus, $w = (27)^{-2/3} = (3)^{-2} = \dfrac{1}{9}$.
The solution set is $\left\{\dfrac{1}{9}\right\}$.

31. Raise each side to the power -2.
So, $t = (7)^{-2} = \dfrac{1}{49}$. The solution set is $\left\{\dfrac{1}{49}\right\}$.

32. Raise each side to the power -2.
So, $t = \left(\dfrac{1}{2}\right)^{-2} = 4$. The solution set is $\{4\}$.

33. Raise each side to the power -2.
Then $s-1 = (2)^{-2} = \dfrac{1}{4}$ and $s = 1+\dfrac{1}{4}$.

The solution set is $\left\{\dfrac{5}{4}\right\}$.

34. Raise each side to the power -2.

Then $s-2 = \left(\dfrac{1}{3}\right)^{-2} = 9$ and the solution

set is $\{11\}$.

35. Since $(x^2-9)(x^2-3)=0$, the solution set is $\left\{\pm 3, \pm\sqrt{3}\right\}$.

36. Since $x^4-7x^2+10=0$, $(x^2-5)(x^2-2)=0$.
The solution set is $\left\{\pm\sqrt{5}, \pm\sqrt{2}\right\}$.

37. Since $(x^2+7)(x^2-1)=0$, the solution set is $\left\{\pm 1, \pm i\sqrt{7}\right\}$.

38. Since $(x^2+3)(x^2-4)=0$, the solution set is $\left\{\pm 2, \pm i\sqrt{3}\right\}$.

39. Since $(x^2+9)(x^2-9)=0$, the solution set is $\{\pm 3, \pm 3i\}$.

40. Since $(x^2+25)(x^2-25)=0$, the solution set is $\{\pm 5, \pm 5i\}$.

41. Let $u = \dfrac{2c-3}{5}$ and $u^2 = \left(\dfrac{2c-3}{5}\right)^2$. Then

$$\begin{aligned} u^2+2u-8 &= 0 \\ (u+4)(u-2) &= 0 \\ u &= -4, 2 \end{aligned}$$

$$\begin{array}{rcl} \dfrac{2c-3}{5} = -4 & \text{or} & \dfrac{2c-3}{5} = 2 \\ 2c-3 = -20 & \text{or} & 2c-3 = 10 \\ c = -\dfrac{17}{2} & \text{or} & c = \dfrac{13}{2}. \end{array}$$

The solution set is $\left\{-\dfrac{17}{2}, \dfrac{13}{2}\right\}$.

42. Let $u = \dfrac{b-5}{6}$ and $u^2 = \left(\dfrac{b-5}{6}\right)^2$. Then

$$\begin{aligned} u^2-u-6 &= 0 \\ (u-3)(u+2) &= 0 \\ u &= 3, -2 \end{aligned}$$

$$\begin{array}{rcl} \dfrac{b-5}{6} = 3 & \text{or} & \dfrac{b-5}{6} = -2 \\ b-5 = 18 & \text{or} & b-5 = -12 \\ b = 23 & \text{or} & b = -7. \end{array}$$

The solution set is $\{23, -7\}$.

43. Let $u = \dfrac{1}{5x-1}$ and $u^2 = \left(\dfrac{1}{5x-1}\right)^2$. Then

$$\begin{aligned} u^2+u-12 &= (u+4)(u-3)=0 \\ u &= -4, 3 \end{aligned}$$

$$\begin{array}{rcl} \dfrac{1}{5x-1} = -4 & \text{or} & \dfrac{1}{5x-1} = 3 \\ 1 = -20x+4 & \text{or} & 1 = 15x-3 \\ x = \dfrac{3}{20} & \text{or} & \dfrac{4}{15} = x. \end{array}$$

The solution set is $\left\{\dfrac{3}{20}, \dfrac{4}{15}\right\}$.

44. Let $u = \dfrac{1}{x-3}$ and $u^2 = \left(\dfrac{1}{x-3}\right)^2$. Then

$$\begin{aligned} u^2+2u-24 &= (u+6)(u-4)=0 \\ u &= -6, 4 \end{aligned}$$

$$\begin{array}{rcl} \dfrac{1}{x-3} = -6 & \text{or} & \dfrac{1}{x-3} = 4 \\ 1 = -6x+18 & \text{or} & 1 = 4x-12 \\ x = \dfrac{17}{6} & \text{or} & \dfrac{13}{4} = x. \end{array}$$

The solution set is $\left\{\dfrac{17}{6}, \dfrac{13}{4}\right\}$.

45. Let $u = v^2-4v$ and $u^2 = \left(v^2-4v\right)^2$. Then

$$\begin{aligned} u^2-17u+60 &= (u-5)(u-12)=0 \\ u &= 5, 12 \end{aligned}$$

$$\begin{array}{rcl} v^2-4v=5 & \text{or} & v^2-4v=12 \\ v^2-4v-5=0 & \text{or} & v^2-4v-12=0 \\ (v-5)(v+1)=0 & \text{or} & (v-6)(v+2)=0. \end{array}$$

The solution set is $\{-2, -1, 5, 6\}$.

46. Let $t = u^2+2u$ and $t^2 = \left(u^2+2u\right)^2$. Then

$$\begin{aligned} t^2-2t-3 &= (t-3)(t+1)=0 \\ t &= 3, -1 \end{aligned}$$

$$\begin{array}{rcl} u^2+2u=3 & \text{or} & u^2+2u=-1 \\ u^2+2u-3=0 & \text{or} & u^2+2u+1=0 \\ (u-1)(u+3)=0 & \text{or} & (u+1)^2=0. \end{array}$$

The solution set is $\{\pm 1, -3\}$.

47. Factor the left-hand side.

$$\begin{aligned} \left(\sqrt{x}-3\right)\left(\sqrt{x}-1\right) &= 0 \\ \sqrt{x} &= 3, 1 \\ x &= 9, 1 \end{aligned}$$

The solution set is $\{1, 9\}$.

48. Factor the left-hand side as $(2\sqrt{x}-5)\left(\sqrt{x}+4\right)=0$. Since $\sqrt{x}$ is nonnegative then only the first factor can be zero and so $\sqrt{x} = \dfrac{5}{2}$. The solution set is $\left\{\dfrac{25}{4}\right\}$.

49. Factor the left-hand side as $\left(\sqrt{q}-4\right)\left(\sqrt{q}-3\right)=0$. Then $\sqrt{q}=3,4$ and the solution set is $\{9, 16\}$.

50. Set the right-hand side to 0 and factor. Then

$$\begin{aligned} h - 2h^{1/2} + 1 &= 0 \\ \left(\sqrt{h} - 1\right)^2 &= 0 \\ \sqrt{h} &= 1 \\ h &= 1 \end{aligned}$$

The solution set is $\{1\}$.

51. Set the right-hand side to 0 and factor.

$$\begin{aligned} x^{2/3} - 7x^{1/3} + 10 &= 0 \\ \left(x^{1/3} - 5\right)\left(x^{1/3} - 2\right) &= 0 \\ x^{1/3} &= 5, 2 \end{aligned}$$

The solution set is $\{8, 125\}$.

52. Factor the left-hand side.

Then $\left(x^{1/4} - 2\right)\left(x^{1/4} - 1\right) = 0$.

Since $x^{1/4} = 1, 2$ the solution set is $\{1, 16\}$.

53. An equivalent statement is

$$\begin{aligned} w^2 - 4 = 3 \quad &\text{or} \quad w^2 - 4 = -3 \\ w^2 = 7 \quad &\text{or} \quad w^2 = 1. \end{aligned}$$

The solution set is $\left\{\pm\sqrt{7}, \pm 1\right\}$.

54. An equivalent statement is

$$\begin{aligned} a^2 - 1 = 1 \quad &\text{or} \quad a^2 - 1 = -1 \\ a^2 = 2 \quad &\text{or} \quad a^2 = 0. \end{aligned}$$

The solution set is $\left\{\pm\sqrt{2}, 0\right\}$.

55. An equivalent statement assuming $5v \geq 0$ is

$$\begin{aligned} v^2 - 3v = 5v \quad &\text{or} \quad v^2 - 3v = -5v \\ v^2 - 8v = 0 \quad &\text{or} \quad v^2 + 2v = 0 \\ v(v - 8) = 0 \quad &\text{or} \quad v(v + 2) = 0 \\ v \quad &= \quad 0, 8, -2. \end{aligned}$$

Since $5v \geq 0$, $v = -2$ is an extraneous root and the solution set is $\{0, 8\}$.

56. An equivalent statement assuming $z \geq 0$ is

$$\begin{aligned} z^2 - 12 = z \quad &\text{or} \quad z^2 - 12 = -z \\ z^2 - z - 12 = 0 \quad &\text{or} \quad z^2 + z - 12 = 0 \\ (z - 4)(z + 3) = 0 \quad &\text{or} \quad (z + 4)(z - 3) = 0 \\ z \quad &= \quad \pm 3, \pm 4. \end{aligned}$$

Since $z \geq 0$, $z = -3, -4$ are extraneous roots and the solution set is $\{3, 4\}$.

57. An equivalent statement is

$$\begin{aligned} x^2 - x - 6 = 6 \quad &\text{or} \quad x^2 - x - 6 = -6 \\ x^2 - x - 12 = 0 \quad &\text{or} \quad x^2 - x = 0 \\ (x - 4)(x + 3) = 0 \quad &\text{or} \quad x(x - 1) = 0. \end{aligned}$$

The solution set is $\{-3, 0, 1, 4\}$.

58. An equivalent statement is

$$\begin{aligned} 2x^2 - x - 2 = 1 \quad &\text{or} \quad 2x^2 - x - 2 = -1 \\ 2x^2 - x - 3 = 0 \quad &\text{or} \quad 2x^2 - x - 1 = 0 \\ (2x - 3)(x + 1) = 0 \quad &\text{or} \quad (2x + 1)(x - 1) = 0. \end{aligned}$$

The solution set is $\left\{\pm 1, \frac{3}{2}, -\frac{1}{2}\right\}$.

59. An equivalent statement is

$$\begin{aligned} x + 5 = 2x + 1 \quad &\text{or} \quad x + 5 = -(2x + 1) \\ 4 = x \quad &\text{or} \quad x = -2. \end{aligned}$$

The solution set is $\{-2, 4\}$.

60. An equivalent statement is

$$\begin{aligned} 3x - 4 = x \quad &\text{or} \quad 3x - 4 = -x \\ 2x = 4 \quad &\text{or} \quad 4x = 4. \end{aligned}$$

The solution set is $\{1, 2\}$.

61. An equivalent statement is

$$\begin{aligned} x - 2 = 5x - 1 \quad &\text{or} \quad x - 2 = -5x + 1 \\ -1 = 4x \quad &\text{or} \quad 6x = 3 \\ -\frac{1}{4} = x \quad &\text{or} \quad x = \frac{1}{2} \end{aligned}$$

Note, $x = -\frac{1}{4}$ is an extraneous root.

The solution set is $\{1/2\}$.

62. An equivalent statement is

$$\begin{aligned} x - 4 = 1 - 4x \quad &\text{or} \quad x - 4 = -1 + 4x \\ 5x = 5 \quad &\text{or} \quad -3 = 3x \\ x = 1 \quad &\text{or} \quad x = -1 \end{aligned}$$

Note, $x = 1$ is an extraneous root. The solution set is $\{-1\}$.

63. An equivalent statement is

$$\begin{array}{rcl} x-4=x-2 & \text{or} & x-4=-x+2 \\ -4=-2 & \text{or} & 2x=6 \\ \textit{inconsistent} & \text{or} & x=3 \end{array}$$

The solution set is $\{3\}$.

64. An equivalent statement is

$$\begin{array}{rcl} 2x-3=2x+7 & \text{or} & 2x-3=-2x-7 \\ -3=7 & \text{or} & 4x=-4 \\ \textit{inconsistent} & \text{or} & x=-1 \end{array}$$

The solution set is $\{-1\}$.

65. Isolate a radical and square both sides.

$$\begin{aligned} \sqrt{16x+1} &= \sqrt{6x+13}-1 \\ 16x+1 &= (6x+13)-2\sqrt{6x+13}+1 \\ 10x-13 &= -2\sqrt{6x+13} \end{aligned}$$

$$\begin{aligned} 100x^2-260x+169 &= 4(6x+13) \\ 100x^2-284x+117 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{284 \pm \sqrt{284^2-4(100)(117)}}{200} \\ x &= \frac{284 \pm 184}{200} \\ x &= \frac{1}{2}, \frac{117}{50} \end{aligned}$$

Checking $x = \frac{117}{50}$ we get $\sqrt{\frac{1922}{50}} - \sqrt{\frac{1352}{50}} > 0$
and so $x = \frac{117}{50}$ is an extraneous root.

The solution set is $\left\{\frac{1}{2}\right\}$.

66. Isolate a radical and square both sides.

$$\begin{aligned} \sqrt{16x+1} &= \sqrt{6x+13}+1 \\ 16x+1 &= (6x+13)+2\sqrt{6x+13}+1 \\ 10x-13 &= 2\sqrt{6x+13} \end{aligned}$$

$$\begin{aligned} 100x^2-260x+169 &= 4(6x+13) \\ 100x^2-284x+117 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{284 \pm \sqrt{284^2-4(100)(117)}}{200} \\ x &= \frac{1}{2}, \frac{117}{50} \end{aligned}$$

Checking $x = \frac{1}{2}$ we get $-1 \neq 1$ and so $x = \frac{1}{2}$ is an extraneous root.
The solution set is $\left\{\frac{117}{50}\right\}$.

67. Factor as a difference of two squares and then as a sum and difference of two cubes.

$$\begin{aligned} (v^3-8)(v^3+8) &= 0 \\ (v-2)(v^2+2v+4)(v+2)(v^2-2v+4) &= 0 \end{aligned}$$

Then $v = \pm 2$ or

$$v = \frac{-2 \pm \sqrt{2^2-16}}{2} \quad \text{or } v = \frac{2 \pm \sqrt{2^2-16}}{2}$$
$$v = \frac{-2 \pm 2i\sqrt{3}}{2} \quad \text{or } v = \frac{2 \pm 2i\sqrt{3}}{2}$$

The solution set is $\left\{\pm 2, -1 \pm i\sqrt{3}, 1 \pm i\sqrt{3}\right\}$.

68. Since $(t^2-1)(t^2+1) = 0$, the solution set is $\{\pm 1, \pm i\}$.

69. Raise both sides to the power 4. Then

$$\begin{aligned} 7x^2-12 &= x^4 \\ 0 &= x^4-7x^2+12 = (x^2-4)(x^2-3) \\ x &= \pm 2, \pm\sqrt{3} \end{aligned}$$

Since the left-hand side of the given equation is nonnegative, $x = -2, -\sqrt{3}$ are extraneous roots. The solution set is $\left\{\sqrt{3}, 2\right\}$.

70. Raise both sides to the power 4.

$$\begin{aligned} 10x^2-1 &= 16x^4 \\ 0 &= 16x^4-10x^2+1 \\ 0 &= (8x^2-1)(2x^2-1) \\ x &= \pm\frac{1}{2\sqrt{2}}, \pm\frac{1}{\sqrt{2}} \end{aligned}$$

Since the left-hand side of the given equation is nonnegative, $x = -\frac{\sqrt{2}}{4}, -\frac{\sqrt{2}}{2}$ are extraneous roots. The solution set is $\left\{\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{2}\right\}$.

