Chapter 1 The Derivative

1.1 The Slope of a Straight Line

1. $y = 3 - 7x$; y-intercept: (0, 3), slope: −7

2. $y = \frac{3x+1}{5} = \frac{3}{5}x + \frac{1}{5}$; y-intercept: $\left(0, \frac{1}{5}\right)$, slope: $\frac{3}{5}$

3. $x = 2y - 3 \Rightarrow y = \frac{x+3}{2} \Rightarrow y = \frac{1}{2}x + \frac{3}{2}$; y-intercept: $\left(0, \frac{3}{2}\right)$, slope: $\frac{1}{2}$

4. $y = 6 \Rightarrow y = 0x + 6$; y-intercept: (0, 6), slope: 0

5. $y = \frac{x}{7} - 5 \Rightarrow y = \frac{1}{7}x - 5$; y-intercept: (0, −5), slope: $\frac{1}{7}$

6. $4x + 9y = -1 \Rightarrow y = \frac{-4x-1}{9} \Rightarrow y = -\frac{4}{9}x - \frac{1}{9}$; y-intercept: $\left(0, -\frac{1}{9}\right)$, slope $= -\frac{4}{9}$

7. slope = −1, (7, 1) on line.
Let $(x, y) = (7, 1)$, $m = -1$.
$y - y_1 = m(x - x_1) \Rightarrow y - 1 = -(x - 7) \Rightarrow$
$y = -x + 8$

8. slope = 2; (1, −2) on line.
Let $(x, y) = (1, -2)$, $m = 2$.
$y - y_1 = m(x - x_1) \Rightarrow y + 2 = 2(x - 1) \Rightarrow$
$y = 2x - 4$

9. slope $= \frac{1}{2}$; (2, 1) on line.
Let $(x_1, y_1) = (2, 1)$; $m = \frac{1}{2}$.
$y - y_1 = m(x - x_1) \Rightarrow y - 1 = \frac{1}{2}(x - 2) \Rightarrow$
$y = \frac{1}{2}x$

10. slope $= \frac{7}{3}$; $\left(\frac{1}{4}, -\frac{2}{5}\right)$ on line.
Let $(x_1, y_1) = \left(\frac{1}{4}, -\frac{2}{5}\right)$; $m = \frac{7}{3}$.
$y - y_1 = m(x - x_1) \Rightarrow y + \frac{2}{5} = \frac{7}{3}\left(x - \frac{1}{4}\right) \Rightarrow$
$y = \frac{7}{3}x - \frac{59}{60}$

11. $\left(\frac{5}{7}, 5\right)$ and $\left(-\frac{5}{7}, -4\right)$ on line.
slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-5}{-\frac{5}{7} - \frac{5}{7}} = \frac{-9}{-\frac{10}{7}} = \frac{63}{10}$
Let $(x_1, y_1) = \left(\frac{5}{7}, 5\right)$, $m = \frac{63}{10}$.
$y - y_1 = m(x - x_1) \Rightarrow y - 5 = \frac{63}{10}\left(x - \frac{5}{7}\right)$

12. $\left(\frac{1}{2}, 1\right)$ and (1, 4) on line.
slope$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{1-\frac{1}{2}} = \frac{3}{\frac{1}{2}} = 6$
Let $(x_1, y_1) = (1, 4)$, $m = 6$.
$y - y_1 = m(x - x_1) \Rightarrow y - 4 = 6(x - 1) \Rightarrow$
$y = 6x - 2$

13. (0, 0) and (1, 0) on line.
slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-0}{1-0} = 0$
$y - 0 = 0(x - 0) \Rightarrow y = 0$

14. $\left(-\frac{1}{2}, -\frac{1}{7}\right)$ and $\left(\frac{2}{3}, 1\right)$ on line.
slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - \left(-\frac{1}{7}\right)}{\frac{2}{3} - \left(-\frac{1}{2}\right)} = \frac{\frac{8}{7}}{\frac{7}{6}} = \frac{48}{49} = m$
Let $(x_1, y_1) = \left(\frac{2}{3}, 1\right)$.
$y - y_1 = m(x - x_1) \Rightarrow y - 1 = \frac{48}{49}\left(x - \frac{2}{3}\right) \Rightarrow$
$y = \frac{48}{49}x + \frac{17}{49}$

15. Horizontal through (2, 9).
Let $(x_1, y_1) = (2, 9)$, $m = 0$ (horizontal line).
$y - y_1 = m(x - x_1) \Rightarrow y - 9 = 0(x - 2) \Rightarrow$
$y = 9$

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16. x-intercept is 1; y-intercept is –3.
The intercepts (1, 0) and (0, –3) are on the line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-0}{0-1} = 3 = m$$

y-intercept $(0, b) = (0, -3)$

$y = mx + b \Rightarrow y = 3x - 3$

17. x-intercept is $-\pi$; y-intercept is 1.
The intercepts $(-\pi, 0)$ and (0, 1) are on the line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{0-(-\pi)} = \frac{1}{\pi}$$

y-intercept $(0, b) = (0, 1)$

$$y = mx + b \Rightarrow y = \frac{x}{\pi} + 1$$

18. Slope = 2; x-intercept is –3.
The x-intercept (–3, 0) is on the line.
Let $(x_1, y_1) = (-3, 0), m = 2.$

$y - y_1 = m(x - x_1) \Rightarrow y - 0 = 2(x + 3) \Rightarrow$
$y = 2x + 6$

19. Slope = −2; x-intercept is –2.
The x-intercept (–2, 0) is on the line.
Let $(x_1, y_1) = (-2, 0), m = -2.$

$y - y_1 = m(x - x_1) \Rightarrow y - 0 = -2(x + 2) \Rightarrow$
$y = -2x - 4$

20. Horizontal through $(\sqrt{7}, 2)$.

Let $(x_1, y_1) = (\sqrt{7}, 2)$, $m = 0$ (horizontal line).

$y - y_1 = m(x - x_1) \Rightarrow y - 2 = 0(x - \sqrt{7}) \Rightarrow$
$y = 2$

21. Parallel to $y = x$; (2, 0) on line.
Let $(x_1, y_1) = (2, 0)$; slope = m = 1.

$y - y_1 = m(x - x_1) \Rightarrow y - 0 = 1(x - 2) \Rightarrow$
$y = x - 2$

22. Parallel to $x + 2y = 0$; (1, 2) on line.

$$x + 2y = 0 \Rightarrow y = -\frac{1}{2}x;\ m = -\frac{1}{2}$$

Let $(x_1, y_1) = (1, 2)$.

$$y - y_1 = m(x - x_1) \Rightarrow y - 2 = -\frac{1}{2}(x - 1) \Rightarrow$$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

23. Parallel to $y = 3x + 7$; x-intercept is 2.
slope = m = 3. Let $(x_1, y_1) = (2, 0)$.

$y - y_1 = m(x - x_1) \Rightarrow y - 0 = 3(x - 2) \Rightarrow$
$y = 3x - 6$

24. Parallel to $y - x = 13$; y-intercept is 0.
$y = x + 13$, slope = m = 1, b = 0.
$y = mx + b \Rightarrow y = x$

25. Perpendicular to $y + x = 0$; (2, 0) on line.
$y + x = 0 \Rightarrow y = -x \Rightarrow \text{slope} = m_1 = -1$
$m_1 \cdot m_2 = -1 \Rightarrow -1 \cdot m_2 = -1 \Rightarrow m_2 = 1$
Let $(x_2, y_2) = (2, 0)$.

$y - y_2 = m_2(x - x_2) \Rightarrow y - 0 = x - 2 \Rightarrow$
$y = x - 2$

26. Perpendicular to $y = -5x + 1$; (1, 5) on line.
slope = $m_1 = -5$

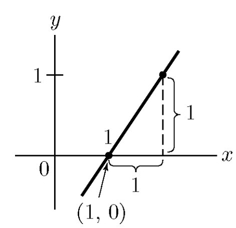
$$m_1 \cdot m_2 = -1 \Rightarrow -5m_2 = -1 \Rightarrow m_2 = \frac{1}{5}$$

Let $(x_2, y_2) = (1, 5)$.

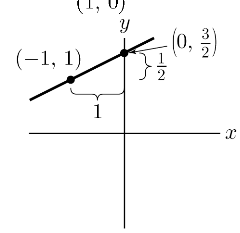
$$y - y_2 = m_2(x - x_2) \Rightarrow y - 5 = \frac{1}{5}(x - 1) \Rightarrow$$

$$y = \frac{1}{5}x + \frac{24}{5}$$

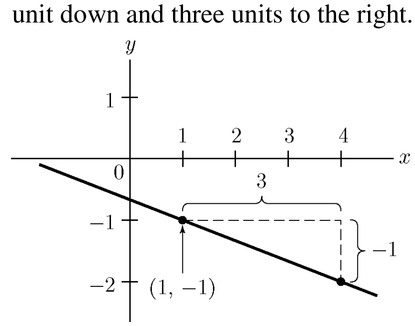
27. Start at (1, 0), then move one unit right and one unit up to (2, 1).



28. Start at (−1, 1), then move one unit up and two units to the right.



29. Start at (1, −1), then move one unit up and three units to the left. Alternatively, move one unit down and three units to the right.



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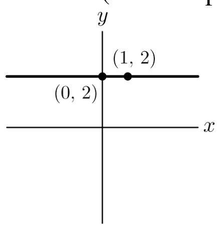
30. Start at (0, 2), then move zero units up and any distance (for example, one unit) right.



31. (a)–(C) x- and y-intercepts are 1.
(b)–(B) x-intercept is 1, y-intercept is –1.
(c)–(D) x- and y-intercepts are –1.
(d)–(A) x-intercept is –1, y-intercept is 1.

32. $x+2y=0 \Rightarrow y=-\frac{1}{2}x \Rightarrow m=-\frac{1}{2}$
The slope of the line through (–1, 2) and (3, b) is also $-\frac{1}{2}$..
$m=-\frac{1}{2}=\frac{b-2}{3-(-1)} \Rightarrow -4=2b-4 \Rightarrow b=0$

33. $m=\frac{1}{3}$, $h=3$
If you move 3 units in the x-direction, then you must move 1 unit in the y-direction to return to the line.

34. $m=2$, $h=\frac{1}{2}$
If you move $\frac{1}{2}$ unit in the x-direction, then you must move $\frac{1}{2}\cdot 2=1$ unit in the y-direction.

35. $m=-3$, $h=.25$
If you move .25 unit in the x-direction, then you must move $-3\cdot .25=-.75$ unit in the y-direction.

36. $m=\frac{2}{3}$, $h=\frac{1}{2}$
If you move $\frac{1}{2}$ unit in the x-direction, then you must move $\frac{1}{2}\cdot\frac{2}{3}=\frac{1}{3}$ unit in the y-direction.

37. Slope = 2, (1, 3) on line.
$x_1=1,\ y_1=3$
If $x = 2$, then $y-3=2(2-1) \Rightarrow y=5$.
If $x = 3$, then $y-3=2(3-1) \Rightarrow y=7$.
If $x = 0$, then $y-3=2(0-1) \Rightarrow y=1$.
The points are (2, 5), (3, 7), and (0, 1).

38. Slope = –3, (2, 2) on line.
$x_1=2,\ y_1=2$
If $x = 3$, then $y-2=-3(3-2) \Rightarrow y=-1$.
If $x = 4$, then $y-2=-3(4-2) \Rightarrow y=-4$.
If $x = 1$, then $y-2=-3(1-2) \Rightarrow y=5$.
The points are (3, –1), (4, –4), and (1, 5).

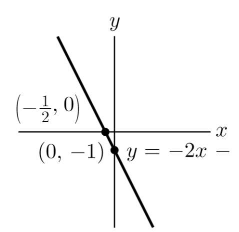
39. $f(1)=0 \Rightarrow (1,0)$ lies on the line.
$f(2)=1 \Rightarrow (2,1)$ lies on the line. Thus, the slope of the line is $\frac{1-0}{2-1}=1$. If $x = 3$ and $y=f(3)$, then $1=\frac{y-1}{3-2} \Rightarrow 1=y-1 \Rightarrow y=2$.
Thus $f(3)=2$.

40. First find the slope of $2x + 3y = 0$.
$2x+3y=0 \Rightarrow 3y=-2x \Rightarrow y=-\frac{2}{3}x \Rightarrow$
$m_1=-\frac{2}{3}$
Now find the slope of the line through (3, 4) and (–1, 2).
$m_2=\frac{y_2-y_1}{x_2-x_1}=\frac{2-4}{-1-3}=\frac{-2}{-4}=\frac{1}{2}$
Since the slopes are not equal, the lines are not parallel.

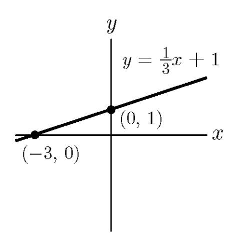
41. l_1

42. l_2

43. Slope = m = –2
y-intercept: (0, –1)
$y = mx + b$
$y = –2x – 1$



44. Slope $=m=\frac{1}{3}$
y-intercept: (0, 1)
$y = mx + b$
$y=\frac{1}{3}x+1$



45. a is the x-coordinate of the point of intersection of $y = –x + 4$ and $y = 2$. Use substitution to find the x-coordinate.
$2=-x+4 \Rightarrow x=2$
So $a = 2$. $f(a)$ is the y-coordinate of the intersection point. So $f(a) = 2$.

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46. a is the x-coordinate of the point of intersection of $y = x$ and $y = \frac{1}{2}x + 1$. Use substitution to find the x-coordinate.
$x = \frac{1}{2}x + 1 \Rightarrow \frac{1}{2}x = 1 \Rightarrow x = 2$
So $a = 2$. $f(a)$ is the y-coordinate of the intersection point. Substituting $x = 2$ into $y = x$ gives $y = 2$. So $f(a) = 2$.

47. $C(x) = 12x + 1100$

a. $C(10) = 12(10) + 1100 = \$1220$

b. The marginal cost is the slope of line. Marginal cost = m = \$12/unit

c. It would cost an additional \$12 to raise the daily production level from 10 units to 11 units.

48. $C(x+1) - C(x)$
$= (12(x+1) + 1100) - (12x + 1100)$
$= 12x + 12 + 1100 - 12x - 1100$
$= 12$
\$12 is the marginal cost. It is the additional cost incurred when the production level of this commodity is increased one unit, from x to $x + 1$, per day.

49. Let x be the number of months since January 1, 2012. Then (0, 4.12) is one point on the line. The slope is .06 since the price increased \$.06 per month. Therefore, $P(x) = .06x + 4.12$ gives the price of gasoline x months after January 1, 2012. On April 1, 2012, 3 months later, the cost of one gallon of gasoline is:
$P(3) = .06(3) + 4.12 = \$4.30$ /gallon. So 15 gallons cost $15 \cdot 4.30 = \$64.50$.
On September 1, 2012, 8 months after January 1, the cost of one gallon of gasoline is:
$P(8) = .06(8) + 4.12 = \$4.60$ /gallon. So, 15 gallons cost $15 \cdot 4.60 = \$69.00$.

50. Let y be the value of monthly exports in millions of dollars. Let x be the number of months since Sept 1, 2003. Since the rate of change of y is constant, we conclude that y is a linear function of x whose slope is equal to its rate of change, $m = 42.5$. On September 1 (when $x = 0$), the value of monthly exports was 0 dollars, since the ban had just ended. So the point (0, 0) is on the graph of y. Using the point-slope form, we have
$y - 0 = 42.5(x - 0) \Rightarrow y = 42.5x$.
The end of December 2003 corresponds to $x = 4$, at which time the exports had reached the value $y = 42.5 \cdot 4 = 170$ million dollars

51. Let x = the cost of order. Then $C(x) = .03x + 5$.

52. a. The points (6.55, .2) and (8, .18) are on the line. The slope of the line is
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{.18 - .2}{8 - 6.55} = -\frac{2}{145}$$
Let $(x_1, y_1) = (8, .18)$. Then, the equation of the line is
$$y - y_1 = m(x - x_1)$$
$$y - .18 = -\frac{2}{145}(x - 8)$$
$$y = -\frac{2}{145}x + \frac{421}{1450}$$
Thus, $Q(x) = -\frac{2}{145}x + \frac{421}{1450}$.

b. Let $Q(x) = .1$ (10 employees per 100) and solve for x.
$$.1 = -\frac{2}{145}x + \frac{421}{1450}$$
$$-\frac{138}{725} = -\frac{2}{145}x \Rightarrow x = 13.8$$
The hourly wage should be \$13.80 in order for the quit ratio to drop to 10 employees per 100.

53. The points (4.10, 1500) and (4.25, 1250) are on the line. The slope of the line is
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1500 - 1250}{4.10 - 4.25} = -\frac{5000}{3}$. Let $(x_1, y_1) = (4.10, 1500)$. The equation of the line is
$$y - y_1 = m(x - x_1)$$
$$y - 1500 = -\frac{5000}{3}(x - 4.10)$$
$$y = -\frac{5000}{3}x + \frac{25,000}{3}$$
$$G(x) = -\frac{5000}{3}x + \frac{25,000}{3}$$
Now find $G(4.35)$:
$$G(4.35) = -\frac{5000}{3}(4.35) + \frac{25,000}{3} \approx 1083.3$$
gallons.

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54. Solve for x:

$$G(x) = 2200 = -\frac{5000}{3}x + \frac{25,000}{3}$$
$$x = -\frac{18,400}{3}\left(-\frac{3}{5000}\right) = 3.68$$

The owner should set the price at \$3.68 in order to sell 2200 gallons per day.

55. **a.** $C(x) = mx + b$

$b = \$1500$ (fixed costs)

Total cost of producing 100 rods is \$2200.

$C(100) = m(100) + 1500 = \$2200 \Rightarrow m = 7$

Thus, $C(x) = 7x + 1500.$

b. Marginal cost at $x = 100$ is $m = \$7$/rod

c. Since the marginal cost = \$7, the cost of raising the daily production level form 100 to 101 rods is \$7. Alternatively, $C(101) - C(100) = 2207 - 2200 = \$7.$

56. Each unit sold increases the pay by 5 dollars. Thus, the slope is her pay per unit sold. The weekly pay is 60 dollars if no units are sold. Thus, the y-intercept is her base pay.

57. If the monopolist wants to sell one more unit of goods, then the price per unit must be lowered by 2 cents. No one will pay 7 dollars or more for a unit of goods.

58. x = degrees Fahrenheit, y = degrees Celsius, so the points (32, 0) and (212, 100) lie on the line.

$$m = \frac{100-0}{212-32} = \frac{100}{180} = \frac{5}{9}$$

Now find b:

$$y = mx + b \Rightarrow 32 = \frac{5}{9}(0) + b \Rightarrow b = -\frac{160}{9}.$$

Thus, $y = \frac{9}{5}x + 32.$ $y = \frac{5}{9}(98.6) - \frac{160}{9} = 37$

98.6°F corresponds to 37°C.

59. The point (0, 1.5) is on the line and the slope is 6 (ml/min). Let y be the amount of drug in the body x minutes from the start of the infusion. Then $y - 1.5 = 6(x-0) \Rightarrow y = 6x + 1.5.$

60. Eliminating 2 ml/hour means that the rate is $-\frac{1}{30}$ ml/min. (The rate given in exercise 59 is in ml/min.) $y = 6x + 1.5 - \frac{1}{30}x = \frac{179}{30}x + 1.5$

61. The diver starts at a depth of 212 ft, which is represented as −212. Thus, the function is $y(t) = 2t - 212.$

62. First we must determine how long it will take the diver to reach 150 feet depth.

$-150 = 2t - 212 \Rightarrow 62 = 2t \Rightarrow t = 31$ sec

The diver must then rest for 5 minutes or $5 \cdot 60 = 300$ sec, which is 331 sec after she started ascending. The remaining depth can be determined by $y = 2(t-331) - 150 = 2t - 812.$

Thus, the function giving depth as a function of time is

$$y(t) = \begin{cases} 2t - 212 & 0 \le t \le 31 \\ -150 & 31 \le t \le 331 \\ 2t - 812 & t \ge 331 \end{cases}$$

The first 62 ft will take the diver 31 sec to ascend. To determine how long it will take the diver to ascend final 150 ft, solve $150 = 2t \Rightarrow t = 75$ sec. Therefore, it will take the diver 31 + 300 + 75 = 406 sec to reach the surface.

63. **a.** $C(x) = 7x + 230$

b. $R(x) = 12x$

64. $C(x) = R(x) \Rightarrow$

$7x + 230 = 12x \Rightarrow 230 = 5x \Rightarrow x = 46$

The business will break even when 46 t-shirts are sold.

65. Let $y = mx + b$ and $y = m'x + b'$ be two distinct lines. We show that these lines are parallel if and only if $m = m'$. Since two lines are parallel if and only if they have no points in common, it suffices to show that $m = m'$ if and only if the equation $mx + b = m'x + b'$. Suppose $m = m'$. Then $mx + b = m'x + b'$ implies $b = b'$; but since the lines are distinct, $b \ne b'$. Thus if $m = m'$, $mx + b = m'x + b'$ has no solution. If $m \ne m'$, then $x = \frac{b'-b}{m-m'}$ is a solution to $mx + b = m'x + b'$. Thus, $mx + b = m'x + b'$ has no solution in x if and only if $m = m'$, and it follows that two distinct lines are parallel if and only if they have the same slope.

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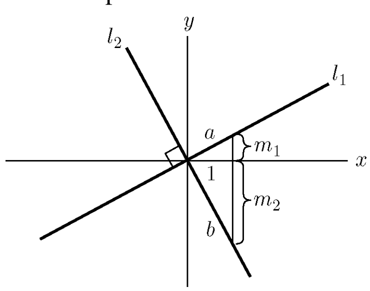
66. Let $\ell_1, \ell_2, m_1, m_2, a$, and b be as in the diagram in the text. Then m_1 is the slope of ℓ_1 and m_2 is the slope of ℓ_2. Using the Pythagorean theorem, we have $m_1^2 + 1^2 = a^2$ and $m_2^2 + 1^2 = b^2$. Since m_2 is negative, the length of the hypotenuse in the large right triangle is $(m_1 - m_2)$. Then,
$(m_1 - m_2)^2 = a^2 + b^2 \Rightarrow$
$m_1^2 - 2m_1m_2 + m_2^2 = a^2 + b^2.$
Substituting, we obtain
$m_1^2 - 2m_1m_2 + m_2^2 = m_1^2 + m_2^2 + 2 \Rightarrow$
$-2m_1m_2 = 2 \Rightarrow m_1m_2 = -1.$ Thus, the product of the slopes is –1.



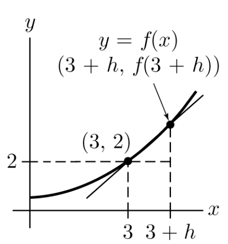
67. Using $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = m$ and the hint,

$$\frac{f(x) - f(x_1)}{x - x_1} = m \Rightarrow f(x) - f(x_1) = m(x - x_1)$$

$$\begin{aligned} f(x) &= m(x - x_1) + f(x_1) \\ &= mx + (-mx_1 + f(x_1)) \end{aligned}$$

Let $b = -mx_1 + f(x_1)$. Then $f(x) = mx + b$.

68. a–c.



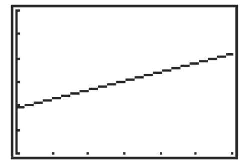
d. $\dfrac{f(3+h) - f(3)}{3 + h - 3} = \dfrac{f(3+h) - 2}{h}$

69. a. (0, 39.5) and (15, 45.2) on line

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{45.2 - 39.5}{15 - 0} = \frac{5.7}{15} = .38$$

$$y - 39.5 = .38(x - 0) \Rightarrow y = .38x + 39.5$$

b.



[0, 30] *by* [30, 60]

c. Every year, .38% more of the world population becomes urban.

d. Using the **TRACE** or **VALUE** feature on a graphing calculator, the point (10, 43.3) is on the line. Thus, 43.3% of the world population was urban in 1990.

e. Graphing the line $y = 50$ and using the **INTERSECT** command, the point (27.63, 50) is on both graphs. In the year 1980 + 27 = 2007, 50% of the world population will be urban.

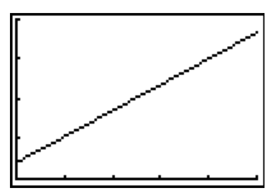
f. From the slope, .38% more of the world population becomes urban each year. Thus, in 5 years the percentage of the world population that is urban has increased by 5(.38%) = 1.9%.

