INSTRUCTOR'S SOLUTIONS MANUAL

MATTHEW G. HUDELSON

BASIC TECHNICAL MATHEMATICS

AND

BASIC TECHNICAL MATHEMATICS WITH CALCULUS

ELEVENTH EDITION

Allyn J. Washington
Dutchess Community College

Richard S. Evans
Corning Community College

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Instructor's Solutions Manual for
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Chapter 1

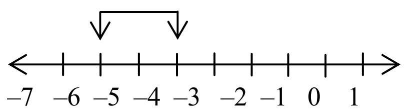
Basic Algebraic Operations

1.1 Numbers

1. The numbers –7 and 12 are integers. They are also rational numbers since they can be written as $\frac{-7}{1}$ and $\frac{12}{1}$.

2. The absolute value of –6 is 6, and the absolute value of –7 is 7. We write these as $|-6| = 6$ and $|-7| = 7$.

3. $-6 < -4$; –6 is to the left of –4.



4. The reciprocal of $\frac{3}{2}$ is $\frac{1}{3/2} = 1 \times \frac{2}{3} = \frac{2}{3}$.

5. 3 is an integer, rational $\left(\frac{3}{1}\right)$, and real.

$\sqrt{-4}$ is imaginary.

6. $\frac{\sqrt{7}}{3}$ is irrational (because $\sqrt{7}$ is an irrational number) and real.

-6 is an integer, rational $\left(\frac{-6}{1}\right)$, and real.

7. $-\frac{\pi}{6}$ is irrational (because π is an irrational number) and real.

$\frac{1}{8}$ is rational and real.

8. $-\sqrt{-6}$ is imaginary.

$-2.33 = \frac{-233}{100}$ is rational and real.

9. $|3| = 3$

$|-3| = 3$

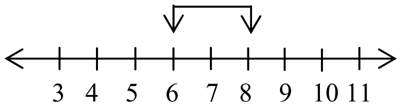
$\left|-\frac{\pi}{2}\right| = \frac{\pi}{2}$

10. $|-0.857| = 0.857$

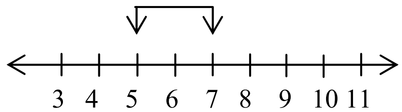
$\left|\sqrt{2}\right| = \sqrt{2}$

$\left|-\frac{19}{4}\right| = \frac{19}{4}$

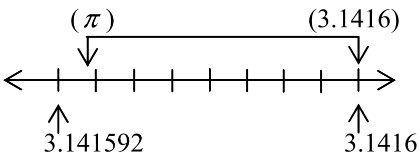
11. $6 < 8$; 6 is to the left of 8.



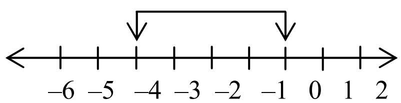
12. $7 > 5$; 7 is to the right of 5.



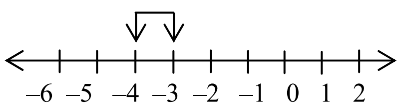
13. $\pi < 3.1416$; $\pi\ (3.1415926\ldots)$ is to the left of 3.1416.



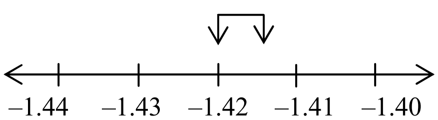
14. $-4 < 0$; -4 is to the left of 0.



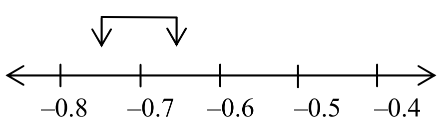
15. $-4 < -|-3|$; -4 is to the left of $-|-3|$, $\left(-|-3| = -(3) = -3\right)$.



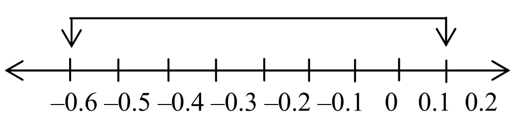
16. $-\sqrt{2} > -1.42$; $(-\sqrt{2} = -(1.414\ldots.) = -1.414\ldots), -\sqrt{2}$ is to the right of -1.42.



17. $-\frac{2}{3} > -\frac{3}{4}; -\frac{2}{3} = -0.666\ldots$ is to the right of $-\frac{3}{4} = -0.75$.



18. $-0.6 < 0.2$; -0.6 is to the left of 0.2.



19. The reciprocal of 3 is $\frac{1}{3}$.

The reciprocal of $-\frac{4}{\sqrt{3}}$ is $-\frac{1}{4/\sqrt{3}} = -\frac{\sqrt{3}}{4}$.

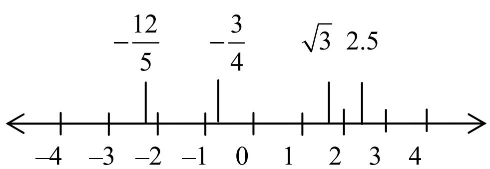
The reciprocal of $\frac{y}{b}$ is $\frac{1}{y/b} = \frac{b}{y}$.

20. The reciprocal of $-\frac{1}{3}$ is $-\frac{1}{1/3} = -\frac{3}{1} = -3$.

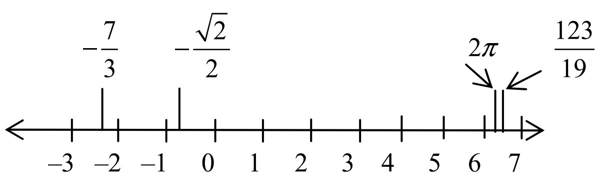
The reciprocal of $0.25 = \frac{1}{4}$ is $\frac{1}{1/4} = \frac{4}{1} = 4$.

The reciprocal of $2x$ is $\frac{1}{2x}$.

21. Find 2.5, $-\frac{12}{5} = -2.4; -\frac{3}{4} = -0.75; \sqrt{3} = 1.732...$



22. Find $-\frac{7}{3} = -2.333...; -\frac{\sqrt{2}}{2} = -\frac{1.414\ldots}{2} = -0.707;\ 2\pi = 2\times 3.14\ldots = 6.28;\ \frac{123}{19} = 6.47$.



23. An absolute value is not always positive, $|0| = 0$ which is not positive.

24. Since $-2.17 = -\frac{217}{100}$, it is rational.

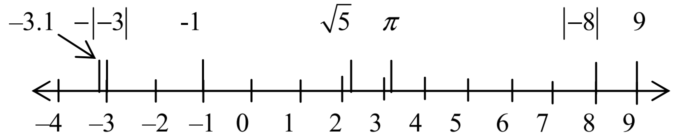
25. The reciprocal of the reciprocal of any positive or negative number is the number itself. The reciprocal of n is $\frac{1}{n}$; the reciprocal of $\frac{1}{n}$ is $\frac{1}{1/n} = 1\cdot\frac{n}{1} = n$.

26. Any repeating decimal is rational, so $2.\overline{72}$ is rational. It turns out that $2.\overline{72} = \frac{30}{11}$.

27. It is true that any nonterminating, nonrepeating decimal is an irrational number.

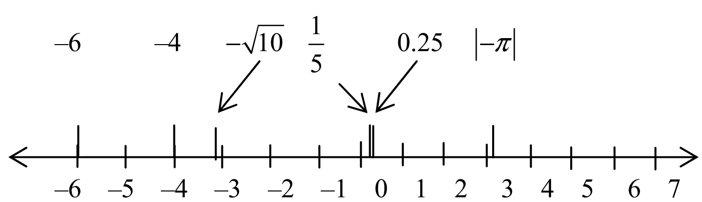
28. No, $|b-a| = |b| - |a|$, as shown below.
If $a > 0$, then $|a| = a$.
If $b > a$ and $a > 0$, then $|b| = b$.
If b $> a$ then $b - a > 0$, then $|b-a| = b - a$.
Therefore, $|b-a| = b - a = |b| - |a|$.
The two sides of the expression are equivalent, one side is not less than the other.

29. List these numbers from smallest to largest: $-1,\ 9,\ \pi = 3.14,\ \sqrt{5} = 2.236,\ |-8| = 8,\ -|-3| = -3,\ -3.1$.



So, from smallest to largest, they are $-3.1,\ -|-3|,\ -1,\ \sqrt{5},\ \pi,\ |-8|,\ 9$.

30. List these numbers from smallest to largest: $\frac{1}{5} = 0.20,\ -\sqrt{10} = -3.16...,\ -|-6| = -6,\ -4,\ 0.25,\ |-\pi| = 3.14...$.



So, from smallest to largest, they are $-|-6|,\ -4,\ -\sqrt{10},\ \frac{1}{5},\ 0.25,\ |-\pi|$.

31. If a and b are positive integers and $b > a$, then

(a) $b - a$ is a positive integer.
(b) $a - b$ is a negative integer.
(c) $\frac{b-a}{b+a}$, the numerator and denominator are both positive, but the numerator is less than the denominator, so the answer is a positive rational number than is less than 1.

32. If a and b are positive integers, then
(a) $a + b$ is a positive integer
(b) a / b is a positive rational number
(c) $a \times b$ is a positive integer

33. **(a)** Is the absolute value of a positive or a negative integer always an integer?
$|x| = x$, so the absolute value of a positive integer is an integer.
$|-x| = x$, so the absolute value of a negative integer is an integer.
(b) Is the reciprocal of a positive or negative integer always a rational number?
If x is a positive or negative integer, then the reciprocal of x is $\frac{1}{x}$. Since both 1 and x are integers, the reciprocal is a rational number.

34. **(a)** Is the absolute value of a positive or negative rational number rational?

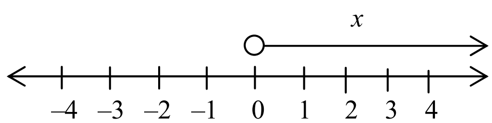
$|x| = x$, so if x is a positive or negative rational number, the absolute value of it is also a rational number.

(b) Is the reciprocal of a positive or negative rational number a rational number?

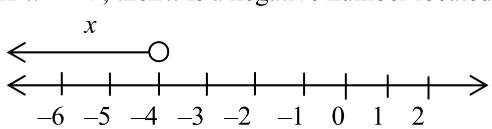
A rational number is a number that can be expressed as a fraction where both the numerator and denominator are integers and the denominator is not zero. So a rational number $\dfrac{\text{integer } a}{\text{integer } b}$ has a reciprocal of

$\dfrac{1}{\dfrac{\text{integer } a}{\text{integer } b}} = \dfrac{\text{integer } b}{\text{integer } a}$, which is also a rational number if integer a is not zero.

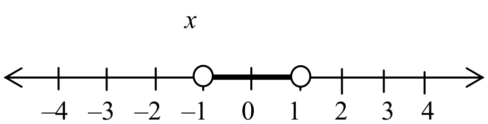
35. **(a)** If $x > 0$, then x is a positive number located to the right of zero on the number line.



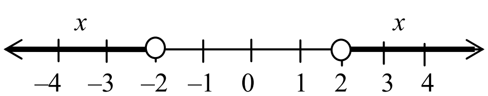
(b) If $x < -4$, then x is a negative number located to the left of –4 on the number line.



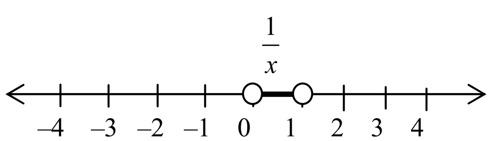
36. **(a)** If $|x| < 1$, then $-1 < x < 1$.



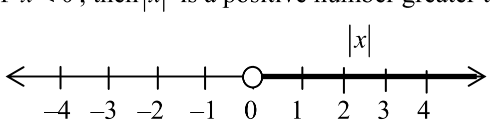
(b) $|x| > 2$, then $x < -2$ or $x > 2$.



37. If $x > 1$, then $\dfrac{1}{x}$ is a positive number less than 1. Or $0 < \dfrac{1}{x} < 1$.



38. If $x < 0$, then $|x|$ is a positive number greater than zero.



39. $a + bj = a + b\sqrt{-1}$ is a real number when $\sqrt{-1}$ is eliminated, which is when $b = 0$. So $a + bj$ is a real number for all real values of a and $b = 0$.

40. The variables are w and t.
The constants are c, 0.1, and 1.

41. $\dfrac{1}{C_T} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$. Find C_T, where $C_1 = 0.0040\,\text{F}$ and $C_2 = 0.0010\,\text{F}$.

$$\frac{1}{C_T} = \frac{1}{0.0040} + \frac{1}{0.0010}$$

$$\frac{1}{C_T} = \frac{1(0.0040) + 1(0.0010)}{0.0040 \times 0.0010}$$

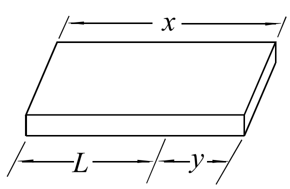
$$C_T = \frac{0.0040 \times 0.0010}{0.0040 + 0.0010} = \frac{0.0000040}{0.0050}$$

$$C_T = 0.00080\ \text{F}$$

42. $|100V| = 100V$
$|-200V| = 200V$
$|-200V| > |100V|$

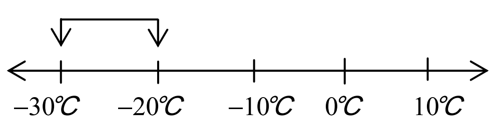
43. $N = \dfrac{a\ \text{bits}}{\text{bytes}} \times \dfrac{1000\ \text{bytes}}{1\ \text{kilobyte}} \times n\,\text{kilobytes}$
$N = 1000\ an\ \text{bits}$

44.



$x =$ length of base in m
$y =$ the shortened length in centimetres.
$100x =$ length of base in cm
$y + L = 100x,$ all dimensions in cm
$L = 100x - y$

45. Yes, $-20\ °\text{C} > -30\ °\text{C}$ because $-30\ °\text{C}$ is found to the left of $-20\ °\text{C}$ on the number line.



46. For $I < 4$ A, $R > 12\ \Omega$.

1.2 Fundamental Operations of Algebra

1. $16-2\times(-2)=16-(-4)=16+4=20$

2. $\frac{-18}{-6}+5-(-2)(3)=3+5-(-6)=8+6=14$

3. $\frac{-12}{8-2}+\frac{5-1}{2(-1)}=\frac{-12}{6}+\frac{4}{-2}=-2+(-2)=-4$

4. $\frac{7\times 6}{0\times 0}=\frac{42}{0}=$ is undefined , not indeterminate.

5. $5+(-8)=5-8=-3$

6. $-4+(-7)=-4-7=-11$

7. $-3+9=6$ or alternatively
$-3+9=+(9-3)=+(6)=6$

8. $18-21=-3$ or alternatively
$18-21=-(21-18)=-(3)=-3$

9. $-19-(-16)=-19+16=-3$

10. $-8-(-10)=-8+10=2$

11. $7(-4)=-(7\times 4)=-28$

12. $-9(3)=-27$

13. $-7(-5)=+(7\times 5)=35$

14. $\frac{-9}{3}=-3$

15. $\frac{-6(20-10)}{-3}=\frac{-6(10)}{-3}=\frac{-60}{-3}=20$

16. $\frac{-28}{-7(5-6)}=\frac{-28}{-7(-1)}=\frac{-28}{7}=-4$

17. $-2(4)(-5)=-8(-5)=40$

18. $-3(-4)(-6)=12(-6)=-72$

19. $2(2-7)\div 10 = 2(-5)\div 10 = -10\div 10 = -1$

20. $\dfrac{-64}{-2|4-8|} = \dfrac{-64}{-2|-4|} = \dfrac{-64}{-2(4)} = \dfrac{-64}{-8} = 8$

21. $16\div 2(-4) = 8(-4) = -32$

22. $-20\div 5(-4) = -4(-4) = 16$

23. $-9-|2-10| = -9-|-8| = -9-8 = -17$

24. $(7-7)\div(5-7) = 0\div(-2) = 0$

25. $\dfrac{17-7}{7-7} = \dfrac{10}{0}$ is undefined

26. $\dfrac{(7-7)(2)}{(7-7)(-1)} = \dfrac{0(2)}{0(-1)} = \dfrac{0}{0}$ is indeterminate

27. $8-3(-4) = 8+12 = 20$

28. $-20+8\div 4 = -20+2 = -18$

29. $-2(-6)+\left|\dfrac{8}{-2}\right| = 12+|-4| = 12+4 = 16$

30. $\dfrac{|-2|}{-2} = \dfrac{2}{-2} = -1$

31.
$$\begin{aligned} 10(-8)(-3)\div(10-50) &= 10(-8)(-3)\div(-40) \\ &= -80(-3)\div(-40) \\ &= 240\div(-40) \\ &= -6 \end{aligned}$$

32. $\dfrac{7-|-5|}{-1(-2)} = \dfrac{7-5}{2} = \dfrac{2}{2} = 1$

33. $\dfrac{24}{3+(-5)} - 4(-9) = \dfrac{24}{-2} + (4\times 9) = -12+36 = 24$

34. $\dfrac{-18}{3} - \dfrac{4-|-6|}{-1} = \dfrac{-18}{3} - \dfrac{4-6}{-1} = -6 - \dfrac{-2}{-1} = -6-2 = -8$

35. $$\begin{aligned}-7-\frac{|-14|}{2(2-3)}-3|6-8| &= -7-\frac{14}{2(-1)}-3|-2| \\ &= -7-\frac{14}{-2}-3(2) \\ &= -7-(-7)-6 \\ &= -7+7-6 \\ &= -6\end{aligned}$$

36. $$\begin{aligned}-7(-3)+\frac{-6}{-3}-|-9| &= +(7\times 3)+2-9 \\ &= 21+2-9 \\ &= 14\end{aligned}$$

37. $$\begin{aligned}\frac{3|-9-2(-3)|}{1-10} &= \frac{3|-9+6|}{-9} \\ &= \frac{3|-3|}{-9} \\ &= \frac{9}{-9} \\ &= -1\end{aligned}$$

38. $\dfrac{20(-12)-40(-15)}{98-|-98|}=\dfrac{-240+600}{98-98}=\dfrac{360}{0}=$ is undefined

39. $6(7)=(7)6$ demonstrates the commutative law of multiplication.

40. $6+8=8+6$ demonstrates the commutative law of addition.

41. $6(3+1)=6(3)+6(1)$ demonstrates the distributive law.

42. $4(5\times\pi)=(4\times 5)\pi$ demonstrates the associative law of multiplication.

43. $3+(5+9)=(3+5)+9$ demonstrates the associative law of addition.

44. $8(3-2)=8(3)-8(2)$ demonstrates the distributive law.

45. $(\sqrt{5}\times 3)\times 9=\sqrt{5}\times(3\times 9)$ demonstrates the associative law of multiplication.

46. $(3\times 6)\times 7=7\times(3\times 6)$ demonstrates the commutative law of multiplication.

47. $-a+(-b)=-a-b$, which is expression (d).

48. $b-(-a)=b+a=a+b$, which is expression (a).

49. $-b-(-a)=-b+a=a-b$, which is expression (b).

50. $-a-(-b)=-a+b=b-a$, which is expression (c).

51. Since $|5-(-2)|=|5+2|=|7|=7$ and $|-5-(-2)|=|-5+2|=|-3|=3$,
$|5-(-2)|>|-5-(-2)|$.

52. Since $|-3-|-7||=|-3-7|=|-10|=10$ and $||-3|-7|=|3-7|=|-4|=4$,
$|-3-|-7|| > ||-3|-7|$.

53. **(a)** The sign of a product of an even number of negative numbers is positive. Example: $-3(-6)=18$

(b) The sign of a product of an odd number of negative numbers is negative.
Example: $-5(-4)(-2)=-40$

54. Subtraction is not commutative because $x-y\neq y-x$. Example: $7-5=2$ does not equal $5-7=-2$

55. Yes, from the definition in Section 1.1, the absolute value of a positive number is the number itself, and the absolute value of a negative number is the corresponding positive number. So for values of *x* where $x>0$ (positive) or $x=0$ (neutral) then $|x|=x$.

Example: $|4|=4$.

The claim that absolute values of negative numbers $|x|=-x$ is also true.

Example: if x is -6, then $|-6|=-(-6)=6$.

56. The incorrect answer was achieved by subtracting before multiplying or dividing which violates the order of operations.
$24-6\div 2\times 3\neq 18\div 2\times 3=9\times 3=27$
The correct value is:
$24-6\div 2\times 3=24-3\times 3=24-9=15$

57. **(a)** $-xy=1$ is true for values of *x* and *y* that are negative reciprocals of each other or $y=-\dfrac{1}{x}$, providing that the number *x* in the denominator is not zero. So if $x=12$, then $y=-\dfrac{1}{12}$ and $-xy=-(12)\left(-\dfrac{1}{12}\right)=1$.

(b) $\dfrac{x-y}{x-y}=1$ is true for all values of *x* and *y*, providing that $x\neq y$ to prevent division by zero.

58. **(a)** $|x+y|=|x|+|y|$ is true for values where both *x* and *y* have the same sign or either are zero:

$|x+y|=|x|+|y|$, when $x\geq 0$ and $y\geq 0$ or when $x\leq 0$ and $y\leq 0$

Example:

$|6+3|=6+3=9$ and

$|6|+|3|=6+3=9$

Also,

$|-6+(-3)|=|-9|=9$

$|-6|+|-3|=6+3=9$

$|x+y|=|x|+|y|$ is not true however, when x and y have opposite signs

$|x+y|\neq|x|+|y|$, when $x>0$ and $y<0$; or $x<0$ and $y>0$.

Example:
$|-21+6| = |-15| = 15,$
$|-21|+|6| = 21+6 = 27 \neq 15$

$|4+(-5)| = |-1| = 1,$
$|4|+|-5| = 4+5 = 9 \neq 1$

(b) In order for $|x-y| = |x|+|y|$ it is necessary that they have opposite signs or either to be zero. Symbolically, $|x-y| = |x|+|y|$ when $x \geq 0$ and $y \leq 0$; or when $x \leq 0$ and $y \geq 0$.

Example:
$|6-(-3)| = 6+3 = 9$ and
$|6|+|-3| = 6+3 = 9$

Example:
$|-11-7| = |-18| = 18$
$|-11|+|-7| = 11+7 = 18$

$|x-y| = |x|+|y|$ is not true, however, when x and y have the same signs.
$|x-y| \neq |x|+|y|$, when x > 0 and $y > 0$; or $x < 0$ and $y < 0$.
Example:
$|21-6| = |15| = 15,$
$|21|+|6| = 27 \neq 15$

59. The total change in the price of the stock is $-0.68+0.42+0.06+(-0.11)+0.02 = -0.29$.

60. The difference in altitude is $-86-(-1396) = 1396-86 = 1310$ m

61. The change in the meter energy reading E would be:
$$E_{change} = E_{used} - E_{generated}$$
$$E_{change} = 2.1 \text{ kW}\cdot\text{h} - 1.5 \text{ kW}\left(3.0 \text{ h}\right)$$
$$E_{change} = 2.1 \text{ kW}\cdot\text{h} - 4.5 \text{ kW}\cdot\text{h}$$
$$E_{change} = -2.4 \text{ kW}\cdot\text{h}$$

62. Assuming that this batting average is for the current season only which is just starting, the number of hits is zero and the total number of at-bats is also zero giving us a $\text{batting average} = \dfrac{\text{number of hits}}{\text{at}-\text{bats}} = \dfrac{0}{0}$ which is indeterminate, not 0.000.

63. The average temperature for the week is:
$$T_{avg} = \frac{-7+(-3)+2+3+1+(-4)+(-6)}{7} \text{ °C}$$
$$T_{avg} = \frac{-7-3+2+3+1-4-6}{7} \text{ °C}$$
$$T_{avg} = \frac{-14}{7} \text{ °C} = -2.0 \text{ °C}$$

64. The vertical distance from the flare gun is

$d=(70)(5)+(-16)(25)$

$d=350+(-400)$

$d=350-400$

$d=-50\text{ m}$

The flare is 50 m below the flare gun.

65. The sum of the voltages is

$V_{sum}=6\text{V}+(-2\text{V})+8\text{V}+(-5\text{V})+3\text{V}$

$V_{sum}=6\text{V}-2\text{V}+8\text{V}-5\text{V}+3\text{V}$

$V_{sum}=10\text{V}$

66. **(a)** The change in the current for the first interval is the second reading – the first reading

$Change_1=-2\text{ lb/in}^2-7\text{ lb/in}^2=-9\text{ lb/in}^2$.

(b) The change in the current for the middle intervals is the third reading – the second reading

$Change_2=-9\text{ lb/in}^2-(-2\text{ lb/in}^2)=-9\text{ lb/in}^2+2\text{ lb/in}^2=-7\text{ lb/in}^2$.

(c) The change in the current for the last interval is the last reading – the third reading

$Change_3=-6\text{ lb/in}^2-(-9\text{ lb/in}^2)=-6\text{ lb/in}^2+9\text{ lb/in}^2=3\text{ lb/in}^2$.

67. The oil drilled by the first well is $100\text{ m}+200\text{ m}=300\text{ m}$ which equals the depth drilled by the second well $200\text{ m}+100\text{ m}=300\text{ m}$.

$100\text{ m}+200\text{ m}=200\text{ m}+100\text{ m}$ demonstrates the commutative law of addition.

68. The first tank leaks $12\frac{\text{L}}{\text{h}}(7\text{ h})=84\text{ L}$ .The second tank leaks $7\frac{\text{L}}{\text{h}}(12\text{h})=84\text{L}$.

$12\times7=7\times12$ demonstrates the commutative law of multiplication.

69. The total time spent browsing these websites is the total time spent browsing the first site on each day + the total time spent browsing the second site on each day

$t=7\text{ days}\times25\,\frac{\text{minutes}}{\text{day}}+7\text{ days}\times15\,\frac{\text{minutes}}{\text{day}}$

$t=175\text{ min}+105\text{ min}$

$t=280\text{ min}$

OR

$t=7\text{ days}\times(25+15)\,\frac{\text{minutes}}{\text{day}}$

$t=7\text{ days}\times40\,\frac{\text{minutes}}{\text{day}}$

$t=280\text{ min}$

which illustrates the distributive law.

70. Distance = rate × time

$$d = \quad 600\frac{\text{km}}{\text{h}} + 50\frac{\text{km}}{\text{h}} \quad 3\text{ h}$$

$$d = 600\frac{\text{km}}{\text{h}}(3\text{h}) + 50\frac{\text{km}}{\text{h}}(3\text{h})$$

$$d = 1800\text{ km} + 150\text{ km} = 1950\text{ km}$$

OR

$$d = \quad 600\frac{\text{km}}{\text{h}} + 50\frac{\text{km}}{\text{h}} \quad 3\text{ h}$$

$$d = \quad 650\frac{\text{km}}{\text{h}} \quad 3\text{ h}$$

$$d = 1950\text{ km}$$

This illustrates the distributive law.

1.3 Calculators and Approximate Numbers

1. 0.390 has three significant digits since the zero is after the decimal. The zero is not necessary as a placeholder and should not be written unless it is significant.

2. 35.303 rounded off to four significant digits is 35.30.

3. In finding the product of the approximate numbers, $2.5 \times 30.5 = 76.25$, but since 2.5 has 2 significant digits, the answer is 76.

4. $38.3 - 21.9(-3.58) = 116.702$ using exact numbers; if we estimate the result, $40 - 20(-4) = 120$.

5. 8 cylinders is exact because they can be counted. 55 km/h is approximate since it is measured.

6. 0.002 mm thick is a measurement and is therefore an approximation. $7.50 is an exact price.

7. 24 hr and 1440 min (60 min/h × 24 h =1140 min) are both exact numbers.

8. 50 keys is exact because you can count them; 50 h of use is approximate since it is a measurement of time.

9. Both 1 cm and 9 g are measured quantities and so they are approximate.

10. The numbers 90 and 75 are exact counts of windows while 15 years is a measurement of time, hence it is approximate.

11. 107 has 3 significant digits; 3004 has 4 significant digits; 1040 has 3 significant digits (the final zero is a placeholder.)

12. 3600 has 2 significant digits; 730 has 2 significant digits; 2055 has 4 significant digits.

13. 6.80 has 3 significant digits since the zero indicates precision; 6.08 has 3 significant digits; 0.068 has 2 significant digits (the zeros are placeholders.)

14. 0.8730 has 4 significant digits; 0.0075 has 2 significant digits; 0.0305 has 3 significant digits.

15. 3000 has 1 significant digit; 3000.1 has 5 significant digits; 3000.10 has 6 significant digits.

16. 1.00 has 3 significant digits since the zeros indicate precision; 0.01 has 1 significant digit since leading zeros are not significant; 0.0100 has 3 significant digits, counting the trailing zeros.

17. 5000 has 1 significant digit; 5000.0 has 5 significant digits; $500\overline{0}$ has 4 significant digits since the bar over the final zero indicates that it is significant.

18. 200 has 1 significant digit; $20\overline{0}$ has 3 significant digits; 200.00 has 5 significant digits.

19. **(a)** 0.010 has more decimal places (3) and is more precise.
(b) 30.8 has more significant digits (3) and is more accurate.

20. **(a)** Both 0.041 and 7.673 have the same precision as they have the same number of decimal places (3).
(b) 7.673 is more accurate because it has more significant digits (4) than 0.041, which has 2 significant digits.

21. **(a)** Both 0.1 and 78.0 have the same precision as they have the same number of decimal places.
(b) 78.0 is more accurate because it has more significant digits (3) than 0.1, which has 1 significant digit.

22. **(a)** 0.004 is more precise because it has more decimal places (3).
(b) 7040 is more accurate because it has more significant digits (3) than 0.004, which has only 1 significant digit.

23. **(a)** 0.004 is more precise because it has more decimal places (3).
(b) Both have the same accuracy as they both have 1 significant digit.

24. The precision and accuracy of $|-8.914|$ and 8.914 are the same.
(a) Both 50.060 and 8.914 have the same precision as they have the same number of decimal places (3).
(b) 50.060 is more accurate because it has more significant digits (5) than 8.914, which has 4 significant digits.

25. **(a)** 4.936 rounded to 3 significant digits is 4.94.
(b) 4.936 rounded to 2 significant digits is 4.9.

26. **(a)** 80.53 rounded to 3 significant digits is 80.5.
(b) 80.53 rounded to 2 significant digits is 81.

27. **(a)** -50.893 rounded to 3 significant digits is -50.9.
(b) -50.893 rounded to 2 significant digits is -51.

28. **(a)** 7.004 rounded to 3 significant digits is 7.00.
(b) 7.004 rounded to 2 significant digits is 7.0.

29. **(a)** 5968 rounded to 3 significant digits is 5970.
(b) 5968 rounded to 2 significant digits is $6\overline{0}00$.

30. **(a)** 30.96 rounded to 3 significant digits is 31.0.
(b) 30.96 rounded to 2 significant digits is 31.

31. **(a)** 0.9449 rounded to 3 significant digits is 0.945.
(b) 0.9449 rounded to 2 significant digits is 0.94.

32. **(a)** 0.9999 rounded to 3 significant digits is 1.00.
(b) 0.9999 rounded to 2 significant digits is 1.0.

33. **(a)** Estimate: $13+1-2=12$
(b) Calculator: $12.78+1.0495-1.633=12.1965$, which is 12.20 to 0.01 precision

34. **(a)** Estimate: $4\times 17=68$
(b) Calculator: $3.64(17.06)=62.0984$, which is 62.1 to 3 significant digits

35. **(a)** Estimate $0.7\times 4-9=-6$
(b) Calculator: $0.6572\times 3.94-8.651=-6.061632$, which is -6.06 to 3 significant digits

36. **(a)** Estimate $40-26\div 4=40-6.5=34$
(b) Calculator: $41.5-26.4\div 3.7=34.3648649$, which is 34 to 2 significant digits

37. **(a)** Estimate $9+(1)(4)=9+4=13$
(b) Calculator: $8.75+(1.2)(3.84)=13.358$, which is 13 to 2 significant digits

38. **(a)** Estimate $30-\dfrac{20}{2}=30-10=20$
(b) Calculator: $28-\dfrac{20.955}{2.2}=18.475$, which is 18 to 2 significant digits

39. **(a)** Estimate $\dfrac{9(15)}{9+15}=\dfrac{135}{24}=6$, to 1 significant digit
(b) Calculator: $\dfrac{8.75(15.32)}{8.75+15.32}=5.569173$, which is 5.57 to 3 significant digits

40. **(a)** Estimate $\dfrac{9(4)}{2+5}=\dfrac{36}{7}=5$, to 1 significant digit
(b) Calculator: $\dfrac{8.97(4.003)}{2.0+4.78}=5.296$, which is 5.3 to 2 significant digits

41. **(a)** Estimate $4.5-\dfrac{2(300)}{400}=3.0$, to 2 significant digits
(b) Calculator: $4.52-\dfrac{2.056(309.6)}{395.2}=2.9093279$, which is 2.91 to 3 significant digits

42. **(a)** Estimate $8+\dfrac{15}{2+2}=12$, to 2 significant digits
(b) Calculator: $8.195+\dfrac{14.9}{1.7+2.1}=12.1160526$, which is 12 to 2 significant digits

43. $0.9788+14.9=15.8788$ since the least precise number in the question has 4 decimal places.

44. $17.311-22.98=-5.669$ since the least precise number in the question has 3 decimal places.

45. $-3.142(65)=-204.23$, which is -204.2 because the least accurate number has 4 significant digits.

46. $8.62\div 1728=0.004988$, which is 0.00499 because the least accurate number has 3 significant digits.

47. With a frequency listed as 2.75 MHz, the least possible frequency is 2.745 MHz, and the greatest possible frequency is 2.755 MHz. Any measurements between those limits would round to 2.75 MHz.

48. For an engine displacement stated at 2400 cm^3, the least possible displacement is 2350 cm^3, and the greatest possible displacement is 2450 cm^3. Any measurements between those limits would round to 2400 cm^3.

49. The speed of sound is $3.25 \text{ mi} \div 15 \text{ s} = 0.21666... \text{ mi/s} = 1144.0... \text{ ft/s}$. However, the least accurate measurement was time since it has only 2 significant digits. The correct answer is 1100 ft/s.

50. $4.4 \text{ s} - 2.72 \text{ s} = 1.68 \text{ s}$, but the answer must be given according to precision of the least precise measurement in the question, so the correct answer is 1.7 s.

51. **(a)** $2.2 + 3.8 \times 4.5 = 2.2 + (3.8 \times 4.5) = 19.3$
(b) $(2.2 + 3.8) \times 4.5 = 6.0 \times 4.5 = 27$

52. **(a)** $6.03 \div 2.25 + 1.77 = (6.03 \div 2.25) + 1.77 = 4.45$
(b) $6.03 \div (2.25 + 1.77) = 6.03 \div 4.02 = 1.5$

53. **(a)** $2 + 0 = 2$
(b) $2 - 0 = 2$
(c) $0 - 2 = -2$
(d) $2 \times 0 = 0$
(e) $2 \div 0 =$ error; from Section 1.2, an equation that has 0 in the denominator is undefined when the numerator is not also 0.

54. **(a)** $2 \div 0.0001 = 20\ 000$; $2 \div 0 =$ error
(b) $0.0001 \div 0.0001 = 1$; $0 \div 0 =$ error
(c) Any number divided by zero is undefined. Zero divided by zero is indeterminate.

55. $\pi = 3.14159265...$
(a) $\pi < 3.1416$
(b) $22 \div 7 = 3.1428$
$\pi < (22 \div 7)$

56. **(a)** $8 \div 33 = 0.2424... = 0.\overline{24}$
(b) $\pi = 3.14159265...$

57. **(a)** $1 \div 3 = 0.333...$ It is a rational number since it is a repeating decimal.
(b) $5 \div 11 = 0.454545...$ It is a rational number since it is a repeating decimal.
(c) $2 \div 5 = 0.400...$ It is a rational number since it is a repeating decimal (0 is the repeating part).

58. $124 \div 990 = 0.12525....$ the calculator may show the answer as 0.1252525253 because it has rounded up for the next 5 that doesn't fit on the screen.

59. $32.4 \text{ MJ} + 26.704 \text{ MJ} + 36.23 \text{ MJ} = 95.334 \text{ MJ}$. The answer must be to the same precision as the least precise measurement. The answer is 95.3 MJ.

60. We would compute $8(68.6) + 5(15.3) = 625.3$ and round to three significant digits for a total weight of 625 lb. The values 8 and 5 are exact.

61. We would compute $12(129) + 16(298.8) = 6328.8$ and round to three significant digits for a total weight of 6330 g. The values 12 and 16 are exact.

62. $V = (15.2\ \Omega + 5.64\ \Omega + 101.23\ \Omega) \times 3.55 \text{ A}$
$V = 122.07\ \Omega \times 3.55 \text{ A}$
$V = 433.3485 \text{ V}$
$V = 433 \text{ V}$ to 3 significant digits

63. $\dfrac{100(40.63+52.96)}{105.30+52.96} = 59.1386\ \% = 59.14\ \%$ to 4 signficiant digits

64. $T = \dfrac{50.45(9.80)}{1+100.9 \div 23} = 91.779$ N = 92 N to 2 significant digits

65. **(a)** Estimate $8 \times 5 - 10 = 30,$ to 1 significant digit.
(b) Calculator: $7.84 \times 4.932 - 11.317 = 27.34988$ which is 27.3 to 3 significant digits.

66. **(a)** Estimate $20 - 50 \div 10 = 15$ to 2 significant digit.
(b) Calculator: $21.6 - 53.14 \div 9.64 = 16.0875519$ which is 16.1 to 3 significant digits.

1.4 Exponents and Unit Conversions

1. $(-x^3)^2 = \left[(-1)x^3\right]^2 = (-1)^2(x^3)^2 = (1)x^6 = x^6$

2. $2x^0 = 2(1) = 2$

3. $x^3x^4 = x^{3+4} = x^7$

4. $y^2y^7 = y^{2+7} = y^9$

5. $2b^4b^2 = 2b^{4+2} = 2b^6$

6. $3k^5k = 3k^{5+1} = 3k^6$

7. $\dfrac{m^5}{m^3} = m^{5-3} = m^2$

8. $\dfrac{2x^6}{-x} = -2x^{6-1} = -2x^5$

9. $\dfrac{-n^5}{7n^9} = -\dfrac{n^{5-9}}{7} = -\dfrac{n^{-4}}{7} = -\dfrac{1}{7n^4}$

10. $\dfrac{3s}{s^4} = 3s^{1-4} = 3s^{-3} = \dfrac{3}{s^3}$

11. $\left(P^2\right)^4 = P^{2(4)} = P^8$

12. $\left(x^8\right)^3 = x^{8(3)} = x^{24}$

13. $\left(aT^2\right)^{30} = a^{30}T^{2(30)} = a^{30}T^{60}$

14. $\left(3r^2\right)^3 = (3)^3r^{2(3)} = 27r^6$

15. $\left(\dfrac{2}{b}\right)^3 = \dfrac{(2)^3}{b^3} = \dfrac{8}{b^3}$

16. $\left(\dfrac{F}{t}\right)^{20} = \dfrac{F^{20}}{t^{20}}$

17. $\left(\dfrac{x^2}{-2}\right)^4 = \dfrac{x^{2(4)}}{(-2)^4} = \dfrac{x^8}{16}$

18. $\left(\dfrac{3}{n^3}\right)^3 = \dfrac{(3)^3}{n^{3(3)}} = \dfrac{27}{n^9}$

19. $(8a)^0 = 1$

20. $-v^0 = -1$

21. $-3x^0 = -3(1) = -3$

22. $-(-2)^0 = -1(1) = -1$

23. $6^{-1} = \dfrac{1}{6^1} = \dfrac{1}{6}$

24. $-w^{-5} = -\dfrac{1}{w^5}$

25. $\dfrac{1}{R^{-2}} = R^2$

26. $\dfrac{1}{-t^{-48}} = -t^{48}$

27. $(-t^2)^7 = \left[(-1)(t^2)\right]^7 = (-1)^7 t^{2(7)} = (-1)t^{14} = -t^{14}$

28. $(-y^3)^5 = \left[(-1)(y^3)\right]^5 = (-1)^5 y^{3(5)} = (-1)y^{15} = -y^{15}$

29. $-\dfrac{L^{-3}}{L^{-5}} = -L^{-3-(-5)} = -L^2$

30. $2i^{40}i^{-70} = 2i^{40+(-70)} = 2i^{-30} = \dfrac{2}{i^{30}}$

31. $\dfrac{2v^4}{(2v)^4} = \dfrac{2v^4}{(2)^4(v^4)} = \dfrac{2v^4}{16v^4} = \dfrac{1}{8}$

32. $\dfrac{x^2x^3}{(x^2)^3}=\dfrac{x^{2+3}}{x^{2(3)}}=\dfrac{x^5}{x^6}=\dfrac{1}{x}$

33. $\dfrac{(n^2)^4}{(n^4)^2}=\dfrac{n^{2(4)}}{n^{4(2)}}=\dfrac{n^8}{n^8}=1$

34. $\dfrac{(3t)^{-1}}{3t^{-1}}=\dfrac{(3)^{-1}t^{-1}}{3t^{-1}}=\dfrac{t}{3(3)t}=\dfrac{1}{9}$

35. $(\pi^0x^2a^{-1})^{-1}=\pi^{0(-1)}x^{2(-1)}a^{-1(-1)}=\pi^0x^{-2}a^1=\dfrac{a}{x^2}$

36. $(3m^{-2}n^4)^{-2}=(3)^{-2}m^{-2(-2)}n^{4(-2)}=3^{-2}m^4n^{-8}=\dfrac{m^4}{9n^8}$

37. $(-8g^{-1}s^3)^2=(-8)^2g^{-1(2)}s^{3(2)}=\dfrac{64s^6}{g^2}$

38. $ax^{-2}(-a^2x)^3=ax^{-2}(-1)^3(a^{2(3)})x^3=-\dfrac{a(a^6)x^3}{x^2}=-a^{1+6}x^{3-2}=-a^7x$

39. $\left(\dfrac{4x^{-1}}{a^{-1}}\right)^{-3}=\dfrac{(4)^{-3}x^{-1(-3)}}{a^{-1(-3)}}=\dfrac{x^3}{64a^3}$

40. $\left(\dfrac{2b^2}{y^5}\right)^{-2}=\dfrac{(2)^{-2}b^{2(-2)}}{y^{5(-2)}}=\dfrac{b^{-4}}{4y^{-10}}=\dfrac{y^{10}}{4b^4}$

41. $\dfrac{15n^2T^5}{3n^{-1}T^6}=\dfrac{5n^{2-(-1)}}{T}=\dfrac{5n^3}{T}$

42. $\dfrac{(nRT^{-2})^{32}}{R^{-2}T^{32}}=\dfrac{n^{32}R^{32-(-2)}T^{-2(32)}}{T^{32}}=\dfrac{n^{32}R^{34}T^{-64}}{T^{32}}=\dfrac{n^{32}R^{34}}{T^{32-(-64)}}=\dfrac{n^{32}R^{34}}{T^{96}}$

43. $7(-4)-(-5)^2=-28-25=-53$

44. $6-|-2|^5-(-2)(8)=6-32-(-16)=6-32+16=-10$

45. $-(-26.5)^2-(-9.85)^3=-(702.25)-(-955.671625)=253.421625$
which gets rounded to 253 because 702.25 and –955.671625 are both accurate to only 3 significant digits due to the original numbers having only 3 significant digits.

46. $-0.711^2-(-|-0.809|)^6=(-1)(0.711)^2-(-0.809)^6=(-1)(0.505521)-(0.2803439122)=-0.7858649122$
which gets rounded to 3 significant digits: –0.786.

47. $\dfrac{3.07(-|-1.86|)}{(-1.86)^4+1.596}=\dfrac{-5.7102}{11.96883216+1.596}=\dfrac{-5.7102}{13.56483216}=-0.420956185$
which gets rounded to 3 significant digits: –0.421.

48. $\dfrac{15.66^2-(-4.017)^4}{1.044(-3.68)}=\dfrac{245.2356-260.379822692}{-3.84192}=\dfrac{-15.144222692}{-3.84192}=3.941837074$
which gets rounded to 3 significant digits: 3.94.

49. $2.38(-60.7)^2-\dfrac{254}{1.17^3}=2.38(3684.49)-\dfrac{254}{1.601613}$
$=8769.0862-158.5901213339$
$=8610.4960786661$
which gets rounded to 3 significant digits: 8610.

50. $4.2(4.6)+\dfrac{0.889}{1.89-1.09^2}$
$=19.32+\dfrac{0.889}{1.89-1.1881}$
$=19.32+\dfrac{0.889}{0.7019}$
$=19.32+0.889880728$
$=20.209880728$
which gets rounded to 2 significant digits: $2\bar{0}$.

51. $\left(\dfrac{1}{x^{-1}}\right)^{-1}=\dfrac{1^{-1}}{x^{-1(-1)}}=\dfrac{1}{x}$, which is the reciprocal of x.

52. $\left(\dfrac{0.2-5^{-1}}{10^{-2}}\right)^0=\left(\dfrac{0.2-\dfrac{1}{5}}{\dfrac{1}{100}}\right)^0=\left(\dfrac{0}{0.01}\right)^0=0^0\neq 1$, since $a^0=1$ requires that $a\neq 0$.

53. If $a^3=5$, then
$a^{12}=a^{3(4)}$
$a^{12}=\left(a^3\right)^4$
$a^{12}=(5)^4$
$a^{12}=625$

54. For any negative value of a, a will be negative, and a^2 will be positive, making all values of $\dfrac{1}{a^2}$ greater than $\dfrac{1}{a}$.
Therefore, it is never the case for negative values of a, $a^{-2}<a^{-1}$.

55. $(x^a\cdot x^{-a})^5=(x^{a-a})^5=(x^0)^5=x^{0(5)}=x^0=1$, provided that $x\neq 0$.

56. $(-y^{a-b}\cdot y^{a+b})^2=((-1)y^{a-b+(a+b)})^2=(-1)^2(y^{2a})^2=y^{2a(2)}=y^{4a}$.

57. $$\left(\frac{kT}{hc}\right)^3 (GkThc)^2 c = \frac{k^3T^3}{h^3c^3}\cdot(G^2k^2T^2h^2c^2)c$$
$$= \frac{k^3T^3}{h^3c^3}\cdot(G^2k^2T^2h^2c^3)$$
$$= \frac{(G^2k^{2+3}T^{2+3}c^{3-3})}{h^1}$$
$$= \frac{G^2k^5T^5}{h}$$

58. $$GmM(mr)^{-1}(r^{-2}) = \frac{GmM}{mr^{1+2}} = \frac{GM}{r^3}$$

59. $$\pi\left(\frac{r}{2}\right)^3\left(\frac{4}{3\pi r^2}\right) = \pi\left(\frac{r^3}{8}\right)\left(\frac{4}{3\pi r^2}\right) = \frac{4r}{24} = \frac{r}{6}$$

60. $$\frac{gM(2\pi fM)^{-2}}{2\pi fC} = \frac{gM}{2\pi fC(2\pi fM)^2}$$
$$= \frac{gM}{2\pi fC(4\pi^2f^2M^2)}$$
$$= \frac{gM}{8\pi^3f^3CM^2}$$
$$= \frac{g}{8\pi^3f^3CM}$$

61. $$2500\left(1+\frac{0.042}{4}\right)^{24} = \$2500(1.0105)^{24}$$
$$= \$2500(1.28490602753)$$
$$= \$3212.26700688$$
$$= \$3212.27$$

62. $$\frac{6.85(1000-20(6.85)^2+(6.85)^3)}{1850} = \frac{6.85(1000-20(46.9225)+321.419125)}{1850}$$
$$= \frac{6.85(1321.419125-938.45)}{1850}$$
$$= \frac{6.85(382.969125)}{1850}$$
$$= \frac{2623.33850625}{1850}$$
$$= 1.418020814$$
$$= 1.42 \text{ cm}$$

63. If $f(n) = \left(1+\frac{1}{n}\right)^n$ then $f(1) = 2^1 = 2.000$, $f(10) = 1.1^{10} = 2.594$, $f(100) = 1.01^{100} = 2.705$, and $f(1000) = 1.001^{1000} = 2.717$.

64. We have 1 TB $= 2^{10}$ GB $= 2^{10}(2^{10}$ MB$) = 2^{10}(2^{10}(2^{20}$ bytes$)) = 2^{10+10+20}$ bytes $= 2^{40}$ bytes

65. $\left(28.2\dfrac{\text{ft}}{\text{s}}\right)(9.81\text{ s}) = 276.642$ ft which is rounded to 277 ft.

66. $\left(40.5\dfrac{\text{mi}}{\text{gal}}\right)(3.7\text{ gal}) = 149.85$ mi which is rounded to 150 mi.

67. $\left(7.25\dfrac{\text{m}}{\text{s}^2}\right)\left(\dfrac{1\text{ ft}}{0.3048\text{ m}}\right)\left(\dfrac{60\text{ s}}{1\text{ min}}\right)^2 = 85{,}629.92\ \dfrac{\text{ft}}{\text{min}^2}$ which is rounded to $85{,}600\ \dfrac{\text{ft}}{\text{min}^2}$.

68. $\left(238\dfrac{\text{kg}}{\text{m}^3}\right)\left(\dfrac{1000\text{ g}}{1\text{ kg}}\right)\left(\dfrac{1\text{ m}}{100\text{ cm}}\right)^3 = 0.238\ \dfrac{\text{g}}{\text{cm}^3}$.

69. $15.7\text{ qt} = 15.7\text{ qt}\times\left(\dfrac{1\text{ L}}{1.057\text{ qt}}\right) = 14.8533586$ L which is rounded to 14.9 L.

70. $7.50\text{ W} = 7.50\text{ W}\times\left(\dfrac{1\text{ hp}}{746.0\text{ W}}\right) = 0.01005362$ hp which is rounded to 0.0101 hp.

71. $245\text{ cm}^2 = 245\text{ cm}^2\times\left(\dfrac{1\text{ in}}{2.54\text{ cm}}\right)^2 = 37.975076\text{ in}^2$ which is rounded to 38.0 in^2.

72. $85.7\text{ mi}^2 = 85.7\text{ mi}^2\times\left(\dfrac{1\text{ km}}{0.6214\text{ mi}}\right)^2 = 221.941401\text{ km}^2$ which is rounded to 222 km^2.

73. $65.2\dfrac{\text{m}}{\text{s}} = 65.2\dfrac{\text{m}}{\text{s}}\times\left(\dfrac{60\text{ s}}{1\text{ min}}\right)\times\left(\dfrac{1\text{ ft}}{0.3048\text{ m}}\right) = 12834.6457\ \dfrac{\text{ft}}{\text{min}}$ which is rounded to $12800\ \dfrac{\text{ft}}{\text{min}}$.

74. $25.0\dfrac{\text{mi}}{\text{gal}} = 25.0\dfrac{\text{mi}}{\text{gal}}\times\left(\dfrac{1\text{ km}}{0.6214\text{ mi}}\right)\times\left(\dfrac{1\text{ gal}}{3.785\text{ L}}\right) = 10.6292562\ \dfrac{\text{km}}{\text{L}}$ which is rounded to $10.6\ \dfrac{\text{km}}{\text{L}}$.

75. $15.6\text{ in} = 15.6\text{ in}\times\left(\dfrac{2.54\text{ cm}}{1\text{ in}}\right) = 39.624$ cm which is rounded to 39.6 cm.

76. $12{,}500\text{ mi} = 12{,}500\text{ mi}\times\left(\dfrac{1\text{ km}}{0.6214\text{ mi}}\right) = 20{,}115.8674$ km which is rounded to 20,100 km.

77. $575{,}000\dfrac{\text{gal}}{\text{day}} = 575{,}000\dfrac{\text{gal}}{\text{day}}\times\left(\dfrac{1\text{ day}}{24\text{ hr}}\right)\times\left(\dfrac{3.785\text{ L}}{1\text{ gal}}\right) = 90{,}682.2917\ \dfrac{\text{L}}{\text{hr}}$ which is rounded to $90{,}700\ \dfrac{\text{L}}{\text{hr}}$.

78. $85\dfrac{\text{gal}}{\text{min}} = 85\dfrac{\text{gal}}{\text{min}}\times\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\times\left(\dfrac{3.785\text{ L}}{1\text{ gal}}\right) = 5.3620833\ \dfrac{\text{L}}{\text{s}}$ which is rounded to $5.4\ \dfrac{\text{L}}{\text{s}}$.

79. $1130\dfrac{\text{ft}}{\text{s}} = 1130\dfrac{\text{ft}}{\text{s}}\times\left(\dfrac{60\text{ s}}{1\text{ min}}\right)\times\left(\dfrac{60\text{ min}}{1\text{ hr}}\right)\times\left(\dfrac{0.3084\text{ m}}{1\text{ ft}}\right)\times\left(\dfrac{1\text{ km}}{1000\text{ m}}\right) = 1254.5712\ \dfrac{\text{km}}{\text{hr}}$ which is rounded to $1250\ \dfrac{\text{km}}{\text{hr}}$.

80. $7200\dfrac{\text{km}}{\text{hr}} = 7200\dfrac{\text{km}}{\text{hr}}\times\left(\dfrac{1\text{ hr}}{60\text{ min}}\right)\times\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\times\left(\dfrac{1000\text{ m}}{1\text{ km}}\right) = 2000\ \dfrac{\text{m}}{\text{s}}$.

81. $14.7\dfrac{\text{lb}}{\text{in}^2} = 14.7\dfrac{\text{lb}}{\text{in}^2}\times\left(\dfrac{4.448\text{ N}}{1\text{ lb}}\right)\times\left(\dfrac{1\text{ in}}{2.54\text{ cm}}\right)^2\times\left(\dfrac{100\text{ cm}}{1\text{ m}}\right)^2 = 101{,}347.883\ \dfrac{\text{N}}{\text{m}^2}$ which is rounded to 101,000 Pa.

82. $62.4\dfrac{\text{lb}}{\text{ft}^3} = 62.4\dfrac{\text{lb}}{\text{ft}^3}\times\ \dfrac{1\text{ kg}}{2.205\text{ lb}}\ \times\ \dfrac{1\text{ ft}}{0.3048\text{ m}}^{3} = 999.381\ \dfrac{\text{kg}}{\text{m}^3}$ which is rounded to 999 $\dfrac{\text{kg}}{\text{m}^3}$.

(The actual value is 1000 $\dfrac{\text{kg}}{\text{m}^3}$.)

1.5 Scientific Notation

1. $8.06\times10^3 = 8060$

2. $750\ 000\ 000\ 000^{-1} = (7.5\times10^{11})^{-1}$
$= 7.5^{-1}\times10^{-11}$
$= 0.1333...\times10^{-11}$
$= 1.33\times10^{-12}$
rounded to 3 significant digits.

3. $4.5\times10^4 = 45{,}000$

4. $6.8\times10^7 = 68{,}000{,}000$

5. $2.01\times10^{-3} = 0.00201$

6. $9.61\times10^{-5} = 0.0000961$

7. $3.23\times10^0 = 3.23\times1 = 3.23$

8. $8\times10^0 = 8\times1 = 8$

9. $1.86\times10 = 18.6$

10. $1\times10^{-1} = 0.1$

11. $4000 = 4\times10^3$

12. $56\ 000 = 5.6\times10^4$

13. $0.0087 = 8.7\times10^{-3}$

14. $0.00074 = 7.4\times10^{-4}$

15. $609{,}000{,}000 = 6.09\times10^8$

16. $10 = 1\times10^{1}$

17. $0.0528 = 5.28\times10^{-2}$

18. $0.0000908 = 9.08\times10^{-5}$

19. $28{,}000(2{,}000{,}000{,}000) = 2.8\times10^{4}(2\times10^{9}) = 5.6\times10^{13}$

20. $50{,}000(0.006) = 5\times10^{4}(6\times10^{-3}) = 300 = 3\times10^{2}$

21. $\dfrac{88{,}000}{0.0004} = \dfrac{8.8\times10^{4}}{4\times10^{-4}} = 2.2\times10^{8}$

22. $\dfrac{0.00003}{6{,}000{,}000} = \dfrac{3\times10^{-5}}{6\times10^{6}} = 5\times10^{-12}$

23. $35{,}600{,}000 = 35.6\times10^{6}$

24. $0.0000056 = 5.6\times10^{-6}$

25. $0.0973 = 97.3\times10^{-3}$

26. $925{,}000{,}000{,}000 = 925\times10^{9}$

27. $0.000000475 = 475\times10^{-9}$

28. $370{,}000 = 370\times10^{3}$

29. $2\times10^{-35} + 3\times10^{-34} = 0.2\times10^{-34} + 3\times10^{-34} = 3.2\times10^{-34}$

30. $5.3\times10^{12} - 3.7\times10^{10} = 530\times10^{10} - 3.7\times10^{10} = 526.3\times10^{10} = 5.263\times10^{12}$

31. $(1.2\times10^{29})^{3} = 1.2^{3}\times10^{29(3)} = 1.728\times10^{87}$

32. $(2\times10^{-16})^{-5} = 2^{-5}\times10^{-16(-5)} = 0.031\ 25\times10^{80} = 3.125\times10^{78}$

33. $1320(649{,}000)(85.3) = 7.3074804\times10^{10}$
which gets rounded to 7.31×10^{10}.

34. $0.0000569(3{,}190{,}000) = 181.511$
which gets rounded to 1.82×10^{2}.

35. $\dfrac{0.0732(6710)}{0.00134(0.0231)} = \dfrac{491.172}{0.000030954} = 1.5867803\times10^{7}$
which gets rounded to 1.59×10^{7}.

36. $\dfrac{0.00452}{2430(97,100)} = \dfrac{0.00452}{235,953,000} = 1.915635741\times10^{-11}$

which gets rounded to 1.92×10^{-11}.

37. $(3.642\times10^{-8})(2.736\times10^{5}) = 9.964512\times10^{-3}$

which gets rounded to 9.965×10^{-3}.

38. $\dfrac{(7.309\times10^{-1})^2}{5.9843(2.5036\times10^{-20})} = \dfrac{0.53421481}{1.49787029\times10^{-19}} = 3.56656723394\times10^{18}$

which gets rounded to 3.567×10^{18}.

39. $\dfrac{(3.69\times10^{-7})(4.61\times10^{21})}{0.0504} = \dfrac{1.70109\times10^{15}}{0.0504} = 3.37517857142\times10^{16}$

which gets rounded to 3.38×10^{16}.

40. $\dfrac{(9.907\times10^{7})(1.08\times10^{12})^2}{(3.603\times10^{-5})(2054)} = \dfrac{(9.907\times10^{7})(1.1664\times10^{24})}{0.07400562} = \dfrac{1.01555248\times10^{32}}{0.07400562} = 1.56143882045\times10^{33}$

which gets rounded to 1.56×10^{33}.

41. $500{,}000{,}000$ tweets $= 5\times10^{8}$ tweets

42. $17{,}200{,}000{,}000$ bytes $= 1.72\times10^{10}$ bytes

43. $0.000003\text{ W} = 3\times10^{-6}\text{ W}$

44. $0.0075\text{ mm} = 7.5\times10^{-3}\text{ mm}$

45. $1{,}200{,}000{,}000\text{ Hz} = 1.2\times10^{9}\text{ Hz}$

46. $1.84\times10^{12} = 1{,}840{,}000{,}000{,}000$

47. $12{,}000{,}000{,}000\text{ m}^2 = 1.2\times10^{10}\text{ m}^2$

48. $3.086\times10^{16}\text{ m} = 30{,}860{,}000{,}000{,}000{,}000\text{ m}$

49. $1.6\times10^{-12}\text{ W} = 0.0000000000016\text{ W}$

50. $2.4\times10^{-43} = 0.00000000000000000000000000000000000000000024$

51. **(a)** $2300 = 2.3\times10^{3} = 2.3\text{ kW}$

(b) $0.23 = 230\times10^{-3} = 230\text{ mW}$

(c) $2{,}300{,}000 = 2.3\times10^{6} = 2.3\text{ MW}$

(d) $0.00023 = 230\times10^{-6} = 230\ \mu\text{W}$

52. **(a)** $8090000 = 8.09\times10^{6} = 8.09\text{ M}\Omega$

(b) $809\,000 = 809\times10^{3} = 809\text{ k}\Omega$

(c) $0.0809 = 80.9\times10^{-3} = 80.9\text{ m}\Omega$

53. **(a)** $\text{googol} = 1\times10^{100} = 10^{100}$

(b) $\text{googolplex} = 10^{\text{googol}} = 10^{10^{100}}$

54. $\text{googol} = 10^{100}$, so to find the ratio $\dfrac{10^{100}}{10^{79}} = 10^{100-79} = 10^{21}$

A googol is 10^{21} times larger than the number of electrons in the universe.

55. $\text{earth's diameter} = \dfrac{\text{sun's diameter}}{110} = \dfrac{1.4\times10^{9}\text{ m}}{110} = 1.27272\times10^{7}$ m which is rounded to 1.3×10^{7} m.

56. $2^{30} = 1{,}073{,}741{,}824 = 1.073741824\times10^{9} \approx 1\times10^{9}$

57. $\dfrac{7.5\times10^{-15}\text{s}}{\text{addition}}\times5.6\times10^{6}\text{additions} = 4.2\times10^{-8}\text{s}$

58. $0.000000039\ \% = 0.00000000039$

$0.00000000039\times0.085\text{ mg} = 3.315\times10^{-11}\text{mg} = 3.3\times10^{-11}\text{mg}$

59. $0.078\text{ s}\times2.998\times10^{8}\,\dfrac{\text{m}}{\text{s}} = 2.3384400\times10^{7}$ m which rounds to 2.3×10^{7} m

60. **(a)** $1\text{ day}\times\dfrac{24\text{ h}}{\text{day}}\times\dfrac{60\text{ min}}{\text{h}}\times\dfrac{60\text{ s}}{\text{min}} = 86400\text{ s} = 8.64\times10^{4}\text{ s}$

(b) $100\text{ year}\times\dfrac{365.25\text{ day}}{\text{year}}\times\dfrac{24\text{ h}}{\text{day}}\times\dfrac{60\text{ min}}{\text{h}}\times\dfrac{60\text{ s}}{\text{min}} = 3\ 155\ 760\ 000\text{ s} = 3.155\ 760\ 0\times10^{9}\text{ s}$

61. $\dfrac{1.66\times10^{-27}\text{ kg}}{\text{amu}}\times\dfrac{1.6\times10^{1}\text{ amu}}{\text{oxygen atoms}}\times1.25\times10^{8}\text{ oxygen atoms} = 3.32\times10^{-18}\text{ kg}$

62. $W = kT^{4}$

$W = 5.7\times10^{-8}\text{ W/K}^{4}\times(3.03\times10^{2}\text{ K})^{4}$

$W = 5.7\times10^{-8}\text{ W/K}^{4}\times8.428892481\times10^{9}\text{ K}^{4}$

$W = 4.80446871417\times10^{2}\text{ W}$

$W = 4.8\times10^{2}\text{ W}$

63. $R = \dfrac{k}{d^{2}} = \dfrac{2.196\times10^{-8}\ \Omega\cdot\text{m}^{2}}{\left(7.998\times10^{-5}\text{m}\right)^{2}} = \dfrac{2.196\times10^{-8}\ \Omega\cdot\text{m}^{2}}{6.396\ 800\ 4\times10^{-9}\text{m}^{2}} = 3.432\ 966\ 268\ 57\ \Omega = 3.433\ \Omega$

64. $\dfrac{1.496\times10^{8}\text{km}}{\text{AU}}\times\dfrac{\text{AU}}{4.99\times10^{2}\text{s}} = 2.99799599198\times10^{5}\text{ km/s} = 2.998\times10^{5}\text{ km/s}$

This is the same speed mentioned in Question 56 as the speed of radio waves.

1.6 Roots and Radicals

1. $-\sqrt[3]{64} = -\sqrt[3]{(4)^3} = -4$

2. $\sqrt{(15)(5)}$
Neither 15 nor 5 is a perfect square, so this expression is not as useful. However, if we further factor the 15 to $\sqrt{(3)(5)(5)} = \sqrt{3(5)^2} = 5\sqrt{3}$, the result can still be obtained.

3. $\sqrt{16\times 9} = \sqrt{144} = \sqrt{12^2} = 12$

4. $-\sqrt{-64}$ is still imaginary because an even root (in this case $n = 2$) of a negative number is imaginary, regardless of the numerical factor placed in front of the root.

5. $\sqrt{49} = \sqrt{7^2} = 7$

6. $\sqrt{225} = \sqrt{(25)(9)} = \sqrt{25}\times\sqrt{9} = 5\times 3 = 15$

7. $-\sqrt{121} = -\sqrt{11^2} = -11$

8. $-\sqrt{36} = -\sqrt{6^2} = -6$

9. $-\sqrt{64} = -\sqrt{8^2} = -8$

10. $\sqrt{0.25} = \sqrt{\dfrac{1}{4}} = \dfrac{\sqrt{1}}{\sqrt{4}} = \dfrac{1}{2} = 0.5$

11. $\sqrt{0.09} = \sqrt{\dfrac{9}{100}} = \dfrac{\sqrt{9}}{\sqrt{100}} = \dfrac{3}{10} = 0.3$

12. $-\sqrt{900} = -\sqrt{(9)(100)} = -\sqrt{9}\times\sqrt{100} = -3\times 10 = -30$

13. $\sqrt[3]{125} = \sqrt[3]{5^3} = 5$

14. $\sqrt[4]{16} = \sqrt[4]{2^4} = 2$

15. $\sqrt[4]{81} = \sqrt[4]{3^4} = 3$

16. $-\sqrt[5]{-32} = -\sqrt[5]{(-2)^5} = -(-2) = 2$

17. $\left(\sqrt{5}\right)^2 = \sqrt{5}\times\sqrt{5} = 5$

18. $\left(\sqrt[3]{31}\right)^3 = \sqrt[3]{31}\times\sqrt[3]{31}\times\sqrt[3]{31} = 31$

19. $\left(-\sqrt[3]{-47}\right)^3 = \left(-1\right)^3\left(\sqrt[3]{-47}\right)^3 = (-1)(-47) = 47$

20. $\left(\sqrt[5]{-23}\right)^5 = -23$

21. $\left(-\sqrt[4]{53}\right)^4 = \left(-1\right)^4\left(\sqrt[4]{53}\right)^4 = (1)(53) = 53$

22. $\sqrt{75} = \sqrt{(25)(3)} = \sqrt{25}\times\sqrt{3} = 5\sqrt{3}$

23. $\sqrt{18} = \sqrt{(9)(2)} = \sqrt{9}\times\sqrt{2} = 3\sqrt{2}$

24. $-\sqrt{32} = -\sqrt{(16)(2)} = -\sqrt{16}\times\sqrt{2} = -4\sqrt{2}$

25. $\sqrt{1200} = \sqrt{(100)(4)(3)} = \sqrt{100}\times\sqrt{4}\times\sqrt{3} = 10\times2\times\sqrt{3} = 20\sqrt{3}$

26. $\sqrt{50} = \sqrt{(25)(2)} = \sqrt{25}\times\sqrt{2} = 5\times\sqrt{2} = 5\sqrt{2}$

27. $2\sqrt{84} = 2\sqrt{(4)(21)} = 2\times\sqrt{4}\times\sqrt{21} = 2\times2\times\sqrt{21} = 4\sqrt{21}$

28. $\dfrac{\sqrt{108}}{2} = \dfrac{\sqrt{(36)(3)}}{2} = \dfrac{\sqrt{36}\times\sqrt{3}}{2} = \dfrac{6\times\sqrt{3}}{2} = 3\sqrt{3}$

29. $\sqrt{\dfrac{80}{|3-7|}} = \sqrt{\dfrac{80}{4}} = \sqrt{20} = \sqrt{4\times5} = \sqrt{4}\times\sqrt{5} = 2\times\sqrt{5} = 2\sqrt{5}$

30. $\sqrt{81\times10^2} = \sqrt{81}\times\sqrt{10^2} = 9\times10 = 90$

31. $\sqrt[3]{-8^2} = \sqrt[3]{-64} = \sqrt[3]{(-4)^3} = -4$

32. $\sqrt[4]{9^2} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$

33. $\dfrac{7^2\sqrt{81}}{(-3)^2\sqrt{49}} = \dfrac{(49)(9)}{(9)(7)} = \dfrac{(49)\cancel{(9)}}{\cancel{(9)}(7)} = 7$

34. $\dfrac{2^5\sqrt[5]{243}}{-3\sqrt{144}} = -\dfrac{32\sqrt[5]{3^5}}{3\sqrt{12^2}} = -\dfrac{(32)\cancel{(3)}}{\cancel{(3)}(12)} = -\dfrac{8}{3}$

35. $\sqrt{36+64} = \sqrt{100} = \sqrt{10^2} = 10$

36. $\sqrt{25+144} = \sqrt{169} = \sqrt{13^2} = 13$

37. $\sqrt{3^2+9^2} = \sqrt{9+81} = \sqrt{90} = \sqrt{(9)(10)} = \sqrt{9}\times\sqrt{10} = 3\sqrt{10}$

38. $\sqrt{8^2-4^2}=\sqrt{64-16}=\sqrt{48}=\sqrt{(16)(3)}=\sqrt{16}\times\sqrt{3}=4\sqrt{3}$

39. $\sqrt{85.4}=9.24121204171$,which is rounded to 9.24

40. $\sqrt{3762}=61.3351449007$,which is rounded to 61.34

41. $\sqrt{0.8152}=0.9028842672$,which is rounded to 0.9029

42. $\sqrt{0.0627}=0.25039968051$,which is rounded to 0.250

43. **(a)** $\sqrt{1296+2304}=\sqrt{3600}=60$, which is expressed as 60.00
(b) $\sqrt{1296}+\sqrt{2304}=36+48=84$, which is expressed as 84.00

44. **(a)** $\sqrt{10.6276+2.1609}=\sqrt{12.7885}=3.57610122899$,which is rounded to 3.57610
(b) $\sqrt{10.6276}+\sqrt{2.1609}=3.26+1.47=4.73$,which is expressed as 4.7300

45. **(a)**
$$\begin{aligned}\sqrt{0.0429^2-0.0183^2}&=\sqrt{0.00184041-0.00033489}\\&=\sqrt{0.00150552}\\&=0.03880103091\\&=0.0388\end{aligned}$$
(b)
$$\begin{aligned}\sqrt{0.0429^2}-\sqrt{0.0183^2}&=0.0429-0.0183\\&=0.0246\end{aligned}$$

46. **(a)**
$$\begin{aligned}\sqrt{3.625^2+0.614^2}&=\sqrt{13.140625+0.376996}\\&=\sqrt{13.517621}\\&=3.67663174658\\&=3.677\end{aligned}$$
(b)
$$\begin{aligned}\sqrt{3.625^2}+\sqrt{0.614^2}&=3.625+0.614\\&=4.239\end{aligned}$$

47. $\sqrt{24s}=\sqrt{(24)(150)}=\sqrt{3600}=60$ mi/h

48.
$$\begin{aligned}\sqrt{Z^2-X^2}&=\sqrt{(5.362\ \Omega)^2-(2.875\ \Omega)^2}\\&=\sqrt{28.751044\ \Omega^2-8.265625\ \Omega^2}\\&=\sqrt{20.485419\ \Omega^2}\\&=4.52608208056\ \Omega\\&=4.526\ \Omega\end{aligned}$$

49. $\sqrt{\dfrac{B}{d}} = \sqrt{\dfrac{2.18\times10^9\ \text{Pa}}{1.03\times10^3\ \text{kg/m}^3}}$

$= \sqrt{2116504.85436\ \dfrac{\text{N/m}^2}{\text{kg/m}^3}}$

$= \sqrt{2116504.85436\ \dfrac{\left(\text{kg}\cdot\text{m/s}^2\right)/\text{m}^2}{\text{kg/m}^3}}$

$= \sqrt{2116504.85436\ \text{m}^2/\text{s}^2}$

$= 1454.82124481\ \text{m/s}$

$= 1450\ \text{m/s}$

50. $\sqrt{40m} = \sqrt{(40)(75)}$

$= \sqrt{3000}$

$= 54.7722557505$

$= 55\ \text{m/s}$

51. $\sqrt{w^2+h^2} = \sqrt{(52.3\ \text{in})^2+(29.3\ \text{in})^2}$

$= \sqrt{2735.29\ \text{in}^2+858.49\ \text{in}^2}$

$= \sqrt{3593.78\ \text{in}^2}$

$= 59.948144258\ \text{in}$

$= 59.9\ \text{in}$

52. $100\left(1-\sqrt{\dfrac{V}{C}}\right) = 100\left(1-\sqrt{\dfrac{24000}{38000}}\right)$

$= 100\left(1-\sqrt{0.63157895}\right)$

$= 100\left(1-0.79471941589\right)$

$= 100(0.2052805841)$

$= 20.52805841\ \%$

$= 21\ \%$

53. $\sqrt{gd} = \sqrt{(9.8)(3500)}$

$= \sqrt{34300}$

$= 185.20259$

$= 190\ \text{m/s}$

54. $\sqrt{1.27\times10^4 h+h^2} = \sqrt{1.27\times10^4(9500)+(9500)^2}$

$= \sqrt{1.2065\times10^8+9.025\times10^7}$

$= \sqrt{2.109\times10^8}$

$= 14522.3965$

which is rounded to 15000 km

55. $\sqrt{a^2} = a$ is not necessarily true for negative values of a because a^2 will be a positive number, regardless whether a is negative or positive.The principal root calculated is assumed to be positive, but there are always two solutions to a square root, $\sqrt{a^2} = \pm a$ since $(+a)^2 = a^2$ and $(-a)^2 = a^2$ (see the introduction to this chapter section),so it is sometimes true and sometimes false for negative values of a, depending on which root solution is desired. If *only principal roots* are considered, then it will *not* be true for negative values of a. For example, $\sqrt{(-4)^2} = \sqrt{16} = 4 \neq -4$.

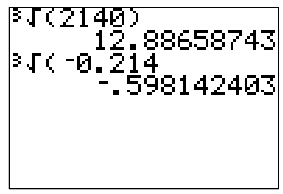
56. **(a)** $x > \sqrt{x}$ when $x > 1$. Any number greater than 1 will have a square root that is smaller than itself. For example, $2 > \sqrt{2} = 1.41$

(b) $x = \sqrt{x}$ when $x = 1$ or $x = 0$ because the only numbers that are their own squares are 0 and 1 (i.e., $0^2 = 0$ and $1^2 = 1$).

(c) $x < \sqrt{x}$ when $0 < x < 1$. Any number between 0 and 1 will have a square root larger than itself. For example, $0.25 < \sqrt{0.25} = 0.5$

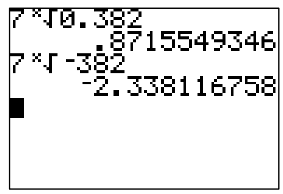
57. **(a)** $\sqrt[3]{2140} = 12.8865874254$,which is rounded to 12.9

(b) $\sqrt[3]{-0.214} = -0.59814240297$,which is rounded to –0.598



58. **(a)** $\sqrt[7]{0.382} = 0.87155493458$,which is rounded to 0.872

(b) $\sqrt[7]{-382} = -2.33811675837$,which is rounded to –2.34



59. $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2(3.1416)\sqrt{0.250(40.52 \times 10^{-6})}}$

$= \dfrac{1}{6.2832\sqrt{10.0625 \times 10^{-6}}}$

$= \dfrac{1}{6.2832(0.003172144385)}$

$= \dfrac{1}{0.0199312175998}$

$= 50.172549$

which is rounded to 50.2 Hz

60. standard deviation = $\sqrt{\text{variance}}$

$= \sqrt{80.5 \text{ kg}^2}$

$= 8.972179222$ kg

which is rounded to 8.97 kg

1.7 Addition and Subtraction of Algebraic Expressions

1. $3x + 2y - 5y = 3x - 3y$

2. $3c - (2b - c) = 3c - 2b + c = -2b + 4c$

3. $$\begin{aligned} 3ax-[(ax-5s)-2ax] &= 3ax-[ax-5s-2ax] \\ &= 3ax-[-ax-5s] \\ &= 3ax+ax+5s \\ &= 4ax+5s \end{aligned}$$

4. $$\begin{aligned} 3a^2b-\{a-[2a^2b-(a+2b)]\} &= 3a^2b-\{a-[2a^2b-a-2b]\} \\ &= 3a^2b-\{a-2a^2b+a+2b\} \\ &= 3a^2b-\{2a-2a^2b+2b\} \\ &= 3a^2b-2a+2a^2b-2b \\ &= 5a^2b-2a-2b \end{aligned}$$

5. $5x+7x-4x=8x$

6. $6t-3t-4t=-t$

7. $2y-y+4x=y+4x$

8. $-4C+L-6C=-10C+L$

9. $3t-4s-3t-s=0t-5s=-5s$

10. $-8a-b+12a+b=4a+0b=4a$

11. $2F-2T-2+3F-T=5F-3T-2$

12. $x-2y-3x-y+z=-2x-3y+z$

13. $a^2b-a^2b^2-2a^2b=-a^2b-a^2b^2$

14. $-xy^2-3x^2y^2+2xy^2=xy^2-3x^2y^2$

15. $2p+(p-6-2p)=2p-6-p=p-6$

16. $5+(3-4n+p)=5+3-4n+p=-4n+p+8$

17. $v-(7-9x+2v)=v-7+9x-2v=-v+9x-7$

18. $-2a-\dfrac{1}{2}(b-a)=-2a-\dfrac{1}{2}b+\dfrac{1}{2}a=-\dfrac{3}{2}a-\dfrac{1}{2}b$

19. $2-3-(4-5a)=-1-4+5a=5a-5$

20. $\sqrt{A}+(h-2\sqrt{A})-3\sqrt{A}=\sqrt{A}+h-2\sqrt{A}-3\sqrt{A}=-4\sqrt{A}+h$

21. $(a-3)+(5-6a)=a-3+5-6a=-5a+2$

22. $(4x-y)-(-2x-4y)=4x-y+2x+4y=6x+3y$

23. $-(t-2u)+(3u-t)=-t+2u+3u-t=-2t+5u$

24. $-2(6x-3y)-(5y-4x)=-12x+6y-5y+4x=-8x+y$

25. $3(2r+s)-(-5s-r)=6r+3s+5s+r=7r+8s$

26. $3(a-b)-2(a-2b)=3a-3b-2a+4b=a+b$

27. $-7(6-3j)-2(j+4)=-42+21j-2j-8=19j-50$

28. $-(5t+a^2)-2(3a^2-2st)=-5t-a^2-6a^2+4st=-7a^2+4st-5t$

29. $$\begin{aligned}-[(4-6n)-(n-3)]&=-[4-6n-n+3]\\&=-[-7n+7]\\&=7n-7\end{aligned}$$

30. $$\begin{aligned}-[(A-B)-(B-A)]&=-[A-B-B+A]\\&=-[2A-2B]\\&=-2A+2B\end{aligned}$$

31. $$\begin{aligned}2[4-(t^2-5)]&=2[4-t^2+5]\\&=2[-t^2+9]\\&=-2t^2+18\end{aligned}$$

32. $$\begin{aligned}-3[-3-\frac{2}{3}(-a-4)]&=-3[-3+\frac{2}{3}a+\frac{8}{3}]\\&=-3[\frac{2}{3}a-\frac{1}{3}]\\&=-2a+1\end{aligned}$$

33. $$\begin{aligned}-2[-x-2a-(a-x)]&=-2[-x-2a-a+x]\\&=-2[-3a]\\&=6a\end{aligned}$$

34. $$\begin{aligned}-2[-3(x-2y)+4y]&=-2[-3x+6y+4y]\\&=-2[-3x+10y]\\&=6x-20y\end{aligned}$$

35. $$\begin{aligned}aZ-[3-(aZ+4)]&=aZ-[3-aZ-4]\\&=aZ-[-aZ-1]\\&=aZ+aZ+1\\&=2aZ+1\end{aligned}$$

36. $$\begin{aligned}9v-[6-(-v-4)+4v]&=9v-[6+v+4+4v]\\&=9v-[5v+10]\\&=9v-5v-10\\&=4v-10\end{aligned}$$

37.
$$\begin{aligned} 5z-\{8-[4-(2z+1)]\} &= 5z-\{8-[4-2z-1]\} \\ &= 5z-\{8-4+2z+1\} \\ &= 5z-\{5+2z\} \\ &= 5z-5-2z \\ &= 3z-5 \end{aligned}$$

38.
$$\begin{aligned} 7y-\{y-[2y-(x-y)]\} &= 7y-\{y-[2y-x+y]\} \\ &= 7y-\{y-[3y-x]\} \\ &= 7y-\{y-3y+x\} \\ &= 7y-\{-2y+x\} \\ &= 7y+2y-x \\ &= -x+9y \end{aligned}$$

39.
$$\begin{aligned} 5p-(q-2p)-[3q-(p-q)] &= 5p-q+2p-[3q-p+q] \\ &= 5p-q+2p-[4q-p] \\ &= 7p-q-4q+p \\ &= 8p-5q \end{aligned}$$

40.
$$\begin{aligned} -(4-\sqrt{LC})-[(5\sqrt{LC}-7)-(6\sqrt{LC}+2)] &= -4+\sqrt{LC}-[5\sqrt{LC}-7-6\sqrt{LC}-2] \\ &= -4+\sqrt{LC}-[-\sqrt{LC}-9] \\ &= -4+\sqrt{LC}+\sqrt{LC}+9 \\ &= 2\sqrt{LC}+5 \end{aligned}$$

41.
$$\begin{aligned} -2\{-(4-x^2)-[3+(4-x^2)]\} &= -2\{-4+x^2-[3+4-x^2]\} \\ &= -2\{-4+x^2-3-4+x^2\} \\ &= -2\{2x^2-11\} \\ &= -4x^2+22 \end{aligned}$$

42.
$$\begin{aligned} -\{-[-(x-2a)-b]-(a-x)\} &= -\{-[-x+2a-b]-a+x\} \\ &= -\{x-2a+b-a+x\} \\ &= -\{-3a+b+2x\} \\ &= 3a-b-2x \end{aligned}$$

43.
$$\begin{aligned} 5V^2-(6-(2V^2+3)) &= 5V^2-(6-2V^2-3) \\ &= 5V^2-(-2V^2+3) \\ &= 5V^2+2V^2-3 \\ &= 7V^2-3 \end{aligned}$$

44.
$$\begin{aligned} -2F+2((2F-1)-5) &= -2F+2(2F-1-5) \\ &= -2F+2(2F-6) \\ &= -2F+4F-12 \\ &= 2F-12 \end{aligned}$$

45. $$\begin{aligned}-(3t-(7+2t-(5t-6))) &= -(3t-(7+2t-5t+6))\\ &= -(3t-(-3t+13))\\ &= -(3t+3t-13)\\ &= -(6t-13)\\ &= -6t+13\end{aligned}$$

46. $$\begin{aligned}a^2-2(x-5-(7-2(a^2-2x)-3x)) &= a^2-2(x-5-(7-2a^2+4x-3x))\\ &= a^2-2(x-5-(7-2a^2+x))\\ &= a^2-2(x-5-7+2a^2-x)\\ &= a^2-2(2a^2-12)\\ &= a^2-4a^2+24\\ &= -3a^2+24\end{aligned}$$

47. $$\begin{aligned}-4[4R-2.5(Z-2R)-1.5(2R-Z)] &= -4[4R-2.5Z+5R-3R+1.5Z]\\ &= -4[6R-Z]\\ &= -24R+4Z\end{aligned}$$

48. $$\begin{aligned}-3\{2.1e-1.3[-f-2(e-5f)]\} &= -3\{2.1e-1.3[-f-2e+10f]\}\\ &= -3\{2.1e-1.3[-2e+9f]\}\\ &= -3\{2.1e+2.6e-11.7f\}\\ &= -3\{4.7e-11.7f\}\\ &= -14.1e+35.1f\end{aligned}$$

49. $3D-(D-d)=3D-D+d=2D+d$

50. $i_1-(2-3i_2)+i_2=i_1-2+3i_2+i_2=i_1+4i_2-2$

51. $$\begin{aligned}B+\frac{4}{3}\alpha \;+2\;\; B-\frac{2}{3}\alpha \;\;-\;\; B+\frac{4}{3}\alpha \;-\; B-\frac{2}{3}\alpha \;&=\; B+\frac{4}{3}\alpha+2B-\frac{4}{3}\alpha \;-\; B+\frac{4}{3}\alpha-B+\frac{2}{3}\alpha\\ &= [3B]-\;\frac{6}{3}\alpha\\ &= 3B-2\alpha\end{aligned}$$

52. $$\begin{aligned}\text{Distance} &= 30\text{ km/h}\times(t-1)\text{h}+40\text{ km/h}\times(t+2)\text{ h}\\ &= 30(t-1)\text{ km}+40(t+2)\text{ km}\\ &= (30t-30+40t+80)\text{ km}\\ &= (70t+50)\text{ km}\end{aligned}$$

53. $$\begin{aligned}\text{Memory} &= x(4\text{ terabytes}) + (x+25)(8\text{ terabytes})\\ &= (4x+8x+200)\text{ terabytes}\\ &= (12x+200)\text{ terabytes}\end{aligned}$$

54. Difference $= 2[(2n+1)(\$30)-(n-2)(\$20)]$
$$= \$\ 2[60n+30-20n+40]$$
$$= \$\ 2[40n+70]$$
$$= \$\ (80n+140)$$

55. **(a)** $(2x^2-y+2a)+(3y-x^2-b)=2x^2-y+2a+3y-x^2-b$
$$=x^2+2y+2a-b$$
(b) $(2x^2-y+2a)-(3y-x^2-b)=2x^2-y+2a-3y+x^2+b$
$$=3x^2-4y+2a+b$$

56. $(3a^2+b-c^3)+(2c^3-2b-a^2)-(4c^3-4b+3)=3a^2+b-c^3+2c^3-2b-a^2-4c^3+4b-3$
$$=2a^2+3b-3c^3-3$$

57. The final y should be added and the final 3 should be subtracted. The correct final answer is $-2x-2y+2$.

58. The final occurrence of $2c$ should be added rather than subtracted, resulting in the final answer of $7a-6b-2c$.

59. $|a-b|=|-(-a+b)|$
$$=|-(b-a)|$$
$$=|-1\times(b-a)|$$
$$=|-1|\times|(b-a)|$$
$$=1\times|b-a|$$
$$=|b-a|$$

60. $(a-b)-c=a-b-c$
However, $a-(b-c)=a-b+c$
Since they are not equivalent, subtraction is not associative.
For example, $(10-5)-2=5-2=3$ is not the same as $10-(5-2)=10-3=7$.

1.8 Multiplication of Algebraic Expressions

1. $2s^3(-st^4)^3(4s^2t)=2s^3(-1)^3s^3t^{12}(4s^2t)$
$$=-2s^6t^{12}(4s^2t)$$
$$=-8s^8t^{13}$$

2. $-2ax(3ax^2-4yz)=(-2ax)(3ax^2)-(-2ax)(4yz)$
$$=(-6a^2x^3)-(-8axyz)$$
$$=-6a^2x^3+8axyz$$

3. $(x-2)(x-3)=x(x)+x(-3)+(-2)(x)+(-2)(-3)$
$$=x^2-3x-2x+6$$
$$=x^2-5x+6$$

4. $(2a-b)^2 = (2a-b)(2a-b)$
$= (2a)(2a)+(2a)(-b)+(2a)(-b)+(-b)(-b)$
$= 4a^2-2ab-2ab+b^2$
$= 4a^2-4ab+b^2$

5. $(a^2)(ax) = a^3x$

6. $(2xy)(x^2y^3) = 2x^3y^4$

7. $-a^2c^2(a^2cx^3) = -a^4c^3x^3$

8. $(-2cs^2)(-4cs)^2 = (-2cs^2)(-4cs)(-4cs)$
$= (-2cs^2)(16c^2s^2)$
$= -32c^3s^4$

9. $(2ax^2)^2(-2ax) = (2ax^2)(2ax^2)(-2ax)$
$= (4a^2x^4)(-2ax)$
$= -8a^3x^5$

10. $(6pq^3)(3pq^2)^2 = (6pq^3)(3pq^2)(3pq^2)$
$= (6pq^3)(9p^2q^4)$
$= 54p^3q^7$

11. $i^2(Ri+2ri) = (i^2)(Ri)+(i^2)(2ri)$
$= i^3R+2i^3r$

12. $2x(-p-q) = (2x)(-p)-(2x)(q)$
$= -2px-2qx$

13. $-3s(s^2-5t) = (-3s)(s^2)+(-3s)(-5t)$
$= -3s^3+15st$

14. $-3b(2b^2-b) = (-3b)(2b^2)+(-3b)(-b)$
$= -6b^3+3b^2$

15. $5m(m^2n+3mn) = (5m)(m^2n)+(5m)(3mn)$
$= 5m^3n+15m^2n$

16. $a^2bc(2ac-3b^2c) = (a^2bc)(2ac)+(a^2bc)(-3b^2c)$
$= 2a^3bc^2-3a^2b^3c^2$

17. $3M(-M-N+2) = (3M)(-M)+(3M)(-N)+(3M)(2)$
$= -3M^2-3MN+6M$

18. $-4c^2(-9gc-2c+g^2) = (-4c^2)(-9cg)+(-4c^2)(-2c)+(-4c^2)(g^2)$
$= 36c^3g+8c^3-4c^2g^2$

19. $$\begin{aligned} xy(tx^2)(x+y^3) &= tx^3y(x+y^3) \\ &= (tx^3y)(x)+(tx^3y)(y^3) \\ &= tx^4y+tx^3y^4 \end{aligned}$$

20. $$\begin{aligned} -2(-3st^3)(3s-4t) &= 6st^3(3s-4t) \\ &= (6st^3)(3s)+(6st^3)(-4t) \\ &= 18s^2t^3-24st^4 \end{aligned}$$

21. $$\begin{aligned} (x-3)(x+5) &= (x)(x)+(x)(5)+(-3)(x)+(-3)(5) \\ &= x^2+5x-3x-15 \\ &= x^2+2x-15 \end{aligned}$$

22. $$\begin{aligned} (a+7)(a+1) &= (a)(a)+(a)(1)+(7)(a)+(7)(1) \\ &= a^2+a+7a+7 \\ &= a^2+8a+7 \end{aligned}$$

23. $$\begin{aligned} (x+5)(2x-1) &= (x)(2x)+(x)(-1)+(5)(2x)+(5)(-1) \\ &= 2x^2-x+10x-5 \\ &= 2x^2+9x-5 \end{aligned}$$

24. $$\begin{aligned} (4t_1+t_2)(2t_1-3t_2) &= (4t_1)(2t_1)+(4t_1)(-3t_2)+(t_2)(2t_1)+(t_2)(-3t_2) \\ &= 8t_1^2-12t_1t_2+2t_1t_2-3t_2^2 \\ &= 8t_1^2-10t_1t_2-3t_2^2 \end{aligned}$$

25. $$\begin{aligned} (y+8)(y-8) &= (y)(y)+(y)(-8)+(8)(y)+(8)(-8) \\ &= y^2-8y+8y-64 \\ &= y^2-64 \end{aligned}$$

26. $$\begin{aligned} (z-4)(z+4) &= (z)(z)+(z)(4)+(-4)(z)+(-4)(4) \\ &= z^2+4z-4z-16 \\ &= z^2-16 \end{aligned}$$

27. $$\begin{aligned} (2a-b)(-2b+3a) &= (2a)(-2b)+(2a)(3a)+(-b)(-2b)+(-b)(3a) \\ &= -4ab+6a^2+2b^2-3ab \\ &= 6a^2-7ab+2b^2 \end{aligned}$$

28. $$\begin{aligned} (-3+4w^2)(3w^2-1) &= (-3)(3w^2)+(-3)(-1)+(4w^2)(3w^2)+(4w)(-1) \\ &= -9w^2+3+12w^4-4w^2 \\ &= 12w^4-13w^2+3 \end{aligned}$$

29. $$\begin{aligned} (2s+7t)(3s-5t) &= (2s)(3s)+(2s)(-5t)+(7t)(3s)+(7t)(-5t) \\ &= 6s^2-10st+21st-35t^2 \\ &= 6s^2+11st-35t^2 \end{aligned}$$

30. $(5p-2q)(p+8q)=(5p)(p)+(5p)(8q)+(-2q)(p)+(-2q)(8q)$
$=5p^2+40pq-2pq-16q^2$
$=5p^2+38pq-16q^2$

31. $(x^2-1)(2x+5)=(x^2)(2x)+(x^2)(5)+(-1)(2x)+(-1)(5)$
$=2x^3+5x^2-2x-5$

32. $(3y^2+2)(2y-9)=(3y^2)(2y)+(3y^2)(-9)+(2)(2y)+(-9)(2)$
$=6y^3-27y^2+4y-18$

33. $(x-2y-4)(x-2y+4)$
$=(x)(x)+(x)(-2y)+(x)(4)+(-2y)(x)+(-2y)(-2y)+(-2y)(4)+(-4)(x)+(-4)(-2y)+(-4)(4)$
$=x^2-2xy+4x-2xy+4y^2-8y-4x+8y-16$
$=x^2+4y^2-4xy-16$

34. $(2a+3b+1)(2a+3b-1)$
$=(2a)(2a)+(2a)(3b)+(2a)(-1)+(3b)(2a)+(3b)(3b)+(3b)(-1)+(1)(2a)+(1)(3b)+(1)(-1)$
$=4a^2+6ab-2a+6ab+9b^2-3b+2a+3b-1$
$=4a^2+9b^2+12ab-1$

35. $2(a+1)(a-9)=2[(a)(a)+(a)(-9)+(1)(a)+(-9)(1)]$
$=2[a^2-9a+a-9]$
$=2[a^2-8a-9]$
$=2a^2-16a-18$

36. $-5(y-3)(y+6)=-5[(y)(y)+(y)(6)+(-3)(y)+(-3)(6)]$
$=-5[y^2+6y-3y-18]$
$=-5[y^2+3y-18]$
$=-5y^2-15y+90$

37. $-3(3-2T)(3T+2)=-3[(3)(3T)+(3)(2)+(-2T)(3T)+(-2T)(2)]$
$=-3[-6T^2+9T-4T+6]$
$=-3[-6T^2+5T+6]$
$=18T^2-15T-18$

38. $2n(-n+5)(6n+5)=2n[(-n)(6n)+(-n)(5)+(5)(6n)+(5)(5)]$
$=2n[-6n^2-5n+30n+25]$
$=2n[-6n^2+25n+25]$
$=-12n^3+50n^2+50n$

39. $2L(L+1)(4-L)=2L[(L)(4)+(L)(-L)+(1)(4)+(1)(-L)]$
$=2L[-L^2+4L-L+4]$
$=2L[-L^2+3L+4]$
$=-2L^3+6L^2+8L$

40. $$\begin{aligned} ax(x+4)(7-x^2) &= ax[(x)(7)+(x)(-x^2)+(4)(7)+(4)(-x^2)] \\ &= ax[-x^3-4x^2+7x+28] \\ &= -ax^4-4ax^3+7ax^2+28ax \end{aligned}$$

41. $$\begin{aligned} (3x-7)^2 &= (3x-7)(3x-7) \\ &= (3x)(3x)+(3x)(-7)+(-7)(3x)+(-7)(-7) \\ &= 9x^2-21x-21x+49 \\ &= 9x^2-42x+49 \end{aligned}$$

42. $$\begin{aligned} (x-3y)^2 &= (x-3y)(x-3y) \\ &= (x)(x)+(x)(-3y)+(-3y)(x)+(-3y)(-3y) \\ &= x^2-3xy-3xy+9y^2 \\ &= x^2-6xy+9y^2 \end{aligned}$$

43. $$\begin{aligned} (x_1+3x_2)^2 &= (x_1+3x_2)(x_1+3x_2) = (x_1)(x_1)+(x_1)(3x_2)+(3x_2)(x_1)+(3x_2)(3x_2) \\ &= x_1^{\ 2}+3x_1x_2+3x_1x_2+9x_2^{\ 2} \\ &= x_1^{\ 2}+6x_1x_2+9x_2^{\ 2} \end{aligned}$$

44. $$\begin{aligned} (-7m-1)^2 &= (-7m-1)(-7m-1) \\ &= (-7m)(-7m)+(-7m)(-1)+(-1)(-7m)+(-1)(-1) \\ &= 49m^2+7m+7m+1 \\ &= 49m^2+14m+1 \end{aligned}$$

45. $$\begin{aligned} (xyz-2)^2 &= (xyz-2)(xyz-2) \\ &= (xyz)(xyz)+(xyz)(-2)+(-2)(xyz)+(-2)(-2) \\ &= x^2y^2z^2-2xyz-2xyz+4 \\ &= x^2y^2z^2-4xyz+4 \end{aligned}$$

46. $$\begin{aligned} (-6x^2+b)^2 &= (-6x^2+b)(-6x^2+b) \\ &= (-6x^2)(-6x^2)+(-6x^2)(b)+(b)(-6x^2)+(b)(b) \\ &= 36x^4-6bx^2-6bx^2+b^2 \\ &= 36x^4-12bx^2+b^2 \end{aligned}$$

47. $$\begin{aligned} 2(x+8)^2 &= 2[(x+8)(x+8)] \\ &= 2[(x)(x)+(x)(8)+(8)(x)+(8)(8)] \\ &= 2[x^2+8x+8x+64] \\ &= 2[x^2+16x+64] \\ &= 2x^2+32x+128 \end{aligned}$$

48. $$\begin{aligned}3(3R-4)^2 &= 3[(3R-4)(3R-4)]\\ &= 3[(3R)(3R)+(3R)(-4)+(-4)(3R)+(-4)(-4)]\\ &= 3[9R^2-12R-12R+16]\\ &= 3[9R^2-24R+16]\\ &= 27R^2-72R+48\end{aligned}$$

49. $$\begin{aligned}(2+x)(3-x)(x-1) &= [(6-2x+3x-x^2)](x-1)\\ &= (x-1)[-x^2+x+6]\\ &= (x)(-x^2)+(x)(x)+(6)(x)+(-1)(-x^2)+(-1)(x)+(-1)(6)\\ &= -x^3+x^2+6x+x^2-x-6\\ &= -x^3+2x^2+5x-6\end{aligned}$$

50. $$\begin{aligned}(-c^2+3x)^3 &= (-c^2+3x)(-c^2+3x)(-c^2+3x)\\ &= [(3x)(3x)-3c^2x-3c^2x+c^4)](-c^2+3x)\\ &= (-c^2+3x)[9x^2-6c^2x+c^4]\\ &= (-c^2)(9x^2)+(-c^2)(-6c^2x)+(-c^2)(+c^4)+(3x)(9x^2)+(3x)(-6c^2x)+(3x)(+c^4)\\ &= -9c^2x^2+6c^4x-c^6+27x^3-18c^2x^2+3c^4x\\ &= -c^6+9c^4x-27c^2x^2+27x^3\end{aligned}$$

51. $$\begin{aligned}3T(T+2)(2T-1) &= 3T[(T)(2T)+(T)(-1)+(2)(2T)+(2)(-1)]\\ &= 3T[2T^2-T+4T-2]\\ &= 3T[2T^2-T+4T-2]\\ &= 3T[2T^2+3T-2]\\ &= 6T^3+9T^2-6T\end{aligned}$$

52. $$\begin{aligned}&[(x-2)^2(x+2)]^2\\ &= [(x-2)(x-2)(x+2)][(x-2)(x-2)(x+2)]\\ &= [(x-2)[(x)(x)+(-2)(x)+(2)(x)+(-2)(2)]][(x-2)[(x)(x)+(-2)(x)+(2)(x)+(-2)(2)]]\\ &= [(x-2)[x^2-2x+2x-4]][(x-2)[x^2-2x+2x-4]]\\ &= [(x-2)[x^2-4]][(x-2)[x^2-4]]\\ &= [(x)(x^2)+(-4)(x)+(-2)(x^2)+(-2)(-4)][(x)(x^2)+(-4)(x)+(-2)(x^2)+(-2)(-4)]\\ &= [x^3-2x^2-4x+8][x^3-2x^2-4x+8]\\ &= (x^3)(x^3)+(x^3)(-2x^2)+(x^3)(-4x)+(x^3)(8)+(-2x^2)(x^3)+(-2x^2)(-2x^2)+(-2x^2)(-4x)+(-2x^2)(8)\\ &\quad +(-4x)(x^3)+(-4x)(-2x^2)+(-4x)(-4x)+(-4x)(8)+(8)(x^3)+(8)(-2x^2)+(8)(-4x)+(8)(8)\\ &= x^6-2x^5-4x^4+8x^3-2x^5+4x^4+8x^3-16x^2-4x^4+8x^3+16x^2-32x+8x^3-16x^2-32x+64\\ &= x^6-4x^5-4x^4+32x^3-16x^2-64x+64\end{aligned}$$

53. **(a)** $(x+y)^2 = (3+4)^2 = 7^2 = 49$

$x^2+y^2 = 3^2+4^2 = 9+16 = 25$

$(x+y)^2 \neq x^2+y^2$

$49 \neq 25$

(b) $(x-y)^2 = (3-4)^2 = (-1)^2 = 1$
$x^2 - y^2 = 3^2 - 4^2 = 9 - 16 = -7$
$(x-y)^2 \neq x^2 - y^2$
$1 \neq -7$

54. One can write $(x+3)^5 = (x+3)(x+3)(x+3)(x+3)(x+3)$ and then perform the multiplications using the rightmost pair of terms at each step.

55.
$$\begin{aligned}(x+y)^3 &= (x+y)(x+y)(x+y)\\ &= (x+y)[(x)(x)+(x)(y)+(y)(x)+(y)(y)]\\ &= (x+y)[x^2+xy+xy+y^2]\\ &= (x+y)[x^2+2xy+y^2]\\ &= (x)(x^2)+(x)(2xy)+(x)(y^2)+(y)(x^2)+(y)(2xy)+(y)(y^2)\\ &= x^3+2x^2y+y^2x+x^2y+2y^2x+y^3\\ &= x^3+3x^2y+3y^2x+y^3\end{aligned}$$

This differs from x^3+y^3 by the presence of $3x^2y+3y^2x$ in the center.

56.
$$\begin{aligned}&(x+y)(x^2-xy+y^2)\\ &= (x)(x^2)+(x)(-xy)+(x)(y^2)+(y)(x^2)+(y)(-xy)+(y)(y^2)\\ &= x^3-x^2y+y^2x+x^2y-y^2x+y^3\\ &= x^3+y^3\end{aligned}$$

57.
$$\begin{aligned}P(1+0.01r)^2 &= P(1+0.01r)(1+0.01r)\\ &= P[(1)(1)+(1)(0.01r)+(0.01r)(1)+(0.01r)(0.01r)]\\ &= P[1+0.01r+0.01r+0.0001r^2]\\ &= 0.0001r^2P+0.02rP+P\end{aligned}$$

58.
$$\begin{aligned}1000(1+0.0025r)^2 &= 1000(1+0.0025r)(1+0.0025r)\\ &= 1000[(1)(1)+(1)(0.0025r)+(0.0025r)(1)+(0.0025r)(0.0025r)]\\ &= 1000[1+0.0025r+0.0025r+0.0000625r^2]\\ &= 1000+2.5r+0.0625r^2\end{aligned}$$

59. The room will be $5+w+5 = w+10$ feet wide and $5+2w+5 = 2w+10$ feet long. Its area is
$$\begin{aligned}(w+10)(2w+10) &= (w)(2w)+(w)(10)+(10)(2w)+(10)(10)\\ &= 2w^2+10w+20w+100\\ &= 2w^2+30w+100\end{aligned}$$

60.
$$\begin{aligned}R &= xp\\ &= x(30-0.01x)\\ &= 30x-0.01x^2\end{aligned}$$

61. $$\begin{aligned}(2R-X)^2-(R^2+X^2) &= (2R-X)(2R-X)-(R^2+X^2)\\ &= [(2R)(2R)+(2R)(-X)+(2R)(-X)+(-X)(-X)]-(R^2+X^2)\\ &= 4R^2-2RX-2RX+X^2-R^2-X^2\\ &= 3R^2-4RX\end{aligned}$$

62. $$\begin{aligned}(2T^3+3)(T^2-T-3) &= (2T^3)(T^2)+(2T^3)(-T)+(2T^3)(-3)+(3)(T^2)+(3)(-T)+(3)(-3)\\ &= 2T^5-2T^4-6T^3+3T^2-3T-9\end{aligned}$$

63. $$\begin{aligned}\text{Number of switches for } n \text{ elements} &= n^2\\ \text{Number of switches for } n+100 \text{ elements} &= (n+100)^2\\ &= (n+100)(n+100)\\ &= (n)(n)+(n)(100)+(100)(n)+(100)(100)\\ &= n^2+100n+100n+10000\\ &= n^2+200n+10000\end{aligned}$$

64. $$\begin{aligned}(T^2-100)(T-10)(T+10) &= (T^2-100)\ \ T^2+10T-10T-100\\ &= (T^2-100)\ \ T^2-100\\ &= T^4-100T^2-100T^2+10\ 000\\ &= T^4-200T^2+10\ 000\end{aligned}$$

65. $$\begin{aligned}(R_1+R_2)^2-2R_2(R_1+R_2) &= \left[(R_1+R_2)(R_1+R_2)\right]-2R_2(R_1+R_2)\\ &= \left[(R_1)(R_1)+(R_1)(R_2)+(R_2)(R_1)+(R_2)(R_2)\right]-2R_1R_2-2R_2{}^2\\ &= \ R_1{}^2+R_1R_2+R_1R_2+R_2{}^2\ \ -2R_1R_2-2R_2{}^2\\ &= R_1{}^2-R_2{}^2\end{aligned}$$

66. $$\begin{aligned}&27x^2-24(x-6)^2-(x-12)^3\\ &= 27x^2-24(x-6)(x-6)-(x-12)(x-12)(x-12)\\ &= 27x^2-24\ \ x^2-6x-6x+36\ \ -(x-12)\ \ x^2-12x-12x+144\\ &= 27x^2-24\ \ x^2-12x+36\ \ -(x-12)\ \ x^2-24x+144\\ &= 27x^2-24x^2+288x-864-\ \ (x)(x^2)+(x)(-24x)+(x)(144)+(-12)(x^2)+(-12)(-24x)+(-12)(144)\\ &= 3x^2+288x-864-x^3+24x^2-144x+12x^2-288x+1728\\ &= -x^3+39x^2-144x+864\end{aligned}$$

1.9 Division of Algebraic Expressions

1. $$\frac{-6a^2xy^2}{-2a^2xy^5}=\left(\frac{-6}{-2}\right)\frac{a^{2-2}x^{1-1}}{y^{5-2}}=\frac{3}{y^3}$$

2. $$\begin{aligned}\frac{4x^3y-8x^3y^2+2x^2y}{2xy^2}&=\frac{4x^3y}{2xy^2}-\frac{8x^3y^2}{2xy^2}+\frac{2x^2y}{2xy^2}\\&=\frac{2x^{3-1}}{y^{2-1}}-4x^{3-1}y^{2-2}+\frac{x^{2-1}}{y^{2-1}}\\&=\frac{2x^2}{y}-4x^2+\frac{x}{y}\end{aligned}$$

3. $$\begin{array}{r@{\,}l} & \phantom{6x^2-{}}3x-2\\ 2x-1\,\big) & 6x^2-7x+2\\ & \underline{6x^2-3x}\\ & -4x+2\\ & \underline{-4x+2}\\ & \phantom{6x^2-4x+{}}0\end{array}$$

4. $$\begin{array}{r@{\,}l} & \phantom{8x^3-4x^2+0x+{}}2x-1\\ 4x^2-1\,\big) & 8x^3-4x^2+0x+3\\ & \underline{8x^3-2x}\\ & -4x^2+2x+3\\ & -\underline{4x^2\phantom{+2x+{}}1}\\ & \phantom{8x^3-4x^2+{}}2x+2\end{array}$$

$$\frac{8x^3-4x^2+3}{4x^2-1}=2x-1+\frac{2x+2}{4x^2-1}$$

5. $$\frac{8x^3y^2}{-2xy}=-4x^{3-1}y^{2-1}=-4x^2y$$

6. $$\frac{-18b^7c^3}{bc^2}=-18b^{7-1}c^{3-2}=-18b^6c$$

7. $$\frac{-16r^3t^5}{-4r^5t}=\frac{4t^{5-1}}{r^{5-3}}=\frac{4t^4}{r^2}$$

8. $$\frac{51mn^5}{17m^2n^2}=\frac{3n^{5-2}}{m^{2-1}}=\frac{3n^3}{m}$$

9. $$\frac{(15x^2y)(2xz)}{10xy}=\frac{30x^3yz}{10xy}=3x^{3-1}y^{1-1}z=3x^2z$$

10. $$\frac{(5sT)(8s^2T^3)}{10s^3T^2}=\frac{40s^3T^4}{10s^3T^2}=4s^{3-3}T^{4-2}=4T^2$$

11. $$\frac{(4a^3)(2x)^2}{(4ax)^2}=\frac{4a^3(4x^2)}{16a^2x^2}=1a^{3-2}x^{2-2}=a$$

12. $\dfrac{12a^2b}{(3ab^2)^2}=\dfrac{12a^2b}{9a^2b^4}=\dfrac{4a^{2-2}}{3b^{4-1}}=\dfrac{4}{3b^3}$

13. $\dfrac{3a^2x+6xy}{3x}=\dfrac{3a^2x}{3x}+\dfrac{6xy}{3x}=\dfrac{3a^2x^{1-1}}{3}+\dfrac{6x^{1-1}y}{3}=a^2+2y$

14. $\dfrac{2m^2n-6mn}{-2m}=\dfrac{2m^2n}{-2m}-\dfrac{6mn}{-2m}=-m^{2-1}n+3m^{1-1}n=-mn+3n$

15. $\dfrac{3rst-6r^2st^2}{3rs}=\dfrac{3rst}{3rs}-\dfrac{6r^2st^2}{3rs}=r^{1-1}s^{1-1}t-2r^{2-1}s^{1-1}t^2=-2rt^2+t$

16. $\dfrac{-5a^2n-10an^2}{5an}=\dfrac{-5a^2n}{5an}-\dfrac{10an^2}{5an}=-a^{2-1}n^{1-1}-2a^{1-1}n^{2-1}=-a-2n$

17.
$$\begin{aligned}\frac{4pq^3+8p^2q^2-16pq^5}{4pq^2}&=\frac{4pq^3}{4pq^2}+\frac{8p^2q^2}{4pq^2}-\frac{16pq^5}{4pq^2}\\&=p^{1-1}q^{3-2}+2p^{2-1}q^{2-2}-4p^{1-1}q^{5-2}\\&=-4q^3+2p+q\end{aligned}$$

18.
$$\begin{aligned}\frac{a^2x_1x_2^2+ax_1^3-ax_1}{ax_1}&=\frac{a^2x_1x_2^2}{ax_1}+\frac{ax_1^3}{ax_1}-\frac{ax_1}{ax_1}\\&=a^{2-1}x_1^{1-1}x_2^2+a^{1-1}x_1^{3-1}-a^{1-1}x_1^{1-1}\\&=ax_2^2+x_1^2-1\end{aligned}$$

19.
$$\begin{aligned}\frac{2\pi fL-\pi fR^2}{\pi fR}&=\frac{2\pi fL}{\pi fR}-\frac{\pi fR^2}{\pi fR}\\&=\frac{2f^{1-1}L}{R}-f^{1-1}R^{2-1}\\&=\frac{2L}{R}-R\end{aligned}$$

20.
$$\begin{aligned}\frac{9(aB)^4-6aB^4}{-3aB^3}&=\frac{9(aB)^4}{-3aB^3}-\frac{6aB^4}{-3aB^3}\\&=-\frac{9a^4B^4}{3aB^3}+\frac{6aB^4}{3aB^3}\\&=-3a^{4-1}B^{4-3}+2a^{1-1}B^{4-3}\\&=-3a^3B+2B\end{aligned}$$

21.
$$\begin{aligned}\frac{-7a^2b+14ab^2-21a^3}{14a^2b^2}&=-\frac{7a^2b}{14a^2b^2}+\frac{14ab^2}{14a^2b^2}-\frac{21a^3}{14a^2b^2}\\&=-\frac{a^{2-2}}{2b^{2-1}}+\frac{b^{2-2}}{a^{2-1}}-\frac{3}{2}a^{3-2}b^{-2}\\&=-\frac{1}{2b}+\frac{1}{a}-\frac{3a}{2b^2}\end{aligned}$$

22. $$\frac{2x^{n+2}+4ax^n}{2x^n}=\frac{2x^{n+2}}{2x^n}+\frac{4ax^n}{2x^n}$$
$$=x^{n-n+2}+2ax^{n-n}$$
$$=x^2+2a$$

23. $$\frac{6y^{2n}-4ay^{n+1}}{2y^n}=\frac{6y^{2n}}{2y^n}-\frac{4ay^{n+1}}{2y^n}$$
$$=3y^{2n-n}-2ay^{n-n+1}$$
$$=3y^n-2ay$$

24. $$\frac{3a(F+T)b^2-(F+T)}{a(F+T)}=\frac{3a(F+T)b^2}{a(F+T)}-\frac{(F+T)}{a(F+T)}$$
$$=\frac{3a^{1-1}\cancel{(F+T)}b^2}{\cancel{(F+T)}}-\frac{\cancel{(F+T)}}{a\cancel{(F+T)}}$$
$$=3b^2-\frac{1}{a}$$

25. $$\begin{array}{r|l} & x+5 \\ \hline x+4 & x^2+9x+20 \\ & \underline{x^2+4x} \\ & 5x+20 \\ & \underline{5x+20} \\ & 0 \end{array}$$

$$\frac{x^2+9x+20}{x+4}=x+5$$

26. $$\begin{array}{r|l} & x+9 \\ \hline x-2 & x^2+7x-18 \\ & \underline{x^2-2x} \\ & 9x-18 \\ & \underline{9x-18} \\ & 0 \end{array}$$

$$\frac{x^2+7x-18}{x-2}=x+9$$

27.
$$\begin{array}{r|l} & 2x+1 \\ \hline x+3 & 2x^2+7x+3 \\ & \underline{2x^2+6x} \\ & x+3 \\ & \underline{x+3} \\ & 0 \end{array}$$

$$\frac{2x^2+7x+3}{x+3}=2x+1$$

28.
$$\begin{array}{r|l} & 3t-4 \\ \hline t-1 & 3t^2-7t+4 \\ & \underline{3t^2-3t} \\ & -4t+4 \\ & \underline{-4t+4} \\ & 0 \end{array}$$

$$\frac{3t^2-7t+4}{t-1}=3t-4$$

29.
$$\begin{array}{r|l} & x-1 \\ \hline x-2 & x^2-3x+2 \\ & \underline{x^2-2x} \\ & -x+2 \\ & \underline{-x+2} \\ & 0 \end{array}$$

$$\frac{x^2-3x+2}{x-2} = x-1$$

30.
$$\begin{array}{r|l} & 2x-7 \\ \hline x+1 & 2x^2-5x-7 \\ & \underline{2x^2+2x} \\ & -7x-7 \\ & \underline{-7x-7} \\ & 0 \end{array}$$

$$\frac{2x^2-5x-7}{x+1}=2x-7$$

31.
$$\begin{array}{r}
4x^2-x-1 \\
2x-3\overline{\big)8x^3-14x^2+x+0} \\
\underline{8x^3-12x^2} \\
-2x^2+x \\
\underline{-2x^2+3x} \\
-2x+0 \\
\underline{-2x+3} \\
-3
\end{array}$$

$$\frac{8x^3-14x^2+x}{2x-3}=4x^2-x-1-\frac{3}{2x-3}$$

32.
$$\begin{array}{r}
3y+2 \\
2y+1\overline{\big)6y^2+7y+6} \\
\underline{6y^2+3y} \\
4y+6 \\
\underline{4y+2} \\
4
\end{array}$$

$$\frac{6y^2+7y+6}{2y+1}=3y+2+\frac{4}{2y+1}$$

33.
$$\begin{array}{r}
Z-2 \\
4Z+3\overline{\big)4Z^2-5Z-7} \\
\underline{4Z^2+3Z} \\
-8Z-7 \\
\underline{-8Z-6} \\
-1
\end{array}$$

$$\frac{4Z^2-5Z-7}{4Z+3}=Z-2-\frac{1}{4Z+3}$$

34.
$$\begin{array}{r}
2x+1 \\
3x-4\overline{\big)6x^2-5x-9} \\
\underline{6x^2-8x} \\
3x-9 \\
\underline{3x-4} \\
-5
\end{array}$$

$$\frac{6x^2-5x-9}{3x-4}=2x+1-\frac{5}{3x-4}$$

35.
$$\begin{array}{r} x^2+x-6 \\ x+2\overline{\big)x^3+3x^2-4x-12} \\ \underline{x^3+2x^2} \\ x^2-4x \\ \underline{x^2+2x} \\ -6x-12 \\ \underline{-6x-12} \\ 0 \end{array}$$

$$\frac{x^3+3x^2-4x-12}{x+2}=x^2+x-6$$

36.
$$\begin{array}{r} x^2+7x+9 \\ 3x-2\overline{\big)3x^3+19x^2+13x-20} \\ \underline{3x^3-2x^2} \\ 21x^2+13x \\ \underline{21x^2-14x} \\ 27x-20 \\ \underline{27x-18} \\ -2 \end{array}$$

$$\frac{3x^3+19x^2+13x-20}{3x-2}=x^2+7x+9-\frac{2}{3x-2}$$

37.
$$\begin{array}{r} 2a^2+8 \\ a^2-2\overline{\big)2a^4+0a^3+4a^2+0a-16} \\ \underline{2a^4-4a^2} \\ 8a^2-16 \\ \underline{8a^2-16} \\ 0 \end{array}$$

$$\frac{2a^4+4a^2-16}{a^2-2}=2a^2+8$$

38.
$$\begin{array}{r} 2T+1 \\ 3T^2-T+2\overline{\big)6T^3+T^2+0T+2} \\ \underline{6T^3-2T^2+4T} \\ 3T^2-4T+2 \\ \underline{3T^2-T+2} \\ -3T \end{array}$$

$$\frac{6T^3+T^2+2}{3T^2-T+2}=2T+1-\frac{3T}{3T^2-T+2}$$

39.

$$\begin{array}{r}
y^2-3y+9 \\
y+3\overline{\smash{\big)}\,y^3+0y^2+0y+27} \\
\underline{y^3+3y^2} \\
-3y^2+0y \\
\underline{-3y^2-9y} \\
9y+27 \\
\underline{9y+27} \\
0
\end{array}$$

$$\frac{y^3+27}{y+3}=y^2-3y+9$$

40.

$$\begin{array}{r}
D^2+D+1 \\
D-1\overline{\smash{\big)}\,D^3+0D^2+0D-1} \\
\underline{D^3-1D^2} \\
D^2+0D \\
\underline{D^2-1D} \\
D-1 \\
\underline{D-1} \\
0
\end{array}$$

$$\frac{D^3-1}{D-1}=D^2+D+1$$

41.

$$\begin{array}{r}
x-y \\
x-y\overline{\smash{\big)}\,x^2-2xy+y^2} \\
\underline{x^2-xy} \\
-xy+y^2 \\
\underline{-xy+y^2} \\
0
\end{array}$$

$$\frac{x^2-2xy+y^2}{x-y}=x-y$$

42.

$$\begin{array}{r}
3r+4R \\
r-3R\overline{\smash{\big)}\,3r^2-5rR+2R^2} \\
\underline{3r^2-9rR} \\
4rR+2R^2 \\
\underline{4rR-12R^2} \\
14R^2
\end{array}$$

$$\frac{3r^2-5rR+2R^2}{r-3R}=3r+4R+\frac{14R^2}{r-3R}$$

43.
$$\begin{array}{r}
t-2 \\
t^2+2t+4\overline{\smash{\big)}t^3+0t^2+0t-8} \\
\underline{t^3+2t^2+4t} \\
-2t^2-4t-8 \\
\underline{-2t^2-4t-8} \\
0
\end{array}$$

$$\frac{t^3-8}{t^2+2t+4}=t-2$$

44.
$$\begin{array}{r}
a^2+2ab+2b^2 \\
a^2-2ab+2b^2\overline{\smash{\big)}a^4+0a^3b+0a^2b^2+0ab^3+b^4} \\
\underline{a^4-2a^3b+2a^2b^2} \\
2a^3b-2a^2b^2+0ab^3 \\
\underline{2a^3b-4a^2b^2+4ab^3} \\
2a^2b^2-4ab^3+b^4 \\
\underline{2a^2b^2-4ab^3+4b^4} \\
-3b^4
\end{array}$$

$$\frac{a^4+b^4}{a^2-2ab+2b^2}=a^2+2ab+2b^2-\frac{3b^4}{a^2-2ab+2b^2}$$

45. We know that $2x+1$ multiplied by $x+c$ will give us $2x^2-9x-5$, so $2x^2-9x-5$ divided by $2x+1$ will give us $x+c$:

$$\begin{array}{r}
x-5 \\
2x+1\overline{\smash{\big)}2x^2-9x-5} \\
\underline{2x^2+x} \\
-10x-5 \\
\underline{-10x-5} \\
0
\end{array}$$

$$x+c=x-5$$
$$c=-5$$

46.
$$\begin{array}{r}
2x-3 \\
3x+4\overline{\smash{\big)}6x^2-x+k} \\
\underline{6x^2+8x} \\
-9x+k \\
\underline{-9x-12} \\
0
\end{array}$$

$$k-(-12)=0$$
$$k+12=0$$
$$k=-12$$

47.
$$
\begin{array}{r}
x^3-x^2+x-1 \\
x+1\overline{)\,x^4+0x^3+0x^2+0x+1} \\
\underline{x^4+x^3} \\
-x^3+0x^2 \\
\underline{-x^3-x^2} \\
x^2+0x \\
\underline{x^2+x} \\
-x+1 \\
\underline{-x-1} \\
2
\end{array}
$$

$$\frac{x^4+1}{x+1}=x^3-x^2+x-1+\frac{2}{x+1}\neq x^3$$

48.
$$
\begin{array}{r}
x^2-xy+y^2 \\
x+y\overline{)\,x^3+0x^2y+0y^2x+0x+y^3} \\
\underline{x^3+x^2\text{y}} \\
-x^2\text{y}+0y^2x \\
\underline{-x^2y-y^2x} \\
y^2x+y^3 \\
\underline{y^2x+y^3} \\
0
\end{array}
$$

$$\frac{x^3+y^3}{x+y}=x^2-xy+y\neq x^2+y^2$$

49.
$$
\begin{aligned}
V_1\ \ 1+\frac{T_2-T_1}{T_1}\ &=V_1\ \ 1+\frac{T_2}{T_1}-\frac{T_1}{T_1} \\
&=V_1\ \ 1+\frac{T_2}{T_1}-1 \\
&=V_1\frac{T_2}{T_1} \\
&=\frac{V_1T_2}{T_1}
\end{aligned}
$$

50.
$$
\begin{array}{r}
3x+2 \\
2x+5\overline{)\,6x^2+19x+10} \\
\underline{6x^2+15x} \\
4x+10 \\
\underline{4x+10} \\
0
\end{array}
$$

The width is $3x+2$

51. $$\frac{8A^5+4A^3\mu^2E^2-A\mu^4E^4}{8A^4}=\frac{8A^5}{8A^4}+\frac{4A^3\mu^2E^2}{8A^4}-\frac{A\mu^4E^4}{8A^4}$$
$$=A^{5-4}+\frac{\mu^2E^2}{2A^{4-3}}-\frac{\mu^4E^4}{8A^{4-1}}$$
$$=A+\frac{\mu^2E^2}{2A}-\frac{\mu^4E^4}{8A^3}$$

52. $$\frac{6R_1+6R_2+R_1R_2}{6R_1R_2}=\frac{6R_1}{6R_1R_2}+\frac{6R_2}{6R_1R_2}+\frac{R_1R_2}{6R_1R_2}$$
$$=\frac{\cancel{R_1}}{\cancel{R_1}R_2}+\frac{\cancel{R_2}}{R_1\cancel{R_2}}+\frac{\cancel{R_1R_2}}{6\cancel{R_1R_2}}$$
$$=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{6}$$

53. $$\frac{GMm[(R+r)-(R-r)]}{2rR}=\frac{GMm[R+r-R+r]}{2rR}$$
$$=\frac{GMm[2r]}{2rR}$$
$$=\frac{GMm\cancel{[2r]}}{\cancel{2r}R}$$
$$=\frac{GMm}{R}$$

54. $$\begin{array}{r|l} & 3T^2-2T-4 \\ \hline T-2 & 3T^3-8T^2+0T+8 \\ & \underline{3T^3-6T^2} \\ & \quad -2T^2+0T \\ & \quad \underline{-2T^2+4T} \\ & \qquad -4T+8 \\ & \qquad \underline{-4T+8} \\ & \qquad\qquad 0 \end{array}$$

55. $$\frac{s^2-2s-2}{s^4+4}^{-1}=\frac{s^4+4}{s^2-2s-2}$$

$$\begin{array}{r}
s^2+2s+6 \\
s^2-2s-2\overline{\big)s^4+0s^3+0s^2+0s+4} \\
\underline{s^4-2s^3-2s^2} \\
2s^3+2s^2+0s \\
\underline{2s^3-4s^2-4s} \\
6s^2+4s+4 \\
\underline{6s^2-12s-12} \\
16s+16
\end{array}$$

$$\frac{s^4+4}{s^2-2s-2}=s^2+2s+6+\frac{16s+16}{s^2-2s-2}$$

56.
$$\begin{array}{r}
t^2-3t+5 \\
2t+100\overline{\big)2t^3+94t^2-290t+500} \\
\underline{2t^3+100t^2} \\
-6t^2-290t \\
\underline{-6t^2-300t} \\
10t+500 \\
\underline{10t+500} \\
0
\end{array}$$

1.10 Solving Equations

1. **(a)**
$$\begin{aligned} x-3&=-12 \\ x-3+3&=-12+3 \\ x&=-9 \end{aligned}$$

(b)
$$\begin{aligned} x+3&=-12 \\ x+3-3&=-12-3 \\ x&=-15 \end{aligned}$$

(c)
$$\begin{aligned} \frac{x}{3}&=-12 \\ 3\ \frac{x}{3}&=3(-12) \\ x&=-36 \end{aligned}$$

(d)
$$3x = -12$$
$$\frac{3x}{3} = \frac{-12}{3}$$
$$x = -4$$

2.
$$7 - 2t = 9$$
$$7 - 7 - 2t = 9 - 7$$
$$-2t = 2$$
$$\frac{-2t}{-2} = \frac{2}{-2}$$
$$t = -1$$

Check:
$$7 - 2t = 9$$
$$7 - 2(-1) = 9$$
$$7 - (-2) = 9$$
$$9 = 9$$

3.
$$x - 7 = 3x - (8 - 6x)$$
$$x - 7 = 3x - 8 + 6x$$
$$x - 7 = 9x - 8$$
$$-8x = -1$$
$$x = \frac{1}{8}$$

4.
$$\frac{L}{3.80} = \frac{7}{4}$$
$$3.80 \quad \frac{L}{3.80} \quad = 3.80 \quad \frac{7}{4}$$
$$L = 6.65 \text{ m}$$

5.
$$x - 2 = 7$$
$$x = 7 + 2$$
$$x = 9$$

6.
$$x - 4 = -1$$
$$x = -1 + 4$$
$$x = 3$$

7.
$$x + 5 = 4$$
$$x = 4 - 5$$
$$x = -1$$

8.
$$s + 6 = -3$$
$$s = -3 - 6$$
$$s = -9$$

9. $\frac{t}{2} = -5$

$t = 2(-5)$

$t = -10$

10. $\frac{x}{-4} = 2$

$x = -4(2)$

$x = -8$

11. $\frac{y-8}{3} = 4$

$y - 8 = 4(3)$

$y = 12 + 8$

$y = 20$

12. $\frac{7-r}{6} = 3$

$7 - r = 6(3)$

$-r = 18 - 7$

$-r = 11$

$r = -11$

13. $4E = -20$

$E = \frac{-20}{4}$

$E = -5$

14. $2x = 12$

$x = \frac{12}{2}$

$x = 6$

15. $5t + 9 = -1$

$5t = -1 - 9$

$t = \frac{-10}{5}$

$t = -2$

16. $5D - 2 = 13$

$5D = 13 + 2$

$D = \frac{15}{5}$

$D = 3$

17.
$$\begin{aligned} 5-2y &= -3 \\ -2y &= -3-5 \\ y &= \frac{-8}{-2} \\ y &= 4 \end{aligned}$$

18.
$$\begin{aligned} -5t+8 &= 18 \\ -5t &= 18-8 \\ t &= \frac{10}{-5} \\ t &= -2 \end{aligned}$$

19.
$$\begin{aligned} 3x+7 &= x \\ x-3x &= 7 \\ -2x &= 7 \\ x &= -\frac{7}{2} \end{aligned}$$

20.
$$\begin{aligned} 6+4L &= 5-3L \\ 4L+3L &= 5-6 \\ 7L &= -1 \\ L &= -\frac{1}{7} \end{aligned}$$

21.
$$\begin{aligned} 2(3q+4) &= 5q \\ 6q+8 &= 5q \\ 6q-5q &= -8 \\ q &= -8 \end{aligned}$$

22.
$$\begin{aligned} 3(4-n) &= -n \\ -n &= 12-3n \\ -n+3n &= 12 \\ 2n &= 12 \\ n &= \frac{12}{2} \\ n &= 6 \end{aligned}$$

23.
$$\begin{aligned} -(r-4) &= 6+2r \\ -r+4 &= 6+2r \\ -r-2r &= 2 \\ -3r &= 2 \\ r &= -\frac{2}{3} \end{aligned}$$

24. $$\begin{aligned} -(x+2)+5 &= 5x \\ 5x &= 5-x-2 \\ 5x+x &= 3 \\ 6x &= 3 \\ x &= \frac{3}{6}=\frac{1}{2} \end{aligned}$$

25. $$\begin{aligned} 8(y-5) &= -2y \\ 8y-40 &= -2y \\ 8y+2y &= 40 \\ 10y &= 40 \\ y &= \frac{40}{10} \\ y &= 4 \end{aligned}$$

26. $$\begin{aligned} 4(7-F) &= -7 \\ 28-4F &= -7 \\ -4F &= -7-28 \\ F &= \frac{-35}{-4}=\frac{35}{4} \end{aligned}$$

27. $$\begin{aligned} 0.1x-0.5(x-2) &= 2 \\ x-5(x-2) &= 2(10) \\ x-5x+10 &= 20 \\ -4x &= 20-10 \\ x &= \frac{10}{-4}=-\frac{5}{2} \end{aligned}$$

28. $$\begin{aligned} 1.5x-0.3(x-4) &= 6 \\ 15x-3(x-4) &= 6(10) \\ 15x-3x+12 &= 60 \\ 12x &= 60-12 \\ x &= \frac{48}{12} \\ x &= 4 \end{aligned}$$

29. $$\begin{aligned} -4-3(1-2p) &= -7+2p \\ -4-3+6p &= -7+2p \\ -7+6p-2p &= -7 \\ 4p &= -7+7 \\ p &= \frac{0}{4} \\ p &= 0 \end{aligned}$$

30.
$$\begin{aligned} 3-6(2-3t) &= t-5 \\ 3-12+18t &= t-5 \\ -9+18t-t &= -5 \\ 17t &= 4 \\ t &= \frac{4}{17} \end{aligned}$$

31.
$$\begin{aligned} \frac{4x-2(x-4)}{3} &= 8 \\ 4x-2x+8 &= 3(8) \\ 2x &= 24-8 \\ x &= \frac{16}{2} \\ x &= 8 \end{aligned}$$

32.
$$\begin{aligned} 2x &= \frac{-5(7-3x)+2}{4} \\ 4(2x) &= -35+15x+2 \\ 8x-15x &= -33 \\ -7x &= -33 \\ x &= \frac{-33}{-7} = \frac{33}{7} \end{aligned}$$

33.
$$\begin{aligned} |x|-9 &= 2 \\ |x| &= 2+9 = 11 \\ x = 11 &\text{ or } x = -11 \end{aligned}$$

34.
$$\begin{aligned} 2-|x| &= 4 \\ -|x| &= 4-2 \\ |x| &= \frac{2}{-1} \\ |x| &= -2 \end{aligned}$$

There is no real solution for x.