71. Raise both sides to the power 3.

$$\begin{aligned} 2+x-2x^2 &= x^3 \\ x^3+2x^2-x-2 &= 0 \\ x^2(x+2)-(x+2) &= (x^2-1)(x+2)=0 \\ x &= \pm 1, -2 \end{aligned}$$

The solution set is $\{\pm 1, -2\}$.

72. Square both sides.

$$\begin{aligned} (48+\sqrt{x})-8\sqrt{48+\sqrt{x}}+16 &= \sqrt{x} \\ 8 &= \sqrt{48+\sqrt{x}} \\ 64 &= 48+\sqrt{x} \\ 16 &= \sqrt{x} \\ 256 &= x \end{aligned}$$

The solution set is $\{256\}$.

73. Let $t = \dfrac{x-2}{3}$ and $t^2 = \left(\dfrac{x-2}{3}\right)^2$. Then

$$\begin{aligned} t^2-2t+10 &= 0 \\ t^2-2t+1 &= -10+1 \\ (t-1)^2 &= -9 \\ t &= 1\pm 3i \\ \frac{x-2}{3} &= 1\pm 3i \\ x-2 &= 3\pm 9i \\ x &= 5\pm 9i. \end{aligned}$$

The solution set is $\{5\pm 9i\}$.

74. Let $t = \dfrac{1}{x+1}$ and $t^2 = \left(\dfrac{1}{x+1}\right)^2$. Then

$$\begin{aligned} t^2-2t+2 &= 0 \\ t^2-2t+1 &= -2+1 \\ (t-1)^2 &= -1 \\ t &= 1\pm i \\ \frac{1}{x+1} &= 1\pm i \\ 1 &= (x+1)(1\pm i) \\ \frac{1}{1\pm i}\cdot\frac{1\mp i}{1\mp i} &= x+1 \\ \frac{1\mp i}{2} &= x+1 \end{aligned}$$

$$x = \frac{1-i}{2}-\frac{2}{2} \quad \text{or} \quad x = \frac{1+i}{2}-\frac{2}{2}.$$

The solution set is $\left\{\dfrac{-1\pm i}{2}\right\}$.

75. Raise both sides to the power $5/2$. Then

$$\begin{aligned} 3u-1 &= \pm 2^{5/2} \\ 3u-1 &= \pm 32^{1/2} \\ 3u &= 1\pm 4\sqrt{2} \end{aligned}$$

The solution set is $\left\{\dfrac{1\pm 4\sqrt{2}}{3}\right\}$.

76. Raise both sides to the power $3/2$.

$$\begin{aligned} 2u+1 &= \pm 3^{3/2} \\ 2u+1 &= \pm 27^{1/2} \\ 2u &= -1\pm 3\sqrt{3} \end{aligned}$$

The solution set is $\left\{\dfrac{-1\pm 3\sqrt{3}}{2}\right\}$.

77. Factor this quadratic type expression.

$$\begin{aligned} (x^2+1)-11\sqrt{x^2+1}+30 &= 0 \\ \left(\sqrt{x^2+1}-5\right)\left(\sqrt{x^2+1}-6\right) &= 0 \\ \sqrt{x^2+1}=5 \ \text{ or } \ \sqrt{x^2+1}=6 & \\ x^2=24 \ \text{ or } \ x^2=35 & \\ x=\pm 2\sqrt{6}, \pm\sqrt{35} & \end{aligned}$$

The solution set is $\left\{\pm\sqrt{35}, \pm 2\sqrt{6}\right\}$.

78. Factor this quadratic type expression.

$$\begin{aligned} (2x^2-3)-3\sqrt{2x^2-3}+2 &= 0 \\ \left(\sqrt{2x^2-3}-2\right)\left(\sqrt{2x^2-3}-1\right) &= 0 \\ \sqrt{2x^2-3}=2 \ \text{ or } \ \sqrt{2x^2-3}=1 & \\ 2x^2-3=4 \ \text{ or } \ 2x^2-3=1 & \\ x^2=\frac{7}{2} \ \text{ or } \ x^2=2 & \\ x=\pm\frac{\sqrt{14}}{2}, \pm\sqrt{2} & \end{aligned}$$

The solution set is $\left\{\pm\dfrac{\sqrt{14}}{2}, \pm\sqrt{2}\right\}$.

79. An equivalent statement is

$$\begin{array}{rcl} x^2-2x=3x-6 & \text{or} & x^2-2x=-3x+6 \\ x^2-5x+6=0 & \text{or} & x^2+x-6=0 \\ (x-3)(x-2)=0 & \text{or} & (x+3)(x-2)=0 \\ x & = & 2,\pm 3 \end{array}$$

The solution set is $\{2,\pm 3\}$.

80. An equivalent statement is

$$\begin{array}{rcl} x^2+5x=3-x^2 & \text{or} & x^2+5x=-3+x^2 \\ 2x^2+5x-3=0 & \text{or} & 5x=-3 \\ (2x-1)(x+3)=0 & \text{or} & x=-\dfrac{3}{5} \end{array}$$

The solution set is $\left\{\dfrac{1}{2},-3,-\dfrac{3}{5}\right\}$.

81. Raise both sides to the power $-5/3$. Then

$$\begin{aligned} 3m+1 &= \left(-\frac{1}{8}\right)^{-5/3} \\ 3m+1 &= \left(-\frac{1}{2}\right)^{-5} \\ 3m+1 &= -32. \end{aligned}$$

The solution set is $\{-11\}$.

82. Raise both sides to the power $-3/5$. Then

$$\begin{aligned} 1-2m &= \left(-\frac{1}{32}\right)^{-3/5} \\ 1-2m &= \left(-\frac{1}{2}\right)^{-3} \\ 1-2m &= -8 \\ -2m &= -9. \end{aligned}$$

The solution set is $\left\{\dfrac{9}{2}\right\}$.

83. An equivalent statement assuming $x-2\geq 0$ is

$$\begin{array}{rcl} x^2-4=x-2 & \text{or} & x^2-4=-x+2 \\ x^2-x-2=0 & \text{or} & x^2+x-6=0 \\ (x-2)(x+1)=0 & \text{or} & (x+3)(x-2)=0 \\ x & = & 2,-1,-3. \end{array}$$

Since $x-2\geq 0$, $x=-1,-3$ are extraneous roots and the solution set is $\{2\}$.

84. An equivalent statement assuming $x^2-4\geq 0$ is

$$\begin{array}{rcl} x^2+7x=x^2-4 & \text{or} & x^2+7x=-x^2+4 \\ 7x=-4 & \text{or} & 2x^2+7x-4=0 \\ x=-\dfrac{4}{7} & \text{or} & (2x-1)(x+4)=0 \\ x & = & -\dfrac{4}{7},\dfrac{1}{2},-4 \end{array}$$

Since $x^2-4\geq 0$, $x=-\dfrac{4}{7},\dfrac{1}{2}$ are extraneous roots and the solution set is $\{-4\}$.

85. Solve for S.

$$\begin{aligned} 21.24+1.25S^{1/2}-9.8(18.34)^{1/3} &= 16.296 \\ 1.25S^{1/2}-25.84397 &\approx -4.944 \\ S^{1/2} &\approx 16.72 \\ S &\approx 279.56 \end{aligned}$$

The maximum sailing area is 279.56 m^2.

86. Solve for D.

$$\begin{aligned} 21.52+1.25(310.64)^{1/2}-9.8D^{1/3} &= 16.296 \\ 21.52+22.03123-9.8D^{1/3} &\approx 16.296 \\ 43.55123-9.8D^{1/3} &\approx 16.296 \\ D^{1/3} &\approx 2.781146 \\ D &\approx 21.51 \end{aligned}$$

The minimum displacement is 21.51 m^3.

87. Solve for x with $C=83.50$.

$$\begin{aligned} 0.5x+\sqrt{8x+5000} &= 83.50 \\ \sqrt{8x+5000} &= 83.50-0.5x \\ 8x+5000 &= 6972.25-83.50x+0.25x^2 \\ 0 &= 0.25x^2-91.50x+1972.25 \end{aligned}$$

$$\begin{aligned} x &= \frac{91.50\pm\sqrt{(-91.50)^2-4(0.25)(1972.25)}}{0.5} \\ x &= \frac{91.50\pm 80}{0.5} \\ x &= 23,343 \end{aligned}$$

Checking $x=343$, the value of the left-hand side of the first equation exceeds 83.50 and so $x=343$ is an extraneous root.
Thus, 23 loaves cost \$83.50.

88. If x is the minimum number of loaves then

$$\begin{aligned} 4.49x - (0.5x + \sqrt{8x+5000}) &= 0 \\ 3.99x &= \sqrt{8x+5000} \\ 3.99^2x^2 - 8x - 5000 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{8 \pm \sqrt{(-8)^2 - 4(3.99)^2(-5000)}}{2(3.99)^2} \\ x &\approx -17, 18 \end{aligned}$$

Note, $x \geq 0$. To make a profit, Juanita has to sell least 18 loaves.

89. Let x and $x+6$ be two numbers. Then

$$\begin{aligned} \sqrt{x+6} - \sqrt{x} &= 1 \\ \sqrt{x+6} &= \sqrt{x}+1 \\ x+6 &= x+2\sqrt{x}+1 \\ 5 &= 2\sqrt{x} \\ 25 &= 4x \\ x &= \frac{25}{4} \end{aligned}$$

Since $\frac{25}{4} + 6 = \frac{49}{4}$, the numbers are $\frac{25}{4}$ and $\frac{49}{4}$.

90. Let x be the length of the short leg. Since $x+1$ is the length of the other leg, by the Pythagorean Theorem the hypotenuse is $\sqrt{x^2+(x+1)^2}$. Then

$$\begin{aligned} x + \sqrt{x^2+(x+1)^2} &= 10 \\ \sqrt{x^2+(x+1)^2} &= 10 - x \\ x^2 + (x^2+2x+1) &= 100 - 20x + x^2 \\ x^2 + 22x - 99 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-22 \pm \sqrt{22^2 - 4(1)(-99)}}{2} \\ x &= \frac{-22 \pm \sqrt{880}}{2} \\ x &= \frac{-22 \pm 4\sqrt{55}}{2}. \end{aligned}$$

The short leg is $-11 + 2\sqrt{55}$ cm.

91. Let x be the length of the short leg. Since $x+7$ is the other leg, by the Pythagorean Theorem the hypotenuse is $\sqrt{x^2+(x+7)^2}$. Then

$$x + (x+7) + \sqrt{x^2+(x+7)^2} = 30$$

$$\begin{aligned} 2x - 23 &= -\sqrt{x^2+(x+7)^2} \\ 4x^2 - 92x + 529 &= 2x^2 + 14x + 49 \\ 2x^2 - 106x + 480 &= 0 \\ x^2 - 53x + 240 &= 0 \\ (x-5)(x-48) &= 0 \\ x &= 5, 48 \end{aligned}$$

Since the perimeter is 30 in., $x = 48$ is an extraneous root. The short leg is $x = 5$ in.

92. Let $x, x+1$, and $2x$ be the lengths of the shortest side, other side, and the hypotenuse, respectively. By the Pythagorean Theorem, we obtain

$$\begin{aligned} x^2 + (x+1)^2 &= (2x)^2 \\ 0 &= 2x^2 - 2x - 1 \\ x &= \frac{2 \pm \sqrt{(-2)^2 - 4(2)(-1)}}{4} \\ x &= \frac{2 \pm 2\sqrt{3}}{4} \\ x &= \frac{1 \pm \sqrt{3}}{2} \end{aligned}$$

The radius is $x = \frac{1+\sqrt{3}}{2}$ ft.

93. Let x be the length of one side of the original square foundation. From the 2100 ft^2 we have

$$\begin{aligned} (x-10)(x+30) &= 2100 \\ x^2 + 20x - 2400 &= 0 \\ (x+60)(x-40) &= 0. \end{aligned}$$

Since x is nonnegative, $x = 40$ and the area of the square foundation is $x^2 = 1600$ ft^2.

94. Let x be the length of one side of the cubic box. From the 119 in.^{2} area, we have

$$\begin{aligned} (x+0.5)(x+6) &= 119 \\ x^2 + 6.5x - 116 &= 0 \end{aligned}$$

$$\begin{aligned}
\frac{-6.5 \pm \sqrt{(6.5)^2 - 4(-116)}}{2} &= x \\
\frac{-6.5 \pm 22.5}{2} &= x \\
-14.5, 8 &= x.
\end{aligned}$$

Since x is positive, $x = 8$ and the original volume is $x^3 = 512$ in.^{3}

95. Solving for d, we find

$$\begin{aligned}
598.9\left(\frac{d}{64}\right)^{-2/3} &= 14.26 \\
\left(\frac{d}{64}\right)^{-2/3} &= \frac{14.26}{598.9} \\
\frac{d}{64} &= \left(\frac{14.26}{598.9}\right)^{-3/2} \\
d &= 64\left(\frac{14.26}{598.9}\right)^{-3/2} \\
d &\approx 17,419.4 \text{ lb.}
\end{aligned}$$

96. Solving for d, we obtain

$$\begin{aligned}
26.1\left(\frac{d}{64}\right)^{-1/3} &= 3.91 \\
\left(\frac{d}{64}\right)^{-1/3} &= \frac{3.91}{26.1} \\
\frac{d}{64} &= \left(\frac{3.91}{26.1}\right)^{-3} \\
d &= 64\left(\frac{3.91}{26.1}\right)^{-3} \\
d &\approx 19,035.8 \text{ lb.}
\end{aligned}$$

97. Let $x^2(x+2)$ be the volume of shrimp to be shipped. Here, x is the length of one side of the square base and $x+2$ is the height. The volume of the styrofoam box is $(x+2)^2(x+4)$. Since the amount of shrimp is one-half of the volume of the styrofoam box, we have

$$\begin{aligned}
2x^2(x+2) &= (x+2)^2(x+4) \\
2x^3 + 4x^2 &= (x^2 + 4x + 4)(x+4) \\
x^3 - 4x^2 - 20x - 16 &= 0.
\end{aligned}$$

By using the Rational Zero Theorem and synthetic division, we obtain

$$x^3 - 4x^2 - 20x - 16 = (x+2)(x^2 - 6x - 8).$$

Note, we have to exclude the zero of $x+2$ which is -2 since a dimension is a positive number. Then by using the method of completing the square, we get

$$\begin{aligned}
x^2 - 6x &= 8 \\
x^2 - 6x + 9 &= 17 \\
(x-3)^2 &= 17 \\
x &= 3 \pm \sqrt{17} \\
x &\approx -1.123, 7.123
\end{aligned}$$

Since $x > 0$, choose $x = 3 + \sqrt{17}$.
Thus, the volume of shrimp to be shipped is $(3+\sqrt{17})^2(5+\sqrt{17}) \approx 462.89$ in.^{3}

98. If x is the length of one side of the cubic container, then the new width and length are $x+3$ and $x-1$, respectively. Then we have

$$\begin{aligned}
x(x+3)(x-1) &= 6 \\
x^3 + 2x^2 - 3x - 6 &= 0 \\
x^2(x+2) - 3(x+2) &= (x^2 - 3)(x+2) = 0 \\
x &= \pm\sqrt{3}, -2
\end{aligned}$$

Since $x > 0$, the height of the cubic container is $x = \sqrt{3}$ m.

