Chapter 1
The Real Number System

1.1 Exponents, Order of Operations, and Inequality

Classroom Examples, Now Try Exercises

1. **(a)** $9^2 = 9 \cdot 9 = 81$

(b) $\left(\frac{1}{2}\right)^4 = \underbrace{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}}_{} = \frac{1}{16}$

$\frac{1}{2}$ is used as a factor 4 times.

N1. **(a)** $6^2 = 6 \cdot 6 = 36$

(b) $\left(\frac{4}{5}\right)^3 = \underbrace{\frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5}}_{} = \frac{64}{125}$

$\frac{4}{5}$ is used as a factor 3 times.

2. **(a)** $10 - 6 \div 2$

$= 10 - 3$ Divide.

$= 7$ Subtract.

(b) $18 + 2(6-3)$

$= 18 + 2(3)$ Subtract inside parentheses.

$= 18 + 6$ Multiply.

$= 24$ Add.

(c) $7 \cdot 6 - 3(8+1)$

$= 7 \cdot 6 - 3(9)$ Add inside parentheses.

$= 42 - 27$ Multiply.

$= 15$ Subtract.

(d) $2 + 3^2 - 5 \cdot 2$

$= 2 + 9 - 5 \cdot 2$ Apply exponents.

$= 2 + 9 - 10$ Multiply.

$= 11 - 10$ Add.

$= 1$ Subtract.

N2. **(a)** $15 - 2 \cdot 6$

$= 15 - 12$ Multiply.

$= 3$ Subtract.

(b) $8 + 2(5-1)$

$= 8 + 2(4)$ Subtract inside parentheses.

$= 8 + 8$ Multiply.

$= 16$ Add.

(c) $6(2+4) - 7 \cdot 5$

$= 6(6) - 7 \cdot 5$ Add inside parentheses.

$= 36 - 35$ Multiply.

$= 1$ Subtract.

(d) $8 \cdot 10 \div 4 - 2^3 + 3 \cdot 4^2$

$= 8 \cdot 10 \div 4 - 8 + 3 \cdot 16$ Apply exponents.

$= 80 \div 4 - 8 + 48$ Multiply.

$= 20 - 8 + 48$ Divide.

$= 12 + 48$ Subtract.

$= 60$ Add.

3. **(a)** $9[(4+8) - 3]$

$= 9[12 - 3]$ Add inside parentheses.

$= 9[9]$ Subtract inside brackets.

$= 81$ Multiply.

(b) $\frac{2(7+8)+2}{3 \cdot 5 + 1}$

$= \frac{2(15)+2}{3 \cdot 5 + 1}$ Add inside parentheses.

$= \frac{30+2}{15+1}$ Multiply.

$= \frac{32}{16}$ Add.

$= 2$ Divide.

N3. **(a)** $7\left[\left(3^2 - 1\right) + 4\right]$

$= 7[(9-1) + 4]$ Apply exponents.

$= 7[8 + 4]$ Subtract inside parentheses.

$= 7[12]$ Add inside brackets.

$= 84$ Multiply.

(b) $\dfrac{9(14-4)-2}{4+3\cdot 6}$

$= \dfrac{9(10)-2}{4+3\cdot 6}$ Subtract inside parentheses.

$= \dfrac{90-2}{4+18}$ Multiply.

$= \dfrac{88}{22}$ Subtract and add.

$= 4$ Divide.

4. **(a)** The statement $12 > 6$ is *true* since 12 is greater than 6. Note that the inequality symbol points to the lesser number.

(b) The statement $28 \neq 4\cdot 7$ is *false* because 28 *is equal to* $4\cdot 7$.

(c) The statement $21 \leq 21$ is *true* since $21 = 21$.

(d) Write the fractions with a common denominator. The statement $\frac{1}{3} < \frac{1}{4}$ is equivalent to the statement $\frac{4}{12} < \frac{3}{12}$. Since 4 is *greater* than 3, the original statement is *false.*

N4. **(a)** The statement $12 \neq 10-2$ is *true* because 12 *is not equal to* 8.

(b) The statement $5 > 4\cdot 2$ is *false* because 5 *is less than* 8.

(c) The statement $7 \leq 7$ is *true* since $7 = 7$.

(d) Write the fractions with a common denominator. The statement $\frac{5}{9} > \frac{7}{11}$ is equivalent to the statement $\frac{55}{99} > \frac{63}{99}$. Since 55 is *less* than 63, the original statement is *false.*

5. **(a)** "Nine is equal to eleven minus two" is written as $9 = 11-2$.

(b) "Fourteen is greater than twelve" is written as $14 > 12$.

(c) "Two is greater than or equal to two" is written as $2 \geq 2$.

N5. **(a)** "Ten is not equal to eight minus two" is written as $10 \neq 8-2$.

(b) "Fifty is greater than fifteen" is written as $50 > 15$.

(c) "Eleven is less than or equal to twenty" is written as $11 \leq 20$.

6. $9 \leq 15$ may be written as $15 \geq 9$.

N6. $8 < 9$ may be written as $9 > 8$.

Exercises

1. False; $3^2 = 3\cdot 3 = 9$.

2. False; 1 raised to *any* power is 1. Here, $1^3 = 1\cdot 1\cdot 1 = 1$.

3. False; a number raised to the first power is that number, so $3^1 = 3$.

4. False; 6^2 means that 6 is used as a factor 2 times, so $6^2 = 6\cdot 6 = 36$.

5. False; the common error leading to 42 is adding 4 to 3 and then multiplying by 6. One must follow the rules for order of operations.

$4+3(8-2)$
$= 4+3(6)$
$= 4+18$
$= 22$

6. False; multiplications and divisions are performed *in order from left to right.*

$12 \div 2\cdot 3$
$= 6\cdot 3$
$= 18$

7. Additions and subtractions are performed in order from left to right.

$18 \underbrace{-}_{1} 2 \underbrace{+}_{2} 3$

8. Multiplications and divisions are performed in order from left to right, and then additions and subtractions are performed in order from left to right.

$28 \underbrace{-}_{2} 6 \underbrace{\div}_{1} 2$

9. Multiplications and divisions are performed in order from left to right, and then additions and subtractions are performed in order from left to right.

$\underbrace{2\cdot 8}_{1} \underbrace{-}_{3} 6 \underbrace{\div}_{2} 3$

10. Multiplications and divisions are performed in order from left to right, and then additions and subtractions are performed in order from left to right. If grouping symbols are present, work within them first, starting with the innermost.
$40\underbrace{+}_{3}\underbrace{6(}_{2}3\underbrace{-}_{1}1)$

11. Multiplications and divisions are performed in order from left to right, and then additions and subtractions are performed in order from left to right. If grouping symbols are present, work within them first, starting with the innermost.
$\underbrace{3\cdot 5}_{2}\underbrace{-}_{4}\underbrace{2(}_{3}4\underbrace{+}_{1}2)$

12. Apply all exponents. Then, multiplications and divisions are performed in order from left to right, and additions and subtractions are performed in order from left to right.
$9\underbrace{-}_{3}\underbrace{2^3}_{1}\underbrace{+}_{4}3\underbrace{\cdot}_{2}4$

13. $3^2 = 3\cdot 3 = 9$

14. $8^2 = 8\cdot 8 = 64$

15. $7^2 = 7\cdot 7 = 49$

16. $4^2 = 4\cdot 4 = 16$

17. $12^2 = 12\cdot 12 = 144$

18. $14^2 = 14\cdot 14 = 196$

19. $4^3 = 4\cdot 4\cdot 4 = 64$

20. $5^3 = 5\cdot 5\cdot 5 = 125$

21. $10^3 = 10\cdot 10\cdot 10 = 1000$

22. $11^3 = 11\cdot 11\cdot 11 = 1331$

23. $3^4 = 3\cdot 3\cdot 3\cdot 3 = 81$

24. $6^4 = 6\cdot 6\cdot 6\cdot 6 = 1296$

25. $4^5 = 4\cdot 4\cdot 4\cdot 4\cdot 4 = 1024$

26. $3^5 = 3\cdot 3\cdot 3\cdot 3\cdot 3 = 243$

27. $\left(\frac{1}{6}\right)^2 = \frac{1}{6}\cdot\frac{1}{6} = \frac{1}{36}$

28. $\left(\frac{1}{3}\right)^2 = \frac{1}{3}\cdot\frac{1}{3} = \frac{1}{9}$

29. $\left(\frac{2}{3}\right)^4 = \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3} = \frac{16}{81}$

30. $\left(\frac{3}{4}\right)^3 = \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4} = \frac{27}{64}$

31. $(0.4)^3 = (0.4)(0.4)(0.4) = 0.064$

32. $(0.5)^4 = (0.5)(0.5)(0.5)(0.5) = 0.0625$

33. $\begin{aligned} 64 \div 4\cdot 2 &= 16\cdot 2 && \text{Divide.} \\ &= 32 && \text{Multiply.} \end{aligned}$

34. $\begin{aligned} 250 \div 5\cdot 2 &= 50\cdot 2 && \text{Divide.} \\ &= 100 && \text{Multiply.} \end{aligned}$

35. $\begin{aligned} 13 + 9\cdot 5 &= 13 + 45 && \text{Multiply.} \\ &= 58 && \text{Add.} \end{aligned}$

36. $\begin{aligned} 11 + 7\cdot 6 &= 11 + 42 && \text{Multiply.} \\ &= 53 && \text{Add.} \end{aligned}$

37. $\begin{aligned} 25.2 - 12.6 \div 4.2 &= 25.2 - 3 && \text{Divide.} \\ &= 22.2 && \text{Subtract.} \end{aligned}$

38. $\begin{aligned} 12.4 - 9.3 \div 3.1 &= 12.4 - 3 && \text{Divide.} \\ &= 9.4 && \text{Subtract.} \end{aligned}$

39. $\begin{aligned} \frac{1}{4}\cdot\frac{2}{3} + \frac{2}{5}\cdot\frac{11}{3} &= \frac{1}{6} + \frac{22}{15} && \text{Multiply.} \\ &= \frac{5}{30} + \frac{44}{30} && \text{LCD} = 30 \\ &= \frac{49}{30}, \text{ or } 1\frac{19}{30} && \text{Add.} \end{aligned}$

40. $\begin{aligned} \frac{9}{4}\cdot\frac{2}{3} + \frac{4}{5}\cdot\frac{5}{3} &= \frac{3}{2} + \frac{4}{3} && \text{Multiply.} \\ &= \frac{9}{6} + \frac{8}{6} && \text{LCD} = 6 \\ &= \frac{17}{6}, \text{ or } 2\frac{5}{6} && \text{Add.} \end{aligned}$

41. $\begin{aligned} 9\cdot 4 - 8\cdot 3 &= 36 - 24 && \text{Multiply.} \\ &= 12 && \text{Subtract.} \end{aligned}$

42. $\begin{aligned} 11\cdot 4 + 10\cdot 3 &= 44 + 30 && \text{Multiply.} \\ &= 74 && \text{Add.} \end{aligned}$

43. $\begin{aligned} 20 - 4\cdot 3 + 5 &= 20 - 12 + 5 && \text{Multiply.} \\ &= 8 + 5 && \text{Subtract.} \\ &= 13 && \text{Add.} \end{aligned}$

44. $18-7\cdot 2+6=18-14+6$ Multiply.
$=4+6$ Subtract.
$=10$ Add.

45. $10+40\div 5\cdot 2=10+8\cdot 2$ Divide.
$=10+16$ Multiply.
$=26$ Add.

46. $12+64\div 8-4=12+8-4$ Divide.
$=20-4$ Add.
$=16$ Subtract.

47. $18-2(3+4)$
$=18-2(7)$ Add inside parentheses.
$=18-14$ Multiply.
$=4$ Subtract.

48. $30-3(4+2)$
$=30-3(6)$ Add inside parentheses.
$=30-18$ Multiply.
$=12$ Subtract.

49. $3(4+2)+8\cdot 3=3\cdot 6+8\cdot 3$ Add.
$=18+24$ Multiply.
$=42$ Add.

50. $9(1+7)+2\cdot 5=9\cdot 8+2\cdot 5$ Add.
$=72+10$ Multiply.
$=82$ Add.

51. $18-4^2+3=18-16+3$ Apply exponents.
$=2+3$ Subtract.
$=5$ Add.

52. $22-2^3+9=22-8+9$ Apply exponents.
$=14+9$ Subtract.
$=23$ Add.

53. $2+3[5+4(2)]=2+3[5+8]$ Multiply.
$=2+3[13]$ Add.
$=2+39$ Multiply.
$=41$ Add.

54. $5+4\left[1+7(3)\right]=5+4[1+21]$ Multiply.
$=5+4[22]$ Add.
$=5+88$ Multiply.
$=93$ Add.

55. $5\left[3+4\left(2^2\right)\right]=5[3+4(4)]$ Apply exponents.
$=5(3+16)$ Multiply.
$=5(19)$ Add.
$=95$ Multiply.

56. $6\left[2+8\left(3^3\right)\right]$
$=6[2+8\cdot 27]$ Apply exponents.
$=6(2+216)$ Multiply.
$=6\cdot 218$ Add.
$=1308$ Multiply.

57. $3^2[(11+3)-4]$
$=3^2[14-4]$ Add inside parentheses.
$=3^2[10]$ Subtract.
$=9[10]$ Apply exponents.
$=90$ Multiply.

58. $4^2[(13+4)-8]$
$=4^2[17-8]$ Add inside parentheses.
$=4^2[9]$ Subtract.
$=16[9]$ Apply exponents.
$=144$ Multiply.

59. Simplify the numerator and denominator separately, and then divide.

$$\frac{6\left(3^2-1\right)+8}{8-2^2}=\frac{6(9-1)+8}{8-4}=\frac{6(8)+8}{4}=\frac{48+8}{4}=\frac{56}{4}=14$$

60. Simplify the numerator and denominator separately, and then divide.

$$\frac{2\left(8^2-4\right)+8}{29-3^3}=\frac{2(64-4)+8}{29-27}=\frac{2(60)+8}{2}=\frac{120+8}{2}=\frac{128}{2}=64$$

61. Simplify the numerator and denominator separately, and then divide.

$$\frac{4(6+2)+8(8-3)}{6(4-2)-2^2}=\frac{4(8)+8(5)}{6(2)-2^2}$$
$$=\frac{4(8)+8(5)}{6(2)-4}$$
$$=\frac{32+40}{12-4}$$
$$=\frac{72}{8}=9$$

62. Simplify the numerator and denominator separately, and then divide.

$$\frac{6(5+1)-9(1+1)}{5(8-6)-2^3}=\frac{6(6)-9(2)}{5(2)-2^3}$$
$$=\frac{36-18}{10-8}$$
$$=\frac{18}{2}=9$$

63. $3\cdot 6+4\cdot 2=60$

Listed below are some possibilities. Use trial and error until you get the desired result.

$(3\cdot 6)+4\cdot 2=18+8=26\neq 60$

$(3\cdot 6+4)\cdot 2=22\cdot 2=44\neq 60$

$3\cdot(6+4\cdot 2)=3\cdot 14=42\neq 60$

$3\cdot(6+4)\cdot 2=3\cdot 10\cdot 2=30\cdot 2=60$

64. $2\cdot 8-1\cdot 3=42$

$2\cdot(8-1)\cdot 3=2\cdot 7\cdot 3=14\cdot 3=42$

65. $10-7-3=6$

$10-(7-3)=10-4=6$

66. $15-10-2=7$

$15-(10-2)=15-8=7$

67. $8+2^2=100$

$(8+2)^2=10^2=10\cdot 10=100$

68. $4+2^2=36$

$(4+2)^2=6^2=6\cdot 6=36$

69. $9\cdot 3-11\leq 16$

$27-11\leq 16$

$16\leq 16$

The statement is true since $16=16$.

70. $6\cdot 5-12\leq 18$

$30-12\leq 18$

$18\leq 18$

The statement is true since $18=18$.

71. $5\cdot 11+2\cdot 3\leq 60$

$55+6\leq 60$

$61\leq 60$

The statement is false since 61 *is greater than* 60.

72. $9\cdot 3+4\cdot 5\geq 48$

$27+20\geq 48$

$47\geq 48$

The statement is false since 47 *is less than* 48.

73. $0\geq 12\cdot 3-6\cdot 6$

$0\geq 36-36$

$0\geq 0$

The statement is true since $0=0$.

74. $10\leq 13\cdot 2-15\cdot 1$

$10\leq 26-15$

$10\leq 11$

The statement is true since $10<11$.

75. $45\geq 2[2+3(2+5)]$

$45\geq 2[2+3(7)]$

$45\geq 2[2+21]$

$45\geq 2[23]$

$45\geq 46$

The statement is false since 45 *is less than* 46.

76. $55\geq 3[4+3(4+1)]$

$55\geq 3[4+3(5)]$

$55\geq 3[4+15]$

$55\geq 3[19]$

$55\geq 57$

The statement is false since 55 *is less than* 57.

77. $[3\cdot 4+5(2)]\cdot 3>72$

$[12+10]\cdot 3>72$

$[22]\cdot 3>72$

$66>72$

The statement is false since 66 *is less than* 72.

78. $2 \cdot [7 \cdot 5 - 3(2)] \le 58$
$2 \cdot [35 - 6] \le 58$
$2[29] \le 58$
$58 \le 58$
The statement is true since $58 = 58$.

79. $\dfrac{3+5(4-1)}{2 \cdot 4+1} \ge 3$
$\dfrac{3+5(3)}{8+1} \ge 3$
$\dfrac{3+15}{9} \ge 3$
$\dfrac{18}{9} \ge 3$
$2 \ge 3$
The statement is false since 2 *is less than* 3.

80. $\dfrac{7(3+1)-2}{3+5 \cdot 2} \le 2$
$\dfrac{7(4)-2}{3+10} \le 2$
$\dfrac{28-2}{13} \le 2$
$\dfrac{26}{13} \le 2$
$2 \le 2$
The statement is true since $2 = 2$.

81. $3 \ge \dfrac{2(5+1)-3(1+1)}{5(8-6)-4 \cdot 2}$
$3 \ge \dfrac{2(6)-3(2)}{5(2)-8}$
$3 \ge \dfrac{12-6}{10-8}$
$3 \ge \dfrac{6}{2}$
$3 \ge 3$
The statement is true since $3 = 3$.

82. $7 \le \dfrac{3(8-3)+2(4-1)}{9(6-2)-11(5-2)}$
$7 \le \dfrac{3(5)+2(3)}{9(4)-11(3)}$
$7 \le \dfrac{15+6}{36-33}$
$7 \le \dfrac{21}{3}$
$7 \le 7$
The statement is true since $7 = 7$.

83. "$5 < 17$" means "five is less than seventeen." The statement is true.

84. "$8 < 12$" means "eight is less than twelve." The statement is true.

85. "$5 \ne 8$" means "five is not equal to eight." The statement is true.

86. "$6 \ne 9$" means "six is not equal to nine." The statement is true.

87. "$7 \ge 14$" means "seven is greater than or equal to fourteen." The statement is false.

88. "$6 \ge 12$" means "six is greater than or equal to twelve." The statement is false.

89. "$15 \le 15$" means "fifteen is less than or equal to fifteen." The statement is true.

90. "$21 \le 21$" means "twenty-one is less than or equal to twenty-one." The statement is true.

91. "$\dfrac{1}{3} = \dfrac{3}{10}$" means "one-third is equal to three-tenths." The statement is false.

92. "$\dfrac{10}{6} = \dfrac{3}{2}$" means "ten-sixths is equal to three-halves." The statement is false.

93. "$2.5 > 2.50$" means "two and five-tenths is greater than two and fifty-hundredths." The statement is false.

94. "$1.80 > 1.8$" means "one and eighty-hundredths is greater than one and eight-tenths." The statement is false.

95. "Fifteen is equal to five plus ten" is written as $15 = 5 + 10$.

96. "Twelve is equal to twenty minus eight" is written as $12 = 20 - 8$.

97. "Nine is greater than five minus four" is written as $9 > 5 - 4$.

98. "Ten is greater than six plus one" is written as $10 > 6 + 1$.

99. "Sixteen is not equal to nineteen" is written as $16 \ne 19$.

100. "Three is not equal to four" is written as $3 \ne 4$.

101. "One-half is less than or equal to two-fourths" is written as $\dfrac{1}{2} \le \dfrac{2}{4}$.

102. "One-third is less than or equal to three-ninths" is written as $\frac{1}{3} \le \frac{3}{9}$.

103. $5 < 20$ becomes $20 > 5$ when the inequality symbol is reversed.

104. $30 > 9$ becomes $9 < 30$ when the inequality symbol is reversed.

105. $\frac{4}{5} > \frac{3}{4}$ becomes $\frac{3}{4} < \frac{4}{5}$ when the inequality symbol is reversed.

106. $\frac{5}{4} < \frac{3}{2}$ becomes $\frac{3}{2} > \frac{5}{4}$ when the inequality symbol is reversed.

107. $2.5 \ge 1.3$ becomes $1.3 \le 2.5$ when the inequality symbol is reversed.

108. $4.1 \le 5.3$ becomes $5.3 \ge 4.1$ when the inequality symbol is reversed.

109. **(a)** Substitute "40" for "age" in the expression for women.
$14.7 - 40 \cdot 0.13$

(b) $14.7 - 40 \cdot 0.13 = 14.7 - 5.2$ Multiply.
$= 9.5$ Subtract.

(c) 85% of 9.5 is $0.85(9.5) = 8.075$.
Walking at 5 mph is associated with 8.0 METs, which is the table value closest to 8.075.

(d) Substitute "55" for "age" in the expression for men.
$14.7 - 55 \cdot 0.11$

$14.7 - 55 \cdot 0.11 = 14.7 - 6.05$ Multiply.
$= 8.65$ Subtract.

85% of 8.65 is $0.85(8.65) = 7.3525$.
Swimming is associated with 7.0 METs, which is the table value closest to 7.3525.

110. Answers will vary.

111. The states that had a number greater than 13.8 are Alaska (16.2), Texas (14.7), California (24.1), and Idaho (17.6).

112. The states that had a number that was at most 14.7 are Texas (14.7), Wyoming (12.5), Maine (12.3), and Missouri (13.8).

113. The states that had a number *not* less than 13.8, which is the same as greater than or equal to 13.8, are Alaska (16.2), Texas (14.7), California (24.1), Idaho (17.6), and Missouri (13.8).

114. The states that had a number less than 13.0 are Wyoming (12.5) and Maine (12.3).

1.2 Variables, Expressions, and Equations

Classroom Examples, Now Try Exercises

1. **(a)** $16p = 16 \cdot p$
$= 16 \cdot 3$ Replace p with 3.
$= 48$ Multiply.

(b) $2p^3 = 2 \cdot p^3$
$= 2 \cdot 3^3$ Replace p with 3.
$= 2 \cdot 27$ Cube 3.
$= 54$ Multiply.

N1. **(a)** $9x = 9 \cdot x$
$= 9 \cdot 6$ Replace x with 6.
$= 54$ Multiply.

(b) $4x^2 = 4 \cdot x^2$
$= 4 \cdot 6^2$ Replace x with 6.
$= 4 \cdot 36$ Square 6.
$= 144$ Multiply.

2. **(a)** $4x + 5y = 4(6) + 5(9)$
$= 24 + 45$ Multiply.
$= 69$ Add.

(b) $\frac{4x - 2y}{x + 1} = \frac{4(6) - 2(9)}{6 + 1}$
$= \frac{24 - 18}{6 + 1}$ Multiply.
$= \frac{6}{7}$ Subtract and add.

(c) $2x^2 + y^2 = 2 \cdot 6^2 + 9^2$
$= 2 \cdot 36 + 81$ Use exponents.
$= 72 + 81$ Multiply.
$= 153$ Add.

N2. **(a)** $3x+4y=3(4)+4(7)$

$=12+28$ Multiply.

$=40$ Add.

(b) $\dfrac{6x-2y}{2y-9}=\dfrac{6(4)-2(7)}{2(7)-9}$

$=\dfrac{24-14}{14-9}$ Multiply.

$=\dfrac{10}{5}=2$ Subtract; reduce.

(c) $4x^2-y^2=4\cdot 4^2-7^2$

$=4\cdot 16-49$ Use exponents.

$=64-49$ Multiply.

$=15$ Subtract.

3. **(a)** Since a number is subtracted *from* 48, write this as $48-x$ when using x as the variable to represent the number.

(b) "Product" indicates multiplication. Using x as the variable to represent the number, "the product of 6 and a number" translates as $6\cdot x$ or $6x$.

(c) "The sum of a number and 5" suggests a number plus 5. Using x as the variable to represent the number, "9 multiplied by the sum of a number and 5" translates as $9(x+5)$.

N3. **(a)** Using x as the variable to represent the number, "the sum of a number and 10" translates as $x+10$, or $10+x$.

(b) "A number divided by 7" translates as $x\div 7$, or $\dfrac{x}{7}$.

(c) "The difference between 9 and a number" translates as $9-x$. Thus, "the product of 3 and the difference between 9 and a number" translates as $3(9-x)$.

4. $8p-11=5$

$8\cdot 2-11\overset{?}{=}5$ Replace p with 2.

$16-11\overset{?}{=}5$ Multiply.

$5=5$ True

The number 2 is a solution of the equation.

N4. $8k+5=61$

$8\cdot 7+5\overset{?}{=}61$ Replace k with 7.

$56+5\overset{?}{=}61$ Multiply.

$61=61$ True

The number 7 is a solution of the equation.