70. a. (20,000, 729) and (50,000, 1380) on line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1380 - 729}{50{,}000 - 20{,}000} = \frac{651}{30{,}000} = .0217$$

$$y - 1380 = .0217(x - 50{,}000) \Rightarrow$$
$$y = .0217x + 295$$

b.



[0, 75000] by [0, 2000]

c. For every increase of \$1 in reported income, the average itemized deductions increase by \$.0217. (Alternatively, an increase of \$100 in reported income corresponds to an average increase of \$2.17 in itemized deductions.)

d. Using the **TRACE** or **VALUE** feature on a graphing calculator, the point (75,000, 1992.5) is on the line. Thus, the average amount of itemized deductions on a return reporting income of \$75,000 is \$1922.50.

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e. Graphing the line $y = 5000$ and using the **INTERSECT** command, the point ($60,138.25, 1600) is on both lines. An average itemized deduction of $1600 corresponds to a reported income of $60,138.25.

f. An increase of $15,000 in income level will correspond to an increase of $15,000(.0217) = $325.50 in itemized deductions.

1.2 The Slope of a Curve at a Point

1. $-\frac{4}{3}$ **2.** 0

3. 1 **4.** 1

5. Small positive slope; large positive slope

6. Zero slope; large negative slope

7. Zero slope; small negative slope

8. $m_A = 1,\ m_B = 8,\ m_C = 0,\ m_D = -6,\ m_E = 0,\ m_F = -\frac{1}{2}$

For 9–20, note that the slope of the line tangent to the graph of $y = x^2$ at the point (x, y) is $2x$.

9. The slope at (–.5, .16) is 2(–.5) = –1. Let $(x_1,\ y_1) = (-.5,\ .16),\ m = -1.$
$y - .16 = -(x - (-.5)) \Rightarrow y - .16 = -(x + .5) \Rightarrow$
$y = -x - .34$

10. The slope at (–2, 4) is $2x = 2(-2) = -4$. Let $(x_1,\ y_1) = (-2, 4),\ m = -4$.
$y - 4 = -4(x - (-2)) \Rightarrow y - 4 = -4(x + 2) \Rightarrow$
$y = -4x - 4$

11. The slope at $\left(\frac{1}{3}, \frac{1}{9}\right)$ is $m = 2x = 2\left(\frac{1}{3}\right) = \frac{2}{3}.$
Let $(x_1,\ y_1) = \left(\frac{1}{3}, \frac{1}{9}\right).$
$y - \frac{1}{9} = \frac{2}{3}\left(x - \frac{1}{3}\right) \Rightarrow y - \frac{1}{9} = \frac{2}{3}x - \frac{2}{9} \Rightarrow$
$y = \frac{2}{3}x - \frac{1}{9}$

12. The slope at (–1.5, 2.25) is $2x = 2(-1.5) = -3$. Let $(x_1,\ y_1) = (-1.5,\ 2.25),\ m = -3$.
$y - 2.25 = -3(x - (-1.5)) \Rightarrow$
$y - 2.25 = -3(x + 1.5) \Rightarrow y = -3x - 2.25$

13. When $x = -\frac{1}{4}$, slope $= 2\left(-\frac{1}{4}\right) = -\frac{1}{2}.$

14. When $x = -.2$, slope $= 2(-.2) = -.4$.

15. When $x = 2.5$, slope $= 2(2.5) = 5$ and $y = (2.5)^2 = 6.25$. Let $(x_1,\ y_1) = (2.5,\ 6.25)$, $m = 5$.
$y - 6.25 = 5(x - 2.5) \Rightarrow y - 6.25 = 5x - 12.5 \Rightarrow$
$y = 5x - 6.25$

16. When $x = 2.1$, slope $= 2(2.1) = 4.2$ and $y = (2.1)^2 = 4.41$. Let $(x_1,\ y_1) = (2.1,\ 4.41)$, $m = 4.2$.
$y - 4.41 = 4.2(x - 2.1) \Rightarrow$
$y - 4.41 = 4.2x - 8.82 \Rightarrow y = 4.2x - 4.41$

17. The slope of the tangent is $2x$, so solve $2x = \frac{7}{2} \Rightarrow x = \frac{7}{4}$. The point is $\left(\frac{7}{4}, \left(\frac{7}{4}\right)^2\right) = \left(\frac{7}{4}, \frac{49}{16}\right).$

18. The slope of the tangent is $2x$, so solve $2x = -6 \Rightarrow x = -3$. The point is $\left(-3, (-3)^2\right) = (-3, 9).$

19. The slope of the line $2x + 3y = 4$ is $-\frac{2}{3}$, so the slope of the tangent line is also $-\frac{2}{3}$. Now solve $2x = -\frac{2}{3} \Rightarrow x = -\frac{1}{3}$. The point is $\left(-\frac{1}{3}, \left(-\frac{1}{3}\right)^2\right) = \left(-\frac{1}{3}, \frac{1}{9}\right).$

20. The slope of the line $3x - 2y = 2$ is $\frac{3}{2}$, so the slope of the tangent line is also $\frac{3}{2}$. Now solve $2x = \frac{3}{2} \Rightarrow x = \frac{3}{4}$. The point is $\left(\frac{3}{4}, \left(\frac{3}{4}\right)^2\right) = \left(\frac{3}{4}, \frac{9}{16}\right).$

21. June 1, 2007: about $68.00
December 1, 2007: about $88.00
The price increased about $20.00
The price was rising on both days.

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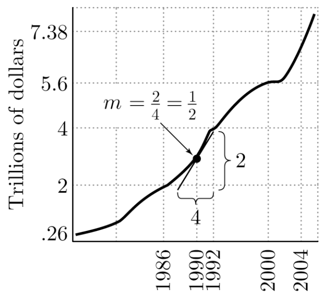
22. No, the price decrease was greater than the price increase.

23. The price of a barrel of oil was \$104.50. It was rising at a rate of about
$$\frac{107-104.50}{2} = \$1.25 \text{ per day.}$$

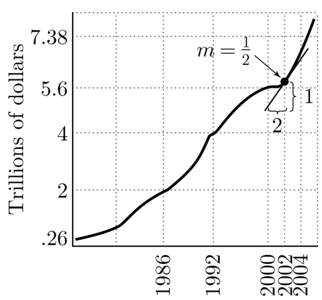
24. The price of a barrel of oil was about \$111. It was rising at a rate of about
$$\frac{111-110.40}{1} \approx \$.60 \text{ per day.}$$

25.



It appears the points (1988, 2) and (1990, 3) are on the tangent line. The slope is
$$m = \frac{3-2}{1990-1988} = .5$$
Therefore, the annual rate of increase of the federal debt in 1990 is approximately \$.5 trillion/year.

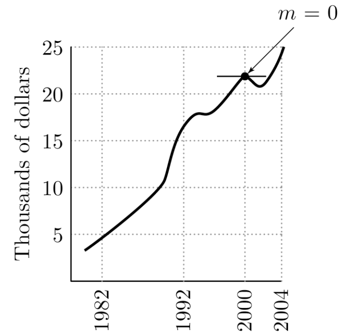
26.



It appears the points (2000, 4.8) and (2002, 5.8) are on the line. The slope is
$m = \dfrac{5.8-4.8}{2002-2000} = \dfrac{1}{2}$. Therefore, the annual rate of increase of the federal debt in 2002 is approximately \$.5 trillion/year.

27. a. In 1950, the debt per capita was approximately \$1000.
In 1990, the debt per capita was approximately \$15,000.
In 2000,the debt per capita was approximately \$21,000.
In 2004, the debt per capita was approximately \$24,000.

b.



The tangent at $x = 2000$ appears to be horizontal, so the slope is 0. Thus, in 2000, the debt per capita held steady at about \$21,000.

28. a. True, the rate of increase in 1980 (the slope of the tangent at $x = 1980$) is greater than 0 while the rate of increase in 2000 is about 0.

b. True, the curve is close to constant up to the mid-1970's and then increases linearly (at a constant rate) from the mid-1970's to the mid- 1980's.

For 29–31, note that the slope of the line tangent to the graph of $y = x^3$ at the point (x, y) is $3x^2$.

29. Slope $= 3x^2$

When $x = 2$, slope $= 3(2)^2 = 12$.

30. Slope $= 3x^2$

When $x = \frac{3}{2}$, slope $= 3\left(\frac{3}{2}\right)^2 = \frac{27}{4}$.

31. Slope $= 3x^2$

When $x = -\frac{1}{2}$, slope $= 3\left(-\frac{1}{2}\right)^2 = \frac{3}{4}$.

32. When $x = -1$, slope $= 3(-1)^2 = 3$.

$y = (-1)^3 = -1$.

Let $(x_1, y_1) = (-1, -1)$.

$y-(-1) = 3\big(x-(-1)\big) \Rightarrow y+1 = 3(x+1) \Rightarrow$
$y = 3x+2$

33. The slope of the line tangent to $y = x^2$ at $x = a$ is $2a$. The slope of $y = 2x - 1$ is 2. Equating these gives $2a = 2 \Rightarrow a = 1$.

So, $f(a) = (1)^2 = 1$, $f'(1) = 2(1) = 2$

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34. The slope of the line tangent to $y = x^2$ at $x = a$ is $2a$. The slope of $y = -x - \frac{1}{4}$ is –1.

Equating these gives $2a = -1 \Rightarrow a = -\frac{1}{2}$.

So, $f(a) = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$, and

$m = f'\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) = -1$

35. The slope of the curve $y = x^3$ at any point is $3x^2$. Solve $3x^2 = \frac{3}{2} \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$.

$x = \frac{1}{\sqrt{2}} \Rightarrow y = \left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{2\sqrt{2}}$.

$x = -\frac{1}{\sqrt{2}} \Rightarrow y = \left(-\frac{1}{\sqrt{2}}\right)^3 = -\frac{1}{2\sqrt{2}}$. The points are $\left(\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{2\sqrt{2}}\right)$.

36. The slope of $y = 2x$ is 2. Solve

$3x^2 = 2 \Rightarrow x^2 = \frac{2}{3} \Rightarrow x = \pm\sqrt{\frac{2}{3}}$.

$x = \sqrt{\frac{2}{3}} \Rightarrow y = \left(\sqrt{\frac{2}{3}}\right)^3 = \frac{2}{3}\sqrt{\frac{2}{3}}$.

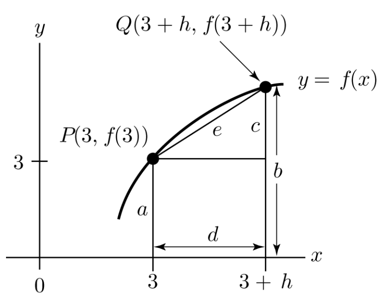
$x = -\sqrt{\frac{2}{3}} \Rightarrow y = \left(-\sqrt{\frac{2}{3}}\right)^3 = -\frac{2}{3}\sqrt{\frac{2}{3}}$. The points are $\left(\sqrt{\frac{2}{3}}, \frac{2}{3}\sqrt{\frac{2}{3}}\right)$ and $\left(-\sqrt{\frac{2}{3}}, -\frac{2}{3}\sqrt{\frac{2}{3}}\right)$.

37. a. $m = \frac{13-4}{5-2} = 3$

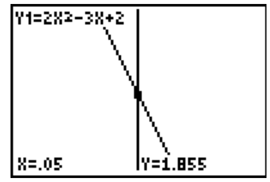
length of d is 13 – 4 = 9

b. The slope of line l increases.

38.



39.



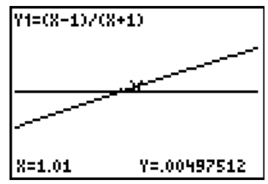
[–.078125, .078125] by [1.927923, 2.084173]

When $x = 0$, $y = 2$. Find a second point on the line using **VALUE**: $x = .05$, $y = 1.855$

$m = \frac{1.855-2}{.05-0} = -2.9$

The actual value of m is −3.

40.



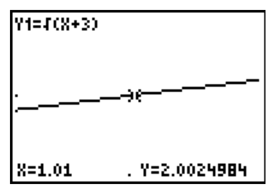
[.93251, 1.08876] by [–.078125, .078125]

When $x = 1$, $y = 0$. Find a second point on the line using **VALUE**: $x = .95$, $y = -.025641$

$m = \frac{.00497512-0}{1.01-1} \approx .5$

The actual value of m is $\frac{1}{2}$.

41.



[.6463, 1.3963] by [1.5, 2.5]

When $x = 1$, $y = 2$

Find a second point on the line using **VALUE**: $x = 1.01$, $y = 2.0024984$

$m = \frac{2.0024984-2}{1.01-1} \approx .25$

The actual value of m is $\frac{1}{2\sqrt{3+1}} = .25$.

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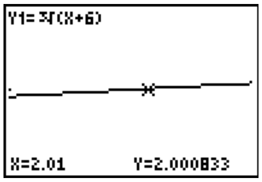
42.



[1.2899, 2.5399] by [1.3679, 2.6179]

When $x = 2$, y=2.
Find a second point on the line using **value**:
$x = 2.01$, $y \approx 2.000833$

$$m = \frac{2.000833 - 2}{2.01 - 2} = .0833$$

The actual value of m is $\frac{1}{12}$.

1.3 The Derivative and Limits

For exercises 1– 16, refer to equations (1) and (2) section 1.3 in the text along with the Power Rule $f'(x) = rx^{r-1}$ for $f(x) = x^r$.

1. $f(x) = 3x + 7,\ f'(x) = 3$

2. $f(x) = -2x,\ f'(x) = -2$

3. $f(x) = \frac{3x}{4} - 2,\ f'(x) = \frac{3}{4}$

4. $f(x) = \frac{2x - 6}{7} = \frac{2x}{7} - \frac{6}{7},\ f'(x) = \frac{2}{7}$

5. $f(x) = x^7,\ f'(x) = 7x^6$

6. $f(x) = x^{-2},\ f'(x) = -2x^{-3} = -\frac{2}{x^3}$

7. $f(x) = x^{2/3},\ f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

8. $f(x) = x^{-1/2},\ f'(x) = -\frac{1}{2}x^{-3/2} = -\frac{1}{2\sqrt{x^3}}$

9. $f(x) = \frac{1}{\sqrt{x^5}} = \frac{1}{x^{5/2}} = x^{-5/2},$
$f'(x) = -\frac{5}{2}x^{-7/2} = -\frac{5}{2x^{7/2}}$

10. $f(x) = \frac{1}{x^3} = x^{-3},\ f'(x) = -3x^{-4} = -\frac{3}{x^4}$

11. $f(x) = \sqrt[3]{x} = x^{1/3},\ f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$

12. $f(x) = \frac{1}{\sqrt[5]{x}} = x^{-1/5},$
$f'(x) = -\frac{1}{5}x^{-6/5} = -\frac{1}{5x^{6/5}}$

13. $f(x) = \frac{1}{x^{-2}} = x^2,\ f'(x) = 2x$

14. $f(x) = \sqrt[7]{x^2} = x^{2/7},\ f'(x) = \frac{2}{7}x^{-5/7} = \frac{2}{7x^{5/7}}$

15. $f(x) = 4^2 = 16,\ f'(x) = 0$

16. $f(x) = \pi,\ f'(x) = 0$

In exercises 17–24, first find the derivative of the function, then evaluate the derivative for the given value of x.

17. $f(x) = x^3$ at $x = \frac{1}{2}$
$f'(x) = 3x^2$
$f'\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 = \frac{3}{4}$

18. $f(x) = x^5$ at $x = \frac{3}{2}$
$f'(x) = 5x^4$
$f'\left(\frac{3}{2}\right) = 5\left(\frac{3}{2}\right)^4 = \frac{405}{16} = 25.3125$

19. $f(x) = \frac{1}{x}$ at $x = \frac{2}{3}$
$f(x) = x^{-1};\ f'(x) = -x^{-2} = -\frac{1}{x^2}$
$f'\left(\frac{2}{3}\right) = -\frac{1}{(2/3)^2} = -\frac{1}{4/9} = -\frac{9}{4}$

20. $f(x) = \frac{1}{3}$ at $x = 2$
$f'(x) = 0 \Rightarrow f'(2) = 0$

21. $f(x) = x + 11$ at $x = 0$
$f'(x) = 1 \Rightarrow f'(0) = 1$

22. $f(x) = x^{1/3}$ at $x = 8$
$f'(x) = \frac{1}{3}x^{-2/3}$
$f'(4) = \frac{1}{3}(8)^{-2/3} = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

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23. $f(x)=\sqrt{x}$ at $x=\frac{1}{16}$

$f(x)=x^{1/2}$

$f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

$f'(16)=\frac{1}{2\sqrt{\frac{1}{16}}}=\frac{1}{2\cdot\frac{1}{4}}=2$

24. $f(x)=\frac{1}{\sqrt[5]{x^2}}$ at $x=32$

$f(x)=x^{-2/5}$

$f'(x)=-\frac{2}{5}x^{-7/5}$

$f'(32)=-\frac{2}{5}(32)^{-7/5}=-\frac{2}{5}\cdot\frac{1}{128}=-\frac{1}{320}$

In exercises 25 and 26, remember that the slope of a curve at a given point is the value of the derivative evaluated at that point.

25. $y=x^4$

slope $=y'=4x^3$

at $x=2$, $y'=4(2)^3=32$

26. $y=x^5$

slope $=y'=5x^4$

at $x=\frac{1}{3}$, $y'=5\left(\frac{1}{3}\right)^4=\frac{5}{81}$

27. $f(x)=x^3$; $f(-5)=(-5)^3=-125$

$f'(x)=3x^2$

$f'(-5)=3(-5)^2=75$

28. $f(x)=2x+6$

$f(0)=2(0)+6=6$

$f'(x)=2\Rightarrow f'(0)=2$

29. $f(x)=x^{1/3}$; $f(8)=8^{1/3}=2$

$f'(x)=\frac{1}{3}x^{-2/3}$

$f'(8)=\frac{1}{3}\cdot 8^{-2/3}=\frac{1}{3}\cdot\frac{1}{4}=\frac{1}{12}$

30. $f(x)=\frac{1}{x^2}=x^{-2}$; $f(1)=\frac{1}{1^2}=1$

$f'(x)=-2x^{-3}=-\frac{2}{x^3}$

$f'(1)=-\frac{2}{1^3}=-2$

31. $f(x)=\frac{1}{x^5}=x^{-5}$

$f(-2)=\frac{1}{(-2)^5}=-\frac{1}{32}$

$f'(x)=-5x^{-6}=-\frac{5}{x^6}$

$f'(-2)=-\frac{5}{(-2)^6}=-\frac{5}{64}$

32. $f(x)=x^{3/2}$

$f(16)=(16)^{3/2}=64$

$f'(x)=\frac{3}{2}x^{1/2}=\frac{3}{2}\sqrt{x}$

$f'(16)=\frac{3}{2}\sqrt{16}=\frac{12}{2}=6$

For exercises 33–40, refer to Example 4 on page 77 in the text.

33. $f(x)=x^3\Rightarrow f'(x)=3x^2$

When $x=-2$, $f(x)=(-2)^3=-8$. The slope of the tangent at $x=-2$ is $f'(-2)=3(-2)^2=12$. Thus, the equation of the tangent at $(-2, -8)$ in point-slope form is $y+8=12(x+2)$.

34. $f(x)=x^2\Rightarrow f'(x)=2x$

When $x=-\frac{1}{2}$, $f(x)=\left(-\frac{1}{2}\right)^2=\frac{1}{4}$. The slope of the tangent at $x=-\frac{1}{2}$ is $f'\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)=-1$. Thus, the equation of the tangent at $\left(-\frac{1}{2},\frac{1}{4}\right)$ in point-slope form is $y-\frac{1}{4}=-\left(x+\frac{1}{2}\right)$.

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35. $f(x)=3x+1 \Rightarrow f'(x)=3$
When $x=4,\ f(x)=3\cdot 4+1=13.$ The slope of the tangent at $x=4$ is $f'(4)=3.$ Thus, the equation of the tangent at $(4,13)$ in point-slope form is $y-13=3(x-4)$ or $y = 3x + 1$ in slope-intercept form.

36. $f(x)=5 \Rightarrow f'(x)=0$
When $x=-2,\ f(x)=5.$ The slope of the tangent at $x=-2$ is $f'(-2)=0.$ Thus, the equation of the tangent at $(4,13)$ in point-slope form is $y-5=0(x+5)$ or $y = 5$.

37. $f(x)=\sqrt{x}=x^{1/2} \Rightarrow f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$
When $x=\frac{1}{9},\ f(x)=\sqrt{\frac{1}{9}}=\frac{1}{3}.$ The slope of the tangent at $x=\frac{1}{9}$ is
$f'\left(\frac{1}{9}\right)=\frac{1}{2}\left(\frac{1}{9}\right)^{-1/2}=\frac{1}{2}(9)^{1/2}=\frac{3}{2}.$ Thus, the equation of the tangent at $\left(\frac{1}{9},\frac{1}{3}\right)$ in point-slope form is $y-\frac{1}{3}=\frac{3}{2}\left(x-\frac{1}{9}\right).$

38. $f(x)=\frac{1}{x}=x^{-1} \Rightarrow f'(x)=-x^{-2}$
When $x=.01,\ f(x)=\frac{1}{.01}=100.$ The slope of the tangent at $x=.01$ is
$f'(.01)=-(.01)^{-2}=-10{,}000.$ Thus, the equation of the tangent at $(.01,\ 100)$ in point-slope form is $y-100=-10{,}000(x-.01).$

39. $f(x)=\frac{1}{\sqrt{x}}=x^{-1/2} \Rightarrow f'(x)=-\frac{1}{2}x^{-3/2}$
When $x=1,\ f(x)=\frac{1}{\sqrt{1}}=1.$ The slope of the tangent at $x=1$ is $f'(1)=-\frac{1}{2}(1)^{-3/2}=-\frac{1}{2}.$
Thus, the equation of the tangent at $(1,1)$ in point-slope form is $y-1=-\frac{1}{2}(x-1).$

40. $f(x)=\frac{1}{x^3}=x^{-3} \Rightarrow f'(x)=-3x^{-4}$
When $x=3,\ f(x)=\frac{1}{3^3}=\frac{1}{27}.$ The slope of the tangent at $x=3$ is
$f'(3)=-3(3)^{-4}=-\frac{3}{81}=-\frac{1}{27}.$
Thus, the equation of the tangent at $\left(3,\ \frac{1}{27}\right)$ in point-slope form is $y-\frac{1}{27}=-\frac{1}{27}(x-3).$

41. Formula 6 states that $y - f(a) = f'(a)(x-a).$
$y = f(x) = x^4 \Rightarrow y' = f'(x) = 4x^3$
For $a=1,\ f(a) = f(1) = 1$ and
$f'(a) = f'(1) = 4.$ Thus, the equation of the tangent at $(1,1)$ in point-slope form is
$y-1 = 4(x-1).$

42. The tangent is perpendicular to $y = 4x+1$, so the slope of the tangent is $m = -\frac{1}{4}$ because the slopes of perpendicular lines is -1.
$f(x)=\frac{1}{x}=x^{-1} \Rightarrow f'(x)=-x^{-2}.$
The slope of the tangent at $x = a$ is
$f'(a)=-a^{-2}$. $-a^{-2}=-\frac{1}{4} \Rightarrow a=\pm 2.$
Therefore, $P=\left(2,\frac{1}{2}\right)$ or $P=\left(-2,-\frac{1}{2}\right).$

43. The slope of the tangent is $m = 2$.
$f(x)=\sqrt{x}=x^{1/2} \Rightarrow f'(x)=\frac{1}{2}x^{-1/2}.$ The slope of the tangent at $x = a$ is
$f'(a)=\frac{1}{2}a^{-1/2}.$ Therefore $P=\left(\frac{1}{16},\frac{1}{4}\right).$
$\frac{1}{4}=2\left(\frac{1}{16}\right)+b \Rightarrow b=\frac{1}{8}.$

44. The slope of the tangent is $m = a$.
$f(x)=x^3 \Rightarrow f'(x)=3x^2$. The slope of the tangent at $(-3, -27)$ is $f'(-3)=3(-3)^2=27$.
Therefore $a = 27$.
$-27=27(-3)+b \Rightarrow b=54$.