99. Let x be the number of hours after 10:00 a.m. so that the distance between Nancy and Edgar is 14 miles greater than the distance between Nancy and William.

By the Pythagorean Theorem, $\sqrt{(5x)^2 + (12x)^2}$ and $\sqrt{(5x)^2 + [4(x+2)]^2}$ are the distances between Nancy and Edgar and Nancy and William, respectively. Then

$$\begin{aligned}
\sqrt{(5x)^2 + (12x)^2} - 14 &= \sqrt{(5x)^2 + [4(x+2)]^2} \\
\sqrt{169x^2} - 14 &= \sqrt{41x^2 + 64x + 64} \\
13x - 14 &= \sqrt{41x^2 + 64x + 64} \\
169x^2 - 364x + 196 &= 41x^2 + 64x + 64 \\
128x^2 - 428x + 132 &= 0 \\
32x^2 - 107x + 33 &= 0
\end{aligned}$$

$x = \dfrac{107 \pm \sqrt{(-107)^2 - 4(32)(33)}}{64} = 3, \dfrac{11}{32}$.

Since the left-hand side of the first equation is negative when $x = \dfrac{11}{32}$, we find that $x = \dfrac{11}{32}$ is an extraneous root. Thus, $x = 3$ hours and the time is 1:00 p.m.

100. $A = \sqrt{0.1^2 + 0.05^2 + 0.02^2} \approx 0.114\%$

Solving for HY, one obtains

$$A^2 = (NL)^2 + (HY)^2 + (NR)^2$$

$$\begin{aligned} A^2 - (NL)^2 - (NR)^2 &= (HY)^2 \\ \sqrt{A^2 - (NL)^2 - (NR)^2} &= HY. \end{aligned}$$

101. (a) $\sqrt[3]{(28.6)(28.3)(46.3)} \approx \$33.5$ billion

(b) Let p be the net income for the fourth quarter. Then

$$\begin{aligned} \sqrt[4]{(28.6)(28.3)(46.3)p} &= 40 \\ p &= \frac{40^4}{(28.6)(28.3)(46.3)} \\ p &\approx \$68.3 \text{ billion.} \end{aligned}$$

102. If Lauren ran 5 miles downstream and swam across for a mile, then this would take her $\dfrac{5}{10} + \dfrac{1}{8} = 0.625$ hours or 37.5 minutes.

The diagonal distance between points A and B is $\sqrt{26}$ miles. Swimming diagonally, it would take her $\dfrac{\sqrt{26}}{8} \approx 0.6374$ hrs. or 38.2 minutes.

If x is the number of miles she ran, then

$$\begin{aligned} \frac{x}{10} + \frac{\sqrt{(5-x)^2+1}}{8} &= \frac{36}{60} \\ \frac{\sqrt{x^2 - 10x + 26}}{8} &= \frac{6-x}{10} \\ 100(x^2 - 10x + 26) &= 64(x^2 - 12x + 36) \\ 36x^2 - 232x + 296 &= 0 \\ 9x^2 - 58x + 74 &= 0 \end{aligned}$$

$x = \dfrac{58 \pm \sqrt{(-58)^2 - 4(9)(74)}}{18} \approx 1.75, 4.69$

To complete the race in 36 minutes, Lauren has to run either 1.75 or 4.69 miles.

103. The weight of the molasses in a cylindrical tank is its volume times its density. Then

$$\begin{aligned} \pi \left(\frac{d}{2}\right)^2 \cdot d \cdot 1600 &= 25,850,060 \\ 400\pi d^3 &= 25,850,060 \\ d &= \sqrt[3]{\frac{25,850,060}{400\pi}} \approx 27.4. \end{aligned}$$

The height of the tank is about 27.4 meters.

104. a) By the Pythagorean Theorem, we obtain

$$h^2 + \left(\frac{b}{2}\right)^2 = 10^2.$$

Solving for h, one finds $h = \dfrac{\sqrt{400 - b^2}}{2}$. From the area of a triangle, we can express the volume V as a function of b:

$$\begin{aligned} V &= \frac{1}{2}bh(40) \\ &= \frac{1}{2}b\left(\frac{\sqrt{400-b^2}}{2}\right)(40) \\ V &= 10b\sqrt{400 - b^2} \end{aligned}$$

b) If $V = 1600$, then b must satisfy

$$\begin{aligned} 1600 &= 10b\sqrt{400 - b^2} \\ 160 &= b\sqrt{400 - b^2} \\ 160^2 &= b^2(400 - b^2) \end{aligned}$$

Then $b^4 - 400b^2 + 160^2 = 0$. Applying the quadratic formula, one obtains

$$\begin{aligned} b^2 &= \frac{400 \pm \sqrt{400^2 - 4(160^2)}}{2} \\ b^2 &= \frac{400 \pm 240}{2} \\ b^2 &= 320, 80 \\ b &= 8\sqrt{5}, 4\sqrt{5}. \end{aligned}$$

If $V = 1600$, then $b = 8\sqrt{5}$ or $b = 4\sqrt{5}$.

c) With a graphing calculator, we find that the maximum volume V is attained when $b = \sqrt{200} \approx 14.14$ ft and $h = \sqrt{50} \approx 7.07$ ft.

105. By choosing an appropriate coordinate system, we can assume the circle is given by

$$(x+r)^2 + (y-r)^2 = r^2$$

where $r > 0$ is the radius of the circle and $(-5, 1)$ is the common point between the block and the circle. Note, the radius is less than 5 feet. Substitute $x = -5$ and $y = 1$. Then we obtain

$$\begin{aligned}
(-5+r)^2 + (1-r)^2 &= r^2 \\
r^2 - 10r + 25 + 1 - 2r + r^2 &= r^2 \\
r^2 - 12r + 26 &= 0.
\end{aligned}$$

The solutions of the last quadratic equation are $r = 6 \pm \sqrt{10}$. Since $r < 5$, the radius of the circle is

$$r = 6 - \sqrt{10} \text{ ft.}$$

106. Assume the circle is given by

$$x^2 + (y-r)^2 = r^2$$

where r is the radius. Suppose the points where the blocks touch the circle are at the points $(-x_2, 1)$ and $(x_1, 2)$ where $x_1, x_2 > 0$. Substitute the points into the equation of the circle. Then $x_1^2 + (2-r)^2 = r^2$ and $x_2^2 + (1-r)^2 = r^2$. From these equations we obtain $x_1^2 + 4 - 4r = 0$ and $x_2^2 + 1 - 2r = 0$. Thus, $x_1 = \sqrt{4r-4}$ and $x_2 = \sqrt{2r-1}$.

Since 6 ft is the distance between the blocks, we obtain

$$\begin{aligned}
x_1 + x_2 &= 6 \\
\sqrt{4r-4} + \sqrt{2r-1} &= 6 \\
6 - \sqrt{2r-1} &= \sqrt{4r-4} \\
36 - 12\sqrt{2r-1} + 2r - 1 &= 4r - 4 \\
-12\sqrt{2r-1} &= 2r - 39 \\
\frac{2r-39}{-12} &= \sqrt{2r-1} \\
\frac{4r^2 - 156r + 39^2}{144} &= 2r - 1 \\
4r^2 - 444r + 1665 &= 0.
\end{aligned}$$

Applying the quadratic formula, we find

$$\begin{aligned}
r &= \frac{444 \pm \sqrt{(-444)^2 - 4(4)(1665)}}{8} \\
&= \frac{111 \pm 12\sqrt{74}}{2} \approx 3.89, 107.11.
\end{aligned}$$

Thus, the radius of the circle is 3.89 in. since 107.11 is an extraneous root.

107. Note, $x - 5 = x + 7$ has no solution.

$$\begin{aligned}
x - 5 &= \pm(x+7) \\
x - 5 &= -(x+7) \\
2x &= -2 \\
x &= -1
\end{aligned}$$

The solution set is $\{-1\}$.

108. If w is the width, then

$$\begin{aligned}
w + (w+3) &= 23 \\
2w &= 20 \\
w &= 10
\end{aligned}$$

The width is 10 ft.

109. Completing the square:

$$\begin{aligned}
(x+4)^2 + (y-5)^2 &= 16 + 25 \\
(x+4)^2 + (y-5)^2 &= 41
\end{aligned}$$

The center is $(-4, 5)$ with radius $R = \sqrt{41}$.

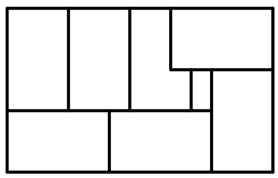
110. The slope is $m = \frac{1-(-2)}{3-(-1)} = \frac{3}{4}$. Then

$$\begin{aligned}
y + 2 &= \frac{3}{4}(x+1) \\
y + 2 &= \frac{3}{4}x + \frac{3}{4} \\
y &= \frac{3}{4}x - \frac{5}{4}
\end{aligned}$$

111. Since $y = \frac{3}{5}x + \frac{11}{5}$, the slope is $\frac{3}{5}$.

112. $2^5 \times \left(10^{-4}\right)^5 = 32 \times 10^{-20} = 3.2 \times 10^{-19}$

113. Using the arrangement below, the maximum area that can be covered is 102 ft^2.



114. Using a common denominator, we find

$$\frac{19}{40} = \frac{95}{200}, \quad \frac{12}{25} = \frac{96}{200}.$$

All together, there are 201 workers.
Further, Chris is a female and Pat is a male.

1.6 Pop Quiz

1. Factoring, we obtain

$$\begin{aligned} x^2(x+1) + (x+1) &= 0 \\ (x+1)(x^2+1) &= 0. \end{aligned}$$

Then the solution set is $\{\pm i, -1\}$.

2. Square both sides of the equation.

$$\begin{aligned} \left(\sqrt{x+4}\right)^2 &= (x-2)^2 \\ x+4 &= x^2 - 4x + 4 \\ 0 &= x^2 - 5x \\ 0 &= x(x-5) \\ x &= 0, 5 \end{aligned}$$

Note, $x = 0$ is an extraneous root.
The solution set is $\{5\}$.

3. Raise both sides to the power -3.

$$\begin{aligned} \left(x^{-2/3}\right)^{-3} &= 4^{-3} \\ x^2 &= \frac{1}{64} \\ x &= \pm\frac{1}{8} \end{aligned}$$

The solution set is $\left\{\pm\frac{1}{8}\right\}$.

4. Factoring, we obtain

$$\begin{aligned} x^4 - 3x^2 - 4 &= 0 \\ (x^2-4)(x^2+1) &= 0. \end{aligned}$$

Then the solution set is $\{\pm i, \pm 2\}$.

5. An equivalent statement is

$$\begin{array}{rcl} x+3 = 2x-5 & \text{or} & x+3 = -2x+5 \\ 8 = x & \text{or} & 3x = 2 \\ x = 8 & \text{or} & x = 2/3 \end{array}$$

The solution set is $\left\{\frac{2}{3}, 8\right\}$.

1.6 Linking Concepts

(a) By using the Pythagorean Theorem, one finds that the cost is $4\sqrt{10^2+30^2} = \$126.49$

(b) Cost is $3(30) + 4(10) = \$130$

(c) By solving

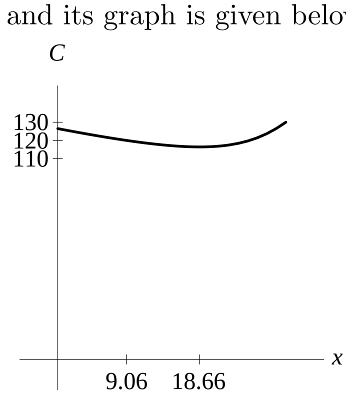
$$\begin{aligned} 3x + 4\sqrt{10^2+(30-x)^2} &= 120 \\ 16(10^2+(30-x)^2) &= (120-3x)^2 \end{aligned}$$

one finds $x \approx 9.06$ or 25.22.

(d) If C is the cost in dollars, then

$$C = 3x + 4\sqrt{10^2+(30-x)^2}$$

and its graph is given below



(e) $x \approx 18.66$ feet

(f) Minimum cost is $C(18.66) = \$116.46$

For Thought

1. True

2. False, since $-2x < -6$ is equivalent to

$$\frac{-2x}{-2} > \frac{-6}{-2}.$$

3. False, since there is a number between any two distinct real numbers.

4. True, since $|-6-6| = |-12| = 12 > -1$.

5. False, $(-\infty, -3) \cap (-\infty, -2) = (-\infty, -3)$.

6. False, $(5, \infty) \cap (-\infty, -3) = \emptyset$.

7. False, no real number satisfies $|x-2| < 0$.

8. False, it is equivalent to $|x| > 3$.

9. False, $|x| + 2 < 5$ is equivalent to $-3 < x < 3$.

10. True

1.7 Exercises

1. interval

2. open

3. closed

4. unbounded

5. compound

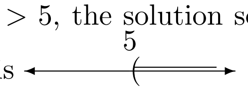
6. intersection

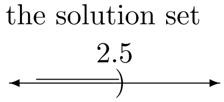
7. $x < 12$ **8.** $x \le -3$

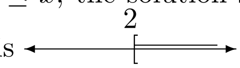
9. $x \ge -7$ **10.** $x > 1.2$

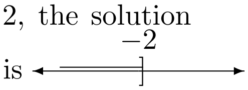
11. $[-8, \infty)$ **12.** $(-\infty, 54)$

13. $(-\infty, \pi/2)$ **14.** $[\sqrt{3}, \infty)$

15. Since $3x > 15$ implies $x > 5$, the solution set is $(5, \infty)$ and the graph is 

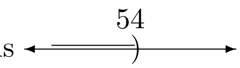
16. Since $2x < 5$ implies $x < 2.5$, the solution set is $(-\infty, 2.5)$ and the graph is 

17. Since $10 \le 5x$ implies $2 \le x$, the solution set is $[2, \infty)$ and the graph is 

18. Since $-4x \ge 8$ implies $x \le -2$, the solution set is $(-\infty, -2]$ and the graph is 

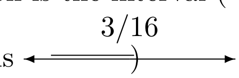
19. Multiply 6 to both sides of the inequality.

$$\begin{aligned} 3x - 24 &< 2x + 30 \\ x &< 54 \end{aligned}$$

The solution is the interval $(-\infty, 54)$ and the graph is 

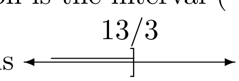
20. Multiplying the inequality by 12, we get

$$\begin{aligned} 6 - 12x &> 4x + 3 \\ 3 &> 16x. \end{aligned}$$

The solution is the interval $(-\infty, 3/16)$ and the graph is 

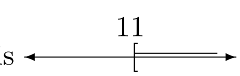
21. Multiplying the inequality by 2, we find

$$\begin{aligned} 7 - 3x &\ge -6 \\ 13 &\ge 3x \\ 13/3 &\ge x. \end{aligned}$$

The solution is the interval $(-\infty, 13/3]$ and the graph is 

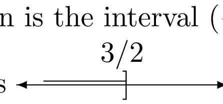
22. Multiply the inequality by 3.

$$\begin{aligned} 5 - x &\le -6 \\ 11 &\le x \end{aligned}$$

The solution is the interval $[11, \infty)$ and the graph is 

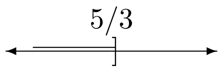
23. Multiply the inequality by -5 and reverse the direction of the inequality.

$$\begin{aligned} 2x - 3 &\le 0 \\ 2x &\le 3 \\ x &\le \frac{3}{2} \end{aligned}$$

The solution is the interval $(-\infty, 3/2]$ and the graph is 

24. Multiply the inequality by -7 and reverse the direction of the inequality.

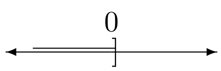
$$\begin{aligned} 5-3x &\geq 0 \\ 5 &\geq 3x \\ 5/3 &\geq x \end{aligned}$$

The solution is the interval $(-\infty, 5/3]$ and the graph is 

25. Multiply the left-hand side.

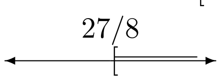
$$\begin{aligned} -6x+4 &\geq 4-x \\ 0 &\geq 5x \\ 0 &\geq x \end{aligned}$$

The solution is the interval $(-\infty, 0]$

and the graph is 

26. Multiply the right-hand side.

$$\begin{aligned} -5x &\leq 3x-27 \\ 27 &\leq 8x \\ 27/8 &\leq x \end{aligned}$$

The solution is the interval $[27/8, \infty)$ and the graph is 

27. Using the portion of the graph below the x-axis, the solution set is $(-\infty, -3.5)$.

28. Using the part of the graph on or above the x-axis, the solution set is $[2.5, \infty)$.

29. Using the part of the graph on or above the x-axis, the solution set is $(-\infty, 1.4]$.

30. Using the portion of the graph below the x-axis, the solution set is $(-3, \infty)$.

31. By taking the part of the line $y = 2x-3$ above the horizontal line $y = 5$ and by using $(4, 5)$, the solution set is $(4, \infty)$.