5. Using x as the variable to represent the number, "three times a number is subtracted from 21, giving 15" translates as $21-3x=15$. Now try each number from the set $\{0, 2, 4, 6, 8, 10\}$.

$x=0$: $21-3(0)\overset{?}{=}15$

$21=15$ False

$x=2$: $21-3(2)\overset{?}{=}15$

$15=15$ True

$x=4$: $21-3(4)\overset{?}{=}15$

$9=15$ False

Similarly, $x=6, 8,$ or 10 result in false statements. Thus, 2 is the only solution.

N5. Using x as the variable to represent the number, "the sum of a number and nine is equal to the difference between 25 and the number" translates as $x+9=25-x$. Now try each number from the set $\{0, 2, 4, 6, 8, 10\}$.

$x=4$: $4+9\overset{?}{=}25-4$

$13=21$ False

$x=6$: $6+9\overset{?}{=}25-6$

$15=19$ False

$x=8$: $8+9\overset{?}{=}25-8$

$17=17$ True

Similarly, $x=0, 2,$ or 10 result in false statements. Thus, 8 is the only solution.

6. **(a)** $\dfrac{3x-1}{5}$ has no equals symbol, so this is an expression.

(b) $\dfrac{3x}{5}=1$ has an equals symbol, so this is an equation.

N6. **(a)** $2x+5=6$ has an equals symbol, so this is an equation.

(b) $2x+5-6$ has no equals symbol, so this is an expression.

Exercises

1. The expression $8x^2$ means $8 \cdot x \cdot x$. The correct choice is B.

2. If $x = 2$ and $y = 1$, then the value of xy is $2 \cdot 1 = 2$. The correct choice is C.

3. The sum of 15 and a number x is represented by the expression $15 + x$. The correct choice is A.

4. There is no equals symbol in $6x + 7$ or $6x - 7$, so those are expressions. $6x = 7$ and $6x - 7 = 0$ have equals symbols, so those are equations.

5. $2x^3 = 2 \cdot x \cdot x \cdot x$, while $2x \cdot 2x \cdot 2x = (2x)^3$. The last expression is equal to $8x^3$.

6. "7 less than a number" is an expression indicating subtraction, $x - 7$, while "7 is less than a number" is a statement relating 7 and x, $7 < x$.

7. The exponent 2 applies only to its base, which is x. (The expression $(5x)^2$ would require multiplying 5 by $x = 4$ first.)

8. An expression cannot be solved—it indicates a series of operations to perform. An expression is simplified; an equation is solved.

9. **(a)** $x + 7 = 4 + 7$
$= 11$

(b) $x + 7 = 6 + 7$
$= 13$

10. **(a)** $x - 3 = 4 - 3$
$= 1$

(b) $x - 3 = 6 - 3$
$= 3$

11. **(a)** $4x = 4(4) = 16$

(b) $4x = 4(6) = 24$

12. **(a)** $6x = 6(4)$
$= 24$

(b) $6x = 6(6)$
$= 36$

13. **(a)** $4x^2 = 4 \cdot 4^2$
$= 4 \cdot 16$
$= 64$

(b) $4x^2 = 4 \cdot 6^2$
$= 4 \cdot 36$
$= 144$

14. **(a)** $5x^2 = 5 \cdot 4^2$
$= 5 \cdot 16$
$= 80$

(b) $5x^2 = 5 \cdot 6^2$
$= 5 \cdot 36$
$= 180$

15. **(a)** $\frac{x+1}{3} = \frac{4+1}{3}$
$= \frac{5}{3}$

(b) $\frac{x+1}{3} = \frac{6+1}{3}$
$= \frac{7}{3}$

16. **(a)** $\frac{x-2}{5} = \frac{4-2}{5}$
$= \frac{2}{5}$

(b) $\frac{x-2}{5} = \frac{6-2}{5}$
$= \frac{4}{5}$

17. **(a)** $\frac{3x-5}{2x} = \frac{3 \cdot 4 - 5}{2 \cdot 4}$
$= \frac{12-5}{8}$
$= \frac{7}{8}$

(b) $\frac{3x-5}{2x} = \frac{3 \cdot 6 - 5}{2 \cdot 6}$
$= \frac{18-5}{12}$
$= \frac{13}{12}$

18. **(a)** $\dfrac{4x-1}{3x}=\dfrac{4(4)-1}{3(4)}$
$=\dfrac{16-1}{12}$
$=\dfrac{15}{12}=\dfrac{5}{4}$

(b) $\dfrac{4x-1}{3x}=\dfrac{4(6)-1}{3(6)}$
$=\dfrac{24-1}{18}$
$=\dfrac{23}{18}$

19. **(a)** $3x^2+x=3\cdot 4^2+4$
$=3\cdot 16+4$
$=48+4=52$

(b) $3x^2+x=3\cdot 6^2+6$
$=3\cdot 36+6$
$=108+6=114$

20. **(a)** $2x+x^2=2(4)+4^2$
$=8+16$
$=24$

(b) $2x+x^2=2(6)+6^2$
$=12+36$
$=48$

21. **(a)** $6.459x=6.459\cdot 4$
$=25.836$

(b) $6.459x=6.459\cdot 6$
$=38.754$

22. **(a)** $3.275x=3.275\cdot 4$
$=13.1$

(b) $3.275x=3.275\cdot 6$
$=19.65$

23. **(a)** $8x+3y+5=8(2)+3(1)+5$
$=16+3+5$
$=19+5$
$=24$

(b) $8x+3y+5=8(1)+3(5)+5$
$=8+15+5$
$=23+5$
$=28$

24. **(a)** $4x+2y+7=4(2)+2(1)+7$
$=8+2+7$
$=17$

(b) $4x+2y+7=4(1)+2(5)+7$
$=4+10+7$
$=21$

25. **(a)** $3(x+2y)=3(2+2\cdot 1)$
$=3(2+2)$
$=3(4)$
$=12$

(b) $3(x+2y)=3(1+2\cdot 5)$
$=3(1+10)$
$=3(11)$
$=33$

26. **(a)** $2(2x+y)=2\left[2(2)+1\right]$
$=2(4+1)$
$=2(5)$
$=10$

(b) $2(2x+y)=2\left[2(1)+5\right]$
$=2(2+5)$
$=2(7)$
$=14$

27. **(a)** $x+\dfrac{4}{y}=2+\dfrac{4}{1}$
$=2+4$
$=6$

(b) $x+\dfrac{4}{y}=1+\dfrac{4}{5}$
$=\dfrac{5}{5}+\dfrac{4}{5}$
$=\dfrac{9}{5}$

28. **(a)** $y+\dfrac{8}{x}=1+\dfrac{8}{2}$
$=1+4$
$=5$

(b) $y+\dfrac{8}{x}=5+\dfrac{8}{1}$
$=5+8$
$=13$

29. **(a)** $\frac{x}{2}+\frac{y}{3}=\frac{2}{2}+\frac{1}{3}$
$=\frac{6}{6}+\frac{2}{6}$
$=\frac{8}{6}=\frac{4}{3}$

(b) $\frac{x}{2}+\frac{y}{3}=\frac{1}{2}+\frac{5}{3}$
$=\frac{3}{6}+\frac{10}{6}$
$=\frac{13}{6}$

30. **(a)** $\frac{x}{5}+\frac{y}{4}=\frac{2}{5}+\frac{1}{4}$
$=\frac{8}{20}+\frac{5}{20}$
$=\frac{13}{20}$

(b) $\frac{x}{5}+\frac{y}{4}=\frac{1}{5}+\frac{5}{4}$
$=\frac{4}{20}+\frac{25}{20}$
$=\frac{29}{20}$

31. **(a)** $\frac{2x+4y-6}{5y+2}=\frac{2(2)+4(1)-6}{5(1)+2}$
$=\frac{4+4-6}{5+2}$
$=\frac{8-6}{7}$
$=\frac{2}{7}$

(b) $\frac{2x+4y-6}{5y+2}=\frac{2(1)+4(5)-6}{5(5)+2}$
$=\frac{2+20-6}{25+2}$
$=\frac{22-6}{27}$
$=\frac{16}{27}$

32. **(a)** $\frac{4x+3y-1}{x}=\frac{4(2)+3(1)-1}{2}$
$=\frac{8+3-1}{2}$
$=\frac{10}{2}$
$=5$

(b) $\frac{4x+3y-1}{x}=\frac{4(1)+3(5)-1}{1}$
$=\frac{4+15-1}{1}$
$=\frac{18}{1}$
$=18$

33. **(a)** $2y^2+5x=2\cdot 1^2+5\cdot 2$
$=2\cdot 1+5\cdot 2$
$=2+10$
$=12$

(b) $2y^2+5x=2\cdot 5^2+5\cdot 1$
$=2\cdot 25+5\cdot 1$
$=50+5$
$=55$

34. **(a)** $6x^2+4y=6(2)^2+4(1)$
$=6(4)+4$
$=24+4$
$=28$

(b) $6x^2+4y=6(1)^2+4(5)$
$=6(1)+4(5)$
$=6+20$
$=26$

35. **(a)** $\frac{3x+y^2}{2x+3y}=\frac{3(2)+1^2}{2(2)+3(1)}$
$=\frac{3(2)+1}{4+3}$
$=\frac{6+1}{7}$
$=\frac{7}{7}$
$=1$

(b) $\dfrac{3x+y^2}{2x+3y}=\dfrac{3(1)+5^2}{2(1)+3(5)}$
$=\dfrac{3(1)+25}{2+15}$
$=\dfrac{3+25}{17}$
$=\dfrac{28}{17}$

36. (a) $\dfrac{x^2+1}{4x+5y}=\dfrac{2^2+1}{4(2)+5(1)}$
$=\dfrac{4+1}{8+5}$
$=\dfrac{5}{13}$

(b) $\dfrac{x^2+1}{4x+5y}=\dfrac{1^2+1}{4(1)+5(5)}$
$=\dfrac{1+1}{4+25}$
$=\dfrac{2}{29}$

37. (a) $0.841x^2+0.32y^2$
$=0.841\cdot 2^2+0.32\cdot 1^2$
$=0.841\cdot 4+0.32\cdot 1$
$=3.364+0.32$
$=3.684$

(b) $0.841x^2+0.32y^2$
$=0.841\cdot 1^2+0.32\cdot 5^2$
$=0.841\cdot 1+0.32\cdot 25$
$=0.841+8$
$=8.841$

38. (a) $0.941x^2+0.25y^2$
$=0.941(2)^2+0.25(1)^2$
$=0.941(4)+0.25(1)$
$=3.764+0.25$
$=4.014$

(b) $0.941x^2+0.25y^2$
$=0.941(1)^2+0.25(5)^2$
$=0.941(1)+0.25(25)$
$=0.941+6.25$
$=7.191$

39. "Twelve times a number" translates as $12\cdot x$, or $12x$.

40. "Fifteen times a number" translates as $15\cdot x$, or $15x$.

41. "Added to" indicates addition. "Nine added to a number" translates as $x+9$.

42. "Six added to a number" translates as $x+6$.

43. "Four subtracted from a number" translates as $x-4$.

44. "Seven subtracted from a number" translates as $x-7$.

45. "A number subtracted from seven" translates as $7-x$.

46. "A number subtracted from four" translates as $4-x$.

47. "The difference between a number and 8" translates as $x-8$.

48. "The difference between 8 and a number" translates as $8-x$.

49. "18 divided by a number" translates as $\dfrac{18}{x}$.

50. "A number divided by 18" translates as $\dfrac{x}{18}$.

51. "The product of 6 and four less than a number" translates as $6(x-4)$.

52. "The product of 9 and five more than a number" translates as $9(x+5)$.

53. $4m+2=6;1$

$4(1)+2\stackrel{?}{=}6$ Let $m=1$.

$4+2\stackrel{?}{=}6$

$6=6$ True

Because substituting 1 for m results in a true statement, 1 is a solution of the equation.

54. $2r+6=8;1$

$2(1)+6\stackrel{?}{=}8$ Let $r=1$.

$2+6\stackrel{?}{=}8$

$8=8$ True

The true result shows that 1 is a solution of the equation.

55. $2y+3(y-2)=14;\ 3$

$$
\begin{aligned}
2\cdot 3+3(3-2) &\overset{?}{=} 14 \qquad \text{Let } y=3.\\
2\cdot 3+3\cdot 1 &\overset{?}{=} 14\\
6+3 &\overset{?}{=} 14\\
9 &= 14 \qquad \text{False}
\end{aligned}
$$

Because substituting 3 for y results in a false statement, 3 is not a solution of the equation.

56. $6x+2(x+3)=14;\ 2$

$$
\begin{aligned}
6(2)+2(2+3) &\overset{?}{=} 14 \qquad \text{Let } x=2.\\
6(2)+2(5) &\overset{?}{=} 14\\
12+10 &\overset{?}{=} 14\\
22 &= 14 \qquad \text{False}
\end{aligned}
$$

The false result shows that 2 is not a solution of the equation.

57. $6p+4p+9=11;\ \frac{1}{5}$

$$
\begin{aligned}
6\left(\frac{1}{5}\right)+4\left(\frac{1}{5}\right)+9 &\overset{?}{=} 11 \quad \text{Let } p=\frac{1}{5}.\\
\frac{6}{5}+\frac{4}{5}+9 &\overset{?}{=} 11\\
\frac{10}{5}+9 &\overset{?}{=} 11\\
2+9 &\overset{?}{=} 11\\
11 &= 11 \quad \text{True}
\end{aligned}
$$

The true result shows that $\frac{1}{5}$ is a solution of the equation.

58. $2x+3x+8=20;\ \frac{12}{5}$

$$
\begin{aligned}
2\left(\frac{12}{5}\right)+3\left(\frac{12}{5}\right)+8 &\overset{?}{=} 20 \qquad \text{Let } x=\frac{12}{5}.\\
\frac{24}{5}+\frac{36}{5}+\frac{40}{5} &\overset{?}{=} 20\\
\frac{100}{5} &\overset{?}{=} 20\\
20 &= 20 \qquad \text{True}
\end{aligned}
$$

The true result shows that $\frac{12}{5}$ is a solution of the equation.

59. $3r^2-2=46;\ 4$

$$
\begin{aligned}
3(4)^2-2 &\overset{?}{=} 46 \qquad \text{Let } r=4.\\
3\cdot 16-2 &\overset{?}{=} 46\\
48-2 &\overset{?}{=} 46\\
46 &= 46 \qquad \text{True}
\end{aligned}
$$

The true result shows that 4 is a solution of the equation.

60. $2x^2+1=19;\ 3$

$$
\begin{aligned}
2(3)^2+1 &\overset{?}{=} 19 \qquad \text{Let } x=3.\\
2\cdot 9+1 &\overset{?}{=} 19\\
18+1 &\overset{?}{=} 19\\
19 &= 19 \qquad \text{True}
\end{aligned}
$$

The true result shows that 3 is a solution of the equation.

61. $\frac{3}{8}x+\frac{1}{4}=1;\ 2$

$$
\begin{aligned}
\frac{3}{8}(2)+\frac{1}{4} &\overset{?}{=} 1 \qquad \text{Let } x=2.\\
\frac{3}{4}+\frac{1}{4} &\overset{?}{=} 1\\
1 &= 1 \qquad \text{True}
\end{aligned}
$$

The true result shows that 2 is a solution of the equation.

62. $\frac{7}{10}x+\frac{1}{2}=4;\ 5$

$$
\begin{aligned}
\frac{7}{10}(5)+\frac{1}{2} &\overset{?}{=} 4 \qquad \text{Let } x=5.\\
\frac{7}{2}+\frac{1}{2} &\overset{?}{=} 4\\
4 &= 4 \qquad \text{True}
\end{aligned}
$$

The true result shows that 5 is a solution of the equation.

63. $0.5(x-4)=80;\ 20$

$$
\begin{aligned}
0.5(20-4) &\overset{?}{=} 80 \qquad \text{Let } x=20.\\
0.5(16) &\overset{?}{=} 80\\
8 &= 80 \qquad \text{False}
\end{aligned}
$$

The false result shows that 20 is not a solution of the equation.

64. $0.2(x-5)=70;\ 40$

$$0.2(40-5) \stackrel{?}{=} 70 \quad \text{Let } x=40.$$
$$0.2(35) \stackrel{?}{=} 70$$
$$7=70 \quad \text{False}$$

The false result shows that 40 is not a solution of the equation.

65. "The sum of a number and 8 is 18" translates as $x+8=18$. Try each number from the given set, $\{2, 4, 6, 8, 10\}$, in turn.

$x+8=18$ Given equation

$2+8=18$ False

$4+8=18$ False

$6+8=18$ False

$8+8=18$ False

$10+8=18$ True

The only solution is 10.

66. "A number minus three equals 1" translates as $x-3=1$. Replace x with each number in the given set. The only true statement results when $x=4$, since $4-3=1$. Thus, 4 is the only solution.

67. "One more than twice a number is 5" translates as $2x+1=5$. Try each number from the given set. The only resulting true equation is $2\cdot 2+1=5$, so the only solution is 2.

68. "The product of a number and 3 is 6" translates as $3x=6$. The only true statement results when $x=2$, since, $3\cdot 2=6$. Thus, 2 is the only solution.

69. "Sixteen minus three-fourths of a number is 13" translates as $16-\frac{3}{4}x=13$. Try each number from the given set, $\{2, 4, 6, 8, 10\}$, in turn.

$16-\frac{3}{4}x=13$ Given equation

$16-\frac{3}{4}(2)=13$ False

$16-\frac{3}{4}(4)=13$ True

$16-\frac{3}{4}(6)=13$ False

$16-\frac{3}{4}(8)=13$ False

$16-\frac{3}{4}(10)=13$ False

The only solution is 4.

70. "The sum of six-fifths of a number and 2 is 14" translates as $\frac{6}{5}x+2=14$. Replace x with each number in the given set. The only true statement results as follows.

$$\frac{6}{5}(10)+2 \stackrel{?}{=} 14 \quad \text{Let } x=10.$$
$$12+2 \stackrel{?}{=} 14$$
$$14=14 \quad \text{True}$$

The only solution is 10.

71. "Three times a number is equal to 8 more than twice the number" translates as $3x=2x+8$. Try each number from the given set.

$3x=2x+8$ Given equation

$3(2)=2(2)+8$ False

$3(4)=2(4)+8$ False

$3(6)=2(6)+8$ False

$3(8)=2(8)+8$ True

$3(10)=2(10)+8$ False

The only solution is 8.

72. "Twelve divided by a number equals $\frac{1}{3}$ times that number" translates as $\frac{12}{x}=\frac{1}{3}x$. The only true statement results as follows.

$$\frac{12}{6} \stackrel{?}{=} \frac{1}{3}(6) \quad \text{Let } x=6.$$
$$2=2 \quad \text{True}$$

The only solution is 6.

73. There is no equals symbol, so $3x+2(x-4)$ is an expression.

74. There is no equals symbol, so $8y-(3y+5)$ is an expression.

75. There is an equals symbol, so $7t+2(t+1)=4$ is an equation.

76. There is an equals symbol, so $9r+3(r-4)=2$ is an equation.

77. There is an equals symbol, so $x+y=9$ is an equation.

78. There is no equals symbol, so $x+y-9$ is an expression.

79. $y = 0.180x - 283$
$= 0.180(1960) - 283$
$= 69.8$
The life expectancy of an American born in 1960 is about 70 years.

80. $y = 0.180x - 283$
$= 0.180(1975) - 283$
$= 72.5$
The life expectancy of an American born in 1975 is about 73 years.

81. $y = 0.180x - 283$
$= 0.180(1995) - 283$
$= 76.1$
The life expectancy of an American born in 1995 is about 76 years.

82. $y = 0.180x - 283$
$= 0.180(2010) - 283$
$= 78.8$
The life expectancy of an American born in 2010 is about 79 years.

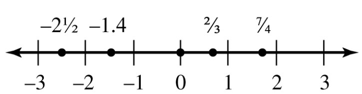
1.3 Real Numbers and the Number Line

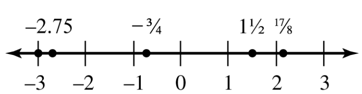
Classroom Examples, Now Try Exercises

1. **(a)** Since Erin spends $\$53$ more than she has in her checking account, her balance is -53.

(b) Since the record high was $134°$ above zero, this temperature is expressed as $134°$.

N1. Since the deepest point is below the water's surface, the depth is -136.

2.


N2.


3. In the set $\left\{\frac{5}{8}, -7, -1\frac{3}{5}, 0, 0.\overline{45}, \sqrt{11}, -\pi\right\}$, $\frac{5}{8}, -7\left(\text{or } \frac{-7}{1}\right), -1\frac{3}{5}\left(\text{or } -\frac{8}{5}\right), 0\left(\text{or } \frac{0}{1}\right)$, and $0.\overline{45}\left(\text{or } \frac{5}{11}\right)$ are rational (since each of these numbers can be written as the quotient of two integers); $\sqrt{11}$ and $-\pi$ are irrational.

N3. **(a)** The whole numbers are 0 and 13.

(b) The integers are -7, 0, and 13.

(c) The rational numbers are $-7, -\frac{4}{5}, 0, 2.7,$ and 13.

(d) The irrational numbers are $\sqrt{3}$ and π.

4. Since -4 lies to the left of -1 on the number line, -4 is less than -1. Therefore, the statement $-4 \geq -1$ is *false*.

N4. Since -8 lies to the right of -9 on the number line, -8 is greater than -9. Therefore, the statement $-8 \leq -9$ is *false*.

5. **(a)** $|32| = 32$

(b) $|-32| = -(-32) = 32$

(c) $-|-32| = -[-(-32)] = -32$

(d) $-|32 - 2| = -|30| = -30$

N5. **(a)** $|4| = 4$

(b) $|-4| = -(-4) = 4$

(c) $-|-4| = -[-(-4)] = -4$

6. The largest positive percent increase is 26.4, so the category is gasoline and the year is 2010 to 2011.

N6. The category appliances is negative in both years.

Exercises

1. The number <u>0</u> is a whole number, but not a natural number.

2. The natural numbers, their additive inverses, and 0 form the set of <u>integers</u>.

3. The additive inverse of every negative number is a *positive* number.

4. If x and y are real numbers with $x > y$, then x lies to the *right* of y on a number line.

5. A rational number is the <u>quotient</u> of two integers with the <u>denominator</u> not equal to 0.

6. Decimal numbers that neither terminate nor repeat are <u>irrational</u> numbers.

7. **(a)** $|-9| = 9$ A
The distance between -9 and 0 on the number line is 9 units.

(b) $-(-9) = 9$ A
The opposite of -9 is 9.

(c) $-|-9| = -(9) = -9$ B

(d) $$\begin{aligned} -|-(-9)| &= -|9| \\ &= -(9) \\ &= -9 \quad \text{(B)} \end{aligned}$$

8. The opposite of -5 is $\underline{5}$, while the absolute value of -5 is $\underline{5}$. The additive inverse of -5 is $\underline{5}$, while the additive inverse of the absolute value of -5 is $\underline{-5}$.

9. The statement "Absolute value is always positive" is not true. The absolute value of 0 is 0, and 0 is not positive. We could say that absolute value is never negative, or absolute value is always nonnegative.

10. If a is negative, $|a| = -a$. This statement is true since both $|a|$ and $-a$ represent the same positive number.

11. The only integer between 3.6 and 4.6 is 4.

12. A rational number between 2.8 and 2.9 is 2.85. There are others.

13. There is only one whole number that is not positive and that is less than 1: the number 0.

14. A whole number greater than 3.5 is 4. There are others.

15. An irrational number that is between $\sqrt{12}$ and $\sqrt{14}$ is $\sqrt{13}$. There are others.

16. The only real number that is neither negative nor positive is 0.

17. True; every natural number is positive.

18. False; 0 is a whole number that is not positive. In fact, it is the *only* whole number that is not positive.

19. True; every integer is a rational number. For example, 5 can be written as $\frac{5}{1}$.

20. True; every rational number is a real number.

21. False; if a number is rational, it cannot be irrational, and vice versa.

22. True; every terminating decimal is a rational number.

23. Three examples of positive real numbers that are not integers are $\frac{1}{2}, \frac{5}{8}$, and $1\frac{3}{4}$. Other examples are $0.7, 4\frac{2}{3}$, and 5.1.

24. Real numbers that are not positive numbers are 0 and all numbers to the left of 0 on the number line. Three examples are $-1, -\frac{3}{4}$, and -5. Other examples are $0, -5, -\sqrt{7}, -1\frac{1}{2}$, and -0.3.

25. Three examples of real numbers that are not whole numbers are $-3\frac{1}{2}, -\frac{2}{3}$, and $\frac{3}{7}$. Other examples are $-4.3, -\sqrt{2}$, and $\sqrt{7}$.

26. Rational numbers that are not integers are all real numbers that can be expressed as a quotient of integers (with nonzero denominators) such that in lowest terms the denominator is not 1. Three examples are $\frac{1}{2}, -\frac{2}{3}$, and $\frac{2}{7}$. Other examples are $-5.6, -4\frac{3}{4}, -\frac{1}{2}, \frac{1}{2}$, and 5.2.

27. Three examples of real numbers that are not rational numbers are $\sqrt{5}$, π, and $-\sqrt{3}$. All irrational numbers are real numbers that are not rational.

28. Rational numbers that are not negative numbers are 0 and all rational numbers to the right of zero on the number line. Three examples are $\frac{2}{3}, \frac{5}{6}$, and $\frac{5}{2}$. Other examples are $0, \frac{1}{2}, 1, 3\frac{1}{4}$, and 5.

