C H A P T E R 1
Functions, Graphs, and Limits

CHAPTER 1
Functions, Graphs, and Limits

Section 1.1 The Cartesian Plane and the Distance Formula

Skills Warm Up

1. $\sqrt{(3-6)^2 + [1-(-5)]^2} = \sqrt{(-3)^2 + 6^2}$
$= \sqrt{9+36}$
$= \sqrt{45}$
$= 3\sqrt{5}$

2. $\sqrt{(-2-0)^2 + [-7-(-3)]^2} = \sqrt{(-2)^2 + (-4)^2}$
$= \sqrt{4+16}$
$= \sqrt{20}$
$= 2\sqrt{5}$

3. $\dfrac{5+(-4)}{2} = \dfrac{1}{2}$

4. $\dfrac{-3+(-1)}{2} = \dfrac{-4}{2} = -2$

5. $\sqrt{27} + \sqrt{12} = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}$

6. $\sqrt{8} - \sqrt{18} = 2\sqrt{2} - 3\sqrt{2} = -\sqrt{2}$

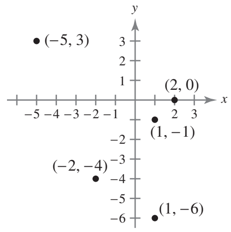
7.
$$\begin{aligned}
\sqrt{(3-x)^2 + (7-4)^2} &= \sqrt{45} \\
\left(\sqrt{(3-x)^2 + (7-4)^2}\right)^2 &= \left(\sqrt{45}\right)^2 \\
(3-x)^2 + (7-4)^2 &= 45 \\
(3-x)^2 + 3^2 &= 45 \\
(3-x)^2 + 9 &= 45 \\
(3-x)^2 &= 36 \\
3-x &= \pm 6 \\
-x &= -3 \pm 6 \\
x &= 3 \mp 6 \\
x &= -3, 9
\end{aligned}$$

8.
$$\begin{aligned}
\sqrt{(6-2)^2 + (-2-y)^2} &= \sqrt{52} \\
\left(\sqrt{(6-2)^2 + (-2-y)^2}\right)^2 &= \left(\sqrt{52}\right)^2 \\
(6-2)^2 + (-2-y)^2 &= 52 \\
4^2 + (-2-y)^2 &= 52 \\
16 + (-2-y)^2 &= 52 \\
(-2-y)^2 &= 36 \\
-2-y &= \pm 6 \\
-y &= \pm 6 + 2 \\
y &= \mp 6 - 2 \\
y &= -8, 4
\end{aligned}$$

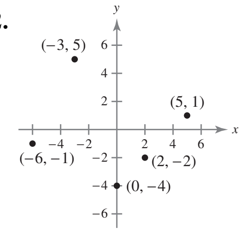
9.
$$\begin{aligned}
\frac{x+(-5)}{2} &= 7 \\
x + (-5) &= 14 \\
x &= 19
\end{aligned}$$

10.
$$\begin{aligned}
\frac{-7+y}{2} &= -3 \\
-7 + y &= -6 \\
y &= 1
\end{aligned}$$

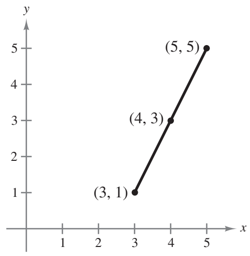
1.



2.



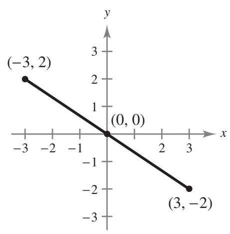
3. (a)



(b) $d = \sqrt{(5-3)^2 + (5-1)^2} = \sqrt{4+16} = 2\sqrt{5}$

(c) Midpoint $= \left(\dfrac{3+5}{2}, \dfrac{1+5}{2}\right) = (4, 3)$

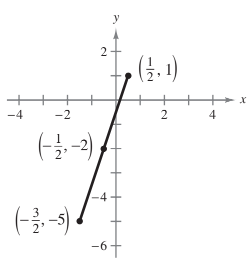
4. (a)



(b) $d = \sqrt{(-3-3)^2 + (2+2)^2} = \sqrt{36+16} = 2\sqrt{13}$

(c) Midpoint $= \left(\dfrac{-3+3}{2}, \dfrac{2+(-2)}{2}\right) = (0, 0)$

5. (a)



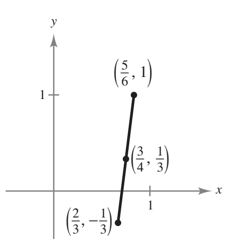
(b) $d = \sqrt{[(3/2)-(1/2)]^2 + (5-1)^2}$

$= \sqrt{4+36}$

$= 2\sqrt{10}$

(c) Midpoint $= \left(\dfrac{(1/2)+(-3/2)}{2}, \dfrac{1+(-5)}{2}\right) = \left(-\dfrac{1}{2}, -2\right)$

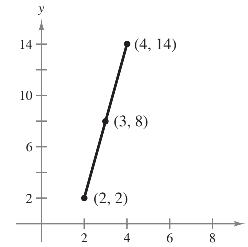
6. (a)



(b) $d = \sqrt{\left(\dfrac{5}{6}-\dfrac{2}{3}\right)^2 + \left(1+\dfrac{1}{3}\right)^2} = \sqrt{\dfrac{1}{36}+\dfrac{16}{9}} = \dfrac{\sqrt{65}}{6}$

(c) Midpoint $= \left(\dfrac{(5/6)+(2/3)}{2}, \dfrac{1-(1/3)}{2}\right) = \left(\dfrac{3}{4}, \dfrac{1}{3}\right)$

7. (a)



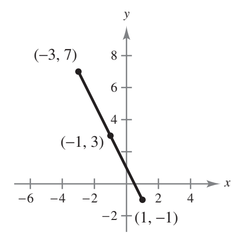
(b) $d = \sqrt{(4-2)^2 + (14-2)^2}$

$= \sqrt{4+144}$

$= 2\sqrt{37}$

(c) Midpoint $= \left(\dfrac{2+4}{2}, \dfrac{2+14}{2}\right) = (3, 8)$

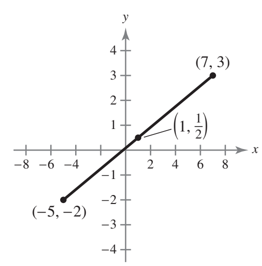
8. (a)



(b) $d = \sqrt{(-3-1)^2 + (7+1)^2} = \sqrt{16+64} = 4\sqrt{5}$

(c) Midpoint $= \left(\dfrac{-3+1}{2}, \dfrac{7-1}{2}\right) = (-1, 3)$

9. (a)



(b) $d = \sqrt{(7-(-5))^2 + (3-(-2)^2)}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{144+25}$

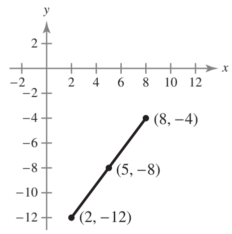
$= \sqrt{169} = 13$

(c) Midpoint $= \left(\dfrac{7+(-5)}{2}, \dfrac{3+(-2)}{2}\right)$

$= \left(\dfrac{2}{2}, \dfrac{1}{2}\right)$

$= \left(1, \dfrac{1}{2}\right)$

10. (a)



(b) $d = \sqrt{(8-2)^2 + \left(-4-(-12)^2\right)}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

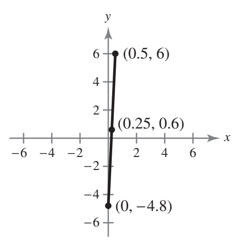
$= \sqrt{100} = 10$

(c) $\text{Midpoint} = \left(\dfrac{2+8}{2}, \dfrac{(-12)+(-4)}{2}\right)$

$= \left(\dfrac{10}{2}, \dfrac{-16}{2}\right)$

$= (5, -8)$

11. (a)



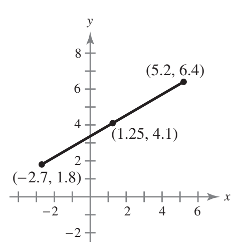
(b) $d = \sqrt{(0.5-0)^2 + \left(6-(-4.8)\right)^2}$

$= \sqrt{0.25 + 116.64}$

$= \sqrt{116.89}$

(c) $\text{Midpoint} = \left(\dfrac{0+0.5}{2}, \dfrac{-4.8+6}{2}\right) = (0.25, 0.6)$

12. (a)



(b) $d = \sqrt{(-2.7-5.2)^2 + (1.8-6.4)^2}$

$= \sqrt{62.41 + 21.16}$

$= \sqrt{83.57}$

(c) $\text{Midpoint} = \left(\dfrac{5.2+(-2.7)}{2}, \dfrac{6.4+1.8}{2}\right)$

$= (1.25, 4.1)$

13. (a) $a = 4$

$b = 3$

$c = \sqrt{(4-0)^2 + (3-0)^2} = \sqrt{16+9} = 5$

(b) $a^2 + b^2 = 16 + 9 = 25 = c^2$

14. (a) $a = \sqrt{(13-1)^2 + (1-1)^2} = \sqrt{144+0} = 12$

$b = \sqrt{(13-13)^2 + (6-1)^2} = \sqrt{0+25} = 5$

$c = \sqrt{(13-1)^2 + (6-1)^2} = \sqrt{144+25} = 13$

(b) $a^2 + b^2 = 144 + 25 = 169 = c^2$

15. (a) $a = 10$

$b = 3$

$c = \sqrt{(7+3)^2 + (4-1)^2} = \sqrt{100+9} = \sqrt{109}$

(b) $a^2 + b^2 = 100 + 9 = 109 = c^2$

16. (a) $a = \sqrt{(6-2)^2 + (-2+2)^2} = \sqrt{16+0} = 4$

$b = \sqrt{(2-2)^2 + (5+2)^2} = \sqrt{0+49} = 7$

$c = \sqrt{(2-6)^2 + (5+2)^2} = \sqrt{16+49} = \sqrt{65}$

(b) $a^2 + b^2 = 16 + 49 = 65 = c^2$

17. $d_1 = \sqrt{(3-0)^2 + (7-1)^2}$

$= \sqrt{9+36}$

$= \sqrt{45}$

$= 3\sqrt{5}$

$d_2 = \sqrt{(4-0)^2 + (-1-1)^2}$

$= \sqrt{16+4}$

$= \sqrt{20}$

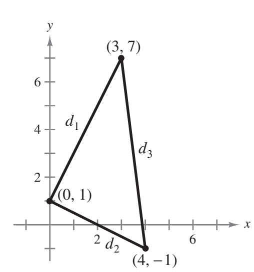
$= 2\sqrt{5}$

$d_3 = \sqrt{(3-4)^2 + \left[7-(-1)\right]^2}$

$= \sqrt{1+64}$

$= \sqrt{65}$

Because ${d_1}^2 + {d_2}^2 = {d_3}^2$, the figure is a right triangle.



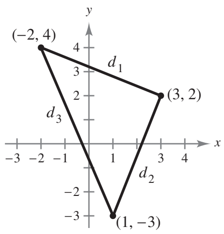
18. $a = \sqrt{(-2-3)^2 + (4-2)^2} = \sqrt{25+4} = \sqrt{29}$

$b = \sqrt{(3-1)^2 + (2+3)^2} = \sqrt{4+25} = \sqrt{29}$

$c = \sqrt{(-2-1)^2 + (4+3)^2} = \sqrt{9+49} = \sqrt{58}$

Because $a = b$ the figure is an isosceles triangle.

[**Note:** It is also a right triangle since $a^2 + b^2 = c^2$.]



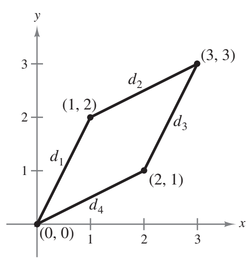
19. $d_1 = \sqrt{(1-0)^2 + (2-0)^2} = \sqrt{1+4} = \sqrt{5}$

$d_2 = \sqrt{(3-1)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$

$d_3 = \sqrt{(2-3)^2 + (1-3)^2} = \sqrt{1+4} = \sqrt{5}$

$d_4 = \sqrt{(0-2)^2 + (0-1)^2} = \sqrt{4+1} = \sqrt{5}$

Because $d_1 = d_2 = d_3 = d_4$, the figure is a parallelogram.



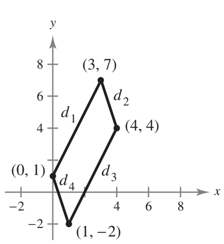
20. $a = \sqrt{(3-0)^2 + (7-1)^2} = \sqrt{9+36} = 3\sqrt{5}$

$b = \sqrt{(3-4)^2 + (7-4)^2} = \sqrt{1+9} = \sqrt{10}$

$c = \sqrt{(4-1)^2 + (4+2)^2} = \sqrt{9+36} = 3\sqrt{5}$

$d = \sqrt{(1-0)^2 + (-2-1)^2} = \sqrt{1+9} = \sqrt{10}$

Because $a = c$ and $b = d$, the figure is a parallelogram.



21. $d = \sqrt{(x-1)^2 + (-4-0)^2} = 5$

$$\sqrt{x^2 - 2x + 17} = 5$$
$$x^2 - 2x + 17 = 25$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x = 4, -2$$

22. $d = \sqrt{(x-2)^2 + (2+1)^2} = 5$

$$\sqrt{x^2 - 4x + 13} = 5$$
$$x^2 - 4x + 13 = 25$$
$$x^2 - 4x - 12 = 0$$
$$(x+2)(x-6) = 0$$
$$x = -2, 6$$

23. $d = \sqrt{(3-0)^2 + (y-0)^2} = 8$

$$\sqrt{9 + y^2} = 8$$
$$9 + y^2 = 64$$
$$y^2 = 55$$
$$y = \pm\sqrt{55}$$

24. $d = \sqrt{(5-5)^2 + (y-1)^2} = 8$

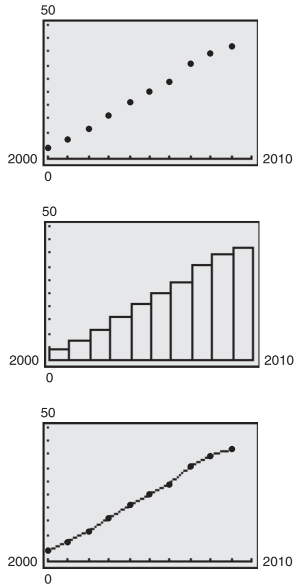
$$\sqrt{(y-1)^2} = 8$$
$$(y-1)^2 = 64$$
$$y - 1 = \pm 8$$
$$y = 1 \pm 8$$
$$y = -7, 9$$

25. $d = \sqrt{(50-12)^2 + (42-18)^2}$

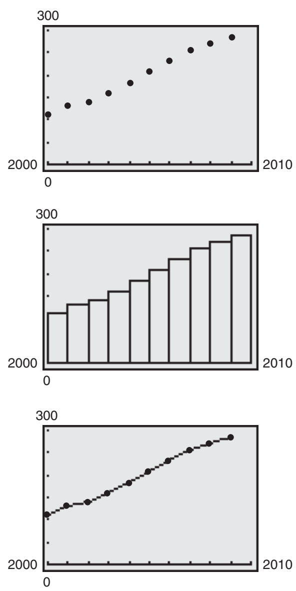
$$= \sqrt{38^2 + 24^2}$$
$$= \sqrt{2020}$$
$$= 2\sqrt{505} \approx 44.9 \text{ yd}$$

26. $d = \sqrt{(33-12)^2 + (37-18)^2}$

$$= \sqrt{21^2 + 19^2}$$
$$= \sqrt{441 + 361}$$
$$= \sqrt{802} \approx 28.3 \text{ yd}$$

27. 

The number of cable high-speed Internet customers increases each year.

28. 

The number of cellular telephone subscribers increases each year.

29. (a) March 2009: 7600

July 2009: 9200

July 2010: 10,500

(b) April 2010: 11,000

May 2010: 10,100

Decrease: $|10{,}100 - 11{,}000| = 900$

Percent decrease: $\dfrac{900}{11{,}000} \approx 0.082 = 8.2\%$

30. (a) 1996: \$120,000

2003: \$180,000

2008: \$200,000

(b) 2001: \$155,000

2002: \$165,000

Increase: $165{,}000 - 155{,}000 = \$10{,}000$

$$\text{Percent increase: } \frac{10{,}000}{155{,}000} \approx 0.065 = 6.5\%$$

31. (a) $\text{Revenue} = \left(\dfrac{2007 + 2009}{2}, \dfrac{329.7 + 538.9}{2}\right) = (2008, 434.3)$

Revenue estimate for 2008: \$434.3 million

$\text{Profit} = \left(\dfrac{2007 + 2009}{2}, \dfrac{19.7 + 30.7}{2}\right) = (2008, 25.2)$

Profit estimate for 2008: \$25.2 million

(b) Actual 2008 revenue: \$422.4 million

Actual 2008 profit: \$24.4 million

(c) Yes, the revenue and profit increased in a linear pattern from 2007 to 2009.

(d) 2007 expense: $329.7 - 19.7 = \$310$ million

2008 expense: $422.4 - 24.4 = \$398$ million

2009 expense: $538.9 - 30.7 = \$508.2$ million

(e) Answers will vary.

32. (a) $\text{Revenue} = \left(\dfrac{2007 + 2009}{2}, \dfrac{6484.5 + 7773.3}{2}\right) = (2008, 7128.9)$

Revenue estimate for 2008: \$7128.9 million

$\text{Profit} = \left(\dfrac{2007 + 2009}{2}, \dfrac{23.7 + 285.6}{2}\right) = (2008, 154.65)$

Profit estimate for 2008: \$154.65 million

(b) Actual 2008 revenue: \$7230.1 million

Actual 2008 profit: \$149.7 million

(c) Yes, the revenue and profit increased in a linear pattern from 2007 to 2009.

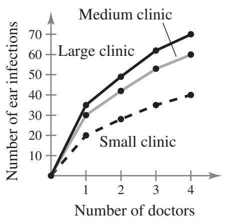
(d) 2007 expense: $6484.5 - 23.7 = \$6460.8$ million

2008 expense: $7230.1 - 149.7 = \$7080.4$ million

2009 expense: $7773.3 - 285.6 = \$7487.7$ million

(e) Answers will vary.

33. (a)



(b) The larger the clinic, the more patients a doctor can treat.

34. (a) 500 pickups were sold in the year 2007.

(b) About 500 pickups were sold in the year 2007.

(c) The number of pickups sold each year is decreasing.

35. The vertex $(-3, -1)$ is translated to $(-6, -6)$.

The vertex $(0, 0)$ is translated to $(-3, -5)$.

The vertex $(-1, -2)$ is translated to $(-4, -7)$.

36. The vertex $(0, 2)$ is translated to $(2, 6)$.

The vertex $(1, 3)$ is translated to $(3, 7)$.

The vertex $(3, 1)$ is translated to $(5, 5)$.

The vertex $(2, 0)$ is translated to $(4, 4)$.

37. Midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

The point one-fourth of the way between (x_1, y_1) and (x_2, y_2) is the midpoint of the line segment from (x_1, y_1) to $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$, which is

$$\left(\frac{x_1 + \frac{x_1 + x_2}{2}}{2}, \frac{y_1 + \frac{y_1 + y_2}{2}}{2}\right) = \left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right).$$

The point three-fourths of the way between (x_1, y_1) and (x_2, y_2) is the midpoint of the line segment from $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ to (x_2, y_2), which is

$$\left(\frac{\frac{x_1 + x_2}{2} + x_2}{2}, \frac{\frac{y_1 + y_2}{2} + y_2}{2}\right) = \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right).$$

Thus,

$\left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right)$, $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$, and $\left(\dfrac{x_1 + 3x_2}{4}, \dfrac{y_1 + 3y_2}{4}\right)$

are the three points that divide the line segment joining (x_1, y_1) and (x_2, y_2) into four equal parts.

38. (a) $\left(\dfrac{3(1) + 4}{4}, \dfrac{3(-2) - 1}{4}\right) = \left(\dfrac{7}{4}, -\dfrac{7}{4}\right)$

$\left(\dfrac{1 + 4}{2}, \dfrac{-2 - 1}{2}\right) = \left(\dfrac{5}{2}, -\dfrac{3}{2}\right)$

$\left(\dfrac{1 + 3(4)}{4}, \dfrac{-2 + 3(-1)}{4}\right) = \left(\dfrac{13}{4}, -\dfrac{5}{4}\right)$

(b) $\left(\dfrac{3(-2) + 0}{4}, \dfrac{3(-3) + 0}{4}\right) = \left(-\dfrac{3}{2}, -\dfrac{9}{4}\right)$

$\left(\dfrac{-2 + 0}{2}, \dfrac{-3 + 0}{2}\right) = \left(-1, -\dfrac{3}{2}\right)$

$\left(\dfrac{-2 + 3(0)}{4}, \dfrac{-3 + 3(0)}{4}\right) = \left(-\dfrac{1}{2}, -\dfrac{3}{4}\right)$

39. To show $\left(\dfrac{2x_1 + x_2}{3}, \dfrac{2y_1 + y_2}{3}\right)$ is a point of trisection of the line segment joining (x_1, y_1) and (x_2, y_2), we must show that $d_1 = \frac{1}{2}d_2$ and $d_1 + d_2 = d_3$.

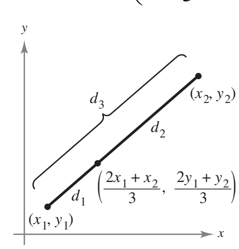
$$\begin{aligned} d_1 &= \sqrt{\left(\frac{2x_1 + x_2}{3} - x_1\right)^2 + \left(\frac{2y_1 + y_2}{3} - y_1\right)^2} \\ &= \sqrt{\left(\frac{x_2 - x_1}{3}\right)^2 + \left(\frac{y_2 - y_1}{3}\right)^2} \\ &= \frac{1}{3}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{aligned}$$

$$\begin{aligned} d_2 &= \sqrt{\left(x_2 - \frac{2x_1 + x_2}{3}\right)^2 + \left(y_2 - \frac{2y_1 + y_2}{3}\right)^2} \\ &= \sqrt{\left(\frac{2x_2 - 2x_1}{3}\right)^2 + \left(\frac{2y_2 - 2y_1}{3}\right)^2} \\ &= \frac{2}{3}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{aligned}$$

$$d_3 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Therefore, $d_1 = \dfrac{1}{2}d_2$ and $d_1 + d_2 = d_3$. The midpoint of the line segment joining $\left(\dfrac{2x_1 + x_2}{3}, \dfrac{2y_1 + y_2}{3}\right)$ and (x_2, y_2) is

$$\begin{aligned} \text{Midpoint} &= \left(\frac{\frac{2x_1 + x_2}{3} + x_2}{2}, \frac{\frac{2y_1 + y_2}{3} + y_2}{2}\right) \\ &= \left(\frac{x_1 + 2x_2}{3}, \frac{y_1 + 2y_2}{3}\right). \end{aligned}$$



40. (a) $\left(\dfrac{2(1)+4}{3}, \dfrac{2(-2)+1}{3}\right) = (2, -1)$

$\left(\dfrac{1+2(4)}{3}, \dfrac{-2+2(1)}{3}\right) = (3, 0)$

(b) $\left(\dfrac{2(-2)+0}{3}, \dfrac{2(-3)+0}{3}\right) = \left(-\dfrac{4}{3}, -2\right)$

$\left(\dfrac{-2+2(0)}{3}, \dfrac{-3+2(0)}{3}\right) = \left(-\dfrac{2}{3}, -1\right)$

Section 1.2 Graphs of Equations

Skills Warm Up

1. $5y - 12 = x$
$$5y = x + 12$$
$$y = \frac{x+12}{5}$$

2. $-y = 15 - x$
$$y = x - 15$$

3. $x^3y + 2y = 1$
$$y(x^3 + 2) = 1$$
$$y = \frac{1}{x^3 + 2}$$

4. $x^2 + x - y^2 - 6 = 0$
$$-y^2 = 6 - x^2 - x$$
$$y^2 = x^2 + x - 6$$
$$y = \sqrt{x^2 + x - 6}$$

5. $(x-2)^2 + (y+1)^2 = 9$
$$(y+1)^2 = 9 - (x-2)^2$$
$$y + 1 = \sqrt{9 - (x-2)^2}$$
$$y = \left(\sqrt{9 - (x-2)^2}\right) - 1$$
$$= \sqrt{9 - (x^2 - 4x + 4)} - 1$$
$$= \sqrt{5 + 4x - x^2} - 1$$

6. $(x+6)^2 + (y-5)^2 = 81$
$$(y-5)^2 = 81 - (x+6)^2$$
$$y - 5 = \sqrt{81 - (x+6)^2}$$
$$y = 5 + \sqrt{81 - (x+6)^2}$$
$$= 5 + \sqrt{81 - (x^2 + 12x + 36)}$$
$$= 5 + \sqrt{45 - 12x - x^2}$$

7. $y = 5(-2) = -10$

8. $y = 3(3) - 4 = 5$

9. $y = 2(2)^2 + 1 = 9$

10. $y = (-4)^2 + 2(-4) - 7 = 1$

11. $x^2 - 3x + 2$
$(x-1)(x-2)$

12. $x^2 + 5x + 6$
$(x+2)(x+3)$

13. $y^2 - 3y + \frac{9}{4}$
$\left(y - \frac{3}{2}\right)^2$

14. $y^2 - 7y + \frac{49}{4}$
$\left(y - \frac{7}{2}\right)^2$

1. The graph of $y = x - 2$ is a straight line with y-intercept at $(0, -2)$. So, it matches (e).

2. The graph of $y = -\frac{1}{2}x + 2$ is a straight line with y-intercept at $(0, 2)$. So, it matches (b).

3. The graph of $y = x^2 + 2x$ is a parabola opening up with vertex at $(-1, -1)$. So, it matches (c).

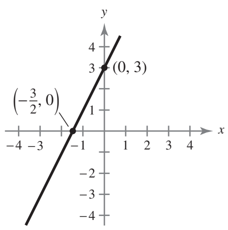
4. The graph of $y = \sqrt{9 - x^2}$ is a semicircle with intercepts $(0, 3)$, $(3, 0)$, and $(-3, 0)$. So, it matches (f).

5. The graph of $y = |x| - 2$ has a y-intercept at $(0, -2)$ and has x-intercepts at $(-2, 0)$ and $(2, 0)$. So, it matches (a).