32. By taking the part of the line $y = 2x-3$ below or on the horizontal line $y = 5$ and by using $(4, 5)$, the solution set is $(-\infty, 4]$.

33. Note, the graph of $y = -3x-7$ is above or on the graph of $y = x+1$ for $x \leq -2$. Thus, the solution set is $(-\infty, -2]$.

34. Note, the graph of $y = x+1$ is above the graph of $y = -3x-7$ for $x > -2$. Thus, the solution set is $(-2, \infty)$.

35. $(-3, \infty)$ **36.** $(-\infty, 6)$

37. $(-3, \infty)$ **38.** $(4, 7)$

39. $(-5, -2)$ **40.** $(2, \infty)$

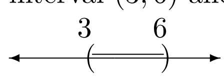
41. $\emptyset$ **42.** $(-\infty, \infty)$

43. $(-\infty, 5]$ **44.** $[3, 7]$

45. Solve each simple inequality and find the intersection of their solution sets.

$$\begin{aligned} x > 3 \quad &\text{and} \quad 0.5x < 3 \\ x > 3 \quad &\text{and} \quad x < 6 \end{aligned}$$

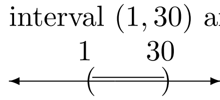
The intersection of these values of x is the interval $(3, 6)$ and the graph is



46. Solve each simple inequality and find the intersection of their solution sets.

$$\begin{aligned} -x < -1 \quad &\text{and} \quad 0.2x < 6 \\ x > 1 \quad &\text{and} \quad x < 30 \end{aligned}$$

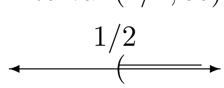
The intersection of these values of x is the interval $(1, 30)$ and the graph is



47. Solve each simple inequality and find the intersection of their solution sets.

$$\begin{aligned} 2x-5 > -4 \quad &\text{and} \quad 2x+1 > 0 \\ x > \frac{1}{2} \quad &\text{and} \quad x > -\frac{1}{2} \end{aligned}$$

The intersection of these values of x is the interval $(1/2, \infty)$ and the graph is



48. Solve each simple inequality and find the intersection of their solution sets.

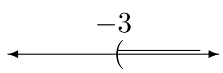
$$\begin{array}{rcl} 4-x>2 & \text{and} & 2x-7>-3 \\ 2>x & \text{and} & 2x>4 \\ x<2 & \text{and} & x>2 \end{array}$$

There is no solution.

49. Solve each simple inequality and find the union of their solution sets.

$$\begin{array}{rcl} -6<2x & \text{or} & 3x>-3 \\ -3<x & \text{or} & x>-1 \end{array}$$

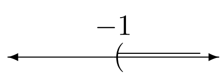
The union of these values of x is $(-3,\infty)$ and

the graph is 

50. Solve each simple inequality and find the union of their solution sets.

$$\begin{array}{rcl} 2x>-2 & \text{or} & x>3 \\ x>-1 & \text{or} & x>3 \end{array}$$

The union of these values of x is $(-1,\infty)$ and

the graph is 

51. Solve each simple inequality and find the union of their solution sets.

$$\begin{array}{rcl} x+1>6 & \text{or} & x<7 \\ x>5 & \text{or} & x<7 \end{array}$$

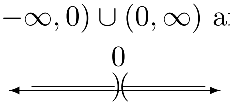
The union of these values of x is $(-\infty,\infty)$ and

the graph is

52. Solve each simple inequality and find the union of their solution sets.

$$\begin{array}{rcl} x+6>6 & \text{or} & 4x-4<3x-4 \\ x>0 & \text{or} & x<0 \end{array}$$

The union of these values of x is $(-\infty,0)\cup(0,\infty)$ and the graph is



53. Solve each simple inequality and find the intersection of their solution sets.

$$\begin{array}{rcl} 2-3x<8 & \text{and} & x-8\le-12 \\ -6<3x & \text{and} & x\le-4 \\ -2<x & \text{and} & x\le-4 \end{array}$$

The intersection is empty.

The solution set is $\emptyset$.

54. Solve each simple inequality and find the intersection of their solution sets.

$$\begin{array}{rcl} 3x-5>10 & \text{and} & 25-2x\ge15 \\ 3x>15 & \text{and} & 10\ge2x \\ x>5 & \text{and} & 5\ge x \end{array}$$

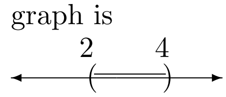
The intersection is empty.

The solution set is $\emptyset$.

55.

$$\begin{array}{ccccc} 6 & < & 3x & < & 12 \\ 2 & < & x & < & 4 \end{array}$$

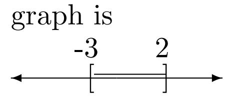
The solution set is the interval $(2,4)$ and the graph is



56.

$$\begin{array}{ccccc} -12 & \le & 4x & \le & 8 \\ -3 & \le & x & \le & 2 \end{array}$$

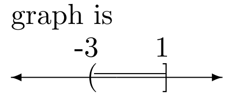
The solution set is the interval $[-3,2]$ and the graph is



57.

$$\begin{array}{ccccc} -6 & \le & -6x & < & 18 \\ 1 & \ge & x & > & -3 \end{array}$$

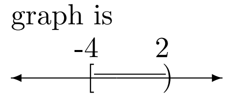
The solution set is the interval $(-3,1]$ and the graph is



58.

$$\begin{array}{ccccc} -18 & < & -9x & \le & 36 \\ 2 & > & x & \ge & -4 \end{array}$$

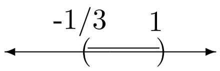
The solution set is the interval $[-4,2)$ and the graph is



59. Solve an equivalent compound inequality.

$$\begin{array}{rcl} -2 < & 3x-1 & < 2 \\ -1 < & 3x & < 3 \\ -\frac{1}{3} < & x & < 1 \end{array}$$

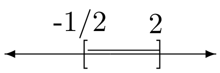
The solution set is the interval $(-1/3, 1)$ and the graph is



60. Solve an equivalent compound inequality.

$$\begin{array}{rcl} -5 \le & 4x-3 & \le 5 \\ -2 \le & 4x & \le 8 \\ -\frac{1}{2} \le & x & \le 2 \end{array}$$

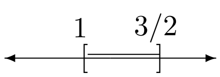
The solution set is the interval $[-1/2, 2]$ and the graph is



61. Solve an equivalent compound inequality.

$$\begin{array}{rcl} -1 \le & 5-4x & \le 1 \\ -6 \le & -4x & \le -4 \\ \frac{3}{2} \ge & x & \ge 1 \end{array}$$

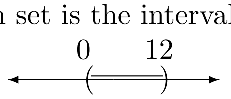
The solution set is the interval $[1, 3/2]$ and the graph is



62. Solve an equivalent compound inequality.

$$\begin{array}{rcl} -6 < & 6-x & < 6 \\ -12 < & -x & < 0 \\ 12 > & x & > 0 \end{array}$$

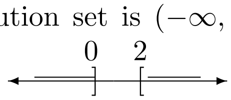
The solution set is the interval $(0, 12)$ and the graph is



63. Solve an equivalent compound inequality.

$$\begin{array}{rcl} x-1 \ge 1 & \text{or} & x-1 \le -1 \\ x \ge 2 & \text{or} & x \le 0 \end{array}$$

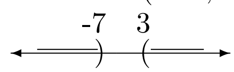
The solution set is $(-\infty, 0] \cup [2, \infty)$ and the graph is



64. Solve an equivalent compound inequality.

$$\begin{array}{rcl} x+2 > 5 & \text{or} & x+2 < -5 \\ x > 3 & \text{or} & x < -7 \end{array}$$

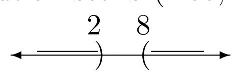
The solution set is $(-\infty, -7) \cup (3, \infty)$ and the graph is



65. Solve an equivalent compound inequality.

$$\begin{array}{rcl} 5-x > 3 & \text{or} & 5-x < -3 \\ 2 > x & \text{or} & 8 < x \end{array}$$

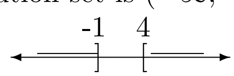
The solution set is $(-\infty, 2) \cup (8, \infty)$ and the graph is



66. Solve an equivalent compound inequality.

$$\begin{array}{rcl} 3-2x \ge 5 & \text{or} & 3-2x \le -5 \\ -2 \ge 2x & \text{or} & 8 \le 2x \\ -1 \ge x & \text{or} & 4 \le x \end{array}$$

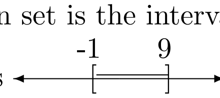
The solution set is $(-\infty, -1] \cup [4, \infty)$ and the graph is



67. Solve an equivalent compound inequality.

$$\begin{array}{rcl} -5 \le & 4-x & \le 5 \\ -9 \le & -x & \le 1 \\ 9 \ge & x & \ge -1 \end{array}$$

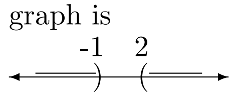
The solution set is the interval $[-1, 9]$ and the graph is



68. Solve an equivalent compound inequality.

$$\begin{array}{rcl} 2x-1 > 3 & \text{or} & 2x-1 < -3 \\ 2x > 4 & \text{or} & 2x < -2 \\ x > 2 & \text{or} & x < -1 \end{array}$$

The solution set is $(-\infty, -1) \cup (2, \infty)$ and the graph is



69. No solution since an absolute value is never negative. The solution set is $\emptyset$.

70. Since an absolute value is always nonnegative, the solution set is the interval $(-\infty, \infty)$and

the graph is

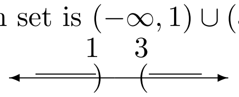
71. No solution since an absolute value is never negative. The solution set is $\emptyset$.

72. Since an absolute value is nonnegative, the solution set is $(-\infty, \infty)$ and the graph is

73. Note, $3|x-2| > 3$ or $|x-2| > 1$.
We solve an equivalent compound inequality.

$$\begin{array}{ccc} x-2>1 & \text{or} & x-2<-1 \\ x>3 & \text{or} & x<1 \end{array}$$

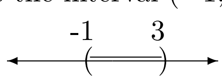
The solution set is $(-\infty, 1) \cup (3, \infty)$ and

the graph is 

74. Solve an equivalent compound inequality.

$$\begin{aligned} 3|x-1| &< 6 \\ |x-1| &< 2 \\ -2 < x-1 &< 2 \\ -1 < x &< 3 \end{aligned}$$

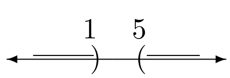
The solution set is the interval $(-1, 3)$

and the graph is 

75. Solve an equivalent compound inequality.

$$\begin{array}{ccc} \dfrac{x-3}{2} > 1 & \text{or} & \dfrac{x-3}{2} < -1 \\ x-3>2 & \text{or} & x-3<-2 \\ x>5 & \text{or} & x<1 \end{array}$$

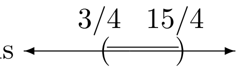
The solution set is the interval $(-\infty, 1) \cup (5, \infty)$

and the graph is 

76. Solve an equivalent compound inequality.

$$\begin{array}{rcl} -3 < & \dfrac{9-4x}{2} & < 3 \\ -6 < & 9-4x & < 6 \\ -15 < & -4x & < -3 \\ \dfrac{15}{4} > & x & > \dfrac{3}{4} \end{array}$$

The solution set is the interval $(3/4, 15/4)$ and

the graph is 

77. $|x| < 5$ **78.** $|x| < 2$

79. $|x| > 3$ **80.** $|x| > 1$

81. Since 6 is the midpoint of 4 and 8, the inequality is $|x-6| < 2$.

82. Since 3 is the midpoint of -3 and 9, the inequality is $|x-3| < 6$.

83. Since 4 is the midpoint of 3 and 5, the inequality is $|x-4| > 1$.

84. Since 2 is the midpoint of -1 and 5, the inequality is $|x-2| > 3$.

85. $|x| \geq 9$ **86.** $|x| < 8$

87. Since 7 is the midpoint, the inequality is $|x-7| \leq 4$.

88. Since 2 is the midpoint, the inequality is $|x-2| \geq 4$.

89. Since 5 is the midpoint, the inequality is $|x-5| > 2$.

90. Since -2 is the midpoint, the inequality is $|x+2| < 1$.

91. Since $x-2 \geq 0$, the solution set is $[2, \infty)$.

92. Since $3x-1 \geq 0$ is equivalent to $x \geq 1/3$, the solution set is $[1/3, \infty)$.

93. Since $2-x > 0$ is equivalent to $2 > x$, the solution set is $(-\infty, 2)$.

94. Since $3-2x > 0$ is equivalent to $3/2 > x$, the solution set is $(-\infty, 3/2)$.

95. Since $|x| \geq 3$ is equivalent to $x \geq 3$ or $x \leq -3$, the solution set is $(-\infty, -3] \cup [3, \infty)$.

96. Since $|x| \leq 5$ is equivalent to $-5 \leq x \leq 5$, the solution set is $[-5, 5]$.

97. If x is the price of a car excluding sales tax then it must satisfy $0 \leq 1.1x + 300 \leq 8000$.

This is equivalent to $0 \leq x \leq \dfrac{7700}{1.1} = 7000$.

The price range of Yolanda's car is the interval $[\$0, \$7000]$.