29. Use the integer 2,259,105 since "increased by 2,259,105" indicates a positive number.

30. Use the integer 207 since "increased by 207" indicates a positive number.

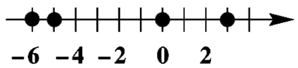
31. Use the integer -3424 since "a decrease of 3424" indicates a negative number.

32. Use the integer -8212 since "a decrease of 8212" indicates a negative number.

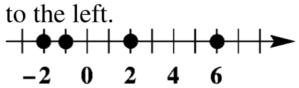
33. Use the rational number 46.77 since "closed up 46.77" indicates a positive number.

34. Use the rational number -79.05 since "closed down 79.05" indicates a negative number.

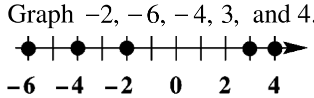
35. Graph $0, 3, -5,$ and -6.
Place a dot on the number line at the point that corresponds to each number. The order of the numbers from smallest to largest is
$-6, -5, 0, 3.$



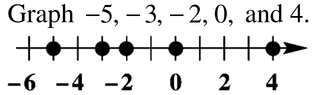
36. Graph $2, 6, -2,$ and -1.
The smallest number, -2, will be the farthest to the left.



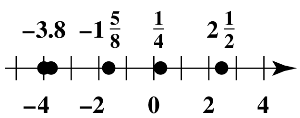
37. Graph $-2, -6, -4, 3,$ and 4.



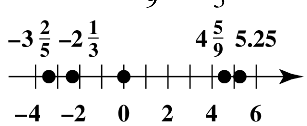
38. Graph $-5, -3, -2, 0,$ and 4.



39. Graph $\frac{1}{4}, 2\frac{1}{2}, -3.8, -4,$ and $-1\frac{5}{8}$.



40. Graph $5.25, 4\frac{5}{9}, -2\frac{1}{3}, 0,$ and $-3\frac{2}{5}$.



41. **(a)** The natural numbers in the given set are 3 and 7, since they are in the natural number set $\{1, 2, 3, \ldots\}$.

(b) The set of whole numbers includes the natural numbers and 0. The whole numbers in the given set are 0, 3, and 7.

(c) The integers are the set of numbers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. The integers in the given set are -9, 0, 3, and 7.

(d) Rational numbers are the numbers that can be expressed as the quotient of two integers, with denominators not equal to 0.
We can write numbers from the given set in this form as follows:

$$-9 = \frac{-9}{1},\ -1\frac{1}{4} = \frac{-5}{4},\ -\frac{3}{5} = \frac{-3}{5},\ 0 = \frac{0}{1},$$
$$0.\overline{1} = \frac{1}{9},\ 3 = \frac{3}{1},\ 5.9 = \frac{59}{10},\ \text{and } 7 = \frac{7}{1}.$$

Thus, the rational numbers in the given set are $-9, -1\frac{1}{4}, -\frac{3}{5}, 0, 0.\overline{1}, 3, 5.9,$ and 7.

(e) Irrational numbers are real numbers that are not rational. $-\sqrt{7}$ and $\sqrt{5}$ can be represented by points on the number line but cannot be written as a quotient of integers. Thus, the irrational numbers in the given set are $-\sqrt{7}$ and $\sqrt{5}$.

(f) Real numbers are all numbers that can be represented on the number line. All the numbers in the given set are real.

42. **(a)** The only natural number in the given set is 3.

(b) The whole numbers in the set are 0 and 3.

(c) The integers in the set are $-5, -1, 0,$ and 3.

(d) The rational numbers are $-5.3, -5, -1, -\frac{1}{9}, 0, 0.\overline{27}, 1.2,$ and 3.

(e) The irrational numbers in the set are $-\sqrt{3}$ and $\sqrt{11}$.

(f) All the numbers in the set are real numbers.

43. **(a)** The additive inverse of -7 is found by changing the sign of -7. The additive inverse of -7 is 7.

(b) The absolute value of -7 is the distance between 0 and -7 on the number line, so $|-7| = 7$.

44. **(a)** The additive inverse of a number is found by changing the sign of a number, so the additive inverse of -4 is 4.

(b) The distance between -4 and 0 on the number line is 4 units, so $|-4| = 4$.

45. **(a)** The additive inverse of 8 is -8.

(**b**) The distance between 0 and 8 on the number line is 8 units, so the absolute value of 8 is 8.

46. (**a**) The additive inverse of 10 is -10.

(**b**) The distance between 10 and 0 on the number line is 10 units, so $|10| = 10$.

47. (**a**) The additive inverse of a number is found by changing the sign of a number, so the additive inverse of $-\frac{3}{4}$ is $\frac{3}{4}$.

(**b**) The distance between $-\frac{3}{4}$ and 0 on the number line is $\frac{3}{4}$ unit, so $\left|-\frac{3}{4}\right| = \frac{3}{4}$.

48. (**a**) The additive inverse of a number is found by changing the sign of a number, so the additive inverse of $-\frac{2}{5}$ is $\frac{2}{5}$.

(**b**) The distance between $-\frac{2}{5}$ and 0 on the number line is $\frac{2}{5}$ unit, so $\left|-\frac{2}{5}\right| = \frac{2}{5}$.

49. (**a**) The additive inverse of a number is found by changing the sign of a number, so the additive inverse of 5.6 is -5.6.

(**b**) The distance between 5.6 and 0 on the number line is 5.6 units, so $|5.6| = 5.6$.

50. (**a**) The additive inverse of a number is found by changing the sign of a number, so the additive inverse of 8.1 is -8.1.

(**b**) The distance between 8.1 and 0 on the number line is 8.1 unit, so $|8.1| = 8.1$.

51. Since -6 is a negative number, its absolute value is the additive inverse of -6 — that is, $|-6| = -(-6) = 6$.

52. $|-14| = -(-14) = 14$

53. $-|12| = -(12) = -12$

54. $-|19| = -(19) = -19$

55. $-\left|-\frac{2}{3}\right| = -\left[-\left(-\frac{2}{3}\right)\right] = -\left[\frac{2}{3}\right] = -\frac{2}{3}$

56. $-\left|-\frac{4}{5}\right| = -\left[-\left(-\frac{4}{5}\right)\right] = -\left[\frac{4}{5}\right] = -\frac{4}{5}$

57. $|6-3| = |3| = 3$

58. $|9-4| = |5| = 5$

59. $-|6-3| = -|3| = -3$

60. $-|9-4| = -|5| = -5$

61. Since -11 is located to the left of -3 on the number line, -11 is the lesser number.

62. Since -13 is located to the left of -8 on the number line, -13 is the lesser number.

63. Since -7 is located to the left of -6 on the number line, -7 is the lesser number.

64. Since -17 is located to the left of -16 on the number line, -17 is the lesser number.

65. Since $|-5| = 5$, 4 is the lesser of the two numbers.

66. Since $|-3| = 3$, $|-3|$ or 3 is the lesser of the two numbers.

67. Since $|-3.5| = 3.5$ and $|-4.5| = 4.5$, $|-3.5|$ or 3.5 is the lesser number.

68. Since $|-8.9| = 8.9$ and $|-9.8| = 9.8$, $|-8.9|$ or 8.9 is the lesser number.

69. Since $-|-6| = -6$ and $-|-4| = -4$, $-|-6|$ is to the left of $-|-4|$ on the number line, so $-|-6|$ or -6 is the lesser number.

70. $-|-2| = -2$ and $-|-3| = -3$, so $-|-3|$ is to the left of $-|-2|$ on the number line; $-|-3|$ or -3 is the lesser number.

71. Since $|5-3| = |2| = 2$ and $|6-2| = |4| = 4$, $|5-3|$ or 2 is the lesser number.

72. Since $|7-2| = |5| = 5$ and $|8-1| = |7| = 7$, $|7-2|$ or 5 is the lesser number.

73. Since -5 is to the *left* of -2 on the number line, -5 is *less than* -2, and the statement $-5 < -2$ is true.

74. Since -8 is to the *left* of -2 on the number line, -8 is *less than* -2, and the statement $-8 > -2$ is false.

75. Since $-(-5) = 5$ and $-4 < 5, -4 \le -(-5)$ is true.

76. Since $-(-3) = 3$ and $-6 \le 3$, $-6 \le -(-3)$ is true.

77. Since $|-6| = 6$ and $|-9| = 9$, and $6 < 9$, $|-6| < |-9|$ is true.

78. Since $|-12| = 12$ and $|-20| = 20$, and $12 < 20$, $|-12| < |-20|$ is true.

79. Since $-|8| = -8$ and $|-9| = -(-9) = 9$, $-|8| < |-9|$, so $-|8| > |-9|$ is false.

80. Since $-|12| = -12$ and $|-15| = -(-15) = 15$, $-|12| < |-15|$, so $-|12| > |-15|$ is false.

81. Since $-|-5| = -5, -|-9| = -9$, and $-5 > -9, -|-5| \ge -|-9|$ is true.

82. Since $-|-12| = -12, -|-15| = -15$, and $-12 > -15, -|-12| \le -|-15|$ is false.

83. Since $|6-5| = |1| = 1$ and $|6-2| = |4| = 4, |6-5| < |6-2|$, so $|6-5| \ge |6-2|$ is false.

84. Since $|13-8| = |5| = 5$ and $|7-4| = |3| = 3$, $|13-8| > |7-4|$, so $|13-8| \le |7-4|$ is false.

85. The number that represents the greatest percentage increase is 7.2, which corresponds to public transportation from 2010 to 2011.

86. The negative number with the largest absolute value in the table is -1.6, so the greatest percentage decrease is communication from 2010 to 2011.

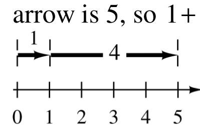
87. The number with the smallest absolute value in the table is -0.3, so the least change corresponds to communication from 2009, to 2010.

88. The industry with two negative entries (representing a decrease for both years) is communication.

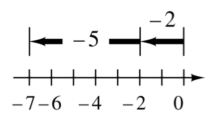
1.4 Adding and Subtracting Real Numbers

Classroom Examples, Now Try Exercises

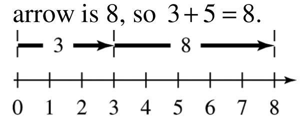
1. **(a)** Start at 0 on a number line. Draw an arrow 1 unit to the right to represent the positive number 1. From the right end of this arrow, draw a second arrow 4 units to the right to represent the addition of a positive number. The number below the end of this second arrow is 5, so $1 + 4 = 5$.



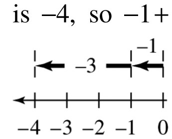
(b) Start at 0 on a number line. Draw an arrow 2 units to the left to represent the negative number -2. From the left end of this arrow, draw a second arrow 5 units to the left to represent the addition of a negative number. The number below the end of this second arrow is -7, so $-2 + (-5) = -7$.



N1. **(a)** Start at 0 on a number line. Draw an arrow 3 units to the right to represent the positive number 3. From the right end of this arrow, draw a second arrow 5 units to the right to represent the addition of a positive number. The number below the end of this second arrow is 8, so $3 + 5 = 8$.



(b) Start at 0 on a number line. Draw an arrow 1 unit to the left to represent the negative number -1. From the left end of this arrow, draw a second arrow 3 units to the left. The number below the end of this second arrow is -4, so $-1 + (-3) = -4$.



2. **(a)** $-15 + (-4) = -19$

The sum of two negative numbers is negative.

(b) $-1.27 + (-5.46) = -6.73$

The sum of two negative numbers is negative.

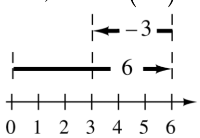
N2. **(a)** $-6+(-11)=-17$

The sum of two negative numbers is negative.

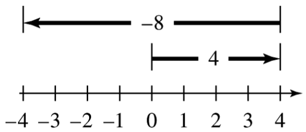
(b) $-\frac{2}{5}+\left(-\frac{1}{2}\right)=-\frac{9}{10}$

The sum of two negative numbers is negative.

3. Start at 0 on a number line. Draw an arrow 6 units to the right. From the right end of this arrow, draw a second arrow 3 units to the left. The number below the end of this second arrow is 3, so $6+(-3)=3$.



N3. Start at 0 on a number line. Draw an arrow 4 units to the right. From the right end of this arrow, draw a second arrow 8 units to the left. The number below the end of this second arrow is -4, so $4+(-8)=-4$.



4. **(a)** Since the numbers have different signs, find the difference between their absolute values: $17-10=7$. Because 17 has the larger absolute value, the sum is negative: $-10+17=7$.

(b) $\frac{3}{4}+\left(-1\frac{3}{8}\right)=-\frac{5}{8}$

(c) $-3.8+9.5=5.7$

(d) $25+(-25)=0$

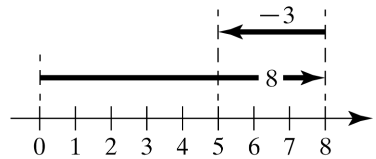
N4. **(a)** Since the numbers have different signs, find the difference between their absolute values: $7-4=3$. Because 7 has the larger absolute value, the sum is negative: $7+(-4)=-3$.

(b) $\frac{2}{3}+\left(-2\frac{1}{9}\right)=-\frac{13}{9}$ or $-1\frac{4}{9}$

(c) $-5.7+3.7=-2$

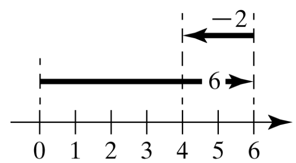
(d) $-10+10=0$

5. Use a number line to find the difference $8-3$.
Step 1 Start at 0 and draw an arrow 8 units to the *right.*
Step 2 From the right end of the first arrow, draw a second arrow 3 units to the *left* to represent the *subtraction*.



The number below the end of the second arrow is 5, so $8-3=5$.

N5. Use a number line to find the difference $6-2$.
Step 1 Start at 0 and draw an arrow 6 units to the *right.*
Step 2 From the right end of the first arrow, draw a second arrow 2 units to the *left* to represent the *subtraction*.



The number below the end of the second arrow is 4, so $6-2=4$.

6. **(a)** $-8-5=-8+(-5)$ Add the opposite.
$=-13$

(b) $-8-(-12)=-8+(12)$ Add the opposite.
$=4$

(c) $\frac{5}{4}-\left(-\frac{3}{7}\right)=\frac{5}{4}+\frac{3}{7}=\frac{35}{28}+\frac{12}{28}=\frac{47}{28}$, or $1\frac{19}{28}$

(d) $7.5-9.2=-1.7$

N6. **(a)** $-5-(-11)=-5+(11)$ Add the opposite.
$=6$

(b) $4-15=4+(-15)$ Add the opposite.
$=-11$

(c) $-\frac{5}{7}-\frac{1}{3}=-\frac{5}{7}+\left(-\frac{1}{3}\right)$ Add the opposite.
$=-\frac{15}{21}+\left(-\frac{7}{21}\right)$
$=-\frac{22}{21}$, or $-1\frac{1}{21}$

(d) $5.25-(-3.24)=8.49$

7. **(a)** $6+\left[(-1-4)-2\right]$
$=6+\left\{\left[-1+(-4)\right]-2\right\}$
$=6+(-5-2)$
$=6+\left[-5+(-2)\right]$
$=6+(-7)$
$=-1$

(b) $\left|-\frac{1}{6}-\left(-\frac{1}{3}\right)\right|-\frac{1}{4}$
$=\left|-\frac{2}{12}-\left(-\frac{4}{12}\right)\right|-\frac{3}{12}$
$=\left|-\frac{2}{12}+\frac{4}{12}\right|-\frac{3}{12}$
$=\left|\frac{2}{12}\right|-\frac{3}{12}$
$=\frac{2}{12}+\left(-\frac{3}{12}\right)$
$=-\frac{1}{12}$

N7. **(a)** $8-[(-3+7)-(3-9)]$
$=8-[(4)-(3+(-9))]$
$=8-[4-(-6)]$
$=8-[4+6]$
$=8-10$
$=8+(-10)$
$=-2$

(b) $3|6-9|-|4-12|$
$=3|6+(-9)|-|4+(-12)|$
$=3|-3|-|-8|$
$=3(3)-8$
$=9-8$
$=1$

8. "7 is increased by the sum of 8 and -3" is written $7+\left[8+(-3)\right]$.
$7+\left[8+(-3)\right]=7+5=12$

N8. "The sum of -3 and 7, increased by 10" is written $(-3+7)+10$.
$(-3+7)+10=4+10=14$

9. **(a)** "The difference between -5 and -12" is written $-5-(-12)$.
$-5-(-12)=-5+12$
$=7$

(b) "-2 subtracted from the sum of 4 and -4" is written $\left[4+(-4)\right]-(-2)$.
$\left[4+(-4)\right]-(-2)=0-(-2)$
$=0+2$
$=2$

N9. **(a)** "The difference between 5 and -8, decreased by 4" is written $[5-(-8)]-4$.
$[5-(-8)]-4=[5+8]-4$
$=13-4$
$=9$

(b) "7 less than -2" is written $-2-7$.
$-2-7=-2+(-7)$
$=-9$

10. The difference between the highest and lowest temperatures is given by
$79-(-56)=79+56$
$=135.$
The difference is $135°$F.

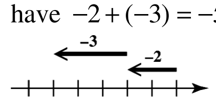
N10. The difference between a gain of 226 yards and a loss of 7 yards is given by
$226-(-7)=226+7$
$=233.$
The difference is 233 yards.

11. Subtract the enrollment number for 1995 from the enrollment number for 2000.
$13.52-12.5=13.52+(-12.5)=1.02$ million
A positive result indicates an increase.

N11. Subtract the enrollment number for 1985 from the enrollment number for 1990.
$11.34-12.39=-1.05$ million
A negative result indicates a decrease.

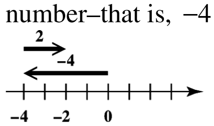
Exercises

1. The sum of two negative numbers will always be a *negative* number. In the illustration, we have $-2+(-3)=-5$.



2. The sum of a number and its opposite will always be zero (0).

3. When adding a positive number and a negative number, where the negative number has the greater absolute value, the sum will be a *negative* number. In the illustration, the absolute value of -4 is larger than the absolute value of 2, so the sum is a negative number–that is, $-4+2=-2$.



4. To simplify the expression $8+\left[-2+(-3+5)\right]$, one should begin by adding $\underline{-3}$ and $\underline{5}$, according to the rules for order of operations.

5. By the definition of subtraction, in order to perform the subtraction $-6-(-8)$, we must add the opposite of $\underline{-8}$ to $\underline{-6}$ to obtain $\underline{2}$.

6. "The difference of 7 and 12" translates as $\underline{7-12}$, while "the difference of 12 and 7" translates as $\underline{12-7}$.

7. The expression $x-y$ would have to be positive since subtracting a negative number from a positive number is the same as adding a positive number to a positive number, which is a positive number.

8. $y-x=y+(-x)$
If x is a positive number and y is a negative number, $y-x$ will be the sum of two negative numbers, which is a negative number.

9. $|x|=x$, since x is a positive number.
$y-|x|=y-x$, which is a negative number. (See Exercise 8.)

10. Since $|y|$ is positive, $x+|y|$ is the sum of two positive numbers, which is positive.

11. The sum of two negative numbers is negative.
$-6+(-2)=-8$

12. Since the numbers have the same sign, add their absolute values: $9+2=11$. Since both numbers are negative, their sum is negative: $-9+(-2)=-11$.

13. Because the numbers have the same sign, add their absolute values: $5+7=12$. Because both numbers are negative, their sum is negative: $-5+(-7)=-12$.

14. Because the numbers have the same sign, add their absolute values: $11+5=16$. Because both numbers are negative, their sum is negative: $-11+(-5)=-16$.

15. To add $6+(-4)$, find the difference between the absolute values of the numbers.
$|6|=6$ and $|-4|=4$
$6-4=2$
Since $|6|>|-4|$, the sum will be positive: $6+(-4)=2$.

16. Since the numbers have different signs, find the difference between their absolute values: $11-8=3$. Since 11 has the larger absolute value, the answer is positive: $11+(-8)=3$.

17. Since the numbers have different signs, find the difference between their absolute values: $6-4=2$. Because -6 has the larger absolute value, the sum is negative: $4+(-6)=-2$.

18. Since the numbers have different signs, find the difference between their absolute values: $7-3=4$. Since -7 has the larger absolute value, the sum is negative: $3+(-7)=-4$.

19. Since the numbers have different signs, find the difference between their absolute values: $16-7=9$. Since -16 has the larger absolute value, the answer is negative: $-16+7=-9$.

20. Since the numbers have different signs, find the difference between their absolute values: $13-6=7$. Since -13 has the larger absolute value, the answer is negative: $-13+6=-7$.

21. $6+(-6)=0$

22. $-11+11=0$

23. $-\frac{1}{3}+\left(-\frac{4}{15}\right)=-\frac{5}{15}+\left(-\frac{4}{15}\right)=-\frac{9}{15}=-\frac{3}{5}$

24. $-\frac{1}{4}+\left(-\frac{5}{12}\right)=-\frac{3}{12}+\left(-\frac{5}{12}\right)=-\frac{8}{12}=-\frac{2}{3}$

25. $-\frac{1}{6}+\frac{2}{3}=-\frac{1}{6}+\frac{4}{6}=\frac{3}{6}=\frac{1}{2}$

26. $-\frac{6}{25}+\frac{19}{20}=-\frac{6\cdot 4}{25\cdot 4}+\frac{19\cdot 5}{20\cdot 5}$
$=-\frac{24}{100}+\frac{95}{100}$
$=\frac{71}{100}$

27. Since $8=2\cdot 2\cdot 2$ and $12=2\cdot 2\cdot 3,$ the LCD is $2\cdot 2\cdot 2\cdot 3=24.$
$\frac{5}{8}+\left(-\frac{17}{12}\right)=\frac{5\cdot 3}{8\cdot 3}+\left(-\frac{17\cdot 2}{12\cdot 2}\right)$
$=\frac{15}{24}+\left(-\frac{34}{24}\right)$
$=-\frac{19}{24}$

28. $\frac{9}{10}+\left(-\frac{11}{8}\right)=\frac{9\cdot 4}{10\cdot 4}+\left(-\frac{11\cdot 5}{8\cdot 5}\right)$
$=\frac{36}{40}+\left(-\frac{55}{40}\right)$
$=-\frac{19}{40}$

29. $2\frac{1}{2}+\left(-3\frac{1}{4}\right)=\frac{5}{2}+\left(-\frac{13}{4}\right)$
$=\frac{10}{4}+\left(-\frac{13}{4}\right)$
$=-\frac{3}{4}$

30. $1\frac{3}{8}+\left(-2\frac{1}{4}\right)=\frac{11}{8}+\left(-\frac{9}{4}\right)$
$=\frac{11}{8}+\left(-\frac{18}{8}\right)$
$=-\frac{7}{8}$

31. $-3.5+12.4=+(12.4-3.5)=8.9$

32. $-12.5+21.3=+(21.3-12.5)=8.8$

33. $-2.34+(-3.67)=-(2.34+3.67)=-6.01$

34. $-1.25+(-6.88)=-(6.88+1.25)=-8.13$

35. $4+[13+(-5)]=4+[8]=12$

36. $6+[12+(-3)]=6+[9]=15$

37. $8+[-2+(-1)]=8+[-3]=5$

38. $12+[-3+(-4)]=12+[-7]=5$

39. $-2+[5+(-1)]=-2+[4]=2$

40. $-8+[9+(-2)]=-8+[7]=-1$

41. $-6+[6+(-9)]=-6+[-3]=-9$

42. $-3+[3+(-8)]=-3+[-5]=-8$

43. $[(-9)+(-3)]+12=[-12]+12=0$

44. $[(-8)+(-6)]+14=[-14]+14=0$

45. $-6.1+[3.2+(-4.8)]=-6.1+[-1.6]$
$=-7.7$

46. $-9.4+[5.8+(-7.9)]=-9.4+[-2.1]$
$=-11.5$

47. $[-3+(-4)]+[5+(-6)]=[-7]+[-1]$
$=-8$

48. $[-8+(-3)]+[4+(-6)]=[-11]+[-2]$
$=-13$

49. $[-4+(-3)]+[8+(-1)]=[-7]+[7]$
$=0$

50. $[-5+(-9)]+[16+(-2)]=[-14]+[14]$
$=0$

51. $[-4+(-6)]+[-3+(-8)]+[12+(-11)]$
$=([-10]+[-11])+[1]$
$=(-21)+1$
$=-20$

52. $[-2+(-11)]+[-12+(-2)]+[18+(-6)]$
$=([-13]+[-14])+[12]$
$=(-27)+12$
$=-15$

53. $4-7=4+(-7)=-3$

54. $8-13=8+(-13)=-5$

55. $5-9=5+(-9)=-4$

56. $6-11=6+(-11)=-5$

57. $-7-1=-7+(-1)=-8$

58. $-9-4=-9+(-4)=-13$

59. $-8-6=-8+(-6)=-14$

60. $-9-5=-9+(-5)=-14$

61. $7-(-2)=7+(2)=9$

62. $9-(-2)=9+(2)=11$

63. $-6-(-2)=-6+(2)=-4$

64. $-7-(-5)=-7+(5)=-2$

65. $$\begin{aligned}2-(3-5)&=2-[3+(-5)]\\&=2-[-2]\\&=2+(2)\\&=4\end{aligned}$$

66. $$\begin{aligned}-3-(4-11)&=-3-[4+(-11)]\\&=-3-[-7]\\&=-3+(7)\\&=4\end{aligned}$$