6. The graph of $y = x^3 - x$ has intercepts at $(0, 0)$, $(1, 0)$, and $(-1, 0)$. So, it matches (d).

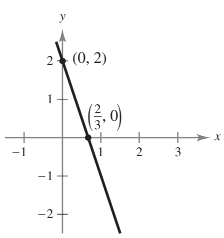
7. $y = 2x + 3$

x	-2	$-\frac{3}{2}$	-1	0	1	2
y	-1	0	1	3	5	7



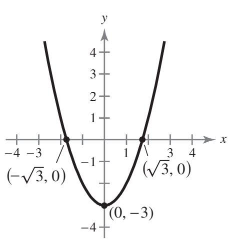
8. $y = -3x + 2$

x	-1	0	$\frac{2}{3}$	1	2
y	5	2	0	-1	-4



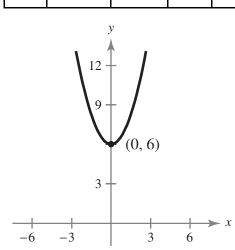
9. $y = x^2 - 3$

x	-2	-1	0	1	2	3
y	1	-2	-3	-2	1	6



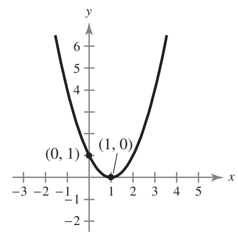
10. $y = x^2 + 6$

x	-2	-1	0	1	2
y	10	7	6	7	10



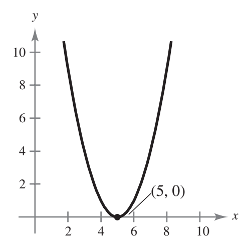
11. $y = (x - 1)^2$

x	-2	-1	0	1	2
y	9	4	1	0	1



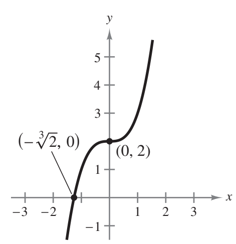
12. $y = (5 - x)^2$

x	3	4	5	6	7
y	4	1	0	1	4



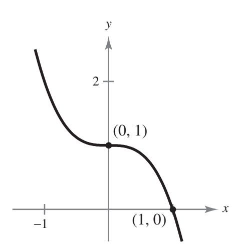
13. $y = x^3 + 2$

x	-2	-1	0	1	2
y	-6	1	2	3	10



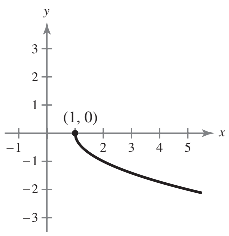
14. $y = 1 - x^3$

x	0	1	-1	2
y	1	0	2	-7



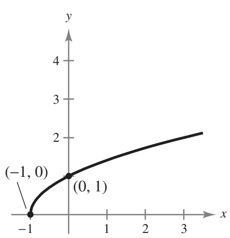
15. $y = -\sqrt{x - 1}$

x	1	2	3	4	5
y	0	-1	-1.41	-1.73	-2



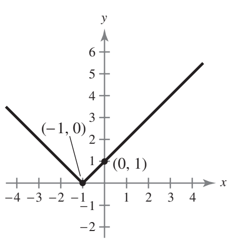
16. $y = \sqrt{x + 1}$

x	-1	0	1	2	3
y	0	1	$\sqrt{2}$	$\sqrt{3}$	4



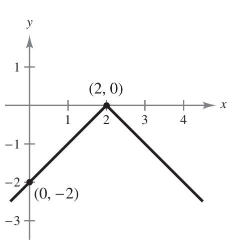
17. $y = |x + 1|$

x	-3	-2	-1	0	1
y	2	1	0	1	2



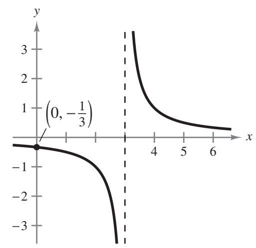
18. $y = -|x - 2|$

x	2	0	1	3	4
y	0	-2	-1	-1	-2



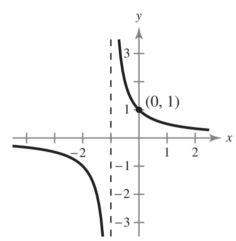
19. $y = \dfrac{1}{x - 3}$

x	-1	0	1	2	2.5	3.5	4	5	6
y	$-\frac{1}{4}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	2	1	$\frac{1}{2}$	$\frac{1}{3}$



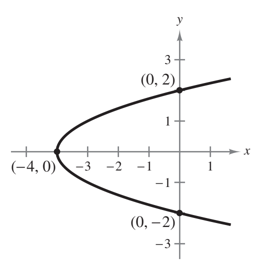
20. $y = \dfrac{1}{x + 1}$

x	-4	-3	-2	-1.5	-0.5	0	1	2
y	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	2	1	$\frac{1}{2}$	$\frac{1}{3}$



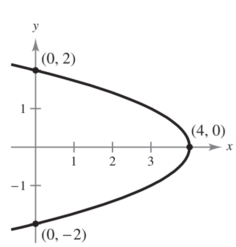
21. $x = y^2 - 4$

x	5	0	-3	-4
y	± 3	± 2	± 1	0



22. $x = 4 - y^2$

x	0	3	4
y	± 2	± 1	0



23. Let $y = 0$. Then,

$$2x - (0) - 3 = 0$$
$$x = \tfrac{3}{2}.$$

Let $x = 0$. Then,

$$2(0) - y - 3 = 0$$
$$y = -3.$$

x-intercept: $\left(\tfrac{3}{2}, 0\right)$

y-intercept: $(0, -3)$

24. Let $y = 0$. Then,

$$4x - 2(0) - 5 = 0$$
$$y = \tfrac{5}{4}.$$

Let $x = 0$. Then,

$$4(0) - 2y - 5 = 0$$
$$y = -\tfrac{5}{2}$$

x-intercept: $\left(\tfrac{5}{4}, 0\right)$

y-intercept: $\left(0, -\tfrac{5}{2}\right)$

25. Let $y = 0$. Then,

$$0 = x^2 + x - 2$$
$$0 = (x + 2)(x - 1)$$
$$x = -2, 1.$$

Let $x = 0$. Then,

$$y = (0)^2 + (0) - 2$$
$$y = -2$$

x-intercepts: $(-2, 0), (1, 0)$

y-intercept: $(0, -2)$

26. Let $y = 0$. Then,

$$0 = x^2 - 4x + 3$$
$$0 = (x - 3)(x - 1)$$
$$x = 1, 3.$$

Let $x = 0$. Then,

$$y = (0)^2 - 4(0) + 3$$
$$y = 3.$$

x-intercepts: $(1, 0), (3, 0)$

y-intercept: $(0, 3)$

27. Let $y = 0$. Then,

$$0 = \sqrt{4 - x^2}$$
$$x^2 = 4$$
$$x = \pm 2.$$

Let $x = 0$. Then,

$$y = \sqrt{4 - (0)^2}$$
$$y = 2.$$

x-intercepts: $(-2, 0), (2, 0)$

y-intercept: $(0, 2)$

28. Let $y = 0$. Then,

$$0 = \sqrt{x + 9}$$
$$(0)^2 = \left(\sqrt{x + 9}\right)^2$$
$$0 = x + 9$$
$$-9 = x$$

Let $x = 0$. Then,

$$y = \sqrt{0 + 9}$$
$$y = \sqrt{9}$$
$$y = 3$$

x-intercept: $(-9, 0)$

y-intercept: $(0, 3)$

29. Let $y = 0$. Then,

$$0 = \frac{x^2 - 4}{x - 2}$$
$$0 = (x - 2)(x + 2)$$
$$x = \pm 2.$$

Let $x = 0$. Then,

$$y = \frac{(0)^2 - 4}{(0) - 2}$$
$$y = 2.$$

x-intercept: Because the equation is undefined when $x = 2$, the only x-intercept is $(-2, 0)$.

y-intercept: $(0, 2)$

30. Let $y = 0$. Then,

$$0 = \frac{x^2 + 3x}{2x}$$
$$0 = x(x + 3)$$
$$x = -3, 0.$$

Let $x = 0$. Then,

$$y = \frac{(0)^2 + 3(0)}{2(0)}$$
$$y = \text{undefined}.$$

x-intercept: Because the equation is undefined when $x = 0$, the only x-intercept is $(-3, 0)$.

y-intercept: Because the equation is undefined when $y = 0$, there is no y-intercept.

31. Let $y = 0$. Then,

$$x^2(0) - x^2 + 4(0) = 0$$
$$x^2 = 0$$
$$x = 0.$$

Let $x = 0$. Then,

$$(0)^2 y - (0)^2 + 4y = 0$$
$$y = 0.$$

x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

32. Let $y = 0$. Then,

$$2x^2(0) + 8(0) - x^2 = 1$$
$$x^2 = -1$$
$$x = \pm\sqrt{-1}.$$

Let $x = 0$. Then,

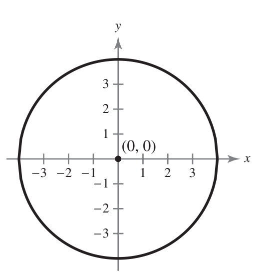
$$2(0)^2 y + 8y - (0)^2 = 1$$
$$y = \tfrac{1}{8}.$$

x-intercept: Because the equation has no real roots when $y = 0$, there is no x-intercept.

y-intercept: $\left(0, \frac{1}{8}\right)$

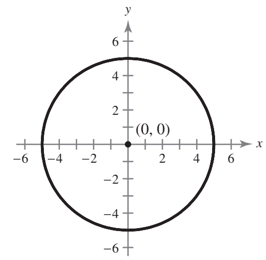
33. $(x - 0)^2 + (y - 0)^2 = 4^2$

$$x^2 + y^2 = 16$$



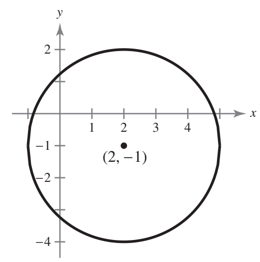
34. $(x - 0)^2 + (y - 0)^2 = 5^2$

$$x^2 + y^2 = 25$$



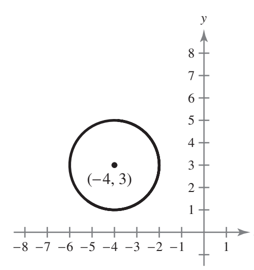
35. $(x - 2)^2 + (y - (-1))^2 = 3^2$

$$(x - 2)^2 + (y + 1)^2 = 9$$



36. $(x - (-4))^2 + (y - 3)^2 = 2^2$

$$(x + 4)^2 + (y - 3)^2 = 4$$

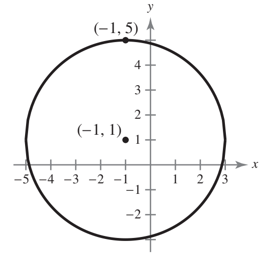


37. The radius is the distance between $(-1, 5)$ and $(-1, 1)$.

$$r = \sqrt{(-1 - (-1))^2 + (5 - 1)^2}$$
$$= \sqrt{0^2 + 4^2}$$
$$= \sqrt{16} = 4$$

Using the center $(-1, 1)$ and the radius $r = 4$:

$$(x - (-1))^2 + (y - 1)^2 = 4^2$$
$$(x + 1)^2 + (y - 1)^2 = 16$$

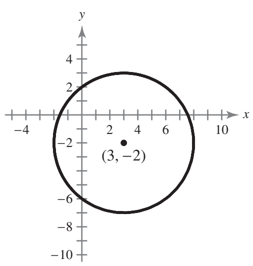


38. The radius is the distance between $(-1, 1)$ and $(3, -2)$.

$$\begin{aligned} r &= \sqrt{(3-(-1))^2 + (-2-1)^2} \\ &= \sqrt{4^2 + (-3)^2} \\ &= \sqrt{16+9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Using the center $(3, -2)$ and the radius $r = 5$:

$$\begin{aligned} (x-3)^2 + (y-(-2))^2 &= 5^2 \\ (x-3)^2 + (y+2)^2 &= 25 \end{aligned}$$



39. The diameter is the distance between $(-6, -8)$ and $(6, 8)$.

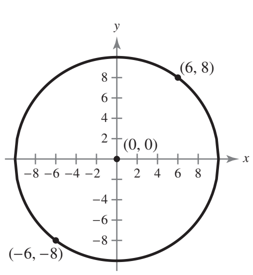
$$\begin{aligned} d &= \sqrt{(6-(-6))^2 + (8-(-8))^2} \\ &= \sqrt{12^2 + 16^2} \\ &= \sqrt{144+256} \\ &= \sqrt{400} \\ &= 20 \end{aligned}$$

The radius is one-half the diameter: $r = \dfrac{20}{2} = 10.$

The center is the midpoint of the diameter:

$$\left(\frac{-6+6}{2}, \frac{-8+8}{2}\right) = (0, 0).$$

$$\begin{aligned} (x-0)^2 + (y-0)^2 &= 10^2 \\ x^2 + y^2 &= 100 \end{aligned}$$



40. The diameter is the distance between $(0, -4)$ and $(6, 4)$.

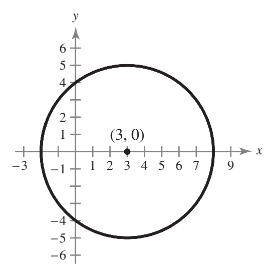
$$\begin{aligned} d &= \sqrt{(6-0)^2 + (4-(-4))^2} \\ &= \sqrt{6^2 + 8^2} \\ &= \sqrt{36+64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

The radius is one-half the diameter: $r = \dfrac{10}{2} = 5.$

The center is the midpoint of the diameter:

$$\left(\frac{0+6}{2}, \frac{4+4}{2}\right) = (3, 0)$$

$$\begin{aligned} (x-3)^2 + (y-0)^2 &= 5^2 \\ (x-3)^2 + y^2 &= 25 \end{aligned}$$



41. Set the two equations equal to each other.

$$\begin{aligned} -x + 2 &= 2x - 1 \\ -3x &= -3 \\ x &= 1 \end{aligned}$$

Substitute $x = 1$ into one of the equations.

$$y = (-1) + 2 = 1$$

The point of intersection is $(1, 1)$.

42. Set the two equations equal to each other.

$$\begin{aligned} -x + 7 &= \tfrac{3}{2}x - 8 \\ -2x + 14 &= 3x - 16 \\ -5x &= -30 \\ x &= 6 \end{aligned}$$

Substitute $x = 6$ into one of the equations.

$$y = (-6) + 7 = 1$$

The point of intersection is $(6, 1)$.

43. Set the two equations equal to each other.

$$\begin{aligned} -x^2 + 15 &= 3x + 11 \\ -x^2 - 3x + 4 &= 0 \\ x^2 + 3x - 4 &= 0 \\ (x + 4)(x - 1) &= 0 \end{aligned}$$

$x + 4 = 0 \qquad x - 1 = 0$

$x = -4 \qquad x = -1$

Substitute $x = -4$: Substitute $x = 1$:

$y = -(-4)^2 + 15 \qquad y = -(1)^2 + 15$

$y = -16 + 15 \qquad y = -1 + 15$

$y = -1 \qquad y = 14$

The points of intersection are $(-4, -1)$ and $(1, 14)$.

44. Set the two equations equal to each other.

$$\begin{aligned} x^2 + 2 &= x + 4 \\ x^2 - x - 2 &= 0 \\ (x - 2)(x + 1) &= 0 \end{aligned}$$

$x - 2 = 0 \qquad x + 1 = 0$

$x = 2 \qquad x = -1$

Substitute $x = 2$: Substitute $x = -1$:

$y = (2)^2 + 2 \qquad y = (-1)^2 + 2$

$y = 6 \qquad y = 3$

The points of intersection are $(2, 6)$ and $(-1, 3)$.

45. By equating the y-values for the two equations, we have

$$\begin{aligned} x^3 &= 2x \\ x^3 - 2x &= 0 \\ x(x^2 - 2) &= 0 \\ x &= 0, \pm\sqrt{2}. \end{aligned}$$

The corresponding y-values are $y = 0$, $y = -2\sqrt{2}$, and $y = 2\sqrt{2}$, so the points of intersection are $(0, 0)$, $(-\sqrt{2}, -2\sqrt{2})$, and $(\sqrt{2}, 2\sqrt{2})$.

46. By equating the y-values for the two equations, we have

$$\begin{aligned} \sqrt{x} &= x \\ x &= x^2 \\ 0 &= x(x - 1) \\ x &= 0, 1. \end{aligned}$$

The corresponding y-values are $y = 0, 1$, so the points of intersection are (0, 0) and (1, 1).

47. By equating the y-values for the two equations, we have

$$\begin{aligned} x^4 - 2x^2 + 1 &= 1 - x^2 \\ x^4 - x^2 &= 0 \\ x^2(x + 1)(x - 1) &= 0 \\ x &= 0, \pm 1. \end{aligned}$$

The corresponding y-values are $y = 1$, 0, and 0, so the points of intersection are $(-1, 0)$, $(0, 1)$, and $(1, 0)$.

48. By equating the y-values for the two equations, we have

$$\begin{aligned} x^3 - 2x^2 + x - 1 &= -x^2 + 3x - 1 \\ x^3 - x^2 - 2x &= 0 \\ x(x + 1)(x - 2) &= 0 \\ x &= 0, -1, 2. \end{aligned}$$

The corresponding y-values are $y = -1, -5$, and 1, so the points of intersection are $(0, -1)$, $(-1, -5)$, and $(2, 1)$.

49. To find the break-even point, set $R = C$.

$$\begin{aligned} 1.55x &= 0.85x + 35{,}000 \\ 0.7x &= 35{,}000 \\ x &= \frac{35{,}000}{0.7} = 50{,}000 \text{ units} \end{aligned}$$

50. To find the break-even point, set $R = C$.

$$\begin{aligned} 35x &= 6x + 500{,}000 \\ 29x &= 500{,}000 \\ x &= \frac{500{,}000}{29} \approx 17{,}242 \text{ units} \end{aligned}$$

51. To find the break-even point, set $R = C$.

$$\begin{aligned} 9950x &= 8650x + 250{,}000 \\ 1300x &= 250{,}000 \\ x &= \frac{250{,}000}{1300} \approx 193 \text{ units} \end{aligned}$$

52. To find the break-even point, set $R = C$.

$$\begin{aligned} 4.9x &= 2.5x + 10{,}000 \\ 2.4x &= 10{,}000 \\ x &= \frac{10{,}000}{2.4} \approx 4167 \text{ units} \end{aligned}$$

53. To find the break-even point, set $R = C$.

$$\begin{aligned} 10x &= 6x + 5000 \\ 4x &= 5000 \\ x &= \frac{5000}{4} \approx 1250 \text{ units} \end{aligned}$$

54. To find the break-even point, set $R = C$.

$$\begin{aligned} 200x &= 130x + 12{,}600 \\ 70x &= 12{,}600 \\ x &= \frac{12{,}600}{70} \approx 180 \text{ units} \end{aligned}$$

55. (a) $C = 11.8x + 15{,}000$

$R = 19.30x$

(b)
$$\begin{aligned} C &= R \\ 11.8x + 15{,}000 &= 19.30x \\ 15{,}000 &= 7.5x \\ x &= 2000 \text{ units} \end{aligned}$$

(c)
$$\begin{aligned} P &= R - C \\ 1000 &= 19.3x - (11.8x + 15{,}000) \\ 16{,}000 &= 7.5x \\ x &\approx 2133.3 \end{aligned}$$

So, 2134 units would yield a profit of \$1000.

56. (a) The cost C_g to drive x miles is the cost of the car itself plus the cost of gasoline per mile. The cost of gasoline per gallon divided by the number of gallons per mile.

$$C_g = 28{,}695 + \frac{2.719}{21}x$$

Similarly, the cost C_h to drive x miles is the cost of the car itself plus the cost of gasoline per mile.

$$C_h = 29{,}720 + \frac{2.719}{34}x$$

(b) To find the break-even point, set the cost equations equal to each other.

$$28{,}695 + \frac{2.719}{21}x = 29{,}720 + \frac{2.719x}{34}$$

Multiply both sides of the equation by $(21)(34)$.

$$\begin{aligned} 20{,}488{,}230 + 92.446x &= 21{,}220{,}080 + 57.099x \\ 35.347x &= 731{,}850 \\ x &= \frac{731{,}850}{35.347} \approx 20{,}705 \text{ mi} \end{aligned}$$

57.
$$\begin{aligned} 240 - 4x &= 135 + 3x \\ 105 &= 7x \\ 15 &= x \end{aligned}$$

Equilibrium point $(x, p) = (15, 180)$

58.
$$\begin{aligned} 190 - 15x &= 75 + 8x \\ 115 &= 23x \\ x &= 5 \end{aligned}$$

Equilibrium point $(x, p) = (5, 115)$

59. (a)

Year	2004	2005	2006	2007	2008	2009
Amount	30	44	54	67	113	313
Model	26	55	48	56	126	308

The model fits the data well. Answers will vary.

(b) Let $t = 14$ (2014).

$$y = 8.148(14)^3 - 139.71(14)^2 + 789.0(14) - 1416 \approx \$4605 \text{ million}$$

60. (a) If less than 10,000 units are sold, the company loses money.

(b) If 10,000 units are sold, the company breaks even.

(c) If more than 10,000 units are sold, the company makes a profit.

61. (a)

Year	2004	2005	2006	2007	2008	2012
Degrees	667.4	692	713.6	732.2	747.8	780.2

(b) Answers will vary.

(c) Let $t = 16$ (2016).

$$y = -1.50(16)^2 + 38.1(16) + 539$$
$$= 764.6 \text{ degrees}$$

The prediction is not valid because the number of associate's degrees should keep increasing over time, and not decrease.

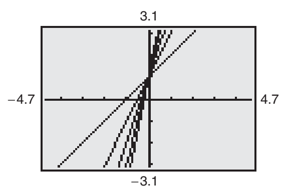
62. (a) and (b)

Year	2005	2006	2007	2008	2009
Transplants (from model)	1411.5	1402.56	1438.34	1518.84	1644.06
Transplants (actual)	1406	1405	1468	1478	1660

(c) For 2015, let $t = 15$ and $y = 3334.5$ or approximately 3335 transplants.

The prediction seems high. Answers will vary.

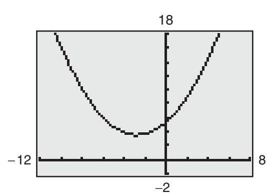
63.



The greater the value of c, the steeper the line.

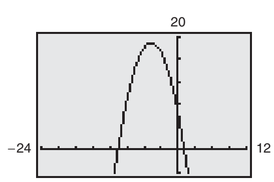
64. If C and R represent the cost and revenue for a business, the break-even point is that value of x for which $C = R$. For example, if $C = 100{,}000 + 10x$ and $R = 20x$, then the break-even point is $x = 10{,}000$ units.

65.



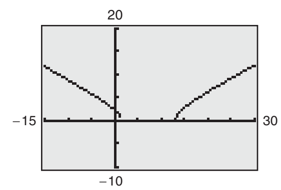
Intercept: $(0, 5.36)$

66.



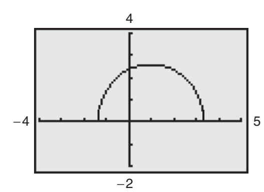
Intercepts: $(0, 6.25)$, $(1.0539, 0)$, $(-10.5896, 0)$

67.



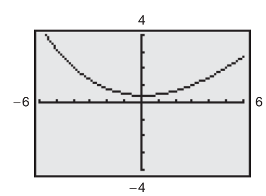
Intercepts: $(1.4780, 0)$, $(12.8553, 0)$, $(0, 2.3875)$

68.



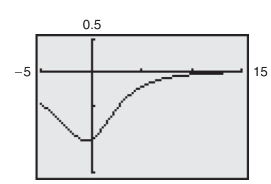
Intercepts: $(3.3256, 0)$, $(-1.3917, 0)$, $(0, 2.3664)$

69.



Intercept: $\left(0, \frac{5}{12}\right) \approx (0, 0.4167)$

70.



Intercepts: $(0, -1)$, $(13.25, 0)$

71. Answers will vary.

Section 1.3 Lines in the Plane and Slope

Skills Warm Up

1. $\dfrac{5-(-2)}{-3-4} = \dfrac{7}{-7} = -1$

2. $\dfrac{-7-(-10)}{4-1} = 1$

3. $-\dfrac{1}{m},\ m = -3$

   $-\dfrac{1}{-3} = \dfrac{1}{3}$

4. $-\dfrac{1}{m},\ m = \dfrac{6}{7}$

   $-\dfrac{1}{\frac{6}{7}} = -\dfrac{7}{6}$

5. $-4x + y = 7$

   $y = 4x + 7$

6. $3x - y = 7$

   $-y = 7 - 3x$

   $y = 3x - 7$

7. $y - 2 = 3(x - 4)$

   $y = 3(x - 4) + 2$

   $y = 3x - 12 + 2$

   $y = 3x - 10$

8. $y - (-5) = -1[x - (-2)]$

   $y + 5 = -x - 2$

   $y = -x - 7$

9. $y - (-3) = \dfrac{4-(-3)}{2-1}(x - 2)$

   $y + 3 = \dfrac{7}{1}(x - 2)$

   $y + 3 = 7x - 14$

   $y = 7x - 17$

10. $y - 1 = \dfrac{-3-1}{-7-(-1)}[x - (-1)]$

    $y - 1 = \dfrac{-4}{-6}(x + 1)$

    $y - 1 = \dfrac{2}{3}(x + 1)$

    $y - 1 = \dfrac{2}{3}x + \dfrac{2}{3}$

    $y = \dfrac{2}{3}x + \dfrac{5}{3}$

---

1. The slope is $m = 1$ because the line rises one unit vertically for each unit the line moves to the right.

2. The slope is 2 because the line rises two units vertically for each unit the line moves to the right.

3. The slope is $m = 0$ because the line is horizontal.

4. The slope is -1 because the line falls one unit vertically for each unit the line moves to the right.

5. $y = x + 7$

   So, the slope is $m = 1$, and the y-intercept is $(0, 7)$.

6. $y = 4x + 3$

   So, the slope is $m = 4$, and the y-intercept is $(0, 3)$.

7. $5x + y = 20$

   $y = -5x + 20$

   So, the slope is $m = -5$, and the y-intercept is $(0, 20)$.

8. $2x + y = 40$

   $y = -2x + 40$

   So, the slope is $m = -2$, and the y-intercept is $(0, 40)$.

9. $7x + 6y = 30$

   $y = -\frac{7}{6}x + 5$

   So, the slope is $m = -\frac{7}{6}$, and the y-intercept is $(0, 5)$.

10. $2x + 3y = 9$

    $y = -\frac{2}{3}x + 3$

    So, the slope is $m = -\frac{2}{3}$, and the y-intercept is $(0, 3)$.

11. $3x - y = 15$

    $y = 3x - 15$

    So, the slope is $m = 3$, and the y-intercept is $(0, -15)$.

12. $2x - 3y = 24$

$$y = \frac{2}{3}x - 8$$

So, the slope is $m = \frac{2}{3}$, and the y-intercept is $(0, -8)$.