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45. **a.** The slope of the tangent line is $m = \frac{1}{8}$ because the slopes of parallel lines are equal.

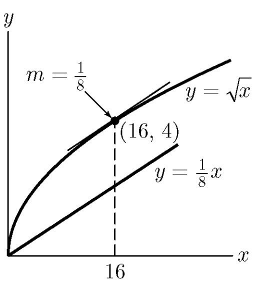
$f(x) = \sqrt{x} = x^{1/2} \Rightarrow f'(x) = \frac{1}{2}x^{-1/2}$. The slope of the tangent is at $x = a$ is

$f'(a) = \frac{1}{2}a^{-1/2}$.

$\frac{1}{2}a^{-1/2} = \frac{1}{8} \Rightarrow a = 16.\ \ f(16) = \sqrt{16} = 4$.

Therefore, the point we are looking for is $(16, 4)$.

b.



46. The slope of the tangent lines is $m = 1$ because the slopes of parallel lines are equal.

$y = f(x) = x^3 \Rightarrow f'(x) = 3x^2$. The slope of the tangent at $x = a$ is $f'(a) = 3a^2$. Thus,

$3a^2 = 1 \Rightarrow a^2 = \frac{1}{3} \Rightarrow a = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$. Now find the values of f at $x = \pm\frac{\sqrt{3}}{3}$:

$$f\left(\frac{\sqrt{3}}{3}\right) = \left(\frac{\sqrt{3}}{3}\right)^3 = \frac{3\sqrt{3}}{27} = \frac{\sqrt{3}}{9}$$

$$f\left(-\frac{\sqrt{3}}{3}\right) = \left(-\frac{\sqrt{3}}{3}\right)^3 = -\frac{3\sqrt{3}}{27} = -\frac{\sqrt{3}}{9}$$

Thus, the two points are $\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{9}\right)$ and $\left(-\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{9}\right)$.

47. A tangent line perpendicular to $y = x$ has slope -1. $f(x) = x^3 \Rightarrow f'(x) = 3x^2$. The slope of the tangent at $x = a$ is $f'(a) = 3a^2$. Thus, $3a^2 = -1$, which has no real solution. Therefore, there is no point on $y = x^3$ where the tangent line is perpendicular to $y = x$.

48. Since the graph passes through the point (2, 3), $f(2) = 3$. The slope of a tangent line at (a, b) equals the value of the derivative for $x = a$, so $y = -2x + 7 \Rightarrow m = -2,\ f'(2) = -2$.

49. $\frac{d}{dx}\left(x^8\right) = 8x^7$

50. $\frac{d}{dx}\left(x^{-3}\right) = -3x^{-4}$

51. $\frac{d}{dx}\left(x^{3/4}\right) = \frac{3}{4}x^{-1/4}$

52. $\frac{d}{dx}\left(x^{-1/3}\right) = -\frac{1}{3}x^{-4/3}$

53. $y = 1 \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(1) = 0$

54. $y = x^{-4} \Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(x^{-4}\right) = -4x^{-5}$

55. $y = x^{1/5},\ \frac{d}{dx}(x^{1/5}) = \frac{1}{5}x^{-4/5}$

56. $y = \frac{x-1}{3} = \frac{1}{3}x - \frac{1}{3} \Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\frac{x-1}{3}\right] = \frac{1}{3}$

57. The tangent line at $x = 6$ is $y = \frac{1}{3}x + 2$, so

$f(6) = \frac{1}{3}(6) + 2 = 4$.

The slope of $y = \frac{1}{3}x + 2$ is $\frac{1}{3}$, so $f'(6) = \frac{1}{3}$.

58. The tangent line at $x = 1$ is $y = 4$, so $f(1) = 4$. The slope of $y = 4$ is 0, so $f'(1) = 0$.

59. $y = f(x) = \sqrt{x} = x^{1/2}$

The slope of the tangent line at $x = a$ is

$f'(a) = \frac{1}{2}a^{-1/2} = \frac{1}{2\sqrt{a}}$.

The slope of the tangent line $y = \frac{1}{4}x + b$ is $\frac{1}{4}$. First, find the value of a. Let $\frac{1}{4} = \frac{1}{2\sqrt{a}}$ and solve for a: $2\sqrt{a} = 4 \Rightarrow \sqrt{a} = 2 \Rightarrow a = 4$

When $x = 4$, $f(4) = \sqrt{4} = 2$.

Let $(x_1, y_1) = (4, 2)$. Then,

$y - 2 = \frac{1}{4}(x - 4) \Rightarrow\ y = \frac{1}{4}x - 1 + 2 \Rightarrow$

$y = \frac{1}{4}x + 1$, so $b = 1$.

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60. $y = \frac{1}{x}$

When $x = 2,\ y = \frac{1}{2}.$

The slope of the tangent line at $x = 2$ is

$f'(2) = -2^{-2} = -\frac{1}{4}.$

To find the equation of the tangent line, let

$(x_1,\ y_1) = \left(2,\ \frac{1}{2}\right)$, and $m = -\frac{1}{4}.$

$y - \frac{1}{2} = -\frac{1}{4}(x-2) \Rightarrow\ y = -\frac{1}{4}x + 1.$

To find the value of a (which is the x-intercept), let $-\frac{1}{4}a + 1 = 0$ and solve for a.

$-\frac{1}{4}a + 1 = 0 \Rightarrow a = 4.$

61. At $x = a$, $y = 2.01a - .51$ or $y = 2.02a - .52$, so $.01a = .01$, and $a = 1$

$y = f(1) = 2.01 - .51 = 1.5.$

$f'(a) = 2$ because the slope of the "smallest" secant line is 2.01.

62. $f'(1) \approx$ the slope of the secant through (1.2, 1.1) and (1, .8).

$f'(1) \approx \frac{1.1 - .8}{1.2 - 1} = \frac{.3}{.2} = 1.5$

63. The coordinates of A are (4, 5). From the graph of the derivative, we see that

$f'(4) = \frac{1}{2}$, so the slope of the tangent line is

$\frac{1}{2}$. By the point-slope formula, the equation of

the tangent line is $y - 5 = \frac{1}{2}(x-4).$

64. The coordinates of P are (2, 1.75). From the graph of the derivative, we see that $f'(2) = .5$, so the slope of the tangent line is

$\frac{1}{2}$. By the point-slope formula, the equation of

the tangent line is $y - 1.75 = .5(x-2).$

65. $f(x) = 2x^2$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{2(x+h)^2 - 2x^2}{h}\\ &= \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h}\\ &= \frac{h(4x+h)}{h} = 4x + h\end{aligned}$$

66. $f(x) = x^2 - 7$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\left[(x+h)^2 - 7\right] - \left(x^2 - 7\right)}{h}\\ &= \frac{x^2 + 2xh + h^2 - 7 - x^2 + 7}{h}\\ &= \frac{h(2x+h)}{h} = 2x + h\end{aligned}$$

67. $f(x) = -x^2 + 2x$

$$\begin{aligned}&\frac{f(x+h)-f(x)}{h}\\ &= \frac{\left[-(x+h)^2 + 2(x+h)\right] - \left(-x^2 + 2x\right)}{h}\\ &= \frac{-x^2 - 2xh - h^2 + 2x + 2h + x^2 - 2x}{h}\\ &= \frac{h(-2x + 2 - h)}{h} = -2x + 2 - h\end{aligned}$$

68. $f(x) = -2x^2 + x + 3$

$$\begin{aligned}&\frac{f(x+h)-f(x)}{h}\\ &= \frac{\left[-2(x+h)^2 + (x+h) + 3\right] - \left(-2x^2 + x + 3\right)}{h}\\ &= \frac{-2x^2 - 4xh - 2h^2 + x + h + 3 + 2x^2 - x - 3}{h}\\ &= \frac{-4xh - 2h^2 + h}{h} = \frac{h(-4x - 2h + 1)}{h}\\ &= -4x + 1 - 2h\end{aligned}$$

69. $f(x) = x^3$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{(x+h)^3 - x^3}{h}\\ &= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}\\ &= \frac{h\left(3x^2 + 3xh + h^2\right)}{h}\\ &= 3x^2 + 3xh + h^2\end{aligned}$$

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70. $f(x) = 2x^3 + x^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{\left[2(x+h)^3+(x+h)^2\right]-\left(2x^3+x^2\right)}{h}$$
$$= \frac{2x^3+6x^2h+6xh^2+2h^3+x^2+2xh+h^2-2x^3-x^2}{h}$$
$$= \frac{h\left(6x^2+6xh+2x+2h^2+h\right)}{h} = 6x^2+2x+6xh+h+2h^2$$

For exercises 71–76, refer to the three-step method on page 79 in the text.

71. $f(x) = -x^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{-(x+h)^2-\left(-x^2\right)}{h}$$
$$= \frac{-x^2-2xh-h^2+x^2}{h}$$
$$= \frac{h(-2x+h)}{h} = -2x+h$$

As h approaches 0, the quantity $-2x + h$ approaches $-2x$. Thus, $f'(x) = -2x$.

72. $f(x) = 3x^2 - 2$

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{\left[3(x+h)^2-2\right]-\left(3x^2-2\right)}{h}$$
$$= \frac{3x^2+6xh+3h^2-2-3x^2+2}{h}$$
$$= \frac{h(6x+3h)}{h} = 6x+3h$$

As h approaches 0, the quantity $6x + 3h$ approaches $6x$. Thus, $f'(x) = 6x$.

73. $f(x) = 7x^2 + x - 1$

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{\left[7(x+h)^2+(x+h)-1\right]-\left(7x^2+x-1\right)}{h}$$
$$= \frac{7x^2+14xh+7h^2+x+h-1-7x^2-x+1}{h}$$
$$= \frac{14xh+7h^2+h}{h} = \frac{h(14x+7h+1)}{h}$$
$$= 14x+1+7h$$

As h approaches 0, the quantity $14x + 1 + 7h$ approaches $14x + 1$. Thus, $f'(x) = 14x + 1$.

74. $f(x) = x + 3$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h+3)-(x+3)}{h}$$
$$= \frac{h}{h} = 1$$

As h approaches 0, the quantity 1 approaches 1. Thus, $f'(x) = 1$.

75. $f(x) = x^3$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^3-x^3}{h}$$
$$= \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$
$$= \frac{h\left(3x^2+3xh+h^2\right)}{h}$$
$$= 3x^2+3xh+h^2$$

As h approaches 0, the quantity $3x^2 + 3xh + h^2$ approaches $3x^2$. Thus, $f'(x) = 3x^2$.

76. $f(x) = 2x^3 - x$

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{\left[2(x+h)^3-(x+h)\right]-\left(2x^3-x\right)}{h}$$
$$= \frac{2x^3+6x^2h+6xh^2+2h^3-x-h-2x^3+x}{h}$$
$$= \frac{h\left(6x^2+6xh+2h^2-1\right)}{h}$$
$$= 6x^2+6xh+2h^2-1$$

As h approaches 0, the quantity $6x^2 + 6xh + 2h^2 - 1$ approaches $6x^2 - 1$. Thus, $f'(x) = 6x^2 - 1$.

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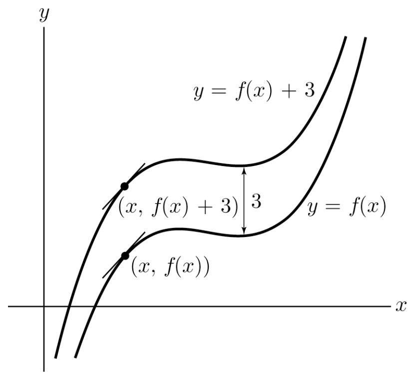
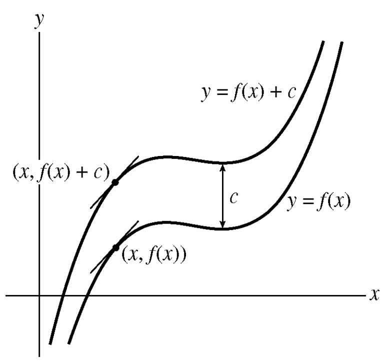
77. a., b.



In both cases the tangent lines are parallel.

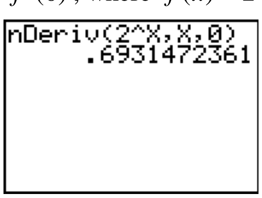
c. A vertical shift in a graph does not change its shape. Therefore, the slope at any point x remains the same for any shift in the y-direction. Since the slope at a given point is the value of the derivative at that point, $\frac{d}{dx}f(x) = \frac{d}{dx}(f(x)+3)$.

78.

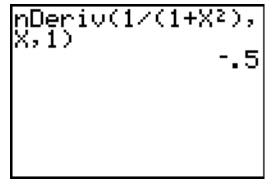


Observe the tangent lines for the two chosen values of x. The tangents are parallel. Thus, the slope of $y = f(x)$ at the point $(x, f(x))$ is equal to the slope of the graph of $y = f(x) + c$ at the point $(x, f(x)+c)$. Since the slope at a given point is the value of the derivative at that point, $\frac{d}{dx}f(x) = \frac{d}{dx}(f(x)+c)$.

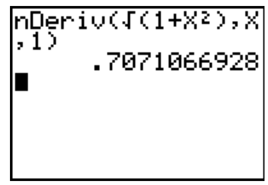
79. $f'(0)$, where $f(x) = 2^x$



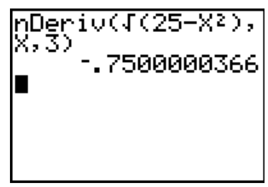
80. $f'(1)$, where $f(x) = \frac{1}{1+x^2}$



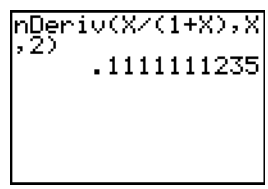
81. $f'(1)$, where $f(x) = \sqrt{1+x^2}$



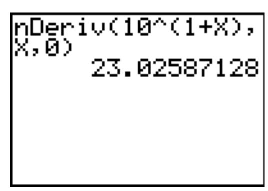
82. $f'(3)$, where $f(x) = \sqrt{25-x^2}$



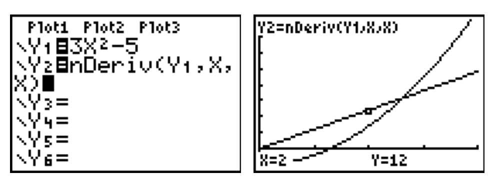
83. $f'(2)$, where $f(x) = \frac{x}{1+x}$



84. $f'(0)$, where $f(x) = 10^{1+x}$



85.



[0, 4] by [−5, 40]

Value of the derivative of Y_1 at $x = 2$ is 12.

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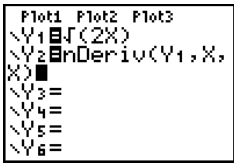
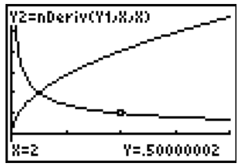
86. 



[0, 4] by [–.5, 3]

Value of the derivative of Y_1 at $x = 2$ is 0.5.

87. $f(x) = \sqrt{x}$, (9, 3)

The slope of the tangent line at $x = a$ is

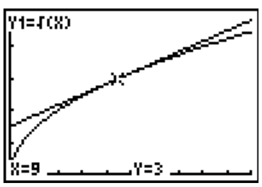
$f'(a) = \frac{1}{2}a^{-1/2} = \frac{1}{2\sqrt{a}}$, so the slope of the

tangent line at $x = 9$ is $\frac{1}{2\sqrt{9}} = \frac{1}{6}$.

To find the equation of the tangent line, let

$(x_1, y_1) = (9, 3)$, and $m = \frac{1}{6}$.

$y - 3 = \frac{1}{6}(x - 9) \Rightarrow y = \frac{1}{6}x + \frac{3}{2}$.



[0, 20] by [0, 5]

88. $f(x) = \frac{1}{x}$, (.5, 2)

The slope of the tangent line at $x = a$ is

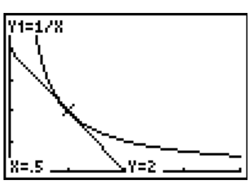
$f'(a) = -\frac{1}{a^2}$, so the slope of the tangent line

at $x = .5$ is $-\frac{1}{.5^2} = -4$.

To find the equation of the tangent line, let

$(x_1, y_1) = (.5, 2)$, and $m = -4$.

$y - 2 = -4(x - .5) \Rightarrow y = -4x + 4$



[0, 2] by [0, 5]

89. $f(x) = \frac{1}{x^2}$, (.5, 4)

The slope of the tangent line at $x = a$ is

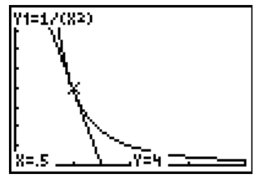
$f'(a) = -\frac{2}{a^3}$, so the slope of the tangent line

at $x = .5$ is $-\frac{2}{(.5)^3} = -16$. To find the equation

of the tangent line, let $(x_1, y_1) = (.5, 4)$, and

$m = -16$.

$y - 4 = -16(x - .5) \Rightarrow y = -16x + 12$



[0, 2] by [0, 8]

90. $f(x) = x^3$, (1, 1)

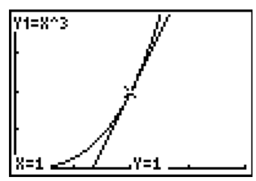
The slope of the tangent line at $x = a$ is

$f'(a) = 3a^2$, so the slope of the tangent line at

$x = 1$ is $3(1)^2 = 3$. To find the equation of the

tangent line, let $(x_1, y_1) = (1, 1)$, and $m = 3$.

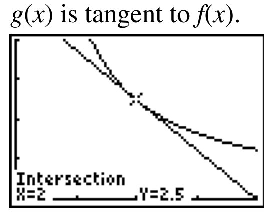
$y - 1 = 3(x - 1) \Rightarrow y = 3x - 2$.



[0, 2] by [0, 2]

91. $f(x) = \frac{5}{x}$, $g(x) = 5 - 1.25x$

To solve graphically, graph the functions and use the **INTERSECT** command to find where $g(x)$ is tangent to $f(x)$.



[0, 4] by [0, 4]

Thus, $a = 2$.

Alternatively, we can solve $\frac{5}{x} = 5 - 1.25x$ to

find the x-value of the intersection of $f(x)$

and $g(x)$.

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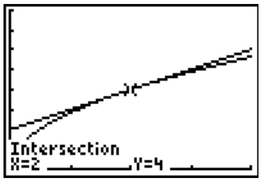
92. $f(x)=\sqrt{8x},\ g(x)=x+2$

To solve graphically, graph the functions and use the **INTERSECT** command to find where $g(x)$ is tangent to $f(x)$.



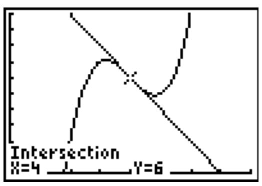
[0, 4] by [0, 8]

Thus, $a = 2$.

Alternatively, we can solve $\sqrt{8x}=x+2$ to find the x-value of the intersection of $f(x)$ and $g(x)$.

93. $f(x)=x^3-12x^2+46x-50,\ g(x)=14-2x$

Graph the functions and use the **INTERSECT** command to find where $g(x)$ is tangent to $f(x)$.



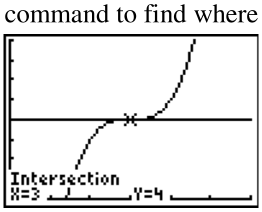
[0, 8] by [0, 10]

Thus, $a = 4$.

Alternatively, we can solve $x^3-12x^2+46x-50=14-2x$ to find the x-value of the intersection of $f(x)$ and $g(x)$.

94. $f(x)=(x-3)^3+4,\ g(x)=4$

Graph the functions and use the **INTERSECT** command to find where $g(x)$ is tangent to $f(x)$.



[0, 6] by [0, 8]

Thus, $a = 3$.

Alternatively, we can solve $(x-3)^3+4=4$ to find the x-value of the intersection of $f(x)$ and $g(x)$.

1.4 Limits and the Derivative

1. There is no limit because as x approaches 3 from the left, $g(x)$ approaches $-\infty$, while as x approaches 3 from the right, $g(x)$ approaches 2.

2. 2

3. 1

4. There is no limit because as x approaches 3 from the left, $g(x)$ approaches 5, while as x approaches 3 from the right, $g(x)$ approaches 3.

5. There is no limit because as x approaches 3 from the left, $g(x)$ approaches 4, while as x approaches 3 from the right, $g(x)$ approaches 5.

6. There is no limit because the values of $g(x)$ become larger and larger as x approaches 3 from the right and from the left, and do not approach a fixed number.

7. $\lim\limits_{x\to 1}(1-6x)=1-6(1)=-5$

8. $\lim\limits_{x\to 2}\dfrac{x}{x-2}$ is undefined.

9. $\lim\limits_{x\to 3}\sqrt{x^2+16}=\sqrt{(3)^2+16}=\sqrt{25}=5$

10. $\lim\limits_{x\to 4}(x^3-7)=4^3-7=57$

11. $\lim\limits_{x\to 5}\dfrac{x^2+1}{5+x}=\dfrac{\lim\limits_{x\to 5}x^2+\lim\limits_{x\to 5}1}{\lim\limits_{x\to 5}5+\lim\limits_{x\to 5}x}=\dfrac{5^2+1}{5+5}=\dfrac{13}{5}$

12.
$$\lim_{x\to 6}\left(\sqrt{6x}+3x-\frac{1}{x}\right)(x^2-4)$$
$$=\left(\lim_{x\to 6}\sqrt{6x}+\lim_{x\to 6}3x-\lim_{x\to 6}\frac{1}{x}\right)\left(\lim_{x\to 6}x^2-\lim_{x\to 6}4\right)$$
$$=\left(6+18-\frac{1}{6}\right)(36-4)=\frac{143}{6}\cdot 32=\frac{2288}{3}$$

13.
$$\lim_{x\to 7}\left(x+\sqrt{x-6}\right)\left(x^2-2x+1\right)$$
$$=\lim_{x\to 7}\left(x+\sqrt{x-6}\right)(x-1)^2$$
$$=\left(\lim_{x\to 7}x+\lim_{x\to 7}\sqrt{x-6}\right)\left(\lim_{x\to 7}x-\lim_{x\to 7}1\right)^2$$
$$=(7+1)(7-1)^2=8\cdot 36=288$$

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14. $$\lim_{x\to 8}\frac{\sqrt{5x-4}-1}{3x^2+2}=\frac{\lim\limits_{x\to 8}\sqrt{5x-4}-\lim\limits_{x\to 8}1}{\lim\limits_{x\to 8}3x^2+\lim\limits_{x\to 8}2}=\frac{6-1}{192+2}=\frac{5}{194}$$

15. $$\lim_{x\to -5}\frac{\sqrt{x^2-5x-36}}{8-3x}=\frac{\left(\lim\limits_{x\to -5}x^2-\lim\limits_{x\to -5}5x-\lim\limits_{x\to -5}36\right)^{1/2}}{\lim\limits_{x\to -5}8-\lim\limits_{x\to -5}3x}=\frac{\left(25-(-25)-36\right)^{1/2}}{8-(-15)}=\frac{\sqrt{14}}{23}$$

16. $$\lim_{x\to 10}(2x^2-15x-50)^{20}=\left(\lim_{x\to 10}2x^2-\lim_{x\to 10}15x-\lim_{x\to 10}50\right)^{20}=(200-150-50)^{20}=0^{20}=0$$

17. $$\lim_{x\to 0}\frac{x^2+3x}{x}=\lim_{x\to 0}\frac{x(x+3)}{x}=\lim_{x\to 0}(x+3)=3$$

18. $$\lim_{x\to 1}\frac{x^2-1}{x-1}=\lim_{x\to 1}\frac{(x-1)(x+1)}{(x-1)}=\lim_{x\to 1}(x+1)=2$$

19. $$\lim_{x\to 2}\frac{-2x^2+4x}{x-2}=\lim_{x\to 2}\frac{-2x(x-2)}{(x-2)}=\lim_{x\to 2}(-2x)=-4$$

20. $$\lim_{x\to 3}\frac{x^2-x-6}{x-3}=\lim_{x\to 3}\frac{(x-3)(x+2)}{(x-3)}=\lim_{x\to 3}(x+2)=5$$

21. $$\lim_{x\to 4}\frac{x^2-16}{4-x}=\lim_{x\to 4}\frac{(x-4)(x+4)}{-(x-4)}=\lim_{x\to 4}(-x-4)=\lim_{x\to 4}(-x)-\lim_{x\to 4}4=-4-4=-8$$

22. $$\lim_{x\to 5}\frac{2x-10}{x^2-25}=\lim_{x\to 5}\frac{2(x-5)}{(x-5)(x+5)}=\frac{\lim\limits_{x\to 5}2}{\lim\limits_{x\to 5}(x+5)}=\frac{2}{5+5}=\frac{1}{5}$$

23. $$\lim_{x\to 6}\frac{x^2-6x}{x^2-5x-6}=\lim_{x\to 6}\frac{x(x-6)}{(x-6)(x+1)}=\frac{\lim\limits_{x\to 6}x}{\lim\limits_{x\to 6}(x+1)}=\frac{6}{6+1}=\frac{6}{7}$$

24. $$\lim_{x\to 7}\frac{x^3-2x^2+3x}{x^2}=\lim_{x\to 7}\frac{x(x^2-2x+3)}{x^2}=\frac{\lim\limits_{x\to 7}(x^2-2x+3)}{\lim\limits_{x\to 7}x}=\frac{49-14+3}{7}=\frac{38}{7}$$

25. $\lim\limits_{x\to 8}\dfrac{x^2+64}{x-8}$ is undefined.