67. $$\begin{aligned}\frac{1}{2}-\left(-\frac{1}{4}\right)&=\frac{1}{2}+\frac{1}{4}\\&=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}\end{aligned}$$

68. $\frac{1}{3}-\left(-\frac{1}{12}\right)=\frac{4}{12}+\frac{1}{12}=\frac{5}{12}$

69. $$\begin{aligned}-\frac{3}{4}-\frac{5}{8}&=-\frac{3}{4}+\left(-\frac{5}{8}\right)\\&=-\frac{6}{8}+\left(-\frac{5}{8}\right)\\&=-\frac{11}{8},\quad\text{or}\quad -1\frac{3}{8}\end{aligned}$$

70. $$\begin{aligned}-\frac{5}{6}-\frac{1}{2}&=-\frac{5}{6}+\left(-\frac{1}{2}\right)\\&=-\frac{5}{6}+\left(-\frac{3}{6}\right)\\&=-\frac{8}{6}\\&=-\frac{4}{3},\quad\text{or}\quad -1\frac{1}{3}\end{aligned}$$

71. $$\begin{aligned}\frac{5}{8}-\left(-\frac{1}{2}-\frac{3}{4}\right)&=\frac{5}{8}-\left[-\frac{1}{2}+\left(-\frac{3}{4}\right)\right]\\&=\frac{5}{8}-\left[-\frac{2}{4}+\left(-\frac{3}{4}\right)\right]\\&=\frac{5}{8}-\left(-\frac{5}{4}\right)\\&=\frac{5}{8}+\frac{5}{4}\\&=\frac{5}{8}+\frac{10}{8}\\&=\frac{15}{8},\quad\text{or}\quad 1\frac{7}{8}\end{aligned}$$

72. $$\begin{aligned}&\frac{9}{10}-\left(\frac{1}{8}-\frac{3}{10}\right)\\&=\frac{9}{10}-\left[\frac{1}{8}+\left(-\frac{3}{10}\right)\right]\\&=\frac{9}{10}-\left[\frac{5}{40}+\left(-\frac{12}{40}\right)\right]\\&=\frac{9}{10}-\left(-\frac{7}{40}\right)\\&=\frac{9}{10}+\frac{7}{40}\\&=\frac{36}{40}+\frac{7}{40}\\&=\frac{43}{40},\quad\text{or}\quad 1\frac{3}{40}\end{aligned}$$

73. $$\begin{aligned}3.4-(-8.2)&=3.4+8.2\\&=11.6\end{aligned}$$

74. $$\begin{aligned}5.7-(-11.6)&=5.7+11.6\\&=17.3\end{aligned}$$

75. $$\begin{aligned}-6.4-3.5&=-6.4+(-3.5)\\&=-9.9\end{aligned}$$

76. $$\begin{aligned}-4.4-8.6&=-4.4+(-8.6)\\&=-13\end{aligned}$$

77. $$\begin{aligned}(4-6)+12&=[4+(-6)]+12\\&=[-2]+12\\&=10\end{aligned}$$

78. $$\begin{aligned}(3-7)+4&=[3+(-7)]+4\\&=[-4]+4\\&=0\end{aligned}$$

79. $(8-1)-12=[8+(-1)]+(-12)$
$=[7]+(-12)$
$=-5$

80. $(9-3)-15=\left[9+(-3)\right]+(-15)$
$=[6]+(-15)$
$=-9$

81. $6-(-8+3)=6-(-5)$
$=6+5$
$=11$

82. $8-(-9+5)=8-(-4)$
$=8+4$
$=12$

83. $2+(-4-8)=2+[-4+(-8)]$
$=2+[-12]$
$=-10$

84. $6+(-9-2)=6+\left[-9+(-2)\right]$
$=6+[-11]$
$=-5$

85. $|-5-6|+|9+2|=|-5+(-6)|+|11|$
$=|-11|+|11|$
$=-(-11)+11$
$=11+11$
$=22$

86. $|-4+8|+|6-1|=|4|+|5|$
$=4+5$
$=9$

87. $|-8-2|-|-9-3|$
$=|-8+(-2)|-|-9+(-3)|$
$=|-10|-|-12|$
$=-(-10)-[-(-12)]$
$=10-[12]$
$=-2$

88. $|-4-2|-|-8-1|$
$=|-4+(-2)|-|-8+(-1)|$
$=|-6|-|-9|$
$=-(-6)-[-(-9)]$
$=6-[9]$
$=-3$

89. $\left(-\frac{3}{4}-\frac{5}{2}\right)-\left(-\frac{1}{8}-1\right)$
$=\left(-\frac{3}{4}-\frac{10}{4}\right)-\left(-\frac{1}{8}-\frac{8}{8}\right)$
$=-\frac{13}{4}-\left(-\frac{9}{8}\right)$
$=-\frac{26}{8}+\frac{9}{8}$
$=-\frac{17}{8}$, or $-2\frac{1}{8}$

90. $\left(-\frac{3}{8}-\frac{2}{3}\right)-\left(-\frac{9}{8}-3\right)$
$=\left(-\frac{9}{24}-\frac{16}{24}\right)-\left(-\frac{9}{8}-\frac{24}{8}\right)$
$=-\frac{25}{24}-\left(-\frac{33}{8}\right)$
$=-\frac{25}{24}+\frac{99}{24}$
$=\frac{74}{24}=\frac{37}{12}$, or $3\frac{1}{12}$

91. $\left(-\frac{1}{2}+0.25\right)-\left(-\frac{3}{4}+0.75\right)$
$=\left(-\frac{1}{2}+\frac{1}{4}\right)-\left(-\frac{3}{4}+\frac{3}{4}\right)$
$=\left(-\frac{2}{4}+\frac{1}{4}\right)-0$
$=-\frac{1}{4}$, or -0.25

92. $\left(-\frac{3}{2}-0.75\right)-\left(0.5-\frac{1}{2}\right)$
$=\left(-\frac{3}{2}-\frac{3}{4}\right)-\left(\frac{1}{2}-\frac{1}{2}\right)$
$=\left(-\frac{6}{4}-\frac{3}{4}\right)-0$
$=-\frac{9}{4}$, or -2.25

93. $-9+[(3-2)-(-4+2)]$
$=-9+[1-(-2)]$
$=-9+[1+2]$
$=-9+3$
$=-6$

94. $-8-\left[(-4-1)+(9-2)\right]$
$=-8-\left[(-4+(-1))+7\right]$
$=-8-[-5+7]$
$=-8-[2]$
$=-8+(-2)$
$=-10$

95. $-3+[(-5-8)-(-6+2)]$
$=-3+[(-5+(-8))-(-4)]$
$=-3+[-13+4]$
$=-3+[-9]$
$=-12$

96. $-4+[(-12+1)-(-1-9)]$
$=-4+\left[(-11)-(-1+(-9))\right]$
$=-4+\left[-11-(-10)\right]$
$=-4+[-11+10]$
$=-4+[-1]$
$=-5$

97. $-9.12+[(-4.8-3.25)+11.279]$
$=-9.12+[(-4.8+(-3.25))+11.279]$
$=-9.12+[-8.05+11.279]$
$=-9.12+3.229$
$=-5.891$

98. $-7.62+\left[(-3.99+1.427)-(-2.8)\right]$
$=-7.62-[-2.563+2.8]$
$=-7.62-[0.237]$
$=-7.62+(-0.237)$
$=-7.857$

99. "The sum of -5 and 12 and 6" is written $-5+12+6$.
$-5+12+6=[-5+12]+6$
$=7+6=13$

100. "The sum of -3 and 5 and -12" is written $-3+5+(-12)$.
$-3+5+(-12)=2+(-12)$
$=-10$

101. "14 added to the sum of -19 and -4" is written $[-19+(-4)]+14$.
$[-19+(-4)]+14=(-23)+14$
$=-9$

102. "-2 added to the sum of -18 and 11" is written $(-18+11)+(-2)$.
$(-18+11)+(-2)=-7+(-2)$
$=-9$

103. "The sum of -4 and -10, increased by 12" is written $[-4+(-10)]+12$.
$[-4+(-10)]+12=-14+12$
$=-2$

104. "The sum of -7 and -13, increased by 14" is written $\left[-7+(-13)\right]+14$.
$[-7+(-13)]+14=-20+14$
$=-6$

105. "$\frac{2}{7}$ more than the sum of $\frac{5}{7}$ and $-\frac{9}{7}$" is written $\left[\frac{5}{7}+\left(-\frac{9}{7}\right)\right]+\frac{2}{7}$.
$\left[\frac{5}{7}+\left(-\frac{9}{7}\right)\right]+\frac{2}{7}=-\frac{4}{7}+\frac{2}{7}$
$=-\frac{2}{7}$

106. "1.85 more than the sum of -1.25 and -4.75" is written $\left[-1.25+(-4.75)\right]+1.85$.
$\left[-1.25+(-4.75)\right]+1.85=-6+1.85$
$=-4.15$

107. "The difference of 4 and -8" is written $4-(-8)$.
$4-(-8)=4+8=12$

108. "The difference of 7 and -14" is written $7-(-14)$. This expression can be simplified as follows.
$7-(-14)=7+14=21$

109. "8 less than -2" is written $-2-8$.
$-2-8=-2+(-8)=-10$

110. "9 less than -13" is written $-13-9$.
$-13-9=-13+(-9)=-22$

111. "The sum of 9 and -4, decreased by 7" is written $[9+(-4)]-7$.
$[9+(-4)]-7=5+(-7)=-2$

112. "The sum of 12 and -7, decreased by 14" is written $\left[12+(-7)\right]-14$.

$$\begin{aligned}[12+(-7)]-14 &= 5-14\\ &= 5+(-14)\\ &= -9\end{aligned}$$

113. "12 less than the difference of 8 and -5" is written $[8-(-5)]-12$.

$$\begin{aligned}[8-(-5)]-12 &= [8+(5)]-12\\ &= 13-12\\ &= 13+(-12)\\ &= 1\end{aligned}$$

114. "19 less than the difference of 9 and -2" is written $\left[9-(-2)\right]-19$.

$$\begin{aligned}[9-(-2)]-19 &= (9+2)-19\\ &= 11-19\\ &= 11+(-19)\\ &= -8\end{aligned}$$

115. $$\begin{aligned}[-4+1]+[-2+(-5)] &= -3+(-7)\\ &= -10\end{aligned}$$

The total score below par is 10, which can be represented by the number -10.

116. $$\begin{aligned}[1+(-3)]+[0+(-2)] &= -2+(-2)\\ &= -4\end{aligned}$$

The total score below par is 4, which can be represented by the number -4.

117. $$\begin{aligned}[-2+9]+[7+(-4)] &= 7+3\\ &= 10\end{aligned}$$

The total score above par is 10, which can be represented by the number $+10$.

118. $$\begin{aligned}[-1+2]+[4+(-3)] &= 1+1\\ &= 2\end{aligned}$$

The total score above par is 2, which can be represented by the number $+2$.

119. $$\begin{aligned}[-5+(-4)]+(-3) &= -9+(-3)\\ &= -12\end{aligned}$$

The total number of seats that New York, Pennsylvania, and Ohio are projected to lose is 12, which can be represented by the signed number -12.

120. $$\begin{aligned}&\left[-3+(-2)\right]+\left[9+5+3+2+2\right]\\ &= [-5]+[14+3+2+2]\\ &= -5+[17+2+2]\\ &= -5+[19+2]\\ &= -5+21\\ &= 16\end{aligned}$$

The algebraic sum of these changes can be represented by the signed number 16.

121. $$\begin{aligned}0+(-130)+(-54) &= -130+(-54)\\ &= -184\end{aligned}$$

Their new altitude is 184 meters below the surface, which can be represented by the signed number -184 m.

122. $$\begin{aligned}34{,}000-2100 &= 34{,}000+(-2100)\\ &= 31{,}900\end{aligned}$$

The new altitude of the plane is 31,900 feet, which can be represented by the signed number $31{,}900$ ft.

123. The lowest temperature is represented by $-29°$F. The highest temperature is represented by $-29+149$, or 120°F.

124. To find the temperature, start with $-4°$F and add 49.

$-4+49=45$

The temperature 2 minutes later was $45°$F.

125. $33°$F lower than $-36°$F can be represented as

$$\begin{aligned}-36-33 &= -36+(-33)\\ &= -69.\end{aligned}$$

The record low in Utah is $-69°$F.

126. $36°$F lower than $-19°$F can be represented as

$$\begin{aligned}-19-36 &= -19+(-36)\\ &= -55.\end{aligned}$$

The record low in Wisconsin is $-55°$F.

127. Add the scores of the four turns to get the final score.

$$\begin{aligned}-19+28+(-5)+13 &= 9+(-5)+13\\ &= 4+13\\ &= 17\end{aligned}$$

Her final score for the four turns was 17.

128. Add the scores for the five turns to get the final score.

$-13+15+(-12)+24+14$
$=2+(-12)+24+14$
$=-10+24+14$
$=14+14$
$=28$

His final score for the five turns was 28.

129. **(a)** $4.4-(-0.5)=4.4+0.5$
$=4.9$

The difference is 4.9%.

(b) Americans spent more money than they earned, which means they had to dip into savings or increase borrowing.

130. $236-(-576)=236+576$
$=812$

The difference is \$812 billion.

131. $4906+788-154$
$=5694-154$ Add.
$=5540$ Subtract.

The average was \$5540.

132. $526+80-12=606-12$ Add.
$=594$ Subtract.

The average was \$594.

133. Sum of withdrawals:
$\$35.84+\$26.14+\$3.12=\$61.98+\$3.12$
$=\$65.10$

Sum of deposits:
$\$85.00+\$120.76=\$205.76$

To obtain the final balance, add the deposits and subtract the withdrawals from the beginning balance.

Final balance $=\$904.89-\$65.10+\$205.76$
$=\$839.79+\$205.76$
$=\$1045.55$

Her account balance at the end of August was \$1045.55.

134. Sum of withdrawals:
$\$41.29+\$13.66+\$84.40=\$54.95+\$84.40$
$=\$139.35$

Sum of deposits:
$\$80.59+\$276.13=\$356.72$

To obtain the final balance, add the deposits and subtract the withdrawals from the beginning balance.

Final balance $=\$904.89-\$139.35+\$356.72$
$=\$765.54+\$356.72$
$=\$1122.26$

His account balance at the end of September was \$1122.26.

135. Linda starts with a debt of \$870.00, or −\$870.00. She returns two items, increasing the amount she has by $35.90+150.00=185.90$. She purchases three items, decreasing the amount she has by
$82.50+10.00+10.00=102.50$.
Finally, add the payment and subtract the finance charge to calculate how much money she has.

−870.00	Amount owed
+185.90	Two return credits
−684.10	
−102.50	Three purchases
−786.60	
+500.00	Payment
−286.60	
−37.23	Finance charge
−323.83	

She still owes \$323.83.

136. Marcial starts with a debt of \$679.00, or −\$679.00. He returns three items, increasing the amount he has by
$36.89+29.40+113.55=179.84$.
He purchases four items, decreasing the amount he has by
$135.78+412.88+20.00+20.00=588.66$.
Finally, add the payment and subtract the finance charge to calculate how much money he has.

−679.00	Amount owed
+179.84	Three return credits
−499.16	
−588.66	Four purchases
−1087.82	
+400.00	Payment
−687.82	
−24.57	Finance charge
−712.39	

He still owes \$712.39.

137. The percent return for 2009 is 26.00% and the percent return for 2010 is 14.97%. Thus, the change in percent returns is $14.97-26.00=-11.03\%$ (a decrease).

138. The percent return for 2010 is 14.97% and the percent return for 2011 is 1.51%. Thus, the change in percent return is $1.51-14.97=-13.46\%$ (a decrease).

139. The percent return for 2011 is 1.51% and the percent return for 2012 is 15.31%. Thus, the change in percent return is $15.31-1.51=13.8\%$ (an increase).

140. The percent return for 2009 is 26.00% and the percent return for 2013 is 13.56%. Thus, the change in percent return is $13.56-26.00=-12.44\%$ (a decrease).

141. $$\begin{aligned}17,400-(-32,995)&=17,400+32,995\\&=50,395\end{aligned}$$
The difference between the height of Mt. Foraker and the depth of the Philippine Trench is 50,395 feet.

142. $$\begin{aligned}14,110-(-23,376)&=14,110+23,376\\&=37,486\end{aligned}$$
The difference between the height of Pikes Peak and the depth of the Java Trench is 37,486 feet.

143. $$\begin{aligned}-23,376-(-24,721)&=-23,376+24,721\\&=1345\end{aligned}$$
The Cayman Trench is 1345 feet deeper than the Java Trench.

144. $$\begin{aligned}-24,721-(-32,995)&=-24,721+32,995\\&=8274\end{aligned}$$
The Philippine Trench is 8274 feet deeper than the Cayman Trench.

145. $$\begin{aligned}14,246-14,110&=14,246+(-14,110)\\&=136\end{aligned}$$
Mt. Wilson is 136 feet higher than Pikes Peak.

146. $$\begin{aligned}&(14,246+14,110)-17,400\\&=28,356+(-17,400)\\&=10,956\end{aligned}$$
If Mt. Wilson and Pikes Peak were stacked one on top of the other, they would be 10,956 feet higher than Mt. Foraker.

1.5 Multiplying and Dividing Real Numbers

Classroom Examples, Now Try Exercises

1. **(a)** $$-16\left(\frac{5}{32}\right)=-\left(\frac{\overset{1}{\cancel{16}}}{1}\cdot\frac{5}{\underset{2}{\cancel{32}}}\right)=-\frac{5}{2}$$

(b) $4.56(-2)=-(4.56\cdot 2)=-9.12$

N1. **(a)** $-11(9)=-(11\cdot 9)=-99$

(b) $3.1(-2.5)=-(3.1\cdot 2.5)=-7.75$

2. **(a)** $$-\frac{3}{4}\left(-\frac{2}{5}\right)=\frac{3}{4}\cdot\frac{2}{5}=\frac{3\cdot 2}{4\cdot 5}=\frac{3\cdot 2}{2\cdot 2\cdot 5}=\frac{3}{10}$$

(b) $-1.2(-1.1)=1.2\cdot 1.1=1.32$

N2. **(a)** $-8(-11)=8\cdot 11=88$

(b) $-\frac{1}{7}\left(-\frac{5}{2}\right)=\frac{1}{7}\cdot\frac{5}{2}=\frac{1\cdot 5}{7\cdot 2}=\frac{5}{14}$

3. **(a)** $\frac{-16}{-2}=-16\left(-\frac{1}{2}\right)=8$

(b) $\frac{-16.4}{2.05}=-16.4\left(\frac{1}{2.05}\right)=-8$

(c) $\frac{1}{4}\div\left(-\frac{2}{3}\right)=\frac{1}{4}\left(-\frac{3}{2}\right)=-\frac{3}{8}$

N3. **(a)** $\frac{-10}{5}=-10\cdot\left(\frac{1}{5}\right)=-2$

(b) $\frac{-1.44}{-0.12}=-1.44\left(-\frac{1}{0.12}\right)=12$

(c) $$-\frac{3}{8}\div\frac{7}{10}=-\frac{3}{\underset{4}{\cancel{8}}}\cdot\frac{\overset{5}{\cancel{10}}}{7}=-\frac{15}{28}$$

4. **(a)** $-3(4)-2(-6)=-12-(-12)$
$=-12+12$
$=0$

(b) $\dfrac{-6(-8)+-3(9)}{-2[4-(-3)]}=\dfrac{48-27}{-2(4+3)}=\dfrac{21}{-2(7)}$
$=\dfrac{21}{-14}=-\dfrac{3}{2}$

N4. **(a)** $-4(6)-(-5)5=-24-(-25)$
$=-24+25$
$=1$

(b) $\dfrac{12(-4)-6(-3)}{-4(7-16)}=\dfrac{-48-(-18)}{-4(-9)}$
$=\dfrac{-48+18}{36}$
$=\dfrac{-30}{36}=-\dfrac{5}{6}$

5. Replace x with -2 and y with -3.
$2x^2-4y^2=2(-2)^2-4(-3)^2$
$=2(4)-4(9)$
$=8-36$
$=-28$

N5. Replace x with -4 and y with -3.
$\dfrac{3x^2-12}{y}=\dfrac{3(-4)^2-12}{-3}$
$=\dfrac{3(16)-12}{-3}$
$=\dfrac{48-12}{-3}$
$=\dfrac{36}{-3}=-12$

6. **(a)** "Three times the difference of 4 and -11" is written $3[4-(-11)]$.
$3[4-(-11)]=3(15)=45$

(b) "Three-fifths of the sum of 2 and -7" is written $\frac{3}{5}[2+(-7)]$.
$\frac{3}{5}[2+(-7)]=\frac{3}{5}(-5)=-3$

N6. **(a)** "Twice the sum of -10 and 7" is written $2(-10+7)$.
$2(-10+7)=2(-3)=-6$

(b) "40% of the difference of 45 and 15" is written $0.40(45-15)$.
$0.40(45-15)=0.4(30)=12$.

7. "The product of -9 and 2, divided by the difference of 5 and -1" is written $\dfrac{-9(2)}{5-(-1)}$.
$\dfrac{-9(2)}{5-(-1)}=\dfrac{-18}{6}=-3$

N7. "The quotient of 21 and the sum of 10 and -7" is written $\dfrac{21}{10+(-7)}$.
$\dfrac{21}{10+(-7)}=\dfrac{21}{3}=7$

8. **(a)** "The quotient of a number and -2 is 6" is written $\dfrac{x}{-2}=6$. Here, x must be a negative number, since the denominator is negative and the quotient is positive. Since $\dfrac{-12}{-2}=6$, the solution is -12.

(b) "Twice a number is -6" is written $2x=-6$. Since $2(-3)=-6$, the solution is -3.

N8. **(a)** "The sum of a number and -4 is 7" is written $x+(-4)=7$. Here, x must be 4 more than 7, so the solution is 11.
$11+(-4)=7, \ldots$

(b) "The difference of -8 and a number is -11" is written $-8-x=-11$. If we start at -8 on a number line, we must move 3 units to the left to get to -11, so the solution is 3.

Exercises

1. The product or the quotient of two numbers with the same sign is greater than 0, since the product or quotient of two positive numbers is positive and the product or quotient of two negative numbers is positive.

2. The product or quotient of two numbers with different signs is less than 0, since the product or quotient is negative.

3. If three negative numbers are multiplied, the product is <u>less than 0</u>, since a negative number times a negative number is a positive number, and that positive number times a negative number is a negative number.

4. If two negative numbers are multiplied and then their product is divided by a negative number, the result is <u>less than 0</u>, since the product is a positive number, and a positive number divided by a negative number is a negative number.

5. If a negative number is squared and the result is added to a positive number, the result is <u>greater than 0</u>, since a negative number squared is a positive number, and a positive number added to another positive number is a positive number.

6. The reciprocal of a negative number is <u>less than 0</u>, since it is just the number one divided by a negative number, which is negative.

7. If three positive numbers, five negative numbers, and zero are multiplied, the product is <u>equal to 0</u>. Since one of the numbers is zero, the product is zero (regardless of what the other numbers are).

8. The cube power of a negative number is <u>less than 0</u>. Remember, a negative number raised to an odd power (like 3) is negative.

9. The quotient formed by any nonzero number divided by 0 is <u>undefined</u>, and the quotient formed by 0 divided by any nonzero number is <u>0</u>. Examples include $\frac{1}{0}$, which is undefined, and $\frac{0}{1}$, which equals 0.

10. Look for the expression that has 0 in the denominator. The expression $\frac{4-4}{4-4}$, or $\frac{0}{0}$, is undefined. The correct response is C.

11. $5(-6) = -(5\cdot 6) = -30$

Note that the product of a positive number and a negative number is negative.

12. $-3(4) = -(3\cdot 4) = -12$

Note that the product of a negative number and a positive number is negative.

13. $-5(-6) = 5\cdot 6 = 30$

Note that the product of two negative numbers is positive.

14. $-3(-4) = 3\cdot 4 = 12$

Note that the product of two negative numbers is positive.

15. $-10(-12) = 10\cdot 12 = 120$

16. $-9(-5) = 9\cdot 5 = 45$

17. $3(-11) = -(3\cdot 11) = -33$

18. $3(-15) = -(3\cdot 15) = -45$

19. $-0.5(0) = 0$

20. $-0.3(0) = 0$

21. $-6.8(0.35) = -(6.8\cdot 0.35) = -2.38$

22. $-4.6(0.24) = -(4.6\cdot 0.24) = -1.104$

23.
$$\begin{aligned} -\frac{3}{8}\cdot\left(-\frac{10}{9}\right) &= \frac{3}{8}\left(\frac{10}{9}\right) \\ &= \frac{3\cdot 10}{8\cdot 9} \\ &= \frac{3\cdot(2\cdot 5)}{(4\cdot 2)\cdot(3\cdot 3)} \\ &= \frac{3\cdot 2\cdot 5}{4\cdot 2\cdot 3\cdot 3} \\ &= \frac{5}{4\cdot 3} = \frac{5}{12} \end{aligned}$$

24.
$$\begin{aligned} -\frac{5}{6}\cdot\left(-\frac{16}{15}\right) &= \frac{5}{6}\cdot\frac{16}{15} \\ &= \frac{5\cdot 16}{6\cdot 15} \\ &= \frac{8}{9} \end{aligned}$$

25.
$$\begin{aligned} \frac{2}{15}\left(-1\frac{1}{4}\right) &= \frac{2}{15}\left(-\frac{5}{4}\right) \\ &= -\frac{2\cdot 5}{15\cdot 4} \\ &= -\frac{2\cdot 5}{3\cdot 5\cdot 2\cdot 2} \\ &= -\frac{1}{3\cdot 2} = -\frac{1}{6} \end{aligned}$$

26. $\frac{3}{7}\left(-1\frac{5}{9}\right)=\frac{3}{7}\left(-\frac{14}{9}\right)$
$$=-\frac{3\cdot(2\cdot7)}{7\cdot(3\cdot3)}$$
$$=-\frac{2}{3}$$

27. $-8\left(-\frac{3}{4}\right)=8\left(\frac{3}{4}\right)=\frac{24}{4}=6$

28. $-6\left(-\frac{2}{3}\right)=6\left(\frac{2}{3}\right)=\frac{12}{3}=4$

29. Using only positive integer factors, 32 can be written as $1\cdot32, 2\cdot16,$ or $4\cdot8.$ Including the negative integer factors, we see that the integer factors of 32 are $-32, -16, -8, -4, -2, -1,$ 1, 2, 4, 8, 16, and 32.