35. $|2x-3| = 5$

$2x-3=5$ or $2x-3=-5$

$$\begin{array}{ll} 2x-3=5 & 2x-3=-5 \\ 2x=5+3 & 2x=-5+3 \\ 2x=8 & 2x=-2 \\ x=\frac{8}{2} & x=\frac{-2}{2} \\ x=4 \quad \text{or} & x=-1 \end{array}$$

36. $|7-x|=1$

$7-x=1$ or $7-x=-1$

$-x=1-7 \qquad -x=-1-7$

$-x=-6 \qquad -x=-8$

$x=6 \quad$ or $\quad x=8$

37. $5.8-0.3(x-6.0)=0.5x$

$$\begin{aligned}0.5x &= 5.8-0.3x+1.8\\ 0.5x+0.3x &= 7.6\\ 0.8x &= 7.6\\ x &= \frac{7.6}{0.8}\\ x &= 9.5\end{aligned}$$

38.

$$\begin{aligned}1.9t &= 0.5(4.0-t)-0.8\\ 1.9t &= 2.0-0.5t-0.8\\ 1.9t+0.5t &= 1.2\\ 2.4t &= 1.2\\ t &= \frac{1.2}{2.4}\\ t &= 0.50\end{aligned}$$

39. $-0.24(C-0.50)=0.63$

$$\begin{aligned}-0.24C+0.12 &= 0.63\\ -0.24C &= 0.63-0.12\\ -0.24C &= 0.51\\ C &= \frac{0.51}{-0.24}\\ C &= -2.125\\ C &= -2.1\end{aligned}$$

40. $27.5(5.17-1.44x)=73.4$

$$\begin{aligned}142.175-39.6x &= 73.4\\ -39.6x &= 73.4-142.175\\ -39.6x &= -68.775\\ x &= \frac{-68.775}{-39.6}\\ x &= 1.736742424\\ x &= 1.74\end{aligned}$$

41. $\frac{x}{2.0}=\frac{17}{6.0}$

$x=2.0\ \frac{17}{6.0}$

$x=5.6666666...$

$x=5.7$

42. $\frac{3.0}{7.0} = \frac{R}{42}$

$R = 42 \;\; \frac{3.0}{7.0}$

$R = 18$

43. $\frac{165}{223} = \frac{13V}{15}$

$\frac{15}{13} \;\; \frac{165}{223} \;\; = \frac{15}{13} \;\; \frac{13V}{15}$

$V = \frac{2475}{2899}$

$V = 0.85374267$

$V = 0.85$

44. $\frac{276x}{17.0} = \frac{1360}{46.4}$

$276x = 17 \;\; \frac{1360}{46.4}$

$x = \frac{498.2758621}{276}$

$x = 1.805347326$

$x = 1.81$

45. **(a)** $2x + 3 = 3 + 2x$

$2x + 3 = 2x + 3$

Is an identity, since it is true for all values of x.

(b) $2x - 3 = 3 - 2x$

$4x = 6$

$x = \frac{6}{4} = \frac{3}{2}$

Is conditional as x has one answer only.

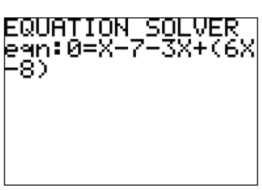
46. There are no values of a that result in a conditional equation. If $a = 0$, then the identity $2x = 2x$ results. If $a \neq 0$, then a contradiction results.

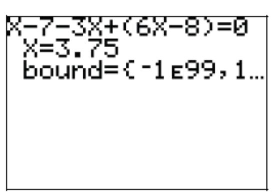
47. $x - 7 = 3x - (6x - 8)$

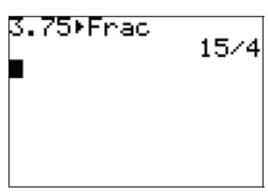
$0 = 3x - 6x + 8 - x + 7$

$0 = -4x + 15$

$x = 3.75$

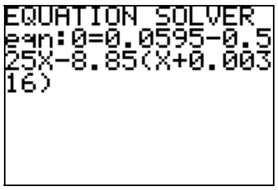


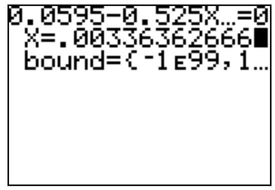




48. $0.0595-0.525i-8.85(i+0.0316)=0$

$$0.595-0.525i-8.85i-0.27966=0$$
$$-9.375i+0.31534=0$$
$$i=0.033636266$$
$$i=0.0336$$





49. $0.03x+0.06(2000-x)=96$

$$0.03x+120-0.06x=96$$
$$-0.03x=96-120$$
$$-0.03x=-24$$
$$x=\frac{-24}{-0.03}$$
$$x=\$800$$

50. $15(5.5+v)=24(5.5-v)$

$$82.5+15v=132-24v$$
$$15v+24v=132-82.5$$
$$39v=49.5$$
$$v=\frac{49.5}{39}$$
$$v=1.269230769 \text{ km/h}$$
$$v=1.3 \text{ km/h}$$

51. $1.1=\frac{(T-76)}{40}$

$$40(1.1)=T-76$$
$$44=T-76$$
$$T=44+76$$
$$T=120\ ^\circ\text{C}$$

52. $1.12V-0.67(10.5-V)=0$

$$1.12V-7.035+0.67V=0$$
$$1.79V-7.035=0$$
$$1.79V=7.035$$
$$V=\frac{7.035}{1.79}$$
$$V=3.930167598 \text{ V}$$
$$V=3.9 \text{ V}$$

53.
$$\begin{aligned}
0.14n + 0.06(2000 - n) &= 0.09(2000)\\
0.14n + 120 - 0.06n &= 180\\
0.14n - 0.06n &= 180 - 120\\
0.08n &= 60\\
n &= \frac{60}{0.08}\\
n &= 750 \text{ L}
\end{aligned}$$

54.
$$\begin{aligned}
210(3x) &= 55.3x + 38.5(8.25 - 3x)\\
630x &= 55.3x + 317.625 - 115.5x\\
630x - 55.3x + 115.5x &= 317.625\\
690.2x &= 317.625\\
x &= \frac{317.625}{690.2}\\
x &= 0.4601927 \text{ m}\\
x &= 0.460 \text{ m}
\end{aligned}$$

55.
$$\begin{aligned}
\frac{x}{350 \text{ mi}} &= \frac{30 \text{ kW}\cdot\text{h}}{107 \text{ mi}}\\
x &= 350 \text{ mi} \times \frac{30 \text{ kW}\cdot\text{h}}{107 \text{ mi}}\\
x &= 98 \text{ kW}\cdot\text{h}
\end{aligned}$$

56.
$$\begin{aligned}
\frac{20 \text{ min}}{250 \text{ cal}} &= \frac{x}{400 \text{ cal}}\\
x &= 400 \text{ cal} \ \frac{20 \text{ min}}{250 \text{ cal}}\\
x &= 32 \text{ min}
\end{aligned}$$

1.11 Formulas and Literal Equations

1.
$$\begin{aligned}
v &= v_0 + at\\
v - v_0 &= at\\
a &= \frac{v - v_0}{t}
\end{aligned}$$

2.
$$\begin{aligned}
W &= \frac{L(wL + 2P)}{8}\\
8W &= L(wL + 2P)\\
8W &= wL^2 + 2LP\\
wL^2 &= 8W - 2LP\\
w &= \frac{8W - 2LP}{L^2}
\end{aligned}$$

3.
$$\begin{aligned} V &= V_0[1+b(T-T_0)] \\ V &= V_0[1+bT-bT_0] \\ V &= V_0+bTV_0-bT_0V_0 \\ bT_0V_0 &= V_0+bTV_0-V \\ T_0 &= \frac{V_0+bTV_0-V}{bV_0} \end{aligned}$$

4.
$$\begin{aligned} V &= V_0+V_0\beta T \\ V-V_0 &= V_0\beta T \\ \beta &= \frac{V-V_0}{V_0T} \end{aligned}$$

5.
$$E = IR$$
$$R = \frac{E}{I}$$

6.
$$pV = nRT$$
$$T = \frac{pV}{nR}$$

7.
$$\begin{aligned} rL &= g_2-g_1 \\ g_1+rL &= g_2 \\ g_1 &= g_2-rL \end{aligned}$$

8.
$$\begin{aligned} W &= S_dT-Q \\ Q+W &= S_dT \\ Q &= S_dT-W \end{aligned}$$

9.
$$\begin{aligned} B &= \frac{nTWL}{12} \\ 12B &= nTWL \\ n &= \frac{12B}{TWL} \end{aligned}$$

10.
$$P = 2\pi Tf$$
$$T = \frac{P}{2\pi f}$$

11.
$$\begin{aligned} p &= p_a+dgh \\ p-p_a &= dgh \\ h &= \frac{p-p_a}{dg} \end{aligned}$$

12. $$\begin{aligned} 2Q &= 2I + A + S \\ 2I &= 2Q - A - S \\ I &= \frac{2Q - A - S}{2} \end{aligned}$$

13. $$\begin{aligned} F_c &= \frac{mv^2}{r} \\ rF_c &= mv^2 \\ r &= \frac{mv^2}{F_c} \end{aligned}$$

14. $$\begin{aligned} P &= \frac{4F}{\pi D^2} \\ P\pi D^2 &= 4F \\ F &= \frac{P\pi D^2}{4} \end{aligned}$$

15. $$\begin{aligned} S_T &= \frac{A}{5T} + 0.05d \\ S_T - 0.05d &= \frac{A}{5T} \\ A &= 5T\left(S_T - 0.05d\right) \end{aligned}$$

16. $$\begin{aligned} u &= -\frac{eL}{2m} \\ eL &= -2mu \\ L &= -\frac{2mu}{e} \end{aligned}$$

17. $$\begin{aligned} ct^2 &= 0.3t - ac \\ ac + ct^2 &= 0.3t \\ ac &= 0.3t - ct^2 \\ a &= \frac{-ct^2 + 0.3t}{c} \end{aligned}$$

18. $$\begin{aligned} 2p + dv^2 &= 2d(C - W) \\ 2p + dv^2 &= 2Cd - 2dW \\ 2Cd &= dv^2 + 2p + 2dW \\ C &= \frac{dv^2 + 2dW + 2p}{2d} \end{aligned}$$

19. $$\begin{aligned} T &= \frac{c + d}{v} \\ c + d &= Tv \\ d &= Tv - c \end{aligned}$$

20.
$$B = \frac{\mu_0 I}{2\pi R}$$
$$BR = \frac{\mu_0 I}{2\pi}$$
$$R = \frac{\mu_0 I}{2\pi B}$$

21.
$$\frac{K_1}{K_2} = \frac{m_1 + m_2}{m_1}$$
$$K_2(m_1 + m_2) = K_1 m_1$$
$$K_2 m_1 + K_2 m_2 = K_1 m_1$$
$$K_2 m_2 = K_1 m_1 - K_2 m_1$$
$$m_2 = \frac{K_1 m_1 - K_2 m_1}{K_2}$$

22.
$$f = \frac{F}{d - F}$$
$$f(d - F) = F$$
$$fd - fF = F$$
$$fd = F + fF$$
$$d = \frac{F + fF}{f}$$

23.
$$a = \frac{2mg}{M + 2m}$$
$$a(M + 2m) = 2gm$$
$$aM + 2am = 2gm$$
$$aM = 2gm - 2am$$
$$M = \frac{2gm - 2am}{a}$$

24.
$$v = \frac{V(m + M)}{m}$$
$$mv = mV + MV$$
$$MV = mv - mV$$
$$M = \frac{mv - mV}{V}$$

25.
$$C_0^2 = C_1^2(1 + 2V)$$
$$C_0^2 = C_1^2 + 2C_1^2 V$$
$$2C_1^2 V = C_0^2 - C_1^2$$
$$V = \frac{C_0^2 - C_1^2}{2C_1^2}$$

26.
$$\begin{aligned} A_1 &= A(M+1) \\ A_1 &= AM + A \\ AM &= A_1 - A \\ M &= \frac{A_1 - A}{A} \end{aligned}$$

27.
$$\begin{aligned} N &= r(A-s) \\ N &= Ar - rs \\ rs + N &= Ar \\ rs &= Ar - N \\ s &= \frac{Ar - N}{r} \end{aligned}$$

28.
$$\begin{aligned} T &= 3(T_2 - T_1) \\ T &= 3T_2 - 3T_1 \\ 3T_1 + T &= 3T_2 \\ 3T_1 &= 3T_2 - T \\ T_1 &= \frac{3T_2 - T}{3} \end{aligned}$$

29.
$$\begin{aligned} T_2 &= T_1 - \frac{h}{100} \\ 100T_2 &= 100T_1 - h \\ h + 100T_2 &= 100T_1 \\ h &= 100T_1 - 100T_2 \end{aligned}$$

30.
$$\begin{aligned} p_2 &= p_1 + rp_1(1-p_1) \\ p_2 - p_1 &= rp_1(1-p_1) \\ r &= \frac{p_2 - p_1}{p_1(1-p_1)} \end{aligned}$$

31.
$$\begin{aligned} Q_1 &= P(Q_2 - Q_1) \\ Q_1 &= PQ_2 - PQ_1 \\ PQ_2 &= Q_1 + PQ_1 \\ Q_2 &= \frac{Q_1 + PQ_1}{P} \end{aligned}$$

32.
$$\begin{aligned} p - p_a &= dg\left(y_2 - y_1\right) \\ y_2 - y_1 &= \frac{p - p_a}{dg} \\ -y_1 &= \frac{p - p_a}{dg} - y_2 \\ y_1 &= y_2 - \frac{p - p_a}{dg} \end{aligned}$$

33.
$$N = N_1T - N_2(1-T)$$
$$N_1T = N + N_2(1-T)$$
$$N_1 = \frac{N + N_2 - N_2T}{T}$$

34.
$$t_a = t_c + (1-h)t_m$$
$$t_a = t_c + t_m - ht_m$$
$$t_a + ht_m = t_c + t_m$$
$$ht_m = t_c + t_m - t_a$$
$$h = \frac{t_c + t_m - t_a}{t_m}$$

35.
$$L = \pi(r_1 + r_2) + 2x_1 + 2x_2$$
$$L = \pi r_1 + \pi r_2 + 2x_1 + 2x_2$$
$$\pi r_1 = L - \pi r_2 - 2x_1 - 2x_2$$
$$r_1 = \frac{L - \pi r_2 - 2x_1 - 2x_2}{\pi}$$

36.
$$I = \frac{VR_2 + VR_1(1+\mu)}{R_1R_2}$$
$$IR_1R_2 = VR_2 + VR_1 + VR_1\mu$$
$$VR_1\mu = IR_1R_2 - VR_2 + VR_1$$
$$\mu = \frac{IR_1R_2 - VR_2 + VR_1}{VR_1}$$

37.
$$P = \frac{V_1(V_2 - V_1)}{gJ}$$
$$gJP = V_1V_2 - V_1^2$$
$$V_1V_2 = V_1^2 + gJP$$
$$V_2 = \frac{V_1^2 + gJP}{V_1}$$

38.
$$W = T(S_1 - S_2) - Q$$
$$W + Q = TS_1 - TS_2$$
$$TS_2 = TS_1 - W - Q$$
$$S_2 = \frac{TS_1 - W - Q}{T}$$

39.
$$C = \frac{2eAk_1k_2}{d(k_1 + k_2)}$$
$$Cd(k_1 + k_2) = 2eAk_1k_2$$
$$e = \frac{Cd(k_1 + k_2)}{2Ak_1k_2}$$

40.
$$d = \frac{3LPx^2 - Px^3}{6EI}$$
$$6dEI = 3LPx^2 - Px^3$$
$$3LPx^2 = 6dEI + Px^3$$
$$L = \frac{6dEI + Px^3}{3Px^2}$$

41.
$$V = C\left(1 - \frac{n}{N}\right)$$
$$V = C - \frac{Cn}{N}$$
$$V + \frac{Cn}{N} = C$$
$$\frac{Cn}{N} = C - V$$
$$Cn = CN - NV$$
$$n = \frac{CN - NV}{C}$$

42.
$$\frac{p}{P} = \frac{AI}{B + AI}$$
$$p(B + AI) = AIP$$
$$pB + AIp = AIP$$
$$pB = AIP - AIp$$
$$B = \frac{AIP - AIp}{p}$$

43.
$$p(C - n) + n = A$$
$$pC - pn + n = A$$
$$(-p + 1)n = A - pC$$
$$(1 - p)n = A - pC$$
$$n = \frac{A - pC}{1 - p}$$
$$n = \frac{13.0\text{ L} - 0.25(15.0\text{ L})}{1 - 0.25}$$
$$T_1 = \frac{13.0\text{ L} - 3.75\text{ L}}{0.75}$$
$$T_1 = \frac{9.25\text{ L}}{0.75}$$
$$T_1 = 12.333333\text{ L}$$
$$T_1 = 12\text{ L}$$

44. $$P_t = P_c(1+0.500m^2)\frac{\pi}{2}$$

$$P_c = \frac{P_t}{1+0.500m^2}$$

$$P_c = \frac{680\text{ W}}{1+0.500(0.925)^2}$$

$$P_c = \frac{680\text{ W}}{1+0.500(0.855625)}$$

$$P_c = \frac{680\text{ W}}{1+0.4278125}$$

$$P_c = \frac{680\text{ W}}{1.4278125}$$

$$P_c = 476.253009\text{ W}$$

$$P_c = 476\text{ W}$$

45. $$F = \frac{9}{5}C + 32$$

$$90.2 = \frac{9}{5}C + 32$$

$$\frac{5}{9}(90.2 - 32) = C$$

$$C = \frac{5}{9}\times 58.2$$

$$C = 32.3^\circ\text{C}$$

46. $$V = \frac{1}{2}L(B+b)$$

$$2V = BL + bL$$

$$bL = 2V - BL$$

$$b = \frac{2V - BL}{L}$$

$$b = \frac{2(38.6\text{ ft}^3) - (2.63\text{ ft}^2)(16.1\text{ ft})}{16.1\text{ ft}}$$

$$b = \frac{77.2\text{ ft}^3 - 42.343\text{ ft}^3}{16.1\text{ ft}}$$

$$b = \frac{34.857\text{ ft}^3}{16.1\text{ ft}}$$

$$b = 2.16503106\text{ ft}^2$$

$$b = 2.16\text{ ft}^2$$

47. $V_1 = \dfrac{VR_1}{R_1 + R_2}$

$V_1(R_1 + R_2) = VR_1$

$R_1 + R_2 = \dfrac{VR_1}{V_1}$

$R_2 = \dfrac{VR_1}{V_1} - R_1$

$R_2 = \dfrac{(12.0 \text{ V})(3.56\ \Omega)}{6.30 \text{ V}} - (3.56\ \Omega)$

$R_2 = 6.780952381\ \Omega - 3.56\ \Omega$

$R_2 = 3.220952381\ \Omega$

$R_2 = 3.22\ \Omega$

48. $\eta = \dfrac{1}{q + p(1-q)}$

$\eta\left[q + p(1-q)\right] = 1$

$\eta q + \eta p(1-q) = 1$

$\eta p(1-q) = 1 - \eta q$

$p = \dfrac{1-\eta q}{\eta(1-q)}$

$p = \dfrac{1-(0.66)(0.83)}{0.66(1-0.83)}$

$p = \dfrac{1-0.5478}{0.66(0.17)}$

$p = \dfrac{0.4522}{0.1122}$

$p = 4.03030303$

$p = 4$ processors

49. $d = v_2t_2 + v_1t_1$

$d = v_2(4 \text{ h}) + v_1(t + 2 \text{ h})$

$tv_1 + v_1(2 \text{ h}) = d - v_2(4 \text{ h})$

$tv_1 = d - v_2(4 \text{ h}) - v_1(2 \text{ h})$

$t = \dfrac{d - v_2(4 \text{ h}) - v_1(2 \text{ h})}{v_1}$

50. $C = x + 15y$

$15y = C - x$

$y = \dfrac{C-x}{15}$

1.12 Applied Word Problems

1. Let x = the number of 25 W lights.
Let $31-x$ = the number of 40 W lights.

$$25x+40(31-x)=1000$$
$$25x+1240-40x=1000$$
$$-15x=1000-1240$$
$$-15x=-240$$
$$x=16$$

There are 16 of the 25 W lights and(31– 16)=15 of the 40 W lights.
Check:

$$25\cdot 16+40(31-16)=1000$$
$$400+40(15)=1000$$
$$400+600=1000$$
$$1000=1000$$

2. Let$x$ = the number of slides with 5 mg.
Let $x-3$ = the number of slides with 6 mg.

$$(5\text{ mg})x=(6\text{ mg})(x-3)$$
$$(5\text{ mg})x=(6\text{ mg})x-18\text{ mg}$$
$$-x=-18$$
$$x=18\text{ slides}$$

There are 18 slides with 5 mg and (18−3)=15 slides with 6 mg.
Check:

$$5\text{ mg}(18)=6\text{ mg}(15)$$
$$90\text{ mg}=90\text{ mg}$$

3. Let t = the time for the shuttle to reach the satellite.

$$(29\ 500\text{ km/h})t=6000\text{ km}+(27\ 100\text{ km/h})t$$
$$(2400\text{ km/h})t=6000\text{ km}$$
$$t=\frac{6000\text{ km}}{2400\text{ km/h}}$$
$$t=2.500\text{ h}$$

It will take the shuttle 2.500 h to reach the satellite.
Check:

$$(29500\text{ km/h})(2.500\text{ h})=6000\text{ km}+(27100\text{ km/h})(2.500\text{ h})$$
$$73750\text{ km}=6000\text{ km}+67750\text{ km}$$
$$73750\text{ km}=73750\text{ km}$$

4. Let x = the number of litres of 50% methanol blend that must be added.

$$0.0600(7250\text{ L})+0.500(x)=0.100(7250\text{ L}+x)$$
$$435\text{ L}+0.500(x)=725\text{ L}+0.100x$$
$$0.400(x)=290\text{ L}$$
$$x=\frac{290\text{ L}}{0.400}$$
$$x=725\text{ L}$$

725 L of the 50% methanol blend must be added.

Check:

$$0.0600(7250\text{ L})+0.500(725\text{ L})=0.100(7250\text{ L}+725\text{ L})$$
$$435\text{ L}+362.5\text{ L}=0.1(7975\text{ L})$$
$$797.5\text{ L}=797.5\text{ L}$$

5. Let x = the cost of the car 6 years ago.
Let x + \$5000 = the cost of the car model today.

$$x+(x+\$5000)=\$49\ 000$$
$$2x=\$44\ 000$$
$$x=\frac{\$44\ 000}{2}$$
$$x=\$22\ 000$$

The cost of the car 6 years ago was \$22 000, and the cost of the today's model is (\$22 000 + 5000) = \$27 000.
Check:

$$\$22\ 000+(\$22\ 000+\$5000)=\$49\ 000$$
$$\$22\ 000+\$27\ 000=\$49\ 000$$
$$\$49\ 000=\$49\ 000$$

6. Let x = the flow from the first stream in m^3/s.
Let x – 1700 ft^3/s = the flow from the second streamin m^3/s.

$$x+(x-1700\text{ ft}^3/\text{s})=\frac{1.98\times10^7\text{ ft}^3}{3600\text{ s}}$$
$$2x-1700\text{ ft}^3/\text{s}=5500\text{ ft}^3/\text{s}$$
$$2x=7200\text{ ft}^3/\text{s}$$
$$x=\frac{7200\text{ ft}^3/\text{s}}{2}$$
$$x=3600\text{ ft}^3/\text{s}$$

The first stream flows $3600\text{ ft}^3/\text{s}$ and the second stream flows 3600 ft^3/s – 1700 ft^3/s = 1900 ft^3/s.
Check:

$$3600\text{ ft}^3/\text{s}+(3600\text{ ft}^3/\text{s}-1700\text{ ft}^3/\text{s})=\frac{1.98\times10^7\text{ ft}^3}{3600\text{ s}}$$
$$7200\text{ ft}^3/\text{s}-1700\text{ ft}^3/\text{s}=5500\text{ ft}^3/\text{s}$$
$$5500\text{ ft}^3/\text{s}=5500\text{ ft}^3/\text{s}$$

7. Let x = the number of cars recycled the first year.
Let x+500000 = the number of cars recycled the second year.

$$x+(x+500000\text{ cars})=6900000\text{ cars}$$
$$2x+500000\text{ cars}=6900000\text{ cars}$$
$$2x=6400000\text{ cars}$$
$$x=\frac{6400000\text{ cars}}{2}$$
$$x=3200000\text{ cars}$$

The first year,3.2 × 10^6 cars were recycled, and the second year (3200000 + 500000)= 3.7×10^6 cars were recycled.
Check:

$$3200000\text{ cars}+(3200000\text{ cars}+500000\text{ cars})=6900000\text{ cars}$$
$$3200000\text{ cars}+3700000\text{ cars}=6900000\text{ cars}$$
$$6900000\text{ cars}=6900000\text{ cars}$$

8. Let x = the number of hits to the website on the first day.
Let 1/2 x = the number of hits on the second day.

$$x + 1/2\,x = 495000 \text{ hits}$$
$$3/2\,x = 495000 \text{ hits}$$
$$x = \frac{495000 \text{ hits}}{3/2}$$
$$x = 330000 \text{ hits}$$

The first day there were 330000 hits, the second day there were 1/2(33000 hits= 165000 hits.
Check:

$$330000 \text{ hits} + 1/2(330000 \text{ hits}) = 495000 \text{ hits}$$
$$330000 \text{ hits} + 165000 \text{ hits} = 495000 \text{ hits}$$
$$495000 \text{ hits} = 495000 \text{ hits}$$

9. Let x = the number acres of land leased for \$200 per acre.
Let 140 – x = the number of acres of land leased for \$300 per acre.

$$\$200 / \text{acre}\; x + \$300 / \text{acre}(140 \text{ acre} - x) = \$37\ 000$$
$$-\$100 / \text{acre}\ (x) = -\$5\ 000$$
$$x = \frac{-\$5000}{-\$100 / \text{acre}}$$
$$x = 50 \text{ acres}$$

There are 50 acres leased at \$200 per acre and (140 acres – 50 acres) = 90 hectares leased for \$300 per hectare.
Check:

$$\$200 / \text{acre}\ (50 \text{ acres}) + \$300 / \text{acre}(140 \text{ acres} - 50 \text{ acres}) = \$37\ 000$$
$$\$10\ 000 + \$27\ 000 = \$37\ 000$$
$$\$37\ 000 = \$37\ 000$$

10. Let x = the first dose in mg.
Let x + 660 mg = the second dose in mg.

$$x + x + 660 \text{ mg} = 2000 \text{ mg}$$
$$2x = 1340 \text{ mg}$$
$$x = \frac{1340 \text{ mg}}{2}$$
$$x = 670 \text{ mg}$$

The first dose is 670 mg, and the second dose is (670 mg + 660 mg) = 1130 mg.
Check:

$$670 \text{ mg} + 670 \text{ mg} + 660 \text{ mg} = 2000 \text{ mg}$$
$$670 \text{ mg} + 1330 \text{ mg} = 2000 \text{ mg}$$
$$2000 \text{ mg} = 2000 \text{ mg}$$

11. Let x = the amount of pollutant after modification in ppm/h.

$$(5 \text{ h})x = (3 \text{ h})150 \text{ ppm/h}$$
$$x = \frac{450 \text{ ppm}}{5 \text{ h}}$$
$$x = 90 \text{ ppm/h}$$

The amount of pollutant after modification is 90 ppm/h. The device reduced emissions by
(150 ppm/h – 90 ppm/h) = 60 ppm/h.
Check:

$$(5 \text{ h})90 \text{ ppm/h} = (3 \text{ h})150 \text{ ppm/h}$$
$$450 \text{ ppm} = 450 \text{ ppm}$$

12. Let $x - 13$ = the number of teeth that the first meshed spur has.
Let x = the number of teeth that the second meshed spur has.
Let $x + 15$ = the number of teeth that the third meshed spur has.

$$\begin{aligned} x-13 \text{ teeth} + x + x + 15 \text{ teeth} &= 107 \text{ teeth} \\ 3x + 2 &= 107 \text{ teeth} \\ 3x &= 105 \text{ teeth} \\ x &= \frac{105 \text{ teeth}}{3} \\ x &= 35 \text{ teeth} \end{aligned}$$

The first spur has (35 – 13)= 22 teeth, the second spur has 35 teeth, and the third spur has (35 + 15) = 50 teeth.
Check:

$$\begin{aligned} 35 \text{ teeth} - 13 \text{ teeth} + 35 \text{ teeth} + 35 \text{ teeth} + 15 \text{ teeth} &= 107 \text{ teeth} \\ 107 \text{ teeth} &= 107 \text{ teeth} \end{aligned}$$

13. Let x = amount paid per month for first six months.
Let $x + 10$ = amount paid per month for final four months.

$$\begin{aligned} (6 \text{ mo})x + (4 \text{ mo})(x + \$10/\text{mo}) &= \$890 \\ (10 \text{ mo})x + \$40 &= \$890 \\ (10 \text{ mo})x &= \$850 \\ x &= \frac{\$850}{10 \text{ mo}} \\ x &= \$85/\text{mo} \end{aligned}$$

The bill was \$85/mo for the first six months and \$95/mo for the next four months.
Check:

$$\begin{aligned} (6 \text{ mo})\$85/\text{mo} + (4 \text{ mo})(\$85/\text{mo} + \$10/\text{mo}) &= \$890 \\ \$510 + (4 \text{ mo})(\$95/\text{mo}) &= \$890 \\ \$510 + \$380 &= \$890 \\ \$890 &= \$890 \end{aligned}$$

14. Let x = amount paid per month for first year.
Let $x + 30$ = amount paid per month for next two years.
Let $(x + 30)+20= x + 50$= amount paid per month for final two years.

$$\begin{aligned} (12 \text{ mo})x + (24 \text{ mo})(x + \$30/\text{mo}) + (24 \text{ mo})(x + \$50/\text{mo}) &= \$7320 \\ (12 \text{ mo})x + (24 \text{ mo})x + \$720 + (24 \text{ mo})x + \$1200 &= \$7320 \\ (60 \text{ mo})x + \$1920 &= \$7320 \\ (60 \text{ mo})x &= \$5400 \\ x &= \frac{\$5400}{60 \text{ mo}} \\ x &= \$90/\text{mo} \end{aligned}$$

For the first year, the bill was \$90/mo, during years 2 and 3, the bill was \$120/mo, and during years 4 and 5, the bill was \$140/mo.
Check:

$$\begin{aligned} (12 \text{ mo})(\$90/\text{mo}) + (24 \text{ mo})(\$90/\text{mo} + \$30/\text{mo}) + (24 \text{ mo})(\$90/\text{mo} + \$50/\text{mo}) &= \$7320 \\ \$1080 + (24 \text{ mo})(\$120/\text{mo}) + (24 \text{ mo})(\$140/\text{mo}) &= \$7320 \\ \$1080 + \$2880 + \$3360 &= \$7320 \\ \$7320 &= \$7320 \end{aligned}$$

15. Let x = the first current in μA .
Let $2x$ = the second current in μA .
Let $x + 9.2$ μA = the third current in μA

$$\begin{aligned} x+2x+x+9.2\ \mu\text{A} &= 0\ \mu\text{A} \\ 4x &= -9.2\ \mu\text{A} \\ x &= \frac{-9.2\ \mu\text{A}}{4} \\ x &= -2.3\ \mu\text{A} \end{aligned}$$

The first current is −2.3 μA ,the second current is 2(−2.3 μA) = −4.6 μA , and the third current is (−2.3 μA +9.2 μA) = 6.9 μA .

Check:

$$\begin{aligned} -2.3\ \mu\text{A}+2(-2.3\ \mu\text{A})+\ (-2.3)\ \mu\text{A}+9.2\ \mu\text{A} &= 0\ \mu\text{A} \\ -2.3\ \mu\text{A}-4.6\ \mu\text{A}-2.3\mu\text{A}+9.2\ \mu\text{A} &= 0\ \mu\text{A} \\ 0\ \mu\text{A} &= 0\ \mu\text{A} \end{aligned}$$

16. Let x = the number of trucks in the first fleet.
Let $x + 5$ = the number of trucks in the second fleet.

$$\begin{aligned} (8\text{ h})x+(6\text{ h})(x+5) &= 198\text{ h} \\ (8\text{ h})x+(6\text{ h})x+30\text{ h} &= 198\text{ h} \\ (14\text{ h})x &= 168\text{ h} \\ x &= \frac{168\text{ h}}{14\text{ h}} \\ x &= 12\text{ trucks} \end{aligned}$$

There are 12 trucks in the first fleet and (12 trucks + 5 trucks) = 17 trucks in the second fleet.
Check:

$$\begin{aligned} (8\text{ h})(12)+(6\text{ h})(12+5) &= 198\text{ h} \\ 96\text{ h}+(6\text{ h})(17) &= 198\text{ h} \\ 96\text{ h}+102\text{ h} &= 198\text{ h} \\ 198\text{ h} &= 198\text{ h} \end{aligned}$$

17. Let x = the length of the first pipeline in km.
Let x + 2.6 km = the length of the 3 other pipelines.

$$\begin{aligned} x+3(x+2.6\text{ km}) &= 35.4\text{ km} \\ x+3x+7.8\text{ km} &= 35.4\text{ km} \\ 4x &= 27.6\text{ km} \\ x &= \frac{27.6\text{ km}}{4} \\ x &= 6.9\text{ km} \end{aligned}$$

The first pipeline is 6.9 km long, and the other three pipelines are each (6.9 km + 2.6 km) = 9.5 km long.
Check:

$$\begin{aligned} 6.9\text{ km}+3(6.9\text{ km}+2.6\text{ km}) &= 35.4\text{ km} \\ 6.9\text{ km}+3(9.5\text{ km}) &= 35.4\text{ km} \\ 6.9\text{ km}+28.5\text{ km} &= 35.4\text{ km} \\ 35.4\text{ km} &= 35.4\text{ km} \end{aligned}$$

18. Let x = the power of the first generator in MW.
Let 750 MW−x = the power of the second generator in MW.

$$0.65x + 0.75(750\text{ MW} - x) = 530\text{ MW}$$
$$0.65x + 562.5\text{ MW} - 0.75x = 530\text{ MW}$$
$$-0.1x = -32.5\text{ MW}$$
$$x = \frac{-32.5\text{ MW}}{-0.1}$$
$$x = 325\text{ MW}$$

The first generator produces 325 MW of power, and the second generator produces (750 MW – 325 MW) = 425 MW of power.
Check:

$$0.65(325\text{ MW}) + 0.75(750\text{ MW} - (325\text{ MW})) = 530\text{ MW}$$
$$211.25\text{ MW} + 0.75(425\text{ MW}) = 530\text{ MW}$$
$$211.25\text{ MW} + 318.75\text{ MW} = 530\text{ MW}$$
$$530\text{ MW} = 530\text{ MW}$$

19. Let x = the number of deluxe systems.
Let $2x$ = the number economy systems.
Let x+75= the number of econo-plus systems.

$$\$140x + \$40(2x) + \$80(x+75) = \$42000$$
$$\$140x + \$80x + \$80x + \$6000 = \$42000$$
$$\$300x = \$36000$$
$$x = 120\text{ systems}$$

There are 120 deluxe systems, 240 economy systems, and 195 econo-plus systems sold.
Check:

$$\$140(120) + \$40(240) + \$80(195) = \$42000$$
$$\$16800 + \$9600 + \$15600 = \$42000$$
$$\$42000 = \$42000$$

20. The amount of lottery winnings after taxes is \$20 000 ×(1−0.25) = \$15 000.
Let x = the amount of money invested at a 40% gain.
Let \$15 000− x = the amount of money invested at a 10% loss.

$$0.40x - 0.10(\$15\ 000 - x) = \$2000$$
$$0.40x - \$1500 + 0.10x = \$2000$$
$$0.50x = \$3500$$
$$x = \frac{\$3500}{0.50}$$
$$x = \$7000$$

The 40% gain investment had \$7000 invested, and the 10% loss investment had (\$15 000− \$7000) = \$8000 invested.
Check:

$$0.40(\$7000) - 0.10(\$15\ 000 - \$7000) = \$2000$$
$$\$2800 - \$1500 + 0.10(\$7000) = \$2000$$
$$\$2800 - \$1500 + \$700 = \$2000$$
$$\$2000 = \$2000$$

21. Let x = the amount of time in seconds between when the start of the trains pass each other to when the end of the trains pass each other.
The total distance the ends must travel in this time is 960 feet. We first convert mi/hr into ft/sec.

$$1\text{ mi/hr} = \frac{5280\text{ ft}}{3600\text{ s}} = \frac{22\text{ ft}}{15\text{ s}} = \frac{22}{15}\text{ ft/s}$$

Therefore, train A travels at 60(22/15)=88 ft/s and train B travels at 40(22/15)=176/3 ft/s.

$$\begin{aligned}(88\text{ ft/s})x + (176/3\text{ ft/s})x &= 960\text{ ft}\\ (440/3\text{ ft/s})x &= 960\text{ ft}\\ x &= \frac{960\text{ ft}}{440/3\text{ ft/s}}\\ x &= \frac{72}{11}\text{ s}\end{aligned}$$

The trains completely pass each other in about 6.55 seconds.
Check:

$$\begin{aligned}(88\text{ ft/s})\frac{72}{11}\text{ s} + (176/3\text{ ft/s})\frac{72}{11}\text{ s} &= 960\text{ ft}\\ 576\text{ ft} + 384\text{ ft} &= 960\text{ ft}\\ 960\text{ ft} &= 960\text{ ft}\end{aligned}$$

22. Let x = the mortgage payment and x/0.23=the monthly income.

$$\begin{aligned}x/0.23 - x &= \$3850\\ x\left(\frac{1}{0.23} - 1\right) &= \$3850\\ x\left(\frac{1}{0.23} - \frac{0.23}{0.23}\right) &= \$3850\\ x\left(\frac{0.77}{0.23}\right) &= \$3850\\ x &= \$3850 \cdot \frac{0.23}{0.77}\\ x &= \$1150\end{aligned}$$

The mortgage payment is \$1150 and the monthly income is \$5000.
Check:

$$\begin{aligned}\$1150/0.23 - \$1150 &= \$3850\\ \$5000 - \$1150 &= \$3850\\ \$3850 &= \$3850\end{aligned}$$

23. Let x = the amount of time the skier spends on the ski lift in minutes.
Let 24 minutes $-x$ = the amount of time the skier spends skiing down the hill in minutes.

$$\begin{aligned}(50\text{ m/min})x &= (150\text{ m/min})(24\text{ min} - x)\\ (50\text{ m/min})x &= 3600\text{ m} - (150\text{ m/min})x\\ (200\text{ m/min})x &= 3600\text{ m}\\ x &= \frac{3600\text{ m}}{200\text{ m/min}}\\ x &= 18\text{ min}\end{aligned}$$

The length of the slope is 18 minutes × 50 m/minute = 900m.

Check:

$$\begin{aligned}(50\text{ m/min})18\text{ min} &= (150\text{ m/min})(24\text{ min}-18\text{ min})\\ 900\text{ m} &= 3600\text{ m}-(150\text{ m/min})(18\text{ min})\\ 900\text{ m} &= 3600\text{ m}-2700\text{ m}\\ 900\text{ m} &= 900\text{ m}\end{aligned}$$

24. Let x = the speed of sound.
Let x – 120 mi/h = speed travelled for 1 h.
Let x + 410 mi/h = the speed travelled for 3 h.

$$\begin{aligned}1\text{ h}(x-120\text{ mi/h})+3\text{ h}(x+410\text{ mi/h}) &= 3990\text{ mi}\\ (1\text{ h})x-(1\text{ h})(120\text{ mi/h})+(3\text{ h})x+(3\text{ h})(410\text{ mi/h}) &= 3990\text{ mi}\\ (1\text{ h})x-120\text{ mi}+(3\text{ h})x+1230\text{ mi} &= 3990\text{ mi}\\ (4\text{ h})x &= 2880\text{ mi}\\ x &= \frac{2880\text{ mi}}{4\text{ h}}\\ x &= 720\text{ mi/h}\end{aligned}$$

The speed of sound is 720 mi/h.
Check:

$$\begin{aligned}1\text{ h}(720\text{ mi/h}-120\text{ mi/h})+3\text{ h}(720\text{ mi/h}+410\text{ mi/h}) &= 3990\text{ mi}\\ (1\text{ h})(600\text{ mi/h})+(3\text{ h})(1130\text{ mi/h}) &= 3990\text{ mi}\\ 600\text{ mi}+3390\text{ mi} &= 3990\text{ mi}\\ 3990\text{ mi} &= 3990\text{ mi}\end{aligned}$$

25. Let x = the speed the train leaving England in km/h.
Let x + 8 km/h = speed of the train leaving France in km/h.
The distance travelled by each train is speed ×time.

$$\begin{aligned}x\left(\frac{17\text{ min}}{60\text{ min/ h}}\right)+(x+8\text{ km/h})\left(\frac{17\text{ min}}{60\text{ min/ h}}\right) &= 50\text{ km}\\ (0.28333\text{ h})x+(x+8\text{ km/h})(0.28333\text{ h}) &= 50\text{ km}\\ (0.28333\text{ h})x+(0.28333\text{ h})x+2.26667\text{ km} &= 50\text{ km}\\ (0.56666\text{ h})x &= 47.73333\text{ km}\\ x &= \frac{47.73333\text{ km}}{0.56666\text{ h}}\\ x &= 84.23529421\text{ km/h}\\ x &= 84.2\text{ km/h}\end{aligned}$$

The train leaving England was travelling at 84.2 km/h, and the train leaving France was travelling at (84.2 km/h + 8 km/h) = 92.2 km/h.
Check:

$$\begin{aligned}84.23529421\text{ km/h}\left(\frac{17\text{ min}}{60\text{ min/ h}}\right)+(84.23529421\text{ km/h}+8\text{ km/h})\left(\frac{17\text{ min}}{60\text{ min/ h}}\right) &= 50\text{ km}\\ 23.86666\text{ km}+(92.23529421\text{ km/h})\left(\frac{17\text{ min}}{60\text{ min/ h}}\right) &= 50\text{ km}\\ 23.86666\text{ km}+26.13333\text{ km} &= 50\text{ km}\\ 50\text{ km} &= 50\text{ km}\end{aligned}$$

26. Let x = time left until the appointment.
Let x– 10.0 min = time taken to get to the appointment travelling at 60.0 mi/h.
Let x– 5.0 min = time taken to get to the appointment travelling at 45.0 mi/h.
The distance travelled by the executive in each scenario is the same. Distance = speed × time

$$60.0\text{ mi/h}\left(x-\frac{10.0\text{ min}}{60\text{ min/h}}\right)=45\text{ mi/h}\left(x-\frac{5.0\text{ min}}{60\text{ min/h}}\right)$$
$$(60.0\text{ mi/h})x-60\text{ mi/h}\left(\frac{10.0\text{ min}}{60\text{ min/h}}\right)=(45\text{ mi/h})x-45\text{ mi/h}\left(\frac{5.0\text{ min}}{60\text{ min/h}}\right)$$
$$(60.0\text{ mi/h})x-10\text{ mi}=(45.0\text{ mi/h})x-3.75\text{ mi}$$
$$(15.0\text{ mi/h})x=6.25\text{ mi}$$
$$x=\frac{6.25\text{ mi}}{15.0\text{ mi/h}}$$
$$x=0.416666667\text{ h}$$
$$x=0.416666667\text{ h x }60\text{ min/h}$$
$$x=25\text{ min}$$

There is 25 minutes left until the executive's appointment.
Check:

$$60.0\text{ mi/h}\left(0.41667\text{ h}-\frac{10.0\text{ min}}{60\text{ min/h}}\right)=45\text{ mi/h}\left(0.41667\text{ h}-\frac{5.0\text{ min}}{60\text{ min/h}}\right)$$
$$60.0\text{ mi/h}(0.25\text{ h})=45\text{ mi/h}(0.33333\text{ h})$$
$$15\text{ mi}=15\text{ mi}$$

27. Let x – 30.0 s = time since the first car started moving in the race in seconds.
Let x= time since the second car started the race in seconds.
The distance travelled by each car will be the same at the point where the first car overtakes the second car.
Distance = speed × time.

$$260.0\text{ ft/s}(x-30.0\text{ s})=240.0\text{ ft/s}(x)$$
$$(260.0\text{ ft/s})x-(260.0\text{ ft/s})(30.0\text{ s})=(240.0\text{ ft/s})x$$
$$(260.0\text{ ft/s})x-7800\text{ ft}=(240.0\text{ ft/s})x$$
$$(20.0\text{ ft/s})x=7800\text{ ft}$$
$$x=\frac{7800\text{ ft}}{20.0\text{ ft/s}}$$
$$x=390\text{ s}$$

The first car will overtake the second car after 390 s. The first car travels 260 ft/s × (390 s – 30 s) = 93600 ft by this point. 8 laps around the track is 2.5 mi/lap. 8 laps × 5280 ft/mi = 105,600 ft, so the first car will already be in the lead at the end of the 8th lap.
Check:

$$260.0\text{ ft/s}(390.0\text{ s}-30.0\text{ s})=240.0\text{ ft/s}(390\text{ s})$$
$$260.0\text{ ft/s}(360.0\text{ s})=240.0\text{ ft/s}(390\text{ s})$$
$$93,600\text{ ft}=93,600\text{ ft}$$

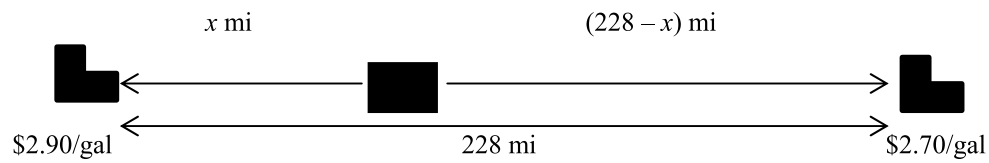
28. Let x = the number of the first chips that is defective 0.50%.
Let 6100 −x = the number of the second chips that is defective 0.80%.

$$0.0050(x)+0.0080(6100\text{ chips}-x)=38\text{ chips}$$
$$(0.0050)x+48.8\text{ chips}-(0.0080)x=38\text{ chips}$$
$$-(0.0030)x=-10.8\text{ chips}$$
$$x=\frac{-10.8\text{ chips}}{-0.0030}$$
$$x=3600\text{ chips}$$

There are 3600 chips that are 0.50% defective and (6100 chips – 3600 chips) = 2500 chips that are defective 0.80%.
Check:

$$0.0050(3600\text{ chips})+0.0080(6100\text{ chips}-3600\text{ chips})=38\text{ chips}$$
$$18\text{ chips}+0.0080(2500\text{ chips})=38\text{ chips}$$
$$18\text{ chips}+20\text{ chips}=38\text{ chips}$$
$$38\text{ chips}=38\text{ chips}$$

29.



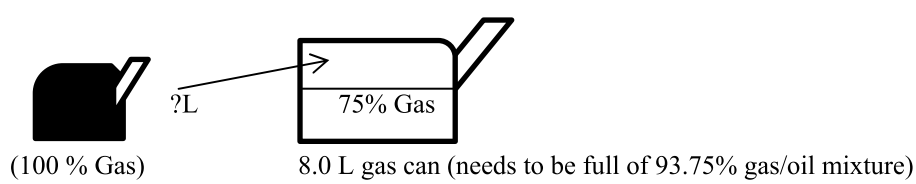
Assuming that the customer is located between the two gasoline distributors:
Let x = the distance in km to the first gasoline distributor that costs \$2.90/gal.
Let 228 mi− x= the distance in km to the second gasoline distributor that costs \$2.70/gal.

$$\$2.90+\$0.002(x)=\$2.70+\$0.002(228-x)$$
$$\$2.90+\$0.002(x)=\$2.70+\$0.456-\$0.002(x)$$
$$\$0.004(x)=\$0.256$$
$$x=\frac{\$0.256}{\$0.004}$$
$$x=64\text{ mi}$$

The customer is 64 mi away from the first gas distributor (\$2.90/gal) and (228 mi – 64 mi)= 164 mi away from the second gas distributor (\$2.70).
Check:

$$\$2.90+\$0.002(64)=\$2.70+\$0.002(228-64)$$
$$\$2.90+\$0.128=\$2.70+\$0.002(164)$$
$$\$3.028=\$2.70+\$0.328$$
$$\$3.028=\$3.028$$

30.



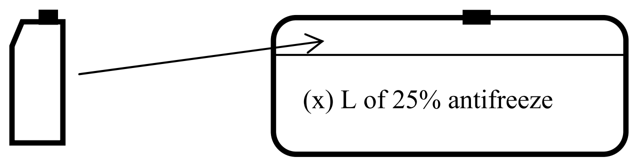
A 15:1 gas/oil mixture is 15/16 gasoline = 93.75%.
Let x = the amount of 100% gasoline added in L.
Let 8.0 L – x = the amount of 75% gasoline mixture in L.

$$1.00(x)+0.75(8.0\text{ L}-x)=0.9375(8.0\text{ L})$$
$$1.00(x)+6.0\text{ L}-0.75(x)=7.5\text{ L}$$
$$0.25(x)=1.5\text{ L}$$
$$x=\frac{1.5\text{ L}}{0.25}$$
$$x=6.0\text{ L}$$

6.0 L of 100% gasoline must be added to the 75% gas/oil mixture to make 8 L of 15:1 gasoline/ oil.
Check:

$$1.00(6.0\text{ L})+0.75(8.0\text{ L}-6.0\text{ L})=0.9375(8.0\text{ L})$$
$$6\text{ L}+0.75(2.0\text{ L})=7.5\text{ L}$$
$$6\text{ L}+1.5\text{ L}=7.5\text{ L}$$
$$7.5\text{ L}=7.5\text{ L}$$

31.



100% Antifreeze 12.0 L radiator (needs to be filled with 50% mixture)

Let x = the amount in L of 25% antifreeze left in radiator
Let 12.0 L – x = the amount of 100% antifreeze added in L.

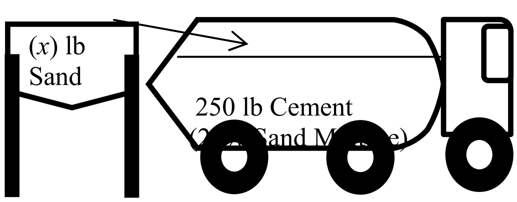
$$0.25(x)+1.00(12.0\text{ L}-x)=0.5(12.0\text{ L})$$
$$0.25(x)+12.0\text{ L}-1.00(x)=6.0\text{ L}$$
$$-0.75(x)=-6.0\text{ L}$$
$$x=\frac{-6.0\text{ L}}{-0.75}$$
$$x=8.0\text{ L}$$

There needs to be 8L of 25% antifreeze left in radiator, so (12.0 L – 8.0 L) = 4.0 L must be drained.

Check:

$$0.25(8.0\text{ L})+1.00(12.0\text{ L}-8.0\text{ L})=0.5(12.0\text{ L})$$
$$2\ .0\text{L}+1.00(4.0\text{ L})=6.0\text{ L}$$
$$2.0\text{ L}+4.0\text{ L}=6.0\text{ L}$$
$$6.0\text{ L}=6.0\text{ L}$$

32.



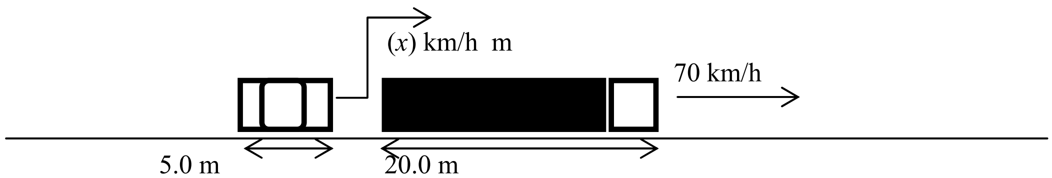
Let x = the amount of sand added.
Let 250 lb + x = the amount in lb of the final 25% sand mixture.

$$\begin{aligned}1.00(x)+0.22(250\text{ lb}) &= 0.25(250\text{ lb}+x)\\ 1.00(x)+55\text{ lb} &= 62.5\text{ lb}+0.25(x)\\ 0.75(x) &= 7.5\text{ lb}\\ x &= \frac{7.5\text{ lb}}{0.75}\\ x &= 10\text{ lb}\end{aligned}$$

Check:

$$\begin{aligned}1.00(10\text{ lb})+0.22(250\text{ lb}) &= 0.25(250\text{ lb}+10\text{ lb})\\ 10\text{ lb}+55\text{ lb} &= 62.5\text{ lb}+2.5\text{ lb}\\ 65\text{ lb} &= 65\text{ lb}\end{aligned}$$

33.



Let x = the speed the car needs to travel in km/h to pass the semi in 10 s.
Speed = distance/time. 10 s is 10s/3600 s/h = 0.002777777 h.

$$x = \frac{\text{distance needed to pass truck + distance travelled by truck in 10s}}{10s}$$

$$x = \frac{0.025\text{ km}+70\text{ km/h}\left(0.0027777\text{ h}\right)}{0.0027777\text{ h}}$$

$$x = \frac{0.025\text{ km}+0.19444\text{ km}}{0.0027777\text{ h}}$$

$$x = \frac{2.19444\text{ km}}{0.0027777\text{ h}}$$

$x = 79$ km/h

The car needs to travel at a speed of 79 km/h to pass the semitrailer in 10s.

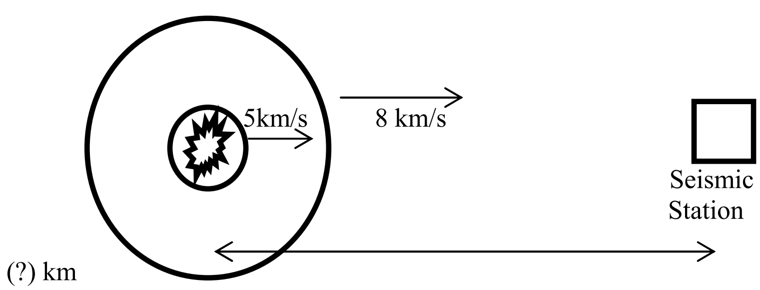
Check:

$$79\text{ km/h} = \frac{0.025\text{ km}+70\text{ km/h}\left(0.0027777\text{ h}\right)}{0.0027777\text{ h}}$$

$$79\text{ km/h} = \frac{0.025\text{ km}+0.19444\text{ km}}{0.0027777\text{ h}}$$

79 km/h = 79 km/h

34.



Let x = the time the first wave takes to travel to the seismic station in s.
Let x + 120 s = the time the first wave takes to travel to the seismic station in s.
Distance =speed × time. The distances travelled by both waves to the seismic station are the same. 2.0 min is (2.0 min × 60 s/min) = 120 s.

$$8.0 \text{ km/s}(x) = 5.0 \text{ km/s}(x + 120 \text{ s})$$
$$8.0 \text{ km/s}(x) = 5.0 \text{ km/s}(x) + (5 \text{ km/s})(120 \text{ s})$$
$$3.0 \text{ km/s}(x) = 600 \text{ km}$$
$$x = \frac{600 \text{ km}}{3.0 \text{ km/s}}$$
$$x = 200 \text{ s}$$

The distance to the seismic station is (200 s × 8.0 km/s) = 1600 km.
Check:

$$8.0 \text{ km/s}(200 \text{ s}) = 5.0 \text{ km/s}(200 \text{ s} + 120 \text{ s})$$
$$1600 \text{ km} = 5.0 \text{ km/s}(320 \text{ s})$$
$$1600 \text{ km} = 1600 \text{ km}$$

Review Exercises

1. False, because $|0| = 0$ which is not a positive value.

2. True. The order of operations dictates performing the division first, then the subtraction.

3. False. The reported answer should have only two significant digits.

4. False. Had the problem been given as $(2a)^3 = 8a^3$, then it would be true.

5. True.

6. False. The left-hand side, $-\sqrt{-4}$, is not a real number, in fact.

7. False. The left-hand side simplifies to $4x - (2x + 3) = 4x - 2x - 3 = 2x - 3$.

8. True.

9. False. The left-hand side simplifies to $\frac{6x+2}{2} = \frac{6x}{2} + \frac{2}{2} = 3x + 1$.

10. True.

11. False. Solving for c yields

$$a - bc = d$$
$$-bc = d - a$$
$$c = \frac{d - a}{-b}$$
$$c = -\frac{d - a}{b}$$

12. False. It is likely that one should set up a phrase such as 'let x be the number of gears of the first type…'

13. $(-2) + (-5) - 3 = -2 - 5 - 3 = -10$

14. $6-8-(-4)=6-8+4=2$

15. $\dfrac{(-5)(6)(-4)}{(-2)(3)}=\dfrac{(20)\cancel{(6)}}{-\cancel{(6)}}=-20$

16. $\dfrac{(-9)(-12)(-4)}{24}=\dfrac{108(-4)}{24}=\dfrac{-432}{24}=-18$

17. $-5-|2(-6)|+\dfrac{-15}{3}=-5-|-12|+(-5)=-5-12-5=-22$

18. $3-5|-3-2|-\dfrac{12}{-4}=3-5|-5|-(-3)=3-5(5)+3=6-25=-19$

19. $\dfrac{18}{3-5}-(-4)^2=\dfrac{18}{-2}-(-4)(-4)=-9-16=-25$

20. $-(-3)^2-\dfrac{-8}{(-2)-|-4|}=-(-3)(-3)-\dfrac{-8}{(-2)-4}=-9-\dfrac{-8}{-6}=-\dfrac{27}{3}-\dfrac{4}{3}=-\dfrac{31}{3}$

21. $\sqrt{16}-\sqrt{64}=\sqrt{(4)(4)}-\sqrt{(8)(8)}=4-8=-4$

22. $-\sqrt{81+144}=-\sqrt{225}=-\sqrt{(5)(5)(3)(3)}=-(3)(5)=-15$

23. $(\sqrt{7})^2-\sqrt[3]{8}=(\sqrt{7})(\sqrt{7})-\sqrt[3]{(2)(2)(2)}=7-2=5$

24. $-\sqrt[4]{16}+(\sqrt{6})^2=-\sqrt[4]{(2)(2)(2)(2)}+(\sqrt{6})(\sqrt{6})=-2+6=4$

25. $(-2rt^2)^2=(-2)^2r^2t^{2x2}=4r^2t^4$

26. $(3a^0b^{-2})^3=(3)^3(1)^3b^{-2\times 3}=27(1)b^{-6}=\dfrac{27}{b^6}$

27. $-3mn^{-5}t(8m^{-3}n^4)=-(3)(8)m^{1-3}n^{-5+4}t=-24m^{-2}n^{-1}t=-\dfrac{24t}{m^2n}$

28. $\dfrac{15p^4q^2r}{5pq^5r}=\dfrac{3p^{4-1}\cancel{r}}{q^{5-2}\cancel{r}}=\dfrac{3p^3}{q^3}$

29. $\dfrac{-16N^{-2}(NT^2)}{-2N^0T^{-1}}=\dfrac{8N^{-2+1}T^{2+1}}{(1)}=\dfrac{8N^{-1}T^3}{(1)}=\dfrac{8T^3}{N}$

30. $\dfrac{-35x^{-1}y(x^2y)}{5xy^{-1}}=\dfrac{-7y^{1+1+1}x^2}{x^{1+1}}=\dfrac{-7y^3\cancel{x^2}}{\cancel{x^2}}=-7y^3$

31. $\sqrt{45}=\sqrt{(5)(3)(3)}=3\sqrt{5}$

32. $\sqrt{9+36}=\sqrt{45}=\sqrt{(5)(3)(3)}=3\sqrt{5}$

33. 8000 has 1 significant digit. Rounded to 2 significant digits, it is $8\bar{0}00$.

34. 21490 has 4 significant digits. Rounded to 2 significant digits, it is 21000.

35. 9.050 has 4 significant digits. Rounded to 2 significant digits, it is 9.0.

36. 0.7000 has 4 significant digits. Rounded to 2 significant digits, it is 0.70.

37. $$\begin{aligned}37.3-16.92(1.067)^2 &= 37.3-16.92(1.138489)\\ &= 37.3-19.26323388\\ &= 18.03676612\end{aligned}$$
which rounds to 18.0.

38. $$\begin{aligned}\frac{8.896\times10^{-12}}{-3.5954-6.0449} &= \frac{8.896\times10^{-12}}{-9.6403}\\ &= -9.227928591\times10^{-13}\end{aligned}$$
which rounds to -9.228×10^{-13}.

39. $$\begin{aligned}\frac{\sqrt{0.1958+2.844}}{3.142(65)^2} &= \frac{\sqrt{3.0398}}{3.142(4225)}\\ &= \frac{1.743502223}{13274.95}\\ &= 0.000131337\end{aligned}$$
which rounds to 1.3×10^{-4}.

40. $$\begin{aligned}\frac{1}{0.03568}+\frac{37\ 466}{29.63^2} &= 28.02690583+\frac{37\ 466}{877.9369}\\ &= 28.02690583+42.67504874\\ &= 70.70195457\end{aligned}$$
which rounds to 70.70, assuming that the 1 is exact.

41. $$\begin{aligned}875\text{ Btu} &= 875\text{ Btu}\times\frac{778.2\text{ ft}\cdot\text{lb}}{1\text{ Btu}}\times\frac{1.356\text{ J}}{1\text{ ft}\cdot\text{lb}}\\ &= 923{,}334.3\text{ J}\end{aligned}$$
which rounds to 923,000 J.

42. $$\begin{aligned}18.4\text{ in} &= 18.4\text{ in}\times\frac{2.54\text{ cm}}{1\text{ in}}\times\frac{1\text{ m}}{100\text{ cm}}\\ &= 0.46736\text{ m}\end{aligned}$$
which rounds to 0.467 m.

43. $$\begin{aligned}65\ \frac{\text{km}}{\text{h}} &= 65\ \frac{\text{km}}{\text{h}}\times\frac{1\text{ hr}}{3600\text{ s}}\times\frac{0.6214\text{ mi}}{1\text{ km}}\times\frac{5280\text{ ft}}{1\text{ mi}}\\ &= 59.2401333\ \frac{\text{ft}}{\text{s}}\end{aligned}$$
which rounds to 59 ft/s.

44. $12.25\ \frac{\text{g}}{\text{L}} = 12.25\ \frac{\text{g}}{\text{L}} \times \frac{28.32\ \text{L}}{1\ \text{ft}^3} \times \frac{1\ \text{kg}}{1000\ \text{g}} \times \frac{2.205\ \text{lb}}{1\ \text{kg}}$

$= 0.7649586\ \frac{\text{lb}}{\text{ft}^3}$

which rounds to 0.7650 lb/ft^3.

45. $225\ \text{hp} = 225\ \text{hp} \times \frac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}} \times \frac{1.356\ \text{J}}{1\ \text{ft}\cdot\text{lb}} \times \frac{60\ \text{s}}{1\ \text{min}}$

$= 10068300\ \frac{\text{J}}{\text{min}}$

which rounds to $10100000 = 1.01\times10^7$ J/min.

46. $89.7\ \frac{\text{lb}}{\text{in}^2} = 89.7\ \frac{\text{lb}}{\text{in}^2} \times \frac{4.448\ \text{N}}{1\ \text{lb}} \times \left(\frac{1\ \text{in}}{2.54\ \text{cm}}\right)^2$

$= 61.8428917\ \frac{\text{N}}{\text{cm}^2}$

which rounds to 61.8 N/cm^2.

47. $a - 3ab - 2a + ab = -2ab - a$

48. $xy - y - 5y - 4xy = -3xy - 6y$

49. $6LC - (3 - LC) = 6LC - 3 + LC = 7LC - 3$

50. $-(2x - b) - 3(-x - 5b) = -2x + b + 3x + 15b = 16b + x$

51. $(2x - 1)(5 + x) = (2x)(5) + (2x)(x) + (-1)(5) + (-1)(x)$

$= 10x + 2x^2 - 5 - x$

$= 2x^2 + 9x - 5$

52. $(C - 4D)(D - 2C) = (C)(D) + (C)(-2C) + (-4D)(D) + (-4D)(-2C)$

$= CD - 2C^2 - 4D^2 + 8CD$

$= -2C^2 + 9CD - 4D^2$

53. $(x + 8)^2 = (x + 8)(x + 8)$

$= (x)(x) + (x)(8) + (8)(x) + (8)(8)$

$= x^2 + 8x + 8x + 64$

$= x^2 + 16x + 64$

54. $(2r - 9s)^2 = (2r - 9s)(2r - 9s)$

$= (2r)(2r) + (2r)(-9s) + (-9s)(2r) + (-9s)(-9s)$

$= 4r^2 - 18rs - 18rs + 81s^2$

$= 4r^2 - 36rs + 81s^2$

55. $\dfrac{2h^3k^2-6h^4k^5}{2h^2k}=\dfrac{2h^3k^2}{2h^2k}-\dfrac{6h^4k^5}{2h^2k}$
$=h^{3-2}k^{2-1}-3h^{4-2}k^{5-1}$
$=-3h^2k^4+hk$

56. $\dfrac{4a^2x^3-8ax^4}{-2ax^2}=\dfrac{4a^2x^3}{-2ax^2}-\dfrac{8ax^4}{-2ax^2}$
$=-2a^{2-1}x^{3-2}+\dfrac{4\cancel{a}x^{4-2}}{\cancel{a}}$
$=4x^2-2ax$

57. $4R-[2r-(3R-4r)]=4R-[2r-3R+4r]$
$=4R-[6r-3R]$
$=4R-6r+3R$
$=7R-6r$

58. $-3b-[3a-(a-3b)]+4a=4a-3b-[3a-a+3b]$
$=4a-3b-[2a+3b]$
$=4a-3b-2a-3b$
$=2a-6b$

59. $2xy-\{3z-[5xy-(7z-6xy)]\}=2xy-\{3z-[5xy-7z+6xy)]\}$
$=2xy-\{3z-[11xy-7z]\}$
$=2xy-\{3z-11xy+7z\}$
$=2xy-\{10z-11xy\}$
$=2xy-10z+11xy$
$=13xy-10z$

60. $x^2+3b+[(b-y)-3(2b-y+z)]=x^2+3b+[b-y-6b+3y-3z)]$
$=x^2+3b+[-5b+2y-3z]$
$=x^2+3b-5b+2y-3z$
$=x^2-2b+2y-3z$

61. $(2x+1)(x^2-x-3)=(2x)(x^2)+(2x)(-x)+(2x)(-3)+(1)(x^2)+(1)(-x)+(1)(-3)$
$=2x^3-2x^2-6x+x^2-x-3$
$=2x^3-x^2-7x-3$

62. $(x-3)(2x^2+1-3x)=(x)(2x^2)+(x)(1)+(x)(-3x)+(-3)(2x^2)+(-3)(1)+(-3)(-3x)$
$=2x^3+x-3x^2-6x^2-3+9x$
$=2x^3-9x^2+10x-3$

63. $$\begin{aligned} -3y(x-4y)^2 &= -3y(x-4y)(x-4y) \\ &= -3y[(x)(x)+(x)(-4y)+(-4y)(x)+(-4y)(-4y)] \\ &= -3y[x^2-4xy-4xy+16y^2] \\ &= -3y[x^2-8xy+16y^2] \\ &= -3x^2y+24xy^2-48y^3 \end{aligned}$$

64. $$\begin{aligned} -s(4s-3t)^2 &= -s(4s-3t)(4s-3t) \\ &= -s[(4s)(4s)+(4s)(-3t)+(-3t)(4s)+(-3t)(-3t)] \\ &= -s[16s^2-12st-12st+9t^2] \\ &= -s[16s^2-24st+9t^2] \\ &= -16s^3+24s^2t-9st^2 \end{aligned}$$

65. $$\begin{aligned} 3p[(q-p)-2p(1-3q)] &= 3p[q-p-2p+6pq] \\ &= 3p[q-3p+6pq] \\ &= 18p^2q-9p^2+3pq \end{aligned}$$

66. $$\begin{aligned} 3x[2y-r-4(x-2r)] &= 3x[2y-r-4x+8r] \\ &= 3x[2y+7r-4x] \\ &= 21rx-12x^2+6xy \end{aligned}$$

67. $$\begin{aligned} \frac{12p^3q^2-4p^4q+6pq^5}{2p^4q} &= \frac{12p^3q^2}{2p^4q}-\frac{4p^4q}{2p^4q}+\frac{6pq^5}{2p^4q} \\ &= \frac{6q^{2-1}}{p^{4-3}}-\frac{2\cancel{p^4q}}{\cancel{p^4q}}+\frac{3q^{5-1}}{p^{4-1}} \\ &= \frac{3q^4}{p^3}+\frac{6q}{p}-2 \end{aligned}$$

68. $$\begin{aligned} \frac{27s^3t^2-18s^4t+9s^2t}{-9s^2t} &= \frac{27s^3t^2}{-9s^2t}-\frac{18s^4t}{-9s^2t}+\frac{9s^2t}{-9s^2t} \\ &= -3s^{3-2}t^{2-1}+\frac{2s^{4-2}\cancel{t}}{\cancel{t}}-\frac{\cancel{9s^2t}}{\cancel{9s^2t}} \\ &= 2s^2-3st-1 \end{aligned}$$

69. $$\begin{array}{r} 2x-5 \\ x+6\overline{\smash{\big)}\,2x^2+7x-30} \\ \underline{2x^2+12x} \\ -5x-30 \\ \underline{-5x-30} \\ 0 \end{array}$$

70.
$$\begin{array}{rl} & 2x-7 \\ 2x+7 \big) & \overline{4x^2+0x-41} \\ & \underline{4x^2+14x} \\ & \quad -14x-41 \\ & \quad \underline{-14x-49} \\ & \qquad\qquad 8 \end{array}$$

$$\frac{4x^2-41}{2x+7}=2x-7+\frac{8}{2x+7}$$

71.
$$\begin{array}{rl} & x^2-2x+3 \\ 3x-1 \big) & \overline{3x^3-7x^2+11x-3} \\ & \underline{3x^3-x^2} \\ & \quad -6x^2+11x \\ & \quad \underline{-6x^2+2x} \\ & \qquad\qquad 9x-3 \\ & \qquad\qquad \underline{9x-3} \\ & \qquad\qquad\quad 0 \end{array}$$

72.
$$\begin{array}{rl} & w^2-w+4 \\ w-3 \big) & \overline{w^3-4w^2+7w-12} \\ & \underline{w^3-3w^2} \\ & \quad -w^2+7w \\ & \quad \underline{-w^2+3w} \\ & \qquad\quad 4w-12 \\ & \qquad\quad \underline{4w-12} \\ & \qquad\qquad\quad 0 \end{array}$$

73.
$$\begin{array}{rl} & 4x^3-2x^2+6x \\ x+3 \big) & \overline{4x^4+10x^3+0x^2+18x-1} \\ & \underline{4x^4+12x^3} \\ & \quad -2x^3+0x^2 \\ & \quad \underline{-2x^3-6x^2} \\ & \qquad\qquad 6\mathrm{x}^2+18x \\ & \qquad\qquad \underline{6\mathrm{x}^2+18x} \\ & \qquad\qquad\qquad 0x-1 \end{array}$$

$$\frac{4x^4+10x^3+18x-1}{x+3}=4x^3-2x^2+6x-\frac{1}{x+3}$$

74. $$\begin{array}{r} 4x^2-6x+2 \\ 2x+3\overline{)8x^3+0x^2-14x+3} \\ \underline{8x^3+12x^2} \\ -12x^2-14x \\ \underline{-12x^2-18x} \\ 4x+3 \\ \underline{4x+6} \\ -3 \end{array}$$

$$\frac{8x^3-14x+3}{2x+3}=4x^2-6x+2-\frac{3}{2x+3}$$

75. $$\begin{aligned} -3\{(r+s-t)-2[(3r-2s)-(t-2s)]\} &= -3\{r+s-t-2[3r-2s-t+2s)]\} \\ &= -3\{r+s-t-2[3r-t]\} \\ &= -3\{r+s-t-6r+2t]\} \\ &= -3\{-5r+s+t\} \\ &= 15r-3s-3t \end{aligned}$$

76. $$\begin{aligned} &(1-2x)(x-3)-(x+4)(4-3x) \\ &=[(1)(x)+(1)(-3)+(-2x)(x)+(-2x)(-3)]-[(x)(4)+(x)(-3x)+(4)(4)+(4)(-3x) \\ &=[x-3-2x^2+6x]-[4x+-3x^2+16+-12x] \\ &=[-2x^2+7x-3]-[-3x^2-8x+16] \\ &=-2x^2+7x-3+3x^2+8x-16 \\ &=x^2+15x-19 \end{aligned}$$

77. $$\begin{array}{r} y^2+5y-1 \\ 2y-1\overline{)2y^3+9y^2-7y+5} \\ \underline{2y^3-1y^2} \\ 10y^2-7y \\ \underline{10y^2-5y} \\ -2y+5 \\ \underline{-2y+1} \\ 4 \end{array}$$

$$\frac{2y^3+9y^2-7y+5}{2y-1}=y^2+5y-1+\frac{4}{2y-1}$$

78. $$\begin{array}{r} 3x+4y \\ 2x-y\overline{)6x^2+5xy-4y^2} \\ \underline{6x^2-3xy} \\ 8xy-4y^2 \\ \underline{8xy-4y^2} \\ 0 \end{array}$$

79. $3x+1=x-8$

$2x=-9$

$x=-\dfrac{9}{2}$

80. $4y-3=5y+7$

$-y=10$

$y=-10$

81. $\dfrac{5x}{7}=\dfrac{3}{2}$

$2(5x)=3(7)$

$10x=21$

$x=\dfrac{21}{10}$

82. $\dfrac{2(N-4)}{3}=\dfrac{5}{4}$

$\dfrac{2N-8}{3}=\dfrac{5}{4}$

$4(2N-8)=3(5)$

$8N-32=15$

$8N=47$

$N=\dfrac{47}{8}$

83. $-6x+5=-3(x-4)$

$-6x+5=-3x+12$

$-3x=7$

$x=-\dfrac{7}{3}$

84. $-2(-4-y)=3y$

$8+2y=3y$

$y=8$

85. $2s+4(3-s)=6$

$2s+12-4s=6$

$-2s=-6$

$s=\dfrac{-6}{-2}$

$s=3$

86.
$$\begin{aligned}2|x|-1&=3\\2|x|&=4\\|x|&=\frac{4}{2}\\|x|&=2\\x&=-2 \text{ and } 2\end{aligned}$$

87.
$$\begin{aligned}3t-2(7-t)&=5(2t+1)\\3t-14+2t&=10t+5\\5t-14&=10t+5\\-5t&=19\\t&=-\frac{19}{5}\end{aligned}$$

88.
$$\begin{aligned}-(8-x)&=x-2(2-x)\\-8+x&=x-4+2x\\-8+x&=3x-4\\-2x&=4\\x&=-\frac{4}{2}\\x&=-2\end{aligned}$$

89.
$$\begin{aligned}2.7+2.0(2.1x-3.4)&=0.1\\2.7+4.2x-6.8&=0.1\\4.2x-4.1&=0.1\\4.2x&=4.2\\x&=\frac{4.2}{4.2}\\x&=1.0\end{aligned}$$

90.
$$\begin{aligned}0.250(6.721-2.44x)&=2.08\\1.68025-0.610x&=2.08\\-0.610x&=0.39975\\x&=-\frac{0.39975}{0.610}\\x&=0.655327868\\x&=0.655\end{aligned}$$

91. 60,000,000,000,000 bytes = 6×10^{13} bytes

92. 25,000 mi/h = 2.5×10^{4} mi/h

93. 15,400,000,000 km = 1.54×10^{10} km

94. 1.02×10^{9} Hz = 1,020,000,000 Hz

95. 2.53×10^{13} mi = 25,300,000,000,000 mi

96. $10^7\text{ ft}^2 = 10{,}000{,}000\text{ ft}^2$

97. $10^{-12}\text{ W/m}^2 = 0.000000000001\text{ W/m}^2$

98. $0.00000015\text{ m} = 1.5 \times 10^{-7}\text{ m}$

99. $1.5 \times 10^{-1}\text{ Bq/L} = 0.15\text{ Bq/L}$

100. $0.00000018\text{ m} = 1.8 \times 10^{-7}\text{ m}$

101. $V = \pi r^2 L$

$$L = \frac{V}{\pi r^2}$$

102.
$$R = \frac{2GM}{c^2}$$
$$c^2 R = 2GM$$
$$G = \frac{c^2 R}{2M}$$

103.
$$P = \frac{\pi^2 EI}{L^2}$$
$$L^2 P = \pi^2 EI$$
$$E = \frac{L^2 P}{\pi^2 I}$$

104.
$$f = p(c-1) - c(p-1)$$
$$f = cp - p - cp + c$$
$$f - c = -p$$
$$p = c - f$$

105.
$$Pp + Qq = Rr$$
$$Qq = Rr - Pp$$
$$q = \frac{Rr - Pp}{Q}$$

106.
$$V = IR + Ir$$
$$IR = V - Ir$$
$$R = \frac{V - Ir}{I}$$

107.
$$d = (n-1)A$$
$$d = An - A$$
$$d + A = An$$
$$n = \frac{d + A}{A}$$

108.
$$\begin{aligned} mu &= (m+M)v \\ mu &= mv + Mv \\ mu - mv &= Mv \\ M &= \frac{mu - mv}{v} \end{aligned}$$

109.
$$\begin{aligned} N_1 &= T(N_2 - N_3) + N_3 \\ N_1 - N_3 &= N_2T - N_3T \\ N_2T &= N_1 - N_3 + N_3T \\ N_2 &= \frac{N_1 - N_3 + N_3T}{T} \end{aligned}$$

110.
$$\begin{aligned} Q &= \frac{kAt(T_2 - T_1)}{L} \\ QL &= kAt(T_2 - T_1) \\ T_2 - T_1 &= \frac{QL}{kAt} \\ -T_1 &= \frac{QL}{kAt} - T_2 \\ T_1 &= T_2 - \frac{QL}{kAt} \end{aligned}$$

111.
$$\begin{aligned} R &= \frac{A(T_2 - T_1)}{H} \\ HR &= AT_2 - AT_1 \\ AT_2 &= HR + AT_1 \\ T_2 &= \frac{HR + AT_1}{A} \end{aligned}$$

112.
$$\begin{aligned} Z^2 \quad 1 - \frac{\lambda}{2a} \quad &= k \\ Z^2 - \frac{Z^2\lambda}{2a} &= k \\ Z^2 - k &= \frac{Z^2\lambda}{2a} \\ 2a(Z^2 - k) &= Z^2\lambda \\ \lambda &= \frac{2aZ^2 - 2ak}{Z^2} \end{aligned}$$

113.
$$\begin{aligned} d &= kx^2[3(a+b) - x] \\ d &= kx^2[3a + 3b - x] \\ d &= 3akx^2 + 3bkx^2 - kx^3 \\ 3akx^2 &= d - 3bkx^2 + kx^3 \\ a &= \frac{d - 3bkx^2 + kx^3}{3kx^2} \end{aligned}$$

114.
$$\begin{aligned}
V &= V_0[1+3a(T_2-T_1)]\\
V &= V_0[1+3aT_2-3aT_1]\\
V &= V_0+3aT_2V_0-3aT_1V_0\\
3aT_2V_0 &= V-V_0+3aT_1V_0\\
T_2 &= \frac{V-V_0+3aT_1V_0}{3aV_0}
\end{aligned}$$

115. $\dfrac{5.25\times10^{13}\text{ bytes}}{6.4\times10^{4}\text{ bytes}}=8.203125\times10^{8}$

which rounds to 8.2×10^8. The newer computer's memory is 8.2×10^8 larger.

116. $t=0.25\sqrt{66}=2.0310096$ s

which rounds to 2.0 s. It would take the person 2.0 s to fall 66 ft.

117. $\dfrac{0.553\text{ km}}{0.442\text{ km}}=1.25113122$

which rounds to 1.25. The CN Tower is 1.25 times taller than the Sears tower.

118. $t=\left(\dfrac{4.8\times10^{3}\text{ cells}}{2650}\right)^2=(1.8113207)^2=3.280882876$ s

which rounds to 3.28 s. It would take the computer 3.28 s to check 4800 memory cells.

119.
$$\begin{aligned}
\frac{R_1R_2}{R_1+R_2} &= \frac{(0.0275\ \Omega)(0.0590\ \Omega)}{0.0275\ \Omega+0.0590\ \Omega}\\
&= \frac{0.0016225\ \Omega^2}{0.0865\ \Omega}\\
&= 0.018757225\ \Omega
\end{aligned}$$

which rounds to $0.0188\ \Omega$. The combined electric resistance is $0.0188\ \Omega$.

120.
$$\begin{aligned}
1.5\times10^{11}\sqrt{\frac{m}{M}} &= 1.5\times10^{11}\sqrt{\frac{5.98\times10^{24}\text{ kg}}{1.99\times10^{30}\text{ kg}}}\\
&= 1.5\times10^{11}\sqrt{0.000003005}\\
&= 1.5\times10^{11}(0.0017335)\\
&= 260\ 025\ 124.4\text{ m}
\end{aligned}$$

which rounds to 2.6×10^8 m. The distance the space craft will be from the earth is 2.6×10^8 m.

121.
$$\begin{aligned}
(x-2a)+3\text{ ft/yd}\cdot(x+2a) &= x-2a+3x+3(2a)\\
&= x-2a+3x+6a\\
&= 4x+4a
\end{aligned}$$

The sum of their length is $4x + 4a$ ft.

122.
$$\begin{aligned}
(Ai-R)(1+i)^2 &= (Ai-R)(1+i)(1+i)\\
&= (Ai-R)[(1)(1)+(1)(i)+(i)(1)+(i)(i)]\\
&= (Ai-R)[i^2+2i+1]\\
&= (Ai)(i^2)+(Ai)(2i)+(Ai)(1)+(-R)(i^2)+(-R)(2i)+(-R)(1)\\
&= Ai^3+2Ai^2+Ai-i^2R-2iR-R
\end{aligned}$$

123. $4(t+h)-2(t+h)^2 = 4t+4h-2(t+h)(t+h)$

$$\begin{aligned} &= 4t+4h-2[(t)(t)+(t)(h)+(h)(t)+(h)(h)] \\ &= 4t+4h-2[t^2+2ht+h^2] \\ &= 4t+4h-2t^2-4ht-2h^2 \\ &= -2t^2-2h^2-4ht+4t+4h \end{aligned}$$

124. $$\begin{aligned} \frac{k^2r-2h^2k+h^2rv^2}{k^2r} &= \frac{k^2r}{k^2r}-\frac{2h^2k}{k^2r}+\frac{h^2rv^2}{k^2r} \\ &= \frac{\cancel{k^2}r}{\cancel{k^2}r}-\frac{2h^2}{k^{2-1}r}+\frac{h^2\cancel{r}v^2}{k^2\cancel{r}} \\ &= 1-\frac{2h^2}{kr}+\frac{h^2v^2}{k^2} \end{aligned}$$

125. $3\times 18\div(9-6)=54\div(3)=18$

$3\times 18\div 9-6=54\div 9-6=6-6=0$

Yes, the removal of the parentheses does affect the answer.

126. $(3\times 18)\div 9-6=54\div 9-6=6-6=0$

$3\times 18\div 9-6=54\div 9-6=6-6=0$

No, the removal of the parentheses does not affect the answer.

127. $$\begin{aligned} x-(3-x) &= 2x-3 \\ x-3+x &= 2x-3 \\ 2x-3 &= 2x-3 \end{aligned}$$

The equation is valid for all values of the unknown, so the equation is an identity.

128. $$\begin{aligned} 7-(2-x) &= x+2 \\ 7-2+x &= x+2 \\ x+5 &= x+2 \\ 5 &= 2 \end{aligned}$$

The equation has no values of the unknown for which it is valid, so the equation is a contradiction.

129. **(a)** $$\begin{aligned} 2|2|-2|4| &= 4-8 \\ &= -4 \end{aligned}$$

(b) $$\begin{aligned} 2|2-(-4)| &= 2|6| \\ &= 12 \end{aligned}$$

130. For $a<0$, $|a|=-a$.

131. Given $3-x=0$,

$$\begin{aligned} |3-x|+7 &= 2x \\ -(3-x)+7 &= 2x \\ -3+x+7 &= 2x \\ 4 &= x \end{aligned}$$

This is consistent with $3-x=0$, so $x=4$.

132. $|x-4|+6=3x$

$$|x-4|=3x-6$$

$$x-4=3x-6 \text{ or } -(x-4)=3x-6$$

$$2=2x \qquad 4-x=3x-6$$

$$x=1 \qquad 10=4x$$

and so the only possible solutions are

$x=1$ or $x=5/2$.

The first possibility, $x=1$, yields $|-3|+6=3$ or $9=3$, which is false.

The second possibility, $x=5/2$, yields $|-3/2|+6=15/2$ or $15/2=15/2$, which is true, and so the only solution is $x=5/2$.

133. $(x-y)^3=(x-y)(x-y)(x-y)$

$$=(-(y-x))(-(y-x))(-(y-x))$$

$$=-(y-x)(y-x)(y-x)$$

$$=-(y-x)^3$$

134. Generally, $(a \div b) \div c \neq a \div (b \div c)$

We demonstrate this using $a=8, b=4, c=2$:

$$(8 \div 4) \div 2 = 2 \div 2 = 1$$

$$8 \div (4 \div 2) = 8 \div 2 = 4$$

Division is not associative.

135. $\dfrac{8\times10^{-3}}{2\times10^{4}}=4\times10^{-7}$

136. $\dfrac{\sqrt{4+36}}{\sqrt{4}}=\dfrac{\sqrt{(2)(2)(10}}{2}=\dfrac{2\sqrt{10}}{2}=\sqrt{10}$

137. $250\text{ hp}=250\text{ hp}\times\dfrac{746.0\text{ W}}{1\text{ hp}}\times\dfrac{1\text{ kW}}{1000\text{ W}}$

$$=186.5\text{ kW}$$

This is rounded to 190 kW.

138. $32\ \dfrac{\text{lb}}{\text{in}^2}=32\ \dfrac{\text{lb}}{\text{in}^2}\times\dfrac{4.448\text{ N}}{1\text{ lb}}\times\dfrac{1\text{ in}}{2.54\text{ cm}}^{2}\times\dfrac{100\text{ cm}}{1\text{ m}}^{2}$

$$=220{,}621.241\ \frac{\text{N}}{\text{m}^2}$$

This is rounded to 220,000 N/m^2.

139. $110\text{ N}\cdot\text{m}=110\text{ N}\cdot\text{m}\times\dfrac{1\text{ lb}}{4.448\text{ N}}\times\dfrac{1\text{ ft}}{0.3048\text{ m}}$

$$=81.1358787\text{ ft}\cdot\text{lb}$$

This is rounded to 81 foot pounds.

140. $1.2\times10^6\ \frac{\text{A}}{\text{m}^2} = 1.2\times10^6\ \frac{\text{A}}{\text{m}^2}\times\frac{1000\text{ mA}}{1\text{ A}}\times\left(\frac{1\text{ m}}{100\text{ cm}}\right)^2$

$= 1.2\times10^5\ \frac{\text{mA}}{\text{cm}^2}$

141. Let x = the cost of the first computer program.
Let x + \$72 = the cost of the second computer program.

$$\begin{aligned} x+(x+\$72) &= \$190 \\ 2x+\$72 &= \$190 \\ 2x &= \$118 \\ x &= \frac{\$118}{2} \\ x &= \$59 \end{aligned}$$

The cost of the first computer program is \$59, and the other program costs (\$59 + \$72) = \$131.
Check: \$59 + \$131 = 190

142. Let x = the cost to run the commercial on the first station.
Let x + \$1100 = the cost to run the commercial on the second station.

$$\begin{aligned} x+(x+\$1100) &= \$9500 \\ 2x+\$1100 &= \$9500 \\ 2x &= \$8400 \\ x &= \frac{\$8400}{2} \\ x &= \$4200 \end{aligned}$$

The cost of the run the commercial on the first station is \$4200, and the cost for the other station is (\$4200 + \$1100) = \$5300.
Check: \$4200 + \$5300 = \$9500

143. Let $2x$ = the amount of oxygen produced in cm^3 by the first reaction.
Let x = the amount of oxygen produced in cm^3 by the second reaction.
Let $4x$ = the amount of oxygen produced in cm^3 by the third reaction.

$$\begin{aligned} 2x+x+4x &= 560\text{ cm}^3 \\ 7x &= 560\text{ cm}^3 \\ x &= \frac{560\text{ cm}^3}{7} \\ x &= 80\text{ cm}^3 \end{aligned}$$

The first reaction produces ($2\times80\text{ cm}^3$) = 160 cm^3 of oxygen, the second reaction produces 80 cm^3 of oxygen, and the third reaction produces ($4\times80\text{ cm}^3$) = 320 cm^3 of oxygen.
Check: $160\text{ cm}^3 + 80\text{ cm}^3 + 320\text{ cm}^3 = 560\text{ cm}^{3*}$

144. Let x = the speed that the river is flowing in mi/h.
Let x + 5.5 mi/h = the speed that the boat travels downstream.
Let $-x$ + 5.5 mi/h = the speed that the boat travels upstream.
The distance that the boat travelled is the same in both experiments. Distance = speed × time.

$$(x+5.5\text{ mi/h})(5.0\text{ h}) = (-x+5.5\text{ mi/h})(8.0\text{ h})$$
$$(5.0\text{ h})(x)+(5.5\text{ mi/h})(5.0\text{ h}) = (8.0\text{ h})(-x)+(5.5\text{ mi/h})(8.0\text{ h})$$
$$(5.0\text{ h})(x)+(27.5\text{ mi}) = (-8.0\text{ h})(x)+(44\text{ mi})$$
$$(13.0\text{ h})x = 16.5\text{ mi}$$
$$x = \frac{16.5\text{ mi}}{13\text{ h}}$$
$$x = 1.269230769\text{ mi/h}$$

which rounds to 1.3 mi/h. The polluted stream is flowing at 1.3 mi/h.
Check:

$$(1.269230769\text{ mi/h}+5.5\text{ mi/h})(5.0\text{ h}) = (-1.269230769\text{ mi/h}+5.5\text{ mi/h})(8.0\text{ h})$$
$$(6.769230769\text{ mi/h})(5.0\text{ h}) = (4.2\text{ mi/h})(8.0\text{ h})$$
$$(33.8\text{ mi}) = (33.8\text{ mi})$$

145. Let x = the resistance in the first resistor in Ω.
Let x + 1200 Ω = the resistance in the second resistor in Ω.
Voltage = current × resistance. 2.4 μA = 2.3 × 10^{-6} A. 12 mV = 0.0120 V

$$(2.4\times10^{-6}\text{A})(x)+(2.4\times10^{-6}\text{A})(x+1200\ \Omega) = 0.0120\text{ V}$$
$$(2.4\times10^{-6}\text{A})(x)+(2.4\times10^{-6}\text{A})(x)+(2.4\times10^{-6}\text{A})(1200\ \Omega) = 0.0120\text{ V}$$
$$(4.8\times10^{-6}\text{A})(x)+(0.00288\text{V}) = 0.0120\text{ V}$$
$$(4.0\times10^{-6}\text{A})(x) = 0.00912\text{ V}$$
$$x = \frac{0.00912\text{ V}}{4.8\times10^{-6}\text{A}}$$
$$x = 1900\ \Omega$$

The first resistor's resistance is 1900 Ω and the second resistor's is (1900 Ω + 1200 Ω) = 3100 Ω.
Check:

$$(2.4\times10^{-6}\text{A})(1900\ \Omega)+(2.4\times10^{-6}\text{A})(1900\ \Omega+1200\ \Omega) = 0.0120\text{ V}$$
$$0.00456\text{ V}+0.00744\text{ V} = 0.0120\text{ V}$$
$$0.0120\text{ V} = 0.0120\text{ V}$$

146. Let x = the concentration of the first pollutant in ppm.
Let $4x$ = the concentration of the second pollutant in ppm.

$$x+4x = 4.0\text{ ppm}$$
$$5x = 4.0\text{ ppm}$$
$$x = \frac{4.0\text{ ppm}}{5}$$
$$x = 0.8\text{ ppm}$$

The concentration of the first pollutant is 0.8 ppm, and the concentration of the second is (4 × 0.8 ppm) = 3.2 ppm.
Check:

$$0.8\text{ ppm}+4(0.8\text{ ppm}) = 4.0\text{ ppm}$$
$$0.8\text{ ppm}+3.2\text{ ppm} = 4.0\text{ ppm}$$
$$4.0\text{ ppm} = 4.0\text{ ppm}$$

147. Let x = the time taken in hours for the crew to build 250 m of road.
The crew works at a rate of 450 m/12 h, which is 37.5 m/h. Time = distance / speed.

$$x = \frac{250 \text{ m}}{37.5 \text{ m/h}}$$

$$x = 6.666666667 \text{ h}$$

which rounds to 6.7 h.

148. Let x = the amount of oil in L in the mixture.
Let $15x$ = the amount of gas in L in the mixture.

$$\begin{aligned} x + 15x &= 6.6 \text{ L} \\ 16x &= 6.6 \text{ L} \\ x &= \frac{6.6 \text{ L}}{16} \\ x &= 0.4125 \text{ L} \end{aligned}$$

which rounds to 0.41 L. There is 0.41 L of oil in the mixture and (15 × 0.41 L) = 6.2 L of gas.
Check:

$$\begin{aligned} 0.4125 \text{ L} + 15(0.4125 \text{ L}) &= 6.6 \text{ L} \\ 0.4125 \text{ L} + 6.1875 \text{ L} &= 6.6 \text{ L} \\ 6.6 \text{ L} &= 6.6 \text{ L} \end{aligned}$$

149.

Let x = the time taken by the second ship in hours.
Let x + 2 h = the amount time taken by the first ship in hours.
The distance travelled adds up to 634km. Distance = speed × time.

$$\begin{aligned} 21.8 \text{ km/h}(x) + 17.4 \text{ km/h}(x + 2 \text{ h}) &= 634 \text{ km} \\ 21.8 \text{ km/h}(x) + 17.4 \text{ km/h}(x) + 17.4 \text{ km/h}(2 \text{ h}) &= 634 \text{ km} \\ 39.2 \text{ km/h}(x) + 34.8 \text{ km} &= 634 \text{ km} \\ 39.2 \text{ km/h}(x) &= 599.2 \text{ km} \\ x &= \frac{599.2 \text{ km}}{39.2 \text{ km/h}} \\ x &= 15.2857 \text{ h} \end{aligned}$$

which rounds to 15.2 h. The ships will pass 15.2 h after the second ship enters the canal.
Check:

$$\begin{aligned} 21.8 \text{ km/h}(15.2857 \text{ h}) + 17.4 \text{ km/h}(15.2857 \text{ h} + 2 \text{ h}) &= 634 \text{ km} \\ 333.23 \text{ km} + 300.77 \text{ km} &= 634 \text{ km} \\ 634 \text{ km} &= 634 \text{ km} \end{aligned}$$

150. Let x = the time take in h for the helicopter to travel from the pond to the fire.
Let 0.5 h $-x$ = the time take in h for the helicopter to travel from the fire to the pond.

30 min / 60 min/h = 0.5 h. The distance travelled by the helicopter is the same for both trips. Distance = speed × time.

$$\begin{aligned} 105\text{ mi/h}(0.5\text{ h } - x) &= 70\text{ mi/h}(x) \\ 52.5\text{ mi } - 105\text{ mi/h}(x) &= 70\text{ mi/h}(x) \\ 52.5\text{ mi } &= 175\text{ mi/h}(x) \\ x &= \frac{52.5\text{ mi}}{175\text{ mi/h}} \\ x &= 0.3\text{ h} \end{aligned}$$

which is reported as 0.30 h to two significant digits. It will take the helicopter 0.30 h to fly from the pond to the fire.
Check:

$$\begin{aligned} 105\text{ mi/h}(0.5\text{ h } - 0.3\text{ h}) &= 70\text{ mi/h}(0.3\text{ h}) \\ 105\text{ mi/h}(0.2\text{ h}) &= 70\text{ mi/h}(0.3\text{ h}) \\ 21\text{ mi } &= 21\text{ mi} \end{aligned}$$

151. Let x = the number of litres of 0.50% grade oil used.
Let 1000L $-\,x$ the number of litres of 0.75% grade oil used.

$$\begin{aligned} 0.005(x) + 0.0075(1000\text{ L} - x) &= 0.0065(1000\text{ L}) \\ 0.005(x) + 7.5\text{ L} - 0.0075(x) &= 6.5\text{ L} \\ -0.0025(x) &= -1.0\text{ L} \\ x &= \frac{-1.0\text{ L}}{-0.0025} \\ x &= 400\text{ L} \end{aligned}$$

It will take 400 L of the 0.50% grade oil and (1000 L – 400 L) = 600 L of the 0.75% grade oil to make 1000 L of 0.65% grade oil.
Check:

$$\begin{aligned} 0.005(400\text{ L}) + 0.0075(1000\text{ L} - 400\text{ L}) &= 0.0065(1000\text{ L}) \\ 2\text{ L} + 4.5\text{ L} &= 6.5\text{ L} \\ 6.5\text{ L} &= 6.5\text{ L} \end{aligned}$$

152. Let x = the amount of rock containing 72 L/Mg of oil .
Let $18000 - x$ = the remaining amount of rock containing 150 L/Mg of oil.

$$\begin{aligned} (72\text{ L/Mg})(x) + (150\text{ L/Mg})(18000\text{ Mg} - x) &= (120\text{ L/Mg})(18000\text{ Mg}) \\ 72\text{ L/Mg}(x) + 2700000\text{ L} - 150\text{L/Mg}(x) &= 2160000\text{ L} \\ -78\text{ L/Mg}(x) &= -540000\text{ L} \\ x &= \frac{-540000\text{ L}}{-78\text{ L/Mg}} \\ x &= 6923.07692\text{ Mg} \end{aligned}$$

which rounds to 6900 Mg. It will take 6900 Mg of 72 L/Mg rock and 11100 Mg of 150 L/Mg rock to make the 18000 Mg of 120 L/Mg rock.
Check:

$$\begin{aligned} (72\text{ L/Mg})(6923.07692\text{ Mg}) + (150\text{ L/Mg})(18000\text{ Mg} - 6923.07692\text{ Mg}) &= (120\text{ L/Mg})(18000\text{ Mg}) \\ 498461.538\text{ L} + 2700000\text{ L} - 1038461.538\text{ L} &= 2160000\text{ L} \\ 2160000\text{ L} &= 2160000\text{ L} \end{aligned}$$

153. Let x = the area of space in ft^2 in the kitchen and bath.

$$\frac{\text{ft}^2 \text{ of tile in the house}}{\text{ft}^2 \text{ in the house}} = 0.25$$

$$\frac{x + 0.15(2200 \text{ ft}^2)}{(x + 2200 \text{ ft}^2)} = 0.25$$

$$x + 330 \text{ ft}^2 = 0.25(x) + (0.25)(2200 \text{ ft}^2)$$

$$x + 330 \text{ ft}^2 = 0.25(x) + 550 \text{ ft}^2$$

$$0.75x = 220 \text{ ft}^2$$

$$x = \frac{220 \text{ ft}^2}{0.75}$$

$$x = 293.33333333 \text{ ft}^2$$

which rounds to 290 ft^2. The kitchen and bath area is 290 ft^2.

Check:

$$\frac{293.33333333 \text{ ft}^2 + 0.15(2200 \text{ ft}^2)}{(293.33333333 \text{ ft}^2 + 2200 \text{ ft}^2)} = 0.25$$

$$\frac{623.3333333 \text{ ft}^2}{2493.3333333 \text{ ft}^2} = 0.25$$

$$0.25 = 0.25$$

154. Let x = the number of grams of 9-karat gold.
Let 200 g – x = the number of grams of 18-karat gold. 9-karat gold is 9/24 gold = 0.375, 18-karat gold is 18/24 gold= 0.75, and 14-karat gold is 14/24 gold = 0.583333333.

$$0.375(x) + 0.75(200 \text{ g} - x) = 0.583333333(200 \text{ g})$$

$$0.375(x) + 150 \text{ g} - 0.75(x) = 116.6666666 \text{ g}$$

$$-0.375(x) = -33.3333334 \text{ g}$$

$$x = \frac{-33.3333334 \text{ g}}{-0.375}$$

$$x = 88.88888907 \text{ g}$$

which rounds to 89 g. There is 89 g of 9-karat gold and (200 g – 89 g) = 111 g of 18-karat gold needed to make 200 g of 14-karat gold.

Check:

$$0.375(88.88888907 \text{ g}) + 0.75(200 \text{ g} - 88.88888907 \text{ g}) = 0.583333333(200 \text{ g})$$

$$33.3333334 \text{ g} + 83.3333332 \text{ g} = 116.6666666 \text{ g}$$

$$116.6666666 \text{ g} = 116.6666666 \text{ g}$$

155.

$$P = P_0 + P_0 rt$$

$$P - P_0 = P_0 rt$$

$$r = \frac{P - P_o}{P_0 t}$$

$$r = \frac{\$7625 - \$6250}{\$6250(4.000 \text{ years})}$$

$$r = \frac{\$1375}{25\ 000}$$

$$r = 0.055$$

The rate is equal to 5.500%.

On the calculator type:

$(7625 - 6250) / (6250 \times 4.000)$

Chapter 2

Geometry

2.1 Lines and Angles

1. $\angle ABE = 90^\circ$ because it is a vertically opposite angle to $\angle CBD$ which is also a right angle.

2. Angles $\angle POR$ and $\angle QOR$ are complementary angles, so sum to 90°

$$\angle POR + \angle QOR = 90^\circ$$
$$35^\circ + \angle QOR = 90^\circ$$
$$\angle QOR = 90^\circ - 35^\circ$$
$$\angle QOR = 55^\circ$$

3. 4 pairs of adjacent angles:

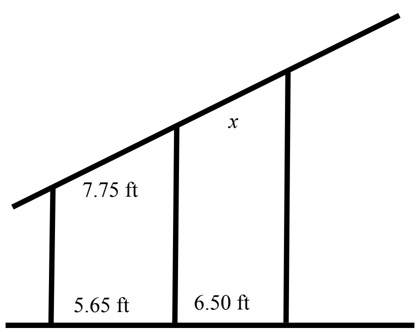
$\angle BOC$ and $\angle COA$ share common ray OC

$\angle COA$ and $\angle AOD$ share common ray OA

$\angle AOD$ and $\angle DOB$ share common ray OD

$\angle DOB$ and $\angle BOC$ share common ray OB

4.



(a) Using Eq. (2.1), we have

$$\frac{7.75}{5.65} = \frac{x}{6.50}$$
$$x = \frac{7.75(6.50)}{5.65}$$

$x = 8.92$ ft (the same answer as Example 5)

(b) More vertical, since the distance along the beam is longer for the same horizontal run, which can only be achieved if the angle increases from horizontal (see sketch).

5. $\angle EBD$ and $\angle DBC$ are acute angles (i.e., $< 90°$).

6. $\angle ABE$ and $\angle CBE$ are right angles (i.e., $= 90°$).

7. $\angle ABC$ is a straight angle (i.e., $= 180°$).

8. $\angle ABD$ is an obtuse angle (i.e., between 90° and 180°) .

9. The complement of $\angle CBD = 65^\circ$ is $\angle DBE$

$$\angle CBD + \angle DBE = 90^\circ$$
$$65^\circ + \angle DBE = 90^\circ$$
$$\angle DBE = 90^\circ - 65^\circ$$
$$\angle DBE = 25^\circ$$

10. The supplement of $\angle CBD = 65^\circ$ is $\angle ABD$

$$\angle CBD + \angle ABD = 180^\circ$$
$$65^\circ + \angle ABD = 180^\circ$$
$$\angle ABD = 180^\circ - 65^\circ$$
$$\angle ABD = 115^\circ$$

11. Sides BD and BC are adjacent to $\angle DBC$.

12. The angle adjacent to $\angle DBC$ is $\angle DBE$ since they share the common side BD, and $\angle DBE$ is acute because it is less than 90°

13. $\angle AOB = \angle AOE + \angle EOB$

but $\angle AOE = 90^\circ$ because it is vertically opposite to $\angle DOF$ a given right angle,

and$\angle EOB = 50^\circ$ because it is vertically opposite to $\angle COF$ a given angle of 50°,

so $\angle AOB = 90^\circ + 50^\circ = 140^\circ$

14. $\angle AOC$ is complementary to $\angle COF$ a given angle of 50°,

$$\angle AOC + \angle COF = 90^\circ$$
$$\angle AOC + 50^\circ = 90^\circ$$
$$\angle AOC = 90^\circ - 50^\circ$$
$$\angle AOC = 40^\circ$$

15. $\angle BOD$ is vertically opposite to $\angle AOC$ a found angle of 40° (see Question 14), so

$$\angle BOD = \angle AOC$$
$$\angle BOD = 40^\circ$$

16. $\angle 1$ is supplementary to 150°, so

$$\angle 1 = 180^\circ - 150^\circ = 30^\circ$$
$$\angle 2 = \angle 1 = 30^\circ$$

$\angle 3$ is supplementary to $\angle 2$, so

$$\angle 3 = 180^\circ - \angle 2$$
$$\angle 3 = 180^\circ - 30^\circ$$
$$\angle 3 = 150^\circ$$

17. $\angle 1$ is supplementary to 150°, so

$\angle 1 = 180^\circ - 150^\circ = 30^\circ$

$\angle 2 = \angle 1 = 30^\circ$

$\angle 4$ is vertically opposite to $\angle 2$, so

$\angle 4 = \angle 2$

$\angle 4 = 30^\circ$

18. $\angle 1$ is supplementary to 150°, so

$\angle 1 = 180^\circ - 150^\circ = 30^\circ$

$\angle 2 = \angle 1 = 30^\circ$

$\angle 5$ is supplementary to $\angle 2$, so

$\angle 5 = 180^\circ - \angle 2$

$\angle 5 = 180^\circ - 30^\circ$

$\angle 5 = 150^\circ$

19. $\angle 1 = 62^\circ$ since they are vertically opposite

20. $\angle 1 = 62^\circ$ since they are vertically opposite

$\angle 2$ is a corresponding angle to $\angle 5$, so

$\angle 2 = \angle 5$

since $\angle 1$ and $\angle 5$ are supplementary angles,

$\angle 5 + \angle 1 = 180^\circ$

$\angle 2 + \angle 1 = 180^\circ$

$\angle 2 = 180^\circ - \angle 1$

$\angle 2 = 180^\circ - 62^\circ$

$\angle 2 = 118^\circ$

21. $\angle 6 = 90 - 62^\circ$ since they are complementary angles

$\angle 6 = 28^\circ$

$\angle 3$ is an alternate-interior angle to $\angle 6$, so

$\angle 3 = \angle 6$

$\angle 3 = 28^\circ$

22. $\angle 3 = 28^\circ$ (see Question 21)

since $\angle 4$ and $\angle 3$ are supplementary angles,

$\angle 4 + \angle 3 = 180^\circ$

$\angle 4 = 180^\circ - \angle 3$

$\angle 4 = 180^\circ - 28^\circ$

$\angle 4 = 152^\circ$

23. $\angle 5 = 180^\circ - 62^\circ$ since they are supplementary angles

$\angle 5 = 118^\circ$

24. $\angle 6 = 90^\circ - 62^\circ$ since they are complementary angles

$\angle 6 = 28^\circ$

25. $\angle EDF = \angle BAD = 44^\circ$ because they are corresponding angles

$\angle BDE = 90^\circ$

$\angle BDF = \angle BDE + \angle EDF$

$\angle BDF = 90^\circ + 44^\circ$

$\angle BDF = 134^\circ$

26. $\angle CBE = \angle BAD = 44^\circ$ because they are corresponding angles

$\angle DBE$ and $\angle CBE$ are complementary so

$\angle DBE + \angle CBE = 90^\circ$

$\angle DBE = 90^\circ - \angle CBE$

$\angle DBE = 90^\circ - 44^\circ$

$\angle DBE = 46^\circ$

and $\angle ABE = \angle ABD + \angle DBE$

$\angle ABE = 90^\circ + 46^\circ$

$\angle ABE = 136^\circ$

27. $\angle CBE = \angle BAD = 44^\circ$ because they are corresponding angles

$\angle DEB$ and $\angle CBE$ are alternate interior angles, so

$\angle DEB = \angle CBE$

$\angle DEB = 44^\circ$

28. $\angle CBE = \angle BAD = 44^\circ$ because they are corresponding angles

$\angle DBE$ and $\angle CBE$ are complementary so

$\angle DBE + \angle CBE = 90^\circ$

$\angle DBE = 90^\circ - \angle CBE$

$\angle DBE = 90^\circ - 44^\circ$

$\angle DBE = 46^\circ$

29. $\angle EDF = \angle BAD = 44^\circ$ because they are corresponding angles

Angles $\angle ADB$, $\angle BDE$, and $\angle EDF$ make a straight angle

$\angle ADB + \angle BDE + \angle EDF = 180^\circ$

$\angle ADB = 180^\circ - \angle BDE - \angle EDF$

$\angle ADB = 180^\circ - 90^\circ - 44^\circ$

$\angle ADB = 46^\circ$

$\angle DFE = \angle ADB$ because they are corresponding angles

$\angle DFE = 46^\circ$

30. $\angle ADE = \angle ADB + \angle BDE$

$\angle ADB = 46^\circ$ (see Question 27)

$\angle BDE = 90^\circ$

$\angle ADE = 46^\circ + 90^\circ$

$\angle ADE = 136^\circ$

31. Using Eq. (2.1),

$$\frac{a}{4.75} = \frac{3.05}{3.20}$$

$$a = 4.75 \cdot \frac{3.05}{3.20}$$

$$a = 4.53 \text{ m}$$

32. Using Eq. (2.1),

$$\frac{b}{6.25} = \frac{3.05}{3.20}$$

$$b = 6.25 \cdot \frac{3.05}{3.20}$$

$$b = 5.96 \text{ m}$$

33. Using Eq. (2.1),

$$\frac{c}{3.20} = \frac{5.05}{4.75}$$

$$c = \frac{(3.20)(5.05)}{4.75}$$

$$c = 3.40 \text{ m}$$

34. Using Eq. (2.1),

$$\frac{d}{6.25} = \frac{5.05}{4.75}$$

$$d = \frac{(6.25)(5.05)}{4.75}$$

$$d = 6.64 \text{ m}$$

35. $\angle BCH = \angle DCG$ is given and $\angle BCH, 50^\circ$, and $\angle DCG$ together form a straight angle.

$\angle BCH + 50^\circ + \angle DCG = 180^\circ$

$2\angle BCH = 130^\circ$

$\angle BCH = 65^\circ$

Since $\angle BCH$ and $\angle GHC$ are alternate-interior angles,

$\angle BCH = \angle GHC.$

Therefore, $\angle GHC = 65^\circ$.

Since $\angle GHC$ and $\angle BHC$ are complementary angles,

$\angle BHC + \angle GHC = 90^\circ$

and so $\angle BHC = 25^\circ$.

By symmetry, $\angle DGC = 25^\circ$.