26. $\lim\limits_{x\to 9}\dfrac{1}{(x-9)^2}=\dfrac{\lim\limits_{x\to 9}1}{\lim\limits_{x\to 9}(x-9)^2}$ is undefined.

27. $\lim\limits_{x\to 0}f(x)=-\dfrac{1}{2}$ and $\lim\limits_{x\to 0}g(x)=\dfrac{1}{2}$.

a. $$\lim_{x\to 0}(f(x)+g(x))=\lim_{x\to 0}f(x)+\lim_{x\to 0}g(x)=-\frac{1}{2}+\frac{1}{2}=0$$

b. $$\lim_{x\to 0}(f(x)-2g(x))=\lim_{x\to 0}f(x)-2\cdot\lim_{x\to 0}g(x)=-\frac{1}{2}-2\cdot\frac{1}{2}=-\frac{3}{2}$$

c. $$\lim_{x\to 0}(f(x)\cdot g(x))=\left[\lim_{x\to 0}f(x)\right]\cdot\left[\lim_{x\to 0}g(x)\right]=\left[-\frac{1}{2}\right]\cdot\left[\frac{1}{2}\right]=-\frac{1}{4}$$

d. Since $\lim\limits_{x\to 0}g(x)\neq 0$,
$$\lim_{x\to 0}\frac{f(x)}{g(x)}=\frac{\lim\limits_{x\to 0}f(x)}{\lim\limits_{x\to 0}g(x)}=\frac{-\frac{1}{2}}{\frac{1}{2}}=-1.$$

28. The limit definition of the derivative is
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}.$$
$$f(x)=mx+b\Rightarrow$$
$$f'(x)=\lim_{h\to 0}\frac{\left[m(x+h)+b\right]-(mx+b)}{h}=\lim_{h\to 0}\frac{mx+mh+b-mx-b}{h}=\lim_{h\to 0}\frac{mh}{h}=\lim_{h\to 0}m=m$$

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29. $f(x)=x^2+1$

$$f'(3)=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to 0}\frac{(3+h)^2+1-(3^2+1)}{h}=\lim_{h\to 0}\frac{9+6h+h^2+1-10}{h}$$
$$=\lim_{h\to 0}\frac{h^2+6h}{h}=\lim_{h\to 0}(h+6)=6$$

30. $f(x)=x^3$

$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{(2+h)^3-2^3}{h}=\lim_{h\to 0}\frac{8+12h+6h^2+h^3-8}{h}$$
$$=\lim_{h\to 0}\frac{h(12+6h+h^2)}{h}=\lim_{h\to 0}(h^2+6h+12)=12$$

31. $f(x)=x^3+3x+1$

$$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{h^3+3h+1-1}{h}=\lim_{h\to 0}\frac{h(h^2+3)}{h}=\lim_{h\to 0}(h^2+3)=3$$

32. $f(x)=x^2+2x+2$

$$f'(0)=\lim_{h\to 0}\frac{(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{h^2+2h+2-2}{h}=\lim_{h\to 0}\frac{h(h+2)}{h}=\lim_{h\to 0}(h+2)=2$$

For exercises 33–36, use the three-step method discussed on page 86 in section 1.4 and illustrated in Examples 5 and 6.

33. $f(x)=x^2+1$

Step 1: $\dfrac{f(x+h)-f(x)}{h}=\dfrac{\left[(x+h)^2+1\right]-\left(x^2+1\right)}{h}$

Step 2: $\dfrac{\left[(x+h)^2+1\right]-\left(x^2+1\right)}{h}=\dfrac{x^2+2xh+h^2+1-x^2-1}{h}=\dfrac{2xh+h^2}{h}=\dfrac{h(2x+h)}{h}=2x+h$

Step 3: $f'(x)=\lim_{h\to 0}(2x+h)=2x$

34. $f(x)=-x^2+2$

Step 1: $\dfrac{f(x+h)-f(x)}{h}=\dfrac{\left[-(x+h)^2+2\right]-\left(-x^2+2\right)}{h}$

Step 2: $\dfrac{\left[-(x+h)^2+2\right]-\left(-x^2+2\right)}{h}=\dfrac{-x^2-2xh-h^2+2+x^2-2}{h}=\dfrac{-2xh-h^2}{h}=\dfrac{h(-2x-h)}{h}=-2x-h$

Step 3: $f'(x)=\lim_{h\to 0}(-2x-h)=-2x$

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35. $f(x)=x^3-1$

Step 1: $\dfrac{f(x+h)-f(x)}{h}=\dfrac{\left[(x+h)^3-1\right]-\left(x^3-1\right)}{h}$

Step 2: $\dfrac{\left[(x+h)^3-1\right]-\left(x^3-1\right)}{h}=\dfrac{x^3+3x^2h+3xh^2+h^3-1-x^3+1}{h}=\dfrac{3x^2h+3xh^2+h^3}{h}$

$=\dfrac{h\left(3x^2+3xh+h^2\right)}{h}=3x^2+3xh+h^2$

Step 3: $f'(x)=\lim\limits_{h\to 0}\left(3x^2+3xh+h^2\right)=3x^2$

36. $f(x)=-3x^2+1$

Step 1: $\dfrac{f(x+h)-f(x)}{h}=\dfrac{\left[-3(x+h)^2+1\right]-\left(-3x^2+1\right)}{h}$

Step 2: $\dfrac{\left[-3(x+h)^2+1\right]-\left(3x^2-1\right)}{h}=\dfrac{-3x^2-6xh-3h^2+1+3x^2-1}{h}=\dfrac{-6xh-3h^2}{h}$

$=\dfrac{h(-6x-h)}{h}=-6x-h$

Step 3: $f'(x)=\lim\limits_{h\to 0}(-6x-h)=-6x$

37. $f(x)=3x+1$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{3(x+h)+1-(3x+1)}{h}=\lim_{h\to 0}\frac{3x+3h+1-3x-1}{h}=\lim_{h\to 0}\frac{3h}{h}=\lim_{h\to 0}3=3$$

38. $f(x)=-x+11$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{-(x+h)+11-(-x+11)}{h}$$
$$=\lim_{h\to 0}\frac{-x-h+11+x-11}{h}=\lim_{h\to 0}\frac{-h}{h}=\lim_{h\to 0}(-1)=-1$$

39. $f(x)=x+\dfrac{1}{x}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)+\frac{1}{x+h}-\left(x+\frac{1}{x}\right)}{h}=\lim_{h\to 0}\frac{x+h+\frac{1}{x+h}-x-\frac{1}{x}}{h}$$
$$=\lim_{h\to 0}\frac{h+\frac{1}{x+h}-\frac{1}{x}}{h}=\lim_{h\to 0}\frac{\frac{h(x+h)(x)+x-(x+h)}{(x+h)(x)}}{h}=\lim_{h\to 0}\frac{\frac{hx^2+h^2x-h}{(x+h)(x)}}{h}$$
$$=\lim_{h\to 0}\frac{h(x^2+hx-1)}{(x+h)(x)}\left(\frac{1}{h}\right)=\lim_{h\to 0}\frac{(x^2+hx-1)}{(x+h)(x)}=\frac{x^2-1}{x^2}=1-\frac{1}{x^2}$$

40. $f(x)=\dfrac{1}{x^2}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\lim_{h\to 0}\frac{\frac{x^2-(x+h)^2}{(x+h)^2(x^2)}}{h}=\lim_{h\to 0}\frac{\frac{x^2-x^2-2xh-h^2}{(x+h)^2(x^2)}}{h}$$
$$=\lim_{h\to 0}\frac{h(-2x-h)}{(x+h)^2(x^2)}\left(\frac{1}{h}\right)=\lim_{h\to 0}\frac{-2x-h}{(x+h)^2(x^2)}=\frac{-2x}{(x^2)(x^2)}=-\frac{2}{x^3}$$

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41. $f(x)=\dfrac{x}{x+1}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{x+h}{x+h+1}-\frac{x}{x+1}}{h}=\lim_{h\to 0}\frac{\frac{(x+h)(x+1)-(x)(x+h+1)}{(x+h+1)(x+1)}}{h}$$
$$=\lim_{h\to 0}\frac{\frac{x^2+xh+x+h-x^2-xh-x}{(x+h+1)(x+1)}}{h}=\lim_{h\to 0}\frac{h}{(x+h+1)(x+1)}\left(\frac{1}{h}\right)$$
$$=\lim_{h\to 0}\frac{1}{(x+h+1)(x+1)}=\frac{1}{(x+1)(x+1)}=\frac{1}{(x+1)^2}$$

42. $f(x)=-1+\dfrac{2}{x-2}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{-1+\frac{2}{x+h-2}-\left(-1+\frac{2}{x-2}\right)}{h}=\lim_{h\to 0}\frac{\frac{2(x-2)-2(x+h-2)}{(x+h-2)(x-2)}}{h}$$
$$=\lim_{h\to 0}\frac{\frac{2x-4-2x-2h+4}{(x+h-2)(x-2)}}{h}=\lim_{h\to 0}\frac{-2h}{(x+h-2)(x-2)}\left(\frac{1}{h}\right)$$
$$=\lim_{h\to 0}\frac{-2}{(x+h-2)(x-2)}=\frac{-2}{(x-2)(x-2)}=-\frac{2}{(x-2)^2}$$

43. $f(x)=\dfrac{1}{x^2+1}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{(x+h)^2+1}-\frac{1}{x^2+1}}{h}=\lim_{h\to 0}\frac{\frac{(x^2+1)-\left((x+h)^2+1\right)}{\left((x+h)^2+1\right)(x^2+1)}}{h}$$
$$=\lim_{h\to 0}\frac{\frac{x^2+1-x^2-2xh-h^2-1}{\left((x+h)^2+1\right)(x^2+1)}}{h}=\lim_{h\to 0}\frac{h(-2x-h)}{\left((x+h)^2+1\right)(x^2+1)}\left(\frac{1}{h}\right)$$
$$=\lim_{h\to 0}\frac{(-2x-h)}{\left((x+h)^2+1\right)(x^2+1)}=\frac{-2x}{(x^2+1)(x^2+1)}=\frac{-2x}{(x^2+1)^2}$$

44. $f(x)=\dfrac{x}{x+2}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{x+h}{x+h+2}-\frac{x}{x+2}}{h}=\lim_{h\to 0}\frac{\frac{(x+h)(x+2)-(x)(x+h+2)}{(x+h+2)(x+2)}}{h}$$
$$=\lim_{h\to 0}\frac{\frac{x^2+xh+2x+2h-x^2-xh-2x}{(x+h+2)(x+2)}}{h}=\lim_{h\to 0}\frac{2h}{(x+h+2)(x+2)}\left(\frac{1}{h}\right)$$
$$=\lim_{h\to 0}\frac{2}{(x+h+2)(x+2)}=\frac{2}{(x+2)(x+2)}=\frac{2}{(x+2)^2}$$

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45. $f(x)=\sqrt{x+2}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}$$
$$=\lim_{h\to 0}\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}\left(\frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}\right)$$
$$=\lim_{h\to 0}\frac{(x+h+2)-(x+2)}{h\left(\sqrt{x+h+2}+\sqrt{x+2}\right)}=\lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h+2}+\sqrt{x+2}\right)}$$
$$=\lim_{h\to 0}\frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}=\frac{1}{\sqrt{x+2}+\sqrt{x+2}}=\frac{1}{2\sqrt{x+2}}$$

46. $f(x)=\sqrt{x^2+1}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h}=\lim_{h\to 0}\frac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h}\left(\frac{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}\right)$$
$$=\lim_{h\to 0}\frac{(x+h)^2+1-(x^2+1)}{h\left(\sqrt{(x+h)^2+1}+\sqrt{x^2+1}\right)}=\lim_{h\to 0}\frac{x^2+2xh+h^2+1-x^2-1}{h\left(\sqrt{(x+h)^2+1}+\sqrt{x^2+1}\right)}$$
$$=\lim_{h\to 0}\frac{h(2x+h)}{h\left(\sqrt{(x+h)^2+1}+\sqrt{x^2+1}\right)}=\lim_{h\to 0}\frac{(2x+h)}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}$$
$$=\frac{2x}{\sqrt{x^2+1}+\sqrt{x^2+1}}=\frac{x}{\sqrt{x^2+1}}$$

47. $f(x)=\dfrac{1}{\sqrt{x}}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}=\lim_{h\to 0}\frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}$$
$$=\lim_{h\to 0}\left(\frac{1}{h}\right)\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\left(\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}\right)=\lim_{h\to 0}\left(\frac{1}{h}\right)\frac{x-x-h}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$
$$=\lim_{h\to 0}\frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}=\frac{-1}{\sqrt{x}\sqrt{x}\left(\sqrt{x}+\sqrt{x}\right)}=\frac{-1}{2x\sqrt{x}}=-\frac{1}{2x^{3/2}}$$

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48. $f(x) = x\sqrt{x}$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)\sqrt{x+h}-x\sqrt{x}}{h}$$
$$= \lim_{h\to 0}\frac{(x+h)\sqrt{x+h}-x\sqrt{x}}{h}\left(\frac{(x+h)\sqrt{x+h}+x\sqrt{x}}{(x+h)\sqrt{x+h}+x\sqrt{x}}\right)$$
$$= \lim_{h\to 0}\frac{(x+h)^2(x+h)-x^2x}{h\left((x+h)\sqrt{x+h}+x\sqrt{x}\right)} = \lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h\left((x+h)\sqrt{x+h}+x\sqrt{x}\right)}$$
$$= \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h\left((x+h)\sqrt{x+h}+x\sqrt{x}\right)} = \lim_{h\to 0}\frac{h\left(3x^2+3xh+h^2\right)}{h\left((x+h)\sqrt{x+h}+x\sqrt{x}\right)}$$
$$= \lim_{h\to 0}\frac{3x^2+3xh+h^2}{(x+h)\sqrt{x+h}+x\sqrt{x}} = \frac{3x^2}{2x\sqrt{x}} = \frac{3}{2}x^{1/2} = \frac{3}{2}\sqrt{x}$$

49. We want to find $f(x)$ such that

$\lim_{h\to 0}\frac{(1+h)^2-1}{h}$ has the same form as

$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$. So,

$f(a+h) = (1+h)^2$ and $f(a) = 1$. Thus,

$f(x) = x^2$ and $a = 1$.

50. We want to find $f(x)$ such that

$\lim_{h\to 0}\frac{(2+h)^3-8}{h}$ has the same form as

$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$. So,

$f(a+h) = (2+h)^3$ and $f(a) = 8$. Thus,

$f(x) = x^3$ and $a = 2$.

51. We want to find $f(x)$ such that

$\lim_{h\to 0}\frac{\frac{1}{10+h}-.1}{h}$ has the same form as

$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$. So,

$f(a+h) = \frac{1}{10+h}$ and $f(a) = .1$. Thus,

$f(x) = x^{-1} = \frac{1}{x}$ and $a = 10$.

52. We want to find $f(x)$ such that

$\lim_{h\to 0}\frac{(64+h)^{1/3}-4}{h}$ has the same form as

$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$. So,

$f(a+h) = (64+h)^{1/3}$ and $f(a) = 4$. Thus,

$f(x) = x^{1/3} = \sqrt[3]{x}$ and $a = 64$.

53. We want to find $f(x)$ such that

$\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}$ has the same form as

$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$. So,

$f(a+h) = \sqrt{9+h}$ and $f(a) = 3$. Thus,

$f(x) = \sqrt{x}$ and $a = 9$.

54. We want to find $f(x)$ such that

$\lim_{h\to 0}\frac{(1+h)^{-1/2}-1}{h}$ has the same form as

$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$. So,

$f(a+h) = (1+h)^{-1/2}$ and $f(a) = 1$. Thus,

$f(x) = x^{-1/2} = \frac{1}{\sqrt{x}}$ and $a = 1$.

55. $f(x) = x^2;\ f'(x) = 2x$

$$f'(2) = \lim_{h\to 0}\frac{(2+h)^2-4}{h} = 4$$

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56. $f(x) = x^3;\ f'(x) = 3x^2$

$$f'(-1) = \lim_{h\to 0} \frac{(-1+h)^3 + 1}{h} = 3$$

57. $f(x) = \sqrt{x};\ f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$$f'(2) = \lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} = \frac{1}{2\sqrt{2}}$$

58. $f(x) = \sqrt{x};\ f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$$f'(4) = \lim_{h\to 0} \frac{\sqrt{4+h} - 2}{h} = \frac{1}{4}$$

59. $f(x) = x^{1/3};\ f'(x) = \frac{1}{3}x^{-2/3}$

$$f'(-8) = \lim_{h\to 0} \frac{(-8+h)^{1/3} + 2}{h} = \frac{1}{12}$$

60. $f(x) = \frac{1}{x};\ f'(x) = -\frac{1}{x^2}$

$$f'(1) = \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{1+h} - 1\right] = -1$$

61. $\lim\limits_{x\to\infty} \frac{1}{x^2} = 0$

62. $\lim\limits_{x\to-\infty} \frac{1}{x^2} = 0$

63. $\lim\limits_{x\to\infty} \frac{5x+3}{3x-2} = \lim\limits_{x\to\infty} \frac{5+\frac{3}{x}}{3-\frac{2}{x}} = \frac{5}{3}$

64. $\lim\limits_{x\to\infty} \frac{1}{x-8} = 0$

65. $\lim\limits_{x\to\infty} \frac{10x+100}{x^2-30} = \lim\limits_{x\to\infty} \frac{10+\frac{100}{x}}{x-\frac{30}{x}} = 0$

66. $\lim\limits_{x\to\infty} \frac{x^2+x}{x^2-1} = \lim\limits_{x\to\infty} \frac{1+\frac{1}{x}}{1-\frac{1}{x^2}} = 1$

67. $\lim\limits_{x\to 0} f(x)$

As x approaches 0 from either side, $f(x)$ approaches $\frac{3}{4}$. So $\lim\limits_{x\to 0} f(x) = \frac{3}{4}$.

68. $\lim\limits_{x\to\infty} f(x)$

As x increases without bound, $f(x)$ approaches 1. So $\lim\limits_{x\to\infty} f(x) = 1$

69. $\lim\limits_{x\to 0} xf(x) = \left[\lim\limits_{x\to 0} x\right]\cdot\left[\lim\limits_{x\to 0} f(x)\right]$

$$= 0\cdot\frac{3}{4} = 0$$

70. $\lim\limits_{x\to\infty} (1+2f(x)) = \lim\limits_{x\to\infty} 1 + 2\cdot\lim\limits_{x\to\infty} f(x)$

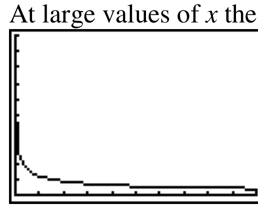
$$= 1 + 2\cdot 1 = 3$$

71. $\lim\limits_{x\to\infty} (1-f(x)) = \lim\limits_{x\to\infty} 1 - \lim\limits_{x\to\infty} f(x) = 1 - 1 = 0$

72. $\lim\limits_{x\to 0} [f(x)]^2 = \left[\lim\limits_{x\to 0} f(x)\right]^2 = \left[\frac{3}{4}\right]^2 = \frac{9}{16}$

73. $\lim\limits_{x\to\infty} \sqrt{25+x} - \sqrt{x}$

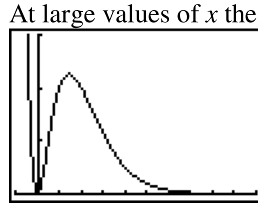
At large values of x the function goes to 0.



[0, 1000] by [0, 10]

74. $\lim\limits_{x\to\infty} \frac{x^2}{2^x}$

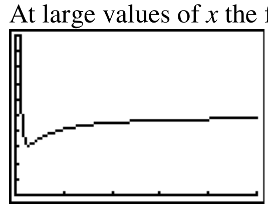
At large values of x the function goes to 0.



[−2, 20] by [0, 1.5]

75. $\lim\limits_{x\to\infty} \frac{x^2-2x+3}{2x^2+1}$

At large values of x the function goes to .5.



[0, 50] by [0, 1]

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76. $\lim_{x\to\infty} \frac{-8x^2+1}{x^2+1}$

At large values of x the function goes to –8.

[0, 20] by [−9, 1]

1.5 Differentiability and Continuity

1. No
2. Yes
3. Yes
4. Yes
5. No
6. No
7. No
8. No
9. Yes
10. Yes
11. No
12. No

13. $f(x) = x^2$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} x^2 = 1$

$f(1) = 1^2 = 1$

Since $\lim_{x\to 1} f(x) = 1 = f(1)$, $f(x)$ is continuous at $x = 1$.

$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{(1+h)^2-(1)^2}{h}$$
$$= \lim_{h\to 0} \frac{1+2h+h^2-1}{h} = \lim_{h\to 0} \frac{h(2+h)}{h} = 2$$

Therefore, $f(x)$ is continuous and differentiable at $x = 1$.

14. $f(x) = \frac{1}{x}$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{1}{x} = 1$

$f(1) = \frac{1}{1} = 1$

Since $\lim_{x\to 1} f(x) = 1 = f(1)$, $f(x)$ is continuous at $x = 1$.

$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{\frac{1}{1+h}-1}{h}$$
$$= \lim_{h\to 0} \frac{1-(1+h)}{1+h}\cdot\frac{1}{h} = \lim_{h\to 0} -\frac{1}{1+h} = -1$$

Therefore, $f(x)$ is continuous and differentiable at $x = 1$.

15. $f(x) = \begin{cases} x+2 & \text{for } -1 \le x \le 1 \\ 3x & \text{for } \quad 1 < x \le 5 \end{cases}$

$\lim_{x\to 1} 3x = 3$

$\lim_{x\to 1} (x+2) = 3$

$f(1) = 1 + 2 = 3$

Since $\lim_{x\to 1} f(x) = 3 = f(1)$, $f(x)$ is continuous at $x = 1$. Since the graph of $f(x)$ at $x = 1$ does not have a tangent line, $f(x)$ is not differentiable at $x = 1$. Therefore, $f(x)$ is continuous but not differentiable at $x = 1$.

16. $f(x) = \begin{cases} x^3 & \text{for } 0 \le x < 1 \\ x & \text{for } 1 \le x \le 2 \end{cases}$

$\lim_{x\to 1} x^3 = 1$

$\lim_{x\to 1} x = 1$

$f(1) = 1$

Since $\lim_{x\to 1} f(x) = 1 = f(1)$, $f(x)$ is continuous at $x = 1$.

Since the graph of $f(x)$ at $x = 1$ does not have a tangent line, $f(x)$ is not differentiable at $x = 1$. Therefore, $f(x)$ is continuous but not differentiable at $x = 1$.

17. $f(x) = \begin{cases} 2x-1 & \text{for } 0 \le x \le 1 \\ 1 & \text{for } 1 < x \end{cases}$

$\lim_{x\to 1} 1 = 1$

$\lim_{x\to 1} (2x-1) = 1$

$f(1) = 2(1) - 1 = 1$

Since $\lim_{x\to 1} f(x) = 1 = f(1)$, $f(x)$ is continuous at $x = 1$.