13. $x = 4$

Because the line is vertical, the slope is undefined. There is no y-intercept.

14. $x + 5 = 0$

$$x = -5$$

Because the line is vertical, the slope is undefined. There is no y-intercept.

15. $y - 4 = 0$

$$y = 4$$

So, the slope is $m = 0$, and the y-intercept is $(0, 4)$.

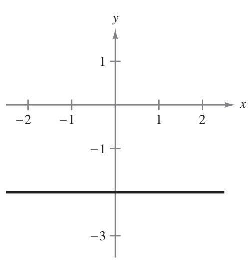
16. $y + 1 = 0$

$$y = -1$$

So, the slope is $m = 0$, and the y-intercept is $(0, -1)$.

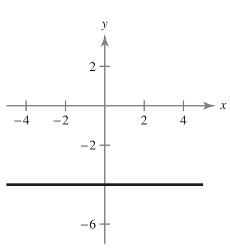
17. $y = -2$

x	-2	-1	0	1
y	-2	-2	-2	-2



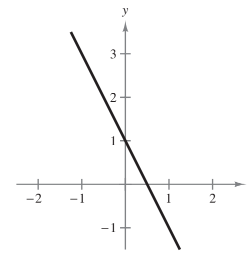
18. $y = -4$

x	-4	-2	0	2
y	-4	-4	-4	-4



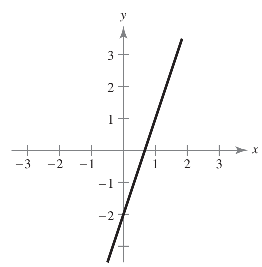
19. $y = -2x + 1$

x	-1	0	1	2
y	3	1	-1	-3



20. $y = 3x - 2$

x	-1	0	1	2
y	-5	-2	1	4

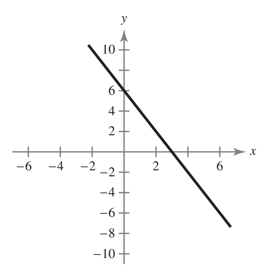


21. $6x + 3y = 18$

$$3y = -6x + 18$$

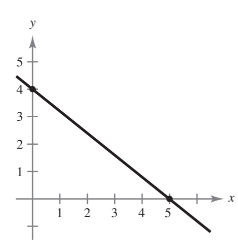
$$y = -2x + 6$$

x	-2	-1	0	3
y	-10	8	6	0



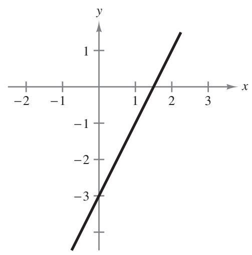
22. $y = -\frac{4}{5}x + 4$

x	0	2	4	5
y	4	$\frac{12}{5}$	$\frac{4}{5}$	0



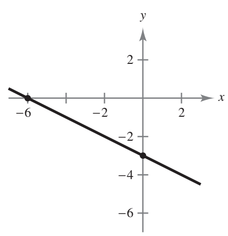
23. $y = 2x - 3$

x	-1	0	1	2
y	-5	-3	-1	1



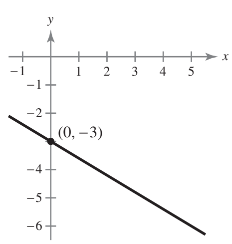
24. $y = -\frac{1}{2}x - 3$

x	-6	-2	0	2
y	0	-2	-3	-4



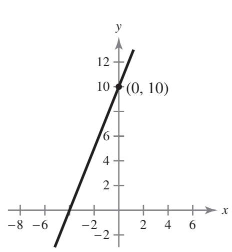
25. $y = -\frac{3}{5}x - 3$

x	0	2	4	5
y	-3	$-\frac{21}{5}$	$-\frac{27}{5}$	-6

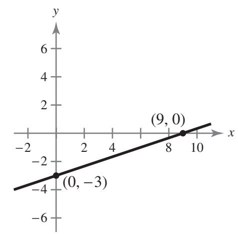


26. $y = \frac{5}{2}x + 10$

x	-4	-2	0	2
y	0	5	10	15

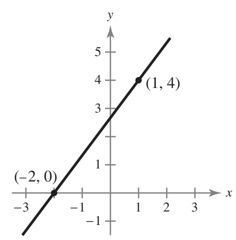


27.



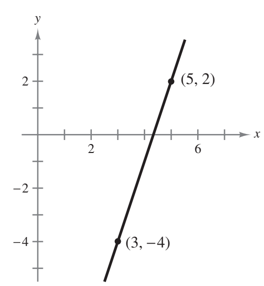
The slope is $m = \dfrac{0 - (-3)}{9 - 0} = \dfrac{1}{3}$.

28.



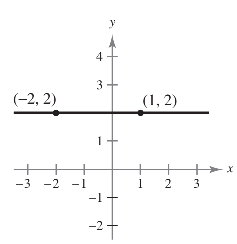
The slope is $m = \dfrac{4 - 0}{1 - (-2)} = \dfrac{4}{3}$.

29.



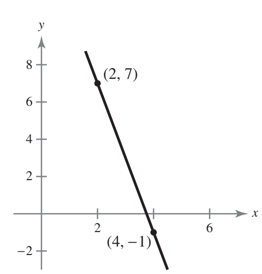
The slope is $m = \dfrac{2 - (-4)}{5 - 3} = 3$.

30.



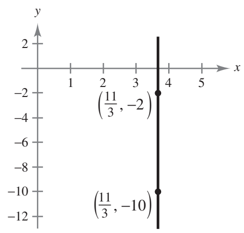
The slope is $m = \dfrac{2 - 2}{1 - (-2)} = 0$.

31.



The slope of $m = \dfrac{7 - (-1)}{2 - 4} = \dfrac{8}{-2} = -4$.

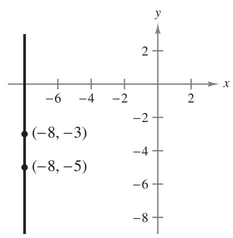
32.



The slope is $m = \dfrac{-10 - (-2)}{\frac{11}{3} - \frac{11}{3}} = \dfrac{-8}{0}$,

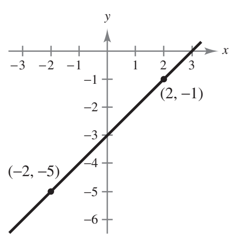
which is undefined.
So, the line is vertical.

33.



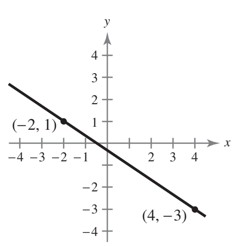
The slope is undefined because $m = \dfrac{-5 - (-3)}{-8 - (-8)}$ and division by zero is undefined. So, the line is vertical.

34.



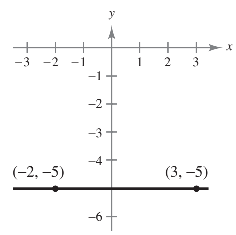
The slope is $m = \dfrac{-1 - (-5)}{2 - (-2)} = \dfrac{4}{4} = 1.$

35.



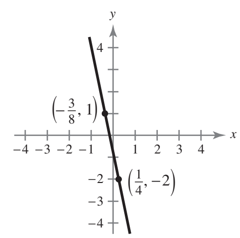
The slope is $m = \dfrac{-3 - 1}{4 - (-2)} = \dfrac{-4}{6} = -\dfrac{2}{3}.$

36.



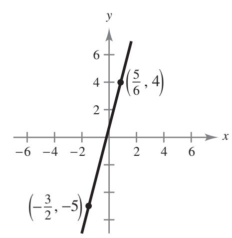
The slope is $m = \dfrac{-5 - (-5)}{-2 - 3} = 0.$

37.



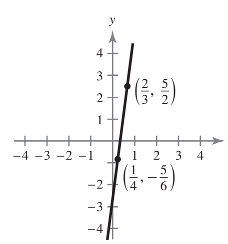
The slope is $m = \dfrac{1 - (-2)}{-\frac{3}{8} - \frac{1}{4}} = \dfrac{3}{-\frac{5}{8}} = -\dfrac{24}{5}.$

38.



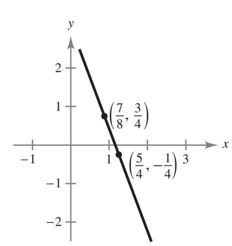
The slope is $m = \dfrac{4 + 5}{(5/6) + (3/2)} = \dfrac{27}{7}.$

39.



The slope is $m = \dfrac{\frac{5}{2} - \left(-\frac{5}{6}\right)}{\frac{2}{3} - \frac{1}{4}} = \dfrac{\frac{10}{3}}{\frac{5}{12}} = \dfrac{10}{3} \cdot \dfrac{12}{5} = 8.$

40.



The slope is $m = \dfrac{(-1/4) - (3/4)}{(5/4) - (7/8)} = -\dfrac{8}{3}.$

41. The equation of this horizontal line is $y = 1$. So, three additional points are $(0, 1)$, $(1, 1)$, and $(3, 1)$.

42. The equation of this horizontal line is $y = -1$. So, three additional points are $(0, -1)$, $(1, -1)$, and $(2, -1)$.

43. The equation of the line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

So, three additional points are $(0, 10)$, $(2, 4)$, and $(3, 1)$.

44. The equation of this line is

$$y + 2 = 2(x - 7)$$
$$y = 2x - 16.$$

So, three additional points are $(0, -16)$, $(1, -14)$, and $(2, -12)$.

45. The equation of this line is

$$y + 4 = \frac{2}{3}(x - 6)$$
$$y = \frac{2}{3}x - 8.$$

So, three additional points are $(3, -6)$, $(9, -2)$, and $(12, 0)$.

46. The equation of this line is

$$y + 6 = -\tfrac{1}{2}(x + 1)$$
$$y = -\tfrac{1}{2}x - \tfrac{13}{2}.$$

So, three additional points are $(1, -7)$, $(3, -8)$, and $(5, -9)$.

47. The equation of this vertical line is $x = -8$. So, three additional points are $(-8, 0)$, $(-8, 2)$, and $(-8, 3)$.

48. The equation of this vertical line is $x = -3$. So, three additional points are $(-3, 0)$, $(-3, 1)$, and $(-3, 2)$.

49. The slope of the line joining $(-2, 1)$ and $(-1, 0)$ is

$$\frac{1 - 0}{-2 - (-1)} = \frac{1}{-1} = -1.$$

The slope of the line joining $(-1, 0)$ and $(2, -2)$ is

$$\frac{0 - (-2)}{-1 - 2} = \frac{2}{-3} = -\frac{2}{3}.$$

Because the slopes are different, the points are not collinear.

50. The slope of the line joining $(-5, 11)$ and $(0, 4)$ is

$$\frac{11 - 4}{-5 - 0} = \frac{7}{-5} = -\frac{7}{5}.$$

The slope of the line joining $(0, 4)$ and $(7, -6)$ is

$$\frac{4 - (-6)}{0 - 7} = -\frac{10}{7}.$$

Because the slopes are different, the points are not collinear.

51. The slope of the line joining $(2, 7)$ and $(-2, -1)$ is

$$\frac{-1 - 7}{-2 - 2} = 2.$$

The slope of the line joining $(0, 3)$ and $(-2, -1)$ is

$$\frac{-1 - 3}{-2 - 0} = 2.$$

Because the slopes are equal and both lines pass through $(-2, -1)$, the three points are collinear.

52. The slope of the line joining $(4, 1)$ and $(-2, -2)$ is

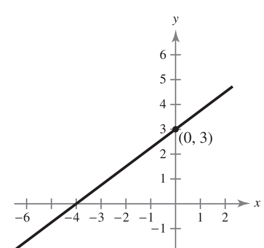
$$\frac{-2 - 1}{-2 - 4} = \frac{1}{2}.$$

The slope of the line joining $(8, 3)$ and $(-2, -2)$ is

$$\frac{-2 - 3}{-2 - 8} = \frac{1}{2}.$$

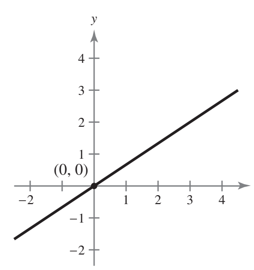
Because the slopes are equal and both lines pass through $(-2, -2)$, the three points are collinear.

53. Using the slope-intercept form, we have $y = \frac{3}{4}x + 3$.

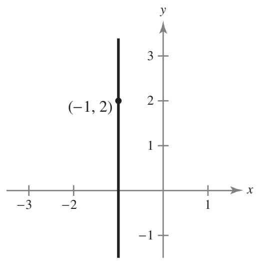


54. Using the slope-intercept form, we have

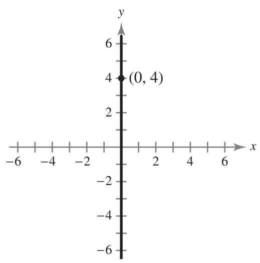
$$y = \tfrac{2}{3}x + 0$$
$$2x - 3y = 0.$$



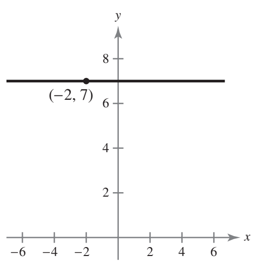
55. Because the slope is undefined, the line is vertical and its equation is $x = -1$.



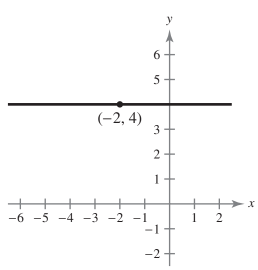
56. Because the slope is undefined, the line is vertical and its equation is $x = 0$.



57. Because the slope is 0, the line is horizontal and its equation is $y = 7$.

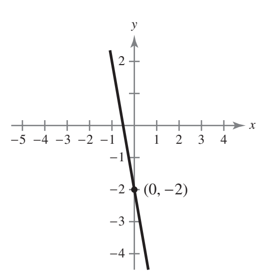


58. Because the slope is 0, the line is horizontal and its equation is $y = 4$.



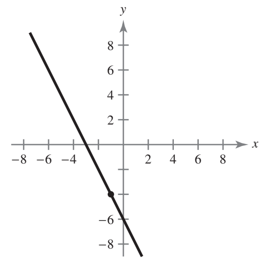
59. Using the point-slope form, we have

$$y + 2 = -4(x - 0)$$
$$y = -4x - 2$$
$$4x + y + 2 = 0.$$



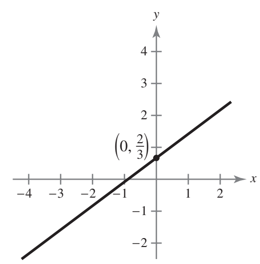
60. Using the point-slope form, we have

$$y + 4 = -2(x + 1)$$
$$y = -2x - 6$$
$$2x + y + 6 = 0.$$



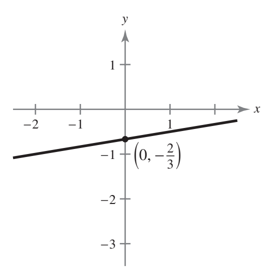
61. Using the point-slope form, we have

$$y - \tfrac{2}{3} = \tfrac{3}{4}(x - 0)$$
$$y = \tfrac{3}{4}x + \tfrac{2}{3}$$
$$0 = 9x - 12y + 8.$$



62. Using the point-slope form, we have

$$y + \tfrac{2}{3} = \tfrac{1}{6}(x - 0)$$
$$6y + 4 = x$$
$$0 = x - 6y - 4.$$

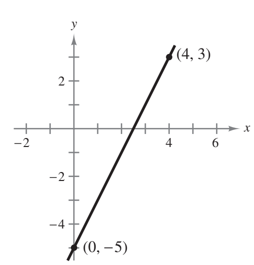


63. The slope of the line is

$$m = \frac{3 - (-5)}{4 - 0} = 2.$$

Using the point-slope form, we have

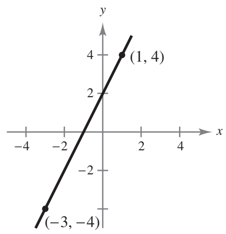
$$y + 5 = 2(x - 0)$$
$$y = 2x - 5$$
$$0 = 2x - y - 5.$$



64. The slope of the line is $m = \dfrac{4 - (-4)}{1 - (-3)} = 2.$

Using the point-slope form, we have

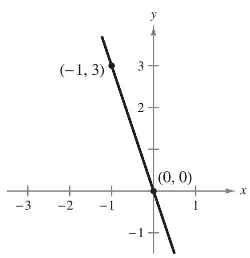
$$y - 4 = 2(x - 1)$$
$$y = 2x + 2$$
$$0 = 2x - y + 2.$$



65. The slope of the line is $m = \dfrac{3 - 0}{-1 - 0} = -3.$

Using the point-slope form, we have

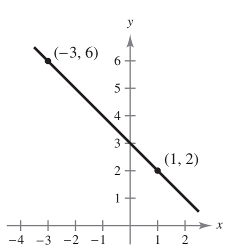
$$y = -3x$$
$$3x + y = 0.$$



66. The slope of the line is $m = \dfrac{2 - 6}{1 - (-3)} = -1.$

Using the point-slope form, we have

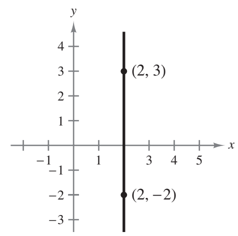
$$y - 2 = -1(x - 1)$$
$$y = -x + 3$$
$$x + y - 3 = 0.$$



67. The slope of the line is $m = \dfrac{-2 - 3}{2 - 2} =$ undefined.

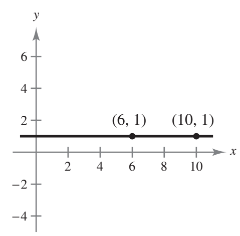
So, the line is vertical, and its equation is

$$x = 2$$
$$x - 2 = 0.$$



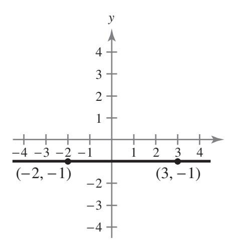
68. The slope of the line is $m = \dfrac{1 - 1}{10 - 6} = 0.$ So, the line is horizontal, and its equation is

$$y = 1$$
$$y - 1 = 0.$$



69. The slope of the line is $m = \dfrac{-1 - (-1)}{-2 - 3} = 0.$ So, the line is horizontal, and its equation is

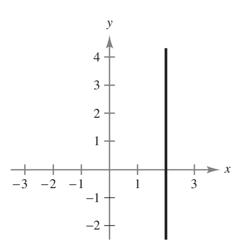
$$y = -1$$
$$y + 1 = 0.$$



70. The slope of the line is $m = \dfrac{-10 - 5}{2 - 2} =$ undefined.

So, the line is vertical, and its equation is

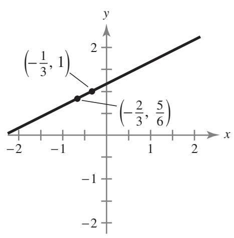
$$x = 2$$
$$x - 2 = 0.$$



71. The slope of the line is $m = \dfrac{1 - 5/6}{(-1/3) + 2/3} = \dfrac{1}{2}$.

Using the point-slope form, we have

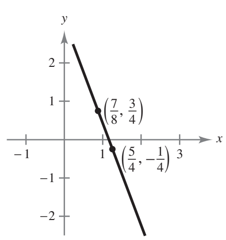
$$y - 1 = \frac{1}{2}\left(x + \frac{1}{3}\right)$$
$$y = \frac{1}{2}x + \frac{7}{6}$$
$$3x - 6y + 7 = 0.$$



72. The slope of the line is $m = \dfrac{(-1/4) - (3/4)}{(5/4) - (7/8)} = -\dfrac{8}{3}$.

Using the point-slope form, we have

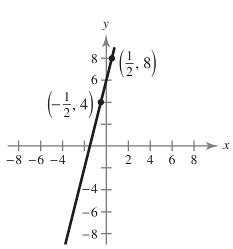
$$y - \frac{3}{4} = -\frac{8}{3}\left(x - \frac{7}{8}\right)$$
$$y - \frac{3}{4} = -\frac{8}{3}x + \frac{7}{3}$$
$$12y - 9 = -32x + 28$$
$$32x + 12y - 37 = 0.$$



73. The slope of the line is $m = \dfrac{8 - 4}{1/2 + 1/2} = 4$.

Using the point-slope form, we have

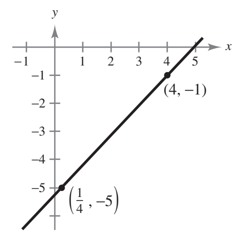
$$y - 8 = 4\left(x - \frac{1}{2}\right)$$
$$y = 4x + 6$$
$$0 = 4x - y + 6.$$



74. The slope is $m = \dfrac{-1 - (-5)}{4 - (1/4)} = \dfrac{16}{15}$.

Using the point-slope form, we have

$$y + 1 = \frac{16}{15}(x - 4)$$
$$15y + 15 = 16x - 64$$
$$15y - 16x + 79 = 0.$$



75. Because the line is vertical, it has an undefined slope, and its equation is

$$x = 3$$
$$x - 3 = 0.$$

76. Because the line is horizontal, it has a slope of $m = 0$, and its equation is

$$y = 0x + (-5)$$
$$y = -5.$$

77. Because the line is parallel to a horizontal line, it has a slope of $m = 0$, and its equation is

$$y = -10.$$

78. Because the line is parallel to a vertical line, it has an undefined slope, and its equation is

$$x = -5.$$

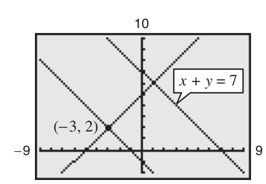
79. Given line: $y = -x + 7$, $m = -1$

(a) Parallel: $m_1 = -1$

$$y - 2 = -1(x + 3)$$
$$x + y + 1 = 0$$

(b) Perpendicular: $m_2 = 1$

$$y - 2 = 1(x + 3)$$
$$x - y + 5 = 0$$



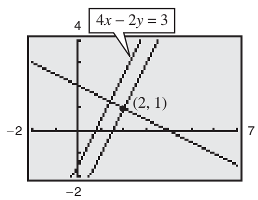
80. Given line: $y = 2x - \frac{3}{2}$, $m = 2$

(a) Parallel: $m_1 = 2$

$$y - 1 = 2(x - 2)$$
$$0 = 2x - y - 3$$

(b) Perpendicular: $m_2 = -\frac{1}{2}$

$$y - 1 = -\frac{1}{2}(x - 2)$$
$$2y - 2 = -x + 2$$
$$x + 2y - 4 = 0$$



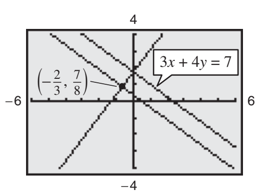
81. Given line: $y = -\frac{3}{4}x + \frac{7}{4}$, $m = -\frac{3}{4}$

(a) Parallel: $m_1 = -\frac{3}{4}$

$$y - \frac{7}{8} = -\frac{3}{4}\left(x + \frac{2}{3}\right) = -\frac{3}{4}x - \frac{1}{2}$$
$$8y - 7 = -6x - 4$$
$$6x + 8y - 3 = 0$$

(b) Perpendicular: $m_2 = \frac{4}{3}$

$$y - \frac{7}{8} = \frac{4}{3}\left(x + \frac{2}{3}\right) = \frac{4}{3}x + \frac{8}{9}$$
$$72y - 63 = 96x + 64$$
$$96x - 72y + 127 = 0$$



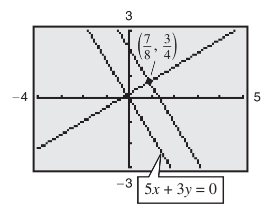
82. Given line: $y = -\frac{5}{3}x$, $m = -\frac{5}{3}$

(a) Parallel: $m_1 = -\frac{5}{3}$

$$y - \frac{3}{4} = -\frac{5}{3}\left(x - \frac{7}{8}\right)$$
$$y - \frac{3}{4} = -\frac{5}{3}x + \frac{35}{24}$$
$$24y - 18 = -40x + 35$$
$$40x + 24y - 53 = 0$$

(b) Perpendicular: $m_2 = \frac{3}{5}$

$$y - \frac{3}{4} = \frac{3}{5}\left(x - \frac{7}{8}\right)$$
$$y - \frac{3}{4} = \frac{3}{5}x - \frac{21}{40}$$
$$40y - 30 = 24x - 21$$
$$0 = 24x - 40y + 9$$



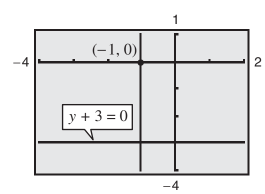
83. Given line: $y = -3$ is horizontal, $m = 0$

(a) Parallel: $m_1 = 0$

$$y - 0 = 0(x + 1)$$
$$y = 0$$

(b) Perpendicular: m_2 is undefined

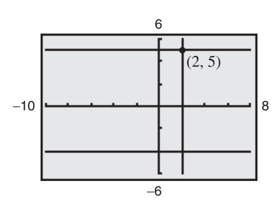
$$x = -1$$



84. Given line: $y + 4 = 0$ is horizontal, $m = 0$

(a) Parallel: $m_1 = 0$, $y - 5 = 0(x - 2)$, $y = 5$

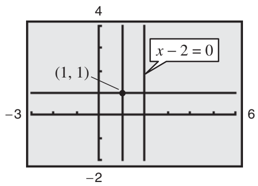
(b) Perpendicular: m_2 is undefined, $x = 2$



85. Given line: $x - 2 = 0$ is vertical, m is undefined

(a) Parallel: m_1 is undefined, $x = 1$

(b) Perpendicular: $m_2 = 0$, $y - 1 = 0(x - 1)$, $y = 1$

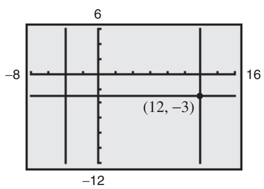


86. Given line: $x + 4 = 0$ is vertical, m is undefined

(a) Parallel: m_1 is undefined, $x = 12$

(b) Perpendicular:

$m_2 = 0$, $y + 3 = 0(x - 12)$, $y = -3$



87. (a) The average salary increased the most from 2006 to 2008 and increased the least from 2002 to 2004.

(b) 1996: $(6, 69{,}277)$ and 2008: $(18, 97{,}486)$

$$m = \frac{97{,}486 - 69{,}277}{18 - 6} = \frac{28{,}209}{12} = \$2350.75/\text{yr}$$

(c) The average salary increased $2350.75 per year over the 12 years between 1996 and 2008.

88. (a) The revenue increased the greatest from 2005 to 2006, and increased the least from 2003 to 2004.

(b) 2003: $(3, 67.8)$ and 2009: $(9, 107.8)$

$$m = \frac{107.8 - 67.8}{9 - 3} = \frac{40}{6} = \$6.667 \text{ billion/yr}$$

(c) From 2003 to 2009, the revenue for Verizon increased at an average rate of $6.667 billion per year.