36. From problem 35, $\angle BHC = 25^\circ$ and $\angle DGC = 25^\circ$.

$$\begin{aligned}\angle AHC &= \angle AHB + \angle BHC\\ &= 20^\circ + 25^\circ\\ &= 45^\circ\\ \angle EGC &= \angle EGD + \angle DGC\\ &= 20^\circ + 25^\circ\\ &= 45^\circ\end{aligned}$$

37. $\angle BCH = \angle DCG$ is given and $\angle BCH, 50^\circ$, and $\angle DCG$ together form a straight angle.

$$\begin{aligned}\angle BCH + 50^\circ + \angle DCG &= 180^\circ\\ 2\angle BCH &= 130^\circ\\ \angle BCH &= 65^\circ\end{aligned}$$

Since $\angle BCH$ and $\angle GHC$ are alternate-interior angles,

$\angle BCH = \angle GHC.$

Therefore, $\angle GHC = 65^\circ$.

Similarly, $\angle HGC = \angle GCD = 65^\circ$.

38. $\angle FGE$ and $\angle EGD$ are complementary angles.

$$\begin{aligned}\angle FGE + \angle EGD &= 90^\circ\\ \angle FGE &= 90^\circ - 20^\circ\\ \angle FGE &= 70^\circ\end{aligned}$$

$\angle FGE$ and $\angle GED$ are alternate-interior angles.

$\angle GED = \angle FGE = 70^\circ$

Similarly, $\angle AHI = \angle BAH = 70^\circ$.

39. $\angle AHF$ and $\angle AHI$ are supplementary angles.

$$\begin{aligned}\angle AHF + \angle AHI &= 180^\circ\\ \angle AHF &= 180^\circ - 70^\circ\\ \angle AHF &= 110^\circ\end{aligned}$$

$\angle EGI$ and $\angle FGE$ are supplementary angles.

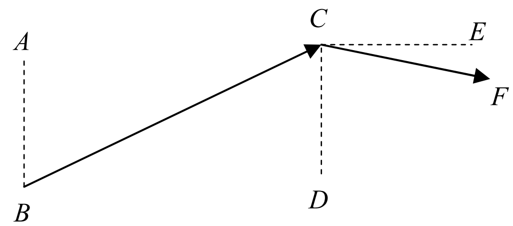
$$\begin{aligned}\angle EGI + \angle FGE &= 180^\circ\\ \angle EGI &= 180^\circ - 70^\circ\\ \angle EGI &= 110^\circ\end{aligned}$$

40. From problem 37, $\angle BCH = \angle GHC = \angle HGC = \angle GCD = 65^\circ$.

Angles supplementary to these have measure $180^\circ - 65^\circ = 115^\circ$.

These supplementary angles are $\angle HCD, \angle CGF, \angle CHI$, and $\angle GCA$.

41.



We are given $\angle ABC = 58°$ and $\angle ECF = 18°$. We draw CD parallel to AB.
Since $\angle ABC$ and $\angle BCD$ are alternate-interior angles, they are equal and so $\angle BCD = 58°$.
Since $\angle ECF$ and $\angle DCF$ are complementary angles,
$\angle ECF + \angle DCF = 90°$. Therefore, $\angle DCF = 90° - 18° = 72°$
Thus, $\angle BCF = \angle BCD + \angle DCF = 58° + 72° = 130°$.

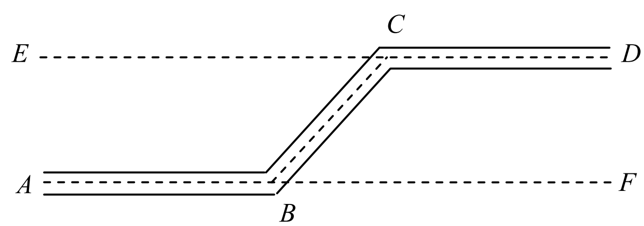
42. **(a)** If the angles are complementary, they sum to 90°. Thus,

$$\begin{aligned} (x+20)+(3x-2) &= 90 \\ 4x+18 &= 90 \\ 4x &= 72 \\ x &= 18 \end{aligned}$$

(b) If the angles are alternate-interior, they are equal. Thus,

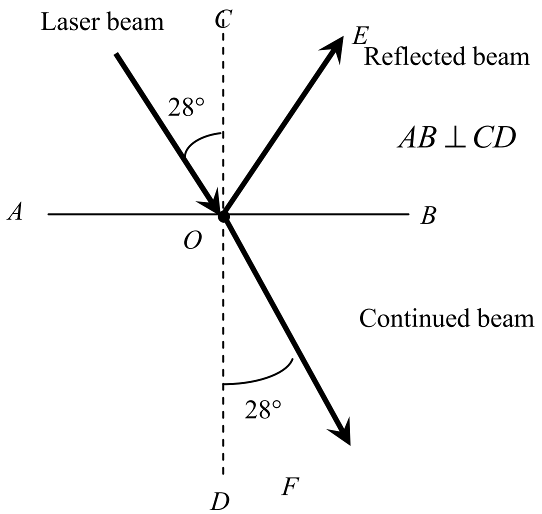
$$\begin{aligned} x+20 &= 3x-2 \\ 18 &= 2x \\ x &= 9 \end{aligned}$$

43.



We are given $\angle CBF = 47°$.
Since $\angle CBF$ and $\angle ECB$ are alternate-interior angles, they are equal and so $\angle ECB = 47°$.
Since $\angle ECB$ and $\angle BCD$ are supplementary angles,
$\angle ECB + \angle BCD = 180°$. Therefore, $\angle BCD = 180° - 47° = 133°$.

44.



$\angle BOD$ is a right angle, so the angle between the surface and the continued beam, $\angle BOF$ satisfies

$$\angle DOF + \angle BOF = 90^\circ$$
$$\angle BOF + 28^\circ = 90^\circ$$
$$\angle BOF = 90^\circ - 56^\circ$$
$$\angle BOF = 34^\circ$$

45. Using Eq. (2.1),

$$\frac{AB}{3} = \frac{BC}{2} =$$
$$AB = \frac{3(2.15)}{2}$$
$$AB = 3.225 \text{ cm}$$
$$AC = AB + BC$$
$$AC = 3.225 \text{ cm} + 2.15 \text{ cm}$$
$$AC = 5.375 \text{ cm}$$
$$AC = 5.38 \text{ cm}$$

46. Using Eq. (2.1),

$$\frac{x}{860} = \frac{590}{550}$$
$$x = \frac{(860)(590)}{550}$$

$x = 922.5454$ m which is rounded to 920 m

47. $\angle 1 + \angle 2 + \angle 3 = 180^\circ$, because $\angle 1$, $\angle 2$, and $\angle 3$ form a straight angle.

48. $\angle 1 = \angle 4$ since they are alternate interior angles

$\angle 3 = \angle 5$ since they are alternate interior angles

$\angle 1 + \angle 2 + \angle 3 = 180^\circ$, because $\angle 1$, $\angle 2$, and $\angle 3$ form a straight angle, so

$\angle 4 + \angle 2 + \angle 5 = 180^\circ$

49. The sum of the angles with vertices at A, B, and D is 180°.
Since those angles are unknown quantities, the sum of
interior angles in a closed triangle is 180°.

50. The angle of elevation and the angle of depression are equal because they are alternate-interior angles.
We do not take into account the curvature of the Earth for this observation.

2.2 Triangles

1. $\angle 5 = 45^\circ$

$\angle 3 = 45^\circ$ since $\angle 3$ and $\angle 5$ are alternate interior angles.

$\angle 1$, $\angle 2$, and $\angle 3$ make a stright angle, so

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$
$$70^\circ + \angle 2 + 45^\circ = 180^\circ$$
$$\angle 2 = 65^\circ$$

2.
$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(61.2)(5.75)$$
$$A = 176 \text{ cm}^2$$

3.
$$AC^2 = AB^2 + BC^2$$
$$AC^2 = 6.25^2 + 3.20^2$$
$$AC = \sqrt{6.25^2 + 3.20^2}$$
$$AC = 7.02 \text{ m}$$

4. Using Eq. (2.1),
$$\frac{h}{3.00} = \frac{24.0}{4.00}$$
$$h = \frac{(3.00)(24.0)}{4.00}$$
$$h = 18.0 \text{ m}$$

5.
$$\angle A + \angle B + \angle C = 180^\circ$$
$$\angle A + 40^\circ + 84^\circ = 180^\circ$$
$$\angle A = 56^\circ$$

6.
$$\angle A+\angle B+\angle C=180^\circ$$
$$\angle A+48^\circ+90^\circ=180^\circ$$
$$\angle A=42^\circ$$

7. This is an isosceles triangle, so the base angles are equal.
$$\angle B=\angle C=66^\circ$$
$$\angle A+\angle B+\angle C=180^\circ$$
$$\angle A+66^\circ+66^\circ=180^\circ$$
$$\angle A=180^\circ-\left(66^\circ+66^\circ\right)$$
$$\angle A=48^\circ$$

8. This is an isosceles triangle, so the base angles are equal.
$$\angle A=\angle B$$
$$\angle A+\angle B+\angle C=180^\circ$$
$$\angle A+\angle A+110^\circ=180^\circ$$
$$2\angle A=70^\circ$$
$$\angle A=35^\circ$$

9.
$$A=\frac{1}{2}bh$$
$$A=\frac{1}{2}(6.3)(2.2)$$
$$A=6.9\text{ ft}^2$$

10.
$$A=\frac{1}{2}bh$$
$$A=\frac{1}{2}(16.0)(9.62)$$
$$A=77.0\text{ mm}^2$$

11. By Hero's formula,
$$p=239+322+415=976\text{ cm}$$
$$s=\frac{976}{2}=488\text{ cm}$$
$$A=\sqrt{s(s-a)(s-b)(s-c)}$$
$$A=\sqrt{488\text{ cm}\cdot(488-239)\text{ cm}\cdot(488-322)\text{ cm}\cdot(488-415)\text{ cm}}$$
$$A=\sqrt{488(249)(166)(73)\text{cm}^4}$$
$A=38{,}372.938\text{ cm}^2$ which is rounded to 38,300 cm^2.

12. By Hero's formula,

$p = 0.862 \text{ in} + 0.535 \text{ in} + 0.684 \text{ in} = 2.081 \text{ in}$

$s = \dfrac{2.081 \text{ in}}{2} = 1.0405 \text{ in}$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{1.0405 \text{ in}(1.0405 \text{ in} - 0.862 \text{ in})(1.0405 \text{ in} - 0.535 \text{ in})(1.0405 \text{ in} - 0.684 \text{ in})}$

$A = 0.18294919 \text{ in}^2$ which is rounded to 0.183 in^2

13. One leg can represent the base, the other leg the height.

$A = \dfrac{1}{2}bh$

$A = \dfrac{1}{2}(3.46)(2.55)$

$A = 4.41 \text{ ft}^2$

14. One leg can represent the base, the other leg the height.

$A = \dfrac{1}{2}bh$

$A = \dfrac{1}{2}(234)(342)$

$A = 40014 \text{ mm}^2$ which rounds to $4.00 \times 10^4 \text{ mm}^2$

15. By Hero's formula,

$s = \dfrac{p}{2} = \dfrac{0.986 + 0.986 + 0.884}{2} \text{ m} = 1.428 \text{ m}$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{1.428(1.428 - 0.986)(1.428 - 0.986)(1.428 - 0.884)}$

$A = 0.390 \text{ m}^2$

16. By Hero's formula,

$s = \dfrac{3(3200)}{2} = 4800 \text{ yd}$

$A = \sqrt{s(s-a)^3}$

$A = \sqrt{4800(4800 - 3200)^3}$

$A = 4,434,050 \text{ yd}^2$ which rounds to 4,400,000 yd^2

17. We add the lengths of the sides to get

$p = 205 + 322 + 415$

$p = 942 \text{ cm}$

18. We add the lengths of the sides to get

$p = 23.5 + 86.2 + 68.4$

$p = 178.1 \text{ in}$

19. We add the lengths of the sides to get
$p = 3(21.5) = 64.5 \text{ cm}$

20. We add the lengths of the sides to get
$p = 2(2.45) + 3.22 = 8.12 \text{ in}$

21.
$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{3^2 + 4^2}$$
$$c = 5 \text{ in}$$

22.
$$c^2 = a^2 + b^2$$
$$b^2 = c^2 - a^2$$
$$b = \sqrt{c^2 - a^2}$$
$$c = \sqrt{13^2 - 5^2}$$
$$c = 12 \text{ yd}$$

23.
$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{13.8^2 + 22.7^2}$$
$$c = 26.6 \text{ ft}$$

24.
$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{2.48^2 + 1.45^2}$$
$$c = 2.87 \text{ m}$$

25.
$$c^2 = a^2 + b^2$$
$$b = \sqrt{c^2 - a^2}$$
$$b = \sqrt{551^2 - 175^2}$$
$$b = 522 \text{ cm}$$

26.
$$c^2 = a^2 + b^2$$
$$a = \sqrt{c^2 - b^2}$$
$$a = \sqrt{0.836^2 - 0.474^2}$$
$$a = 0.689 \text{ in}$$

27. All interior angles in a triangle add to 180°
$$23^\circ + \angle B + 90^\circ = 180^\circ$$
$$\angle B = 180^\circ - 90^\circ - 23^\circ$$
$$\angle B = 67^\circ$$

28. $c^2 = a^2 + b^2$

$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{38.4^2 + 90.5^2}$$
$$c = 98.3 \text{ cm}$$

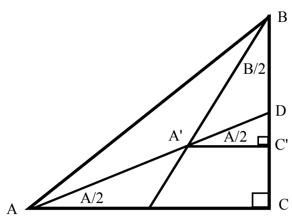
29. Length c is found in Question 26, $c = 98.309\ 77$ cm

$$p = 98.309\ 77 + 90.5 + 38.4 = 227.2 \text{ cm}$$

30. $A = \frac{1}{2}bh$

$$A = \frac{1}{2}(90.5)(38.4)$$
$$A = 1740 \text{ cm}^2$$

31.



$$\Delta ADC \sim \Delta A'DC'$$
$$\angle DA'C' = A/2$$
$$\angle BA'D = \angle \text{ between bisectors}$$

From $\Delta BA'C'$, and all angles in a triangle must sum to 180°

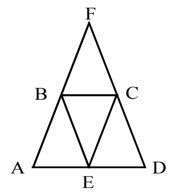
$$\frac{B}{2} + (\angle BA'D + A/2) + 90^\circ = 180^\circ$$
$$\angle BA'D = 90^\circ - \left(\frac{A}{2} + \frac{B}{2}\right)$$
$$\angle BA'D = 90^\circ - \left(\frac{A+B}{2}\right)$$

But ΔABC is a right triangle, and all angles in a triangle must sum to 180°,

so $A + B = 90^\circ$

$$\angle BA'D = 90^\circ - \frac{90^\circ}{2}$$
$$\angle BA'D = 45^\circ$$

32.



$\angle A = \angle D$ since ΔAFD is isosceles.

Since $AF = FD$ $(\Delta AFD$ is isosceles$)$ and since B and C are midpoints,

$AB = CD$

$AE = DE$ because E is a midpoint of AD,

so if two of the three sides are identical, the last side is the same too.

so $\Delta ABE = \Delta ECD$

Therefore, $BE = EC$ from which it follows that the inner ΔBCE is isosceles.

Also, since $AB = CD = FB = FC$

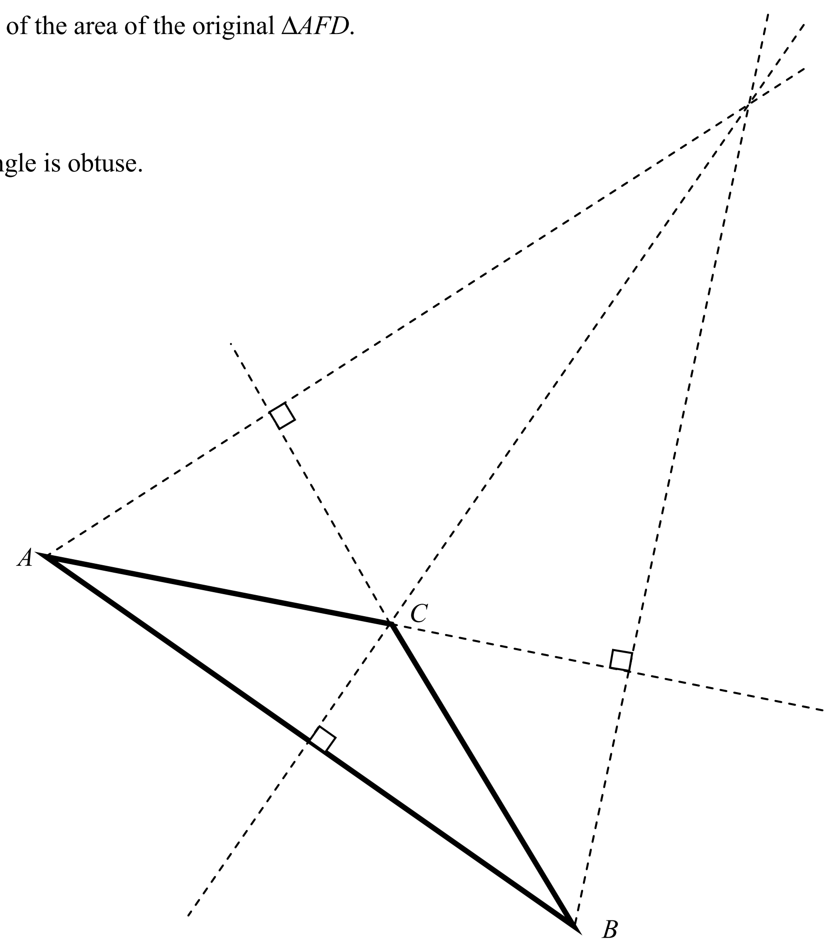
$\Delta ABE = \Delta ECD = \Delta BFC = \Delta BCE$

and all four triangles are similar triangles to the original ΔAFD

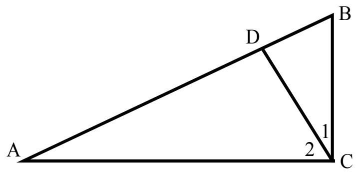
So, ΔBCE is also 1/4 of the area of the original ΔAFD.

33. An equilateral triangle.

34. Yes, if one of the angles of the triangle is obtuse.
For example, see ΔABC below.



35.

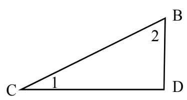


$\angle A + \angle B = 90^\circ$

$\angle 1 + \angle B = 90^\circ$

$\angle A = \angle 1$

redraw ΔBDC as

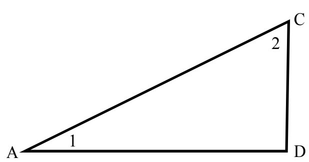


$\angle 1 + \angle 2 = 90^\circ$

$\angle 1 + \angle B = 90^\circ$

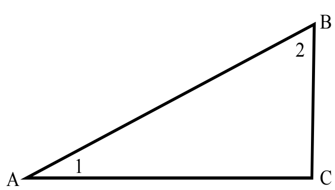
$\angle 2 = \angle B$

and ΔADC as



ΔBDC and ΔADC are similar.

36.



Comparing the original triangle to the two smaller triangles (see Question 35) shows that all three are similar.

37. $\angle LMK$ and $\angle OMN$ are vertically opposite angles and thus equal.
Since each triangle has a right angle, the remaining angle in each triangle must be the same.
$\angle KLM = \angle MON$.
The triangles ΔMKL and ΔMNO have all the same angles, so therefore the triangles are similar:
$\Delta MKL \sim \Delta MNO$

38. $\angle ACB = \angle ADC = 90^\circ$

$\angle DAC = \angle BAC$ since they share the common vertex *A*.

Since all angles in any triangle sum to 180°,

$\angle DCA = 180 - 90 - \angle BAC$,

$\angle ABC = 180 - 90 - \angle BAC$,

Therefore, all the angles in ΔACB and ΔADC are equivalent, so

$\Delta ACB \sim \Delta ADC$

39.
$$KM = KN - MN$$
$$KM = 15 - 9$$
$$KM = 6$$

Since $\Delta MKL \sim \Delta MNO$

$$\frac{LM}{KM} = \frac{OM}{MN}$$
$$\frac{LM}{6} = \frac{12}{9}$$
$$LM = \frac{(6)(12)}{9}$$
$$LM = 8$$

40. Since $\Delta ADC \sim \Delta ACB$

$$\frac{AB}{AC} = \frac{AC}{AD}$$
$$\frac{AB}{12} = \frac{12}{9}$$
$$AB = \frac{(12)(12)}{9}$$
$$AB = 16$$

41. $s = \frac{p}{2} = \frac{6+25+29}{2} = 30$

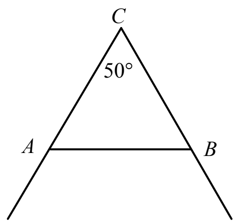
By Hero's formula,

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$A = \sqrt{30(30-6)(30-25)(30-29)}$$
$$A = \sqrt{3600} = 60$$
$$p = 6 + 25 + 29 = 60$$

and so a triangle with sides 6, 25, 29 is perfect.

42. The ramp has a horizontal run of at least 80.0 in. and so, using the Pythagorean formula, the length of the ramp should be at least $\sqrt{4.0^2 + 80.0^2} = \sqrt{6416} \approx 80.1$ in.

43.



ΔABC is isosceles,

so $\angle CAB = \angle CBA$

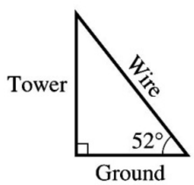
But all interior angles in a triangle sum to 180°

$\angle CAB + \angle CBA + 50^\circ = 180^\circ$

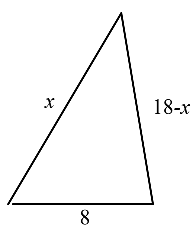
$2\angle CAB = 130^\circ$

$\angle CAB = 65^\circ$

44. All interior angles in a triangle sum to 180° and so the angle between the tower and the wire $= 180^\circ - 90^\circ - 52^\circ = 38^\circ$



45.



The tree could break at any point between 5 and 13 feet to have the top land 8 feet away from the base.

46. $s = \frac{p}{2} = \frac{3(1600)}{2} = 2400 \text{ km}$

By Hero's formula,

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{2400(2400-1600)^3}$

$A = 1{,}100{,}000 \text{ km}^2$

47. One leg can represent the base, the other leg the height.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(8.0)(15)$$

$$A = 60 \text{ ft}^2$$

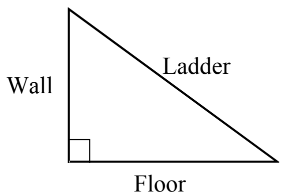
48. $c^2 = a^2 + b^2$

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{750^2 + 550^2}$$

$$c = 930 \text{ m}$$

49.



The base of the window is leg b satisfying

$b = \sqrt{c^2 - a^2}, c = 10, a = 6$

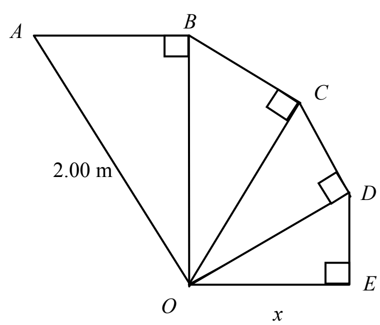
$b = \sqrt{100 - 36} = 8$

The top of the window is $b + 2.5 = 10.5$ ft.

and so the ladder's new length must be

$l = \sqrt{6^2 + 10.5^2} = 12.1$ ft.

50.



On ΔABO

the idea that the side opposite the 30° angle is half the hypotenuse gives

$AB = 1.00$ m

Using Pythagorean theorem gives

$$AO^2 = AB^2 + BO^2$$

$$BO = \sqrt{AO^2 - AB^2}$$

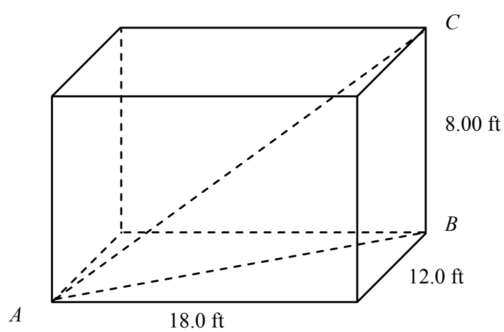
$$BO = \sqrt{2^2 - 1^2}$$

$$BO = \sqrt{3} \text{ m}$$

Using an identical technique on each successive triangle moving clockwise,

$$BC = \frac{\sqrt{3}}{2} \text{ m}$$
$$CO = \sqrt{3 - \tfrac{3}{4}}$$
$$CO = 1.50 \text{ m}$$
$$CD = 0.750 \text{ m}$$
$$DO = \sqrt{1.50^2 - (0.750)^2}$$
$$DO = 1.30 \text{ m}$$
$$DE = 0.650 \text{ m}$$
$$x = \sqrt{1.30^2 - (0.650)^2}$$
$$x = 1.125 \text{ m}$$
$$x = 1.12 \text{ m}$$

51.



Diagonal AB

$$AB = \sqrt{18^2 + 12^2} = \sqrt{468} \text{ ft}$$

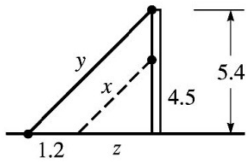
Diagonal AC

$$AC = \sqrt{AB^2 + 8^2}$$
$$AC = \sqrt{468 + 64} \text{ ft}$$
$$AC = \sqrt{532} \text{ ft}$$
$$AC = 23.1 \text{ ft}$$

52. By Eq. (2.1),

$$\frac{45.6 \text{ cm}}{x} = \frac{1.20 \text{ cm}}{1.00 \text{ m}}$$
$$x = \frac{45.6 \text{ cm } (1.00 \text{ m})}{1.20 \text{ cm}}$$
$$x = 38.0 \text{ m}$$

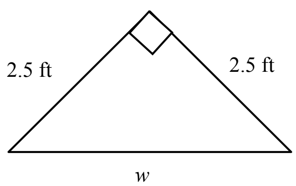
53.



By Eq. (2.1),

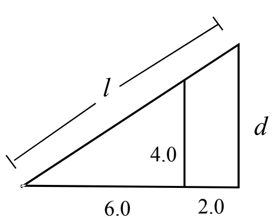
$$\frac{z}{4.5} = \frac{1.2}{0.9}$$
$$z = \frac{(4.5)(1.2)}{0.9}$$
$$z = 6.0 \text{ m}$$
$$x^2 = z^2 + 4.5^2$$
$$x = \sqrt{56.25} \text{ m}$$
$$x = 7.5 \text{ m}$$
$$y^2 = (1.2+6)^2 + 5.4^2$$
$$x = \sqrt{81.0} \text{ m}$$
$$y = 9.0 \text{ m}$$

54.



$$w^2 = 2.5^2 + 2.5^2$$
$$w = \sqrt{12.5} \text{ ft}$$
$$w = 3.5 \text{ ft}$$

55.



By Eq. (2.1),

$$\frac{d}{4.0} = \frac{8.0}{6.0}$$
$$d = \frac{8.0(4.0)}{6.0}$$
$$d = 5.333 \text{ ft}$$
$$l^2 = 8.0^2 + 5.333^2$$
$$l = \sqrt{8.0^2 + 5.333^2}$$
$$l = 9.6 \text{ ft}$$

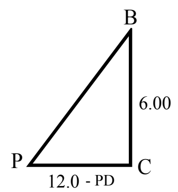
56. $\frac{ED}{AB} = \frac{DC}{BC}$

$\frac{ED}{80.0} = \frac{312}{50.0}$

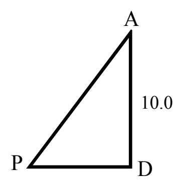
$ED = \frac{(80.0)(312)}{50.0}$

$ED = 499 \text{ ft}$

57. Redraw ΔBCP as



ΔAPD is



since $\Delta BCP \sim \Delta ADP$

$\frac{6.00}{12.0 - PD} = \frac{10.0}{PD}$

$6PD = 120 - 10PD$

$16PD = 120$

$PD = 7.50 \text{ km}$

$PC = 12.0 - PD$

$PC = 4.50 \text{ km}$

$l = PB + PA$

$l = \sqrt{4.50^2 + 6.00^2} + \sqrt{7.50^2 + 10.0^2}$

$l = 7.50 + 12.5$

$l = 20.0 \text{ km}$

58. Original area:

$$A_o = \frac{1}{2}bh$$
$$A_o = \frac{1}{2}x(x-12)$$

New area:

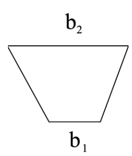
$$A_n = \frac{1}{2}bh$$
$$A_n = \frac{1}{2}x(x-12+16)$$
$$A_n = \frac{1}{2}x(x+4)$$

If the new area is 160 cm^2 larger than the original,

$$A_n - A_o = 160$$
$$\frac{1}{2}x(x+4) - \frac{1}{2}x(x-12) = 160$$
$$\frac{1}{2}x^2 + 2x - \frac{1}{2}x^2 + 6x = 160$$
$$8x = 160$$
$$x = 20 \text{ cm is the orignal width}$$
$$d = x - 12$$
$$d = 8 \text{ cm is the original depth}$$

2.3 Quadrilaterals

1.



trapezoid

2. $L = 4s + 2w + 2l$
$L = 4(21) + 2(21) + 2(36)$
$L = 198$ in

3. $A_1 = \frac{1}{2}bh = \frac{1}{2}(72)(55) = 1980 = 2000 \text{ ft}^2$
$A_2 = bh = 72(55) = 3960 = 4000 \text{ ft}^2$
$A_3 = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(55)(72+35)$
$A_3 = 2942.5 = 2900 \text{ ft}^2$
$A_{tot} = 1980 + 3960 + 2942.5 = 8900 \text{ ft}^2$

4.
$$\begin{aligned} 2(w+3.0)+2w &= 26.4 \\ 2w+6.0+2w &= 26.4 \\ 4w &= 20.4 \\ w &= 5.1 \text{ mm} \\ w+3.0 &= 8.1 \text{ mm} \end{aligned}$$

5. $p = 4s = 4(85) = 340 \text{ m}$

6. $p = 4(2.46) = 9.84 \text{ ft}$

7. $p = 2(9.200) + 2(7.420) = 33.24 \text{ in}$

8. $p = 2(142) + 2(126) = 536 \text{ cm}$

9. $p = 2l + 2w = 2(3.7) + 2(2.7) = 12.8 \text{ m}$

10. $p = 2(27.3) + 2(14.2) = 83.0 \text{ mm}$

11. $p = 0.362 + 0.730 + 0.440 + 0.612 = 2.144 \text{ ft}$

12. $p = 272 + 392 + 223 + 672 = 1559 \text{ cm}$

13. $A = s^2 = 6.4^2 = 41 \text{ mm}^2$

14. $A = 15.6^2 = 243 \text{ ft}^2$

15. $A = lw = 8.35(2.81) = 23.5 \text{ in}^2$

16. $A = lw = 142(126) = 17\ 900 \text{ cm}^2$

17. $A = bh = 3.7(2.5) = 9.2 \text{ m}^2$

18. $A = bh = 27.3(12.6) = 344 \text{ in}^2$

19.
$$\begin{aligned} A &= \tfrac{1}{2}h(b_1 + b_2) \\ &= \tfrac{1}{2}(0.298)(0.612 + 0.730) \\ &= 0.200 \text{ ft}^2 \end{aligned}$$

20.
$$\begin{aligned} A &= \tfrac{1}{2}h(b_1 + b_2) \\ &= \tfrac{1}{2}(201)(392 + 672) \\ &= 107\ 000 \text{ cm}^2 \end{aligned}$$

21. $p = 2b + 4a$

22. $p = a + b + b + a + (b - a) + (b - a)$

$p = 2a + 2b + 2b - 2a$

$p = 4b$

23. $A = bh + a^2$

24. $A = \frac{1}{2}a[b + b - a] + \frac{1}{2}a[b + b - a]$

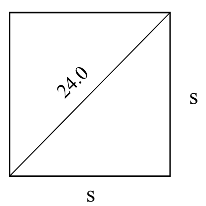
$A = ab - \frac{1}{2}a^2 + ab - \frac{1}{2}a^2$

$A = 2ab - a^2$

25. The parallelogram is a rectangle.

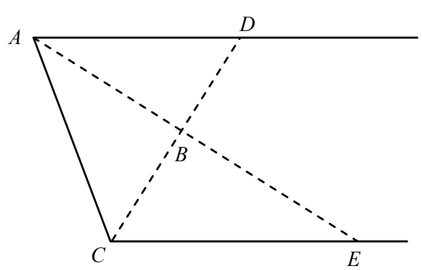
26. The triangles are congruent. Corresponding sides and angles are equal.

27.



$$s^2 + s^2 = 24.0^2$$
$$2s^2 = 576$$
$$s^2 = \frac{576}{2}$$
$$A = s^2 = 288 \text{ cm}^2$$

28.



$\angle DAE$ and $\angle CEA$ are alternate interior angles, and so

$\angle DAB = \angle DAE = \angle CEA = \angle CEB$

$\angle CAB = \frac{1}{2}\angle CAD = \angle DAE$ because of the angle bisector AE

$\angle CAB = \angle DAB$

$\angle ADC$ and $\angle ECD$ are alternate interior angles, and so

$$\angle ADB = \angle ADC = \angle ECD = \angle ECB$$

$\angle ACB = \frac{1}{2}\angle ACE = \angle ECD$ because of the angle bisector AE

$$\angle ACB = \angle ADB$$

$$\angle ABD = 180° - \angle DAB - \angle ADB$$

$$\begin{aligned}\angle ABC &= 180° - \angle CAB - \angle ACB \\ &= 180° - \angle DAB - \angle ADB \\ &= \angle ABD\end{aligned}$$

$\angle ABC$ and $\angle ABD$ are supplementary angles, so

$$\begin{aligned}\angle ABC + \angle ABD &= 180° \\ 2\angle ABC &= 180° \\ \angle ABC &= 90°\end{aligned}$$

29. The diagonal divides the quadrilateral into two triangles. The sum of the interior angles in each triangle is 180° and so the sum of the interior angles in the quadrilateral must be 360°.

30. **(a)**

$$\begin{aligned}S &= 180(n-2) \\ \frac{S}{180} &= n-2 \\ n &= \frac{S}{180}+2\end{aligned}$$

(b) If $S = 3600$ then $n = \dfrac{3600}{180} + 2 = 22$.

31. Area = $a(b+c) = ab + ac$.
This illustrates the distributive property.

32. Area = $(a+b)^2 = a^2 + 2ab + b^2$.
This illustrates the expansion of $(a+b)^2$.

33. The diagonals of a rhombus bisect the angles formed by pairs of adjacent sides of the rhombus. This also implies that if one diagonal is horizontal, then the sides on either side of this diagonal form the same angle above and below the diagonal. This corresponds perfectly to the handle of the jack being horizontal and the sides of the jack being positioned as they appear in figure 2.70.

34. The side of the rhombus has length $l = \sqrt{16^2 + 12^2} = 20$ mm and so the length of the wire is 4(20)=80 mm.

35. **(a)** For the perimeter of the courtyard

$$s=\frac{p}{4}=\frac{320}{4}=80.0 \text{ m}$$

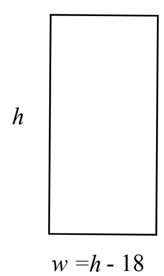
For the outer edge of the walkway, each side will be

$$x=80.0+6.00=86.0 \text{ m}$$
$$p=4x$$
$$p=4(86.0)$$
$$p=344 \text{ m}$$

(b) We find the area of the walkway by subtracting the area of the courtyard from the area bounded by the outer edge of the walkway.

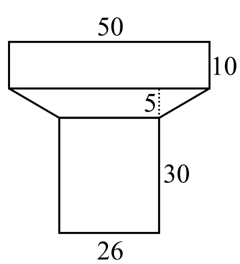
Area $=86^2-80^2=996 \text{ m}^2$.

36.



$$p=2h+2(h-18)$$
$$180=2h+2h-36$$
$$216=4h$$
$$h=54 \text{ in}$$
$$w=54-18$$
$$w=36 \text{ in}$$

37.



The total area is

$$26(30)+\tfrac{1}{2}(5)(26+50)+50(10)=780+190+500=1470 \text{ ft}^2$$

38. $h=\sqrt{3.93^2-1.80^2}$

$$=3.50 \text{ ft}$$
$$A=bh$$
$$A=1.80(3.50)$$
$$A=6.30 \text{ ft}^2$$

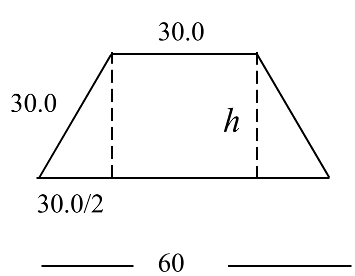
39. The trapezoid has lower base 28 ft and upper base 16 ft, making the lower side 12 ft longer than the upper side. This means that a right triangle in each corner can be built with hypotenuse c of 10 ft and horizontal leg (base b) of 6 ft. This means that the height of the trapezoid is

$$c^2 = b^2 + h^2$$
$$h = \sqrt{c^2 - b^2}$$
$$h = \sqrt{10^2 - 6^2}$$
$$h = 8$$
$$A_{\text{paint}} = 2(\text{area of trapezoid} - \text{area of window})$$
$$A_{\text{paint}} = 2\left(\tfrac{1}{2}h(b_1 + b_2) - lw\right)$$
$$A_{\text{paint}} = 2\left(\tfrac{1}{2}\cdot 8\cdot(28+16) - 3.5(12)\right)$$
$$A_{\text{paint}} = 268 \text{ ft}^2$$
$$V_{\text{paint}} = 268 \text{ ft}^2 \times \frac{1 \text{ gal}}{320 \text{ ft}^2}$$
$$V_{\text{paint}} = 0.84 \text{ gal of paint (to two significant digits)}$$

40. The number of pixels is $1080 \times 1920 = 2073600$.
The area of the screen is $15.8 \times 28.0 = 442.4 \text{ in}^2$.
The number of pixels per square inch is
$$\frac{2073600 \text{ pixels}}{442.4 \text{ in}^2} = 4687 \frac{\text{pixels}}{\text{in}^2} \text{ which rounds to } 4690 \frac{\text{pixels}}{\text{in}^2}$$

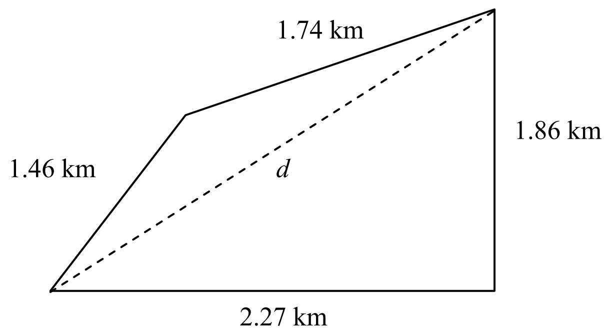
41. Let h be the height. Then the width is $w = 1.60h$.
The diagonal is 43.3 in and so
$$43.3^2 = h^2 + (1.60h)^2$$
$$1874.89 = 3.56h^2$$
$$h^2 = 526.6545$$
$h = 22.94895$ which rounds to 22.9 in
$w = 36.71833$ which rounds to 36.7 in

42.



$$30^2 = 15^2 + h^2$$
$$h = \sqrt{30^2 - 15^2}$$
$$h = 25.98076 \text{ in}$$
$$A = \text{area of 6 identical trapezoids}$$
$$A = 6\left[\tfrac{1}{2}h(b_1 + b_2)\right]$$
$$A = 3(25.98076)(30.0 + 60.0)$$
$$A = 7010 \text{ in}^2$$

43.



$$d = \sqrt{2.27^2 + 1.86^2}$$
$$d = 2.934706 \text{ km}$$

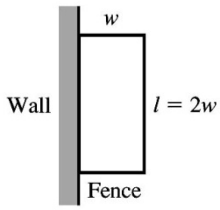
For the right triangle,

$$A = \tfrac{1}{2}bh$$
$$A = \tfrac{1}{2}(2.27)(1.86)$$
$$A = 2.1111 \text{ km}^2$$

For obtuse triangle,

$$s = \frac{1.46 + 1.74 + d}{2}$$
$$s = \frac{1.46 + 1.74 + 2.934706}{2}$$
$$s = 3.06735 \text{ km}$$
$$A = \sqrt{s(s - 1.46)(s - d)(s - 1.74)}$$
$$A = \sqrt{3.06735(3.06735 - 1.46)(3.06735 - 2.934706)(3.06735 - 1.74)}$$
$$A = 0.931707 \text{ km}^2$$
$$A_{\text{quadrilateral}} = \text{Sum of areas of two triangles}$$
$$A = 2.1111 \text{ km}^2 + 0.931707 \text{ km}^2$$
$$A = 3.04 \text{ km}^2$$

44.

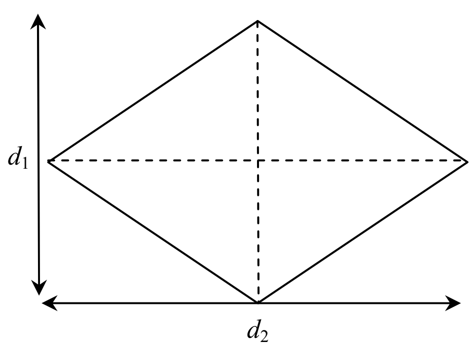


Total cost = cost of wall + cost of fence

$$13200 = 50(2w) + 5(2w) + 5w + 5w$$
$$13200 = 120w$$
$$w = 110 \text{ m}$$
$$l = 2w$$
$$l = 220 \text{ m}$$

45. 360°. A diagonal divides a quadrilateral into two triangles, and the sum of the interior angles of each triangle is 180°.

46.



The rhombus consists of four triangles, the areas of which are equal since the sides are all consistently $\frac{1}{2}d_1$ and $\frac{1}{2}d_2$

$$A = 4\left(\tfrac{1}{2}bh\right)$$
$$A = 4\left(\tfrac{1}{2}\left(\tfrac{1}{2}d_2\right)\left(\tfrac{1}{2}d_1\right)\right)$$
$$A = \frac{1}{2}d_1d_2$$

2.4 Circles

1. $\angle OAB + OBA + \angle AOB = 180^\circ$

$$\angle OAB + 90^\circ + 72^\circ = 180^\circ$$
$$\angle OAB = 18^\circ$$

2. $A = \pi r^2 = \pi(2.4)^2$

$A = 18 \text{ km}^2$

3. $p = 2s + \frac{2\pi s}{4} = 2s + \frac{\pi s}{2}$

$p = 2(3.25) + \frac{\pi(3.25)}{2}$

$p = 11.6$ in.

$A = \frac{\pi s^2}{4} = \frac{\pi(3.25)}{4}$

$A = 8.30\text{ in}^2$

4. $\overset{\frown}{AC} = 2 \cdot \angle ABC$

$= 2(25^\circ)$

$= 50^\circ$

5. (a) AD is a secant line.

(b) AF is a tangent line.

6. (a) EC and BC are chords.

(b) $\angle ECO$ is an inscribed angle.

7. (a) $AF \perp OE$.

(b) $\triangle OCE$ is isosceles.

8. (a) EC and $\overset{\frown}{EC}$ enclose a segment.

(b) Radii OE and OB together with arc $\overset{\frown}{EB}$ enclose a sector with an acute central angle.

9. $c = 2\pi r = 2\pi(275) = 1730$ ft

10. $c = 2\pi r = 2\pi(0.563) = 3.54$ m

11. $c = \pi d = \pi(23.1) = 72.6$ mm

12. $c = \pi d = \pi(8.2) = 26$ in

13. $A = \pi r^2 = \pi(0.0952)^2 = 0.0285\text{ yd}^2$

14. $A = \pi r^2 = \pi(45.8)^2 = 6590\text{ cm}^2$

15. $A = \pi(d/2)^2 = \pi(2.33/2)^2 = 4.26\text{ m}^2$

16. $A = \frac{1}{4}\pi d^2 = \frac{1}{4}\pi(1256)^2 = 1\ 239\ 000\text{ ft}^2$

17. $r = \frac{c}{2\pi} = \frac{40.1}{2\pi} = 6.38211 \text{ cm}$

$A = \pi r^2 = \pi(6.38211)^2 = 127.96 \text{ cm}^2$

which rounds to 128 cm^2.

18. $r = \frac{c}{2\pi} = \frac{147}{2\pi} = 23.39578 \text{ m}$

$A = \pi r^2 = \pi(23.39578)^2 = 1719.6 \text{ m}^2$

which rounds to 1720 m^2.

19. $\angle CBT = 90^\circ - \angle ABC = 90^\circ - 65^\circ = 25^\circ$

20. $\angle BCT = 90^\circ$, any angle such as $\angle BCA$ inscribed in a semicircle is a right angle and $\angle BCT$ is supplementary to $\angle BCA$.

21. A tangent to a circle is perpendicular to the radius drawn to the point of contact. Therefore,

$\angle ABT = 90^\circ$

$\angle CBT = \angle ABT - \angle ABC = 90^\circ - 65^\circ = 25^\circ$;

$\angle CAB = 25^\circ$

22. $\angle BTC = 65^\circ$; $\angle CBT = 25^\circ$ since it is complementary to $\angle ABC = 65^\circ$.

$(\angle CBT = 25^\circ) + \angle BTC = 90^\circ$

Therefore $\angle BTC = 65^\circ$

23. $\overset{\frown}{BC} = 2(60^\circ) = 120^\circ$

24.
$$\overset{\frown}{BC} = 2(60^\circ) = 120^\circ$$
$$\overset{\frown}{AB} + 80^\circ + 120^\circ = 360^\circ$$
$$\overset{\frown}{AB} = 160^\circ$$

25. $\angle ABC = (1/2)(80^\circ) = 40^\circ$ since the measure of an inscribed angle is one-half its intercepted arc.

26. $\angle ACB = \frac{1}{2}(160^\circ) = 80^\circ$

27. $22.5^\circ = 22.5^\circ\left(\frac{\pi \text{ rad}}{180^\circ}\right) = 0.393 \text{ rad}$

28. $60.0^\circ = 60.0^\circ \cdot \frac{\pi \text{ rad}}{180^\circ} = 1.05 \text{ rad}$

29. $125.2^\circ = 125.2\left(\frac{\pi \text{ rad}}{180^\circ}\right) = 2.185 \text{ rad}$

30. $323.0^\circ = 323.0^\circ \cdot \frac{\pi \text{ rad}}{180^\circ} = 5.64 \text{ rad}$

31. $\text{Perimeter} = \frac{1}{4}(2\pi r) + 2r = \frac{\pi r}{2} + 2r$

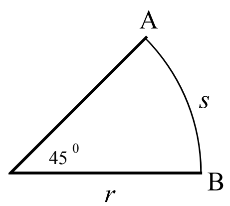
32. $\text{Perimeter} = a + b + \frac{1}{4} \cdot 2\pi r + r$

33. $\text{Perimeter} = \frac{\pi r}{2} + \sqrt{r^2 + r^2} = \frac{\pi r}{2} + r\sqrt{2}$

34. $\text{Area} = \frac{1}{2}(ar) + \frac{1}{4}\pi r^2$

35. All are on the same diameter.

36.

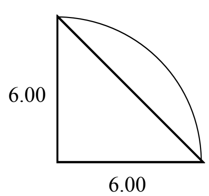


$$\widehat{AB} = 45^\circ$$

$$\frac{s}{2\pi r} = \frac{45^\circ}{360^\circ}$$

$$s = \frac{\pi}{4} \cdot r$$

37.



A of sector = A of quarter circle − A of triangle

$$A = \frac{1}{4} \cdot \pi (6.00)^2 - \frac{1}{2}(6.00)(6.00)$$

$$A = 10.3 \text{ in}^2$$

38. $\angle ACB = \angle DCE$ (vertical angles)
$\angle BAC = \angle DEC$ and
$\angle ABC = \angle CDE$ (alternate interior angles)
The triangles are similar since
corresponding angles are equal.

39. $c = 2\pi r$

$\pi = \dfrac{c}{2r}$

$d = 2r$

$\pi = \dfrac{c}{d}$

The value π is the ratio of the circumference of a circle to its diameter.

40. The Indiana bill declared

$\dfrac{c}{d} = \dfrac{4}{5/4} = \dfrac{16}{5} = 3.2$

This is only a crude approximation to the true value of $\pi = 3.1415926...$

41. A of sector = A of quarter circle − A of triangle

$A = \dfrac{1}{4} \cdot \pi r^2 - \dfrac{1}{2} r^2$

$A = \quad \dfrac{\pi}{4} - \dfrac{1}{2} \quad r^2$

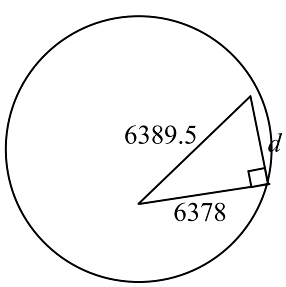
42. height of sector = radius of circle − height of triangle

Let k be the height of the triangle. Then $k^2 + k^2 = r^2$ and so $k\sqrt{2} = r$ or $k = \dfrac{\sqrt{2}}{2} r.$

$h = r - \dfrac{\sqrt{2}}{2} r.$

$h = \quad 1 - \dfrac{\sqrt{2}}{2} \quad r$

43.



The plane is 6378+11.5=6389.5 km above the center of the Earth.
This is the hypotenuse of a right triangle with one of the legs measuring 6378 km. Thus, the other leg (of length d) measures

$d = \sqrt{6389.5^2 - 6378^2} = 383.18$ km

and so one can see out 383.2 km (neglecting the curvature of the earth.)

44. The top of the tower is 6378000+346=6378346 m above the center of the Earth. This is the hypotenuse of a right triangle with one of the legs measuring 6378000 m. Thus, the other leg (of length d) measures

$d = \sqrt{6378346^2 - 6378000^2} = 66,435.65$ m

and so one can see out 66,440 m (neglecting the curvature of the earth.)

45. We calculate $\sqrt{68^2 + 58^2} = 89.3756$ km which is farther than the 85 km range of the radio station. A clear signal cannot be received at this distance.

46. Area(ledge)=Area(outer boundary)-Area(pool)

$$\begin{aligned} A &= \pi(12.6)^2 - \pi(12.0)^2 \\ &= 46.37 \text{ m}^2 \end{aligned}$$

47. $c = 2\pi r$

$= 2\pi(3960 \text{ mi})$

$=24881$ mi

The circumference is 24900 miles (to three significant digits.)

48. $11(2\pi r) = 109$

$r = 1.58$ mm

49. $$\frac{A_{\text{basketball}}}{A_{\text{hoop}}} = \frac{\pi\left(\frac{12.0}{2}\right)^2}{\pi\left(\frac{18.0}{2}\right)^2} = \frac{0.444}{1}$$

50. $$\text{flow rate} = \frac{\text{volume}}{\text{time}} = \frac{\pi r_1^2 L}{t}$$

$$2 \cdot \text{flow rate} = \frac{\pi r_2^2}{t} = \frac{2\pi r_1^2}{t}$$

$$r_2^2 = 2 \cdot r_1^2$$

$$r_2 = \sqrt{2} r_1$$

51. $c = 112$

$c = \pi d$

$d = c / \pi$

$= 112 / \pi$

$= 35.7$ in

52. $\text{Stress} = \dfrac{\text{Force}}{\text{Area}}$

$$= \frac{5250}{\pi(4.00^2) - \pi(2.25^2)}$$

$$= \frac{5250 \text{ lb}}{34.3611 \text{ in}^2}$$

$$= 153 \ \frac{\text{lb}}{\text{in}^2}$$

53. Let D = diameter of large conduit, then

$D = 3d$, where d = diameter of smaller conduit

$$F = \frac{\text{area large conduit}}{\text{area 7 small conduits}}$$

$$= \frac{7\pi \dfrac{d^2}{4}}{\pi \dfrac{D^2}{4}}$$

$$= \frac{7d^2}{D^2} = \frac{7d^2}{(3d)^2} = \frac{7d^2}{9d^2}$$

$$F = \frac{7}{9}$$

The smaller conduits occupy $\dfrac{7}{9}$ of the larger conduits.

54. A of room $= A$ of rectangle $+ \dfrac{3}{4} A$ of circle

$$A = 24(35) + \frac{3}{4}\pi(9.0)^2$$

$$A = 1030.85 \text{ ft}^2$$

This rounds to $1.00 \times 10^3 \text{ ft}^2$

55. $\text{Length} = (2)\dfrac{3}{4}(2\pi)(5.5) + (4)(5.5) = 73.8 \text{ in}$

56. The total cross-sectional area of the two smaller pipes is $2 \times \pi(25.0/2)^2 = 312.5\pi \text{ mm}^2$.
The large pipe's cross-sectional area must then be

$\pi r^2 = 312.5\pi$

and so its radius is $r = \sqrt{312.5} = 17.6777$, implying its diameter is twice this, or $d = 35.3554$ mm. This rounds to 35.4 mm.

57. Horizontally and opposite to original direction

58. Let A be the left end point at which the dashed lines intersect and C be the center of the gear. Draw a line from C bisecting the 20° angle. Call the intersection of this line and the extension of the upper dashed line B, then

$$\frac{360^\circ}{24\text{ teeth}} = \frac{15^\circ}{\text{tooth}} \Rightarrow \angle ACB = 7.5^\circ$$

$$\angle ABC = 180^\circ - \frac{20^\circ}{2} = 170^\circ$$

$$\angle\frac{1}{2}x + \angle ABC + \angle ACB = 180^\circ$$

$$\angle\frac{1}{2}x + 170^\circ + 7.5^\circ = 180^\circ$$

$$\angle\frac{1}{2}x = 2.5^\circ$$

$$x = 5^\circ$$

2.5 Measurement of Irregular Areas

1. The use of smaller intervals improves the approximation since the total omitted area or the total extra area is smaller. Also, since the number of intervals would be 10 (an even number) Simpson's Rule could be employed to achieve a more accurate estimate.

2. Using data from the south end as stated gives only five intervals. Therefore, the trapezoidal rule must be used since Simpson's rule cannot be used for an odd number of intervals.

3. Simpson's rule should be more accurate in that it accounts better for the arcs between points on the curve, and since the number of intervals (6) is even, Simpson's Rule can be used.

4. The calculated area would be too high since each trapezoid would include more area than under the curve. The shape of the curve is such that a straight line approximation for the curve will always overestimate the area below the curve (the curve dips below the straight line approximation).

5. The area is exact for areas whose boundaries are line segments.

6. There is no impediment to using Simpson’s rule here, since the number of intervals (4) is even.

7. $A_{trap} = \frac{h}{2}[y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n]$

$A_{trap} = \frac{2.0}{2}\left[0.0 + 2(6.4) + 2(7.4) + 2(7.0) + 2(6.1) + 2(5.2) + 2(5.0) + 2(5.1) + 0.0\right]$

$A_{trap} = 84.4 = 84 \text{ m}^2$ to two significant digits

8. $A_{\text{simp}} = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)$

$A_{\text{simp}} = \frac{2}{3}\left[0 + 4(6.4) + 2(7.4) + 4(7.0) + 2(6.1) + 4(5.2) + 2(5.0) + 4(5.1) + 0\right]$

$A_{\text{simp}} = 87.8667 \text{ m}^2 = 88 \text{ m}^2$ (to two significant digits)

9. $A_{\text{simp}} = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)$

$A_{\text{simp}} = \frac{1.00}{3}\left[0 + 4(0.52) + 2(0.75) + 4(1.05) + 2(1.15) + 4(1.00) + 0.62\right]$

$A_{\text{simp}} = 4.9 \text{ ft}^2$

10. $A_{\text{trap}} = \frac{h}{2}[y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n]$

$A_{\text{trap}} = \frac{1.00}{2}\left[0 + 2(0.52) + 2(0.75) + 2(1.05) + 2(1.15) + 2(1.00) + 0.62\right]$

$A_{\text{trap}} = 4.78 \text{ ft}^2 = 4.8 \text{ ft}^2$ (rounded to 2 significant digits)

11. $A_{\text{trap}} = \frac{h}{2}[y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n]$

$A_{\text{trap}} = \frac{0.5}{2}\left[0.6 + 2(2.2) + 2(4.7) + 2(3.1) + 2(3.6) + 2(1.6) + 2(2.2) + 2(1.5) + 0.8\right]$

$A_{\text{trap}} = 9.8 \text{ mi}^2$

12. $A_{\text{simp}} = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)$

$A_{\text{simp}} = \frac{0.5}{3}\left[0.6 + 4(2.2) + 2(4.7) + 4(3.1) + 2(3.6) + 4(1.6) + 2(2.2) + 4(1.5) + 0.8\right]$

$A_{\text{simp}} = 9.3333 \text{ mi}^2 = 9.3 \text{ mi}^2$ (rounded to 2 significant digits)

13. We can use Simpson's rule (which is usually more accurate) because we have an even number of intervals.

$A_{\text{Simp}} = \frac{h}{3}[y_0 + 4y_1 + 2y_2 + ... + 4y_{n-1} + y_n]$

$A_{\text{Simp}} = \frac{10.0}{2}\left[38 + 4(24) + 2(25) + 4(17) + 2(34) + 4(29) + 2(36) + 4(34) + 30\right]$

$A_{\text{Simp}} = \left(3370 \text{ mm}^2\right)\left(\frac{23 \text{ km}}{10 \text{ mm}}\right)^2$

$A_{\text{Simp}} = 17{,}827 \text{ km}^2$

We round this to $18{,}000 \text{ km}^2$ (two significant digits.)

14. $A_{\text{trap}} = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right]$

$$A_{\text{trap}} = \frac{4.0}{2}\left[110 + 2(109.6) + 2(108) + 2(107.2) + 2(102.4) + 2(98.0) + 2(91.6) + 2(22.5)\right.$$
$$\left. + 2(84.0) + 2(74.4) + 2(62.2) + 2(43.4) + 0.00\right]$$

$A_{\text{trap}} = 53{,}106\ \text{m}^2$

which is rounded to $53{,}100\ \text{m}^2$.

15. $A_{\text{trap}} = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right]$

$$A_{\text{trap}} = \frac{45}{2}\left[230 + 2(290) + 2(330) + 2(350) + 2(390) + 2(410) + 2(420) + 2(360) + 170\right]$$

$A_{\text{trap}} = 123{,}750\ \text{ft}^2$

which is rounded to $120{,}000\ \text{ft}^2$.

16. $A_{\text{simp}} = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)$

$$A_{\text{simp}} = \frac{45}{3}\left[230 + 4(290) + 2(330) + 4(350) + 2(390) + 4(410) + 2(420) + 4(360) + 170\right]$$

$A_{\text{simp}} = 124{,}800\ \text{ft}^2$

which is rounded to $120{,}000\ \text{ft}^2$

17. $A_{\text{simp}} = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)$

$$A_{\text{simp}} = \frac{50}{3}\left[5 + 4(12) + 2(17) + 4(21) + 2(22) + 4(25) + 2(26) + 4(16) + 2(10) + 4(8) + 0\right]$$

$A_{\text{simp}} = 8050\ \text{ft}^2 = 8.0 \times 10^3\ \text{ft}^2$

18. $A_{\text{trap}} = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right]$

$$A_{\text{trap}} = \frac{2.0}{2}\ 3.5 + 2(6.0) + 2(7.6) + 2(10.8) + 2(16.2) + 2(18.2) + 2(19.0) + 2(17.8) + 2(12.5) + 8.2$$

$A_{\text{trap}} = 228.7\ \text{in}^2$

$A_{\text{circles}} = 2\ \ \frac{\pi d^2}{4}$

$A_{\text{circles}} = \frac{\pi(2.50\ \text{in})^2}{2} = 9.817477\ \text{in}^2$

$A_{\text{total}} = 228.7\ \text{in}^2 - 9.817477\ \text{in}^2$

$A_{\text{total}} = 218.88\ \text{in}^2 = 220\ \text{in}^2$

19. $A_{\text{trap}} = \frac{h}{2}[y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n]$

$$A_{\text{trap}} = \frac{0.500}{2}\left[0.0 + 2(1.732) + 2(2.000) + 2(1.732) + 0.0\right]$$

$$A_{\text{trap}} = 2.73 \text{ cm}^2$$

This value is less than 3.14 cm^2 because all of the trapezoids are inscribed.

20. $A_{\text{trap}} = \frac{h}{2}[y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n]$

$$A_{\text{trap}} = \frac{0.250}{2}\left[\begin{array}{l} 0.000 + 2(1.323) + 2(1.732) + 2(1.936) + 2(2.000) \\ +2(1.936) + 2(1.732) + 2(1.323) + 0.000 \end{array}\right]$$

$$A_{\text{trap}} = 3.00 \text{ cm}^2$$

The trapezoids are smaller so they can get closer to the boundary, and less area is missed from the calculation.

21. $A_{\text{simp}} = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)$

$$A_{\text{simp}} = \frac{0.500}{3}\left[0.000 + 4(1.732) + 2(2.000) + 4(1.732) + 0.000\right]$$

$$A_{\text{simp}} = 2.98 \text{ cm}^2$$

The ends of the areas are curved so they can get closer to the boundary, including more area in the calculation.

22. $A_{\text{simp}} = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)$

$$A_{\text{simp}} = \frac{0.250}{3}\left[\begin{array}{l} 0.000 + 4(1.323) + 2(1.732) + 4(1.936) + 2(2.000) \\ +4(1.936) + 2(1.732) + 4(1.323) + 0.000 \end{array}\right]$$

$$A_{\text{simp}} = 3.08 \text{ cm}^2$$

The intervals are smaller so they can get closer to the boundary.

2.6 Solid Geometric Figures

1. $V_1 = lwh$

$$V_2 = (2l)(w)(3h)$$

$$V_2 = 6lwh$$

$$V_2 = 6V_1$$

The volume increases by a factor of 6.

2. $s^2 = r^2 + h^2$

$h = \sqrt{s^2 - r^2}$

$h = \sqrt{17.5^2 - 11.9^2}$

$h = 12.8 \text{ cm}$

3. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{11.9 \text{ cm}}{2}\right)^2 \left(2(10.4 \text{ cm})\right)$

$V = 771 \text{ cm}^3$

4. $V = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \pi(40.0)^2\left(\frac{122}{2}\right) + \frac{2}{3}\pi(40.0)^3$

$V = 440{,}661 \text{ ft}^3$

$V = 441{,}000 \text{ ft}^3$ (rounded to three significant digits.)

5. $V = s^3$

$V = (6.95 \text{ ft})^3$

$V = 336 \text{ ft}^3$

6. $V = \pi r^2 h$

$V = \pi(23.5 \text{ cm})^2 (48.4 \text{ cm})$

$V = 83971.3 \text{ cm}^3$

$V = 8.40 \times 10^4 \text{ cm}^3$

7. $A = 2\pi r^2 + 2\pi rh$

$A = 2\pi(689)^2 + 2\pi(689)(233)$

$A = 3\ 991\ 444 \text{ m}^2$

$A = 3.99 \times 10^6 \text{ m}^2$

8. $A = 4\pi r^2$

$A = 4\pi(0.067 \text{ in})^2$

$A = 0.056 \text{ in}^2$

9. $V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(1.037 \text{ yd})^3$

$V = 4.671 \text{ yd}^3$

10. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(25.1\text{ m})^2(5.66\text{ m})$

$V = 3730\text{ m}^3$

11. $S = \pi rs$

$S = \pi(78.0\text{ cm})(83.8\text{ cm})$

$S = 20\,534.71\text{ cm}^2$

$S = 20\,500\text{ cm}^2$

12. $S = \frac{1}{2}ps$

$S = \frac{1}{2}(345\text{ ft})(272\text{ ft})$

$S = 46\,900\text{ ft}^2$

13. $V = \frac{1}{3}Bh$

$V = \frac{1}{3}(0.76\text{ in})^2(1.30\text{ in})$

$V = .250\,293\text{ in}^3$

$V = 2.50\times 10^{-1}\text{ in}^3$

14. $V = Bh$

$V = (29.0\text{ cm})^2(11.2\text{ cm})$

$V = 9419.2\text{ cm}^3$

$V = 9420\text{ cm}^3$

15. $V = \frac{1}{3}\pi h(R^2 + Rr + r^2)$

$V = \frac{1}{3}\pi(45.1)(37.3^2 + (37.3)(28.2) + 28.2^2)$

$V = 152{,}944\text{ mm}^3$

$V = 153{,}000\text{ mm}^3$

16. $S = \pi(R + r)s$

$S = \pi(3.42 + 2.69)3.25$

$S = 62.3842\text{ m}^2$

$S = 62.4\text{ m}^2$

17. $S = ph$

$S = (3\times 1.092\text{ m})(1.025\text{ m})$

$S = 3.358\text{ m}^2$

18. $S = 2\pi rh$

$S = 2\pi \; \frac{d}{2} \; h$

$S = \pi(250 \text{ ft})(347 \text{ ft})$

$S = 272\ 533 \text{ ft}^2$

$S = 270\ 000 \text{ ft}^2$

$S = 2.7 \times 10^5 \text{ ft}^2$

19. $V = \frac{1}{2} \; \frac{4}{3}\pi r^3$

$V = \frac{2\pi}{3} \; \frac{d}{2}^{\;3}$

$V = \frac{2\pi}{3} \; \frac{0.65 \text{ yd}}{2}^{\;3}$

$V = 0.0718957 \text{ yd}^3$

$V = 0.0719 \text{ yd}^3 = 7.19 \times 10^{-2} \text{ yd}^3$

20. To analyze the right triangle formed by the center of the pyramid base, the top of the pyramid, and any lateral facelength s, notice that the bottom of that triangle has width of half the square base side length.

$b = \frac{22.4}{2} = 11.2$

$s^2 = h^2 + b^2$

$h = \sqrt{s^2 - b^2}$

$h = \sqrt{14.2^2 - 11.2^2}$

$h = 8.72926 \text{ m}$

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}(22.4 \text{ m})^2 (8.72926 \text{ m})$

$V = 1459.998 \text{ m}^3$

$V = 1460 \text{ m}^3$

21. $s^2 = h^2 + r^2$

$s = \sqrt{h^2 + r^2}$

$s = \sqrt{0.274^2 + 3.39^2}$

$s = 3.401055 \text{ cm}$

$A = \pi r^2 + \pi rs$

$A = \pi(3.39 \text{ cm})^2 + \pi(3.39 \text{ cm})(3.401055 \text{ cm})$

$A = 72.3 \text{ cm}^2$

22. There are four triangles in this shape, all having the same area.
Using Hero's formula for each triangle:

$s=\frac{1}{2}(a+b+c)$

$s=\frac{1}{2}(3\times 3.67\text{ dm})$

$s=5.505\text{ dm}$

$A=\sqrt{s(s-a)(s-b)(s-c)}$

$A=\sqrt{5.505(1.835)^3}$

$A=\sqrt{5.505(1.835)^3}$

$A=5.832205\text{ dm}^2$

The total surface area A consists of four of these triangles,

$A=4\times 5.832205\text{ dm}^2$

$A=23.3\text{ dm}^2$

Or, we could determine the lateral side length h (triangle heights) from the Pythagorean Theorem

$a^2=h^2+\left(\frac{a}{2}\right)^2$

$h=\sqrt{3.67^2-\left(\frac{3.67}{2}\right)^2}$

$h=3.17831\text{ dm}$

There are four triangles of the same area, so total surface area is:

$A=4\times\frac{1}{2}bh$

$A=2(3.67\text{ dm})(3.17831\text{ dm})$

$A=23.3\text{ dm}^2$

23. $V=\frac{4}{3}\pi r^3$

$V=\frac{4}{3}\pi\left(\frac{d}{2}\right)^3$

$V=\frac{4}{3}\pi\frac{d^3}{8}$

$V=\frac{1}{6}\pi d^3$

24. $A=A_{\text{flat}}+A_{\text{curved}}$

$A=\pi r^2+\frac{1}{2}\cdot 4\pi r^2$

$A=\pi r^2+2\pi r^2$

$A=3\pi r^3$

25. Let r = radius of cone,
Let h = height of the cone

$$\frac{V_{\text{cylinder}}}{V_{\text{cone}}} = \frac{\pi(2r)^2 \frac{h}{2}}{\frac{1}{3}\pi r^2 h}$$

$$\frac{V_{\text{cylinder}}}{V_{\text{cone}}} = \frac{2\pi r^2 h}{\frac{1}{3}\pi r^2 h}$$

$$\frac{V_{\text{cylinder}}}{V_{\text{cone}}} = 6$$

26. $$A_{conebase} = \frac{1}{4}A$$

$$\pi r^2 = \frac{1}{4}\left(\pi r^2 + \pi rs\right)$$

$$4\pi r^2 = \pi r^2 + \pi rs$$

$$3\pi r^2 = \pi rs$$

$$\frac{r}{s} = \frac{1}{3}$$

27. $$\frac{A_{final}}{A_{original}} = \frac{4\pi(2r)^2}{4\pi r^2}$$

$$\frac{A_{final}}{A_{original}} = \frac{16\pi r^2}{4\pi r^2}$$

$$\frac{A_{final}}{A_{original}} = 4$$

28. w = weight density $\times$ volume

$$w = \gamma V$$

$$w = \frac{62.4\text{ lb}}{1\text{ ft}^3}(1.00\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)(1.00\text{ mi}^2)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)^2$$

$$w = 1.45\times10^8\text{ lb}$$

29. $$A = A_{base} + A_{ends} + A_{sides}$$

$$A = 2lw + 2wh + 2lh$$

$$A = 2(12.0)(9.50) + 2(9.50)(8.75) + 2(12.0)(8.75)$$

$$A = 604\text{ in}^2$$

30. The volume of pool can be represented by a trapezoidal right prism

$$V = A_{trapezoid} \times width$$

$$V = \frac{1}{2}h(b_1 + b_2)\cdot w$$

$$V = \frac{1}{2}(78.0)(8.75 + 3.50)\cdot(50.0)$$

$$V = 23{,}887.5\text{ ft}^3$$

which is rounded to 23,900 ft^3

31. $V = \pi r^2 h$

$V = \pi\left(\frac{d}{2}\right)^2 h$

$V = \frac{\pi}{4}(4.0 \text{ ft})^2 (750 \text{ mi})\frac{5280 \text{ ft}}{1 \text{ mi}}$

$V = 49{,}762{,}827 \text{ ft}^3$

$V = 5.0 \times 10^7 \text{ ft}^3$

32. $V = \frac{1}{3}h(a^2 + ab + b^2)$

$V = \frac{1}{3}0.750((2.50)^2 + (2.50)(3.25) + (3.25)^2)$

$V = 6.234375 \text{ m}^3$

$V = 6.23 \text{ m}^3$

33. $V = \frac{1}{3}\pi h(R^2 + Rr + r^2)$

$h = 62.5 \text{ m}, R = \frac{3.88}{2} = 1.94 \text{ m}, r = \frac{1.90}{2} = 0.95 \text{ m}$

$V = \frac{1}{3}\pi(62.5)(1.94^2 + (1.94)(0.95) + 0.95^2)$

$V = 426.02 \text{ m}^3$

$V = 426 \text{ m}^3$

34. There are three rectangles and two triangles in this shape.
The triangles have hypotenuse

$c^2 = a^2 + b^2$

$c = \sqrt{3^2 + 4^2}$

$c = 5.00 \text{ cm}$

$A = A_{\text{rectangles}} + A_{\text{triangles}}$

$A = (8.50)(5.00) + (8.50)(3.00) + (8.50)(4.00) + 2\ \frac{1}{2}\ (4.00)(3.00)$

$A = 114 \text{ cm}^2$

35. $V = \frac{1}{3}BH$

$V = \frac{1}{3}(250^2)(160)$

$V = 3{,}333{,}333 \text{ yd}^3$

$V = 3.3 \times 10^6 \text{ yd}^3$

36. Here, $h = 3.50$ in and $r = \dfrac{3.60}{2}$=1.80 in. Use the Pythagorean Thoerem to find the slant height.

$$s^2 = h^2 + r^2$$
$$s = \sqrt{h^2 + r^2}$$
$$s = \sqrt{3.50^2 + 1.80^2}$$
$$s = 3.9357 \text{ in}$$
$$S = \pi rs$$
$$S = \pi(1.80 \text{ in})(3.9357 \text{ in})$$
$$S = 22.3 \text{ in}^2$$

37. $V = \dfrac{4}{3}\pi r^3$

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$
$$V = \frac{4}{3}\pi\left(\frac{165}{2}\right)^3$$
$$V = 2{,}352{,}071 \text{ ft}^3$$
$$V = 2.35\times10^6 \text{ ft}^3$$

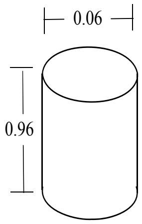
38. $V = \dfrac{4}{3}\pi r^3 + \pi r^2 h$

$$r = \frac{d}{2} = \frac{4.00}{2} = 2.00 \text{ ft}$$
$$V = \frac{4}{3}\pi(2.00)^3 + \pi(2.00)^2(6.50)$$
$$V = 115 \text{ ft}^3$$

39. The lateral side length can be determined from the Pythagorean Theorem

$$s^2 = 8.0^2 + h^2$$
$$s = \sqrt{8.0^2 + 40.0^2}$$
$$s = 40.792 \text{ mm}$$
$$A = x^2 + \frac{1}{2}ps$$
$$A = 16^2 + \frac{1}{2}(4\times16)(40.792)$$
$$A = 1560 \text{ mm}^2$$

40.



Let n = number of revolutions of the lateral surface area S

$$n \cdot S = 76$$
$$n \cdot 2\pi rh = 76$$
$$n = \frac{76}{2\pi\left(\frac{d}{2}\right)h}$$
$$n = \frac{76}{\pi dh}$$
$$n = \frac{76\ \text{m}^2}{\pi(0.60\ \text{m})(0.96\ \text{m})}$$
$$n = 42 \text{ revolutions}$$

41.

$$c = 2\pi r$$
$$29.8 = 2\pi r$$
$$r = \frac{29.8}{2\pi}$$
$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi\left(\frac{29.8}{2\pi}\right)^3$$
$$V = 447\ \text{in}^3$$

42.

$$S = 2\pi rh$$
$$S = 2\pi \ \frac{d}{2} \ h$$
$$S = \pi dh$$

Assuming the label overlaps on both ends of the can by 0.25 in

$$S = \pi(3.00)(4.25 + 0.25 + 0.25)$$
$$S = 44.8\ \text{in}^2$$

43.

$$V = V_{cylinder} + V_{cone}$$
$$V = \pi r_{cylinder}{}^2 h_{cylinder} + \frac{1}{3}\pi r_{cone}{}^2 h_{cone}$$
$$V = \pi(0.625/2)^2(2.75) + \frac{1}{3}\pi(1.25/2)^2(0.625)$$
$$V = 1.09935\ \text{in}^3$$
$$V = 1.10\ \text{cm}^3$$

44. We need the radius r of the semicircle.

The perimeter is $p = 2r + \frac{1}{2} \cdot 2\pi r = r(2+\pi)$

$$\text{and so } r = \frac{p}{2+\pi};\ p = 18 \text{ m} = 1800 \text{ cm}$$

$$A_{\text{semicircle}} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \left(\frac{p}{2+\pi}\right)^2$$

$$V = A_{\text{semicircle}} h$$

$$V = \frac{1}{2}\pi \left(\frac{p}{2+\pi}\right)^2 h$$

$$V = \frac{1}{2}\pi \left(\frac{1800}{2+\pi}\right)^2 (7.5)$$

$$V = 1{,}443{,}879 \text{ cm}^3$$

$$V = 1.4 \times 10^6 \text{ cm}^3$$

45. $V_{\text{new}} = V_{\text{old}} - 0.08\ V_{\text{old}} = 0.92\ V_{\text{old}}$

$$V_{\text{new}} = \frac{4}{3}\pi r_{\text{new}}^3;\ V_{\text{old}} = \frac{4}{3}\pi r_{\text{old}}^3$$

$$r_{\text{new}}^3 = \frac{3V_{\text{new}}}{4\pi}$$

$$= \frac{3(0.92\ V_{\text{old}})}{4\pi}$$

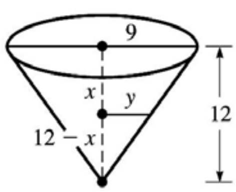
$$= 0.92 \frac{3V_{\text{old}}}{4\pi}$$

$$r_{\text{new}}^3 = 0.92\ r_{\text{old}}^3$$

$$r_{\text{new}} = \sqrt[3]{0.92}\ r_{\text{old}} = 0.9726\ r_{\text{old}} = (1 - 0.0274)\ r_{\text{old}}$$

and so the radius has decreased by 2.7% (rounded to two significant digits.)

46.



$$\frac{9}{12} = \frac{y}{12-x}$$

$$y = \frac{3}{4}(12-x)$$

To achieve half the volume of the cone

$$\frac{V_{cone}}{2} = V_{\text{fluid}}$$

$$\frac{\frac{1}{3}\pi r_{cone}{}^2 h_{cone}}{2} = \frac{1}{3}\pi r_{fluid}{}^2 h_{fluid}$$

$$\frac{\frac{1}{3}\pi\left(9^2\right)12}{2} = \frac{1}{3}\pi\ \ \frac{3}{4}(12-x)\ \ ^2\ (12-x)$$

$$\frac{9^2 \cdot 12}{2} = \ \ \frac{3}{4}(12-x)\ \ ^2 \cdot (12-x)$$

$$486 = 0.5625(12-x)^3$$

$$864 = (12-x)^3$$

$$\sqrt[3]{864} = 12 - x$$

$$x = 12 - \sqrt[3]{864}$$

$$x = 2.48 \text{ cm}$$

Review Exercises

1. This is false; the angles are supplementary, not complementary.

2. This is true. $15 = \sqrt{225} = \sqrt{9^2 + 12^2}$

3. This is false. The sides of length a could be consecutive, forming a kite shape.

4. This is true.

5. This is true. There is an even number of intervals, making Simpson’s rule applicable.

6. This is true.

7. $\angle CGH$ and given angle 148° are corresponding angles, so
$\angle CGH = 148^\circ$
$\angle CGE$ and $\angle CGH$ are supplementary angles so
$\angle CGE + \angle CGH = 180^\circ$
$\angle CGE = 180^\circ - 148^\circ$
$\angle CGE = 32^\circ$

8. $\angle CGE = 32^\circ$ from Question 1
$\angle CGE$ and $\angle EGF$ are complementary angles so
$\angle CGE + \angle EGF = 90^\circ$
$\angle EGF = 90^\circ - 32^\circ$
$\angle EGF = 58^\circ$

9. $\angle CGE = 32^\circ$ from Question 1

$\angle CGE$ and $\angle DGH$ are vertically opposite angles

$\angle DGH = \angle CGE$

$\angle DGH = 32^\circ$

10. $\angle CGE = 32^\circ$ from Question 1

$\angle EGI = \angle CGE + 90^\circ$

$\angle EGI = 32^\circ + 90^\circ$

$\angle EGI = 122^\circ$

11. $c^2 = a^2 + b^2$

$c = \sqrt{12^2 + 35^2}$

$c = \sqrt{1369}$

$c = 37$

12. $c^2 = a^2 + b^2$

$c = \sqrt{14^2 + 48^2}$

$c = \sqrt{2500}$

$c = 50$

13. $c^2 = a^2 + b^2$

$c = \sqrt{400^2 + 580^2}$

$c = \sqrt{496\ 400}$

$c = 704.55659815$

$c = 700$

14. $c^2 = a^2 + b^2$

$a^2 = c^2 - b^2$

$a = \sqrt{6500^2 - 5600^2}$

$a = \sqrt{10890000}$

$a = 3300$

15. $a^2 = c^2 - b^2$

$a = \sqrt{0.736^2 - 0.380^2}$

$c = \sqrt{0.397296}$

$c = 0.630314$

$c = 0.630$

16. $a^2 = c^2 - b^2$

$a = \sqrt{128^2 - 25.1^2}$

$c = \sqrt{15753.99}$

$c = 125.514899$

$c = 126$

17. $c^2 = a^2 + b^2$

$$a^2 = c^2 - b^2$$
$$a = \sqrt{52.9^2 - 38.3^2}$$
$$a = \sqrt{1331.52}$$
$$a = 36.4899986$$
$$a = 36.5$$

18. $c^2 = a^2 + b^2$

$$b^2 = c^2 - a^2$$
$$b = \sqrt{0.885^2 - 0.782^2}$$
$$b = \sqrt{0.171701}$$
$$b = 0.41436819$$
$$b = 0.414$$

19. $p = 3s$

$$p = 3(8.5 \text{ mm})$$
$$p = 25.5 \text{ mm}$$

20. $p = 4s$

$$p = 4(15.2 \text{ in})$$
$$p = 60.8 \text{ in}$$

21. $A = \frac{1}{2}bh$

$$A = \frac{1}{2}(0.125 \text{ ft})(0.188 \text{ ft})$$
$$A = 0.0118 \text{ ft}^2$$

22. $s = \frac{1}{2}(a + b + c)$

$$s = \frac{1}{2}(175 + 138 + 119)$$
$$s = 216 \text{ cm}$$
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$A = \sqrt{216(216-175)(216-138)(216-119)}$$
$$A = \sqrt{216(41)(78)(97)}$$
$$A = \sqrt{67\ 004\ 496}$$
$$A = 8185.627404 \text{ cm}^2$$
$$A = 8190 \text{ cm}^2$$

23. $c = 2\pi r$

$c = \pi d$

$c = \pi(74.8 \text{ mm})$

$c = 234.99 \text{ mm}$

$c = 235 \text{ mm}$

24. $p = 2l + 2w$

$p = 2(2980 \text{ yd}) + 2(1860 \text{ yd})$

$p = 9680 \text{ yd}$

25. $A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(34.2 \text{ in})(67.2 \text{ in} + 126.7 \text{ in})$

$A = 3315.69 \text{ in}^2$

$A = 3320 \text{ in}^2$

26. $A = \pi r^2$

$A = \pi \quad \frac{d}{2} \quad ^2$

$A = \frac{\pi d^2}{4}$

$A = \frac{\pi(0.328 \text{ m})^2}{4}$

$A = .08449627601 \text{ m}^2$

$A = .0845 \text{ m}^2$

27. $V = Bh$

$V = \frac{1}{2}bl \cdot h$

$V = \frac{1}{2}(26.0 \text{ cm} \times 34.0 \text{ cm})(14.0 \text{ cm})$

$V = 6188 \text{ cm}^3$

$V = 6190 \text{ cm}^3$

28. $V = \pi r^2 h$

$V = \pi(36.0 \text{ in})^2 (2.40 \text{ in})$

$V = 9771.60979 \text{ in}^3$

$V = 9770 \text{ in}^3$

29.
$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}\left(3850\text{ ft}^2\right)\left(125\text{ ft}\right)$$
$$V = 160416.6667\text{ ft}^3$$
$$V = 1.60 \times 10^5\text{ ft}^3$$

30.
$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi\left(\frac{2.21\text{ mm}}{2}\right)^3$$
$$V = 5.651652404\text{ mm}^3$$
$$V = 5.65\text{ mm}^3$$

31.
$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi\left(32.4\text{ cm}\right)^2(50.7\text{ cm})$$
$$V = 55734.8\text{ cm}^3$$
$$V = 55700\text{ cm}^3$$

32.
$$V = \frac{1}{3}\pi h(R^2 + Rr + r^2)$$
$$V = \frac{1}{3}\pi(4.890)\left(2.336^2 + (2.336)(2.016) + 2.016^2\right)$$
$$V = 72.87163\text{ ft}^3$$
$$V = 72.87\text{ ft}^3$$

33.
$$A = 6s^2$$
$$A = 6\left(0.520\text{ m}\right)^2$$
$$A = 1.6224\text{ m}^2$$
$$A = 1.62\text{ m}^2$$

34.
$$A = 2\pi r^2 + 2\pi rh$$
$$A = 2\pi\left(\frac{d}{2}\right)^2 + 2\pi\left(\frac{d}{2}\right)h$$
$$A = \frac{\pi d^2}{2} + \pi dh$$
$$A = \frac{\pi(12.0\text{ ft})^2}{2} + \pi(12.0\text{ ft})(58.0\text{ ft})$$
$$A = 2412.743158\text{ ft}^2$$
$$A = 2410\text{ ft}^2$$

35. $s^2 = r^2 + h^2$

$s = \sqrt{2.56^2 + 12.3^2}$

$s = \sqrt{157.8436}$

$s = 12.56358229 \text{ in}$

$S = \pi rs$

$S = \pi(2.56 \text{ in})(12.56358229 \text{ in})$

$S = 101.042324 \text{ in}^2$

$S = 101 \text{ in}^2$

36. $A = 4\pi r^2$

$A = 4\pi \left(\frac{12\ 760 \text{ km}}{2}\right)^2$

$A = 511\ 506\ 576 \text{ km}^2$

$A = 5.115 \times 10^8 \text{ km}^2$

37. $\angle BTA = \frac{50^\circ}{2} = 25^\circ$

38. $\angle TBA = 90^\circ$ since an angle inscribed in a semicircle is 90°

$\angle BTA = 25^\circ$ from Question 29

All angles in ΔBTA must sum to 180°

$\angle TAB + \angle BTA + \angle TBA = 180^\circ$

$\angle TAB = 180^\circ - 90^\circ - 25^\circ$

$\angle TAB = 65^\circ$

39. $\angle BTC$ is a complementary angle to $\angle BTA$

$\angle BTA = 25^\circ$ from Question 29

$\angle BTC + \angle BTA = 90^\circ$

$\angle BTC = 90^\circ - 25^\circ$

$\angle BTC = 65^\circ$

40. $\angle ABT = 90^\circ$ since any angle inscribed in a semi-circle is 90°

41. $\angle ABE$ and $\angle ADC$ are corresponding angles since $\Delta ABE \sim \Delta ADC$

$\angle ABE = \angle ADC$

$\angle ABE = 53^\circ$

42. $AD^2 = AC^2 + CD^2$

$AD = \sqrt{(4+4)^2 + 6^2}$

$AD = \sqrt{100}$

$AD = 10$

43. since $\Delta ABE \sim \Delta ADC$

$$\frac{BE}{CD}=\frac{AB}{AD}$$
$$\frac{BE}{6}=\frac{4}{10}$$
$$BE=\frac{6(4)}{10}$$
$$BE=2.4$$

44. since $\Delta ABE \sim \Delta ADC$

$$\frac{AE}{AC}=\frac{AB}{AD}$$
$$\frac{AE}{8}=\frac{4}{10}$$
$$AE=\frac{4(8)}{10}$$
$$AE=3.2$$

45. p = base of triangle + hypotenuse of triangle + semicircle perimeter

$$p=b+\sqrt{b^2+(2a)^2}+\frac{1}{2}\pi(2a)$$
$$p=b+\sqrt{b^2+4a^2}+\pi a$$

46. p = perimeter of semicircle + 4 square lengths

$$p=\frac{1}{2}(2\pi s)+4s$$
$$p=\pi s+4s$$

47. A = area of triangle + area of semicircle

$$A=\frac{1}{2}b(2a)+\frac{1}{2}\cdot\pi(a)^2$$
$$A=ab+\frac{1}{2}\pi a^2$$

48. A = area of semicircle + area square

$$A=\frac{1}{2}(\pi s^2)+s^2$$

49. A square is a rectangle with four equal sides.
A rectangle is a parallelogram with perpendicular intersecting sides so a square is a parallelogram.
A rhombus is a parallelogram with four equal sides and since a square is a parallelogram, a square is a rhombus.

50. If two triangles share two angles that are the same, then the third angle must also be the same in both triangles.
The triangles are similar to each other because they all have the same angles, and the sides must be proportional.

51. $A = \pi r^2$

If the radius of the circle is multplied by $n,$ then the area of the new circle is:

$A = \pi(nr)^2$

$A = \pi(n^2 r^2)$

$A = n^2(\pi r^2)$

The area of the circle is multiplied by $n^2,$ when the radius is multplied by $n.$

Any plane geometric figure scaled by n in each dimension will increase its area by $n^2.$

52. $V = s^3$

If the length of a cube's side is multplied by $n,$ then the volume of the new cube is:

$V = (ns)^3$

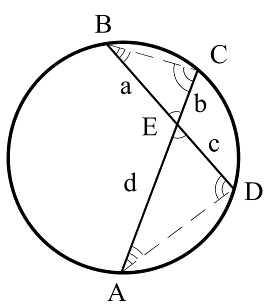
$V = (n^3 s^3)$

$V = n^3(s^3)$

The volume of the cube is multiplied by $n^3,$ when the length of the side is multplied by $n.$

This will be true of any geometric figure scaled by n in all dimensions.

53.



$\angle BEC = \angle AED,$ since they are vertically opposite angles

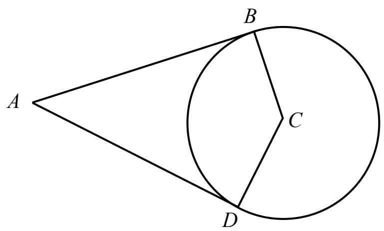
$\angle BCA = \angle ADB,$ both are inscribed in $\overparen{AB}$

$\angle CBD = \angle CAD,$ both are inscribed in $\overparen{CD}$

which shows $\Delta AED \sim \Delta BEC$

$\dfrac{a}{d} = \dfrac{b}{c}$

54.



We are given $\angle BAD = 36^{\circ}$.

The two angles $\angle ABC$ and $\angle ADC$ of the quadrilateral at the point where the tangents touch the circle are each 90°.

The four angles of the quadrilateral will add up to 360°.

$$\angle ABC + \angle ADC + \angle BAD + \angle BCD = 360^{\circ}$$
$$90^{\circ} + 90^{\circ} + 36^{\circ} + \angle BCD = 360^{\circ}$$
$$\angle BCD = 144^{\circ}$$

55. The three angles of the triangle will add up to 180°.

If the tip of the isosceles triangle is 32°, find the other two equal angles.

$2(\text{base angle}) + 32^{\circ} = 180^{\circ}$

$2(\text{base angle}) = 148^{\circ}$

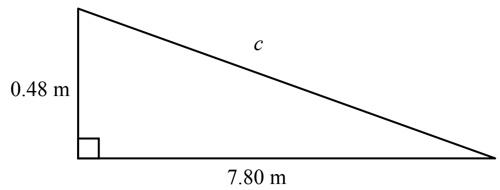
$\text{base angle} = 74^{\circ}$

56. The two volumes are equal

$$V_{sphere} = V_{sheet}$$
$$\frac{4}{3}\pi r_{sphere}{}^{3} = \pi r_{sheet}{}^{2} t$$
$$\frac{4}{3}\;\frac{d_{sphere}}{2}^{3} = \frac{d_{sheet}}{2}^{2}\; t$$
$$\frac{d_{sphere}{}^{3}}{6} = \frac{d_{sheet}{}^{2}}{4} t$$
$$t = \frac{2}{3}\cdot\frac{d_{sphere}{}^{3}}{d_{sheet}{}^{2}}$$
$$t = \frac{2}{3}\cdot\frac{(1.50\text{ in})^{3}}{(14.0\text{ in})^{2}}$$
$$t = 0.011479591\text{ in}$$
$$t = 0.0115\text{ in}$$

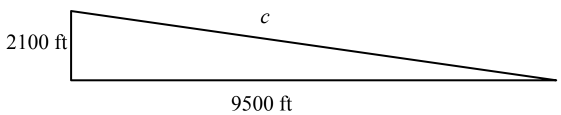
The sphere is flattened to make a 0.0115 inch thick circular sheet.

57.



$$c^2 = a^2 + b^2$$
$$c = \sqrt{(0.48\text{ m})^2 + (7.80\text{ m})^2}$$
$$c = \sqrt{61.0704\text{ m}^2}$$
$$c = 7.814755274\text{ m}$$
$$c = 7.81\text{ m}$$

58.



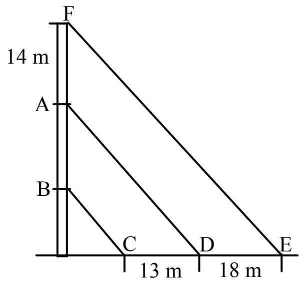
$$c^2 = a^2 + b^2$$
$$c = \sqrt{(2100\text{ ft})^2 + (9500\text{ ft})^2}$$
$$c^2 = \sqrt{94,660,000\text{ ft}^2}$$
$$c = 9729.33708\text{ ft}$$
$$c = 9700\text{ ft}$$

59. Let s be the side length of the square and each triangle. The total perimeter is $6s$. We have, using the Pythagorean theorem, $s^2 + s^2 = 2.4^2$ or $2s^2 = 5.76$. Therefore, $s^2 = 2.88$ and so $s = \sqrt{2.88} = 1.697056$. The perimeter is $6(1.697056) = 10.18233$ which is 1.0×10^1 rounded to two places.

60. A = Area of square + Area of 4 semi-circles

$$A = s^2 + 4\ \frac{\pi r^2}{2}$$
$$A = s^2 + 2\pi r^2$$
$$A = s^2 + 2\pi\ \frac{s}{2}^{2}$$
$$A = s^2 + \frac{\pi s^2}{2}$$
$$A = (4.50\text{ cm})^2 + \frac{\pi(4.50\text{ cm})^2}{2}$$
$$A = 52.05862562\text{ cm}^2$$
$$A = 52.1\text{ cm}^2$$

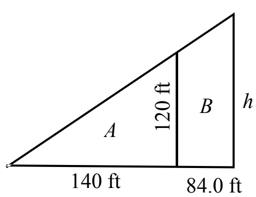
61.



Since line segments BC, AD, and EF are parallel,
the segments AB and CD are proportional to AF and DE

$$\frac{AB}{CD} = \frac{AF}{DE}$$
$$\frac{AB}{13 \text{ m}} = \frac{14 \text{ m}}{18 \text{ m}}$$
$$AB = \frac{13 \text{ m}(14 \text{ m})}{18 \text{ m}}$$
$$AB = 10.111111 \text{ m}$$
$$AB = 10 \text{ m}$$

62.



Since the triangles are similar, their sides are proportional.

$$\frac{h}{140+84} = \frac{120}{140}$$
$$h = \frac{120(224)}{140}$$
$$h = 192 \text{ ft}$$

Lot A is a triangle

$$A_A = \frac{1}{2}(140 \text{ ft})(120 \text{ ft}) = 8400 \text{ ft}^2$$

Lot B is a trapezoid

$$A_B = \frac{1}{2}(120 \text{ m} + 192 \text{ ft})(84.0 \text{ ft}) = 13\,100 \text{ ft}^2$$

63. Since the triangles are proportional

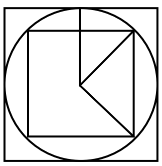
$$\frac{BF}{AE} = \frac{MB}{AM}$$

$$\frac{BF}{1.6\text{ m}} = \frac{4.5\text{ m}}{1.2\text{ m}}$$

$$BF = \frac{4.5\text{ m}(1.6\text{ m})}{1.2\text{ m}}$$

$$BF = 6.0\text{ m}$$

64.



Let r denote the radius of the circle. The side length of the larger square is $2r$ and, using the Pythagorean theorem, the side length of the smaller square s satisfies $r^2 + r^2 = s^2$ or $s^2 = 2r^2$ and so $s = r\sqrt{2}$.

(a) $A_{\text{large}} = (2r)^2 = 4r^2$

$A_{\text{small}} = (r\sqrt{2})^2 = 2r^2$

and so

$$\frac{A_{\text{large}}}{A_{\text{small}}} = \frac{4r^2}{2r^2} = 2$$

(b) $p_{\text{large}} = 4 \cdot 2r = 8r$

$p_{\text{small}} = 4 \cdot r\sqrt{2} = 4\sqrt{2}r$

and so

$$\frac{p_{\text{large}}}{p_{\text{small}}} = \frac{8r}{4\sqrt{2}r} = \sqrt{2}$$

65. The longest distance between points on the photograph is

$c^2 = a^2 + b^2$

$c = \sqrt{(8.00\text{ in})^2 + (10.00\text{ in})^2}$

$c = \sqrt{164\text{ in}^2}$

$c = 12.806248\text{ in}$

Find the distance in km represented by the longest measure on the map

$x = (12.806248\text{ in})\,\dfrac{18\,450}{1}\;\dfrac{1\text{ ft}}{12\text{ in}}\;\dfrac{1\text{ mi}}{5280\text{ ft}}$

$x = 3.7290922\text{ mi}$

$x = 3.73\text{ mi}$

66. $MA = \dfrac{\pi {r_L}^2}{\pi {r_S}^2}$

d_L = diameter of large piston in cm

d_S = diameter of small piston in cm

$MA = \dfrac{\pi\left(\dfrac{d_L}{2}\right)^2}{\pi\left(\dfrac{d_S}{2}\right)^2}$

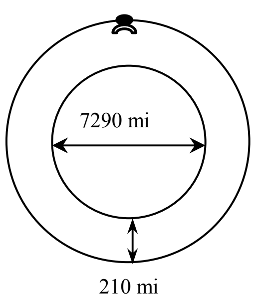
$MA = \left(\dfrac{d_L}{d_s}\right)^2$

$MA = \left(\dfrac{3.10}{2.25}\right)^2$

$MA = 1.898271605$

$MA = 1.90$

67.



The diameter of the satellite's orbit is Earth diameter plus two times its distance from the surface of Earth.

$c = \pi D$

$c = \pi\big(7920\text{ mi} + 2(210\text{ mi})\big)$

$c = 26{,}200\text{ mi}$

68. $c = 2\pi r$

$$r = \frac{c}{2\pi}$$

$$A = \pi r^2$$

$$A = \pi \left(\frac{c}{2\pi}\right)^2$$

$$A = \frac{c^2}{4\pi}$$

$$A = \frac{(651\text{ m})^2}{4\pi}$$

$$A = 33{,}725\text{ m}^2$$

$$A = 33{,}700\text{ m}^2$$

69. Area of the drywall is the area of the rectangle subtract the two circular cutouts.

$$A = lw - 2\left(\pi r^2\right)$$

$$A = lw - 2\left(\frac{\pi d^2}{4}\right)$$

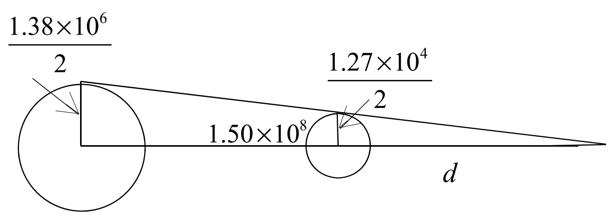
$$A = lw - \frac{\pi d^2}{2}$$

$$A = (4.0\text{ ft})(8.0\text{ ft}) - \frac{\pi(1.0\text{ ft})^2}{2}$$

$$A = 30.42920367\text{ ft}^2$$

$$A = 3.0\times 10^1\text{ ft}^2$$

70.



The triangles are similar so,

$$\frac{d}{\frac{12\ 700}{2}} = \frac{d + 150\ 000\ 000}{\frac{1\ 380\ 000}{2}}$$

$$\frac{d}{6350} = \frac{d + 150\ 000\ 000}{690\ 000}$$

$$690\ 000\, d = 6350\ (d + 150\ 000\ 000)$$

$$690\ 000\, d = 6350\, d + 952\ 500\ 000\ 000$$

$$683\ 650\, d = 952\ 500\ 000\ 000$$

$$d = \frac{952\ 500\ 000\ 000}{683\ 650}$$

$$d = 1\ 393\ 256.783\text{ km}$$

$$d = 1.39\times 10^6\text{ km}$$

71. $A = \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + ... + 2y_{n-2} + 4y_{n-1} + y_n\right]$

$$A = \frac{250}{3}\begin{bmatrix} 220 + 4(530) + 2(480) + 4(320 + 190 + 260) \\ +2(510) + 4(350) + 2(730) + 4(560) + 240 \end{bmatrix}$$

$$A = \frac{250}{3}(12\ 740)$$

$$A = 1\ 061\ 666 \text{ m}^2$$

$$A = 1.1 \times 10^6 \text{ m}^2$$

72. $V = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n\right]$

$$V = \frac{250}{2}\left[560 + 2(1780) + 2(4650) + 2(6730) + 2(5600) + 2(6280) + 2(2260) + 230\right]$$

$$V = \frac{250}{2}(55\ 390)$$

$$V = 6\ 923\ 750 \text{ ft}^3$$

$$V = 6.92 \times 10^6 \text{ ft}^3$$

73. $V = \pi r^2 h$

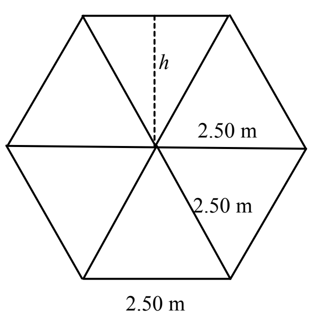
$$V = \frac{\pi d^2}{4}h$$

$$V = \frac{\pi(4.3 \text{ m})^2}{4}(13 \text{ m})$$

$$V = 188.7861565 \text{ m}^3$$

$$V = 189 \text{ m}^3$$

74.



Area of cross-section is the area of six equilateral triangles with sides of 2.50 m each

Using Hero's formula,

$$s = \tfrac{1}{2}(a + b + c)$$

$$s = \tfrac{1}{2}(2.5 + 2.5 + 2.5)$$

$$s = 3.75 \text{ m}$$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{3.75(3.75-2.5)^3}$

$A = 2.70633\ \text{m}^2$

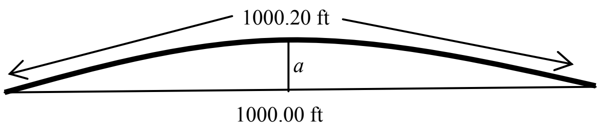
$V = \text{area of cross section} \times \text{height}$

$V = 6\ \ 2.70633\ \text{m}^2\ \ (6.75\ \text{m})$

$V = 109.6063402\ \text{m}^3$

$V = 1.10 \times 10^2\ \text{m}^3$

75.



$c^2 = a^2 + b^2$

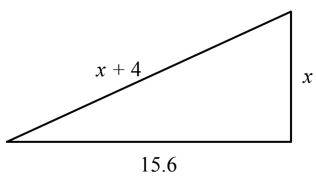
$a^2 = c^2 - b^2$

$a = \sqrt{(500.10\ \text{ft})^2 - (500.00\ \text{ft})^2}$

$a = \sqrt{100.01\ \text{ft}^2}$

$a = 10.000\ \text{ft}$

76.



$c = \text{length of guy wire}$

$(x+4)^2 = x^2 + 15.6^2$

$x^2 + 8x + 16 = x^2 + 243.36$

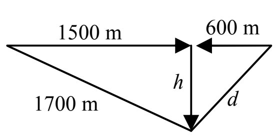
$8x = 227.36$

$x = 28.42\ \text{ft}$

$c = x + 4 = 32.42\ \text{ft}$

$c = 32.4\ \text{ft}$

77.



$h = \text{distance walked south}$

$1700^2 = h^2 + 1500^2$

$h = \sqrt{1700^2 - 1500^2}$

$h = 800\ \text{m}$

$d^2 = 600^2 + h^2$

$d = \sqrt{600^2 + 800^2}$

$d = 1000\ \text{m}$

78. w = width, $l = w + 44$ = length

$p = 2w + 2l$

$288 = 2w + 2(w + 44)$

$288 = 4w + 88$

$4w = 200$

$w = 50$

$l = w + 44 = 94$

The court is 50 feet wide and 94 feet long.

79. $V = V_{cylinder} + V_{dome}$

$V = \pi r^2 h + \frac{1}{2}\ \frac{4}{3}\pi r^3$

Note the height of the cylinder is the total height less the radius of the hemisphere. This radius is $\frac{2.50}{2} = 1.25$ ft.

$V = \pi\left(1.25 \text{ ft}\right)^2\left(4.75 \text{ ft} - 1.25 \text{ ft}\right) + \frac{2}{3}\pi\left(1.25 \text{ ft}\right)^3$

$V = 21.2712 \text{ ft}^3$

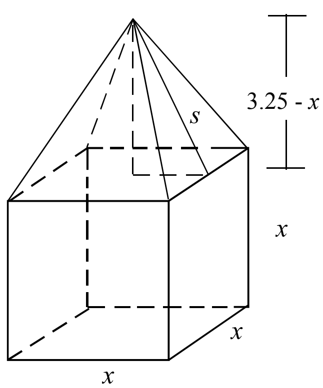
Convert ft^3 to gal,

$V = 21.2712 \text{ ft}^3\ \frac{7.48 \text{ gal}}{1 \text{ ft}^3}$

$V = 159.1085779 \text{ gal}$

$V = 159 \text{ gal}$

80.



Given $x = 2.50$ m

Find the lateral height s of the pyramid's triangles

$s^2 = a^2 + b^2$

$s^2 = \left(3.25 - x\right)^2 + \ \frac{x}{2}\ ^2$

$s^2 = \left(0.75 \text{ m}\right)^2 + \left(1.25 \text{ m}\right)^2$

$s = \sqrt{2.125 \text{ m}^2}$

$s = 1.457737974 \text{ m}$

tent surface area = surface area of pyramid + surface area of cube
tent surface area = 4 triangles + 4 squares

$$A = 4\left(\frac{1}{2}xs\right) + 4x^2$$

$$A = 2(2.50\text{ m})(1.457738\text{ m}) + 4(2.50\text{ m})^2$$

$$A = 32.2887\text{ m}^2$$

$$A = 32.3\text{ m}^2$$

81.

$$\frac{w}{h} = \frac{16}{9}$$

$$w = \frac{16h}{9}$$

$$152^2 = w^2 + h^2$$

$$23104 = \left(\frac{16h}{9}\right)^2 + h^2$$

$$23104 = \frac{256}{81}h^2 + h^2$$

$$23104 = \frac{337}{81}h^2$$

$$h^2 = \frac{81(23104)}{337}$$

$$h = \sqrt{5553.18694\text{ cm}^2}$$

$$h = 74.519708\text{ cm}$$

$$h = 74.5\text{ cm}$$

$$w = \frac{16h}{9}$$

$$w = \frac{16(74.519708\text{ cm})}{9}$$

$$w = 132.4794816\text{ cm}$$

$$w = 132\text{ cm}$$

82. Using the Pythagorean theorem,

$$(5k+2)^2 = (4k+3)^2 + (3k-1)^2$$

$$25k^2 + 20k + 4 = 16k^2 + 24k + 9 + 9k^2 - 6k + 1$$

$$2k = 6$$

$$k = 3$$

and so the edges measure 8 ft, 15 ft, and 17 ft, respectively.

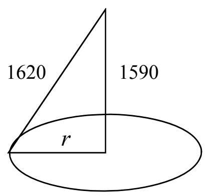
83. Let r be the common radius and h be the height of the cylinder.
Since the volumes are equal,

$$\frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \pi r^2 h$$

or

$$r = \frac{3}{2}h.$$

84.



$r^2 = 1620^2 - 1590^2$

$r^2 = 96\ 300\ \text{km}^2$

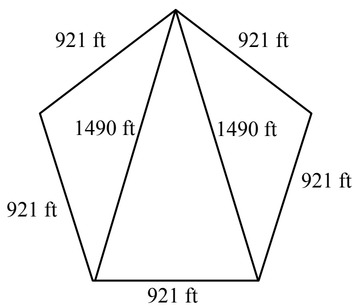
$A = \pi r^2$

$A = \pi(96\ 300\ \text{km}^2)$

$A = 302\ 535.3725\ \text{km}^2$

$A = 303\ 000\ \text{km}^2$

85.



The area is the sum of the areas of three triangles, one with sides 921, 1490, and 1490 and two with sides 921, 921, and 1490. The semi-perimeters are given by

$s_1 = \dfrac{921 + 921 + 1490}{2} = 1666$

$s_2 = \dfrac{1490 + 1490 + 921}{2} = 1950.5$

$A = 2\sqrt{1666(1666-921)(1666-921)(1666-1490)} + \sqrt{1950.5(1950.5-1490)(1950.5-1490)(1950.5-921)}$

$A = 806826\ \text{ft}^2 + 652{,}553\ \text{ft}^2 = 1{,}459{,}379\ \text{ft}^2$

$A = 1.46 \times 10^6\ \text{ft}^2$

Chapter 3

Functions and Graphs

3.1 Introduction to Functions

1. $f(x) = 3x - 7$

$f(-2) = 3(-2) - 7 = -13$

2. $g(t) = \dfrac{t^2}{2t+1}$

$g(-a^2) = \dfrac{(-a^2)}{2(-a^2)+1} = \dfrac{a^4}{1-2a^2}$

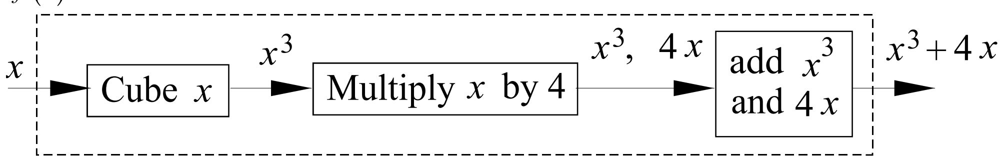
3. $f(T) = 10.0 + 0.10T + 0.001T^2$

$f(T-10) = 10.0 + 0.10(T-10) + 0.001(T-10)^2$

$f(T-10) = 10.0 + 0.10T - 1 + 0.001T^2 - 0.02T + 0.1$

$f(T-10) = 9.1 + 0.08T + 0.001T^2$

4. $f(x) = x^3 + 4x$



5. (a) $A(r) = \pi r^2$

(b) $A(d) = \pi\left(\dfrac{d}{2}\right)^2 = \dfrac{\pi d^2}{4}$

6. (a) $c(r) = 2\pi r$

(b) $c(d) = 2\pi \; \dfrac{d}{2} \; = \pi d$

7. $V = \frac{4}{3}\pi r^3$

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

$$V = \frac{4}{3}\pi\frac{d^3}{8}$$

$$V = \frac{\pi d^3}{6}$$

$$\frac{6V}{\pi} = d^3$$

$$d(V) = \sqrt[3]{\frac{6V}{\pi}}$$

8. $A = 6e^2$

$$e^2 = \frac{A}{6}$$

$$e(A) = \sqrt{\frac{A}{6}}$$

9. To get A as a function of the diagonal d of a square, we note that d satisfies

$s^2 + s^2 = d^2$

$2s^2 = d^2$

$d = s\sqrt{2}$

where s is its side length

and so $s = \frac{d}{\sqrt{2}}$

$A = s^2 = \left(\frac{d}{\sqrt{2}}\right)^2 = \frac{d^2}{2}$

$A(d) = \frac{d^2}{2}$

To get d as a function of A, starting with $A = \frac{d^2}{2}$,

$d^2 = 2A$

$d = \sqrt{2A}$

$d(A) = \sqrt{2A}$

10. $p(s) = 4s$

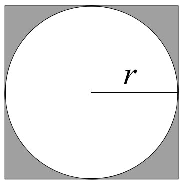
$$s = \frac{p}{4}$$

$$s(p) = \frac{p}{4}$$

11. $A = \text{area of square} - \text{area of circle}$

$$A = (2r)^2 - \pi r^2$$

$$A(r) = 4r^2 - \pi r^2$$



12. semiperimeter $= \dfrac{s+s+s}{2} = \dfrac{3s}{2}$

Using Hero's formula:

$$A = \sqrt{\frac{3s}{2}\left(\frac{3s}{2} - s\right)\left(\frac{3s}{2} - s\right)\left(\frac{3s}{2} - s\right)}$$

$$A = \sqrt{\frac{3s}{2}\left(\frac{s}{2}\right)^3}$$

$$A = \sqrt{\frac{3s^4}{16}}$$

$$A(s) = \frac{s^2}{4}\sqrt{3}$$

13. $f(x) = 2x + 1$

$$f(3) = 2(3) + 1 = 7$$

$$f(-5) = 2(-5) + 1 = -9$$

14. $f(x) = -x^2 - 9$

$$f(2) = -(2)^2 - 9 = -4 - 9 = -13$$

$$f(-2) = -(-2)^2 - 9 = -4 - 9 = -13$$

15. $f(x) = 6$

$$f(-2) = 6$$

$$f(0.4) = 6$$

16. $f(T) = 7.2 - 2.5|T|$

$$f(2.6) = 7.2 - 2.5|2.6| = 7.2 - 2.5(2.6) = 0.7$$

$$f(-4) = 7.2 - 2.5|-4| = 7.2 - 2.5(4) = -2.8$$

17. $\phi(x)=\dfrac{6-x^2}{2x}$

$\phi(2\pi)=\dfrac{6-(2\pi)^2}{2(2\pi)}=\dfrac{6-4\pi^2}{4\pi}=\dfrac{3-2\pi^2}{2\pi}=-2.66$

$\phi(-2)=\dfrac{6-(-2)^2}{2(-2)}=\dfrac{2}{-4}=-\dfrac{1}{2}$

18. $H(q)=\dfrac{8}{q}+2\sqrt{q}$

$H(4)=\dfrac{8}{4}+2\sqrt{4}=2+2(2)=2+4=6$

$H((-0.4)^2)=H(0.16)=\dfrac{8}{0.16}+2\sqrt{0.16}=50+2(0.4)=50.8$

19. $g(t)=at^2-a^2t$

$g\left(\frac{1}{3}\right)=a\left(\frac{1}{3}\right)^2-a^2\left(\frac{1}{3}\right)=a\left(\frac{1}{9}\right)-a^2\left(\frac{1}{3}\right)=\dfrac{a}{9}-\dfrac{a^2}{3}$

$g(a)=a\left(a^2\right)-a^2(a)=a^3-a^3=0$

20. $s(y)=6\sqrt{y+11}-3$

$s(-2)=6\sqrt{-2+11}-3=6\sqrt{9}-3=6(3)-3=15$

$s\left(a^2\right)=6\sqrt{a^2+11}-3$

21. $K(s)=3s^2-s+6$

$K(-s+2)=3(-s+2)^2-(-s+2)+6=3s^2-12s+12+s-6=3s^2-11s+6$

$K(-s)+2=\left(3(-s)^2-(-s)+6\right)+2=3s^2+s+8$

22. $T(t)=5t+7$

$T(2t+a-1)=5(2t+a-1)+7=10t+5a-5+7=10t+5a+2$

$T(2t)+a-1=\left(5(2t)+7\right)+a-1=10t+7+a-1=10t+a+6$

23. $f(x)=8x+3$

$$\begin{aligned} f(3x)-3f(x)&=8(3x)+3-3(8x+3)\\ &=24x+3-24x-9\\ &=-6 \end{aligned}$$

24. $f(x)=2x^2+1$

$$\begin{aligned} f(x+2)-\left[f(x)+2\right]&=2(x+2)^2+1-\left[2x^2+1+2\right]\\ &=2\left(x^2+4x+4\right)+1-2x^2-3\\ &=2x^2+8x+8-2x^2-2\\ &=8x+6 \end{aligned}$$

25. $f(x) = 5x^2 - 3x$

$f(3.86) = 5(3.86)^2 - 3(3.86) = 62.918 = 62.9$

$f(-6.92) = 5(-6.92)^2 - 3(-6.92) = 260.192 = 2.60 \times 10^2$

26. $g(t) = \sqrt{t + 1.0604} - 6t^3$

$g(0.9261) = \sqrt{0.9261 + 1.0604} - 6(0.9261)^3 = -3.35624777 = -3.356$

27. $F(H) = \dfrac{2H^2}{H + 36.85}$

$F(-84.466) = \dfrac{2(-84.466)^2}{-84.466 + 36.85} = -299.6683953 = -299.67$

28. $f(x) = \dfrac{x^4 - 2.0965}{6x}$

$f(1.9654) = \dfrac{(1.9654)^4 - 2.0965}{6(1.9654)} = 1.0875396 = 1.0875$

29. $f(x) = 3x^2 - 4$

30. $f(x) = 5(x^3 + 2)$

31. $f(x) = x^3 - 7x$

32. $f(x) = x^2 + \dfrac{x}{4}$

33. $f(x) = x^2 + 2$

Square the value of the input and add 2 to the result.

34. $f(x) = 2x - 6$

Multiply the value of the input by 2 and subtract 6 from the result.

35. $g(y) = 6y - y^3$

Multiply the value of the input variable by 6,
cube the input, and
subtract the second result from the first.

36. $\phi(s) = 8 - 5s + s^2$

Multiply the input by 5 and square the input.
Then subtract the first result from 8 and then add the
second result to the difference.

37. $R(r) = 3(2r+5)$

Double the value of the input and then add 5.

Multiply this result by 3.

38. $f(z) = \dfrac{4z}{5-z}$

Multiply the input by 4,

subtract the input from 5,

then divide the first result by the second.

39. $A = 5e^2$

$f(e) = 5e^2$

40. $d = \sqrt{1000^2 + x^2}$

$f(x) = \sqrt{1000^2 + x^2}$

41. $A = 5200 - 120t$

$f(t) = 5200 - 120t$

42. $R = \dfrac{10R_c}{10 + R_c}$

$f(R_c) = \dfrac{10R_c}{10 + R_c}$

43. $s = f(t) = 17.5 - 4.9t^2$

$f(12) = 17.5 - 4.9(1.2)^2 = 10.4$ m

44. $L = f(s) = 100\left(1 + 0.0003s^2\right)$

$f(15) = 100\left(1 + 0.0003(15)^2\right) = 106.75$

$= 110$ ft (rounded to two significant digits)

45. $d = f(v) = v + 0.05v^2$

$f(30) = 30 + 0.05(30)^2 = 75$ ft

$f(2v) = 2v + 0.05(2v)^2 = 2v + 0.2v^2$

$f(60) = 60 + 0.05(60)^2 = 240$ ft

$f(60) = f(2(30)) = 2(30) + 0.2(30)^2 = 240$ ft

46. $P = f(R) = \dfrac{200R}{(100+R)^2}$

$f(R+10) = \dfrac{200(R+10)}{100+(R+10)^{\ 2}}$

$f(R+10) = \dfrac{200R+2000}{(110+R)^2}$

$f(10R) = \dfrac{200(10R)}{(100+(10R))^2}$

$f(10R) = \dfrac{2000R}{(100+10R)^2}$

47.
$$L = 1.2Q^2 + 1.5Q$$
$$f(Q) = 1.2Q^2 + 1.5Q$$

For a flow of 450 gal/min, $Q = 4.5$ and so

$$L = f(4.5) = 1.2(4.5)^2 + 1.5(4.5)$$
$$L = 31.05 \text{ lb/in}^2$$
$L = 31 \text{ lb/in}^2$ rounded to two significant digits

48. For the original price,

$C = D + 0.07D = 1.07D$

$C(D) = 1.07D$

For the 19% off sale price, one pays tax on the final sale price, so

$$\begin{aligned} S(D) &= C(D - 0.19D) \\ &= 1.07(D - 0.19D) \\ &= 1.07(0.81D) \\ S(D) &= 0.8667D \end{aligned}$$

49. distance = rate $\times$ time

For the first leg,

$d_1 = 55t$

For the second leg,

$d_2 = 65(t+1)$

The total distance traveled is

$d_1 + d_2 = 55t + 65(t+1)$

$f(t) = 120t + 65$

50. **(a)** For the lease option,

$P(m) = 2000 + 450m + 18000$

$P(m) = 450m + 20000$

(b) The difference between the two options is

$$\begin{aligned} P(36) - 35000 &= 450(36) + 20000 - 35000 \\ &= \$1200 \end{aligned}$$

51. **(a)** $f[f(x)]$ means "the function of the function of x."

To evaluate, replace x in the function by the function itself.

(b) $f(x) = 2x^2$

$f[f(x)] = f\ \ 2x^2\ \ = 2(2x^2)^2 = 8x^4$

52. $f(x) = x$

$g(x) = x^2$

(a) $f[g(x)] = f[x^2] = x^2$

(b) $g[f(x)] = g[x] = x^2$

For this pair of functions,

$f[g(x)] = g[f(x)]$

3.2 More about Functions

1. $f(x) = -x^2 + 2$ is defined for all real values of x.

Domain: all real numbers $\mathbb{R}$ or $(-\infty, \infty)$

Since x^2 cannot be negative, the maximum value of $f(x)$ is 2.

Range: all real numbers $f(x) \le 2,$ or $(-\infty, 2]$

2. $f(x) = 16\sqrt{x} + \dfrac{1}{x-1}.$

To ensure there are no negative values in the square root, $x \ge 0.$

$\dfrac{1}{x-1}$ requires $x \ne 1,$ since the function is undefined there.

Domain: all real numbers $x \ge 0,\ x \ne 1,$ or $[0,1)$ and $(1, \infty).$

3. $f(t) = \begin{cases} 8 - 2t, & 0 \le t \le 4 \text{ s} \\ 0\ , & t > 4 \text{ s} \end{cases}$

$f(2) = 8 - 2(2) = 4$ mA

$f(5) = 0$ mA

4. $0.20m + 0.40n = 200$

$0.40n = 200 - 0.20m$

$n = 500 - 0.50m$

$f(m) = 500 - 0.50m$

Since neither m nor n can be negative,

Domain: all values $0 \le m \le 1000$ g or [0 g, 1000 g]

Range: all values $0 \le n \le 500$ g or [0 g, 500 g]

5. $f(x) = x + 5$

Domain: all real numbers $\mathbb{R}$ or $(-\infty,\ \infty)$

Range: all real numbers $\mathbb{R}$ or $(-\infty,\ \infty)$

6. $g(u) = 3 - 4u^2$

There are no restrictions on the value of u.

Domain: all real numbers $\mathbb{R}$, or $(-\infty,\ \infty)$

However, since u^2 is never negative, the maximum value for $g(u)$ is 3.

Range: all real numbers $g(u) \le 3$, or $(-\infty,\ 3]$

7. $G(R) = \dfrac{3.2}{R}$ is not defined for $R = 0$, but is defined elsewhere.

Domain: all real numbers $R \ne 0$, or $(-\infty, 0)$ and $(0, \infty)$

No value of R can produce $G(R) = 0$, so

Range: all real numbers $G(R) \ne 0$, or $(-\infty, 0)$ and $(0, \infty)$

8. $F(r) = \sqrt{r+4}$

$F(r)$ is not defined for real numbers r less than -4.