Since the graph of $f(x)$ at $x = 1$ does not have a tangent line, $f(x)$ is not differentiable at $x = 1$. Therefore, $f(x)$ is continuous but not differentiable at $x = 1$.

18. $f(x) = \begin{cases} x & \text{for } x \ne 1 \\ 2 & \text{for } x = 1 \end{cases}$

$\lim_{x\to 1} x = 1$

$f(1) = 2$

Since $\lim_{x\to 1} f(x) = 1 \ne 2 = f(1)$, $f(x)$ is not continuous at $x = 1$. By Theorem 1, since $f(x)$ is not continuous at $x = 1$, it is not differentiable.

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19. $f(x) = \begin{cases} \dfrac{1}{x-1} & \text{for } x \neq 1 \\ 0 & \text{for } x = 1 \end{cases}$

$\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \dfrac{1}{x-1}$ is undefined. Since $\lim\limits_{x\to 1} f(x)$ does not exist, $f(x)$ is not continuous at $x = 1$. By Theorem 1, since $f(x)$ is not continuous at $x = 1$, it is not differentiable.

20. $f(x) = \begin{cases} x-1 & \text{for } 0 \le x < 1 \\ 1 & \text{for } x = 1 \\ 2x-2 & \text{for } x > 1 \end{cases}$

$\lim\limits_{x\to 1}(x-1) = 0 = \lim\limits_{x\to 1}(2x-2)$, but $f(1) = 1$, so $f(x)$ is not continuous at $x = 1$. Therefore, $f(x)$ is not differentiable at $x = 1$.

21. $\dfrac{x^2-7x+10}{x-5} = \dfrac{(x-5)(x-2)}{x-5} = x-2$, so define $f(5) = 5 - 2 = 3$.

22. $\dfrac{x^2+x-12}{x+4} = \dfrac{(x+4)(x-3)}{x+4} = x-3$ so define $f(-4) = -4 - 3 = -7$.

23. $\dfrac{x^3-5x^2+4}{x^2}$

It is not possible to define $f(x)$ at $x = 0$ and make $f(x)$ continuous.

24. $\dfrac{x^2+25}{x-5}$

It is not possible to define $f(x)$ at $x = 5$ and make $f(x)$ continuous.

25. $\dfrac{(6+x)^2-36}{x} = \dfrac{\left(x^2+12x+36\right)-36}{x} = \dfrac{x^2+12x}{x} = x+12$

So, define $f(0) = 12$.

26. $\dfrac{\sqrt{9+x}-\sqrt{9}}{x} \cdot \dfrac{\sqrt{9+x}+\sqrt{9}}{\sqrt{9+x}+\sqrt{9}} = \dfrac{1}{\sqrt{9+x}+\sqrt{9}}$, so define $f(0) = \dfrac{1}{\sqrt{9+0}+\sqrt{9}} = \dfrac{1}{3+3} = \dfrac{1}{6}$.

27. **a.** The function $T(x)$ is a piecewise-defined function.
For $0 \le x \le 27{,}050$, $T(x) = .15x$.
For $27{,}050 < x \le 65{,}550$, we have
$T(x) = .15 \cdot 27{,}050 + .275 \cdot (x - 27{,}050) = .275x - 3381.25$
For $65{,}550 < x \le 136{,}750$, we have
$T(x) = .275 \cdot 65{,}550 - 3381.25 + .305(x - 65{,}550) = .305x - 5347.75$
All together, the function is

$$T(x) = \begin{cases} .15x & \text{for } 0 \le x \le 27{,}050 \\ .275x - 3381.25 & \text{for } 27{,}050 < x \le 65{,}550 \\ .305x - 5347.75 & \text{for } 65{,}550 < x \le 136{,}750 \end{cases}$$

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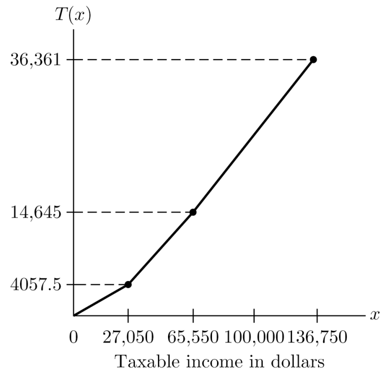
b.



c. $T(65{,}550)$ is the maximum tax you will pay for income below the third tax bracket. $T(27{,}050)$ is the maximum tax for income below the second tax bracket. The maximum tax on the portion of income in the second tax bracket is $T(65{,}550) - T(27{,}050) = 10{,}587.5$ dollars.

28. a. The function $T(x)$ is a piecewise-defined function.
For $0 \le x \le 27{,}050$, $T(x) = .15x$
For $27{,}050 < x \le 65{,}550$, we have

$$T(x) = .15 \cdot 27,050 + .275(x - 27,050) \\ = .275x - 3381.25$$

For $65{,}550 < x \le 136{,}750$, we have

$$T(x) = 43057.50 + .275(65,550 - 27,050) + .305(x - 65,550) \\ = .305x - 5347.75$$

For $136{,}750 < x \le 297{,}350$, we have

$$T(x) = .305 \cdot 136,750 - 5347.75 + .355(x - 136,750) \\ = .355x - 12,185.25$$

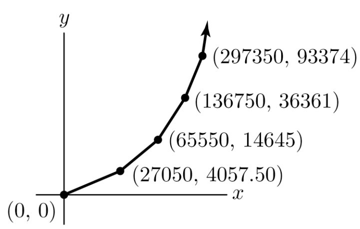
For $297{,}350 < x$, we have

$$T(x) = .355 \cdot 297,350 - 12,185.25 + .391(x - 297,350) \\ = .391x - 22,889.85$$

All together, we have

$$T(x) = \begin{cases} .15x & \text{for } 0 \le x \le 27,050 \\ .275x - 3381.25 & \text{for } 27,050 < x \le 65,550 \\ .305x - 5347.75 & \text{for } 65,550 < x \le 136,750 \\ .355x - 12,185.25 & \text{for } 136,750 < x \le 297,350 \\ .391x - 22,889.85 & \text{for } x > 297,350 \end{cases}$$

b.



c. $T(297{,}350) - T(136{,}750)$
$= 93{,}374 - 36{,}361 = \$57{,}013$

29. a. The function $R(x)$ is a piecewise function
For $0 \le x \le 100$, $R(x) = 2.50 + .07x$.
For $x > 100$, we have

$$R(x) = 2.50 + .07 \cdot 100 + .04(x - 100) \\ = 5.50 + .04x$$

All together, we have

$$R(x) = \begin{cases} .07x + 2.50 & \text{for } 0 \le x \le 100 \\ .04x + 5.50 & \text{for } x > 100 \end{cases}$$

b. Let $P(x)$ be the profit on x copies.
For $0 \le x \le 100$,
$P(x) = 2.50 + .07x - .03x = 2.50 + .04x$
For $x > 100$,
$P(x) = 5.50 + .04x - .03x = 5.50 + .01x$
All together, we have

$$P(x) = \begin{cases} .04x + 2.50 & \text{for } 0 \le x \le 100 \\ .01x + 5.50 & \text{for } x > 100 \end{cases}$$

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30. a. For $0 \le x \le 50$, $R(x) = .10x$.
For $x > 50$, $R(x) = .10(50) + .05(x - 50)$
$= 2.50 + .05x$
All together, we have

$$R(x) = \begin{cases} .10x & \text{for } 0 \le x \le 50 \\ .05x + 2.50 & \text{for } x > 50 \end{cases}$$

b. Let $P(x)$ be the profit on x copies.
For $0 \le x \le 50$, $P(x) = .10x - .03x = .07x$
For $x > 50$, $P(x) = 2.50 + 0.5x - .03x$
$= 2.50 + .02x$
All together, we have

$$P(x) = \begin{cases} .07x & \text{for } 0 \le x \le 50 \\ .02x + 2.50 & \text{for } x > 50 \end{cases}$$

31. a. The rate of sales is the slope of the line connecting the points (8, 4) and (10, 10).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-4}{10-8} = 3$$

The rate of sales between 8 a.m. and 10 a.m. is about \$3000 per hour.

b. We need to find the 2-hour period with the greatest slope. Looking at the graph gives 3 possibilities:
8 a.m. – 10 a.m., $m = 3$ (from part a)
12 p.m. – 2 p.m.,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16-12}{14-12} = 2$$

6 p.m. – 8 p.m., $m = \dfrac{22-18}{8-6} = 2$

The interval from 8 a.m. to 10 a.m. has the greatest rate, which is \$3000 per hour.

32. a. Find the slope for each 2-hour interval.

Midnight–2 a.m., $m = \dfrac{\frac{1}{2} - 0}{2} = \dfrac{1}{4} = .25$

2 a.m.–4 a.m., $m = \dfrac{1 - \frac{1}{2}}{2} = \dfrac{1}{4} = .25$

4 a.m.– 6 a.m., $m = \dfrac{2-1}{2} = \dfrac{1}{2} = .5$

6 a.m.– 8 a.m., $m = \dfrac{4-2}{2} = 1$

8 a.m.– 10 a.m., $m = \dfrac{10-4}{2} = 3$

10 a.m.– noon, $m = \dfrac{12-10}{2} = 1$

noon–2 p.m., $m = \dfrac{16-12}{2} = 2$

2 p.m.–4 p.m., $m = \dfrac{17-16}{2} = \dfrac{1}{2} = .5$

4 p.m.–6 p.m., $m = \dfrac{18-17}{2} = \dfrac{1}{2} = .5$

6 p.m.–8 p.m., $m = \dfrac{22-18}{2} = 2$

8 p.m.–10 p.m., $m = \dfrac{22.5-22}{2} = \dfrac{1}{4} = .25$

10 p.m.–midnight,
$m = \dfrac{23-22.5}{2} = \dfrac{1}{4} = .25$

The intervals midnight–2 a.m., 2 a.m–4 a.m., 8 p.m–10 p.m., and 10 p.m–midnight have a sales rate of \$250 per hour.
The intervals 4 a.m.–6 a.m., 2 p.m.–4 p.m., and 4 p.m.–6 p.m. have a sales rate of \$500 per hour.
The intervals 6 a.m.–8 a.m. and 10 a.m–noon, have a sales rate of \$1000 per hour.
The intervals noon–2 p.m. and 6 p.m–8 p.m. both have a sales rate of \$2000 per hour.

b. 4,000 – 0 = \$4,000 between midnight and 8 a.m.
10,000 – 4,000 = \$6,000 between 8 a.m. and 10 a.m.
The sales between 8 a.m. and 10 a.m. are 50% more than the sales between midnight and 8 a.m.

33. For the function to be continuous, $\lim\limits_{x \to 0} f(x)$ must exist and equal $f(0)$. Therefore $\lim\limits_{x \to 0} (x + a) = \lim\limits_{x \to 0} 1$, so $a = 1$.

34. For the function to be continuous, $\lim\limits_{x \to 0} f(x)$ must exist and equal $f(0)$. Therefore,

$$\begin{aligned} \lim_{x \to 0} 2(x - a) &= \lim_{x \to 0} x^2 + 1 \\ 2(-a) &= (0)^2 + 1 \\ -2a &= 1 \\ a &= -\frac{1}{2} \end{aligned}$$

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1.6 Some Rules for Differentiation

1. $y = 6x^3$

$$\frac{dy}{dx} = \frac{d}{dx}\left(6x^3\right) = 18x^2$$

2. $y = 3x^4$

$$\frac{dy}{dx} = \frac{d}{dx}\left(3x^4\right) = 12x^3$$

3. $y = 3\sqrt[3]{x} = 3x^{1/3}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(3x^{1/3}\right) = \frac{1}{3}\cdot 3x^{-2/3} = x^{-2/3} = \frac{1}{x^{2/3}} = \frac{1}{\sqrt[3]{x^2}}$$

4. $y = \frac{1}{3x^3} = \frac{1}{3}x^{-3}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^{-3}\right) = -x^{-4} = -\frac{1}{x^4}$$

5. $y = \frac{x}{2} - \frac{2}{x} = \frac{1}{2}x - 2x^{-1}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}x - 2x^{-1}\right) = \frac{d}{dx}\left(\frac{1}{2}x\right) - \frac{d}{dx}\left(2x^{-1}\right) = \frac{1}{2} + 2x^{-2} = \frac{1}{2} + \frac{2}{x^2}$$

6. $f(x) = 12 + \frac{1}{7^3}$

$$\frac{d}{dx}\left(12 + \frac{1}{7^3}\right) = \frac{d}{dx}(12) + \frac{d}{dx}\left(\frac{1}{7^3}\right) = 0 + 0 = 0$$

7. $f(x) = x^4 + x^3 + x$

$$\frac{d}{dx}\left(x^4 + x^3 + x\right) = \frac{d}{dx}x^4 + \frac{d}{dx}x^3 + \frac{d}{dx}x = 4x^3 + 3x^2 + 1$$

8. $y = 4x^3 - 2x^2 + x + 1$

$$\frac{dy}{dx} = \frac{d}{dx}\left(4x^3 - 2x^2 + x + 1\right) = \frac{d}{dx}\left(4x^3\right) - \frac{d}{dx}\left(2x^2\right) + \frac{d}{dx}(x) + \frac{d}{dx}(1) = 12x^2 - 4x + 1$$

9. $y = (2x+4)^3$

$$\frac{dy}{dx} = \frac{d}{dx}(2x+4)^3 = 3(2x+4)^2\frac{d}{dx}(2x+4) = 3(2x+4)^2(2) = 6(2x+4)^2$$

10. $y = \left(x^2 - 1\right)^3$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 - 1\right)^3 = 3\left(x^2 - 1\right)^2\frac{d}{dx}\left(x^2 - 1\right) = 3\left(x^2 - 1\right)^2(2x) = 6x\left(x^2 - 1\right)^2$$

11. $y = \left(x^3 + x^2 + 1\right)^7$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 + x^2 + 1\right)^7 = 7\left(x^3 + x^2 + 1\right)^6\frac{d}{dx}\left(x^3 + x^2 + 1\right) = 7\left(x^3 + x^2 + 1\right)^6\left(3x^2 + 2x\right)$$

12. $y = \left(x^2 + x\right)^{-2}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 + x\right)^{-2} = -2\left(x^2 + x\right)^{-3}\frac{d}{dx}\left(x^2 + x\right) = -2\left(x^2 + x\right)^{-3}(2x + 1)$$

13. $y = \frac{4}{x^2} = 4x^{-2}$

$$\frac{dy}{dx} = 4\frac{d}{dx}x^{-2} = -\frac{8}{x^3}$$

14. $y = 4(x^2 - 6)^{-3}$

$$\frac{dy}{dx} = \frac{d}{dx}4\left(x^2 - 6\right)^{-3} = -12\left(x^2 - 6\right)^{-4}\frac{d}{dx}\left(x^2 - 6\right) = -12\left(x^2 - 6\right)^{-4}(2x) = -\frac{24x}{\left(x^2 - 6\right)^4}$$

15. $y = 3\sqrt[3]{2x^2 + 1} = 3\left(2x^2 + 1\right)^{1/3}$

$$\frac{dy}{dx} = \frac{d}{dx}3\left(2x^2 + 1\right)^{1/3} = 3\cdot\frac{1}{3}\left(2x^2 + 1\right)^{-2/3}\frac{d}{dx}\left(2x^2 + 1\right) = (2x^2 + 1)^{-2/3}(4x) = (4x)\left(2x^2 + 1\right)^{-2/3}$$

16. $y = 2\sqrt{x+1} = 2(x+1)^{1/2}$

$$\frac{dy}{dx} = \frac{d}{dx}2(x+1)^{1/2} = (x+1)^{-1/2}\frac{d}{dx}(x+1) = (x+1)^{-1/2}(1) = (x+1)^{-1/2} = \frac{1}{\sqrt{x+1}}$$

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17. $y = 2x + (x+2)^3$

$$\frac{dy}{dx} = \frac{d}{dx}\left(2x + (x+2)^3\right) = \frac{d}{dx}2x + \frac{d}{dx}(x+2)^3$$
$$= 2 + 3(x+2)^2\frac{d}{dx}(x+2) = 2 + 3(x+2)^2(1)$$
$$= 2 + 3(x+2)^2$$

18. $y = (x-1)^3 + (x+2)^4$

$$\frac{dy}{dx} = \frac{d}{dx}\left((x-1)^3 + (x+2)^4\right)$$
$$= \frac{d}{dx}(x-1)^3 + \frac{d}{dx}(x+2)^4$$
$$= 3(x-1)^2\frac{d}{dx}(x-1) + 4(x+2)^3\frac{d}{dx}(x+2)$$
$$= 3(x-1)^2(1) + 4(x+2)^3(1)$$
$$= 3(x-1)^2 + 4(x+2)^3$$

19. $y = \frac{1}{5x^5} = \frac{1}{5}x^{-5}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{5}x^{-5}\right) = \frac{1}{5}(-5)x^{-6} = -\frac{1}{x^6}$$

20. $y = (x^2+1)^2 + 3(x^2-1)^2$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\left(x^2+1\right)^2 + 3\left(x^2-1\right)^2\right)$$
$$= \frac{d}{dx}\left(x^2+1\right)^2 + \frac{d}{dx}3\left(x^2-1\right)^2$$
$$= 2\left(x^2+1\right)\frac{d}{dx}\left(x^2+1\right) + 6\left(x^2-1\right)\frac{d}{dx}\left(x^2-1\right)$$
$$= 2\left(x^2+1\right)(2x) + 6\left(x^2-1\right)(2x)$$
$$= 4x(x^2+1) + 12x\left(x^2-1\right)$$

21. $y = \frac{1}{x^3+1} = \left(x^3+1\right)^{-1}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3+1\right)^{-1} = -\left(x^3+1\right)^{-2}\frac{d}{dx}\left(x^3+1\right)$$
$$= -\left(x^3+1\right)^{-2}\left(3x^2\right) = -3x^2\left(x^3+1\right)^{-2}$$
$$= -\frac{3x^2}{\left(x^3+1\right)^2}$$

22. $y = \frac{2}{x+1} = 2(x+1)^{-1}$

$$\frac{dy}{dx} = \frac{d}{dx}2(x+1)^{-1} = -2(x+1)^{-2}\frac{d}{dx}(x+1)$$
$$= -2(x+1)^{-2}(1) = -2(x+1)^{-2} = -\frac{2}{(x+1)^2}$$

23. $y = x + \frac{1}{x+1} = x + (x+1)^{-1}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x + (x+1)^{-1}\right) = \frac{d}{dx}x + \frac{d}{dx}(x+1)^{-1}$$
$$= 1 + \left(-(x+1)^{-2}\frac{d}{dx}(x+1)\right)$$
$$= 1 + \left(-(x+1)^{-2}(1)\right) = 1 - (x+1)^{-2}$$

24. $y = 2\sqrt[4]{x^2+1} = 2\left(x^2+1\right)^{1/4}$

$$\frac{dy}{dx} = \frac{d}{dx}2\left(x^2+1\right)^{1/4} = \frac{1}{2}\left(x^2+1\right)^{-3/4}\frac{d}{dx}\left(x^2+1\right)$$
$$= \frac{1}{2}\left(x^2+1\right)^{-3/4}(2x) = x\left(x^2+1\right)^{-3/4}$$

25. $f(x) = 5\sqrt{3x^3+x} = 5(3x^3+x)^{1/2}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[5\left(3x^3+x\right)^{1/2}\right] = 5\frac{d}{dx}\left(3x^3+x\right)^{1/2}$$
$$= 5\cdot\frac{1}{2}\left(3x^3+x\right)^{-1/2}\cdot\frac{d}{dx}\left(3x^3+x\right)$$
$$= \frac{5\left(9x^2+1\right)}{2\sqrt{3x^3+x}} = \frac{45x^2+5}{2\sqrt{3x^3+x}}$$

26. $y = \frac{1}{x^3+x+1} = (x^3+x+1)^{-1}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3+x+1\right)^{-1}$$
$$= -\left(x^3+x+1\right)^{-2}\frac{d}{dx}\left(x^3+x+1\right)$$
$$= -\left(x^3+x+1\right)^{-2}\left(3x^2+1\right) = -\frac{3x^2+1}{\left(x^3+x+1\right)^2}$$

27. $y = 3x + \pi^3$

$$\frac{dy}{dx} = \frac{d}{dx}\left(3x+\pi^3\right) = \frac{d}{dx}3x + \frac{d}{dx}\pi^3 = 3$$

28. $y = \sqrt{1+x^2} = \left(1+x^2\right)^{1/2}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(1+x^2\right)^{1/2} = \frac{1}{2}\left(1+x^2\right)^{-1/2}\cdot\frac{d}{dx}\left(1+x^2\right)$$
$$= \frac{1}{2}\left(1+x^2\right)^{-1/2}\cdot 2x = \frac{x}{\sqrt{1+x^2}}$$

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29. $y=\sqrt{1+x+x^2}=\left(1+x+x^2\right)^{1/2}$

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}\left(1+x+x^2\right)^{1/2}\\&=\frac{1}{2}\left(1+x+x^2\right)^{-1/2}\cdot\frac{d}{dx}\left(1+x+x^2\right)\\&=\frac{1}{2}\left(1+x+x^2\right)^{-1/2}\cdot(1+2x)=\frac{1+2x}{2\sqrt{1+x+x^2}}\end{aligned}$$

30. $y=\frac{1}{2x+5}=(2x+5)^{-1}$

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}(2x+5)^{-1}\\&=(-1)\cdot(2x+5)^{-2}\cdot\frac{d}{dx}(2x+5)\\&=-(2x+5)^{-2}\cdot 2=-\frac{2}{(2x+5)^2}\end{aligned}$$

31. $y=\frac{2}{1-5x}=2(1-5x)^{-1}$

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}\left[2(1-5x)^{-1}\right]=2\cdot\frac{d}{dx}\left[(1-5x)^{-1}\right]\\&=2\cdot(-1)(1-5x)^{-2}\cdot\frac{d}{dx}(1-5x)\\&=-2(1-5x)^{-2}\cdot(-5)=10(1-5x)^{-2}\end{aligned}$$

32. $y=\frac{7}{\sqrt{1+x}}=7(1+x)^{-1/2}$

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}\left[7(1+x)^{-1/2}\right]\\&=7\left(-\frac{1}{2}\right)(1+x)^{-3/2}\cdot\frac{d}{dx}(1+x)\\&=7\cdot\left(-\frac{1}{2}\right)\cdot(1+x)^{-3/2}\cdot 1=-\frac{7}{2(1+x)^{3/2}}\end{aligned}$$

33. $y=\frac{45}{1+x+\sqrt{x}}=45\left(1+x+x^{1/2}\right)^{-1}$

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}\left[45\left(1+x+x^{1/2}\right)^{-1}\right]\\&=45(-1)\left(1+x+x^{1/2}\right)^{-2}\frac{d}{dx}\left(1+x+x^{1/2}\right)\\&=-45\left(1+x+\sqrt{x}\right)^{-2}\left(1+\frac{1}{2}x^{-1/2}\right)\end{aligned}$$

34. $y=(1+x+x^2)^{11}$

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}\left(1+x+x^2\right)^{11}\\&=11\left(1+x+x^2\right)^{10}\frac{d}{dx}\left(1+x+x^2\right)\\&=11\left(1+x+x^2\right)^{10}(1+2x)\end{aligned}$$

35. $y=x+1+\sqrt{x+1}=x+1+(x+1)^{1/2}$

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}\left(x+1+(x+1)^{1/2}\right)\\&=\frac{d}{dx}x+\frac{d}{dx}1+\frac{d}{dx}(x+1)^{1/2}\\&=1+\frac{1}{2}(x+1)^{-1/2}\end{aligned}$$

36. $y=\pi^2 x$

$$\frac{dy}{dx}=\frac{d}{dx}\left(\pi^2 x\right)=\pi^2$$

37. $f(x)=\left(\frac{\sqrt{x}}{2}+1\right)^{3/2}$

$$\begin{aligned}\frac{d}{dx}\left(\frac{\sqrt{x}}{2}+1\right)^{3/2}&=\frac{3}{2}\left(\frac{\sqrt{x}}{2}+1\right)^{1/2}\cdot\frac{d}{dx}\left(\frac{\sqrt{x}}{2}+1\right)\\&=\frac{3}{2}\left(\frac{\sqrt{x}}{2}+1\right)^{1/2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot x^{-1/2}\\&=\frac{3}{2}\left(\frac{\sqrt{x}}{2}+1\right)^{1/2}\left(\frac{1}{4}x^{-1/2}\right)\\&=\frac{3}{8\sqrt{x}}\left(\frac{\sqrt{x}}{2}+1\right)^{1/2}\end{aligned}$$

38. $y=\left(x-\frac{1}{x}\right)^{-1}=\left(x-x^{-1}\right)^{-1}$

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}\left[\left(x-x^{-1}\right)^{-1}\right]\\&=(-1)\left(x-x^{-1}\right)^{-2}\frac{d}{dx}\left(x-x^{-1}\right)\\&=-\left(x-x^{-1}\right)^{-2}\left(1-(-1)x^{-2}\right)\\&=-\left(x-\frac{1}{x}\right)^{-2}\left(1+\frac{1}{x^2}\right)\\&=-\frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2}=-\frac{x^2+1}{\left(x^2-1\right)^2}\end{aligned}$$

39. $f(x)=3x^2-2x+1,\ (1, 2)$

$$f'(x)=\frac{d}{dx}(3x^2-2x+1)=6x-2$$

slope $=f'(1)=6(1)-2=4$

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40. $f(x)=x^{10}+1+\sqrt{1-x},\ (0,2)$

$$f'(x)=\frac{d}{dx}\left(x^{10}+1+\sqrt{1-x}\right)$$
$$=10x^9+\frac{d}{dx}(1-x)^{1/2}$$
$$=10x^9+\left(\frac{1}{2}(1-x)^{-1/2}\cdot(-1)\right)$$
$$=10x^9-\frac{1}{2\sqrt{1-x}}$$

slope $= f'(0)=10(0)^9-\frac{1}{2\sqrt{1-0}}=-\frac{1}{2}$

41. $y=x^3+3x-8$

$$y'=\frac{d}{dx}\left(x^3+3x-8\right)=3x^2+3$$

slope $= f'(2)=3(2)^2+3=15$

42. $y=x^3+3x-8$

$y'=3x^2+3,$ at $x=2,\ y'=15$

To find the equation of the tangent line, let $(x_1,\ y_1)=(2,\ 6)$ and the slope = 15.