30. The integer factors of 36 are $-36, -18, -12, -9, -6, -4, -3, -2, -1, 1, 2, 3, 4, 6, 9, 12, 18,$ and 36.

31. The integer factors of 40 are $-40, -20, -10, -8, -5, -4, -2, -1, 1, 2, 4, 5, 8, 10, 20,$ and 40.

32. The integer factors of 50 are $-50, -25, -10, -5, -2, -1, 1, 2, 5, 10, 25,$ and 50.

33. The integer factors of 31 are $-31, -1, 1,$ and 31.

34. The integer factors of 17 are $-17, -1, 1,$ and 17.

35. $\frac{15}{5}=\frac{5\cdot3}{5}=\frac{3}{1}=3$

36. $\frac{35}{5}=\frac{7\cdot5}{5}=\frac{7}{1}=7$

37. $\frac{-42}{6}=-\frac{2\cdot3\cdot7}{2\cdot3}=-7$
Note that the quotient of two numbers having different signs is negative.

38. $\frac{-28}{7}=-\frac{4\cdot7}{7}=-4$
Note that the quotient of two numbers having different signs is negative.

39. $\frac{-32}{-4}=\frac{4\cdot8}{4}=8$
Note that the quotient of two numbers having the same sign is positive.

40. $\frac{-35}{-5}=\frac{5\cdot7}{5}=7$
Note that the quotient of two numbers having the same sign is positive.

41. $\frac{96}{-16}=-\frac{6\cdot16}{16}=-6$

42. $\frac{38}{-19}=-\frac{2\cdot19}{19}=-2$

43. $\frac{-8.8}{2.2}=-\frac{4(2.2)}{2.2}=-4$

44. $\frac{-4.6}{0.23}=-20$

45. Dividing by a fraction (in this case, $-\frac{1}{8}$) is the same as multiplying by the reciprocal of the fraction (in this case, $-\frac{8}{1}$).
Note that dividing by a number with absolute value between 0 and 1 gives us a number larger than the original numerator.
$$\left(-\frac{4}{3}\right)\div\left(-\frac{1}{8}\right)=\left(-\frac{4}{3}\right)\cdot\left(-\frac{8}{1}\right)$$
$$=\frac{4\cdot8}{3\cdot1}$$
$$=\frac{32}{3},\ \text{or}\ 10\frac{2}{3}$$

46. Dividing by a fraction (in this case, $-\frac{1}{3}$) is the same as multiplying by the reciprocal of the fraction (in this case, $-\frac{3}{1}$).
$$\left(-\frac{6}{5}\right)\div\left(-\frac{1}{3}\right)=\left(-\frac{6}{5}\right)\cdot\left(-\frac{3}{1}\right)$$
$$=\frac{6\cdot3}{5\cdot1}$$
$$=\frac{18}{5},\ \text{or}\ 3\frac{3}{5}$$

47. $-\frac{5}{6}\div\frac{8}{9}=-\frac{5}{6}\cdot\frac{9}{8}$
$$=-\frac{5\cdot9}{6\cdot8}$$
$$=-\frac{45}{48},\ \text{or}\ -\frac{15}{16}$$

48. $$\begin{aligned} -\frac{7}{10} \div \frac{3}{4} &= -\frac{7}{10} \cdot \frac{4}{3} \\ &= -\frac{7 \cdot 4}{10 \cdot 3} \\ &= -\frac{28}{30}, \text{ or } -\frac{14}{15} \end{aligned}$$

49. $\frac{0}{-5} = 0$, because 0 divided by any nonzero number is 0.

50. $\frac{0}{-9} = 0$, because 0 divided by any nonzero number is 0.

51. $\frac{11.5}{0}$ is undefined, because we cannot divide by 0.

52. $\frac{15.2}{0}$ is undefined, because we cannot divide by 0.

53. $$\begin{aligned} 7 - 3 \cdot 6 &= 7 - 18 \\ &= -11 \end{aligned}$$

54. $$\begin{aligned} 8 - 2 \cdot 5 &= 8 - 10 \\ &= -2 \end{aligned}$$

55. $$\begin{aligned} -10 - (-4)(2) &= -10 - (-8) \\ &= -10 + 8 \\ &= -2 \end{aligned}$$

56. $$\begin{aligned} -11 - (-3)(6) &= -11 - (-18) \\ &= -11 + 18 \\ &= 7 \end{aligned}$$

57. $$\begin{aligned} -7(3 - 8) &= -7[3 + (-8)] \\ &= -7(-5) = 35 \end{aligned}$$

58. $$\begin{aligned} -5(4 - 7) &= -5[4 + (-7)] \\ &= -5(-3) = 15 \end{aligned}$$

59. $$\begin{aligned} 7 + 2(4 - 1) &= 7 + 2(3) \\ &= 7 + 6 \\ &= 13 \end{aligned}$$

60. $$\begin{aligned} 5 + 3(6 - 4) &= 5 + 3(2) \\ &= 5 + 6 \\ &= 11 \end{aligned}$$

61. $$\begin{aligned} -4 + 3(2 - 8) &= -4 + 3(-6) \\ &= -4 - 18 \\ &= -22 \end{aligned}$$

62. $$\begin{aligned} -8 + 4(5 - 7) &= 8 + 4(-2) \\ &= -8 - 8 \\ &= -16 \end{aligned}$$

63. $$\begin{aligned} (12 - 14)(1 - 4) &= (-2)(-3) \\ &= 6 \end{aligned}$$

64. $$\begin{aligned} (8 - 9)(4 - 12) &= (-1)(-8) \\ &= 8 \end{aligned}$$

65. $$\begin{aligned} (7 - 10)(10 - 4) &= (-3)(6) \\ &= -18 \end{aligned}$$

66. $$\begin{aligned} (5 - 12)(19 - 4) &= -7(15) \\ &= -105 \end{aligned}$$

67. $$\begin{aligned} (-2 - 8)(-6) + 7 &= (-10)(-6) + 7 \\ &= 60 + 7 \\ &= 67 \end{aligned}$$

68. $$\begin{aligned} (-9 - 4)(-2) + 10 &= (-13)(-2) + 10 \\ &= 26 + 10 \\ &= 36 \end{aligned}$$

69. $$\begin{aligned} 3(-5) + |3 - 10| &= -15 + |-7| \\ &= -15 + 7 \\ &= -8 \end{aligned}$$

70. $$\begin{aligned} 4(-8) + |4 - 15| &= -32 + |-11| \\ &= -32 + 11 \\ &= -21 \end{aligned}$$

71. $$\begin{aligned} \frac{-5(-6)}{9 - (-1)} &= \frac{30}{10} \\ &= \frac{3 \cdot 10}{10} = 3 \end{aligned}$$

72. $$\begin{aligned} \frac{-12(-5)}{7 - (-5)} &= \frac{60}{12} \\ &= \frac{5 \cdot 12}{12} = 5 \end{aligned}$$

73. $$\begin{aligned} \frac{-21(3)}{-3 - 6} &= \frac{-63}{-3 + (-6)} \\ &= \frac{-63}{-9} = 7 \end{aligned}$$

74. $\dfrac{-40(3)}{-2-3}=\dfrac{-120}{-5}$
$=\dfrac{5\cdot 24}{5}=24$

75. $\dfrac{-10(2)+6(2)}{-3-(-1)}=\dfrac{-20+12}{-3+1}$
$=\dfrac{-8}{-2}=4$

76. $\dfrac{-12(4)+5(3)}{-14-(-3)}=\dfrac{-48+15}{-14+3}$
$=\dfrac{-33}{-11}=3$

77. $\dfrac{3^2-4^2}{7(-8+9)}=\dfrac{9-16}{7(1)}=\dfrac{-7}{7}=-1$

78. $\dfrac{5^2-7^2}{2(3+3)}=\dfrac{25-49}{2(6)}$
$=\dfrac{-24}{2(6)}$
$=-\dfrac{2\cdot 2\cdot 6}{2\cdot 6}$
$=-2$

79. $\dfrac{8(-1)-|(-4)(-3)|}{-6-(-1)}=\dfrac{-8-|12|}{-6+1}$
$=\dfrac{-8-12}{-5}$
$=\dfrac{-20}{-5}=4$

80. $\dfrac{-27(-2)-|6\cdot 4|}{-2(3)-2(2)}=\dfrac{54-|24|}{-6-4}$
$=\dfrac{54-24}{-10}$
$=\dfrac{30}{-10}$
$=-3$

81. $\dfrac{-13(-4)-(-8)(-2)}{(-10)(2)-4(-2)}$
$=\dfrac{52-16}{-20-(-8)}$
$=\dfrac{36}{-20+8}$
$=\dfrac{36}{-12}=-3$

82. $\dfrac{-5(2)+[3(-2)-4]}{-3-(-1)}=\dfrac{-10+[-6+(-4)]}{-3+(1)}$
$=\dfrac{-10+[-10]}{-2}$
$=\dfrac{-20}{-2}=10$

83. $3+2\times 4\div 2-3\times 7-4+47$
$=3+\left(\dfrac{2\times 4}{2}\right)-(3\times 7)-4+47$
$=3+\left(\dfrac{8}{2}\right)-(21)-4+47$
$=3+4-21-4+47$
$=29$

84. The incorrect answer, 92, was obtained by performing all of the operations from left to right rather than following the rules for order of operations. The multiplications and divisions need to be done in order, before the additions and subtractions.

85. $5x-2y+3a=5(6)-2(-4)+3(3)$
$=30-(-8)+9$
$=30+8+9$
$=38+9$
$=47$

86. $6x-5y+4a=6(6)-5(-4)+4(3)$
$=36+20+12$
$=56+12$
$=68$

87. $(2x+y)(3a)=[2(6)+(-4)][3(3)]$
$=[12+(-4)](9)$
$=(8)(9)$
$=72$

88. $$(5x-2y)(-2a) = [5(6)-2(-4)][-2(3)] = (30+8)(-6) = (38)(-6) = -228$$

89. $$\left(\frac{1}{3}x-\frac{4}{5}y\right)\left(-\frac{1}{5}a\right) = \left[\frac{1}{3}(6)-\frac{4}{5}(-4)\right]\left[-\frac{1}{5}(3)\right] = \left[2-\left(-\frac{16}{5}\right)\right]\left(-\frac{3}{5}\right) = \left(2+\frac{16}{5}\right)\left(-\frac{3}{5}\right) = \left(\frac{10}{5}+\frac{16}{5}\right)\left(-\frac{3}{5}\right) = \left(\frac{26}{5}\right)\left(-\frac{3}{5}\right) = -\frac{78}{25}$$

90. $$\left(\frac{5}{6}x+\frac{3}{2}y\right)\left(-\frac{1}{3}a\right) = \left[\frac{5}{6}(6)+\frac{3}{2}(-4)\right]\left[-\frac{1}{3}(3)\right] = [5+(-6)](-1) = (-1)(-1) = 1$$

91. $$(-5+x)(-3+y)(3-a) = (-5+6)[-3+(-4)][3-3] = (1)(-7)(0) = 0$$

92. $$(6-x)(5+y)(3+a) = (6-6)[5+(-4)](3+3) = 0(1)(6) = 0$$

93. $$-2y^2+3a = -2(-4)^2+3(3) = -2(16)+9 = -32+9 = -23$$

94. $$5x-4a^2 = 5(6)-4(3)^2 = 30-4(9) = 30-36 = -6$$

95. $$\frac{2y-x}{a-3} = \frac{2(-4)-(6)}{3-3} = \frac{(-8)-6}{0} = \frac{-14}{0}$$
The expression is undefined.

96. $$\frac{xy+8a}{x-6} = \frac{6(-4)+8(3)}{6-6} = \frac{-24+24}{0} = \frac{0}{0}$$
The expression is undefined.

97. "The product of -9 and 2, added to 9" is written $9+(-9)(2)$.
$$9+(-9)(2) = 9+(-18) = -9$$

98. "The product of 4 and -7, added to -12" is written $-12+4(-7)$.
$$-12+4(-7) = -12+(-28) = -40$$

99. "Twice the product of -1 and 6, subtracted from -4" is written $-4-2[(-1)(6)]$.
$$-4-2[(-1)(6)] = -4-2(-6) = -4-(-12) = -4+12 = 8$$

100. "Twice the product of -8 and 2, subtracted from -1" is written $-1-2(-8)(2)$.
$$-1-2[(-8)(2)] = -1-2(-16) = -1-(-32) = -1+32 = 31$$

101. "Nine subtracted from the product of 1.5 and -3.2" is written $(1.5)(-3.2)-9$.

$$\begin{aligned}(1.5)(-3.2)-9 &= -4.8-9\\ &= -4.8+(-9)\\ &= -13.8\end{aligned}$$

102. "Three subtracted from the product of 4.2 and -8.5" is written $(4.2)(-8.5)-3$.

$$\begin{aligned}(4.2)(-8.5)-3 &= -35.7-3\\ &= -35.7+(-3)\\ &= -38.7\end{aligned}$$

103. "The product of 12 and the difference of 9 and -8" is written $12[9-(-8)]$.

$$\begin{aligned}12[9-(-8)] &= 12[9+8]\\ &= 12(17) = 204\end{aligned}$$

104. "The product of -3 and the difference of 3 and -7" is written $-3[3-(-7)]$.

$$\begin{aligned}-3[3-(-7)] &= -3[3+7]\\ &= -3(10) = -30\end{aligned}$$

105. "The quotient of -12 and the sum of -5 and -1" is written $\dfrac{-12}{-5+(-1)}$, and

$$\frac{-12}{-5+(-1)} = \frac{-12}{-6} = 2.$$

106. "The quotient of -20 and the sum of -8 and -2" is written $\dfrac{-20}{-8+(-2)}$, and

$$\frac{-20}{-8+(-2)} = \frac{-20}{-10} = 2.$$

107. "The sum of 15 and -3, divided by the product of 4 and -3" is written $\dfrac{15+(-3)}{4(-3)}$, and

$$\frac{15+(-3)}{4(-3)} = \frac{12}{-12} = -1.$$

108. "The sum of -18 and -6, divided by the product of 2 and -4" is written $\dfrac{-18+(-6)}{2(-4)}$, and $\dfrac{-18+(-6)}{2(-4)} = \dfrac{-24}{-8} = 3.$

109. "Two-thirds of the difference of 8 and -1" is written $\frac{2}{3}[8-(-1)]$, and

$$\frac{2}{3}[8-(-1)] = \frac{2}{3}[8+(1)] = \frac{2}{3}[9] = 6.$$

110. "Three-fourths of the sum of -8 and 12" is written $\frac{3}{4}(-8+12)$, and

$$\frac{3}{4}(-8+12) = \frac{3}{4}(4) = 3.$$

111. "20% of the product of -5 and 6" is written $0.20(-5\cdot 6)$, and

$$0.20(-5\cdot 6) = 0.20(-30) = -6.$$

112. "30% of the product of -8 and 5" is written $0.30(-8\cdot 5)$, and

$$0.30(-8\cdot 5) = 0.30(-40) = -12.$$

113. "The sum of $\frac{1}{2}$ and $\frac{5}{8}$, times the difference of $\frac{3}{5}$ and $\frac{1}{3}$" is written $\left(\frac{1}{2}+\frac{5}{8}\right)\left(\frac{3}{5}-\frac{1}{3}\right)$, and

$$\begin{aligned}\left(\frac{1}{2}+\frac{5}{8}\right)\left(\frac{3}{5}-\frac{1}{3}\right) &= \left(\frac{4}{8}+\frac{5}{8}\right)\left(\frac{9}{15}-\frac{5}{15}\right)\\ &= \frac{9}{8}\left(\frac{4}{15}\right)\\ &= \frac{3\cdot 3\cdot 4}{2\cdot 4\cdot 3\cdot 5} = \frac{3}{10}.\end{aligned}$$

114. "The sum of $\frac{3}{4}$ and $\frac{1}{2}$, times the difference of $\frac{2}{3}$ and $\frac{1}{6}$" is written $\left(\frac{3}{4}+\frac{1}{2}\right)\left(\frac{2}{3}-\frac{1}{6}\right)$, and

$$\begin{aligned}\left(\frac{3}{4}+\frac{1}{2}\right)\left(\frac{2}{3}-\frac{1}{6}\right) &= \left(\frac{3}{4}+\frac{2}{4}\right)\left(\frac{4}{6}-\frac{1}{6}\right)\\ &= \frac{5}{4}\left(\frac{3}{6}\right)\\ &= \frac{5\cdot 3}{4\cdot 2\cdot 3} = \frac{5}{8}.\end{aligned}$$

115. "The product of $-\frac{1}{2}$ and $\frac{3}{4}$, divided by $-\frac{2}{3}$," is written $\frac{-\frac{1}{2}\left(\frac{3}{4}\right)}{-\frac{2}{3}}$. Simplifying gives us

$$\frac{-\frac{1}{2}\left(\frac{3}{4}\right)}{-\frac{2}{3}}=\frac{-\frac{3}{8}}{-\frac{2}{3}}$$
$$=-\frac{3}{8}\cdot\left(-\frac{3}{2}\right)$$
$$=\frac{9}{16}.$$

116. "The product of $-\frac{2}{3}$ and $-\frac{1}{5}$, divided by $\frac{1}{7}$" is written $\frac{-\frac{2}{3}\left(-\frac{1}{5}\right)}{\frac{1}{7}}$. Simplifying gives us

$$\frac{-\frac{2}{3}\left(-\frac{1}{5}\right)}{\frac{1}{7}}=\frac{\frac{2}{15}}{\frac{1}{7}}$$
$$=\frac{2}{15}\cdot\frac{7}{1}$$
$$=\frac{14}{15}.$$

117. "The quotient of a number and 3 is -3" is written $\frac{x}{3}=-3$. The solution is -9, since $\frac{-9}{3}=-3$.

118. "The quotient of a number and 4 is -1" is written $\frac{x}{4}=-1$. The solution is -4, since $\frac{-4}{4}=-1$.

119. "6 less than a number is 4" is written $x-6=4$. The solution is 10, since $10-6=4$.

120. "7 less than a number is 2" is written $x-7=2$. The solution is 9, since $9-7=2$.

121. "When 5 is added to a number, the result is -5" is written $x+5=-5$. The solution is -10, since $-10+5=-5$.

122. "When 6 is added to a number, the result is -3" is written $x+6=-3$. The solution is -9, since $-9+6=-3$.

123. Add the numbers and divide by 5.

$$\frac{(23+18+13)+[(-4)+(-8)]}{5}$$
$$=\frac{54-12}{5}$$
$$=\frac{42}{5}, \text{ or } 8\frac{2}{5}$$

124. Add the numbers and divide by 5.

$$\frac{(18+12)+0+[(-4)+(-10)]}{5}$$
$$=\frac{30-14}{5}$$
$$=\frac{16}{5}, \text{ or } 3\frac{1}{5}$$

125. Add the numbers and divide by 4.

$$\frac{(29+8)+[(-15)+(-6)]}{4}$$
$$=\frac{37-21}{4}$$
$$=\frac{16}{4}=4$$

126. Add the numbers and divide by 4.

$$\frac{(34+9)+[(-17)+(-2)]}{4}$$
$$=\frac{43-19}{4}$$
$$=\frac{24}{4}=6$$

127. Add the integers from -10 to 14.

$(-10)+(-9)+\cdots+14=50$

[The 3 dots indicate that the pattern continues.]
There are 25 integers from -10 to 14 (10 negative, zero, and 14 positive). Thus, the average is $\frac{50}{25}=2$.

128. Add the even integers from -18 to 4.

$(-18)+(-16)+\cdots+4=-84$

There are 12 integers from -18 to 4 (9 negative, zero, and 2 positive). Thus, the average is $\frac{-84}{12}=-7$.

129. **(a)** 3,473,986 is divisible by 2 because its last digit, 6, is divisible by 2.

(b) 4,336,879 is not divisible by 2 because its last digit, 9, is not divisible by 2.

130. **(a)** 4,799,232 is divisible by 3 because the sum of its digits, $4+7+9+9+2+3+2=36$, is divisible by 3.

(b) 2,443,871 is not divisible by 3 because the sum of its digits, $2+4+4+3+8+7+1=29$, is not divisible by 3.

131. **(a)** 6,221,464 is divisible by 4 because the number formed by its last two digits, 64, is divisible by 4.

(b) 2,876,335 is not divisible by 4 because the number formed by its last two digits, 35, is not divisible by 4.

132. **(a)** 3,774,595 is divisible by 5 because its last digit, 5, is divisible by 5.

(b) 9,332,123 is not divisible by 5 because its last digit, 3, is not divisible by 5.

133. **(a)** 1,524,822 is divisible by 2 because its last digit, 2, is divisible by 2. It is also divisible by 3 because the sum of its digits, $1+5+2+4+8+2+2=24$, is divisible by 3. Because 1,524,822 is divisible by both 2 and 3, it is divisible by 6.

(b) 2,873,590 is divisible by 2 because its last digit, 0, is divisible by 2. However, it is not divisible by 3 because the sum of its digits, $2+8+7+3+5+9+0=34$, is not divisible by 3. Because 2,873,590 is not divisible by both 2 and 3, it is not divisible by 6.

134. **(a)** 2,923,296 is divisible by 8 because the number formed by its last three digits, 296, is divisible by 8.

(b) 7,291,623 is not divisible by 8 because the number formed by its last three digits, 623, is not divisible by 8.

135. **(a)** 4,114,107 is divisible by 9 because the sum of its digits, $4+1+1+4+1+0+7=18$, is divisible by 9.

(b) 2,287,321 is not divisible by 9 because the sum of its digits, $2+2+8+7+3+2+1=25$, is not divisible by 9.

136. **(a)** 4,253,520 is divisible by 3 because the sum of its digits, $4+2+5+3+5+2+0=21$, is divisible by 3. It is also divisible by 4 because the number formed by its last two digits, 20, is divisible by 4. Because 4,253,520 is divisible by both 3 and 4, it is divisible by 12.

(b) 4,249,474 is not divisible by 3 because the sum of its digits, $4+2+4+9+4+7+4=34$, is not divisible by 3. Because a number is not divisible by 12 unless it is divisible by both 3 and 4, this is sufficient to show that the number is not divisible by 12.

Summary Exercises Performing Operations with Real Numbers

1. $$\begin{aligned}14-3\cdot 10&=14-30\\&=14+(-30)\\&=-16\end{aligned}$$

2. $$\begin{aligned}-3(8)-4(-7)&=-24-(-28)\\&=-24+28\\&=4\end{aligned}$$

3. $$\begin{aligned}(3-8)(-2)-10&=(-5)(-2)-10\\&=10-10\\&=0\end{aligned}$$

4. $$\begin{aligned}-6(7-3)&=-6(4)\\&=-24\end{aligned}$$

5. $$\begin{aligned}7+3(2-10)&=7+3(-8)\\&=7-24\\&=-17\end{aligned}$$

6. $$\begin{aligned}-4[(-2)(6)-7]&=-4[-12-7]\\&=-4[-19]\\&=76\end{aligned}$$

7. $$\begin{aligned}(-4)(7)-(-5)(2)&=(-28)-(-10)\\&=-28+(10)\\&=-18\end{aligned}$$

8. $$\begin{aligned}-5[-4-(-2)(-7)]&=-5[-4-(14)]\\&=-5[-18]\\&=90\end{aligned}$$

9. $40-(-2)[8-9]=40-(-2)[-1]$
$=40-(2)$
$=38$

10. $\dfrac{5(-4)}{-7-(-2)}=\dfrac{-20}{-7+2}$
$=\dfrac{-20}{-5}=4$

11. $\dfrac{-3-(-9+1)}{-7-(-6)}=\dfrac{-3-(-8)}{-7+6}$
$=\dfrac{-3+8}{-1}$
$=\dfrac{5}{-1}=-5$

12. $\dfrac{5(-8+3)}{13(-2)+(-7)(-3)}=\dfrac{5(-5)}{-26+21}$
$=\dfrac{-25}{-5}=5$

13. $\dfrac{6^2-8}{-2(2)+4(-1)}=\dfrac{36-8}{-4+(-4)}$
$=\dfrac{28}{-8}$
$=-\dfrac{4\cdot 7}{2\cdot 4}=-\dfrac{7}{2},\quad \text{or}\quad -3\dfrac{1}{2}$

14. $\dfrac{16(-8+5)}{15(-3)+(-7-4)(-3)}=\dfrac{16(-3)}{-45+(-11)(-3)}$
$=\dfrac{-48}{-45+33}$
$=\dfrac{-48}{-12}=4$

15. $\dfrac{9(-6)-3(8)}{4(-7)+(-2)(-11)}=\dfrac{-54-24}{-28+22}$
$=\dfrac{-78}{-6}=13$

16. $\dfrac{2^2+4^2}{5^2-3^2}=\dfrac{4+16}{25-9}$
$=\dfrac{20}{16}=\dfrac{5}{4},\ \text{or}\ 1\dfrac{1}{4}$

17. $\dfrac{(2+4)^2}{(5-3)^2}=\dfrac{(6)^2}{(2)^2}$
$=\dfrac{36}{4}=9$

18. $\dfrac{4^3-3^3}{-5(-4+2)}=\dfrac{64-27}{-5(-2)}$
$=\dfrac{37}{10},\ \text{or}\ 3\dfrac{7}{10}$

19. $\dfrac{-9(-6)+(-2)(27)}{3(8-9)}=\dfrac{(54)+(-54)}{3(-1)}$
$=\dfrac{0}{-3}=0$

20. $|-4(9)|-|-11|=|-36|-11$
$=36-11$
$=25$

21. $\dfrac{6(-10+3)}{15(-2)-3(-9)}=\dfrac{6(-7)}{(-30)-(-27)}$
$=\dfrac{-42}{-30+27}$
$=\dfrac{-42}{-3}$
$=14$

22. $\dfrac{3^2-5^2}{(-9)^2-9^2}=\dfrac{9-25}{81-81}$
$=\dfrac{-16}{0}$, which is undefined.