Domain: all real numbers $r \ge -4$, or $[-4, \infty)$

Range cannot be negative if we consider only principal roots of $r + 4$.

Range: all real numbers $F(r) \ge 0$, or $[0, \infty)$

9. $f(s) = \sqrt{s-2}$

$f(s)$ is not defined for $s < 2$.

Domain: all real values $s \ge 2$, or $[2, \infty)$.

Since $\sqrt{s-2}$ is never negative

Range: all real numbers $f(s) \ge 0$, or $[0,\ \infty)$

10. $T(t) = 2t^4 + t^2 - 1$

$T(t)$ is defined for all values of t.

Domain: all real numbers $\mathbb{R}$ or $(-\infty,\ \infty)$

Since t^4 and t^2 are both always positive, $T(t)$ will always be greater or equal to -1.

Range: all real numbers $T(t) \ge -1$, or $[-1,\ \infty)$

11. $H(h) = 2h + \sqrt{h} + 1$

The root $\sqrt{h}$ is not defined for values $h < 0$.

Domain: all real numbers $h \ge 0$, or $[0,\ \infty)$.

Since the restricted domain makes all h values at least zero,

the minimum value for $H(h)$ is 1.

Range: all real numbers $H(h) \ge 1$, or $[1,\ \infty)$.

12. $f(x) = \dfrac{-6}{\sqrt{2-x}}$

The terms in the square root must be positive to avoid both a square root of a negative number and a division by zero error, so $2-x>0$ or $x<2$.

Domain: all real numbers $x<2$, or $(-\infty,\ 2)$

If we consider only principal roots, the denominator is always positive and the numerator is negative so $f(x)<0$.

Range: all real numbers $f(x)<0$, or $(-\infty,\ 0)$

13. $y = |x-3|$

Domain: all real numbers $\mathbb{R}$ or $(-\infty,\ \infty)$

Absolute value is never negative, so

Range: all real numbers $y \geq 0$, or $[0,\ \infty)$

14. $y = x + |x| = \begin{cases} x + x = 2x,\ x \geq 0 \\ x + (-x) = 0,\ x < 0 \end{cases}$

There are no restrictions to evaluating either x or its abolute value, so

Domain: all real numbers $\mathbb{R}$ or $(-\infty,\ \infty)$

The function value is either zero or it is positive, so

Range: all real numbers $y \geq 0$, or $[0,\ \infty)$

15. $Y(y) = \dfrac{y+1}{\sqrt{y-2}}$

The square root requires $y-2 \geq 0$ or $y \geq 2$ to avoid the square root of a negative, and to avoid a division by zero, $y \neq 2$ is required. We need both conditions to be satisfied at the same time.

Domain: all real values $y > 2$, or $(2, \infty)$

16. $f(n) = \dfrac{n^2}{6-2n}$

Division by zero is undefined, so the domain must be restricted so the denominator is not 0, or $6-2n \neq 0$, which is $n \neq 3$.

Domain: all real numbers $n \neq 3$, or $(-\infty,\ 3)$ and $(3, \infty)$

17. $f(D) = \sqrt{D} + \dfrac{1}{D-2}$

Division by zero is undefined, so the domain must be restricted to exclude any value for which $D-2$ is equal to zero. In this case, $D \neq 2$.

Also, $\sqrt{D}$ is undefined for $D<0$ and so negative numbers are excluded from the domain.

Domain: all non-negative real numbers $D \neq 2$ or $[0, 2)$ and $(2, \infty)$.

18. $g(x) = \dfrac{\sqrt{x-2}}{x-3}$

Division by zero is undefined, so the domain must be restricted so $x \neq 3$.

Square roots aren't defined for negative values, so $x \geq 2$.

Domain: all real numbers $x \geq 2$ and $x \neq 3$, or $[2,3)$, and $(3, \infty)$

19.

$$x = \frac{3}{y-1}$$
$$x(y-1) = 3$$
$$y - 1 = \frac{3}{x}$$
$$y = \frac{3}{x} + 1$$
$$f(x) = \frac{3}{x} + 1$$

The domain is all nonzero x, or $(-\infty, 0)$ and $(0, \infty)$.

20. For $f(x) = \dfrac{1}{\sqrt{x}}$ we have

$$f(x+4) = \frac{1}{\sqrt{x+4}}$$

The domain excludes values of x that make the denominator zero or that require taking the square root of a negative quantity. It is necessary that $x > -4$

The domain is all $x > -4$, or $(-4, \infty)$.

21. $F(t) = 3t - t^2$ for $t \leq 2$

$F(1) = 3(1) - 1^2 = 2$

$F(2) = 3(2) - 2^2 = 6 - 4 = 2$

$F(3)$ does not exist since function is not defined at that location.

22.

$$h(s) = \begin{cases} 2s & \text{for } s < -1 \\ s+1 & \text{for } s \geq -1 \end{cases}$$

$h(-8) = 2(-8) = -16$ (since $-8 < -1$)

$h(-0.5) = -0.5 + 1 = 0.5$ (since $-0.5 \geq -1$)

23.

$$f(x) = \begin{cases} x+1 & \text{for } x < 1 \\ \sqrt{x+3} & \text{for } x \geq 1 \end{cases}$$

$f(1) = \sqrt{1+3} = \sqrt{4} = 2$ (since $1 \geq 1$)

$f(-0.25) = -0.25 + 1 = 0.75$ (since $-0.25 < 1$)

24. $g(x)=\begin{cases}\dfrac{1}{x} & \text{for } x\neq 0\\ 0 & \text{for } x=0\end{cases}$

$g(0.2)=\dfrac{1}{0.2}=5 \quad (\text{since } \tfrac{1}{5}\neq 0)$

$g(0)=0 \qquad (\text{since } 0=0)$

25. Distance d is expressed in mi.

$d(t)=40t+55\cdot 2$

$d(t)=40t+110$

26. Cost C is expressed in dollars.

$C(r)=\$3/\text{m}^2\times\text{area}$

$C(r)=3(2\pi rh+2\pi r^2)$

$C(r)=6\pi(2r)+6\pi r^2$

$C(r)=12\pi r+6\pi r^2$

27. Weight w is expressed in Mg,

$w(t)=5500-2t$

28. Profit is expressed in dollars.

$p(c)=\text{Income}-\text{Expenses}$

$p(c)=100c-100(3)$

$p(c)=100c-300$

29. For every metre over 1000 m in altitude, an additional 0.5 kg is added to the mass, if mass is expressed in kilograms, and the altitude is restricted to be greater than 1000 m,

$m(h)=110+0.5(h-1000)$

$m(h)=0.5h-390 \quad \text{for } h>1000\text{ m}$

30. n is expressed in L.

$n(x)=x(0.50)+100(0.70)$

$n(x)=0.5x+70$

31. For any length up to 50 ft in length, it costs \$500. For every additional foot, \$5 is charged. But with the domain restriction that $l>50$ ft,

If cost C is in dollars,

$C(l)=500+5(l-50)$

$C(l)=5l+250$

32. $MA(h)=\dfrac{8}{h}$

33. (a)

$$x(0.10) + y(0.40) = 1200$$

$$y(x) = \frac{1200 - 0.1x}{0.4}$$

(b)

$$y(400) = \frac{1200 - 0.1(400)}{0.4}$$

$$y(400) = 2900 \text{ L}$$

34. (a) For any income up to \$30 000 there is no tax.

For any income over \$30 000, the tax rate of 5% is charged.

If tax T is in dollars,

$$T(I) = \begin{cases} 0 & \text{for } I \le \$30\ 000 \\ 0.05(I - 30\ 000) & \text{for } I > \$30\ 000 \end{cases}$$

(b) $T(25\ 000) = \$0$

$T(45\ 000) = 0.05(45\ 000 - 30\ 000) = \$750$

35. Profit is expressed in dollars.

$$p = \text{Profit from cell phones} + \text{Profit from Blu-ray players}$$

$$2850 = 15x + 30y$$

$$30y = 2850 - 15x$$

$$y(x) = \frac{2850 - 15x}{30}$$

or

$$y(x) = 95 - \frac{1}{2}x$$

36. Cost C is expressed in dollars.

$$C(x) = 15 + 18x$$

$$C(x) = 375$$

$$15 + 18x = 375$$

$$18x = 360$$

$$x = 20$$

37. For the square, with the length of a side s,

$$p = 4s$$

$$s = \frac{p}{4}$$

$$A_{\text{square}} = (s)^2$$

$$A_{\text{square}} = \left(\frac{p}{4}\right)^2$$

$$A_{\text{square}} = \frac{p^2}{16}$$

For the circle, the circumference will be whatever is left over after the square perimeter is cut from the wire, $60 - p$

$$c = 60 - p$$
$$2\pi r = 60 - p$$
$$r = \frac{60 - p}{2\pi}$$
$$A_{\text{circle}} = \pi r^2$$
$$A_{\text{circle}} = \pi \left(\frac{60 - p}{2\pi}\right)^2$$
$$A_{\text{circle}} = \frac{(60 - p)^2}{4\pi}$$

Thus, the total area if both figures A will be a function of square perimeter p,

$$A(p) = A_{\text{square}} + A_{\text{circle}}$$
$$A(p) = \frac{p^2}{16} + \frac{(60 - p)^2}{4\pi}$$

38. Area A of the cross section can be expressed a function of diameter d.
The total area is the sum of a square of side length d and a circle of diameter d.

$$A(d) = A_{circle} + A_{square}$$
$$A(d) = \pi\left(\frac{d}{2}\right)^2 + d^2$$
$$A(d) = \frac{\pi d^2}{4} + d^2$$

39. The distance d from the helicopter is a function of the height h of the helicopter, related by the Pythagorean theorem.

$$h^2 + 120^2 = d^2$$
$$d(h) = \sqrt{h^2 + 14\ 400}$$

since distance above ground is nonnegative,

Domain: all real values $h \geq 0$, or $(0, \infty)$

Distance d is a minium of 120 m since d is 120 m when h is 0, and when h increases, d increases.

Range: all real values $d \geq 120$ m, or $(120, \infty)$

40. The radius of the circle is decreased by x cm, when it was originally 6 in, so

$$r = 6 - x$$
$$A = \pi r^2$$
$$A(x) = \pi(6 - x)^2$$

since x represents the decrease in the radius and it must be greater than or equal to zero and less than or equal to six.

Domain: all real values $0 \leq x \leq 6$ in, or [0, 6 in]

Using the end point values for the domain interval for x,

Range: all real values $0 \leq A \leq 36\pi$ in^2, or [0, 36π in^2]

41. distance = velocity $\times$ time

$300 = v \cdot (t-3)$

$v(t) = \dfrac{300}{t-3}$

In this example, negative time and speed have no meaning.

Cannot divide by zero, so $t \neq 3$.

Domain: all real values $t > 3$, or $(3, \infty)$

Since all $t > 3$, this forces v to always be positive.

No object with mass can reach the speed of light c (3×10^8 m/s), but in practical terms the upper limit for speed is probably less than that of sound (around 330 m/s), and most real trucks would have a limit much lower than that.

Range: all real numbers $0 < v < c$, or $(0, c)$

42. $A = lw$

$8 = lw$

$l(w) = \dfrac{8}{w}$

Negative width or length is meaningless, and cannot divide by zero, restricting $w > 0$.

Domain: all real numbers $w > 0$, or $(0, \infty)$

Range: all real numbers $l > 0$, or $(0, \infty)$

43. $f = \dfrac{1}{2\pi\sqrt{C}}$

To avoid a division by zero error, $C \neq 0$.

And to avoid a negative in a square root error, $C > 0$.

Domain: all real values $C > 0$, or $(0, \infty)$

44. The sum of the distances is 550 mi.

$x + y = 550$

$y(x) = 550 - x$

Both x and y are restricted in that they cannot be negative, nor can either exceed the total 550 mi.

Domain: all real numbers $0 \leq x \leq 550$ mi, or [0, 550 mi]

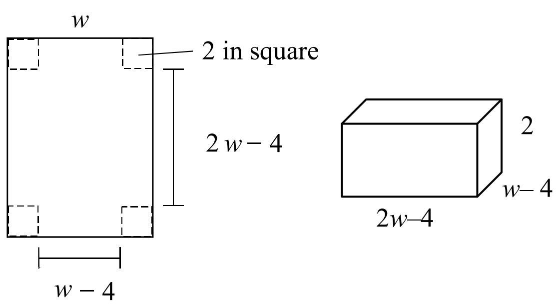
45. For every metre over 1000 m in altitude, an additional 0.5 kg is added to the mass, but up to that altitude, mass is a constant 110 kg. If mass is expressed in kilograms, and from Exercise 29, when altitude is restricted to be greater than 1000 m, $m(h) = 0.5h - 390$

$$m(h) = \begin{array}{ll} 110 & \text{for } 0 \leq h \leq 1000 \text{ m} \\ 0.5h - 390 & \text{for } h > 1000 \text{ m} \end{array}$$

46. For any length up to 50 ft in length, it costs \$500.
For every additional feet, \$5 is charged. From Exercise 31,
with the domain restriction that $l > 50$ ft, $C(l) = 5l + 250$
If cost C is in dollars,

$$C(l) = \begin{cases} 500 & \text{for } 0 \le l \le 50 \text{ ft} \\ 5l + 250 & \text{for } l > 50 \text{ ft} \end{cases}$$

47.

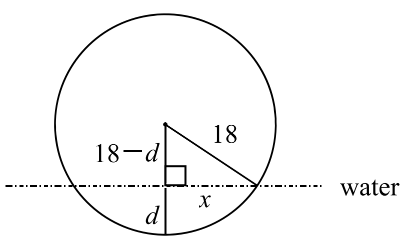


(a) $V = lwh$

$$V(w) = (2w-4)(w-4)(2)$$
$$V(w) = (2w^2 - 8w - 4w + 16)(2)$$
$$V(w) = (2w^2 - 12w + 16)(2)$$
$$V(w) = 4w^2 - 24w + 32$$

(b) The width has to be larger than 4 in to allow two 2-in cutout squares to be removed from its length and still produce a box.
Domain: $w > 4$ in, or (4 in, ∞)

48.



(a) From the diagram above (side-view),
let x = radius of circle of intersection
of buoy and water. Using the Pythagorean theorem,

$$18^2 = (18-d)^2 + x^2$$
$$18^2 = 18^2 - 36d + d^2 + x^2$$
$$x^2 = 36d - d^2$$
$$x = \sqrt{36d - d^2}$$

When viewed from above, the circular buoy will intersect the water with a circular cross section or radius x.

c = circumference of circle of intersection

$$c = 2\pi r$$

$$c(d) = 2\pi\sqrt{36d - d^2}$$

(b) If the buoy has a single point of contact with the water surface, the depth is 0. The same is true when the buoy is just barely completely submerged, therefore, the restriction for depth is

Domain: all real values $0 \le d \le 36$ in, or [0, 36 in].

When the buoy is completely out of the water, the radius of the intersection circle is 0

When the buoy is completely submerged, the radius is also 0.

When the buoy is exactly half-submerged, the radius of the circle of intersection with the water surface will be the same as the radius of the buoy, 18 in.

$$c_{\max} = 2\pi\sqrt{36(18) - 18^2}$$

$$c_{\max} = 113 \text{ in}$$

Range: all real values $0 \le c \le 113$ in, or [0, 113 in]

49. $f(x+2) = |x|$

In order to evaluate $f(0)$, the value of x must be -2.

$$f(-2+2) = f(0)$$

So, when $x = -2$ the value of the function will be:

$$f(0) = |-2|$$

$$f(0) = 2$$

50. $f(x) = x^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2}{h}$$

$$\frac{f(x+h) - f(x)}{h} = 2x + h, \quad \text{where } h \ne 0.$$

51. $f(x) = |x| + |x-2|$,

If $x \le 0$ then $f(x) = |x| + |x-2| = -x + 2 - x = 2 - 2x \ge 2$

(recalling that x is not positive and so $-2x \ge 0$.)

If $0 \le x \le 2$ then $f(x) = |x| + |x-2| = x + 2 - x = 2$.

If $2 \le x$ then $f(x) = |x| + |x-2| = x + x - 2 = 2x - 2 \ge 2$.

Range: all real values $f(x) \ge 2$, or $[2, \infty)$

52. $f(x)=\sqrt{x-1}$ has domain $x\ge 1$ to avoid a negative in the square root.

$g(x)=x^2$ has all real numbers as domain.

$g(f(x))=(f(x))^2$

$g(f(x))=\left(\sqrt{x-1}\right)^2$

$g(f(x))=x-1$

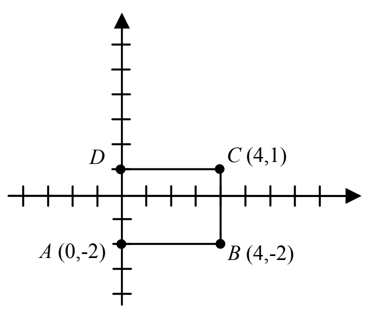
Although this function looks as though it should have domain of all real values of x,
The domain of $g(f(x))$ is restricted to all x in domain of $f(x)$, because $f(x)$ must be defined before it can be substituted into $g(x)$. Therefore,

Domain: all real values $x\ge 1$, or $[1, \infty)$.

3.3 Rectangular Coordinates

1. $A(0,-2)$, $B(4,-2)$, $C(4,1)$

The vertices of base AB both have y-coordinates of -2, which means the base CD which must be parallel, has the same y-coordinates for its vertices. Since at point C the y-coordinate is 1, then at D, y must also be 1. In the same way, the x-coordinates of the left side must both be -1. Therefore the fourth vertex is $D(0, 1)$.

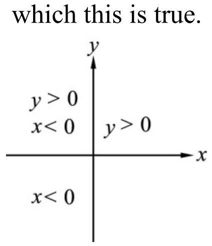


2. Where are all points (x, y) for which $x<0$ and $y>0$?

$x<0$ which means x is negative (left of the y-axis)

$y>0$ which means y is positive (above the x-axis)

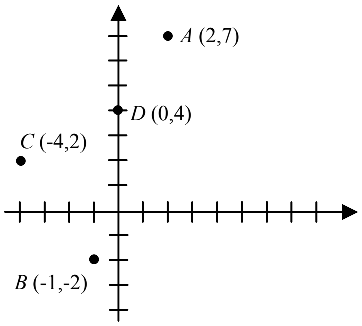
Therefore, (x, y) is in the second quadrant, the only location for which this is true.



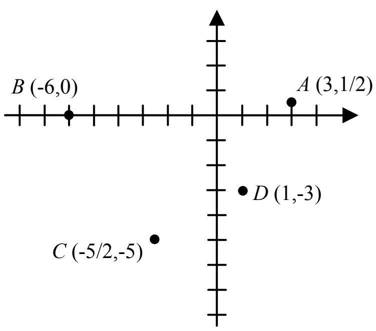
3. $A(2, 1)$, $B(-1, 2)$, $C(-2, -3)$

4. $D(3, -2), E(-3.5, 0.5), F(0, -4)$

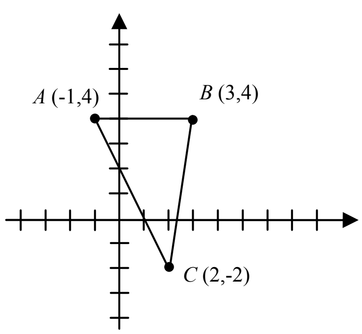
5.



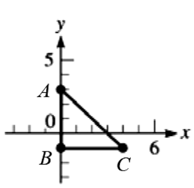
6.



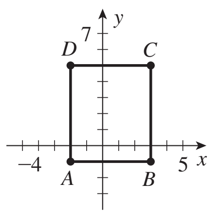
7. Joining the points in the order ΔABC forms a triangle.



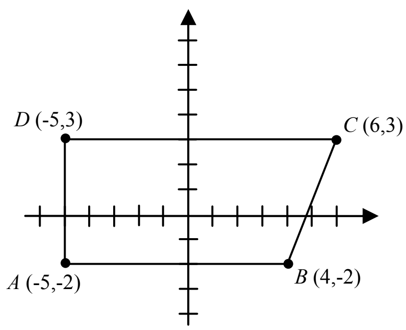
8. ΔABC forms an isosceles right triangle



9. Figure *ABCD* forms a rectangle.



10. Figure *ABCD* forms a trapezoid.



11. $A(6,\,3),\ B(-1,\,3),\ C(-1,\,-2)$

The vertices of base *AB* both have *y*-coordinates of 3, which means the base *CD* which must be parallel, having the same *y*-coordinates for its vertices. Since at point *C*, the *y*-coordinate is -2, then at *D*, *y* must also be -2.

Similarly, the *x*-coordinates of the right side must both be 6.

Therefore, the fourth vertex is $D(6,\,-2)$.

12. Since this is an equilateral triangle and we know the base is $(2,\,1),\ (7,\,1)$ it means the base is horizontal. The third vertex must be equidistant between 2 and 7. So $x = \dfrac{2+7}{2} = \dfrac{9}{2}$.

13. In order for the *x*-axis to be the perpendicular bisector of the line segment joining $P(3,\ 6)$ and $Q(x,\ y)$, *Q* must be located the same distance on the opposite side of the *x*-axis from *P*, and at the same *x*-coordinate so that the segment *PQ* is vertical (perpendicular to the *x*-axis). Therefore the point is $Q(3,\,-6)$.

14. $P(-4,\ 1)$ and $Q(x,\ y)$ form a line segment PQ. For it to be bisected by the origin, point Q must have the same coordinate values as P, but with opposite sign since they are on the opposite side of the origin, so the point is $Q(4,\ -1)$.

15. If $x < 0$ then the point lies in either the second or third quadrant.
If $y > 0$ then the point lies in either the first or second quadrant.
For both of these to occur, the point must lie in the second quadrant.

16. If $x > 0$ then the point lies in either the first or fourth quadrant.
If $y < 0$ then the point lies in either the third or fourth quadrant.
For both of these to occur, the point must lie in the fourth quadrant.

17. If $x < 0$ then the point lies in either the second or third quadrant.
If $y < 0$ then the point lies in either the third or fourth quadrant.
For both of these to occur, the point must lie in the third quadrant.

18. If $x > 0$ then the point lies in either the first or fourth quadrant.
If $y > 0$ then the point lies in either the first or second quadrant.
For both of these to occur, the point must lie in the first quadrant.

19. All points with x-coordinate of 1 are on a vertical line 1 unit to the right of the y-axis, passing through $(1,\ 0)$. The equation of this vertical line is $x = 1$.

20. All points with y-coordinate of -3 are on a horizontal line 3 units below the x-axis, passing through $(0,\ -3)$. The equation of this horizontal line is $y = -3$.

21. All points with y-coordinate of 3 are on a horizontal line 3 units above the x-axis, passing through $(0,\ 3)$. The equation of this horizontal line is $y = 3$.

22. All points $(-2,\ y)$ and $(2,\ y)$ have $|x| = 2$.
y can be any real number, and these are points on a two lines parallel to the y-axis, 2 units to the left and 2 units to the right, respectively.

23. All points whose x-coordinates equal their y-coordinates are on a 45° line through the origin. The equation of this line is $y = x$ and it bisects the first and third quadrants.

24. When the x-coordinates equal the negative of the y-coordinates ordinates, then $x = -y$. These pairs lie on a line formed by points of form $(-y, y)$. These points form a line angled at 135° from the positive x-axis, bisecting quadrants two and four.

25. The x-coordinate of all points on the y-axis is zero, since all points on that line have form $(0, y)$, where y can be any real number.

26. The y-coordinate of all points on the x-axis is zero, since all points on that line have form $(x, 0)$, where x can be any real number.

27. All points for which $x > 0$ are in Quadrant I and Quadrant IV to the right of the y-axis.

28. All points for which $y < 0$ are in Quadrant III and Quadrant IV. All points are below the x-axis.

29. All points for which $x < -1$ are in Quadrant II and Quadrant III to the left of the line $x = -1$, which is parallel to the y-axis, one unit to the left of the y-axis.

30. All points for which $y > 4$ are in Quadrant I and Quadrant II above the line $y = 4$, which is parallel to the x-axis, four units above the x-axis.

31. If $xy > 0$ the product of the coordinates x and y must be positive. Therefore, either both coordinates are positive, $x > 0$ and $y > 0$, which is Quadrant I, or both coordinates are negative, $x < 0$ and $y < 0$, which is Quadrant III.

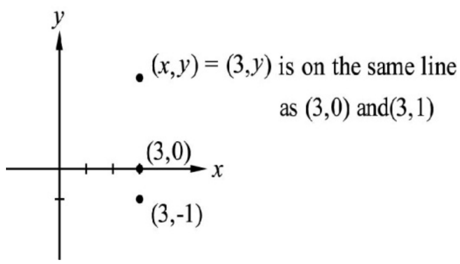
32. If $\frac{y}{x} < 0$ the division of the coordinates x and y must be negative.
Therefore, the sign of each coordinate must be opposite.
Therefore $x > 0$ and $y < 0$, which is Quadrant IV,
or $x < 0$ and $y > 0$, which is Quadrant II.

33. If $xy = 0$ then the product of the coordinates x and y must be zero.
Therefore, either coordinate may be zero.
For $x = 0$ points lie on the y-axis , and
for $y = 0$ points lie on the x-axis.
So, all points where $xy = 0$ lie on the x- or y - axis.

34. If $x < y$ then the y-coordinates of all points must be higher than the x-coordinates. All points whose x-coordinates equal their y-coordinates are on a 45° line through the origin, equation $y = x$ which bisects the first and third quadrants. We have $x < y$ for all points above the line $y = x$.

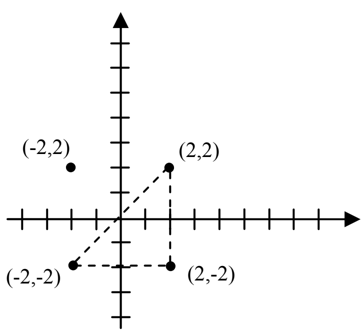
35. If (a, b) is in Quadrant II, then $a < 0$, and $b > 0$. Point $(a, -b)$ will then have a negative x-coordinate, and a negative y-coordinate, so must lie in Quadrant III.

36.



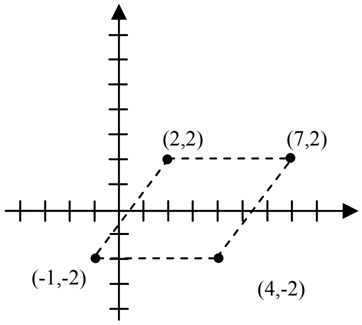
The points (3, 0) and (3, −1) have the same x-coordinate, so any point on a line passing through those two points must also have an x-coordinate of 3. Therefore $(x,\ y) = (3,\ y)$ is on the same line.

37.



The distance between (2, 2) and (−2, −2) can be found using the Pythagorean theorem applied to the right triangle whose third vertex is at (2, −2). This triangle has legs of length 4 and 4 and so its hypotenuse has length $\sqrt{4^2+4^2} = \sqrt{32} = 4\sqrt{2}$.

38.



The distance between $(-1, -2)$ and $(4, -2)$ is 5 as is the distance between $(7, 2)$ and $(2, 2)$. The distance between $(4, -2)$ and $(7, 2)$ as well as the distance between $(2, 2)$ and $(-1, -2)$, can be found as the length of the hypotenuse of a right triangle with legs 3 and 4. Calling this distance d, using the Pythagorean Theorem,

$d^2 = 3^2 + 4^2$

$d = \sqrt{9+16}$

$d = \sqrt{25}$

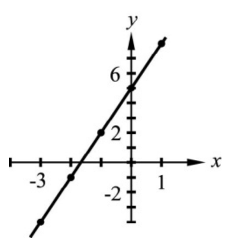
$d = 5$

It follows that the figure is a rhombus.

3.4 The Graph of a Function

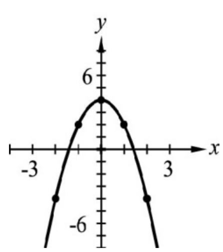
1. $f(x) = 3x + 5$

x	y
-3	-4
-2	-1
-1	2
0	5
1	8



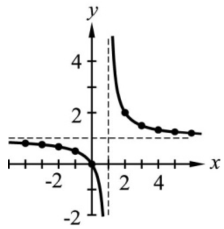
2. $f(x) = 4 - 2x^2$

x	y
-2	-4
-1	2
0	4
1	2



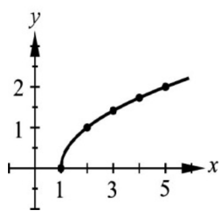
3. $f(x)=1+\dfrac{1}{x-1}$

x	y
–4	4 / 5
–3	3 / 4
–2	2 / 3
–1	1 / 2
0	0



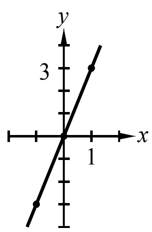
4. $f(x)=\sqrt{x-1}$

x	y
1	0
2	1
3	$\sqrt{2}$
4	$\sqrt{3}$
5	2



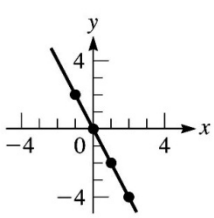
5. $y=3x$

x	y
–1	–3
0	0
1	3



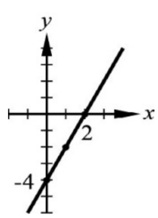
6. $y=-2x$

x	y
–1	2
0	0
1	–2
2	–4



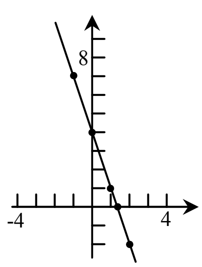
7. $y=2x-4$

x	y
0	–4
1	–2
2	0



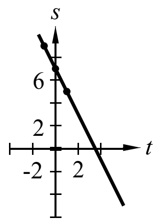
8. $y=4-3x$

x	y
–1	7
0	4
1	1
4 / 3	0
2	–2



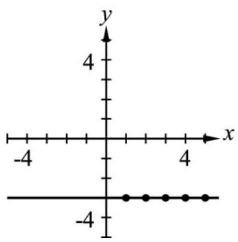
9. $s = 7 - 2t$

s	t
−1	9
0	7
1	5



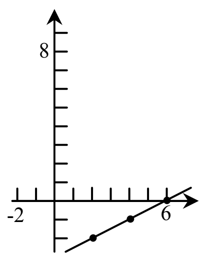
10. $y = -3$

x	y
1	−3
2	−3
3	−3
4	−3
5	−3



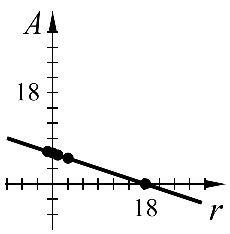
11. $y = \frac{1}{2}x - 3$

x	y
2	−2
4	−1
6	0



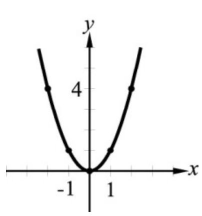
12. $A = 6 - \frac{1}{3}r$

r	A
−1	6.3
0	6
1	5.6
3	5
18	0



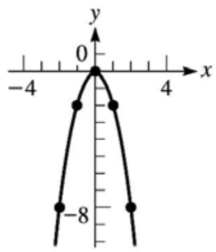
13. $y = x^2$

x	y
−2	4
−1	1
0	0
1	1
2	4



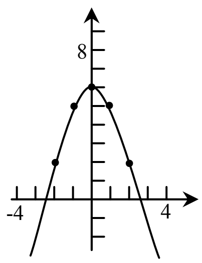
14. $y = -2x^2$

x	y
–2	–8
–1	–2
0	0
1	–2
2	–8



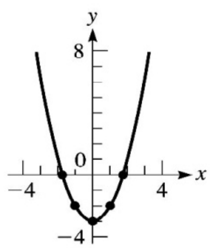
15. $y = 6 - x^2$

x	y
–2	2
–1	5
0	6
1	5
2	2



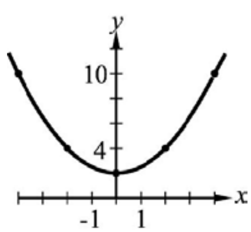
16. $y = x^2 - 3$

x	y
–1	–2
0	–3
1	–2
$\pm\sqrt{3}$	0



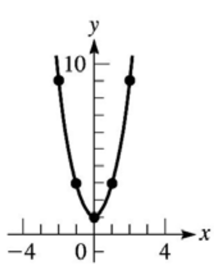
17. $y = \frac{1}{2}x^2 + 2$

x	y
–4	10
–2	4
0	2
2	4
4	10



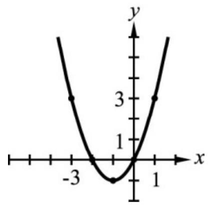
18. $y = 2x^2 + 1$

x	y
–2	9
–1	3
0	1
1	3
2	9



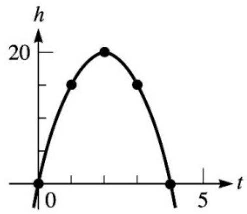
19. $y = x^2 + 2x$

x	y
–3	3
–2	0
–1	–1
0	0
1	3



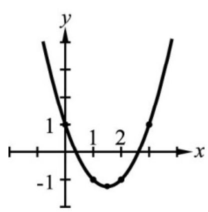
20. $h = 20t - 5t^2$

t	h
0	0
1	15
2	20
3	15
4	0



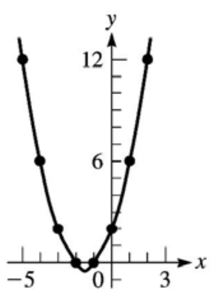
21. $y = x^2 - 3x + 1$

x	y
3	1
2	–1
1.5	–1.25
1	–1
0	1



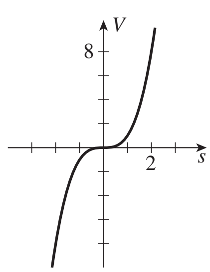
22. $y = 2 + 3x + x^2$

x	y
–3	2
–2	0
–1.5	–0.25
–1	0
0	2
1	6
2	12



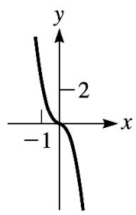
23. $V = e^3$

e	V
–2	–8
–1	–1
0	0
1	1
2	8



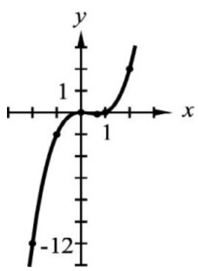
24. $y = -2x^3$

x	y
–2	16
–1	2
0	0
1	–2
2	–16



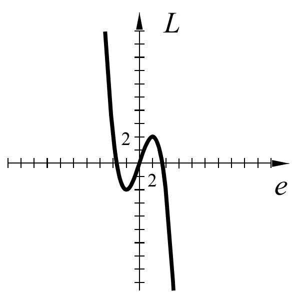
25. $y = x^3 - x^2$

x	y
–2	–12
–1	–2
0	0
2 / 3	–4 / 27
1	0
2	4



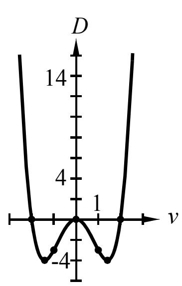
26. $L = 3e - e^3$

e	L
–3	18
–2	2
–1	–2
0	0
1	2
2	–2
3	–18



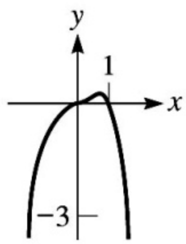
27. $D = v^4 - 4v^2$

v	D
–3	45
–2	0
–1.414	–4
–1	–3
0	0
1	–3
1.414	–4
2	0
3	45



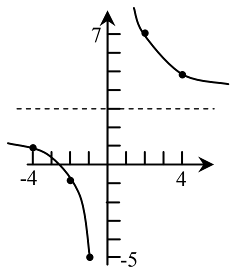
28. $y = x^3 - x^4$

x	y
–2	–24
–1	–2
0	0
0.5	0.06
1	0
2	–8



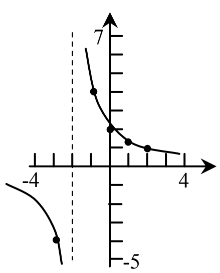
29. $P = \dfrac{8}{V} + 3$

V	P
–4	1
–2	–1
–1	–5
2	7
4	5



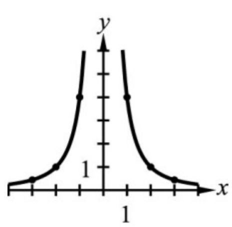
30. $y = \dfrac{4}{x+2}$

x	y
–3	–4
–1	4
0	2
1	4 / 3
2	1



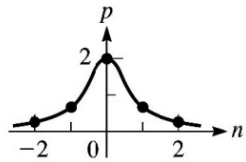
31. $y = \dfrac{4}{x^2}$

x	y
–3	0.444
–2	1
–1	4
1	4
2	1
3	0.444



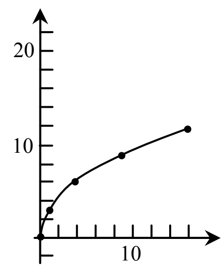
32. $p = \dfrac{1}{n^2 + 0.5}$

p	n
–2	0.22
–1	0.67
0	2
1	0.67
2	0.22



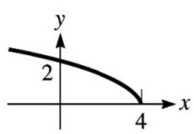
33. $y = \sqrt{9x}$

x	y
0	0
1	3
4	6
9	9
16	12



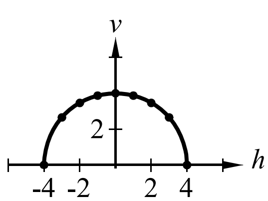
34. $y = \sqrt{4 - x}$

x	y
–5	3
0	2
3	1
4	0



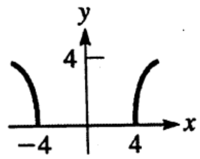
35. $v = \sqrt{16 - h^2}$

h	v
–4	0
–3	2.646
–2	3.464
–1	3.873
0	4
1	3.873
2	3.464
3	2.646
4	0



36. $y=\sqrt{x^2-16}$

x	y
−6	4.5
−5	3
−4	0
4	0
5	3
6	4.5



37. The domain is all real numbers, or $(-\infty,\infty)$.
The range is all real numbers at least -3, or $[-3,\infty)$.

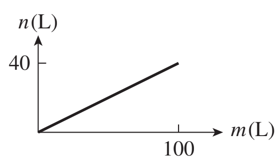
38. The domain is all real numbers, or $(-\infty,\infty)$.
The range is all real numbers, or $(-\infty,\infty)$.

39. The domain is all real numbers at least 2, or $[2,\infty)$.
The range is all real numbers at least 0, or $[0,\infty)$.

40. The domain is $[-5,5]$.
The range is $[0,3]$.

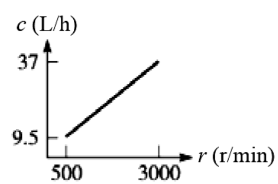
41. $n=0.40m$

m (L)	n (L)
10	4
50	20
80	32



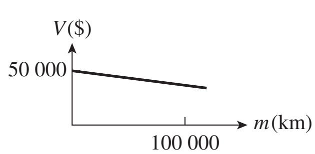
42. $c=0.011r+4.0$

r (r/min)	c (L/h)
500	9.5
1000	15
2000	26



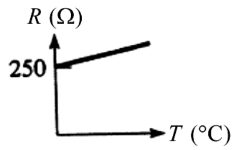
43. $V=50\ 000-0.2m$

m (km)	V ($)
0	50 000
30 000	44 000
60 000	38 000



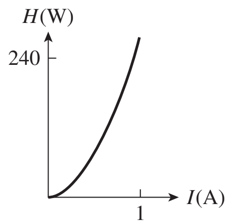
44. $R = 250(1 + 0.0032T)$

T (°C)	R (Ω)
0	250
100	330
200	410



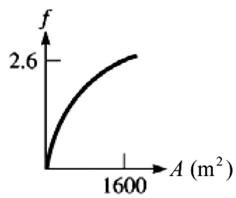
45. $H = 240I^2$

I (A)	H (W)
0	0
0.2	9.6
0.4	38.4
0.6	86.4
0.8	153.6



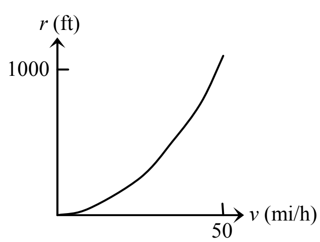
46. $f = 0.065\sqrt{A}$

A (m^2)	f
0	0
400	1.3
900	1.95
1600	2.6
2500	3.25



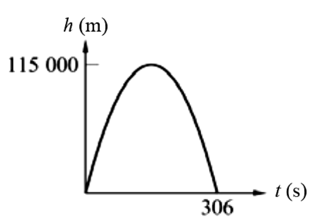
47. $r = 0.42v^2$

v (mi/h)	r (ft)
0	0
10	42
20	168
40	672
50	1050



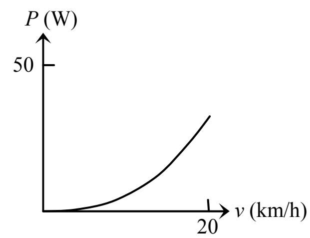
48. $h = 1500t - 4.9t^2$

t (s)	h (m)
0	0
10	14,510
50	62,750
100	101,000
200	104,000
300	9,000
306	0



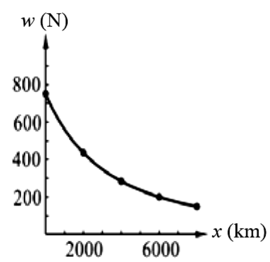
49. $P = 0.004v^3$

v (km/h)	P (W)
0	0
5	0.5
10	4.0
15	13.5
20	32.0



50. $w = 750\left(\dfrac{6400}{6400+x}\right)^2$

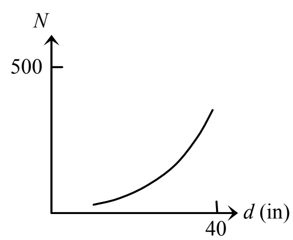
x (km)	w (N)
0	750
2000	435.37
4000	284.02
6000	199.79
8000	148.5



51. $V = 0.22d^2 - 0.71d$

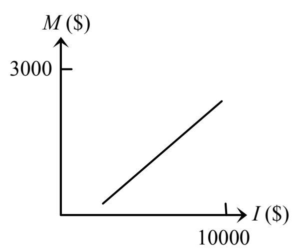
d (in)	N
10	15
20	74
30	180
40	320

(all output values are
to two significant digits)



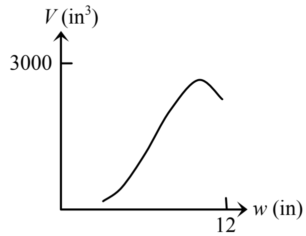
52. $M = 0.25(I - E)$

I (\$)	M (\$)
2000	350
4000	850
6000	1350
8000	1850
10000	2350



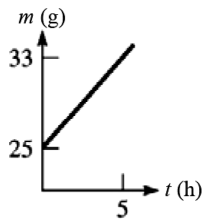
53. Since $l + w + h = 45$ and $l = 2w$ are given,
$h = 45 - w - l = 45 - 32$ and
$V = lwh = 2w^2(45 - 3w) = 90w^2 - 6w^3$.
$V(w) = 90w^2 - 6w^3$

w (in)	V (in^3)
3	648
6	1944
9	2916
12	2592

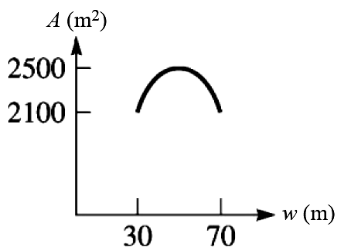


54. $m(t) = 1.6t + 25.0$

t (h)	m (g)
0	25
1	26.6
3	29.8
5	33.0



55.
$$P = 2l + 2w$$
$$200 = 2l + 2w$$
$$l = \frac{200 - 2w}{2}$$
$$l = 100 - w$$
$$A = lw$$
$$A = (100 - w)w$$

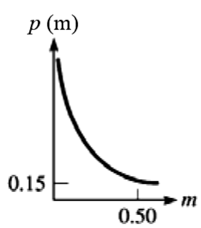


$A(w) = 100w - w^2$ for $30 \le w \le 70$

w (m)	30	40	50	60	70
A (m^2)	2100	2400	2500	2400	2100

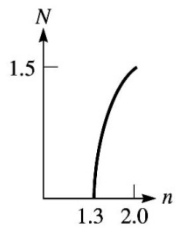
56. $p = \dfrac{0.05(1 + m)}{m}$

m	p (m)
0.1	0.55
0.2	0.3
0.3	0.216
0.4	0.175



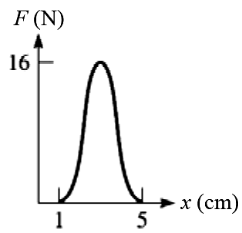
57. $N = \sqrt{n^2 - 1.69}$

n	N
1.3	0
1.5	0.748
1.7	1.095
2.0	1.520



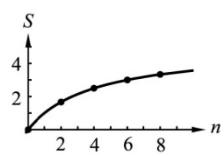
58. $F = x^4 - 12x^3 + 46x^2 - 60x + 25$

x (cm)	F (N)
1	0
1.5	3.1
3	16
3.5	14.1
4	9
5	0



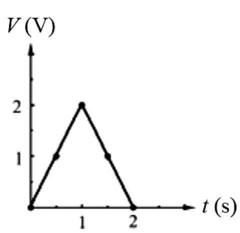
59. $S = \dfrac{5n}{4+n}$

n	S
0	0
2	$5/3$
4	$5/2$
6	3
8	$10/3$



60. $V = \begin{cases} 2t, & 0 \le t < 1 \\ 4 - 2t, & 1 \le t \le 2 \end{cases}$

t (s)	V (V)
0	0
$1/2$	1
1	2
$3/2$	1
2	0



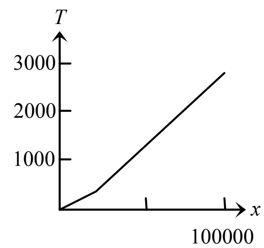
61. $(1, 2)$ on the graph means if we evaluate the function at $x = 1$ it gives $f(1) = 2$ which says nothing about $f(2)$, the function evaluated at $x = 2$, which may or may not equal 1.

62. $f(x) = 2x^2 + 0.5$

$$A = lw$$
$$A = f(0.5)(0.6 - 0.5)$$
$$A = (2 \cdot 0.5^2 + 0.5)(0.1)$$
$$A = 1(0.1)$$
$$A = 0.1$$

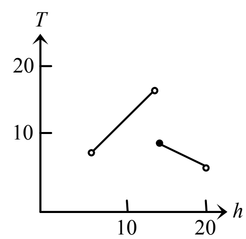
63. $T = \begin{cases} 0.02x, & 0 < x \le 20000 \\ 400 + 0.03(x - 20000), & x > 20000 \end{cases}$

x	T
10000	200
20000	400
30000	700
50000	1300
70000	1900
90000	2500



64. $T = \begin{cases} 2 + h, & 6 < h < 14 \\ 16 - 0.5h, & 14 \le h < 20 \end{cases}$

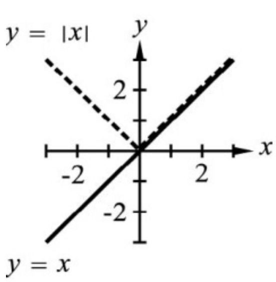
h (hours)	T (°C)
8	10
10	12
12	14
14	9
16	8
18	7



65.

x	y
−2	2
−1	1
0	0
1	1
2	2

$y = x$ is the same as $y = |x|$ for $x \ge 0$.
$y = |x|$ is the same as $y = -x$ for $x < 0$.

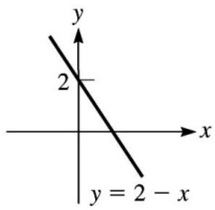


For negative values of x, $y = |x|$ becomes $y = -x$.

66. The graphs differ because the absolute value does not allow the graph to go below the x-axis.

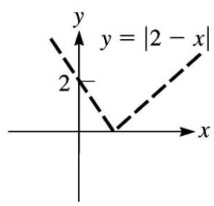
$y = 2 - x$

x	y
–1	3
0	2
1	1
2	0
3	–1



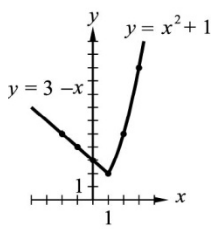
$y = |2 - x|$

x	y
–2	4
1	1
2	0
3	1



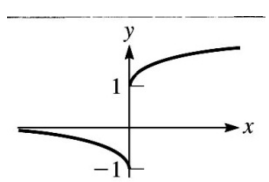
67. $f(x) = \begin{cases} 3 - x & \text{for } x < 1 \\ x^2 + 1 & \text{for } x \geq 1 \end{cases}$

x	y
–2	5
–1	4
0	3
1	2
2	5
3	10



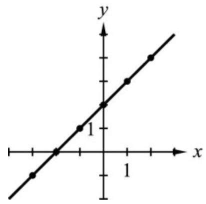
68. $f(x) = \begin{cases} \dfrac{1}{x-1} & x < 0 \\ \sqrt{x+1} & x \geq 0 \end{cases}$

x	y
–2	–0.3
–1	–0.5
–0.1	–0.9
0	1
1	1.4
3	2



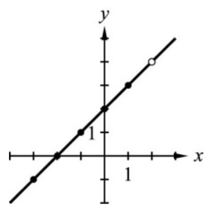
69. **(a)** $y = x + 2$

x	y
−3	−1
−2	0
−1	1
0	2
1	3
2	4



(b) $y = \dfrac{x^2 - 4}{x - 2}$

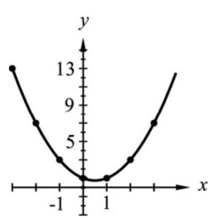
x	y
−3	−1
−2	0
−1	1
0	2
1	3
2	undefined



The graphs of $y = x + 2$ and $y = \dfrac{x^2 - 4}{x - 2}$ are identical except the second curve is undefined at a single point $x = 2$. It turns out that if you factor the second function (see Chapter 6), it will reduce to the first function everywhere except at $x = 2$ where it is undefined.

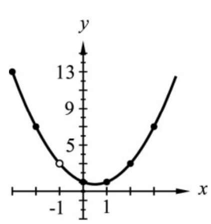
70. **(a)** $y = x^2 - x + 1$

x	y
−3	13
−2	7
−1	3
0	1
1	1
2	3



(b) $y = \dfrac{x^3 + 1}{x + 1}$

x	y
−3	13
−2	7
−1	undefined
0	1
1	1
2	3



The graphs of $y = x^2 - x + 1$ and $y = \dfrac{x^3 + 1}{x + 1}$ are identical except the second curve is undefined at a single point $x = -1$. It turns out that if you factor the second function (see Chapter 6), it will reduce to the first function everywhere except at $x = -1$ where it is undefined.

71. The graph passes the vertical line test and is, therefore, a function.

72. Some vertical lines will intercept the graph at multiple points, when $x < 0$. The graph is that of a relation.

73. No. Some vertical lines will intercept the graph at multiple points when $x > 0$. The graph is that of a relation.

74. Any vertical line will intercept the graph at only one point. The graph is that of a function.

3.5 Graphs on the Graphing Calculator

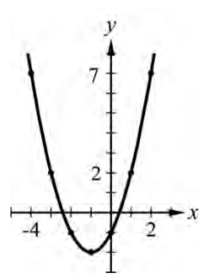
1. $x^2 + 2x = 1$

$x^2 + 2x - 1 = 0$

Graph the following function, and estimate solutions.

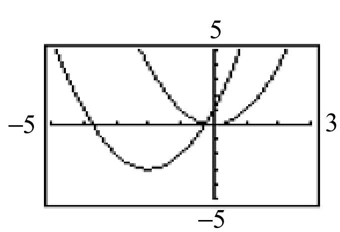
$y = x^2 + 2x - 1$

x	y
−4	7
−3	2
−2	−1
−1	−2
0	−1
1	2
2	7

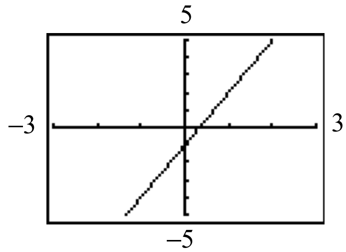


$x = -2.4$, $x = 0.4$

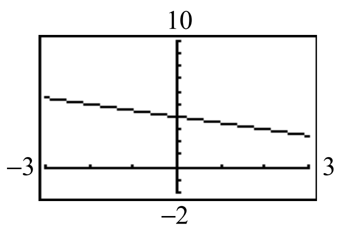
2. To shift the function $y = x^2$ two units left and three units down, add 2 to x and add -3 to the resulting function, thus, $y = (x+2)^2 - 3$ is the required equation. Shown are the graphs of both $y = x^2$ and $y = (x+2)^2 - 3$



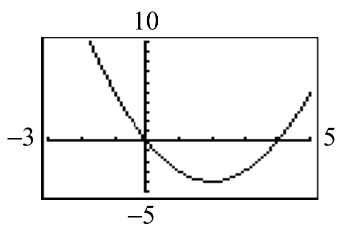
3. $y = 3x - 1$



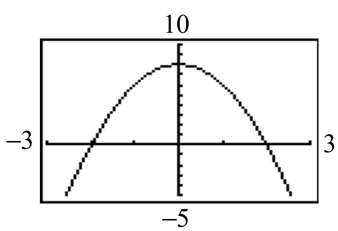
4. $y = 4 - 0.5x$



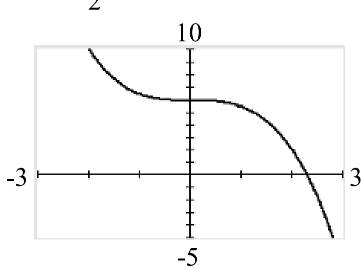
5. $y = x^2 - 4x$



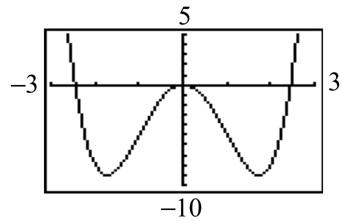
6. $y = 8 - 2x^2$



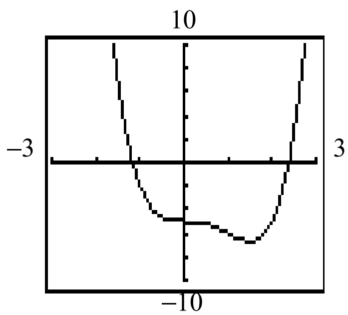
7. $y = 6 - \frac{1}{2}x^3$



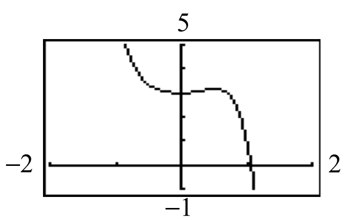
8. $y = x^4 - 6x^2$



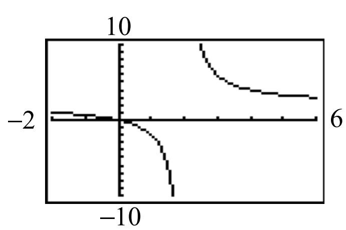
9. $y = x^4 - 2x^3 - 5$



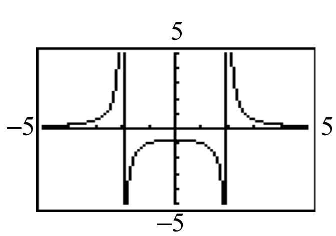
10. $y = x^2 - 3x^5 + 3$



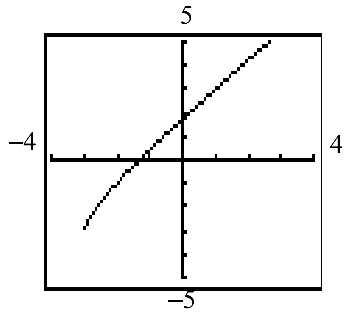
11. $y = \dfrac{2x}{x-2}$



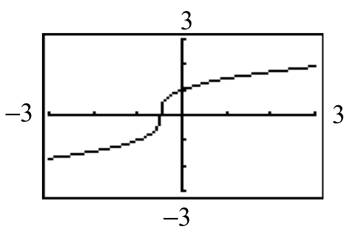
12. $y = \dfrac{3}{x^2 - 4}$



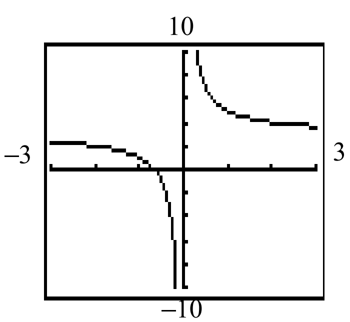
13. $y = x + \sqrt{x+3}$



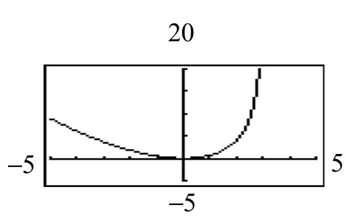
14. $y = \sqrt[3]{2x+1}$



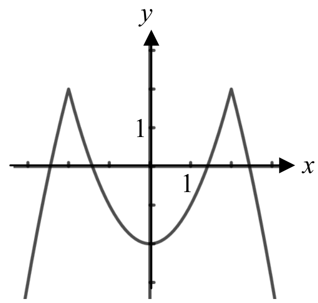
15. $y = 3 + \dfrac{2}{x}$



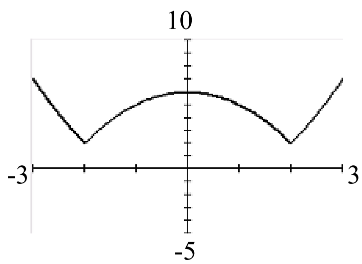
16. $y = \dfrac{x^2}{\sqrt{3-x}}$



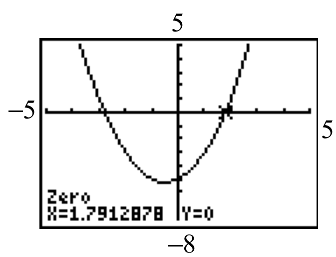
17. $y = 2 - \left|x^2 - 4\right|$

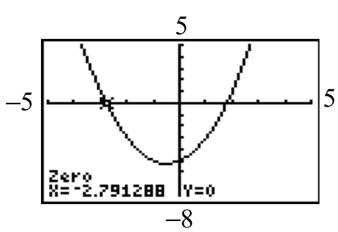


18. $y = \left|4 - x^2\right| + 2$



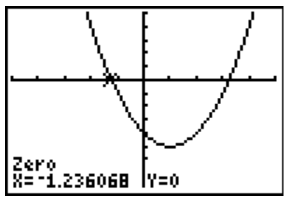
19. To solve $x^2 + x - 5 = 0$, graph $y = x^2 + x - 5$ and use the zero feature to solve.

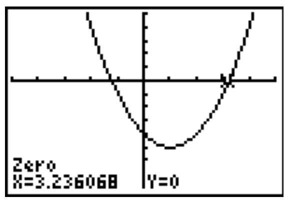




Solutions $x = -2.791$, $x = 1.791$

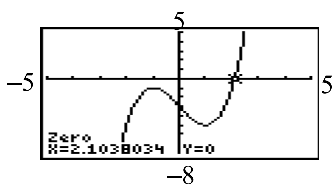
20. To solve $v^2 - 2v - 4 = 0$, graph $y = x^2 - 2x - 4$ and use the zero feature to solve.





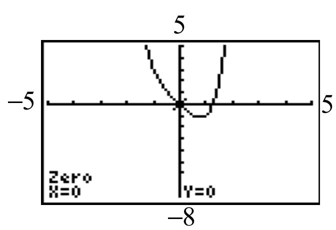
Solutions $v = -1.236$, $v = 3.236$

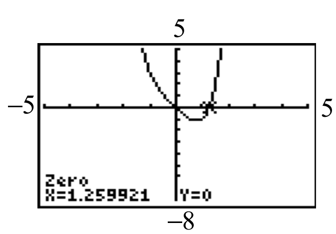
21. To solve $x^3 - 3 = 3x$ or $x^3 - 3x - 3 = 0$, graph $y = x^3 - 3x - 3$ and use the zero feature to solve.



Solution $x = 2.104$

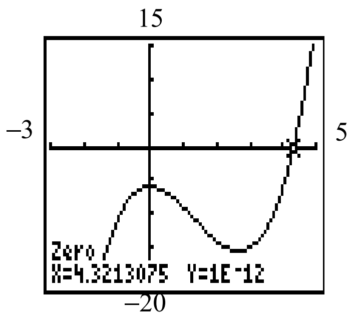
22. To solve $x^4 - 2x = 0$, graph $y = x^4 - 2x$ and use the zero feature to solve.





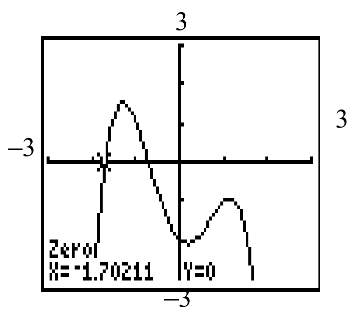
Solutions $x = 0.000$, $x = 1.260$

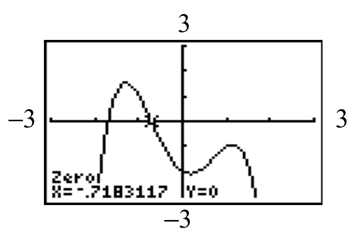
23. To solve $s^3 - 4s^2 = 6$, or $s^3 - 4s^2 - 6 = 0$, graph $y = x^3 - 4x^2 - 6$ and use the zero feature to solve.



Solution $s = 4.321$

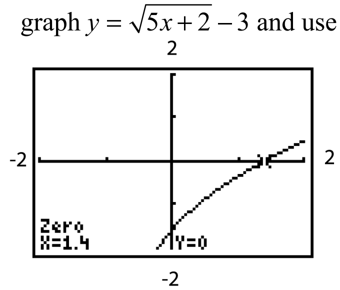
24. To solve $3x^2 - x^4 = 2 + x$, or $-x^4 + 3x^2 - x - 2 = 0$, graph $y = -x^4 + 3x^2 - x - 2$ and use the zero feature to solve.





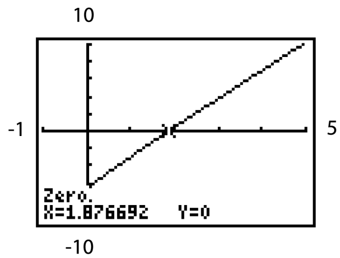
Solutions $x = -1.702,\ x = -0.718$

25. To solve $\sqrt{5R+2} = 3$ or $\sqrt{5R+2} - 3 = 0$ graph $y = \sqrt{5x+2} - 3$ and use the zero feature to solve.



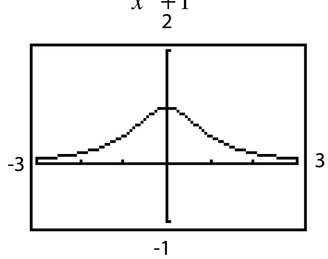
Solution $R = 1.400$

26. To solve $\sqrt{x}+3x=7$ or $\sqrt{x}+3x-7=0,$
graph $y=\sqrt{x}+3x-7$ and use the zero feature to solve.



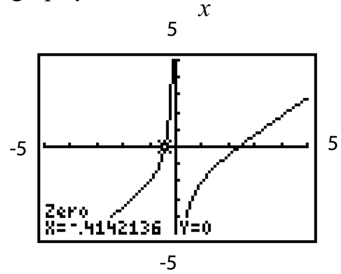
Solution $x=1.877$

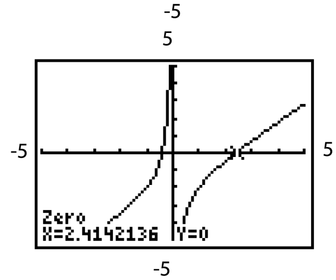
27. To solve $\dfrac{1}{x^2+1}=0,$ graph $y=\dfrac{1}{x^2+1}.$



Since the graph does not cross the x-axis, the equation
$\dfrac{1}{x^2+1}=0$ has no solution.

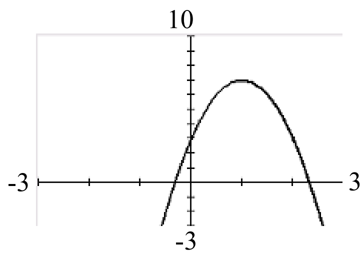
28. To solve $T-2=\dfrac{1}{T}$ or $T-2-\dfrac{1}{T}=0,$
graph $y=x-2-\dfrac{1}{x}$ and use the zero feature to solve.



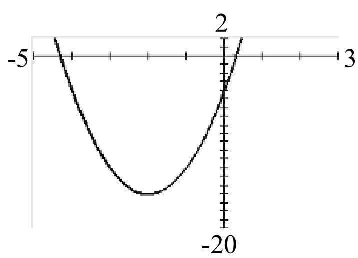


Solutions $T=-0.414,\ T=2.414$

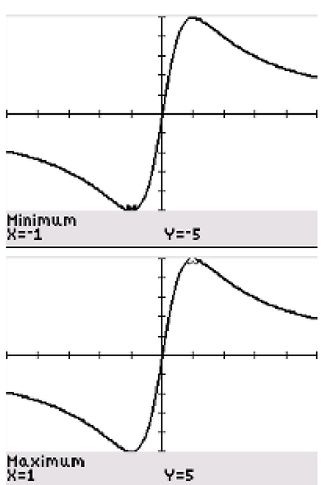
29. From the graph, $y = -4x^2 + 8x + 3$ has
Range: all real values $y \le 7$.



30. From the graph, $y = 3x^2 + 12x - 4$ has
Range: all real values $y \ge -16$.

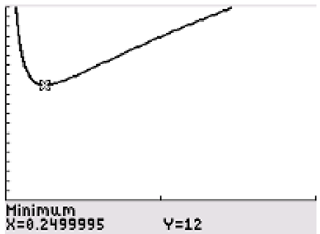


31. From the graph, $y = \dfrac{10x}{1+x^2}$ has
Range: all real values $-5 \le y \le 5$.



32. From the graph, $y = 16\sqrt{x} + \frac{1}{x}$ has

Range: all real values $y \geq 12$.

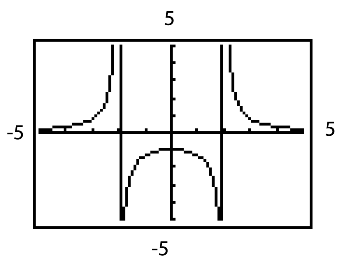


33. From the graph, $y = \frac{4}{x^2 - 4}$ has

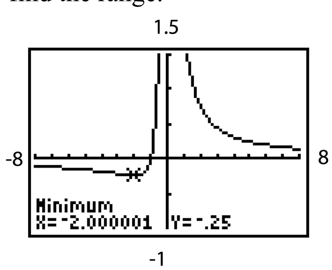
Range: all real values $y > 0$ when $x < -2$ or $x > 2$

Range: all real values $y \leq -1$ when $-2 < x < 2$

$x = \pm 2$ are vertical asymptotes for the function.



34. Graph $y = \frac{x+1}{x^2}$ and use the minimum feature to find the range.

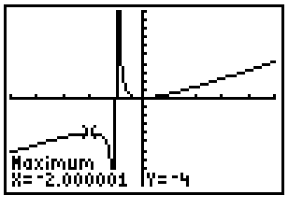


Range: all real values $y \geq -0.25$, or $[-0.25, \infty)$

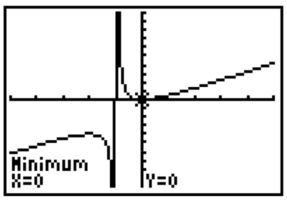
$x = 0$ is a vertical asymptote for the function

35. Graph $y = \dfrac{x^2}{x+1}$ and use the minimum and maximum feature to find the range.

10

-5  5

10

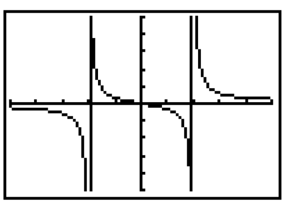
-5  5

-10

Range: all real values $y \le -4$ when $x < -1$
Range: all real values $y \ge 0$ when $x > -1$
$x = -1$ is a vertical asymptote for the function.

36. Graph $y = \dfrac{x}{x^2 - 4}$.

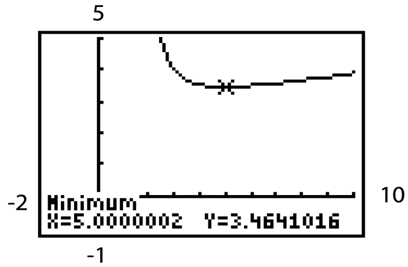
5

-5  5

-5

Range: all real numbers $y < 0$ when $x < -2$
Range: all real numbers when $-2 < x < 2$
Range: all real numbers $y > 0$ when $x > 2$
$x = \pm 2$ are vertical asymptotes for the function

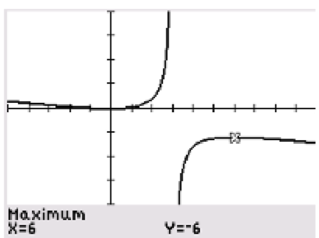
37. To find range of $Y(y)=\dfrac{y+1}{\sqrt{y-2}}$ graph $y=\dfrac{x+1}{\sqrt{x-2}}$ on the graphing calculator and use the minimum feature.

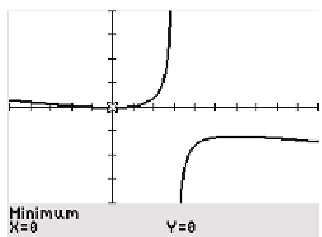


Range: all real values $Y(y)\geq 3.464$

$x=2$ is a vertical asymptote for the function.

38. To find range of $f(n)=\dfrac{n^2}{6-2n}$ graph $y=\dfrac{x^2}{6-2x}$.

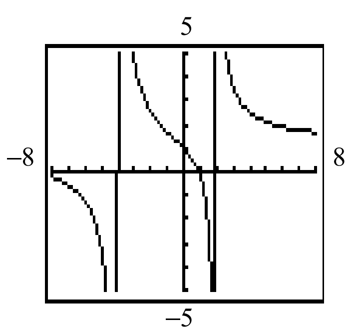




Range: all real numbers $f(n)\leq -6$ or $f(n)\geq 0$ using the maximum and minimum graphing tool on the calculator.

39. To find range of $f(D)=\dfrac{D^2+8D-8}{D(D-2)+4(D-2)}=\dfrac{D^2+8D-8}{(D+4)(D-2)}$,

graph $y=\dfrac{x^2+8x-8}{(x+4)(x-2)}$.



Range: all real numbers $y<1$ when $x<-4$

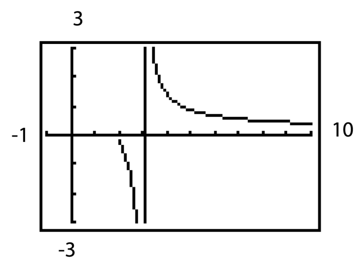
Range: all real numbers when $-4<x<2$

Range: all real numbers $y>1$ when $x>2$

$y=1$ is a horizontal asymptote for the function

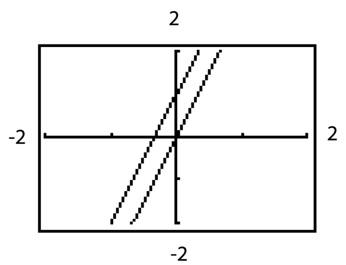
$x=-4$ and $x=2$ are vertical asymptotes for the function

40. To find range of $g(x)=\dfrac{\sqrt{x-2}}{x-3}$, graph $y=\dfrac{\sqrt{x-2}}{x-3}$.

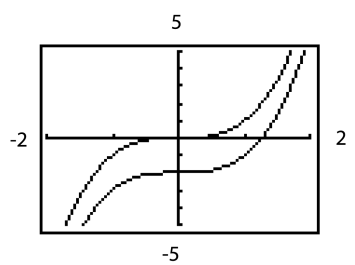


Range: all real numbers $g(x) \le 0$ when $2 \le x < 3$
Range: all real numbers $g(x) > 0$ when $x > 3$
$x = 3$ is a vertical asymptote for the function

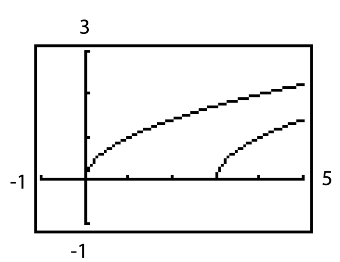
41. function: $y = 3x$
function shifted up 1: $y = 3x + 1$



42. function: $y = x^3$
function shifted down 2: $y = x^3 - 2$

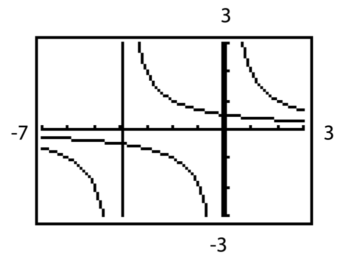


43. function: $y = \sqrt{x}$
function shifted right 3: $y = \sqrt{x-3}$



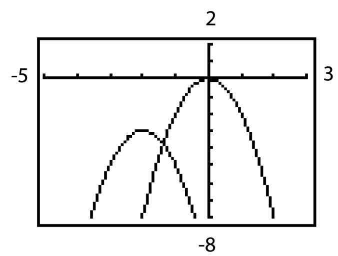
44. function: $y=\frac{2}{x}$

function shifted left 4: $y=\frac{2}{x+4}$



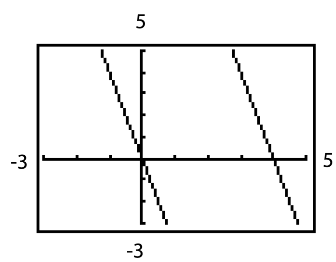
45. function: $y=-2x^2$

function shifted down 3, left 2: $y=-2(x+2)^2-3$



46. function: $y=-4x$

function shifted up 4, right 3: $y=-4(x-3)+4$

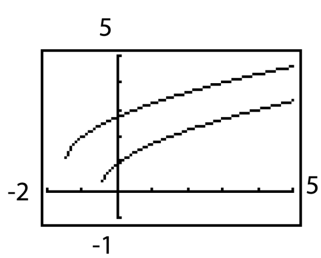


47. function: $y=\sqrt{2x+1}$

function shifted up 1, left 1:

$y=\sqrt{2(x+1)+1}+1$

$y=\sqrt{2x+3}+1$

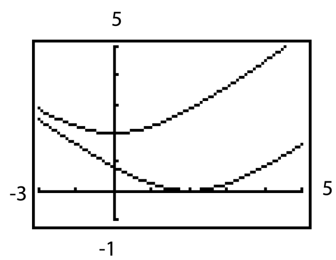


48. function: $y=\sqrt{x^2+4}$

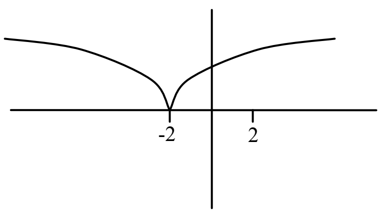
function shifted down 2, right 2:

$y=\sqrt{(x-2)^2+4}-2$

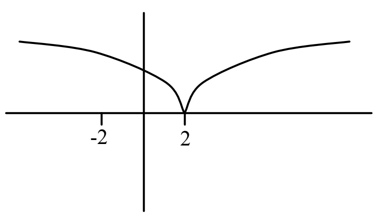
$y=\sqrt{x^2-4x+8}-2$



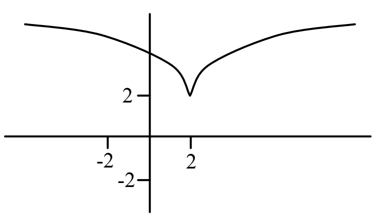
49.



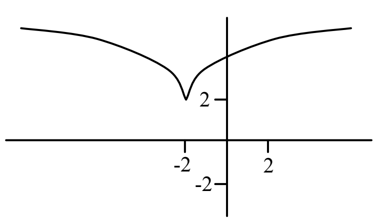
50.



51.



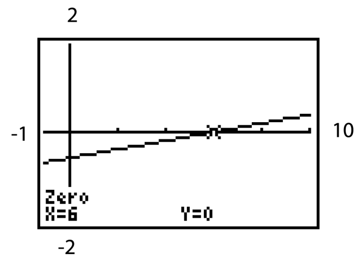
52.



53. $i = 0.01v - 0.06,$ graph $y = 0.01x - 0.06$

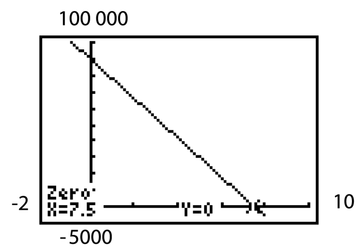
We need to solve for $i = 0$, so use the zero feature to solve.

From the graph, $v = 6.00$ V for $i = 0$.



54. $V = 90\ 000 - 12\ 000t$, graph $y = 90\ 000 - 12\ 000x$

and use the zero feature to solve.



The computer will be fully depreciated after 7.50 yrs.

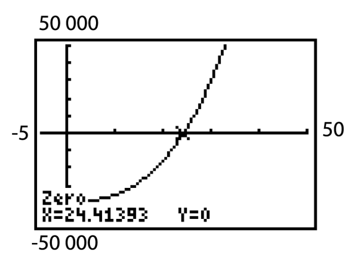
55. Let s = length of the inside edge of first cooler

$s^3 + (s+5)^3 = 40,000$

$s^3 + (s+5)^3 - 40,000 = 0$

To solve, graph $y = x^3 + (x+5)^3 - 40,000$

and use the zero feature to solve.

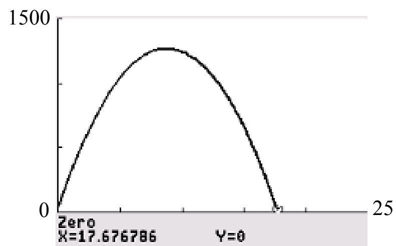


$s = 24.4$ cm

$s + 5 = 29.4$ cm

The edges of the coolers are 24.4 cm and 29.4 cm.

56. $h = 50 + 280t - 16t^2$, at ground level $h = 0$,
so graph $y = 50 + 280x - 16x^2$
and use the zero feature to solve.



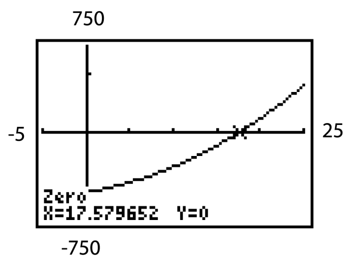
The rocket will be at ground level after 17.7 s.

57.

$$\begin{aligned} &\text{Let } w = \text{width of the panel} \\ &\text{Let } l = w + 12 \\ &A = lw \\ &520 = (w+12)w \\ &520 = w^2 + 12w \end{aligned}$$

$w^2 + 12w - 520 = 0$

So, graph $y = x^2 + 12x - 520$ and use the zero feature to solve.



The approximate dimensions are $w = 17.6$ cm, $l = 29.6$ cm.

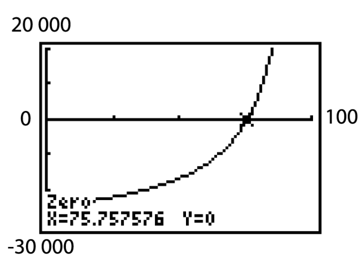
58.

$$C = \frac{8000x}{100 - x}$$

$$25\ 000 = \frac{8000x}{100 - x}$$

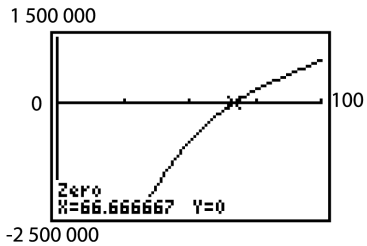
$$0 = \frac{8000x}{100 - x} - 25\ 000$$

graph $y = \dfrac{8000x}{100 - x} - 25\ 000$ and use the zero feature to solve.



75.8 % of pollutants can be removed for \$25 000.

59. To solve $9x^3 - 2400x^2 + 240\ 000x - 8\ 000\ 000 = 0$,
graph $y = 9x^3 - 2400x^2 + 240\ 000x - 8\ 000\ 000$
and use the zero feature to solve.

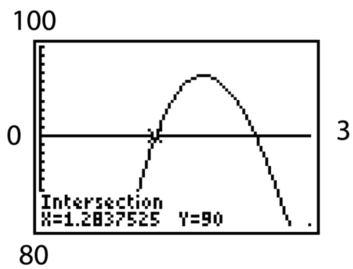


The light sources are 66.7 m apart.

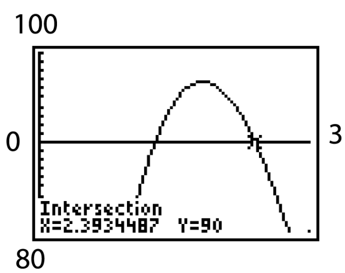
60. A length of $2x$ is removed from the width and length,
and a height of x results when the edges are folded up.

$$V = x(10-2x)(12-2x)$$
$$90 = x(10-2x)(12-2x)$$
$$0 = x(10-2x)(12-2x)-90$$

graph $y = x(10-2x)(12-2x)-90$ and use the zero feature to solve.

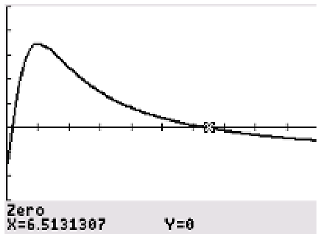


These two graphs have the wrong Y-scale, the function graphed has omitted the -90 term, so the results are 90 units too high for Y. The answers are correct for the X-variable.



The storage bin will hold 90.0 cm^3 for $x = 1.28$ cm or $x = 2.39$ cm

61. Graph $y = \dfrac{10.0x}{x^2 + 1.00} - 1.50$
and use the zero feature to solve.

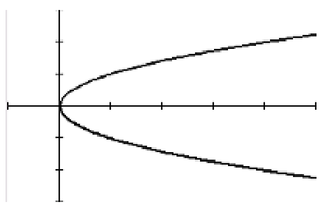


From the graph $x = 6.51$ hours.
We ignore the zero that is between 0 and 1 since that corresponds to when the concentration first increases above 1.50 mg/L.

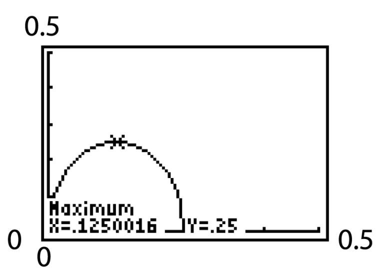
62. $y^2 = x$

$y = \pm\sqrt{x}$

Graph both $y_1 = \sqrt{x}$ and $y_2 = -\sqrt{x}$.



63. $s = \sqrt{t - 4t^2}$, graph $y = \sqrt{x - 4x^2}$ and use the maximum feature to solve.

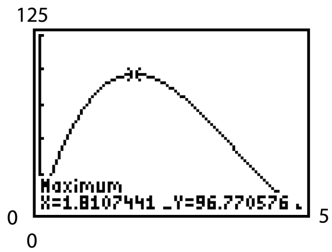


The maximum cutting speed is 0.250 cm/min.

64. $V = x(10-2x)(12-2x)$

graph $y = x(10-2x)(12-2x)$

and use the maximum feature to solve.

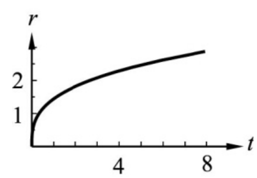


The maximum possible capacity is 96.8 cm^3,

if the size of the cutouts is $x = 1.81$ cm.

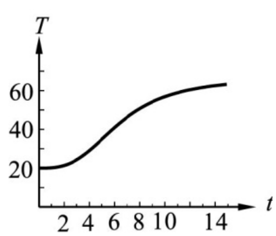
65. **(a)** Since V is increasing at a constant rate and $V = \frac{4}{3}\pi r^3$ it seems reasonable that r^3 should be increasing at a constant rate. Thus, r should be proportional to the cube root of time, $r = k\sqrt[3]{t}$. The graph will look similar to the graph shown below.

(b) $r = \sqrt[3]{3t}$ would be a typical situation.

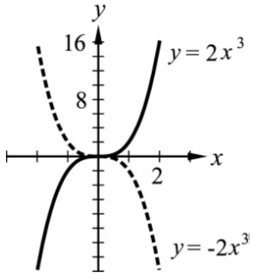


66. **(a)** It seems reasonable that the water would have essentially a constant temperature for period of time, followed by a rising temperature, and then a leveling off of temperature. The graph is similar to the one below.

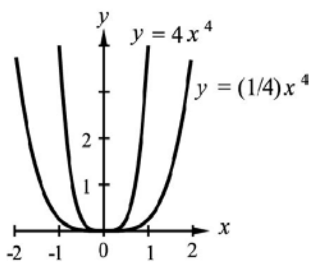
(b) The graph of $T = \dfrac{t^3 + 80}{0.015t^3 + 4}$, shown below, reflects such a situation.



67. Comparing the curve with a positive c to the curve with a negative c they are reflected about the x-axis (every positive value becomes negative and vice-versa).

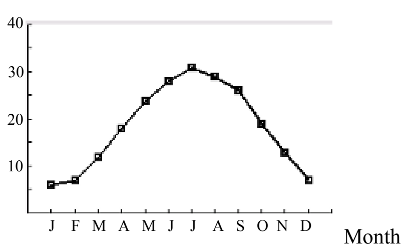


68. As shown in the graph below, a larger c produces a narrower graph (i.e. the function rises more rapidly than compared to a smaller c).

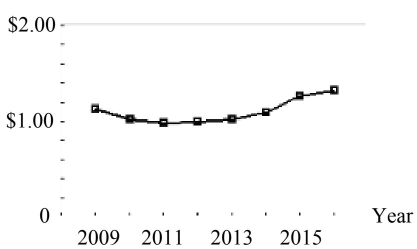


3.6 Graphs of Functions Defined by Tables of Data

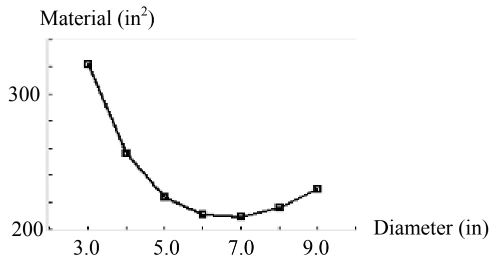
1.



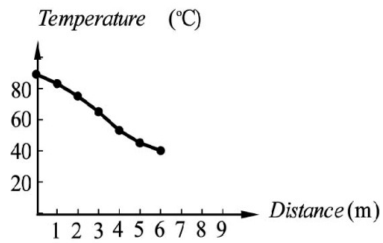
2.



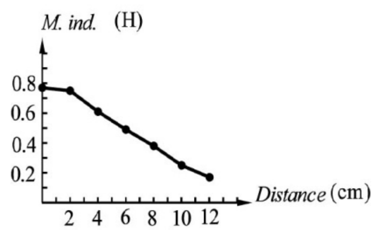
3.


Material (in²)
300
200
3.0
5.0
7.0
9.0
Diameter (in)

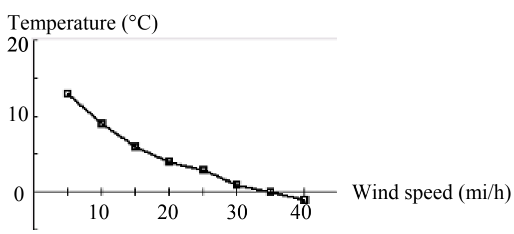
4.


Temperature (°C)
80
60
40
20
1 2 3 4 5 6 7 8 9
Distance (m)

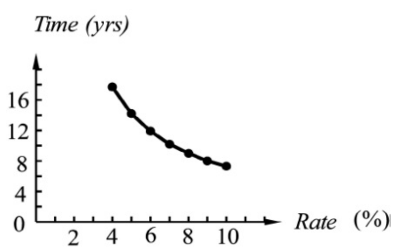
5.


M. ind. (H)
0.8
0.6
0.4
0.2
2 4 6 8 10 12
Distance (cm)

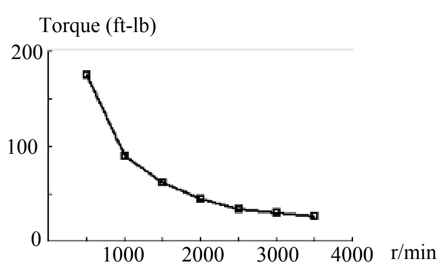
6.


Temperature (°C)
20
10
0
10
20
30
40
Wind speed (mi/h)

7.


Time (yrs)
16
12
8
4
0
2 4 6 8 10
Rate (%)

8.


Torque (ft-lb)
200
100
0
1000
2000
3000
4000
r/min

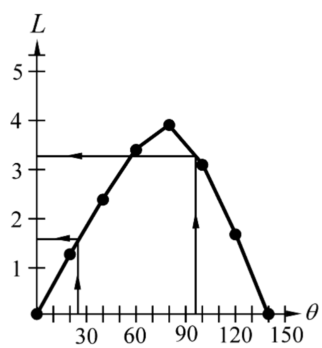
9. Estimate 0.3 of the interval between 4 and 5 on the t-axis and mark it. Draw a line vertically from this point to the graph. From the point where it intersects the graph, draw a horizontal line to the T-axis which it crosses at 132°C. Therefore for $t = 4.3$ min, $T = 132^\circ$C

10. Estimate 0.8 of the interval between 1 and 2 on the t-axis and mark it. Draw a line vertically from this point to the graph. From the point where it intersects the graph, draw a horizontal line to the T-axis which it crosses at 139.4°C.
Therefore for $t = 1.8$ min, $T = 139.4^\circ$C.

11. Draw a line horizontally from $T = 145.0^\circ$C to the point where it intersects the graph. Draw a vertical line from this point to the t-axis, which it crosses between 0 and 1. Estimate $t = 0.7$ min.
So, for $T = 145.0^\circ$C, $t = 0.7$ min

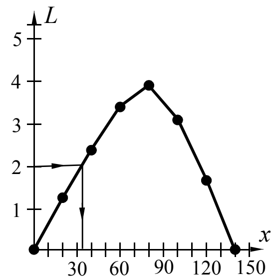
12. Draw a line horizontally from $T = 133.5^\circ$C to the point where it intersects the graph. Draw a vertical line from this point to the t-axis, which it crosses between 3 and 4.
Estimate for $T = 133.5^\circ$C, $t = 3.7$ min.

13.



(a) For $\theta = 25^\circ$, $L = 1.5$ mm
(b) For $\theta = 96^\circ$, $L = 3.2$ mm

14. For $L = 20$ mm, $\theta = 34^{\circ}$



15.

$$2\left[\begin{array}{l} 1.2\left[\begin{array}{ll}8.0 & 0.38\\ 9.2 & ?\end{array}\right]x \\ \quad 10.0 \quad 0.25\end{array}\right]-0.13$$

$$\frac{1.2}{2}=\frac{x}{-0.13}$$
$$x=\frac{(1.2)(-0.13)}{2}$$
$$x=-0.078$$
$$M.ind.=0.38+(-0.078)$$
$$M.ind.=0.30 \text{ H}$$

16.

$$5\left[\begin{array}{l} 2\left[\begin{array}{ll}10 & 9\\ 12 & ?\end{array}\right]x \\ \quad 15 \quad 6\end{array}\right]-3$$

$$\frac{2}{5}=\frac{x}{-3}$$
$$x=\frac{(2)(-3)}{5}$$
$$x=-1.2$$
$$T=9+(-1.2)=-7.8^{\circ}\text{F}$$

17.

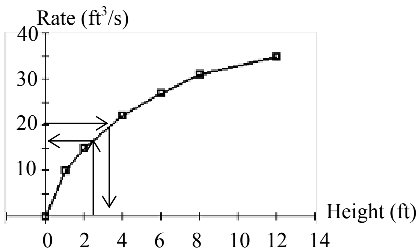
$$1.2\left[\begin{array}{l} 0.2\left[\begin{array}{ll}10.2 & 7\\ 10.0 & ?\end{array}\right]x \\ \quad 9.0 \quad 8\end{array}\right]1$$

$$\frac{0.2}{1.2}=\frac{x}{1}$$
$$x=\frac{0.2}{1.2}$$
$$x=0.1666$$
$$r=7+0.1666$$
$$r=7.2\%$$

18. $$500\left[\begin{array}{c} 300\left[\begin{array}{cc} 2000 & 45 \\ 2300 & ? \end{array}\right]x \\ \begin{array}{cc} 2500 & 34 \end{array} \end{array}\right]-11$$

$$\frac{300}{500}=\frac{x}{-11}$$
$$x=-\frac{11(300)}{500}$$
$$x=-6.6$$
$$T=45+(-6.6)$$
$$T=38\ \text{N}\cdot\text{m}$$

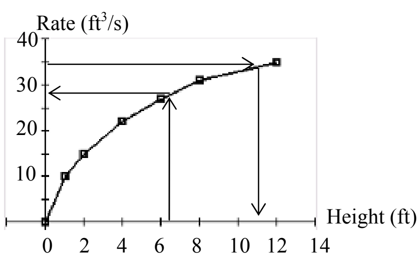
19.



(a) For $R=20\ \text{ft}^3/\text{s},\ H=3.4\ \text{ft}$

(b) For $H=2.5\ \text{ft},\ R=17\ \text{ft}^3/\text{s}$

20.



(a) For $R=34\ \text{ft}^3/\text{s},\ H=11\ \text{ft}$

(b) For $H=6.4\ \text{ft},\ R=28\ \text{ft}^3/\text{s}$

21. $$1.0\left[\begin{array}{c} 0.7\left[\begin{array}{cc} 1.0 & 10 \\ 1.7 & ? \end{array}\right]x \\ \begin{array}{cc} 2.0 & 15 \end{array} \end{array}\right]5$$

$$\frac{0.7}{1.0}=\frac{x}{5}$$
$$x=3.5$$
$$R=10+3.5$$
$$R=13.5\ \text{ft}^3/\text{s}$$

22. $$5\left[\begin{array}{l} 3\left[\begin{array}{ll} 22 & 4.0 \\ 25 & ? \end{array}\right] x \\ \quad\;\; \begin{array}{ll} 27 & 6.0 \end{array} \end{array}\right] 2.0$$

$$\frac{3}{5}=\frac{x}{2}$$
$$x=1.2$$
$$H=4.0+1.2$$
$$H=5.2 \text{ ft}$$

23. $$10\left[\begin{array}{l} 6\left[\begin{array}{ll} 30 & 0.30 \\ 36 & ? \end{array}\right] x \\ \quad\;\; \begin{array}{ll} 40 & 0.37 \end{array} \end{array}\right] 0.07$$

$$\frac{6}{10}=\frac{x}{0.07}$$
$$x=\frac{6(0.07)}{10}$$
$$x=0.042$$
$$f=0.30+0.042$$
$$f=0.34$$

24. $$10\left[\begin{array}{l} 2\left[\begin{array}{ll} 50 & 0.44 \\ 52 & ? \end{array}\right] x \\ \quad\;\; \begin{array}{ll} 60 & 0.50 \end{array} \end{array}\right] 0.06$$

$$\frac{2}{10}=\frac{x}{0.06}$$
$$x=\frac{2(0.06)}{10}$$
$$x=0.012$$
$$f=0.44+0.012$$
$$f=0.45$$

25. $$0.05\left[\begin{array}{l} 0.03\left[\begin{array}{ll} 0.56 & 70 \\ 0.59 & ? \end{array}\right] x \\ \qquad\;\; \begin{array}{ll} 0.61 & 80 \end{array} \end{array}\right] 10$$

$$\frac{0.03}{0.05}=\frac{x}{10}$$
$$x=\frac{0.03(10)}{0.05}$$
$$x=6$$
$$A=70+6$$
$$A=76 \text{ m}^2$$

26.

$$0.08\left[\begin{array}{ll} 0.05\left[\begin{array}{ll} 0.22 & 20 \\ 0.27 & ? \end{array}\right]x & \\ 0.30 & 30 \end{array}\right]10$$

$$\frac{0.05}{0.08}=\frac{x}{10}$$

$$x=\frac{0.05(10)}{0.08}$$

$$x=6.25$$

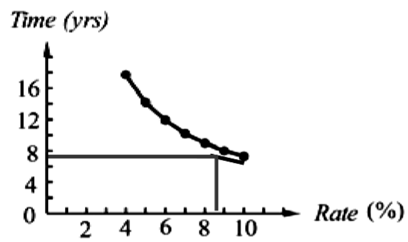
$$A=20+6.25$$

$$A=26\ \text{m}^2$$

27. The graph is extended using a straight line segment.

$T\approx 130.3^{\circ}\text{C}$ for $t=5.3$ min

28.



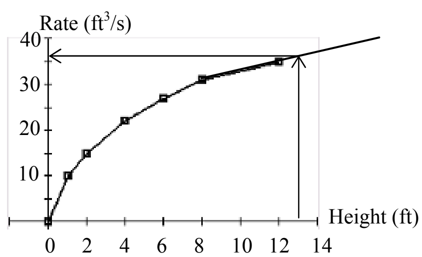
The graph is extended using a straight line segment.

The estimated value of time for $R=10.4\%$ is 7.0 years

29. The graph is extended using a straight line segment.

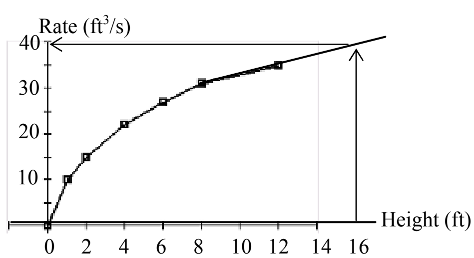
The estimated value of the rate of discharge is

$35.5\ \text{ft}^3/\text{s}$ for $H=13$ ft



30. The graph is extended using a straight line segment.

The estimated value of R is 39 ft^3/s for $H=16$ ft



Review Exercises

1. It is false that for any function $f(x)$, $f(-x) = -f(x)$. Consider $f(x) = x^2$. Then $f(-x) = (-x)^2 = x^2$ and $-f(x) = -x^2$ which is not equal to x^2 for any $x \neq 0$.

2. The statement is false. The domain includes $x = 1$ as well.

3. The statement is false. The point $(-2, 2)$ is in quadrant II.

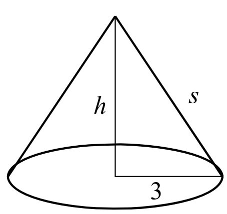
4. The statement is false. If $x = 4$ then $y = 2$ or $y = -2$. There is no single value of y that corresponds to this value of x.

5. The statement is true. If $(2, 0)$ is an x-intercept, then $f(2) = 0$ and so $2f(2) = 2(0) = 0$ as well.

6. The statement is true.

7.
$$A = \pi r^2$$
$$\text{If } r = 2 \text{ m/s} \times t$$
$$A(t) = \pi(2t)^2$$
$$A(t) = 4\pi t^2$$

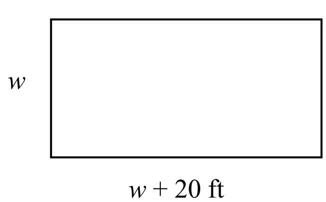
8.



$$s^2 = 3^2 + h^2$$
$$s^2 = 9 + h^2$$
$$s = \sqrt{9 + h^2}$$
$$S = \pi r s$$
$$S(h) = \pi(3)\sqrt{9 + h^2}$$
$$S(h) = 3\pi\sqrt{h^2 + 9}$$

9.
$$0.05x + 0.04y = 2000$$
$$0.04y = 2000 - 0.05x$$
$$y(x) = \frac{2000 - 0.05x}{0.04}$$

10.



$$C = \$20/\text{ft}\left(w + 20 \text{ ft}\right) + \$10/\text{ft}(w) + \$10/\text{ft}\left(w + 20 \text{ ft}\right) + \$10/\text{ft}\left(w\right)$$
$$C = \$20/\text{ft}\left(w\right) + \$400 + \$10/\text{ft}(w) + \$10/\text{ft}(w) + \$200 + \$10/\text{ft}(w)$$
$$C(w) = \$50/\text{ft}\left(w\right) + \$600$$

11. $f\left(x\right) = 7x - 5$
$$f\left(3\right) = 7\left(3\right) - 5 = 21 - 5 = 16$$
$$f\left(-6\right) = 7\left(-6\right) - 5 = -42 - 5 = -47$$

12. $g\left(I\right) = 8 - 3I$
$$g\left(\frac{1}{6}\right) = 8 - 3\left(\frac{1}{6}\right) = 8 - \frac{1}{2} = \frac{16}{16} - \frac{1}{2} = \frac{15}{2}$$
$$g\left(-4\right) = 8 - 3\left(-4\right) = 8 + 12 = 20$$

13. $H\left(h\right) = \sqrt{1 - 2h}$
$$H\left(-4\right) = \sqrt{1 - 2\left(-4\right)} = \sqrt{1 + 8} = \sqrt{9} = 3$$
$$H\left(2h\right) + 2 = \sqrt{1 - 2\left(2h\right)} + 2 = \sqrt{1 - 4h} + 2$$

14. $\phi\left(v\right) = \dfrac{6v - 9}{\left|v + 1\right|}$
$$\phi\left(-2\right) = \frac{6\left(-2\right) - 9}{\left|\left(-2\right) + 1\right|} = \frac{-12 - 9}{\left|-1\right|} = \frac{-21}{1} = -21$$
$$\phi\left(v + 1\right) = \frac{6\left(v + 1\right) - 9}{\left|\left(v + 1\right) + 1\right|} = \frac{6v + 6 - 9}{\left|v + 2\right|} = \frac{6v - 3}{\left|v + 2\right|}$$

15. $F\left(x\right) = x^3 + 2x^2 - 3x$
$$F\left(3 + h\right) - F\left(3\right) = \left(3 + h\right)^3 + 2\left(3 + h\right)^2 - 3\left(3 + h\right) - \left(\left(3\right)^3 + 2\left(3\right)^2 - 3\left(3\right)\right)$$
$$F\left(3 + h\right) - F\left(3\right) = (3 + h)(9 + 6h + h^2) + 2(9 + 6h + h^2) - 9 - 3h - (27 + 2(9) - 9)$$
$$F\left(3 + h\right) - F\left(3\right) = (27 + 18h + 3h^2 + 9h + 6h^2 + h^3) + 18 + 12h + 2h^2 - 9 - 3h - 27 - 18 + 9$$
$$F\left(3 + h\right) - F\left(3\right) = h^3 + 11h^2 + 36h$$

16. $f\left(x\right) = 3x^2 - 2x + 4$
$$\frac{f\left(x + h\right) - f\left(x\right)}{h} = \frac{3\left(x + h\right)^2 - 2\left(x + h\right) + 4 - \left(3x^2 - 2x + 4\right)}{h}$$
$$\frac{f\left(x + h\right) - f\left(x\right)}{h} = \frac{3(x^2 + 2xh + h^2) - 2x - 2h + 4 - 3x^2 + 2x - 4}{h}$$
$$\frac{f\left(x + h\right) - f\left(x\right)}{h} = \frac{3x^2 + 6xh + 3h^2 - 2h - 3x^2}{h}$$
$$\frac{f\left(x + h\right) - f\left(x\right)}{h} = \frac{6xh + 3h^2 - 2h}{h}$$
$$\frac{f\left(x + h\right) - f\left(x\right)}{h} = 6x + 3h - 2$$

17.
$$f(x) = 4 - 5x$$
$$f(2x) - 2f(x) = 4 - 5(2x) - 2(4 - 5x)$$
$$f(2x) - 2f(x) = 4 - 10x - 8 + 10x$$
$$f(2x) - 2f(x) = -4$$

18.
$$f(x) = 1 - 9x^2$$
$$\left[f(x)\right]^2 - f\left(x^2\right) = \left(1 - 9x^2\right)^2 - \left(1 - 9\left(x^2\right)^2\right)$$
$$\left[f(x)\right]^2 - f\left(x^2\right) = 1 - 18x^2 + 81x^4 - 1 + 9x^4$$
$$\left[f(x)\right]^2 - f\left(x^2\right) = 90x^4 - 18x^2$$

19.
$$f(x) = 8.07 - 2x$$
$$f(5.87) = 8.07 - 2(5.87) = 8.07 - 11.74 = -3.67$$
$$f(-4.29) = 8.07 - 2(-4.29) = 8.07 + 8.58 = 16.65 = 16.6$$

20.
$$g(x) = 7x - x^2$$
$$g(45.81) = 7(45.81) - (45.81)^2 = 320.67 - 2098.5561 = -1777.8861 = -1778$$
$$g(-21.85) = 7(-21.85) - (21.85)^2 = -152.95 - 477.4225 = -630.3725 = -630.4$$

21.
$$G(S) = \frac{S - 0.087629}{(3.0125)S}$$
$$G(0.17427) = \frac{0.17427 - 0.087629}{(3.0125)(0.17427)} = \frac{0.086641}{0.524988375} = 0.16503413 = 0.16503$$
$$G(0.053206) = \frac{0.053206 - 0.087629}{3.0125(0.053206)} = \frac{-0.034423}{0.160283075} = -0.214763785 = -0.21476$$

22.
$$h(t) = \frac{t^2 - 4t}{t^3 + 564}$$
$$h(8.91) = \frac{(8.91)^2 - 4(8.91)}{(8.91)^3 + 564} = \frac{79.3881 - 35.64}{707.347\ 971 + 564} = \frac{43.7481}{1271.347\ 971} = 0.034\ 410\ 799 = 0.0344$$
$$h(-4.91) = \frac{(-4.91)^2 - 4(-4.91)}{(-4.91)^3 + 564} = \frac{24.1081 + 19.64}{-118.370\ 771 + 564} = \frac{43.7481}{445.629\ 229} = 0.098\ 171\ 523 = 0.0982$$

23. $f(x) = x^4 + 1$

There are no restrictions to the value of x, so

Domain: all real numbers, or $(-\infty, \infty)$

Because x^4 is never negative, the minimum value of $f(x)$ is 1.

Range: all real numbers $f(x) \geq 1$, or $[1, \infty)$

24. $G(z)=\dfrac{-4}{z^3}$

To avoid a division by zero error, $z \neq 0$, so

Domain: all real numbers $z \neq 0$, or $(-\infty,0)$ and $(0,\infty)$

No value of z will produce $G(z)=0$

Range: all real numbers $G(z) \neq 0,$ or $(-\infty,0)$ and $(0,\infty)$

25. $g(t)=\dfrac{8}{\sqrt{t+4}}$

To avoid a division by zero error $t \neq -4$

To avoid a negative root, $t \geq -4$, so

Domain: all real numbers $t > -4$, or $(-4,\infty)$

No value of t will produce $g(t)=0,$ and if we consider only principal roots,

Range: all real numbers $g(t) > 0,$ or $(0,\infty)$

26. $F(y)=1-2\sqrt{y}$

To avoid a negative root, $y \geq 0$, so

Domain: all real numbers $y \geq 0$, or $[0,\infty)$

Using principal roots, no value of y will produce $F(y) > 1$

Range: all real numbers $F(y) \leq 1,$ or $(-\infty,\ 1]$

27. $f(n)=1+\dfrac{2}{(n-5)^2}$

To avoid a division by zero error, $n \neq 5$, so

Domain: all real numbers $n \neq 5$, or $(-\infty,5)$ and $(5,\infty)$

Since $(n-5)^2 > 0,$ for all $n \neq 5,$ no value of n will produce $f(n) \leq 1$

Range: all real numbers $f(n) > 1,$ or $(1,\infty)$

28. $F(x)=3-|x|$

There are no restrictions on the value of $x,$ so

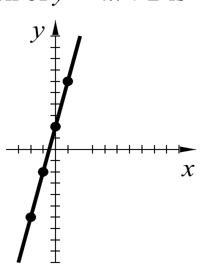
Domain: all real numbers or $(-\infty,\infty)$

No value of x will produce $F(x) > 3$

Range: all real numbers $F(x) \leq 3,$ or $(-\infty,\ 3]$

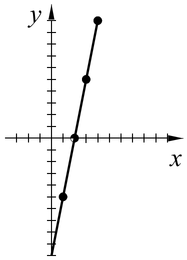
29. The graph of $y=4x+2$ is

x	y
-2	-6
-1	-2
0	2
1	6
2	10



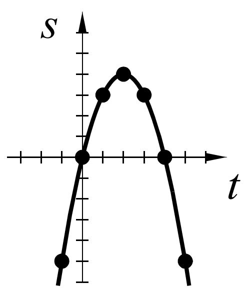
30. The graph of $y = 5x - 10$ is

x	y
1	−5
2	0
3	5
4	10



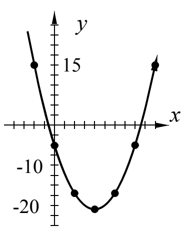
31. The graph of $s = 4t - t^2$ is

t	s
−1	−5
0	0
1	3
2	4
3	3
4	0
5	−5



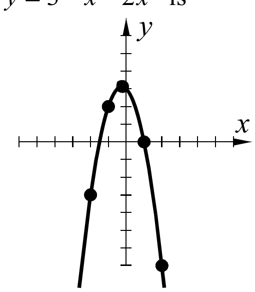
32. The graph of $y = x^2 - 8x - 5$ is

x	y
−1	4
0	−5
1	−12
2	−17
3	−20



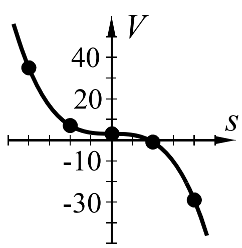
33. The graph of $y = 3 - x - 2x^2$ is

x	y
−3	−12
−2	−3
−1	2
0	3
1	0
2	−7



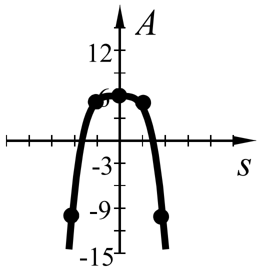
34. The graph of $V = 3 - 0.5s^3$ is

s	V
−4	35
−2	7
0	3
2	−1
4	−29



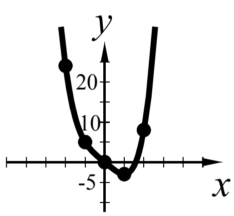
35. The graph of $A = 6 - s^4$ is

A	s
–2	–10
–1	5
0	6
1	5
0	–10



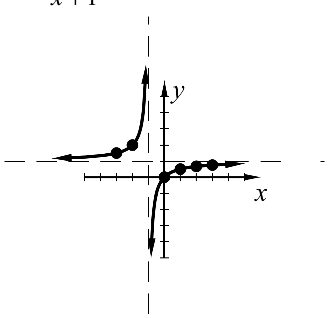
36. The graph of $y = x^4 - 4x$ is

x	y
–2	24
–1	5
0	0
1	–3
2	8



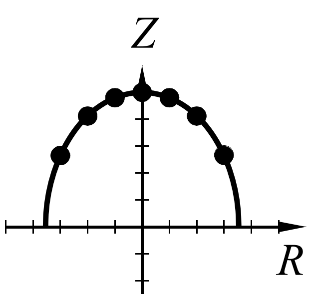
37. The graph of $y = \dfrac{x}{x+1}$ is

x	y
–3	1.5
–2	2
0	0
1	1 / 2
2	2 / 3
3	3 / 4

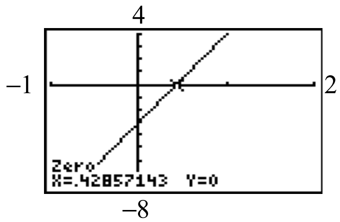


38. The graph of $Z = \sqrt{25 - 2R^2}$ is

R	Z
–3	$\sqrt{7}$
–2	$\sqrt{17}$
–1	$\sqrt{23}$
0	5
1	$\sqrt{23}$
2	$\sqrt{17}$
3	$\sqrt{7}$

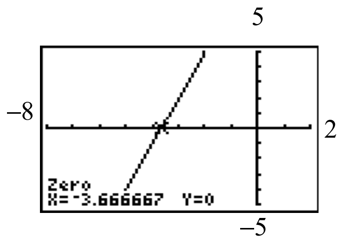


39. To solve $7x-3=0$, graph $y=7x-3$ and use the zero feature to solve.



Solution $x=0.43$

40. To solve $3x+11=0$, graph $y=3x+11$ and use the zero feature to solve.

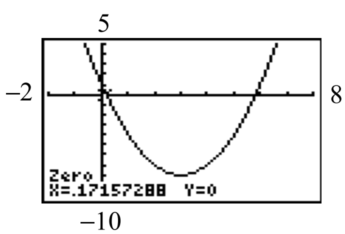


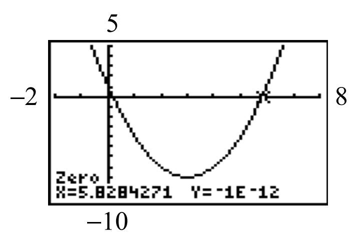
Solution $x=-3.67$

41. $x^2+1=6x$

$x^2-6x+1=0$

Graph $y=x^2-6x+1$ and use the zero feature to solve.



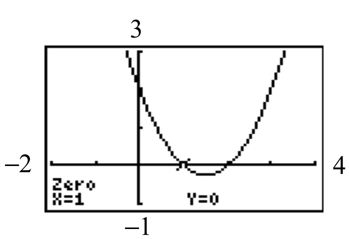


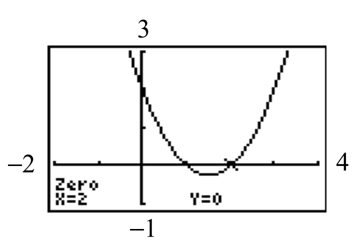
Solutions $x=0.17$ and $x=5.83$

42. $3t-2=t^2$

$t^2-3t+2=0$

Graph $y=x^2-3x+2$ and use the zero feature to solve.



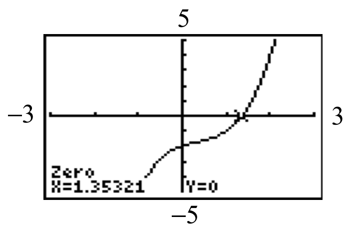


Solutions $x=1.00$ and $x=2.00$

43. $x^3 - x^2 = 2 - x$

$x^3 - x^2 + x - 2 = 0$

Graph $y = x^3 - x^2 + x - 2$ and use the zero feature to solve.

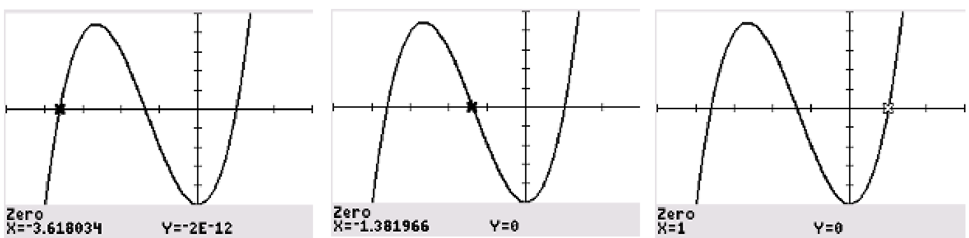


Solution $x = 1.35$

44. $5 - x^3 = 4x^2$

$x^3 + 4x^2 - 5 = 0$

Graph $y = x^3 + 4x^2 - 5$ and use the zero feature to solve.

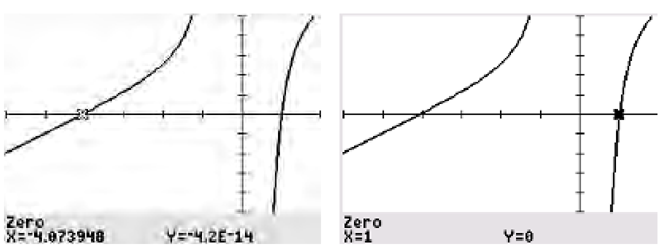


Solutions are $x = -3.62,\ x = -1.38,$ and $x = 1.$

45. $\dfrac{5}{v^3} = v + 4$

$v + 4 - \dfrac{5}{v^3} = 0$

Graph $y = x + 4 - \dfrac{5}{x^3}$ and use the zero feature to solve.

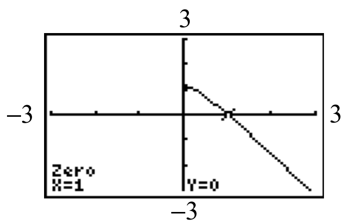


Solutions $x = -4.07$ and $x = 1.00$

46. $\sqrt{x} = 2x - 1$

$\sqrt{x} - 2x + 1 = 0$

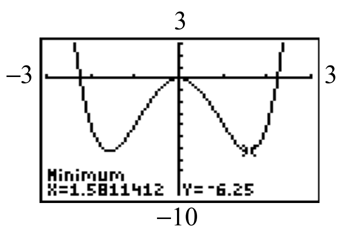
Graph $y = \sqrt{x} - 2x + 1$ and use the zero feature to solve.



Solution $x = 1.00$

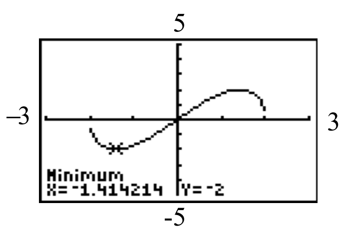
47. Graph $y = x^4 - 5x^2$ and use the minimum feature,

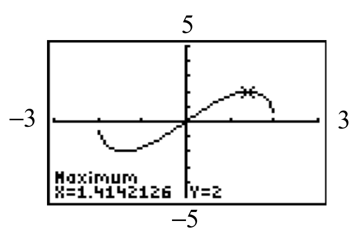
Range: all real values $y \geq -6.25$ or $[-6.25, \infty)$



48. Graph $y = x\sqrt{4 - x^2}$ and use the minimum and maximum feature.

Range: all real values $-2 \leq y \leq 2$ or $[-2, 2]$

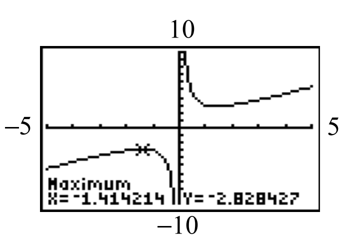


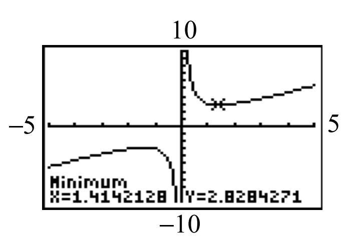


49. If $A = w + \dfrac{2}{w}$, then graph $y = x + \dfrac{2}{x}$ and use the minimum and maximum feature.

Range: all real values $A \leq -2.83$ when $w < 0$ or $(-\infty, -2.83]$

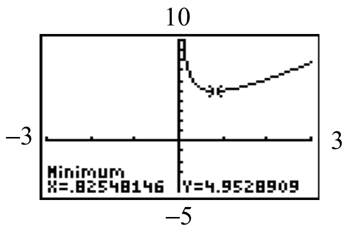
Range: all real values $A \geq 2.83$ when $w > 0$ or $[2.83, \infty)$





50. Graph $y = 2x + \dfrac{3}{\sqrt{x}}$ and use the minimum feature.

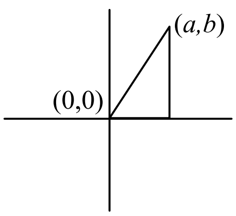
Range: all real values $y \geq 4.95$ or $[4.95, \infty)$



51. When a and b have opposite signs, then $A(a,\ b)$ and $B(b,\ a)$ will be in different quadrants.

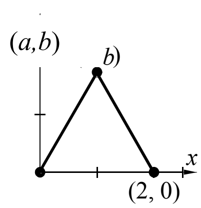
If $A(a,\ b) = A(2,\ -3)$ it is in Quadrant IV but $B(b,\ a) = B(-3,\ 2)$ is in Quadrant II.

52.



Using the Pythagorean theorem, the distance from $(0,\ 0)$ to $(a,\ b) = \sqrt{a^2 + b^2}$

53.



We know the x-component of the third vertex is halfway between
the other two points because it is an equilateral triangle, and the base is horizontal.

so $a = \dfrac{2-0}{2} = 1$

The y-component of the vertex is the b of the Pythagorean theorem,
and we know $c = 2$ since all sides of the triangle are the same length.

$b^2 = c^2 - a^2$

$b^2 = 2^2 - 1^2$

$b^2 = 4 - 1$

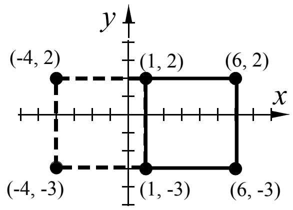
$b^2 = 3$

$b = \sqrt{3}$

$b = \pm\sqrt{3}$

$\left(1,\ \pm\sqrt{3}\right)$ are the coordinates of the third vertex

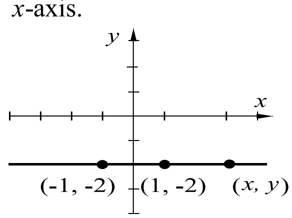
54.



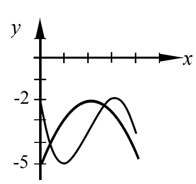
The length of a side of the square is $= 2-(-3)=5$
and the other vertices are $(6,\ 2),\ (6,\ -3)$ or $(-4,\ 2),\ (-4,\ -3)$

55. $\left|\dfrac{y}{x}\right|>0$ for $\dfrac{y}{x}\neq 0,$ so $y\neq 0$ to make the fraction nonzero,
and $x\neq 0$ to avoid a division by zero error.
$\left|\dfrac{y}{x}\right|>0$ for all values of $(x,\ y)$ that are not on x-axis or y-axis.