$y-6=15(x-2)\Rightarrow y=15x-24$

43. $y=f(x)=(x^2-15)^6$

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^2-15\right)^6$$
$$=6\left(x^2-15\right)^5\cdot\frac{d}{dx}\left(x^2-15\right)$$
$$=6\left(x^2-15\right)^5\cdot 2x=12x\left(x^2-15\right)^5$$

$f'(x)=12x\left(x^2-15\right)^5$

slope $= f'(4)=12(4)(16-15)^5=48$

$f(4)=(4^2-15)^6=1$

Let $(x_1,\ y_1)=(4,\ 1)$, slope = 48.

$y-1=48(x-4)\Rightarrow y=48x-191$

44. $y=f(x)=\dfrac{8}{x^2+x+2}$

$$f(2)=\frac{8}{2^2+2+2}=1$$

$$f'(x)=\frac{d}{dx}8\left(x^2+x+2\right)^{-1}$$
$$=8(-1)\left(x^2+x+2\right)^{-2}\cdot\frac{d}{dx}\left(x^2+x+2\right)$$
$$=-8\left(x^2+x+2\right)^{-2}(2x+1)$$
$$=\frac{-8(2x+1)}{\left(x^2+x+2\right)^2}$$

slope $= f'(2)=\dfrac{-8(4+1)}{(4+2+2)^2}=-\dfrac{40}{64}=-\dfrac{5}{8}$

Let $(x_1,\ y_1)=(2,\ 1)$.

$y-1=-\frac{5}{8}(x-2)\Rightarrow y=-\frac{5x}{8}+\frac{10}{8}+1\Rightarrow$

$y=-\frac{5x}{8}+\frac{9}{4}$

45. $f(x)=(3x^2+x-2)^2$

a.
$$\frac{d}{dx}\left(3x^2+x-2\right)^2$$
$$=2\left(3x^2+x-2\right)\cdot\frac{d}{dx}\left(3x^2+x-2\right)$$
$$=2\left(3x^2+x-2\right)(6x+1)$$
$$=36x^3+18x^2-22x-4$$

b.
$$\left(3x^2+x-2\right)\left(3x^2+x-2\right)$$
$$=9x^4+3x^3-6x^2+3x^3$$
$$+x^2-2x-6x^2-2x+4$$
$$=9x^4+6x^3-11x^2-4x+4$$
$$\frac{d}{dx}(9x^4+6x^3-11x^2-4x+4)$$
$$=36x^3+18x^2-22x-4$$

46.
$$\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}[f(x)+(-g(x))]$$
$$=\frac{d}{dx}f(x)+\frac{d}{dx}(-g(x))\quad\text{(sum rule)}$$
$$=\frac{d}{dx}f(x)+\frac{d}{dx}(-1)g(x)\quad\text{(const. mult. rule)}$$
$$=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

47. $f(1)=.6(1)+1=1.6$, so $g(1)=3f(1)=4.8$.
$f'(1)=.6$ (slope of the line),
$g'(1)=3f'(1)=1.8$

48. $h(1)=f(1)+g(1)=-.4(1)+2.6+.26(1)+1.1$
$=3.56$
$h'(1)=f'(1)+g'(1)=-.4+.26=-.14$

49. $h(5)=3f(5)+2g(5)=3(2)+2(4)=14$
$h'(5)=3f'(5)+2g'(5)=3(3)+2(1)=11$

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50. $f(x) = 2[g(x)]^3$

$f'(x) = 6[g(x)]^2 g'(x)$

$f(3) = 2[g(3)]^3 = 2(2)^3 = 16$

$f'(3) = 6(2)^2 4 = 96$

51. $f(x) = 5\sqrt{g(x)}$

$f'(x) = \frac{5}{2\sqrt{g(x)}} g'(x)$

$f(1) = 5\sqrt{g(1)} = 5\sqrt{4} = 10$

$f'(1) = \frac{5}{2\sqrt{g(1)}} g'(1) = \frac{5}{2\sqrt{4}} 3 = \frac{15}{4}$

52. $h(1) = \left[f(1)\right]^2 + \sqrt{g(1)} = 1^2 + \sqrt{4} = 3$

$h'(x) = 2\left[f(x)\right] f'(x) + \frac{1}{2}\left[g(x)\right]^{-1/2} g'(x)$

$h'(1) = 2\left[f(1)\right] f'(1) + \frac{1}{2}\left[g(1)\right]^{-1/2} g'(1)$

$= 2(1)(-1) + \frac{1}{2}(4)^{-1/2}(4) = -1$

53. $\frac{dy}{dx} = x^2 - 8x + 18 = 3,$ since the slope of $6x - 2y = 1$ is 3.

$x^2 - 8x + 15 = 0 \Rightarrow (x-5)(x-3) = 0 \Rightarrow$
$x = 5$ or $x = 3$

Fit 3 and 5 back into the equation to get the points (3, 49) and $\left(5, \frac{161}{3}\right)$.

54. $\frac{dy}{dx} = 3x^2 - 12x - 34 = 2$

$3x^2 - 12x - 36 = 0$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = -2$ or $x = 6$

Put -2 and 6 back into the equation to get the points (–2, 27) and (6, –213).

55. $y = f(x)$

slope $= \frac{3-5}{0-4} = \frac{1}{2}$

Let $(x_1, y_1) = (4, 5)$.

$y - 5 = \frac{1}{2}(x-4) \Rightarrow y = \frac{1}{2}x + 3$

$f(4) = \frac{1}{2}(4) + 3 = 2 + 3 = 5$

$f'(4) = \frac{1}{2}$

56. $y = f(x) = \frac{1}{2}x^2 - 4x + 10$

$f(6) = \frac{1}{2}(6)^2 - 4(6) + 10 = 4$

The slope of the tangent line at (6, 4) is

$f'(x) = \frac{d}{dx}\left(\frac{1}{2}x^2 - 4x + 10\right) = \frac{1}{2}\cdot 2x - 4 = x - 4$

$f'(6) = 6 - 4 = 2$

Now find the equation of the tangent line. Let $(x_1, y_1) = (6, 4)$.

$y - 4 = 2(x-6) \Rightarrow y = 2x - 8$

To find the value of b, let $x = 0$ and solve for y.
$y = 2(0) - 8 = -8$

1.7 More About Derivatives

1. $f(t) = \left(t^2+1\right)^5$

$\frac{d}{dt}\left(t^2+1\right)^5 = 5\left(t^2+1\right)^4 \cdot \frac{d}{dt}\left(t^2+1\right)$

$= 5\left(t^2+1\right)^4 (2t)$

$= 10t\left(t^2+1\right)^4$

2. $f(P) = P^3 + 3P^2 - 7P + 2$

$\frac{d}{dP}\left(P^3 + 3P^2 - 7P + 2\right) = 3P^2 + 6P - 7$

3. $v(t) = 4t^2 + 11\sqrt{t} + 1 = 4t^2 + 11t^{1/2} + 1$

$\frac{d}{dt}\left(4t^2 + 11t^{1/2} + 1\right) = 8t + \frac{11}{2}t^{-1/2}$

4. $g(y) = y^2 - 2y + 4$

$\frac{d}{dy}\left(y^2 - 2y + 4\right) = 2y - 2$

5. $y = T^5 - 4T^4 + 3T^2 - T - 1$

$\frac{dy}{dT} = \frac{d}{dT}\left(T^5 - 4T^4 + 3T^2 - T - 1\right)$

$= 5T^4 - 16T^3 + 6T - 1$

6. $x = 16t^2 + 45t + 10$

$\frac{dx}{dt} = \frac{d}{dt}\left(16t^2 + 45t + 10\right) = 32t + 45$

7. $\frac{d}{dP}\left(3P^2 - \frac{1}{2}P + 1\right) = 6P - \frac{1}{2}$

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8. $$\frac{d}{ds}\sqrt{s^2-1}=\frac{d}{ds}\left(s^2-1\right)^{1/2}$$
$$=\frac{1}{2}\left(s^2-1\right)^{-1/2}\frac{d}{ds}\left(s^2-1\right)$$
$$=\frac{1}{2}\left(s^2-1\right)^{-1/2}(2s)=\frac{s}{\sqrt{s^2-1}}$$

9. $$\frac{d}{dt}(a^2t^2+b^2t+c^2)=2a^2t+b^2+0$$
$$=2a^2t+b^2$$

10. $$\frac{d}{dP}\left(T^2+3P\right)^3=3\left(T^2+3P\right)^2\frac{d}{dP}\left(T^2+3P\right)$$
$$=9\left(T^2+3P\right)^2$$

11. $y=x+1$
$y'=1$
$y''=0$

12. $y=(x+12)^3$
$y'=3(x+12)^2$
$y''=6(x+12)=6x+72$

13. $y=\sqrt{x}=x^{1/2}$
$y'=\frac{1}{2}x^{-1/2}$
$y''=-\frac{1}{4}x^{-3/2}$

14. $y=100$
$y'=0$
$y''=0$

15. $y=\sqrt{x+1}=(x+1)^{1/2}$
$y'=\frac{1}{2}(x+1)^{-1/2}$
$y''=-\frac{1}{2}\left(\frac{1}{2}\right)(x+1)^{-3/2}=-\frac{1}{4}(x+1)^{-3/2}$

16. $v=2t^2+3t+11$
$v'=4t+3$
$v''=4$

17. $f(r)=\pi r^2$
$f'(r)=2\pi r$
$f''(r)=2\pi$

18. $y=\pi^2+3x^2$
$y'=6x$
$y''=6$

19. $f(P)=(3P+1)^5$
$$f'(P)=5(3P+1)^4\cdot\frac{d}{dP}(3P+1)$$
$$=5(3P+1)^4\cdot 3=15(3P+1)^4$$
$$f''(P)=60(3P+1)^3\cdot\frac{d}{dP}(3P+1)$$
$$=60(3P+1)^3\cdot 3=180(3P+1)^3$$

20. $T=(1+2t)^2+t^3$
$T'=2\cdot(1+2t)\cdot 2+3t^2=4+8t+3t^2$
$T''=8+6t$

21. $$\left.\frac{d}{dx}(2x+7)^2\right|_{x=1}=\left[2(2x+7)\frac{d}{dx}(2x+7)\right]_{x=1}$$
$$=\left[4(2x+7)\right]\Big|_{x=1}$$
$$=4(2(1)+7)=36$$

22. $$\left.\frac{d}{dt}\left(t^2+\frac{1}{t+1}\right)\right|_{t=0}$$
$$=\left.\frac{d}{dt}\left(t^2+(t+1)^{-1}\right)\right|_{t=0}$$
$$=\left.\left[2t+(-1)(t+1)^{-2}\frac{d}{dt}(t+1)\right]\right|_{t=0}$$
$$=\left.2t-\frac{1}{(t+1)^2}\right|_{t=0}$$
$$=2(0)-\frac{1}{(0+1)^2}=-1$$

23. $$\left.\frac{d}{dz}(z^2+2z+1)^7\right|_{z=-1}$$
$$=\left.\left[7(z^2+2z+1)^6\frac{d}{dz}(z^2+2z+1)\right]\right|_{z=-1}$$
$$=\left.7(2z+2)(z^2+2z+1)^6\right|_{z=-1}$$
$$=7\left(2(-1)+2\right)\left((-1)^2+2(-1)+1\right)$$
$$=0$$

24. $$\left.\frac{d^2}{dx^2}\left(3x^4+4x^2\right)\right|_{x=2}$$
$$\frac{d}{dx}\left(3x^4+4x^2\right)=12x^3+8x$$
$$\frac{d}{dx}\left(12x^3+8x\right)=36x^2+8$$
$$\left.\frac{d^2}{dx^2}\left(3x^4+4x^2\right)\right|_{x=2}=36(2)^2+8=152$$

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25. $\dfrac{d^2}{dx^2}(3x^3 - x^2 + 7x - 1)\Big|_{x=2}$

$\dfrac{d}{dx}\left(3x^3 - x^2 + 7x - 1\right) = 9x^2 - 2x + 7$

$\dfrac{d}{dx}\left(9x^2 - 2x + 7\right) = 18x - 2$

$\dfrac{d^2}{dx^2}(3x^3 - x^2 + 7x - 1)\Big|_{x=2} = 18(2) - 2 = 34$

26. $\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)\Big|_{x=1}$, where $y = x^3 + 2x - 11$

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(x^3 + 2x - 11\right) = 3x^2 + 2$

$\dfrac{d}{dx}\left(3x^2 + 2\right)\Big|_{x=1} = 6x\big|_{x=1} = 6(1) = 6$

27. $f'(1)$ and $f''(1)$, when $f(t) = \dfrac{1}{2+t}$.

$f'(t) = (-1)(2+t)^{-2}$, $f'(1) = -(2+1)^{-2} = -\dfrac{1}{9}$

$f''(t) = (-2)(-1)(2+t)^{-3}$,

$f''(1) = 2(2+1)^{-3} = \dfrac{2}{3^3} = \dfrac{2}{27}$

28. $g'(0)$ and $g''(0)$, when $g(T) = (T+2)^3$.

$g'(T) = 3(T+2)^2 \Rightarrow g'(0) = 3(0+2)^2 = 12$
$g''(T) = 6(T+2) \Rightarrow g''(0) = 6(0+2) = 12$

29. $\dfrac{d}{dt}\left(\dfrac{d\upsilon}{dt}\right)\Big|_{t=32}$, where $\upsilon(t) = 3t^3 + \dfrac{4}{t}$

$\dfrac{d\upsilon}{dt} = \dfrac{d}{dt}\left(3t^3 + \dfrac{4}{t}\right) = 9t^2 - \dfrac{4}{t^2}$

$\dfrac{d}{dt}\left(9t^2 - \dfrac{4}{t^2}\right)\Big|_{t=2} = \left(18t + \dfrac{8}{t^3}\right)\Big|_{t=2}$

$= 18 \cdot 2 + \dfrac{8}{2^3} = 36 + 1 = 37$

30. $\dfrac{d}{dt}\left(\dfrac{d\upsilon}{dt}\right)$, where $\upsilon = 2t^2 + \dfrac{1}{t+1}$

$\dfrac{d\upsilon}{dt} = \dfrac{d}{dt}\left(2t^2 + (t+1)^{-1}\right)$

$= 4t + (-1)(t+1)^{-2}\dfrac{d}{dt}(t+1)$

$= 4t - (t+1)^{-2}$

$\dfrac{d}{dt}\left(\dfrac{d\upsilon}{dt}\right) = \dfrac{d}{dt}\left(4t - (t+1)^{-2}\right)$

$= 4 - (-2)(t+1)^{-3}\dfrac{d}{dt}(t+1)$

$= 4 + \dfrac{2}{(t+1)^3}$

31. $R = 1000 + 80x - .02x^2$, for $0 \le x \le 2000$

$\dfrac{dR}{dx} = 80 - .04x$

$\dfrac{dR}{dx}\Big|_{x=1500} = 80 - .04(1500) = 20$

32. $V = 20\left(1 - \dfrac{100}{100+t^2}\right),\ 0 \le t \le 24$

$V = 20 - 2000\left(100 + t^2\right)^{-1}$

$\dfrac{dV}{dt} = 2000\left(100 + t^2\right)^{-2} \cdot \dfrac{d}{dt}\left(100 + t^2\right)$

$= 2000\left(100 + t^2\right)^{-2} \cdot 2t$

$= 4000t\left(100 + t^2\right)^{-2}$

$\dfrac{dV}{dt}\Big|_{t=10} = \dfrac{4000(10)}{\left(100 + 10^2\right)^2} = 1$

33. a. $f(x) = x^5 - x^4 + 3x$

$f'(x) = 5x^4 - 4x^3 + 3$

$f''(x) = 20x^3 - 12x^2$

$f'''(x) = 60x^2 - 24x$

b. $f(x) = 4x^{5/2}$

$f'(x) = 10x^{3/2}$

$f''(x) = 15x^{1/2}$

$f'''(x) = \dfrac{15}{2}x^{-1/2} = \dfrac{15}{2\sqrt{x}}$

34. a. $f(t) = t^{10}$

$f'(t) = 10t^9$

$f''(t) = 90t^8$

$f'''(t) = 720t^7$

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b. $f(z)=\dfrac{1}{z+5}=(z+5)^{-1}$

$f'(z)=-(z+5)^{-2}$

$f''(z)=2(z+5)^{-3}$

$f'''(z)=-6(z+5)^{-4}=-\dfrac{6}{(z+5)^4}$

35. $s=Tx^2+3xP+T^2$

a. $\dfrac{ds}{dx}=\dfrac{d}{dx}\left(Tx^2+3xP+T^2\right)=2Tx+3P$

b. $\dfrac{ds}{dP}=\dfrac{d}{dP}\left(Tx^2+3xP+T^2\right)=3x$

c. $\dfrac{ds}{dT}=\dfrac{d}{dT}\left(Tx^2+3xP+T^2\right)=x^2+2T$

36. $s=7x^2y\sqrt{z}$

a. $\dfrac{d^2s}{dx^2}=\dfrac{d^2}{dx^2}\left(7x^2y\sqrt{z}\right)=\dfrac{d}{dx}\left(14xy\sqrt{z}\right)$
$=14y\sqrt{z}$

b. $\dfrac{d^2s}{dy^2}=\dfrac{d^2}{dy^2}\left(7x^2y\sqrt{z}\right)=\dfrac{d}{dy}\left(7x^2\sqrt{z}\right)=0$

c. $\dfrac{ds}{dz}=\dfrac{d}{dz}\left(7x^2y\sqrt{z}\right)=\dfrac{7}{2}x^2yz^{-1/2}=\dfrac{7x^2y}{2\sqrt{z}}$

37. $C(50)=5000$ means that it costs \$5000 to manufacture 50 bicycles in one day.
$C'(50)=45$ means that it costs an additional \$45 to make the 51st bicycle.

38. $C(51)\approx C(50)+C'(50)=5000+45=\$5045$

39. $R(x)=3x-.01x^2$, $R'(x)=3-.02x$

a. $R'(20)=3-.02(20)=\$2.60$ per unit

b. $R(x)=3x-.01x^2=200\Rightarrow$.
$x=100$ or $x=200$ units

40. A $\to$ d, B $\to$ b, C $\to$ a, D $\to$ c

41. a. When 1200 chips are produced per day, the revenue is \$22,000 $\Rightarrow$ $R(12)=22$, and the marginal revenue is \$.75 per chip $\Rightarrow$ $R'(12)=\$.075$ thousand / unit (\$.75 per chip = \$75 per unit = \$.075 thousand/unit)

b. Marginal Profit = Marginal Revenue − Marginal Cost
$P'(12)=R'(12)-C'(12)=.75-1.5$
$=-\$.75$ per chip

42. Since $C(12)=14$ and $R(12)=22$, then $P(12)-C(12)=8$. Since $P'(12)=-.075$, then $P(13)\approx 8-.075=7.925$.Thus, it is profitable to raise the production level to 1300.

43. a. The sales at the end of January reached \$120,560, so $S(1)=\$120,560$.
Sales rising at a rate of \$1500/month means that $S'(1)=\$1500$.

b. At the end of March, the sales for the month dropped to \$80,000, so $S(3)=\$80,000$. Sales falling by about \$200/day means that
$S'(30)=-\$200(30)=-\$6000$.

44. a. $S(10)=3+\dfrac{9}{(10+1)^2}\approx\$3.07438$ thousand

$S'(10)=\dfrac{-18}{(10+1)^3}\approx-\$0.013524$ thousand/day

b. Rate of change of sales on January 2: −\$0.667 thousand/day (down \$667/day), Rate of change of sales on January 10: −\$0.013524 thousand/day (down \$13/day). Although sales are still down on January 10, the rate at which the sales are down is increasing, and since the rates are negative, this implies sales are getting better.

45. a. $S(10)=3+\dfrac{9}{11^2}\approx\$3.074$ thousand

$S'(10)=\dfrac{-18}{11^3}\approx\$-.0135$ thousand/day

b. $S(11)\approx S(10)+S'(10)$
$\approx 3.07438+(-.013524)$
$\approx\$3.061$ thousand

$S(11)=3+\dfrac{9}{12^2}=\$3.0625$ thousand

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46. **a.** $T(1) = \frac{24}{5} + \frac{36}{5(3(1)+1)^2} = \$5.25$ thousand

$$T'(x) = \frac{d}{dx}\left(\frac{24}{5} + \frac{36}{5}(3x+1)^{-2}\right)$$
$$= -\frac{72}{5}(3x+1)^{-3}(3)$$
$$= -\frac{216}{5}(3x+1)^{-3}$$
$$T'(1) = -\frac{216}{5}(3(1)+1)^{-3}$$
$$= -\$0.675 \text{ thousand/day}$$
$$S(1) = 3 + \frac{9}{4} = \$5.25 \text{ thousand}$$
$$S'(1) = -\frac{18}{8} = -\$2.25 \text{ thousand/day}$$

b. According to both functions, the sales were $5250 on January 1. Although, the rate at which sales fell on that date differ. $T(x)$ gives a much smaller rate of sales dropping then $S(x)$.

47. **a.** When $8000 is spent on advertising, 1200 computers were sold , so $A(8) = 12$, and sales rising at the rate of 50 computers for each $1000 spent on advertising means that $A'(8) = .5$.

b. $A(9) \approx A(8) + A'(8) = 12.5$ (hundred)
$= 1250$ computers

48. $S(n)$: The number of video games sold on day n since the item was released.
$S'(n)$: The rate at which the video games are being sold on day n since the item was released.
$S(n) + S'(n)$: The approximate number of video games sold on day n + 1 since the item was released.

49. Federal debt at the end of 1999 = $D(4)$

$$D(4) = 4.95 + .402(4) - .1067(4)^2 + .0124(4)^3 - .00024(4)^4$$
$$= \$5.58296 \text{ trillion}$$

Rate of increase at the end of 1999 = $D'(4)$

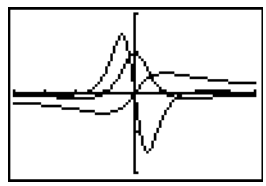
$$D'(x) = .402 - .2134x + .0372x^2 - .00096x^3$$
$$D'(4) = .402 - .2134(4) + .0372(4)^2 - .00096(4)^3$$
$$= \$.08216 \text{ trillion/year}$$

50. **a.** $D(8) = \$6.70296$ trillion
$D(6) = \$5.88816$ trillion
No, the federal debt was not twice as large by the end of 2003 than the end of 2001.

b. $D'(8) = \$.58408$ trillion/year
$D'(6) = \$.25344$ trillion/year
Yes, it is true that by the end of 2003 the federal debt was increasing at a rate that was more than twice the rate at the end of 2001.