98. Let x and $x+0.10$ be the price of a hamburger and a Big Salad, respectively, in dollars. Then the price for 10 hamburgers and 5 Big Salads with tax and tip satisfy

$$\begin{aligned} 37.94 < 1.08\left[10x + 5(x+0.10)\right] + 5 &\leq 46.04 \\ 32.94 < 1.08[15x + 0.50] &\leq 41.04 \\ 30.50 < 15x + 0.50 &\leq 38 \\ 30 < 15x &\leq 37.50 \\ 2 < x &\leq 2.5 \end{aligned}$$

The price range of a hamburger is $(\$2, \$2.50]$.

99. Let x be Lucky's score on the final exam.

$$\begin{aligned} 79 &\leq \frac{65+x}{2} \leq 90 \\ 158 &\leq 65 + x \leq 180 \\ 93 &\leq x \leq 115 \end{aligned}$$

Since $x \leq 100$, the final exam score must lie in $[93, 100]$.

100. Let x be Felix' score on the third test.

$$\begin{aligned} \frac{52+64+x}{3} &\geq 70 \\ 116 + x &\geq 210 \\ x &\geq 94 \end{aligned}$$

Since $x \leq 100$, the third test score must lie in $[94, 100]$.

101. Let x be Ingrid's final exam score. Since $\dfrac{2x+65}{3}$ is her weighted average, we obtain

$$\begin{aligned} 79 &< \frac{2x+65}{3} < 90 \\ 237 &< 2x + 65 < 270 \\ 172 &< 2x < 205 \\ 86 &< x < 102.5 \end{aligned}$$

Since $x \leq 100$, Ingrid's final exam score must lie in $(86, 100]$.

102. Let x be Elizabeth's score on the final exam. Her weighted average must satisfy (which is solved by multiplying both sides by 9)

$$\begin{aligned} \frac{2}{3}x + \frac{1}{3} \cdot \frac{64+75+80}{3} &> 70 \\ 6x + 219 &> 630 \\ 6x &> 411 \\ x &> 68.5 \end{aligned}$$

Since $x \leq 100$, the final exam score must lie in $(68.5, 100]$.

103. If h is the height of the box, then

$$\begin{aligned} 40 + 2(30) + 2h &\leq 130 \\ 100 + 2h &\leq 130 \\ 2h &\leq 30. \end{aligned}$$

The range of the height is $(0 \text{ in.}, 15 \text{ in.}]$.

104. Let b be the number of times he would have to bat.

a) Assume he has to get a hit every time.

$$\begin{aligned} \frac{97+b}{387+b} &> 0.300 \\ 97 + b &> (387+b)(0.3) \\ 97 + b &> 116.1 + 0.3b \\ 0.7b &> 19.1 \\ b &> 27.3 \end{aligned}$$

Lopez must bat 28 or more times (and get a hit each time) to average over 0.300.

b) Assume he got a hit 50% of the time.

$$\begin{aligned} \frac{97+0.5b}{387+b} &> 0.300 \\ 97 + 0.5b &> (387+b)(0.3) \\ 97 + 0.5b &> 116.1 + 0.3b \\ 0.2b &> 19.1 \\ b &> 95.5 \end{aligned}$$

He must bat 96 or more times.

105. By substituting $N = 50$ and $w = 27$ into $r = \dfrac{Nw}{n}$ we find $r = \dfrac{1350}{n}$. Moreover if $n = 14$, then $r = \dfrac{1350}{14} = 96.4 \approx 96$.
Similarly, the other gear ratios are the following.

n	14	17	20	24	29
r	96	79	68	56	47

Yes, the bicycle has a gear ratio for each of the four types.

106. **(a)** Substituting $w = 27$ and $n = 17$ into $r = \dfrac{Nw}{n}$, we obtain

$$\begin{aligned} 60 < &\ \frac{27N}{17} < 80 \\ 1020 < &\ 27N < 1360 \\ 37.8 < &\ N < 50.4. \end{aligned}$$

The number N of teeth in the chainring must be in the range 38 through 50.

(b) Substituting $N = 40$ and $w = 26$ into $r = \dfrac{Nw}{n}$, we obtain

$$\begin{aligned} &60 < \frac{1040}{n} < 75 \\ &60n < 1040 \quad \text{and} \quad 1040 < 75n \\ &n < 17.3 \quad \text{and} \quad 13.9 < n. \end{aligned}$$

The number n of teeth in the cog must be in the range 14 through 17.

107. Let x be the price of a CL 600.

a) $|x - 130,645| > 10,000$

b) The above inequality is equivalent to

$$\begin{aligned} x - 130,645 > 10,000 \quad &\text{or} \quad x - 130,645 < -10,000 \\ x > 140,645 \quad &\text{or} \quad x < 120,645. \end{aligned}$$

Thus, the price of a CL 600 is less than \$120,645 or more than \$140,645.

108. Let x be the price of the Saturn.

a) $|x - 21,195| < 5100$

b) An equivalent inequality is

$$\begin{aligned} -5100 &< x - 21,195 < 5100 \\ 16,095 &< x < 26,295. \end{aligned}$$

The price range of the Saturn is

$$16,095 < x < 26,295.$$

109. If x is the actual temperature, then

$$\begin{aligned} \left|\frac{x-35}{35}\right| &< 0.01 \\ -0.35 < x - 35 &< 0.35 \\ 34.65 < x &< 35.35. \end{aligned}$$

The actual temperature must lie in the interval $(34.65°, 35.35°)$.

110. If x is the actual length, then

$$\begin{aligned} \left|\frac{x-100}{100}\right| &< 0.005 \\ -0.5 < x - 100 &< 0.5 \\ 99.5 < x &< 100.5. \end{aligned}$$

The actual length must lie in the interval $(99.5 \text{ m}, 100.5 \text{ m})$.

111. If c is the actual circumference, then $c = \pi d$ and

$$\begin{aligned} |\pi d - 7.2| &\le 0.1 \\ -0.1 \le \pi d - 7.2 &\le 0.1 \\ 7.1 \le \pi d &\le 7.3 \\ 2.26 \le d &\le 2.32. \end{aligned}$$

The actual diameter must lie in the interval $[2.26 \text{ cm}, 2.32 \text{ cm}]$.

112. If A is the actual area, then $A = \pi r^2$ and

$$\begin{aligned} \left|\pi r^2 - 15\right| &\le 0.5 \\ -0.5 \le \pi r^2 - 15 &\le 0.5 \\ 14.5 \le \pi r^2 &\le 15.5 \\ 4.62 \le r^2 &\le 4.93 \\ 2.15 \le r &\le 2.22. \end{aligned}$$

The actual radius must lie in the interval $[2.15 \text{ ft}, 2.22 \text{ ft}]$.

113. a) The inequality $|a - 40,584| < 3000$ is equivalent to

$$\begin{array}{rcl} -3000 < & a - 40,584 & < 3000 \\ 37,584 < & a & < 43,584. \end{array}$$

The states within this range are Colorado, Iowa, and Vermont.

b) The inequality $|a - 40,584| > 5000$ is equivalent to

$$\begin{array}{rcl} a - 40,584 > 5000 & \text{or} & a - 40,584 < -5000 \\ a > 45,584 & \text{or} & a < 35,584 \end{array}$$

The states satisfying the inequality are Alabama, Georgia, Maryland, New Jersey, and South Carolina.

114. Her total cost for taking x flats of strawberries to market is $300+4200+2.40x$. If revenue must exceed cost, then

$$\begin{array}{rcl} 11x & > & 4500 + 2.40x \\ 8.60x & > & 4500 \\ x & > & \dfrac{4500}{8.60} \approx 523.3. \end{array}$$

She must sell more than 523 flats.

115.

$$\begin{array}{rcl} x(x+2) & = & 0 \\ x & = & 0, -2 \end{array}$$

The solution set is $\{-2, 0\}$.

116. Use the method of completing the square.

$$\begin{array}{rcl} x^2 + 2x + 1 & = & 9 + 1 \\ (x+1)^2 & > & 10 \\ x + 1 & > & \pm\sqrt{10} \end{array}$$

The solution set is $\{-1 - \sqrt{10}, -1 + \sqrt{10}\}$.

117. Since the slope of $2x - y = 1$ is 2, the slope of a perpendicular line is $-\frac{1}{2}$. Then

$$\begin{array}{rcl} y + 4 & = & -\dfrac{1}{2}(x - 3) \\ -2y - 8 & > & x - 3 \\ -5 & > & x + 2y. \end{array}$$

The standard form is $x + 2y = -5$.

118. The distance is

$$\sqrt{(2-(-3))^2 + (8-5)^2} = \sqrt{25+9} = \sqrt{34}$$

and the midpoint is

$$\left(\frac{-3+2}{2}, \frac{5+8}{2}\right) = \left(\frac{-1}{2}, \frac{13}{2}\right).$$

119. Solving for y, we find

$$\begin{array}{rcl} 3y - ay & = & w + 9 \\ y(3-a) & = & w + 9 \\ y & = & \dfrac{w+9}{3-a} \end{array}$$

120. Since $2x - 9 = 0$, the solution set is $\{9/2\}$.

121. Consider the list of 6 digit numbers from 000,000 through 999,999. There are 1 million 6 digit numbers in this list for a total of 6 million digits. Each of the ten digits 0 through 9 occurs with the same frequency in this list. So there are 600,000 of each in this list. In particular there are 600,000 ones in the list. You need one more to write 1,000,000. So there are 600,001 ones used in writing the numbers 1 through 1 million.

122. Let $\triangle ABC$ be a right triangle with vertices at $A(2,7)$, $B(0,-3)$, and $C(6,1)$. Notice, the midpoints of the sides of $\triangle ABC$ are $(3,-1)$, $(4,4)$ and $(1,2)$. The area of $\triangle ABC$ is

$$\begin{array}{rcl} \dfrac{1}{2}\overline{AC} \times \overline{BC} & = & \dfrac{1}{2}\sqrt{4^2+6^2}\sqrt{6^2+4^2} \\ & = & \dfrac{1}{2}(52) \ = \ 26. \end{array}$$

1.7 Pop Quiz

1. $[\sqrt{2}, \infty)$ since $x \geq \sqrt{2}$

2. Since $6 < 2x$ or $3 < x$, the solution set is $(3, \infty)$.

3. $[-1, \infty)$

4. Since $x > 6$ and $x < 9$, the solution set is $(6, 9)$.

5. Since $x > 6$ or $x < -6$, the solution set is $(-\infty, -6) \cup (6, \infty)$.

6. Solving an equivalent compound inequality, we obtain

$$\begin{array}{rcccl} -2 & \leq & x-1 & \leq & 2 \\ -1 & \leq & x & \leq & 3 \end{array}$$

The solution set is $[-1, 3]$.

1.7 Linking Concepts

a) If n is the number of copies made during 5 years, then the cost of renting is

$$C = 6300 + 0.08n \text{ dollars.}$$

Note, $6300 = 105(60)$.

b) If n is the number of copies made during 5 years, then the cost of buying is

$$C = 8000 + 0.04n \text{ dollars.}$$

Note, $8000 = 6500 + 25(60)$.

c) Since the cost of renting exceeds \$10,000,

$$\begin{array}{rcl} 6300 + 0.08n & > & 10,000 \\ 0.08n & > & 3,700 \\ n & > & 46,250. \end{array}$$

The cost of renting will exceed \$10,000 if they make over 46,250 copies.

d) Since the cost of buying exceeds \$10,000,

$$\begin{array}{rcl} 8000 + 0.04n & > & 10,000 \\ 0.04n & > & 2,000 \\ n & > & 50,000. \end{array}$$

The cost of buying will exceed \$10,000 if they make over 50,000 copies.

e) If the cost of renting and buying differ by less than \$1000, then

$$|(6300 + 0.08n) - (8000 + 0.04n)| < 1000.$$

Solving, we find

$$\begin{array}{rcl} |0.04n - 1700| & < & 1000 \\ -1000 < 0.04n - 1700 & < & 1000 \\ 700 < 0.04n & < & 2700 \\ 17,500 < n & < & 67,500. \end{array}$$

The cost of renting and buying will differ by less than \$1000 if the number of copies lies in the range $(17,500, 67,500)$.

f) If the cost of renting and buying are equal, then

$$\begin{array}{rcl} 6300 + 0.08n & = & 8000 + 0.04n \\ 0.04n & = & 1700 \\ n & = & 42,500. \end{array}$$

The cost of renting and buying will be the same if 42,500 copies are made during five years.

g) If at most 42,500 copies are made during five years, then the cost to the company will be smaller if they purchase a copy machine (the better plan).

Chapter 1 Review Exercises

1. Since $3x = 2$, the solution set is $\{2/3\}$.

2. Since $3x-5 = 5x+35$ is equivalent to $-40 = 2x$, the solution set is $\{-20\}$.

3. Multiply by 60 to get $30y - 20 = 15y + 12$, or $15y = 32$. The solution set is $\{32/15\}$.

4. Multiply by 40 to get $20 - 8w = 10w - 5$, or $25 = 18w$. The solution set is $\{25/18\}$.

5. Multiply by $x(x-1)$ to get $2x - 2 = 3x$. The solution set is $\{-2\}$.

6. Multiply by $(x+1)(x-3)$ and get $5x - 15 = 2x + 2$. Then $3x = 17$. The solution set is $\{17/3\}$.

7. Multiply by $(x-1)(x-3)$ and get $-2x - 3 = x - 2$. Then $-1 = 3x$. The solution set is $\{-1/3\}$.

8. Multiplying by $(x-8)(x-4)$, we get $-x - 12 = -x - 56$, an inconsistent equation. The solution set is $\emptyset$.

9. The distance is $\sqrt{(-3-2)^2 + (5-(-6))^2} = \sqrt{(-5)^2 + 11^2} = \sqrt{25 + 121} = \sqrt{146}$.

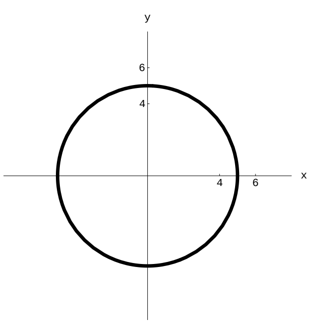
The midpoint is $\left(\frac{-3+2}{2}, \frac{5-6}{2}\right) = \left(-\frac{1}{2}, -\frac{1}{2}\right)$.

10. Distance is $\sqrt{(-1-(-2))^2+(1-(-3))^2} = \sqrt{1^2+4^2} = \sqrt{17}$. The midpoint is $\left(\dfrac{-1-2}{2}, \dfrac{1-3}{2}\right) = (-1.5, -1)$

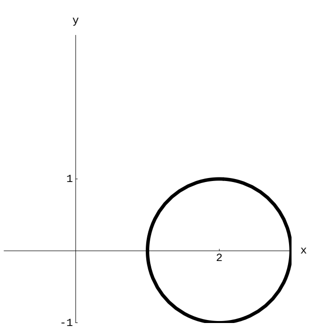
11. Distance is $\sqrt{\left(\dfrac{1}{2}-\dfrac{1}{4}\right)^2+\left(\dfrac{1}{3}-1\right)^2} = \sqrt{\left(\dfrac{1}{4}\right)^2+\left(-\dfrac{2}{3}\right)^2} = \sqrt{\dfrac{1}{16}+\dfrac{4}{9}} = \sqrt{\dfrac{73}{144}} = \dfrac{\sqrt{73}}{12}$. Midpoint is $\left(\dfrac{1/2+1/4}{2}, \dfrac{1/3+1}{2}\right) = \left(\dfrac{3/4}{2}, \dfrac{4/3}{2}\right) = \left(\dfrac{3}{8}, \dfrac{2}{3}\right)$.