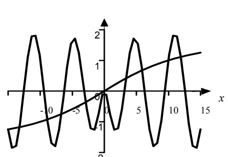
56. Since two points have y-coordinate of -2, then all points on the line have $y=-2$ for all values of x. This is a horizontal line 2 units below the x-axis.



57. There are many possibilities. Two are shown.



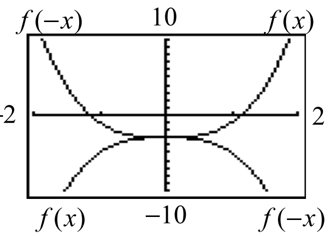
58. There are many possibilities. Two are shown.



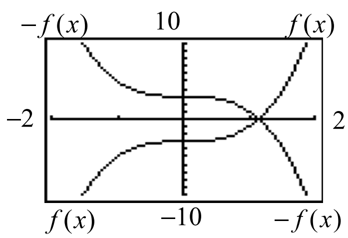
59. $y=\sqrt{x-1}$ shifted left 2 and up 1 is
$y=\sqrt{(x+2)-1}+1$
$y=\sqrt{x+1}+1$

60. $y = 3 - 2x$ shifted right 1 and down 3 is
$y = 3 - 2(x-1) - 3$
$y = -2x + 2$

61. $f(x) = 2x^3 - 3,$
$f(-x) = 2(-x)^3 - 3 = -2x^3 - 3.$
The graphs are reflections of each other across the y-axis.

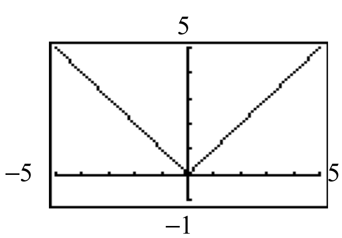


62. $f(x) = 2x^3 - 3,$
$-f(x) = -2x^3 + 3$
The graphs are reflections of each other across the x-axis.

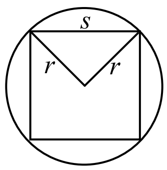


63. $(2, 3), (5, -1), (-1, 3)$
None of these points are vertically above one another
(i.e. none have the same x-coordinate with different y-coordinates.
Therefore this passes the vertical line test, and the three points
could all lie on the same function.

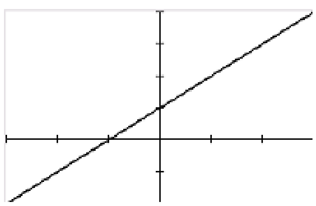
64. For $f(x) = |x|$ and $g(x) = \sqrt{x^2}$ are the same because for all values of x,
$f(x)$ is equal to positive x. And if we consider only principal roots, $g(x)$
will be equal to positive x. The graphs are the same, $f(x) = g(x)$ for all real x.



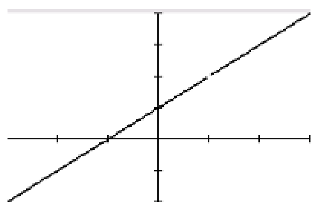
65. Let r be the radius of the circle. Then since the triangle formed by two radii and the side of the square is a right isosceles triangle, we have $r^2 + r^2 = s^2$ by the Pythagorean theorem. Therefore, $2r^2 = s^2$, or $r^2 = \frac{1}{2}s^2$. The area of the circle is $A = \pi r^2 = \frac{1}{2}\pi s^2$.



66. The two functions are not the same since the domain of f is all real numbers while that of g is all $x \neq 1$. The functions agree at all values of $x \neq 1$.



$y = f(x)$



$y = g(x)$

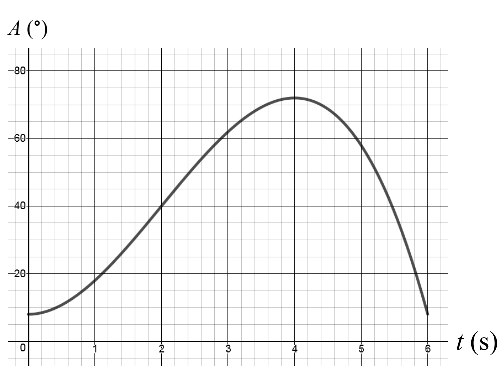
67.
$$I = f(m) = 12.5\sqrt{1 + 0.5m^2}$$
$$f(0.550) = 12.5\sqrt{1 + 0.5(0.550)^2}$$
$$f(0.550) = 12.5\sqrt{1.15125}$$
$$f(0.550) = 12.5(1.072963187)$$
$$f(0.550) = 13.41203983$$
$$f(0.550) = 13.4$$

68. $p = f(t) = f(0.4) = \dfrac{100(0.4)}{0.4 + 1.5} = \dfrac{40}{1.9} = 21.0526316 = 21\%$

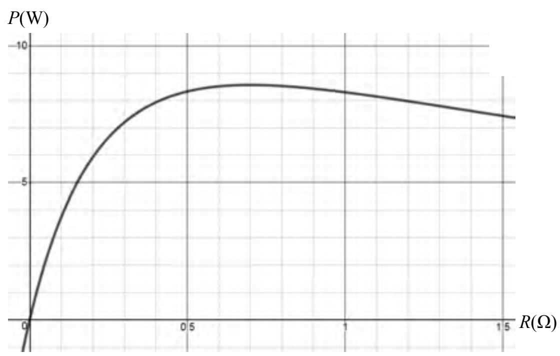
69. $A = 8.0 + 12t^2 - 2t^3$

Graph $y = 8.0 + 12x^2 - 2x^3$ for $0 \le x \le 6$ and use the maximum feature.

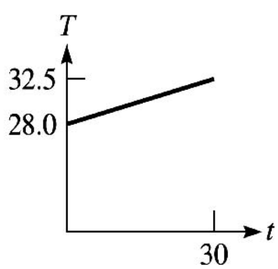
The maximum angle is 72°.



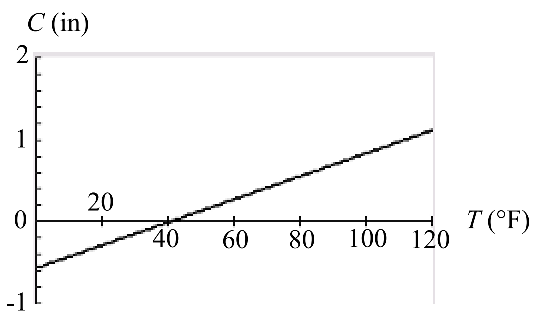
70. $P = \dfrac{24R}{R^2 + 1.4R + 0.49}$, graph $y = \dfrac{24x}{x^2 + 1.4x + 0.49}$ and use the maximum feature.
The maximum power is 8.57 W.



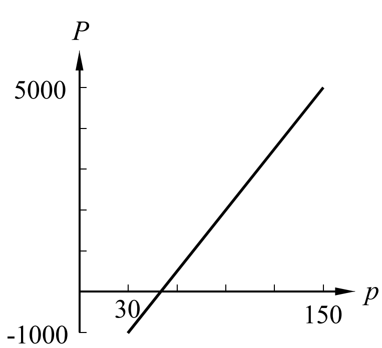
71. $T = f(t) = 28.0 + 0.15t,\ 0 \le t \le 30$



72. $C = f(T) = 0.014(T - 40) = 0.014T - 0.56,\ 0°\text{F} \le T \le 120\ °\text{F}$



73. $P = f(p) = 50(p - 50) = 50p - 2500,\ \ \$30 \le p \le \$150$



74. $N = f(t) = 7000t + 500,$

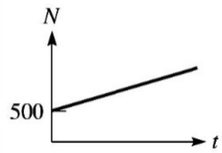
The tank will be filled at

$100\ 000 = 7000t + 500$

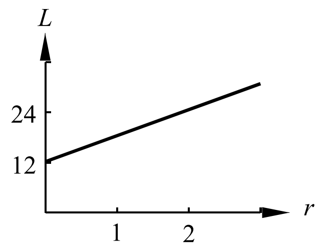
$7000t = 99\ 500$

$t = 14.21428571 = 14.2 \text{ h}$

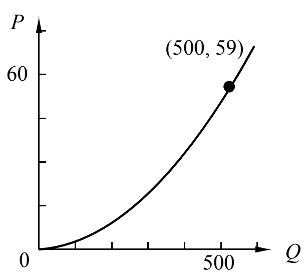
Range: all real values $0 \le t \le 14.2$ h



75. $L = f(r) = 2\pi r + 12$



76. $P = f(Q) = 0.000\,21Q^2 + 0.013Q$



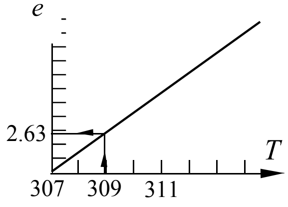
77. $e = f(T) = \dfrac{100\left(T^4 - 307^4\right)}{307^4}$

$f(309) = \dfrac{100\left(309^4 - 307^4\right)}{307^4}$

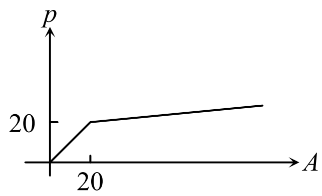
$f(309) = \dfrac{100(233\ 747\ 360)}{8\ 882\ 874\ 001}$

$f(309) = 2.631438428$

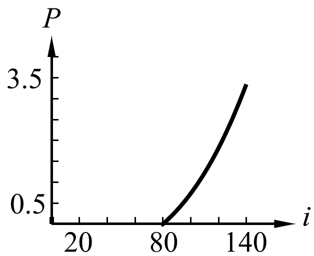
$f(309) = 2.63\%$



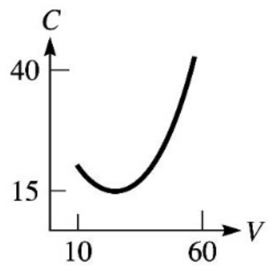
78. $p = \begin{cases} A \\ 20 + 0.10(A - 20) \end{cases}$



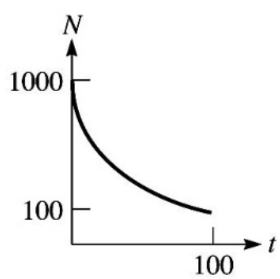
79. $P = f(i) = \left(1.5 \times 10^{-6}\right) i^3 - 0.77,\ 80 \le i \le 140$



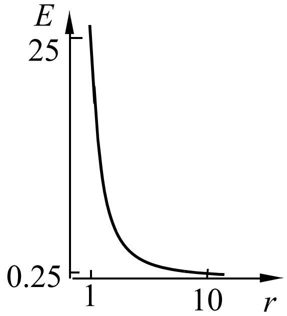
80. $C = f(v) = 0.025v^2 - 1.4v + 35,\ \ 10 \le v \le 60$



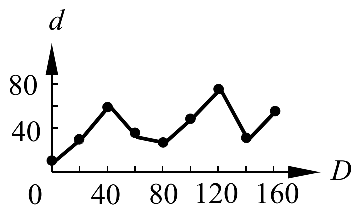
81. $N = f(t) = \dfrac{1000}{\sqrt{t+1}}$



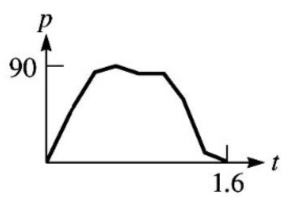
82. $E = f(r) = 25/r^2$.
r is restricted by $r \neq 0$ to avoid a division by a zero error
The domain is (0,10]



83. $d = f(D)$



84. $p = f(t)$



85. Since $f(47)$ lies between $f(40) = 11.1$ ft and $f(60) = 11.7$ ft, we interpolate between 40 and 60.
The distance between 40 and 60 is 20. The distance between 40 and 47 is 7.
The distance between the corresponding distances is $11.7 - 11.1 = 0.6$

	7	40	11.1	x	
20		47	?		0.6
		60	11.7		

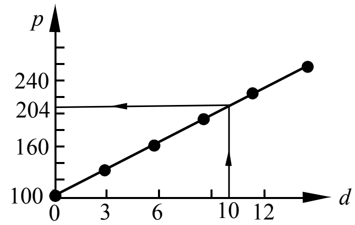
so, $\dfrac{x}{7} = \dfrac{0.6}{20}$

$x = \dfrac{(0.6)(7)}{20}$

$x = 0.21$

$f(47) = 11.1 + 0.21 = 11.31$ ft.

86.



Check using linear interpolation:

Since $f(10)$ lies between the 9 m and 12 m

The distance between 9 and 12 is 3. The distance between 9 and 10 is 1.

The distance between the corresponding presuures is $225-193=32$

$$3\left[\begin{array}{l} 1\left[\begin{array}{ll} 9 & 193 \\ 10 & ? \end{array}\right]x \\ \quad\;\; 12 \quad 225 \end{array}\right]32$$

so, $\frac{x}{1}=\frac{32}{3}$

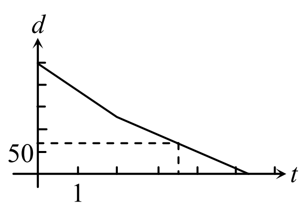
$x=10.6667$

$f(10)=193+10.6667=204$ kPa

87. $d=f(t)=250-60t$, Domain: all real values $0\le t\le 2$;

$d=f(t)=130-40(t-2)$, Domain: all real values $2<t<\frac{21}{4}=5.25$

The person is 70 mi from home after 3.5 h.



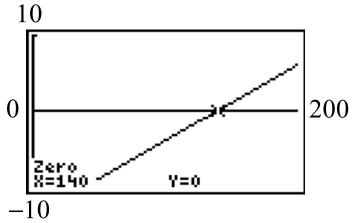
88. $120(0.3)+x(0.1)=50$

$36+0.1x=50$

$0.1x-14=0$

Graph $y=0.1x-14$ and use the zero feature to solve.

140 L of second cleaner must be used.

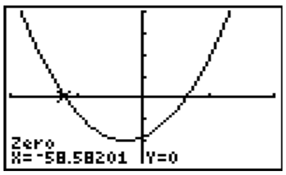


89. $$s = 135 + 4.9T + 0.19T^2$$
$$500 = 135 + 4.9T + 0.19T^2$$
$$0.19T^2 + 4.9T - 365 = 0$$

Graph $y = 0.19x^2 + 4.9x - 365$ and use the zero feature to solve.

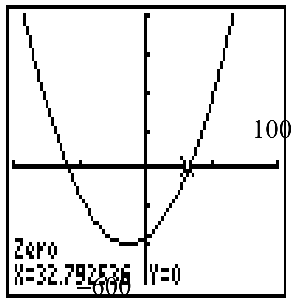
Then $T = 32.8$ °C, the negative temperature is likely too low to be physical.

800

−100  100

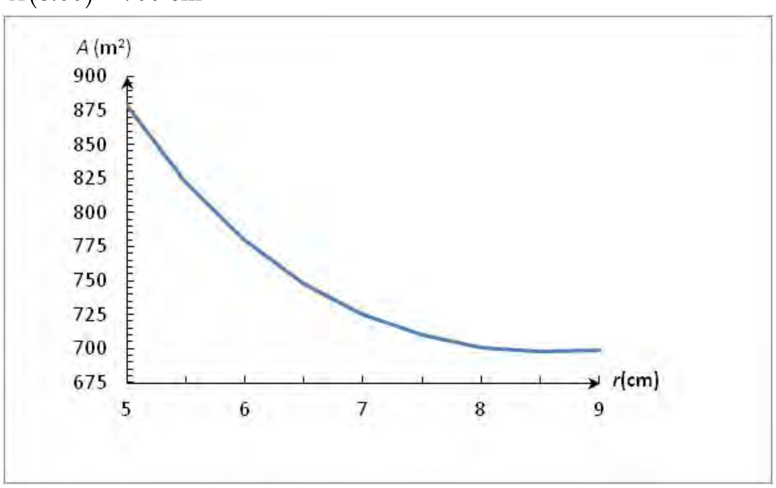
−600

800

−100  100

−600

90. $$V = \pi r^2 h = 2000$$
$$h = \frac{2000}{\pi r^2}$$
$$A = \pi r^2 + 2\pi r h$$
$$A = \pi r^2 + 2\pi r \ \frac{2000}{\pi r^2}$$
$$A(r) = \pi r^2 + \frac{4000}{r}$$
$$A(6.00) = 780 \text{ cm}^2$$
$$A(7.00) = 725 \text{ cm}^2$$
$$A(8.00) = 700 \text{ cm}^2$$



91. $v = 7.6x - 2.1x^2$, and when $v = 5.6$ ft/s,

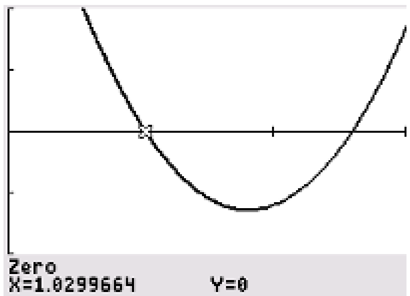
$5.6 = 7.6x - 2.1x^2$

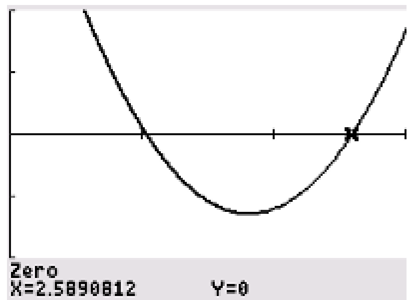
$2.1x^2 - 7.6x + 5.6 = 0$

Graph $y = 2.1x^2 - 7.6x + 5.6,\ 0 \le x \le 3.0$

and use the zero feature to evaluate.

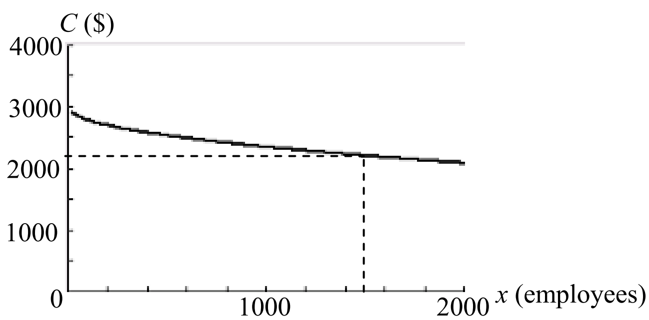
Solutions: $x = 1.03$ ft and $x = 2.59$ ft





92. $C(x) = 3000 - 20\sqrt{x-1}$

$C(1500) = \$2225.66$



93. $A(r) = \pi(r + 45.0)^2$

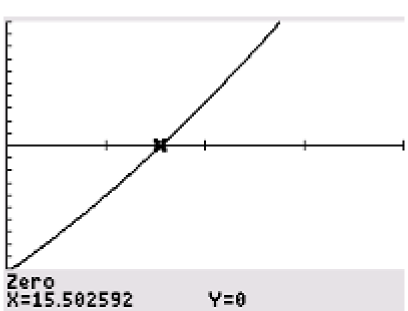
$11500 = \pi(r + 45.0)^2$

$\pi(r + 45.0)^2 - 11500 = 0$

We solve for r using the zero feature

on the graphing calculator.

We find $x = 15.5$ ft.



94.

$$2\ \frac{4}{3}\pi r^3 \ = \frac{4}{3}\pi(r+1.00)^3$$

$$\frac{8}{3}\pi r^3 = \frac{4}{3}\pi(r+1.00)(r^2+2r+1.00)$$

$$\frac{8}{3}\pi r^3 = \frac{4}{3}\pi\left(r^3+2r^2+r+r^2+2r+1.00\right)$$

$$\frac{8}{3}\pi r^3 = \frac{4}{3}\pi\left(r^3+3r^2+3r+1.00\right)$$

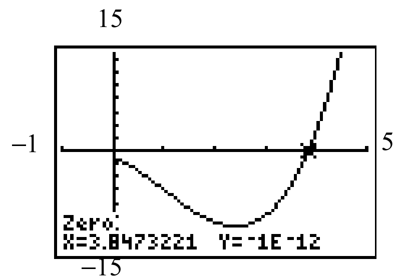
$$\frac{\frac{8}{3}\pi r^3}{\frac{4}{3}\pi} = \frac{\frac{4}{3}\pi\left(r^3+3r^2+3r+1.00\right)}{\frac{4}{3}\pi}$$

$$2r^3 = r^3+3r^2+3r+1.00$$

$$r^3-3r^2-3r-1.00=0$$

Graph $y = x^3-3x^2-3x-1.00,\ x>0,$
and use the zero feature to solve.

Solution: $r = 3.85$ mm and
$r+1.00 = 4.85$ mm.



95.

$$T = \frac{4t^2}{t+2} - 20,\ t \ge 0$$

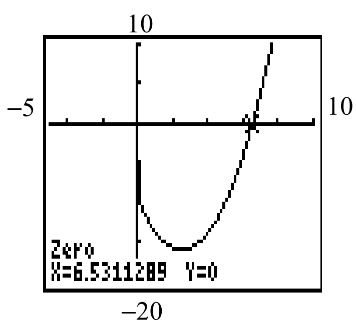
$$0 = \frac{4t^2}{t+2} - 20$$

$$4t^2 - 20(t+2) = 0$$

$$4t^2 - 20t - 40 = 0$$

$$t^2 - 5t - 10 = 0$$

Graph $y = x^2 - 5x - 10$ for $x \ge 0$ and use the zero feature to solve.
Solution $t = 6.53$ hours



96. (1) $R_T = \dfrac{R_1R_2}{R_1+R_2}$

(2) $R_2 = R_1 + 2.00$

Substitute equation 2 into 1

$$R_T = f(R_1) = \frac{R_1\left(R_1+2.00\right)}{R_1+\left(R_1+2.00\right)}$$

$$f(R_1) = \frac{R_1^{\ 2}+2.00R_1}{2R_1+2.00}$$

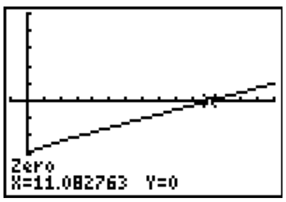
If R_T or $f(R_1) = 6.0\ \Omega$ then:

$$6.0 = \frac{R_1^{\ 2} + 2.00R_1}{2R_1 + 2.00}$$

$$\frac{R_1^{\ 2} + 2.00R_1}{2R_1 + 2.00} - 6.00 = 0$$

Graph $y_1 = \dfrac{x^2 + 2.00x}{2x + 2.00} - 6.00,$ where $x > 0,$ and use the zero feature to solve.

$R_1 = 11.1\ \Omega$



97. Volume is a function of radius and height,
so we can write the volume equation

$V = \pi r^2 h = 250.0\text{ cm}^3$

$$h = \frac{250.0}{\pi r^2}$$

The area is the surface area of the bottom and sides of the cup (no top)

$A = A_{\text{base}} + A_{\text{side}}$

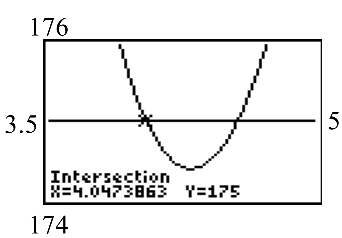
$A = \pi r^2 + 2\pi rh$

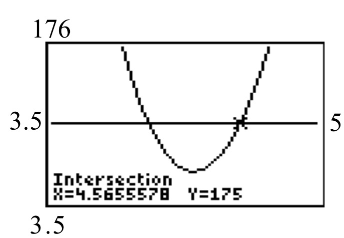
$$A = \pi r^2 + 2\pi r \cdot \frac{250.0}{\pi r^2}$$

$$A = \pi r^2 + \frac{500.0}{r}$$

If $A = 175.0\text{ cm}^2$, then solve for r:

Graph $y = \pi r^2 + \dfrac{500.0}{x},$ for $x > 0,$ and $y_2 = 175.0,$ for $x > 0$
and use the intersect feature to solve.





Solutions: $x = 4.047$ cm or $x = 4.566$ cm

Chapter 4

Trigonometric Functions

4.1 Angles

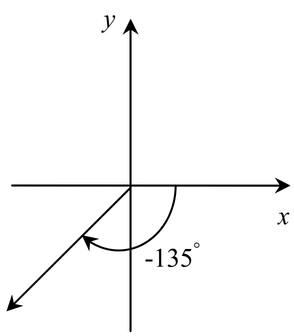
1. $145.6^\circ + 2\left(360^\circ\right) = 865.6^\circ$, or

$145.6^\circ - 2\left(360^\circ\right) = -574.4^\circ$

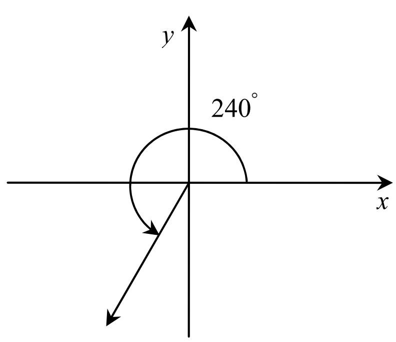
2. $35' = 35'\left(\dfrac{1^\circ}{60'}\right) = 0.58^\circ$ (to nearest 0.01°)

$17^\circ 35' = 17.58^\circ$

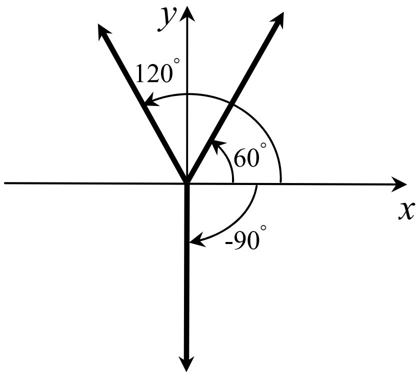
3. $225^\circ - 360^\circ = -135^\circ$



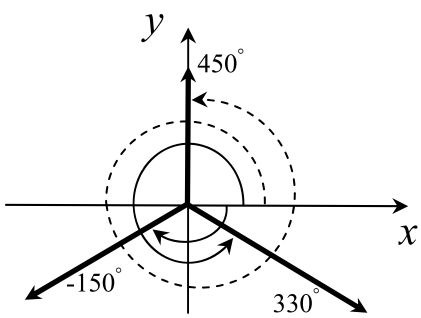
4. $-120^\circ + 360^\circ = 240^\circ$



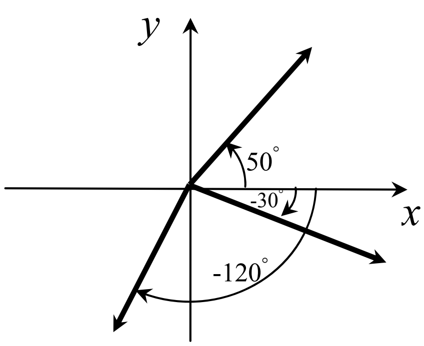
5. $60^\circ, 120^\circ, -90^\circ$



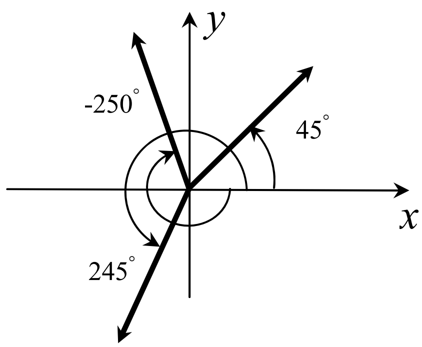
6. $330^\circ, -150^\circ, 450^\circ$



7. $50^\circ, -360^\circ, -30^\circ$



8. $45^\circ, 245^\circ, -250^\circ$



9. positive: $125^\circ + 360^\circ = 485^\circ$
negative: $125^\circ - 360^\circ = -235^\circ$

10. positive: $173^\circ + 360^\circ = 533^\circ$
negative: $173^\circ - 360^\circ = -187^\circ$

11. positive: $-150^\circ + 360^\circ = 210^\circ$
negative: $-150^\circ - 360^\circ = -510^\circ$

12. positive: $462^\circ - 360^\circ = 102^\circ$
negative: $462^\circ - 2(360^\circ) = -258^\circ$

13. positive: $278.1^\circ + 360^\circ = 638.1^\circ$
negative: $278.1^\circ - 360^\circ = -81.9^\circ$

14. positive: $-197.6^\circ + 360^\circ = 162.4^\circ$
negative: $-197.6^\circ - 360^\circ = -557.6^\circ$

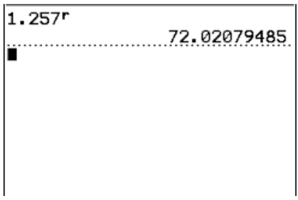
15. $0.675 \text{ rad} = 0.675 \text{ rad}\left(\dfrac{180^\circ}{\pi \text{ rad}}\right) = 38.67^\circ$

16. $0.838 \text{ rad} = 0.838 \text{ rad}\left(\dfrac{180^\circ}{\pi \text{ rad}}\right) = 48.01^\circ$

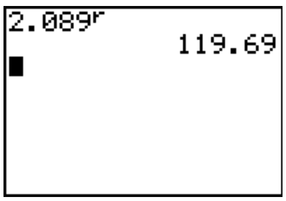
17. $4.447 \text{ rad} = 4.447 \text{ rad}\left(\dfrac{180^\circ}{\pi \text{ rad}}\right) = 254.79^\circ$

18. $-3.642 \text{ rad} = -3.642 \text{ rad}\left(\dfrac{180^\circ}{\pi \text{ rad}}\right) = -208.67^\circ$

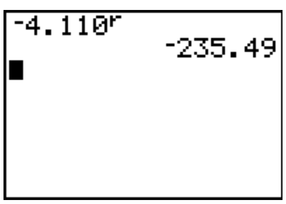
19. With calculator in Degree mode, enter 1.257^r (ANGLE menu #3 on TI-83+)
$1.257 \text{ rad} = 72.02^\circ$



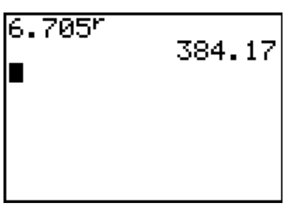
20. With calculator in Degree mode, enter 2.089^r (ANGLE menu #3 on TI-83+)
$2.089 \text{ rad} = 119.69^\circ$



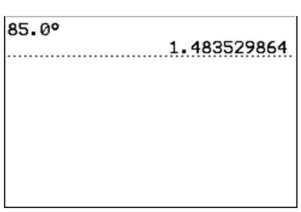
21. With calculator in Degree mode, enter -4.110^r (ANGLE menu #3 on TI-83+)
$-4.110 \text{ rad} = -235.49^\circ$



22. With calculator in Degree mode, enter 6.705^r (ANGLE menu #3 on TI-83+)
$6.705 \text{ rad} = 384.17^\circ$

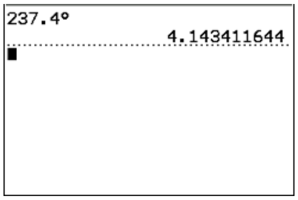


23. With calculator in Radian mode, enter 85.0° (ANGLE menu #1 on TI-83+)
$85.0^\circ = 1.48 \text{ rad}$



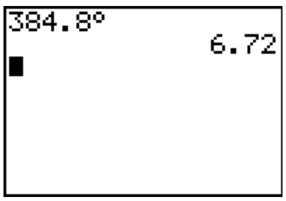
24. With calculator in Radian mode, enter 237.4° (ANGLE menu #1 on TI-83+)

$237.4^{\circ} = 4.143$ rad



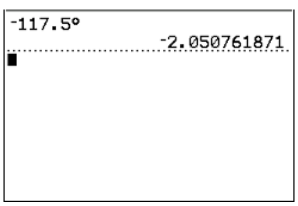
25. With calculator in Radian mode, enter 384.8° (ANGLE menu #1 on TI-83+)

$384.8^{\circ} = 6.72$ rad



26. With calculator in Radian mode, enter -117.5° (ANGLE menu #1 on TI-83+)

$-117.5^{\circ} = -2.051$ rad



27. $47.50^{\circ} = 47^{\circ} + 0.50^{\circ}\left(\dfrac{60'}{1^{\circ}}\right) = 47^{\circ} + 30' = 47^{\circ}30'$

28. $715.80^{\circ} = 715^{\circ} + 0.80^{\circ}\left(\dfrac{60'}{1^{\circ}}\right) = 715^{\circ} + 48' = 715^{\circ}48'$

29. $-5.62^{\circ} = -5^{\circ} - 0.62^{\circ}\left(\dfrac{60'}{1^{\circ}}\right) = -5^{\circ} - 37' = -5^{\circ}37'$

30. $142.87^{\circ} = 142^{\circ} + 0.87^{\circ}\left(\dfrac{60'}{1^{\circ}}\right) = 142^{\circ} + 52' = 142^{\circ}52'$

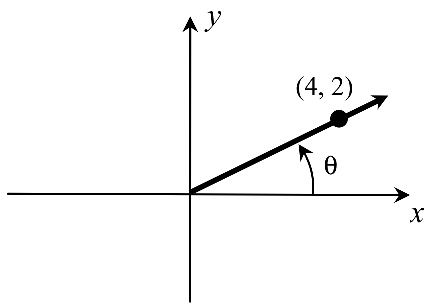
31. $15^{\circ}12' = 15^{\circ} + 12'\left(\dfrac{1^{\circ}}{60'}\right) = 15^{\circ} + 0.20^{\circ} = 15.20^{\circ}$

32. $517^{\circ}39' = 517^{\circ} + 39'\left(\dfrac{1^{\circ}}{60'}\right) = 517^{\circ} + 0.65^{\circ} = 517.65^{\circ}$

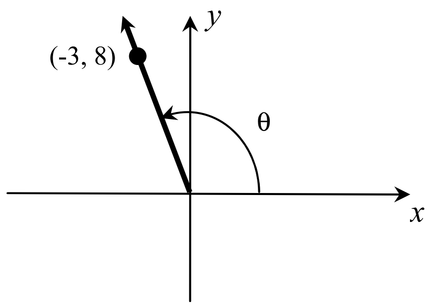
33. $301^{\circ}16' = 301^{\circ} + 16'\left(\dfrac{1^{\circ}}{60'}\right) = 301^{\circ} + 0.27^{\circ} = 301.27^{\circ}$

34. $-94^\circ 47' = -94^\circ - 47'\left(\dfrac{1^\circ}{60'}\right) = -94^\circ - 0.78^\circ = -94.78^\circ$

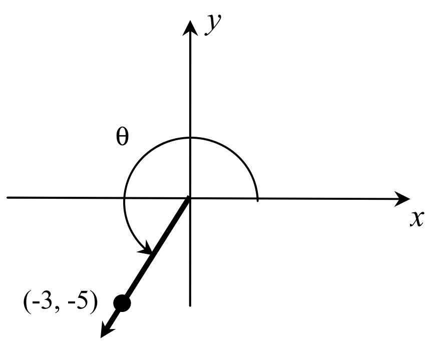
35. Angle in standard position terminal side passing through $(4, 2)$.



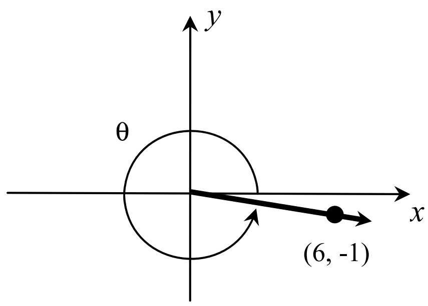
36. Angle in standard position terminal side passing through $(-3, 8)$.



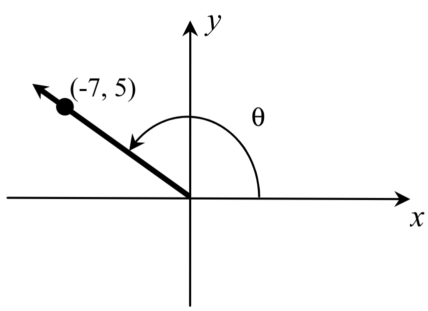
37. Angle in standard position terminal side passing through $(-3, -5)$.



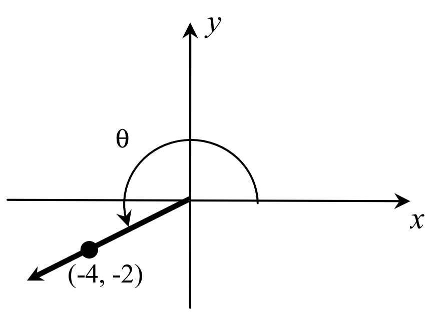
38. Angle in standard position terminal side passing through $(6, -1)$.



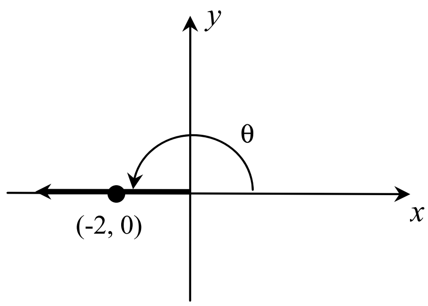
39. Angle in standard position terminal side passing through $(-7, 5)$.



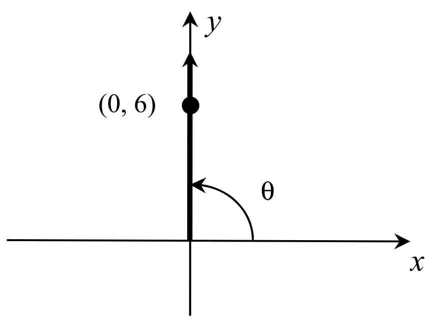
40. Angle in standard position terminal side passing through $(-4, -2)$.



41. Angle in standard position terminal side passing through $(-2, 0)$.



42. Angle in standard position terminal side passing through $(0, 6)$.



43. 31° lies in Quadrant I, so it is a first-quadrant angle.
310° lies in Quadrant IV, so it is a fourth-quadrant angle.

44. 180° lies between Quandrant II and Quandrant III, so it is a quadrantal angle.
92° lies in Quadrant II, so it is a second-quadrant angle.

45. 435°, coterminal with 75°, lies in Quadrant I, so it is a first-quadrant angle.
-270° lies between Quandrant III and Quandrant IV, so it is a quadrantal angle.

46. -5° lies in Quadrant IV, so it is a fourth-quadrant angle.
265° lies in Quadrant III, so it is a third-quadrant angle.

47. $1 \text{ rad} = 57.3^\circ$ lies in Quadrant I, so it is a first-quadrant angle.
$2 \text{ rad} = 114.6^\circ$ lies in Quadrant II, so it is a second-quadrant angle.

48. $3 \text{ rad} = 171.9^\circ$ lies in Quadrant II, so it is a second-quadrant angle.
$-3\pi \text{ rad} = -540^\circ$, coterminal with 180°, lies between Quandrant II and Quandrant III, so it is a quadrantal angle.

49. 4 rad = 229.2° lies in Quadrant III, so it is a third-quadrant angle.

$\frac{\pi}{3}$ rad = 60° lies in Quadrant I, so it is a first-quadrant angle.

50. 12 rad = 687.5° which is coterminal with 327.5° lies in Quadrant IV, so it is a fourth-quadrant angle.

-2 rad = -114.6° lies in Quadrant III, so it is a third-quadrant angle.

51. $21^\circ 42' 36'' = 21^\circ + 42' \ \frac{1^\circ}{60'} \ + 36'' \ \frac{1'}{60''} \ \frac{1^\circ}{60'}$

$21^\circ 42' 36'' = 21^\circ + 0.7^\circ + 0.01^\circ$

$21^\circ 42' 36'' = 21.710^\circ$

52. $-107^\circ 16' 23'' = -107^\circ - 16' \ \frac{1^\circ}{60'} \ - 23'' \ \frac{1'}{60''} \ \frac{1^\circ}{60'}$

$-107^\circ 16' 23'' = -107^\circ - 0.266667^\circ - 0.006389^\circ$

$-107^\circ 16' 23'' = -107.273^\circ$

53. Convert 86.274° to dms format

$0.274^\circ \ \frac{60'}{1^\circ} \ = 16.44'$

$0.44' \ \frac{60''}{1'} \ = 26''$

so $86.274^\circ = 86^\circ 16' 26''$

54. Convert 257.019° to dms format

$0.019^\circ \ \frac{60'}{1^\circ} \ = 1.14'$

$0.14' \ \frac{60''}{1'} \ = 8''$

so $257.019^\circ = 257^\circ 1' 8''$

55. Since a complete revolution clockwise is $-360°$,

3.5 complete revolutions clockwise is $3.5 \times (-360°) = -1260°$.

56. A complete revolution counterclockwise is 2π radians,

so 15.6 revolutions counterclockwise is $15.6 \times 2\pi = 98.0$ radians

4.2 Defining the Trigonometric Functions

1. For a point (4, 3) on the terminal side,

$x = 4,\ y = 3$

$r = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

$\sin\theta = \frac{y}{r} = \frac{3}{5}$ $\qquad \csc\theta = \frac{r}{y} = \frac{5}{3}$

$\cos\theta = \frac{x}{r} = \frac{4}{5}$ $\qquad \sec\theta = \frac{r}{x} = \frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{3}{4}$ $\qquad \cot\theta = \frac{x}{y} = \frac{4}{3}$

2. $\sin\theta = \frac{y}{r} = \frac{4}{7}$

Use $y = 4,\ r = 7$, so

$x = \sqrt{r^2 - y^2} = \sqrt{7^2 - 4^2} = \sqrt{33}$

$\sin\theta = \frac{y}{r} = \frac{4}{7}$ $\qquad \csc\theta = \frac{r}{y} = \frac{7}{4}$

$\cos\theta = \frac{x}{r} = \frac{\sqrt{33}}{7}$ $\qquad \sec\theta = \frac{r}{x} = \frac{7}{\sqrt{33}}$

$\tan\theta = \frac{y}{x} = \frac{4}{\sqrt{33}}$ $\qquad \cot\theta = \frac{x}{y} = \frac{\sqrt{33}}{4}$

3. For a point (6, 8) on the terminal side,

$x = 6,\ y = 8$

$r = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

$\sin\theta = \frac{y}{r} = \frac{8}{10} = \frac{4}{5}$ $\qquad \csc\theta = \frac{r}{y} = \frac{5}{4}$

$\cos\theta = \frac{x}{r} = \frac{6}{10} = \frac{3}{5}$ $\qquad \sec\theta = \frac{r}{x} = \frac{5}{3}$

$\tan\theta = \frac{y}{x} = \frac{8}{6} = \frac{4}{3}$ $\qquad \cot\theta = \frac{x}{y} = \frac{3}{4}$

4. For a point (5, 12) on the terminal side,

$x = 5,\ y = 12$

$r = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$

$\sin\theta = \frac{y}{r} = \frac{12}{13}$ $\qquad \csc\theta = \frac{r}{y} = \frac{13}{12}$

$\cos\theta = \frac{x}{r} = \frac{5}{13}$ $\qquad \sec\theta = \frac{r}{x} = \frac{13}{5}$

$\tan\theta = \frac{y}{x} = \frac{12}{5}$ $\qquad \cot\theta = \frac{x}{y} = \frac{5}{12}$

5. For a point (15, 8) on the terminal side,

$x = 15,\ y = 8$

$r = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$

$\sin\theta = \frac{y}{r} = \frac{8}{17} \qquad \csc\theta = \frac{r}{y} = \frac{17}{8}$

$\cos\theta = \frac{x}{r} = \frac{15}{17} \qquad \sec\theta = \frac{r}{x} = \frac{17}{15}$

$\tan\theta = \frac{y}{x} = \frac{8}{15} \qquad \cot\theta = \frac{x}{y} = \frac{15}{8}$

6. For a point (240, 70) on the terminal side,

$x = 240,\ y = 70$

$r = \sqrt{240^2 + 70^2} = \sqrt{62,500} = 250$

$\sin\theta = \frac{y}{r} = \frac{70}{250} = \frac{7}{25} \qquad \csc\theta = \frac{r}{y} = \frac{25}{7}$

$\cos\theta = \frac{x}{r} = \frac{240}{250} = \frac{24}{25} \qquad \sec\theta = \frac{r}{x} = \frac{25}{24}$

$\tan\theta = \frac{y}{x} = \frac{70}{240} = \frac{7}{24} \qquad \cot\theta = \frac{x}{y} = \frac{24}{7}$

7. For a point (0.09, 0.40) on the terminal side,

$x = 0.09,\ y = 0.40$

$r = \sqrt{0.09^2 + 0.40^2} = \sqrt{0.1681} = 0.41$

$\sin\theta = \frac{y}{r} = \frac{0.40}{0.41} = \frac{40}{41} \qquad \csc\theta = \frac{r}{y} = \frac{41}{40}$

$\cos\theta = \frac{x}{r} = \frac{0.09}{0.41} = \frac{9}{41} \qquad \sec\theta = \frac{r}{x} = \frac{41}{9}$

$\tan\theta = \frac{y}{x} = \frac{0.40}{0.09} = \frac{40}{9} \qquad \cot\theta = \frac{x}{y} = \frac{9}{40}$

8. For a point (1.1, 6.0) on the terminal side,

$x = 1.1,\ y = 6.0$

$r = \sqrt{1.1^2 + 6.0^2} = \sqrt{37.21} = 6.1$

$\sin\theta = \frac{y}{r} = \frac{6.0}{6.1} = \frac{60}{61} \qquad \csc\theta = \frac{r}{y} = \frac{61}{60}$

$\cos\theta = \frac{x}{r} = \frac{1.1}{6.1} = \frac{11}{61} \qquad \sec\theta = \frac{r}{x} = \frac{61}{11}$

$\tan\theta = \frac{y}{x} = \frac{6.0}{1.1} = \frac{60}{11} \qquad \cot\theta = \frac{x}{y} = \frac{11}{60}$

9. For a point (1.2, 3.5) on the terminal side,

$x = 1.2,\ y = 3.5$

$r = \sqrt{1.3^2 + 3.5^2} = \sqrt{13.69} = 3.7$

$\sin\theta = \frac{y}{r} = \frac{3.5}{3.7} = \frac{35}{37}$ $\qquad \csc\theta = \frac{r}{y} = \frac{37}{35}$

$\cos\theta = \frac{x}{r} = \frac{1.2}{3.7} = \frac{12}{37}$ $\qquad \sec\theta = \frac{r}{x} = \frac{37}{12}$

$\tan\theta = \frac{y}{x} = \frac{3.5}{1.2} = \frac{35}{12}$ $\qquad \cot\theta = \frac{x}{y} = \frac{12}{35}$

10. For a point (1.2, 0.9) on the terminal side,

$x = 1.2,\ y = 0.9$

$r = \sqrt{1.2^2 + 0.9^2} = \sqrt{2.25} = 1.5$

$\sin\theta = \frac{y}{r} = \frac{0.9}{1.5} = \frac{3}{5}$ $\qquad \csc\theta = \frac{r}{y} = \frac{5}{3}$

$\cos\theta = \frac{x}{r} = \frac{1.2}{1.5} = \frac{4}{5}$ $\qquad \sec\theta = \frac{r}{x} = \frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{0.9}{1.2} = \frac{3}{4}$ $\qquad \cot\theta = \frac{x}{y} = \frac{4}{3}$

11. For a point $(1, \sqrt{15})$ on the terminal side,

$x = 1,\ y = \sqrt{15}$

$r = \sqrt{1^2 + \left(\sqrt{15}\right)^2} = \sqrt{16} = 4$

$\sin\theta = \frac{y}{r} = \frac{\sqrt{15}}{4}$ $\qquad \csc\theta = \frac{r}{y} = \frac{4}{\sqrt{15}}$

$\cos\theta = \frac{x}{r} = \frac{1}{4}$ $\qquad \sec\theta = \frac{r}{x} = 4$

$\tan\theta = \frac{y}{x} = \sqrt{15}$ $\qquad \cot\theta = \frac{x}{y} = \frac{1}{\sqrt{15}}$

12. For a point $(\sqrt{3}, 2)$ on the terminal side,

$x = \sqrt{3},\ y = 2$

$r = \sqrt{\left(\sqrt{3}\right)^2 + 2^2} = \sqrt{3+4} = \sqrt{7}$

$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{7}}$ $\qquad \csc\theta = \frac{r}{y} = \frac{\sqrt{7}}{2}$

$\cos\theta = \frac{x}{r} = \frac{\sqrt{3}}{\sqrt{7}}$ $\qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{7}}{\sqrt{3}}$

$\tan\theta = \frac{y}{x} = \frac{2}{\sqrt{3}}$ $\qquad \cot\theta = \frac{x}{y} = \frac{\sqrt{3}}{2}$

13. For a point (7, 7) on the terminal side,

$x = 7,\ y = 7$

$r = \sqrt{7^2 + 7^2} = \sqrt{98} = 7\sqrt{2}$

$\sin\theta = \frac{y}{r} = \frac{7}{7\sqrt{2}} = \frac{1}{\sqrt{2}}$ $\qquad \csc\theta = \frac{r}{y} = \sqrt{2}$

$\cos\theta = \frac{x}{r} = \frac{7}{7\sqrt{2}} = \frac{1}{\sqrt{2}}$ $\qquad \sec\theta = \frac{r}{x} = \sqrt{2}$

$\tan\theta = \frac{y}{x} = \frac{7\sqrt{2}}{7\sqrt{2}} = 1$ $\qquad \cot\theta = \frac{x}{y} = 1$

14. For a point (840, 130) on the terminal side,

$x = 840,\ y = 130$

$r = \sqrt{840^2 + 130^2} = \sqrt{722500} = 850$

$\sin\theta = \frac{y}{r} = \frac{130}{850} = \frac{13}{85}$ $\qquad \csc\theta = \frac{r}{y} = \frac{85}{13}$

$\cos\theta = \frac{x}{r} = \frac{840}{850} = \frac{84}{85}$ $\qquad \sec\theta = \frac{r}{x} = \frac{85}{84}$

$\tan\theta = \frac{y}{x} = \frac{130}{840} = \frac{13}{84}$ $\qquad \cot\theta = \frac{x}{y} = \frac{84}{13}$

15. For a point (50, 20) on the terminal side,

$x = 50,\ y = 20$

$r = \sqrt{50^2 + 20^2} = \sqrt{2900} = 10\sqrt{29}$

$\sin\theta = \frac{y}{r} = \frac{20}{10\sqrt{29}} = \frac{2}{\sqrt{29}}$ $\qquad \csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{2}$

$\cos\theta = \frac{x}{r} = \frac{50}{10\sqrt{29}} = \frac{5}{\sqrt{29}}$ $\qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{29}}{5}$

$\tan\theta = \frac{y}{x} = \frac{20}{50} = \frac{2}{5}$ $\qquad \cot\theta = \frac{x}{y} = \frac{5}{2}$

16. For a point $(1, \frac{1}{2})$ on the terminal side,

$x = 1,\ y = \frac{1}{2}$

$r = \sqrt{1^2 + \frac{1}{2}^2} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$

$\sin\theta = \frac{y}{r} = \frac{\frac{1}{2}}{\frac{\sqrt{5}}{2}} = \frac{1}{\sqrt{5}}$ $\qquad \csc\theta = \frac{r}{y} = \sqrt{5}$

$\cos\theta = \frac{x}{r} = \frac{1}{\frac{\sqrt{5}}{2}} = \frac{2}{\sqrt{5}}$ $\qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{2}$

$\tan\theta = \frac{y}{x} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$ $\qquad \cot\theta = \frac{x}{y} = 2$

17. For a point (0.687, 0.943) on the terminal side,

$x = 0.687,\ y = 0.943$

$r = \sqrt{0.687^2 + 0.943^2} = \sqrt{1.361218} = 1.17$

$\sin\theta = \frac{y}{r} = \frac{0.943}{1.17} = 0.808$ $\csc\theta = \frac{r}{y} = \frac{1.17}{0.943} = 1.24$

$\cos\theta = \frac{x}{r} = \frac{0.687}{1.17} = 0.589$ $\sec\theta = \frac{r}{x} = \frac{1.17}{0.687} = 1.70$

$\tan\theta = \frac{y}{x} = \frac{0.943}{0.687} = 1.37$ $\cot\theta = \frac{x}{y} = \frac{0.687}{0.943} = 0.729$

18. For a point (37.65, 21.87) on the terminal side,

$x = 37.65,\ y = 21.87$

$r = \sqrt{37.65^2 + 21.87^2} = \sqrt{1895.8194} = 43.54$

$\sin\theta = \frac{y}{r} = \frac{21.87}{43.54} = 0.5023$ $\csc\theta = \frac{r}{y} = \frac{43.54}{21.87} = 1.991$

$\cos\theta = \frac{x}{r} = \frac{37.65}{43.54} = 0.8647$ $\sec\theta = \frac{r}{x} = \frac{43.54}{37.65} = 1.156$

$\tan\theta = \frac{y}{x} = \frac{21.87}{37.65} = 0.5809$ $\cot\theta = \frac{x}{y} = \frac{37.65}{21.87} = 1.722$

19. $\cos\theta = \frac{x}{r} = \frac{12}{13}$

Use $x = 12,\ r = 13$, so

$y = \sqrt{r^2 - x^2} = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5$

$\sin\theta = \frac{y}{r} = \frac{5}{13}$

$\cot\theta = \frac{x}{y} = \frac{12}{5}$

20. $\sin\theta = \frac{y}{r} = \frac{1}{2}$

Use $y = 1,\ r = 2$, so

$x = \sqrt{2^2 - 1^2} = \sqrt{4 - 1} = \sqrt{3}$

$\cos\theta = \frac{x}{r} = \frac{\sqrt{3}}{2}$

$\csc\theta = \frac{r}{y} = \frac{2}{1} = 2$

21. $\tan\theta = \dfrac{y}{x} = \dfrac{2}{1}$

Use $y = 2,\ x = 1$, so

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{5}}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

22. $\sec\theta = \dfrac{r}{x} = \dfrac{\sqrt{5}}{2}$

Use $r = \sqrt{5},\ x = 2$, so

$$y = \sqrt{r^2 - x^2} = \sqrt{5 - 2^2} = \sqrt{5 - 4} = 1$$

$$\tan\theta = \frac{y}{x} = \frac{1}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{2}{\sqrt{5}}$$

23. $\sin\theta = \dfrac{y}{r} = \dfrac{0.750}{1}$

Use $y = 0.750$ and $r = 1$

$$x = \sqrt{r^2 - y^2} = \sqrt{1^2 - 0.750^2} = \sqrt{0.4375}$$

$$\cot\theta = \frac{x}{y} = \frac{\sqrt{0.4375}}{0.750} = 0.882$$

$$\csc\theta = \frac{r}{y} = \frac{1}{0.750} = 1.33$$

24. $\cos\theta = \dfrac{x}{r} = \dfrac{0.0326}{1}$

Use $x = 0.0326$ and $r = 1$

$$y = \sqrt{r^2 - x^2} = \sqrt{1^2 - 0.0326^2} = \sqrt{0.99893724}$$

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{0.99893724}}{1} = 0.999$$

$$\tan\theta = \frac{y}{x} = \frac{\sqrt{0.99893724}}{0.0326} = 30.7$$

25. $\cot\theta = \dfrac{x}{y} = \dfrac{0.254}{1}$

Use $x = 0.254,\ y = 1$

$$r = \sqrt{x^2 + y^2} = \sqrt{0.254^2 + 1^2} = \sqrt{1.064516}$$

$$\cos\theta = \frac{x}{r} = \frac{0.254}{\sqrt{1.064516}} = 0.246$$

$$\tan\theta = \frac{y}{x} = \frac{1}{0.254} = 3.94$$

26. $\csc\theta = \frac{r}{y} = \frac{1.20}{1}$

Use $r = 1.20,\ y = 1$

$$x = \sqrt{r^2 - y^2} = \sqrt{1.20^2 - 1^2} = \sqrt{0.44}$$

$$\sec\theta = \frac{r}{x} = \frac{1.20}{\sqrt{0.44}} = 1.81$$

$$\cos\theta = \frac{x}{r} = \frac{\sqrt{0.44}}{1.20} = 0.553$$

27. For $(3, 4)$, $x = 3,\ y = 4,\ r = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$,

$$\sin\theta = \frac{y}{r} = \frac{4}{5} \text{ and } \tan\theta = \frac{y}{x} = \frac{4}{3}$$

For $(6, 8)$, $x = 6,\ y = 8,\ r = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$,

$$\sin\theta = \frac{y}{r} = \frac{8}{10} = \frac{4}{5} \text{ and } \tan\theta = \frac{y}{x} = \frac{8}{6} = \frac{4}{3}$$

For $(4.5, 6)$, $x = 4.5,\ y = 6,\ r = \sqrt{4.5^2 + 6^2} = \sqrt{56.25} = 7.5$,

$$\sin\theta = \frac{y}{r} = \frac{6}{7.5} = \frac{4}{5} \text{ and } \tan\theta = \frac{y}{x} = \frac{6}{4.5} = \frac{4}{3}$$

28. For $(5, 12)$, $x = 5,\ y = 12,\ r = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$,

$$\cos\theta = \frac{x}{r} = \frac{5}{13} \text{ and } \cot\theta = \frac{x}{y} = \frac{5}{12}$$

For $(15, 36)$, $x = 15,\ y = 36,\ r = \sqrt{15^2 + 36^2} = \sqrt{1521} = 39$,

$$\cos\theta = \frac{x}{r} = \frac{15}{39} = \frac{5}{13} \text{ and } \cos\theta = \frac{x}{y} = \frac{15}{36} = \frac{5}{12}$$

For $(7.5, 18)$, $x = 7.5,\ y = 18,\ r = \sqrt{7.5^2 + 18^2} = \sqrt{380.25} = 19.5$,

$$\cos\theta = \frac{x}{r} = \frac{7.5}{19.5} = \frac{15}{39} = \frac{5}{13} \text{ and } \cot\theta = \frac{x}{y} = \frac{7.5}{18} = \frac{15}{36} = \frac{5}{12}$$

29. For $(0.3, 0.1)$, $x = 0.3,\ y = 0.1,\ r = \sqrt{0.3^2 + 0.1^2} = \sqrt{0.1}$

$$\tan\theta = \frac{y}{x} = \frac{0.1}{0.3} = \frac{1}{3} \text{ and}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{0.1}}{0.3} = \frac{\sqrt{10}}{3}$$

For $(9, 3)$, $x = 9,\ y = 3,\ r = \sqrt{9^2 + 3^2} = \sqrt{90} = \sqrt{9(10)} = 3\sqrt{10}$

$$\tan\theta = \frac{y}{x} = \frac{3}{9} = \frac{1}{3} \text{ and}$$

$$\sec\theta = \frac{r}{x} = \frac{3\sqrt{10}}{9} = \frac{\sqrt{10}}{3}$$

For $(33, 11)$, $x = 33,\ y = 11,\ r = \sqrt{33^2 + 11} = \sqrt{1210} = \sqrt{121(10)} = 11\sqrt{10}$

$$\tan\theta = \frac{y}{x} = \frac{11}{33} = \frac{1}{3} \text{ and}$$

$$\sec\theta = \frac{r}{x} = \frac{11\sqrt{10}}{33} = \frac{\sqrt{10}}{3}$$

30. For $(40, 30)$, $x = 40,\ y = 30,\ r = \sqrt{40^2 + 30^2} = \sqrt{2500} = 50$

$$\csc\theta = \frac{r}{y} = \frac{50}{30} = \frac{5}{3} \text{ and } \cos\theta = \frac{x}{r} = \frac{40}{50} = \frac{4}{5}$$

For $(56,\ 42)$, $x = 56,\ y = 42,\ r = \sqrt{56^2 + 42^2} = \sqrt{4900} = 70$

$$\csc\theta = \frac{r}{y} = \frac{70}{42} = \frac{5}{3} \text{ and } \cos\theta = \frac{x}{r} = \frac{56}{70} = \frac{4}{5}$$

For $(36,\ 27)$, $x = 36,\ y = 27,\ r = \sqrt{36^2 + 27^2} = \sqrt{2025} = 45$

$$\csc\theta = \frac{r}{y} = \frac{45}{27} = \frac{5}{3} \text{ and } \cos\theta = \frac{x}{r} = \frac{36}{45} = \frac{4}{5}$$

31. A 0° angle, placed in standard position, intersects the unit circle at the point $(1, 0)$. Therefore, $\cos\ 0° = 1$.

32. A 0° angle, placed in standard position, intersects the unit circle at the point $(1, 0)$. Therefore, $\sin 0° = 0$.

33. We use $x = 0.28, y = 0.96$ to obtain

$\sin\theta = y = 0.96$

$\csc\theta = \dfrac{1}{y} = \dfrac{1}{0.96} = 1.04$

34. We use $x = 0.96, y = 0.28$ to obtain

$\cos\theta = x = 0.28$

$\sec\theta = \dfrac{1}{x} = \dfrac{1}{0.28} = 3.57$

35. If A is acute, it intersects the unit circle at some point (x, y) in the first quadrant. We have, for $0 < x < 1$, that $\dfrac{1}{x} > 1$ and so for $y > 0$,

$\tan A = \dfrac{y}{x} > y = \sin A.$

Hence $\tan A$ is greater.

36. If A is acute, it intersects the unit circle at some point (x, y) in the first quadrant. If $A > 45°$, we have $y > x$ as well and so

$\tan A = \dfrac{y}{x} > 1 > \dfrac{x}{y} = \cot A.$

Hence $\cot A$ is less.

37. If $\tan\theta = \frac{y}{x} = \frac{3}{4}$

Use $y = 3,\ x = 4,\ r = \sqrt{x^2 + y^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

$\sin\theta = \frac{y}{r} = \frac{3}{5}$ and $\cos\theta = \frac{x}{r} = \frac{4}{5}$

$\sin^2\theta + \cos^2\theta = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$

$\sin^2\theta + \cos^2\theta = \frac{9}{25} + \frac{16}{25}$

$\sin^2\theta + \cos^2\theta = \frac{25}{25}$

$\sin^2\theta + \cos^2\theta = 1$

38. If $\sin\theta = \frac{y}{r} = \frac{2}{3}$

Use $y = 2,\ r = 3,\ x = \sqrt{r^2 - y^2} = \sqrt{3^2 - 2^2} = \sqrt{5}$

$\sec\theta = \frac{r}{x} = \frac{3}{\sqrt{5}}$ and $\tan\theta = \frac{y}{x} = \frac{2}{\sqrt{5}}$

$\sec^2\theta - \tan^2\theta = \left(\frac{3}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)^2$

$\sec^2\theta - \tan^2\theta = \frac{9}{5} - \frac{4}{5}$

$\sec^2\theta - \tan^2\theta = \frac{5}{5}$

$\sec^2\theta - \tan^2\theta = 1$

39. $\sin\theta = \frac{y}{r} = \frac{y}{1}$

Use $y = y,\ r = 1,\ x = \sqrt{r^2 - y^2} = \sqrt{1 - y^2}$

$\cos\theta = \frac{x}{r}$

$\cos\theta = \frac{\sqrt{1 - y^2}}{1}$

$\cos\theta = \sqrt{1 - y^2}$

40. $\cos\theta = \frac{x}{r} = \frac{x}{1}$

Use $x = x,\ r = 1,\ y = \sqrt{r^2 - x^2} = \sqrt{1 - x^2}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\sqrt{1 - x^2}}{x}$

41. $\cos\theta = \dfrac{x}{r}$

$\dfrac{1}{\cos\theta} = \dfrac{r}{x} = \sec\theta$

42. $\sin\theta = \dfrac{y}{r}$ and $\cos\theta = \dfrac{x}{r}$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{y/r}{x/r}$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{y}{x}$

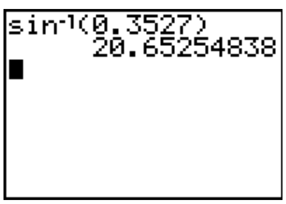
$\dfrac{\sin\theta}{\cos\theta} = \tan\theta$

4.3 Values of the Trigonometric Functions

1. $\sin\theta = 0.3527$

$\theta = \sin^{-1}(0.3527)$

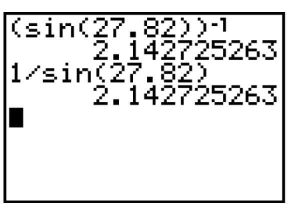
$\theta = 20.65^\circ$



2. $\csc 27.82^\circ = \dfrac{1}{\sin 27.82^\circ}$

$\csc 27.82^\circ = \left(\sin 27.82^\circ\right)^{-1}$

$\csc 27.82^\circ = 2.143$



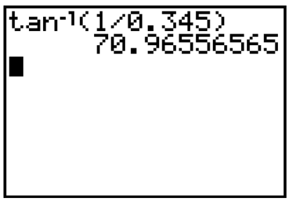
3. $\cot\theta = 0.345$

$$\tan\theta = \frac{1}{\cot\theta}$$

$$\tan\theta = \frac{1}{0.345}$$

$$\theta = \tan^{-1}\frac{1}{0.345}$$

$$\theta = 71.0^\circ$$



4. Find $\tan\theta$ if $\sec\theta = 2.504$

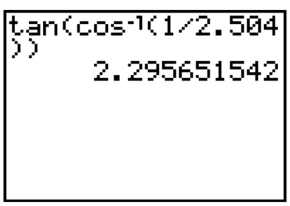
$$\sec\theta = \frac{1}{2.504} = \frac{1}{\cos\theta}$$

$$\cos\theta = \frac{1}{2.504}$$

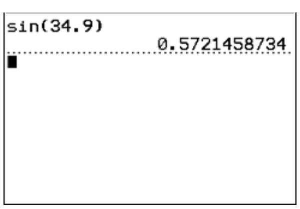
$$\theta = \cos^{-1}\left(\frac{1}{2.504}\right)$$

$$\tan\theta = \tan\left(\cos^{-1}\left(\frac{1}{2.504}\right)\right)$$

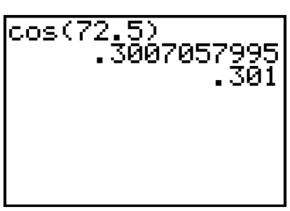
$$\tan\theta = 2.296$$



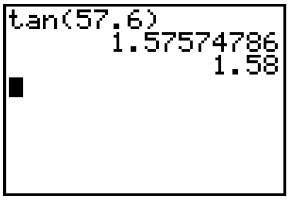
5. $\sin 34.9^\circ = 0.572$



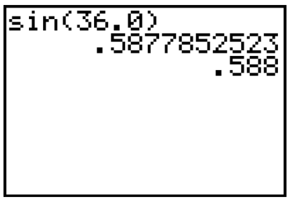
6. $\cos 72.5^\circ = 0.301$



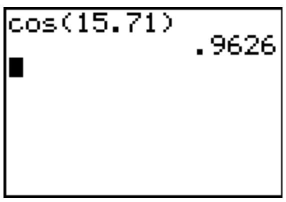
7. $\tan 57.6^\circ = 1.58$



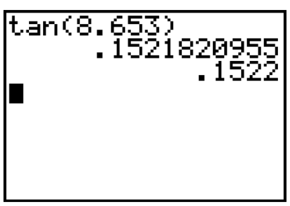
8. $\sin 36.0^\circ = 0.588$



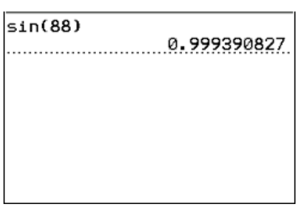
9. $\cos 15.71^\circ = 0.9626$



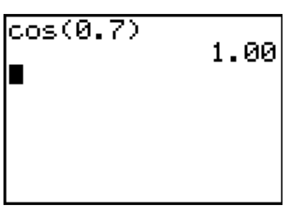
10. $\tan 8.653^\circ = 0.1522$



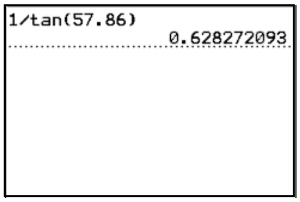
11. $\sin 88^\circ = 1.00$



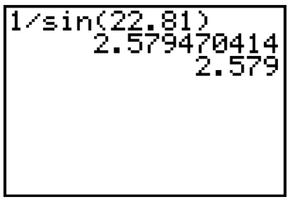
12. $\cos 0.7^\circ = 1.00$



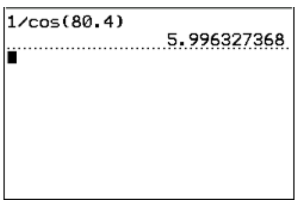
13. $\cot 57.86^\circ = 0.6283$



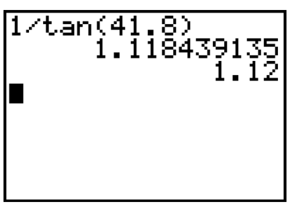
14. $\csc 22.81^\circ = 2.579$



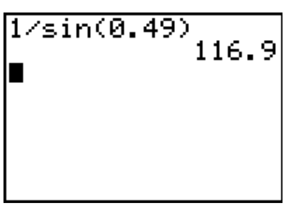
15. $\sec 80.4^\circ = 6.00$



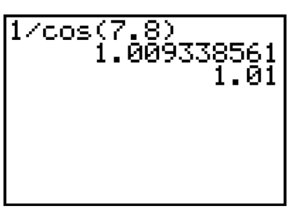
16. $\cot 41.8^\circ = 1.12$



17. $\csc 0.49^\circ = 116.9$

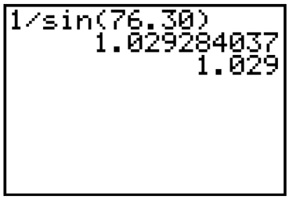


18. $\sec 7.8^\circ = 1.01$



19. $\cot 85.96^\circ = 0.07063$

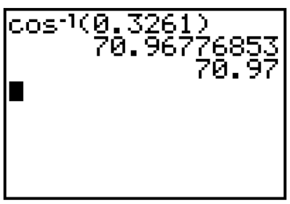
20. $\csc 76.30^\circ = 1.029$



21. $\cos\theta = 0.3261$

$$\theta = \cos^{-1}(0.3261)$$

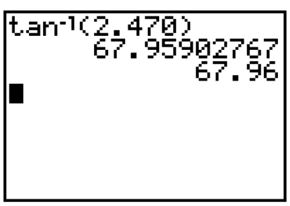
$$\theta = 70.97^\circ$$



22. $\tan\theta = 2.470$

$$\theta = \tan^{-1}(2.470)$$

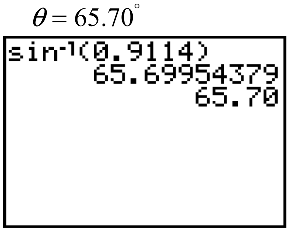
$$\theta = 67.96^\circ$$



23. $\sin\theta = 0.9114$

$$\theta = \sin^{-1}(0.9114)$$

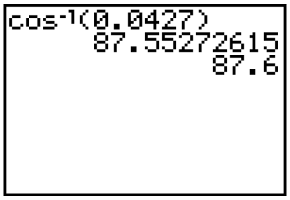
$$\theta = 65.70^\circ$$



24. $\cos\theta = 0.0427$

$\theta = \cos^{-1}(0.0427)$

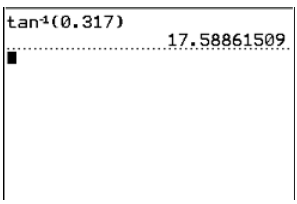
$\theta = 87.6^\circ$



25. $\tan\theta = 0.317$

$\theta = \tan^{-1}(0.317)$

$\theta = 17.6^\circ$

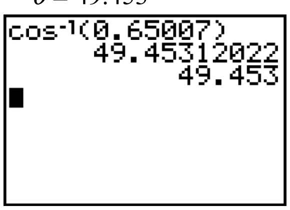


26. $\sin\theta = 1.09$ has no solution. There is no angle whose sine is greater than 1.

27. $\cos\theta = 0.65007$

$\theta = \cos^{-1}(0.65007)$

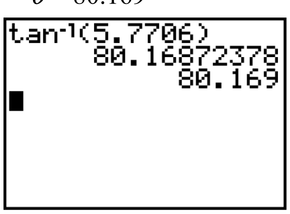
$\theta = 49.453^\circ$



28. $\tan\theta = 5.7706$

$\theta = \tan^{-1}(5.7706)$

$\theta = 80.169^\circ$

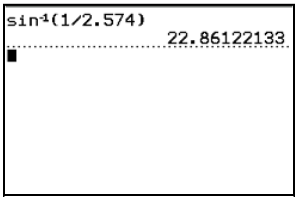


29. $\csc\theta = 2.574$

$\sin\theta = 1/2.574$

$\theta = \sin^{-1}(1/2.574)$

$\theta = 22.86^\circ$

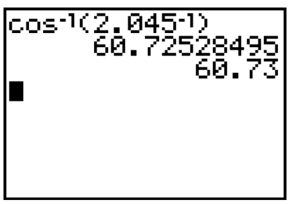


30. $\sec\theta = 2.045$

$\cos\theta = 1/2.045$

$\theta = \cos^{-1}(1/2.045)$

$\theta = 60.73^\circ$

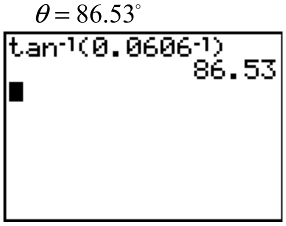


31. $\cot\theta = 0.06060$

$\tan\theta = 1/0.06060$

$\theta = \tan^{-1}(1/0.06060)$

$\theta = 86.53^\circ$

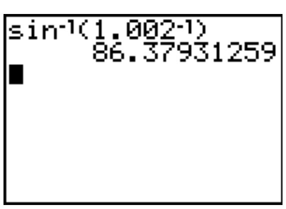


32. $\csc\theta = 1.002$

$\sin\theta = 1/1.002$

$\theta = \sin^{-1}(1/1.002)$

$\theta = 86.38^\circ$



33. $\sec\theta = 0.305$

$\cos\theta = 1/0.305$

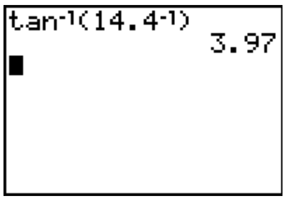
This has no solution. There is no angle whose cosine is greater than 1.

34. $\cot\theta = 14.4$

$\tan\theta = 1/14.4$

$\theta = \tan^{-1}(1/14.4)$

$\theta = 3.97^\circ$

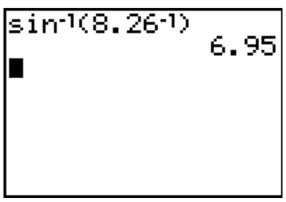


35. $\csc\theta = 8.26$

$\sin\theta = 1/8.26$

$\theta = \sin^{-1}(1/8.26)$

$\theta = 6.95^\circ$

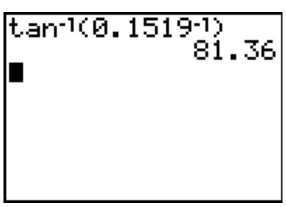


36. $\cot\theta = 0.1519$

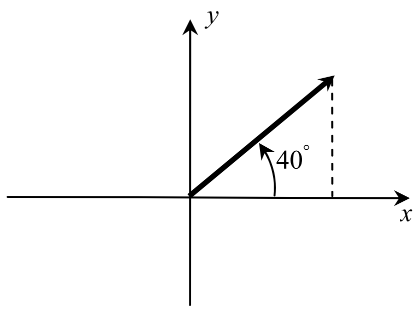
$\tan\theta = 1/0.1519$

$\theta = \tan^{-1}(1/0.1519)$

$\theta = 81.36^\circ$



37.



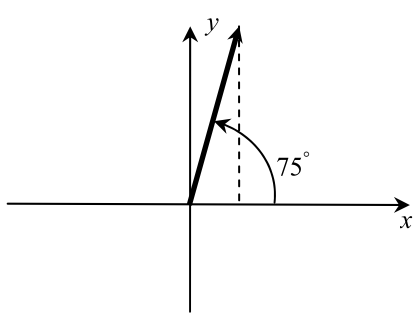
Answers may vary. One set of measurements gives $x = 7.6$ and $y = 6.5$.

$\sin 40^\circ = \dfrac{6.5}{10} = 0.65$ $\quad\csc 40^\circ = \dfrac{10}{6.5} = 1.5$

$\cos 40^\circ = \dfrac{7.6}{10} = 0.76$ $\quad\sec 40^\circ = \dfrac{10}{7.6} = 1.3$

$\tan 40^\circ = \dfrac{6.5}{7.6} = 0.86$ $\quad\cot 40^\circ = \dfrac{7.6}{6.5} = 1.2$

38.



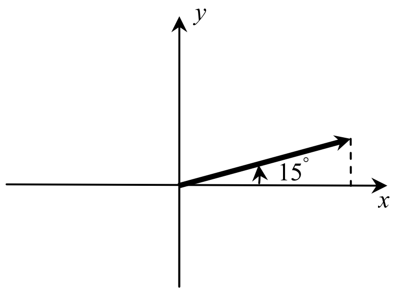
Answers may vary. One set of measurements gives $x = 2.6$ and $y = 9.7$.

$\sin 75^\circ = \dfrac{9.7}{10} = 0.97$ $\quad\csc 75^\circ = \dfrac{10}{9.7} = 1.0$

$\cos 75^\circ = \dfrac{2.6}{10} = 0.26$ $\quad\sec 75^\circ = \dfrac{10}{2.6} = 3.8$

$\tan 75^\circ = \dfrac{9.7}{2.6} = 3.7$ $\quad\cot 75^\circ = \dfrac{2.6}{9.7} = 0.27$

39.



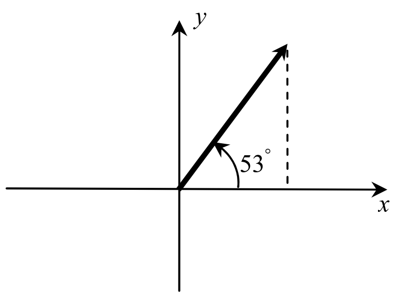
Answers may vary. One set of measurements gives $x = 9.7$ and $y = 2.6$.

$\sin 15^\circ = \dfrac{2.6}{10} = 0.26$ $\qquad$ $\csc 15^\circ = \dfrac{10}{2.6} = 3.8$

$\cos 15^\circ = \dfrac{9.7}{10} = 0.97$ $\qquad$ $\sec 15^\circ = \dfrac{10}{9.7} = 1.0$

$\tan 15^\circ = \dfrac{2.6}{9.7} = 0.27$ $\qquad$ $\cot 15^\circ = \dfrac{9.7}{2.6} = 3.7$

40.



Answers may vary. One set of measurements gives $x = 6.0$ and $y = 8.0$.

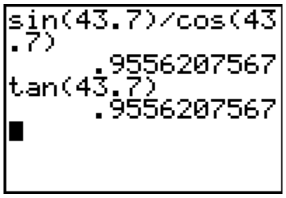
$\sin 53^\circ = \dfrac{8.0}{10} = 0.80$ $\qquad$ $\csc 53^\circ = \dfrac{10}{8.0} = 1.2$

$\cos 53^\circ = \dfrac{6.0}{10} = 0.60$ $\qquad$ $\sec 53^\circ = \dfrac{10}{6.0} = 1.7$

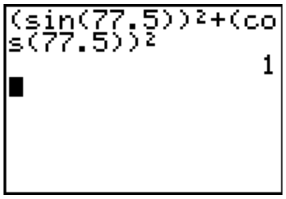
$\tan 53^\circ = \dfrac{8.0}{6.0} = 1.3$ $\qquad$ $\cot 53^\circ = \dfrac{6.0}{8.0} = 0.75$

41. $\dfrac{\sin 43.7^\circ}{\cos 43.7^\circ} = \tan 43.7^\circ$

$0.956 = 0.956$

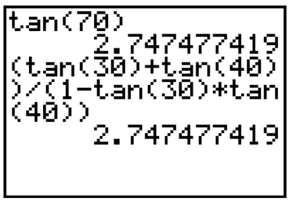


42. $\sin^2 77.5^\circ + \cos^2 77.5^\circ = 1$



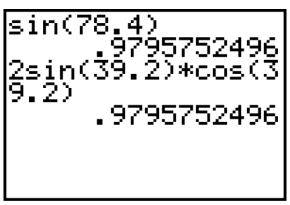
43. $\tan 70^\circ = \dfrac{\tan 30^\circ + \tan 40^\circ}{1 - \tan 30^\circ\left(\tan 40^\circ\right)}$

$2.7 = 2.7$



44. $\sin 78.4^\circ = 2\left(\sin 39.2^\circ\right)\left(\cos 39.2^\circ\right)$

$0.980 = 0.980$



45. Since $\sin\theta = \dfrac{y}{r}$, and since y is always less than or equal to r,

$y \le r$

$\dfrac{y}{r} \le 1$

$\sin\theta \le 1$

The minimum value of y is 0, so

$\sin\theta \ge 0$

Together,

$0 \le \sin\theta \le 1$

46. For any acute angle θ, $\tan\theta$ can equal any positive real number because it is the ratio of two positive numbers, either of which can take on any positive value.

47. $\cos\theta = \dfrac{x}{r}$

If $\theta = 0^\circ$ then x is at a maximum, since the radius vector points to the right. As θ increases from 0° to 90°, the radius vector rotates counterclockwise, becoming more vertical, which shrinks x. If $\theta = 90^\circ$ then $x = 0$, since the radius vector points vertically upward. Therefore, $\cos\theta$ must decrease as θ increases from 0° to 90°.

48. Since $\sec\theta = \dfrac{r}{x}$, and since r is always greater than or equal to x,

$$r \ge x$$
$$\frac{r}{x} \ge 1$$
$$\sec\theta \ge 1$$

49. Since $\sin\theta = \dfrac{y}{r}$ and $\cos\theta = \dfrac{x}{r}$ and $x, y \ge 0$

$$\sin\theta + \cos\theta = \frac{y+x}{r}$$
$$\begin{aligned}(\sin\theta+\cos\theta)^2 &= \left(\frac{y+x}{r}\right)^2\\ &= \frac{x^2+2xy+y^2}{r^2}\\ &= \frac{x^2+2xy+y^2}{x^2+y^2}\\ &= 1+\frac{2xy}{x^2+y^2} \ge 1\end{aligned}$$

implying $\sin\theta + \cos\theta \ge 1$.

50. Since $\sin\theta = \dfrac{y}{r}$ and $\cos\theta = \dfrac{x}{r}$ and $x, y \ge 0$

If $\theta < 45$, then $y < x$ and so $\sin\theta < \cos\theta$.

51. Given $\tan\theta = 1.936$

$$\theta = \tan^{-1}1.936$$
$$\sin\theta = \sin\left(\tan^{-1}1.936\right)$$
$$\sin\theta = 0.8885$$

52. $\sin\theta = 0.6725$

$$\theta = \sin^{-1}0.6725$$
$$\cos\theta = \cos\left(\sin^{-1}0.6725\right)$$
$$\cos\theta = 0.7401$$

53. $\sec\theta = 1.3698$

$\cos\theta = 1.3698^{-1}$

$\theta = \cos^{-1}\left(1.3698^{-1}\right)$

$\tan\theta = \tan\left(\cos^{-1}\left(1.3698^{-1}\right)\right)$

$\tan\theta = 0.93614$

54. $\cos\theta = 0.1063$

$\theta = \cos^{-1} 0.1063$

$\sin\theta = \sin\left(\cos^{-1} 0.1063\right)$

$\csc\theta = \dfrac{1}{\sin\left(\cos^{-1}\left(0.1063\right)\right)}$

$\csc\theta = 1.006$

55. $\sin 45° = 1.33\ \sin\theta_r$

$\sin\theta_r = \dfrac{\sin 45°}{1.33} = 0.531659$

$\theta_r = \sin^{-1}\left(0.531659\right) = 32.12°$

56. $x = \dfrac{h}{\tan\theta}$

$= \dfrac{24.0 \text{ in.}}{\tan(15.0°)}$

$= \dfrac{24.0 \text{ in.}}{0.26794919}$

$= 89.6 \text{ in.}$

57. $l = a\left(\sec\theta + \csc\theta\right)$

$l = 28.0 \text{ cm}\left(\sec 34.5^{\circ} + \csc 34.5^{\circ}\right)$

$l = 28.0 \text{ cm}\left(\dfrac{1}{\cos 34.5^{\circ}} + \dfrac{1}{\sin 34.5^{\circ}}\right)$

$l = 83.4 \text{ cm}$

58. $L = 70.0 + 30\cos\theta$

$L = 70.0 + 30\cos 54.5^{\circ}$

$L = 87.4 \text{ dB}$

59. $\sin\theta = \dfrac{1.50\lambda}{d}$

$\sin\theta = \dfrac{1.5\left(200 \text{ m}\right)}{400 \text{ m}}$

$\theta = \sin^{-1}\dfrac{1.5\left(200\right)}{400}$

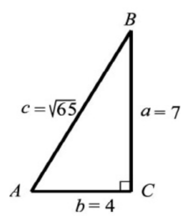
$\theta = 48.6^{\circ}$

60.

$$v = h\tan\theta$$
$$1.52 = 2.35\tan\theta$$
$$\tan\theta = \frac{1.52}{2.35}$$
$$\theta = \tan^{-1}\frac{1.52}{2.35}$$
$$\theta = 32.9^\circ$$

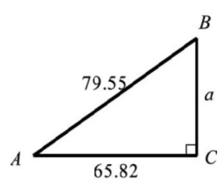
4.4 The Right Triangle

1.



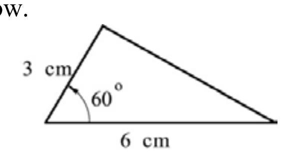
$$\sin A = \frac{7}{\sqrt{65}} = 0.868$$
$$\cos A = \frac{4}{\sqrt{65}} = 0.496$$
$$\tan A = \frac{7}{4} = 1.75$$
$$\sin B = \frac{4}{\sqrt{65}} = 0.496$$
$$\cos B = \frac{4}{\sqrt{65}} = 0.868$$
$$\tan B = \frac{4}{7} = 0.571$$

2.

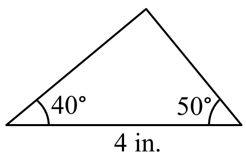


$$a = \sqrt{c^2 - b^2}$$
$$a = \sqrt{79.55^2 - 65.82^2}$$
$$a = 44.68$$
$$\cos A = \frac{65.82}{79.55}$$
$$A = \cos^{-1}\frac{65.82}{79.55}$$
$$A = 34.17^\circ$$
$$B = 90^\circ - 34.17^\circ$$
$$B = 55.83^\circ$$

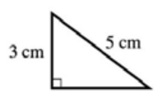
3. A 60° angle between sides of 3 cm and 6 cm determines the unique triangle shown in the figure below.



4. Once the given parts are in place, two more sides must be included. In order to make the angles 40° and 50° respectively, only two unique lengths for the sides will close up the space to form a triangle.



5. Once the given parts are in place one side remains to be included. Only one length and one direction of the line will close up the space to form a triangle.



6. Once the given parts are in place only two unique lengths and with the required directions of the lines will enclose the space to form a triangle.

7. Given $A = 77.8^\circ,\ a = 6700$

$$\sin A = \frac{a}{c}$$

$$c = \frac{a}{\sin A}$$

$$c = \frac{6700}{\sin 77.8^\circ}$$

$$c = 6850$$

$\tan A = \dfrac{a}{b}$

$b = \dfrac{a}{\tan A}$

$b = \dfrac{6700}{\tan 77.8^\circ}$

$b = 1450$

$B = 90^\circ - 77.8^\circ$

$B = 12.2^\circ$

8. Given $A = 18.4^\circ$, $c = 0.0897$

$\sin A = \dfrac{a}{c}$

$a = c \sin A$

$a = 0.0897 \sin 18.4^\circ$

$a = 0.0283$

$\cos A = \dfrac{b}{c}$

$b = c \cos A$

$b = 0.0897 \cos 18.4^\circ$

$b = 0.0851$

$B = 90^\circ - 18.4^\circ$

$B = 71.6^\circ$

9. Given $a = 150$, $c = 345$

$b = \sqrt{c^2 - a^2}$

$b = \sqrt{345^2 - 150^2}$

$b = 311$

$\sin A = \dfrac{a}{c}$

$A = \sin^{-1} \dfrac{a}{c}$

$A = \sin^{-1} \dfrac{150}{345}$

$A = 25.8^\circ$

$\cos B = \dfrac{a}{c}$

$B = \cos^{-1}\left(\dfrac{a}{c}\right)$

$B = \cos^{-1}\left(\dfrac{150}{345}\right)$

$B = 64.2^\circ$

10. Given $a = 932,\ c = 1240$

$$b = \sqrt{c^2 - a^2}$$
$$b = \sqrt{1240^2 - 932^2}$$
$$b = 818$$

$$\sin A = \frac{a}{c}$$
$$A = \sin^{-1}\left(\frac{932}{1240}\right)$$
$$A = 48.7^\circ$$
$$\cos B = \frac{a}{c}$$
$$B = \cos^{-1}\left(\frac{932}{1240}\right)$$
$$B = 41.3^\circ$$

11. Given $B = 32.1^\circ,\ c = 238$

$$\sin B = \frac{b}{c}$$
$$b = c \sin\ B$$
$$b = 238 \sin 32.1^\circ$$
$$b = 126$$
$$\cos B = \frac{a}{c}$$
$$a = c \cos B$$
$$a = 238 \cos 32.1^\circ$$
$$a = 202$$
$$A = 90^\circ - 32.1^\circ$$
$$A = 57.9^\circ$$

12. Given $B = 64.3^\circ,\ b = 0.652$

$$\sin B = \frac{b}{c}$$
$$c = \frac{b}{\sin B}$$
$$c = \frac{0.652}{\sin 64.3^\circ}$$
$$c = 0.724$$
$$\tan B = \frac{b}{a}$$
$$a = \frac{b}{\tan B}$$
$$a = \frac{0.652}{\tan 64.3^\circ}$$
$$a = 0.314$$
$$A = 90^\circ - 64.3^\circ$$
$$A = 25.7^\circ$$

13. Given $b = 82.0,\ c = 881$

$$a = \sqrt{c^2 - b^2}$$
$$a = \sqrt{881^2 - 82.0^2}$$
$$a = 877 \text{ (rounded to three significant digits.)}$$

$$\cos A = \frac{b}{c}$$
$$A = \cos^{-1}\ \frac{82.0}{881}$$
$$A = 84.7^\circ$$
$$\sin B = \frac{b}{c}$$
$$B = \sin^{-1}\ \frac{82.0}{881}$$
$$B = 5.34^\circ$$

14. Given $a = 5920,\ b = 4110$

$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{5920^2 + 4110^2}$$
$$c = 7210$$
$$\tan A = \frac{a}{b}$$
$$A = \tan^{-1}\ \frac{5920}{4110}$$
$$A = 55.2^\circ$$
$$\tan B = \frac{b}{a}$$
$$B = \tan^{-1}\ \frac{4110}{5920}$$
$$B = 34.8^\circ$$

15. Given $A = 32.10^\circ,\ c = 56.85$

$$\sin A = \frac{a}{c}$$
$$a = c \sin A$$
$$a = 56.85 \sin 32.10^\circ$$
$$a = 30.21$$
$$\cos A = \frac{b}{c}$$
$$b = c \cos A$$
$$b = 56.85 \cos 32.10^\circ$$
$$b = 48.16$$
$$B = 90^\circ - 32.10^\circ$$
$$B = 57.90^\circ$$

16. Given $B = 12.60^\circ$, $c = 184.2$

$$\sin B = \frac{b}{c}$$
$$b = c\sin B$$
$$b = 184.2\sin 12.60^\circ$$
$$b = 40.18$$
$$\cos B = \frac{a}{c}$$
$$a = c\cos B$$
$$a = 184.2\cos 12.60^\circ$$
$$a = 179.8$$
$$A = 90^\circ - 12.60^\circ$$
$$A = 77.40^\circ$$

17. Given $a = 56.73$, $b = 44.09$

$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{56.73^2 + 44.09^2}$$
$$c = 71.85$$
$$\tan A = \frac{a}{b}$$
$$A = \tan^{-1}\left(\frac{56.73}{44.09}\right)$$
$$A = 52.15^\circ$$
$$\tan B = \frac{b}{a}$$
$$B = \tan^{-1}\left(\frac{44.09}{56.73}\right)$$
$$B = 37.85^\circ$$

18. Given $a = 9.908$, $c = 12.63$

$$b = \sqrt{c^2 - a^2}$$
$$b = \sqrt{12.63^2 - 9.908^2}$$
$$b = 7.833$$
$$\sin A = \frac{a}{c}$$
$$A = \sin^{-1}\left(\frac{9.908}{12.63}\right)$$
$$A = 51.67^\circ$$
$$\cos B = \frac{a}{c}$$
$$B = \cos^{-1}\left(\frac{9.908}{12.63}\right)$$
$$B = 38.33^\circ$$

19. Given $B = 37.5^\circ,\ a = 0.862$

$$\cos B = \frac{a}{c}$$
$$c = \frac{a}{\cos B}$$
$$c = \frac{0.862}{\cos 37.5^\circ}$$
$$c = 1.09$$
$$\tan B = \frac{b}{a}$$
$$b = a \tan\ B$$
$$b = 0.862 \tan 37.5^\circ$$
$$b = 0.661$$
$$A = 90^\circ - 37.5^\circ$$
$$A = 52.5^\circ$$

20. Given $A = 87.25^\circ,\ b = 8.450$

$$\cos A = \frac{b}{c}$$
$$c = \frac{b}{\cos A}$$
$$c = \frac{8.450}{\cos 87.25^\circ}$$
$$c = 176.1$$
$$\tan A = \frac{a}{b}$$
$$a = b \tan A$$
$$a = 8.450 \tan 87.25^\circ$$
$$a = 175.9$$
$$B = 90^\circ - 87.25^\circ$$
$$B = 2.75^\circ$$

21. Given $B = 74.18^\circ,\ b = 1.849$

$$\sin B = \frac{b}{c}$$
$$c = \frac{b}{\sin B}$$
$$c = \frac{1.849}{\sin 74.18^\circ}$$
$$c = 1.922$$
$$\tan B = \frac{b}{a}$$
$$a = \frac{b}{\tan B}$$
$$a = \frac{1.849}{\tan 74.18^\circ}$$
$$a = 0.5239$$
$$A = 90^\circ - 74.18^\circ$$
$$A = 15.82^\circ$$

22. Given $A = 51.36^\circ,\ a = 3692$

$$\sin A = \frac{a}{c}$$
$$c = \frac{a}{\sin A}$$
$$c = \frac{3692}{\sin 51.36^\circ}$$
$$c = 4727$$
$$\tan A = \frac{a}{b}$$
$$b = \frac{a}{\tan A}$$
$$b = \frac{3692}{\tan 51.36^\circ}$$
$$b = 2952$$
$$B = 90^\circ - 51.36^\circ$$
$$B = 38.64^\circ$$

23. Given $a = 591.87,\ b = 264.93$

$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{591.87^2 + 264.93^2}$$
$$c = 648.46$$

$\tan A = \dfrac{a}{b}$

$A = \tan^{-1}\left(\dfrac{591.87}{264.93}\right)$

$A = 65.883^\circ$

$\tan B = \dfrac{b}{a}$

$B = \tan^{-1}\left(\dfrac{264.93}{591.87}\right)$

$B = 24.117^\circ$

24. Given $b = 2.9507,\ c = 50.864$

$a = \sqrt{c^2 - b^2}$

$a = \sqrt{50.864^2 - 2.9507^2}$

$a = 50.778$

$\cos A = \dfrac{b}{c}$

$A = \cos^{-1}\ \dfrac{2.9507}{50.864}$

$A = 86.674^\circ$

$\sin B = \dfrac{b}{c}$

$B = \sin^{-1}\ \dfrac{2.9507}{50.864}$

$B = 3.326^\circ$

25. Given $A = 2.975^\circ,\ b = 14.592$

$\cos A = \dfrac{b}{c}$

$c = \dfrac{b}{\cos A}$

$c = \dfrac{14.592}{\cos 2.975^\circ}$

$c = 14.61$

$\tan A = \dfrac{a}{b}$

$a = b \tan A$

$a = 14.592 \tan 2.975^\circ$

$a = 0.7584$

$B = 90^\circ - 2.975^\circ$

$B = 87.025^\circ$

26. Given $B = 84.942^\circ$, $a = 7413.5$

$$\cos B = \frac{a}{c}$$
$$c = \frac{a}{\cos B}$$
$$c = \frac{7413.5}{\cos 84.942^\circ}$$
$$c = 84\ 087$$
$$\tan B = \frac{b}{a}$$
$$b = a\tan B$$
$$b = 7413.5\tan 84.942^\circ$$
$$b = 83\ 760 = 8.3760\times 10^4$$
$$A = 90^\circ - 84.942^\circ$$
$$A = 5.058^\circ$$

27. Given $B = 9.56^\circ$, $c = 0.0973$

$$\sin B = \frac{b}{c}$$
$$b = c\sin B$$
$$b = 0.0973\sin 9.56^\circ$$
$$b = 0.0162$$
$$\cos B = \frac{a}{c}$$
$$a = c\cos B$$
$$a = 0.0973\cos 9.56^\circ$$
$$a = 0.0959$$
$$A = 90^\circ - 9.56^\circ$$
$$A = 80.44^\circ$$

28. Given $a = 1.28$, $b = 16.3$

$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{1.28^2 + 16.3^2}$$
$$c = 16.4$$
$$\tan A = \frac{a}{b}$$
$$A = \tan^{-1}\left(\frac{1.28}{16.3}\right)$$
$$A = 4.49^\circ$$
$$\tan B = \frac{b}{a}$$
$$B = \tan^{-1}\left(\frac{16.3}{1.28}\right)$$
$$B = 85.5^\circ$$

29. The given information does not determine a unique triangle. An infinte number of solutions exist for b, c, A, and B.

30. The given information does not determine a unique triangle. An infinite number of triangles exist with the same interior angles 25.7°, 64.3°, and 90.0°, all similar triangles.

31. $\sin 61.7^\circ = \dfrac{3.92}{x}$

$$x = \frac{3.92}{\sin 61.7^\circ}$$
$$x = 4.45$$

32. $\tan A = \dfrac{19.7}{36.3}$

$$A = \tan^{-1}\left(\frac{19.7}{36.3}\right)$$
$$A = 28.5^\circ$$

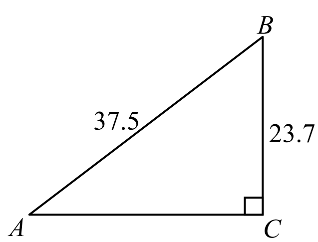
33. $\cos A = \dfrac{0.6673}{0.8742}$

$$A = \cos^{-1}\left(\frac{0.6673}{0.8742}\right)$$
$$A = 40.24^\circ$$

34. $\sin 22.45^\circ = \dfrac{x}{7265}$

$$x = 7265 \sin 22.45^\circ$$
$$x = 2774$$

35.

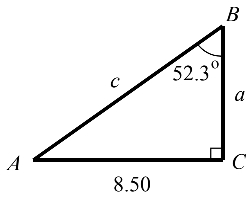


$\sin A = \dfrac{23.7}{37.5}$

$$A = \sin^{-1}\left(\frac{23.7}{37.5}\right)$$
$$A = 39.2^\circ$$
$$B = 90^\circ - 39.2^\circ$$
$$B = 50.8^\circ$$

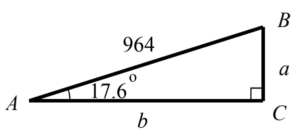
A is the smaller acute angle.

36.



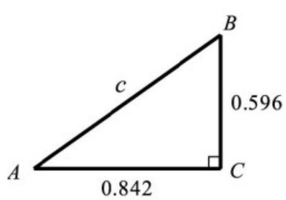
$$\tan 52.3^\circ = \frac{8.50}{a}$$
$$a = \frac{8.50}{\tan 52.3^\circ}$$
$$a = 6.57$$

37.



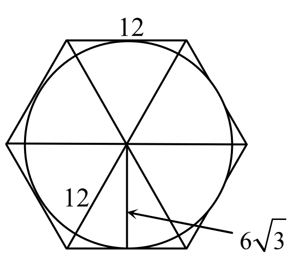
$$\cos 17.6^\circ = \frac{b}{964}$$
$$b = 964\cos 17.6^\circ$$
$$b = 919$$

38.



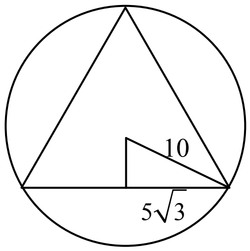
$$\tan B = \frac{0.842}{0.596}$$
$$B = \tan^{-1}\left(\frac{0.842}{0.596}\right)$$
$$B = 54.7^\circ$$

39.



Since the perimeter of the hexagon is 72, each side has length $\frac{72}{6} = 12$. The hexagon is formed from six equilateral triangles of side length 12. The radius of the circle is the height of any of these triangles; this height is $12\sin 60° = 6\sqrt{3}$. Thus, the area of the circle is $\pi(6\sqrt{3})^2 = 108\pi$.

40.



Since the circumference is 20π, the radius is 10.
This radius is the hypotenuse of a right triangle with
$30°$ and $60°$ angles. The long leg has length $10\cos 30° = 5\sqrt{3}$;
this is half the length of one side of the equilateral triangle and
so the triangle's perimeter is $6\times 5\sqrt{3} = 30\sqrt{3}$.

41. If h is the height, then

$$350 = h\cos 78.85°$$
$$h = \frac{350}{\cos 78.85°}$$
$$h = 1810 \text{ ft.}$$

42. The legs are $a = 1536$ and $b = 2048$. The hypotenuse (in pixels) is

$$c = \sqrt{1536^2 + 2048^2} = 2560.$$

The angle A then satisfies $\cos A = \frac{b}{c}$ or

$$A = \cos^{-1}\frac{2048}{2560} = 36.87°$$

The length and width are

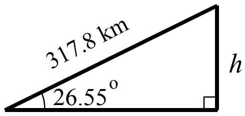
$l = 9.7\cos 36.87° = 7.8$ in; $w = 9.7\sin 36.87° = 5.8$ in.

4.5 Applications of Right Triangles

1.

$$\frac{1850 \text{ ft}}{d} = \cos 27.9^{\circ}$$
$$d = \frac{1850 \text{ ft}}{\cos 27.9^{\circ}}$$
$$d = 2090 \text{ ft}$$

2.



$$d = rt$$

$$d = (6355 \text{ km/h})(3.000 \text{ min})\left(\frac{1}{60 \text{ min}}\right)$$

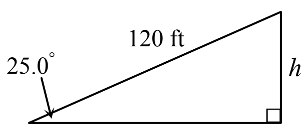
$$d = 317.8 \text{ km}$$

$$\sin 26.55^\circ = \frac{\text{h}}{317.8}$$

$$h = 317.8 \sin 26.55^\circ$$

$$h = 142 \text{ km}$$

3.

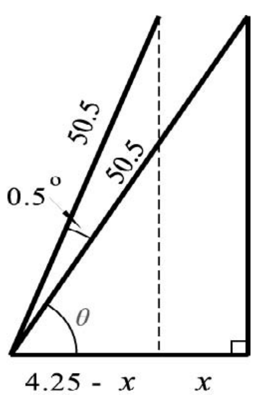


$$\sin 25.0^\circ = \frac{h}{120 \text{ ft}}$$

$$h = 120 \text{ ft } (\sin 25.0^\circ)$$

$$h = 50.7 \text{ ft}$$

4.



$$\cos\theta = \frac{4.25}{50.5}$$

$$\theta = \cos^{-1}\frac{4.25}{50.5}$$

$$\theta = 85.2^\circ$$

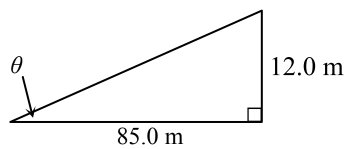
$$\cos(85.2^\circ + 0.5^\circ) = \frac{4.25 - x}{50.5}$$

$$4.25 - x = 50.5\cos(85.7^\circ)$$

$$x = 4.25 \text{ m} - 3.81 \text{ m}$$

$$x = 0.44 \text{ m}$$

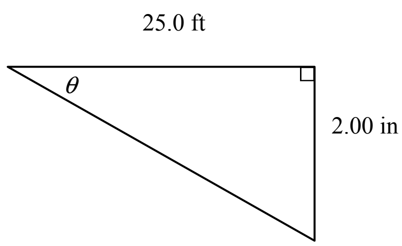
5.



$$\tan\theta = \frac{12.0}{85.0} = 0.1411765$$
$$\theta = \tan^{-1}(0.1411765)$$
$$\theta = 8.04°$$

6. $$\sin 13.0^\circ = \frac{h}{1.25 \text{ m}}$$
$$h = 1.25 \text{ m} \left(\sin 13.0^\circ\right)$$
$$h = 0.281 \text{ m}$$

7.



$$25.0 \text{ ft}\left(\frac{12 \text{ in}}{1 \text{ ft}}\right) = 300 \text{ in}$$
$$\tan\theta = \frac{2.00}{300}$$
$$\theta = \tan^{-1}\frac{2.00}{300}$$
$$\theta = 0.382^\circ$$

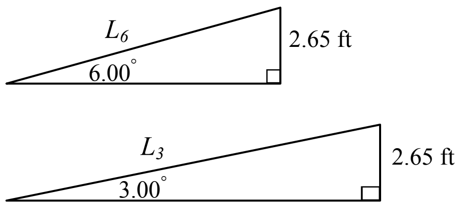
8. $$\tan A = \frac{4.63}{12.60}$$
$$A = \tan^{-1}\left(\frac{4.63}{12.60}\right)$$
$$A = 20.2^\circ$$

9. $$Z = \sqrt{12.0^2 + 15.0^2} = 19.2$$
$$\tan\phi = \frac{15.0}{12.0}$$
$$\phi = \tan^{-1}\left(\frac{15.0}{12.0}\right)$$
$$\phi = 51.34^\circ$$

10. $\cos 37.28° = \dfrac{875.0}{AB}$

$$AB = \frac{875.0}{\cos 37.28°} = 1099.68 \text{ ft}$$
$$DE = AB - 28.74 - 34.16 = 1036.78 \text{ ft}$$

11.



$\sin 6.00° = \dfrac{2.65 \text{ ft}}{L_6}$

$L_6 = \dfrac{2.65 \text{ ft}}{\sin 6.00°}$

$L_6 = 25.35 \text{ ft}$

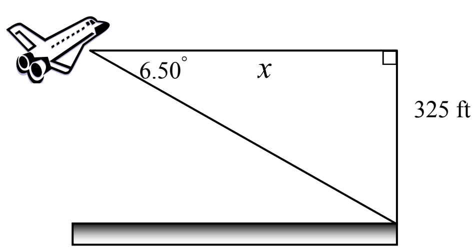
$\sin 3.00° = \dfrac{2.65 \text{ ft}}{L_3}$

$L_3 = \dfrac{2.65 \text{ ft}}{\sin 3.00°}$

$L_3 = 50.63 \text{ ft}$

The ramp must be
$50.63 - 25.35 = 25.28$ feet longer.

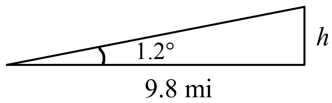
12.



$\tan 6.50° = \dfrac{325 \text{ ft}}{x}$

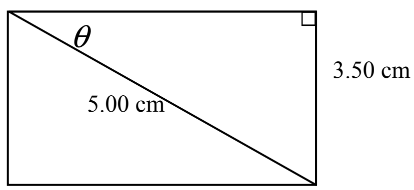
$$x = \frac{325 \text{ ft}}{\tan 6.50°}$$
$$x = 2850 \text{ ft}$$

13.



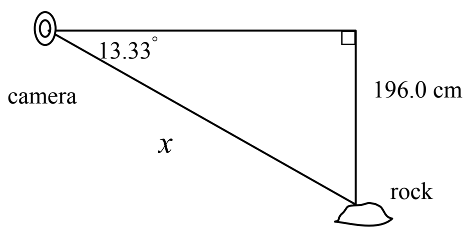
$$\tan 1.2^\circ = \frac{h}{9.8 \text{ mi}}$$
$$h = 9.8 \text{ mi}\left(\tan 1.2^\circ\right)$$
$$h = 0.205 \text{ mi} = 1080 \text{ ft}$$

14.



$$\sin\theta = \frac{3.50}{5.00}$$
$$\theta = \sin^{-1}\frac{3.50}{5.00}$$
$$\theta = 44.4^\circ$$

15.

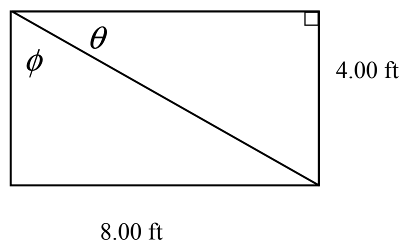


$$\sin 13.33^\circ = \frac{196.0 \text{ cm}}{x}$$
$$x = \frac{196.0 \text{ cm}}{\sin 13.33^\circ}$$
$$x = 850.1 \text{ cm}$$

16. $\tan 16^\circ = \dfrac{h}{5200 \text{ ft}}$

$$h = 5200 \text{ ft}\left(\tan 16^\circ\right)$$
$$h = 1490 \text{ ft}$$

17.



$$\tan\phi = \frac{8.00}{4.00}$$

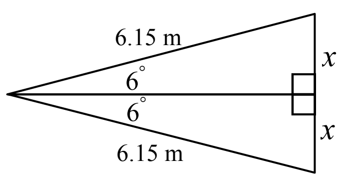
$$\phi = \tan^{-1}\left(\frac{8.00}{4.00}\right) = 63.4^\circ$$

$$\tan\theta = \frac{4.00}{8.00}$$

$$\theta = \tan^{-1}\left(\frac{4.00}{8.00}\right) = 26.6^\circ$$

The angles are 26.6°, 63.4° between the two pieces.

18.



The circle is broken in to 30 parts, so each section is $\frac{360^\circ}{30}$ or 12° wide. This forms two right triangles, each with hypotenuse equal to radius of circular observation area. Since $d = 12.3$ m, then $r = 6.15$ m

$$\sin 6^\circ = \frac{x}{6.15 \text{ m}}$$

$$x = 6.15 \text{ m}\left(\sin 6^\circ\right)$$

$$x = 0.643 \text{ m}$$

The length of the straight section is $2x$.

$$2x = 1.29 \text{ m}$$

19. $\tan\theta = \frac{6.75}{15.5}$

$$\theta = \tan^{-1}\ \frac{6.75}{15.5}$$

$$\theta = 23.5^\circ$$

20. The distance the street light is above the bend is

$$\sin 20.0^\circ = \frac{d}{12.5\text{ ft}}$$

$$d = 12.5\text{ ft}\left(\sin 20.0^\circ\right)$$

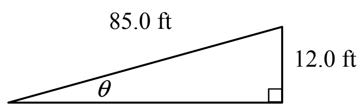
$$d = 4.275\text{ ft}$$

Height of the light above the street

$$h = 28.0\text{ ft} + 4.275\text{ ft}$$

$$h = 32.3\text{ ft}$$

21.

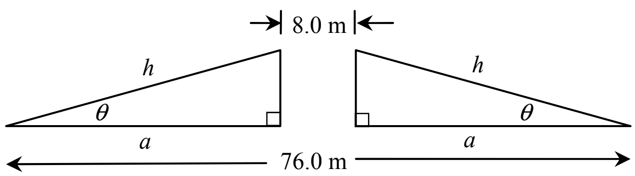


$$\sin\theta = \frac{12.0\text{ ft}}{85.0\text{ ft}}$$

$$\theta = \sin^{-1}\frac{12.0}{85.0}$$

$$\theta = 8.12^\circ$$

22.



The width of each right triangle will be

$$a = \frac{76.0\text{ m} - 8.0\text{ m}}{2}$$

When the bridge is flat, the ends must meet in the middle,
so the hypotenuse of each right triangle must be half the span,

$$h = \frac{76.0\text{ m}}{2}$$

$$\cos A = \frac{a}{h}$$

$$\cos A = \frac{\frac{76-8}{2}}{\frac{76}{2}}$$

$$A = \cos^{-1}\frac{34}{38}$$

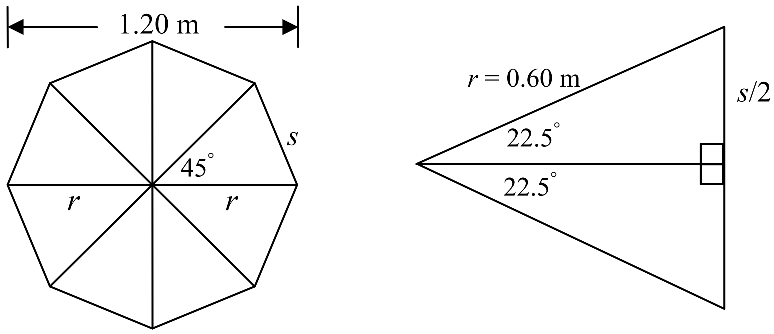
$$A = 26.5^\circ$$

23. $\tan\theta = \dfrac{6.0\text{ m}}{100\text{ m}}$

$$\theta = \tan^{-1}0.060$$

$$\theta = 3.4^\circ$$

24.



The octagon has eight sides, each side has $\frac{360^\circ}{8 \text{ sides}}$ or $\frac{45^\circ}{\text{side}}$,

where each piece makes two right triangles, of angle $\frac{45^\circ}{2}$ or 22.5°

with the hypotenuse being the radius of the octagon, and
opposite side being half the octagon side length.

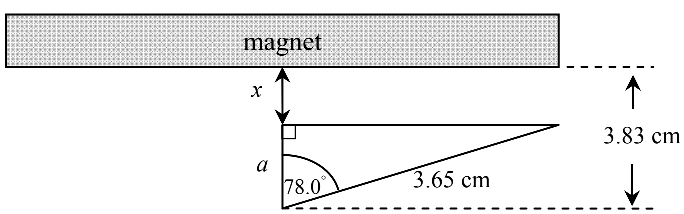
$\sin 22.5^\circ = \frac{s/2}{0.6 \text{ m}}$

$s = 2(\sin 22.5^\circ)(0.6 \text{ m})$

$s = 0.45922 \text{ m}$

$p = 8s = 3.67 \text{ m}$

25.



$\cos 78.0^\circ = \frac{a}{3.65 \text{ cm}}$

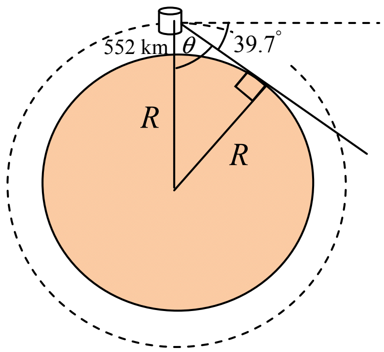
$a = 3.65 \text{ cm} \left(\cos 78.0^\circ\right)$

$a = 0.759 \text{ cm}$

$x = 3.83 \text{ cm} - 0.759 \text{ cm}$

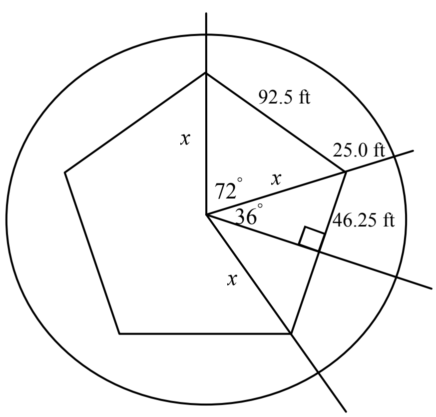
$x = 3.07 \text{ cm}$

26.



$$\begin{aligned}\theta &= 90^\circ - 39.7^\circ\\ \theta &= 50.3^\circ\\ \sin 50.3^\circ &= \frac{R}{R+552}\\ (R+552)\sin 50.3^\circ &= R\\ 0.76940R + 424.71 &= R\\ 424.71 &= 0.23060R\\ R &= 1841.8 \text{ km}\\ R &= 1840 \text{ km}\end{aligned}$$

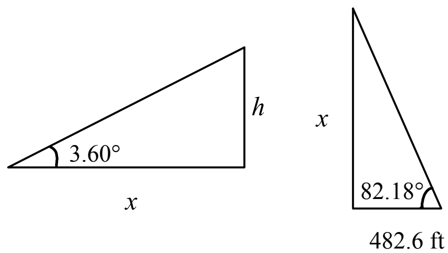
27.



Each of the five triangles in the pentagon has a central angle $\frac{360^\circ}{5}$ or 72°. Radii drawn from the center of the pentagon (which is also the center of the circle) through adjacent vertices of the pentagon form an isosceles triangle with base 92.5 m and equal sides x. A perpendicular bisector from the center of the pentagon to the base of this isosceles triangle forms a right triangle with hypotenuse x and base 46.25 m. The central angle of this right triangle is $\frac{72^\circ}{2}$ or 36°. Thus,

$$\sin 36^\circ = \frac{46.25}{x}$$
$$x = \frac{46.25}{\sin 36^\circ}$$
$$x = 78.685 \text{ m}$$
$$\text{and } C = 2\pi(x+25)$$
$$C = 2\pi(78.685+25)$$
$$C = 651 \text{ m}$$

28.



If x is the perpendicular distance to the boat's path,

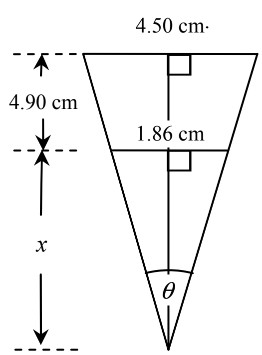
$$\tan 82.18^\circ = \frac{x}{482.6 \text{ ft}}$$
$$x = 482.6 \text{ ft}\left(\tan 82.18^\circ\right)$$
$$x = 3513.943 \text{ ft}$$

To find the cliff height h,

$$\tan 3.60^\circ = \frac{h}{3513.943 \text{ ft}}$$
$$h = 3513.943 \text{ ft}\left(\tan 3.60^\circ\right)$$
$$h = 221.1 \text{ ft}$$

The cliff is approximately 221.1 ft high.

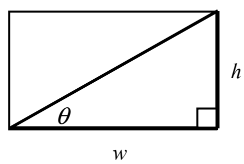
29.



Let x be the vertical distance from the bottom of the taper to the vertex of angle θ. These are similar triangles.

$$\frac{x}{1.86} = \frac{x+4.90}{4.50}$$
$$4.50x = 1.86(x+4.90)$$
$$4.50x = 1.86x + 9.114$$
$$2.64x = 9.114$$
$$x = 3.45 \text{ cm}$$
$$\tan\tfrac{\theta}{2} = \frac{1.86/2}{x}$$
$$\tfrac{\theta}{2} = \tan^{-1}\frac{0.93}{3.45227}$$
$$\tfrac{\theta}{2} = 15.077^{\circ}$$
$$\theta = 30.2^{\circ}$$

30.



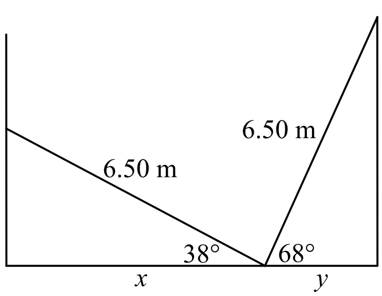
$$\frac{w}{h} = \frac{16}{9}$$
$$\frac{h}{w} = \frac{9}{16}$$
$$\tan\theta = \frac{h}{w}$$
$$\theta = \tan^{-1}\left(\tfrac{9}{16}\right)$$
$$\theta = 29.4^{\circ}$$

31. The radius of the sun r satisfies

$$\tan\frac{0.522°}{2} = \frac{r}{94{,}500{,}000 \text{ mi}}$$
$$r = 94{,}500{,}000 \text{ mi} \times \tan(0.261°)$$
$$r = 430{,}479 \text{ mi}$$

and so the diameter of the sun is $d = 2r = 861{,}000$ mi.

32.



$$x + y = 6.50 \text{ m}(\cos 38°) + 6.50 \text{ m}(\cos 68°)$$
$$= 7.557 \text{ m}$$

The alley is 7.56 m wide.

33. The angles α and β satisfy

$$\tan\alpha = \frac{65.3}{224}; \tan\beta = \frac{65.3}{302}$$

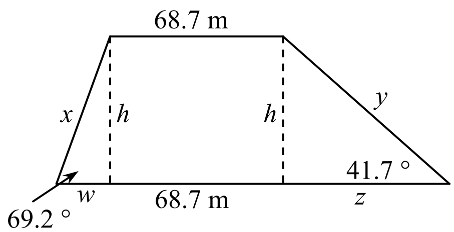
and so

$$\alpha - \beta = \tan^{-1}\frac{65.3}{224} - \tan^{-1}\frac{65.3}{302}$$

$$= 16.25234669° - 12.20095859°$$

$$= 4.05°$$

34.



We relate the quantites w, x, y, z :

$$\cos 69.2° = \frac{w}{x}; w = x\cos 69.2°$$

$$\cos 41.7° = \frac{z}{y}; z = y\cos 41.7°$$

$$w + z + 68.7 = 148$$

$$x\cos 69.2° + y\cos 41.7° = 148 - 68.7$$

$$0.3551x + 0.7466y = 79.3$$

And to introduce h :

$$\sin 69.2° = \frac{w}{x}; h = x\sin 69.2°$$

$$\sin 41.7° = \frac{z}{y}; h = y\sin 41.7°$$

$$x\sin 69.2° = y\sin 41.7°$$

$$y = \frac{\sin 69.2°}{\sin 41.7°}x = 1.405266x$$

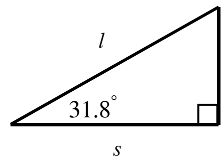
$$0.3551x + 0.7466(1.405266)x = 79.3$$

$$1.40427x = 79.3$$

$$x = 56.5 \text{ m}$$

$$y = 1.405266 \times 56.5 = 79.4 \text{ m}$$

35.