51. $f(x) = \frac{x}{1+x^2}$

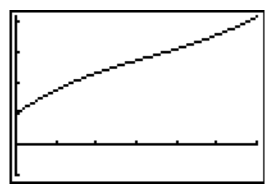
$Y_1 = \frac{X}{1+X^2}$; $Y_2 = \text{nDeriv}(Y_1, X, X)$
$Y_3 = \text{nDeriv}(Y_2, X, X)$



[–4, 4] by [–2, 2]

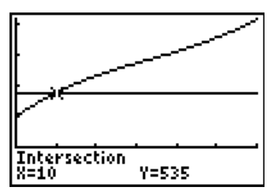
52. $C(x) = .005x^3 - .5x^2 + 28x + 300$

a.



[0, 60] by [–300, 1260]

b. $C(x) = 535$



Graphing the line y = 535 and using the **INTERSECT** command, the point (10, 535) is on both graphs. A level of production of 10 items has a cost of $535.

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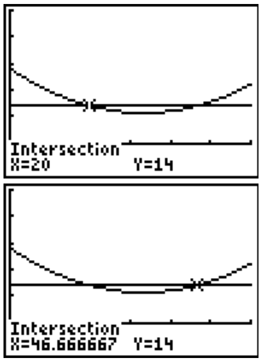
c. $C'(x) = 14$

Graph the derivative:

$Y_1 = .015X^2 - X + 28$ and $Y_2 = 14$.



[0, 60] by [−10, 50]

Using the **INTERSECT** command, the points (20, 14) and $\left(46\frac{2}{3}, 14\right)$ are on both graphs. The marginal cost will be \$14 at production levels of 20 items and 47 items.

1.8 The Derivative as a Rate of Change

1. $f(x) = 4x^2$

a. Over $1 \le x \le 2$,

$$\frac{f(b)-f(a)}{b-a} = \frac{4(2)^2 - 4(1)^2}{2-1} = \frac{16-4}{1} = 12$$

over $1 \le x \le 1.5$,

$$\frac{f(b)-f(a)}{b-a} = \frac{4(1.5)^2 - 4(1)^2}{1.5-1} = \frac{9-4}{.5} = 10$$

over $1 \le x \le 1.1$,

$$\frac{f(b)-f(a)}{b-a} = \frac{4(1.1)^2 - 4(1)^2}{1.1-1} = \frac{4.84-4}{.1} = 8.4$$

b. $f'(x) = 8x \Rightarrow f'(1) = 8$

2. $f(x) = -\frac{6}{x}$

a. Over $1 \le x \le 2$,

$$\frac{f(b)-f(a)}{b-a} = \frac{-\frac{6}{2} - \left(-\frac{6}{1}\right)}{2-1} = \frac{-3+6}{1} = 3$$

over $1 \le x \le 1.5$,

$$\frac{f(b)-f(a)}{b-a} = \frac{-\frac{6}{1.5} - \left(-\frac{6}{1}\right)}{1.5-1} = \frac{-4+6}{.5} = 4$$

over $1 \le x \le 1.2$,

$$\frac{f(b)-f(a)}{b-a} = \frac{-\frac{6}{1.2} - \left(-\frac{6}{1}\right)}{1.2-1} = \frac{-5+6}{.2} = 5$$

b. $f'(x) = \frac{6}{x^2} \Rightarrow f'(1) = \frac{6}{1^2} = 6$

3. $f(t) = t^2 + 3t - 7$

a. Over $5 \le x \le 6$,

$$\frac{f(b)-f(a)}{b-a} = \frac{6^2 + 3(6) - 7 - (5^2 + 3(5) - 7)}{6-5} = 36 + 18 - 7 - 25 - 15 + 7 = 14$$

b. $f'(t) = 2t + 3$

$f'(5) = 2(5) + 3 = 13$

4. $f(t) = 3t + 2 - \frac{12}{t}$

a. Over $2 \le x \le 3$,

$$\frac{f(b)-f(a)}{b-a} = \frac{3(3) + 2 - \frac{12}{3} - \left(3(2) + 2 - \frac{12}{2}\right)}{3-2} = 9 + 2 - 4 - 6 - 2 + 6 = 5$$

b. $f'(t) = 3 + \frac{12}{t^2}$

$f'(2) = 3 + \frac{12}{2^2} = 3 + 3 = 6$

5. a. $f(1) = 14, f(5) = 7$

$$\frac{f(b)-f(a)}{b-a} = \frac{7-14}{5-1} = -\frac{7}{4}$$

b. Slope of tangent line at $t = 9$:

$$\frac{10-5}{11-5} = \frac{5}{6}$$

The yield was rising at the rate of $\frac{5}{6}$ percent per year on January 1, 1989.

c. The graph is clearly steeper in 1980 than in 1989, so the percentage yield was rising faster on January 1, 1980.

6. a. $f(20) = 150, f(60) = 300$

$$\frac{f(b)-f(a)}{b-a} = \frac{300-150}{60-20} = \frac{150}{40} = \frac{15}{4} = 3.75 \text{ acres per year}$$

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b. The tangent line passes through (80,400) and (40, 150). The slope of tangent line at $x = 50$ is $\frac{400-150}{80-40} = \frac{250}{40} = \frac{25}{4} = 6.25$
The mean farm size was increasing at the rate of 6.25 acres per year on January 1, 1950.

c. The graph is steeper in 1960 than in 1980, so the mean farm size was rising faster on January 1, 1960.

7. $s(t) = 2t^2 + 4t$

a. $s'(t) = 4t + 4$
$s'(6) = 4(6) + 4 = 28$ km/hr

b. $s(6) = 2(6)^2 + 4(6) = 72 + 24 = 96$ km

c. When does $s'(t) = 6$?
$s'(t) = 4t + 4$
$6 = 4t + 4 \Rightarrow t = \frac{1}{2}$
The object is traveling at the rate of 6 km/hr when $t = \frac{1}{2}$ hr.

8. $f(t) = -3t^2 + 32t + 100$
Over $3 \le t \le 4$,
$$\frac{f(b)-f(a)}{b-a}$$
$$= \frac{-3(4)^2 + 32(4) + 100 - (-3(3)^2 + 32(3) + 100)}{4-3}$$
$= -48 + 128 + 100 + 27 - 96 - 100$
$= 11$ units/day
$f'(t) = -6t + 32$
$f'(2) = -6(2) + 32 = 20$ units/day

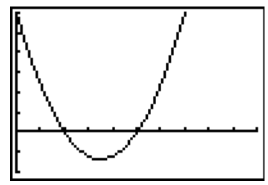
9. $f(t) = 60t + t^2 - \frac{1}{12}t^3$
$f'(t) + 60 + 2t - \frac{1}{4}t^2$
$f'(2) = 60 + 2(2) - \frac{1}{4}(2)^2 = 60 + 4 - 1$
$= 63$ units/hour

10. $f(t) = 5t + \sqrt{t}$
$f'(t) = 5 + \frac{1}{2\sqrt{t}}$
$f'(4) = 5 + \frac{1}{2\sqrt{4}} = 5 + \frac{1}{4} = 5.25$ gallons/hour

11. $s(t) = 2t^3 - 21t^2 + 60t,\ t \ge 0$

a. $v(t) = s'(t) = 6t^2 - 42t + 60$
$v(3) = 6(3)^2 - 42(3) + 60 = -12$ ft/sec
$v(6) = 6(6)^2 - 42(6) + 60 = 24$ ft/sec

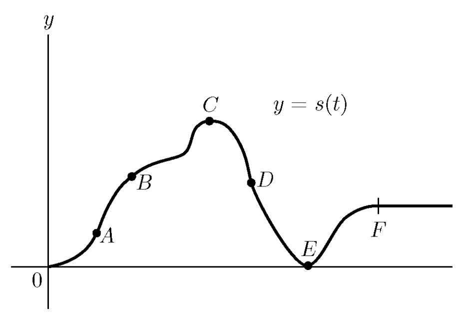
b. To find when the particle is moving in the positive direction, solve $v(t) > 0$.
$6t^2 - 42t + 60 > 0$
$t^2 - 7t + 10 = 0$
$(t-5)(t-2) = 0 \Rightarrow t = 5$ or $t = 2$
Looking at the graph of $v(t)$, we see that the velocity is positive for $0 \le t < 2$ or $t > 5$.



[0, 10] by [−20, 60]

c. The total distance is not $s(7)$ because the particle moves in the negative direction for $2 < t < 5$. The total distance is $s(2) + (s(2) - s(5)) + (s(7) - s(5))$.
$s(2) = 2(2)^3 - 21(2)^2 + 60(2) = 52$
$s(5) = 2(5)^3 - 21(5)^2 + 60(5) = 25$
$s(7) = 2(7)^3 - 21(7)^2 + 60(7) = 77$
$s(2) + (s(2) - s(5)) + (s(7) - s(5))$
$= 52 + (52 - 25) + (77 - 25) = 131$ ft

12.



a. The slope of the tangent line at A is steeper than the slope of the tangent line at B, so the car was going faster at A.

b. Since the car was going slower at B than at A, the car's acceleration was negative at B.

c. The tangent line at C is horizontal, with slope 0, so the car's velocity is 0.

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d. The slope of the tangent line at D is negative, so the car's velocity is negative. In other words, the car is traveling backwards at D.

e. At E, the car is back at its starting point and is at rest. (Its velocity is 0.)

f. After F, the car's velocity is 0.

13. $s(t) = 160t - 16t^2$

a. $s'(t) = 160 - 32t$

$s'(0) = 160 - 32(0) = 160$ ft/sec

b. $s'(2) = 160 - 32(2) = 160 - 64 = 96$ ft/sec

c. $s''(t) = -32$

$s''(3) = -32$ ft/sec^2

d. When will $s(t) = 0$?

$160t - 16t^2 = 0 \Rightarrow 16t(10 - t) = 0 \Rightarrow$
$t = 0$ sec or $t = 10$ sec
The rocket will hit the ground after 10 sec.

e. What is $s'(t)$ when $t = 10$?

$s'(10) = 160 - 32(10) = 160 - 320$
$= -160$ ft/sec

14. $s(t) = t^2 + t$

a. When will $s(t) = 20$?

$20 = t^2 + t \Rightarrow t^2 + t - 20 = 0 \Rightarrow$
$(t-4)(t+5) = 0 \Rightarrow t = 4$ or $t = -5$

t must be positive, so the helicopter takes 4 seconds to rise 20 feet.

b. $s'(t) = 2t + 1$

$s'(4) = 2(4) + 1 = 9$ feet/second

$s''(t) = 2$

$s''(4) = 2$ feet/second^2

15. A. The velocity of the ball after 3 seconds is the first derivative evaluated at $t = 3$, or $s'(3)$. The solution is **b**.

B. To find when the velocity will be 3 feet per second, set $s'(t) = 3$ and solve for t. The solution is **d**.

C. The average velocity during the first 3 seconds can be found from:

$$\frac{f(b) - f(a)}{b - a} = \frac{s(3) - s(0)}{3}$$

The solution is **f**.

D. The ball will be 3 feet above the ground when, for some value a, $s(a) = 3$. The solution is **e**.

E. The ball will hit the ground when $s(t) = 0$. Solve for t. The solution is **a**.

F. The ball will be $s(3)$ feet high after 3 seconds. The solution is **c**.

G. The ball travels $s(3) - s(0)$ feet during the first 3 seconds. The solution is **g**.

16. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{47.4 - 45}{1.05 - 1} = \dfrac{2.4}{.05} = 48$ mi/hr

To estimate the speed at time 1 hour, calculate the average speed in a small interval near one hour:

$$\frac{f(b) - f(a)}{b - a} = \frac{f(1.01) - f(1)}{1.01 - 1} = \frac{45.4 - 45}{.01}$$
$$= \frac{.4}{.01} = 40 \text{ mi/hr}$$

17. $s(t) = t^2 + 3t + 2$

a. $s'(t) = 2t + 3$

$s'(6) = 2(6) + 3 = 12 + 3 = 15$ feet/second

b. No; the positive velocity indicates the object is moving away from the reference point.

c. The object is 6 feet from the reference point when $s(t) = 6$.

$s(t) = t^2 + 3t + 2 = 6$

$t^2 + 3t - 4 = 0 \Rightarrow (t+4)(t-1) = 0 \Rightarrow$
$t = -4$ or $t = 1$
Time is positive, so $t = 1$ second. The velocity at this time is:
$s'(1) = 2(1) + 3 = 5$ feet/second

18. a. If the car travels at a steady speed, the distance of the car from New York will increase at a constant rate. The distance function will be a straight line with a positive slope. The answer is **b**.

b. If the car is stopped, the value of the distance function will not change, so the function will be a straight line with slope of 0. The answer is **c**.

c. If the car is backing up, its distance function will have a negative slope. The answer is **d**.

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d. If the car is accelerating, its velocity is increasing, so the slopes of tangents to the distance curve are increasing. The answer is **a**.

e. If the car is decelerating, its velocity is decreasing, so the slopes of the tangents to the distance curve are decreasing. The answer is **e**.

19. $f(100) = 5000$
$f'(100) = 10$
$f(a+h) - f(a) \approx f'(a)\cdot h$
$f(a+h) \approx f'(a)\cdot h + f(a)$

a. $101 = 100 + 1$
$$f(100+1) \approx f'(100)\cdot 1 + f(100)$$
$$\approx 10 + 5000 \approx 5010$$

b. $100.5 = 100 + .5$
$$f(100+.5) \approx f'(100)\cdot .5 + f(100)$$
$$\approx 10\cdot .5 + 5000 \approx 5005$$

c. $99 = 100 + (-1)$
$$f(100+(-1)) \approx f'(100)\cdot(-1) + f(100)$$
$$\approx 10\cdot(-1) + 5000 \approx 4990$$

d. $98 = 100 + (-2)$
$$f(100+(-2)) \approx f'(100)\cdot(-2) + f(100)$$
$$\approx 10(-2) + 5000 \approx 4980$$

e. $99.75 = 100 + (-.25)$
$$f(100+(-.25))$$
$$\approx f'(100)\cdot(-.25) + f(100)$$
$$\approx 10(.25) + 5000 \approx 4997.5$$

20. $f(25) = 10$
$f'(25) = -2$
$f(a+h) \approx f'(a)\cdot h + f(a)$

a. $27 = 25 + 2$
$$f(25+2) \approx f'(25)\cdot 2 + f(25)$$
$$\approx -2\cdot 2 + 10 \approx 6$$

b. $26 = 25 + 1$
$$f(25+1) \approx f'(25)\cdot 1 + f(25)$$
$$\approx -2\cdot 1 + 10 \approx 8$$

c. $25.25 = 25 + .25$
$$f(25+.25) \approx f'(25)\cdot .25 + f(25)$$
$$\approx -2\cdot .25 + 10 \approx 9.5$$

d. $24 = 25 + (-1)$
$$f(25+(-1)) \approx f'(25)\cdot(-1) + f(25)$$
$$\approx -2\cdot(-1) + 10 \approx 12$$

e. $23.5 = 25 + (-1.5)$
$$f(25+(-1.5)) \approx f'(25)\cdot(-1.5) + f(25)$$
$$\approx 2\cdot(-1.5) + 10 \approx 13$$

21. $f(4) = 120;\ f'(4) = -5$
Four minutes after it has been poured, the temperature of the coffee is 120°. At that time, its temperature is decreasing by 5° per minute.
At 4.1 minutes: $4.1 = 4 + .1$
$$f(4+.1) \approx f'(4)\cdot .1 + f(4)$$
$$\approx -5\cdot .1 + 120 \approx 119.5°$$

22. $f(3) = 2;\ f'(3) = -.5$
Three hours after it is injected, the amount of the drug present in the bloodstream is 2 mg. At that time, the concentration of the drug is decreasing by .5 mg/hour.
At 3.5 hours: $3.5 = 3 + .5$
$$f(3+.5) \approx f'(3)\cdot .5 + f(3)$$
$$\approx -.5\cdot .5 + 2 \approx 1.75 \text{ mg}$$

23. $f(10{,}000) = 200{,}000;\ f'(10{,}000) = -3$
When the price of a car is \$10,000, 200,000 cars are sold. At that price, the number of cars sold decreases by 3 for each dollar increase in the price.

24. $f(100{,}000) = 3{,}000{,}000;\ f'(100{,}000) = 30$
When \$100,000 is spent on advertising, 3,000,000 toys are sold. For every dollar increase in advertising from that amount, 30 more toys are sold.

25. $f(12) = 60;\ f'(12) = -2$
When the price of a computer is \$1200, 60,000 computers will be sold. At that price, the number of computers sold decreases by 2000 for every \$100 increase in price.
$$f(12.5) \approx f(12) + .5f'(12)$$
$$= 60 + .5(-2) = 59$$
About 59,000 computers will be sold if the price increases to \$1250.

26. $C(2000) = 50{,}000;\ C'(2000) = 10$
When 2000 radios are manufactured, the cost to manufacture them is \$50,000. For every additional radio manufactured, there is an additional cost of \$10.
At 1998 radios: $1998 = 2000 + (-2)$
$$C(2000+(-2)) = C'(2000)\cdot(-2) + C(2000)$$
$$\approx 10\cdot(-2) + 50{,}000 \approx \$49{,}980$$

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27. $P(100) = 90{,}000$; $P'(100) = 1200$

The profit from manufacturing and selling 100 luxury cars is $90,000. Each additional car made and sold creates an additional profit of $1200.

At 99 cars: $99 = 100 + (-1)$

$$\begin{aligned} f(100+(-1)) &\approx f'(100)\cdot(-1) + f(100) \\ &\approx 1200(-1) + 90{,}000 \\ &\approx \$88{,}800 \end{aligned}$$

28. a. $f(100) = 16$; $f'(100) = .25$

The value of the company is $16 per share 100 days since the company went public. At 100 days since the company went public, the value is increasing at a rate of $.25/day.

b. $f(101) \approx f(100) + f'(100) = .25 + 16$
$= \$16.25/\text{share}$

29. $C(x) = 6x^2 + 14x + 18$

a. $C'(x) = 12x + 14$

$C'(5) = 12(5) + 14 = \$74$ thousand/unit

b. $C(5.25) \approx C'(5)(.25) + C(5)$
$= 74(.25) + 238$
$= \$256.5$ thousand

c. Solve

$6x^2 + 14x + 18 = -x^2 + 37x + 38$

$7x^2 - 23x - 20 = 0$

$(x-4)(7x+5) = 0 \Rightarrow x = 4 \text{ or } x = -\frac{5}{7}$

The break even point is $x = 4$ items.

d. $R'(x) = -2x + 37$
$R'(4) = \$29$ thousand/unit

$C'(x) = 12x + 14$
$C'(4) = \$62$ thousand/unit

No, the company should not increase production beyond $x = 4$ items. The additional cost is greater than the additional revenue generated and the company will lose money.

30. $f(.9) \approx f'(1)(-.1) + f(1)$

$f(x) = (1+x^2)^{-1}$; $f(1) = .5$

$f'(x) = -(1+x^2)^{-2}(2x)$;

$f'(1) = -.5 \Rightarrow f(.9) \approx (-.5)(.1) + .5 = .55$

The function will increase by .05.

31. a. $f(7) \approx \$500$ billion

b. $f'(7) \approx \$50$ billion/year

c. $f(t) = 1000$ at $t = 14$, or 1994.

d. $f'(t) = 100$ at $t = 14$, or 1994.

32. a. Find $s(3.5) = 60$ feet.

b. Find $s'(2) = 20$ feet/second.

c. Find $s''(1) = 10$ feet/second^2.

d. Find $s(t) = 120$; $t = 5.5$ seconds.

e. Find $s'(t) = 20$; $t = 7$ seconds.

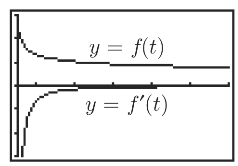
f. Find maximum $s'(t)$; at $t = 4.5$ seconds, $s'(t) = 30$ feet/second.
$s(4.5) = 90$ feet

33. $f(t) = .36 + .77(t - .5)^{-.36}$

a. Graph:

$Y_1 = .36 + .77(X - .5)^{-.36}$

$Y_2 = \text{nDeriv}(Y_1, X, X)$



[.5, 6] by [−3, 3]

b. Evaluate at $t = 4$.
$f(4) \approx .85$ seconds

c. Graphing the line $y = .8$ and using the **INTERSECT** command, the point (5.23, .8) is on both graphs. After 5 days the judgment time was about .8 seconds.

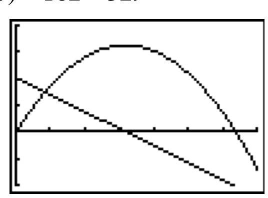
d. Evaluate $f'(t)$ at $t = 4$.
$f'(4) \approx -.05$ seconds/day

e. Graphing the line $y = -.08$ and $f'(t)$, and using the **INTERSECT** command, the point (2.994, –.08) is on both graphs. After 3 days the judgment time was changing at the rate of –.08 seconds per day.

34. $s(t) = 102t - 16t^2$

$s'(t) = 102 - 32t$

a.



[0, 7] by [−100, 200]

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b. Evaluate $s(t)$ at $t = 2$.
$s(2) = 140$

c. Graphing the line $y = 110$ and using the Intersect command, the point (5, 110) is on both graphs. During the descent, at 5 seconds the ball has a height of 110 feet.

d. $s'(6) = -90$

e. When is $s'(t) = 70$?
Graphing the lines $y = 70$ and $s'(t)$ and using the **INTERSECT** command, the point (1, 70) is on both graphs. The velocity is 70 feet/second at 1 second.

f. When is $s(t) = 0$?
Using the **ROOT** command, at 6.375 $s'(6.375) = -102$ feet/second.

Chapter 1 Fundamental Concept Check Exercises

1. The slope of a nonvertical line is the rate of change of the line. It is given by the ratio $\frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are any two distinct points on the line. The slope measures the steepness of the line.

2. The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$, where m is the lsope and (x_1, y_1) is a point on the line.

3. First find the slope using the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$. Then, use the point-slope form of the equation with any one of the two given points.

4. Slopes of parallel lines are equal. Slopes of perpendicular lines are the opposite reciprocal of each other.

5. The slope of $f(x)$ at the point $(2, f(2))$ is the slope of the tangent line to the graph of the tangent line to the graph of $y = f(x)$ at the point $(2, f(2))$.

6. $f'(2)$ represents the slope of the tangent line to the graph of $y = f(x)$ at the point $(2, f(2))$.

7. The derivative or slope formula for a line function $f(x) = mx + b$ is m. For a constant function, $m = 0$, so the derivative is 0.

8. Power rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\frac{d}{dx}(x^7) = 7x^6$$

Constant-multiple rule:

$$\frac{d}{dx}[k \cdot f(x)] = k\frac{d}{dx}[f(x)]$$

$$\frac{d}{dx}(-3x^4) = (-3)\frac{d}{dx}(x^4) = (-3)(4x^3) = -12x^3$$

Sum rule:

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

$$\frac{d}{dx}\left[x + \frac{1}{x}\right] = \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{1}{x}\right) = 1 - \frac{1}{x^2}$$

9. The slope of the secant line through $P = (2, f(2))$ and $Q = (2 + h, f(2 + h))$ is $\frac{f(2+h) - f(2)}{(2+h) - 2} = \frac{f(2+h) - f(2)}{h}$. As h approaches 0, the point Q approaches P and the slope of the secant line approaches the slope of the tangent line $f'(2)$.

10. $\lim_{x \to 2} f(x) = 3$ means that the values of $f(x)$ become arbitrarily close to 3 as x approaches 2. An example is $\lim_{x \to 2}(2x - 1) = 4 - 1 = 3$.

11. $f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$.

12. $\lim_{x \to \infty} f(x) = 3$ means that the values of $f(x)$ become arbitrarily close to 3 as x approaches infinity; that is, as x becomes very large in positive values. $\lim_{x \to -\infty} f(x) = 3$ means that the values of $f(x)$ become arbitrarily close to 3 as x approaches minus infinity; that is, as x becomes very large in negative values.