12. Distance is $\sqrt{(0.5-(-1.2))^2+(0.2-2.1)^2} = \sqrt{1.7^2+(-1.9)^2} = \sqrt{6.5} \approx 2.5495$.
Midpoint is $\left(\dfrac{0.5-1.2}{2}, \dfrac{0.2+2.1}{2}\right) = \left(\dfrac{-0.7}{2}, \dfrac{2.3}{2}\right) = (-0.35, 1.15)$.

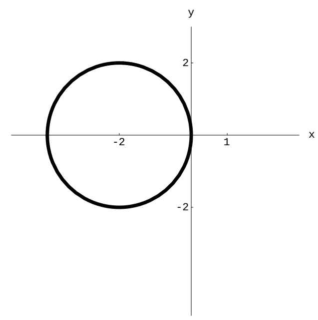
13. Circle with radius 5 and center at the origin.



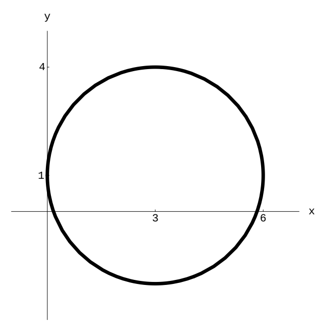
14. Circle with radius 1 and center $(2, 0)$.



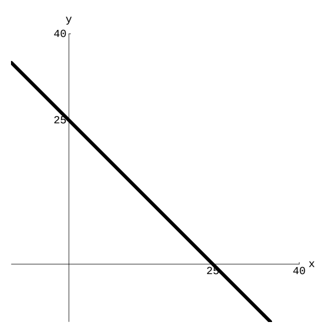
15. Equivalently, by using the method of completing the square, the circle is given by $(x+2)^2+y^2=4$. It has radius 2 and center $(-2, 0)$.



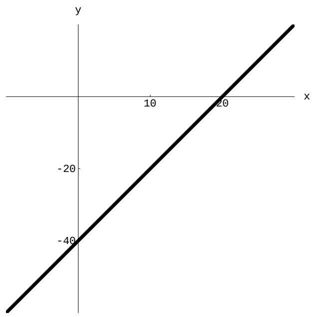
16. Equivalently, by using the method of completing the square, the equation is $(x-3)^2+(y-1)^2=9$. It has radius 3 and center $(3, 1)$.



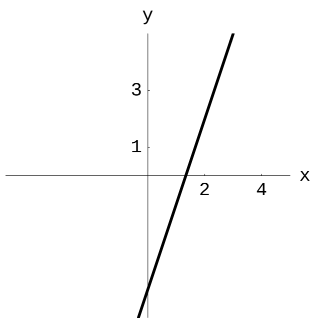
17. The line $y=-x+25$ has intercepts $(0, 25)$, $(25, 0)$.



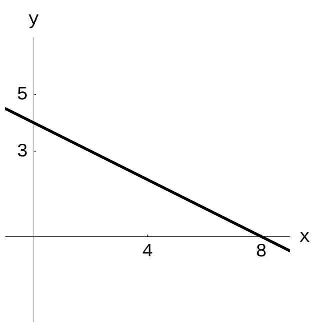
18. The line $y = 2x - 40$ has intercepts $(0, -40)$, $(20, 0)$.



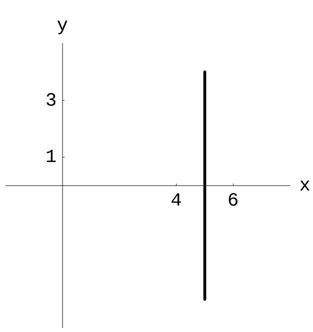
19. The line $y = 3x - 4$ has intercepts $(0, -4)$, $(4/3, 0)$.



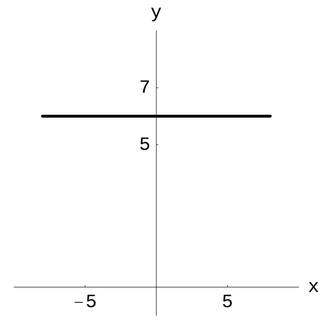
20. The line $y = -\dfrac{1}{2}x + 4$ has intercepts $(0, 4)$, $(8, 0)$.



21. Vertical line $x = 5$ has intercept $(5, 0)$.



22. Horizontal line $y = 6$ has intercept $(0, 6)$.



23. Simplify $(x - (-3))^2 + (y - 5)^2 = \left(\sqrt{3}\right)^2$. The standard equation is $(x+3)^2+(y-5)^2 = 3$.

24. Using the method of completing the square, we get

$$\begin{aligned} x^2 - x + y^2 + 2y &= 1 \\ \left(x - \frac{1}{2}\right)^2 + (y+1)^2 &= 1 + \frac{1}{4} + 1 \\ \left(x - \frac{1}{2}\right)^2 + (y+1)^2 &= \frac{9}{4}. \end{aligned}$$

The radius is $\dfrac{3}{2}$ and the center is $\left(\dfrac{1}{2}, -1\right)$.

25. Substitute $y = 0$ in $3x - 4y = 12$. Then $3x = 12$ or $x = 4$. The x-intercept is $(4, 0)$. Substitute $x = 0$ in $3x - 4y = 12$ to get $-4y = 12$ or $y = -3$. The y-intercept is $(0, -3)$.

26. When we substitute $x = 0$ into $y = 5$, we get $y = 5$. Thus, the y-intercept is $(0, 5)$.

27. $\dfrac{2 - (-6)}{-1 - 3} = \dfrac{8}{-4} = -2$

28. Solving for y, we get $y = \dfrac{3}{4}x - \dfrac{9}{4}$.

The slope is $\dfrac{3}{4}$.

29. Note, $m = \dfrac{-1 - 3}{5 - (-2)} = -\dfrac{4}{7}$. Solving for y in

$y - 3 = -\dfrac{4}{7}(x + 2)$, we obtain $y = -\dfrac{4}{7}x + \dfrac{13}{7}$.

30. Note, $m = \dfrac{-1-(-3)}{2-(-1)} = \dfrac{2}{3}$.

The slope-intercept form is derived below.

$$\begin{aligned} y+1 &= \frac{2}{3}(x-2) \\ 3y+3 &= 2(x-2) \\ -2x+3y &= -4-3 \\ 2x-3y &= 7 \end{aligned}$$

31. Note, the slope of $3x+y=-5$ is -3.
The standard form for the line through $(2,-4)$ with slope $\dfrac{1}{3}$ is derived below.

$$\begin{aligned} y+4 &= \frac{1}{3}(x-2) \\ 3y+12 &= x-2 \\ -x+3y &= -14 \\ x-3y &= 14 \end{aligned}$$

32. Note, the slope of $2x-3y=5$ is $\dfrac{2}{3}$.

The standard form for the line through $(2,-5)$ with slope $\dfrac{2}{3}$ is obtained below.

$$\begin{aligned} y+5 &= \frac{2}{3}(x-2) \\ y &= \frac{2}{3}x - \frac{4}{3} - 5 \\ y &= \frac{2}{3}x - \frac{19}{3} \end{aligned}$$

33. Since $2x-6=3y$, $y=\dfrac{2}{3}x-2$.

34. Since $y-2=\dfrac{1}{x}$, $y=2+\dfrac{1}{x}$.

35. Note, $y(x-3)=1$. Then $y=\dfrac{1}{x-3}$.

36. Note, $y(x^2-9)=1$. Thus, $y=\dfrac{1}{x^2-9}$.

37. Note, $by=-ax+c$. Then $y=-\dfrac{a}{b}x+\dfrac{c}{b}$ provided $b\neq 0$.

38. Multiply by $2xy$ to obtain $2x=2y+xy$.
Then factor as $2x=y(2+x)$. Thus, $y=\dfrac{2x}{x+2}$.

39. The discriminant of x^2-4x+2 is $(-4)^2-4(2)=8$. There are two distinct real solutions.

40. The discriminant of y^2-3y+2 is $(-3)^2-4(2)=1$. There are two distinct real solutions.

41. The discriminant is $(-20)^2-4(4)(25)=0$. Only one real solution.

42. The discriminant is $(-3)^2-4(2)(10)=-71$. There are no real solutions.

43. Since $x^2=5$, the solution set is $\{\pm\sqrt{5}\}$.

44. Since $x^2=\dfrac{54}{3}=18$, the solution set is $\{\pm 3\sqrt{2}\}$.

45. Since $x^2=-8$, the solution set is $\{\pm 2i\sqrt{2}\}$.

46. Since $x^2=-27$, the solution set is $\{\pm 3i\sqrt{3}\}$.

47. Since $x^2=-\dfrac{2}{4}$, the solution set is $\left\{\pm i\dfrac{\sqrt{2}}{2}\right\}$.

48. Since $x^2=-\dfrac{6}{9}$, the solution set is $\left\{\pm i\dfrac{\sqrt{6}}{3}\right\}$.

49. Since $x-2=\pm\sqrt{17}$, we get $x=2\pm\sqrt{17}$.
The solution set is $\left\{2\pm\sqrt{17}\right\}$.

50. Since $2x-1=\pm 3$, we obtain $x=\dfrac{1\pm 3}{2}$.
The solution set is $\{-1,2\}$.

51. Since $(x+3)(x-4)=0$, the solution set is $\{-3,4\}$.

52. Since $(2x-1)(x-5)=0$, the solution set is $\left\{\dfrac{1}{2},5\right\}$.

53. We apply the method of completing the square.

$$\begin{aligned} b^2-6b+10 &= 0 \\ b^2-6b+9 &= -10+9 \\ (b-3)^2 &= -1 \\ b-3 &= \pm i \end{aligned}$$

The solution set is $\{3\pm i\}$.

54. We apply the method of completing the square.

$$\begin{aligned} 4t^2 - 16t &= -17 \\ t^2 - 4t &= -\frac{17}{4} \\ t^2 - 4t + 4 &= -\frac{1}{4} \\ (t-2)^2 &= -\frac{1}{4} \\ t - 2 &= \pm i\frac{1}{2} \end{aligned}$$

The solution set is $\left\{2 \pm \frac{1}{2}i\right\}$.

55. We apply the method of completing the square.

$$\begin{aligned} s^2 - 4s &= -1 \\ s^2 - 4s + 4 &= -1 + 4 \\ (s-2)^2 &= 3 \\ s - 2 &= \pm\sqrt{3} \end{aligned}$$

The solution set is $\left\{2 \pm \sqrt{3}\right\}$.

56. We use factoring.

$$\begin{aligned} 3z^2 - 2z - 1 &= 0 \\ (3z+1)(z-1) &= 0 \\ 3z + 1 = 0 \quad &\text{or} \quad z - 1 = 0 \end{aligned}$$

The solution set is $\left\{-\frac{1}{3}, 1\right\}$.

57. Use the quadratic formula to solve $4x^2 - 4x - 5 = 0$.

$$\begin{aligned} x &= \frac{4 \pm \sqrt{(-4)^2 - 4(4)(-5)}}{2(4)} \\ &= \frac{4 \pm \sqrt{96}}{8} \\ &= \frac{4 \pm 4\sqrt{6}}{8} \\ &= \frac{1 \pm \sqrt{6}}{2} \end{aligned}$$

The solution set is $\left\{\frac{1 \pm \sqrt{6}}{2}\right\}$.

58. Use the quadratic formula to solve $9x^2 - 30x + 23 = 0$.

$$\begin{aligned} x &= \frac{30 \pm \sqrt{(-30)^2 - 4(9)(23)}}{2(9)} \\ &= \frac{30 \pm \sqrt{72}}{18} \\ &= \frac{30 \pm 6\sqrt{2}}{18} \\ &= \frac{5 \pm \sqrt{2}}{3} \end{aligned}$$

The solution set is $\left\{\frac{5 \pm \sqrt{2}}{3}\right\}$.

59. Subtracting 1 from both sides, we find

$$\begin{aligned} x^2 - 2x + 1 &= -1 \\ (x-1)^2 &= -1 \\ x - 1 &= \pm i. \end{aligned}$$

The solution set is $\{1 \pm i\}$.

60. Subtracting 1 from both sides, we find

$$\begin{aligned} x^2 - 4x + 4 &= -1 \\ (x-2)^2 &= -1 \\ x - 2 &= \pm i. \end{aligned}$$

The solution set is $\{2 \pm i\}$.

61. Multiplying by $2x(x-1)$, we obtain

$$\begin{aligned} 2(x-1) + 2x &= 3x(x-1) \\ 0 &= 3x^2 - 7x + 2 \\ 0 &= (x-2)(3x-1). \end{aligned}$$

The solution set is $\left\{\frac{1}{3}, 2\right\}$.

62. Multiplying by $2(x-2)(x+2)$, we obtain

$$\begin{aligned} 4(x+2) - 6(x-2) &= x^2 - 4 \\ 0 &= x^2 + 2x - 24 \\ 0 &= (x-4)(x+6). \end{aligned}$$

The solution set is $\{-6, 4\}$.

63. Solve an equivalent statement

$$\begin{aligned} 3q - 4 = 2 \quad &\text{or} \quad 3q - 4 = -2 \\ 3q = 6 \quad &\text{or} \quad 3q = 2. \end{aligned}$$

The solution set is $\{2/3, 2\}$.

64. Solving $|2v-1|=3$, we find

$$\begin{array}{rcl} 2v-1=3 & \text{or} & 2v-1=-3 \\ v=2 & \text{or} & v=-1. \end{array}$$

The solution set is $\{-1,2\}$.

65. We obtain

$$\begin{aligned} |2h-3| &= 0 \\ 2h-3 &= 0 \\ h = \frac{3}{2}. \end{aligned}$$

The solution set is $\left\{\frac{3}{2}\right\}$.

66. We find

$$\begin{aligned} |x-3| &= 0 \\ x-3 &= 0 \\ x &= 3. \end{aligned}$$

The solution set is $\{3\}$.

67. No solution since absolute values are nonnegative. The solution set is $\emptyset$.

68. No solution, absolute values are nonnegative. The solution set is $\emptyset$.

69. Solve an equivalent statement assuming $3v \geq 0$.

$$\begin{array}{rcl} 2v-1=3v & \text{or} & 2v-1=-3v \\ -1=v & \text{or} & 5v=1 \end{array}$$

Since $3v \geq 0$, $v=-1$ is an extraneous root.

The solution set is $\{1/5\}$.

70. Solve an equivalent statement.

$$\begin{array}{rcl} 2h-3=h & \text{or} & 2h-3=-h \\ h=3 & \text{or} & 3h=3 \end{array}$$

The solution set is $\{1,3\}$.

71. Let $w=x^2$ and $w^2=x^4$.

$$\begin{aligned} w^2+7w &= 18 \\ (w+9)(w-2) &= 0 \\ w &= -9,2 \\ x^2=-9 \quad &\text{or} \quad x^2=2. \end{aligned}$$

Since $x^2=-9$ has no real solution, the solution set is $\{\pm\sqrt{2}\}$.