The circumference s of a cross-section of the rail is half of the circumference of a circle of radius 12.8 m

$$s = \frac{1}{2}\ 2\pi(11.8 + 1.0)$$

$$s = 12.8\pi$$

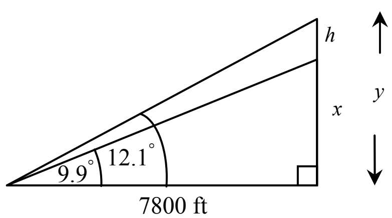
Stretching the handrail straight gives a triangle as shown above.

$$\cos 31.8^\circ = \frac{s}{l}$$

$$l = \frac{12.8\pi}{\cos 31.8^\circ}$$

$$l = 47.3 \text{ m}$$

36.



$$\tan 9.9^\circ = \frac{x}{7800}$$

$$x = 7800 \tan 9.9^\circ$$

$$\tan 12.1^\circ = \frac{y}{7800}$$

$$y = 7800 \tan 12.1^\circ$$

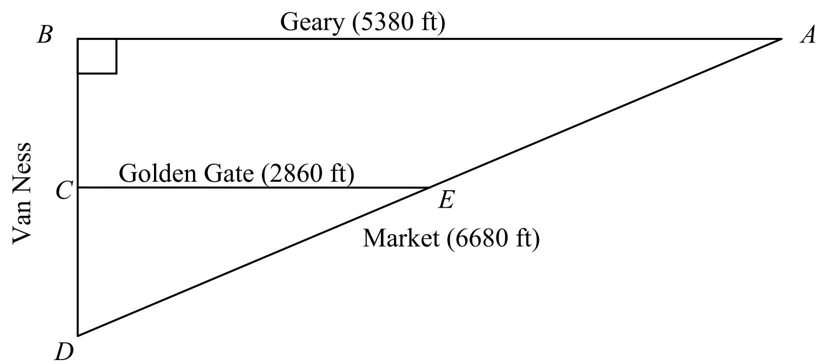
Antenna height is h.

$$h = y - x$$

$$h = 7800 \tan 12.1^\circ - 7800 \tan 9.9^\circ$$

$$h = 310 \text{ ft}$$

37.



$BD = \sqrt{6680^2 - 5380^2} = 4000$

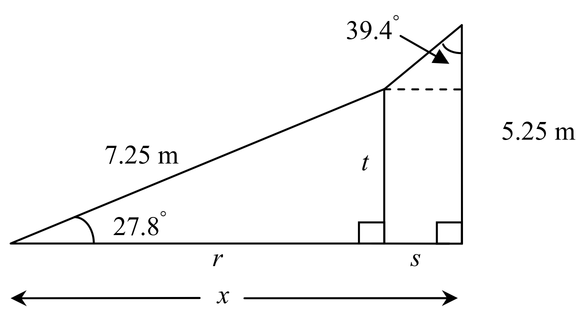
Triangles ΔABD and ΔECD are similar.

$$\frac{2860}{5380} = \frac{CD}{BD}$$

$$CD = 4000 \times \frac{2860}{5380} = 2126.387 \text{ ft}$$

Intersections C and D are 2130 ft apart.

38.



$$\cos 27.8^\circ = \frac{r}{7.25}$$

$$r = 7.25 \cos 27.8^\circ$$

$$r = 6.4132 \text{ m}$$

$$\sin 27.8^\circ = \frac{t}{7.25}$$

$$t = 7.25 \sin 27.8^\circ$$

$$t = 3.3813 \text{ m}$$

$$5.25 - 3.3813 = 1.8687 \text{ m}$$

$$\tan 39.4^\circ = \frac{s}{1.8687}$$

$$s = 1.8687 \tan 39.4^\circ$$

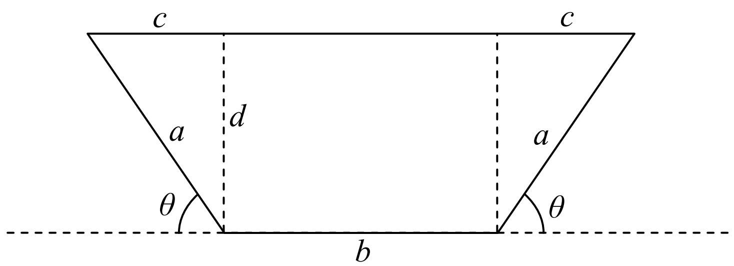
$$s = 1.5350 \text{ m}$$

Therefore, $x = s + r$

$$x = 6.4132 + 1.5350$$

$$x = 7.95 \text{ m}$$

39.

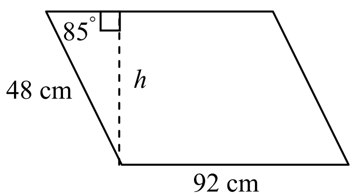


$c = a\ \cos\theta; d = a\ \sin\theta$

The area $A = bd + 2(\frac{1}{2}cd)$

$A = ab\ \sin\theta + a^2\ \sin\theta\ \cos\theta$

40.



$$\sin 85^\circ = \frac{h}{48\text{ cm}}$$

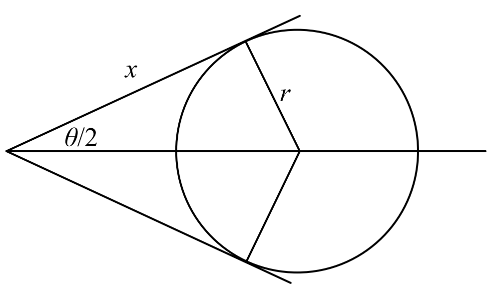
$$h = 48\text{ cm}\ \cdot \sin 85^\circ$$

$$A = bh$$

$$A = 92\text{ cm}\left(48\text{ cm} \cdot \sin 85^\circ\right)$$

$$A = 4400\text{ cm}^2$$

41.



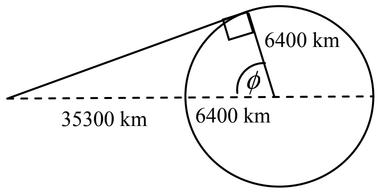
$$\tan\frac{\theta}{2} = \frac{r}{x}$$

$$r = x\tan\frac{\theta}{2}$$

And so the diameter d is

$$d = 2r = 2x\tan\frac{\theta}{2}$$

42.

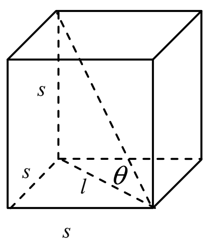


$$\cos\phi = \frac{6400}{6400 + 35\ 300}$$

$$\phi = \cos^{-1}\left(\frac{6400}{6400 + 35\ 300}\right)$$

$$\phi = 81.2^\circ$$

43.



Let s is the length of the edge of the cube, then the diagonal on the base has length

$$l = \sqrt{s^2 + s^2}$$

$$l = \sqrt{2s^2}$$

$$l = s\sqrt{2}$$

If θ = angle between the base and a diagonal of the cube, then

$$\tan\theta = \frac{s}{s\sqrt{2}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\theta = 35.3^\circ$$

44. $\tan\left(\frac{\theta}{2}\right) = \frac{\frac{42.5}{2}}{375}$

$$\frac{\theta}{2} = \tan^{-1}\left(\frac{21.25}{375}\right)$$

$$\frac{\theta}{2} = 3.2433^\circ$$

$$\theta = 6.49^\circ$$

Review Exercises

1. This is false. A standard position angle of 205° is a third-quadrant angle.

2. This is true. Scaling the coordinates of a terminal point by a positive quantity has no effect on the trigonometric functions.

3. This is false. The cosecant function is the reciprocal of the sine function, rather than the inverse function of the sine function.

4. This is true. The two legs of a right triangle with a 45° angle have the same length.

5. This is true.

6. This is true. The secant function is the reciprocal of the cosine function.

7. positive: $47.0^\circ + 360^\circ = 407.0^\circ$
negative: $47.0^\circ - 360^\circ = -313.0^\circ$

8. positive: $338.8^\circ + 360^\circ = 698.8^\circ$
negative: $338.8^\circ - 360^\circ = -21.2^\circ$

9. positive: $-217.5^\circ + 360^\circ = 142.5^\circ$
negative: $-217.5^\circ - 360^\circ = -577.5^\circ$

10. positive: $-0.72^\circ + 360^\circ = 359.28^\circ$
negative: $-0.72^\circ - 360^\circ = -360.72^\circ$

11. $31^\circ 54' = 31^\circ + 54'\left(\dfrac{1^\circ}{60'}\right) = 31^\circ + 0.90^\circ = 31.90^\circ$

12. $574^\circ 45' = 574^\circ + 45'\left(\dfrac{1^\circ}{60'}\right) = 574^\circ + 0.75^\circ = 574.75^\circ$

13. $-83^\circ 21' = -\left(83^\circ + 21'\left(\dfrac{1^\circ}{60'}\right)\right) = -83^\circ - 0.35^\circ = -83.35^\circ$

14. $321^\circ 27' = 321^\circ + 27'\left(\dfrac{1^\circ}{60'}\right) = 321^\circ + 0.45^\circ = 321.45^\circ$

15. $17.5^\circ = 17^\circ + 0.5^\circ\left(\dfrac{60'}{1^\circ}\right) = 17^\circ 30'$

16. $-65.4^\circ = -\left(65^\circ + 0.4^\circ\left(\dfrac{60'}{1^\circ}\right)\right) = -65^\circ 24'$

17. $749.75^\circ = 749^\circ + 0.75^\circ\left(\dfrac{60'}{1^\circ}\right) = 749^\circ 45'$

18. $126.05^\circ = 126^\circ + 0.05^\circ\left(\dfrac{60'}{1^\circ}\right) = 126^\circ 3'$

19. $x = 24,\ \ y = 7$

$r = \sqrt{x^2 + y^2} = \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625} = 25$

$\sin\,\theta = \dfrac{y}{r} = \dfrac{7}{25}$ $\qquad$ $\csc\,\theta = \dfrac{r}{y} = \dfrac{25}{7}$

$\cos\,\theta = \dfrac{x}{r} = \dfrac{24}{25}$ $\qquad$ $\sec\,\theta = \dfrac{r}{x} = \dfrac{25}{24}$

$\tan\,\theta = \dfrac{y}{x} = \dfrac{7}{24}$ $\qquad$ $\cot\,\theta = \dfrac{x}{y} = \dfrac{24}{7}$

20. $x = 5,\ \ y = 4$

$r = \sqrt{x^2 + y^2} = \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41}$

$\sin\,\theta = \dfrac{y}{r} = \dfrac{4}{\sqrt{41}}$ $\qquad$ $\csc\,\theta = \dfrac{r}{y} = \dfrac{\sqrt{41}}{4}$

$\cos\,\theta = \dfrac{x}{r} = \dfrac{5}{\sqrt{41}}$ $\qquad$ $\sec\,\theta = \dfrac{r}{x} = \dfrac{\sqrt{41}}{5}$

$\tan\,\theta = \dfrac{y}{x} = \dfrac{4}{5}$ $\qquad$ $\cot\,\theta = \dfrac{x}{y} = \dfrac{5}{4}$

21. $x = 48,\ \ y = 48$

$r = \sqrt{x^2 + y^2} = \sqrt{48^2 + 48^2} = \sqrt{2(48^2)} = 48\sqrt{2}$

$\sin\,\theta = \dfrac{y}{r} = \dfrac{48}{48\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ $\qquad$ $\csc\,\theta = \dfrac{r}{y} = \sqrt{2}$

$\cos\,\theta = \dfrac{x}{r} = \dfrac{48}{48\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ $\qquad$ $\sec\,\theta = \dfrac{r}{x} = \sqrt{2}$

$\tan\,\theta = \dfrac{y}{x} = \dfrac{48}{48} = 1$ $\qquad$ $\cot\,\theta = \dfrac{x}{y} = 1$

22. $x = 0.36,\ \ y = 0.77$

$r = \sqrt{x^2 + y^2} = \sqrt{0.36^2 + 0.77^2} = 0.85$

$\sin\,\theta = \dfrac{y}{r} = \dfrac{0.77}{0.85} = \dfrac{77}{85}$ $\qquad$ $\csc\,\theta = \dfrac{r}{y} = \dfrac{85}{77}$

$\cos\,\theta = \dfrac{x}{r} = \dfrac{0.36}{0.85} = \dfrac{36}{85}$ $\qquad$ $\sec\,\theta = \dfrac{r}{x} = \dfrac{85}{36}$

$\tan\,\theta = \dfrac{y}{x} = \dfrac{0.77}{0.36} = \dfrac{77}{36}$ $\qquad$ $\cot\,\theta = \dfrac{x}{y} = \dfrac{36}{77}$

23. $\sin\theta = \dfrac{y}{r} = \dfrac{5}{13}$, so $y = 5$, $r = 13$

$$x = \sqrt{r^2 - y^2} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12$$

$$\cos\theta = \frac{x}{r} = \frac{12}{13} = 0.923$$

$$\cot\theta = \frac{x}{y} = \frac{12}{5} = 2.40$$

24. $\cos\theta = \dfrac{x}{r} = \dfrac{3}{8}$, so $x = 3$, $r = 8$

$$y = \sqrt{r^2 - x^2} = \sqrt{8^2 - 3^2} = \sqrt{64 - 9} = \sqrt{55}$$

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{55}}{8} = 0.927$$

$$\tan\theta = \frac{y}{x} = \frac{\sqrt{55}}{3} = 2.47$$

25. $\tan\theta = \dfrac{y}{x} = \dfrac{2}{1}$, so $y = 2$, $x = 1$

$$r = \sqrt{x^2 + y^2} = \sqrt{2^2 + 1^2} = \sqrt{4 + 1} = \sqrt{5}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{5}} = 0.447$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{5}}{2} = 1.12$$

26. $\cot\theta = \dfrac{x}{y} = \dfrac{40}{1}$, so $x = 40$, $y = 1$

$$r = \sqrt{x^2 + y^2} = \sqrt{40^2 + 1^2} = \sqrt{1600 + 1} = \sqrt{1601}$$

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{1601}} = 0.0250$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{1601}}{40} = 1.00$$

27. $\cos\theta = \dfrac{x}{r} = \dfrac{0.327}{1}$, so $x = 0.327$, $r = 1$

$$y = \sqrt{r^2 - x^2} = \sqrt{1 - 0.327^2} = 0.945$$

$$\sin\theta = \frac{y}{r} = \frac{0.945}{1} = 0.945$$

$$\csc\theta = \frac{r}{y} = \frac{1}{0.945} = 1.058$$

28. $\csc\theta = \dfrac{r}{y} = \dfrac{3.41}{1}$, so $r = 3.41,\ \ y = 1$

$x = \sqrt{r^2 - y^2} = \sqrt{3.41^2 - 1^2} = \sqrt{10.6281} = 3.26$

$\tan\theta = \dfrac{y}{x} = \dfrac{1}{3.26} = 0.307$

$\sec\theta = \dfrac{r}{x} = \dfrac{3.41}{3.07} = 1.11$

29. $\sin 72.1^\circ = 0.952594403 = 0.952$

30. $\cos 40.3^\circ = 0.762668329 = 0.763$

31. $\tan 85.68^\circ = 13.2377697 = 13.24$

32. $\sin 0.91^\circ = 0.015881828 = 0.016$

33. $\sec 36.2^\circ = \dfrac{1}{\cos 36.2^\circ} = 1.2392183 = 1.24$

34. $\csc 82.4^\circ = \dfrac{1}{\sin 82.4^\circ} = 1.00886231 = 1.01$

35.
$$\begin{aligned}\left(\cot 7.06^\circ\right)\left(\sin 7.06^\circ\right) - \cos 7.06^\circ &= \frac{1}{\tan\ 7.06^\circ}\sin 7.06^\circ - \cos 7.06^\circ \\ &= 0.992418 - 0.992418 \\ &= 0.00\end{aligned}$$

36.
$$\begin{aligned}\left(\sec 79.36^\circ\right)\left(\sin 79.36^\circ\right) - \tan 79.36^\circ &= \frac{1}{\cos 79.36^\circ}\cdot\sin 79.36^\circ - \tan 79.36^\circ \\ &= 5.3229 - 5.3229 \\ &= 0.000\end{aligned}$$

37. $\cos\theta = 0.850$

$\theta = \cos^{-1}(0.850)$

$\theta = 31.8^\circ$

38. $\sin\theta = 0.63052$

$\theta = \sin^{-1}(0.63052)$

$\theta = 39.088^\circ$

39. $\tan\theta = 1.574$

$\theta = \tan^{-1}(1.574)$

$\theta = 57.57^\circ$

40. $\cos\theta = 0.0135$

$\theta = \cos^{-1}(0.0135)$

$\theta = 89.2^\circ$

41. $\csc\theta = \frac{1}{\sin\theta} = 4.713$

$$\sin\theta = \frac{1}{4.713}$$
$$\theta = \sin^{-1}\left(\frac{1}{4.713}\right)$$
$$\theta = 12.25^\circ$$

42. $\cot\theta = \frac{1}{\tan\theta} = 0.7561$

$$\tan\theta = \frac{1}{0.7561}$$
$$\theta = \tan^{-1}\left(\frac{1}{0.7561}\right)$$
$$\theta = 52.91^\circ$$

43. $\sec\theta = \frac{1}{\cos\theta} = 34.2$

$$\cos\theta = \frac{1}{34.2}$$
$$\theta = \cos^{-1}\left(\frac{1}{34.2}\right)$$
$$\theta = 88.3^\circ$$

44. $\csc\theta = \frac{1}{\sin\theta} = 1.92$

$$\sin\theta = \frac{1}{1.92}$$
$$\theta = \sin^{-1}\left(\frac{1}{1.92}\right)$$
$$\theta = 31.4^\circ$$

45. $\cot\theta = \frac{1}{\tan\theta} = 7.117$

$$\tan\theta = \frac{1}{7.117}$$
$$\theta = \tan^{-1}\left(\frac{1}{7.117}\right)$$
$$\theta = 7.998^\circ$$

46. $\sec\theta = \frac{1}{\cos\theta} = 1.006$

$$\cos\theta = \frac{1}{1.006}$$
$$\theta = \cos^{-1}\left(\frac{1}{1.006}\right)$$
$$\theta = 6.261^\circ$$

47. There is no θ for which $\sin\theta = 1.030$.

48. $\tan\theta = 0.0052$

$$\theta = \tan^{-1}(0.0052)$$
$$\theta = 0.298^\circ$$

49. $\sin\theta = \frac{2}{5} = \frac{y}{r}$

$$y = 2, r = 5, x = \sqrt{r^2 - y^2} = \sqrt{25-4} = \sqrt{21}$$

The point where the terminal side intersects is $\left(\frac{x}{r}, \frac{y}{r}\right)$

or $\left(\frac{\sqrt{21}}{5}, \frac{2}{5}\right)$.

50. $\tan\theta = \frac{3}{1} = \frac{y}{x}$

$$y = 3, x = 1, r = \sqrt{x^2 + y^2} = \sqrt{1+9} = \sqrt{10}$$

The point where the terminal side intersects is $\left(\frac{x}{r}, \frac{y}{r}\right)$

or $\left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$.

51. Given $A = 17.0^\circ$, $b = 6.00$

$$B = 90.0^\circ - 17.0^\circ$$
$$B = 73.0^\circ$$

$$\tan A = \frac{a}{b}$$
$$a = b\tan A$$
$$a = (6.00)\tan 17.0^\circ$$
$$a = 1.83$$

$$\cos A = \frac{b}{c}$$
$$c = \frac{b}{\cos A}$$
$$c = \frac{6.00}{\cos 17.0^\circ}$$
$$c = 6.27$$

52. Given $B = 68.1^\circ,\ a = 1080$

$$A = 90^\circ - 68.1^\circ$$
$$A = 21.9^\circ$$
$$\tan B = \frac{b}{a}$$
$$b = a\tan B$$
$$b = (1080)\tan 68.1^\circ$$
$$b = 2690$$
$$\cos B = \frac{a}{c}$$
$$c = \frac{a}{\cos B}$$
$$c = \frac{1080}{\cos 68.1^\circ}$$
$$c = 2900 = 2.90\times 10^3$$

53. Given $a = 81.0,\ b = 64.5$

$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{81.0^2 + 64.5^2}$$
$$c = 104$$
$$\tan A = \frac{a}{b}$$
$$A = \tan^{-1}\ \frac{81.0}{64.5}$$
$$A = 51.5^\circ$$
$$\tan B = \frac{b}{a}$$
$$B = \tan^{-1}\ \frac{64.5}{81.0}$$
$$B = 38.5^\circ$$

54. Given $a = 106,\ c = 382$

$$b = \sqrt{c^2 - a^2}$$
$$b = \sqrt{382^2 - 106^2}$$
$$b = 367$$
$$\sin A = \frac{a}{c}$$
$$A = \sin^{-1}\ \frac{106}{382}$$
$$A = 16.1^\circ$$
$$\cos B = \frac{a}{c}$$
$$B = \cos^{-1}\ \frac{106}{382}$$
$$B = 73.9^\circ$$

55. Given $A = 37.5^\circ$, $a = 12.0$

$$B = 90^\circ - 37.5^\circ$$
$$B = 52.5^\circ$$
$$\tan A = \frac{a}{b}$$
$$b = \frac{a}{\tan A}$$
$$b = \frac{12.0}{\tan 37.5^\circ}$$
$$b = 15.6$$
$$\sin A = \frac{a}{c}$$
$$c = \frac{a}{\sin A}$$
$$c = \frac{12.0}{\sin 37.5^\circ}$$
$$c = 19.7$$

56. Given $B = 85.7^\circ$, $b = 852.44$

$$A = 90^\circ - 85.7^\circ$$
$$A = 4.3^\circ$$
$$\sin B = \frac{b}{c}$$
$$c = \frac{b}{\sin B}$$
$$c = \frac{852.44}{\sin 85.7^\circ}$$
$$c = 854.85$$
$$\cos B = \frac{a}{c}$$
$$a = c \cos B$$
$$a = 854.85 \cos 85.7^\circ$$
$$a = 64.10$$

57. Given $b = 6.508$, $c = 7.642$

$$a = \sqrt{c^2 - b^2}$$
$$a = \sqrt{7.642^2 - 6.508^2}$$
$$a = 4.006$$

$$\cos A = \frac{b}{c}$$
$$A = \cos^{-1} \frac{6.508}{7.642}$$
$$A = 31.61^{\circ}$$
$$\sin B = \frac{b}{c}$$
$$B = \sin^{-1} \frac{6.508}{7.642}$$
$$B = 58.39^{\circ}$$

58. Given $a = 0.721,\ b = 0.144$

$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{0.721^2 + 0.144^2}$$
$$c = 0.735$$
$$\tan A = \frac{a}{b}$$
$$A = \tan^{-1}\left(\frac{0.721}{0.144}\right)$$
$$A = 78.7^{\circ}$$
$$\tan B = \frac{b}{a}$$
$$B = \tan^{-1}\left(\frac{0.144}{0.721}\right)$$
$$B = 11.3^{\circ}$$

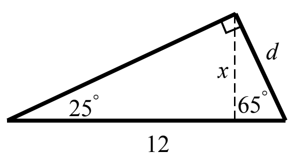
59. Given $A = 49.67^{\circ},\ c = 0.8253$

$$B = 90^{\circ} - 49.67^{\circ}$$
$$B = 40.33^{\circ}$$
$$\sin A = \frac{a}{c}$$
$$a = c \sin A$$
$$a = 0.8253 \sin 49.67^{\circ}$$
$$a = 0.6292$$
$$\cos A = \frac{b}{c}$$
$$b = c \cos A$$
$$b = 0.8253 \cos 49.67^{\circ}$$
$$b = 0.5341$$

60. Given $B = 4.38^\circ,\ b = 5682$

$$A = 90^\circ - 4.38^\circ$$
$$A = 85.62^\circ$$
$$\tan B = \frac{b}{a}$$
$$a = \frac{b}{\tan B}$$
$$a = \frac{5682}{\tan 4.38^\circ}$$
$$a = 74\ 200$$
$$\sin B = \frac{b}{c}$$
$$c = \frac{b}{\sin B}$$
$$c = \frac{5682}{\sin 4.38^\circ}$$
$$c = 74\ 400$$

61.



$$\sin 25^\circ = \frac{d}{12}$$
$$d = 12\sin 25^\circ$$
$$d = 5.0714$$
$$90^\circ - 25^\circ = 65^\circ$$
$$\sin 65^\circ = \frac{x}{d}$$
$$x = d\sin 65^\circ$$
$$x = 5.0714\sin 65^\circ$$
$$x = 4.6$$

62. Method 1:

$$\sin 31^\circ = \frac{x}{2}$$
$$x = 2\sin 31^\circ$$
$$x = 1.03$$

Method 2:

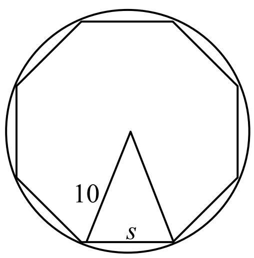
$$90^\circ - 31^\circ = 59^\circ$$
$$\cos 59^\circ = \frac{x}{2}$$
$$x = 2\cos 59^\circ$$
$$x = 1.03$$

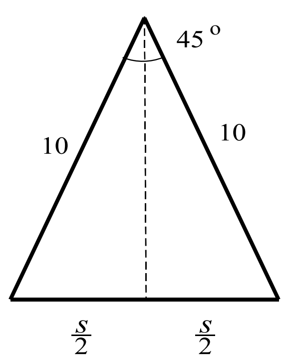
Method 3:

$$\cos 31^\circ = \frac{b}{2}$$
$$b = 2\cos 31^\circ$$
$$b = 1.7143$$
$$x = \sqrt{2^2 - (1.7143)^2}$$
$$x = 1.03$$

Method 1 is the easiest

63.



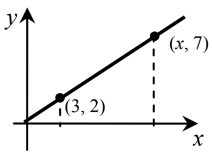


Each side s of the octagon is the base of an isosceles triangle with vertex angle of $\frac{360^\circ}{8}$ or 45°.

$$\sin 22.5^\circ = \frac{\frac{s}{2}}{10}$$
$$\frac{s}{2} = 10\sin 22.5^\circ$$
$$s = 20\sin 22.5^\circ$$
$$p = 8s$$
$$p = 8\left(20\sin 22.5^\circ\right)$$
$$p = 61.2$$

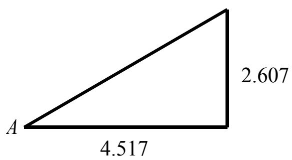
64. Since $\sin\theta = \frac{y}{r}$ for any given r, y increases as θ increases from 0° (terminal arm to the right) to 90° (terminal arm straight up). Since the denominator r does not change, the ratio must increase.

65.



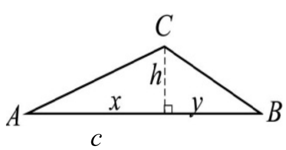
$$\frac{x}{3} = \frac{7}{2}$$
$$x = \frac{3(7)}{2}$$
$$x = 10.5$$

66.



$$\tan A = \frac{2.607}{4.517}$$
$$A = \tan^{-1}\left(\frac{2.607}{4.517}\right)$$
$$A = 29.99^\circ$$

67.

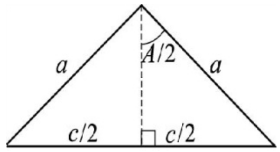


$$\cot A = \frac{x}{h}$$
$$x = h\cot A$$
$$\cot B = \frac{y}{h}$$
$$y = h\cot B$$
$$c = x + y$$

Substitute equations for x and y into equation for c

$$c = h\cot A + h\cot B$$

68.

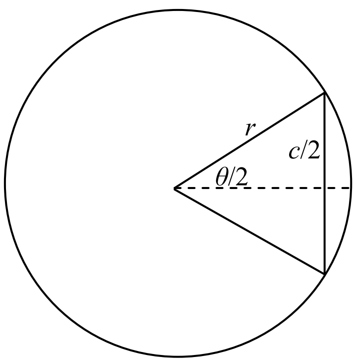


$$\sin\left(\tfrac{A}{2}\right) = \frac{\frac{c}{2}}{a}$$

$$\frac{c}{2} = a\sin\left(\tfrac{A}{2}\right)$$

$$c = 2a\sin\left(\tfrac{A}{2}\right)$$

69.

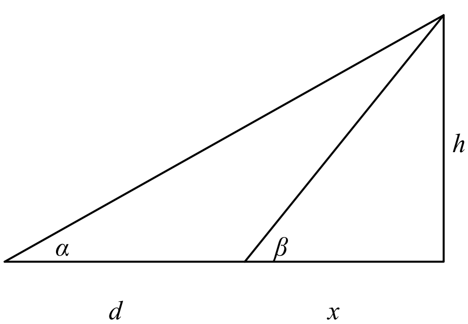


$$\sin\frac{\theta}{2} = \frac{c\,/\,2}{r}$$

$$\frac{c}{2} = r\sin\frac{\theta}{2}$$

$$c = 2r\sin\frac{\theta}{2}$$

70.



$$\cot\alpha = \frac{d+x}{h}$$

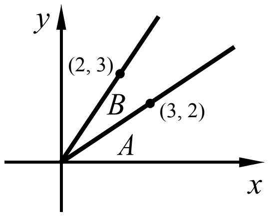
$$\cot\beta = \frac{x}{h}$$

$$x = h\cot\beta$$

$$d + x = h\cot\alpha$$

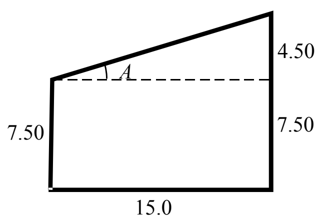
$$d = h\cot\alpha - h\cot\beta$$

71.



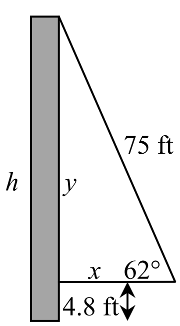
$$\tan A = \tfrac{2}{3}$$
$$A = \tan^{-1} \tfrac{2}{3}$$
$$A = 33.7^\circ$$
$$\tan(A+B) = \tfrac{3}{2}$$
$$A+B = \tan^{-1} \tfrac{3}{2}$$
$$A+B = 56.3^\circ$$
$$B = 56.3^\circ - A$$
$$B = 56.3^\circ - 33.7^\circ$$
$$B = 22.6^\circ$$

72.



$$\tan A = \frac{4.50}{15.0}$$
$$A = \tan^{-1}\left(\frac{4.50}{15.0}\right)$$
$$A = 16.7^\circ$$

73.



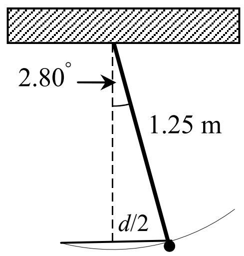
$y = (75\text{ ft})(\sin 62°) = 66.2\text{ ft}$

$h = y + 4.8\text{ ft} = 71\text{ ft}$

$x = (75\text{ ft})(\cos 62°) = 35.2\text{ ft}$

The ladder reaches 71 feet high up the building and the fire truck must be 35.2 feet away from the building.

74.



d = distance between extreme positions

$$\sin 2.80^\circ = \frac{\frac{d}{2}}{1.25\text{ m}}$$

$$d = 2\left(1.25\text{ m}\cdot\sin 2.80^\circ\right)$$

$$d = 0.122\text{ m}$$

75. $e = E\cos\alpha$

$$\cos\alpha = \frac{e}{E}$$

$$\alpha = \cos^{-1}\left(\frac{56.9}{339}\right)$$

$$\alpha = 80.3^\circ$$

76. $A = \frac{1}{2}d_1 d_2 \sin\theta$

$$A = \frac{1}{2}(320\text{ ft})(440\text{ ft})\sin 72.0^\circ = 66954.3787\text{ ft}^2$$

$A = 67000\text{ ft}^2$ rounded to two significant digits.

77. $\tan\theta = \frac{v^2}{gr}$

$$\theta = \tan^{-1}\frac{v^2}{gr}$$

$$\theta = \tan^{-1}\frac{(80.7\text{ ft/s})^2}{(950\text{ ft})(32.2\text{ ft/s}^2)}$$

$$\theta = 12.0^\circ$$

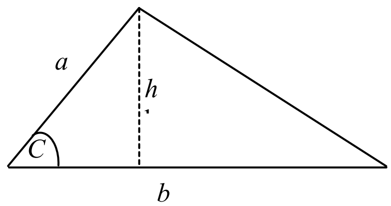
78. $S = P\sec\theta$

$$S = \frac{P}{\cos\theta}$$

$$S = \frac{12.0\text{ V}\cdot\text{A}}{\cos 29.4^\circ}$$

$$S = 13.8\text{ V}\cdot\text{A}$$

79.



(a) A triangle with angle C included between sides a and b, has an altitude h

$$\sin C = \frac{h}{a}$$
$$h = a\sin C$$

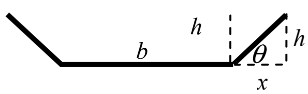
The area is $A = \frac{1}{2}bh$

$$A = \tfrac{1}{2}b(a\sin C)$$
$$A = \tfrac{1}{2}ab\sin C$$

(b) The area of the tract is

$$A = \tfrac{1}{2}(31.96\text{ m})(47.25\text{ m})\sin 64.09^\circ$$
$$A = 679.2\text{ m}^2$$

80.



(a) $\cot\theta = \frac{x}{h}$

$$x = h\cot\theta$$

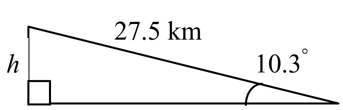
Area consists of a central rectangle and two triangles.

$$A = bh + 2\cdot\frac{1}{2}xh$$
$$A = bh + h\cot\theta\cdot h$$
$$A = bh + h^2\cot\theta$$

(b) $A = (12.6\text{ ft})(4.75\text{ ft}) + (4.75\text{ ft})^2\cot 37.2^\circ$

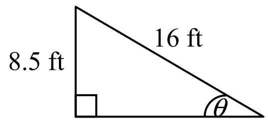
$$A = (12.6\text{ ft})(4.75\text{ ft}) + \frac{(4.75\text{ ft})^2}{\tan 37.2^\circ}$$
$$A = 89.6\text{ ft}^2$$

81.



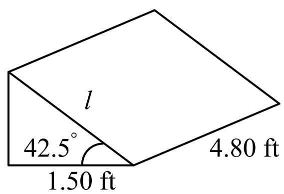
$$\sin 10.3^\circ = \frac{h}{27.5\text{ km}}$$
$$h = 27.5\text{ km}\cdot\sin 10.3^\circ$$
$$h = 4.92\text{ km}$$

82.



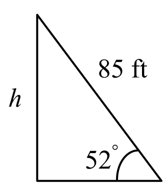
$$\sin\theta = \frac{8.5}{16}$$
$$\theta = \sin^{-1}\frac{8.5}{16}$$
$$\theta = 32^{\circ}$$

83.



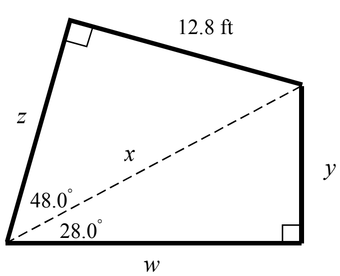
$$\cos 42.5^{\circ} = \frac{1.50\text{ ft}}{l}$$
$$l = \frac{1.50\text{ ft}}{\cos 42.5^{\circ}}$$
$$A = lw$$
$$A = \frac{1.50\text{ ft}}{\cos 42.5^{\circ}}(4.80\text{ ft})$$
$$A = 9.77\text{ ft}^2$$

84.



$$\sin 52.0^{\circ} = \frac{h}{85\text{ ft}}$$
$$h = 85\text{ ft}\cdot\sin 52.0^{\circ}$$
$$h = 67\text{ ft}$$

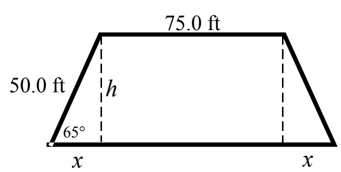
85.



$$76.0^{\circ} - 28.0^{\circ} = 48.0^{\circ}$$
$$\tan 48.0^{\circ} = \frac{12.8}{z}$$
$$z = \frac{12.8}{\tan 48.0^{\circ}}$$
$$z = 11.525\text{ ft}$$

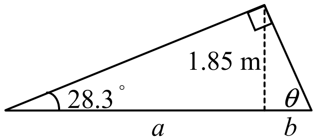
$$\sin 48.0^\circ = \frac{12.8}{x}$$
$$x = \frac{12.8}{\sin 48.0^\circ}$$
$$x = 17.224 \text{ ft}$$
$$\sin 28.0^\circ = \frac{y}{x}$$
$$y = x\sin 28.0^\circ$$
$$y = 17.224 \text{ ft} \cdot \sin 28.0^\circ$$
$$y = 8.086 \text{ ft}$$
$$\cos 28.0^\circ = \frac{w}{x}$$
$$w = x\cos 28.0^\circ$$
$$w = 17.224 \text{ ft} \cdot \cos 28.0^\circ$$
$$w = 15.208 \text{ ft}$$
$$A = \frac{1}{2}(12.8 \text{ ft})z + \frac{1}{2}wy$$
$$A = \frac{1}{2}(12.8 \text{ ft})(11.525 \text{ ft}) + \frac{1}{2}(15.208 \text{ ft})(8.086 \text{ ft})$$
$$A = 135 \text{ ft}^2$$

86.



$$\sin 65.0^\circ = \frac{h}{50.0 \text{ ft}}$$
$$h = 50.0 \text{ ft} \cdot \sin 65.0^\circ$$
$$h = 45.315 \text{ ft}$$
$$\cos 65.0^\circ = \frac{x}{50.0 \text{ ft}}$$
$$x = 50.0 \text{ ft} \cdot \cos 65.0^\circ$$
$$x = 21.131 \text{ ft}$$
$$V = A \cdot l$$
$$V = \frac{1}{2}h(b_1 + b_2) \cdot l$$
$$V = \frac{1}{2}(45.315 \text{ ft})\left[75.0 \text{ ft} + (75.0 \text{ ft} + 2(21.131 \text{ ft}))\right](5280 \text{ ft})$$
$$V = 23{,}000{,}000 \text{ ft}^3$$

87.



$$\tan 28.3^\circ = \frac{1.85}{a}$$

$$a = \frac{1.85}{\tan 28.3^\circ}$$

$$\theta = 90.0^\circ - 28.3^\circ$$

$$\theta = 61.7^\circ$$

$$\tan 61.7^\circ = \frac{1.85}{b}$$

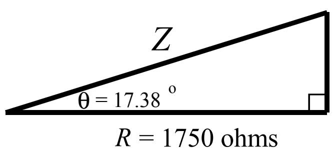
$$b = \frac{1.85}{\tan 61.7^\circ}$$

$$d = a + b$$

$$d = \frac{1.85}{\tan 28.3^\circ} + \frac{1.85}{\tan 61.7^\circ}$$

$$d = 4.43 \text{ m}$$

88.



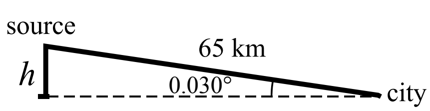
$$\cos\theta = \frac{R}{Z}$$

$$Z = \frac{R}{\cos\theta}$$

$$Z = \frac{1750}{\cos 17.38^\circ}$$

$$Z = 1830\ \Omega$$

89.



$$\sin 0.030^\circ = \frac{h}{65 \text{ km}}$$

$$h = (65 \text{ km}) \sin 0.030^\circ$$

$$h = 0.034033918 \text{ km}\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)$$

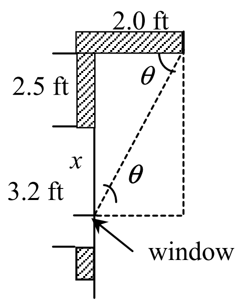
$$h = 34 \text{ m}$$

90. $\tan 76^\circ = \dfrac{\text{ceiling}}{950\text{ m}}$

$\text{ceiling} = (950\text{ m})\tan 76^\circ$

$\text{ceiling} = 3800\text{ m}$

91.



Let x = length of window through which sun does not shine.

$\tan 65^\circ = \dfrac{x+2.5}{2.0}$

$x = 2.0\tan 65^\circ - 2.5$

$x = 1.789\text{ ft}$

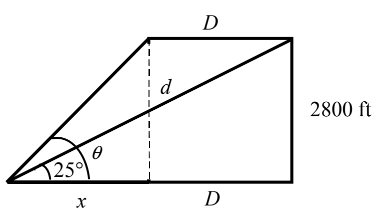
To find the fraction f of the window shaded,

$f = \dfrac{1.789\text{ ft}}{3.2\text{ ft}}$

$f = 0.55907$

$f = 56\%$

92.



$\sin 25^\circ = \dfrac{2800}{d}$

$d = \dfrac{2800}{\sin 25^\circ}$

$d = 6625.36\text{ ft}$

$d = 1130t$

$t = \dfrac{d}{1130}$

$t = \dfrac{6625.36\text{ ft}}{1130\text{ ft/s}}$

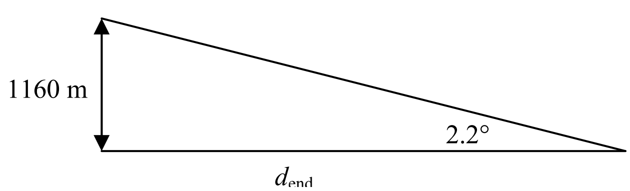
$t = 5.86315\text{ s}$

$$D = 660\text{ ft/s}\cdot t$$
$$D = 660\text{ ft/s}(5.86315\text{ s})$$
$$D = 3870\text{ ft}$$
$$\tan 25^\circ = \frac{2800}{x+D}$$
$$x + D = \frac{2800}{\tan 25^\circ}$$
$$x = \frac{2800\text{ ft}}{\tan 25^\circ} - 3870\text{ ft}$$
$$x = 2134.9375\text{ ft}$$
$$\tan\theta = \frac{2800}{x}$$
$$\theta = \tan^{-1}\ \frac{2800\text{ ft}}{2134.9375\text{ ft}}$$
$$\theta = 52.675^\circ$$
$$\theta = 53^\circ$$

93. $\sin 21.8^\circ = \dfrac{d}{14.2\text{ in}}$

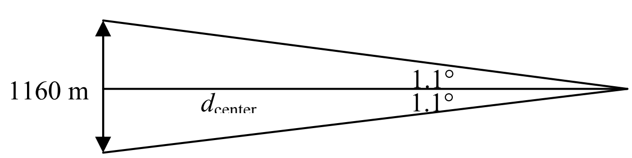
$$d = (14.2\text{ in})\sin 21.8^\circ$$
$$d = 5.2734\text{ in}$$
$$\sin 31.0^\circ = \frac{d}{x}$$
$$x = \frac{d}{\sin 31.0^\circ}$$
$$x = \frac{5.2734\text{ in}}{\sin 31.0^\circ}$$
$$x = 10.2\text{ in}$$

94.



Line of sight toward an endpoint of the span,

$$\tan 2.2^\circ = \frac{1160\text{ m}}{d_{\text{end}}}$$
$$d_{\text{end}} = \frac{1160\text{ m}}{\tan 2.2^\circ}$$
$$d_{\text{end}} = 30195.65\text{ m}$$



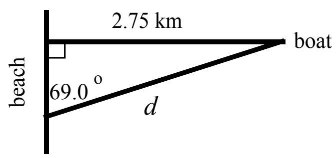
Line of sight toward the center of the span,

$$\tan 1.1^\circ = \frac{1160/2\text{ m}}{d_{\text{center}}}$$

$$d_{\text{center}} = \frac{580\text{ m}}{\tan 1.1^\circ}$$

$$d_{\text{center}} = 30206.79\text{ m}$$

95.



$$\sin 69.0^\circ = \frac{2.75\text{ km}}{d}$$

$$d = \frac{2.75\text{ km}}{\sin 69.0^\circ}$$

$$d = 2.9456\text{ km}$$

$$d = vt$$

$$t = \frac{d}{v}$$

$$t = \frac{2.9456\text{ km}}{37.5\text{ km/h}}$$

$$t = 0.078551\text{ h}\cdot\left(\frac{60\text{ min}}{1\text{ h}}\right)$$

$$t = 4.71\text{ min}$$

96. length of side piece = l

$$\sin 80.0^\circ = \frac{2.25\text{ m}}{l}$$

$$l = \frac{2.25\text{ m}}{\sin 80.0^\circ}$$

$$l = 2.28\text{ m}$$

length of base = $b = 2.25 + 2x$

$$\tan 80.0^\circ = \frac{2.25\text{ m}}{x}$$

$$x = \frac{2.25\text{ m}}{\tan 80.0^\circ}$$

$$x = 0.39674\text{ m}$$

$$b = 2.25\text{ m} + 2(0.396735706\text{ m})$$

$$b = 3.0435\text{ m}$$

$$A = \frac{1}{2}h(b_1 + b_2)$$

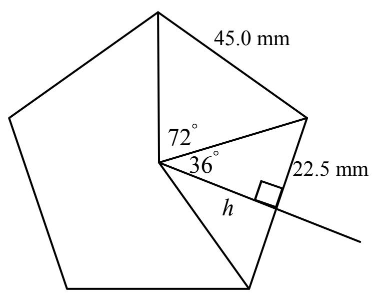
$$A = \frac{1}{2}(2.25\text{ m})(2.25\text{ m} + 3.0435\text{ m})$$

$$A = 5.96\text{ m}^2$$

97. $\sin\dfrac{0.00200^\circ}{2} = \dfrac{d/2}{52\,500 \text{ km}}$

$$d = 2(52500 \text{ km})(\sin 0.00100^\circ)$$
$$d = 1.83 \text{ km}$$

98.



Each of the five triangles in the pentagon has a central angle $\frac{360^\circ}{5}$ or 72°.
Radii drawn from the center of the pentagon through adjacent vertices of the pentagon form an isosceles triangle with base 45.0 mm.
A perpendicular bisector from the center of the pentagon to the base of this isosceles triangle forms a right triangle with height h and base 22.5 mm.
The central angle of this right triangle is $\frac{72^\circ}{2}$ or 36°. Thus,

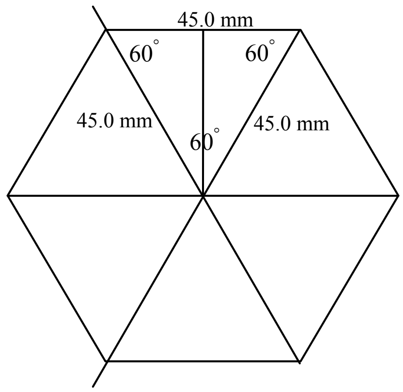
$$\tan 36^\circ = \frac{22.5}{h}$$
$$h = \frac{22.5}{\tan 36^\circ}$$
$$h = 30.969 \text{ mm}$$

The area of each triangle in the pentagon is

$$A = \tfrac{1}{2}bh$$
$$A = \tfrac{1}{2}(45.0 \text{ mm})(30.969 \text{ mm})$$
$$A = 696.79 \text{ mm}^2$$

Each pentagon has five triangles, and there are 12 pentagons on the ball, so the surface area of all the pentagons is

$$A_{\text{pentagons}} = (12)(5)(696.79 \text{ mm}^2)$$
$$A_{\text{pentagons}} = 41808 \text{ mm}^2$$



Each of the six triangles in the hexagon has a central angle $\frac{360^\circ}{6}$ or 60°.
Radii drawn from the center of the pentagon through adjacent vertices of the pentagon form an equilateral triangle with sides 45.0 mm.

$$\tan 30^\circ = \frac{22.5}{h}$$

$$h = \frac{22.5}{\tan 30^\circ}$$

$$h = 38.971 \text{ mm}$$

The area of each triangle in the pentagon is

$$A = \tfrac{1}{2}bh$$

$$A = \tfrac{1}{2}(45.0 \text{ mm})(38.971 \text{ mm})$$

$$A = 876.85 \text{ mm}^2$$

Each hexagon has six triangles, and there are 20 hexagons on the ball, so the surface area of all the hexagons is

$$A_{\text{hexagons}} = (20)(6)(876.85 \text{ mm}^2)$$

$$A_{\text{hexaagons}} = 105222 \text{ mm}^2$$

Total surface area of ball

$$A = 41\ 808 \text{ mm}^2 + 105\ 222 \text{ mm}^2$$

$$A = 147\ 000 \text{ mm}^2$$

Since this is the area of a flat surface it approximates the area of the spherical soccer ball which is given by

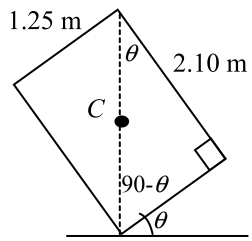
$$A = 4\pi r^2$$

$$A = 4\pi \cdot \left(\frac{222 \text{ mm}}{2}\right)^2$$

$$A = 155\ 000 \text{ mm}^2$$

The flat surface approximation does not account for the curved surface of the soccer ball.

99.

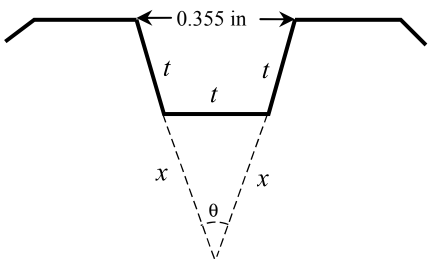


$$\tan\theta = \frac{1.25}{2.10}$$

$$\theta = \tan^{-1}\frac{1.25}{2.10}$$

$$\theta = 30.8^\circ$$

100.



There are similar triangles here.

$$\frac{t}{x} = \frac{0.355}{x+t}$$

But $t = 0.180$ in

$$\frac{0.180}{x} = \frac{0.355}{x+0.180}$$

$$0.355x = 0.180(x+0.180)$$

$$0.355x = 0.180x + 0.0324$$

$$0.175x = 0.0324$$

$$x = \frac{0.0324}{0.175}$$

$$x = 0.18514 \text{ in}$$

$$\sin\left(\tfrac{\theta}{2}\right) = \frac{t/2}{x}$$

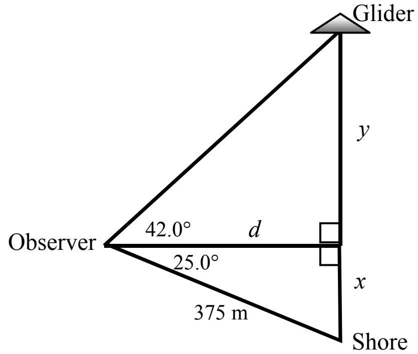
$$\sin\left(\tfrac{\theta}{2}\right) = \frac{0.180/2}{0.185142857}$$

$$\frac{\theta}{2} = \sin^{-1}(0.48611)$$

$$\frac{\theta}{2} = 29.085^\circ$$

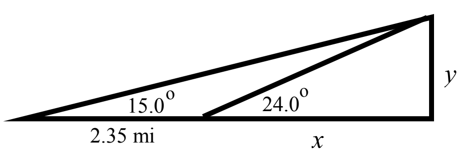
$$\theta = 58.2^\circ$$

101.



$$\cos 25.0^\circ = \frac{d}{375}$$
$$d = 375\cos 25.0^\circ$$
$$d = 339.87 \text{ m}$$
$$\sin 25.0^\circ = \frac{x}{375}$$
$$x = 375\sin 25.0^\circ$$
$$x = 158.48 \text{ m}$$
$$\tan 42.0^\circ = \frac{y}{d}$$
$$y = (339.87 \text{ m})\tan 42.0^\circ$$
$$x = 306.02 \text{ m}$$
$$x + y = 306.02 \text{ m} + 158.48 \text{ m}$$
$$x + y = 464 \text{ m}$$

102.



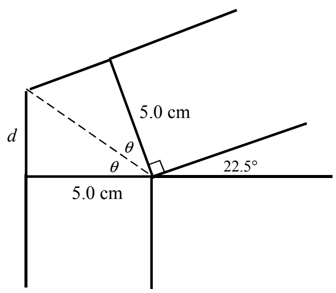
$$\tan 15.0^\circ = \frac{y}{x + 2.35}$$
$$y = (x + 2.35)\tan 15.0^\circ$$
$$\tan 24.0^\circ = \frac{y}{x}$$
$$y = x\tan 24.0^\circ$$

Since the ballon height is the same for each observer,

$$y = y$$

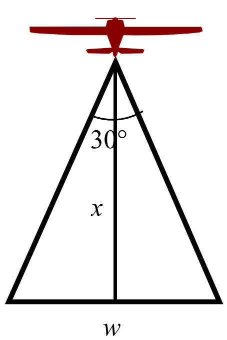
$$x\tan 24.0^\circ = (x+2.35)\tan 15.0^\circ$$
$$0.44523x = 0.26795x + 0.62968$$
$$0.17728x = 0.62968$$
$$x = \frac{0.62968}{0.17728}$$
$$x = 3.5519 \text{ mi}$$
$$\tan 24.0^\circ = \frac{y}{x}$$
$$y = x\tan 24.0^\circ$$
$$y = (3.5519 \text{ mi})\tan 24.0^\circ$$
$$y = 1.58 \text{ mi}$$

103.



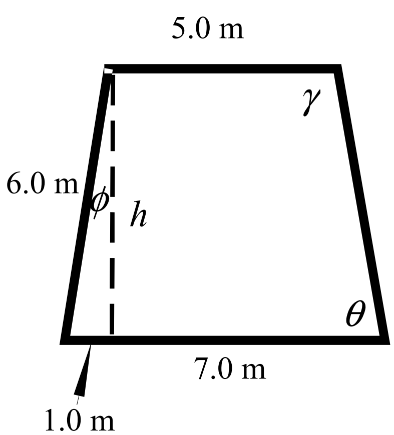
$$2\theta + 90^\circ + 22.5^\circ = 180^\circ$$
$$2\theta = 67.5^\circ$$
$$\theta = 33.75^\circ$$
$$\tan 33.75^\circ = \frac{d}{5.0 \text{ cm}}$$
$$d = 5.0 \text{ cm} \cdot \tan 33.75^\circ$$
$$d = 3.3409 \text{ cm}$$
$$l = 5.0 \text{ cm} + 65.0 \text{ cm} + 3.3409 \text{ cm}$$
$$l = 73.3 \text{ cm}$$

104.



$$x = \sqrt{25^2 + 75^2}$$
$$x = \sqrt{625 + 5625}$$
$$x = \sqrt{6250}$$
$$x = 79.05694 \text{ ft}$$
$$\tan 15^\circ = \frac{w/2}{x}$$
$$w = 2x \tan 15^\circ$$
$$w = 2(79.05694 \text{ ft}) \tan 15^\circ$$
$$w = 42.366487 \text{ ft}$$
$$w = 42 \text{ ft}$$

105.



base angle

$$\cos\theta = \frac{1.0}{6.0}$$
$$\theta = \cos^{-1}\tfrac{1}{6} = 80.4^\circ$$

upper angle

$$\sin\phi = \frac{1.0}{6.0}$$
$$\phi = \sin^{-1}\tfrac{1}{6} = 9.59^\circ$$
$$\gamma = 90^\circ + 9.59^\circ$$
$$\gamma = 99.6^\circ$$
$$\tan\phi = \frac{1.0}{h}$$
$$h = \frac{1.0}{\tan 9.59^\circ}$$
$$h = 5.9161 \text{ m}$$
$$A = \tfrac{1}{2}h(b_1 + b_2)$$
$$A = \tfrac{1}{2}(5.9161 \text{ m})(7.0 \text{ m} + 5.0 \text{ m})$$
$$A = 35 \text{ m}^2$$

Chapter 5

Systems of Linear Equations; Determinants

5.1 Linear Equations and Graphs of Linear Functions

1. $2x - y - 4 = 0$

For $x = -3$, the solution is

$$2(-3) - y - 4 = 0$$
$$-6 - y - 4 = 0$$
$$-10 = y$$

The point $(-3, -10)$ must lie on the graph of the line.

2. We wish to find the slope of the line through $(-1, -2)$ and $(3, -1)$.

We have $y_1 = -2$ and so

$$m = \frac{-1-(-2)}{3-(-1)} = \frac{1}{4}$$

3. Changing the sign from $+$ to $-$ results in the equation $2x - 3y = 4$.

Solving for y, we have $y = \frac{2}{3}x - \frac{4}{3}$. The slope is $\frac{2}{3}$ and the y-intercept is $(0, -\frac{4}{3})$.

4. Changing the sign from $-$ to $+$ results in the equation $2x + 3y = 6$. Setting $x = 0$ gives us $3y = 6$ or $y = 2$. Thus, $(0, 2)$ is the y-intercept. Setting $y = 0$ gives us $2x = 6$ or $x = 3$. Thus, $(3, 0)$ is the x-intercept.

5. The equation $8x - 3y = 12$ is linear, being of the form $ax + by = c$ where $a = 8, b = -3$ and $c = 12$.

6. The equation $2v + 3t^2 = 60$ is not linear because of the appearance of t^2, a nonlinear term.

7. The equation $2l + 3w = 4lw$ is not linear because of the appearance of lw, a nonlinear term.

8. The equation $I_1 + I_2 = I_2$ is linear in the variables I_1 and I_2 because it can be transformed into $I_1 = 0$; here, $a = 1, b = 0$ and $c = 0$.

9. $2x + 3y = 9$

The coordinates of the point $(3, 1)$ do satisfy the equation since

$2(3) + 3(1) = 6 + 3 = 9$

The coordinates of the point $\left(5, \frac{1}{3}\right)$ do not satisfy the equation since

$2(5) + 3\left(\frac{1}{3}\right) = 10 + 1 = 11 \neq 9.$

10. $5x+2y=1$

The coordinates of the point $(0.2, -1)$ do not satisfy the equation since

$5(0.2)+2(-1)=1-2=-1\neq 1$

The coordinates of the point $(1, -2)$ do satisfy the equation since

$5(1)+2(-2)=5-4=1$

11. $-3x+5y=13$

The coordinates of the point $(-1, 2)$ do satisfy the equation since

$-3(-1)+5(2)=3+10=13$

The coordinates of the point $(4, 5)$ do satisfy the equation since

$-3(4)+5(5)=-12+25=13$

12. $4y-x=-10$

The coordinates of the point $(2, -2)$ do not satisfy the equation since

$4(-2)-5(2)=-8-10=-18\neq -10$

The coordinates of the point $(2, 2)$ do not satisfy the equation since

$4(2)-5(2)=8-10=-2\neq -10$

13.
$$\begin{aligned} 3x-2y&=12 \\ \text{If } x&=2, \\ 3(2)-2y&=12 \\ 2y&=6-12 \\ 2y&=-6 \\ y&=-3 \\ \text{If } x&=-3, \\ 3(-3)-2y&=12 \\ 2y&=-9-12 \\ 2y&=-21 \\ y&=-\tfrac{21}{2} \end{aligned}$$

14.
$$\begin{aligned} 6y-5x&=60 \\ \text{If } x&=-10 \\ 6y-5(-10)&=60 \\ 6y&=60-50 \\ 6y&=10 \\ y&=\tfrac{5}{3} \\ \text{If } x&=8 \\ 6y-5(8)&=60 \\ 6y&=60+40 \\ 6y&=100 \\ y&=\tfrac{50}{3} \end{aligned}$$

15.
$$\begin{aligned}
x-4y&=2\\
\text{If } x&=3\\
3-4y&=2\\
-4y&=-1\\
y&=\tfrac{1}{4}\\
\text{If } x&=-0.4\\
-0.4-4y&=2\\
-4y&=2.4\\
y&=-0.6
\end{aligned}$$

16.
$$\begin{aligned}
3x-2y&=9\\
\text{If } x&=\tfrac{2}{3}\\
3\left(\tfrac{2}{3}\right)-2y&=9\\
2-2y&=9\\
-2y&=7\\
y&=-\tfrac{7}{2}\\
\text{If } x&=-3\\
3(-3)-2y&=9\\
-9-2y&=9\\
-2y&=18\\
y&=-9
\end{aligned}$$

17. $m=\dfrac{8-0}{3-1}=\dfrac{8}{2}=4$

18. $m=\dfrac{(-7)-1}{2-3}=\dfrac{-8}{-1}=8$

19. $m=\dfrac{17-2}{(-4)-(-1)}=\dfrac{15}{-3}=-5$

20. $m=\dfrac{10-(-2)}{6-(-1)}=\dfrac{12}{7}$

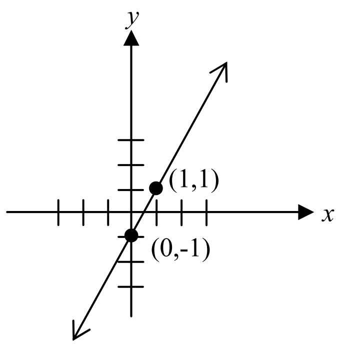
21. $m=\dfrac{(-5)-(-3)}{(-2)-5}=\dfrac{-2}{-7}=\dfrac{2}{7}$

22. $m=\dfrac{(-4)-4}{(-7)-(-3)}=\dfrac{-8}{-4}=2$

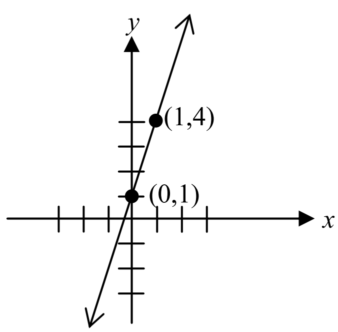
23. $m=\dfrac{0.2-0.5}{(-0.2)-0.4}=\dfrac{-0.3}{-0.6}=0.5$

24. $m = \dfrac{4.2 - 3.4}{1.2 - (-2.8)} = \dfrac{0.8}{4.0} = 0.2$

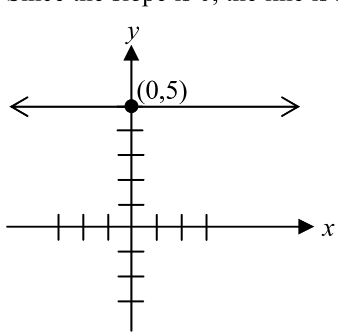
25. The line has slope 2 and passes through $(0, -1)$.
Since the slope is 2, the line rises 2 units for each unit of run going from left to right.
Thus, $(1, 1)$ is another point on the line.



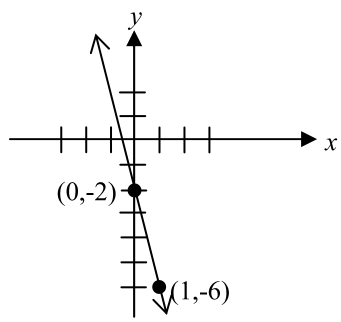
26. The line has slope 3 and passes through $(0, 1)$.
Since the slope is 3, the line rises 3 units for each unit of run going from left to right.
Thus, $(1, 4)$ is another point on the line.



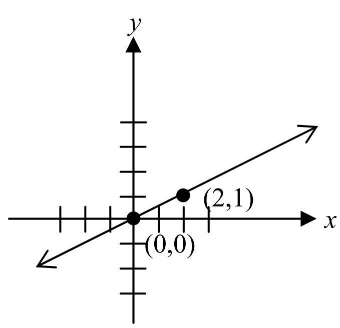
27. The line has slope 0 and passes through $(0, 5)$.
Since the slope is 0, the line is horizontal.



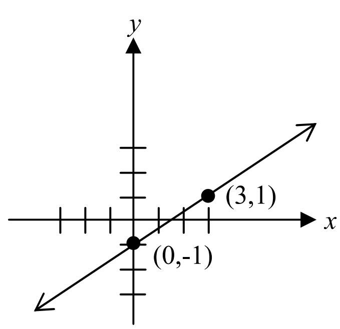
28. The line has slope -4 and passes through $(0,-2)$.
Since the slope is -4, the line drops 4 units for each unit of run going from left to right.
Thus, $(1,-6)$ is another point on the line.



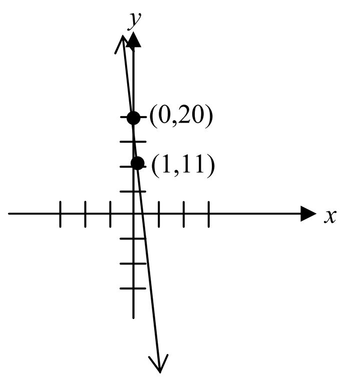
29. The line has slope $\frac{1}{2}$ and passes through $(0,0)$.
Since the slope is $\frac{1}{2}$, the line rises 1 unit for each 2 units of run going from left to right.
Thus, $(2,1)$ is another point on the line.



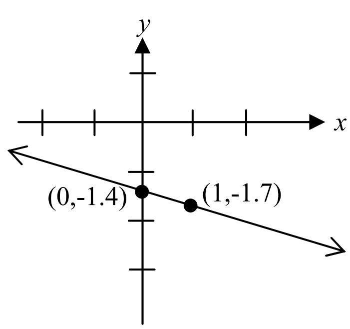
30. The line has slope $\frac{2}{3}$ and passes through $(0,-1)$.
Since the slope is $\frac{2}{3}$, the line rises 2 unit for each 3 units of run going from left to right.
Thus, $(3,1)$ is another point on the line.



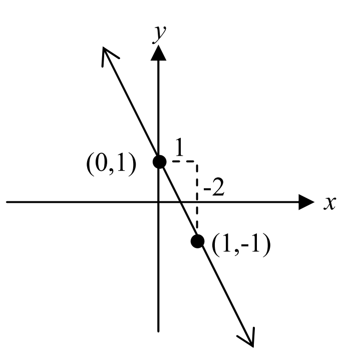
31. The line has slope -9 and passes through $(0, 20)$.
Since the slope is -9, the line drops 9 units for each unit of run going from left to right.
Thus, $(1, 11)$ is another point on the line.



32. The line has slope -0.3 and passes through $(0, -1.4)$.
Since the slope is -0.3, the line drops 0.3 units for each unit of run going from left to right.
Thus, $(1, -1.7)$ is another point on the line.



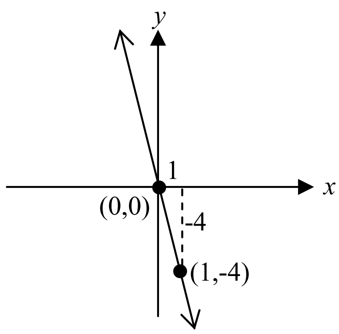
33. $y = -2x + 1$, compare to $y = mx + b$
$m = -2$, $b = 1$
Plot the y-intercept point $(0, 1)$. Since the slope is $-2/1$, from this point go right 1 unit and down 2 units, and plot a second point. Sketch a line passing through these two points.



34. $y = -4x$, compare to $y = mx + b$

$m = -4$, $b = 0$

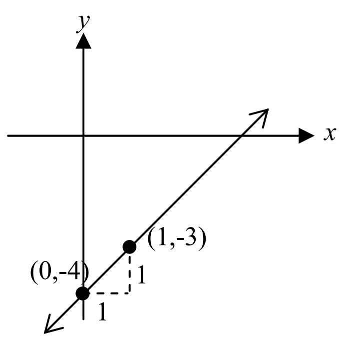
Plot the y-intercept $(0,0)$. Since the slope of the line is $-4/1$, from this point go right 1 unit, and down 4 units, and plot a second point. Sketch a line passing through these two points.



35. $y = x - 4$, compare to $y = mx + b$

$m = 1$, $b = -4$

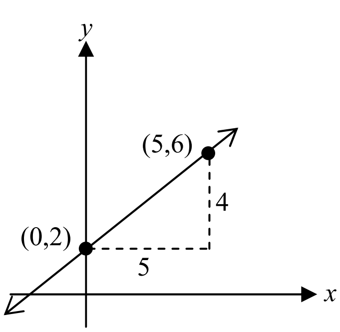
Plot the y-intercept $(0, -4)$. Since the slope of the line is $1/1$, from this point go to the right 1 unit, and up 1 unit and plot a second point. Sketch a line passing through these two points.



36. $y = \frac{4}{5}x + 2$, compare to $y = mx + b$

$m = \frac{4}{5}$, $b = 2$

Plot the y-intercept $(0, 2)$. Since the slope of the line is $4/5$, from this point go right 5 units and up 4 units and plot a second point. Sketch a line passing through these two points.



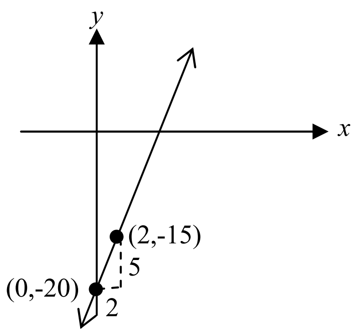
37. $5x - 2y = 40$

$2y = 5x - 40$

$y = \frac{5}{2}x - 20$, compare to $y = mx + b$

$m = \frac{5}{2}$, $b = -20$

Plot the y-intercept point $(0, -20)$. Since the slope is 5/2, from this point go right 2 units and up 5 units, and plot a second point. Sketch a line passing through these two points.

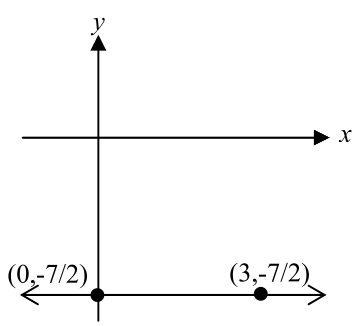


38. $-2y = 7$

$y = -\frac{7}{2}$, compare to $y = mx + b$

$m = 0$, $b = -\frac{7}{2}$

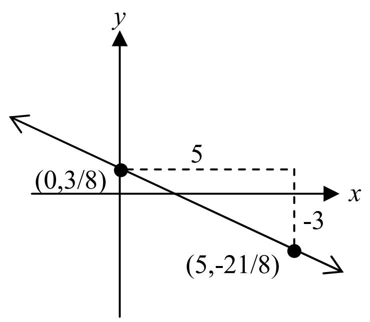
Plot the y-intercept $\left(0, -\frac{7}{2}\right)$. Since the slope is zero, from this point go any number of units to the right (in this case arbitrarily choose 3 units right), and then go 0 units up and plot a second point. Sketch a line passing through these two points.



39. $24x+40y=15$

$$40y=-24x+15$$
$$y=-\tfrac{24}{40}x+\tfrac{15}{40}$$
$$y=-\tfrac{3}{5}x+\tfrac{3}{8}, \text{ compare to } y=mx+b$$
$$m=-\tfrac{3}{5},\ b=\tfrac{3}{8}$$

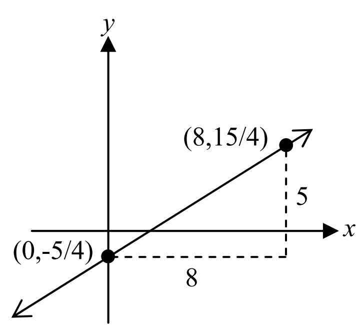
Plot the y-intercept $\left(0, \tfrac{3}{8}\right)$. Since the slope is $-3/5$, from this point go right 5 units and down 3 units and plot a second point. Sketch a line passing through these two points.



40. $1.5x-2.4y=3.0$

$$2.4y=1.5x-3.0$$
$$y=\tfrac{1.5}{2.4}x-\tfrac{3.0}{2.4}$$
$$y=\tfrac{5}{8}x-\tfrac{5}{4}, \text{ compare to } y=mx+b$$
$$m=\tfrac{5}{8},\ b=-\tfrac{5}{4}$$

Plot the y-intercept $\left(0, -\tfrac{5}{4}\right)$. The slope is 5/8, so from that point go right 8 units and up 5 units, and plot a second point. Sketch a line passing through these two points.



41. $x + 2y = 4$

For y-int, set $x = 0$

$0 + 2y = 4$

$2y = 4$

$y = 2$ y-int is (0, 2)

For x-int, set $y = 0$

$x + 0 = 4$

$x = 4$ x-int is (4, 0)

Plot the x-intercept point $(4, 0)$ and the y-intercept point $(0, 2)$.

Sketch a line passing through these two points.

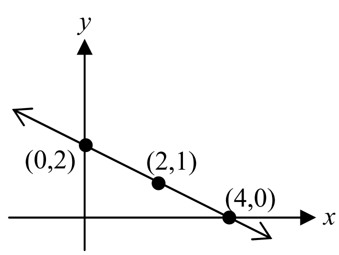
A third point is found as a check.

Let $x = 2$

$2 + 2y = 4$

$2y = 2$

$y = 1.$ Therefore the point $(2, 1)$ should lie on the line.



42. $3x + y = 3$

For y-int, set $x = 0$

$0 + y = 3$

$y = 3$ y-int is (0, 3)

For x-int, set $y = 0$

$3x + 0 = 3$

$x = 1$ x-int is (1, 0)

Plot the x-intercept point $(1, 0)$ and the y-intercept point $(0, 3)$.

Sketch a line passing through these two points.

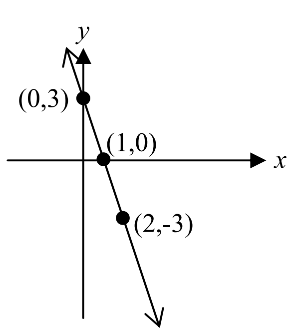
A third point is found as a check.

Let $x = 2$

$3(2) + y = 3$

$y = 3 - 6$

$y = -3.$ Therefore the point $(2, -3)$ should lie on the line.



43. $4x - 3y = 12$

For y-int, set $x = 0$

$0 - 3y = 12$

$y = -4$ y-int is $(0, -4)$

For x-int, set $y = 0$

$4x + 0 = 12$

$x = 3$ x-int is $(3, 0)$

Plot the x-intercept point $(3, 0)$ and the y-intercept point $(0, -4)$.

Sketch a line passing through these two points.

A third point is found as a check.

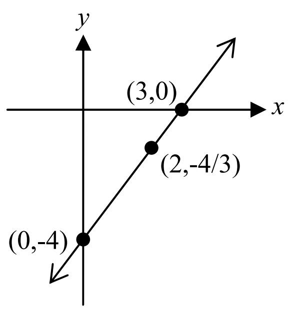
Let $x = 2$

$4(2) - 3y = 12$

$-3y = 4$

$y = -\frac{4}{3}$

Therefore the point $\left(2, -\frac{4}{3}\right)$ should lie on the line.



44. $5y - x = 5$

For y-int, set $x = 0$

$5y - 0 = 5$

$y = 1$ y-int is $(0, 1)$

For x-int, set $y = 0$

$0 - x = 5$

$x = -5$ x-int is $(-5, 0)$

Plot the x-intercept point $(-5, 0)$ and the y-intercept point $(0, 1)$.

Sketch a line passing through these two points.

A third point is found as a check.

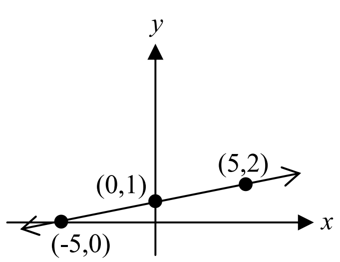
Let $x = 5$

$5y - (5) = 5$

$5y = 10$

$y = 2$

Therefore the point $(5, 2)$ should lie on the line.



45. $y = 3x + 6$

For y-int, set $x = 0$

$y = 0 + 6$

$y = 6$ y-int is (0, 6)

For x-int, set $y = 0$

$0 = 3x + 6$

$x = -2$ x-int is (− 2, 0)

Plot the x-intercept point $(-2, 0)$ and the y-intercept point $(0, 6)$.

Sketch a line passing through these two points.

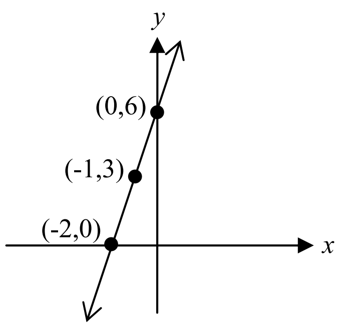
A third point is found as a check.

Let $x = -1$

$y = 3(-1) + 6$

$y = 3$

Therefore the point $(-1, 3)$ should lie on the line.



46. $y = -2x - 4$

For y-int, set $x = 0$

$y = 0 - 4$

$y = -4$ y-int is (0, − 4)

For x-int, set $y = 0$

$0 = -2x - 4$

$x = -2$ x-int is (− 2, 0)

Plot the x-intercept point $(-2, 0)$ and the y-intercept point $(0, -4)$.

Sketch a line passing through these two points.

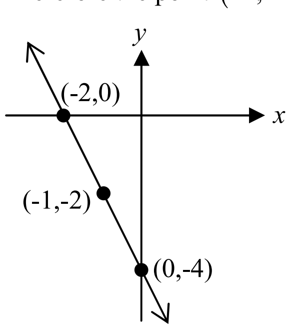
A third point is found as a check.

Let $x = -1$

$y = -2(-1) - 4$

$y = -2$

Therefore the point $(-1, -2)$ should lie on the line.



47. $12x + y = 30$

$y = -12x + 30$

For y-int, set $x = 0$

$y = 0 + 30$

$y = 30$ y-int is (0, 30)

For x-int, set $y = 0$

$0 = -12x + 30$

$x = \frac{30}{12}$

$x = \frac{5}{2}$ x-int is ($\frac{5}{2}$, 0)

Plot the x-intercept point $\left(\frac{5}{2}, 0\right)$ and the y-intercept point $(0, 30)$.

Sketch a line passing through these two points.

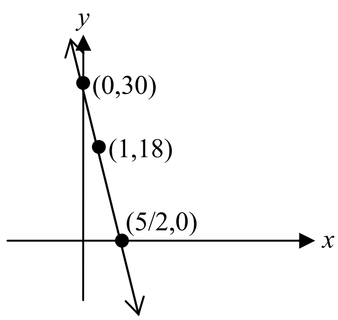
A third point is found as a check.

Let $x = 1$

$y = -12(1) + 30$

$y = 18$

Therefore the point $(1, 18)$ should lie on the line.



48. $y = 0.25x + 4.5$

For y-int, set $x = 0$

$y = 0 + 4.5$

$y = 4.5$ y-int is (0, 4.5)

For x-int, set $y = 0$

$0 = 0.25x + 4.5$

$x = -\frac{4.5}{0.25}$

$x = -18$ x-int is (− 18, 0)

Plot the x-intercept point $(-18, 0)$ and the y-intercept point $(0, 4.5)$.

Sketch a line passing through these two points.

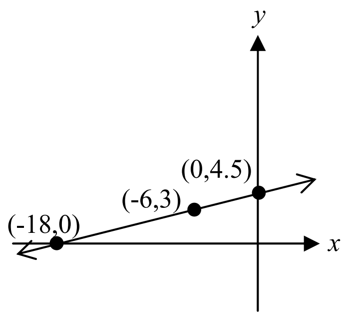
A third point is found as a check.

Let $x = -6$

$y = 0.25(-6) + 4.5$

$y = 3$

Therefore the point $(-6, 3)$ should lie on the line.



49. Using the LinReg(ax+b) feature of the TI-84, we find the regression line is $y = 32.500x - 84.714$. Substituting $x = 4.5$ yields $y = 61.536$ which is rounded to a prediction of $s = 62$ kN/mm.

50. Using the LinReg(ax+b) feature of the TI-84, we find the regression line is $y = 4.200x - 16.400$. Substituting $x = 16$ yields $y = 50.800$ which is rounded to a prediction of $p = 51\%$.

51. Using the LinReg(ax+b) feature of the TI-84, we find the regression line is $y = 5.689x - 45.582$. Substituting $x = 15.0$ yields $y = 39.753$ which is rounded to a prediction of $v = 39.8 \text{ ft}^3$.

52. Using the LinReg(ax+b) feature of the TI-84, we find the regression line is $y = 0.00650x + 0.10357$. Substituting $x = 38$ yields $y = 0.35057$ which is rounded to a prediction of $f = 0.35$.

53. Given the line $\frac{x}{a} + \frac{y}{b} = 1,$

For y-int, set $x = 0$

$$\frac{0}{a} + \frac{y}{b} = 1$$

$$\frac{y}{b} = 1$$

$$y = b \quad \text{The } y\text{-int is } (0, b)$$

For x-int, set $y = 0$

$$\frac{x}{a} + \frac{0}{b} = 1$$

$$\frac{x}{a} = 1$$

$$x = a \quad \text{The } x\text{-int is } (a, 0)$$

54. Given the line $y = mx + b$,

For y-int, set $x = 0$

$$y = m \cdot 0 + b$$

$$y = b \quad \text{The } y\text{-int is } (0, b)$$

For x-int, set $y = 0$

$$0 = mx + b$$

$$mx = -b$$

$$x = -\frac{b}{m} \quad \text{The } x\text{-int is } \left(-\tfrac{b}{m},\ 0\right)$$

55. If the points lie on the same line, that line would have slope

$$m = \frac{-3-(-2)}{3-1} = -\frac{1}{2}$$

calculated using the two points $(1, -2)$ and $(3, -3)$. On the other hand, the line would have slope

$$m = \frac{-7-(-6)}{11-7} = -\frac{1}{4}$$

calculated using the two points $(7, -6)$ and $(11, -7)$. A line cannot have two different slopes, so the points cannot lie on the same line.

56. The line through $(-2, 5)$ and $(3, 7)$ has slope

$$m = \frac{7-5}{3-(-2)} = \frac{2}{5}.$$

To find an equation $y = mx + b$ for the line, we substitute $y = 5, x = -2, m = \frac{2}{5}$ obtaining

$$5 = \frac{2}{5}(-2) + b$$

$$5 + \frac{4}{5} = b$$

$$b = \frac{29}{5}$$

and so an equation for the line is $y = \frac{2}{5}x + \frac{29}{5}$.

(a) When $x = -12$ we have $y = \frac{2}{5}(-12) + \frac{29}{5} = 1$ and so the point $(-12, 1)$ is on the line.

(b) When $y = -3$ we have

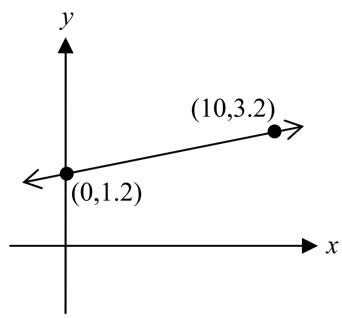
$$-3 = \frac{2}{5}x + \frac{29}{5}$$

$$-\frac{44}{5} = \frac{2}{5}x$$

$$x = -22$$

and so the point $(-22, -3)$ is on the line.

57. For the line $d = 0.2l + 1.2,$ we have a slope of 0.2 and an intercept of $(0,1.2)$. When $l = 10$ we have $d = 0.2 \cdot 10 + 1.2 = 3.2$ and so $(10,3.2)$ is another point on the line.



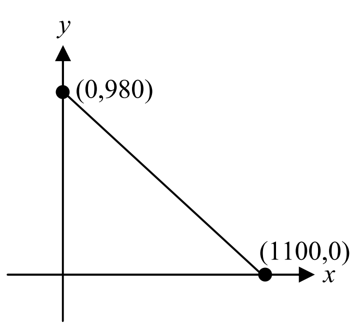
58. For the line $0.85x + 0.93y = 910,$ we use intercepts.
Setting $y = 0$ gives

$$0.85x + 0.93 \cdot 0 = 910$$
$$0.85x = 910$$
$$x = \frac{910}{0.85} \approx 1100$$

and so $(1100,0)$ is an intercept.
Setting $x = 0$ gives

$$0.85 \cdot 0 + 0.93y = 910$$
$$0.93y = 910$$
$$y = \frac{910}{0.93} \approx 980$$

and so $(0,980)$ is an intercept.



59. For the line $4I_1 - 5I_2 = 2,$ we use intercepts.
Setting $I_2 = 0$ gives

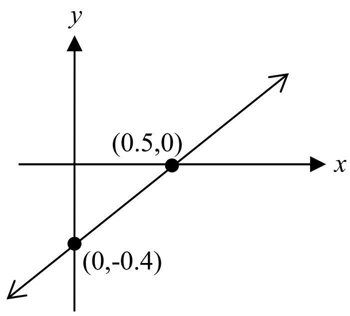
$$4I_1 - 5 \cdot 0 = 2$$
$$4I_1 = 2$$
$$I_1 = \frac{1}{2}$$

and so $(\frac{1}{2},0) = (0.5,0)$ is an intercept.

Setting $I_1 = 0$ gives

$$4 \cdot 0 - 5I_2 = 2$$
$$-5I_2 = 2$$
$$I_2 = -\frac{2}{5}$$

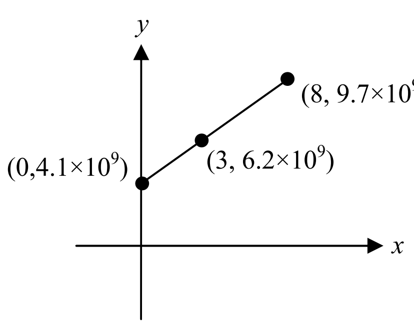
and so $(0, -\frac{2}{5}) = (0, -0.4)$ is an intercept.



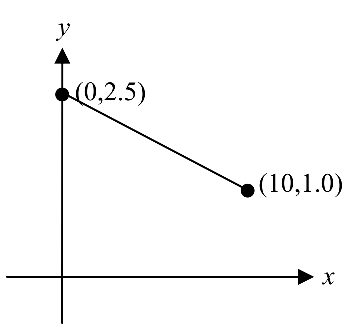
60. We let t be the number of years after 2009 and N be the number of text messages.
When $t = 0$ we have $N = 4.1 \times 10^9$ giving us $(0, 4.1 \times 10^9)$ as an intercept; also when $t = 3$ we have $N = 6.2 \times 10^9$ and so $(3, 6.2 \times 10^9)$ is on the line.
Thus,

$$m = \frac{6.2 \times 10^9 - 4.1 \times 10^9}{3 - 0} = \frac{2.1 \times 10^9}{3} = 7.0 \times 10^8 \text{ and } b = 4.1 \times 10^9$$

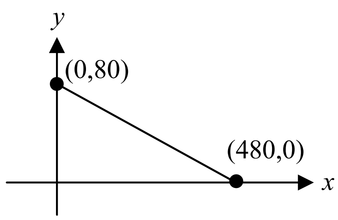
and so $N = 7.0 \times 10^8 t + 4.1 \times 10^9$.
At $t = 8$ we have $N = 7.0 \times 10^8 \cdot 8 + 4.1 \times 10^9 = 9.7 \times 10^9$.



61. Letting t be time in minutes and h be altitude in kilometers,
we are told $h = 2.5$ at $t = 0$ and so $(0, 2.5)$ is the $y-$intercept.
After 10 minutes, the plane has descended $150 \cdot 10 = 1500\text{m} = 1.5$ km.
Thus, when $t = 10, h = 2.5 - 1.5 = 1.0$ and so $(10, 1.0)$ is another point on the line.



62. Letting d be horizontal distance in meters and h be height in meters, we are told $h = 80$ at $d = 0$ and so $(0, 80)$ is the $y-$intercept. When $d = 480$ we have $h = 0$ and so $(480, 0)$ is the $x-$ intercept.



5.2 Systems of Equations and Graphical Solutions

1. In Example 2, we check to see if F_1=20, F_2=40 is a solution to

$$2F_1 + 4F_2 = 200$$
$$F_2 = 2F_1$$

$$2(20) + 4(40) \stackrel{?}{=} 200 \qquad 40 \stackrel{?}{=} 2(20)$$
$$200 = 200 \qquad 40 = 40$$

and we do in fact have a solution to the system of equations.

2. For the line $2x + 5y = 10$

Let $x = 0$ to find the y-int

$5y = 10$

$y = 2 \quad y\text{-int is } (0, 2)$

Let $y = 0$ to find the x-int

$2x = 10$

$x = 5 \quad x\text{-int is } (5, 0)$

Let $x = 1$ to find a third point

$2(1) + 5y = 10$

$5y = 8$

$y = \frac{8}{5} \quad \text{A third point is } \left(1, \frac{8}{5}\right)$

For the line $3x + y = 6$

Let $x = 0$ to find the y-int

$y = 6 \quad y\text{-int is } (0, 6)$

Let $y = 0$ to find the x-int

$3x = 6$

$x = 2 \quad x\text{-int is } (2, 0)$

Let $x = 1$ to find a third point

$$3(1) + y = 6$$

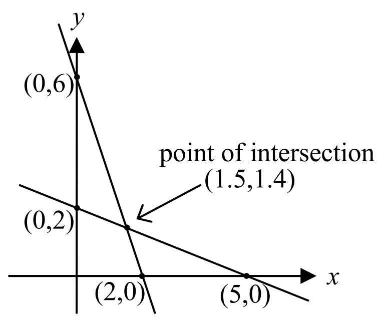
$y = 3$ A third point is $(1, 3)$

From the graph the solution is approximately

$x = 1.5,\ y = 1.4$

Checking both equations,

$$\begin{aligned} 2x + 5y &= 10 \\ 2(1.5) + 5(1.4) &= 10 \\ 10 &= 10 \end{aligned} \qquad \begin{aligned} 3x + y &= 6 \\ 3(1.5) + 1.4 &= 6 \\ 5.9 &\approx 6 \end{aligned}$$



3. If the 6 is changed to a 2 in the first equation of example 7, we obtain the dependent system

$$x = 2y + 2$$
$$6y = 3x - 6$$

because these two equations determine the same line.

4. **(a)** The equations are identical, but if the 9 in the first equation is changed, the function will keep the same slope, but have a different intercept, producing an inconsistent system.

(b) To make the system consistent, the slope of one of the lines must change, and this can be achieved by changing the 1 or 3 in equation 1.

5.
$$x - y = 5$$
$$2x + y = 7$$

If the values $x = 4$ and $y = -1$ satisfy both equations, they are a solution.

$$4 - (-1) = 4 + 1 = 5$$
$$2(4) + (-1) = 8 - 1 = 7$$

Therefore the given values are a solution.

6.
$$2x + y = 8$$
$$3x - y = -13$$

If the values $x = -1$ and $y = 10$ satisfy both equations, they are a solution.

$$2(-1) + 10 = -2 + 10 = 8$$
$$3(-1) - 10 = -3 - 10 = -13$$

Therefore the given values are a solution.

7.
$$A + 5B = -7$$
$$3A - 4B = -4$$

If the values $A = -2$ and $B = 1$ satisfy both equations, they are a solution.

$$-2 + 5(1) = 3 \neq -7$$
$$3(-2) - 4(1) = -10 \neq -4$$

Since neither equation is satisfied, the given values are not a solution.

8.
$$3y - 6x = 4$$
$$6x - 3y = -4$$

If the values $x = \frac{1}{3}$ and $y = 2$ satisfy both equations, they are a solution.

$$3(2) - 6(\tfrac{1}{3}) = 6 - 2 = 4$$
$$6\left(\tfrac{1}{3}\right) - 3(2) = 2 - 6 = -4$$

Since both equations are satisfied, the given values are a solution.

9.
$$2x - 5y = 0$$
$$4x + 10y = 4$$

If the values $x = \frac{1}{2}$ and $y = -\frac{1}{5}$ satisfy both equations, they are a solution.

$$2\left(\tfrac{1}{2}\right) - 5\left(-\tfrac{1}{5}\right) = 1 + 1 = 2 \neq 0$$
$$4\left(\tfrac{1}{2}\right) + 10\left(-\tfrac{1}{5}\right) = 2 - 2 = 0 \neq 4$$

Neither equation is satisfied, therefore the given values are not a solution.

10.
$$6i_1 + i_2 = 5$$
$$3i_1 - 4i_2 = -1$$

If the values $i_1 = 1$ and $i_2 = -1$ satisfy both equations, they are a solution.

$$6(1) + (-1) = 6 - 1 = 5$$
$$3(1) - 4(-1) = 3 + 4 = 7 \neq -1$$

The second equation is not satisfied, they are not a solution.

11. $3x - 2y = 2.2$

$5x + y = 2.8$

If the values $x = 0.6$ and $y = -0.2$ satisfy both equations, they are a solution.

$3(0.6) - 2(-0.2) = 1.8 + 0.4 = 2.2$

$5(0.6) + (-0.2) = 3 - 0.2 = 2.8$

Both equations are satisfied, therefore they are a solution.

12. $7t - s = 3.2$

$2s + t = 2.5$

If the values $s = -1.1$ and $t = 0.3$ satisfy both equations, they are a solution.

$7(0.3) - (-1.1) = 2.1 + 1.1 = 3.2$

$2(-1.1) + 0.3 = -2.2 + 0.3 = -1.9 \neq 2.5$

The second equation is not satisfied, so they are not a solution.

13. $y = -x + 4$ and $y = x - 2$

The slope of the first line is -1, and the y-intercept is 4.

The slope of the second line is 1 and the y- intercept is -2.

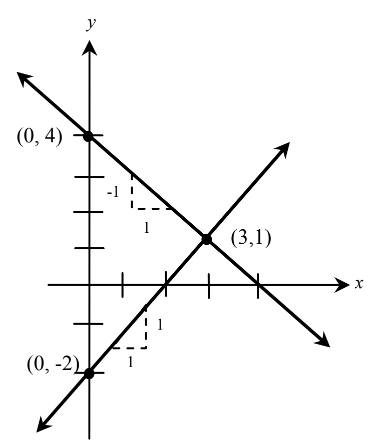
From the graph, the point of intersection is approximately at $(3, 1)$.

Therefore, the solution of the system of equations is

$x = 3.0,\ y = 1.0.$

Check:

$y = -x + 4$	$y = x - 2$
$1.0 = -3.0 + 4$	$1.0 = 3.0 - 2$
$1.0 = 1.0$	$1.0 = 1.0$



14. $y = \frac{1}{2}x - 1$ and $y = -x + 8$

The slope of the first line is $1/2$, and the y-intercept is -1.

The slope of the second line is -1, and the y-intercept is 8.

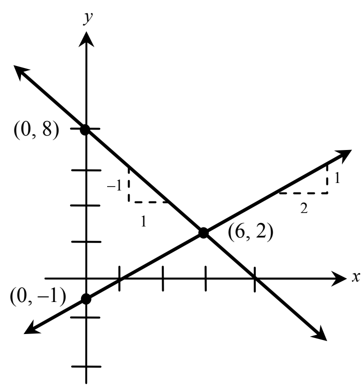
From the graph, the point of intersection is approximately $(6, 2)$.

Therefore, the solution of the system of equations is

$x = 6.0,\ y = 2.0.$

Check:

$$y = \tfrac{1}{2}x - 1 \qquad\qquad y = -x + 8$$
$$2.0 = \tfrac{1}{2}(6.0) - 1 \qquad\qquad 2.0 = -6.0 + 8$$
$$2.0 = 2.0 \qquad\qquad 2.0 = 2.0$$

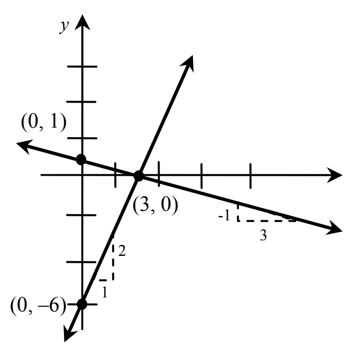


15. $y = 2x - 6$ and $y = -\frac{1}{3}x + 1$

The slope of the first line is 2, and the y-intercept is -6.
The slope of the second line is $-1/3$ and the y-intercept is 1.
From the graph, the point of intersection is $(3, 0)$. Therefore, the solution of the system of equations is
$x = 3.0,\ y = 0.0.$

Check:

$$y = 2x - 6 \qquad\qquad y = -\tfrac{1}{3}x + 1$$
$$0.0 = 2(3.0) - 6 \qquad\qquad 0.0 = -\tfrac{1}{3}(3.0) + 1$$
$$0.0 = 0.0 \qquad\qquad 0.0 = 0.0$$



16. $2y = x - 8$ and $y = 2x + 2$

The slope of the first line is 1/2, and the y-intercept is -4.

The slope of the second line is 2 and the y-intercept is 2.

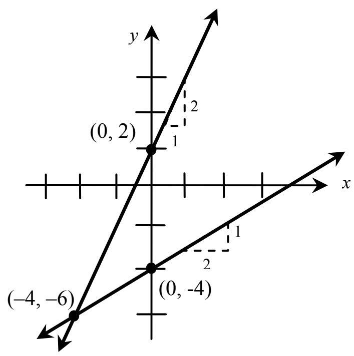
From the graph, the point of intersection is $(-4, -6)$.

Therefore, the solution of the system of equations is

$x = -4.0,\ y = -6.0$

Check:

$$\begin{array}{cc} 2y = x - 8 & y = 2x + 2 \\ 2(-6.0) = -4.0 - 8 & -6.0 = 2(-4.0) + 2 \\ -12.0 = -12.0 & -6.0 = -6.0 \end{array}$$



17. For line $3x + 2y = 6$

Let $x = 0$ to find the y-int

$2y = 6$

$y = 3$ y-int is $(0, 3)$

Let $y = 0$ to find the x-int

$3x = 6$

$x = 2$ x-int is $(2, 0)$

Let $x = 4$ to find a third point

$3(4) + 2y = 6$

$2y = -6$

$y = -3$ A third point is $(4, -3)$

For line $x - 3y = 3$

Let $x = 0$ to find the y-int

$-3y = 3$

$y = -1$ y-int is $(0, -1)$

Let $y = 0$ to find the x-int

$x = 3$ x-int is $(3, 0)$

Let $x = 1$ to find a third point

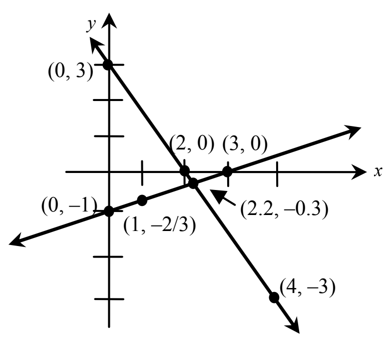
$$1 - 3y = 3$$
$$-3y = 2$$
$$y = -\tfrac{2}{3} \quad \text{A third point is } \left(1, -\tfrac{2}{3}\right)$$

From the graph the solution is approximately $(2.2, -0.3)$

$x = 2.2,\ y = -0.3$

Checking both equations,

$$\begin{aligned} 3x + 2y &= 6 & x - 3y &= 3 \\ 3(2.2) + 2(-0.3) &= 6 & 2.2 - 3(-0.3) &= 3 \\ 6.0 &= 6 & 3.1 &\approx 3 \end{aligned}$$



18. For line $4R - 3V = -8$

Let $R = 0$ to find the V-int

$$-3V = -8$$
$$V = \tfrac{8}{3} \quad V\text{-int is } \left(0, \tfrac{8}{3}\right)$$

Let $V = 0$ to find the R-int

$$4R = -8$$
$$R = -2 \quad R\text{-int is } (-2, 0)$$

Let $R = 1$ to find a third point

$$4(1) - 3V = -8$$
$$-3V = -12$$
$$V = 4 \quad \text{A third point is } (1,\ 4)$$

For line $6R + V = 6$

Let $R = 0$ to find the V-int

$$V = 6 \quad V\text{-int is } (0, 6)$$

Let $V = 0$ to find the R-int

$$6R = 6$$
$$R = 1 \quad R\text{-int is } (1, 0)$$

Let $R = 2$ to find a third point

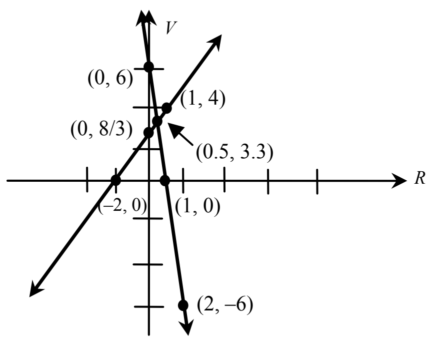
$$6(2)+V=6$$
$$V=-6 \quad \text{A third point is } (2, -6)$$

From the graph the solution is approximately $(0.5, 3.3)$

$R = 0.5, V = 3.3$

Checking both equations,

$$4R-3V=-8 \qquad 6R+V=6$$
$$4(0.5)-3(3.3)=-8 \qquad 6(0.5)+3.3=3$$
$$-7.9\approx -8 \qquad 6.3\approx 6$$



19. For line $2x-5y=10$

Let $x=0$ to find the y-int

$$-5y=10$$
$$y=-2 \quad y\text{-int is } (0, -2)$$

Let $y=0$ to find the x-int

$$2x=10$$
$$x=5 \quad x\text{-int is } (5, 0)$$

Let $x=2.5$ to find a third point

$$2(2.5)-5y=10$$
$$-5y=5$$
$$y=-1 \quad \text{A third point is } (2.5, -1)$$

For line $3x+4y=-12$

Let $x=0$ to find the y-int

$$4y=-12$$
$$y=-3 \quad y\text{-int is } (0, -3)$$

Let $y=0$ to find the x-int

$$3x=-12$$
$$x=-4 \quad x\text{-int is } (-4, 0)$$

Let $x=-2$ to find a third point

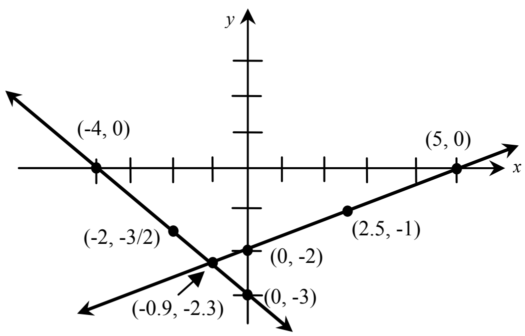
$$3(-2)+4y=-12$$
$$4y=-6$$
$$y=-\tfrac{3}{2} \quad \text{A third point is } \left(-2, -\tfrac{3}{2}\right)$$

From the graph the solution is approximately $(-0.9, -2.3)$

$x = -0.9,\ y = -2.3$

Checking both equations,

$$2x - 5y = 10 \qquad 3x + 4y = -12$$
$$2(-0.9) - 5(-2.3) = 10 \qquad 3(-0.9) + 4(-2.3) = -12$$
$$9.7 \approx 10 \qquad -11.9 \approx -12$$



20. For line $-5x + 3y = 15$

Let $x = 0$ to find the y-int

$3y = 15$

$y = 5$ $\quad y$-int is $(0, 5)$

Let $y = 0$ to find the x-int

$-5x = 15$

$x = -3$ $\quad x$-int is $(-3, 0)$

Let $x = 1$ to find a third point

$-5(1) + 3y = 15$

$3y = 20$

$y = \frac{20}{3}$ $\quad$ A third point is $\left(1,\ \frac{20}{3}\right)$

Let $x = 0$ to find the y-int

$7y = 14$

$y = 2$ $\quad y$-int is $(0, 2)$

Let $y = 0$ to find the x-int

$2x = 14$

$x = 7$ $\quad x$-int is $(7, 0)$

Let $x = 5$ to find a third point

$2(5) + 7y = 14$

$7y = 4$

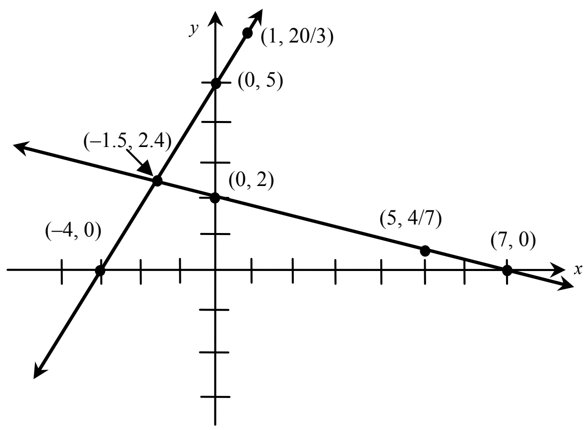
$y = \frac{4}{7}$ $\quad$ A third point is $\left(5,\ \frac{4}{7}\right)$

From the graph the solution is approximately $(-1.5,\ 2.4)$

$x = -1.5,\ y = 2.4$

Checking both equations,

$$\begin{aligned} -5x + 3y &= 15 & 2x + 7y &= 14 \\ -5(-1.5) + 3(2.4) &= 15 & 2(-1.5) + 7(2.4) &= 14 \\ 14.7 &\approx 15 & 13.8 &\approx 14 \end{aligned}$$



21. $s - 4t = 8$ and $2s = t + 4$

$s = 4t + 8$ $\quad s = \frac{1}{2}t + 2$

The slope of the first line is 4, and the s-intercept is 8.

The slope of the second line is 1/2 and the s-intercept is 2.

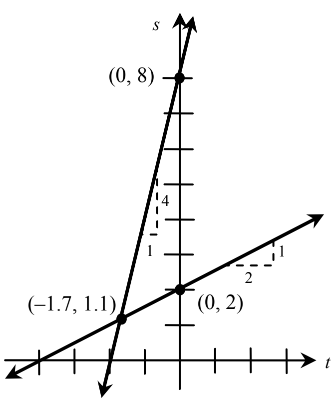
From the graph, the point of intersection is $(-1.7, 1.1)$.

Therefore, the solution of the system of equations is

$t = -1.7,\ s = 1.1.$

Check:

$$\begin{aligned} s &= 4t + 8 & s &= \tfrac{1}{2}t + 2 \\ 1.1 &= 4(-1.7) + 8 & 1.1 &= \tfrac{1}{2}(-1.7) + 2 \\ 1.1 &\approx 1.2 & 1.1 &\approx 1.2 \end{aligned}$$



22. $y = 4x - 6$ and $y = 2x + 4$

The slope of the first line is 4, and the y-intercept is -6.

The slope of the second line is 2 and the y-intercept is 4.

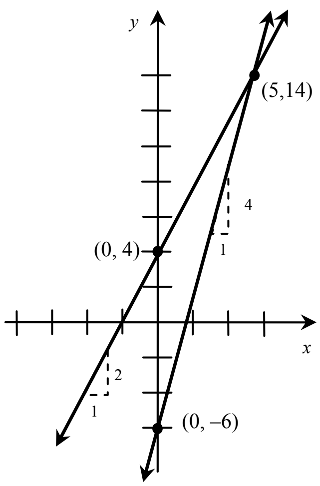
From the graph, the point of intersection is $(5, 14)$.

Therefore, the solution of the system of equations is

$x = 5,\ y = 14$

Check:

$$\begin{array}{ll} y = 4x - 6 & y = 2x + 4 \\ 14 = 4(5) - 6 & 14 = 2(5) + 4 \\ 14 = 14 & 14 = 14 \end{array}$$



23. $y = -x + 3$ and $y = -2x + 3$

The slope of the first line is -1, and the y-intercept is 3.

The slope of the second line is -2, and the y-intercept is 3.

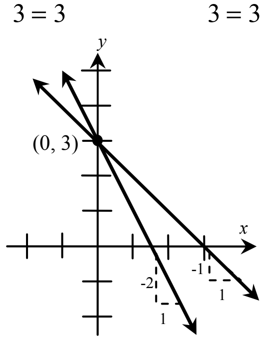
From the graph, the point of intersection is $(0, 3)$.

Therefore, the solution of the system of equations is

$x = 0,\ y = 3$

Check:

$$\begin{array}{ll} y = -x + 3 & y = -2x + 3 \\ 3 = 0 + 3 & 3 = 0 + 3 \\ 3 = 3 & 3 = 3 \end{array}$$



24. $p-6=6v$ and $v=3-3p$

$p=6v+6$ $\quad p=-\frac{1}{3}v+1$

The slope of the first line is 6, and the p-intercept is 6.

The slope of the second line is $-1/3$, and the p-intercept is 1.

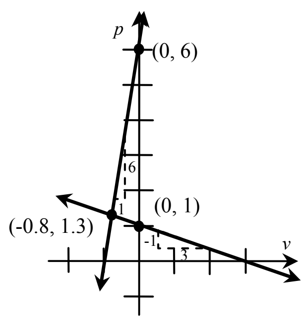
From the graph, the point of intersection is $(-0.8, 1.3)$.

Therefore, the solution of the system of equations is

$v=-0.8,\ p=1.3$

Check:

$$\begin{array}{ll} p-6=6v & v=3-3p \\ 1.3-6=6(-0.8) & -0.8=3-3(1.3) \\ -4.7\approx-4.8 & -0.8\approx-0.9 \end{array}$$



25. $x-4y=6$ and $2y=x+4$

$4y=x-6$ $\quad y=\frac{1}{2}x+2$

$y=\frac{1}{4}x-\frac{3}{2}$

The slope of the first line is 1/4, and the y-intercept is $-3/2$.

The slope of the second line is 1/2 and the y-intercept is 2.

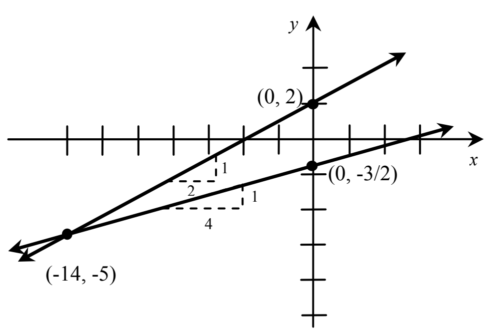
From the graph, the point of intersection is $(-14, -5)$.

Therefore, the solution of the system of equations is

$x=-14.0,\ y=-5.0$

Check:

$$\begin{array}{rr} x-4y=6 & 2y=x+4 \\ -14-4(-5)=6 & 2(-5)=-14+4 \\ 6=6 & -10=-10 \end{array}$$



26. For the line $x + y = 3$

Let $x = 0$ to find the y-int

$y = 3 \quad y\text{-int is } (0, 3)$

Let $y = 0$ to find the x-int

$x = 3 \quad x\text{-int is } (3, 0)$

Let $x = 1$ to find a third point

$1 + y = 3$

$y = 2 \quad \text{A third point is } (1, 2)$

For the line $3x - 2y = 14$

Let $x = 0$ to find the y-int

$-2y = 14$

$y = -7 \quad y\text{-int is } (0, -7)$

Let $y = 0$ to find the x-int

$3x = 14$

$x = \frac{14}{3} \quad x\text{-int is } \left(\frac{14}{3}, 0\right)$

Let $x = 2$ to find a third point

$3(2) - 2y = 14$

$-2y = 8$

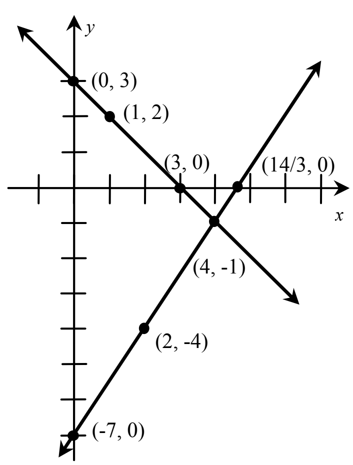
$y = -4 \quad \text{A third point is } (2, -4)$

From the graph the solution is approximately $(4, -1)$

$x = 4.0, \ y = -1.0$

Checking both equations,

$$\begin{array}{rr} x + y = 3 & 3x - 2y = 14 \\ 4.0 + (-1.0) = 3 & 3(4.0) - 2(-1.0) = 14 \\ 3.0 = 3 & 14.0 = 14 \end{array}$$



27. For the line $-2r_1 + 2r_2 = 7$

$$\text{Let } r_1 = 0 \text{ to find the } r_2\text{-int}$$
$$2r_2 = 7$$
$$r_2 = \tfrac{7}{2} \qquad r_2\text{-int is } \left(0, \tfrac{7}{2}\right)$$
$$\text{Let } r_2 = 0 \text{ to find the } r_1\text{-int}$$
$$-2r_1 = 7$$
$$r_1 = -\tfrac{7}{2} \qquad r_1\text{-int is } \left(-\tfrac{7}{2}, 0\right)$$
$$\text{Let } r_1 = 1 \text{ to find a third point}$$
$$-2(1) + 2r_2 = 7$$
$$2r_2 = 9$$
$$r_2 = \tfrac{9}{2} \quad \text{A third point is } \left(1, \tfrac{9}{2}\right)$$

For the line $4r_1 - 2r_2 = 1$

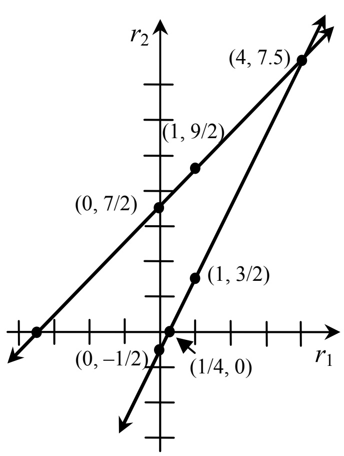
$$\text{Let } r_1 = 0 \text{ to find the } r_2\text{-int}$$
$$-2r_2 = 1$$
$$r_2 = -\tfrac{1}{2} \qquad r_2\text{-int is } \left(0, -\tfrac{1}{2}\right)$$
$$\text{Let } r_2 = 0 \text{ to find the } r_1\text{-int}$$
$$4r_1 = 1$$
$$r_1 = \tfrac{1}{4} \qquad r_1\text{-int is } \left(\tfrac{1}{4}, 0\right)$$
$$\text{Let } r_1 = 1 \text{ to find a third point}$$
$$4(1) - 2r_2 = 1$$
$$-2r_2 = -3$$
$$r_2 = \tfrac{3}{2} \quad \text{A third point is } \left(1, \tfrac{3}{2}\right)$$

From the graph the solution is approximately $(4, 7.5)$

$$r_1 = 4.0,\ r_2 = 7.5$$

Checking both equations,

$$-2r_1 + 2r_2 = 7 \qquad 4r_1 - 2r_2 = 1$$
$$-2(4.0) + 2(7.5) = 7 \qquad 4(4.0) - 2(7.5) = 1$$
$$7.0 = 7 \qquad 1.0 = 1$$



28. For the line $2x - 3y = -5$

Let $x = 0$ to find the y-int

$-3y = -5$

$y = \frac{5}{3}$ y-int is $\left(0, \frac{5}{3}\right)$

Let $y = 0$ to find the x-int

$2x = -5$

$x = -\frac{5}{2}$ x-int is $\left(-\frac{5}{2}, 0\right)$

Let $x = 5$ to find a third point

$2(5) - 3y = -5$

$-3y = -15$

$y = 5$ A third point is $(5, 5)$

For the line $3x + 2y = 12$

Let $x = 0$ to find the y-int

$2y = 12$

$y = 6$ y-int is $(0, 6)$

Let $y = 0$ to find the x-int

$3x = 12$

$x = 4$ x-int is $(4, 0)$

Let $x = 1$ to find a third point

$3(1) + 2y = 12$

$2y = 9$

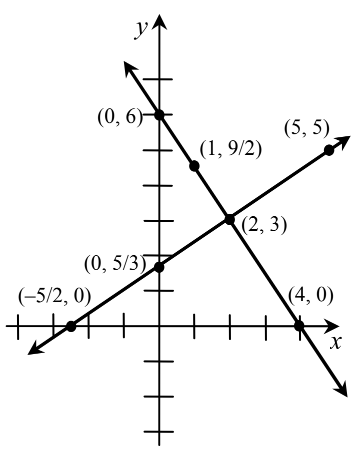
$y = \frac{9}{2}$ A third point is $\left(1, \frac{9}{2}\right)$

From the graph the solution is approximately $(2, 3)$

$x = 2.0,\ y = 3.0$

Checking both equations,

$2x - 3y = -5$	$3x + 2y = 12$
$2(2.0) - 3(3.0) = -5$	$3(2.0) + 2(3.0) = 12$
$-5.0 = -5$	$12.0 = 12$



29. $x = 4y + 2$ and $3y = 2x + 3$

$y = \dfrac{x-2}{4}$ $\quad y = \dfrac{2x+3}{3}$

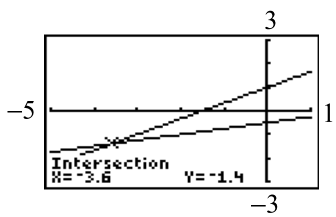
On a graphing calculator let

$y_1 = \dfrac{x-2}{4}$ and $y_2 = \dfrac{2x+3}{3}$

Using the intersect feature, the point of intersection is $(-3.6, -1.4)$, and the solution of the system of equations is

$x = -3.600$

$y = -1.400$



30. $1.2x - 2.4y = 4.8$ and $3.0x = -2.0y + 7.2$

$y = 0.5x - 2.0$ $\quad y = -1.5x + 3.6$

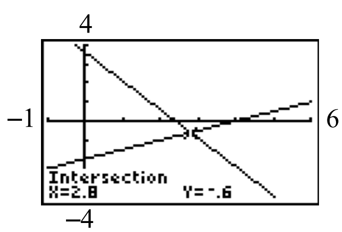
On a graphing calculator let

$y_1 = 0.5x - 2.0$ and $y_2 = -1.5x + 3.6$

Using the intersect feature, the point of intersection is $(2.8, -0.6)$, and the solution of the system of equations is

$x = 2.800$

$y = -0.600$



31. $4.0x - 3.5y = 1.5$ and $1.4y + 0.2x = 1.4$

$$y = \frac{4.0x - 1.5}{3.5} \qquad y = \frac{-0.2x + 1.4}{1.4}$$

$$y = \frac{8x - 3}{7} \qquad y = -\frac{1}{7}x + 1$$

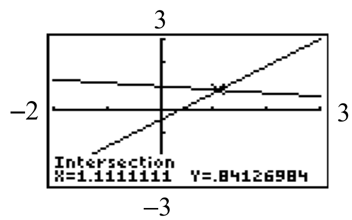
On a graphing calculator let

$y_1 = \frac{8x - 3}{7}$ and $y_2 = -\frac{1}{7}x + 1$

Using the intersect feature, the point of intersection is $(1.111,\ 0.841)$, and the solution of the system of equations is

$x = 1.111$

$y = 0.841$



32. $5F - 2T = 7$ and $3F + 4T = 8$

$$T = \frac{5F - 7}{2} \qquad T = \frac{-3F + 8}{4}$$

$$T = 2.5F - 3.5 \qquad T = -0.75F + 2$$

On a graphing calculator use with

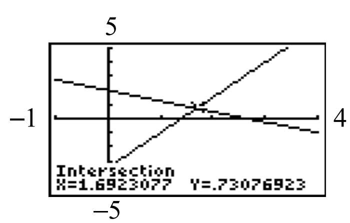
$x = F$ and $y = T$,

$y_1 = 2.5x - 3.5$ and $y_2 = -0.75x + 2.0$

Using the intersect feature, the point of intersection is $(1.692,\ 0.731)$, and the solution of the system of equations is

$F = 1.692$

$T = 0.731$



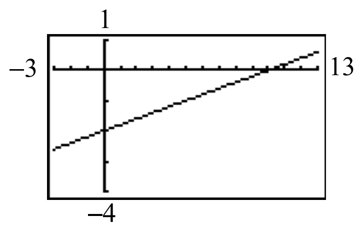
33. $x-5y=10$ and $2x-10y=20$

$y=\dfrac{x-10}{5}$ $\qquad$ $y=\dfrac{x-10}{5}$

On a graphing calculator let

$y_1=\dfrac{x-10}{5}$ and $y_2=\dfrac{x-10}{5}$

From the graph the lines are the same. The system is dependent.



34. $18x-3y=7$ and $2y=1+12x$

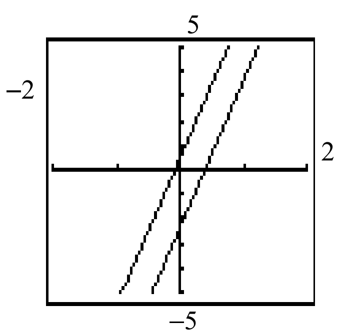
$y=\dfrac{18x-7}{3}$ $\qquad$ $y=\dfrac{12x+1}{2}$

On a graphing calculator let

$y_1=6x-7/3$ and $y_2=6x+1/2$

From the graph we see the lines never cross; they are parallel lines.

The system is inconsistent.



35. $1.9v = 3.2t$ and $1.2t - 2.6v = 6$

$$v = \frac{3.2t}{1.9} \qquad v = \frac{1.2t - 6}{2.6}$$

On a graphing calculator use

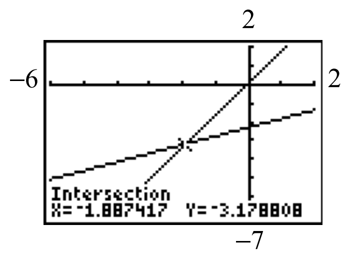
$x = t$ and $y = v$,

$y_1 = 3.2x / 1.9$ and $y_2 = (1.2x - 6) / 2.6$

Using the intersect feature, the point of intersection is $(-1.887, -3.179)$, and the solution to the system of equations is

$t = -1.887$

$v = -3.179$



36. $3y = 14x - 9$ and $12x + 23y = 0$

$$y = \frac{14x - 9}{3} \qquad y = \frac{-12x}{23}$$

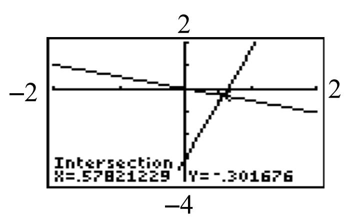
On a graphing calculator let

$y_1 = \dfrac{14x - 9}{3}$ and $y_2 = -\dfrac{12x}{23}$

Using the intersect feature, the point of intersection is $(0.578, -0.302)$.