89. $(0, 32)$, $(100, 212)$

$$F - 32 = \frac{212 - 32}{100 - 0}(C - 0)$$

$$F = 1.8C + 32 = \frac{9}{5}C + 32$$

or

$$C = \frac{5}{9}(F - 32)$$

90. Use $C = \frac{5}{9}(F - 32)$.

(a) If $F = 102.2°\text{F}$, then $C = \frac{5}{9}(102.2 - 32) = 39°\text{C}$.

(b) If $F = 76°\text{F}$, then $C = \frac{5}{9}(76 - 32) = 24.4°\text{C}$.

91. (a) 2004: $(4, 5511)$ and 2009: $(9, 5655)$

$$m = \frac{5655 - 5511}{9 - 4} = \frac{144}{5} = 28.8$$

$$y - y_1 = m(t - t_1)$$
$$y - 5511 = 28.8(t - 4)$$
$$y - 5511 = 28.8t - 115.2$$
$$y = 28.8t + 5395.8$$

The slope is 28.8 and indicates the population increases 28.8 thousand per year from 2004 to 2009.

(b) Let $t = 6$.

$$y = 28.8(6) + 5395.8$$
$$y = 5568.6$$

The population was 5568.6 thousand or 5,568,600 in 2006.

(c) The actual population was 5,572,000.

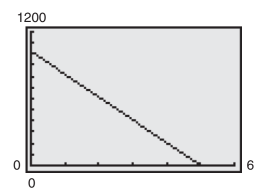
The model's estimate was very close to the actual population.

(d) The model could possibly be used to predict the population in 2015 if the population continues to grow at the same linear rate.

92. (a) The equipment depreciates $\frac{1025}{5} = \$205$ per year,

so the value is $y = 1025 - 205t$, where $0 \le t \le 5$.

(b)



(c) When $t = 3$, the value is $410.00.

(d) The value is $600 when $t = 2.07$ years.

93. (a) 2004: $(4, 9937)$ and 2009: $(9, 12{,}175)$

$$m = \frac{12{,}175 - 9937}{9 - 4} = \frac{2238}{5} = 447.6$$

$$y - y_1 = m(t - t_1)$$
$$y - 9937 = 447.6(t - 4)$$
$$y - 9937 = 447.6t - 1790.4$$
$$y = 447.6t + 8146.6$$

(b) Let $t = 6$.

$$y = 447.6(6) + 8146.6 = \$10{,}832.2 \text{ billion}$$

(c) Let $t = 11$.

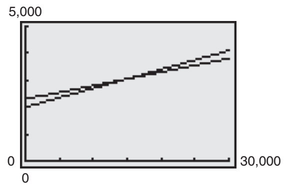
$$y = 447.6(11) + 8146.6 = \$13{,}070.2 \text{ billion}$$

(d) Answers will vary.

94. (a) Current wage: $W_c = 0.07s + 2000$

New offer wage: $W_N = 0.05s + 2300$

(b)



The lines intersect at $(15{,}000, 3050)$. If you sell \$15,000, then both jobs would yield wages of \$3050.

(c) No. Your current job would yield wages of \$3400 as compared to the new job, which would yield wages of \$3300 if your sales are \$20,000.

95. (a) $C = 50x + 350{,}000$

$R = 120x$

(b) $P = R - C$

$= 120x - (50x + 350{,}000)$

$= 70x - 350{,}000$

(c) If $x = 13{,}000$, then the profit is

$P = 70(13{,}000) - 350{,}000$

$= \$560{,}000.$

96. (a) Matches (ii); $y = -10x + 100.$

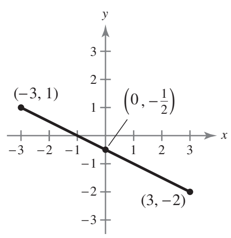
(b) Matches (iii); $y = 1.50x + 12.50.$

(c) Matches (i); $y = 0.51x + 30.$

(d) Matches (iv); $y = -100x + 600.$

Chapter 1 Quiz Yourself

1. (a)



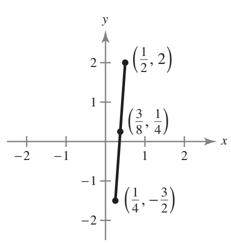
(b) $d = \sqrt{(-3 - 3)^2 + (1 - (-2))^2}$

$= \sqrt{36 + 9}$

$= 3\sqrt{5}$

(c) Midpoint $= \left(\dfrac{-3 + 3}{2}, \dfrac{1 - 2}{2}\right) = \left(0, -\dfrac{1}{2}\right)$

2. (a)



(b) $d = \sqrt{\left(\dfrac{1}{2} - \dfrac{1}{4}\right)^2 + \left(2 - \left(-\dfrac{3}{2}\right)\right)^2}$

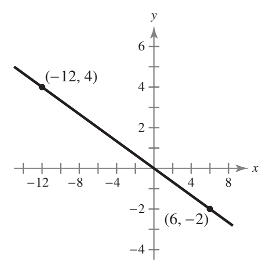
$= \sqrt{\dfrac{1}{16} + \dfrac{49}{4}}$

$= \sqrt{\dfrac{197}{16}}$

$= \dfrac{1}{4}\sqrt{197}$

(c) Midpoint $= \left(\dfrac{\frac{1}{2} + \frac{1}{4}}{2}, \dfrac{2 - \frac{3}{2}}{2}\right) = \left(\dfrac{3}{8}, \dfrac{1}{4}\right)$

3. (a)



(b) $d = \sqrt{(6 - (-12))^2 + (-2 - 4)^2}$

$= \sqrt{18^2 + (-6)^2}$

$= \sqrt{324 + 36}$

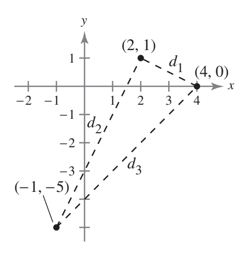
$= \sqrt{360}$

$= 6\sqrt{10}$

≈ 18.97

(c) Midpoint $= \left(\dfrac{-12 + 6}{2}, \dfrac{4 + (-2)}{2}\right) = (-3, 1)$

4.



$a = \sqrt{(2 - 4)^2 + (1 - 0)^2} = \sqrt{5}$

$b = \sqrt{(2 - (-1))^2 + (1 - (-5))^2} = 3\sqrt{5}$

$c = \sqrt{(-1 - 4)^2 + (-5 - 0)^2} = 5\sqrt{2}$

$a^2 + b^2 = (\sqrt{5})^2 + (3\sqrt{5})^2 = (5\sqrt{2})^2 = c^2$

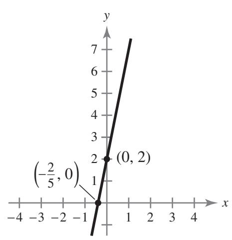
5. $(2007, 9534)$ and $(2009, 9829)$

$$\text{Midpoint} = \left(\frac{2007 + 2009}{2}, \frac{9534 + 9829}{2}\right)$$
$$= (2008, 9681.5)$$

The population in 2008 was approximately 9681.5 thousand or 9,681,500.

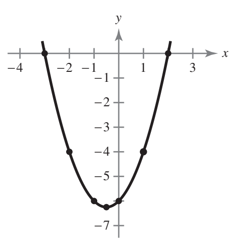
6. $y = 5x + 2$

x	$-\frac{2}{5}$	0	$\frac{1}{5}$	1
y	0	2	3	7



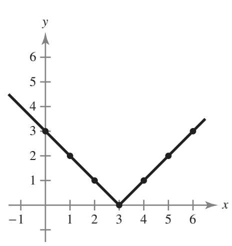
7. $y = x^2 + x - 6$

x	-3	-2	-1	-0.5	0	1	2
y	0	-4	-6	-6.25	-6	-4	0



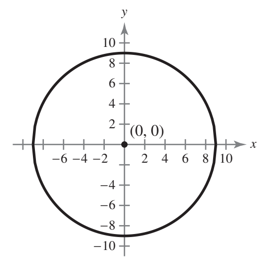
8. $y = |x - 3|$

x	0	1	2	3	4	5	6
y	3	2	1	0	1	2	3



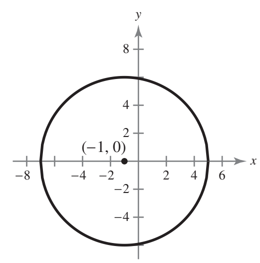
9. $(x - 0)^2 + (y - 0)^2 = 9^2$

$$x^2 + y^2 = 81$$



10. $(x - (-1))^2 + (y - 0)^2 = 6^2$

$$(x + 1)^2 + y^2 = 36$$

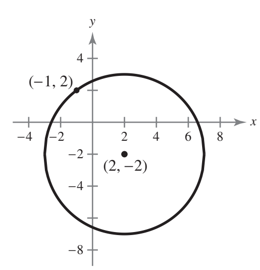


11. The radius is the distance between $(2, -2)$ and $(-1, 2)$.

$$r = \sqrt{(-1 - 2)^2 + (2 - (-2)^2)}$$
$$r = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Using the center $(2, -2)$ and radius $r = 5$:

$$(x - 2)^2 + (y - (-2))^2 = 5^2$$
$$(x - 2)^2 + (y + 2)^2 = 25$$



12. $C = 4.55x + 12{,}500$

$R = 7.19x$

$R = C$

$7.19x = 4.55x + 12{,}500$

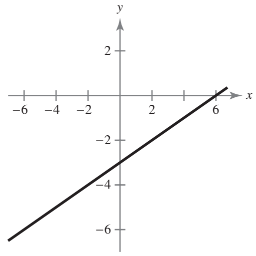
$2.64x = 12{,}500$

$x \approx 4734.8$

The company must sell 4735 units to break even.

13. $y = mx + b$

$y = \frac{1}{2}x - 3$

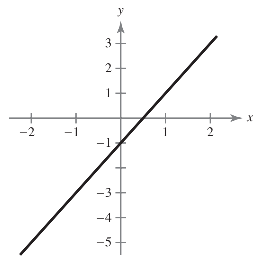


14. $y - y_1 = m(x - x_1)$

$y - 1 = 2(x - 1)$

$y - 1 = 2x - 2$

$y = 2x - 1$

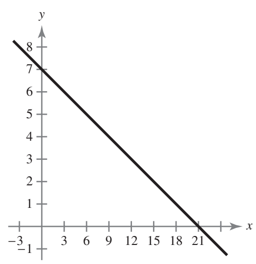


15. $y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{3}(x - 6)$

$y - 5 = -\frac{1}{3}x + 2$

$y = -\frac{1}{3}x + 7$

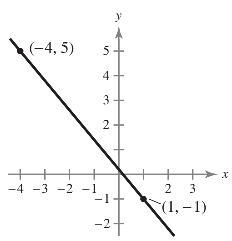


16. $(1, -1), (-4, 5)$

$m = \dfrac{5 + 1}{-4 - 1} = -\dfrac{6}{5}$

$y + 1 = -\dfrac{6}{5}(x - 1)$

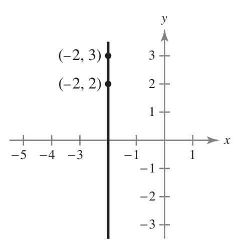
$y = -\dfrac{6}{5}x + \dfrac{1}{5}$



17. $(-2, 3), (-2, 2)$

$m = \dfrac{2 - 3}{-2 + 2} =$ undefined

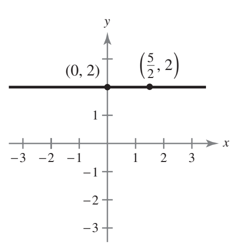
Because the slope is undefined, the line is vertical and its equation is $x = -2$.



18. $\left(\dfrac{5}{2}, 2\right), (0, 2)$

$m = \dfrac{2 - 2}{0 - \frac{5}{2}} = 0$

Because the slope is 0, the line is horizontal and its equation is $y = 2$.



19. Given line: $y = -\frac{1}{4}x - \frac{1}{2}$, $m = -\frac{1}{4}$

(a) Parallel: $m_1 = -\frac{1}{4}$

$y + 5 = -\frac{1}{4}(x - 3)$

$y = -\frac{1}{4}x - \frac{17}{4}$

(b) Perpendicular: $m_2 = 4$

$y + 5 = 4(x - 3)$

$y = 4x - 17$

20. Let $t = 7$ correspond to 2007.

$(7, 1{,}330{,}000), (11, 1{,}800{,}000)$

$$m = \frac{1{,}800{,}000 - 1{,}330{,}000}{11 - 7} = \frac{470{,}000}{4} = 117{,}500$$

$y - 1{,}330{,}000 = 117{,}500(x - 7)$

$y - 1{,}330{,}000 = 117{,}500x - 822{,}500$

$y = 117{,}500x + 507{,}500$

For 2015, let $t = 15$.

$y = 117{,}500(15) + 507{,}500 = \$2{,}270{,}000$

For 2018, let $t = 18$.

$y = 117{,}500(18) + 507{,}500 = \$2{,}622{,}500$

21. The daily cost equals the cost for lodging and meals plus the cost per mile driven, x.

$C = 175 + 0.55x$

22. (a) Let $t = 9$ correspond to 2009 and S equal salary.

2009: $(9, 34{,}600)$ and 2011: $(11, 37{,}800)$

$m = \dfrac{37{,}800 - 34{,}600}{11 - 9} = \dfrac{3200}{2} = 1600$

$S - S_1 = m(t - t_1)$

$S - 34{,}600 = 1600(t - 9)$

$S - 34{,}600 = 1600t - 14{,}400$

$S = 1600t + 20{,}200$

(b) For 2015, let $t = 15$.

$S = 1600(15) + 20{,}200$

$= \$44{,}200$

Section 1.4 Functions

Skills Warm Up

1. $5(-1)^2 - 6(-1) + 9 = 5(1) + 6 + 9 = 20$

2. $(-2)^3 + 7(-2)^2 - 10 = -8 + 7(4) - 10 = -18 + 28 = 10$

3. $(x - 2)^2 + 5x - 10 = x^2 - 4x + 4 + 5x - 10 = x^2 + x - 6$

4. $$\begin{aligned}(3 - x) + (x + 3)^3 &= (3 - x) + (x + 3)(x^2 + 6x + 9)\\ &= (3 - x) + x^3 + 3x^2 + 6x^2 + 18x + 9x + 27\\ &= x^3 + 9x^2 + 26x + 30\end{aligned}$$

5. $\dfrac{1}{1 - (1 - x)} = \dfrac{1}{1 - 1 + x} = \dfrac{1}{x}$

6. $1 + \dfrac{x - 1}{x} = \dfrac{x}{x} + \dfrac{x - 1}{x} = \dfrac{x + x - 1}{x} = \dfrac{2x - 1}{x}$

7. $$\begin{aligned}2x + y - 6 &= 11\\ y &= -2x + 17\end{aligned}$$

8. $$\begin{aligned}5y - 6x^2 - 1 &= 0\\ 5y &= 6x^2 + 1\\ y &= \frac{6x^2 + 1}{5}\\ &= \frac{6}{5}x^2 + \frac{1}{5}\end{aligned}$$

9. $$\begin{aligned}(y - 3)^2 &= 5 + (x + 1)^2\\ y - 3 &= \sqrt{5 + (x + 1)^2}\\ y - 3 &= \sqrt{5 + x^2 + 2x + 1}\\ y &= \sqrt{x^2 + 2x + 6} + 3\end{aligned}$$

10. $$\begin{aligned}y^2 - 4x^2 &= 2\\ y^2 &= 2 + 4x^2\\ y &= \sqrt{2 + 4x^2}\end{aligned}$$

11. $$\begin{aligned}x &= \frac{2y - 1}{4}\\ 4x &= 2y - 1\\ 4x + 1 &= 2y\\ \frac{4x + 1}{2} &= y\\ 2x + \frac{1}{2} &= y\end{aligned}$$

12. $$\begin{aligned}x &= \sqrt[3]{2y - 1}\\ x^3 &= 2y - 1\\ -2y &= -x^3 - 1\\ y &= \tfrac{1}{2}x^3 + \tfrac{1}{2}\end{aligned}$$

1. $$\begin{aligned}x^2 + y^2 &= 16\\ y^2 &= 16 - x^2\\ y &= \pm\sqrt{16 - x^2}\end{aligned}$$

y is *not* a function of x since there are two values of y for some x.

2. $y = \pm\sqrt{4 - x}$

y is *not* a function of x since there are two values of y for some x.

3. $$\begin{aligned}\tfrac{1}{2}x - 6y &= -3\\ y &= \tfrac{1}{12}x + \tfrac{1}{2}\end{aligned}$$

y *is* a function of x since there is only one value of y for each x.

4. $y = \dfrac{3x + 5}{2}$

y *is* a function of x since there is only one value of y for each x.

5. $y = 4 - x^2$

y *is* a function of x since there is only one value of y for each x.

6. $$\begin{aligned}x^2 + y^2 + 2x &= 0\\ y^2 &= -x^2 - 2x\\ y &= \pm\sqrt{-x^2 - 2x}\end{aligned}$$

y is *not* a function of x since there are two values of y for some x.

7. $y = |x + 2|$

y *is* a function of x since there is only one value of y for each x.

8. $y(x^2 + 4) = x^2$

$$y = \frac{x^2}{x^2 + 4}$$

y *is* a function of x since there is only one value of y for each x. [**Note:** It is not a one-to-one function.]

9. y is *not* a function of x.

10. y *is* a function of x.

11. y *is* a function of x.

12. y is *not* a function of x.

13. Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

14. Domain: $\left[\frac{3}{2}, \infty\right)$

Range: $[0, \infty)$

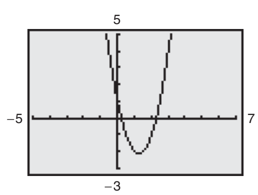
15. Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

16. Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

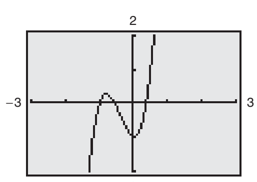
17.



Domain: $(-\infty, \infty)$

Range: $[-2.125, \infty)$

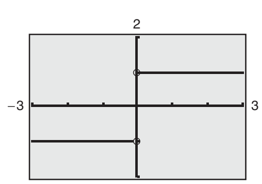
18.



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

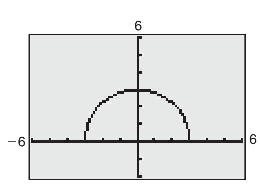
19.



Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $\{-1, 1\}$

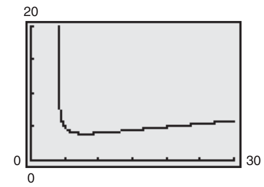
20.



Domain: $[-3, 3]$

Range: $[0, 3]$

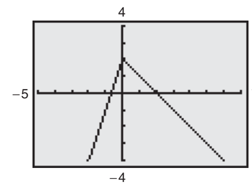
21.



Domain: $(4, \infty)$

Range: $[4, \infty)$

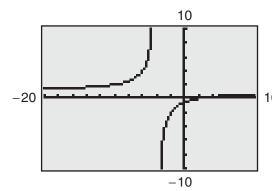
22.



Domain: $(-\infty, \infty)$

Range: $(-\infty, 2]$

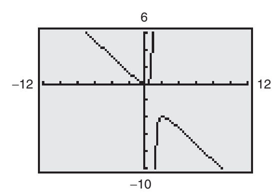
23.



Domain: $(-\infty, -4) \cup (-4, \infty)$

Range: $(-\infty, 1) \cup (1, \infty)$

24.



Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, -4] \cup [0, \infty)$

25. $f(x) = 3x - 2$

(a) $f(0) = 3(0) - 2 = -2$

(b) $f(5) = 3(5) - 2 = 13$

(c) $f(x - 1) = 3(x - 1) - 2 = 3x - 3 - 2 = 3x - 5$

26. $f(x) = x^2 - 4x + 1$

(a) $f(-1) = (-1)^2 - 4(-1) + 1 = 6$

(b) $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1 = -\frac{3}{4}$

(c) $$\begin{aligned} f(c + 2) &= (c + 2)^2 - 4(c + 2) + 1 \\ &= c^2 + 4c + 4 - 4c - 8 + 1 \\ &= c^2 - 3 \end{aligned}$$

27. $g(x) = \dfrac{1}{x}$

(a) $g\left(\dfrac{1}{4}\right) = \dfrac{1}{\frac{1}{4}} = 4$

(b) $g(-4) = \dfrac{1}{-4} = -\dfrac{1}{4}$

(c) $g(x + 4) = \dfrac{1}{x + 4}$

28. $f(x) = |x| + 4$

(a) $f(-2) = |-2| + 4 = 6$

(b) $f(2) = |2| + 4 = 6$

(c) $f(x + 2) = |x + 2| + 4$

29. $\dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

$$= \frac{(x + \Delta x)^2 - 5(x + \Delta x) + 2 - (x^2 - 5x + 2)}{\Delta x}$$

$$= \frac{[x^2 + 2x\Delta x + (\Delta x)^2 - 5x + 5\Delta x + 2] - [x^2 - 5x + 2]}{\Delta x}$$

$$= \frac{2x\Delta x + (\Delta x)^2 + 5\Delta x}{\Delta x}$$

$$= 2x + \Delta x + 5,\ \Delta x \neq 0$$

30. $\dfrac{h(2 + \Delta x) - h(2)}{\Delta x}$

$$= \frac{(2 + \Delta x)^2 + (2 + \Delta x) + 3 - (2^2 + 2 + 3)}{\Delta x}$$

$$= \frac{\left[4 + 4\Delta x + (\Delta x)^2 + 2 + \Delta x + 3\right] - [4 + 2 + 3]}{\Delta x}$$

$$= \frac{4\Delta x + (\Delta x)^2 + \Delta x}{\Delta x}$$

$$= 4 + \Delta x + 1,\ \Delta x \neq 0$$

31. $\dfrac{g(x + \Delta x) - g(x)}{\Delta x}$

$$= \frac{\sqrt{x + \Delta x + 1} - \sqrt{x + 1}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x + 1} + \sqrt{x + 1}}{\sqrt{x + \Delta x + 1} + \sqrt{x + 1}}$$

$$= \frac{(x + \Delta x + 1) - (x + 1)}{\Delta x\left[\sqrt{x + \Delta x + 1} + \sqrt{x + 1}\right]}$$

$$= \frac{1}{\sqrt{x + \Delta x + 1} + \sqrt{x + 1}},\ \Delta x \neq 0$$

32. $\dfrac{f(x) - f(2)}{x - 2} = \dfrac{\dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt{2}}}{x - 2}$

$$= \frac{\dfrac{1}{\sqrt{x}} \cdot \dfrac{2\sqrt{x}}{2\sqrt{x}} - \dfrac{1}{\sqrt{2}} \cdot \dfrac{x\sqrt{2}}{x\sqrt{2}}}{x - 2}$$

$$= \frac{\dfrac{2\sqrt{x} - x\sqrt{2}}{2x}}{x - 2}$$

$$= \frac{2\sqrt{x} - x\sqrt{2}}{2x(x - 2)}$$

33. $\dfrac{f(x + \Delta x) - f(x)}{\Delta x} = \dfrac{\dfrac{1}{x + \Delta x - 2} - \dfrac{1}{x - 2}}{\Delta x}$

$$= \frac{(x - 2) - (x + \Delta x - 2)}{(x + \Delta x - 2)(x - 2)\Delta x}$$

$$= \frac{-1}{(x + \Delta x - 2)(x - 2)},\ \Delta x \neq 0$$

34. $\dfrac{f(x + \Delta x) - f(x)}{\Delta x} = \dfrac{\dfrac{1}{x + \Delta x + 4} - \dfrac{1}{x + 4}}{\Delta x}$

$$= \frac{(x + 4) - (x + \Delta x + 4)}{\Delta x[x + \Delta x + 4][x + 4]}$$

$$= \frac{-1}{(x + \Delta x + 4)(x + 4)},\ \Delta x \neq 0$$

35. (a) $f(x) + g(x) = 2x - 5 + 5 = 2x$

(b) $f(x) - g(x) = 2x - 5 - 5 = 2x - 10$

(c) $f(x) \cdot g(x) = (2x - 5)(5) = 10x - 25$

(d) $f(x)/g(x) = \dfrac{2x - 5}{5}$

(e) $f(g(x)) = f(5) = 2(5) - 5 = 5$

(f) $g(f(x)) = g(2x - 5) = 5$

36. (a) $f(x) + g(x) = x^2 + 5 + \sqrt{1 - x},\ x \leq 1$

(b) $f(x) - g(x) = x^2 + 5 - \sqrt{1 - x},\ x \leq 1$

(c) $f(x) \cdot g(x) = \left(\sqrt{1 - x}\right)(x^2 + 5),\ x \leq 1$

(d) $f(x)/g(x) = \dfrac{x^2 + 5}{\sqrt{1 - x}},\ x \leq 1$

(e) $f(g(x)) = 6 - x,\ x \leq 1$

(f) $g(f(x))$ is undefined

37. (a) $f(x) + g(x) = x^2 + 1 + x - 1 = x^2 + x$

(b) $f(x) - g(x) = x^2 + 1 - x + 1 = x^2 - x + 2$

(c) $f(x) \cdot g(x) = (x^2 + 1)(x - 1) = x^3 - x^2 + x - 1$

(d) $f(x)/g(x) = \dfrac{x^2 + 1}{x - 1}, x \neq 1$

(e) $f(g(x)) = (x - 1)^2 + 1 = x^2 - 2x + 2$

(f) $g(f(x)) = x^2 + 1 - 1 = x^2$

38. (a) $f(x) + g(x) = \dfrac{x^4 + x^3 + x}{x + 1}, x \neq -1$

(b) $f(x) - g(x) = \dfrac{x - x^3 - x^4}{x + 1}, x \neq -1$

(c) $f(x) \cdot g(x) = \dfrac{x^4}{x + 1}, x \neq -1$

(d) $f(x)/g(x) = \dfrac{x}{x^4 + x^3}, x \neq 0, -1$

(e) $f(g(x)) = \dfrac{x^3}{x^3 + 1}, x \neq -1$

(f) $g(f(x)) = \dfrac{x^3}{(x + 1)^3}, x \neq -1$

39. (a) $f(g(1)) = f(1^2 - 1) = f(0) = 0$

(b) $g(f(1)) = g(\sqrt{1}) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = x - 1, x \geq 0$