72. Let $w=x^{-1}$ and $w^2=x^{-2}$.

$$\begin{aligned} 2w^2+5w-12 &= 0 \\ (w+4)(2w-3) &= 0 \\ w &= -4, 3/2 \\ x^{-1}=-4 \quad &\text{or} \quad x^{-1}=\frac{3}{2}. \end{aligned}$$

The solution set is $\{-1/4, 2/3\}$.

73. Isolate a radical and square both sides.

$$\begin{aligned} \sqrt{x+6} &= \sqrt{x-5}+1 \\ x+6 &= x-5+2\sqrt{x-5}+1 \\ 5 &= \sqrt{x-5} \\ 25 &= x-5 \end{aligned}$$

The solution set is $\{30\}$.

74. Square both sides

$$\begin{aligned} 2x-1 &= x-1+2\sqrt{x-1}+1 \\ x-1 &= 2\sqrt{x-1} \\ x^2-2x+1 &= 4(x-1) \\ x^2-6x+5 &= 0 \\ (x-5)(x-1) &= 0 \end{aligned}$$

The solution set is $\{1,5\}$.

75. Let $w=\sqrt[4]{y}$ and $w^2=\sqrt{y}$.

$$\begin{aligned} w^2+w-6 &= 0 \\ (w+3)(w-2) &= 0 \\ w=-3 \quad &\text{or} \quad w=2 \\ y^{1/4}=-3 \quad &\text{or} \quad y^{1/4}=2 \end{aligned}$$

Since $y^{1/4}=-3$ has no real solution, the solution set is $\{16\}$.

76. Let $w=\sqrt[3]{x}$ and $w^2=\sqrt[3]{x^2}$.

$$\begin{aligned} w^2+w-2 &= 0 \\ (w+2)(w-1) &= 0 \\ \sqrt[3]{x}=-2 \quad &\text{or} \quad \sqrt[3]{x}=1 \\ x &= -8,1 \end{aligned}$$

The solution set is $\{1,-8\}$

77. Let $w = x^2$ and $w^2 = x^4$.

$$\begin{aligned} w^2 - 3w - 4 &= 0 \\ (w+1)(w-4) &= 0 \\ x^2 = -1 \quad \text{or} \quad x^2 &= 4 \end{aligned}$$

Since $x^2 = -1$ has no real solution, the solution set is $\{\pm 2\}$.

78. Let $w = y - 1$ and $w^2 = (y-1)^2$.

$$\begin{aligned} w^2 - w - 2 &= 0 \\ (w+1)(w-2) &= 0 \\ w = -1 \quad &\text{or} \quad w = 2 \\ y - 1 = -1 \quad &\text{or} \quad y - 1 = 2 \end{aligned}$$

The solution set is $\{0, 3\}$.

79. Raise to the power $3/2$ and get $x - 1 = \pm(4^{1/2})^3$. So, $x = 1 \pm 8$. The solution set is $\{-7, 9\}$.

80. Raise to the power -2 and get $2x - 3 = (1/2)^{-2}$. Then $2x - 3 = 4$. The solution set is $\{7/2\}$.

81. No solution since $(x+3)^{-3/4}$ is nonnegative.

82. Since $(x-3)^{1/4} = \dfrac{1}{2}$, we get $x - 3 = \dfrac{1}{16}$.

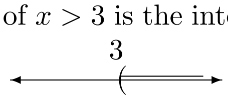
The solution set is $\left\{\dfrac{49}{16}\right\}$.

83. Since $3x - 7 = 4 - x$, we obtain $4x = 11$. The solution set is $\{11/4\}$.

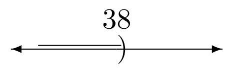
84. Raise both sides to the power 6.

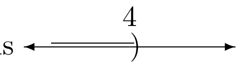
$$\begin{aligned} (x+1)^2 &= 4x + 9 \\ x^2 - 2x - 8 &= 0 \\ (x-4)(x+2) &= 0 \\ x &= 4, -2. \end{aligned}$$

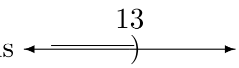
Checking $x = -2$, we get $-1 = 1$ and so $x = -2$ is an extraneous root. The solution set is $\{4\}$.

85. The solution set of $x > 3$ is the interval $(3, \infty)$ and the graph is 

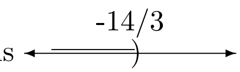
86. The solution set of $6x - 18 < 5x + 20$ is the interval $(-\infty, 38)$ and the graph is



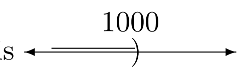
87. The solution set of $8 > 2x$ is the interval $(-\infty, 4)$ and the graph is 

88. The solution set of $13 > x$ is the interval $(-\infty, 13)$ and the graph is 

89. Since $-\dfrac{7}{3} > \dfrac{1}{2}x$, the solution set is

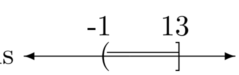
$(-\infty, -14/3)$ and the graph is 

90. The solution set of $940 > 0.94x$ is the interval $(-\infty, 1000)$ and the

graph is 

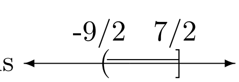
91. After multiplying the inequality by 2 we have

$$\begin{aligned} -4 < x - 3 &\le 10 \\ -1 < x &\le 13. \end{aligned}$$

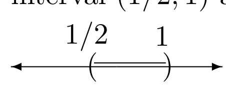
The solution set is the interval $(-1, 13]$ and the graph is 

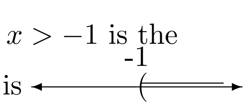
92. Multiplying the inequality by 4, we find

$$\begin{aligned} -4 \le 3 - 2x &< 12 \\ -7 \le -2x &< 9 \\ \frac{7}{2} \ge x &> -\frac{9}{2}. \end{aligned}$$

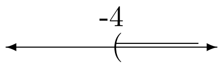
The solution set is the interval $(-9/2, 7/2]$ and the graph is 

93. The solution set of $\dfrac{1}{2} < x$ and $x < 1$ is the interval $(1/2, 1)$ and the graph is



94. The solution set of $x > -2$ and $x > -1$ is the interval $(-1, \infty)$ and the graph is 

95. The solution set of $x > -4$ or $x > -1$ is the interval $(-4, \infty)$ and the graph is

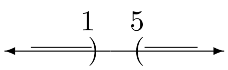


96. The solution set of $-5 < x$ or $x < 6$ is the interval $(-\infty, \infty)$ and the graph is

97. Solving an equivalent statement, we get

$$\begin{array}{lcl} x-3>2 & \text{or} & x-3<-2 \\ x>5 & \text{or} & x<1. \end{array}$$

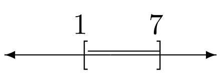
The solution set is $(-\infty, 1) \cup (5, \infty)$ and the graph is



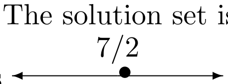
98. Solving an equivalent statement, we obtain

$$\begin{array}{rl} -3 & \le 4-x \le 3 \\ 7 & \ge x \ge 1. \end{array}$$

The solution set is the interval $[1, 7]$ and the graph is



99. Since an absolute value is nonnegative, $2x - 7 = 0$. The solution set is $\{7/2\}$ and the graph is



100. No solution since absolute values are nonnegative. The solution set is $\emptyset$.

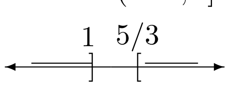
101. Since absolute values are nonnegative, the solution set is $(-\infty, \infty)$ and

the graph is

102. Solving an equivalent inequality, we find

$$\begin{array}{rcl} 4-3x \ge 1 & \text{or} & 4-3x \le -1 \\ 1 \ge x & \text{or} & 5/3 \le x. \end{array}$$

The solution set is $(-\infty, 1] \cup [5/3, \infty)$ and the graph is



103. The solution set is $\{10\}$ since the x-intercept is $(10, 0)$.

104. The solution set is $\{-30, 26\}$ since the x-intercepts are $(-30, 0)$ and $(26, 0)$.

105. Since the x-intercept is $(8, 0)$ and the y-values are negative in quadrants 3 and 4, the solution set is $(-\infty, 8)$.

106. Since the x-intercept is $(30, 0)$ and the y-values are positive in quadrants 1 and 2, the solution set is $(-\infty, 30]$.

107. Let x be the length of one side of the square. Since dimensions of the base are $8 - 2x$ and $11 - 2x$, we obtain

$$\begin{aligned} (11-2x)(8-2x) &= 50 \\ 4x^2 - 38x + 38 &= 0 \\ 2x^2 - 19x + 19 &= 0 \\ x &= \frac{19 \pm \sqrt{209}}{4}. \end{aligned}$$

But $x = \dfrac{19 + \sqrt{209}}{4} \approx 8.36$ is too big.

Then $x = \dfrac{19 - \sqrt{209}}{4} \approx 1.14$ in.

108. Let x be the number of hours since 9:00 a.m.

$$\begin{aligned} (x+1)\frac{1}{12} + x\frac{1}{8} &= 1 \\ 8x + 8 + 12x &= 96 \\ 20x &= 88 \\ x &= \frac{22}{5} = 4.4 \end{aligned}$$

They will finish in 4.4 hrs. or at 1:24 p.m.

109. Let x be the number of hours it takes Lisa or Taro to drive to the restaurant. Since the sum of the driving distances is 300, we obtain $300 = 50x + 60x$. Thus, $x = \dfrac{300}{110} \approx 2.7272$ and Lisa drove $50(2.7272) \approx 136.4$ miles.

110. Let $x+10$ and x be the average driving speeds of Lisa and Taro. Since Taro drove an hour less than Lisa and *time* = *distance* ÷ *speed*,

$$\begin{aligned} \frac{100}{x} + 1 &= \frac{200}{x+10} \\ 100x + 1000 + x^2 + 10x &= 200x \\ x^2 - 90x + 1000 &= 0 \\ x &= 45 \pm 5\sqrt{41}. \end{aligned}$$

Note $x+10 = 55 \pm 5\sqrt{41} \approx 87.02, 22.98$. Lisa's possible speeds are 87.02 mph and 22.98 mph.

111. Let x and $8000 - x$ be the number of fish in Homer Lake and Mirror Lake, respectively. Then

$$\begin{aligned} 0.2x + 0.3(8000 - x) &= 0.28(8000) \\ -0.1x + 2400 &= 2240 \\ 1600 &= x. \end{aligned}$$

There were originally 1600 fish in Homer Lake.

112. Let x be the number of representatives after redistricting. Then

$$\begin{aligned} \frac{22}{x} - 0.05 &= \frac{18}{x-4} \\ 18x &= 22x - 88 - 0.05(x^2 - 4x) \\ 0.05x^2 - 4.2x + 88 &= 0. \end{aligned}$$

By using the quadratic formula, we have

$$\begin{aligned} x &= \frac{4.2 \pm \sqrt{(-4.2)^2 - 4(0.05)(88)}}{0.1} \\ x &= 40, 44. \end{aligned}$$

Since after redistricing the pro-gambling representatives still did not constitute a majority, there are $x = 44$ representatives in the house after redistricting.

113. Let x be the distance she hiked in the northern direction. At 4 mph and 8 hr, she hiked 32 miles. Then she hiked $32-x$ miles in the eastern direction. By the Pythagorean Theorem, we obtain

$$\begin{aligned} x^2 + (32 - x)^2 &= (4\sqrt{34})^2 \\ 2x^2 - 64x + 480 &= 0 \\ 2(x - 20)(x - 12) &= 0 \\ x &= 20, 12. \end{aligned}$$

Since the eastern direction was the shorter leg of the journey, the northern direction was 20 miles.

114. After substituting $L = 20$ m, we use the method of completing the square to solve for the width W.

$$\begin{aligned} \frac{20}{W} &= \frac{W}{20 - W} \\ 400 - 20W &= W^2 \\ 400 &= W^2 + 20W \\ 400 + 100 &= (W + 10)^2 \\ \sqrt{500} &= W + 10 \\ 10\sqrt{5} - 10 &= W \end{aligned}$$

If $L = 20$ m, then $W = 10\sqrt{5} - 10 \approx 12.36$ m.

Next, we substitute $W = 8$ m.

$$\begin{aligned} \frac{L}{8} &= \frac{8}{L - 8} \\ L^2 - 8L &= 64 \\ (L - 4)^2 &= 64 + 16 \\ L - 4 &= \sqrt{80} \end{aligned}$$

If $W = 8$ m, then $L = 4 + 4\sqrt{5} \approx 12.94$ m.

115. Let x and $x + 50$ be the cost of a haircut at Joe's and Renee's, respectively. Since 5 haircuts at Joe's is less than one haircut at Renee's, we have

$$5x < x + 50.$$

Thus, the price range of a haircut at Joe's is $x < \$12.50$ or $(0, \$12.50)$.

116. Let x be the selling price.

Then $x - 0.06x \geq 120,000$. The minimum selling price is $\dfrac{120,000}{0.94} \approx \$127,659.57$.

117. Let x and $x + 2$ be the width and length of a picture frame in inches, respectively. Since there are between 32 and 50 inches of molding, we get

$$\begin{aligned} 32 &< 2x + 2(x + 2) < 50 \\ 32 &< 4x + 4 < 50 \\ 28 &< 4x < 46 \\ 7 \text{ in.} &< x < 11.5 \text{ in.} \end{aligned}$$

The set of possible widths is (7 in, 11.5 in).

118. The number of gallons of gas saved in a year is $\dfrac{10^{12}}{27.5} - \dfrac{10^{12}}{29.5} \approx 2.47 \times 10^9$.

119. If the average gas mileage is increased from 29.5 mpg to 31.5 mpg, then the amount of gas saved is

$$\frac{10^{12}}{29.5} - \frac{10^{12}}{31.5} \approx 2.15 \times 10^9 \quad \text{gallons.}$$

Suppose the mileage is increased to x from 29.5 mpg. Then x must satisfy

$$\begin{aligned} \frac{10^{12}}{29.5} - \frac{10^{12}}{x} &= \frac{10^{12}}{27.5} - \frac{10^{12}}{29.5} \\ \frac{1}{29.5} - \frac{1}{x} &= \frac{1}{27.5} - \frac{1}{29.5} \\ -\frac{1}{x} &\approx -0.031433 \\ x &\approx 31.8. \end{aligned}$$

The mileage must be increased to 31.8 mpg.

120. Let x be the thickness in yards. From the volume one gets $\frac{12}{3} \cdot \frac{54}{3} x = 40$.

The solution is $x = \frac{5}{9}$ and the thickness is $\frac{5}{9} \times 36 = 20$ in.

121. a) The line is given by

$$y \approx 17.294x - 34,468$$

where x is the year and y is the number of millions of cell users. We use three decimal places.

b) If $x = 2016$, the number of millions of cell users is

$$y \approx 17.294(2016) - 34,468 \approx 397.$$

There will be 397 million cell users in 2016.

122. a) The line is given by

$$y \approx 340.94x - 680,371$$

where x is the year and y is the number of millions of worldwide cell users.

b) If $x = 2020$, then

$$y \approx 340.94(2020) - 680,371 \approx 8,328.$$

There will be 8,328 million worldwide cell users in 2020.