The solution to the system of equations is

$x = 0.578$, $y = -0.302$



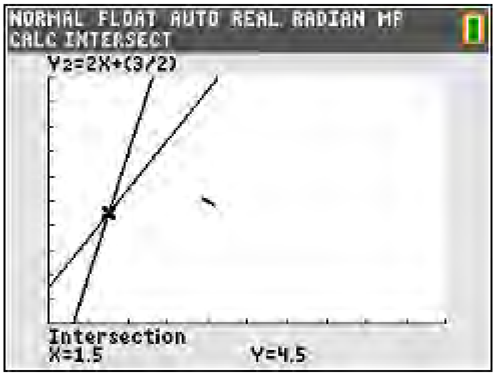
37. $5x = y + 3$ and $4x = 2y - 3$

$$y = 5x - 3 \qquad y = 2x + \frac{3}{2}$$

On a graphing calculator using the intersect feature, the point of intersection is $(1.5, 4.5)$, and the solution to the system of equations is

$x = 1.500$

$y = 4.500$



38. $0.75u + 0.67v = 5.9$ and $2.1u - 3.9v = 4.8$

$v = \dfrac{5.9 - 0.75u}{0.67}$ $\qquad$ $v = \dfrac{2.1u - 4.8}{3.9}$

On a graphing calculator use

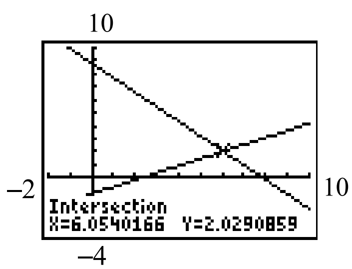
$x = u$ and $y = v$,

$y_1 = \dfrac{5.9 - 0.75x}{0.67}$ and $y_2 = \dfrac{2.1x - 4.8}{3.9}$

Using the intersect feature, the point of intersection is $(6.054, 2.029)$, and the solution to the system of equations is

$u = 6.054$

$v = 2.029$



39. $7R = 18V + 13$ and $-1.4R + 3.6V = 2.6$

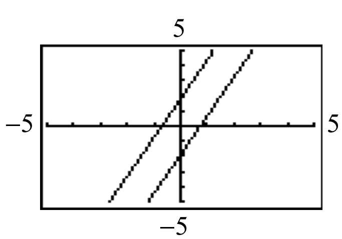
$R = \dfrac{18V + 13}{7}$ $\qquad$ $R = \dfrac{3.6V - 2.6}{1.4}$

On a graphing calculator use

$x = V$ and $y = R$,

$y_1 = \dfrac{18x + 13}{7}$ and $y_2 = \dfrac{3.6x - 2.6}{1.4}$

From the graph, the lines are parallel, there is no solution, the system is inconsistent.



40. $y = 6x + 2$ and $12x - 2y = -4$

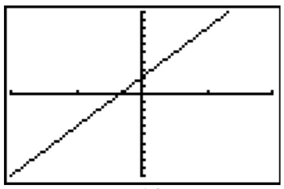
$y = 6x + 2 \qquad y = \dfrac{12x + 4}{2}$

$y = 6x + 2 \qquad y = 6x + 2$

On a graphing calculator let

$y_1 = 6x + 2$ and $y_2 = 6x + 2$

From the graph, we see the lines are the same; they have all points in common. The system is dependent.



41. We verify whether $p = 260$ and $w = 40$ satisfy the equations

$p + w = 300$ and $p - w = 220$:

$260 + 40 = 300$ and $260 - 40 = 220$

and so the two speeds are in fact $p = 260$ and $w = 40$.

42. We verify whether $a = 4.00\ \Omega/°\text{C}$ and $b = 1160\ \Omega$ satisfy the equations

$1200 = 10.0a + b$ and $1280 = 50.0a + b$:

$1200 = 10(4.00) + 1160$ but $1280 \neq 1260 = 50.0(4.00) + 1160$

and so the constants are not $a = 4.00\ \Omega/°\text{C}$ and $b = 1160\ \Omega$.

43. We verify whether $F_1 = 45$ and $F_2 = 28$ satisfy the equations

$0.80\ F_1 + 0.50F_2 = 50$ and $0.60\ F_1 - 0.87F_2 = 12$:

$0.80(45) + 0.50(28) = 36 + 14 = 50$ but $0.60\ (45) - 0.87(28) = 27 - 24.36 = 2.64 \neq 12$

and so $F_1 = 45$ and $F_2 = 28$ do not satisfy both equations.

44. We verify whether $x = \$1200$ and $y = \$800$ satisfy the equations

$x + y = 2000$ and $0.040x + 0.050y = 92$:

$1200 + 800 = 2000$ but $0.040(1200) + 0.050(800) = 88 \neq 92$

and so $x = \$1200$ and $y = \$800$ do not satisfy both equations.

45. $0.8T_1 - 0.6T_2 = 12$ and $0.6T_1 + 0.8T_2 = 68$

$$T_2 = \frac{0.8T_1 - 12}{0.6} \qquad T_2 = \frac{68 - 0.6T_1}{0.8}$$

On a graphing calculator use

$x = T_1$ and $y = T_2$ and

let $y_1 = \frac{0.8x-12}{6}$ and $y_2 = \frac{68-0.6x}{0.8}$.

Using the intersect feature, the point of intersection is $(50, 47)$.

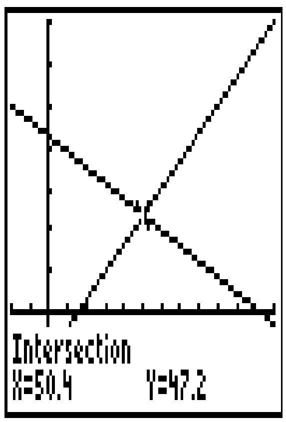
The tensions (to the nearest 1 N) are

$T_1 = 50$ N

$T_2 = 47$ N

140

−20 120



−40

46. Let x = compact car width

Let y = full-size car width

Let $y = 0$ to find the x-int

$12x = 201.0$

$x = 16.8$ x-int is $(16.8, 0)$

From the graph, the point of intersection is approximately $(8.5, 11.0)$.

the solution to the system of equations (to the nearest 0.1 m) is

$x = 8.5$ ft

$y = 11.0$ ft

Checking both equations,

$$16x + 6y = 202.0 \qquad 12x + 9y + 1.0 = 202.0$$

$$16(8.5) + 6(11.0) = 202.0 \qquad 12(8.5) + 9(11.0) + 0.2 = 202.0$$

$$202.0 = 202.0 \qquad 202.0 = 202.0$$

For line $16x+6y=202.0$

Let $x=0$ to find the y-int

$6y=202.0$

$y=33.7$ y-int is $(0, 33.7)$

Let $y=0$ to find the x-int

$16x=202.0$

$x=12.6$ x-int is $(12.6, 0)$

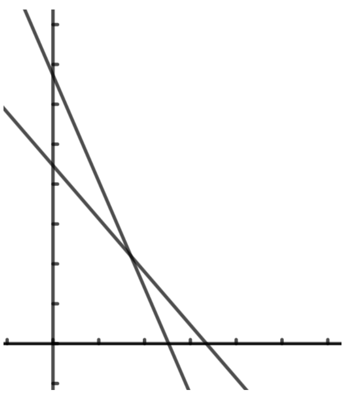
For the line $12x+9y+1.0=202.0$

$12x+9y=201.0$

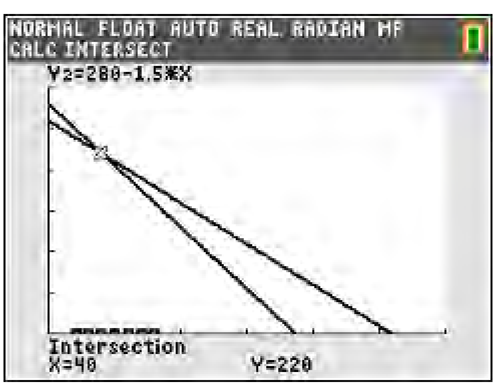
Let $x=0$ to find the y-int

$9y=201.0$

$y=21.3$ y-int is $(0, 21.3)$



47. We let p be the speed of the plane in still air and w be the speed of the tailwind. The equations are $780=3p+3w$ and $700=2.5p+2.5(1.5w)$
or
$780=3p+3w$ and $700=2.5p+3.75w$
Solving these for p, we have
$p=260-w$ and $p=280-1.5w$
which we plot on a graphing calculator. The intersection is at $p=220$ and $w=40$ and so the plane's speed in still air is 220 km/hr and the wind speed is 40 km/hr.



48. For the line $2i_1 + 6(i_1 + i_2) = 12$

$$8i_1 + 6i_2 = 12$$

Let $i_1 = 0$ to find the i_2-int

$$6i_2 = 12$$

$$i_2 = 2 \quad i_2\text{-int is } (0, 2)$$

Let $i_2 = 0$ to find the i_1-int

$$8i_1 = 12$$

$$i_1 = 1.5 \quad i_1\text{-int is } (1.5, 0)$$

Let $i_1 = 3$ to find a third point

$$8(3) + 6i_2 = 12$$

$$6i_2 = -12$$

$$i_2 = -2 \quad \text{A third point is } (3, -2)$$

For the line $4i_2 + 6(i_1 + i_2) = 12$

$$6i_1 + 10i_2 = 12$$

Let $i_1 = 0$ to find the i_2-int

$$10i_2 = 12$$

$$i_2 = 1.2 \quad i_2\text{-int is } (0, 1.2)$$

Let $i_2 = 0$ to find the i_1-int

$$6i_1 = 12$$

$$i_1 = 2 \quad i_1\text{-int is } (2, 0)$$

Let $i_1 = -1$ to find a third point

$$6(-1) + 10i_2 = 12$$

$$10i_2 = 18$$

$$i_2 = 1.8 \quad \text{A third point is } (-1, 1.8)$$

From the graph the solution is approximately $(1.1, 0.5)$, and the solution to the system of equations is

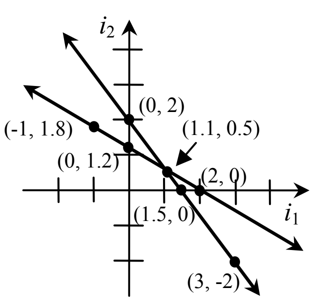
$$i_1 = 1.1 \text{ A}, i_2 = 0.5 \text{ A}$$

Checking both equations,

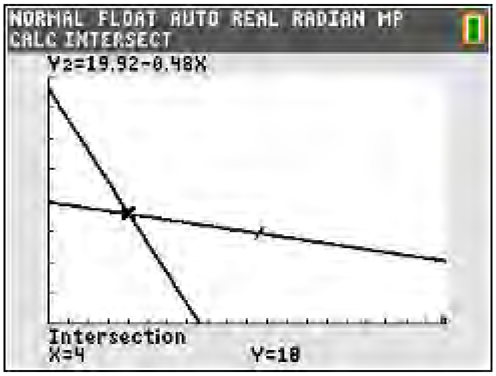
$$2i_1 + 6(i_1 + i_2) = 12 \qquad 4i_2 + 6(i_1 + i_2) = 12$$

$$2(1.1) + 6(1.1 + 0.5) = 12 \qquad 4(0.5) + 6(1.1 + 0.5) = 1$$

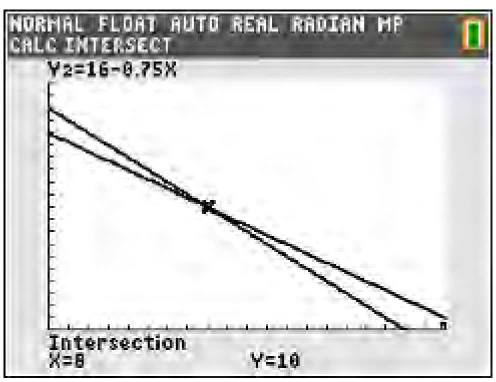
$$11.8 \approx 12 \qquad 11.6 \approx 12$$



49. We let p be the cost per sheet of plywood and f be the cost of one framing stud . The equations are $304 = 8p + 40f$ and $498 = 25p + 12f$.
Solving these for p, we have
$p = 38 - 5f$ and $p = 19.92 - 0.48f$
which we plot on a graphing calculator. The intersection is at $p = 18$ and $f = 4$ and so each sheet of plywood costs \$18.00 and each framing stud costs \$4.00.



50. We let c and h be the number of gallons of gas used in the city and the highway, respectively.
The equations are
$c + h = 18$ and $21c + 28h = 448$.
Solving these for h, we have
$h = 18 - c$ and $h = 16 - 0.75c$
which we plot on a graphing calculator. The intersection is at $c = 8$ and $h = 10$. The number of miles driven in the city is 8×21=168 and the number of miles driven on the highway is 10×28=280.



51. In order to have no solution, the two lines must have the same slope but different $y-$intercepts.
This occurs when $m = 3$ and $b \neq -7$.

52. **(a)** In order to have one solution, the two lines must have different slopes.
This occurs when $m \neq -3$. The variable b may assume any value.

(b) In order to have an unlimited number of solutions, the two lines must have the same slopes and $y-$intercepts. This occurs when $m = -3$ and $b = 8$.

5.3 Solving Systems of Two Linear Equations in Two Unknowns Algebraically

1.
$$x - 3y = 6$$
$$x = 3y + 6 \qquad \text{(A)}$$
$$2x - 3y = 3 \qquad \text{(B)}$$
$$2(3y+6) - 3y = 3 \qquad \text{substitute } x \text{ from (A) into (B)}$$
$$6y + 12 - 3y = 3$$
$$3y = -9$$
$$y = -3$$
$$x = 3(-3) + 6 \qquad \text{substitute } -3 \text{ for } y \text{ into (A)}$$
$$x = -3$$
The solution to the system is
$x = -3,\ y = -3$

2.
$$x - 3y = 6 \qquad \text{(A)}$$
$$2x - 3y = 3 \qquad \text{(B)}$$
If we subtract Eq. (A) − Eq. (B)
$$\begin{array}{rr} & x - 3y = 6 \\ - & 2x - 3y = 3 \\ \hline & -x \qquad = 3 \end{array}$$
$$x = -3$$
$$-3 - 3y = 6 \qquad \text{substitute } -3 \text{ for } x \text{ into (A)}$$
$$-3y = 9$$
$$y = -3$$
The solution to the system is
$x = -3,\ y = -3$

3.
$$3x - 2y = 4 \qquad \text{Equation (A)}$$
$$2x + 3y = 2 \qquad \text{Equation (B)}$$
If we add $3 \times$ Eq. (A) $+\ 2 \times$ Eq. (B)
$$\begin{array}{rr} & 9x - 6y = 12 \\ + & 4x + 6y = \ \ 4 \\ \hline & 13x \qquad = 16 \end{array}$$
$$x = \frac{16}{13}$$
$$3\ \frac{16}{13} - 2y = 4 \qquad \text{substitute } \frac{16}{13} \text{ for } x \text{ into (A)}$$
$$\frac{48}{13} - 2y = \frac{52}{13}$$
$$y = -\frac{2}{13}$$
The solution to the system is
$x = \frac{16}{13},\ y = -\frac{2}{13}$

4.

$$4x = 2y + 4$$

$$4x - 2y = 4 \quad \text{Equation (A)}$$

$$-y + 2x - 2 = 0$$

$$2x - y = 2 \quad \text{Equation (B)}$$

If we subtract Eq. (A) $-\,2\times$Eq. (B)

$$\begin{array}{r} 4x - 2y = 4 \\ -\quad 4x - 2y = 4 \\ \hline 0 = 0 \end{array}$$

The system is dependent.

5.

$$x = y + 3 \quad \text{Equation (A)}$$

$$x - 2y = 5 \quad \text{Equation (B)}$$

$$(y + 3) - 2y = 5 \quad \text{substitute } x \text{ from (A) into (B)}$$

$$-y = 2$$

$$y = -2$$

$$x = -2 + 3 \quad \text{substitute } -2 \text{ for } y \text{ into (A)}$$

$$x = 1$$

The solution to the system is

$x = 1,\ y = -2$

6.

$$x = 2y + 1 \quad \text{Equation (A)}$$

$$2x - 3y = 4 \quad \text{Equation (B)}$$

$$2(2y + 1) - 3y = 4 \quad \text{substitute } x \text{ from (A) into (B)}$$

$$4y - 3y = 4 - 2$$

$$y = 2$$

$$x = 2(2) + 1 \quad \text{substitute } 2 \text{ for } y \text{ into (A)}$$

$$x = 5$$

The solution to the system is

$x = 5,\ y = 2$

7.

$$p = V - 4 \quad \text{Equation (A)}$$

$$V + p = 10 \quad \text{Equation (B)}$$

$$V + (V - 4) = 10 \quad \text{substitute } p \text{ from (A) into (B)}$$

$$2V = 14$$

$$V = 7$$

$$p = 7 - 4 \quad \text{substitute } 7 \text{ for } V \text{ into (A)}$$

$$p = 3$$

The solution to the system is

$p = 3,\ V = 7$

8.

$y = 2x + 10$ Equation (A)

$2x + y = -2$ Equation (B)

$2x + (2x + 10) = -2$ substitute y from (A) into (B)

$4x = -12$

$x = -3$

$y = 2(-3) + 10$ substitute -3 for x into (A)

$y = 4$

The solution to the system is

$x = -3,\ y = 4$

9.

$x + y = -5$

$y = -x - 5$ Equation (A)

$2x - y = 2$ Equation (B)

$2x - (-x - 5) = 2$ substitute y from (A) into (B)

$3x = -3$

$x = -1$

$y = -(-1) - 5$ substitute -1 for x into (A)

$y = -4$

The solution to the system is

$x = -1,\ y = -4$

10.

$3x + y = 1$

$y = 1 - 3x$ Equation (A)

$3x - 2y = 16$ Equation (B)

$3x - 2(1 - 3x) = 16$ substitute y from (A) into (B)

$3x - 2 + 6x = 16$

$9x = 18$

$x = 2$

$y = 1 - 3(2)$ substitute 2 for x into (A)

$y = -5$

The solution to the system is

$x = 2,\ y = -5$

11.

$2x + 3y = 7$ Equation (A)

$6x - y = 1$

$y = 6x - 1$ Equation (B)

$2x + 3(6x - 1) = 7$ substitute y from (B) into (A)

$2x + 18x - 3 = 7$

$20x = 10$

$x = \frac{1}{2}$

$y = 6\left(\frac{1}{2}\right) - 1$ substitute $\frac{1}{2}$ for x into (B)

$y = 2$

The solution to the system is

$x = \frac{1}{2},\ y = 2$

12.

$2s + 2t = 1$

$t = \dfrac{1 - 2s}{2}$ Equation (A)

$4s - 2t = 17$ Equation (B)

$4s - 2\left(\dfrac{1 - 2s}{2}\right) = 17$ substitute t from (A) into (B)

$4s - 1 + 2s = 17$

$6s = 18$

$s = 3$

$t = \dfrac{1 - 2(3)}{2}$ substitute 3 for s into (A)

$t = -\frac{5}{2}$

The solution to the system is

$s = 3,\ t = -\frac{5}{2}$

13.

$33x + 2y = 34$

$y = \dfrac{34 - 33x}{2}$ Equation (A)

$40y = 9x + 11$

$40y - 9x = 11$ Equation (B)

$40\left(\dfrac{34 - 33x}{2}\right) - 9x = 11$ substitute y from (A) into (B)

$680 - 660x - 9x = 11$

$-669x = -669$

$x = 1$

$y = \dfrac{34 - 33(1)}{2}$ substitute 1 for x into (A)

$y = \frac{1}{2}$

The solution to the system is

$x = 1,\ y = \frac{1}{2}$

14. $3A+3B=-1$

$A=\dfrac{-1-3B}{3}$ Equation (A)

$5A=-6B-1$

$5A+6B=-1$ Equation (B)

$5\left(\dfrac{-1-3B}{3}\right)+6B=-1$ substitute A from (A) into (B)

$-\frac{5}{3}-5B+6B=-1$

$B=\frac{2}{3}$

$A=\dfrac{-1-3\left(\frac{2}{3}\right)}{3}$ substitute $\frac{2}{3}$ for B into (A)

$A=-1$

The solution to the system is

$A=-1,\ B=\frac{2}{3}$

15. $x+2y=5$ Equation (A)

$x-2y=1$ Equation (B)

If we add Eq. (A) + Eq. (B)

$$\begin{array}{rl} & x+2y=5 \\ + & x-2y=1 \\ \hline & 2x \quad\ \ =6 \end{array}$$

$x=3$

$3+2y=5$ substitute 3 for x into (A)

$2y=2$

$y=1$

The solution to the system is

$x=3,\ y=1$

16. $x+3y=7$ Equation (A)

$2x+3y=5$ Equation (B)

If we subtract Eq. (A) − Eq. (B)

$$\begin{array}{rl} & x+3y=7 \\ - & 2x+3y=5 \\ \hline & -x \quad\ \ =2 \end{array}$$

$x=-2$

$-2+3y=7$ substitute -2 for x into (A)

$3y=9$

$y=3$

The solution to the system is

$x=-2,\ y=3$

17. $2x-3y=4$ Equation (A)

$2x+y=-4$ Equation (B)

If we subtract Eq. (A) − Eq. (B)

$$\begin{array}{r} 2x-3y=\ \ 4 \\ -\quad 2x+\ \ y=-4 \\ \hline -4y=8 \end{array}$$

$y=-2$

$2x+(-2)=-4$ substitute -2 for y into (B)

$2x=-2$

$x=-1$

The solution to the system is

$x=-1,\ y=-2$

18. $R-4r=17$ Equation (A)

$3R+4r=3$ Equation (B)

If we add Eq. (A) + Eq. (B)

$$\begin{array}{r} R-4r=17 \\ +\quad 3R+4r=\ \ 3 \\ \hline 4R\qquad =20 \end{array}$$

$R=5$

$5-4r=17$ substitute 5 for R into (A)

$-4r=12$

$r=-3$

The solution to the system is

$r=-3,\ R=5$

19. $12t+9y=14$ Equation (A)

$6t=\ 7y-16$

$6t-7y=-16$ Equation (B)

If we subtract Eq. (A) $-\,2\times$ Eq. (B)

$$\begin{array}{r} 12t\ +9y=\ 14 \\ -\quad 12t-14y=-32 \\ \hline 23y=46 \end{array}$$

$y=2$

$6t=7(2)-16$ substitute 2 for y into equation that led to (B)

$6t=-2$

$t=-\frac{1}{3}$

The solution to the system is

$t=-\frac{1}{3},\ y=2$

20. $3x - y = 3$ Equation (A)

$4x = 3y + 14$

$4x - 3y = 14$ Equation (B)

If we subtract $3 \times$ Eq. (A) – Eq. (B)

$$\begin{array}{rl} & 9x - 3y = 9 \\ - & 4x - 3y = 14 \\ \hline & 5x \quad = -5 \end{array}$$

$x = -1$

$3(-1) - y = 3$ substitute -1 for x into (A)

$y = -6$

The solution to the system is

$x = -1,\ y = -6$

21. $v + 2t = 7$ Equation (A)

$2v + 4t = 9$ Equation (B)

If we subtract $2 \times$ Eq. (A) – Eq. (B)

$$\begin{array}{rl} & 2v + 4t = 14 \\ - & 2v + 4t = 9 \\ \hline & 0 = 5 \end{array}$$

The system of equations is inconsistent.

22. $3x - y = 5$ Equation (A)

$-9x + 3y = -15$ Equation (B)

If we add $3 \times$ Eq. (A) + Eq. (B)

$$\begin{array}{rl} & 9x + 3y = 15 \\ + & -9x + 3y = -15 \\ \hline & 0 = 0 \end{array}$$

The system of equations is dependent.

These two lines are the same line.

23. $2x - 3y - 4 = 0$

$2x - 3y = 4$ Equation (A)

$3x + 2 = 2y$

$3x - 2y = -2$ Equation (B)

If we subtract $3 \times$ Eq. (A) – $2 \times$ Eq. (B)

$$\begin{array}{rl} & 6x - 9y = 12 \\ - & 6x - 4y = -4 \\ \hline & -5y = 16 \end{array}$$

$y = -\frac{16}{5}$

$3x+2=2\left(-\frac{16}{5}\right)$ substitute $-\frac{16}{5}$ for y into equation that led to (B)

$3x=-\frac{32}{5}-\frac{10}{5}$

$x=-\frac{42/3}{5}$

$x=-\frac{14}{5}$

The solution to the system is

$x=-\frac{14}{5},\ y=-\frac{16}{5}$

24. $3i_1+5=-4i_2$

$3i_1+4i_2=-5$ Equation (A)

$3i_2=5i_1-2$

$-5i_1+3i_2=-2$ Equation (B)

If we add $5\times$ Eq. (A) $+\,3\times$ Eq. (B)

$$\begin{array}{r} 15i_1+20i_2=-25 \\ +\quad -15i_1+9i_2=-6 \\ \hline 29i_2=-31 \end{array}$$

$i_2=-\frac{31}{29}$

$3i_1+4\left(-\frac{31}{29}\right)=-5$ substitute $-\frac{31}{29}$ for i_2 into (A)

$3i_1=-\frac{145}{29}+\frac{124}{29}$

$3i_1=-\frac{21}{29}$

$i_1=-\frac{7}{29}$

The solution to the system is

$i_1=-\frac{7}{29},\ i_2=-\frac{31}{29}$

25. $2x-y=5$ Equation (A)

$6x+2y=-5$ Equation (B)

If we add $2\times$ Eq. (A) + Eq. (B)

$$\begin{array}{r} 4x-2y=\ 10 \\ +\quad 6x+2y=-5 \\ \hline 10x\qquad=5 \end{array}$$

$x=\frac{1}{2}$

$2\ \frac{1}{2}-y=5$ substitute $\frac{1}{2}$ for x into (A)

$y=1-5$

$y=-4$

The solution to the system is

$x=\frac{1}{2},\ y=-4$

26. $3x+2y=4$ Equation (A)

$6x-6y=13$ Equation (B)

If we subtract $2\times$ Eq. (A) − Eq. (B)

$$\begin{array}{r} 6x+4y=8 \\ -\quad 6x-6y=13 \\ \hline 10y=-5 \end{array}$$

$y=-\frac{1}{2}$

$3x+2\left(-\frac{1}{2}\right)=4$ substitute $-\frac{1}{2}$ for y into (A)

$3x=5$

$x=\frac{5}{3}$

The solution to the system is

$x=\frac{5}{3},\ y=-\frac{1}{2}$

27. $6x+3y+4=0$ Equation (A)

$5y=-9x-6$

$y=\dfrac{-9x-6}{5}$ Equation (B)

$6x+3\ \dfrac{-9x-6}{5}\ +4=0$ substitute y from (B) into (A)

$6x-\frac{27}{5}x-\frac{18}{5}+4=0$

$\dfrac{30-27}{5}x=\dfrac{18-20}{5}$

$3x=-2$

$x=-\frac{2}{3}$

$y=\dfrac{-9\left(-\frac{2}{3}\right)-6}{5}$ substitute $-\frac{2}{3}$ for x into (B)

$y=0$

The solution to the system is

$x=-\frac{2}{3},\ y=0$

28. $1 + 6q = 5p$

$-5p + 6q = -1$ Equation (A)

$3p - 4q = 7$ Equation (B)

If we add $2 \times$ Eq. (A) $+ 3 \times$ Eq. (B)

$$\begin{array}{rl} & -10p + 12q = -2 \\ + & \quad 9p - 12q = 21 \\ \hline & \quad -p \qquad\;\; = 19 \end{array}$$

$p = -19$

$-5(-19) + 6q = -1$ substitute -19 for p into (A)

$6q = -96$

$q = -16$

The solution to the system is

$p = -19,\ q = -16$

29. $15x + 10y = 11$ Equation (A)

$20x - 25y = 7$ Equation (B)

If we add $5 \times$ Eq. (A) $+ 2 \times$ Eq. (B)

$$\begin{array}{rl} & 75x + 50y = 55 \\ + & 40x - 50y = 14 \\ \hline & 115x \qquad = 69 \end{array}$$

$x = \frac{69}{115} = \frac{3}{5}$

$15\ \frac{3}{5} + 10y = 11$ substitute $\frac{3}{5}$ for x into (A)

$10y = 2$

$y = \frac{1}{5}$

The solution to the system is

$x = \frac{3}{5},\ y = \frac{1}{5}$

30. $2x + 6y = -3$ Equation (A)

$-6x - 18y = 5$ Equation (B)

If we add $3 \times$ Eq. (A) + Eq. (B)

$$\begin{array}{rl} & 6x + 18y = -9 \\ + & -6x - 18y = 5 \\ \hline & \qquad\qquad 0 = -4 \end{array}$$

The system of equations is inconsistent.

The lines are parallel.

31.
$$12V + 108 = -84C$$
$$12V + 84C = -108 \qquad \text{Equation (A)}$$
$$36C + 48V + 132 = 0$$
$$48V + 36C = -132 \qquad \text{Equation (B)}$$

If we subtract $4 \times$ Eq. (A) $-$ Eq. (B)
$$\begin{array}{rl} & 48V + 336C = -432 \\ - & 48V + 36C = -132 \\ \hline & \qquad 300C = -300 \end{array}$$
$$C = -1$$
$$48V + 36(-1) = -132 \qquad \text{substitute } -1 \text{ for } C \text{ into (B)}$$
$$48V = -96$$
$$V = -2$$

The solution to the system is
$$V = -2,\ C = -1$$

32.
$$66x + 66y = -77 \qquad \text{Equation (A)}$$
$$33x - 132y = 143 \qquad \text{Equation (B)}$$

If we subtract Eq. (A) $- 2 \times$ Eq. (B)
$$\begin{array}{rl} & 66x + 66y = -77 \\ - & 66x - 264y = 286 \\ \hline & \qquad 330y = -363 \end{array}$$
$$y = -\tfrac{363}{330} = -\tfrac{11}{10}$$
$$66x + 66\left(-\tfrac{11}{10}\right) = -77 \qquad \text{substitute } -\tfrac{11}{10} \text{ for } y \text{ into (A)}$$
$$66x = -77 + 72.6$$
$$x = -\tfrac{4.4}{66} = -\tfrac{1}{15}$$

The solution to the system is
$$x = -\tfrac{1}{15},\ y = -\tfrac{11}{10}$$

33.
$$44A = 1 - 15B$$
$$A = \frac{1 - 15B}{44} \qquad \text{Equation (A)}$$
$$5B = 22 + 7A \qquad \text{Equation (B)}$$
$$5B = 22 + 7\ \frac{1 - 15B}{44} \qquad \text{substitute } A \text{ from (A) into (B)}$$
$$5B = 22 + \frac{7}{44} - \frac{105}{44}B$$

$$\frac{220+105}{44}B = \frac{968+7}{44}$$
$$325B = 975$$
$$B = 3$$
$$A = \frac{1-15(3)}{44} \quad \text{substitute 3 for } B \text{ into (A)}$$
$$A = -1$$

The solution to the system is

$A = -1,\ B = 3$

34. $60x - 40y = 80$ Equation (A)

$2.9x - 2.0y = 8.0$ Equation (B)

If we subtract $\frac{1}{20}$ Eq. (A) − Eq. (B)

$$\begin{array}{rl} & 3.0x - 2.0y = 4.0 \\ - & 2.9x - 2.0y = 8.0 \\ \hline & 0.1x \qquad\quad = -4.0 \end{array}$$

$$x = -40$$
$$3.0(-40) - 2.0y = 4.0 \quad \text{substitute } -40 \text{ for } x \text{ into (A)}$$
$$-2.0y = 124$$
$$y = -62$$

The solution to the system is

$x = -40,\ y = -62$

35. $2b = 6a - 16$

$b = 3a - 8$ Equation (A)

$33a = 4b + 39$ Equation (B)

$33a = 4(3a - 8) + 39$ substitute b from (A) into (B)

$33a = 12a - 32 + 39$

$21a = 7$

$a = \frac{1}{3}$

$b = 3\left(\frac{1}{3}\right) - 8$ substitute $\frac{1}{3}$ for a into (A)

$b = -7$

The solution to the system is

$a = \frac{1}{3},\ b = -7$

36.

$$30P = 55 - Q$$

$$P = \frac{55-Q}{30} \quad \text{Equation (A)}$$

$$19P + 14Q + 32 = 0 \quad \text{Equation (B)}$$

$$19\left(\frac{55-Q}{30}\right) + 14Q + 32 = 0 \quad \text{substitute } P \text{ from (A) into (B)}$$

$$\frac{1045}{30} - \frac{19}{30}Q + 14Q + 32 = 0$$

$$\frac{-19+420}{30}Q = \frac{-960-1045}{30}$$

$$401Q = -2005$$

$$Q = -5$$

$$P = \frac{55-(-5)}{30} \quad \text{substitute } -5 \text{ for } Q \text{ into (A)}$$

$$P = 2$$

The solution to the system is

$P = 2,\ Q = -5$

37.

$$0.3x - 0.7y = 0.4 \quad \text{multiply every term by 10}$$

$$3x - 7y = 4 \quad \text{Equation (A)}$$

$$0.2x + 0.5y = 0.7 \quad \text{multiply every term by 10}$$

$$2x + 5y = 7 \quad \text{Equation (B)}$$

If we subtract $2 \times$ Eq. (A) $- 3 \times$ Eq. (B)

$$\begin{array}{rl} & 6x - 14y = 8 \\ - & 6x + 15y = 21 \\ \hline & -29y = -13 \end{array}$$

$$y = \tfrac{13}{29}$$

$$3x - 7\left(\tfrac{13}{29}\right) = 4 \quad \text{substitute } \tfrac{13}{29} \text{ for } y \text{ into (A)}$$

$$3x = \tfrac{116+91}{29} = \tfrac{207}{29}$$

$$x = \tfrac{69}{29}$$

The solution to the system is

$x = \tfrac{69}{29},\ y = \tfrac{13}{29}$

38. $250R+225Z=400$ divide every term by 25

$10R+9Z=16$ Equation (A)

$375R-675Z=325$ divide every term by 25

$15R-27Z=13$ Equation (B)

If we add $3\times$ Eq. (A) + Eq. (B)

$$\begin{array}{rr} & 30R+27Z=48 \\ + & 15R-27Z=13 \\ \hline & 45R=61 \end{array}$$

$R=\frac{61}{45}$

$10\ \frac{61}{45}\ +9Z=16$ substitute $\frac{61}{45}$ for R into (A)

$9Z=\frac{720-610}{45}=\frac{110}{45}$

$9Z=\frac{22}{9}$

$Z=\frac{22}{81}$

The solution to the system is

$R=\frac{61}{45},\ Z=\frac{22}{81}$

39. $40s-30t=60$ divide every term by 10

$4s-3t=6$ Equation (A)

$20s-40t=-50$ divide every term by 10

$2s-4t=-5$ Equation (B)

If we subtract Eq. (A) $-\,2\times$ Eq. (B)

$$\begin{array}{rr} & 4s-3t=6 \\ - & 4s-8t=-10 \\ \hline & 5t=16 \end{array}$$

$t=\frac{16}{5}$

$4s-3\left(\frac{16}{5}\right)=6$ substitute $\frac{16}{5}$ for t into (A)

$4s=\frac{30+48}{5}=\frac{78}{5}$

$s=\frac{78}{20}=\frac{39}{10}$

The solution to the system is

$s=\frac{39}{10},\ t=\frac{16}{5}$

40. $0.060x + 0.048y = -0.084$ multiply every term by 1000

$60x + 48y = -84$ divide every term by 12

$5x + 4y = -7$ Equation (A)

$0.065x - 0.13y = 0.078$ multiply every term by 1000

$65x - 130y = 78$ divide every term by 13

$5x - 10y = 6$ Equation (B)

If we subtract Eq. (A) – Eq. (B)

$$\begin{array}{r} 5x + \ 4y = -7 \\ - \quad 5x - 10y = 6 \\ \hline 14y = -13 \end{array}$$

$y = -\frac{13}{14}$

$5x - 10\left(-\frac{13}{14}\right) = 6$ substitute $-\frac{13}{14}$ for y into (B)

$5x = \frac{84-130}{14}$

$x = -\frac{46}{70} = -\frac{23}{35}$

The solution to the system is

$x = -\frac{23}{35}$, $y = -\frac{13}{14}$

41. $\frac{x}{3} + \frac{2y}{3} = 2$ Multiply by 3

$x + 2y = 6$ Equation (A)

$\frac{x}{2} - 2y = \frac{5}{2}$ Multiply by 2

$x - 4y = 5$ Equation (B)

If we subtract Eq. (A) – Eq. (B)

$$\begin{array}{r} x + 2y = 6 \\ - \quad x - 4y = 5 \\ \hline 6y = 1 \end{array}$$

and so $y = \frac{1}{6}$.

$x + 2(\frac{1}{6}) = 6$ Substitute $\frac{1}{6}$ for y into (A)

$x = 6 - 2(\frac{1}{6})$

$x = \frac{17}{3}$

The solution to the system is

$x = \frac{17}{3}$, $y = \frac{1}{6}$

42. $\frac{2x}{5}-\frac{y}{5}=1$ Multiply by 5

$2x-y=5$ Equation (A)

$\frac{3x}{4}-y=\frac{5}{4}$ Multiply by 4

$3x-4y=5$ Equation (B)

If we subtract $4\times$ Eq. (A) − Eq. (B)

$$\begin{array}{rl} & 8x-4y=20 \\ - & 3x-4y=5 \\ \hline & 5x=15 \end{array}$$

and so $x=3$.

$2(3)-y=5$ Substitute $\frac{1}{6}$ for y into (A)

$y=2(3)-5$

$y=1$

The solution to the system is

$x=3,\ y=1$

43. We want a and b for $f(x)=ax+b$.

Substitute $x=2$: $1=f(2)=a(2)+b$

$2a+b=1$ Equation (A)

Substitute $x=-1$: $-5=f(-1)=a(-1)+b$

$-a+b=-5$ Equation (B)

If we subtract Eq. (A) − Eq. (B)

$$\begin{array}{rl} & 2a+b=1 \\ - & -a+b=-5 \\ \hline & 3a=6 \end{array}$$

and so $a=2$.

$2(2)+b=1$ Substitute 2 for a into (A)

$b=1-2(2)$

$b=-3$

The solution to the system is

$a=2,\ b=-3$

and so

$f(x)=2x-3$

44. We want a and b for $f(x)=ax+b$.

Substitute $x=6$: $-1=f(6)=a(6)+b$

$6a+b=-1$ Equation (A)

Substitute $x=-6$: $11=f(-6)=a(-6)+b$

$-6a+b=11$ Equation (B)

If we subtract Eq. (A) − Eq. (B)

$$\begin{array}{rr} & 6a+b=-1 \\ - & -6a+b=11 \\ \hline & 12a=-12 \end{array}$$

and so $a=-1$.

$-1(6)+b=-1$ Substitute -1 for a into (A)

$b=-1-(-1(6))$

$b=5$

The solution to the system is

$a=-1,\ b=5$

and so

$f(x)=-x+5$

45. (a) Solving for x and substituting:

$2x+y=4$

$2x=4-y$

$x=2-\frac{1}{2}y$ Equation (A)

$3x-4y=-5$ Equation (B)

$3(2-\frac{1}{2}y)-4y=-5$ substitute for x into (B)

$6-\frac{11}{2}y=-5$

$-\frac{11}{2}y=-11$

$y=2$

$x=2-\frac{1}{2}(2)$ substitute for y into (A)

$x=1$

The solution to the system is

$x=1,\ y=2$

(b) Solving for y and substituting:

$2x+y=4$

$y=4-2x$ Equation (A)

$3x-4y=-5$ Equation (B)

$3x-4(4-2x)=-5$ substitute for x into (B)

$11x-16=-5$

$11x=11$

$x=1$

$y=4-2(1)$ substitute for x into (A)

$y=2$

The solution to the system is

$x=1,\ y=2$

46. Let $a=\dfrac{1}{x+y}$ and $b=\dfrac{1}{x-y}$.

The system becomes

$5a+2b=3$ Equation (A)

$20a-2b=2$ Equation (B)

If we add Eq. (A)+Eq. (B)

$$\begin{array}{r} 5a+2b=3 \\ +\quad 20a-2b=2 \\ \hline 25a=5 \end{array}$$

and so $a=\frac{1}{5}$, implying $\frac{1}{x+y}=\frac{1}{5}$ or $x+y=5$.

$5(\frac{1}{5})+2b=3$ Substitute $\frac{1}{5}$ for a into (A)

$1+2b=3$

$2b=2$

and so $b=1$, implying $\frac{1}{x-y}=1$ or $x-y=1$.

$x+y=5$ Equation (C)

$x-y=1$ Equation (D)

If we add Eq. (C)+Eq. (D)

$$\begin{array}{r} x+y=5 \\ +\quad x-y=1 \\ \hline 2x=6 \end{array}$$

and so $x=3$. Substituting this into (C) yields $3+y=5$, or $y=2$.

The solution to the system is

$x=3,\ y=2$

47. $V_1+V_2=15$

$V_2=15-V_1$ Equation (A)

$V_1-V_2=3$ Equation (B)

$V_1-(15-V_1)=3$ substitute V_2 from (A) into (B)

$2V_1=18$

$V_1=9$

$V_2=15-9$ substitute 9 for V_1 into (A)

$V_2=6$

The solution to the system is

$V_1=9\text{ V},\ V_2=6\text{ V}$

48.

$$L + 3x = 18$$
$$L = 18 - 3x \qquad \text{Equation (A)}$$
$$L + 5x = 22 \qquad \text{Equation (B)}$$
$$(18 - 3x) + 5x = 22 \qquad \text{substitute } L \text{ from (A) into (B)}$$
$$2x = 4$$
$$x = 2$$
$$L = 18 - 3(2) \qquad \text{substitute 2 for } x \text{ into (A)}$$
$$L = 12$$

The solution to the system is

$x = 2$ cm/N, $L = 12$ cm

49.

$$x + y = 10\ 000$$
$$y = 10\ 000 - x \qquad \text{Equation (A)}$$
$$0.0180x + 0.0100y = 0.0150(10\ 000)$$
$$0.0180x + 0.0100y = 150 \qquad \text{Equation (B)}$$
$$0.0180x + 0.0100(10\ 000 - x) = 150 \qquad \text{substitute } y \text{ from (A) into (B)}$$
$$0.0080x = 50$$
$$x = 6250$$
$$y = 10\ 000 - 6250 \qquad \text{substitute 6250 for } x \text{ into (A)}$$
$$y = 3750$$

The solution to the system is

$x = 6250$ L, $y = 3750$ L

50.

$$x + y + 200 = 1200$$
$$y = 1000 - x \qquad \text{Equation (A)}$$
$$0.060x + 0.150y + 0.200(200) = 0.120(1200)$$
$$0.060x + 0.150y = 104 \qquad \text{Equation (B)}$$
$$0.060x + 0.150(1000 - x) = 104 \qquad \text{substitute } y \text{ from (A) into (B)}$$
$$-0.090x = -46$$
$$x = 511$$
$$y = 1000 - 511 \qquad \text{substitute 511 for } x \text{ into (A)}$$
$$y = 489$$

The solution to the system is

$x = 511$ mL, $y = 489$ mL

51. Let x = number of regular email messages
Let y = number of spam messages

$x + y = 78$ Equation (A)
$y = 4x - 2$ Equation (B)
$x + (4x - 2) = 78$ substitute y from (B) into (A)
$5x = 80$
$x = 16$ regular messages
$y = 4(16) - 2$ substitute 16 for x into (B)
$y = 62$ spam messages

52. Let x = length of piece one
Let y = length of piece two

$x + y = 150$ Equation (A)
$y = 4x$ Equation (B)
$x + 4x = 150$ substitute y from (B) into (A)
$5x = 150$
$x = 30$ m length of short piece
$y = 4(30)$ substitute 30 for x into (B)
$y = 120$ m length of long piece

53. Let W_f = weight supported by front wheels
Let W_r = weight supported by rear wheels

$W_r + W_f = 17\ 700$ Equation (A)

$$\frac{W_r}{W_f} = 0.847$$

$W_r = 0.847W_f$ Equation (B)
$0.847\ W_f + W_f = 17\ 700$ substitute W_r from (B) into (A)
$1.847W_f = 17\ 700$
$W_f = 9583.108$ N
$W_f = 9580$ N weight supported by front wheels
$W_r = 0.847(9583.108)$ substitute 9580 for W_f into (B)
$W_r = 8116.892$ N
$W_r = 8120$ N weight supported by rear wheels

54. Let Q_1 = flow rate in sprinkler 1
Let Q_2 = flow rate in sprinkler 2

$Q_1 + Q_2 = 980$ Equation (A)
$Q_1 = 0.65Q_2$ Equation (B)
$0.65Q_2 + Q_2 = 980$ substitute Q_1 from (B) into (A)
$1.65Q_2 = 980$
$Q_2 = 593.939$ L/h
$Q_2 = 590$ L/h flow rate in sprinkler 2
$Q_1 = 0.65(593.939)$ substitute 9580 for W_f into (B)
$Q_1 = 386.061$ L/h
$Q_1 = 390$ L/h flow rate in sprinkler 1

55. Let t_1 = time of flight for rocket 1
Let t_2 = time of light for heat-seeking rocket 2
The rockets will meet at the same distance travelled by both, and distance is velocity $\times$ time,
$(d = vt)$

$2000t_1 = 3200t_2$ Equation (A)

The time elapsed by the first rocket will be larger

$t_1 = t_2 + 12$ Equation (B)
$2000(t_2 + 12) = 3200t_2$ substitute t_1 from (B) into (A)
$24000 = 1200t_2$
$t_2 = 20$ s time elapsed for heat-seeking rocket 2
$t_1 = 20 + 12$ substitute 20 for t_2 into (B)
$t_1 = 32$ s time elapsed for rocket 1

56. Let F_1 = force 1
Let F_2 = force 2
The *moments* must balance for each configuration

$F_1(3) = 20(1) + F_2(5)$
$3F_1 = 20 + 5F_2$ Equation (A)
$F_1(2) = 20(1) + F_2(3)$
$2F_1 = 20 + 3F_2$
$F_1 = 10 + 1.5F_2$ Equation (B)
$3(10 + 1.5F_2) = 20 + 5F_2$ substitute F_1 from (B) into (A)
$30 + 4.5F_2 = 20 + 5F_2$
$0.5F_2 = 10$
$F_2 = 20$ N
$F_1 = 10 + 1.5(20)$ substitute 20 for F_2 into (B)
$F_1 = 40$ N

57. Let x = number of offices renting at \$900 per month

Let y = number of offices renting at \$1250 per month

$x + y = 54$

$y = 54 - x$ Equation (A)

$900x + 1250y = 55\ 600$ Equation (B)

$900x + 1250(54 - x) = 55\ 600$ substitute y from (A) into (B)

$900x + 67\ 500 - 1250x = 55\ 600$

$-350x = -11\ 900$

$x = 34$

$y = 54 - 34$ substitute 34 for x into (A)

$y = 20$

There are 34 offices renting at \$900 per month
and 20 offices renting at \$1250 per month.

58. Let p = speed of the plane

Let w = speed of the wind

$p - w = 140$ Equation (A)

$p + w = 240$ Equation (B)

$2p = 380$ (A) + (B)

$p = 190$

$190 - w = 140$ substitute $p = 190$ into (A)

$w = 50$

The plane's speed is $p = 190$ mi/h and the wind speed is $w = 50$ mi/h.

59. Let a = votes received by candidate A

Let b = votes received by candidate B

$a - b = 2000$ Equation (A)

$(b + 0.010a) - 0.990a = 1000$ Transfer 1% of A's votes to B

$b - 0.980a = 1000$ Equation (B)

$0.020a = 3000$ (A) + (B)

$a = 150000$

$150000 - b = 2000$ substitute $a = 150000$ into (A)

$b = 148000$

Candidate A received $a = 150000$ votes and candidate B received $b = 148000$ votes.

60. Let t_1 = time for sound to travel through air

Let t_2 = time for sound to travel through water

The signls travel the same distance and distance is velocity $\times$ time.

$(d = vt)$

$1100t_1 = 5000t_2$ Equation (A)

The time through air was larger by 30 s,

$t_1 = t_2 + 30$ Equation (B)

$1100(t_2 + 30) = 5000t_2$ substitute t_1 from (B) into (A)

$33000 = 3900t_2$

$t_2 = 8.46$ s time elapsed through water

$t_1 = 8.46 + 30$ substitute 8.46 for t_2 into (B)

$t_1 = 38.5$ s time elapsed through air

The distance traveled can be found by using either side of equation (A)

(remember $d = vt$)

$d = 1100t_1 = 1100(38.46) = 42300$ ft

$d = 5000t_2 = 5000(8.46) = 42300$ ft

61. Let x = windmill power capacity (in kW)

Let y = gas generator power capacity (in kW)

Energy produced = power × time

In a 10-day period, there are 240 h.

For the first 10-day period:

$(0.450x)240 + y(240) = 3010$

$108x + 240y = 3010$

$y = \dfrac{3010 - 108x}{240}$ Equation (A)

For the second 10-day period:

$(0.720x)240 + y(240 - 60) = 2900$

$172.8x + 180y = 2900$ Equation (B)

$172.8x + 180\ \dfrac{3010 - 108x}{240} = 2900$ substitute y from (A) into (B)

$172.8x + 2257.5 - 81x = 2900$

$91.8x = 642.5$

$x = 7.00$ kW substitute 7.00 for x into (A)

$y = \dfrac{3010 - 108(7.00)}{240}$

$y = 9.39$ kW

The windmill power capacity is 7.00 kW, and the gas generator capacity is 9.39 kW.

62. Let x = volume of 40% solution

Let y = volume of 85% solution

$x + y = 20$

$y = 20 - x$ Equation (A)

$0.40x + 0.85y = 0.60(20)$

$0.40x + 0.85y = 12$ Equation (B)

$0.40x + 0.85(20 - x) = 12$ substitute y from (A) into (B)

$-0.45x = -5$

$x = 11$ L

$y = 20 - 11.1$ substitute 11 for x into (A)

$y = 9$ L

11 L of the 40% solution should be mixed with 9 L of the 85% solution.

63. Let x = sales this month (in $)

Let y = sales last month (in $)

$x = y + 8000$

$x - y = 8000$ Equation (A)

$x + y = 2y + 4000$

$x - y = 4000$ Equation (B)

Subtracting Equation A – Equation B yields

$0 = 4000$

This is an inconsistent system, which means that both statements from the sales report cannot be true. There could be an error in the sales figures, and/or the conclusion is in error.

64. The report can be summarized in the two equations

$F_1 = 2F_2$ Equation (A)

$2(F_1 + F_2) - 6F_2 = 6$ Equation (B)

Substituting (A) into (B) yields

$2(2F_2 + F_2) - 6F_2 = 6$

or

$0 = 6$

This is an inconsistent system, which means that both statements from the report cannot be true. There must be an error in either statement or both.

65. $ax + y = c$
$bx + y = d$
In order to create a unique solution, the lines must have different slopes. Putting both equations into slope-intercept form,
$y = -ax + c$ has slope $-a$
$y = -bx + d$ has slope $-b$
To make the slopes different, $a \neq b$.

66. $ax + y = c$
$bx + y = d$

(a) In order to create an inconsistent system, the lines must be parallel. Therefore, they should have the same slopes, but different intercepts. Putting both equations into slope-intercept form,
$y = -ax + c$ has slope $-a$, y-intercept c
$y = -bx + d$ has slope $-b$, y-intercept d
To make the system inconsistent, $a = b$ and $c \neq d$.

(b) In order to create a dependent system, the lines must be identical. Therefore, they should have the same slopes, and the same intercepts. Putting both equations into slope-intercept form,
$y = -ax + c$ has slope $-a$, y-intercept c
$y = -bx + d$ has slope $-b$, y-intercept d
To make the system dependent, $a = b$ and $c = d$.

67. When $x = -3$,
$\left(x, \tfrac{1}{3}x - 3\right) = \left(-3, \tfrac{1}{3}(-3) - 3\right) = (-3, -4)$
When $x = 9$,
$\left(x, \tfrac{1}{3}x - 3\right) = \left(9, \tfrac{1}{3}(9) - 3\right) = (9, 0)$

68. $y = \dfrac{1}{3}x - 3$
Rearrange equation for x.
$\dfrac{1}{3}x = y + 3$
$x = 3y + 9$
The solution for an arbitrary y is $(3y + 9, y)$.

5.4 Solving Systems of Two Linear Equations in Two Unknowns by Determinants

Note to students: In all questions where solving a system of equations is required, substitution of the solutions into *both* equations serves as a check.

1. $\begin{vmatrix} 4 & -6 \\ 3 & 17 \end{vmatrix} = 4(17) - 3(-6) = 68 + 18 = 86$

2. $\begin{vmatrix} -4 & -6 \\ 3 & 17 \end{vmatrix} = -4(17) - 3(-6) = -68 + 18 = -50$

3. $2x - y = 1$

$5x - 2y = -11$

$$x = \frac{\begin{vmatrix} 1 & -1 \\ -11 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}} = \frac{1(-2) - (-11)(-1)}{2(-2) - 5(-1)} = \frac{-2 - 11}{-4 + 5} = \frac{-13}{1} = -13$$

$$y = \frac{\begin{vmatrix} 2 & 1 \\ 5 & -11 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}} = \frac{2(-11) - 5(1)}{1} = \frac{-22 - 5}{1} = -27$$

4. $x + y = 18\ 000$

$0.055x + 0.030y = 830$

$$x = \frac{\begin{vmatrix} 18000 & 1 \\ 830 & 0.030 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.055 & 0.030 \end{vmatrix}} = \frac{18000(0.030) - 830(1)}{1(0.030) - 0.055(1)} = \frac{540 - 830}{-0.025} = \frac{-290}{-0.025} = 11600$$

$$y = \frac{\begin{vmatrix} 1 & 18000 \\ 0.055 & 830 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.055 & 0.030 \end{vmatrix}} = \frac{1(830) - 0.055(18000)}{-0.025} = \frac{830 - 990}{-0.025} = \frac{-160}{-0.025} = 6400$$

The two investments are \$11 600 and \$6 400

5. $\begin{vmatrix} 8 & 3 \\ 4 & 1 \end{vmatrix} = (8)(1) - (3)(4) = 8 - 12 = -4$

6. $\begin{vmatrix} -1 & 3 \\ 2 & 6 \end{vmatrix} = -1(6) - 2(3) = -6 - 6 = -12$

7. $\begin{vmatrix} 3 & -5 \\ 7 & -2 \end{vmatrix} = 3(-2) - 7(-5) = -6 + 35 = 29$

8. $\begin{vmatrix} -4 & 7 \\ 1 & -3 \end{vmatrix} = -4(-3) - 1(7) = 12 - 7 = 5$

9. $\begin{vmatrix} 15 & -9 \\ 12 & 0 \end{vmatrix} = (15)(0) - (12)(-9) = 0 - (-108) = 108$

10. $\begin{vmatrix} -20 & -15 \\ -8 & -6 \end{vmatrix} = -20(-6) - (-8)(-15) = 120 - 120 = 0$

11. $\begin{vmatrix} -20 & 110 \\ -70 & -80 \end{vmatrix} = -20(-80) - (-70)(110) = 1600 + 7700 = 9300$

12. $\begin{vmatrix} -6.5 & 12.2 \\ -15.5 & 34.6 \end{vmatrix} = -6.5(34.6) - (-15.5)(12.2) = -224.9 + 189.1 = -35.8$

13. $\begin{vmatrix} 0.75 & -1.32 \\ 0.15 & 1.18 \end{vmatrix} = 0.75(1.18) - (0.15)(-1.32) = 0.885 + 0.198 = 1.083$

14. $\begin{vmatrix} 0.20 & -0.05 \\ 0.28 & 0.09 \end{vmatrix} = 0.20(0.09) - (0.28)(-0.05) = 0.018 + 0.014 = 0.032$

15. $\begin{vmatrix} -8 & -4 \\ -32 & 16 \end{vmatrix} = -8(16) - (-32)(-4) = -128 - 128 = -256$

16. $\begin{vmatrix} 43 & -7 \\ -81 & 16 \end{vmatrix} = 43(16) - (-81)(-7) = 688 - 567 = 121$

17. $\begin{vmatrix} 2 & a-1 \\ a+2 & a \end{vmatrix} = 2(a) - (a+2)(a-1) = 2a - (a^2 + a - 2) = -a^2 + a + 2$

18. $\begin{vmatrix} x+y & y-x \\ 2x & 2y \end{vmatrix} = (x+y)(2y) - (2x)(y-x) = 2xy + 2y^2 - (2xy - 2x^2) = 2x^2 + 2y^2$

19. $x + 2y = 5$

$x - 2y = 1$

$$x = \frac{\begin{vmatrix} 5 & 2 \\ 1 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & -2 \end{vmatrix}} = \frac{5(-2) - 1(2)}{1(-2) - 1(2)} = \frac{-12}{-4} = 3$$

$$y = \frac{\begin{vmatrix} 1 & 5 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & -2 \end{vmatrix}} = \frac{1(1) - 1(5)}{-4} = \frac{-4}{-4} = 1$$

20. $x+3y=7$

$2x+3y=5$

$$x=\frac{\begin{vmatrix}7 & 3\\ 5 & 3\end{vmatrix}}{\begin{vmatrix}1 & 3\\ 2 & 3\end{vmatrix}}=\frac{7(3)-5(3)}{1(3)-2(3)}=\frac{6}{-3}=-2$$

$$y=\frac{\begin{vmatrix}1 & 7\\ 2 & 5\end{vmatrix}}{\begin{vmatrix}1 & 3\\ 2 & 3\end{vmatrix}}=\frac{1(5)-2(7)}{-3}=\frac{-9}{-3}=3$$

21. $2x-3y=4$

$2x+y=-4$

$$x=\frac{\begin{vmatrix}4 & -3\\ -4 & 1\end{vmatrix}}{\begin{vmatrix}2 & -3\\ 2 & 1\end{vmatrix}}=\frac{4(1)-(-4)(-3)}{2(1)-2(-3)}=\frac{-8}{8}=-1$$

$$y=\frac{\begin{vmatrix}2 & 4\\ 2 & -4\end{vmatrix}}{\begin{vmatrix}2 & -3\\ 2 & 1\end{vmatrix}}=\frac{2(-4)-2(4)}{8}=\frac{-16}{8}=-2$$

22. Rearrange so the variables are in the same positions.

$R-4r=17$

$3R+4r=3$

$$R=\frac{\begin{vmatrix}17 & -4\\ 3 & 4\end{vmatrix}}{\begin{vmatrix}1 & -4\\ 3 & 4\end{vmatrix}}=\frac{17(4)-3(-4)}{1(4)-3(-4)}=\frac{68+12}{4+12}=\frac{80}{16}=5$$

$$r=\frac{\begin{vmatrix}1 & 17\\ 3 & 13\end{vmatrix}}{\begin{vmatrix}1 & -4\\ 3 & 4\end{vmatrix}}=\frac{1(3)-3(17)}{16}=\frac{-48}{16}=-3$$

23. Write both equations in standard form.

$12t + 9y = 14$

$6t - 7y = -16$

$$t = \frac{\begin{vmatrix} 14 & 9 \\ -16 & -7 \end{vmatrix}}{\begin{vmatrix} 12 & 9 \\ 6 & -7 \end{vmatrix}} = \frac{14(-7) - (-16)(9)}{12(-7) - 6(9)} = \frac{46}{-138} = -\frac{1}{3}$$

$$y = \frac{\begin{vmatrix} 12 & 14 \\ 6 & -16 \end{vmatrix}}{\begin{vmatrix} 12 & 9 \\ 6 & -7 \end{vmatrix}} = \frac{12(-16) - 6(14)}{-138} = \frac{-276}{-138} = 2$$

24. Write both equations in standard form.

$3x - y = 3$

$4x - 3y = 14$

$$x = \frac{\begin{vmatrix} 3 & -1 \\ 14 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 4 & -3 \end{vmatrix}} = \frac{3(-3) - 14(-1)}{3(-3) - 4(-1)} = \frac{-9 + 14}{-9 + 4} = \frac{5}{-5} = -1$$

$$y = \frac{\begin{vmatrix} 3 & 3 \\ 4 & 14 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 4 & -3 \end{vmatrix}} = \frac{3(14) - 4(3)}{-5} = \frac{30}{-5} = -6$$

25. $v + 2t = 7$

$2v + 4t = 9$

$$v = \frac{\begin{vmatrix} 7 & 2 \\ 9 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}} = \frac{7(4) - 9(2)}{1(4) - 2(2)} = \frac{28 - 18}{4 - 4} = \frac{10}{0} = \text{inconsistent}$$

26. $3x - y = 5$

$-9x + 3y = -15$

$$x = \frac{\begin{vmatrix} 5 & -1 \\ -15 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ -9 & 3 \end{vmatrix}} = \frac{5(3) - (-15)(-1)}{3(3) - (-9)(-1)} = \frac{15 - 15}{9 - 9} = \frac{0}{0} = \text{dependent}$$

27. Rewrite both equations in standard form.

$2x-3y=4$

$3x-2y=-2$

$$x=\frac{\begin{vmatrix}4 & -3\\ -2 & -2\end{vmatrix}}{\begin{vmatrix}2 & -3\\ 3 & -2\end{vmatrix}}=\frac{4(-2)-(-2)(-3)}{2(-2)-3(-3)}=-\frac{14}{5}$$

$$y=\frac{\begin{vmatrix}2 & 4\\ 3 & -2\end{vmatrix}}{\begin{vmatrix}2 & -3\\ 3 & -2\end{vmatrix}}=\frac{2(-2)-3(4)}{5}=-\frac{16}{5}$$

28. Rewrite both equations in standard form.

$3i_1+4i_2=-5$

$5i_1-3i_2=2$

$$i_1=\frac{\begin{vmatrix}-5 & 4\\ 2 & -3\end{vmatrix}}{\begin{vmatrix}3 & 4\\ 5 & -3\end{vmatrix}}=\frac{-5(-3)-2(4)}{3(-3)-5(4)}=\frac{15-8}{-9-20}=-\frac{7}{29}$$

$$i_2=\frac{\begin{vmatrix}3 & -5\\ 5 & 2\end{vmatrix}}{\begin{vmatrix}3 & 4\\ 5 & -3\end{vmatrix}}=\frac{3(2)-5(-5)}{-29}=\frac{6+25}{-29}=-\frac{31}{29}$$

29. $0.3x-0.7y=0.4$

$0.2x+0.5y=0.7$

$$x=\frac{\begin{vmatrix}0.4 & -0.7\\ 0.7 & 0.5\end{vmatrix}}{\begin{vmatrix}0.3 & -0.7\\ 0.2 & 0.5\end{vmatrix}}=\frac{0.4(0.5)-0.7(-0.7)}{0.3(0.5)-0.2(-0.7)}=\frac{0.69}{0.29}=\frac{69}{29}$$

$$y=\frac{\begin{vmatrix}0.3 & 0.4\\ 0.2 & 0.7\end{vmatrix}}{\begin{vmatrix}0.3 & -0.7\\ 0.2 & 0.5\end{vmatrix}}=\frac{0.3(0.7)-0.2(0.4)}{0.3(0.5)-0.2(-0.7)}=\frac{0.13}{0.29}=\frac{13}{29}$$

30. $250R + 225Z = 400$

$375R - 675Z = 325$

$$R = \frac{\begin{vmatrix} 400 & 225 \\ 325 & -675 \end{vmatrix}}{\begin{vmatrix} 250 & 225 \\ 375 & -675 \end{vmatrix}} = \frac{400(-675) - 325(225)}{250(-675) - 375(225)} = \frac{-343\ 125}{-253\ 125} = \frac{61}{45}$$

$$Z = \frac{\begin{vmatrix} 250 & 400 \\ 375 & 325 \end{vmatrix}}{\begin{vmatrix} 250 & 225 \\ 375 & -675 \end{vmatrix}} = \frac{250(325) - 375(400)}{-253\ 125} = \frac{-68\ 750}{-253\ 125} = \frac{22}{81}$$

31. $40s - 30t = 60$

$20s - 40t = -50$

$$s = \frac{\begin{vmatrix} 60 & -30 \\ -50 & -40 \end{vmatrix}}{\begin{vmatrix} 40 & -30 \\ 20 & -40 \end{vmatrix}} = \frac{60(-40) - (-50)(-30)}{40(-40) - 20(-30)} = \frac{-3900}{-1000} = \frac{39}{10}$$

$$t = \frac{\begin{vmatrix} 40 & 60 \\ 20 & -50 \end{vmatrix}}{\begin{vmatrix} 40 & -30 \\ 20 & -40 \end{vmatrix}} = \frac{40(-50) - 20(60)}{-1000} = \frac{-3200}{-1000} = \frac{16}{5}$$

32. $0.060x + 0.048y = -0.084$

$0.013x - 0.065y = -0.078$

$$x = \frac{\begin{vmatrix} -0.084 & 0.048 \\ -0.078 & -0.065 \end{vmatrix}}{\begin{vmatrix} 0.060 & 0.048 \\ 0.013 & -0.065 \end{vmatrix}} = \frac{-0.084(-0.065) - (-0.078)(0.048)}{0.060(-0.065) - 0.013(0.048)} = \frac{0.009204}{-0.004524} = -\frac{59}{29}$$

$$y = \frac{\begin{vmatrix} 0.060 & -0.084 \\ 0.013 & -0.078 \end{vmatrix}}{\begin{vmatrix} 0.060 & 0.048 \\ 0.013 & -0.065 \end{vmatrix}} = \frac{0.060(-0.078) - 0.013(-0.084)}{-0.004524} = \frac{-0.003588}{-0.004524} = \frac{23}{29}$$

33. $301x - 529y = 1520$

$385x - 741y = 2540$

$$x = \frac{\begin{vmatrix} 1520 & -529 \\ 2540 & -741 \end{vmatrix}}{\begin{vmatrix} 301 & -529 \\ 385 & -741 \end{vmatrix}} = \frac{217340}{-19376} = -11.2$$

$$y = \frac{\begin{vmatrix} 301 & 1520 \\ 385 & 2540 \end{vmatrix}}{\begin{vmatrix} 301 & -529 \\ 385 & -741 \end{vmatrix}} = \frac{179340}{-19376} = -9.26$$

34. $0.25d + 0.63n = -0.37$

$-0.61d - 1.80n = 0.55$

$$d = \frac{\begin{vmatrix} -0.37 & 0.63 \\ 0.55 & -1.80 \end{vmatrix}}{\begin{vmatrix} 0.25 & 0.63 \\ -0.61 & -1.80 \end{vmatrix}} = \frac{-0.37(-1.80) - 0.55(0.63)}{0.25(-1.80) - (-0.61)(0.63)} = \frac{0.3195}{-0.0657} = -4.9$$

$$n = \frac{\begin{vmatrix} 0.25 & -0.37 \\ -0.61 & 0.55 \end{vmatrix}}{\begin{vmatrix} 0.25 & 0.63 \\ -0.61 & -1.80 \end{vmatrix}} = \frac{0.25(0.55) - (-0.61)(-0.37)}{-0.0657} = \frac{-0.0882}{-0.0657} = 1.3$$

35. Write both equations in standard form.

$8.4x + 1.2y = -10.8$

$3.5x + 4.8y = -12.9$

$$x = \frac{\begin{vmatrix} -10.8 & 1.2 \\ -12.9 & 4.8 \end{vmatrix}}{\begin{vmatrix} 8.4 & 1.2 \\ 3.5 & 4.8 \end{vmatrix}} = \frac{-10.8(4.8) - (-12.9)(1.2)}{8.4(4.8) - 3.5(1.2)} = \frac{-36.36}{36.12} = -1.0$$

$$y = \frac{\begin{vmatrix} 8.4 & -10.8 \\ 3.5 & -12.9 \end{vmatrix}}{\begin{vmatrix} 8.4 & 1.2 \\ 3.5 & 4.8 \end{vmatrix}} = \frac{8.4(-12.9) - 3.5(-10.8)}{36.12} = \frac{-70.56}{36.12} = -2.0$$

36. $6541x + 4397y = -7732$

$3309x - 8755y = 7622$

$$x = \frac{\begin{vmatrix} -7732 & 4397 \\ 7622 & -8755 \end{vmatrix}}{\begin{vmatrix} 6541 & 4397 \\ 3309 & -8755 \end{vmatrix}} = \frac{-7732(-8755) - 7622(4397)}{6541(-8755) - 3309(4397)} = \frac{34{,}179{,}726}{-71{,}816{,}128} = -0.4759$$

$$y = \frac{\begin{vmatrix} 6541 & -7732 \\ 3309 & 7622 \end{vmatrix}}{\begin{vmatrix} 6541 & 4397 \\ 3309 & -8755 \end{vmatrix}} = \frac{6541(7622) - 3309(-7732)}{-71{,}816{,}128} = \frac{75{,}440{,}690}{-71{,}816{,}128} = -1.050$$

37. For $c = d = 0$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ 0 & 0 \end{vmatrix} = a(0) - 0(b) = 0$$

38. Original $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Interchanging rows yields $\begin{vmatrix} c & d \\ a & b \end{vmatrix}$ and then interchanging columns

yields $\begin{vmatrix} d & c \\ b & a \end{vmatrix} = da - cb.$

Change in value $= ad - bc - (da - cb) = 0$

There is no change in value.

39. If $a = kb$, $c = kd$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} kb & b \\ kd & d \end{vmatrix} = kb(d) - kd(b) = kbd - kbd = 0$$

40. If a and c are doubled $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ becomes

$$\begin{vmatrix} 2a & b \\ 2c & d \end{vmatrix} = 2ad - 2bc = 2(ad - bc) = 2\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

If a and c are doubled the value of the determinant doubles.

41. Rewrite both equations in standard form.

$$F_1 + F_2 = 21$$
$$2F_1 - 5F_2 = 0$$

$$F_1 = \frac{\begin{vmatrix} 21 & 1 \\ 0 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & -5 \end{vmatrix}} = \frac{21(-5) - 0(1)}{1(-5) - 2(1)} = \frac{-105}{-7} = 15 \text{ N}$$

$$F_2 = \frac{\begin{vmatrix} 1 & 21 \\ 2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & -5 \end{vmatrix}} = \frac{1(0) - 2(21)}{-7} = \frac{-42}{-7} = 6.0 \text{ N}$$

42. $A = \frac{1}{2}\left(\begin{vmatrix} 12.79 & 67.21 \\ 0.00 & 12.30 \end{vmatrix} + \begin{vmatrix} 67.21 & 53.05 \\ 12.30 & 47.12 \end{vmatrix} + \begin{vmatrix} 53.05 & 10.09 \\ 47.12 & 53.11 \end{vmatrix} + \begin{vmatrix} 10.09 & 12.79 \\ 53.11 & 0.00 \end{vmatrix}\right)$

$$A = \frac{1}{2}\left(12.79(12.30) - 0 + 67.21(47.12) - 12.03(53.05) + 53.05(53.11) - 47.12(10.09) + 0 - 53.11(12.79)\right)$$

$$A = \frac{1}{2}[4334.505]$$

$$A = 2167 \text{ m}^2$$

43. $x + y = 36.0$

$0.250x + 0.375y = 11.2$

$$x = \frac{\begin{vmatrix} 36.0 & 1 \\ 11.2 & 0.375 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.250 & 0.375 \end{vmatrix}} = \frac{36.0(0.375) - 11.2(1)}{0.375 - 0.250} = \frac{2.30}{0.125} = 18.4 \text{ gal}$$

$$y = \frac{\begin{vmatrix} 1 & 36.0 \\ 0.250 & 11.2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.250 & 0.375 \end{vmatrix}} = \frac{1(11.2) - 0.250(36.0)}{0.125} = \frac{2.20}{0.125} = 17.6 \text{ gal}$$

44. $52I_1 - 27I_2 = -420$

$-27I_1 + 76I_2 = 210$

$$I_1 = \frac{\begin{vmatrix} -420 & -27 \\ 210 & 76 \end{vmatrix}}{\begin{vmatrix} 52 & -27 \\ -27 & 76 \end{vmatrix}} = \frac{-420(76) - 210(-27)}{52(76) - (-27)(-27)} = \frac{-26\ 250}{3223} = -8.1 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 52 & -420 \\ -27 & 210 \end{vmatrix}}{\begin{vmatrix} 52 & -27 \\ -27 & 76 \end{vmatrix}} = \frac{52(210) - (-27)(-420)}{3223} = \frac{-420}{3223} = -0.13 \text{ A}$$

45. Let x = number of three bedroom homes

Let y = number of four bedroom homes

$2\ 000x + 3\ 000y = 560\ 000$

$25\ 000x + 35\ 000y = 6\ 800\ 000$

$$x = \frac{\begin{vmatrix} 560\ 000 & 3\ 000 \\ 6\ 800\ 000 & 35\ 000 \end{vmatrix}}{\begin{vmatrix} 2\ 000 & 3\ 000 \\ 25\ 000 & 35\ 000 \end{vmatrix}} = \frac{560\ 000(35\ 000) - 6\ 800\ 000(3\ 000)}{35\ 000(2\ 000) - (25\ 000)(3\ 000)} = \frac{-800\ 000\ 000}{-5\ 000\ 000} = 160$$

$$y = \frac{\begin{vmatrix} 2\ 000 & 560\ 000 \\ 25\ 000 & 6\ 800\ 000 \end{vmatrix}}{\begin{vmatrix} 2\ 000 & 3\ 000 \\ 25\ 000 & 35\ 000 \end{vmatrix}} = \frac{6\ 800\ 000(2\ 000) - 560\ 000(25\ 000)}{-5\ 000\ 000} = 80$$

46. v_1 = rate of faster jogger (in mi/h)

v_2 = rate of slower jogger (in mi/h)

Remember $d = vt$

When they jog toward each other they cover a total distance of 2 mi.

$$v_1\left(\frac{12}{60}\right) + v_2\left(\frac{12}{60}\right) = 2$$

$$0.2v_1 + 0.2v_2 = 2$$

When they jog in the same direction, the faster jogger covers a distance of 2 mi + distance the slower jogger runs.

$$v_1(2) = 2 + v_2(2)$$

$$2v_1 - 2v_2 = 2$$

$$v_1 = \frac{\begin{vmatrix} 2 & 0.2 \\ 2 & -2 \end{vmatrix}}{\begin{vmatrix} 0.2 & 0.2 \\ 2 & -2 \end{vmatrix}} = \frac{-4 - 0.4}{-0.4 - 0.4} = \frac{-4.4}{-0.8} = 5.5 \text{ mi/h}$$

$$v_2 = \frac{\begin{vmatrix} 0.2 & 2 \\ 2 & 2 \end{vmatrix}}{\begin{vmatrix} 0.2 & 0.2 \\ 2 & -2 \end{vmatrix}} = \frac{0.4 - 4}{-0.8} = \frac{-3.6}{-0.8} = 4.5 \text{ mi/h}$$

47. x = number of phones

y = number of detectors

$$x + y = 320$$

$$110x + 160y = 40\ 700$$

$$x = \frac{\begin{vmatrix} 320 & 1 \\ 40700 & 160 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 110 & 160 \end{vmatrix}} = \frac{320(160) - 40700}{160 - 110} = \frac{10500}{50} = 210$$

$$y = \frac{\begin{vmatrix} 1 & 320 \\ 110 & 40700 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 110 & 160 \end{vmatrix}} = \frac{40700 - 110(320)}{50} = \frac{5500}{50} = 110$$

48. Let a_1 = the number of the first type of carburetors

Let a_2 = the number of the second type of carburetors

Assembly : $15a_1 + 12a_2 = 222$

Testing : $2a_1 + 3a_2 = 45$

$$a_1 = \frac{\begin{vmatrix} 222 & 12 \\ 45 & 3 \end{vmatrix}}{\begin{vmatrix} 15 & 12 \\ 2 & 3 \end{vmatrix}} = \frac{666-540}{45-24} = \frac{126}{21} = 6$$

$$a_2 = \frac{\begin{vmatrix} 15 & 222 \\ 2 & 45 \end{vmatrix}}{\begin{vmatrix} 15 & 12 \\ 2 & 3 \end{vmatrix}} = \frac{675-444}{21} = \frac{231}{21} = 11$$

49. Let L = the length of the rectangle in meters

Let w = the width of the rectangle in meters

$$L = 1.62w$$
$$2L + 2w = 4.20$$

Rewrite these in standard form

$$L - 1.62w = 0$$
$$2L + 2w = 4.20$$

$$L = \frac{\begin{vmatrix} 0 & -1.62 \\ 4.20 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & -1.62 \\ 2 & 2 \end{vmatrix}} = \frac{0-4.20(-1.62)}{1(2)-2(-1.62)} = \frac{6.804}{5.24} = 1.30 \text{ m}$$

$$w = \frac{\begin{vmatrix} 1 & 0 \\ 2 & 4.20 \end{vmatrix}}{\begin{vmatrix} 1 & -1.62 \\ 2 & 2 \end{vmatrix}} = \frac{4.20-0}{5.24} == 0.80 \text{ m}$$

50. Let m = the number of men before the last passengers boarded

Let w = the number of women before the last passengers boarded

We can write the two ratios as

$$\frac{m}{w} = \frac{5}{7}$$
$$\frac{m+1}{w+2} = \frac{7}{10}$$

Clear the denominators

$$7m = 5w$$
$$10m + 10 = 7w + 14$$

Rewrite these as linear equations in standard form

$$7m - 5w = 0$$
$$10m - 7w = 4$$

$$m = \frac{\begin{vmatrix} 0 & -5 \\ 4 & -7 \end{vmatrix}}{\begin{vmatrix} 7 & -5 \\ 10 & -7 \end{vmatrix}} = \frac{0 - 4(-5)}{7(-7) - 10(-5)} = \frac{20}{1} = 20$$

$$w = \frac{\begin{vmatrix} 7 & 0 \\ 10 & 4 \end{vmatrix}}{\begin{vmatrix} 7 & -5 \\ 10 & -7 \end{vmatrix}} = \frac{7(4) - 0}{1} == 28$$

There were 20 men and 28 women on the bus before the last three passengers boarded.

51. Let s = fixed salary (in \$)

Let c = sales commission (%)

$s + 70000c = 6200$ first month

$s + 45000c = 4700$ second month

$$s = \frac{\begin{vmatrix} 6200 & 70000 \\ 4700 & 45000 \end{vmatrix}}{\begin{vmatrix} 1 & 70000 \\ 1 & 45000 \end{vmatrix}} = \frac{6200(45\ 000) - 4700(70\ 000)}{45\ 000 - 70\ 000} = \frac{-50\ 000\ 000}{-25\ 000} = 2000$$

$$c = \frac{\begin{vmatrix} 1 & 6200 \\ 1 & 4700 \end{vmatrix}}{\begin{vmatrix} 1 & 70000 \\ 1 & 45000 \end{vmatrix}} = \frac{4700 - 6200}{-25\ 000} = \frac{-1500}{-25\ 000} = 0.06$$

The fixed salary is \$2000, and the commission percentage is 6.0%

52. Let v_c =velocity of child (in m/s)

Let v_w =velocity of walkway (in m/s)

Remember $d = vt$

If child and walkway are moving in same direction

$$(v_c + v_w)(20.0) = 65$$
$$20.0v_c + 20.0v_w = 65.0$$

If child and walkway are moving in opposite direction

$$(v_c - v_w)(52.0) = 65.0$$
$$52.0v_c - 52.0v_w = 65.0$$

$$v_c = \frac{\begin{vmatrix} 65.0 & 20.0 \\ 65.0 & -52.0 \end{vmatrix}}{\begin{vmatrix} 20.0 & 20.0 \\ 52.0 & -52.0 \end{vmatrix}} = \frac{65.0(-52.0) - 65.0(20.0)}{20.0(-52.0) - 52.0(20.0)} = \frac{-4680}{-2080} = 2.25 \text{ m/s}$$

$$v_w = \frac{\begin{vmatrix} 20.0 & 65.0 \\ 52.0 & 65.0 \end{vmatrix}}{\begin{vmatrix} 20.0 & 20.0 \\ 52.0 & -52.0 \end{vmatrix}} = \frac{20.0(65.0) - 52.0(65.0)}{-2080} = \frac{-2080}{-2080} = 1.00 \text{ m/s}$$

53. Let t_1 = time taken by drug boat

Let t_2 = time taken by Coast Guard

$$24 \text{ min} = 24 \text{ min} \left(\frac{1 \text{ h}}{60 \text{ min}}\right) = 0.40 \text{ h}$$

We know the drug boat had a 0.40 h head start

$$t_1 = t_2 + 0.40$$
$$t_1 - t_2 = 0.40$$

Remember $d = vt$, and the total distance travelled by each boat is the same

$$42t_1 = 50t_2$$
$$42t_1 - 50t_2 = 0$$

$$t_1 = \frac{\begin{vmatrix} 0.4 & -1 \\ 0 & -50 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 42 & -50 \end{vmatrix}} = \frac{0.4(-50) - 0}{-50 - (-42)} = \frac{-20}{-8} = 2.5 \text{ h}$$

$$t_2 = \frac{\begin{vmatrix} 1 & 0.4 \\ 42 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 42 & -50 \end{vmatrix}} = \frac{0 - 42(0.4)}{-8} = \frac{-16.8}{-12} = 2.1 \text{ h}$$

54. Let x = mass of the 94.0% silver alloy (in g)

Let y = mass of the 85.0% silver alloy (in g)

$$x + y = 100$$
$$0.94x + 0.85y = 0.925(100)$$

$$x = \frac{\begin{vmatrix} 100 & 1 \\ 92.5 & 0.85 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.94 & 0.85 \end{vmatrix}} = \frac{100(0.85) - 92.5}{0.85 - 0.94} = \frac{-7.5}{-0.09} = 83.3$$

$$y = \frac{\begin{vmatrix} 1 & 100 \\ 0.94 & 92.5 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.94 & 0.85 \end{vmatrix}} = \frac{92.5 - 0.94(100)}{0.85 - 0.94} = \frac{-1.5}{-0.09} = 16.7$$

The solution is 83.3 g of the 1st alloy and 16.7 g of the 2nd alloy.

55. Using the tangent function we know

$$\tan(21.4°) = 0.392 = \frac{h}{d} \text{ or } 0.392d = h$$

$$\tan(15.5°) = 0.277 = \frac{h}{d + 345} \text{ or } 0.277(d + 345) = h$$

We rewrite these in standard form.

$h - 0.392d = 0$

$h - 0.277d = 95.6$

$$h = \frac{\begin{vmatrix} 0 & -0.392 \\ 95.6 & -0.277 \end{vmatrix}}{\begin{vmatrix} 1 & -0.392 \\ 1 & -0.277 \end{vmatrix}} = \frac{0 - 95.6(-0.392)}{-0.277 - (-0.392)} = \frac{37.5}{0.115} = 326 \text{ ft.}$$

$$d = \frac{\begin{vmatrix} 1 & 0 \\ 1 & 95.6 \end{vmatrix}}{\begin{vmatrix} 1 & -0.392 \\ 1 & -0.277 \end{vmatrix}} = \frac{95.6}{0.115} = 831 \text{ ft.}$$

56. Let v_s = velocity of sound in steel

Let v_a = velocity of sound in air

The speed of sound in steel is 15900 ft/s faster than the speed of sound in air.

$v_s = v_a + 15900$

$v_s - v_a = 15900$

The distances traveled by each sound are the same, and remember

$d = vt$

$0.0120v_s = 0.180v_a$

$0.0120v_s - 0.180v_a = 0$

$$v_s = \frac{\begin{vmatrix} 15900 & -1 \\ 0 & -0.180 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 0.0120 & -0.180 \end{vmatrix}} = \frac{15900(-0.180) - 0}{-0.180 + 0.0120} = \frac{-2862}{-0.168} = 17000 \text{ ft/s}$$

$$v_a = \frac{\begin{vmatrix} 1 & 15900 \\ 0.0120 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 0.0120 & -0.180 \end{vmatrix}} = \frac{0 - 0.0120(15900)}{-0.168} = \frac{-191}{-0.168} = 1136 \text{ ft/s}$$

The speed of sound in steel is 17000 ft/s.

The speed of sound in air is 1100 ft/s (to conform with significant figures)

5.5 Solving Systems of Three Linear Equations in Three Unknowns Algebraically

Note to students: In all questions where solving a system of equations is required, substitution of the solutions into *all three* equations serves as a check.

1.

$$
\begin{array}{lrl}
(1) & 4x + y + 3z = & 1 \\
(2) & 2x - 2y + 6z = & 12 \\
(3) & -6x + 3y + 12z = & -14 \\
\hline
(4) & 8x + 2y + 6z = & 2 \quad \text{(1) multiplied by 2} \\
(2) & 2x - 2y + 6z = & 12 \quad \text{add} \\
\hline
(5) & 10x \qquad + 12z = & 14 \\
\\
(6) & 12x + 3y + 9z = & 3 \quad \text{(1) multiplied by 3} \\
(3) & -6x + 3y + 12z = & -14 \quad \text{subtract} \\
\hline
(7) & 18x \qquad - 3z = & 17 \\
\\
(8) & 72x \qquad - 12z = & 68 \quad \text{(7) multiplied by 4} \\
(5) & 10x \qquad + 12z = & 14 \quad \text{add} \\
\hline
(9) & 82x \qquad = & 82 \\
 & x = & 1 \\
(11) & 18(1) - 3z = & 17 \quad \text{substituting } x = 1 \text{ into (7)} \\
 & -3z = & -1 \\
 & z = & \frac{1}{3} \\
(12) & 4(1) + y + 3\,\frac{1}{3} = & 1 \quad \text{substitute } x = 1 \text{ and } z = \frac{1}{3} \text{ into (1)} \\
 & 4 + y + 1 = & 1 \\
 & y = & -4
\end{array}
$$

The solution is $x = 1$, $y = -4$, $z = \frac{1}{3}$

2.

$$
\begin{array}{lrl}
(1) & 4x + y + 3z = & 1 \\
(2) & 8x \qquad + 9z = & 10 \\
(3) & -6x + 3y + 12z = & -4 \\
\hline
(1) & 12x + 3y + 9z = & 3 \quad \text{(1) multiplied by 3} \\
(3) & -6x + 3y + 12z = & -4 \quad \text{subtract} \\
\hline
(4) & 18x \qquad - 3z = & 7
\end{array}
$$

$$\begin{array}{lrcll}
(5) & 54x & -9z = & 21 & \text{(4) multiplied by 3} \\
(2) & 8x & +9z = & 10 & \text{add} \\
\hline
(6) & 62x & = & 31 & \\
(7) & x & = & \frac{1}{2} &
\end{array}$$

$$\begin{array}{lrl}
(8) & 8\left(\frac{1}{2}\right)+9z=10 & \text{substituted } x=\frac{1}{2} \text{ into (2)} \\
& 4+9z=10 & \\
& 9z=6 & \\
& z=\frac{2}{3} & \\
(9) & 4\left(\frac{1}{2}\right)+y+3\left(\frac{2}{3}\right)=1 & \text{substituted } x=\frac{1}{2},\ z=\frac{2}{3} \text{ into (1)} \\
& 2+y+2=1 & \\
& y=-3 &
\end{array}$$

The solution is $x=\frac{1}{2}$, $y=-3$, $z=\frac{2}{3}$.

3.

$$\begin{array}{lrl}
(1) & x+y+z=2 & \\
(2) & x\quad -z=1 & \\
(3) & x+y\quad =1 & \\
\hline
(4) & 2x+y\quad =3 & \text{add (1) + (2)} \\
(3) & x+y\quad =1 & \text{subtract} \\
\hline
(5) & x=2 & \\
(6) & 2-z=1 & \text{substitute 2 for } x \text{ into (2)} \\
& z=1 & \\
(7) & 2+y=1 & \text{substitute 2 for } x \text{ into (3)} \\
& y=-1 &
\end{array}$$

The solution is $x=2$, $y=-1$, $z=1$.

4.

$$\begin{array}{lrl}
(1) & x+y-z=-3 & \\
(2) & x\quad +z=2 & \\
(3) & 2x-y+2z=3 & \\
\hline
(4) & 3x+z=0 & \text{Add (1) + (3)} \\
(2) & x+z=2 & \text{subtract} \\
\hline
(5) & 2x=-2 & \\
& x=-1 & \\
(6) & -1+z=2 & \text{substitute } -1 \text{ for } x \text{ into (2)} \\
& z=3 & \\
(7) & -1+y-3=-3 & \text{substitute } -1 \text{ for } x, \text{ and 3 for } z \text{ into (1)} \\
& y=1 &
\end{array}$$

The solution is $x=-1$, $y=1$, $z=3$.

5.

$$\begin{array}{lrl}
(1) & 2x+3y+\ \ z=2 & \\
(2) & -x+2y+3z=-1 & \\
(3) & -3x-3y+\ \ z=0 & \\
\hline
(4) & 5x+6y\qquad =2 & \text{Subtract (1) } - \text{ (3)} \\
 & & \\
(5) & 6x+9y+3z=6 & \text{multiply (1) by 3} \\
(2) & -x+2y+3z=-1 & \text{subtract} \\
\hline
(6) & 7x+7y\qquad =7 & \\
 & & \\
(7) & 5x+5y=5 & \text{multiply (6) by 5/7} \\
(4) & 5x+6y=2 & \text{subtract} \\
\hline
 & -y=3 & \\
 & y=-3 & \\
(8) & 7x+7(-3)=7 & \text{substitute } -3 \text{ for } y \text{ into (6)} \\
 & 7x=28 & \\
 & x=4 & \\
(9) & -3(4)-3(-3)+z=0 & \text{substitute 4 for } x, \text{ and } -3 \text{ for } y \text{ into (3)} \\
 & z=3 &
\end{array}$$

The solution is $x=4,\ y=-3,\ z=3$.

6.

$$\begin{array}{lrl}
(1) & 2x+\ \ y-\ \ z=\ \ 4 & \\
(2) & 4x-3y-2z=-2 & \\
(3) & 8x-2y-3z=\ \ 3 & \\
\hline
(4) & 4x+2y-2z=8 & \text{multiply (1) by 2} \\
(2) & 4x-3y-2z=-2 & \text{subtract} \\
\hline
 & 5y=10 & \\
 & y=2 & \\
(5) & 6x+3y-3z=12 & \text{multiply (1) by 3} \\
(3) & 8x-2y-3z=\ \ 3 & \text{subtract} \\
\hline
(6) & -2x+5y=9 & \\
(7) & -2x+5(2)=9 & \text{substitute 2 for } y \text{ into (6)} \\
 & -2x=-1 & \\
 & x=\frac{1}{2} & \\
\hline
(8) & 2\left(\frac{1}{2}\right)+2-z=\ \ 4 & \text{substitute } \frac{1}{2} \text{ for } x, \text{ and 2 for } y \text{ into (1)} \\
 & z=-1 &
\end{array}$$

The solution is $x=\frac{1}{2},\ y=2,\ z=-1.$

7.

$$\begin{array}{lrl}
(1) & 5l+6w-3h= & 6 \\
(2) & 4l-7w-2h= & -3 \\
(3) & 3l+w-7h= & 1 \\
\hline
(4) & 18l+6w-42h= & 6 \quad \text{multiply (3) by 6} \\
(1) & 5l+6w-3h= & 6 \quad \text{subtract} \\
\hline
(5) & 13l-39h= & 0 \\
(6) & l-3h= & 0 \quad \text{divide (5) by 13} \\
 & & \\
(7) & 21l+7w-49h= & 7 \quad \text{multiply (3) by 7} \\
(2) & 4l-7w-2h= & -3 \quad \text{add} \\
\hline
(8) & 25l-51h= & 4 \\
(9) & 25l-75h= & 0 \quad \text{multiply (6) by 25, then subtract from (8)} \\
\hline
(10) & 24h= & 4 \\
 & h= & \frac{1}{6} \\
(11) & l-3\left(\frac{1}{6}\right)= & 0 \quad \text{substitute } \tfrac{1}{6} \text{ for } h \text{ into (6)} \\
 & l= & \frac{1}{2} \\
(12) & 3\left(\frac{1}{2}\right)+w-7\left(\frac{1}{6}\right)= & 1 \quad \text{substitute } \tfrac{1}{2} \text{ for } l, \text{ and } \tfrac{1}{6} \text{ for } h \text{ into (3)} \\
 & \frac{9}{6}+w-\frac{7}{6}= & \frac{6}{6} \\
 & w= & \frac{4}{6}=\frac{2}{3}
\end{array}$$

The solution is $l=\frac{1}{2},\ w=\frac{2}{3},\ h=\frac{1}{6}.$

8.

$$\begin{array}{lrl}
(1) & 3r+s-t= & 2 \\
(2) & r-2s+t= & 0 \\
(3) & 4r-s+t= & 3 \\
\hline
(4) & 7r= & 5 \quad \text{add (1) and (3)} \\
 & r= & \frac{5}{7} \\
\hline
(5) & 4r-s= & 2 \quad \text{add (1) and (2)} \\
(6) & 4\left(\frac{5}{7}\right)-s= & 2 \quad \text{substitute } \tfrac{5}{7} \text{ for } r \text{ into (5)} \\
 & s= & \frac{20}{7}-\frac{14}{7} \\
 & s= & \frac{6}{7} \\
(7) & r-2s+t= & 0 \quad \text{substitute } \tfrac{5}{7} \text{ for } r, \text{ and } \tfrac{6}{7} \text{ for } s \text{ into (2)} \\
 & \frac{5}{7}-2\left(\frac{6}{7}\right)+t= & 0 \\
 & t= & \frac{7}{7}=1
\end{array}$$

The solution is $r=\frac{5}{7},\ s=\frac{6}{7},\ t=1.$

9.

(1)	$2x-2y+3z=5$	
(2)	$2x+y-2z=-1$	
(3)	$4x-y-3z=0$	
(4)	$6x-5z=-1$	add (2) and (3)
(5)	$4x+2y-4z=-2$	multiply (2) by 2
(1)	$2x-2y+3z=5$	add
(6)	$6x-z=3$	
(4)	$6x-5z=-1$	subtract
	$4z=4$	
	$z=1$	
(7)	$6x-1=3$	substitute 1 for z into (6)
	$x=\frac{4}{6}=\frac{2}{3}$	
(8)	$2\left(\frac{2}{3}\right)+y-2(1)=-1$	substitute $\frac{2}{3}$ for x, and 1 for z into (2)
	$\frac{4}{3}+y-\frac{6}{3}=-\frac{3}{3}$	
	$y=-\frac{1}{3}$	

The solution is $x=\frac{2}{3}$, $y=-\frac{1}{3}$, $z=1$.

10.

(1)	$2u+2v+3w=0$	
(2)	$3u+v+4w=21$	
(3)	$-u-3v+7w=15$	
(4)	$-2u-6v+14w=30$	multiply (3) by 2
(1)	$2u+2v+3w=0$	add
(5)	$-4v+17w=30$	
(6)	$-3u-9v+21w=45$	multiply (3) by 3
(2)	$3u+v+4w=21$	add
(7)	$-8v+25w=66$	
(8)	$-8v+34w=60$	multiply (5) by 2, then subtract from (7)
(9)	$-9w=6$	
	$w=-\frac{2}{3}$	
(10)	$-4v+17\left(-\frac{2}{3}\right)=30$	substitute $-\frac{2}{3}$ for w into (5)
	$-4v=\frac{90}{3}+\frac{34}{3}=\frac{124}{3}$	
	$v=-\frac{31}{3}$	
(11)	$-u-3\left(-\frac{31}{3}\right)+7\left(-\frac{2}{3}\right)=15$	substitute $-\frac{2}{3}$ for w, and $-\frac{31}{3}$ for v into (3)
	$-u+\frac{93}{3}-\frac{14}{3}=\frac{45}{3}$	
	$u=\frac{34}{3}$	

The solution is $u=\frac{34}{3}$, $v=-\frac{31}{3}$, $w=-\frac{2}{3}$.

11.

(1)	$3x - 7y + 3z = 6$	
(2)	$3x + 3y + 6z = 1$	
(3)	$5x - 5y + 2z = 5$	
(4)	$6x - 14y + 6z = 12$	multiply (1) by 2
(2)	$3x + 3y + 6z = 1$	subtract
(5)	$3x - 17y = 11$	
(6)	$15x - 15y + 6z = 15$	multiply (3) by 3
(2)	$3x + 3y + 6z = 1$	subtract
(7)	$12x - 18y = 14$	
(8)	$12x - 68y = 44$	multiply (5) by 4, then subtract from (7)
(9)	$50y = -30$	
	$y = -\frac{3}{5}$	
(10)	$3x - 17\left(-\frac{3}{5}\right) = 11$	substitute $-\frac{3}{5}$ for y into (5)
	$3x = \frac{55}{5} - \frac{51}{5} = \frac{4}{5}$	
	$x = \frac{4}{15}$	
(11)	$5\left(\frac{4}{15}\right) - 5\left(-\frac{3}{5}\right) + 2z = 5$	substitute $-\frac{3}{5}$ for y, and $\frac{4}{15}$ for x into (3)
	$\frac{4}{3} + 3 + 2z = 5$	
	$2z = \frac{2}{3}$	
	$z = \frac{1}{3}$	

The solution is $x = \frac{4}{15}$, $y = -\frac{3}{5}$, $z = \frac{1}{3}$.

12.

(1)	$18x + 24y + 4z = 46$	
(2)	$63x + 6y - 15z = -75$	
(3)	$-90x + 30y - 20z = -55$	
(4)	$90x + 120y + 20z = 230$	multiply (1) by 5
(3)	$-90x + 30y - 20z = -55$	add
(5)	$150y = 175$	
	$y = \frac{7}{6}$	
(6)	$270x + 360y + 60z = 690$	multiply (1) by 15
(7)	$252x + 24y - 60z = -300$	multiply (2) by 4, add to (6)
(8)	$522x + 384y = 390$	
(9)	$522x + 384\left(\frac{7}{6}\right) = 390$	substitute $\frac{7}{6}$ for y into (8)
	$522x = -58$	
	$x = -\frac{1}{9}$	
(11)	$18\left(-\frac{1}{9}\right) + 24\left(\frac{7}{6}\right) + 4z = 46$	substitute $-\frac{1}{9}$ for x, and $\frac{7}{6}$ for y into (1)
	$-2 + 28 + 4z = 46$	
	$4z = 20$	
	$z = 5$	

The solution is $x = -\frac{1}{9}$, $y = \frac{7}{6}$, $z = 5$.

13.

(1)	$10x + 15y - 25z = 35$	
(2)	$40x - 30y - 20z = 10$	
(3)	$16x - 2y + 8z = 6$	
(4)	$20x + 30y - 50z = 70$	multiply (1) by 2
(2)	$40x - 30y - 20z = 10$	add
(5)	$60x - 70z = 80$	
(6)	$240x - 30y + 120z = 90$	multiply (3) by 15
(2)	$40x - 30y - 20z = 10$	subtract
(7)	$200x + 140z = 80$	
(8)	$120x - 140z = 160$	multiply (5) by 2, then add to (7)
(9)	$320x = 240$	
	$x = \frac{3}{4}$	
(10)	$60\left(\frac{3}{4}\right) - 70z = 80$	substitute $\frac{3}{4}$ for x into (5)
	$-70z = 35$	
	$z = -\frac{1}{2}$	
(11)	$16\left(\frac{3}{4}\right) - 2y + 8\left(-\frac{1}{2}\right) = 6$	substitute $\frac{3}{4}$ for x, and $-\frac{1}{2}$ for z into (3)
	$12 - 2y - 4 = 6$	
	$-2y = -2$	
	$y = 1$	

The solution is $x = \frac{3}{4}$, $y = 1$, $z = -\frac{1}{2}$.

14.

$$\begin{array}{lrl}
(1) & 2i_1 - 4i_2 - 4i_3 = 3 & \\
(2) & 3i_1 + 8i_2 + 2i_3 = -11 & \\
(3) & 4i_1 + 6i_2 - i_3 = -8 & \\
\hline
(4) & 8i_1 + 12i_2 - 2i_3 = -16 & \text{multiply (3) by 2} \\
(2) & 3i_1 + 8i_2 + 2i_3 = -11 & \text{add} \\
\hline
(5) & 11i_1 + 20i_2 = -27 &
\end{array}$$

$$\begin{array}{lrl}
(6) & 6i_1 + 16i_2 + 4i_3 = -22 & \text{multiply (2) by 2} \\
(1) & 2i_1 - 4i_2 - 4i_3 = 3 & \text{add} \\
\hline
(7) & 8i_1 + 12i_2 = -19 &
\end{array}$$

$$\begin{array}{lrl}
(8) & 33i_1 + 60i_2 = -81 & \text{multiply (5) by 3} \\
(9) & 40i_1 + 60i_2 = -95 & \text{multiply (7) by 5, subtract from (8)} \\
\hline
 & -7i_1 = 14 & \\
 & i_1 = -2 & \\
(10) & 8(-2) + 12i_2 = -19 & \text{substitute } -2 \text{ for } i_1 \text{ into (7)} \\
 & 12i_2 = -3 & \\
 & i_2 = -\tfrac{1}{4} & \\
(11) & 4(-2) + 6\left(-\tfrac{1}{4}\right) - i_3 = -8 & \text{substitute } -2 \text{ for } i_1, \text{ and } -\tfrac{1}{4} \text{ for } i_2 \text{ into (3)} \\
 & -8 - \tfrac{3}{2} - i_3 = -8 & \\
 & i_3 = -\tfrac{3}{2} &
\end{array}$$

The solution is $i_1 = -2,\ i_2 = -\frac{1}{4},\ i_3 = -\frac{3}{2}$.

15. In standard form, the equations are

$-x + 3y - 2z = 7$

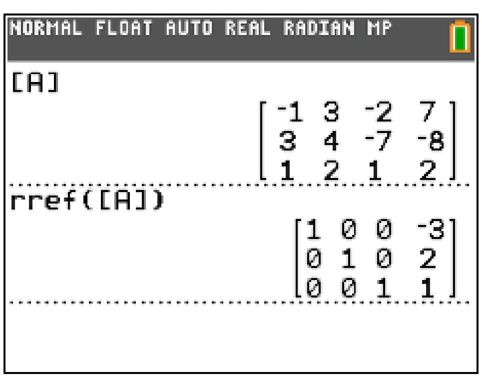
$3x + 4y - 7z = -8$

$x + 2y + z = 2$

which are encoded in the augmented matrix

−1	3	−2	7
3	4	−7	−8
1	2	1	2

Using the rref feature on a calculator, we find $x = -3, y = 2, z = 1$.



16. In standard form, the equations are

$3a + 2b + c = 20$

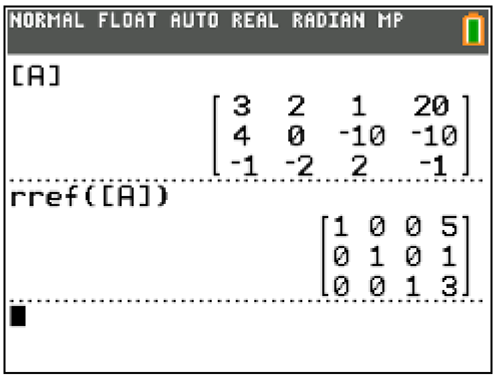
$4a + 0b - 10c = -10$

$-a - 2b + 2c = -1$

which are encoded in the augmented matrix

$$\begin{array}{cccc} 3 & 2 & 1 & 20 \\ 4 & 0 & -10 & -10 \\ -1 & -2 & 2 & -1 \end{array}$$

Using the rref feature on a calculator, we find $a = 5, b = 1, c = 3$.



17. In standard form, the equations are

$-I_1 + I_2 - I_3 = 0$

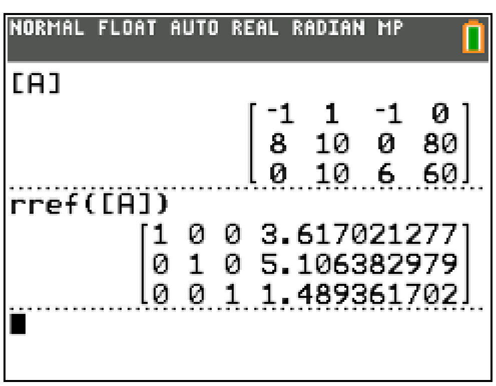
$8.00I_1 + 10.0I_2 + 0.00I_3 = 80.0$

$0.00I_1 + 10.0I_2 + 6.00I_3 = 60.0$

which are encoded in the augmented matrix

$$\begin{array}{cccc} -1 & 1 & -1 & 0 \\ 8.00 & 10.0 & 0 & 80.0 \\ 0 & 10.0 & 6.00 & 60.0 \end{array}$$

Using the rref feature on a calculator, we find $I_1 = 3.62, I_2 = 5.11, I_3 = 1.49$ to three significant figures.



18. In standard form, the equations are

$1.21x + 1.32y + 1.20z = 6.81$

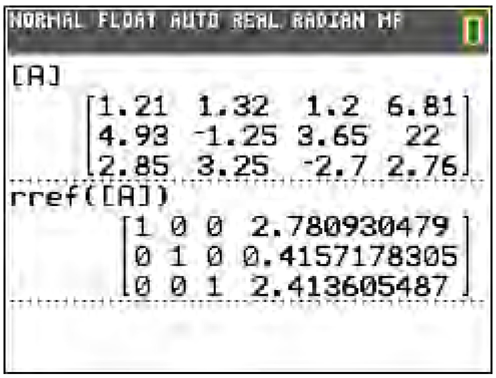
$4.93x - 1.25y + 3.65z = 22.0$

$2.85x + 3.25y - 2.70z = 2.76$

which are encoded in the augmented matrix

$$\begin{matrix} 1.21 & 1.32 & 1.20 & 6.81 \\ 4.93 & -1.25 & 3.65 & 22.0 \\ 2.85 & 3.25 & -2.70 & 2.76 \end{matrix}$$

Using the rref feature on a calculator, we find $x = -2.78, y = 0.416, z = 2.41.$



19. By substituting $x = 1$ into $f(x) = ax^2 + bx + c,$ we obtain $3 = a + b + c.$

By substituting $x = -2$ into $f(x) = ax^2 + bx + c,$ we obtain $15 = 4a - 2b + c.$

By substituting $x = 3$ into $f(x) = ax^2 + bx + c,$ we obtain $5 = 9a + 3b + c.$

(1)	$a + b + c = 3$	
(2)	$4a - 2b + c = 15$	
(3)	$9a + 3b + c = 5$	
	$3a - 3b = 12$	subtract (1) from (2)
(4)	$a - b = 4$	divide by 3
	$5a + 5b = -10$	subtract (2) from (3)
(5)	$a + b = -2$	divide by 5
	$2a = 2$	add (4) and (5)
(6)	$a = 1$	solve for *a*
	$1 + b = -2$	substitute (6) into 5
(7)	$b = -3$	solve for *b*
	$1 - 3 + c = 3$	substitute (6) and (7) into (1)
(8)	$c = 5$	solve for *c*

The function is $f(x) = x^2 - 3x + 5.$

20. By substituting $x = 1$ into $f(x) = ax^2 + bx + c$, we obtain $-3 = a + b + c$.

By substituting $x = -3$ into $f(x) = ax^2 + bx + c$, we obtain $-35 = 9a - 3b + c$.

By substituting $x = 3$ into $f(x) = ax^2 + bx + c$, we obtain $-11 = 9a + 3b + c$.

(1)	$a + b + c = -3$	
(2)	$9a - 3b + c = -35$	
(3)	$9a + 3b + c = -11$	
	$6b = 24$	subtract (2) from (3)
(4)	$b = 4$	solve for b
	$8a - 4b = -32$	subtract (1) from (2)
(5)	$2a - b = -8$	divide by 4
	$2a = -4$	add (4) and (5)
(6)	$a = -2$	solve for a
	$-2 + 4 + c = -3$	substitute (4) and (6) into (1)
(7)	$c = -5$	solve for c

The function is $f(x) = -2x^2 + 4x - 5$.

21.

(1)	$P + M + I = 1150$	
(2)	$P - 4I = -100$	
(3)	$P - 6M = 50$	
(4)	$4P + 4M + 4I = 4600$	multiply (1) by 4
(2)	$P - 4I = -100$	add
(5)	$5P + 4M = 4500$	
(6)	$5P - 30M = 250$	multiply (3) by 5, then subtract from (5)
(7)	$34M = 4250$	
	$M = 125$	
(8)	$P - 6(125) = 50$	substitute 125 for M into (3)
	$P = 800$	
(9)	$800 + 125 + I = 1150$	substitute 125 for M and 800 for P into (1)
	$I = 225$	

The solution is $P = 800$ h, $M = 125$ h, $I = 225$ h.

22.

(1)	$r_1 + r_2 + r_3 = 14000$	
(2)	$r_1 + 2r_2 = 13000$	
(3)	$3r_1 + 3r_2 + 2r_3 = 36000$	
(4)	$2r_1 + 2r_2 + 2r_3 = 28000$	multiply (1) by 2
(3)	$3r_1 + 3r_2 + 2r_3 = 36000$	subtract
(5)	$-r_1 - r_2 = -8000$	
(2)	$r_1 + 2r_2 = 13000$	add
(6)	$r_2 = 5000$	
(7)	$r_1 + 2(5000) = 13000$	substitute 5000 for r_2 into (2)
	$r_1 = 3000$	
(8)	$3000 + 5000 + r_3 = 14000$	substitute 3000 for r_1, 5000 for r_2 into (1)
	$r_3 = 6000$	

The solution is $r_1 = 3000$ L/h, $r_2 = 5000$ L/h, $r_3 = 6000$ L/h.

23.

(1)	$0.707F_1 - 0.800F_2 = 0$	
(2)	$0.707F_1 + 0.600F_2 - F_3 = 10.0$	
(3)	$3.00F_2 - 3.00F_3 = 20.0$	
(4)	$-1.400F_2 + F_3 = -10.0$	subtract (1) − (2)
(5)	$-4.200F_2 + 3F_3 = -30.0$	multiply (4) by 3
(3)	$3.00F_2 - 3.00F_3 = 20.0$	add
(6)	$-1.200F_2 = -10.0$	
	$F_2 = 8.33$	
(7)	$3.00(8.33) - 3.00F_3 = 20.0$	substitute 8.33 for F_2 into (3)
	$-3.00F_3 = -5.00$	
	$F_3 = 1.67$	
(9)	$0.707F_1 - 0.800(8.33) = 0$	substitute 8.33 for F_2 into (1)
	$0.707F_1 = 6.67$	
	$F_1 = 9.43$	

The solution is $F_1 = 9.43$ N, $F_2 = 8.33$ N, $F_3 = 1.67$ N.

24.

(1)	$4.0I_1 \qquad - 3.0I_3 = 12$	
(2)	$6.0I_2 + 4.0I_3 = 12$	
(3)	$1.0I_1 - 2.0I_2 + 3.0I_3 = 0$	
(4)	$4.0I_1 - 8.0I_2 + 12.0I_3 = 0$	multiply (3) by 4
(1)	$4.0I_1 \qquad - 3.0I_3 = 12$	subtract
(5)	$-8.0I_2 + 15.0I_3 = -12$	
(6)	$8.0I_2 + 5.33I_3 = 16$	multiply (2) by 4/3, add to (5)
(7)	$20.33I_3 = 4$	
	$I_3 = 0.1967$	
(8)	$4.0I_1 - 3.0(0.1967) = 12$	substitute 0.1967 for I_3 into (1)
	$4.0I_1 = 12.59016$	
	$I_1 = 3.1475$	
(9)	$6.0I_2 + 4.0(0.1967) = 12$	substitute 0.1967 for I_3 into (2)
	$6.0I_2 = 11.2131$	
	$I_2 = 1.8689$	

The solution is $I_1 = 3.15$ A, $I_2 = 1.87$ A, $I_3 = 0.197$ A.

25.

(1*a*)	$A + (A + B) = 90$	from left triangle
(1*b*)	$2A + B = 90$	
(2*a*)	$A + 2B = C$	from angle C
(2*b*)	$A + 2B - C = 0$	
(3)	$A + B + C = 180$	from large (outside) triangle
(4)	$2A + 3B = 180$	add (2b) and (3)
(1*b*)	$2A + B = 90$	subtract
(5)	$2B = 90$	
	$B = 45.0$	
(6)	$2A + 45 = 90$	substitute 45 for B into (1b)
	$2A = 45$	
	$A = 22.5$	
(7)	$22.5 + 45 + C = 180$	substitute 22.5 for A, 45 for B into (3)
	$C = 112.5$	

The solution is $A = 22.5^\circ$, $B = 45.0^\circ$, $C = 112.5^\circ$.

26. $C = av^2 + bv + c$

(1)	$100a + 10b + c = 28$	
(2)	$2500a + 50b + c = 22$	
(3)	$6400a + 80b + c = 24$	
(4)	$-2400a - 40b = 6$	subtract $(1) - (2)$
(5)	$-3900a - 30b = -2$	subtract $(2) - (3)$
(6)	$-7200a - 120b = 18$	multiply (4) by 3
(7)	$15600a + 120b = 8$	multiply (5) by -4, add to (6)
(8)	$8400a = 26$	
	$a = 0.0030952$	
(9)	$-2400(0.0030952) - 40b = 6$	substitute 0.0030952 for a into (4)
	$b = \frac{13.42857}{-40}$	
	$b = -0.33571$	
(10)	$100(0.0030952) + 10(-0.33571) + c = 28$	substitute a and b into (1)
	$c = 31.048$	

The solution is $a = 0.00310,\ b = -0.336,\ c = 31.0.$

$C = 0.00310v^2 - 0.336v + 31.0$

27. Let A, B, C denote the initial vote counts for the corresponding candidates.

$A = B + 200$

$A = C + 500$

$B + 0.01A = (C + 0.02A) + 100$

We convert these equations into standard form

(1)	$A - B = 200$	
(2)	$A - C = 500$	
(3)	$-0.01A + B - C = 100$	
(4)	$0.99A - C = 300$	add (1) and (3)
	$0.01A = 200$	subtract (4) from (2)
(5)	$A = 20000$	solve for A
	$20000 - B = 200$	substitute (5) into (1)
(6)	$B = 19800$	solve for B
	$20000 - C = 500$	substitute (5) into (2)
(7)	$C = 19500$	solve for C

Finally, we adjust for the final vote counts, giving B 1.0% of A's initial count, or 200 more votes and giving C 2.0% of A's initial count, or 400 more votes. Also, A's count goes down by 600 votes. Thus, A ends up with 19400 votes, B with 20000 votes, and C with 19900 votes.

28. Let x, y, z represent amounts invested at 5.00%, 6.00%, and 6.50%, respectively.

(1)	$x + y + z = 22500$	
(2)	$0.0500x + 0.0600y + 0.0650z = 1308$	
(3)	$0.0500x - 0.0600y = 0$	
(4)	$0.0500x + 0.0500y + 0.0500z = 1125$	multiply (1) by 0.0500
(5)	$0.1200y + 0.0650z = 1308$	subtract (3) from (2)
(6)	$0.0100y + 0.0150z = 183$	subtract (4) from (2)
	$-0.1150z = -888$	multiply (6) by 12, then subtract from (5)
(7)	$z = 7721.74$	solve for z
	$0.0100y + 0.0150(7721.74) = 183$	substitute (7) into (6)
(8)	$y = 6717.39$	solve for y
(8)	$x + 6717.39 + 7721.74 = 22500$	substitute (7) and (8) into (1)
	$x = 8060.87$	solve for x

Thus, \$8060.87 was invested at 5.00%, \$6717.39 was invested at 6.00% and \$7721.74 was invested at 6.50%.

29. Let x, y, z represent numbers of MA, MS, and PhD degrees, respectively.

(1)	$x + y + z = 420$	
(2)	$x - y - z = 100$	
(3)	$y - 3z = 0$	
(4)	$2x = 520$	add (1) and (2)
(5)	$x = 260$	solve for x
	$2y + 2z = 320$	subtract (2) from (1)
(6)	$y + z = 160$	divide by 2
	$4z = 160$	subtract (3) from 6
(7)	$z = 40$	solve for z
(8)	$y + 40 = 160$	substitute (7) into (6)
	$y = 120$	solve for y

Thus, 260 MA degrees, 120 MS degrees, and 40 PhD degrees were awarded.

30.

(1)	$A+B+C=8.0$	combined capacity 8.0 TB
(2*a*)	$A \quad +C=B+0.2$	A and C total 0.2 TB more than B
(2*b*)	$A-B+C=0.2$	
(3*a*)	$2(A+B) \; =3C$	twice (A+B) is 3×C
(3*b*)	$2A+2B-3C=0$	
(4)	$2B=7.8$	subtract (1) $-$(2b)
	$B=3.9$	
(5)	$2A+2B+2C=16.0$	multiply (1) by 2
(3*b*)	$2A+2B-3C=0$	subtract
(6)	$5C=16.0$	
	$C=3.2$	
(7)	$A+3.9+3.2=8.0$	substitute B and C into (1)
	$A=0.9$	

The solution is $A=0.900$ TB, $B=3.90$ TB, $C=3.20$ TB.

31. Let x = mass of fertilizer 1 (20-30-50)

Let y = mass of fertilizer 2 (10-20-70)

Let z = mass of fertilizer 3 (0-30-70)

(1)	$0.20x+0.10y \qquad = 0.12(200) = 24$	potassium
(2)	$0.30x+0.20y+0.30z = 0.25(200) = 50$	nitrogen
(3)	$0.50x+0.70y+0.70z = 0.63(200) = 126$	phosphorus
(4)	$0.70x+0.4667y+0.70z = 116.667$	multiply (2) by 7/3
(3)	$0.50x+ \quad 0.70y+0.70z = 126$	subtract
(5)	$0.20x-0.2333y=-9.3333$	
(1)	$0.20x+ \quad 0.10y = 24$	subtract
(6)	$-0.3333y=-33.33333$	
	$y=100$	
(7)	$0.20x+0.10(100) = 24$	substitute y into (2)
	$x = 70$	
(8)	$0.30(70)+0.20(100)+0.30z = 50$	substitute x and y into (5)
	$0.30z = 9$	
	$z = 30$	

The solution is x = 70.0 kg, $y=100.0$ kg, z = 30.0 kg.

32. There are three intersections, with three unknowns.

(1*a*)	$x+y+300=800$	Intersection 1 (bottom left)
(1*b*)	$x+y\qquad=500$	
(2*a*)	$y+400=z+200$	Intersection 2 (bottom right)
(2*b*)	$y-\quad z=-200$	
(3)	$x\quad+\quad z=700$	Intersection 3 (top)
(4)	$y-z=-200$	subtract (1b) $-$(3)
(2*b*)	$y-z=-200$	
	$0=0$	

The equations (4) and (2b) are the same equation, so the system is dependent. Two of the three equations contain the same information, there is no unique solution.

33.

(1)	$x-2y-\ 3z=2$	
(2)	$x-4y-13z=14$	
(3)	$-3x+5y+\ 4z=0$	
(4)	$2y+10z=-12$	subtract $(1)-(2)$
(5)	$y+5z=-6$	divide (4) by 2
(6)	$3x-6y-9z=6$	multiply (1) by 3
(3)	$-3x+5y+4z=0$	add
(7)	$-y-5z=6$	
(8)	$y+5z=-6$	divide (7) by -1
(5)	$y+5z=-6$	subtract
	$0=0$	

The system is dependent, there are an infinite number of solutions.
One possible solution, if we let $z=0$, $y=-6$ from Eq. (8) or Eq. (5).
Substituting into Eq. (1),

$$x-2(-6)-0=2$$
$$x=-10$$

A possible solution is $x=-10,\ y=-6,\ z=0$

34.

(1)	$x-2y-3z=2$	
(2)	$x-4y-13z=14$	
(3)	$-3x+5y+4z=2$	
(4)	$2y+10z=-12$	subtract $(1)-(2)$
(5)	$y+5z=-6$	divide (4) by 2
(6)	$3x-6y-9z=6$	multiply (1) by 3
(3)	$-3x+5y+4z=2$	add
(7)	$-y-5z=4$	
(8)	$y+5z=-4$	divide (7) by -1
(5)	$y+5z=-6$	subtract
	$0\neq 2$	

The system is inconsistent, there are no solutions.

35.

(1)	$3x+3y-2z=2$	
(2)	$2x-y+z=1$	
(3)	$x-5y+4z=-3$	
(4)	$3x-15y+12z=-9$	multiply (3) by 3
(1)	$3x+3y-2z=2$	subtract
(5)	$-18y+14z=-11$	
(6)	$2x-10y+8z=-6$	multiply (3) by 2
(2)	$2x-y+z=1$	subtract
(7)	$-9y+7z=-7$	
(8)	$18y-14z=14$	multiply (7) by -2
(5)	$-18y+14z=-11$	add
	$0\neq 3$	

The system is inconsistent, there are no solutions.

36.

(1)	$3x+y-z=-3$	
(2)	$x+y-3z=-5$	
(3)	$-5x-2y+3z=-7$	
(4)	$3x+3y-9z=-15$	multiply (2) by 3
(1)	$3x+y-z=-3$	subtract
(5)	$2y-8z=-12$	
(6)	$5x+5y-15z=-25$	multiply (2) by 5
(3)	$-5x-2y+3z=-7$	add
(7)	$3y-12z=-32$	
(8)	$3y-12z=-18$	multiply (5) by $3/2$, subtract from (7)
	$0\neq -14$	

The system is inconsistent, there are no solutions.

5.6 Solving Systems of Three Linear Equations in Three Unknowns by Determinants

Note to students: In all questions where solving a system of equations is required, substitution of the solutions into *all* equations serves as a check.

1. $\begin{vmatrix} -2 & 3 & -1 \\ 1 & 5 & 4 \\ 2 & -1 & 5 \end{vmatrix}\begin{matrix} -2 & 3 \\ 1 & 5 \\ 2 & -1 \end{matrix}$

$= -2(5)(5)+3(4)(2)+(-1)(1)(-1)-2(5)(-1)-(-1)(4)(-2)-5(1)(3)$

$= -50+24+1+10-8-15$

$= -38$

This is the same determinant as that of Example 1 except the sign has changed.

Interchanging a single pair of rows alters the determinant in sign only.