40. (a) $f(g(2)) = f(3) = \dfrac{1}{3}$

(b) $g(f(2)) = g\left(\dfrac{1}{2}\right) = -\dfrac{3}{4}$

(c) $f(g(-3)) = f(8) = \dfrac{1}{8}$

(d) $g\left(f\left(\dfrac{1}{\sqrt{2}}\right)\right) = g(\sqrt{2}) = 1$

(e) $f(g(x)) = \dfrac{1}{x^2 - 1}, x \neq \pm 1$

(f) $g(f(x)) = \dfrac{1}{x^2} - 1, x \neq 0$

41. $f(x) = 4x$

$f^{-1}(x) = \frac{1}{4}x$

$f(f^{-1}(x)) = 4\left(\frac{1}{4}x\right) = x$

$f^{-1}(f(x)) = \frac{1}{4}(4x) = x$

42. $f(x) = \frac{1}{3}x$

$f^{-1}(x) = 3x$

$f(f^{-1}(x)) = \frac{1}{3}(3x) = x$

$f^{-1}(f(x)) = 3\left(\frac{1}{3}x\right) = x$

43. $f(x) = x + 12$

$f^{-1}(x) = x - 12$

$f(f^{-1}(x)) = (x - 12) + 12 = x$

$f^{-1}(f(x)) = (x + 12) - 12 = x$

44. $f(x) = x - 3$

$f^{-1}(x) = x + 3$

$f(f^{-1}(x)) = (x + 3) - 3 = x$

$f^{-1}(f(x)) = (x - 3) + 3 = x$

45.
$$\begin{aligned} f(x) &= 2x - 3 = y \\ 2y - 3 &= x \\ 2y &= x + 3 \\ y &= \frac{x + 3}{2} \\ f'(x) &= \frac{1}{2}x + \frac{3}{2} \end{aligned}$$

46.
$$\begin{aligned} f(x) &= 7 - x = y \\ 7 - y &= x \\ -y &= x - 7 \\ y &= -x + 7 \\ f'(x) &= -x + 7 \end{aligned}$$

47.
$$\begin{aligned} f(x) &= \tfrac{3}{2}x + 1 = y \\ \tfrac{3}{2}y + 1 &= x \\ 3y + 2 &= 2x \\ 3y &= 2x - 2 \\ y &= \tfrac{2}{3}x - \tfrac{2}{3} \\ f^{-1}(x) &= \tfrac{2}{3}x - \tfrac{2}{3} \end{aligned}$$

48. $f(x) = -6x - 4 = y$

$-6y - 4 = x$

$-6y = x + 4$

$y = -\frac{1}{6}x - \frac{2}{3}$

$f^{-1}(x) = -\frac{1}{6}x - \frac{2}{3}$

49. $f(x) = x^5 = y$

$y^5 = x$

$y = \sqrt[5]{x}$

$f^{-1}(x) = \sqrt[5]{x}$

50. $f(x) = x^3 = y$

$y^3 = x$

$y = \sqrt[3]{x}$

$f^{-1}(x) = \sqrt[3]{x}$

51. $f(x) = \dfrac{1}{x} = y$

$\dfrac{1}{y} = x$

$y = \dfrac{1}{x}$

$f^{-1}(x) = \dfrac{1}{x}$

52. $f(x) = -\dfrac{2}{x} = y$

$-\dfrac{2}{y} = x$

$-\dfrac{1}{y} = \dfrac{1}{2}x$

$y = -\dfrac{2}{x}$

$f^{-1}(x) = -\dfrac{2}{x}$

53. $f(x) = \sqrt{9 - x^2} = y, 0 \le x \le 3$

$\sqrt{9 - y^2} = x$

$9 - y^2 = x^2$

$y^2 = 9 - x^2$

$y = \sqrt{9 - x^2}$

$f^{-1}(x) = \sqrt{9 - x^2}, \quad 0 \le x \le 3$

54. $f(x) = \sqrt{x^2 - 4} = y, \ x \ge 2$

$\sqrt{y^2 - 4} = x$

$y^2 = x^2 + 4$

$y = \sqrt{x^2 + 4}, \ x \ge 0$

$f^{-1}(x) = \sqrt{x^2 + 4}, \ x \ge 0$

55. $f(x) = x^{2/3} = y, \ x \ge 0$

$y^{2/3} = x$

$y = x^{3/2}$

$f^{-1}(x) = x^{3/2}$

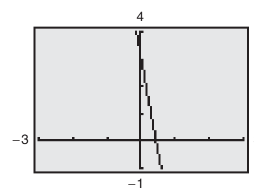
56. $f(x) = x^{3/5} = y$

$y^{3/5} = x$

$y = x^{5/3}$

$f^{-1}(x) = x^{5/3}$

57.



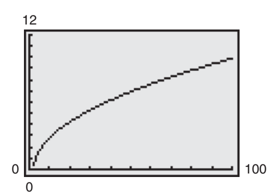
$f(x) = 3 - 7x$ is one-to-one.

$y = 3 - 7x$

$x = 3 - 7y$

$y = \dfrac{3 - x}{7}$

58.



$f(x) = \sqrt{x - 2}$ is one-to-one.

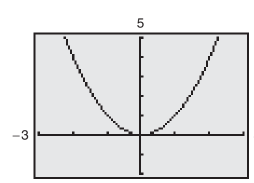
$y = \sqrt{x - 2}$

$x = \sqrt{y - 2}$

$x^2 = y - 2$

$y = x^2 + 2, \ x \ge 0$

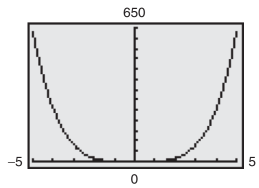
59.



$f(x) = x^2$

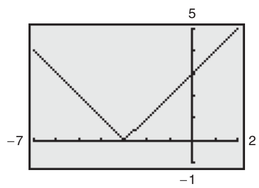
f is *not* one-to-one because $f(1) = 1 = f(-1)$.

60.



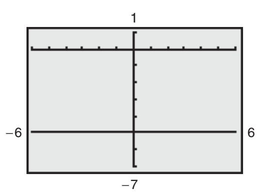
$f(x) = x^4$ is *not* one-to-one because
$f(2) = 16 = f(-2)$.

61. $f(x) = |x + 3|$

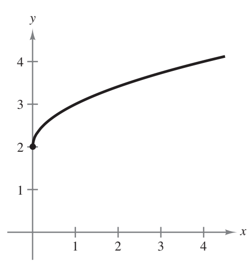


f is *not* one-to-one because $f(-5) = 2 = f(-1)$.

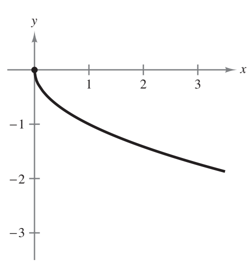
62. $f(x) = -5$ is *not* one-to-one because
$f(1) = -5 = f(-1)$.



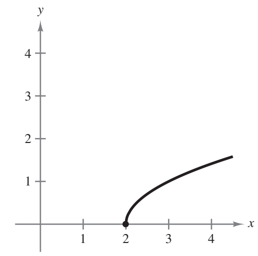
63. (a) $y = \sqrt{x} + 2$



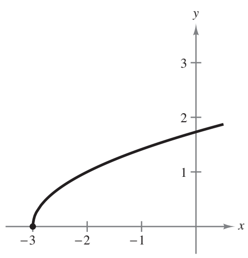
(b) $y = -\sqrt{x}$



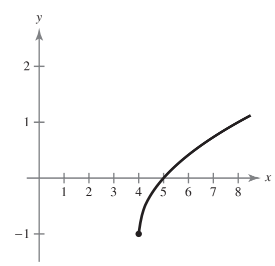
(c) $y = \sqrt{x - 2}$



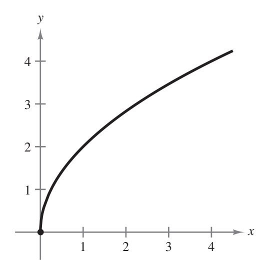
(d) $y = \sqrt{x + 3}$



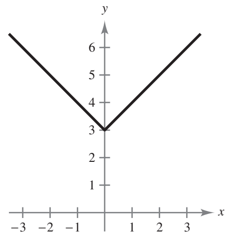
(e) $y = \sqrt{x - 4} - 1$



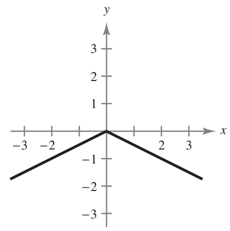
(f) $y = 2\sqrt{x}$



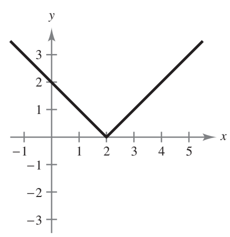
64. (a)



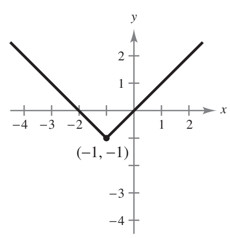
(b)



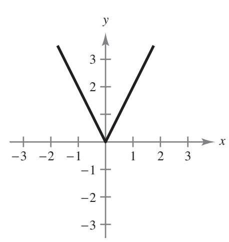
(c)



(d)



(e)



65. (a) Shifted three units to the left: $y = (x + 3)^2$

(b) Shifted six units to the left, three units downward, and reflected: $y = -(x + 6)^2 - 3$

66. (a) Stretched by a factor of $\frac{1}{8}$: $y = \frac{1}{8}x^3$

(b) Stretched by a factor of 2, and reflected: $y = -2x^3$

67. (a) 2000: \$120 billion
2003: 170 billion
2007: \$225 billion

(b) 2000: \$121.3 billion
2003: \$173.8 billion
2007: \$225.6 billion

The model fits the data well.

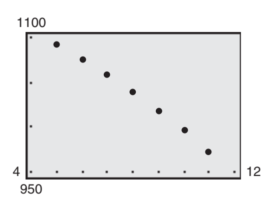
68. (a) T is a function of t because for any value of t (time of day), there is exactly one value of T (temperature in the house).

(b) $T(4) \approx 60°\text{F}$
$T(15) \approx 72°\text{F}$

(c) H is a horizontal shift of T, 1 hour to the right or 1 hour later.

(d) H is a vertical shift of T, 1 degree upward or 1 degree increase in setting.

69.
$$\begin{aligned} R_{\text{TOTAL}} &= R_1 + R_2 \\ &= 690 - 8t - 0.8t^2 + 458 + 0.78t \\ &= -0.8t^2 - 7.22t + 1148,\ t = 5, 6, \ldots, 11 \end{aligned}$$



70. $B(t) - D(t) = -0.151t^3 - 8.00t^2 - 85.0t + 4176 - \left(-1.25t^2 + 38.5t + 2136\right) = -0.151t^3 - 6.75t^2 - 123.5t + 2040$

$B(t) - D(t)$ is the function that yields the increase and/or decrease in people living in the United States from 1990 to 2007.

71. (a) $C = 1.95x + 6000$

(b) $\overline{C} = \dfrac{C}{x} = \dfrac{1.95x + 6000}{x} = 1.95 + \dfrac{6000}{x}$

(c)
$$\begin{aligned} 1.95 + \frac{6000}{x} &< 4.95 \\ \frac{6000}{x} &< 3 \\ \frac{6000}{3} &< x \text{ because } x > 0. \\ 2000 &< x \end{aligned}$$

Must sell 2000 units before the average cost per unit falls below the selling price.

72. (a) $1 + 0.01x = \dfrac{14.75}{p}$

$$x = \frac{(14.75/p) - 1}{0.01}$$
$$= \frac{14.75 - p}{0.01p}$$
$$= \frac{100(14.75 - p)}{p}$$
$$= \frac{1475}{p} - 100$$

(b) $x = \dfrac{100(14.75 - 10)}{10} = 47.5 \approx 48$ units

73. (a) $C(x) = 70x + 500$

$x(t) = 40t$

$$C(x(t)) = 70(40t) + 500$$
$$= 2800t + 500$$

C is the weekly cost per t hours of production.

(b) $C(x(4)) = 2800(4) + 500 = \$11{,}700$

(c)
$$C(x(t)) = 18{,}000$$
$$2800t + 500 = 18{,}000$$
$$2800t = 17{,}500$$
$$t = \frac{17{,}500}{2800} = 6.25 \text{ hr}$$

74. (a) Revenue $= R = rn = [15 - 0.05(n - 80)]n = 19n - 0.05n^2$

(b)

n	100	125	150	175	200	225	250
R	1400	1593.75	1725	1793.75	1800	1743.75	1625

(c) The revenue increases and then decreases as n gets larger, so it is not a good formula for the bus company to use.

75. (a) Cost $= C = 98{,}000 + 12.30x$

(b) Revenue $= R = 17.98x$

(c) Profit $= R - C = 17.98x - (12.30x + 98{,}000) = 5.68x - 98{,}000$

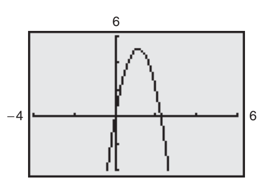
76. (a) If $0 \le x \le 100$, then $p = 90$. If $100 < x \le 1600$, then $p = 90 - 0.01(x - 100) = 91 - 0.01x$. If $x > 1600$, then $p = 75$. Thus,

$$p = \begin{cases} 90, & 0 \le x \le 100 \\ 91 - 0.01x, & 100 < x \le 1600. \\ 75, & x > 1600 \end{cases}$$

(b) $P = px - 60x$

$$P = \begin{cases} 90x - 60x, & 0 \le x \le 100 \\ (91 - 0.01x)x - 60x, & 100 < x \le 1600 \\ 75x - 60x, & x > 1600 \end{cases}$$
$$= \begin{cases} 30x, & 0 \le x \le 100 \\ 31x - 0.01x^2, & 100 < x \le 1600 \\ 15x, & x > 1600 \end{cases}$$

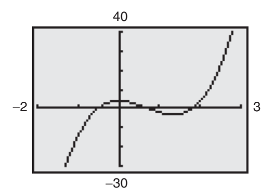
77. $f(x) = 9x - 4x^2$



Zeros: $x(9 - 4x) = 0 \Rightarrow x = 0, \frac{9}{4}$

The function is not one-to-one.

78.

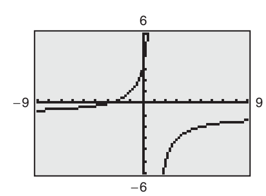


$h(x) = 6x^3 - 12x^2 + 4$

Zeros: $x \approx -0.5419, 0.7224, 1.7925$

The function is not one-to-one.

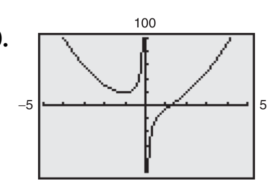
79. $g(t) = \dfrac{t + 3}{1 - t}$



Zero: $t = -3$

The function is one-to-one.

80.

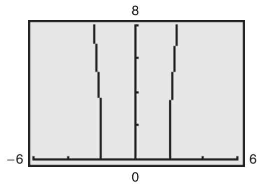


$f(x) = 2\left(3x^2 - \dfrac{6}{x}\right)$

Zero: $x = 1.2599$

The function is not one-to-one.

81. $g(x) = x^2\sqrt{x^2 - 4}$

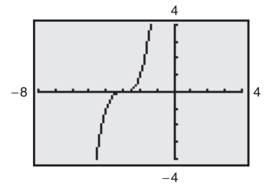


Domain: $|x| \geq 2$

Zeros: $x = \pm 2$

The function is not one-to-one.

82.



$f(x) = (x + 3)^3$

Zero: $x = -3$

The function is one-to-one.

83. Answers will vary.

Section 1.5 Limits

Skills Warm Up

1. $\dfrac{2x^3 + x^2}{6x} = \dfrac{x^2(2x + 1)}{6x} = \dfrac{x(2x + 1)}{6} = \dfrac{1}{6}x(2x + 1)$

2. $\dfrac{x^5 + 9x^4}{x^2} = \dfrac{x^4(x + 9)}{x^2} = x^3(x + 9)$

3. $\dfrac{x^2 - 3x - 28}{x - 7} = \dfrac{(x - 7)(x + 4)}{x - 7} = x + 4$

4. $\dfrac{x^2 + 11x + 30}{x + 5} = \dfrac{(x + 6)(x + 5)}{x + 5} = x + 6$

5. $f(x) = x^2 - 3x + 3$

(a) $f(-1) = (-1)^2 - 3(-1) + 3 = 1 + 3 + 3 = 7$

(b) $f(c) = c^2 - 3c + 3$

(c) $f(x + h) = (x + h)^2 - 3(x + h) + 3$
$= x^2 + 2xh + h^2 - 3x - 3h + 3$

6. $f(x) = \begin{cases} 2x - 2, & x < 1 \\ 3x + 1, & x \geq 1 \end{cases}$

(a) $f(-1) = 2(-1) - 2 = -2 - 2 = -4$

(b) $f(3) = 3(3) + 1 = 9 + 1 = 10$

(c) $f(t^2 + 1) = 3(t^2 + 1) + 1$
$= 3t^2 + 3 + 1$
$= 3t^2 + 4$

7. $f(x) = x^2 - 2x + 2$

$$\frac{f(1 + h) - f(1)}{h}$$
$$= \frac{(1 + h)^2 - 2(1 + h) + 2 - (1^2 - 2(1) + 2)}{h}$$
$$= \frac{1 + 2h + h^2 - 2 - 2h + 2 - 1 + 2 - 2}{h}$$
$$= \frac{h^2}{h}$$
$$= h$$

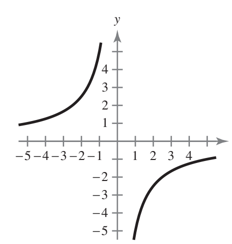
8. $f(x) = 4x$

$$\frac{f(2 + h) - f(2)}{h} = \frac{4(2 + h) - 4(2)}{h}$$
$$= \frac{8 + 4h - 8}{h}$$
$$= \frac{4h}{h}$$
$$= 4$$

9. $h(x) = -\dfrac{5}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

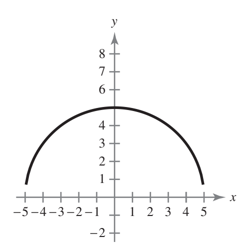
Range: $(-\infty, 0) \cup (0, \infty)$



10. $g(x) = \sqrt{25 - x^2}$

Domain: $[-5, 5]$

Range: $[0, 5]$

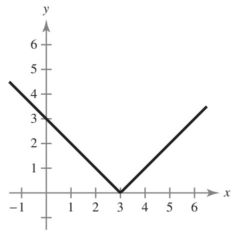


Skills Warm Up —*continued*—

11. $f(x) = |x - 3|$

Domain: $(-\infty, \infty)$

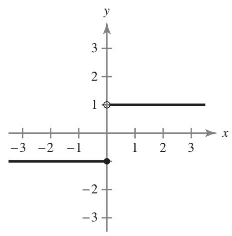
Range: $[0, \infty)$



12. $f(x) = \dfrac{|x|}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $y = -1,\ y = 1$



13.
$$\begin{aligned} 9x^2 + 4y^2 &= 49 \\ 4y^2 &= 49 - 9x^2 \\ y^2 &= \frac{49 - 9x^2}{4} \\ y &= \frac{\pm\sqrt{49 - 9x^2}}{2} \end{aligned}$$

Not a function of x (fails the vertical line test).

14.
$$\begin{aligned} 2x^2y + 8x &= 7y \\ 2x^2y - 7y &= -8x \\ y(2x^2 - 7) &= -8x \\ y &= -\frac{8x}{2x^2 - 7} \end{aligned}$$

Yes, y is a function of x.

1. (a) $\lim_{x\to 0} f(x) = 1$

(b) $\lim_{x\to -1} f(x) = 3$

2. (a) $\lim_{x\to 1} f(x) = -2$

(b) $\lim_{x\to 3} f(x) = 0$

3. (a) $\lim_{x\to 0} g(x) = 1$

(b) $\lim_{x\to -1} g(x) = 3$

4. (a) $\lim_{x\to -2} h(x) = -5$

(b) $\lim_{x\to 0} h(x) = -3$

5.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	8.8	8.98	8.998	?	9.002	9.02	9.2

$\lim_{x\to 2}(2x + 5) = 9$

6.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	−1.09	−1.0099	−1.000999	?	−0.998999	−0.9899	−0.89

$\lim_{x\to 2}(x^2 - 3x + 1) = -1$

7.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	0.2564	0.2506	0.2501	?	0.2499	0.2494	0.2439

$\lim_{x\to 2} \dfrac{x - 2}{x^2 - 4} = \dfrac{1}{4} = 0.25$

8.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	1.111	1.010	1.001	?	0.999	0.990	0.909

$\lim_{x\to 2} \dfrac{x - 2}{x^2 - 3x + 2} = 1$

9.

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.5132	0.5013	0.5001	?	0.4999	0.4988	0.4881

$$\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x} = 0.5$$

10.

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.3581	0.3540	0.3536	?	0.3535	0.3531	0.3492

$$\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x} = \frac{1}{2\sqrt{2}} \approx 0.3536$$

11.

x	–4.1	–4.01	–4.001	–4	–3.999	–3.99	–3.9
$f(x)$	2.5	25	250	?	–250	–25	–2.5

The limit does not exist.

12.

x	–2.1	–2.01	–2.001	–2	–1.999	–1.99	–1.9
$f(x)$	–2.5	–25	–250	?	250	25	2.5

The limit does not exist.

13. $\lim_{x\to 3} 6 = 6$

14. $\lim_{x\to 5} 4 = 4$

15. $\lim_{x\to -2} x = -2$

16. $\lim_{x\to 10} x = 10$

17. $\lim_{x\to 7} x^2 = (7)^2 = 49$

18. $\lim_{x\to 3} x^3 = (3)^3 = 27$

19. $\lim_{x\to 16} \sqrt{x} = \sqrt{16} = 4$

20. $\lim_{x\to -1} \sqrt[3]{x} = \sqrt[3]{-1} = -1$

21. (a) $\lim_{x\to c}\left[f(x)+g(x)\right] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$
$= 3 + 9$
$= 12$

(b) $\lim_{x\to c}\left[f(x)g(x)\right] = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right]$
$= 3 \cdot 9$
$= 27$

(c) $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{3}{9} = \frac{1}{3}$

22. (a) $\lim_{x\to c}\left[f(x)+g(x)\right] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$
$= \frac{3}{2} + \frac{1}{2}$
$= 2$

(b) $\lim_{x\to c}\left[f(x)\cdot g(x)\right] = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right]$
$= \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$
$= \frac{3}{4}$

(c) $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{3/2}{1/2} = 3$

23. (a) $\lim_{x\to c}\sqrt{f(x)} = \sqrt{16} = 4$

(b) $\lim_{x\to c}\left[3f(x)\right] = 3(16) = 48$

(c) $\lim_{x\to c}\left[f(x)\right]^2 = 16^2 = 256$

24. (a) $\lim_{x\to c}\sqrt{f(x)} = \sqrt{9} = 3$

(b) $\lim_{x\to c}\left(3f(x)\right) = 3(9) = 27$

(c) $\lim_{x\to c}\left[f(x)\right]^2 = 9^2 = 81$

25. $\lim_{x\to -3}(2x+5) = \lim_{x\to -3} 2x + \lim_{x\to -3} 5 = 2(-3) + 5 = -1$

26. $\lim_{x\to 0}(3x - 2) = \lim_{x\to 0} 3x - \lim_{x\to 0} 2 = 3(0) - 2 = -2$

27. $\lim_{x\to 1}(1 - x^2) = \lim_{x\to 1} 1 - \lim_{x\to 1} x^2 = 1 - 1^2 = 0$

28. $\lim_{x\to 2}(-x^2 + x - 2) = -\lim_{x\to 2} x^2 + \lim_{x\to 2} x - \lim_{x\to 2} 2$
$= -4 + 2 - 2 = -4$

29. $\lim_{x\to 3}\sqrt{x + 6} = \sqrt{3 + 6} = 3$

30. $\lim_{x\to 4}\sqrt[3]{x + 4} = \sqrt[3]{4 + 4} = 2$

31. $\lim_{x\to -3}\dfrac{2}{x + 2} = \dfrac{2}{-3 + 2} = -2$

32. $\lim_{x\to -2}\dfrac{3x + 1}{2 - x} = \dfrac{3(-2) + 1}{2 - (-2)} = \dfrac{-5}{4} = -\dfrac{5}{4}$

33. $\lim_{x\to -2}\dfrac{x^2 - 1}{2x} = \dfrac{(-2)^2 - 1}{2(-2)} = \dfrac{3}{-4} = -\dfrac{3}{4}$

34. $\lim_{x\to 7}\dfrac{5x}{x + 2} = \dfrac{5(7)}{7 + 2} = \dfrac{35}{9}$

35. $\lim_{x\to 5}\dfrac{\sqrt{x + 11} + 6}{x} = \dfrac{\sqrt{5 + 11} + 6}{5}$
$= \dfrac{\sqrt{16} + 6}{5}$
$= \dfrac{4 + 6}{5} = \dfrac{10}{5} = 2$

36. $\lim_{x\to 12}\dfrac{\sqrt{x - 3} - 2}{x} = \dfrac{\sqrt{12 - 3} - 2}{12}$
$= \dfrac{\sqrt{9} - 2}{12} = \dfrac{3 - 2}{12} = \dfrac{1}{12}$

37. $\lim_{x\to -3}\dfrac{x^2 - 9}{x + 3} = \lim_{x\to -3}\dfrac{(x + 3)(x - 3)}{x + 3}$
$= \lim_{x\to -3}(x - 3) = -6$

38. $\lim_{x\to -1}\dfrac{2x^2 - x - 3}{x + 1} = \lim_{x\to -1}\dfrac{(x + 1)(2x - 3)}{x + 1}$
$= \lim_{x\to -1}(2x - 3) = -5$

39. $\lim_{x\to 2}\dfrac{2 - x}{x^2 - 4} = \lim_{x\to 2}\dfrac{-(x - 2)}{(x + 2)(x - 2)}$
$= \lim_{x\to 2}\dfrac{-1}{x + 2} = -\dfrac{1}{4}$

40. $\lim_{t\to 1}\dfrac{t^2 + t - 2}{t^2 - 1} = \lim_{t\to 1}\dfrac{(t - 1)(t + 2)}{(t + 1)(t - 1)}$
$= \lim_{t\to 1}\dfrac{t + 2}{t + 1} = \dfrac{3}{2}$

41. $\lim_{x\to -2}\dfrac{x^3 + 8}{x + 2} = \lim_{x\to -2}\dfrac{(x + 2)(x^2 - 2x + 4)}{x + 2}$
$= \lim_{x\to -2}(x^2 - 2x + 4) = 12$

42. $\lim_{x\to -1}\dfrac{x^3 + 1}{x + 1} = \lim_{x\to -1}\dfrac{(x + 1)(x^2 - x + 1)}{x + 1}$
$= \lim_{x\to -1}(x^2 - x + 1) = 3$

43. $\lim_{\Delta x\to 0}\dfrac{2(x + \Delta x) - 2x}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{2x + 2\Delta x - 2x}{\Delta x}$
$= \lim_{\Delta x\to 0} 2 = 2$