23. $\dfrac{(-10)^2+10^2}{-10(5)}=\dfrac{100+100}{-50}$
$=\dfrac{200}{-50}=-4$

24. $-\dfrac{3}{4}\div\left(-\dfrac{5}{8}\right)=-\dfrac{3}{4}\cdot\left(-\dfrac{8}{5}\right)$
$=\dfrac{3\cdot 2\cdot 4}{4\cdot 5}$
$=\dfrac{3\cdot 2}{5}=\dfrac{6}{5},\quad \text{or}\quad 1\dfrac{1}{5}$

25. $\dfrac{1}{2}\div\left(-\dfrac{1}{2}\right)=\dfrac{1}{2}\cdot\left(-\dfrac{2}{1}\right)$
$=-\dfrac{2}{2}=-1$

26. $\dfrac{8^2-12}{(-5)^2+2(6)}=\dfrac{64-12}{25+12}$

$=\dfrac{52}{37}$, or $1\dfrac{15}{37}$

27. $\left[\dfrac{5}{8}-\left(-\dfrac{1}{16}\right)\right]+\dfrac{3}{8}=\left[\dfrac{10}{16}+\dfrac{1}{16}\right]+\dfrac{6}{16}$

$=\left[\dfrac{11}{16}\right]+\dfrac{6}{16}$

$=\dfrac{17}{16}$, or $1\dfrac{1}{16}$

28. $\left(\dfrac{1}{2}-\dfrac{1}{3}\right)-\dfrac{5}{6}=\left(\dfrac{3}{6}-\dfrac{2}{6}\right)-\dfrac{5}{6}$

$=\left(\dfrac{1}{6}\right)-\dfrac{5}{6}$

$=-\dfrac{4}{6}$

$=-\dfrac{2}{3}$

29. $-0.9(-3.7)=0.9(3.7)$

$=3.33$

30. $-5.1(-0.2)=5.1(0.2)$

$=1.02$

31. $|-2(3)+4|-|-2|=|-6+4|-2$

$=|-2|-2$

$=2-2=0$

32. $40+2[-5-3]=40+2[-8]$

$=40-16$

$=24$

33. $-x+y-3a=-(-2)+3-3(4)$

$=2+3-12$

$=5-12$

$=-7$

34. $(x-y)-(a-2y)=(-2-3)-(4-2\cdot 3)$

$=(-5)-(4-6)$

$=-5-(-2)$

$=-5+2$

$=-3$

35. $\left(\dfrac{1}{2}x+\dfrac{2}{3}y\right)\left(-\dfrac{1}{4}a\right)$

$=\left(\dfrac{1}{2}(-2)+\dfrac{2}{3}(3)\right)\left(-\dfrac{1}{4}(4)\right)$

$=(-1+2)(-1)$

$=(1)(-1)$

$=-1$

36. $\dfrac{2x+3y}{a-xy}=\dfrac{2(-2)+3(3)}{4-(-2)(3)}$

$=\dfrac{-4+9}{4-(-6)}$

$=\dfrac{5}{4+6}$

$=\dfrac{5}{10}=\dfrac{1}{2}$

37. $\dfrac{x^2-y^2}{x^2+y^2}=\dfrac{(-2)^2-3^2}{(-2)^2+3^2}$

$=\dfrac{4-9}{4+9}$

$=\dfrac{-5}{13}=-\dfrac{5}{13}$

38. $-x^2+3y=-(-2)^2+3(3)$

$=-(4)+9$

$=5$

39. $\left(\dfrac{x}{y}\right)^3=\left(\dfrac{-2}{3}\right)^3=\left(-\dfrac{2}{3}\right)\left(-\dfrac{2}{3}\right)\left(-\dfrac{2}{3}\right)$

$=-\dfrac{8}{27}$

40. $\left(\dfrac{a}{x}\right)^2=\left(\dfrac{4}{-2}\right)^2=(-2)^2=(-2)(-2)=4$

1.6 Properties of Real Numbers

Classroom Examples, Now Try Exercises

1. **(a)** $x+2=2+\underline{x}$

(b) $5x=x\cdot\underline{5}$

N1. **(a)** $7+(-3)=-3+\underline{7}$

(b) $(-5)4=4\cdot\underline{(-5)}$

2. **(a)** $-5+(2+8)=\underline{(-5+2)}+8$

(b) $10[(-8)\cdot(-3)]=\underline{[10\cdot(-8)]\cdot(-3)}$

N2. **(a)** $-9+(3+7)=\underline{(-9+3)}+7$

(b) $5[(-4)\cdot 9]=\underline{[5\cdot(-4)]\cdot 9}$

3. $(2\cdot 4)6=(4\cdot 2)6$

While the same numbers are grouped inside the two pairs of parentheses, the order of the numbers has been changed. This illustrates a commutative property.

N3. $5+(7+6)=5+(6+7)$

While the same numbers are grouped inside the two pairs of parentheses, the order of the numbers has been changed. This illustrates a commutative property.

4. **(a)** $43+26+17+24+6$

$=(43+17)+(26+24)+6$

$=60+50+6$

$=110+6$

$=116$

(b) $\frac{1}{2}(67)(2)=\frac{1}{2}(2)(67)=1(67)=67$

N4. **(a)** $8+54+7+6+32$

$=(8+32)+(54+6)+7$

$=40+60+7$

$=100+7$

$=107$

(b) $5(37)(20)=5(20)(37)=100(37)=3700$

5. **(a)** $5+\underline{0}=5$ Additive identity

(b) $\underline{1}\cdot\frac{1}{3}=\frac{1}{3}$ Multiplicative identity

N5. **(a)** $\frac{2}{5}\cdot\underline{1}=\frac{2}{5}$ Multiplicative identity

(b) $8+\underline{0}=8$ Additive identity

6. **(a)** $\frac{36}{48}=\frac{3\cdot 12}{4\cdot 12}$ Factor.

$=\frac{3}{4}\cdot\frac{12}{12}$ Write as a product.

$=\frac{3}{4}\cdot 1$ Divide.

$=\frac{3}{4}$ Identity property

(b) $\frac{3}{8}-\frac{5}{24}=\frac{3}{8}\cdot 1-\frac{5}{24}$ Identity property

$=\frac{3}{8}\cdot\frac{3}{3}-\frac{5}{24}$ Multiply by $\frac{3}{3}$.

$=\frac{9}{24}-\frac{5}{24}$ Multiply.

$=\frac{4}{24}$ Subtract.

$=\frac{1}{6}$ Reduce.

N6. **(a)** $\frac{16}{20}=\frac{4\cdot 4}{5\cdot 4}$ Factor.

$=\frac{4}{5}\cdot\frac{4}{4}$ Write as a product.

$=\frac{4}{5}\cdot 1$ Divide.

$=\frac{4}{5}$ Identity property

(b) $\frac{2}{5}+\frac{3}{20}=\frac{2}{5}\cdot 1+\frac{3}{20}$ Identity property

$=\frac{2}{5}\cdot\frac{4}{4}+\frac{3}{20}$ Multiply by $\frac{4}{4}$.

$=\frac{8}{20}+\frac{3}{20}$ Multiply.

$=\frac{11}{20}$ Add.

7. **(a)** $\underline{-6}+6=0$ Inverse property

(b) $-\frac{1}{9}\cdot\underline{(-9)}=1$ Inverse property

N7. **(a)** $10+\underline{(-10)}=0$ Inverse property

(b) $-9\cdot\underline{\left(-\frac{1}{9}\right)}=1$ Inverse property

8. $\frac{1}{2}+3y+\left(-\frac{1}{2}\right)$

$=\left(\frac{1}{2}+3y\right)+\left(-\frac{1}{2}\right)$ Order of operations

$=\left(3y+\frac{1}{2}\right)+\left(-\frac{1}{2}\right)$ Commutative property

$=3y+\left[\frac{1}{2}+\left(-\frac{1}{2}\right)\right]$ Associative property

$=3y+0$ Inverse property

$=3y$ Identity property

N8. $-\frac{1}{3}x+7+\frac{1}{3}x$

$=\left(-\frac{1}{3}x+7\right)+\frac{1}{3}x$ Order of operations

$=\left[7+\left(-\frac{1}{3}x\right)\right]+\frac{1}{3}x$ Commutative property

$=7+\left[\left(-\frac{1}{3}x\right)+\frac{1}{3}x\right]$ Associative property

$=7+0$ Inverse property

$=7$ Identity property

9. **(a)** $4(3+7)=4\cdot 3+4\cdot 7$ Distributive property
$=12+28$ Multiply.
$=40$ Add.

(b) $5(2m-3)=5(2m)-5(3)$
$=10m-15$

(c) $-6(x+y-z)=-6x+(-6)y+(-6)(-z)$
$=-6x-6y+6z$

N9. **(a)** $2(p+5)=2(p)+2(5)$ Distributive prop.
$=2p+10$ Multiply.

(b) $-5(4x+1)=-5\cdot 4x+(-5\cdot 1)$
$=-20x-5$

(c) $6(2r+t-5z)=6(2r)+6t+6(-5z)$
$=12r+6t-30z$

10. **(a)** $-(-5y+8)=-1(-5y+8)$
$=5y-8$

(b) $-(x-y-z)=-1(x-y-z)$
$=-x+y+z$

N10. **(a)** $-(2-r)=-1(2-r)$
$=-2+r$

(b) $-(2x-5y-7)=-1(2x-5y-7)$
$=-2x+5y+7$

Exercises

1. **(a)** B, since 0 is the identity element for addition.

(b) F, since 1 is the identity element for multiplication.

(c) C, since $-a$ is the additive inverse of a.

(d) I, since $\frac{1}{a}$ is the multiplicative inverse, or reciprocal, of any nonzero number a.

(e) B, since 0 is the only number that is equal to its negative—that is, $0=-0$.

(f) D and F, since -1 has reciprocal $\frac{1}{(-1)}=-1$ and 1 has a reciprocal $\frac{1}{(1)}=1$—that is, -1 and 1 are their own multiplicative inverses.

(g) B, since the multiplicative inverse of a number a is $\frac{1}{a}$ and the only number that we cannot divide by is 0.

(h) A, because the equation $(5\cdot 4)\cdot 3=(3\cdot 4)\cdot 5$ is true by the associative property.

(i) G, since we can consider $(5\cdot 4)$ to be one number, $(5\cdot 4)\cdot 3$ is the same as $3\cdot(5\cdot 4)$ by the commutative property.

(j) H, because $5(4+3)$ is the same as $5\cdot 4+5\cdot 3$ by the distributive property.

2. The commutative property allows us to change the *order* of the addends in a sum or the factors in a product.
The associative property allows us to change the *grouping* of the addends in a sum or the factors in a product.

3. “Washing your face” and “brushing your teeth” are commutative.

4. “Putting on your left sock” and “putting on your right sock” are commutative.

5. “Preparing a meal” and “eating a meal” are not commutative.

6. "Starting a car" and "driving away in a car" are not commutative.

7. "Putting on your socks" and "putting on your shoes" are not commutative.

8. "Getting undressed" and "taking a shower" are not commutative.

9. "(Foreign sales) clerk" is a clerk dealing with foreign sales, whereas "foreign (sales clerk)" is a sales clerk who is foreign.

10. "(Defective merchandise) counter" is a location at which we would return an item that does not work, whereas "defective (merchandise counter)" is a broken place where items are bought and sold.

11. $$\begin{aligned} 25-(6-2) &= 25-(4) \\ &= 21 \\ (25-6)-2 &= 19-2 \\ &= 17 \end{aligned}$$
Since $21 \neq 17$, this example shows that subtraction is not associative.

12. $$\begin{aligned} 180 \div (15 \div 3) &= 180 \div 5 \\ &= 36 \\ (180 \div 15) \div 3 &= 12 \div 3 \\ &= 4 \end{aligned}$$
Since $36 \neq 4$, this example shows that division is not associative.

13. In general, a number and its additive inverse have *opposite* signs. A number and its multiplicative inverse have *the same* signs.

Number	**Additive Inverse**	**Multiplicative Inverse**
5	-5	$\frac{1}{5}$
-10	10	$-\frac{1}{10}$
$-\frac{1}{2}$	$\frac{1}{2}$	-2
$\frac{3}{8}$	$-\frac{3}{8}$	$\frac{8}{3}$
$x\,(x \neq 0)$	$-x$	$\frac{1}{x}$
$-y\,(y \neq 0)$	y	$-\frac{1}{y}$

14. Jack recognized the identity property of addition.

15. $-15+9=9+\underline{(-15)}$ by the commutative property of addition.

16. $6+(-2)=-2+\underline{6}$ by the commutative property of addition.

17. $-8\cdot 3=\underline{3}\cdot(-8)$ by the commutative property of multiplication.

18. $-12\cdot 4=4\cdot\underline{(-12)}$ by the commutative property of multiplication.

19. $(3+6)+7=3+(\underline{6}+7)$ by the associative property of addition.

20. $(-2+3)+6=-2+(\underline{3}+6)$ by the associative property of addition.

21. $7\cdot(2\cdot 5)=(\underline{7}\cdot 2)\cdot 5$ by the associative property of multiplication.

22. $8\cdot(6\cdot 4)=(8\cdot\underline{6})\cdot 4$ by the associative property of multiplication.

23. $4+15=15+4$
The order of the two numbers has been changed, so this is an example of the commutative property of addition: $a+b=b+a$.

24. $3+12=12+3$
The order of the two numbers has been changed, so this is an example of the commutative property of addition: $a+b=b+a$.

25. $5\cdot(13\cdot 7)=(5\cdot 13)\cdot 7$
The numbers are in the same order but grouped differently, so this is an example of the associative property of multiplication: $(ab)c=a(bc)$.

26. $-4(2\cdot 6)=(-4\cdot 2)\cdot 6$
The numbers are in the same order but grouped differently, so this is an example of the associative property of multiplication: $(ab)c=a(bc)$.

27. $-6+(12+7)=(-6+12)+7$
The numbers are in the same order but grouped differently, so this is an example of the associative property of addition: $(a+b)+c=a+(b+c)$.

28. $(-8+13)+2=-8+(13+2)$
The numbers are in the same order but grouped differently, so this is an example of the associative property of addition: $(a+b)+c=a+(b+c)$.

29. $-9+9=0$
The sum of the two numbers is 0, so they are additive inverses (or opposites) of each other. This is an example of the additive inverse property: $a+(-a)=0$.

30. $1+(-1)=0$
The sum of the two numbers is 0, so they are additive inverses (or opposites) of each other. This is an example of the additive inverse property: $a+(-a)=0$.

31. $\frac{2}{3}\left(\frac{3}{2}\right)=1$
The product of the two numbers is l, so they are multiplicative inverses (or reciprocals) of each other. This is an example of the multiplicative inverse property: $a\cdot\frac{1}{a}=1(a\neq 0)$.

32. $\frac{5}{8}\left(\frac{8}{5}\right)=1$
The product of the two numbers is l, so they are multiplicative inverses (or reciprocals) of each other. This is an example of the multiplicative inverse property: $a\cdot\frac{1}{a}=1\,(a\neq 0)$.

33. $1.75+0=1.75$
The sum of a number and 0 is the original number. This is an example of the identity property of addition: $a+0=a$.

34. $-8.45+0=-8.45$
The sum of a number and 0 is the original number. This is an example of the identity property of addition: $a+0=a$.

35. $(4+17)+3=3+(4+17)$
The order of the numbers has been changed, but the grouping has not, so this is an example of the commutative property of addition: $a+b=b+a$.

36. $(-8+4)+12=12+(-8+4)$
The order of the numbers has been changed, but the grouping has not, so this is an example of the commutative property of addition: $a+b=b+a$.

37. $2(x+y)=2x+2y$
The number 2 outside the parentheses is "distributed" over the x and y. This is an example of the distributive property.

38. $9(t+s)=9t+9s$
The number 9 outside the parentheses is "distributed" over the t and s. This is an example of the distributive property.

39. $-\frac{5}{9}=-\frac{5}{9}\cdot\frac{3}{3}=-\frac{15}{27}$

$\frac{3}{3}$ is a form of the number 1. We use it to rewrite $-\frac{5}{9}$ as $-\frac{15}{27}$. This is an example of the identity property of multiplication.

40. $-\frac{7}{12}=-\frac{7}{12}\cdot\frac{7}{7}=-\frac{49}{84}$

$\frac{7}{7}$ is a form of the number 1. We use it to rewrite $-\frac{7}{12}$ as $-\frac{49}{84}$. This is an example of the identity property of multiplication.

41. $4(2x)+4(3y)=4(2x+3y)$

This is an example of the distributive property. The number 4 is "distributed" over $2x$ and $3y$.

42. $6(5t)-6(7r)=6(5t-7r)$

This is an example of the distributive property. The number 6 is "distributed" over $5t$ and $7r$.

43. $$\begin{aligned}97+13+3+37&=(97+3)+(13+37)\\&=100+50\\&=150\end{aligned}$$

44. $$\begin{aligned}49+199+1+1&=(49+1)+(199+1)\\&=50+200\\&=250\end{aligned}$$

45. $$\begin{aligned}1999+2+1+8&=(1999+1)+(2+8)\\&=2000+10\\&=2010\end{aligned}$$

46. $$\begin{aligned}2998+3+2+17&=(2998+2)+(3+17)\\&=3000+20\\&=3020\end{aligned}$$

47. $$\begin{aligned}159+12+141+88&=(159+141)+(12+88)\\&=300+100\\&=400\end{aligned}$$

48. $$\begin{aligned}&106+8+(-6)+(-8)\\&=[106+(-6)]+[8+(-8)]\\&=100+0\\&=100\end{aligned}$$

49. $$\begin{aligned}&843+627+(-43)+(-27)\\&=[843+(-43)]+[627+(-27)]\\&=800+600\\&=1400\end{aligned}$$

50. $$\begin{aligned}&1846+1293+(-46)+(-93)\\&=[1846+(-46)]+[1293+(-93)]\\&=1800+1200\\&=3000\end{aligned}$$

51. $5(47)(2)=5(2)(47)=10(47)=470$

52. $2(79)(5)=2(5)(79)=10(79)=790$

53. $$\begin{aligned}-4\cdot5\cdot93\cdot5&=-4\cdot5\cdot5\cdot93\\&=-20\cdot5\cdot93\\&=-100\cdot93\\&=-9300\end{aligned}$$

54. $$\begin{aligned}2\cdot25\cdot67\cdot(-2)&=-2\cdot2\cdot25\cdot67\\&=-4\cdot25\cdot67\\&=-100\cdot67\\&=-6700\end{aligned}$$

55. $6t+8-6t+3$

$=6t+8+(-6t)+3$	Def. of subtraction
$=(6t+8)+(-6t)+3$	Order of operations
$=(8+6t)+(-6t)+3$	Commutative property
$=8+[6t+(-6t)]+3$	Associative property
$=8+0+3$	Inverse property
$=(8+0)+3$	Order of operations
$=8+3$	Identity property
$=11$	Add.

56. $9r+12-9r+1$

$=9r+12+(-9r)+1$	Def. of subtraction
$=(9r+12)+(-9r)+1$	Order of operations
$=(12+9r)+(-9r)+1$	Commutative property
$=12+[9r+(-9r)]+1$	Associative property
$=12+0+1$	Inverse property
$=(12+0)+1$	Order of operations
$=12+1$	Identity property
$=13$	Add.

57. $\frac{2}{3}x-11+11-\frac{2}{3}x$

$=\frac{2}{3}x+(-11)+11+\left(-\frac{2}{3}x\right)$

$=\left[\frac{2}{3}x+(-11)\right]+11+\left(-\frac{2}{3}x\right)$

$=\frac{2}{3}x+(-11+11)+\left(-\frac{2}{3}x\right)$ Associative prop.

$=\frac{2}{3}x+0+\left(-\frac{2}{3}x\right)$ Inverse property

$=\left(\frac{2}{3}x+0\right)+\left(-\frac{2}{3}x\right)$

$=\frac{2}{3}x+\left(-\frac{2}{3}x\right)$ Identity property

$=0$ Inverse property

58. $\frac{1}{5}y+4-4-\frac{1}{5}y$

$=\frac{1}{5}y+4+(-4)+\left(-\frac{1}{5}y\right)$

$=\left(\frac{1}{5}y+4\right)+(-4)+\left(-\frac{1}{5}y\right)$

$=\frac{1}{5}y+\left[4+(-4)\right]+\left(-\frac{1}{5}y\right)$ Associative prop.

$=\frac{1}{5}y+0+\left(-\frac{1}{5}y\right)$ Inverse property

$=\left(\frac{1}{5}y+0\right)+\left(-\frac{1}{5}y\right)$

$=\frac{1}{5}y+\left(-\frac{1}{5}y\right)$ Identity property

$=0$ Inverse property

59. $\left(\frac{9}{7}\right)(-0.38)\left(\frac{7}{9}\right)$

$=\left[\left(\frac{9}{7}\right)(-0.38)\right]\left(\frac{7}{9}\right)$ Order of operations

$=\left[(-0.38)\left(\frac{9}{7}\right)\right]\left(\frac{7}{9}\right)$ Commutative property

$=(-0.38)\left[\left(\frac{9}{7}\right)\left(\frac{7}{9}\right)\right]$ Associative property

$=(-0.38)(1)$ Inverse property

$=-0.38$ Identity property

60. $\left(\frac{4}{5}\right)(-0.73)\left(\frac{5}{4}\right)$

$=\left[\left(\frac{4}{5}\right)(-0.73)\right]\left(\frac{5}{4}\right)$ Order of operations

$=\left[(-0.73)\left(\frac{4}{5}\right)\right]\left(\frac{5}{4}\right)$ Commutative property

$=(-0.73)\left[\left(\frac{4}{5}\right)\left(\frac{5}{4}\right)\right]$ Associative property

$=(-0.73)(1)$ Inverse property

$=-0.73$ Identity property

61. $t+(-t)+\frac{1}{2}(2)$

$=t+(-t)+1$ Inverse property

$=\left[t+(-t)\right]+1$ Order of operations

$=0+1$ Inverse property

$=1$ Identity property

62. $w+(-w)+\frac{1}{4}(4)$

$=w+(-w)+1$ Inverse property

$=\left[w+(-w)\right]+1$ Order of operations

$=0+1$ Inverse property

$=1$ Identity property

63. When distributing a negative number over a quantity, be careful not to "lose" a negative sign. The problem should be worked in the following way.

$-3(4-6)=-3(4)-3(-6)$

$=-12+18$

$=6$

64. In the third line, -1 must also be distributed to the number 4.

$-(3x+4)=-1(3x+4)$

$=-1(3x)+(-1)4$

$=-3x-4$

65. $\frac{3}{4}=\frac{3}{4}\cdot 1=\frac{3}{4}\cdot\frac{3}{3}=\frac{9}{12}$

To rewrite $\frac{3}{4}$ as $\frac{9}{12}$, use the fact that $\frac{3}{3}$ is another name for the multiplicative identity element, 1.

66. $\frac{9}{12}=\frac{3}{3}\cdot\frac{3}{4}=1\cdot\frac{3}{4}=\frac{3}{4}$

To rewrite $\frac{9}{12}$ as $\frac{3}{4}$, use the fact that $\frac{3}{3}$ is another name for the multiplicative identity element, 1.