123. Let a be the age in years and p be the percentage. The equation of the line passing through $(20, 0.23)$ and $(50, 0.47)$ is

$$p = 0.008a + 0.07.$$

If $a = 65$, then $p = 0.008(65) + 0.07 \approx 0.59$. Thus, the percentage of body fat in a 65-year old woman is 59%.

124. Using a calculator, the regression line passing through $(2004, 19.79)$ and $(2008, 19.30)$ is

$$y \approx -0.1225x + 265.28$$

where x is the year and y is the number of seconds.

If $x = 2012$, then

$$y \approx -0.1225(2012) + 265.28 \approx 18.81.$$

In 2012, a winning time prediction is 18.81 sec.

125. a) Using a calculator, the regression line is given by

$$y \approx 3.52x + 48.03$$

where $x = 0$ corresponds to 2000.

b) If $x = 17$, then the average price of a prescription in 2017 is

$$y \approx 3.52(17) + 48.03 = \$107.87.$$

126. a) Using a calculator, the regression line is given by

$$y \approx 145.7x + 2614.4$$

where $x = 0$ corresponds to 2000.

b) If $x = 15$, then the predicted number of millions of prescriptions in 2012 is

$$y \approx 145.7(15) + 2614.4 \approx 4800 \text{ million.}$$

127. Circle A is given by

$$(x-1)^2 + (y-1)^2 = 1.$$

Draw a right triangle with sides 1 and x, and with hypotenuse 3 such that the hypotenuse has as endpoints the centers of circles A and

B. Here, x is the horizontal distance between the centers of A and B. Since

$$1 + x^2 = 9$$

we obtain $x = 2\sqrt{2}$. Then the center of B is $(1 + 2\sqrt{2}, 2)$. Thus, circle B is given by

$$\left(x - 1 - 2\sqrt{2}\right)^2 + (y - 2)^2 = 4.$$

Let r and (a, r) be the radius and center of circle C. Draw a right triangle with sides $a - 1$ and $1 - r$, and with hypotenuse $1 + r$ such that the hypotenuse has as endpoints the centers of circles A and C. Then

$$(1 + r)^2 = (1 - r)^2 + (a - 1)^2.$$

Next, draw a right triangle with sides $1+2\sqrt{2}-a$ and $2-r$, and with hypotenuse $2+r$ such that the hypotenuse has as endpoints the centers of circles B and C. Then

$$(2 + r)^2 = (2 - r)^2 + (1 + 2\sqrt{2} - a)^2.$$

The solution of the two previous equations are

$$a = 5 - 2\sqrt{2}, \quad r = 6 - 4\sqrt{2}.$$

Hence, circle C is given by

$$(x - 5 + 2\sqrt{2})^2 + (y - 6 + 4\sqrt{2})^2 = (6 - 4\sqrt{2})^2$$

128. Let x be the top speed of the hiker, and let d be the length of the tunnel. The time it takes the hiker to cover one-fourth of the tunnel is

$$\frac{d/4}{x}.$$

Let y be the number of miles that the tunnel is ahead of the train when the hiker spots the train. Since the hiker can return to the entrance of the tunnel just before the train enters the tunnel, we obtain.

$$\frac{y}{30} = \frac{d/4}{x}.$$

Similarly, if the hiker runs towards the other end of the tunnel then

$$\frac{y + d}{30} = \frac{3d/4}{x}.$$

Thus,

$$\frac{d/4}{x} + \frac{d}{30} = \frac{3d/4}{x}.$$

Dividing by d, we find

$$\frac{1}{4x} + \frac{1}{30} = \frac{3}{4x}.$$

Solving for x, we obtain $x = 15$ mph which is the top speed of the hiker.

Chapter 1 Test

1. Since $2x - x = -6 - 1$, the solution set is $\{-7\}$.

2. Multiplying the original equation by 6, we get $3x - 2x = 1$. The solution set is $\{1\}$.

3. Since $x^2 = \dfrac{2}{3}$, one obtains $x = \pm\dfrac{\sqrt{2}}{\sqrt{3}} = \pm\dfrac{\sqrt{6}}{3}$.

The solution set is $\left\{\pm\dfrac{\sqrt{6}}{3}\right\}$.

4. By completing the square, we obtain

$$\begin{aligned} x^2 - 6x &= -1 \\ x^2 - 6x + 9 &= -1 + 9 \\ (x - 3)^2 &= 8 \\ x - 3 &= \pm\sqrt{8}. \end{aligned}$$

The solution set is $\{3 \pm 2\sqrt{2}\}$.

5. Since $x^2 - 9x + 14 = (x - 2)(x - 7) = 0$, the solution set is $\{2, 7\}$.

6. After cross-multiplying, we get

$$\begin{aligned} (x - 1)(x - 6) &= (x + 3)(x + 2) \\ x^2 - 7x + 6 &= x^2 + 5x + 6 \\ -7x &= 5x \\ 0 &= 12x \end{aligned}$$

The solution set is $\{0\}$.

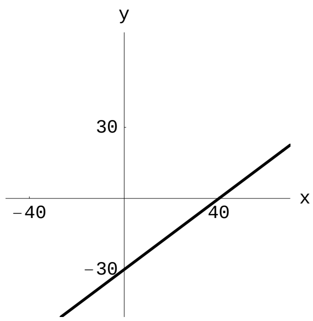
7. We use the method of completing the square.

$$\begin{aligned} x^2 - 2x &= -5 \\ x^2 - 2x + 1 &= -5 + 1 \\ (x - 1)^2 &= -4 \\ x - 1 &= \pm 2i \end{aligned}$$

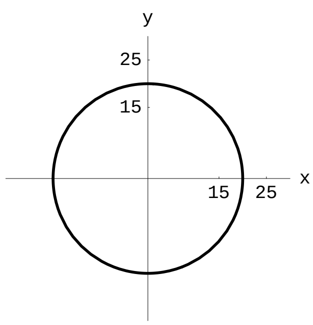
The solution set is $\{1 \pm 2i\}$.

8. Since $x^2 = -1$, the solution set is $\{\pm i\}$.

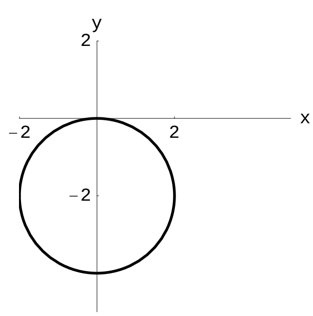
9. The line $3x - 4y = 120$ passes through $(0, -30)$ and $(40, 0)$.



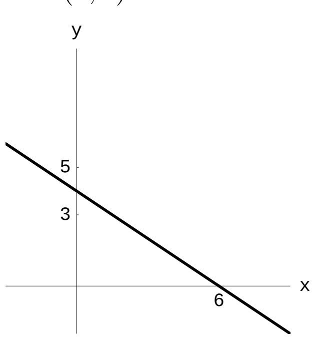
10. Circle with center $(0, 0)$ and radius 20.



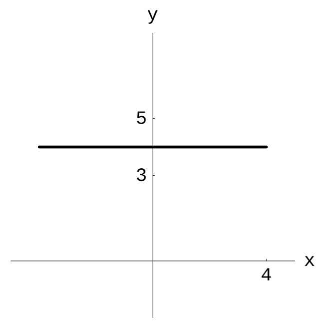
11. By using the method of completing the square, we obtain $x^2+(y+2)^2 = 4$. A circle with center $(0, -2)$ and radius 2.



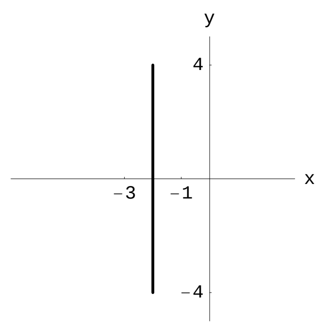
12. The line $y = -\dfrac{2}{3}x + 4$ passes through $(0, 4)$ and $(6, 0)$.



13. The horizontal line $y = 4$.



14. The vertical line $x = -2$.



15. Since $y = \dfrac{3}{5}x - \dfrac{8}{5}$, the slope is $\dfrac{3}{5}$.

16. $\dfrac{-4-6}{5-(-3)} = \dfrac{-10}{8} = -\dfrac{5}{4}$

17. We rewrite $2x - 3y = 6$ as $y = \dfrac{2}{3}x - 2$. Note, the slope of $y = \dfrac{2}{3}x - 2$ is $\dfrac{2}{3}$. Then we use $m = -\dfrac{3}{2}$ and the point $(1, -2)$.

$$\begin{aligned} y + 2 &= -\frac{3}{2}(x-1) \\ y &= -\frac{3}{2}x + \frac{3}{2} - 2. \end{aligned}$$

The perpendicular line is $y = -\dfrac{3}{2}x - \dfrac{1}{2}$.

18. The slope is $\dfrac{-1-2}{3-0} = -1$. We use the point $(3, -4)$.

$$\begin{aligned} y + 4 &= -(x-3) \\ y &= -x + 3 - 4. \end{aligned}$$

The parallel line is $y = -x - 1$.

19. $\sqrt{(-3-2)^2 + (1-4)^2} = \sqrt{25+9} = \sqrt{34}$

20. $\left(\dfrac{-1+1}{2}, \dfrac{1+0}{2}\right) = \left(0, \dfrac{1}{2}\right)$

21. Since the discriminant is negative, namely, $(-5)^2 - 4(1)(9) = -11$, then there are no real solutions.

22. We solve for y.

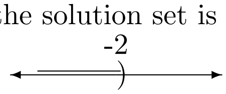
$$\begin{aligned} 5-4 &= 3xy+2y \\ 1 &= y(3x+2) \\ y &= \frac{1}{3x+2} \end{aligned}$$

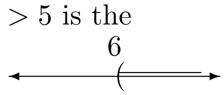
23. If we raise each side of $(x-3)^{-2/3} = \dfrac{1}{3}$ to the power -3, then we obtain $(x-3)^2 = 27$. Thus, $x = 3 \pm \sqrt{27}$ and the solution set is $\left\{3 \pm 3\sqrt{3}\right\}$.

24. Isolate a radical and square each side.

$$\begin{aligned} \sqrt{x}-1 &= \sqrt{x-7} \\ x-2\sqrt{x}+1 &= x-7 \\ \sqrt{x} &= 4 \end{aligned}$$

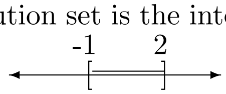
The solution set is $\{16\}$.

25. Since $-4 > 2x$, the solution set is $(-\infty, -2)$ and the graph is 

26. The solution set to $x > 6$ and $x > 5$ is the interval $(6, \infty)$ and the graph is 

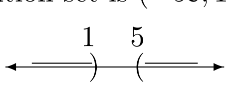
27. Solving an equivalent statement, we obtain

$$\begin{aligned} -3 &\le 2x-1 \le 3 \\ -2 &\le 2x \le 4 \\ -1 &\le x \le 2. \end{aligned}$$

The solution set is the interval $[-1, 2]$ and the graph is 

28. We rewrite $|x-3| > 2$ without any absolute values. Then

$$\begin{aligned} x-3 > 2 &\quad \text{or} \quad x-3 < -2 \\ x > 5 &\quad \text{or} \quad x < 1. \end{aligned}$$

The solution set is $(-\infty, 1) \cup (5, \infty)$ and the graph is 

29. If x is the original length of one side of the square, then

$$\begin{aligned} (x+20)(x+10) &= 999 \\ x^2+30x+200 &= 999 \\ x^2+30x-799 &= 0 \\ \frac{-30 \pm \sqrt{900+4(799)}}{2} &= x \\ \frac{-30 \pm 64}{2} &= x \\ 17, -47 &= x. \end{aligned}$$

Thus, $x = 17$ and the original area is $17^2 = 289$ ft^2.

30. Let x be the number of gallons of the 20% solution. From the concentrations,

$$\begin{aligned} 0.3(10+x) &= 0.5(10)+0.2x \\ 3+0.3x &= 5+0.2x \\ 0.1x &= 2 \\ x &= 20. \end{aligned}$$

Then 20 gallons of the 20% solution are needed.

31. a) Using a calculator, the regression line is given by

$$y \approx 18.4x + 311$$

where $x = 0$ corresponds to 1997 and y is the median price of a home in thousands of dollars.

b) If $x = 18$, then

$$y \approx 18.4(18) + 311 \approx 642.$$

The predicted median price in 2015 is $642,000.

32. If $D = 10$, then $T = 0.07(10)^{3/2} \approx 2.2$ hr.
If $T = 4$, then $D = \left(\frac{4}{0.07}\right)^{2/3} \approx 14.8$ mi.

Tying It All Together

1. $7x$ **2.** $30x^2$ **3.** $\dfrac{2}{2x} + \dfrac{1}{2x} = \dfrac{3}{2x}$

4. x^2+6x+9 **5.** $6x^2+x-2$

6. $\dfrac{x^2+2xh+h^2-x^2}{h} = \dfrac{2xh+h^2}{h} = 2x+h$

7. $\dfrac{x+1}{(x-1)(x+1)} + \dfrac{x-1}{(x+1)(x-1)} = \dfrac{2x}{x^2-1}$

8. $x^2+3x+\dfrac{9}{4}$

9. An identity, the solution set is $(-\infty,\infty)$ or $\mathbb{R}$.

10. Since $30x^2-11x = x(30x-11) = 0$, the solution set is $\left\{0, \dfrac{11}{30}\right\}$.

11. Since $\dfrac{3}{2x} = \dfrac{3}{2x}$, the solution set is $(-\infty,0)\cup(0,\infty)$.

12. Subtract x^2+9 from $x^2+6x+9 = x^2+9$ and get $6x=0$. The solution set is $\{0\}$.

13. Since $(2x-1)(3x+2)=0$, the solution set is $\left\{-\dfrac{2}{3}, \dfrac{1}{2}\right\}$.

14. Multiply the equation by $8(x+1)(x-1)$.

$$\begin{aligned}
8x+8+8x-8 &= 5(x^2-1)\\
0 &= 5x^2-16x-5\\
x &= \frac{16\pm\sqrt{356}}{10}\\
x &= \frac{16\pm 2\sqrt{89}}{10}
\end{aligned}$$

The solution set is $\left\{\dfrac{8\pm\sqrt{89}}{5}\right\}$.

15. Since $7x-7x^2 = 7x(1-x) = 0$, the solution set is $\{0,1\}$.

16. Since $7x-7 = 7(x-1) = 0$, the solution set is $\{1\}$.

17. 0 **18.** $-1-3+2=-2$

19. $-\dfrac{1}{8} - 3\left(\dfrac{2}{8}\right) + \dfrac{16}{8} = \dfrac{9}{8}$

20. $-\dfrac{1}{27} - 3\left(\dfrac{3}{27}\right) + \dfrac{54}{27} = \dfrac{44}{27}$

21. $-1-3-4=-8$

22. $-4+6-4=-2$ **23.** -4

24. $-0.25+1.5-4=-2.75$

25. conditional

26. identity

27. inconsistent

28. distributive

29. associative

30. irrational

31. additive, multiplicative

32. difference

33. quotient

34. additive inverse, opposite