44. $\lim_{\Delta x\to 0}\dfrac{4(x + \Delta x) - 5 - (4x - 5)}{\Delta x} = \lim_{\Delta x\to 0}\dfrac{4\Delta x}{\Delta x} = 4$

45. $\lim_{\Delta t\to 0}\dfrac{(t + \Delta t)^2 - 5(t + \Delta t) - (t^2 - 5t)}{\Delta t} = \lim_{\Delta t\to 0}\dfrac{t^2 + 2t(\Delta t) + (\Delta t)^2 - 5t - 5(\Delta t) - t^2 + 5t}{\Delta t}$
$= \lim_{\Delta t\to 0}\dfrac{2t(\Delta t) + (\Delta t)^2 - 5(\Delta t)}{\Delta t}$
$= \lim_{\Delta t\to 0} 2t + (\Delta t) - 5$
$= 2t - 5$

46. $\lim_{\Delta t\to 0}\dfrac{(t + \Delta t)^2 - 4(t + \Delta t) + 2 - (t^2 - 4t + 2)}{\Delta t} = \lim_{\Delta t\to 0}\dfrac{t^2 + 2t\Delta t + (\Delta t)^2 - 4t - 4\Delta t - t^2 + 4t}{\Delta t}$
$= \lim_{\Delta t\to 0}\dfrac{2t\Delta t + (\Delta t)^2 - 4\Delta t}{\Delta t}$
$= \lim_{\Delta t\to 0}(2t + \Delta t - 4)$
$= 2t - 4$

47. $$\lim_{x\to4}\frac{\sqrt{x+5}-3}{x-4} = \lim_{x\to4}\frac{\sqrt{x+5}-3}{x-4}\cdot\frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$$
$$= \lim_{x\to4}\frac{(x+5)-9}{(x-4)\sqrt{x+5}+3}$$
$$= \lim_{x\to4}\frac{x-4}{(x-4)\left(\sqrt{x+5}+3\right)}$$
$$= \lim_{x\to4}\frac{1}{\sqrt{x+5}+3} = \frac{1}{6}$$

48. $$\lim_{x\to3}\frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to3}\frac{\sqrt{x+1}-2}{x-3}\cdot\lim_{x\to3}\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$$
$$= \lim_{x\to3}\frac{(x+1)-4}{(x-3)\left(\sqrt{x+1}+2\right)}$$
$$= \lim_{x\to3}\frac{x-3}{(x-3)\left(\sqrt{x+1}+2\right)}$$
$$= \lim_{x\to3}\frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$$

49. $$\lim_{x\to0} x = \frac{\sqrt{x+5}-\sqrt{5}}{x} = \lim_{x\to0}\frac{\sqrt{x+5}-\sqrt{5}}{x}\cdot\frac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}}$$
$$= \lim_{x\to0}\frac{(x+5)-5}{x\left(\sqrt{x+5}+\sqrt{5}\right)}$$
$$= \lim_{x\to0}\frac{x}{x\left(\sqrt{x+5}+\sqrt{5}\right)}$$
$$= \lim_{x\to0}\frac{1}{\sqrt{x+5}+\sqrt{5}} = \frac{1}{2\sqrt{5}}$$

50. $$\lim_{x\to0} x = \frac{\sqrt{x+2}-\sqrt{2}}{x} = \lim_{x\to0}\frac{\sqrt{x+2}-\sqrt{2}}{x}\cdot\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}$$
$$= \lim_{x\to0}\frac{(x+2)-2}{x\left(\sqrt{x+2}+\sqrt{2}\right)}$$
$$= \lim_{x\to0}\frac{x}{x\left(\sqrt{x+2}+\sqrt{2}\right)}$$
$$= \lim_{x\to0}\frac{1}{\sqrt{x+2}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

51. $\lim_{x\to2^-}(4-x) = 2$

$\lim_{x\to2^+}(4-x) = 2$

So, $\lim_{x\to2} f(x) = 2$

52. $\lim_{x\to1^-}\left(x^2+2\right) = 3$

$\lim_{x\to1^+}\left(x^2+2\right) = 3$

So, $\lim_{x\to1} f(x) = 3$

53. $\lim_{x\to3^-} f(x) = \lim_{x\to3^-}\left(\frac{1}{3}x-2\right) = -1$

$\lim_{x\to3^+} f(x) = \lim_{x\to3^+}(-2x+5) = -1$

So, $\lim_{x\to3} f(x) = -1$.

54. $\lim_{s\to1^-} f(s) = \lim_{s\to1^-} s = 1$

$\lim_{s\to1^+} f(s) = \lim_{s\to1^+}(1-s) = 0$

So, $\lim_{s\to1} f(s)$ does not exist.

55. $\lim_{x \to -4} \dfrac{2}{x+4} = \dfrac{2}{0}$

The limit does not exist.

56. $\lim_{x \to 5} \dfrac{4}{x-5} = \dfrac{4}{0}$

The limit does not exist.

57. $\lim_{x \to 2} \dfrac{x-2}{x^2 - 4x + 4} = \lim_{x \to 2} \dfrac{x-2}{(x-2)(x-2)}$

$= \lim_{x \to 2} \dfrac{1}{x-2}$

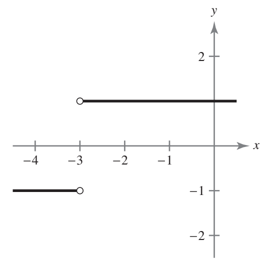
The limit does not exist.

58. $\lim_{t \to 4} \dfrac{t+4}{t^2 - 16} = \lim_{t \to 4} \dfrac{t+4}{(t+4)(t-4)}$

$= \lim_{t \to 4} \dfrac{1}{t-4}$

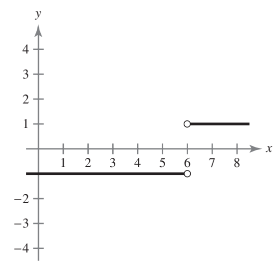
The limit does not exist.

59.



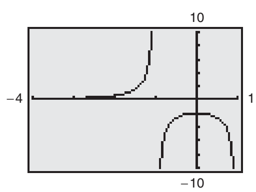
$\lim_{x \to -3^-} \dfrac{|x+3|}{x+3} = -1$ and $\lim_{x \to -3^+} \dfrac{|x+3|}{x+3} = 1$

60.



$\lim_{x \to 6^-} \dfrac{|x-6|}{x-6} = -1$ and $\lim_{x \to 6^+} \dfrac{|x-6|}{x-6} = 1$

61.

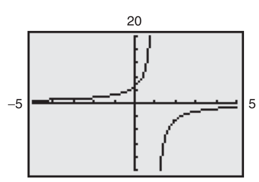


$\lim_{x \to 1^-} \dfrac{2}{x^2 - 1} = -\infty$

x	0	0.5	0.9	0.99	0.999	0.9999	1
$f(x)$	–2	–2.67	–10.53	–100.5	–1000.5	–10,000.5	undefined

Because $f(x) = \dfrac{2}{x^2 - 1}$ decreases without bound as x tends to 1 from the left, the limit is $-\infty$.

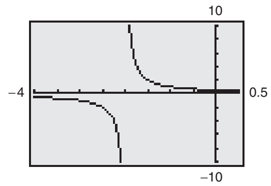
62.



$\lim_{x \to 1^+} \dfrac{5}{1-x} = -\infty$

x	2	1.5	1.1	1.01	1.001	1.0001	1
$f(x)$	–5	–10	–50	–500	–5000	–50,000	undefined

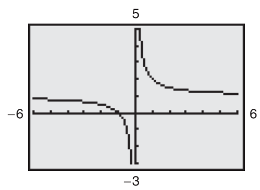
Because $f(x) = \dfrac{5}{1-x}$ decreases without bound as x tends to 1 from the right, the limit is $-\infty$.

63. 

$$\lim_{x \to -2^-} \frac{1}{x+2} = -\infty$$

x	-3	-2.5	-2.1	-2.01	-2.001	-2.0001	-2
$f(x)$	-1	-2	-10	-100	-1000	$-10{,}000$	undefined

Because $f(x) = \dfrac{1}{x+2}$ decreases without bound as x tends to -2 from the left, the limit is $-\infty$.

64. 

$$\lim_{x \to 0^-} \frac{x+1}{x} = -\infty$$

x	-1	-0.5	-0.1	-0.01	-0.001	-0.0001	0
$f(x)$	0	-1	-9	-99	-999	-9999	undefined

Because $f(x) = \dfrac{x+1}{x}$ decreases without bound as x tends to 0 from the left, the limit is $-\infty$.

65. (a) $\lim_{x \to 3^+} f(x) = 1$

(b) $\lim_{x \to 3^-} f(x) = 1$

(c) $\lim_{x \to 3} f(x) = 1$

66. (a) $\lim_{x \to -2^+} f(x) = -2$

(b) $\lim_{x \to -2^-} f(x) = -2$

(c) $\lim_{x \to -2} f(x) = -2$

67. (a) $\lim_{x \to 3^+} f(x) = 0$

(b) $\lim_{x \to 3^-} f(x) = 0$

(c) $\lim_{x \to 3} f(x) = 0$

68. (a) $\lim_{x \to -2^+} f(x) = 2$

(b) $\lim_{x \to -2^-} f(x) = 2$

69. (a) $\lim_{x \to 3^+} f(x) = 3$

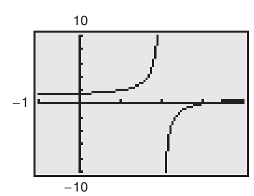
(b) $\lim_{x \to 3^-} f(x) = -3$

(c) $\lim_{x \to 3} f(x)$ does not exist.

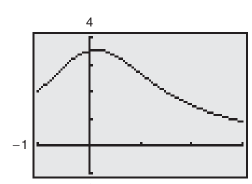
70. (a) $\lim_{x \to -1^+} f(x) = 0$

(b) $\lim_{x \to -1^-} f(x) = 2$

(c) $\lim_{x \to -1} f(x)$ does not exist.

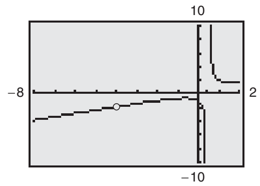
71. 

$\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{x^2 - 4x + 4}$ does not exist.

72. 

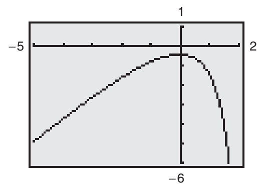
$$\lim_{x \to 1} \frac{x^2 + 6x - 7}{x^3 - x^2 + 2x - 2} \approx 2.667$$

73.



$$\lim_{x\to -4} \frac{x^3 + 4x^2 + x + 4}{2x^2 + 7x - 4} \approx -1.889$$

74.



$$\lim_{x\to -2} \frac{4x^3 + 7x^2 + x + 6}{3x^2 - x - 14} \approx -1.615$$

75. $C = \dfrac{25p}{100 - p}, \; 0 \le p < 100$

(a) $C(50) = \dfrac{25(50)}{100 - 50} = \$25$ thousand

(b) Find p for $C = 100$.

$$\begin{aligned} 100 &= \frac{25p}{100 - p} \\ 100(100 - p) &= 25p \\ 10{,}000 - 100p &= 25p \\ 10{,}000 &= 125p \\ 80 &= p, \text{ or } 80\% \end{aligned}$$

(c) $\displaystyle\lim_{p\to 100^-} = \lim_{p\to 100^-} \frac{25p}{100 - p} = \infty$

The cost function increases without bound as x approaches 100 from the left. Therefore, according to the model, it is not possible to remove 100% of the pollutants.

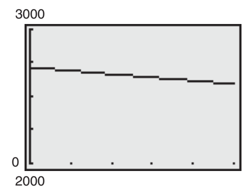
76. (a) $\displaystyle\lim_{x\to 50} C$ does not exist. The two one-sided limits are not equal.

$\displaystyle\lim_{x\to 50^-} C \approx \$7.50$ and $\displaystyle\lim_{x\to 50^+} C \approx \$5$

(b) $\displaystyle\lim_{x\to 150} C \approx \$10.50$

(c) It would be less expensive to make 201 copies, since $\displaystyle\lim_{x\to 200^-} C \approx \$14$ and $\displaystyle\lim_{x\to 200^+} C \approx \$10$.

77. (a)



(b) When $x = 0.25$: $A = 2685.06$

When $x = \frac{1}{365}$: $A = 2717.91$

(c) Using the *zoom* and t*race* features, $\displaystyle\lim_{x\to 0^+} A \approx \$2718.28$. Because x, the length of the compounding period, is approaching 0, this limit represents the balance with continuous compounding.

Section 1.6 Continuity

Skills Warm Up

1. $\dfrac{x^2 + 6x + 8}{x^2 - 6x - 16} = \dfrac{(x + 4)(x + 2)}{(x - 8)(x + 2)} = \dfrac{x + 4}{x - 8}$

2. $\dfrac{x^2 - 5x - 6}{x^2 - 9x + 18} = \dfrac{(x - 6)(x + 1)}{(x - 6)(x - 3)} = \dfrac{x + 1}{x - 3}$

3.
$$\begin{aligned} \frac{2x^2 - 2x - 12}{4x^2 - 24x + 36} &= \frac{2(x^2 - x - 6)}{4(x^2 - 6x + 9)} \\ &= \frac{2(x - 3)(x + 2)}{4(x - 3)(x - 3)} \\ &= \frac{x + 2}{2(x - 3)} \end{aligned}$$

4.
$$\begin{aligned} \frac{x^3 - 16x}{x^3 + 2x^2 - 8x} &= \frac{x(x^2 - 16)}{x(x^2 + 2x - 8)} \\ &= \frac{x(x^2 - 16)}{x(x + 4)(x - 2)} \\ &= \frac{x(x + 4)(x - 4)}{x(x + 4)(x - 2)} \\ &= \frac{x - 4}{x - 2} \end{aligned}$$

5.
$$\begin{aligned} x^2 + 7x &= 0 \\ x(x + 7) &= 0 \\ x &= 0 \\ x + 7 = 0 &\Rightarrow x = -7 \end{aligned}$$

Skills Warm Up —*continued*—

6. $x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$

$x + 5 = 0 \Rightarrow x = -5$

$x - 1 = 0 \Rightarrow x = 1$

7. $3x^2 + 8x + 4 = 0$

$(3x + 2)(x + 2) = 0$

$3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$

$x + 2 = 0 \Rightarrow x = -2$

8. $x^3 + 5x^2 - 24x = 0$

$x(x^2 + 5x - 24) = 0$

$x(x - 3)(x + 8) = 0$

$x = 0$

$x - 3 = 0 \Rightarrow x = 3$

$x + 8 = 0 \Rightarrow x = -8$

9. $\lim_{x\to 3}(2x^2 - 3x + 4) = 2(3^2) - 3(3) + 4$

$= 2(9) - 9 + 4$

$= 13$

10. $\lim_{x\to -2}(3x^3 - 8x + 7) = 3(-2)^3 - 8(-2) + 7$

$= 3(-8) + 16 + 7$

$= -24 + 23$

$= -1$

1. Continuous; The function is a polynomial.

2. Continuous; The function is a polynomial.

3. Not continuous; The rational function is not defined at $x = \pm 4$.

4. Not continuous; The rational function is not defined at $x = \pm 3$.

5. Continuous; The rational function's domain is the entire real line.

6. Continuous; The rational function's domain is the entire real line.

7. Not continuous; The rational function is not defined at $x = 3$ or $x = 5$.

8. Not continuous; The rational function is not defined at $x = 1$ or $x = 5$.

9. Not continuous; The rational function is not defined at $x = \pm 2$.

10. Not continuous; The rational function is not defined at $x = \pm 4$.

11. $f(x) = \dfrac{x^2 - 1}{x}$ is continuous on $(-\infty, 0)$ and $(0, \infty)$ because the domain of f consists of all real numbers except $x = 0$. There is a discontinuity at $x = 0$ because $f(0)$ is not defined and $\lim_{x\to 0} f(x)$ does not exist.

12. $f(x) = \dfrac{1}{x^2 - 4}$ is continuous on $(-\infty, -2)$, $(-2, 2)$ and $(2, \infty)$ because the domain of f consists of all real numbers except $x = \pm 2$. There are discontinuities at $x = \pm 2$ because $f(2)$ and $f(-2)$ are not defined and $\lim_{x\to 2} f(x)$ and $\lim_{x\to -2} f(x)$ do not exist.

13. $f(x) = \dfrac{x^2 - 1}{x + 1}$ is continuous on $(-\infty, -1)$ and $(-1, \infty)$ because the domain of f consists of all real numbers except $x = -1$. There is a discontinuity at $x = -1$ because $f(-1)$ is not defined and $\lim_{x\to -1} f(x) \neq f(-1)$.

14. $f(x) = \dfrac{x^3 - 8}{x - 2}$ is continuous on $(-\infty, 2)$ and $(2, \infty)$ because the domain of f consists of all real numbers except $x = 2$. There is a discontinuity at $x = 2$ because $f(2)$ is not defined and $\lim_{x\to 2} f(x) \neq f(2)$.

15. $f(x) = x^2 - 2x + 1$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers.

16. $f(x) = 3 - 2x - x^2$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers.

17. $f(x) = \dfrac{x}{x^2 - 1} = \dfrac{x}{(x+1)(x-1)}$ is continuous on $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$ because the domain of f consists of all real numbers except $x = \pm 1$. There are discontinuities at $x = \pm 1$ because $f(1)$ and $f(-1)$ are not defined and $\lim_{x\to 1} f(x)$ and $\lim_{x\to -1} f(x)$ do not exist.

18. $f(x) = \dfrac{x-3}{x^2-9}$ is continuous on $(-\infty, -3)$, $(-3, 3)$, and $(3, \infty)$ because the domain of f consists of all real numbers except $x = \pm 3$. There are discontinuities at $x = \pm 3$ because $f(-3)$ and $f(3)$ are not defined, $\lim_{x\to -3}$ does not exist, and $\lim_{x\to 3} f(x) \neq f(3)$.

19. $f(x) = \dfrac{x}{x^2+1}$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers.

20. $f(x) = \dfrac{6}{x^2+3}$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers.

21. $f(x) = \dfrac{x-5}{x^2-9x+20} = \dfrac{x-5}{(x-5)(x-4)}$ is continuous on $(-\infty, 4)$, $(4, 5)$, and $(5, \infty)$ because the domain of f consists of all real numbers except $x = 4$ and $x = 5$. There is a discontinuity at $x = 4$ and $x = 5$ because $f(4)$ and $f(5)$ are not defined and $\lim_{x\to 4} f(x)$ does not exist and $\lim_{x\to 5} f(x) \neq f(5)$.

22. $f(x) = \dfrac{x-1}{x^2+x-2} = \dfrac{x-1}{(x-1)(x+2)}$ is continuous on $(-\infty, -2)$, $(-2, 1)$, and $(1, \infty)$ because the domain of f consists of all real number except $x = -2$ and $x = 1$. There is discontinuity at $x = -2$ and $x = 1$ because $f(-2)$ and $f(1)$ are not defined, $\lim_{x\to -2} f(x)$ does not exist, and $\lim_{x\to 1} f(x) \neq f(1)$.

23. $f(x) = \sqrt{4-x}$ is continuous on $(-\infty, 4]$ because the domain of f consists of all real $x \le 4$.

24. $f(x) = \sqrt{x-1}$ is continuous on $[1, \infty)$ because the domain of f consists of all real $x \ge 1$.

25. $f(x) = \sqrt{x} + 2$ is continuous on $[0, \infty)$ because the domain of f consists of all real $x \ge 0$.

26. $f(x) = 3 - \sqrt{x}$ is continuous on $[0, \infty)$ because the domain of f consists of all real $x \ge 0$.

27. $f(x) = \begin{cases} -2x+3, & -1 \le x \le 1 \\ x^2, & 1 < x \le 3 \end{cases}$ is continuous on $[-1, 3]$.

28. $f(x) = \begin{cases} \frac{1}{2}x+1, & -3 \le x \le 2 \\ 3-x, & 2 < x \le 4 \end{cases}$ is continuous on $[-3, 2)$, $(2, 4]$. f is discontinuous at $x = 2$ because $\lim_{x\to 2} f(x)$ does not exist.

$\lim_{x\to 2^-} f(x) = 2$ and $\lim_{x\to 2^+} f(x) = 1$.

29. $f(x) = \begin{cases} 3+x, & x \le 2 \\ x^2+1, & x > 2 \end{cases}$

is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers, $f(2)$ is defined, $\lim_{x\to 2} f(x)$ exists, and $\lim_{x\to 2} f(x) = f(2)$.

30. $f(x) = \begin{cases} x^2-4, & x \le 2 \\ 3x+1, & x > 2 \end{cases}$

is continuous on $(-\infty, 0)$ and $(0, \infty)$. There is a discontinuity at $x = 0$ because $\lim_{x\to 0} f(x)$ does not exist.

31. $f(x) = \dfrac{|x+1|}{x+1}$ is continuous on $(-\infty, -1)$ and $(-1, \infty)$ because the domain of f consists of all real numbers except $x = -1$. There is a discontinuity at $x = -1$ because $f(-1)$ is not defined, and $\lim_{x\to -1} f(x)$ does not exist.

32. $f(x) = \dfrac{|4-x|}{4-x}$ is continuous on $(-\infty, 4)$ and $(4, \infty)$ because the domain of f consists of all real numbers except $x = 4$. There is a discontinuity at $x = 4$ because $f(4)$ is not defined and $\lim_{x\to 4} f(x)$ does not exist.

33. $f(x) = x\sqrt{x+3}$ is continuous on $[-3, \infty)$.

34. $f(x) = \dfrac{x+1}{\sqrt{x}}$ is continuous on $(0, \infty)$.

35. $f(x) = [\![2x]\!] + 1$ is continuous on all intervals of the form $\left(\frac{1}{2}c, \frac{1}{2}c + \frac{1}{2}\right)$, where c is an integer. That is, f is continuous on $\ldots, \left(-\frac{1}{2}, 0\right), \left(0, \frac{1}{2}\right), \left(\frac{1}{2}, 1\right), \ldots$. f is not continuous at all points $\frac{1}{2}c$, where c is an integer. There are discontinuities at $x = \dfrac{c}{2}$, where c is an integer, because $\lim\limits_{x \to c/2} f(x)$ does not exist.

36. $f(x) = \dfrac{[\![x]\!]}{2} + x$ is continuous on all intervals of the form $(c, c + 1)$ where c is an integer. There are discontinuities at all integer values c because $\lim\limits_{x \to c} f(x)$ does not exist.

37. $f(x) = [\![x - 1]\!]$ is continuous on all intervals $(c, c + 1)$. There are discontinuities at $x = c$, where c is an integer, because $\lim\limits_{x \to c} f(x)$ does not exist.

38. $f(x) = x - [\![x]\!]$ is continuous on all intervals $(c, c + 1)$. There are discontinuities at all integer values c because $\lim\limits_{x \to c} f(x)$ does not exist.

39. $h(x) = f(g(x)) = f(x - 1) = \dfrac{1}{\sqrt{x - 1}}, \ x > 1$

h is continuous on its entire domain $(1, \infty)$.

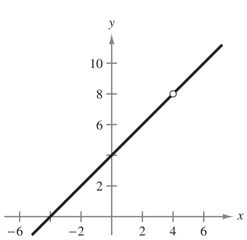
40. $h(x) = f(g(x)) = f(x^2 + 5)$

$$= \frac{1}{(x^2 + 5) - 1} = \frac{1}{x^2 + 4}$$

h is continuous on $(-\infty, \infty)$.

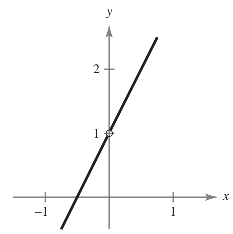
41. $f(x) = \dfrac{x^2 - 16}{x - 4} = \dfrac{(x + 4)(x - 4)}{x - 4} = x + 4, \ x \neq 4$

f has a removable discontinuity at $x = 4$; Continuous on $(-\infty, 4)$ and $(4, \infty)$.



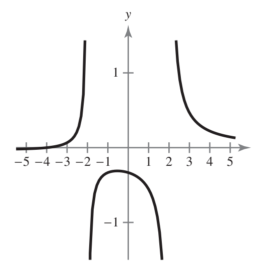
42. $f(x) = \dfrac{2x^2 + x}{x} = \dfrac{x(2x + 1)}{x}$

f has a removable discontinuity at $x = 0$; Continuous on $(-\infty, 0)$ and $(0, \infty)$.



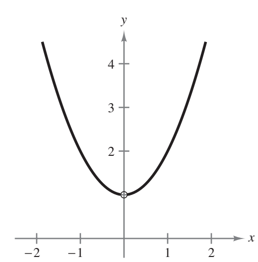
43. $f(x) = \dfrac{x + 4}{3x^2 - 12}$

Continuous on $(-\infty, 2), (-2, 2)$, and $(2, \infty)$.



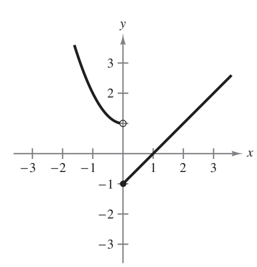
44. $f(x) = \dfrac{x^3 + x}{x} = \dfrac{x(x^2 + 1)}{x} = x^2 + 1, \ x \neq 0$

f has a removable discontinuity at $x = 0$; Continuous on $(-\infty, 0)$ and $(0, \infty)$.



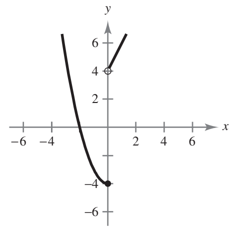
45. $f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$

f has a nonremovable discontinuity at $x = 0$; Continuous on $(-\infty, 0)$ and $(0, \infty)$.



46. $f(x) = \begin{cases} x^2 - 4, & x \le 0 \\ 2x + 4, & x > 0 \end{cases}$

f has a nonremovable discontinuity at $x = 0$; Continuous on $(-\infty, 0)$ and $(0, \infty)$.



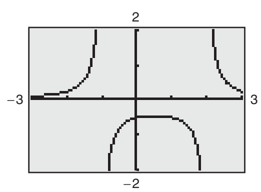
47. Continuous on $[-1, 5]$ because $f(x) = x^2 - 4x - 5$ is a polynomial.

48. Continuous on $[-2, 2]$ because $f(x) = \dfrac{5}{x^2 + 1}$ is defined on the entire interval.

49. Continuous on $[1, 2)$ and $(2, 4]$ because $f(x) = \dfrac{1}{x - 2}$ has a nonremovable discontinuity at $x = 2$.

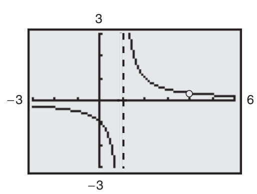
50. Continuous on $(1, 3)$ and $(3, 4]$ because $f(x) = \dfrac{x}{(x - 1)(x - 3)}$ has nonremovable discontinuities at $x = 1$ and $x = 3$.