67. $5(9+8)=5\cdot 9+5\cdot 8$
$=45+40$
$=85$

68. $6(11+8)=6\cdot 11+6\cdot 8$
$=66+48$
$=114$

69. $4(t+3)=4\cdot t+4\cdot 3$
$=4t+12$

70. $5(w+4)=5\cdot w+5\cdot 4$
$=5w+20$

71. $7(z-8)=7[z+(-8)]$
$=7z+7(-8)$
$=7z-56$

72. $8(x-6)=8[x+(-6)]$
$=8x+8(-6)$
$=8x-48$

73. $-8(r+3)=-8(r)+(-8)(3)$
$=-8r+(-24)$
$=-8r-24$

74. $-11(x+4)=-11(x)+(-11)(4)$
$=-11x-44$

75. $-\frac{1}{4}(8x+3)$
$=-\frac{1}{4}(8x)+\left(-\frac{1}{4}\right)(3)$
$=\left[\left(-\frac{1}{4}\right)\cdot 8\right]x-\frac{3}{4}$
$=-2x-\frac{3}{4}$

76. $-\frac{1}{3}(9x+5)$
$=-\frac{1}{3}(9x)+\left(-\frac{1}{3}\right)(5)$
$=\left[\left(-\frac{1}{3}\right)\cdot 9\right]x-\frac{5}{3}$
$=-3x-\frac{5}{3}$

77. $-5(y-4)=-5(y)+(-5)(-4)$
$=-5y+20$

78. $-9(g-4)=-9(g)+(-9)(-4)$
$=-9g+36$

79. $2(6x+5)=2(6x)+2(5)$
$=12x+10$

80. $3(3x+4)=3(3x)+3(4)$
$=9x+12$

81. $-3(2x-5)=(-3)(2x)+(-3)(-5)$
$=-6x+15$

82. $-4(3x-2)=(-4)(3x)+(-4)(-2)$
$=-12x+8$

83. $-6(8x+1)=(-6)(8x)+(-6)(1)$
$=-48x-6$

84. $-5(4x+1)=(-5)(4x)+(-5)(1)$
$=-20x-5$

85. $-\frac{4}{3}(12y+15z)$
$=-\frac{4}{3}(12y)+\left(-\frac{4}{3}\right)(15z)$
$=\left[\left(-\frac{4}{3}\right)\cdot 12\right]y+\left[\left(-\frac{4}{3}\right)\cdot 15\right]z$
$=-16y+(-20)z$
$=-16y-20z$

86. $-\frac{2}{5}(10b+20a)$
$$\begin{aligned}&=-\frac{2}{5}(10b)+\left(-\frac{2}{5}\right)(20a)\\&=\left[\left(-\frac{2}{5}\right)\cdot 10\right]b+\left[\left(-\frac{2}{5}\right)\cdot 20\right]a\\&=-4b+(-8a)\\&=-4b-8a\end{aligned}$$

87. $8(3r+4s-5y)$
$$\begin{aligned}&=8(3r)+8(4s)+8(-5y) && \text{Dist. prop.}\\&=(8\cdot 3)r+(8\cdot 4)s+[8(-5)]y && \text{Assoc. prop.}\\&=24r+32s-40y && \text{Multiply.}\end{aligned}$$

88. $2(5u-3v+7w)$
$$\begin{aligned}&=2(5u)+2(-3v)+2(7w) && \text{Dist. prop.}\\&=(2\cdot 5)u+[2(-3)]v+(2\cdot 7)w && \text{Assoc. prop.}\\&=10u-6v+14w && \text{Multiply.}\end{aligned}$$

89. $-3(8x+3y+4z)$
$$\begin{aligned}&=-3(8x)+(-3)(3y)+(-3)(4z) && \text{Dist. prop.}\\&=(-3\cdot 8)x+(-3\cdot 3)y+(-3\cdot 4)z && \text{Assoc. prop.}\\&=-24x-9y-12z && \text{Multiply.}\end{aligned}$$

90. $-5(2x-5y+6z)$
$$\begin{aligned}&=-5(2x)+(-5)(-5y)+(-5)(6z) && \text{Dist. prop.}\\&=(-5\cdot 2)x+[-5(-5)]y+(-5\cdot 6)z && \text{Assoc. prop.}\\&=-10x+25y-30z && \text{Multiply.}\end{aligned}$$

91. $-(4t+3m)$
$$\begin{aligned}&=-1(4t+3m) && \text{Identity property}\\&=-1(4t)+(-1)(3m) && \text{Distributive property}\\&=(-1\cdot 4)t+(-1\cdot 3)m && \text{Associative property}\\&=-4t-3m && \text{Multiply.}\end{aligned}$$

92. $-(9x+12y)$
$$\begin{aligned}&=-1(9x+12y) && \text{Identity property}\\&=-1(9x)+(-1)(12y) && \text{Distributive property}\\&=(-1\cdot 9)x+(-1\cdot 12)y && \text{Associative property}\\&=-9x-12y && \text{Multiply.}\end{aligned}$$

93. $-(-5c-4d)$
$$\begin{aligned}&=-1(-5c-4d) && \text{Identity property}\\&=-1(-5c)+(-1)(-4d) && \text{Distributive property}\\&=(-1\cdot -5)c+(-1\cdot 4)d && \text{Associative property}\\&=5c+4d && \text{Multiply.}\end{aligned}$$

94. $-(-13x-15y)$
$$\begin{aligned}&=-1(-13x-15y)\\&=-1(-13x)+(-1)(-15y)\\&=(-1\cdot -13)x+(-1\cdot -15)y\\&=13x+15y\end{aligned}$$

95. $-(-q+5r-8s)$
$$\begin{aligned}&=-1(-q+5r-8s)\\&=-1(-q)+(-1)(5r)+(-1)(-8s)\\&=(-1\cdot -1)q+(-1\cdot 5)r+(-1\cdot -8)s\\&=q-5r+8s\end{aligned}$$

96. $-(-z+5w-9y)$
$$\begin{aligned}&=-1(-z+5w-9y)\\&=-1(-z)+(-1)(5w)+(-1)(-9y)\\&=(-1\cdot -1)z+(-1\cdot 5)w+(-1\cdot -9)y\\&=z-5w+9y\end{aligned}$$

1.7 Simplifying Expressions

Classroom Examples, Now Try Exercises

1. **(a)**
$$\begin{aligned}5(4x-3y)&=5(4x)-5(3y)\\&=(5\cdot 4)x-(5\cdot 3)y\\&=20x-15y\end{aligned}$$

(b)
$$\begin{aligned}-(7-6k)+9&=-1(7-6k)+9\\&=-1(7)-1(-6k)+9\\&=-7+6k+9\\&=-7+9+6k\\&=2+6k\end{aligned}$$

N1. **(a)**
$$\begin{aligned}3(2x-4y)&=3(2x)-3(4y)\\&=(3\cdot 2)x-(3\cdot 4)y\\&=6x-12y\end{aligned}$$

(b) $-4-(-3y+5)=-4-1(-3y+5)$
$=-4-1(-3y)-1(5)$
$=-4+3y+(-5)$
$=-4+(-5)+3y$
$=-9+3y,\ \text{or}\ \ 3y-9$

2. **(a)** $5z+9z-4z=(5+9-4)z=10z$

(b) $4r-r=4r-1r=(4-1)r=3r$

(c) $8p+8p^2$ cannot be simplified. $8p$ and $8p^2$ are unlike terms and cannot be combined.

N2. **(a)** $4x+6x-7x=(4+6-7)x=3x$

(b) $z+z=1z+1z=(1+1)z=2z$

(c) $4p^2-3p^2=(4-3)p^2=1p^2,\ \text{or}\ \ p^2$

3. **(a)** $-(3+5k)+7k=-1(3+5k)+7k$
$=-1(3)-1(5k)+7k$
$=-3-5k+7k$
$=-3+2k$

(b) $7z-2-(1+z)=7z-2-1(1+z)$
$=7z-2-1-z$
$=6z-3$

N3. **(a)** $5k-6-(3-4k)$
$=5k-6-1(3-4k)$
$=5k-6-1(3)-1(-4k)$
$=5k-6-3+4k$
$=9k-9$

(b) $\frac{1}{4}x-\frac{2}{3}(x-9)=\frac{1}{4}x-\frac{2}{3}(x)-\frac{2}{3}(-9)$
$=\frac{3}{12}x-\frac{8}{12}x+6$
$=-\frac{5}{12}x+6$

4. "Three times a number, subtracted from the sum of the number and 8" is written $(x+8)-3x$.
$(x+8)-3x=x+8-3x$
$=-2x+8$

N4. "Twice a number, subtracted from the sum of the number and 5" is written $(x+5)-2x$.
$(x+5)-2x=x+5-2x$
$=-x+5,\ \text{or}\ \ 5-x$

Exercises

1. $-(6x-3)=-1(6x-3)$
$=-1(6x)-1(-3)$
$=-6x+3$
The correct response is B.

2. The numerical coefficient of $5x^3y^7$ is 5. The correct response is A.

3. Examples A, B, and D are pairs of *unlike* terms, since either the variables or their powers are different. Example C is a pair of *like* terms, since both terms have the same variables (r and y) and the same exponents (both variables are to the first power). Note that we can use the commutative property to rewrite $6yr$ as $6ry$.

4. "Six times a number" translates as $6x$, and "the product of eleven and the number" translates as $11x$. Thus, the correct translation of "six times a number, subtracted from the product of eleven and the number" is B, $11x-6x$.

5. The student made a sign error when applying the distributive property.
$7x-2(3-2x)=7x-2(3)-2(-2x)$
$=7x-6+4x$
$=11x-6$
The correct answer is $11x-6$.

6. The student incorrectly started by adding $3+2$. First, 2 must be multiplied by $4x-5$.
$3+2(4x-5)=3+2(4x)+2(-5)$
$=3+8x-10$
$=8x-7$

7. $4r+19-8=4r+11$

8. $7t+18-4=7t+14$

9. $7(3x-4y)=7(3x)+7(-4y)$
$=21x-28y$

10. $8(2p-9q)=8(2p)+8(-9q)$
$=16p-72q$

11. $5+2(x-3y)=5+2(x)+2(-3y)$
$=5+2x-6y$

12. $8+3(s-6t)=8+3s+3(-6t)$
$=8+3s-18t$

13. $-2-(5-3p)=-2-1(5-3p)$
$=-2-1(5)-1(-3p)$
$=-2-5+3p$
$=-7+3p$

14. $-10-(7-14r)$
$=-10-1(7-14r)$
$=-10-1(7)-1(-14r)$
$=-10-7+14r$
$=-17+14r$

15. $6+(4-3x)-8=6+4-3x-8$
$=10-3x-8$
$=10-8-3x$
$=2-3x$

16. $-12+(7-8x)+6=-12+7-8x+6$
$=-5-8x+6$
$=-5+6-8x$
$=1-8x$

17. The numerical coefficient of the term $-12k$ is -12.

18. The numerical coefficient of the term $-11y$ is -11.

19. The numerical coefficient of the term $3m^2$ is 3.

20. The numerical coefficient of the term $9n^6$ is 9.

21. Because xw can be written as $1 \cdot xw$, the numerical coefficient of the term xw is 1.

22. Because pq can be written as $1 \cdot pq$, the numerical coefficient of the term pq is 1.

23. Since $-x=-1x$, the numerical coefficient of the term $-x$ is -1.

24. Since $-t=-1t$, the numerical coefficient of the term $-t$ is -1.

25. Since $\frac{x}{2}=\frac{1}{2}x$, the numerical coefficient of the term $\frac{x}{2}$ is $\frac{1}{2}$.

26. Since $\frac{x}{6}=\frac{1}{6}x$, the numerical coefficient of the term $\frac{x}{6}$ is $\frac{1}{6}$.

27. Since $\frac{2x}{5}=\frac{2}{5}x$, the numerical coefficient of the term $\frac{2x}{5}$ is $\frac{2}{5}$.

28. Since $\frac{8x}{9}=\frac{8}{9}x$, the numerical coefficient of the term $\frac{8x}{9}$ is $\frac{8}{9}$.

29. Since $-0.5x^3=-0.5 \cdot x^3$, the numerical coefficient of the term $-0.5x^3$ is -0.5.

30. Since $-1.75x^2=-1.75 \cdot x^2$, the numerical coefficient of the term $-1.75x^2$ is -1.75.

31. The numerical coefficient of the term 10 is 10.

32. The numerical coefficient of the term 15 is 15.

33. $8r$ and $-13r$ are like terms since they have the same variable with the same exponent (which is understood to be 1).

34. $-7x$ and $12x$ are like terms since they have the same variable with the same exponent (which is understood to be 1).

35. $5z^4$ and $9z^3$ are unlike terms. Although both have the variable z, the exponents are not the same.

36. $8x^5$ and $-10x^3$ are unlike terms. Although both have the variable x, the exponents are not the same.

37. All numerical terms (constants) are considered like terms, so 4, 9, and -24 are like terms.

38. All numerical terms (constants) are considered like terms, so 7, 17, and -83 are like terms.

39. x and y are unlike terms because they do not have the same variable.

40. t and s are unlike terms because they do not have the same variable.

41. $7y+6y=(7+6)y$
$=13y$

42. $5m+2m=(5+2)m$
$=7m$

43. $-6x-3x=(-6-3)x$
$=-9x$

44. $-4z-8z=(-4-8)z$
$=-12z$

45. $12b+b=12b+1b$
$=(12+1)b$
$=13b$

46. $19x+x=19x+1x$
$=(19+1)x$
$=20x$

47. $3k+8+4k+7=3k+4k+8+7$
$=(3+4)k+15$
$=7k+15$

48. $15z+1+4z+2=15z+4z+1+2$
$=(15+4)z+3$
$=19z+3$

49. $-5y+3-1+5+y-7$
$=(-5y+1y)+(3+5)+(-1-7)$
$=(-5+1)y+(8)+(-8)$
$=-4y+8-8$
$=-4y$

50. $2k-7-5k+6-1+2$
$=(2k-5k)+(-7+6-1+2)$
$=(2-5)k+(0)$
$=-3k$

51. $-2x+3+4x-17+20$
$=(-2x+4x)+(3-17+20)$
$=(-2+4)x+6$
$=2x+6$

52. $r-6-12r-4+16$
$=(1r-12r)+(-6-4+16)$
$=(1-12)r+(6)$
$=-11r+6$

53. $16-5m-4m-2+2m$
$=(16-2)+(-5m-4m+2m)$
$=14+(-5-4+2)m$
$=14-7m$

54. $6-3z-2z-5-2z$
$=(6-5)+(-3z-2z-2z)$
$=1+(-3-2-2)z$
$=1-7z$

55. $-10+x+4x-7-4x$
$=(-10-7)+(1x+4x-4x)$
$=-17+(1+4-4)x$
$=-17+1x$
$=-17+x$

56. $-p+10p-3p-4-5p$
$=(-1p+10p-3p-5p)+(-4)$
$=(-1+10-3-5)p-4$
$=1p-4$
$=p-4$

57. $1+7x+11x-1+5x$
$=(1-1)+(7x+11x+5x)$
$=0+(7+11+5)x$
$=23x$

58. $-r+2-5r-2+4r$
$=(-1r-5r+4r)+(2-2)$
$=(-1-5+4)r+(0)$
$=-2r$

59. $-\frac{4}{3}+2t+\frac{1}{3}t-8-\frac{8}{3}t$
$=\left(2t+\frac{1}{3}t-\frac{8}{3}t\right)+\left(-\frac{4}{3}-8\right)$
$=\left(2+\frac{1}{3}-\frac{8}{3}\right)t+\left(-\frac{4}{3}-8\right)$
$=\left(\frac{6}{3}+\frac{1}{3}-\frac{8}{3}\right)t+\left(-\frac{4}{3}-\frac{24}{3}\right)$
$=-\frac{1}{3}t-\frac{28}{3}$

60. $-\frac{5}{6}+8x+\frac{1}{6}x-7-\frac{7}{6}$
$=\left(8x+\frac{1}{6}x\right)+\left(-\frac{5}{6}-7-\frac{7}{6}\right)$
$=\left(\frac{48}{6}x+\frac{1}{6}x\right)+\left(-\frac{5}{6}-\frac{42}{6}-\frac{7}{6}\right)$
$=\frac{49}{6}x-\frac{54}{6}$
$=\frac{49}{6}x-9$

61. $6y^2+11y^2-8y^2=(6+11-8)y^2$
$=9y^2$

62. $-9m^3+3m^3-7m^3=(-9+3-7)m^3$
$=-13m^3$

63. $2p^2+3p^2-8p^3-6p^3$
$=(2p^2+3p^2)+(-8p^3-6p^3)$
$=(2+3)p^2+(-8-6)p^3$
$=5p^2-14p^3$ or $-14p^3+5p^2$

64. $5y^3+6y^3-3y^2-4y^2$
$=(5y^3+6y^3)+(-3y^2-4y^2)$
$=(5+6)y^3+(-3-4)y^2$
$=11y^3-7y^2$

65. $2(4x+6)+3=2(4x)+2(6)+3$
$=8x+12+3$
$=8x+15$

66. $4(6y+9)+7=4(6y)+4(9)+7$
$=24y+36+7$
$=24y+43$

67. $-6-4(y-7)$
$=-6-4(y)+(-4)(-7)$ Distributive prop.
$=-6-4y+28$
$=-4y+22$

68. $-4-5(t-13)$
$=-4-5(t)+(-5)(-13)$ Distributive prop.
$=-4-5t+65$
$=-5t+61$

69. $13p+4(4-8p)=13p+4(4)+4(-8p)$
$=13p+16-32p$
$=-19p+16$

70. $5x+3(7-2x)=5x+3(7)+3(-2x)$
$=5x+21-6x$
$=-x+21$

71. $3t-5-2(2t-4)=3t-5-2(2t)-2(-4)$
$=3t-5-4t+8$
$=-t+3$

72. $8p+6-3(3p-1)=8p+6-3(3p)-3(-1)$
$=8p+6-9p+3$
$=-p+9$

73. $100[0.05(x+3)]$
$=[100(0.05)](x+3)$ Associative property
$=5(x+3)$
$=5(x)+5(3)$ Distributive prop.
$=5x+15$

74. $100[0.06(x+5)]$
$=[100(0.06)](x+5)$ Associative prop.
$=6(x+5)$
$=6(x)+6(5)$ Distributive prop.
$=6x+30$

75. $10[0.3(5-3x)]$
$=[10(0.3)](5-3x)$ Associative prop.
$=3(5-3x)$
$=3(5)+3(-3x)$ Distributive prop.
$=15-9x$

76. $10[0.5(8-2z)]$
$=[10(0.5)](8-2z)$ Associative prop.
$=5(8-2z)$
$=5(8)+5(-2z)$ Distributive prop.
$=40-10z$

77. $-5(5y-9)+3(3y+6)$
$=-5(5y)+(-5)(-9)+3(3y)+3(6)$
$=-25y+45+9y+18$
$=(-25y+9y)+(45+18)$
$=(-25+9)y+63$
$=-16y+63$

78. $-3(2t+4)+8(2t-4)$
$=-3(2t)+(-3)(4)+8(2t)+8(-4)$
$=-6t-12+16t-32$
$=(-6t+16t)+(-12-32)$
$=(-6+16)t+(-44)$
$=10t-44$

79. $2(5r+3)-3(2r-3)$
$= 2(5r)+2(3)+(-3)(2r)+(-3)(-3)$
$= 10r+6-6r+9$
$= (10r-6r)+(6+9)$
$= (10-6)r+(15)$
$= 4r+15$

80. $3(2y-5)-4(5y-7)$
$= 3(2y)+3(-5)+(-4)(5y)+(-4)(-7)$
$= 6y-15-20y+28$
$= (6y-20y)+(-15+28)$
$= (6-20)y+(13)$
$= -14y+13$

81. $8(2k-1)-(4k-3)$
$= 8(2k)+8(-1)+(-1)(4k)+(-1)(-3)$
$= 16k-8-4k+3$
$= (16k-4k)+(-8+3)$
$= (16-4)k+(-5)$
$= 12k-5$

82. $6(3p-2)-(5p+1)$
$= 6(3p)+6(-2)+(-1)(5p)+(-1)(1)$
$= 18p-12-5p-1$
$= (18p-5p)+(-12-1)$
$= (18-5)p+(-13)$
$= 13p-13$

83. $-\frac{4}{3}(y-12)-\frac{1}{6}y$
$= -\frac{4}{3}y-\frac{4}{3}(-12)-\frac{1}{6}y$
$= -\frac{4}{3}y+16-\frac{1}{6}y$
$= -\frac{4}{3}y-\frac{1}{6}y+16$
$= \left(-\frac{8}{6}-\frac{1}{6}\right)y+16$
$= -\frac{3}{2}y+16$

84. $-\frac{7}{5}(t-15)-\frac{1}{2}t = -\frac{7}{5}t-\frac{7}{5}(-15)-\frac{1}{2}t$
$= -\frac{14}{10}t+21-\frac{5}{10}t$
$= -\frac{19}{10}t+21$

85. $\frac{1}{2}(2x+4)-\frac{1}{3}(9x-6)$
$= \frac{1}{2}(2x)+\frac{1}{2}(4)+\left(-\frac{1}{3}\right)(9x)+\left(-\frac{1}{3}\right)(-6)$
$= x+2-3x+2$
$= -2x+4$

86. $\frac{1}{4}(8x+16)-\frac{1}{5}(20x-15)$
$= \frac{1}{4}(8x)+\frac{1}{4}(16)+\left(-\frac{1}{5}\right)(20x)+\left(-\frac{1}{5}\right)(-15)$
$= 2x+4-4x+3$
$= -2x+7$

87. $-\frac{2}{3}(5x+7)-\frac{1}{3}(4x+8)$
$= \left(-\frac{2}{3}\right)5x+\left(-\frac{2}{3}\right)7+\left(-\frac{1}{3}\right)4x+\left(-\frac{1}{3}\right)8$
$= -\frac{10x}{3}-\frac{14}{3}-\frac{4x}{3}-\frac{8}{3}$
$= -\frac{14x}{3}-\frac{22}{3}$

88. $-\frac{3}{4}(7x+9)-\frac{1}{4}(5x+7)$
$= \left(-\frac{3}{4}\right)7x+\left(-\frac{3}{4}\right)9+\left(-\frac{1}{4}\right)5x+\left(-\frac{1}{4}\right)7$
$= -\frac{21x}{4}-\frac{27}{4}-\frac{5x}{4}-\frac{7}{4}$
$= -\frac{13x}{2}-\frac{17}{2}$

89. $-7.5(2y+4)-2.9(3y-6)$
$= -7.5(2y)-7.5(4)-2.9(3y)-2.9(-6)$
$= -15y-30-8.7y+17.4$
$= -23.7y-12.6$

90. $8.4(6t-6)+2.4(9-3t)$
$= 8.4(6t)+8.4(-6)+2.4(9)+2.4(-3t)$
$= 50.4t-50.4+21.6-7.2t$
$= 43.2t-28.8$

91. $-2(-3k+2)-(5k-6)-3k-5$
$=-2(-3k)+(-2)(2)-1(5k-6)-3k-5$
$=6k-4+(-1)(5k)+(-1)(-6)-3k-5$
$=6k-4-5k+6-3k-5$
$=-2k-3$

92. $-2(3r-4)-(6-r)+2r-5$
$=-2(3r)+(-2)(-4)-1(6-r)+2r-5$
$=-6r+8+(-1)(6)+(-1)(-r)+2r-5$
$=-6r+8-6+r+2r-5$
$=-3r-3$

93. $-4(-3x+3)-(6x-4)-2x+1$
$=-4(-3x+3)-1(6x-4)-2x+1$
$=12x-12-6x+4-2x+1$
$=(12x-6x-2x)+(-12+4+1)$
$=4x-7$

94. $-5(8x+2)-(5x-3)-3x+17$
$=-5(8x+2)-1(5x-3)-3x+17$
$=-40x-10-5x+3-3x+17$
$=-48x+10$

95. $(4x+8)+(3x-2)$
$=4x+8+3x-2$
$=7x+6$

96. $(10t-8)+(8t+5)$
$=10t-8+8t+5$
$=18t-3$

97. $(5x+1)-(x-7)$
$=5x+1-x+7$
$=4x+8$

98. $(2x-3)-(3x-5)$
$=2x-3-3x+5$
$=-x+2$

99. "Five times a number, added to the sum of the number and three" is written $(x+3)+5x$.
$(x+3)+5x=x+3+5x$
$=(x+5x)+3$
$=6x+3$

100. "Six times a number, added to the sum of the number and six" is written $(x+6)+6x$.
$(x+6)+6x=x+6+6x$
$=(x+6x)+6$
$=7x+6$

101. "A number multiplied by -7, subtracted from the sum of 13 and six times the number" is written $(13+6x)-(-7x)$.
$(13+6x)-(-7x)=13+6x+7x$
$=13+13x$

102. "A number multiplied by 5, subtracted from the sum of 14 and eight times the number" is written $(14+8x)-5x$.
$(14+8x)-5x=14+8x-5x$
$=14+3x$

103. "Six times a number added to -4, subtracted from twice the sum of three times the number and 4" is written $2(3x+4)-(-4+6x)$.
$2(3x+4)-(-4+6x)$
$=2(3x+4)-1(-4+6x)$
$=6x+8+4-6x$
$=6x+(-6x)+8+4$
$=0+12=12$

104. "Nine times a number added to 6, subtracted from triple the sum of 12 and 8 times the number" is written $3(12+8x)-(6+9x)$.
$3(12+8x)-(6+9x)$
$=3(12+8x)-1(6+9x)$
$=36+24x-6-9x$
$=30+15x$

105. For gizmos, the fixed cost is \$1000 and the variable cost is \$5 per gizmo, so the cost to produce x gizmos is $1000+5x$ (dollars).

106. For gadgets, the fixed cost is \$750 and the variable cost is \$3 per gadget, so the cost to produce y gadgets is $750+3y$ (dollars).