Examples are $\lim_{x \to \infty}\left(3 + \frac{1}{x}\right) = 3 + 0 = 3$ and

$$\lim_{x \to -\infty}\left(3 + \frac{1}{x}\right) = 3 - 0 = 3$$

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13. $f(x)$ is continuous at $x = 2$ if $\lim_{x\to 2} f(x) = f(2)$. An example of a function that is not continuous at $x = 2$ is

$$f(x) = \begin{cases} 1 & \text{if } x \ge 2 \\ -1 & \text{if } x < 2 \end{cases}.$$

14. $f(x)$ is differentiable at $x = 2$ if $f'(2)$ exists. If f is differentiable, then it has to be continuous. The function given in exercise 13 is an example of a function that is not differentiable at $x = 2$ because it is not continuous at $x = 2$.

15. General power rule:

$$\frac{d}{dx}\left[(g(x))^n\right] = n(g(x))^{n-1}\frac{d}{dx}\left[g(x)\right]$$

For example:

$$\frac{d}{dx}\left[\left(x^2+2x-3\right)^9\right]$$
$$= 9\left(x^2+2x-3\right)^8 \frac{d}{dx}\left[x^2+2x-3\right]$$
$$= 9\left(x^2+2x-3\right)^8(2x+2)$$

16. The first derivative of $f(x)$ at $x = 2$ can be represented by $f'(2)$ and $\left.\frac{d}{dx}\left[f(x)\right]\right|_{x=2}$. The second derivative can be represented by $f''(2)$ and $\left.\frac{d^2}{dx^2}\left[f(x)\right]\right|_{x=2}$.

17. The average rate of change of a function over an interval is given by $\dfrac{f(b)-f(a)}{b-a}$.

18. The average rate of change approaches the instantaneous rate of change as the size of the interval $b - a$ approaches 0.

19. If $s(t)$ represents the position of an object, then the first derivative $s'(t)$ represents the velocity at time t, $v(t)$. Acceleration at time t, $a(t)$, is represented by the first derivative of $v(t)$, $v'(t)$ or the second derivative of $s(t)$, $s''(t)$. Thus, $v(t) = s'(t)$ and $a(t) = v'(t) = s''(t)$.

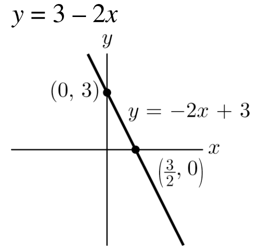
20. $f(a+h) - f(a) \approx f'(a)\cdot h$

21. The marginal cost is the additional cost that is incurred when the production level is increased by 1 unit. If $C(x)$ is a cost function, then the marginal cost function is given by the first derivative, $C'(x)$.

22. Unit of measure for $f'(x) =$ unit of measure for $f(x)$ per unit of measure for x. For example if $C(x)$ is the cost in dollars of manufacturing x units of a certain item, then the marginal cost, is measured in dollars per unit item.

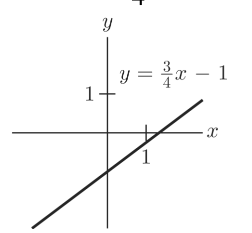
Chapter 1 Review Exercises

1. Let $(x_1, y_1) = (0, 3)$.
$y - 3 = -2(x - 0)$
$y = 3 - 2x$

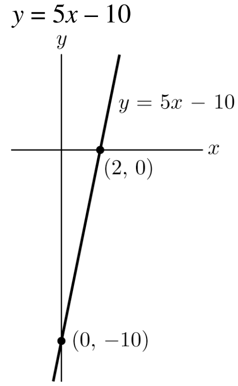


2. Let $(x_1, y_1) = (0, -1)$.

$$y - (-1) = \frac{3}{4}(x - 0) \Rightarrow y = \frac{3}{4}x - 1$$



3. Let $(x_1, y_1) = (2, 0)$.
$y - 0 = 5(x - 2)$
$y = 5x - 10$



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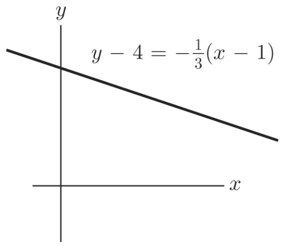
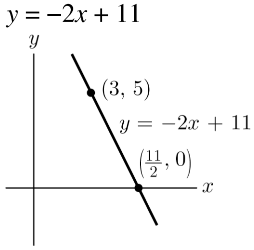
4. Let $(x_1, y_1) = (1, 4)$.

$$y - 4 = -\frac{1}{3}(x - 1)$$

$$y = -\frac{x}{3} + \frac{13}{3}$$



5. $y = -2x$, slope $= -2$
Let $(x_1, y_1) = (3, 5)$.
$y - 5 = -2(x - 3)$
$y = 11 - 2x$ or
$y = -2x + 11$



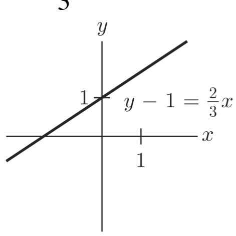
6. $-2x + 3y = 6$

$y = 2 + \frac{2}{3}x$, slope $= \frac{2}{3}$

Let $(x_1, y_1) = (0, 1)$.

$$y - 1 = \frac{2}{3}(x - 0)$$

$$y = \frac{2}{3}x + 1$$

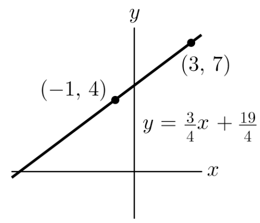


7. slope $= \dfrac{7 - 4}{3 - (-1)} = \dfrac{3}{4}$

Let $(x_1, y_1) = (3, 7)$.

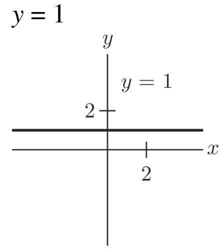
$$y - 7 = \frac{3}{4}(x - 3)$$

$$y = \frac{3}{4}x + \frac{19}{4}$$



8. slope $= \dfrac{1 - 1}{5 - 2} = 0$

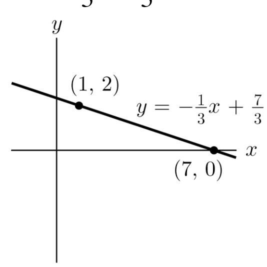
Let $(x_1, y_1) = (2, 1)$.
$y - 1 = 0(x - 2)$
$y = 1$



9. Slope of $y = 3x + 4$ is 3, thus a perpendicular line has slope of $-\frac{1}{3}$. The perpendicular line through (1, 2) is

$$y - 2 = \left(-\frac{1}{3}\right)(x - 1)$$

$$y = -\frac{1}{3}x + \frac{7}{3}$$



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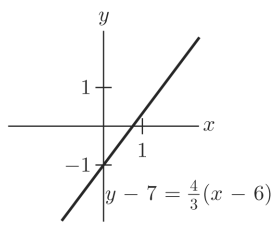
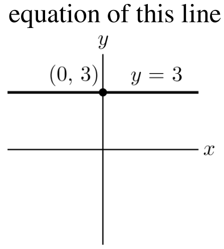
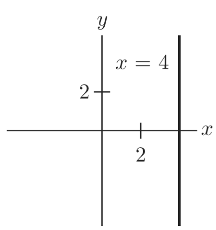
10. Slope of $3x + 4y = 5$ is $-\frac{3}{4}$ since $y = -\frac{3}{4}x + \frac{5}{4}$, thus a perpendicular line has slope of $\frac{4}{3}$. The perpendicular line through (6, 7) is $y - 7 = \frac{4}{3}(x - 6)$ or $y = \frac{4}{3}x - 1$.



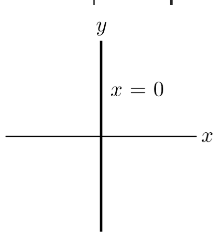
11. The equation of the x-axis is $y = 0$, so the equation of this line is $y = 3$.



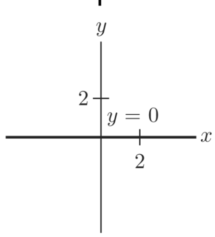
12. The equation of the y-axis is $x = 0$, so 4 units to the right is $x = 4$.



13.



14.



15. $y = x^7 + x^3;\ y' = 7x^6 + 3x^2$

16. $y = 5x^8;\ y' = 40x^7$

17. $y = 6\sqrt{x} = 6x^{1/2};\ y' = 3x^{-1/2} = \frac{3}{\sqrt{x}}$

18. $y = x^7 + 3x^5 + 1;\ y' = 7x^6 + 15x^4$

19. $y = \frac{3}{x} = 3x^{-1};\ y' = -3x^{-2} = -\frac{3}{x^2}$

20. $y = x^4 - \frac{4}{x} = x^4 - 4x^{-1}$

$y' = 4x^3 + 4x^{-2} = 4x^3 + \frac{4}{x^2}$

21. $y = (3x^2 - 1)^8$

$y' = 8(3x^2 - 1)^7(6x) = 48x(3x^2 - 1)^7$

22. $y = \frac{3}{4}x^{4/3} + \frac{4}{3}x^{3/4}$

$y' = x^{1/3} + x^{-1/4}$

23. $y = \frac{1}{5x - 1} = (5x - 1)^{-1}$

$\frac{dy}{dx} = -(5x - 1)^{-2}(5) = -\frac{5}{(5x - 1)^2}$

24. $y = (x^3 + x^2 + 1)^5$

$y' = 5(x^3 + x^2 + 1)^4(3x^2 + 2x)$

25. $y = \sqrt{x^2 + 1} = (x^2 + 1)^{1/2}$

$y' = \frac{1}{2}(x^2 + 1)^{-1/2}(2x)$

$= \frac{1}{2}x(x^2 + 1)^{-1/2} = \frac{x}{\sqrt{x^2 + 1}}$

26. $y = \frac{5}{7x^2 + 1} = 5(7x^2 + 1)^{-1}$

$\frac{dy}{dx} = -5(7x^2 + 1)^{-2}(14x) = -\frac{70x}{(7x^2 + 1)^2}$

27. $f(x) = \frac{1}{\sqrt[4]{x}} = x^{-1/4}$

$f'(x) = -\frac{1}{4}x^{-5/4} = -\frac{1}{4x^{5/4}}$

28. $f(x) = (2x + 1)^3$

$f'(x) = 3(2x + 1)^2(2) = 6(2x + 1)^2$

29. $f(x) = 5;\ f'(x) = 0$

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30. $f(x)=\dfrac{5x}{2}-\dfrac{2}{5x}=\dfrac{5}{2}x-\dfrac{2}{5}x^{-1}$

$f'(x)=\dfrac{5}{2}+\dfrac{2}{5}x^{-2}=\dfrac{5}{2}+\dfrac{2}{5x^2}$

31. $f(x)=[x^5-(x-1)^5]^{10}$

$f'(x)=10[x^5-(x-1)^5]^9[5x^4-5(x-1)^4]$

32. $f(t)=t^{10}-10t^9;\ f'(t)=10t^9-90t^8$

33. $g(t)=3\sqrt{t}-\dfrac{3}{\sqrt{t}}=3t^{1/2}-3t^{-1/2}$

$g'(t)=\dfrac{3}{2}t^{-1/2}+\dfrac{3}{2}t^{-3/2}$

34. $h(t)=3\sqrt{2};\ h'(t)=0$

35. $f(t)=\dfrac{2}{t-3t^3}=2(t-3t^3)^{-1}$

$f'(t)=-2(t-3t^3)^{-2}(1-9t^2)=\dfrac{-2(1-9t^2)}{(t-3t^3)^2}$

$=\dfrac{2(9t^2-1)}{(t-3t^3)^2}$

36. $g(P)=4P^{.7};\ g'(P)=2.8P^{-.3}$

37. $h(x)=\dfrac{3}{2}x^{3/2}-6x^{2/3};\ h'(x)=\dfrac{9}{4}x^{1/2}-4x^{-1/3}$

38. $f(x)=\sqrt{x+\sqrt{x}}=(x+x^{1/2})^{1/2}$

$f'(x)=\dfrac{1}{2}(x+x^{1/2})^{-1/2}\left(1+\dfrac{1}{2}x^{-1/2}\right)$

$=\dfrac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\dfrac{1}{2\sqrt{x}}\right)$

39. $f(t)=3t^3-2t^2$

$f'(t)=9t^2-4t$

$f'(2)=36-8=28$

40. $V(r)=15\pi r^2$

$V'(r)=30\pi r$

$V'\left(\dfrac{1}{3}\right)=10\pi$

41. $g(u)=3u-1$

$g(5)=15-1=14$

$g'(u)=3$

$g'(5)=3$

42. $h(x)=-\dfrac{1}{2};\quad h(-2)=-\dfrac{1}{2}$

$h'(x)=0;\quad h'(-2)=0$

43. $f(x)=x^{5/2};\quad f'(x)=\dfrac{5}{2}x^{3/2}$

$f''(x)=\dfrac{15}{4}x^{1/2};\quad f''(4)=\dfrac{15}{2}$

44. $g(t)=\dfrac{1}{4}(2t-7)^4$

$g'(t)=(2t-7)^3(2)=2(2t-7)^3$

$g''(t)=6(2t-7)^2(2)=12(2t-7)^2$

$g''(3)=12[2(3)-7]^2=12$

45. $y=(3x-1)^3-4(3x-1)^2$

slope $=y'=3(3x-1)^2(3)-8(3x-1)(3)$

$=9(3x-1)^2-24(3x-1)$

When $x=0$, slope $=9+24=33$.

46. $y=(4-x)^5$

slope $=y'=5(4-x)^4(-1)=-5(4-x)^4$

When $x=5$, slope $=-5$.

47. $\dfrac{d}{dx}(x^4-2x^2)=4x^3-4x$

48. $\dfrac{d}{dt}(t^{5/2}+2t^{3/2}-t^{1/2})$

$=\dfrac{5}{2}t^{3/2}+3t^{1/2}-\dfrac{1}{2}t^{-1/2}$

49. $\dfrac{d}{dP}\left(\sqrt{1-3P}\right)=\dfrac{d}{dP}(1-3P)^{1/2}$

$=\dfrac{1}{2}(1-3P)^{-1/2}(-3)$

$=-\dfrac{3}{2}(1-3P)^{-1/2}$

50. $\dfrac{d}{dn}(n^{-5})=-5n^{-6}$

51. $\dfrac{d}{dz}(z^3-4z^2+z-3)\Big|_{z=-2}=(3z^2-8z+1)\Big|_{z=-2}$

$=12+16+1=29$

52. $\dfrac{d}{dx}(4x-10)^5\Big|_{x=3}=[5(4x-10)^4(4)]\Big|_{x=3}$

$=[20(4x-10)^4]\Big|_{x=3}=320$

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53. $\frac{d^2}{dx^2}(5x+1)^4 = \frac{d}{dx}[4(5x+1)^3(5)]$
$= 60(5x+1)^2(5) = 300(5x+1)^2$

54. $\frac{d^2}{dt^2}\left(2\sqrt{t}\right) = \frac{d^2}{dt^2}2t^{1/2} = \frac{d}{dt}t^{-1/2} = -\frac{1}{2}t^{-3/2}$

55. $\frac{d^2}{dt^2}(t^3+2t^2-t)\Big|_{t=-1} = \frac{d}{dt}(3t^2+4t-1)\Big|_{t=-1}$
$= (6t+4)\big|_{t=-1} = -2$

56. $\frac{d^2}{dP^2}(3P+2)\Big|_{P=4} = \frac{d}{dP}3\Big|_{P=4} = 0\big|_{P=4} = 0$

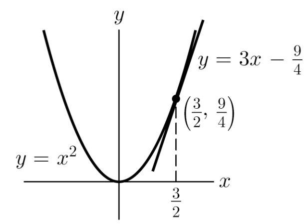
57. $\frac{d^2y}{dx^2}(4x^{3/2}) = \frac{dy}{dx}(6x^{1/2}) = 3x^{-1/2}$

58. $\frac{d}{dt}\left(\frac{1}{3t}\right) = \frac{d}{dt}\left(\frac{1}{3}t^{-1}\right) = -\frac{1}{3}t^{-2}$ or $-\frac{1}{3t^2}$
$\frac{d}{dt}\left(-\frac{1}{3}t^{-2}\right) = \frac{2}{3}t^{-3}$ or $\frac{2}{3t^3}$

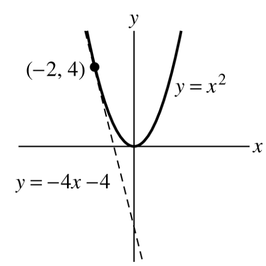
59. $f(x) = x^3 - 4x^2 + 6$
slope $= f'(x) = 3x^2 - 8x$
When $x = 2$, slope $= 3(2)^2 - 8(2) = -4$.
When $x = 2$, $y = 2^3 - 4(2)^2 + 6 = -2$.
Let $(x_1, y_1) = (2, -2)$.
$y - (-2) = -4(x-2) \Rightarrow y = -4x + 6$

60. $y = \frac{1}{3x-5} = (3x-5)^{-1}$
$y' = -(3x-5)^{-2}(3) = -\frac{3}{(3x-5)^2}$
When $x = 1$, slope $= -\frac{3}{(3(1)-5)^2} = -\frac{3}{4}$.
When $x = 1$, $y = \frac{1}{3(1)-5} = -\frac{1}{2}$.
Let $(x_1, y_1) = \left(1, -\frac{1}{2}\right)$.
$y - \left(-\frac{1}{2}\right) = -\frac{3}{4}(x-1) \Rightarrow y = -\frac{3}{4}x + \frac{1}{4}$

61. $y = x^2$
slope $= y' = 2x$
When $x = \frac{3}{2}$, slope $= 2\left(\frac{3}{2}\right) = 3$.
Let $(x_1, y_1) = \left(\frac{3}{2}, \frac{9}{4}\right)$.
$y - \frac{9}{4} = 3\left(x - \frac{3}{2}\right) \Rightarrow y = 3x - \frac{9}{4}$



62. $y = x^2$
slope $= y' = 2x$
When $x = -2$, slope $= 2(-2) = -4$.
Let $(x_1, y_1) = (-2, 4)$.
$y - 4 = -4(x+2) \Rightarrow y = -4x - 4$



63. $y = 3x^3 - 5x^2 + x + 3$
slope $= y' = 9x^2 - 10x + 1$
When $x = 1$, slope $= 9(1)^2 - 10(1) + 1 = 0$.
When $x = 1$, $y = 3(1)^3 - 5(1)^2 + 1 + 3 = 2$.
Let $(x_1, y_1) = (1, 2)$.
$y - 2 = 0(x-1) \Rightarrow y = 2$

64. $y = (2x^2 - 3x)^3$
slope $= y' = 3(2x^2 - 3x)^2(4x-3)$
When $x = 2$,
slope $= 3\left(2(2)^2 - 3(2)\right)^2\left(4(2) - 3\right) = 60$.
When $x = 2$, $y = \left(2(2)^2 - 3(2)\right)^3 = 8$.
Let $(x_1, y_1) = (2, 8)$.
$y - 8 = 60(x-2) \Rightarrow y = 60x - 112$

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65. The line has slope –1 and contains the point (5, 0).
$y-0=-1(x-5) \Rightarrow y=-x+5$
$f(2) = -2 + 5 = 3$
$f'(2)=-1$

66. The tangent line contains the points (0, 2) and (a, a^3) and has slope $=3a^2$. Thus,
$\dfrac{a^3-2}{a}=3a^2 \Rightarrow a^3-2=3a^3 \Rightarrow -2=2a^3 \Rightarrow$
$a=-1$

67. $s'(t)=-32t+32$
The binoculars will hit the ground when $s(t) = 0$, i.e.,
$s(t)=-16t^2+32t+128=0$
$-16(t^2-2t-8)=0$
$-16(t-4)(t+2) = 0$
$t = 4$ or $t = -2$
$s'(4) = -32(4) + 32 = -96$ feet/sec.
Therefore, when the binoculars hit the ground, they will be falling at the rate of 96 feet/sec.

68. $40t+t^2-\dfrac{1}{15}t^3$ tons is the total output of a coal mine after t hours. The rate of output is $40+2t-\dfrac{1}{5}t^2$ tons per hour. At $t = 5$, the rate of output is $40+2(5)-\dfrac{1}{5}(5)^2=45$ tons/hour.

69. 11 feet

70. $\dfrac{s(4)-s(1)}{4-1}=\dfrac{6-1}{4-1}=\dfrac{5}{3}$ ft/sec

71. Slope of the tangent line is $\dfrac{5}{3}$ so $\dfrac{5}{3}$ ft/sec.

72. $t = 6$, since $s(t)$ is steeper at $t = 6$ than at $t = 5$.

73. $C(x)=.1x^3-6x^2+136x+200$

a. $C(21) - C(20)$
$=.1(21)^3-6(21)^2+136(21)+200$
$\quad -(.1(20)^3-6(20)^2+136(20)+200)$
$= 1336.1 - 1320 = \$16.10$

b. $C'(x)=.3x^2-12x+136$
$C'(20)=.3(20)^2-12(20)+136=\$16$

74. $f(235) = 4600$
$f'(235)=-100$
$f(a+h)\approx f'(a)\cdot h+f(a)$

a. $237 = 235 + 2$
$f(235+2)\approx f'(235)\cdot 2+f(235)$
$\approx -100\cdot 2+4600$
≈ 4400 riders

b. $234 = 235 + (-1)$
$f(235+(-1))\approx f'(235)\cdot(-1)+f(235)$
$\approx -100\cdot(-1)+4600$
≈ 4700 riders

c. $240 = 235 + 5$
$f(235+5)\approx f'(235)\cdot 5+f(235)$
$\approx -100\cdot 5+4600$
≈ 4100 riders

d. $232 = 235 + (-3)$
$f(235+(-3))\approx f'(235)\cdot(-3)+f(235)$
$\approx -100\cdot(-3)+4600$
≈ 4900 riders

75. $h(12.5) - h(12) \approx h'(12)(.5)$
$= (1.5)(.5) = .75$ in.

76. $f\left(7+\dfrac{1}{2}\right)-f(7)\approx f'(7)\cdot\dfrac{1}{2}$
$=(25.06)\cdot\dfrac{1}{2}=12.53$
\$12.53 is the additional money earned if the bank paid $7\dfrac{1}{2}\%$ interest.

77. $\displaystyle\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x+2)(x-2)}{x-2}$
$\displaystyle=\lim_{x\to 2}(x+2)=2+2=4$

78. The limit does not exist.

79. The limit does not exist.

80. $\displaystyle\lim_{x\to 5}\frac{x-5}{x^2-7x+2}=\frac{5-5}{25-35+2}=0$

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81. $f'(5) = \lim_{h\to 0} \dfrac{f(5+h) - f(5)}{h}$

If $f(x) = \dfrac{1}{2x}$, then

$$\begin{aligned} f(5+h) - f(5) &= \frac{1}{2(5+h)} - \frac{1}{2(5)} \\ &= \frac{1}{2(5+h)} \cdot \frac{5}{5} - \frac{1}{2(5)} \cdot \left(\frac{5+h}{5+h}\right) \\ &= \frac{5-(5+h)}{10(5+h)} = \frac{-h}{10(5+h)} \end{aligned}$$

Thus,

$$\begin{aligned} f'(5) &= \lim_{h\to 0} [f(5+h) - f(5)] \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \frac{-h}{10(5+h)} \cdot \frac{1}{h} \\ &= \lim_{h\to 0} \frac{-1}{10(5+h)} = -\frac{1}{50} \end{aligned}$$

82. $f'(3) = \lim_{h\to 0} \dfrac{f(3+h) - f(3)}{h}$

If $f(x) = x^2 - 2x + 1$, then

$$\begin{aligned} &f(3+h) - f(3) \\ &= (3+h)^2 - 2(3+h) + 1 - (9 - 6 + 1) \\ &= h^2 + 4h. \end{aligned}$$

Thus,

$$\begin{aligned} f'(3) &= \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{h^2 + 4h}{h} \\ &= \lim_{h\to 0} (h+4) = 4. \end{aligned}$$

83. The slope of a secant line at (3, 9)

84. $\dfrac{\frac{1}{2+h} - \frac{1}{2}}{h} = \dfrac{\frac{2-2-h}{2(2+h)}}{h} = \dfrac{-1}{2(2+h)}$

As $h \to 0$, $\dfrac{-1}{2(2+h)} \to -\dfrac{1}{4}$.

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