51.



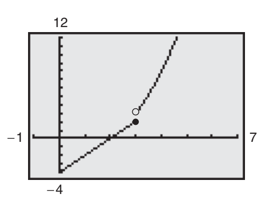
From the graph, you can see that $h(2)$ and $h(-1)$ are not defined, so *h* is not continuous at $x = 2$ and $x = -1$.

52.



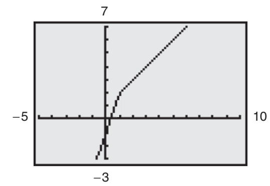
From the graph, you can see that $k(1)$ is not defined, so *k* is not continuous at $x = 1$. [**Note:** There is a hole at $x = 4$.]

53.



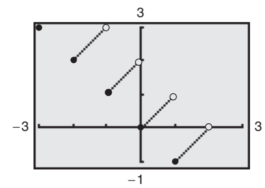
From the graph, you can see that $\lim\limits_{x\to 3} f(x)$ does not exist, so *f* is not continuous at $x = 3$.

54.



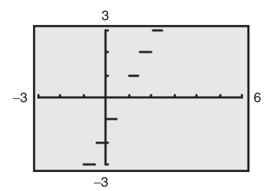
f is continuous on $(-\infty, \infty)$.

55.



From the graph, you can see that $\lim\limits_{x\to c} \left(x - 2[\![x]\!]\right)$, where *c* is an integer, does not exist. So *f* is not continuous at all integers *c*.

56.



From the graph, you can see that $\lim\limits_{x\to c/2} [\![2x - 1]\!]$, where *c* is an integer, does not exist. So *f* is not continuous at all integers *c*.

57. $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} x^3 = 8$

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} ax^2 = 4a$

So, $8 = 4a$ and $a = 2$.

58. $\lim\limits_{x\to -1^-} f(x) = 2$

$\lim\limits_{x\to -1^+} f(x) = -a + b$

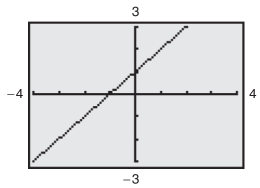
$\lim\limits_{x\to 3^-} f(x) = 3a + b$

$\lim\limits_{x\to 3^+} f(x) = -2$

So,

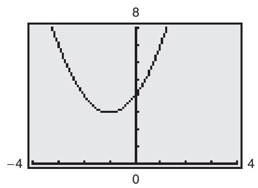
$$\begin{aligned} -a + b &= 2 \\ 3a + b &= -2 \\ \hline -4a \quad &= 4 \\ a &= -1 \\ b &= 1. \end{aligned}$$

59.



$f(x) = \dfrac{x^2 + x}{x} = \dfrac{x(x + 1)}{x}$ appears to be continuous on $[-4, 4]$. But it is not continuous at $x = 0$ (removable discontinuity). Examining a function analytically can reveal removable discontinuities that are difficult to find just from analyzing its graph.

60.



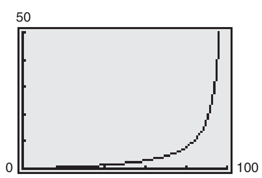
$f(x) = \dfrac{x^3 - 8}{x - 2} = \dfrac{(x - 2)(x^2 + 2x + 4)}{(x - 2)}$ appears to be continuous on $[-4, 4]$. But, it is not continuous at $x = 2$ (removable discontinuity). Examining a function analytically can reveal removable discontinuities that are difficult to find just from analyzing its graph.

61. (a) $[0, 100)$; Negative x-values and values greater than 100 do not make sense in this context. Also, $C(100)$ is undefined.

(b) C is continuous on its domain because all rational functions are continuous on their domains.

(c) For $x = 75$,

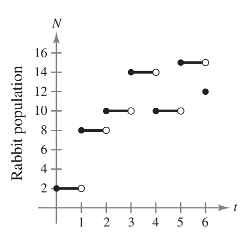
$$C = \frac{2(75)}{100 - 75} = \frac{150}{25} = 6 \text{ million dollars.}$$



62. (a) The graph of G is not continuous on day 8 and day 22.

(b) On these days, the person fills his or her gas tank.

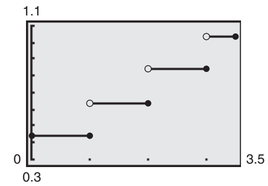
63.



There are nonremovable discontinuities at $t = 1$, 2, 3, 4, 5, and 6.

64. Yes, a linear model is a continuous function. No, actual revenue would probably not be continuous because revenue is usually recorded over larger units of time (hourly, daily, or monthly). In these cases, the revenue may jump between different units of time.

65. (a)



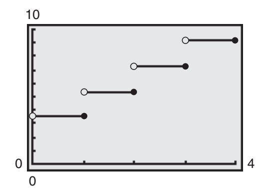
Discontinuities at $x = 1, x = 2, x = 3$

Explanations will vary.

(b) $C(2.5) = \$0.84$

66. (a) $C(x) = 3.50 - 1.90[\![1 - x]\!], x > 0$

(b)



C is not continuous at all integers.

67. (a) The graph has nonremovable discontinuities at $t = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \ldots$.

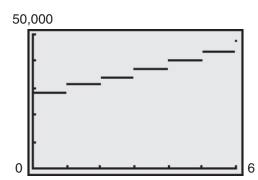
(b) Let $t = 2$.

$A = 7500(1.015)^{[\![4 \cdot 2]\!]} \approx \$8448.69$

(c) Let $t = 7$.

$A = 7500(1.015)^{[\![4 \cdot 7]\!]} \approx \$11{,}379.17$

68. (a)

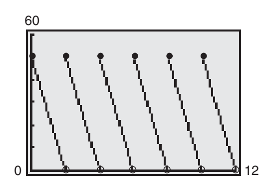


Nonremovable discontinuities at $t = 1, 2, 3, 4, 5$; s is not continuous at $t = 1, 2, 3, 4,$ or 5.

(b) For $t = 5$, $S = \$43{,}850{,}78$.

The salary during the fifth year is \$43,850.78.

69. (a)

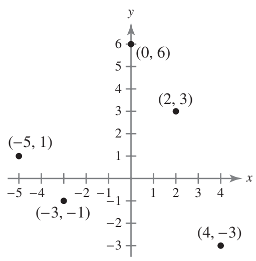


Nonremovable discontinuities at $t = 2, 4, 6, 8, \ldots$; N is not continuous at $t = 2, 4, 6, 8, \ldots$.

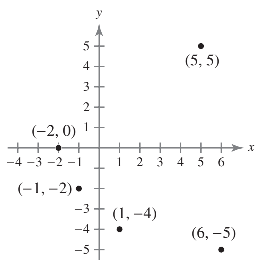
(b) $N \to 0$ when $t \to 2^-, 4^-, 6^-, 8^-, \ldots$, so the inventory is replenished every two months.

Review Exercises for Chapter 1

1.



2.



3. Distance $= \sqrt{(0-5)^2 + (0-2)^2}$
$= \sqrt{25+4}$
$= \sqrt{29}$

4. Distance $= \sqrt{(1-4)^2 + (2-3)^2}$
$= \sqrt{9+1}$
$= \sqrt{10}$

5. Distance $= \sqrt{\left[-1-(-4)\right]^2 + (3-6)^2}$
$= \sqrt{9+9}$
$= 3\sqrt{2}$

6. Distance $= \sqrt{\left[6-(-3)\right]^2 + (8-7)^2}$
$= \sqrt{81+1}$
$= \sqrt{82}$

7. $d = \sqrt{\left(\frac{3}{4} - \frac{1}{4}\right)^2 + \left(-6-(-8)\right)^2}$

$= \sqrt{\left(\frac{1}{2}\right)^2 + (2)^2} = \sqrt{\frac{1}{4}+4} = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$

8. $d = \sqrt{\left(4-(-0.6)\right)^2 + (-1.8-3)^2}$

$= \sqrt{(4.6)^2 + (-4.8)^2} = \sqrt{21.16 + 23.04} = \sqrt{44.2} \approx 6.65$

9. Midpoint $= \left(\frac{5+9}{2}, \frac{6+2}{2}\right) = (7, 4)$

10. Midpoint $= \left(\frac{0-4}{2}, \frac{0+8}{2}\right) = (-2, 4)$

11. Midpoint $= \left(\frac{-10-6}{2}, \frac{4+8}{2}\right) = (-8, 6)$

12. Midpoint $= \left(\frac{7-3}{2}, \frac{-9+5}{2}\right) = (2, -2)$

13. Midpoint $= \left(\frac{-1+6}{2}, \frac{\frac{1}{5}+\frac{3}{5}}{2}\right)$

$= \left(\frac{5}{2}, \frac{2}{5}\right)$

14. Midpoint $= \left(\frac{6(-3.2)}{2}, \frac{1.2+5}{2}\right)$

$= \left(\frac{2.8}{2}, \frac{6.2}{2}\right)$

$= (1.4, 3.1)$

15. $P = R - C$. The tallest bars represent revenues. The middle bars represent costs. The bars on the left of each group represent profits because $P = R - C$.

16. 2005: $R \approx \$6.0$ billion
$C \approx \$4.5$ billion
$P \approx \$1.5$ billion
2006: $R \approx \$10.5$ billion
$C \approx \$8.0$ billion
$P \approx \$3.0$ billion
2007: $R \approx \$16.5$ billion
$C \approx \$12.5$ billion
$P \approx \$4.5$ billion
2008: $R \approx \$21.5$ billion
$C \approx \$16.5$ billion
$P \approx \$5.5$ billion
2009: $R \approx \$23.5$ billion
$C \approx \$17.5$ billion
$P \approx \$6.5$ billion

17. $(1, 3)$ translates to $(-2, 7)$.

$(2, 4)$ translates to $(-1, 8)$.

$(4, 1)$ translates to $(1, 5)$.

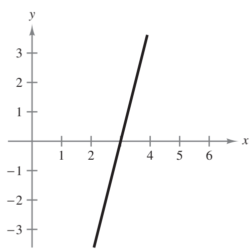
18. $(-2, 1)$ translates to $(2, 0)$.

$(-1, 2)$ translates to $(3, 1)$.

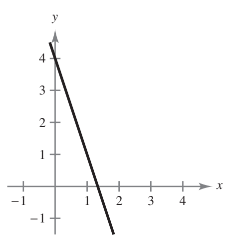
$(1, 0)$ translates to $(5, -1)$.

$(0, -1)$ translates to $(4, -2)$.

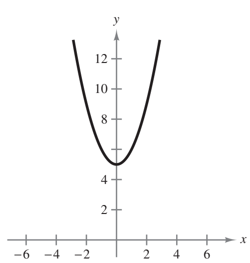
19. $y = 4x - 12$



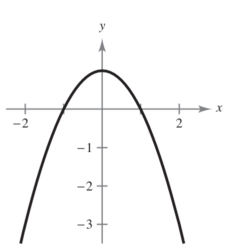
20. $y = 4 - 3x$



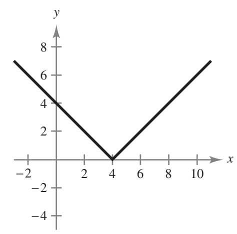
21. $y = x^2 + 5$



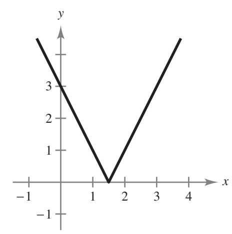
22. $y = 1 - x^2$



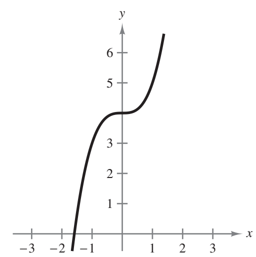
23. $y = |4 - x|$



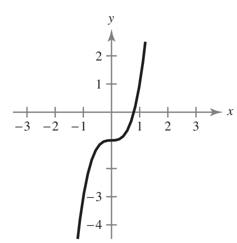
24. $y = |2x - 3|$



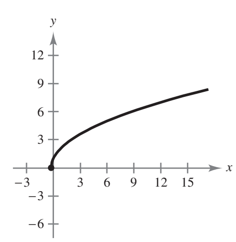
25. $y = x^3 + 4$



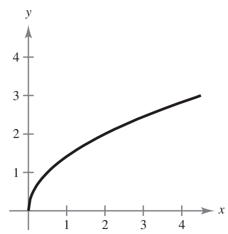
26. $y = 2x^3 - 1$



27. $y = \sqrt{4x + 1}$



28. $y = \sqrt{2x}$



29. Let $y = 0$. Then,

$$4x + 0 + 3 = 0$$
$$x = -\tfrac{3}{4}.$$

Let $x = 0$. Then,

$$4(0) + y + 3 = 0$$
$$y = -3.$$

x-intercept: $\left(-\tfrac{3}{4}, 0\right)$

y-intercept: $(0, -3)$

30. Let $y = 0$. Then,

$$3x - (0) + 6 = 0$$
$$3x = -6$$
$$x = -2.$$

Let $x = 0$. Then,

$$3(0) - y + 6 = 0$$
$$-y = -6$$
$$y = 6.$$

x-intercept: $(-2, 0)$

y-intercept: $(0, 6)$

31. Let $y = 0$. Then,

$$0 = x^2 + 2x - 8$$
$$0 = (x + 4)(x - 2)$$
$$x + 4 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -4 \qquad\qquad x = 2.$$

Let $x = 0$. Then,

$$y = (0)^2 + 2(0) - 8$$
$$y = -8.$$

x-intercepts: $(-4, 0)$, $(2, 0)$

y-intercept: $(0, -8)$

32. Let $y = 0$. Then,

$$0 = (x - 1)^3 + 2(x - 1)^2$$
$$0 = (x - 1)^2(x + 1)$$
$$x = \pm 1.$$

Let $x = 0$. Then,

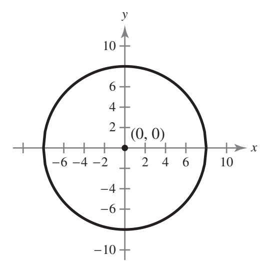
$$y = (0 - 1)^3 + 2(0 - 1)^2$$
$$y = 1.$$

x-intercepts: $(-1, 0)$, $(1, 0)$

y-intercept: $(0, 1)$

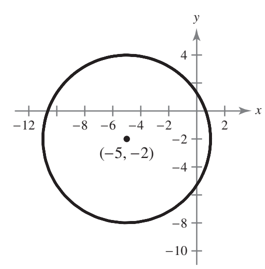
33. $(x - 0)^2 + (y - 0)^2 = 8^2$

$$x^2 + y^2 = 64$$



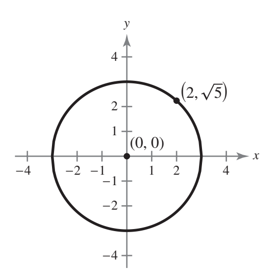
34. $(x - (-5))^2 + (y - (-2))^2 = 6^2$

$$(x + 5)^2 + (y + 2)^2 = 36$$



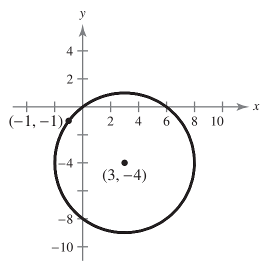
35. $(x - 0)^2 + (y - 0)^2 = r^2$

$$x^2 + y^2 = r^2$$
$$2^2 + \left(\sqrt{5}\right)^2 = r^2$$
$$9 = r^2$$
$$x^2 + y^2 = 9$$



36. $(x-3)^2 + (y-(-4))^2 = r^2$

$$\begin{aligned}(x-3)^2 + (y+4)^2 &= r^2\\ (-1-3)^2 + (-1+4)^2 &= r^2\\ (-4)^2 + (3)^2 &= r^2\\ 16+9 &= r^2\\ 25 &= r^2\\ (x-3)^2 + (y+4)^2 &= 25\end{aligned}$$



37. $y = 2x + 13$ and $y = -5x - 1$

Set the two equations equal to each other.

$$\begin{aligned}2x + 13 &= -5x - 1\\ 7x &= -14\\ x &= -2\end{aligned}$$

Substitute $x = -2$ into one of the equations.

$$y = 2(-2) + 13 = 9$$

38. $y = x^2 - 5$ and $y = x + 1$

Set the two equations equal to each other.

$$\begin{aligned}x^2 - 5 &= x + 1\\ x^2 - x - 6 &= 0\\ (x-3)(x+2) &= 0\\ x - 3 = 0 \quad &\text{or} \quad x + 2 = 0\\ x = 3 \quad & \quad\quad x = -2\end{aligned}$$

Substitute $x = 3$ and $x = -2$ into one of the equations.

$$y = (-2)^2 - 5 = -1$$

$$y = (3)^2 - 5 = 4$$

Points of intersection: $(3, 4)$ and $(-2, -1)$

39. By equating the y-values for the two equations, we have

$$\begin{aligned}x^3 &= x\\ x(x^2 - 1) &= 0\\ x &= -1, 0, 1\end{aligned}$$

The corresponding y-values are $y = -1$, $y = 0$, and $y = 1$, so the points of intersection are $(-1, -1)$, $(0, 0)$, and $(1, 1)$.

40. $y = -x^2 + 4$ and $y = 2x - 1$

Set the two equations equal to each other.

$$\begin{aligned}-x^2 + 4 &= 2x - 1\\ -x^2 - 2x + 5 &= 0\\ x^2 + 2x - 5 &= 0\end{aligned}$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 + 20}}{2}$$

$$x = \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 \pm 2\sqrt{6}}{2} = -1 \pm \sqrt{6}$$

Substitute $x = -1 + \sqrt{6}$ and $x = -1 - \sqrt{6}$ into one of the equations.

$$\begin{aligned}y &= -\left(-1 + \sqrt{6}\right)^2 + 4\\ &= -3 + 2\sqrt{6}\end{aligned}$$

$$\begin{aligned}y &= -\left(-1 - \sqrt{6}\right)^2 + 4\\ &= -3 - 2\sqrt{6}\end{aligned}$$

Points of intersection: $\left(-1 + \sqrt{6}, -3 + 2\sqrt{6}\right) \approx (1.45, 1.90)$

$\left(-1 - \sqrt{6}, -3 - 2\sqrt{6}\right) \approx (-3.45, -7.90)$

41. (a) $C = 200 + 2x + 8x = 200 + 10x$

$R = 14x$

(b)
$$\begin{aligned} C &= R \\ 200 + 10x &= 14x \\ 200 &= 4x \\ x &= 50 \text{ shirts} \end{aligned}$$

$(x, R) = (x, C) = (50, 700)$

42. (a) $C = 6000 + 6.50x$

$R = 13.90x$

(b)
$$\begin{aligned} C &= R \\ 6000 + 6.5x &= 13.9x \\ 6000 &= 7.4x \\ x &\approx 810.81, \text{ or } 811 \text{ units} \end{aligned}$$

43.
$$\begin{aligned} p = 91.4 - 0.009x &= 6.4 + 0.008x \\ 85 &= 0.017x \\ x &= 5000 \text{ units} \end{aligned}$$

Equilibrium point $(x, p) = (5000, 46.40)$

44. (a)

Year	2003	2004	2005	2006	2007	2008
Wind (actual)	115	142	178	264	341	546
Wind (model)	112.23	145.84	182.25	244.68	356.35	540.48

The model fits the data well for the years 2003 through 2008.

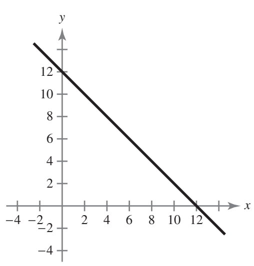
(b) Let $t = 14$.

$y \approx 4467.24$ trillion Btu

45. $y = -x + 12$

Slope: $m = -1$

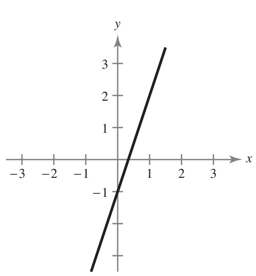
y-intercept: $(0, 12)$



46. $y = 3x - 1$

Slope: $m = 3$

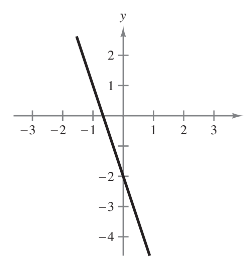
y-intercept: $(0, -1)$



47.
$$\begin{aligned} 3x + y &= -2 \\ y &= -3x - 2 \end{aligned}$$

Slope: $m = -3$

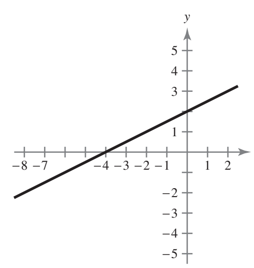
y-intercept: $(0, -2)$



48.
$$\begin{aligned} 2x - 4y &= -8 \\ -4y &= -2x - 8 \\ y &= \tfrac{1}{2}x + 2 \end{aligned}$$

Slope: $m = \frac{1}{2}$

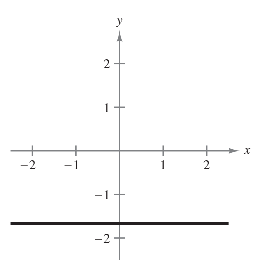
y-intercept: $(0, 2)$



49. $y = -\frac{5}{3}$

Slope: $m = 0$ (horizontal line)

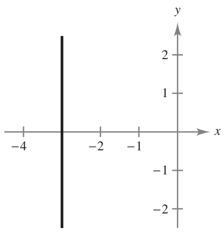
y-intercept: $\left(0, -\frac{5}{3}\right)$



50. $x = -3$

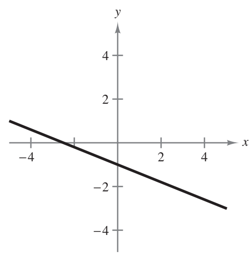
Slope: undefined (vertical line)

No y-intercept



51. $-2x - 5y - 5 = 0$

$$5y = -2x - 5$$
$$y = -\tfrac{2}{5}x - 1$$

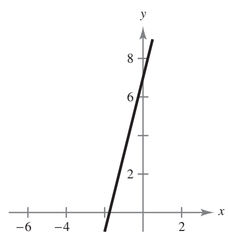


Slope: $m = -\frac{2}{5}$

y-intercept: $(0, -1)$

52. $3.2x - 0.8y + 5.6 = 0$

$$8y = 32x + 56$$
$$y = 4x + 7$$



Slope: $m = 4$

y-intercept: $(0, 7)$

53. Slope $= \dfrac{6 - 0}{7 - 0} = \dfrac{6}{7}$

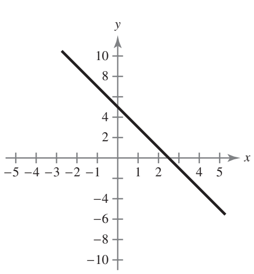
54. Slope $= \dfrac{7 - 5}{-5 - (-1)} = -\dfrac{1}{2}$

55. Slope $= \dfrac{17 - (-3)}{10 - (-11)} = \dfrac{20}{21}$

56. Slope $= \dfrac{-3 - (-3)}{-1 - (-11)} = 0$ (horizontal line)

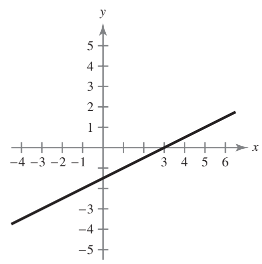
57. $y - (-1) = -2(x - 3)$

$$y + 1 = -2x + 6$$
$$y = -2x + 5$$



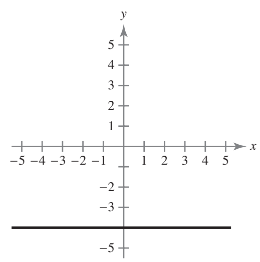
58. $y - (-3) = \frac{1}{2}(x - (-3))$

$$y + 3 = \tfrac{1}{2}x + \tfrac{3}{2}$$
$$y = \tfrac{1}{2}x - \tfrac{3}{2}$$



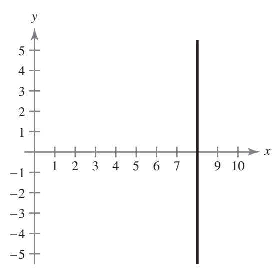
59. $m = 0$: horizontal line through $(1.5, -4)$

$y = -4$



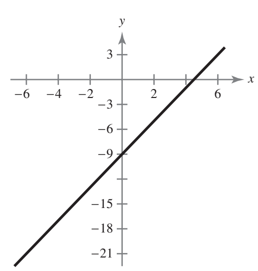
60. m is undefined: vertical line through $(8, 2)$

$x = 8$



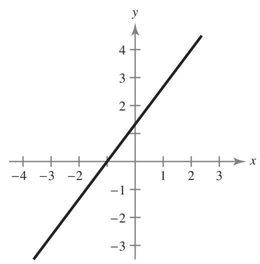
61. $m = \dfrac{5 - (-7)}{7 - 1} = \dfrac{12}{6} = 2$

$$y - (-7) = 2(x - 1)$$
$$y + 7 = 2x - 2$$
$$y = 2x - 9$$



62. $m = \dfrac{12 - 4}{8 - 2} = \dfrac{8}{6} = \dfrac{4}{3}$

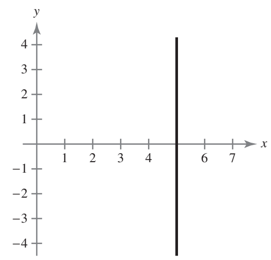
$$y - 4 = \frac{4}{3}(x - 2)$$
$$y - 4 = \frac{4}{3}x - \frac{8}{3}$$
$$y = \frac{4}{3}x + \frac{4}{3}$$



63. $m = \dfrac{14 - 7}{5 - 5} = \dfrac{7}{0} \Rightarrow m$ is undefined.