107. The total cost to make x gizmos and y gadgets is $1000+5x+750+3y$ (dollars).

108. $1000+5x+750+3y$
$=(1000+750)+5x+3y$
$=1750+5x+3y,$
so the total cost to make x gizmos and y gadgets is $1750+5x+3y$ (dollars).

Chapter 1 Review Exercises

1. $5^4=5\cdot5\cdot5\cdot5=625$

2. $\left(\frac{3}{5}\right)^3=\frac{3}{5}\cdot\frac{3}{5}\cdot\frac{3}{5}=\frac{27}{125}$

3. $\left(\frac{1}{8}\right)^2=\left(\frac{1}{8}\right)\left(\frac{1}{8}\right)$
$=\frac{1}{64}$

4. $(0.1)^3=(0.1)(0.1)(0.1)$
$=0.001$

5. $8\cdot5-13=40-13=27$

6. $16+12\div4-2=16+(12\div4)-2$
$=16+3-2$
$=19-2$
$=17$

7. $20-2(5+3)=20-2(8)$
$=20-16$
$=4$

8. $7\left[3+6\left(3^2\right)\right]=7\left[3+6(9)\right]$
$=7(3+54)$
$=7(57)$
$=399$

9. $\frac{9\left(4^2-3\right)}{4\cdot5-17}=\frac{9(16-3)}{20-17}$
$=\frac{9(13)}{3}$
$=\frac{3\cdot3\cdot13}{3}=39$

10. $\frac{6(5-4)+2(4-2)}{3^2-(4+3)}=\frac{6(1)+2(2)}{9-(4+3)}$
$=\frac{6+4}{9-7}$
$=\frac{10}{2}=5$

11. $12\cdot3-6\cdot6=36-36=0$
Since $0=0$ is true, so is $0\le0$, and therefore, the statement "$12\cdot3-6\cdot6\le0$" is true.

12. $3[5(2)-3]=3(10-3)=3(7)=21$
Therefore, the statement "$3[5(2)-3]>20$" is true.

13. $4^2-8=16-8=8$
Since $9\le8$ is false, the statement "$9\le4^2-8$" is false.

14. "Thirteen is less than seventeen" is written $13<17$.

15. "Five plus two is not equal to ten" is written $5+2\ne10$.

16. "Two-thirds is greater than or equal to four-sixths" is written $\frac{2}{3}\ge\frac{4}{6}$.

17. $2x+6y=2(6)+6(3)$
$=12+18=30$

18. $4(3x-y)=4[3(6)-3]$
$=4(18-3)$
$=4(15)=60$

19. $\frac{x}{3}+4y=\frac{6}{3}+4(3)$
$=2+12=14$

20. $\frac{x^2+3}{3y-x}=\frac{6^2+3}{3(3)-6}$
$=\frac{36+3}{9-6}$
$=\frac{39}{3}=13$

21. "Six added to a number" translates as $x+6$.

22. "A number subtracted from eight" translates as $8-x$.

23. "Nine subtracted from six times a number" translates as $6x-9$.

24. "Three-fifths of a number added to 12" translates as $12+\frac{3}{5}x$.

25. $5x+3(x+2)=22;\ 2$

$$\begin{aligned} 5x+3(x+2) &= 5(2)+3(2+2) \quad \text{Let } x=2. \\ &= 5(2)+3(4) \\ &= 10+12=22 \end{aligned}$$

Since the left side and the right side are equal, 2 is a solution of the given equation.

26. $\frac{t+5}{3t}=1;\ 6$

$$\begin{aligned} \frac{t+5}{3t} &= \frac{6+5}{3(6)} \quad \text{Let } t=6. \\ &= \frac{11}{18} \end{aligned}$$

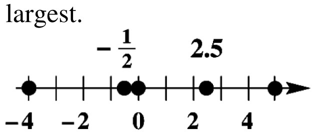
Since the left side, $\frac{11}{18}$, is not equal to the right side, 6 is not a solution of the equation.

27. "Six less than twice a number is 10" is written $2x-6=10$.
Letting x equal 0, 2, 4, 6, and 10 results in a false statement, so those values are not solutions. Since $2(8)-6=16-6=10$, the solution is 8.

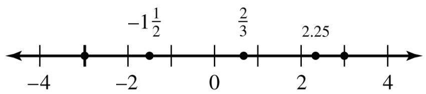
28. "The product of a number and 4 is 8" is written $4x=8$. Since $4(2)=8$, the solution is 2.

29. $-4, -\frac{1}{2}, 0, 2.5, 5$

Graph these numbers on a number line. They are already arranged in order from smallest to largest.



30. $-3, -1\frac{1}{2}, \frac{2}{3}, 2.25, 3$



31. Since $\frac{4}{3}$ is the quotient of two integers, it is a rational number. Since all rational numbers are also real numbers, $\frac{4}{3}$ is a real number.

32. Since the decimal representation of $0.\overline{63}$ repeats, it is a rational number. Since all rational numbers are also real numbers, $0.\overline{63}$ is a real number.

33. Since 19 is a natural number, it is also a whole number and an integer. We can write it as $\frac{19}{1}$, so it is a rational number and, hence, a real number.

34. Since the decimal representation of $\sqrt{6}$ does not terminate or repeat, it is an irrational number. Since all irrational numbers are also real numbers, $\sqrt{6}$ is a real number.

35. Since any negative number is less than any positive number, -10 is the lesser number.

36. Since -9 is to the left of -8 on the number line, -9 is the lesser number.

37. To compare these fractions, use a common denominator.

$-\frac{2}{3}=-\frac{8}{12}, -\frac{3}{4}=-\frac{9}{12}$

Since $-\frac{9}{12}$ is to the left of $-\frac{8}{12}$ on the number line, $-\frac{3}{4}$ is the lesser number.

38. Since $-|23|=-23$ and $-23<0$, $-|23|$ is the lesser number.

39. The statement $12>-13$ is true since 12 is to the right of -13 on the number line.

40. The statement $0>-5$ is true since 0 is to the right of -5 on the number line.

41. The statement $-9<-7$ is true since -9 is to the left of -7 on the number line.

42. The statement $-13\geq-13$ is true since $-13=-13$.

43. **(a)** The opposite of the number -9 is its negative—that is, $-9(-9)=9$.

(b) Since $-9<0$, the absolute value of the number -9 is $|-9|=-(-9)=9$.

44. **(a)** $-0=0$

(b) $|0|=0$

45. **(a)** $-(6)=-6$

(b) $|6|=6$

46. **(a)** $-\left(-\frac{5}{7}\right)=\frac{5}{7}$

(b) $\left|-\frac{5}{7}\right|=-\left(-\frac{5}{7}\right)=\frac{5}{7}$

47. $|-12|=-(-12)=12$

48. $-|3|=-3$

49. $-|-19|=-[-(-19)]=-19$

50. $-|9-2|=-|7|=-7$

51. $-10+4=-6$

52. $14+(-18)=-4$

53. $-8+(-9)=-17$

54. $\frac{4}{9}+\left(-\frac{5}{4}\right)=\frac{4\cdot 4}{9\cdot 4}+\left(-\frac{5\cdot 9}{4\cdot 9}\right)$ LCD $=36$

$$=\frac{16}{36}+\left(-\frac{45}{36}\right)$$
$$=-\frac{29}{36}$$

55. $-13.5+(-8.3)=-21.8$

56. $(-10+7)+(-11)=(-3)+(-11)$
$$=-14$$

57. $[-6+(-8)+8]+[9+(-13)]$
$$=\{[-6+(-8)]+8\}+(-4)$$
$$=[(-14)+8]+(-4)$$
$$=(-6)+(-4)=-10$$

58. $(-4+7)+(-11+3)+(-15+1)$
$$=(3)+(-8)+(-14)$$
$$=[3+(-8)]+(-14)$$
$$=(-5)+(-14)=-19$$

59. $-7-4=-7+(-4)=-11$

60. $-12-(-11)=-12+(11)=-1$

61. $5-(-2)=5+(2)=7$

62. $-\frac{3}{7}-\frac{4}{5}=-\frac{3\cdot 5}{7\cdot 5}-\frac{4\cdot 7}{5\cdot 7}$
$$=-\frac{15}{35}-\frac{28}{35}\qquad \text{LCD}=35$$
$$=-\frac{15}{35}+\left(-\frac{28}{35}\right)$$
$$=-\frac{43}{35},\text{ or }-1\frac{8}{35}$$

63. $2.56-(-7.75)=2.56+(7.75)$
$$=10.31$$

64. $(-10-4)-(-2)=[-10+(-4)]+2$
$$=(-14)+(2)$$
$$=-12$$

65. $(-3+4)-(-1)=(-3+4)+1$
$$=1+1$$
$$=2$$

66. $-(-5+6)-2=-(1)+(-2)$
$$=-1+(-2)$$
$$=-3$$

67. "19 added to the sum of -31 and 12" is written
$$(-31+12)+19=(-19)+19$$
$$=0.$$

68. "13 more than the sum of -4 and -8" is written
$$[-4+(-8)]+13=-12+13$$
$$=1.$$

69. "The difference between -4 and -6" is written
$$-4-(-6)=-4+6$$
$$=2.$$

70. "Five less than the sum of 4 and -8" is written
$$\begin{aligned}[4+(-8)]-5 &= (-4)+(-5)\\ &= -9.\end{aligned}$$

71. $-23.75+50.00=26.25$
He now has a positive balance of \$26.25.

72. $-26+16=-10$
The high temperature was $-10°$F.

73.
$$\begin{aligned}-28+13-14 &= (-28+13)-14\\ &= (-28+13)+(-14)\\ &= -15+(-14)\\ &= -29\end{aligned}$$
His present financial status is $-\$29$.

74.
$$\begin{aligned}-3-7 &= -3+(-7)\\ &= -10\end{aligned}$$
The new temperature is $-10°$.

75.
$$\begin{aligned}8-12+42 &= [8+(-12)]+42\\ &= -4+42\\ &= 38\end{aligned}$$
The total net yardage is 38.

76. To get the closing value for the previous day, we can add the amount it was down to the amount at which it closed.
$14,810.31+30.64=14,840.95$

77. $(-12)(-3)=36$

78.
$$\begin{aligned}15(-7) &= -(15\cdot 7)\\ &= -105\end{aligned}$$

79.
$$\begin{aligned}-\frac{4}{3}\left(-\frac{3}{8}\right) &= \frac{4}{3}\cdot\frac{3}{8}\\ &= \frac{4\cdot 3}{3\cdot 4\cdot 2}\\ &= \frac{1}{2}\end{aligned}$$

80. $(-4.8)(-2.1)=10.08$

81.
$$\begin{aligned}5(8-12) &= 5[8+(-12)]\\ &= 5(-4)=-20\end{aligned}$$

82.
$$\begin{aligned}(5-7)(8-3) &= [5+(-7)][8+(-3)]\\ &= (-2)(5)=-10\end{aligned}$$

83.
$$\begin{aligned}2(-6)-(-4)(-3) &= -12-(12)\\ &= -12+(-12)\\ &= -24\end{aligned}$$

84. $3(-10)-5=-30+(-5)=-35$

85. $\dfrac{-36}{-9}=\dfrac{4\cdot 9}{9}=4$

86. $\dfrac{220}{-11}=-\dfrac{20\cdot 11}{11}=-20$

87. $-\dfrac{1}{2}\div\dfrac{2}{3}=-\dfrac{1}{2}\cdot\dfrac{3}{2}=-\dfrac{3}{4}$

88. $\dfrac{-33.9}{-3}=\dfrac{11.3\cdot(-3)}{-3}=11.3$

89.
$$\begin{aligned}\frac{-5(3)-1}{8-4(-2)} &= \frac{-15+(-1)}{8-(-8)}\\ &= \frac{-16}{8+8}\\ &= \frac{-16}{16}=-1\end{aligned}$$

90.
$$\begin{aligned}\frac{5(-2)-3(4)}{-2[3-(-2)]-1} &= \frac{-10-12}{-2(3+2)-1}\\ &= \frac{-10+(-12)}{-2(5)-1}\\ &= \frac{-22}{-10+(-1)}\\ &= \frac{-22}{-11}=2\end{aligned}$$

91.
$$\begin{aligned}\frac{10^2-5^2}{8^2+3^2-(-2)} &= \frac{100-25}{64+9+2}\\ &= \frac{75}{75}=1\end{aligned}$$

92.
$$\begin{aligned}\frac{(0.6)^2+(0.8)^2}{(-1.2)^2-(-0.56)} &= \frac{0.36+0.64}{1.44+0.56}\\ &= \frac{1.00}{2.00}=0.5\end{aligned}$$

93.
$$\begin{aligned}6x-4z &= 6(-5)-4(-3)\\ &= -30-(-12)\\ &= -30+12=-18\end{aligned}$$

94. $5x + y - z = 5(-5) + (4) - (-3)$
$= (-25 + 4) + 3$
$= -21 + 3 = -18$

95. $5x^2 = 5(-5)^2$
$= 5(25)$
$= 125$

96. $z^2(3x - 8y) = (-3)^2[3(-5) - 8(4)]$
$= 9(-15 - 32)$
$= 9[-15 + (-32)]$
$= 9(-47) = -423$

97. "Nine less than the product of -4 and 5" is written
$-4(5) - 9 = -20 + (-9)$
$= -29.$

98. "Five-sixths of the sum of 12 and -6" is written
$\frac{5}{6}[12 + (-6)] = \frac{5}{6}(6)$
$= 5.$

99. "The quotient of 12 and the sum of 8 and -4" is written
$\frac{12}{8 + (-4)} = \frac{12}{4} = 3.$

100. "The product of -20 and 12, divided by the difference of 15 and -15" is written
$\frac{-20(12)}{15 - (-15)} = \frac{-240}{15 + 15}$
$= \frac{-240}{30} = -8.$

101. "8 times a number is -24" is written $8x = -24.$
If $x = -3,$ $8x = 8(-3) = -24.$ The solution is $-3.$

102. "The quotient of a number and 3 is -2" is written $\frac{x}{3} = -2.$ If $x = -6,$ $\frac{x}{3} = \frac{-6}{3} = -2.$ The solution is $-6.$

103. The statement $6 + 0 = 6$ is an example of an identity property.

104. The statement $5 \cdot 1 = 5$ is an example of an identity property.

105. The statement $-\frac{2}{3}\left(-\frac{3}{2}\right) = 1$ is an example of an inverse property.

106. The statement $17 + (-17) = 0$ is an example of an inverse property.

107. The statement $5 + (-9 + 2) = [5 + (-9)] + 2$ is an example of an associative property.

108. The statement $w(xy) = (wx)y$ is an example of an associative property.

109. The statement $3(x + y) = 3x + 3y$ is an example of the distributive property.

110. The statement $(1 + 2) + 3 = 3 + (1 + 2)$ is an example of a commutative property.

111. $7(y + 2) = 7y + 7 \cdot 2$
$= 7y + 14$

112. $-12(4 - t) = -12(4) - (-12)(t)$
$= -48 + 12t$

113. $3(2s + 5y) = 3(2s) + 3(5y)$
$= 6s + 15y$

114. $-(-4r + 5s) = -1(-4r + 5s)$
$= (-1)(-4r) + (-1)(5s)$
$= 4r - 5s$

115. $2m + 9m = (2 + 9)m$
$= 11m$

116. $15p^2 - 7p^2 + 8p^2$
$= (15 - 7 + 8)p^2$
$= 16p^2$

117. $5p^2 - 4p + 6p + 11p^2$
$= (5 + 11)p^2 + (-4 + 6)p$
$= 16p^2 + 2p$

118. $-2(3k - 5) + 2(k + 1)$
$= -6k + 10 + 2k + 2$
$= -4k + 12$

119. $7(2m + 3) - 2(8m - 4)$
$= 14m + 21 - 16m + 8$
$= (14 - 16)m + 29$
$= -2m + 29$

120. $-(2k+8)-(3k-7)$
$= -1(2k+8)-1(3k-7)$
$= -2k-8-3k+7$
$= -5k-1$

121. "Seven times a number, subtracted from the product of -2 and three times the number" is written $-2(3x)-7x = -6x-7x = -13x.$

122. "A number multiplied by 8, added to the sum of 5 and four times the number" is written $(5+4x)+8x = 5+(4x+8x) = 5+12x.$

Chapter 1 Mixed Review Exercises

1. Complete the 1st row of the table.

Number	Absolute Value	Additive Inverse	Multiplicative Inverse
-3	3	3	$-\frac{1}{3}$

2. Complete the 2nd row of the table.

Number	Absolute Value	Additive Inverse	Multiplicative Inverse
12	12	-12	$\frac{1}{12}$

3. Complete the 3rd row of the table.

Number	Absolute Value	Additive Inverse	Multiplicative Inverse
$-\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{3}{2}$

4. Complete the 4th row of the table.

Number	Absolute Value	Additive Inverse	Multiplicative Inverse
0.2	0.2	-0.2	5

5. The repeating decimal $0.\overline{6}$ is a rational number. All rational numbers are real numbers, so it is also a real number.

6. $(x+6)^3 - y^3$
$= (-2+6)^3 - (3)^3$
$= (4)^3 - (27)$
$= 64 - 27$
$= 37$

7. $$\frac{6(-4)+2(-12)}{5(-3)+(-3)} = \frac{-24+(-24)}{-15+(-3)} = \frac{-48}{-18} = \frac{8\cdot 6}{3\cdot 6} = \frac{8}{3}, \quad \text{or} \quad 2\frac{2}{3}$$

8. $$\frac{3}{8}-\frac{5}{12} = \frac{3\cdot 3}{8\cdot 3}-\frac{5\cdot 2}{12\cdot 2} = \frac{9}{24}-\frac{10}{24} = \frac{9}{24}+\left(-\frac{10}{24}\right) = -\frac{1}{24}$$

9. $$\frac{8^2+6^2}{7^2+1^2} = \frac{64+36}{49+1} = \frac{100}{50} = 2$$

10. $$-\frac{12}{5}\div\frac{9}{7} = -\frac{12}{5}\cdot\frac{7}{9} = -\frac{12\cdot 7}{5\cdot 9} = -\frac{3\cdot 4\cdot 7}{5\cdot 3\cdot 3} = -\frac{28}{15}, \quad \text{or} \quad -1\frac{13}{15}$$

11. $2\frac{5}{6}-4\frac{1}{3}=\frac{17}{6}-\frac{13}{3}$
$=\frac{17}{6}-\frac{13\cdot 2}{3\cdot 2}$
$=\frac{17}{6}-\frac{26}{6}$
$=\frac{17}{6}+\left(-\frac{26}{6}\right)$
$=-\frac{9}{6}=-\frac{3}{2},\quad \text{or}\quad -1\frac{1}{2}$

12. $\left(\frac{5}{6}\right)^2=\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)$
$=\frac{25}{36}$

13. $\left[(-2)+7-(-5)\right]+\left[-4-(-10)\right]$
$=\left\{\left[(-2)+7\right]-(-5)\right\}+(-4+10)$
$=(5+5)+6$
$=10+6=16$

14. $-16(-3.5)-7.2(-3)$
$=56-\left[(7.2)(-3)\right]$
$=56-(-21.6)$
$=56+21.6$
$=77.6$

15. $-8+\left[(-4+17)-(-3-3)\right]$
$=-8+\left\{(13)-\left[-3+(-3)\right]\right\}$
$=-8+\left[13-(-6)\right]$
$=-8+(13+6)$
$=-8+19=11$

16. $-4(2t+1)-8(-3t+4)$
$=-4(2t)-4(1)-8(-3t)-8(4)$
$=-8t-4+24t-32$
$=16t-36$

17. $5x^2-12y^2+3x^2-9y^2$
$=\left(5x^2+3x^2\right)+\left(-12y^2-9y^2\right)$
$=(5+3)x^2+(-12-9)y^2$
$=8x^2-21y^2$

18. $(-8-3)-5(2-9)$
$=\left[-8+(-3)\right]-5\left[2+(-9)\right]$
$=-11-5(-7)$
$=-11-(-35)$
$=-11+35=24$

19. $118-165=118+(-165)$
$=-47$
The lowest temperature ever recorded in Iowa was -47°F.

20. $14,494-(-282)=14,494+282$
$=14,776$
The difference in elevation is 14,776 ft.

Chapter 1 Test

1. $4\left[-20+7(-2)\right]=4\left[-20+(-14)\right]$
$=4(-34)=-136$
Since $-136\le 135$, the statement "$4\left[-20+7(-2)\right]\le 135$" is true.

2. $\left(\frac{1}{2}\right)^2+\left(\frac{2}{3}\right)^2=\left(\frac{1}{2}+\frac{2}{3}\right)^2$
$\frac{1}{4}+\frac{4}{9}=\left(\frac{7}{6}\right)^2$
$\frac{25}{36}\ne\frac{49}{36}$
Since $\frac{25}{36}\ne\frac{49}{36}$ the statement
$\left(\frac{1}{2}\right)^2+\left(\frac{2}{3}\right)^2=\left(\frac{1}{2}+\frac{2}{3}\right)^2$ is false.

3. The number $-\frac{2}{3}$ can be written as a quotient of two integers with denominator not 0, so it is a rational number. Since all rational numbers are real numbers, it is also a real number.

4. Simplify $-|-8|$.
$-|-8|=-\left[-(-8)\right]$
$=-(8)$
$=-8$
The number -8 is less than 6, and thus $-|-8|$ is the lesser number.

5. "The quotient of -6 and the sum of 2 and -8" is written $\frac{-6}{2+(-8)}$, and $\frac{-6}{2+(-8)} = \frac{-6}{-6} = 1$.

6. If a and b are both negative, $a+b$ is negative and $a \cdot b$ is positive. A positive number divided by a negative number is negative, and thus $\frac{a+b}{a \cdot b}$ is negative.

7. $-2-(5-17)+(-6)$
$= -2-[5+(-17)]+(-6)$
$= -2-(-12)+(-6)$
$= (-2+12)+(-6)$
$= 10+(-6) = 4$

8. $-5\frac{1}{2}+2\frac{2}{3} = -\frac{11}{2}+\frac{8}{3}$
$= -\frac{11 \cdot 3}{2 \cdot 3}+\frac{8 \cdot 2}{3 \cdot 2}$
$= -\frac{33}{6}+\frac{16}{6}$
$= -\frac{17}{6}$, or $-2\frac{5}{6}$

9. $-6.2-[-7.1+(2.0-3.1)]$
$= -6.2-[-7.1+(2.0-3.1)]$
$= -6.2-[-7.1-1.1]$
$= -6.2-[-8.2]$
$= -6.2+8.2$
$= 2$

10. $4^2+(-8)-(2^3-6)$
$= 16+(-8)-(8-6)$
$= [16+(-8)]-2$
$= 8-2 = 6$

11. $(-5)(-12)+4(-4)+(-8)^2$
$= (-5)(-12)+4(-4)+64$
$= [60+(-16)]+64$
$= 44+64 = 108$

12. $\frac{30(-1-2)}{-9[3-(-2)]-12(-2)}$
$= \frac{30(-3)}{-9(5)-(-24)}$
$= \frac{-90}{-45+24}$
$= \frac{-90}{-21}$
$= \frac{30 \cdot 3}{7 \cdot 3} = \frac{30}{7}$, or $4\frac{2}{7}$

13. $3x-4y^2$
$= 3(-2)-4(4^2)$ Let $x=-2, y=4$.
$= 3(-2)-4(16)$
$= -6-64 = -70$

14. $\frac{5x+7y}{3(x+y)}$
$= \frac{5(-2)+7(4)}{3(-2+4)}$ Let $x=-2, y=4$.
$= \frac{-10+28}{3(2)}$
$= \frac{18}{6} = 3$

15. The difference between the highest and lowest elevations is
$6960-(-40) = 6960+40 = 7000$ meters.

16. 4 saves (3 points per save)
+3 wins (3 points per win)
+2 losses (-2 points per loss)
+1 blown save (-2 points per blown save)
$= 4(3)+3(3)+2(-2)+1(-2)$
$= 12+9-4-2$
$= 15$ points
He has a total of 15 points.

17. $2.45-3.54 = 2.45+(-3.54) = -1.09$
As a signed number, the federal budget deficit is $-\$1.09$ trillion.

18. $3x+0=3x$ illustrates an identity property. The correct response is D.

19. $(5+2)+8 = 8+(5+2)$ illustrates a commutative property because the order of the numbers is changed, but the grouping is not. The correct response is A.

20. $-3(x+y) = -3x + (-3y)$ illustrates the distributive property. The correct response is E.

21. $-5 + (3+2) = (-5+3) + 2$ illustrates an associative property because the grouping of the numbers is changed, but the order is not. The correct response is B.

22. $-\frac{5}{3}\left(-\frac{3}{5}\right) = 1$ illustrates an inverse property. The correct response is C.

23. $3(x+1) = 3 \cdot x + 3 \cdot 1$
$= 3x + 3$
The distributive property is used to rewrite $3(x+1)$ as $3x+3$.

24. **(a)** $-6[5+(-2)] = -6(3) = -18$

(b) $-6[5+(-2)] = -6(5) + (-6)(-2)$
$= -30 + 12 = -18$

(c) The distributive property assures us that the answers must be the same, because $a(b+c) = ab + ac$ for all a, b, c.

25. $8x + 4x - 6x + x + 14x$
$= (8+4-6+1+14)x$
$= 21x$

26. $5(2x-1) - (x-12) + 2(3x-5)$
$= 5(2x-1) - 1(x-12) + 2(3x-5)$
$= 10x - 5 - x + 12 + 6x - 10$
$= (10-1+6)x + (-5+12-10)$
$= 15x - 3$