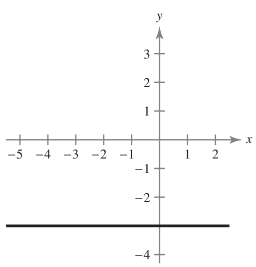
Vertical line through $(5, 7)$

$x = 5$



64. $m = \dfrac{-3 - (-3)}{-2 - 4} = \dfrac{0}{-6} = 0 \Rightarrow$ horizontal line through $(4, -3)$

$y = -3$



65. (a) $$y - 6 = \frac{7}{8}[x - (-3)]$$
$$y = \frac{7}{8}x + \frac{69}{8}$$
$$7x - 8y + 69 = 0$$

(b) $4x + 2y = 7 \Rightarrow y = -2x + \dfrac{7}{2}$; slope $= -2$

$$y - 6 = -2[x - (-3)]$$
$$y = -2x$$
$$2x + y = 0$$

(c) The line through $(0, 0)$ and $(-3, 6)$ has slope

$$\frac{6}{-3} = -2$$
$$y = -2x$$
$$2x + y = 0$$

(d) $3x - 2y = 2 \Rightarrow y = \dfrac{3}{2}x - 1$

Slope of perpendicular is $-\dfrac{2}{3}$.

$$y - 6 = -\frac{2}{3}[x - (-3)]$$
$$y = -\frac{2}{3}x + 4$$
$$2x + 3y - 12 = 0$$

66. (a) Slope $= 0$

$$y = -3$$
$$y + 3 = 0$$

(b) Slope undefined

$$x = 1$$
$$x - 1 = 0$$

(c) $-4x + 5y = -3$

$$y = \tfrac{4}{5}x - \tfrac{3}{5}$$
$$y - (-3) = \tfrac{4}{5}(x - 1)$$
$$y = \tfrac{4}{5}x - \tfrac{19}{5}$$
$$-\tfrac{4}{5}x + y + \tfrac{19}{5} = 0$$

(d) $5x - 2y = 3$

$$y = \tfrac{5}{2}x - \tfrac{3}{2}$$

Slope of perpendicular is $-\tfrac{2}{5}$.

$$y - (-3) = -\tfrac{2}{5}(x - 1)$$
$$y = -\tfrac{2}{5}x - \tfrac{13}{5}$$
$$\tfrac{2}{5}x + y + \tfrac{13}{5} = 0$$

67. $(32, 750), (37, 700)$

$$m = \frac{750 - 700}{32 - 37} = \frac{50}{-5} = -10$$

(a) $x - 750 = -10(p - 32)$

$$x = -10p + 1070$$

(b) If $p = 34.50$, $x = -10(34.50) + 1070 = 725$ units

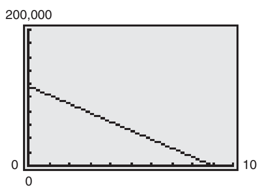
(c) If $p = 42.00$, $x = -10(42.00) + 1070 = 650$ units

68. $(0, 117{,}000), (9, 0)$

$$m = \frac{117{,}000}{-9} = -13{,}000$$

(a) $v = -13{,}000(t - 9) = -13{,}000t + 117{,}000$

(b) Graphing utility



(c) $v(4) = \$65{,}000$

(d) $v = 84{,}000$ when $t \approx 2.54$ years

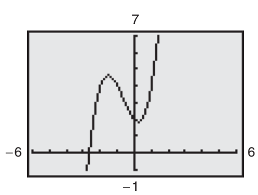
69. Yes, $y = -x^2 + 2$ is a function of x.

70. No, $x^2 + y^2 = 4$ is not a function of x.

71. No, $y^2 - \frac{1}{4}x^2 = 4$ is not a function of x.

72. Yes, $y = |x + 4|$ is a function of x.

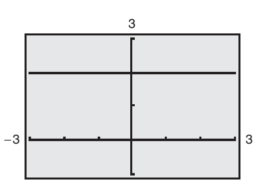
73.



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

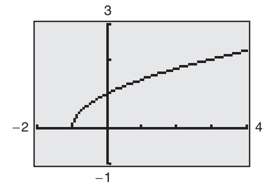
74.



Domain: $(-\infty, \infty)$

Range: $\{2\}$

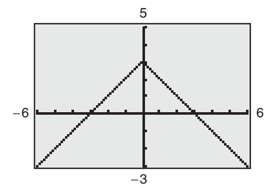
75.



Domain: $[-1, \infty)$

Range: $[0, \infty)$

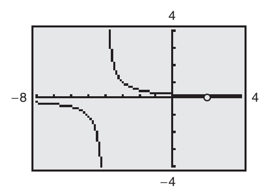
76.



Domain: $(-\infty, \infty)$

Range: $(-\infty, 3]$

77.



$$f(x) = \frac{x - 3}{x^2 + x - 12}$$

$$= \frac{(x - 3)}{(x - 3)(x + 4)}$$

$$= \frac{1}{x + 4}, \ x \neq 3$$

Domain: $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

Range: $(-\infty, 0) \cup \left(0, \frac{1}{7}\right) \cup \left(\frac{1}{7}, \infty\right)$

78. $f(x) = \begin{cases} 6 - x, & x < 2 \\ 3x - 2, & x \geq 2 \end{cases}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

79. $f(x) = 3x + 4$

(a) $f(1) = 3(1) + 4 = 7$

(b) $f(-5) = 3(-5) + 4 = -11$

(c) $f(x + 1) = 3(x + 1) + 4 = 3x + 7$

80. $f(x) = x^2 + 4x + 3$

(a) $f(0) = (0)^2 + 4(0) + 3 = 3$

(b) $f(3) = (3)^2 + 4(3) + 3 = 24$

(c) $f(x-1) = (x-1)^2 + 4(x-1) + 3$
$= x^2 - 2x + 1 + 4x - 4 + 3$
$= x^2 + 2x$

81. (a) $f(x) + g(x) = (1 + x^2) + (2x - 1)$
$= x^2 + 2x$

(b) $f(x) - g(x) = (1 + x^2) - (2x - 1)$
$= x^2 - 2x + 2$

(c) $f(x)g(x) = (1 + x^2)(2x - 1)$
$= 2x^3 - x^2 + 2x - 1$

(d) $\dfrac{f(x)}{g(x)} = \dfrac{1 + x^2}{2x - 1}$

(e) $f(g(x)) = f(2x - 1)$
$= 1 + (2x - 1)^2$
$= 4x^2 - 4x + 2$

(f) $g(f(x)) = g(1 + x^2)$
$= 2(1 + x^2) - 1$
$= 2x^2 + 1$

82. (a) $f(x) + g(x) = 2x - 3 + \sqrt{x + 1}$

(b) $f(x) - g(x) = 2x - 3 - \sqrt{x + 1}$

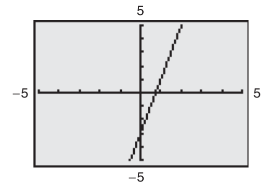
(c) $f(x)g(x) = (2x - 3)\sqrt{x + 1}$

(d) $\dfrac{f(x)}{g(x)} = \dfrac{2x - 3}{\sqrt{x + 1}}$

(e) $f(g(x)) = f(\sqrt{x + 1})$
$= 2\sqrt{x + 1} - 3$

(f) $g(f(x)) = g(2x - 3)$
$= \sqrt{(2x - 3) + 1}$
$= \sqrt{2x - 2}$

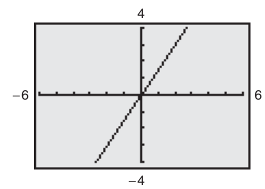
83.



$f(x)$ is one-to-one.

$f(x) = 4x - 3 = y$
$4y - 3 = x$
$4y = x + 3$
$y = \frac{1}{4}x + \frac{3}{4}$
$f^{-1}(x) = \frac{1}{4}x + \frac{3}{4}$

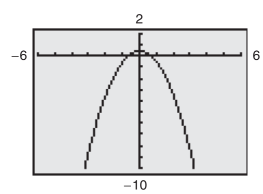
84.



$f(x)$ is one-to-one.

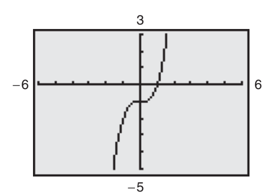
$f(x) = \frac{3}{2}x = y$
$\frac{3}{2}y = x$
$3y = 2x$
$y = \frac{2}{3}x$
$f^{-1}(x) = \frac{2}{3}x$

85.



$f(x)$ does not have an inverse function.

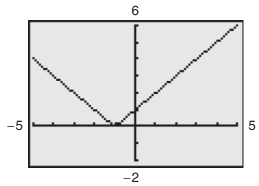
86.



$f(x)$ is one-to-one.

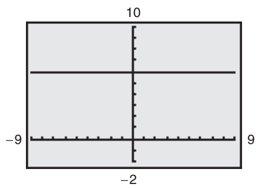
$f(x) = x^3 - 1 = y$
$y^3 - 1 = x$
$y^3 = x + 1$
$y = \sqrt[3]{x + 1}$
$f^{-1}(x) = \sqrt[3]{x + 1}$

87.



$f(x)$ does not have an inverse function.

88.



$f(x)$ does not have an inverse function.

89.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	0.6	0.96	0.996	?	1.004	1.04	1.4

$\lim_{x\to 1}(4x - 3) = 1$

90.

x	2.9	2.99	2.999	3
$f(x)$	0.2564	0.2506	0.2501	?

x	3.001	3.01	3.1
$f(x)$	0.2499	0.2494	0.2439

$\lim_{x\to 3}\dfrac{x-3}{x^2-2x-3} = 0.25$

91.

x	−0.1	−0.01	−0.001	0
$f(x)$	35.71	355.26	3550.71	?

x	0.001	0.01	0.1
$f(x)$	−3550.31	−354.85	−35.30

$\lim_{x\to 0}\dfrac{\sqrt{x+6}-6}{x}$ does not exist.

92.

x	6.9	6.99	6.999	7
$f(x)$	−1.47	−14.33	−142.90	?

x	7.001	7.01	7.1
$f(x)$	142.82	14.24	1.39

$\lim_{x\to 7}\dfrac{\dfrac{1}{x-7}-\dfrac{1}{7}}{x}$ does not exist.

93. $\lim_{x\to 3} 8 = 8$

94. $\lim_{x\to 4} x^2 = (4)^2 = 16$

95. $\lim_{x\to 2}(5x - 3) = 5(2) - 3 = 7$

96. $\lim_{x\to 5} = (2x + 4) = 2(5) + 4 = 14$

97. $\lim_{x\to -1}\dfrac{x+3}{6x+1} = \dfrac{-1+3}{6(-1)+1} = -\dfrac{2}{5}$

98. $\lim_{t\to 3}\dfrac{t}{t+5} = \dfrac{3}{3+5} = \dfrac{3}{8}$

99. $\lim_{t\to 0^-}\dfrac{t^2+1}{t} = -\infty$

$\lim_{t\to 0^+}\dfrac{t^2+1}{t} = \infty$

$\lim_{t\to 0}\dfrac{t^2+1}{t}$ does not exist.

100. $\lim_{t\to 2^-}\dfrac{t+1}{t-2} = -\infty$

$\lim_{t\to 2^+}\dfrac{t+1}{t-2} = \infty$

$\lim_{t\to 2}\dfrac{t+1}{t-2}$ does not exist.

101.
$$\lim_{x\to -2}\frac{x+2}{x^2-4} = \lim_{x\to -2}\frac{x+2}{(x+2)(x-2)} = \lim_{x\to -2}\frac{1}{x-2} = -\frac{1}{4}$$

102.
$$\lim_{x\to 3^-}\frac{x^2-9}{x-3} = \lim_{x\to 3^-}\frac{(x-3)(x+3)}{x-3} = \lim_{x\to 3^-}(x+3) = 6$$

103. $\lim_{x\to 0^+}\left(x - \dfrac{1}{x}\right) = \lim_{x\to 0^+}\dfrac{x^2-1}{x} = -\infty$

104. $\lim_{x\to 1/2}\dfrac{2x-1}{6x-3} = \lim_{x\to 1/2}\dfrac{1}{3} = \dfrac{1}{3}$

105.
$$\lim_{x\to 0}\frac{[1/(x-2)]-1}{x} = \lim_{x\to 0}\frac{1-(x-2)}{x(x-2)} = \lim_{x\to 0}\frac{3-x}{x(x-2)}$$
does not exist.

106. $\displaystyle\lim_{s\to 0}\frac{\left(1/\sqrt{1+s}\right)-1}{s} = \lim_{s\to 0}\frac{1-\sqrt{1+s}}{s\sqrt{1+s}}\cdot\frac{1+\sqrt{1+s}}{1+\sqrt{1+s}}$

$$= \lim_{s\to 0}\frac{1-(1+s)}{s\sqrt{1+s}\left(1+\sqrt{1+s}\right)}$$

$$= \lim_{s\to 0}\frac{-1}{\sqrt{1+s}\left(1+\sqrt{1+s}\right)}$$

$$= -\frac{1}{2}$$

107. $\displaystyle\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} x+5, & x \neq 0 \\ 3, & x = 0 \end{cases}$

$\displaystyle\lim_{x\to 0} f(x) = \lim_{x\to 0}(x+5) = 0+5 = 5$

108. $\displaystyle\lim_{x\to -2} f(x)$, where $f(x) = \begin{cases} \frac{1}{2}x+5, & x < -2 \\ -x+2, & x \geq -2 \end{cases}$

$\displaystyle\lim_{x\to -2} f(x) = -x+2 = -(-2)+4 = 4$

109. $\displaystyle\lim_{\Delta x\to 0}\frac{(x+\Delta x)^3-(x+\Delta x)-\left(x^3-x\right)}{\Delta x} = \lim_{\Delta x\to 0}\frac{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3-x-\Delta x-x^3+x}{\Delta x}$

$$= \lim_{\Delta x\to 0}\frac{3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3-\Delta x}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\left[3x^2+3x\Delta x+(\Delta x)^2-1\right]$$

$$= 3x^2-1$$

110. $\displaystyle\lim_{\Delta x\to 0}\frac{\left[1-(x+\Delta x)^2\right]-\left(1-x^2\right)}{\Delta x} = \lim_{\Delta x\to 0}\frac{1-x^2-2x\Delta x-(\Delta x)^2-1+x^2}{\Delta x}$

$$= \lim_{\Delta x\to 0}\frac{\Delta x(-2x-\Delta x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0}(-2x-\Delta x) = -2x$$

111. $f(x) = x+b$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real x.

112. $f(x) = x^2+3x+2$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real x.

113. $f(x) = \dfrac{1}{(x+4)^2}$ is continuous on the intervals $(-\infty, -4)$ and $(-4, \infty)$ because the domain of *f* consists of all real numbers except $x = -4$. There is a discontinuity at $x = -4$ because $f(4)$ is not defined.

114. $f(x) = \dfrac{x+2}{x}$ is continuous on the intervals $(-\infty, 0)$ and $(0, \infty)$ because the domain of *f* consists of all real numbers except $x = 0$. There is a discontinuity at $x = 0$ because $f(0)$ is not defined.

115. $f(x) = \dfrac{3}{x+1}$ is continuous on the intervals $(-\infty, -1)$ and $(-1, \infty)$ because the domain of *f* consists of all real numbers except $x = -1$. There is a discontinuity at $x = -1$ because $f(-1)$ is not defined.

116. $f(x) = \dfrac{x+1}{2x+2}$ is continuous on the intervals $(-\infty, -1)$ and $(-1, \infty)$ because the domain of *f* consists of all real number except $x = -1$. There is a discontinuity at $x = -1$ because $f(-1)$ is not defined.

117. $f(x) = [\![x+3]\!]$ is continuous on all intervals of the form $(c, c+1)$, where *c* is an integer. There are discontinuities at all integer values *c* because $\displaystyle\lim_{x\to c} f(x)$ does not exist.

118. $f(x) = [\![x]\!]-2$ is continuous on all intervals of the form $(c, c+1,)$ where *c* is an integer. There are discontinuities at all integer values *c* because $\displaystyle\lim_{x\to c} f(x)$ does not exist.

119. $f(x) = \begin{cases} x, & x \le 0 \\ x+1, & x > 0 \end{cases}$ is continuous on the intervals $(-\infty, 0)$ and $(0, \infty)$. There is a discontinuity at $x = 0$ because $\lim_{x\to 0} f(x)$ does not exist.

120. $f(x) = \begin{cases} x, & x \le 0 \\ x^2, & x > 0 \end{cases}$ is continuous on $(-\infty, \infty)$ because $f(0)$ is defined, $\lim_{x\to 0} f(x)$ exists, and $\lim_{x\to 0} f(x) = f(0)$.

121. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (-x+1) = -2$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (ax - 8) = 3a - 8$

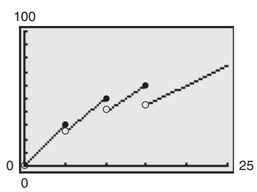
So, $-2 = 3a - 8$ and $a = 2$.

122. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x+1) = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2x + a) = 2 + a$

So, $2 = 2 + a$ and $a = 0$.

123. (a)



Explanations will vary. The function is defined for all values of x greater than zero. The function is not continuous at $x = 5$, $x = 10$, and $x = 15$.

(b) $C(10) = 4.99(10) = \$49.90$

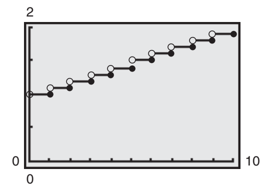
124. $\lim_{t\to 2^-} S(t) = 30.80$

$\lim_{t\to 2^+} S(t) = 33.88$

The limit of S as t approaches 2 does not exist.

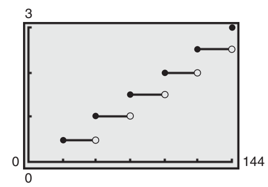
125. (a) $C(t) = \begin{cases} 1 + 0.1[\![t]\!], & t > 0,\ t \text{ not an integer} \\ 1 + 0.1[\![t-1]\!], & t > 0,\ t \text{ an integer} \end{cases}$

(b)



C is not continuous at $t = 1, 2, 3, \ldots$.

126. (a)



The function is not continuous at $x = 24n$, where n is a positive integer.

(b) When $x = 1500$, $A = \$31$.

Chapter 1 Test Yourself

1. (a) $d = \sqrt{(-4-1)^2 + (4-(-1))^2}$

$= \sqrt{(-5)^2 + (5)^2} = \sqrt{50} = 5\sqrt{2}$

(b) Midpoint $= \left(\dfrac{1+(-4)}{2}, \dfrac{-1+4}{2}\right) = \left(-\dfrac{3}{2}, \dfrac{3}{2}\right)$

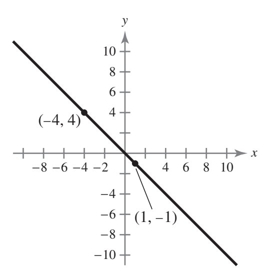
(c) $m = \dfrac{4-(-1)}{-4-1} = \dfrac{5}{-5} = -1$

(d) $y - (-1) = -1(x - 1)$

$y + 1 = -x + 1$

$y = -x$

(e)



2. (a) $d = \sqrt{\left(0 - \frac{5}{2}\right)^2 + (2 - 2)^2}$

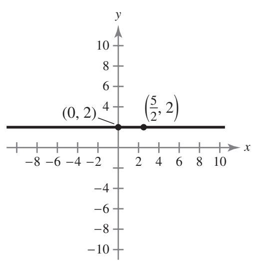
$= \sqrt{\left(-\frac{5}{2}\right)^2 + 0^2} = \frac{5}{2}$

(b) $\text{Midpoint} = \left(\frac{\frac{5}{2} + 0}{2}, \frac{2 + 2}{2}\right) = \left(\frac{5}{4}, 2\right)$

(c) $m = \frac{2 - 2}{0 - \frac{5}{2}} = 0$

(d) Horizontal line: $y = 2$

(e)



3. (a) $d = \sqrt{(-4 - 2)^2 + (1 - 3)^2}$

$= \sqrt{(-6)^2 + (-2)^2} = \sqrt{40}$

$= 2\sqrt{10}$

(b) $\text{Midpoint} = \left(\frac{2 + (-4)}{2}, \frac{3 + 1}{2}\right)$

$= \left(-\frac{2}{2}, \frac{4}{2}\right)$

$= (-1, 2)$

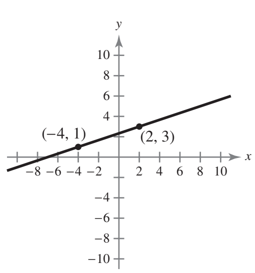
(c) $m = \frac{1 - 3}{-4 - 2} = \frac{-2}{-6} = \frac{1}{3}$

(d) $y - 3 = \frac{1}{3}(x - 2)$

$y - 3 = \frac{1}{3}x - \frac{2}{3}$

$y = \frac{1}{3}x + \frac{7}{3}$

(e)



4. $65 - 2.1x = 43 + 1.9x$

$-4x = -22$

$x = 5.5$

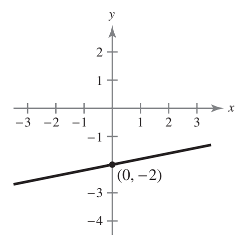
Equilibrium point $(x, p) = (5.5, 5500)$

The equilibrium point occurs when the demand and supply are each 5500 units.

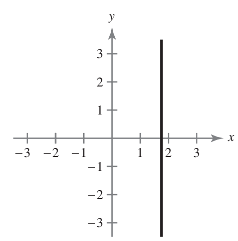
5. $m = \frac{1}{5}$

When $x = 0$: $y = \frac{1}{5}(0) - 2 = -2$

y-intercept: $(0, -2)$



6. The line is vertical, so its slope is undefined, and it has no y-intercept.

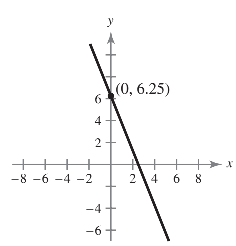


7. $y = -2.5x + 6.25$

$m = -2.5$

When $x = 0$: $y = -2.5(0) + 6.25 = 6.25$

y-intercept: $(0, 6.25)$



8. The slope of the given line $-4x + y = 8 \Rightarrow y = 4x + 8$ is $m = 4$. The slope of the line perpendicular is $m = -\frac{1}{4}$.

Using the point $(-1, -7)$ and $m = -\frac{1}{4}$, the equation is:

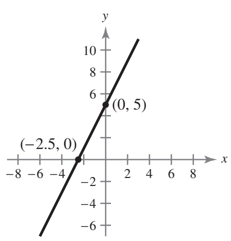
$$\begin{aligned} y - (-7) &= -\tfrac{1}{4}(x - (-1)) \\ y + 7 &= -\tfrac{1}{4}(x + 1) \\ y + 7 &= -\tfrac{1}{4}x - \tfrac{1}{4} \\ y &= -\tfrac{1}{4}x - \tfrac{29}{4} \end{aligned}$$

9. The slope of the given line $5x - 2y = 8 \Rightarrow -2y = -5x + 8 \Rightarrow y = \frac{5}{2}x - 4$ is $m = \frac{5}{2}$. The slope of the line parallel is $m = \frac{5}{2}$.

Using the point $(2, 1)$ and $m = \frac{5}{2}$, the equation is:

$$\begin{aligned} y - 1 &= \tfrac{5}{2}(x - 2) \\ y - 1 &= \tfrac{5}{2}x - 5 \\ y &= \tfrac{5}{2}x - 4 \end{aligned}$$

10. (a)



(b) Domain: $(-\infty, \infty)$

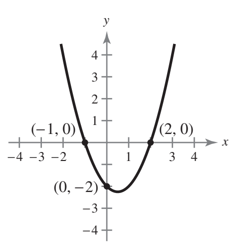
Range: $(-\infty, \infty)$

(c)

x	-3	-2	3
$f(x)$	-1	1	11

(d) The function is one-to-one.

11. (a)



(b) Domain: $(-\infty, \infty)$

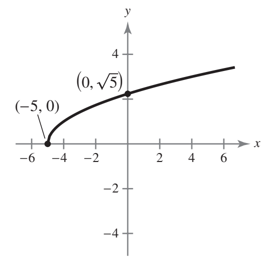
Range: $\left[-\frac{9}{4}, \infty\right)$

(c)

x	-3	-2	3
$f(x)$	10	4	4

(d) The function is not one-to-one.

12. (a)



(b) Domain: $[-5, \infty)$

Range: $[0, \infty)$

(c)

x	-3	-2	3
$f(x)$	$\sqrt{2}$	$\sqrt{3}$	$2\sqrt{2}$

(d) The function is one-to-one.

13.
$$\begin{aligned} f(x) = 4x + 6 &= y \\ 4y + 6 &= x \\ 4y &= x - 6 \\ y &= \tfrac{1}{4}x - \tfrac{3}{2} \\ f^{-1}(x) &= \tfrac{1}{4}x - \tfrac{3}{2} \end{aligned}$$

14.
$$\begin{aligned} f(x) = \sqrt[3]{8 - 3x} &= y \\ \sqrt[3]{8 - 3y} &= x \\ 8 - 3y &= x^3 \\ -3y &= x^3 - 8 \\ y &= -\tfrac{1}{3}x^3 + \tfrac{8}{3} \\ f^{-1}(x) &= -\tfrac{1}{3}x^3 + \tfrac{8}{3} \end{aligned}$$

15. $\displaystyle\lim_{x \to 0} \frac{x - 2}{x + 2} = \frac{0 - 2}{0 + 2} = -1$

16.

x	4.9	4.99	4.999	5.001	5.01	5.1
$f(x)$	–99	–999	–9999	10,001	1001	101

$$\lim_{x \to 5^-} \frac{x + 5}{x - 5} = -\infty$$

$$\lim_{x \to 5^+} \frac{x + 5}{x - 5} = \infty$$

$\displaystyle\lim_{x \to 5} f(x)$ does not exist.

17. $\lim_{x\to-3}\dfrac{x^2+2x-3}{x^2+4x+3} = \lim_{x\to-3}\dfrac{(x-1)(x+3)}{(x+1)(x+3)}$

$= \lim_{x\to-3}\dfrac{x-1}{x+1}$

$= 2$

18.

x	–0.01	–0.001	–0.0001
$f(x)$	0.16671	0.16667	0.16667

x	0.0001	0.001	0.01
$f(x)$	0.16667	0.16666	0.16662

$\lim_{x\to0}\dfrac{\sqrt{x+9}-3}{x} \approx 0.16667$

19. $f(x) = \dfrac{x^2-16}{x-4}$ is continuous on the intervals $(-\infty, 4)$ and $(4, \infty)$ because the domain of f consists of all real numbers except $x = 4$. There is a discontinuity at $x = 4$ because $f(4)$ is not defined.

20. $f(x) = \sqrt{5-x}$ is continuous on the interval $(5, \infty)$ because the domain of f consists of all $x > 5$.

21. $\lim_{x\to1^-} f(x) = \lim_{x\to1^-}(1-x) = 0$

$\lim_{x\to1^+} f(x) = \lim_{x\to1^+}\left(x-x^2\right) = 0$

So, $\lim_{x\to1} f(x) = 0$.

Because $f(1)$ is defined, $\lim_{x\to1} f(x)$ exists, and $\lim_{x\to1} f(x) = f(1)$, the function is continuous on the interval $(-\infty, \infty)$.

22. (a)

t	4	5	6	7	8	9
y (actual)	8149	7591	7001	7078	8924	14,265
y (model)	8113.4	7696.8	6936.4	6995.8	9038.3	14,227.2

The model fits the data well because the actual values are close to the values given by the model.

(b) Let $t = 14$.

$y \approx 128{,}087.392$ or 128,087,392 workers