2. $3x+2y-5z=-3$

$2x-3y-\ \ z=11$

$5x-2y+7z=11$

$$x=\frac{\begin{vmatrix} -3 & 2 & -5 \\ 11 & -3 & -1 \\ 11 & -2 & 7 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & -5 \\ 2 & -3 & -1 \\ 5 & -2 & 7 \end{vmatrix}}=\frac{63+(-22)+110-165-(-6)-154}{-63+(-10)+20-75-6-28}=\frac{-162}{-162}=1$$

$$y=\frac{\begin{vmatrix} 3 & -3 & -5 \\ 2 & 11 & -1 \\ 5 & 11 & 7 \end{vmatrix}}{-162}=\frac{231+15+(-110)-(-275)-(-33)-(-42)}{-162}=\frac{486}{-162}=-3$$

$$z=\frac{\begin{vmatrix} 3 & 2 & -3 \\ 2 & -3 & 11 \\ 5 & -2 & 11 \end{vmatrix}}{-36}=\frac{-99+110+12-45-(-66)-44}{-162}=\frac{0}{-162}=0$$

Check

$3x+2y-5z=-3$	$2x-3y-\ \ z=11$	$5x-2y+7z=11$
$3(1)+2(-3)-5(0)=-3$	$2(1)-3(-3)-0=11$	$5(1)-2(-3)+7(0)=11$
$-3=-3$	$11=11$	$11=11$

3. $\begin{vmatrix} 5 & 4 & -1 \\ -2 & -6 & 8 \\ 7 & 1 & 1 \end{vmatrix}\begin{matrix} 5 & 4 \\ -2 & -6 \\ 7 & 1 \end{matrix} = -30 + 224 + 2 - 42 - 40 - (-8) = 122$

4. $\begin{vmatrix} -7 & 0 & 0 \\ 2 & 4 & 5 \\ 1 & 4 & 2 \end{vmatrix}\begin{matrix} -7 & 0 \\ 2 & 4 \\ 1 & 4 \end{matrix} = -56 + 0 + 0 - 0 - (-140) - 0 = 84$

5. $\begin{vmatrix} 8 & 9 & -6 \\ -3 & 7 & 2 \\ 4 & -2 & 5 \end{vmatrix}\begin{matrix} 8 & 9 \\ -3 & 7 \\ 4 & -2 \end{matrix} = 280 + 72 + (-36) - (-168) - (-32) - (-135) = 651$

6. $\begin{vmatrix} -2 & 4 & -1 \\ 5 & -1 & 4 \\ 4 & -8 & 2 \end{vmatrix}\begin{matrix} -2 & 4 \\ 5 & -1 \\ 4 & -8 \end{matrix} = 4 + 64 + 40 - 4 - 64 - 40 = 0$

7. $\begin{vmatrix} -8 & -4 & -6 \\ 5 & -1 & 0 \\ 2 & 10 & -1 \end{vmatrix}\begin{matrix} -8 & -4 \\ 5 & -1 \\ 2 & 10 \end{matrix} = -8 + 0 + (-300) - 12 - 0 - 20 = -340$

8. $\begin{vmatrix} 10 & 2 & -7 \\ -2 & -3 & 6 \\ 6 & 5 & -2 \end{vmatrix}\begin{matrix} 10 & 2 \\ -2 & -3 \\ 6 & 5 \end{matrix} = 60 + 72 + 70 - 126 - 300 - 8 = -232$

9. $\begin{vmatrix} 4 & -3 & -11 \\ -9 & 2 & -2 \\ 0 & 1 & -5 \end{vmatrix}\begin{matrix} 4 & -3 \\ -9 & 2 \\ 0 & 1 \end{matrix} = -40 + 0 + 99 - 0 - (-8) - (-135) = 202$

10. $\begin{vmatrix} 9 & -2 & 0 \\ -1 & 3 & -6 \\ -4 & -6 & -2 \end{vmatrix}\begin{matrix} 9 & -2 \\ -1 & 3 \\ -4 & -6 \end{matrix} = -54 + (-48) + 0 - 0 - 324 - (-4) = -422$

11. $\begin{vmatrix} 25 & 18 & -50 \\ -15 & 24 & -12 \\ -20 & 55 & -22 \end{vmatrix}\begin{matrix} 25 & 18 \\ -15 & 24 \\ -20 & 55 \end{matrix}$

$= 25(24)(-22) + 18(-12)(-20) + (-50)(-15)(55) - (-20)(24)(-50) - 55(-12)(25) - (-22)(-15)(18)$

$= -13200 + 4320 + 41250 - 24000 - (-16500) - 5940$

$= 18930$

12. $\begin{vmatrix} 20 & 0 & -15 \\ -4 & 30 & 1 \\ 6 & -1 & 40 \end{vmatrix}\begin{matrix} 20 & 0 \\ -4 & 30 \\ 6 & -1 \end{matrix} = 24\,000 + 0 + (-60) - (-2700) - (-20) - 0 = 26\ 660$

13. $\begin{vmatrix} 0.1 & -0.2 & 0 \\ -0.5 & 1 & 0.4 \\ -2 & 0.8 & 2 \end{vmatrix} \begin{matrix} 0.1 & -0.2 \\ -0.5 & 1 \\ -2 & 0.8 \end{matrix} = 0.2 + 0.16 + 0 - 0 - 0.032 - 0.2 = 0.128$

14. $\begin{vmatrix} 0.25 & -0.54 & -0.42 \\ 1.20 & 0.35 & 0.28 \\ -0.50 & 0.12 & -0.44 \end{vmatrix} \begin{matrix} 0.25 & -0.54 \\ 1.20 & 0.35 \\ -0.50 & 0.12 \end{matrix}$

$= 0.25(0.35)(-0.44) + (-0.54)(0.28)(-0.50) + (-0.42)(1.20)(0.12)$
$\quad - (-0.50)(0.35)(-0.42) - (0.12)(0.28)(0.25) - (-0.44)(1.20)(-0.54)$
$= -0.0385 + 0.0756 + (-0.06048) - 0.0735 - 0.0084 - 0.28512$
$= -0.3904$

15. $2x + 3y + z = 4$
$3x - z = -3$
$x - 2y + 2z = -5$

$$x = \frac{\begin{vmatrix} 4 & 3 & 1 \\ -3 & 0 & -1 \\ -5 & -2 & 2 \end{vmatrix} \begin{matrix} 4 & 3 \\ -3 & 0 \\ -5 & -2 \end{matrix}}{\begin{vmatrix} 2 & 3 & 1 \\ 3 & 0 & -1 \\ 1 & -2 & 2 \end{vmatrix} \begin{matrix} 2 & 3 \\ 3 & 0 \\ 1 & -2 \end{matrix}} = \frac{0 + 15 + 6 - 0 - 8 - (-18)}{0 + (-3) + (-6) - 0 - 4 - 18} = \frac{31}{-31} = -1$$

$$y = \frac{\begin{vmatrix} 2 & 4 & 1 \\ 3 & -3 & -1 \\ 1 & -5 & 2 \end{vmatrix} \begin{matrix} 2 & 4 \\ 3 & -3 \\ 1 & -5 \end{matrix}}{-31} = \frac{-12 + (-4) + (-15) - (-3) - 10 - 24}{-31} = \frac{-62}{-31} = 2$$

$$z = \frac{\begin{vmatrix} 2 & 3 & 4 \\ 3 & 0 & -3 \\ 1 & -2 & -5 \end{vmatrix} \begin{matrix} 2 & 3 \\ 3 & 0 \\ 1 & -2 \end{matrix}}{-31} = \frac{0 + (-9) + (-24) - 0 - 12 - (-45)}{-31} = \frac{0}{-31} = 0$$

Solution: $x = -1,\ y = 2,\ z = 0.$

16. $4x + y + z = 2$

$2x - y - z = 4$

$3y + z = 2$

$$x = \frac{\left|\begin{array}{rrr|rr} 2 & 1 & 1 & 2 & 1 \\ 4 & -1 & -1 & 4 & -1 \\ 2 & 3 & 1 & 2 & 3 \end{array}\right.}{\left|\begin{array}{rrr|rr} 4 & 1 & 1 & 4 & 1 \\ 2 & -1 & -1 & 2 & -1 \\ 0 & 3 & 1 & 0 & 3 \end{array}\right.} = \frac{-2+(-2)+12-(-2)-(-6)-4}{-4+0+6-0-(-12)-2} = \frac{12}{12} = 1$$

$$y = \frac{\left|\begin{array}{rrr|rr} 4 & 2 & 1 & 4 & 2 \\ 2 & 4 & -1 & 2 & 4 \\ 0 & 2 & 1 & 0 & 2 \end{array}\right.}{12} = \frac{16+0+4-0-(-8)-4}{12} = \frac{24}{12} = 2$$

$$z = \frac{\left|\begin{array}{rrr|rr} 4 & 1 & 2 & 4 & 1 \\ 2 & -1 & 4 & 2 & -1 \\ 0 & 3 & 2 & 0 & 3 \end{array}\right.}{12} = \frac{-8+0+12-0-48-4}{12} = \frac{-48}{12} = -4$$

Solution: $x = 1,\ y = 2,\ z = -4$.

17. $x + y + z = 2$

$x \quad - z = 1$

$x + y \quad = 1$

$$x = \frac{\left|\begin{array}{rrr|rr} 2 & 1 & 1 & 2 & 1 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{array}\right.}{\left|\begin{array}{rrr|rr} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{array}\right.} = \frac{0+(-1)+1-0-(-2)-0}{0+(-1)+1-0-(-1)-0} = \frac{2}{1} = 2$$

$$y = \frac{\left|\begin{array}{rrr|rr} 1 & 2 & 1 & 1 & 2 \\ 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{array}\right.}{1} = \frac{0+(-2)+1-1-(-1)-0}{1} = \frac{-1}{1} = -1$$

$$z = \frac{\left|\begin{array}{rrr|rr} 1 & 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right.}{1} = \frac{0+1+2-0-1-1}{1} = \frac{1}{1} = 1$$

Solution: $x = 2,\ y = -1,\ z = 1$.

18.
$$\begin{aligned} x+y-\ z&=-3\\ x\qquad +\ z&=\ 2\\ 2x-y+2z&=\ 3 \end{aligned}$$

$$x=\frac{\begin{vmatrix}-3&1&-1\\2&0&1\\3&-1&2\end{vmatrix}\begin{matrix}-3&1\\2&0\\3&-1\end{matrix}}{\begin{vmatrix}1&1&-1\\1&0&1\\2&-1&2\end{vmatrix}\begin{matrix}1&1\\1&0\\2&-1\end{matrix}}=\frac{0+3+2-0-3-4}{0+2+1-0-(-1)-2}=\frac{-2}{2}=-1$$

$$y=\frac{\begin{vmatrix}1&-3&-1\\1&2&1\\2&3&2\end{vmatrix}\begin{matrix}1&-3\\1&2\\2&3\end{matrix}}{2}=\frac{4+(-6)+(-3)-(-4)-3-(-6)}{2}=\frac{2}{2}=1$$

$$z=\frac{\begin{vmatrix}1&1&-3\\1&0&2\\2&-1&3\end{vmatrix}\begin{matrix}1&1\\1&0\\2&-1\end{matrix}}{2}=\frac{0+4+3-0-(-2)-3}{2}=\frac{6}{2}=3$$

Solution: $x=-1,\ y=1,\ z=3$

19.
$$\begin{aligned} 2x+3y+\ z&=\ 2\\ -x+2y+3z&=-1\\ -3x-3y+\ z&=\ 0 \end{aligned}$$

$$x=\frac{\begin{vmatrix}2&3&1\\-1&2&3\\0&-3&1\end{vmatrix}\begin{matrix}2&3\\-1&2\\0&-3\end{matrix}}{\begin{vmatrix}2&3&1\\-1&2&3\\-3&-3&1\end{vmatrix}\begin{matrix}2&3\\-1&2\\-3&-3\end{matrix}}=\frac{4+0+3-0-(-18)-(-3)}{4+(-27)+3-(-6)-(-18)-(-3)}=\frac{28}{7}=4$$

$$y=\frac{\begin{vmatrix}2&2&1\\-1&-1&3\\-3&0&1\end{vmatrix}\begin{matrix}2&2\\-1&-1\\-3&0\end{matrix}}{7}=\frac{-2+(-18)+0-3-0-(-2)}{7}=\frac{-21}{7}=-3$$

$$z=\frac{\begin{vmatrix}2&3&2\\-1&2&-1\\-3&-3&0\end{vmatrix}\begin{matrix}2&3\\-1&2\\-3&-3\end{matrix}}{7}=\frac{0+9+6-(-12)-6-0}{7}=\frac{21}{7}=3$$

Solution: $x=4,\ y=-3,\ z=3$

20. $2x + y - z = 4$
$4x - 3y - 2z = -2$
$8x - 2y - 3z = 3$

$$x = \frac{\begin{vmatrix} 4 & 1 & -1 \\ -2 & -3 & -2 \\ 3 & -2 & -3 \end{vmatrix} \begin{matrix} 4 & 1 \\ -2 & -3 \\ 3 & -2 \end{matrix}}{\begin{vmatrix} 2 & 1 & -1 \\ 4 & -3 & -2 \\ 8 & -2 & -3 \end{vmatrix} \begin{matrix} 2 & 1 \\ 4 & -3 \\ 8 & -2 \end{matrix}} = \frac{36 + (-6) + (-4) - 9 - 16 - 6}{18 + (-16) + 8 - 24 - 8 - (-12)} = \frac{-5}{-10} = \frac{1}{2}$$

$$y = \frac{\begin{vmatrix} 2 & 4 & -1 \\ 4 & -2 & -2 \\ 8 & 3 & -3 \end{vmatrix} \begin{matrix} 2 & 4 \\ 4 & -2 \\ 8 & 3 \end{matrix}}{-10} = \frac{12 + (-64) + (-12) - 16 - (-12) - (-48)}{-10} = \frac{-20}{-10} = 2$$

$$z = \frac{\begin{vmatrix} 2 & 1 & 4 \\ 4 & -3 & -2 \\ 8 & -2 & 3 \end{vmatrix} \begin{matrix} 2 & 1 \\ 4 & -3 \\ 8 & -2 \end{matrix}}{-10} = \frac{-18 + (-16) + (-32) - (-96) - 8 - 12}{-10} = \frac{10}{-10} = -1$$

Solution: $x = \frac{1}{2}$, $y = 2$, $z = -1$

21. $5l + 6w - 3h = 6$
$4l - 7w - 2h = -3$
$3l + w - 7h = -1$

$$l = \frac{\begin{vmatrix} 6 & 6 & -3 \\ -3 & -7 & -2 \\ 1 & 1 & -7 \end{vmatrix} \begin{matrix} 6 & 6 \\ -3 & -7 \\ 1 & 1 \end{matrix}}{\begin{vmatrix} 5 & 6 & -3 \\ 4 & -7 & -2 \\ 3 & 1 & -7 \end{vmatrix} \begin{matrix} 5 & 6 \\ 4 & -7 \\ 3 & 1 \end{matrix}} = \frac{294 + (-12) + 9 - 21 - (-12) - 126}{245 + (-36) + (-12) - 63 - (-10) - (-168)} = \frac{156}{312} = \frac{1}{2}$$

$$w = \frac{\begin{vmatrix} 5 & 6 & -3 \\ 4 & -3 & -2 \\ 3 & 1 & -7 \end{vmatrix} \begin{matrix} 5 & 6 \\ 4 & -3 \\ 3 & 1 \end{matrix}}{312} = \frac{105 + (-36) + (-12) - 27 - (-10) - (-168)}{312} = \frac{208}{312} = \frac{2}{3}$$

$$h = \frac{\begin{vmatrix} 5 & 6 & 6 \\ 4 & -7 & -3 \\ 3 & 1 & 1 \end{vmatrix} \begin{matrix} 5 & 6 \\ 4 & -7 \\ 3 & 1 \end{matrix}}{312} = \frac{-35 + (-54) + 24 - (-126) - (-15) - 24}{312} = \frac{52}{312} = \frac{1}{6}$$

Solution: $l = \frac{1}{2}$, $w = \frac{2}{3}$, $h = \frac{1}{6}$

22. $3r + s - t = 2$
$r - 2s + t = 0$
$4r - s + t = 3$

$$r = \frac{\begin{vmatrix} 2 & 1 & -1 \\ 0 & -2 & 1 \\ 3 & -1 & 1 \end{vmatrix}\begin{matrix} 2 & 1 \\ 0 & -2 \\ 3 & -1 \end{matrix}}{\begin{vmatrix} 3 & 1 & -1 \\ 1 & -2 & 1 \\ 4 & -1 & 1 \end{vmatrix}\begin{matrix} 3 & 1 \\ 1 & -2 \\ 4 & -1 \end{matrix}} = \frac{-4+3+0-6-(-2)-0}{-6+4+1-8-(-3)-1} = \frac{-5}{-7} = \frac{5}{7}$$

$$s = \frac{\begin{vmatrix} 3 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}\begin{matrix} 3 & 2 \\ 1 & 0 \\ 4 & 3 \end{matrix}}{-7} = \frac{0+8+(-3)-0-9-2}{-7} = \frac{-6}{-7} = \frac{6}{7}$$

$$t = \frac{\begin{vmatrix} 3 & 1 & 2 \\ 1 & -2 & 0 \\ 4 & -1 & 3 \end{vmatrix}\begin{matrix} 3 & 1 \\ 1 & -2 \\ 4 & -1 \end{matrix}}{-7} = \frac{-18+0+(-2)-(-16)-0-3}{-7} = \frac{-7}{-7} = 1$$

Solution: $r = \frac{5}{7}$, $s = \frac{6}{7}$, $t = 1$.

23. $2x - 2y + 3z = 5$
$2x + y - 2z = -1$
$4x - y - 3z = 0$

$$x = \frac{\begin{vmatrix} 5 & -2 & 3 \\ -1 & 1 & -2 \\ 0 & -1 & -3 \end{vmatrix}\begin{matrix} 5 & -2 \\ -1 & 1 \\ 0 & -1 \end{matrix}}{\begin{vmatrix} 2 & -2 & 3 \\ 2 & 1 & -2 \\ 4 & -1 & -3 \end{vmatrix}\begin{matrix} 2 & -2 \\ 2 & 1 \\ 4 & -1 \end{matrix}} = \frac{-15+0+3-0-10-(-6)}{-6+16+(-6)-12-4-12} = \frac{-16}{-24} = \frac{2}{3}$$

$$y = \frac{\begin{vmatrix} 2 & 5 & 3 \\ 2 & -1 & -2 \\ 4 & 0 & -3 \end{vmatrix}\begin{matrix} 2 & 5 \\ 2 & -1 \\ 4 & 0 \end{matrix}}{-24} = \frac{6+(-40)+0-(-12)-0-(-30)}{-24} = \frac{8}{-24} = -\frac{1}{3}$$

$$z = \frac{\begin{vmatrix} 2 & -2 & 5 \\ 2 & 1 & -1 \\ 4 & -1 & 0 \end{vmatrix}\begin{matrix} 2 & -2 \\ 2 & 1 \\ 4 & -1 \end{matrix}}{-24} = \frac{0+8+(-10)-20-2-0}{-24} = \frac{-24}{-24} = 1$$

Solution: $x = \frac{2}{3}$, $y = -\frac{1}{3}$, $z = 1$.

24. ` $2u+2v+3w=0$
$3u+\ v+4w=21$
$-u-3v+7w=15$

$$u=\frac{\begin{vmatrix}0&2&3\\21&1&4\\15&-3&7\end{vmatrix}\begin{matrix}0&2\\21&1\\15&-3\end{matrix}}{\begin{vmatrix}2&2&3\\3&1&4\\-1&-3&7\end{vmatrix}\begin{matrix}2&2\\3&1\\-1&-3\end{matrix}}=\frac{0+120+(-189)-45-0-294}{14+(-8)+(-27)-(-3)-(-24)-42}=\frac{-408}{-36}=\frac{34}{3}$$

$$v=\frac{\begin{vmatrix}2&0&3\\3&21&4\\-1&15&7\end{vmatrix}\begin{matrix}2&0\\3&21\\-1&15\end{matrix}}{-36}=\frac{294+0+135-(-63)-120-0}{-36}=\frac{372}{-36}=-\frac{31}{3}$$

$$w=\frac{\begin{vmatrix}2&2&0\\3&1&21\\-1&-3&15\end{vmatrix}\begin{matrix}2&2\\3&1\\-1&-3\end{matrix}}{-36}=\frac{30+(-42)+0-0-(-126)-90}{-36}=\frac{24}{-36}=-\frac{2}{3}$$

Solution : $u=\frac{34}{3}, v=-\frac{31}{3}, w=-\frac{2}{3}$

25. $3x-7y+3z=6$
$3x+3y+6z=1$
$5x-5y+2z=5$

$$x=\frac{\begin{vmatrix}6&-7&3\\1&3&6\\5&-5&2\end{vmatrix}\begin{matrix}6&-7\\1&3\\5&-5\end{matrix}}{\begin{vmatrix}3&-7&3\\3&3&6\\5&-5&2\end{vmatrix}\begin{matrix}3&-7\\3&3\\5&-5\end{matrix}}=\frac{36+(-210)+(-15)-45-(-180)-(-14)}{18+(-210)+(-45)-45-(-90)-(-42)}=\frac{-40}{-150}=\frac{4}{15}$$

$$y=\frac{\begin{vmatrix}3&6&3\\3&1&6\\5&5&2\end{vmatrix}\begin{matrix}3&6\\3&1\\5&5\end{matrix}}{-150}=\frac{6+180+45-15-90-36}{-150}=\frac{90}{-150}=-\frac{3}{5}$$

$$z=\frac{\begin{vmatrix}3&-7&6\\3&3&1\\5&-5&5\end{vmatrix}\begin{matrix}3&-7\\3&3\\5&-5\end{matrix}}{-150}=\frac{45+(-35)+(-90)-90-(-15)-(-105)}{-150}=\frac{-50}{-150}=\frac{1}{3}$$

Solution: $x=\frac{4}{15},\ y=-\frac{3}{5},\ z=\frac{1}{3}$

26.

$$\begin{aligned} 18x+24y+4z&=46\\ 63x+6y-15z&=-75\\ -90x+30y-20z&=-55 \end{aligned}$$

$$x=\frac{\left|\begin{array}{rrr|rr} 46 & 24 & 4 & 46 & 24\\ -75 & 6 & -15 & -75 & 6\\ -55 & 30 & -20 & -55 & 30 \end{array}\right.}{\left|\begin{array}{rrr|rr} 18 & 24 & 4 & 18 & 24\\ 63 & 6 & -15 & 63 & 6\\ -90 & 30 & -20 & -90 & 30 \end{array}\right.}$$

$$x=\frac{-5520+19800+(-9000)-(-1320)-(-20700)-36000}{-2160+32400+7560-(-2160)-(-8100)-(-30240)}$$

$$x=\frac{-8700}{78300}=-\frac{1}{9}$$

$$y=\frac{\left|\begin{array}{rrr|rr} 18 & 46 & 4 & 18 & 46\\ 63 & -75 & -15 & 63 & -75\\ -90 & -55 & -20 & -90 & -55 \end{array}\right.}{78300}$$

$$y=\frac{27000+62100+(-13860)-27000-14850-(-57960)}{78300}$$

$$y=\frac{91350}{78300}=\frac{7}{6}$$

$$z=\frac{\left|\begin{array}{rrr|rr} 18 & 24 & 46 & 18 & 24\\ 63 & 6 & -75 & 63 & 6\\ -90 & 30 & -55 & -90 & 30 \end{array}\right.}{78300}$$

$$z=\frac{-5940+162000+86940-(-24840)-(-40500)-(-83160)}{78300}$$

$$z=\frac{391500}{78300}=5$$

Solution: $x=-\frac{1}{9}$, $y=\frac{7}{6}$, $z=5$.

27.

$$\begin{aligned} p+2q+2r&=0\\ 2p+6q-3r&=-1\\ 4p-3q+6r&=-8 \end{aligned}$$

$$p=\frac{\left|\begin{array}{rrr|rr} 0 & 2 & 2 & 0 & 2\\ -1 & 6 & -3 & -1 & 6\\ -8 & -3 & 6 & -8 & -3 \end{array}\right.}{\left|\begin{array}{rrr|rr} 1 & 2 & 2 & 1 & 2\\ 2 & 6 & -3 & 2 & 6\\ 4 & -3 & 6 & 4 & -3 \end{array}\right.}=\frac{0+48+6-(-96)-0-(-12)}{36+(-24)+(-12)-48-9-24}=\frac{162}{-81}=-2$$

$$q=\frac{\begin{vmatrix}1&0&2\\2&-1&-3\\4&-8&6\end{vmatrix}\begin{matrix}1&0\\2&-1\\4&-8\end{matrix}}{-81}=\frac{-6+0+(-32)-(-8)-24-0}{-81}=\frac{-54}{-81}=\frac{2}{3}$$

$$r=\frac{\begin{vmatrix}1&2&0\\2&6&-1\\4&-3&-8\end{vmatrix}\begin{matrix}1&2\\2&6\\4&-3\end{matrix}}{-81}=\frac{-48+(-8)+0-0-3-(-32)}{-81}=\frac{-27}{-81}=\frac{1}{3}$$

Solution: $p=-2,\ q=\frac{2}{3},\ r=\frac{1}{3}$

28.
$$\begin{aligned}9x+12y+2z&=23\\21x+2y-5z&=-25\\-18x+6y-4z&=-11\end{aligned}$$

$$x=\frac{\begin{vmatrix}23&12&2\\-25&2&-5\\-11&6&-4\end{vmatrix}\begin{matrix}23&12\\-25&2\\-11&6\end{matrix}}{\begin{vmatrix}9&12&2\\21&2&-5\\-18&6&-4\end{vmatrix}\begin{matrix}9&12\\21&2\\-18&6\end{matrix}}$$

$$x=\frac{-184+660+(-300)-(-44)-(-690)-1200}{-72+1080+252-(-72)-(-270)-(-1008)}=\frac{-290}{2610}=-\frac{1}{9}$$

$$y=\frac{\begin{vmatrix}9&23&2\\21&-25&-5\\-18&-11&-4\end{vmatrix}\begin{matrix}9&23\\21&-25\\-18&-11\end{matrix}}{2610}$$

$$y=\frac{900+2070+(-462)-900-495-(-1932)}{2610}=\frac{3045}{2610}=\frac{7}{6}$$

$$z=\frac{\begin{vmatrix}9&12&23\\21&2&-25\\-18&6&-11\end{vmatrix}\begin{matrix}9&12\\21&2\\-18&6\end{matrix}}{2610}$$

$$z=\frac{-198+5400+2898-(-828)-(-1350)-(-2772)}{2610}=\frac{13050}{2610}=5$$

Solution: $x=-\frac{1}{9},\ y=\frac{7}{6},\ z=5.$

29.
$$\begin{vmatrix}-2&1&1\\2&3&1\\6&5&1\end{vmatrix}\begin{matrix}-2&1\\2&3\\6&5\end{matrix}=-6+6+10-18-(-10)-2=0$$

and so the points (–2,1), (2,3), and (6,5) are collinear.

30. $$\begin{vmatrix} -8 & 20 & 1 \\ -3 & 6 & 1 \\ 1 & -8 & 1 \end{vmatrix}\begin{matrix} -8 & 20 \\ -3 & 6 \\ 1 & -8 \end{matrix} = -48+20+24-6-64-(-60) = -14 \neq 0$$

and so the points $(-8, 20)$, $(-3, 6)$, and $(1, -8)$ are not collinear.

31. $$\begin{vmatrix} 3 & 6 & 5 \\ 2 & 4 & 1 \\ 7 & 9 & 8 \end{vmatrix} = 96+42+90-140-27-96 = -35$$

The value changes from 35 to -35.
Interchanging one row changes the sign of the determinant.

32. $$\begin{vmatrix} 2 & 4 & 1 \\ 2 & 4 & 1 \\ 7 & 9 & 8 \end{vmatrix} = 64+28+18-28-18-64 = 0$$

The determinant is 0 when two rows are the same.

33. $$\begin{vmatrix} 2 & 4 & 1 \\ 5 & 10 & 6 \\ 7 & 9 & 8 \end{vmatrix} = 160+168+45-70-108-160 = 35$$

Adding a multiple of one row to another does not
change the value of the determinant.

34. $$\begin{vmatrix} 4 & 8 & 2 \\ 3 & 6 & 5 \\ 7 & 9 & 8 \end{vmatrix} = 192+280+54-84-180-192 = 70$$

The value changes from 35 to 70.
If a row is multiplied by a factor of 2,
the determinant is multiplied by the same factor 2.

35. $$\begin{aligned} A \qquad\quad - 0.60F &= 80 \\ B - 0.80F &= 0 \\ 6.0A \qquad - \quad 10F &= 0 \end{aligned}$$

$$A = \frac{\begin{vmatrix} 80 & 0 & -0.60 \\ 0 & 1 & -0.80 \\ 0 & 0 & -10 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -0.60 \\ 0 & 1 & -0.80 \\ 6.0 & 0 & -10 \end{vmatrix}} = \frac{-800+0+0-0-0-0}{-10+0+0-(-3.6)-0-0} = \frac{-800}{-6.4} = 125$$

$$B = \frac{\begin{vmatrix} 1 & 80 & -0.60 \\ 0 & 0 & -0.80 \\ 6.0 & 0 & -10 \end{vmatrix}}{-6.4} = \frac{0+(-384)+0-0-0-0}{-6.4} = 60$$

$$F = \frac{\begin{vmatrix} 1 & 0 & 80 \\ 0 & 1 & 0 \\ 6.0 & 0 & 0 \end{vmatrix}}{-6.4} = \frac{0+0+0-480-0-0}{-6.4} = 75$$

Solution: $A = 125$ N, $B = 60.0$ N, $C = 75.0$ N

36. $19I_1 - 12I_2 = 60$

$12I_1 - 18I_2 + 6.0I_3 = 0$

$6.0I_2 - 18I_3 = 0$

$$I_1 = \frac{\begin{vmatrix} 60 & -12 & 0 \\ 0 & -18 & 6.0 \\ 0 & 6.0 & -18 \end{vmatrix}}{\begin{vmatrix} 19 & -12 & 0 \\ 12 & -18 & 6.0 \\ 0 & 6.0 & -18 \end{vmatrix}} = \frac{19440+0+0-0-2160-0}{6156+0+0-0-684-2592} = \frac{17280}{2880} = 6$$

$$I_2 = \frac{\begin{vmatrix} 19 & 60 & 0 \\ 12 & 0 & 6.0 \\ 0 & 0 & -18 \end{vmatrix}}{2880} = \frac{0+0+0-0-0-(-12960)}{2880} = 4.5$$

$$I_3 = \frac{\begin{vmatrix} 19 & -12 & 60 \\ 12 & -18 & 0 \\ 0 & 6.0 & 0 \end{vmatrix}}{2880} = \frac{0+0+4320-0-0-0}{2880} = 1.5$$

Solution: $I_1 = 6.00$ A, $I_2 = 4.50$ A, $I_3 = 1.50$ A

37. $s_0 + 2v_0 + 2a = 20$

$s_0 + 4v_0 + 8a = 54$

$s_0 + 6v_0 + 18a = 104$

$$s_0 = \frac{\begin{vmatrix} 20 & 2 & 2 \\ 54 & 4 & 8 \\ 104 & 6 & 18 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 2 \\ 1 & 4 & 8 \\ 1 & 6 & 18 \end{vmatrix}} = \frac{1440+1664+648-832-960-1944}{72+16+12-8-48-36} = \frac{16}{8} = 2$$

$$v_0 = \frac{\begin{vmatrix} 1 & 20 & 2 \\ 1 & 54 & 8 \\ 1 & 104 & 18 \end{vmatrix}}{8} = \frac{972+160+208-108-832-360}{8} = \frac{40}{8} = 5$$

$$a = \frac{\begin{vmatrix} 1 & 2 & 20 \\ 1 & 4 & 54 \\ 1 & 6 & 104 \end{vmatrix}}{8} = \frac{416+108+120-80-324-208}{8} = \frac{32}{8} = 4$$

Solution: $s_0 = 2.00$ m, $v_0 = 5.00$ m/s, $a = 4.00$ m/s^2

38. $\theta = f(t) = at^3 + bt^2 + ct$

$$\begin{aligned} a + \quad b + \quad c &= 19.0 \\ 27.0a + 9.0b + 3.0c &= 30.9 \\ 125.0a + 25.0b + 5.0c &= 19.8 \end{aligned}$$

$$a = \frac{\begin{vmatrix} 19.0 & 1.0 & 1.0 \\ 30.9 & 9.0 & 3.0 \\ 19.8 & 25.0 & 5.0 \end{vmatrix}}{\begin{vmatrix} 1.0 & 1.0 & 1.0 \\ 27.0 & 9.0 & 3.0 \\ 125.0 & 25.0 & 5.0 \end{vmatrix}} = \frac{855.0+59.4+772.5-178.2-1425.0-154.5}{45+375+675-1125-75-135} = \frac{-70.8}{-240} = 0.2951$$

$$b = \frac{\begin{vmatrix} 1.0 & 19.0 & 1.0 \\ 27.0 & 30.9 & 3.0 \\ 125.0 & 19.8 & 5.0 \end{vmatrix}}{-240} = \frac{154.5+7125+534.6-3862.5-59.4-2565}{-240} = \frac{1327.2}{-240} = -5.53$$

$$c = \frac{\begin{vmatrix} 1.0 & 1.0 & 19.0 \\ 27.0 & 9.0 & 30.9 \\ 125.0 & 25.0 & 19.8 \end{vmatrix}}{-240} = \frac{178.2+3862.5+12825-772.5-534.6-21375}{-240} = \frac{-5816.4}{-240} = 24.235$$

$\theta = f(t) = 0.295t^3 - 5.53t^2 + 24.2t$

39. Let $x = \text{m}\angle A$

Let $y = \text{m}\angle B$

Let $z = \text{m}\angle C$

$$\begin{aligned} 2x + y + z &= 360 \\ x - y - z &= 0 \\ x + 2y + z &= 280 \end{aligned}$$

$$x=\frac{\begin{vmatrix}360 & 1 & 1\\ 0 & -1 & -1\\ 280 & 2 & 1\end{vmatrix}}{\begin{vmatrix}2 & 1 & 1\\ 1 & -1 & -1\\ 1 & 2 & 1\end{vmatrix}}=\frac{-360-280+0-(-280)-(-720)-0}{-2-1+2-(-1)-(-4)-1}=\frac{360}{3}=120$$

$$y=\frac{\begin{vmatrix}2 & 360 & 1\\ 1 & 0 & -1\\ 1 & 280 & 1\end{vmatrix}}{3}=\frac{0-360+280-0-(-560)-360}{3}=\frac{120}{3}=40$$

$$z=\frac{\begin{vmatrix}2 & 1 & 360\\ 1 & -1 & 0\\ 1 & 2 & 280\end{vmatrix}}{3}=\frac{-560+0+720-(-360)-0-280}{3}=\frac{240}{3}=80$$

and so $\angle A = 120°, \angle B = 40°,$ and $\angle C = 80°.$

40. Let x = number of par-3 holes
Let y = number of par-4 holes
Let z = number of par-5 holes

$$\begin{aligned} x+y+z&=18\\ y-2z&=0\\ 3x+4y+5z&=70\end{aligned}$$

$$x=\frac{\begin{vmatrix}18 & 1 & 1\\ 0 & 1 & -2\\ 70 & 4 & 5\end{vmatrix}}{\begin{vmatrix}1 & 1 & 1\\ 0 & 1 & -2\\ 3 & 4 & 5\end{vmatrix}}=\frac{90+(-140)+0-70-(-144)-0}{5+(-6)+0-3-(-8)-0}=\frac{24}{4}=6 \text{ par-3 holes}$$

$$y=\frac{\begin{vmatrix}1 & 18 & 1\\ 0 & 0 & -2\\ 3 & 70 & 5\end{vmatrix}}{4}=\frac{0+(-108)+0-0-(-140)-0}{4}=\frac{32}{4}=8 \text{ par-4 holes}$$

$$z=\frac{\begin{vmatrix}1 & 1 & 18\\ 0 & 1 & 0\\ 3 & 4 & 70\end{vmatrix}}{4}=\frac{70+0+0-54-0-0}{4}=\frac{16}{4}=4 \text{ par-5 holes}$$

41. $V = f(T) = a + bT + cT^2$

$a + 2.0b + \ 4.0c = 6.4$

$a + 4.0b + 16.0c = 8.6$

$a + 6.0b + 36.0c = 11.6$

$$a = \frac{\begin{vmatrix} 6.4 & 2.0 & 4.0 \\ 8.6 & 4.0 & 16.0 \\ 11.6 & 6.0 & 36.0 \end{vmatrix}}{\begin{vmatrix} 1 & 2.0 & 4.0 \\ 1 & 4.0 & 16.0 \\ 1 & 6.0 & 36.0 \end{vmatrix}} = \frac{921.6 + 371.2 + 206.4 - 185.6 - 614.4 - 619.2}{144 + 32 + 24 - 16 - 96 - 72} = \frac{80}{16} = 5$$

$$b = \frac{\begin{vmatrix} 1 & 6.4 & 4.0 \\ 1 & 8.6 & 16.0 \\ 1 & 11.6 & 36.0 \end{vmatrix}}{16} = \frac{309.6 + 102.4 + 46.4 - 34.4 - 185.6 - 230.4}{16} = \frac{8}{16} = \frac{1}{2}$$

$$c = \frac{\begin{vmatrix} 1 & 2.0 & 6.4 \\ 1 & 4.0 & 8.6 \\ 1 & 6.0 & 11.6 \end{vmatrix}}{16} = \frac{46.4 + 17.2 + 38.4 - 25.6 - 51.6 - 23.2}{16} = \frac{1.6}{16} = \frac{1}{10}$$

$V = f(T) = 5.00 + 0.500T + 0.100T^2$

42. Let a = number of containers of type A

Let b = number of containers of type B

Let c = number of containers of type C

$a + \ b + \ c = 2500$

$200a + 400b + 600c = 1100000$

$4a + 6b + 7c = 15000$

$$x = \frac{\begin{vmatrix} 2500 & 1 & 1 \\ 1100000 & 400 & 600 \\ 15000 & 6 & 7 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 200 & 400 & 600 \\ 4 & 6 & 7 \end{vmatrix}} = \frac{7000000 + 9000000 + 6600000 - 6000000 - 9000000 - 7700000}{2800 + 2400 + 1200 - 1600 - 3600 - 1400} = \frac{-100000}{-200} = 500$$

$$y = \frac{\begin{vmatrix} 1 & 2500 & 1 \\ 200 & 1100000 & 600 \\ 4 & 15000 & 7 \end{vmatrix}}{-200} = \frac{7700000 + 6000000 + 3000000 - 9000000 - 3500000 - 4400000}{-200} = \frac{-200000}{-200} = 1000$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 2500 \\ 200 & 400 & 1100000 \\ 4 & 6 & 15000 \end{vmatrix}}{-200} = \frac{6000000 + 4400000 + 3000000 - 4000000 - 6600000 - 3000000}{-200} = \frac{-200000}{-200} = 1000$$

There were 500 containers of type A, 1000 containers of type B, and 1000 containers of type C.

43. Let x = percent of nickel

Let y = percent of iron

Let z = percent of molybdenum

$$\begin{aligned} x + y + z &= 100 \\ x - 5y &= -1 \\ y - 3z &= 1 \end{aligned}$$

$$x = \frac{\begin{vmatrix} 100 & 1 & 1 \\ -1 & -5 & 0 \\ 1 & 1 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -5 & 0 \\ 0 & 1 & -3 \end{vmatrix}} = \frac{1500 + 0 + (-1) - (-5) - 0 - 3}{15 + 0 + 1 - 0 - 0 - (-3)} = \frac{1501}{19} = 79\ \%\text{ nickel}$$

$$y = \frac{\begin{vmatrix} 1 & 100 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -3 \end{vmatrix}}{19} = \frac{3 + 0 + 1 - 0 - 0 - (-300)}{19} = \frac{304}{19} = 16\ \%\text{ iron}$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 100 \\ 1 & -5 & -1 \\ 0 & 1 & 1 \end{vmatrix}}{19} = \frac{-5 + 0 + 100 - 0 - (-1) - 1}{19} = \frac{95}{19} = 5\ \%\text{ molybdenum}$$

44. Let s = salaries

Let h = hardware

Let c = computer time

$$\begin{aligned} s + h + c &= 750000 \\ s - h - c &= 0 \\ h - 2c &= 0 \end{aligned}$$

$$s = \frac{\begin{vmatrix} 750000 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 1 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & 1 & -2 \end{vmatrix}} = \frac{1500000 + 0 + 0 - 0 - (-750000) - 0}{2 - 0 + 1 - 0 - (-1) - (-2)} = \frac{2250000}{6} = 375000$$

$$h = \frac{\begin{vmatrix} 1 & 750000 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & -2 \end{vmatrix}}{6} = \frac{0+0+0-0-0-(-1500000)}{6} = \frac{1500000}{6} = 250000$$

$$c = \frac{\begin{vmatrix} 1 & 1 & 750000 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{vmatrix}}{6} = \frac{0+0+750000-0-0-0}{6} = \frac{750000}{6} = 125000$$

and so salaries are budgeted at \$375,000, hardware at \$250,000, and computer time at \$125,000.

45. Let v_c = velocity of the car (in mi/h)
Let v_j = velocity of the jet (in mi/h)
Let v_t = velocity of the taxi (in mi/h)
Remeber $d = vt$

$$\begin{aligned} v_j &= 12v_c \\ v_c &= v_t + 15 \\ 1.10v_c + 1.95v_j + 0.52v_t &= 1140 \end{aligned} \qquad \begin{aligned} 12v_c - v_j \phantom{{}-v_t} &= 0 \\ v_c \phantom{{}-v_j} - v_t &= 15 \\ 1.10v_c + 1.95v_j + 0.52v_t &= 1140 \end{aligned}$$

$$v_c = \frac{\begin{vmatrix} 0 & -1 & 0 \\ 15 & 0 & -1 \\ 1140 & 1.95 & 0.52 \end{vmatrix}}{\begin{vmatrix} 12 & -1 & 0 \\ 1 & 0 & -1 \\ 1.10 & 1.95 & 0.52 \end{vmatrix}} = \frac{0+1140+0-0-0-(-7.8)}{0+1.10+0-0-(-23.4)-(-0.52)} = \frac{1147.8}{25.02} = 45.9$$

$$v_j = \frac{\begin{vmatrix} 12 & 0 & 0 \\ 1 & 15 & -1 \\ 1.10 & 1140 & 0.52 \end{vmatrix}}{25.02} = \frac{93.6+0+0-0-(-13680)-0}{25.02} = \frac{13773.6}{25.02} = 551$$

$$v_t = \frac{\begin{vmatrix} 12 & -1 & 0 \\ 1 & 0 & 15 \\ 1.10 & 1.95 & 1140 \end{vmatrix}}{25.02} = \frac{0+(-16.5)+0-0-351-(-1140)}{25.02} = \frac{772.5}{25.02} = 30.9$$

The average speeds for the trip were
$v_c = 45.9$ mi/h, $v_j = 551$ mi/h, $v_t = 30.9$ mi/h.

46. Let x = volume of mixture 1 (5-20-75)
Let y = volume of mixture 2 (0-5-95)
Let z = volume of mixture 3 (10-5-85)
For medication 1: $0.05x + 0y + 0.1z = 0.06(500)$
For medication 2: $0.2x + 0.05y + 0.05z = 0.08(500)$
For water: $0.75x + 0.95y + 0.85z = 0.86(500)$

$$\begin{aligned} 0.05x + 0y + 0.1z &= 30 \\ 0.2x + 0.05y + 0.05z &= 40 \\ 0.75x + 0.95y + 0.85z &= 430 \end{aligned}$$

$$x = \frac{\begin{vmatrix} 30 & 0 & 0.1 \\ 40 & 0.05 & 0.05 \\ 430 & 0.95 & 0.85 \end{vmatrix}}{\begin{vmatrix} 0.05 & 0 & 0.1 \\ 0.2 & 0.05 & 0.05 \\ 0.75 & 0.95 & 0.85 \end{vmatrix}} = \frac{1.275 + 0 + 3.8 - 2.15 - 1.425 - 0}{0.002125 + 0 + 0.019 - 0.00375 - 0.002375 - 0}$$

$$x = \frac{1.5}{0.015} = 100 \text{ mL of mixture 1}$$

$$y = \frac{\begin{vmatrix} 0.05 & 30 & 0.1 \\ 0.2 & 40 & 0.05 \\ 0.75 & 430 & 0.85 \end{vmatrix}}{0.015} = \frac{1.7 + 1.125 + 8.6 - 3 - 1.075 - 5.1}{0.015}$$

$$y = \frac{2.25}{0.015} = 150 \text{ mL of mixture 2}$$

$$z = \frac{\begin{vmatrix} 0.05 & 0 & 30 \\ 0.2 & 0.05 & 40 \\ 0.75 & 0.95 & 430 \end{vmatrix}}{0.015} = \frac{1.075 + 0 + 5.7 - 1.125 - 1.9 - 0}{0.015}$$

$$z = \frac{3.75}{0.015} = 250 \text{ mL of mixture 3}$$

Chapter 5 Review Exercises

1. This is false.
We substitute $x = 2, y = -3$ into $4x - 3y = -1,$ obtaining

$$\begin{aligned} 4(2) - 3(-3) &= -1 \\ 8 + 9 &= -1 \\ 17 &= -1 \end{aligned}$$

which is a false statement.

2. This is true.

We compute the slope:

$$\frac{0-(-3)}{2-0}=\frac{3}{2}$$

3. This is false.

There are infinitely many solutions including $x = 1, y = 2$.

This system is dependent.

4. This is false.

Instead of adding the left sides and right sides, they should be subtracted in order to eliminate the variable x. Alternatively, multiply each term of the second equation by -2 and then add corresponding sides.

5. This is false.

We compute the determinant:

$$\begin{vmatrix} 2 & 4 \\ -1 & 3 \end{vmatrix} = (2)(3)-(4)(-1)=10.$$

6. This is true.

The proposed method successfully eliminates the variable y in the resulting equations.

7. This is false.

We compute the determinant, expanding along the first row:

$$\begin{aligned} \begin{vmatrix} 1 & 0 & 3 \\ 0 & 4 & -1 \\ -1 & 7 & 0 \end{vmatrix} &= 1((4)(0)-(-1)(7))+0((-1)(-1)+(0)(0))+3((0)(7)-(4)(-1)) \\ &= 1(7)+0(1)+3(4) \\ &= 19 \end{aligned}$$

8. This is false.

Such a system is said to be inconsistent.

9. $\begin{vmatrix} -2 & 5 \\ 3 & 1 \end{vmatrix} = (-2)(1)-(5)(3)=-17.$

10. $\begin{vmatrix} 40 & 10 \\ -20 & -60 \end{vmatrix} = (40)(-60)-(10)(-20)=-2200.$

11. $\begin{vmatrix} -18 & 33 \\ -21 & 44 \end{vmatrix} = (-18)(44)-(33)(-21)=-792+693=-99.$

12. $\begin{vmatrix} 0.91 & -1.2 \\ 0.73 & -5.0 \end{vmatrix} = (0.91)(-5.0) - (-1.2)(0.73) = -4.55 + 0.876 = -3.674.$

The determinant is -3.7 to the required precision.

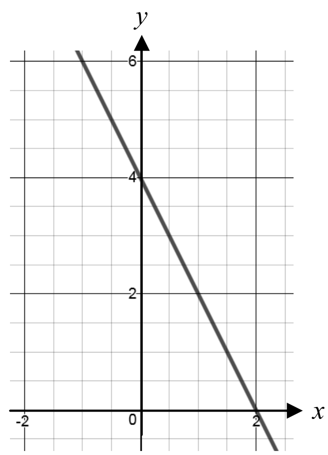
13. $m = \dfrac{0-(-8)}{2-4} = \dfrac{8}{-2} = -4$

14. $m = \dfrac{4-(-5)}{-4-(-1)} = \dfrac{9}{-3} = -3$

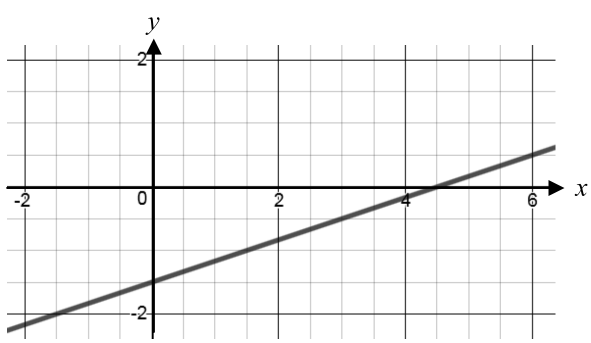
15. $m = \dfrac{-40-(-20)}{-30-40} = \dfrac{-20}{-70} = \dfrac{2}{7}$

16. $m = \dfrac{-\frac{7}{2}-\frac{1}{2}}{1-(-6)} = \dfrac{-3}{7} = -\dfrac{3}{7}$

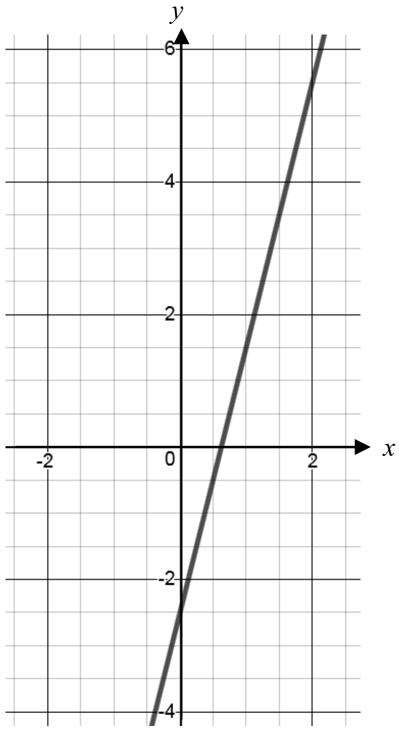
17. $m = -2, b = 4$



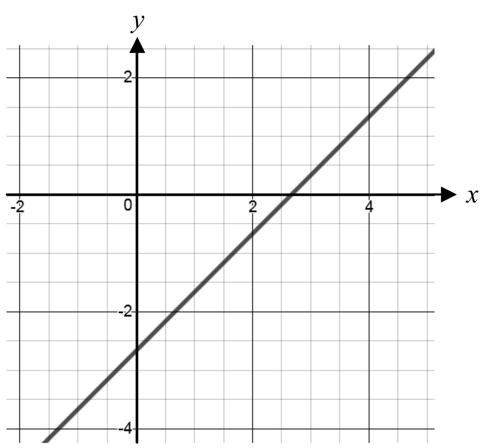
18. $m = \dfrac{1}{3}, b = -\dfrac{3}{2}$



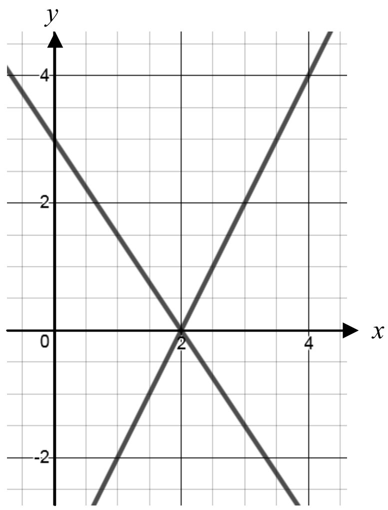
19. $m = 4, b = -\frac{5}{2}$



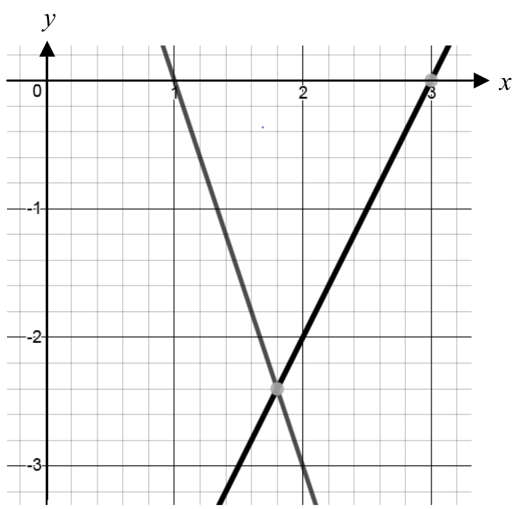
20. $m = 1, b = -\frac{8}{3}$



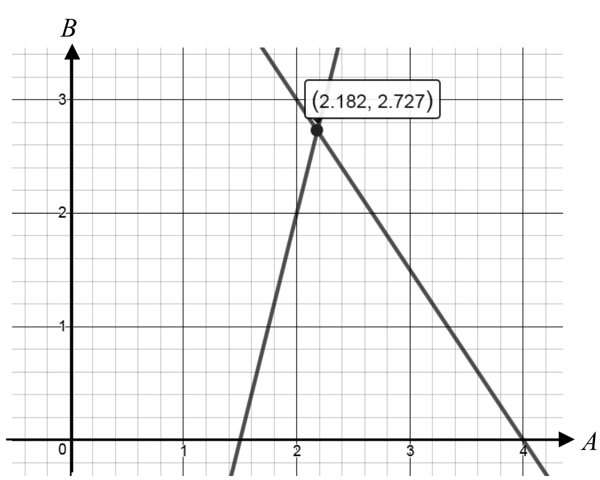
21. $x = 2, y = 0$



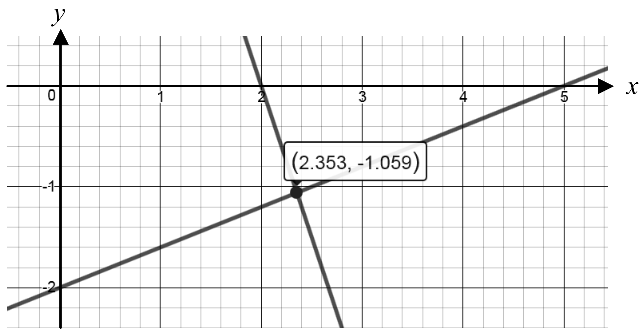
22. $x = 1.8, y = -2.4$



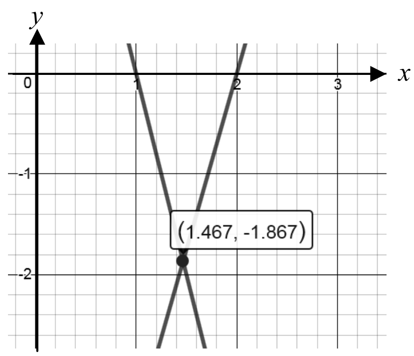
23. $A = 2.182, B = 2.727$



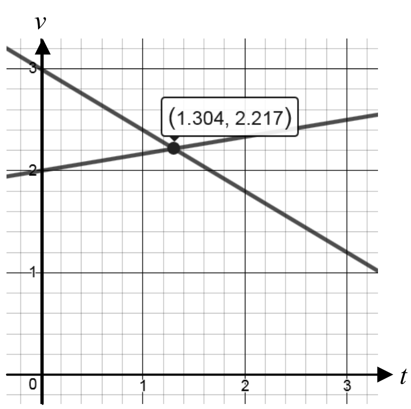
24. $x = 2.353, y = -1.059$



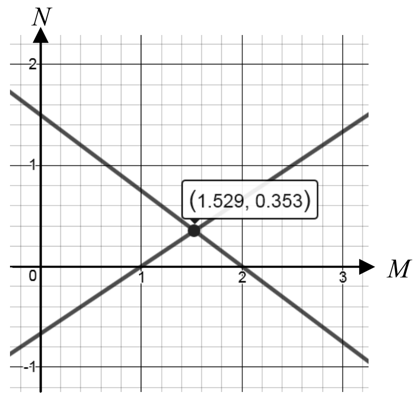
25. $x = 1.467, y = -1.867$



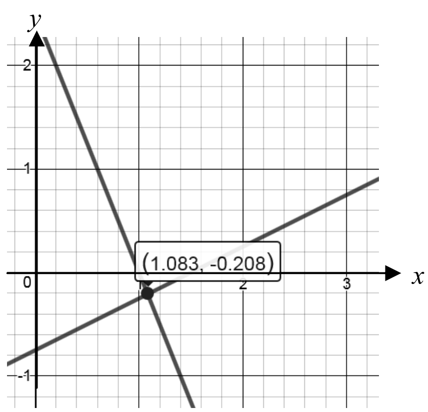
26. $t = 1.304, v = 2.217$



27. $M = 1.529, N = 0.353$



28. $x = 1.083, y = -0.208$



29. $x + 2y = 5 \quad \text{(I)}$
$x + 3y = 7 \quad \text{(II)}$
Subtract (I) from (II) to eliminate x :
$y = 2$
Substitute $y = 2$ into (I):
$x + 4 = 5 \rightarrow x = 1$
Solution: $x = 1, y = 2$

30. $2x - y = 7 \quad \text{(I)}$
$x + y = 2 \quad \text{(II)}$
Add (I) to (II) to eliminate y :
$3x = 9 \rightarrow x = 3$
Substitute $x = 3$ into (I):
$6 - y = 7 \rightarrow y = -1$
Solution: $x = 3, y = -1$

31. $4x + 3y = -4 \quad \text{(I)}$

$y = 2x - 3 \quad \text{(II)}$

Substitute $y = 2x - 3$ into (I) to eliminate y :

$4x + 3(2x - 3) = -4$

$10x - 9 = -4$

$x = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ into (I):

$2 + 3y = -4 \to y = -2$

Solution: $x = \frac{1}{2}, y = -2$

32. $r = -3s - 2 \quad \text{(I)}$

$-2r - 9s = 2 \quad \text{(II)}$

Substitute $r = -3s - 2$ into (II) to eliminate r :

$-2(-3s - 2) - 9s = 2$

$-3s + 4 = 2$

$s = \frac{2}{3}$

Substitute $s = \frac{2}{3}$ into (I):

$r = -2 - 2 \to r = -4$

Solution: $r = -4, s = \frac{2}{3}$

33. $10i - 27v = 29 \quad \text{(I)}$

$40i + 33v = 69 \quad \text{(II)}$

Multiply (I) by 4:

$40i - 108v = 116 \quad \text{(III)}$

Subtract (III) from (II) to eliminate i :

$141v = -37 \to v = -\frac{1}{3}$

Substitute $v = -\frac{1}{3}$ into (I):

$10i + 9 = 29 \to i = 2$

Solution: $i = 2, v = -\frac{1}{3}$

34. $3x - 6y = 5 \quad \text{(I)}$

$2y + 7x = 4 \quad \text{(II)}$

Multiply (II) by 3 and rearrange:

$21x + 6y = 12 \quad \text{(III)}$

Add (I) to (III) to eliminate y :

$24x = 17 \to x = \frac{17}{24}$

Substitute $x = \frac{17}{24}$ into (I):

$\frac{17}{8} - 6y = 5$

$-6y = \frac{23}{8}$

$y = -\frac{23}{48}$

Solution: $x = \frac{17}{24}, y = -\frac{23}{48}$

35. $7x = 2y - 6$ (I)

$7y = 12 - 4x$ (II)

Multiply (I) by 7, multiply (II) by 2, and rearrange:

$49x - 14y = -42$ (III)

$8x + 14y = 24$ (IV)

Add (III) to (IV) to eliminate y :

$57x = -18 \rightarrow x = -\frac{6}{19}$

Substitute $x = -\frac{6}{19}$ into (II):

$7y = 12 + \frac{24}{19} = \frac{252}{19} \rightarrow y = \frac{36}{19}$

Solution: $x = -\frac{6}{19}, y = \frac{36}{19}$

36. $3R = 8 - 5I$ (I)

$6I = 8R + 11$ (II)

Multiply (I) by 8, multiply (II) by 3, and rearrange:

$24R + 40I = 64$ (III)

$-24R + 18I = 33$ (IV)

Add (III) to (IV) to eliminate R :

$58I = 97 \rightarrow I = \frac{97}{58}$

Substitute $I = \frac{97}{58}$ into (I):

$3R = 8 - \frac{485}{58} = -\frac{21}{58} \rightarrow R = -\frac{7}{58}$

Solution: $I = \frac{97}{58}, y = R = -\frac{7}{58}$

37. $90x - 110y = 40$ (I)

$30x - 15y = 25$ (II)

Multiply (II) by 3:

$90x - 45y = 75$ (III)

Subtract (I) from (III) to eliminate x :

$65y = 35 \rightarrow y = \frac{7}{13}$

Substitute $y = \frac{7}{13}$ into (II):

$30x - \frac{105}{13} = 25$

$30x = \frac{430}{13} \rightarrow x = \frac{43}{39}$

Solution: $x = \frac{43}{39}, y = \frac{7}{13}$

38. $0.42x - 0.56y = 1.26$ (I)

$0.98x - 1.40y = -0.28$ (II)

Multiply (I) by 5 and (II) by 2:

$2.10x - 2.80y = 6.30$ (III)

$1.96x - 2.80y = -0.56$ (IV)

Subtract (IV) from (III) to eliminate y :

$0.14x = 6.86 \to x = 49$

Substitute $x = 49$ into (I):

$20.58 - 0.56y = 1.26$

$0.56y = 19.32$

$y = 34.5$

Solution: $x = 49, y = 34.5$

39. $x + 2y = 5$ (I)

$x + 3y = 7$ (II)

Using Cramer's rule:

$$x = \frac{\begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}} = \frac{(5)(3) - (2)(7)}{(1)(3) - (2)(1)} = 1$$

$$y = \frac{\begin{vmatrix} 1 & 5 \\ 1 & 7 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}} = \frac{(1)(7) - (5)(1)}{(1)(3) - (2)(1)} = 2$$

40. $2x - y = 7$ (I)

$x + y = 2$ (II)

Using Cramer's rule:

$$x = \frac{\begin{vmatrix} 7 & -1 \\ 2 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{(7)(1) - (-1)(2)}{(2)(1) - (-1)(2)} = 3$$

$$y = \frac{\begin{vmatrix} 2 & 7 \\ 1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{(2)(2) - (7)(1)}{(2)(1) - (-1)(2)} = -1$$

41. $4x + 3y = -4$ (I)

$-2x + y = -3$ (II,rearranged)

Using Cramer's rule:

$$x = \frac{\begin{vmatrix} -4 & 3 \\ -3 & 1 \end{vmatrix}}{\begin{vmatrix} 4 & 3 \\ -2 & 1 \end{vmatrix}} = \frac{(-4)(1) - (3)(-3)}{(4)(1) - (3)(-2)} = \frac{5}{10} = \frac{1}{2}$$

$$y = \frac{\begin{vmatrix} 4 & -4 \\ -2 & -3 \end{vmatrix}}{\begin{vmatrix} 4 & 3 \\ -2 & 1 \end{vmatrix}} = \frac{(4)(-3) - (-4)(-2)}{(4)(1) - (3)(-2)} = \frac{-20}{10} = -2$$

42. $r + 3s = -2$ (I, rearranged)

$-2r - 9s = 2$ (II)

Using Cramer's rule:

$$r = \frac{\begin{vmatrix} -2 & 3 \\ 2 & -9 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ -2 & -9 \end{vmatrix}} = \frac{(-2)(-9) - (3)(2)}{(1)(-9) - (3)(-2)} = \frac{12}{-3} = -4$$

$$s = \frac{\begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ -2 & -9 \end{vmatrix}} = \frac{(1)(2) - (-2)(-2)}{(1)(-9) - (3)(-2)} = \frac{-2}{-3} = \frac{2}{3}$$

43. $10i - 27v = 29$ (I)

$40i + 33v = 69$ (II)

Using Cramer's rule:

$$i = \frac{\begin{vmatrix} 29 & -27 \\ 69 & 33 \end{vmatrix}}{\begin{vmatrix} 10 & -27 \\ 40 & 33 \end{vmatrix}} = \frac{(29)(33) - (-27)(69)}{(10)(33) - (-27)(40)} = \frac{2820}{1410} = 2$$

$$v = \frac{\begin{vmatrix} 10 & 29 \\ 40 & 69 \end{vmatrix}}{\begin{vmatrix} 10 & -27 \\ 40 & 33 \end{vmatrix}} = \frac{(10)(69) - (29)(40)}{(10)(33) - (-27)(40)} = \frac{-470}{1410} = -\frac{1}{3}$$

44. $3x - 6y = 5$ (I)

$7x + 2y = 4$ (II, rearranged)

Using Cramer's rule:

$$x = \frac{\begin{vmatrix} 5 & -6 \\ 4 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & -6 \\ 7 & 2 \end{vmatrix}} = \frac{(5)(2) - (-6)(4)}{(3)(2) - (-6)(7)} = \frac{34}{48} = -\frac{17}{24}$$

$$y = \frac{\begin{vmatrix} 3 & 5 \\ 7 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & -6 \\ 7 & 2 \end{vmatrix}} = \frac{(3)(4) - (5)(7)}{(3)(2) - (-6)(7)} = \frac{-23}{48} = -\frac{23}{48}$$

45. $7x - 2y = -6$ (I, rearranged)

$4x + 7y = 12$ (II, rearranged)

Using Cramer's rule:

$$x = \frac{\begin{vmatrix} -6 & -2 \\ 12 & 7 \end{vmatrix}}{\begin{vmatrix} 7 & -2 \\ 4 & 7 \end{vmatrix}} = \frac{(-6)(7) - (-2)(12)}{(7)(7) - (-2)(4)} = \frac{-18}{57} = -\frac{6}{19}$$

$$y = \frac{\begin{vmatrix} 7 & -6 \\ 4 & 12 \end{vmatrix}}{\begin{vmatrix} 7 & -2 \\ 4 & 7 \end{vmatrix}} = \frac{(7)(12) - (-6)(4)}{(7)(7) - (-2)(4)} = \frac{108}{57} = \frac{36}{19}$$

46. $5I + 3R = 8$ (I, rearranged)

$6I - 8R = 11$ (II, rearranged)

Using Cramer's rule:

$$I = \frac{\begin{vmatrix} 8 & 3 \\ 11 & -8 \end{vmatrix}}{\begin{vmatrix} 5 & 3 \\ 6 & -8 \end{vmatrix}} = \frac{(8)(-8) - (3)(11)}{(5)(-8) - (3)(6)} = \frac{-97}{-58} = \frac{97}{58}$$

$$R = \frac{\begin{vmatrix} 5 & 8 \\ 6 & 11 \end{vmatrix}}{\begin{vmatrix} 5 & 3 \\ 6 & -8 \end{vmatrix}} = \frac{(5)(11) - (8)(6)}{(5)(-8) - (3)(6)} = \frac{7}{-58} = -\frac{7}{58}$$

47. $90x - 110y = 40 \quad \text{(I)}$

$30x - 15y = 25 \quad \text{(II)}$

Using Cramer's rule:

$$x = \frac{\begin{vmatrix} 40 & -110 \\ 25 & -15 \end{vmatrix}}{\begin{vmatrix} 90 & -110 \\ 30 & -15 \end{vmatrix}} = \frac{(40)(-15) - (-110)(25)}{(90)(-15) - (-110)(30)} = \frac{2150}{1950} = \frac{43}{39}$$

$$y = \frac{\begin{vmatrix} 90 & 40 \\ 30 & 25 \end{vmatrix}}{\begin{vmatrix} 90 & -110 \\ 30 & -15 \end{vmatrix}} = \frac{(90)(25) - (40)(30)}{(90)(-15) - (-110)(30)} = \frac{1050}{1950} = \frac{7}{13}$$

48. $0.42x - 0.56y = 1.26 \quad \text{(I)}$

$0.98x - 1.40y = -0.28 \quad \text{(II)}$

Using Cramer's rule:

$$x = \frac{\begin{vmatrix} 1.26 & -0.56 \\ -0.28 & -1.40 \end{vmatrix}}{\begin{vmatrix} 0.42 & -0.56 \\ 0.98 & -1.40 \end{vmatrix}} = \frac{(1.26)(-1.40) - (-0.56)(-0.28)}{(0.42)(-1.40) - (-0.56)(0.98)} = \frac{-1.9208}{-0.0392} = 49$$

$$y = \frac{\begin{vmatrix} 0.42 & 1.26 \\ 0.98 & -0.28 \end{vmatrix}}{\begin{vmatrix} 0.42 & -0.56 \\ 0.98 & -1.40 \end{vmatrix}} = \frac{(0.42)(-0.28) - (1.26)(0.98)}{(0.42)(-1.40) - (-0.56)(0.98)} = \frac{-1.3524}{-0.0392} = 34.5$$

49. Exercise 41 is well-suited for substitution since the variable y is already isolated in the second equation.

50. Exercise 39 is well-suited for subtraction since the variable x has the same coefficient on the left-hand sides of both equations.

51. Exercise 45 is better-suited for determinants once the equations are rearranged. Without scaling or further rearrangements, solving using substitution, addition or subtraction is not as readily accomplished.

52. If the slopes are distinct, then there is a unique solution.
If the slopes are the same and the intercepts are distinct, then the system is inconsistent.
If both the slopes are the same and the intercepts are the same, then the system is dependent.

53.
$$\begin{vmatrix} 4 & -1 & 8 \\ -1 & 6 & -2 \\ 2 & 1 & -1 \end{vmatrix} = (4)(6)(-1) - (4)(-2)(1) + (-1)(-2)(2) - (-1)(-1)(-1) + (8)(-1)(1) - (8)(6)(2)$$
$$= -24 + 8 + 4 + 1 - 8 - 96$$
$$= -115$$

54. $\begin{vmatrix} -500 & 0 & -500 \\ 250 & 300 & -100 \\ -300 & 200 & 200 \end{vmatrix} = (-500)(300)(200) - (-500)(-100)(200) + (0)(-100)(-300)$

$- (0)(250)(200) + (-500)(250)(200) - (-500)(300)(-300)$

$= -30{,}000{,}000 - 10{,}000{,}000 + 0 - 0 - 25{,}000{,}000 - 45{,}000{,}000$

$= -110{,}000{,}000$

55. $\begin{vmatrix} -2.2 & -4.1 & 7.0 \\ 1.2 & 6.4 & -3.5 \\ -7.2 & 2.4 & -1.0 \end{vmatrix} = (-2.2)(6.4)(-1.0) - (-2.2)(-3.5)(2.4) + (-4.1)(-3.5)(-7.2)$

$- (-4.1)(1.2)(-1.0) + (7.0)(1.2)(2.4) - (7.0)(6.4)(-7.2)$

$= 230.08$

which to the required precision is 230.

56. $\begin{vmatrix} 30 & 22 & -12 \\ 0 & -34 & 44 \\ 35 & -41 & -27 \end{vmatrix} = (30)(-34)(-27) - (30)(44)(-41) + (22)(44)(35)$

$- (22)(0)(-27) + (-12)(0)(-41) - (-12)(-34)(35)$

$= 101{,}260$

57. $2x + y + z = 4 \quad \text{(I)}$

$x - 2y - z = 3 \quad \text{(II)}$

$3x + 3y - 2z = 1 \quad \text{(III)}$

Add (I) and (II) to produce (IV); Add 2(I) and (III) to produce (V):

$3x - y = 7 \quad \text{(IV)}$

$7x + 5y = 9 \quad \text{(V)}$

Add 5(IV) and (V):

$22x = 44 \to x = 2$

Substitute $x = 2$ into (IV):

$6 - y = 7 \to y = -1$

Substitute $x = 2$ and $y = -1$ into (I):

$4 - 1 + z = 4 \to z = 1$

The solution is $x = 2, y = -1, z = 1$.

58. $x + 2y + z = 2$ (I)

$3x - 6y + 2z = 2$ (II)

$2x \quad - z = 8$ (III)

Add (I) and (III) to produce (IV); Add (II) and 2(III) to produce (V):

$3x + 2y = 10$ (IV)

$7x - 6y = 18$ (V)

Add 3(IV) and (V):

$16x = 48 \to x = 3$

Substitute $x = 3$ into (V):

$21 - 6y = 18 \to y = \frac{1}{2}$

Substitute $x = 3$ into (III):

$6 - z = 8 \to z = -2$

The solution is $x = 3, y = \frac{1}{2}, z = -2$.

59. $2r + s + 2t = 8$ (I)

$3r - 2s - 4t = 5$ (II)

$-2r + 3s + 4t = -3$ (III)

Add 2(I) and (II) to produce (IV); Add (II) and (III) to produce (V):

$7r = 21 \to r = 3$ (IV)

$r + s = 2 \to s = -1$ (V)

Substitute $r = 3$ and $s = -1$ into (I):

$6 - 1 + 2t = 8 \to t = \frac{3}{2}$

The solution is $r = 3, s = -1, t = \frac{3}{2}$.

60. $4u + 4v - 2w = -4$ (I)

$20u - 15v + 10w = -10$ (II)

$24u - 12v - 9w = 39$ (III)

Scale (I) by $\frac{1}{4}$, (II) by $\frac{1}{5}$, and (III) by $\frac{1}{3}$:

$2u + 2v - w = -2$ (I')

$4u - 3v + 2w = -2$ (II')

$8u - 4v - 3w = 13$ (III')

Add 2(I') and (II') to obtain (IV); add -3(I') and (III') to obtain (V):

$8u + v = -6$ (IV)

$2u - 10v = 19$ (V)

Add 10(IV) and (V):

$82u = -41 \to u = -\frac{1}{2}$

Substitute $u = -\frac{1}{2}$ into (IV):

$-4 + v = -6 \to v = -2$

Substitute $u = -\frac{1}{2}$ and $v = -2$ into (I'):

$-1 - 4 - w = -2 \to w = -3$

The solution is $u = -\frac{1}{2}, v = -2, w = -3$.

61. $3.6x + 5.2y - z = -2.2$ (I)

$3.2x - 4.8y + 3.9z = 8.1$ (II)

$6.4x + 4.1y + 2.3z = 5.1$ (III)

Add 3.9(I) and (II) to produce (IV); Add 2.3(I) and (III) to produce (V):

$17.24x + 15.48y = -0.48$ (IV)

$14.68x + 16.06y = 0.04$ (V)

Add 16.06(IV) and -15.48(V):

$49.628x = -8.328 \rightarrow x = -0.1678085$

Substitute $x = -0.1678085$ into (V):

$-2.46342878 + 16.06y = 0.04 \rightarrow y = 0.1558798$

Substitute $x = -0.1678085$ and $y = 0.1558798$ into (I):

$-0.6041106 + 0.8105747 - z = -2.2 \rightarrow z = 2.4064641$

The solution is $x = -0.17, y = 0.16, z = 2.4$.

62. $32t + 24u + 63v = 32$ (I)

$42t - 31u + 19v = 132$ (II, rearranged)

$48t + 12u + 11v = 0$ (III)

Add -19(I) and 63(II) to obtain (IV); add -11(I) and 63(III) to obtain (V):

$2038t - 2409u = 7708$ (IV)

$2672t + 492u = -352$ (V)

Add 492(IV) and 2409(V):

$7439544t = 2944368 \rightarrow t = 0.39577264$

Substitute $t = 0.39577264$ into (IV):

$806.584649 - 2409u = 7708 \rightarrow u = -2.86484656$

Substitute $t = 0.39577264$ and $u = -2.86484656$ into (I):

$12.6647245 - 68.7563173 + 63v = 32 \rightarrow v = 1.39827925$

The solution is $t = 0.40, u = -2.9, v = 1.4$.

63. $5x + y - 4z = -5$

$3x - 5y - 6z = -20$

$x - 3y + 8z = -27$

Solution: $x = -3, y = 4, z = -1.5$

64. $x + y + z = 80$

$2x - 3y = -20$

$2x + 3z = 115$

Solution: $x = 35, y = 30, z = 15$

65. $2x + y + z = 4$ (I)
$x - 2y - z = 3$ (II)
$3x + 3y - 2z = 1$ (III)

$$x = \frac{\begin{vmatrix} 4 & 1 & 1 \\ 3 & -2 & -1 \\ 1 & 3 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 3 & 3 & -2 \end{vmatrix}} = \frac{44}{22} = 2;\ y = \frac{\begin{vmatrix} 2 & 4 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 3 & 3 & -2 \end{vmatrix}} = \frac{-22}{22} = -1;\ z = \frac{\begin{vmatrix} 2 & 1 & 4 \\ 1 & -2 & 3 \\ 3 & 3 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 3 & 3 & -2 \end{vmatrix}} = \frac{22}{22} = 1$$

The solution is $x = 2, y = -1, z = 1$.

66. $x + 2y + z = 2$ (I)
$3x - 6y + 2z = 2$ (II)
$2x \quad - z = 8$ (III)

$$x = \frac{\begin{vmatrix} 2 & 2 & 1 \\ 2 & -6 & 2 \\ 8 & 0 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 1 \\ 3 & -6 & 2 \\ 2 & 0 & -1 \end{vmatrix}} = \frac{96}{32} = 3;\ y = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 2 & 8 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 1 \\ 3 & -6 & 2 \\ 2 & 0 & -1 \end{vmatrix}} = \frac{16}{32} = \frac{1}{2};\ z = \frac{\begin{vmatrix} 1 & 2 & 2 \\ 3 & -6 & 2 \\ 2 & 0 & 8 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 1 \\ 3 & -6 & 2 \\ 2 & 0 & -1 \end{vmatrix}} = \frac{-64}{32} = -2$$

The solution is $x = 3, y = \frac{1}{2}, z = -2$.

67. $2r + s + 2t = 8$ (I)
$3r - 2s - 4t = 5$ (II)
$-2r + 3s + 4t = -3$ (III)

$$r = \frac{\begin{vmatrix} 8 & 1 & 2 \\ 5 & -2 & -4 \\ -3 & 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 2 \\ 3 & -2 & -4 \\ -2 & 3 & 4 \end{vmatrix}} = \frac{42}{14} = 3; s = \frac{\begin{vmatrix} 2 & 8 & 2 \\ 3 & 5 & -4 \\ -2 & -3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 2 \\ 3 & -2 & -4 \\ -2 & 3 & 4 \end{vmatrix}} = \frac{-14}{14} = -1; t = \frac{\begin{vmatrix} 2 & 1 & 8 \\ 3 & -2 & 5 \\ -2 & 3 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 2 \\ 3 & -2 & -4 \\ -2 & 3 & 4 \end{vmatrix}} = \frac{21}{14} = \frac{3}{2}$$

Solution: $r = 3, s = -1, t = \frac{3}{2}$.

68. $4u + 4v - 2w = -4$ (I)
$20u - 15v + 10w = -10$ (II)
$24u - 12v - 9w = 39$ (III)

$$u = \frac{\begin{vmatrix} -4 & 4 & -2 \\ -10 & -15 & 10 \\ 39 & -12 & -9 \end{vmatrix}}{\begin{vmatrix} 4 & 4 & -2 \\ 20 & -15 & 10 \\ 24 & -12 & -9 \end{vmatrix}} = \frac{-1230}{2460}; v = \frac{\begin{vmatrix} 4 & -4 & -2 \\ 20 & -10 & 10 \\ 24 & 39 & -9 \end{vmatrix}}{\begin{vmatrix} 4 & 4 & -2 \\ 20 & -15 & 10 \\ 24 & -12 & -9 \end{vmatrix}} = \frac{-4920}{2460}; w = \frac{\begin{vmatrix} 4 & 4 & -4 \\ 20 & -15 & -10 \\ 24 & -12 & 39 \end{vmatrix}}{\begin{vmatrix} 4 & 4 & -2 \\ 20 & -15 & 10 \\ 24 & -12 & -9 \end{vmatrix}} = \frac{-7380}{2460};$$

The solution is $u = -\frac{1}{2}, v = -2, w = -3$.

69. $3.6x + 5.2y - z = -2.2$ (I)
$3.2x - 4.8y + 3.9z = 8.1$ (II)
$6.4x + 4.1y + 2.3z = 5.1$ (III)

$$x = \frac{\begin{vmatrix} -2.2 & 5.2 & -1.0 \\ 8.1 & -4.8 & 3.9 \\ 5.1 & 4.1 & 2.3 \end{vmatrix}}{\begin{vmatrix} 3.6 & 5.2 & -1.0 \\ 3.2 & -4.8 & 3.9 \\ 6.4 & 4.1 & 2.3 \end{vmatrix}} = \frac{8.328}{-49.628}; y = \frac{\begin{vmatrix} 3.6 & -2.2 & -1.0 \\ 3.2 & 8.1 & 3.9 \\ 6.4 & 5.1 & 2.3 \end{vmatrix}}{\begin{vmatrix} 3.6 & 5.2 & -1.0 \\ 3.2 & -4.8 & 3.9 \\ 6.4 & 4.1 & 2.3 \end{vmatrix}} = \frac{-7.736}{-49.628}; z = \frac{\begin{vmatrix} 3.6 & 5.2 & -2.2 \\ 3.2 & -4.8 & 8.1 \\ 6.4 & 4.1 & 5.1 \end{vmatrix}}{\begin{vmatrix} 3.6 & 5.2 & -1.0 \\ 3.2 & -4.8 & 3.9 \\ 6.4 & 4.1 & 2.3 \end{vmatrix}} = \frac{-119.428}{-49.628}$$

The solution is $x = -0.17, y = 0.16, z = 2.4$

70. $32t + 24u + 63v = 32$ (I)
$42t - 31u + 19v = 132$ (II, rearranged)
$48t + 12u + 11v = 0$ (III)

$$t = \frac{\begin{vmatrix} 32 & 24 & 63 \\ 132 & -31 & 19 \\ 0 & 12 & 11 \end{vmatrix}}{\begin{vmatrix} 32 & 24 & 63 \\ 42 & -31 & 19 \\ 48 & 12 & 11 \end{vmatrix}} = \frac{46736}{118088}; u = \frac{\begin{vmatrix} 32 & 32 & 63 \\ 42 & 132 & 19 \\ 48 & 0 & 11 \end{vmatrix}}{\begin{vmatrix} 32 & 24 & 63 \\ 42 & -31 & 19 \\ 48 & 12 & 11 \end{vmatrix}} = \frac{-338304}{118088}; v = \frac{\begin{vmatrix} 32 & 24 & 32 \\ 42 & -31 & 132 \\ 48 & 12 & 0 \end{vmatrix}}{\begin{vmatrix} 32 & 24 & 63 \\ 42 & -31 & 19 \\ 48 & 12 & 11 \end{vmatrix}} = \frac{165120}{118088};$$

The solution is $t = 0.40, u = -2.9, v = 1.4.$

71. $3 = \begin{vmatrix} 2 & 5 \\ 1 & x \end{vmatrix} = 2x - 5$
$x = 4$

72. $7 = \begin{vmatrix} -1 & x \\ 3 & 4 \end{vmatrix} = -4 - 3x$
$x = -\frac{11}{3}$

73. $5 = \begin{vmatrix} x & 1 & 2 \\ 0 & -1 & 3 \\ -2 & 2 & 1 \end{vmatrix} = x(-1-6) + 1(-6-0) + 2(0-2) = -7x - 10$
$x = -\frac{15}{7}$

74. $-3 = \begin{vmatrix} 1 & 2 & -1 \\ -2 & 3 & x \\ -1 & 2 & -2 \end{vmatrix} = 1(-6-2x) + 2(-x-4) + (-1)(-4+3) = -4x - 13$
$x = -\frac{5}{2}$

75. Regression line equation: $y = -0.00698x + 54.6$
Estimated mpg is $-0.00698(3500) + 54.6 = 30.1$

76. Regression line equation: $y = -44.9x + 102$
Estimated pulse rate is $-44.9(0.25) + 102 = 90.7$

77. The system is dependent or inconsistent (here, it is dependent) if
$$0 = \begin{vmatrix} 3 & -k \\ 1 & 2 \end{vmatrix} = 6 + k, \text{ or if } k = -6.$$

78. The system is dependent or inconsistent (here, it is dependent) if
$$0 = \begin{vmatrix} 5 & 20 \\ 2 & k \end{vmatrix} = 5k - 40, \text{ or if } k = 8.$$

79. The system is dependent or inconsistent (here, it is inconsistent) if
$$0 = \begin{vmatrix} k & -2 \\ 4 & 6 \end{vmatrix} = 6k + 8, \text{ or if } k = -\frac{4}{3}.$$

80. The system is dependent or inconsistent (here, it is inconsistent) if
$$0 = \begin{vmatrix} 2 & -5 \\ k & 10 \end{vmatrix} = 20 + 5k, \text{ or if } k = -4.$$

81. $F_1 + 2.0F_2 = 280$ (I)
$0.87F_1 - F_3 = 0$ (II)
$3.0F_1 - 4.0F_2 = 600$ (III)
$$F_1 = \frac{\begin{vmatrix} 280 & 2 & 0 \\ 0 & 0 & -1 \\ 600 & 4 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 0 \\ 0.87 & 0 & -1 \\ 3 & 4 & 0 \end{vmatrix}} = \frac{-80}{-2}; F_2 = \frac{\begin{vmatrix} 1 & 280 & 0 \\ 0.87 & 0 & -1 \\ 3 & 600 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 0 \\ 0.87 & 0 & -1 \\ 3 & 4 & 0 \end{vmatrix}} = \frac{-240}{-2}; F_3 = \frac{\begin{vmatrix} 1 & 2 & 280 \\ 0.87 & 0 & 0 \\ 3 & 4 & 600 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 0 \\ 0.87 & 0 & -1 \\ 3 & 4 & 0 \end{vmatrix}} = \frac{-69.6}{-2};$$
The solution is $F_1 = 40, F_2 = 120, F_3 = 35.$

82. $i_1 + i_2 + i_3 = 0$ (I)
$5.20i_1 - 3.25i_2 = 1.88$ (II)
$3.25i_2 - 2.62i_3 = -3.35$ (III)
$$i_1 = \frac{\begin{vmatrix} 0 & 1 & 1 \\ 1.88 & -3.25 & 0 \\ -3.35 & 3.25 & -2.62 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 5.20 & -3.25 & 0 \\ 0 & 3.25 & -2.62 \end{vmatrix}} = \frac{0.148}{39.039}; i_2 = \frac{\begin{vmatrix} 1 & 0 & 1 \\ 5.20 & 1.88 & 0 \\ 0 & -3.35 & -2.62 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 5.20 & -3.25 & 0 \\ 0 & 3.25 & -2.62 \end{vmatrix}} = \frac{-22.346}{39.039}; i_3 = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 5.20 & -3.25 & 1.88 \\ 0 & 3.25 & -3.35 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 5.20 & -3.25 & 0 \\ 0 & 3.25 & -2.62 \end{vmatrix}} = \frac{22.198}{39.039};$$
The solution is $i_1 = 0.00379, i_2 = -0.572, i_3 = 0.569.$

83. $p_1 + p_2 = 1 \quad \text{(I)}$

$p_1 = p_2 - 0.16 \quad \text{(II)}$

Substitute (II) into (I):

$2p_2 - 0.16 = 1$

$p_2 = \dfrac{1.16}{2} = 0.58$

$p_1 = 1 - 0.58 = 0.42$

We have $p_1 = 42\%$ and $p_2 = 58\%$.

84. Let x and y denote the fuel burned at 80% efficiency and 70% efficienty, respectively

The system of equations becomes

$$x + y = 150000$$
$$0.80x + 0.70y = 114000$$

Using Cramer's rule,

$$x = \frac{\begin{vmatrix} 150000 & 1 \\ 114000 & 0.70 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.80 & 0.70 \end{vmatrix}} = \frac{-9000}{-0.10}; y = \frac{\begin{vmatrix} 1 & 150000 \\ 0.80 & 114000 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.80 & 0.70 \end{vmatrix}} = \frac{-6000}{-0.10}$$

$x = 90000$ Btu and $y = 60000$ Btu

85. Let x be the sales in a month. At parity, we have

$0.10x = 2400 + 0.04x$

$0.06x = 2400$

$x = 40000$

The monthly sales at parity is \$40000

and the sales reps would make 10% of this amount, or \$4000.

86. Let x and y denote the amounts invested at 6.00% and 5.00%, respectively.

The system of equations becomes

$$x + y = 20900$$
$$0.06x + 0.05y = 1170$$

Using Cramer's rule,

$$x = \frac{\begin{vmatrix} 20900 & 1 \\ 1170 & 0.05 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.06 & 0.05 \end{vmatrix}} = \frac{-125}{-0.01}; y = \frac{\begin{vmatrix} 1 & 20900 \\ 0.06 & 1170 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.06 & 0.05 \end{vmatrix}} = \frac{-84}{-0.01}$$

$x = 12500$ and $y = 8400$

We have \$12500 invested at 6.00% and \$8400 invested at 5.00%.

87. Let x and y denote the amounts with 6.0% and 2.4% concentrations of copper, respectively.

The system of equations becomes

$$x + y = 42$$
$$0.06x + 0.024y = 2$$

Using Cramer's rule,

$$x = \frac{\begin{vmatrix} 42 & 1 \\ 2 & 0.024 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.06 & 0.024 \end{vmatrix}} = \frac{-0.992}{-0.036}; y = \frac{\begin{vmatrix} 1 & 42 \\ 0.06 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.06 & 0.024 \end{vmatrix}} = \frac{-0.52}{-0.036}$$

$x = 27.555$ and $y = 14.444$

We need 28 tons of 6.0% copper ore and 14 tons of 2.4% copper ore to produce 2 tons of copper.

88. Let c_1, c_2, and c_3 denote the circumferences from largest to smallest, respectively.

Let $r_i = \frac{40}{c_i}$ be the number of revolutions corresponding to each circumference.

The system of equations becomes

$r_2 = r_1 + 1$ (I)

$r_3 = 2r_2$ (II)

$r_3 = r_1 + 6$ (III)

Substitute (I) into (II):

$r_3 = 2r_1 + 2$ (IV)

Subtract (III) from (IV):

$0 = r_1 - 4 \rightarrow r_1 = 4, r_2 = 5, r_3 = 10$

The circumferences are

$c_1 = \frac{40}{4} = 10$ ft, $c_2 = \frac{40}{5} = 8$ ft, $c_3 = \frac{40}{10} = 4$ ft.

89. At $x = 0, T = 14 = \dfrac{a}{100} + b,$ or $a + 100b = 1400$ (I)

At $x = 900, T = 10 = \dfrac{a}{1000} + b,$ or $a + 1000b = 10000$ (II)

Subtracting (I) from (II),

$900b = 8600 \rightarrow b = 9.556$

and $a = 1400 - 100(9.556) = 444.4$

We have $a = 440$ and $b = 9.6$.

90. Letting a, b, c be the three angles, we know

$a + b + c = 180$

$a + b = c$

and so $2c = 180,$ or $c = 90.$

The sides of the triangular parcel form a right triangle.

91. Let x and y denote the amounts with 18.0 gal/ton and 30.0 gal/ton concentrations of copper, respectively.

The system of equations becomes

$$x + y = 42$$
$$18.0x + 30.0y = 1050$$

Using Cramer's rule,

$$x = \frac{\begin{vmatrix} 42 & 1 \\ 1050 & 30 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 18 & 30 \end{vmatrix}} = \frac{210}{12}; y = \frac{\begin{vmatrix} 1 & 42 \\ 18 & 1050 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 18 & 30 \end{vmatrix}} = \frac{294}{12}$$

$x = 17.5$ and $y = 24.5$

We need 17.5 tons of 18 gal/ton shale and 24.5 tons of 30 gal/ton shale to produce 1050 gallons of oil.

92. Letting a, b, c be the number of orders charged at \$4, \$6, and \$8, respectively.

The system of equations is then

$$a + b + c = 384 \quad \text{(I)}$$
$$4a + 6b + 8c = 2160 \quad \text{(II)}$$
$$a - 2c = 12 \quad \text{(III)}$$

We subtract (III) from (I) to obtain (IV) and subtract 4(III) from (II) to obtain (V):

$$b + 3c = 372 \quad \text{(IV)}$$
$$6b + 16c = 2112 \quad \text{(V)}$$

We subtract (V) from 6(IV):

$2c = 120 \to c = 60$

$a = 12 + 2c = 132$

$b = 384 - 192 = 192$

A total of 132 shipments at \$4, 192 shipments at \$6, and 60 shipments at \$8 were delivered.

93. Let x and y denote the numbers of 800 ft^2 and 1100 ft^2 offices, respectively.

The system of equations becomes

$$800x + 1100y = 49200$$
$$900x + 1250y = 55600$$

Using Cramer's rule,

$$x = \frac{\begin{vmatrix} 49200 & 1100 \\ 55600 & 1250 \end{vmatrix}}{\begin{vmatrix} 800 & 1100 \\ 900 & 1250 \end{vmatrix}} = \frac{340000}{10000}; y = \frac{\begin{vmatrix} 800 & 49200 \\ 900 & 55600 \end{vmatrix}}{\begin{vmatrix} 800 & 1100 \\ 900 & 1250 \end{vmatrix}} = \frac{200000}{10000}$$

$x = 34$ and $y = 20$

There are 34 smaller and 20 larger offices.

94. Let the desired equation be $p = mn + b$ where we must determine m and b.
We have
$31.7 = m(0) + b = b$
$34.0 = m(4) + b = 4m + 31.7$
$4m = 2.3$
$m = 0.575$
The desired equation is $p = 0.575n + 31.7$
In 2020, $n = 10$ and $p = 37.45$, so 37.5% of persons 25 to 29 years old will have completed four years of college in 2020.

95. Let v be the velocity of the shuttle and w be the velocity of the satellite relative to the shuttle. We have
$v + w = 24200$
$v - w = 21400$
Summing these, $2v = 45600 \to v = 22800$ km/hr and
$22800 - w = 21400 \to w = 1400$ km/hr.

96. The system of equations becomes
$337.5 = 10a + b$ (I)
$346.6 = 25a + b$ (II)
Subtracting (I) from (II) yields
$9.1 = 15a \to a = 0.606667$
$b = 337.5 - 6.06667 = 331.4333$
The desired equation is
$v = 0.6067T + 331.4$

97. The system of equations becomes
$R_1 + 9R_2 = 14$ (I)
$9R_1 + R_2 = 6$ (II)
Using Cramer's rule,

$$R_1 = \frac{\begin{vmatrix} 14 & 9 \\ 6 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 9 \\ 9 & 1 \end{vmatrix}} = \frac{-40}{-80}; R_2 = \frac{\begin{vmatrix} 1 & 14 \\ 9 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 9 \\ 9 & 1 \end{vmatrix}} = \frac{-120}{-80}$$

$R_1 = 0.5\Omega$ and $R_2 = 1.5\Omega$

98. Let l be the length of a panel and w be the width of a panel.

The system of equations is

$l = w + 0.5$ (I)

$16l + 15w = 132$ (II)

Substituting (I) into (II) yields:

$16(w + 0.5) + 15w = 132$

$31w = 124$

$w = 4.0$ ft.

$l = 4.5$ ft.

99. Considering moments, the system of equations is

$8L = 2w$

$8(4L) = 2w + 12(20)$

or, rearranging,

$w = 4L$

$2w + 240 = 32L$

Substituting, we have

$8L + 240 = 32L \to L = 10$ lb.

$w = 40$ lb.

100. Let m_1 and m_2 be the amounts of the fuel mixtures. We have

$m_1 + m_2 = 10.0$ (eq. I: liters of mixture)

$0.02m_1 + 0.08m_2 = 0.40$ (eq. II: liters of oil)

Divide (II) by 0.02:

$m_1 + 4m_2 = 20.0$ (III)

Subtract (I) from (III):

$3m_2 = 10 \to m_2 = 3.3333$ L

$m_1 = 10.0 - m_2 = 6.6667$ L

We need 6.7 L of the 2% oil concentration mixture and 3.3 L of the 8% oil concentration mixture.

101. Let a, b, c be the measures of angles A, B, C respectively.

We have

$a + b + c = 180$

$a = 2b - 55$

$c = b - 25$

Substitute the latter two equations into the first:

$(2b - 55) + b + (b - 25) = 180$

$4b - 80 = 180$

$b = 65$

$c = 65 - 25 = 40$

$a = 130 - 55 = 75$

The three angles have measures $a = 75°, b = 65°, c = 40°$.

102. Let a, b, c be the numbers produced of models A, B, C respectively.

We have

$a + b + c = 97000$

$a = 4c$

$b = c + 7000$

Substitute the latter two equations into the first:

$4c + (c + 7000) + c = 97000$

$6c = 90000$

$c = 15000$

$a = 4c = 60000$

$b = c + 7000 = 22000$

Production: 60000 units of model A, 22 units of model B, 15 units of model C.

103. Let a, b, c be the memory required by programs A, B, C respectively.

We have

$a + b + c = 140 \quad \text{(I)}$

$a + 3b + 2c = 236 \quad \text{(II)}$

$2a + b + 3c = 304 \quad \text{(III)}$

Subtract (I) from (II) and subtract 2(I) from (III):

$2b + c = 96 \quad \text{(IV)}$

$-b + c = 24 \quad \text{(V)}$

Add 2(V) to (IV)

$3c = 144$

$c = 48$

$b = 24$

$a = 68$

Program A requires 68 MB of memory, program B requires 24 MB of memory, and program C requires 48 MB of memory.

104. Let a, b, c be the masses of silver, copper, and hydrogen produced, respectively.

We have

$a + b + c = 1.750 \quad \text{(I)}$

$a = 3.4b \quad \text{(II)}$

$b + 70c = a - 0.037 \quad \text{(III)}$

Substitute (II) into (I) and (III):

$4.4b + c = 1.750 \quad \text{(IV)}$

$-2.4b + 70c = -0.037 \quad \text{(V)}$

Subtract (V) from 70(IV):

$310.4b = 122.537$

$b = 0.39477126$

$a = 3.4b = 1.34222229$

$c = 1.750 - a - b = 0.01300645$

There are 1.34 grams of silver, 0.395 grams of copper, and 0.0130 grams of hydrogen produced.

105. Let a and b represent the weights of gold and silver in air, respectively.

The system of equations becomes

$a + b = 6.0 \quad \text{(I)}$

$0.947a + 0.9b = 5.6 \quad \text{(II)}$

Subtracting 0.9(I) from (II) yields

$0.047a = 0.2 \rightarrow a = 4.255$

$b = 6.0 - a = 1.745$

The gold weighs 4.3 N and the silver weighs 1.7 N.

106. Let w and i represent days worked and days idle.

The system of equations becomes

$w + i = 48 \quad \text{(I)}$

$24w - 12i = 504 \quad \text{(II)}$

Adding 12(I) to (II) yields

$36w = 1080 \rightarrow w = 30$

$i = 48 - w = 18$

The laborer worked 30 days and was idle 18 days.

107. Let a and b represent the volume of water per hour that the pumps can remove.

The system of equations becomes

$2.2a + 2.7b = 1100$ (I)

$1.4a + 2.5b = 840$ (II)

Since the coefficients are not obvious multiples of one another, substitution is unlikely to be the most efficient method. Also, addition or subtraction steps are unlikely to lead to a quick solution. One suggestion would be to scale each equation by a factor of 10 to remove decimals, obtaining

$22a + 27b = 11000$

$14a + 25b = 8400$

Now, Cramer's rule can be used:

$$a = \frac{\begin{vmatrix} 11000 & 27 \\ 8400 & 25 \end{vmatrix}}{\begin{vmatrix} 22 & 27 \\ 14 & 25 \end{vmatrix}} = \frac{48200}{172} = 280.2326$$

$$b = \frac{\begin{vmatrix} 22 & 11000 \\ 14 & 8400 \end{vmatrix}}{\begin{vmatrix} 22 & 27 \\ 14 & 25 \end{vmatrix}} = \frac{30800}{172} = 179.0698$$

So the first pump removes 280 ft^3 per hour

and the second pump removes 180 ft^3 per hour.

Chapter 6

Factoring and Fractions

6.1 Factoring: Greatest Common Factor and Difference of Squares

1. $4ax^2 - 2ax = 2ax(2x) - 2ax(1)$
$= 2ax(2x-1)$

2. $4ax^2 + 2ax = B$
$a(4x^2 + 2x) = B$ Factor out a
$a = \dfrac{B}{4x^2 + 2x}$ Divide both sides by $4x^2 + 2x$

3. $5x^2 - 45 = 5(x^2 - 9)$
$= 5(x+3)(x-3)$

4. $2x + 2y + ax + ay = 2(x+y) + a(x+y)$
$= (x+y)(2+a)$

5. $(T+6)(T-6) = T^2 - 6^2$
$= T^2 - 36$

6. $(s+5t)(s-5t) = s^2 - 5st + 5st - 25t^2$
$= s^2 - 25t^2$

7. $(4x-5y)(4x+5y) = 16x^2 + 20xy - 20xy - 25y^2$
$= 16x^2 - 25y^2$

8. $(3v-7y)(3v+7y) = 9v^2 + 21vy - 21vy - 49y^2$
$= 9v^2 - 49y^2$

9. $7x + 7y = 7(x+y)$

10. $3a - 3b = 3(a-b)$

11. $5a - 5 = 5(a-1)$

12. $2x^2 + 2 = 2(x^2+1)$

13. $3x^2 - 9x = 3x(x-3)$

14. $20s + 4s^2 = 4s(5+s)$

15. $7b^2h - 28b = 7b(bh-4)$

16. $5a^2 - 20ax = 5a(a - 4x)$

17. $72n^3 + 24n = 24n(3n^2 + 1)$

18. $90p^3 - 15p^2 = 15p^2(6p - 1)$

19. $2x + 4y - 8z = 2(x + 2y - 4z)$

20. $23a - 46b + 69c = 23(a - 2b + 3c)$

21. $3ab^2 - 6ab + 12ab^3 = 3ab(b - 2 + 4b^2)$

22. $4pq - 14q^2 - 16pq^2 = 2q(2p - 7q - 8pq)$

23. $12pq^2 - 8pq - 28pq^3 = 4pq(3q - 2 - 7q^2)$

24. $27a^2b - 24ab - 9a = 3a(9ab - 8b - 3)$

25. $2a^2 - 2b^2 + 4c^2 - 6d^2 = 2(a^2 - b^2 + 2c^2 - 3d^2)$

26. $5a + 10ax - 5ay + 20az = 5a(1 + 2x - y + 4z)$

27. $x^2 - 9 = x^2 - 3^2$
$x^2 - 9 = (x + 3)(x - 3)$

28. $r^2 - 25 = r^2 - 5^2$
$r^2 - 25 = (r + 5)(r - 5)$

29. $100 - 4A^2 = (10)^2 - (2A)^2$
$100 - 4A^2 = (10 + 2A)(10 - 2A)$

30. $49 - Z^4 = 7^2 - (Z^2)^2$
$49 - Z^4 = (7 + Z^2)(7 - Z^2)$

31. $36a^4 + 1$ is already in prime factored form, there are no common factors, and although it is a sum of squares, that is not a special product.

32. $324z^2 + 4$ is already in prime factored form, there are no common factors, and although it is a sum of squares, that is not a special product.

33. $$\begin{aligned}162s^2 - 50t^2 &= 2\left(81s^2 - 25t^2\right)\\ &= 2((9s)^2 - (5t)^2)\\ &= 2(9s+5t)(9s-5t)\end{aligned}$$

34. $$\begin{aligned}36s^2 - 121t^4 &= (6s)^2 - \left(11t^2\right)^2\\ &= \left(6s+11t^2\right)\left(6s-11t^2\right)\end{aligned}$$

35. $$\begin{aligned}144n^2 - 169p^4 &= (12n)^2 - \left(13p^2\right)^2\\ &= \left(12n+13p^2\right)\left(12n-13p^2\right)\end{aligned}$$

36. $36a^2b^2 + 169c^2$ is already in prime factored form. It is a sum of squares, but that is not a special product result.

37. $$\begin{aligned}(x+y)^2 - 9 &= ((x+y)+3)((x+y)-3)\\ &= (x+y+3)(x+y-3)\end{aligned}$$

38. $$\begin{aligned}(a-b)^2 - 1 &= [(a-b)+1][(a-b)-1]\\ &= (a-b+1)(a-b-1)\end{aligned}$$

39. $$\begin{aligned}8 - 2x^2 &= 2\left(4 - x^2\right)\\ &= 2(2+x)(2-x)\end{aligned}$$

40. $$\begin{aligned}5a^4 - 125a^2 &= 5a^2\left(a^2 - 25\right)\\ &= 5a^2(a+5)(a-5)\end{aligned}$$

41. $$\begin{aligned}300x^2 - 2700z^2 &= 300\left(x^2 - 9z^2\right)\\ &= 300(x+3z)(x-3z)\end{aligned}$$

42. $$\begin{aligned}28x^2 - 700y^2 &= 7\left(4x^2 - 100y^2\right)\\ &= 7(2x+10y)(2x-10y)\end{aligned}$$

43. $$\begin{aligned}2(I-3)^2 - 8 &= 2\left[(I-3)^2 - 4\right]\\ &= 2[(I-3)+2][(I-3)-2]\\ &= 2(I-1)(I-5)\end{aligned}$$

44. $$\begin{aligned}a(x+2)^2 - ay^2 &= a\left[(x+2)^2 - y^2\right]\\ &= a[(x+2)+y][(x+2)-y]\\ &= a(x+2+y)(x+2-y)\end{aligned}$$

45. $x^4 - 16 = (x^2 + 4)(x^2 - 4)$
$= (x^2 + 4)(x + 2)(x - 2)$

46. $81 - y^4 = (9 + y^2)(9 - y^2)$
$= (9 + y^2)(3 + y)(3 - y)$

47. $x^{10} - x^2 = x^2(x^8 - 1)$
$= x^2(x^4 + 1)(x^4 - 1)$
$= x^2(x^4 + 1)(x^2 + 1)(x^2 - 1)$
$= x^2(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$

48. $2x^4 - 8y^4 = 2(x^4 - 4y^4)$
$= 2(x^2 + 2y^2)(x^2 - 2y^2)$

49. Solve $2a - b = ab + 3$ for a.
$$\begin{aligned} 2a - ab &= b + 3 \\ a(2 - b) &= b + 3 \\ a &= \frac{b + 3}{2 - b} \end{aligned}$$

50. Solve $n(x + 1) = 5 - x$ for x.
$$\begin{aligned} n(x + 1) &= 5 - x \\ nx + n &= 5 - x \\ nx + x &= 5 - n \\ x(n + 1) &= 5 - n \\ x &= \frac{5 - n}{n + 1} \end{aligned}$$

51. Solve $3 - 2s = 2(3 - st)$ for s.
$$\begin{aligned} 3 - 2s &= 2(3 - st) \\ 3 - 2s &= 6 - 2st \\ 2st - 2s &= 6 - 3 \\ 2s(t - 1) &= 3 \\ s &= \frac{3}{2(t - 1)} \end{aligned}$$

52. Solve $k(2-y)=y(2k-1)$ for y.

$$\begin{aligned} k(2-y) &= y(2k-1) \\ 2k-ky &= 2ky-y \\ 2k &= 3ky-y \\ y(3k-1) &= 2k \\ y &= \frac{2k}{3k-1} \end{aligned}$$

53. Solve $(x+2k)(x-2)=x^2+3x-4k$ for k.

$$\begin{aligned} (x+2k)(x-2) &= x^2+3x-4k \\ x^2-2x+2kx-4k &= x^2+3x-4k \\ 2kx &= 5x \\ k &= \frac{5}{2} \end{aligned}$$

54. Solve $(2x-3k)(x+1)=2x^2-x-3$ for k.

$$\begin{aligned} (2x-3k)(x+1) &= 2x^2-x-3 \\ 2x^2+2x-3kx-3k &= 2x^2-x-3 \\ 3kx+3k &= 3x+3 \\ k(3x+3) &= (3x+3) \\ k &= \frac{(3x+3)}{(3x+3)} \\ k &= 1 \end{aligned}$$

55. $$\begin{aligned} 3x-3y+bx-by &= 3(x-y)+b(x-y) \\ &= (x-y)(3+b) \end{aligned}$$

56. $$\begin{aligned} am+an+cn+cm &= (am+an)+(cn+cm) \\ &= a(m+n)+c(n+m) \\ &= (m+n)(a+c) \end{aligned}$$

57. $$\begin{aligned} a^2+ax-ab-bx &= (a^2+ax)-(ab+bx) \\ &= a(a+x)-b(a+x) \\ &= (a+x)(a-b) \end{aligned}$$

58. $$\begin{aligned} 2y-y^2-6y^4+12y^3 &= (2y-y^2)+(-6y^4+12y^3) \\ &= y(2-y)+6y^3(-y+2) \\ &= (2-y)(y+6y^3) \\ &= (2-y)y(1+6y^2) \\ &= y(2-y)(1+6y^2) \end{aligned}$$

59. $x^3 + 3x^2 - 4x - 12 = x^2(x+3) - 4(x+3)$
$= (x+3)(x^2 - 4)$
$= (x+3)(x+2)(x-2)$

60. $S^3 - 5S^2 - S + 5 = (S^3 - 5S^2) - 1(S-5)$
$= S^2(S-5) - 1(S-5)$
$= (S-5)(S^2 - 1)$
$= (S-5)(S+1)(S-1)$

61. $x^2 - y^2 + x - y = (x^2 - y^2) + (x - y)$
$= (x+y)(x-y) + (x-y)$
$= (x-y)(x+y+1)$

62. $4p^2 - q^2 + 2p + q = (4p^2 - q^2) + (2p + q)$
$= (2p+q)(2p-q) + (2p+q)$
$= (2p+q)(2p-q+1)$

63. $\dfrac{8^9 - 8^8}{7} = \dfrac{8^8(8-1)}{7}$
$= \dfrac{8^8(7)}{7}$
$= 8^8$
$= 16\ 777\ 216$

64. $\dfrac{5^9 - 5^7}{7^2 - 5^2} = \dfrac{5^7(5^2 - 1)}{(7+5)(7-5)}$
$= \dfrac{5^7(5+1)(5-1)}{(7+5)(7-5)}$
$= 5^7 \dfrac{6(4)}{(12)(2)}$
$= 78\ 125$

65. Since $n^2 + n = n(n+1)$, this is the product of two consecutive integers of which one must be even. Therefore, the product is even.

66. Since $n^3 - n = n(n^2 - 1)$

$$= n(n+1)(n-1)$$
$$= (n-1)(n)(n+1),$$

this is the product of three consecutive integers one of which must be a multiple of 2 and one of which must be a multiple of 3. Therefore, the product is a multiple of 6.

67. $2Q^2 + 2 = 2(Q^2 + 1)$

68. $4d^2D^2 - 4d^3D - d^4 = d^2\left(4D^2 - 4Dd - d^2\right)$

69. $81s - s^3 = s(81 - s^2)$

$$= s(9+s)(9-s)$$

70. $12(4-x^2) - 2x(4-x^2) - (4-x^2)^2 = (4-x^2)(12 - 2x - (4-x^2))$

$$= (4-x^2)(8 - 2x + x^2)$$

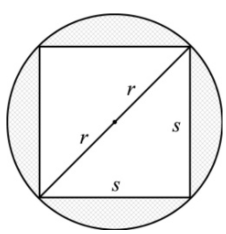
71. $rR^2 - r^3 = r\left(R^2 - r^2\right)$

$$= r(R+r)(R-r)$$

72. $p_1R^2 - p_1r^2 - p_2R^2 + p_2r^2 = p_1\left(R^2 - r^2\right) - p_2\left(R^2 - r^2\right)$

$$= (p_1 - p_2)\left(R^2 - r^2\right)$$
$$= (p_1 - p_2)(R+r)(R-r)$$

73.



Using Pythagoras' Theorem

$$s^2 + s^2 = (2r)^2$$
$$2s^2 = 4r^2$$
$$s^2 = 2r^2$$

Area of square $= s^2 = 2r^2$

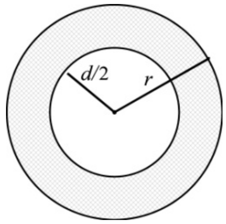
Area of circle $= \pi r^2$

Area left = Area of circle − Area of square

Area left $= \pi r^2 - 2r^2$

Area left $= r^2(\pi - 2)$

74.



$$\text{Area between} = \pi r^2 - \pi\left(\frac{d}{2}\right)^2$$

$$\text{Area between} = \pi\left(r^2 - \left(\frac{d}{2}\right)^2\right)$$

$$\text{Area between} = \pi\left(r + \frac{d}{2}\right)\left(r - \frac{d}{2}\right)$$

75.

$$\begin{aligned}
\text{Surface area of outer} &= 4\pi r_2^2 \\
\text{Surface area of inner} &= 4\pi r_1^2 \\
\text{Difference of surface areas} &= 4\pi r_2^2 - 4\pi r_1^2 \\
&= 4\pi(r_2^2 - r_1^2) \\
&= 4\pi(r_2 + r_1)(r_2 - r_1)
\end{aligned}$$

76.

$$\begin{aligned}
\text{Original kinetic energy} &= \tfrac{1}{2}m_0 v_1^2 \\
\text{Final kinetic energy} &= \tfrac{1}{2}m_0 v_2^2 \\
\text{Difference of kinetic energies} &= \tfrac{1}{2}m_0 v_1^2 - \tfrac{1}{2}m_0 v_2^2 \\
&= \tfrac{1}{2}m_0(v_1^2 - v_2^2) \\
&= \tfrac{1}{2}m_0(v_1 + v_2)(v_1 - v_2)
\end{aligned}$$

77. Solve $i_1R_1 = (i_2 - i_1)R_2$ for i_1.

$$\begin{aligned}
i_1R_1 &= (i_2 - i_1)R_2 \\
i_1R_1 &= i_2R_2 - i_1R_2 \\
i_1R_1 + i_1R_2 &= i_2R_2 \\
i_1(R_1 + R_2) &= i_2R_2 \\
i_1 &= \frac{i_2R_2}{R_1 + R_2}
\end{aligned}$$

78. Solve $nV + n_1v = n_1V$ for n_1.

$$\begin{aligned}
nV + n_1v &= n_1V \\
nV &= n_1V - n_1v \\
nV &= n_1(V - v) \\
n_1 &= \frac{nV}{V - v}
\end{aligned}$$

79. Solve $3BY + 5Y = 9BS$ for B.

$$\begin{aligned} 3BY + 5Y &= 9BS \\ 5Y &= 9BS - 3BY \\ 5Y &= B(9S - 3Y) \\ B &= \frac{5Y}{9S - 3Y} \end{aligned}$$

80. Solve $Sq + Sp = Spq + p$ for q.

$$\begin{aligned} Sq + Sp &= Spq + p \\ Sq - Spq &= p - Sp \\ q(S - Sp) &= p - Sp \\ q &= \frac{p - Sp}{S - Sp} \end{aligned}$$

81. Solve $ER = AtT_0 - AtT_1$ for t

$$\begin{aligned} ER &= AtT_0 - AtT_1 \\ ER &= At\left(T_0 - T_1\right) \\ t &= \frac{ER}{A\left(T_0 - T_1\right)} \end{aligned}$$

82. Solve $R = kT_2^4 - kT_1^4$ for k

$$\begin{aligned} R &= kT_2^4 - kT_1^4 \\ R &= k\left(T_2^4 - T_1^4\right) \\ k &= \frac{R}{\left(T_2^4 - T_1^4\right)} \\ k &= \frac{R}{\left(T_2^2 + T_1^2\right)\left(T_2^2 - T_1^2\right)} \\ k &= \frac{R}{\left(T_2^2 + T_1^2\right)\left(T_2 + T_1\right)\left(T_2 - T_1\right)} \end{aligned}$$

6.2 Factoring Trinomials

1. $x^2 + 4x + 3 = (x + 3)(x + 1)$

2. $x^2 - 7x - 8 = (x - 8)(x + 1)$

3. $2x^2 - 11x + 5 = (2x - 1)(x - 5)$

4. $$\begin{aligned} 2x^2 + 6x - 36 &= 2\left(x^2 + 3x - 18\right) \\ &= 2(x + 6)(x - 3) \end{aligned}$$

5. $(x-7)^2 = x^2 - 2(7)x + 7^2$
$= x^2 - 14x + 49$

6. $(y+6)^2 = y^2 + 2(6)y + 6^2$
$= y^2 + 12y + 36$

7. $(a+3b)^2 = a^2 + 2(a)(3b) + (3b)^2$
$= a^2 + 6ab + 9b^2$

8. $(2n+5m)^2 = (2n)^2 + 2(2n)(5m) + (5m)^2$
$= 4n^2 + 20nm + 25m^2$

9. $x^2 + 4x + 3 = (x+1)(x+3)$

10. $x^2 - 5x - 6 = (x-6)(x+1)$

11. $s^2 - s - 42 = (s-7)(s+6)$

12. $a^2 + 14a - 32 = (a+16)(a-2)$

13. $t^2 + 5t - 24 = (t+8)(t-3)$

14. $r^3 - 11r^2 + 18r = r(r-9)(r-2)$

15. $x^2 + 8x + 16 = (x+4)(x+4)$
$= (x+4)^2$

16. $D^2 + 8D + 16 = (D+4)(D+4)$
$= (D+4)^2$

17. $a^2 - 6ab + 9b^2 = (a-3b)(a-3b)$
$= (a-3b)^2$

18. $b^2 - 12bc + 36c^2 = (b-6c)(b-6c)$
$= (b-6c)^2$

19. $3x^2 - 5x - 2 = (3x+1)(x-2)$
(because $-5x = -6x + x$)

20. $6n^2 - 39n - 21 = 3(2n^2 - 13n - 7)$
$= 3(2n+1)(n-7)$
(because $-13n = -14n + n$)

21. $12y^2-32y-12=4(3y^2-8y-3)$
$=4(3y+1)(y-3)$
because $-8y=y-9y$

22. $25x^2+45x-10=5(5x^2+9x-2)$
$=5(5x-1)(x+2)$
(because $9x=10x-x$)

23. $2s^2+13s+11=(2s+11)(s+1)$
(because $13s=2s+11s$)

24. $5-12y+7y^2=7y^2-12y+5$
$=(7y-5)(y-1)$
(because $-12y=-7y-5y$)

25. $3z^2-19z+6=(3z-1)(z-6)$
(because $-19z=-18z-z$)

26. $10R^4-6R^2-2=2(5R^4-3R^2-2)$
$=2(5R^2+2)(R^2-1)$
$=2(5R^2+2)(R+1)(R-1)$
(because $-3R^2=-5R^2+2R^2$)

27. $2t^2+7t-15=(2t-3)(t+5)$
(because $7t=10t-3t$)

28. $20-20n+3n^2=3n^2-20n+20$ is prime (cannot be further factored)

29. $3t^2-7tu+4u^2=(3t-4u)(t-u)$
(because $-7tu=-4tu-3tu$)

30. $3x^2+xy-14y^2=(3x+7y)(x-2y)$
(because $xy=-6xy+7xy$)

31. $6x^2+x-5=(6x-5)(x+1)$
(because $x=6x-5x$)

32. $2z^2+13z-5$ is prime (cannot be further factored)

33. $9x^2+7xy-2y^2=(x+y)(9x-2y)$
(because $7xy=9xy-2xy$)

34. $4r^2 + 11rs - 3s^2 = (4r - s)(r + 3s)$

(because $11rs = 12rs - rs$)

35. $$\begin{aligned} 12m^2 + 60m + 75 &= 3(4m^2 + 20m + 25) \\ &= 3(2m + 5)(2m + 5) \\ &= 3(2m + 5)^2 \end{aligned}$$

(because $20m = 10m + 10m$)

36. $$\begin{aligned} 48q^2 + 72q + 27 &= 3(16q^2 + 24q + 9) \\ &= 3(4q + 3)(4q + 3) \\ &= 3(4q + 3)^2 \end{aligned}$$

(because $24q = 12q + 12q$)

37. $$\begin{aligned} 8x^2 - 24x + 18 &= 2(4x^2 - 12x + 9) \\ &= 2(2x - 3)(2x - 3) \\ &= 2(2x - 3)^2 \end{aligned}$$

(because $-12x = -6x - 6x$)

38. $$\begin{aligned} 3a^2c^2 - 6ac + 3 &= 3(a^2c^2 - 2ac + 1) \\ &= 3(ac - 1)(ac - 1) \\ &= 3(ac - 1)^2 \end{aligned}$$

(because $-2ac = -ac - ac$)

39. $9t^2 - 15t + 4 = (3t - 4)(3t - 1)$

(because $-15t = -3t - 12t$)

40. $6t^4 + t^2 - 12 = (3t^2 - 4)(2t^2 + 3)$

(because $t^2 = -8t^2 + 9t^2$)

41. $8b^6 + 31b^3 - 4 = (8b^3 - 1)(b^3 + 4)$

(because $31b^3 = -b^3 + 32b^3$)

But the first factor is actually a difference of cubes
(discussed in Section 6.4)
So this could be further factored to

$$\begin{aligned} 8b^6 + 31b^3 - 4 &= ((2b)^3 - 1^3)(b^3 + 4) \\ &= (2b - 1)(4b^2 + 2b + 1)(b^3 + 4) \end{aligned}$$

42. $12n^4 + 8n^2 - 15 = (6n^2 - 5)(2n^2 + 3)$

(because $8n^2 = 18n^2 - 10n^2$)

43. $4p^2 - 25pq + 6q^2 = (4p - q)(p - 6q)$
(because $-25pq = -24pq - pq$)

44. $12x^2 + 4xy - 5y^2 = (2x - y)(6x + 5y)$
(because $4xy = 10xy - 6xy$)

45. $12x^2 + 47xy - 4y^2 = (12x - y)(x + 4y)$
(because $47xy = -xy + 48xy$)

46. $8r^2 - 14rs - 9s^2 = (2r + s)(4r - 9s)$
(because $-14rs = -18rs + 4rs$)

47.
$$\begin{aligned} 12 - 14x + 2x^2 &= 2x^2 - 14x + 12 \\ &= 2(x^2 - 7x + 6) \\ &= 2(x - 1)(x - 6) \end{aligned}$$

48.
$$\begin{aligned} 6y^2 - 33y - 18 &= 3(2y^2 - 11y - 6) \\ &= 3(2y + 1)(y - 6) \end{aligned}$$
(because $-12y + y = -11y$)

49.
$$\begin{aligned} 4x^5 + 14x^3 - 8x &= 2x(2x^4 + 7x^2 - 4) \\ &= 2x(2x^2 - 1)(x^2 + 4) \end{aligned}$$
because $8x^2 - x^2 = 7x^2$

50.
$$\begin{aligned} 12B^2 + 22BH - 4H^2 &= 2(6B^2 + 11BH - 2H^2) \\ &= 2(6B - H)(B + 2H) \end{aligned}$$
because $12BH - BH = 11BH$

51.
$$\begin{aligned} ax^3 + 4a^2x^2 - 12a^3x &= ax(x^2 + 4ax - 12a^2) \\ &= ax(x + 6a)(x - 2a) \end{aligned}$$

52.
$$\begin{aligned} 15x^2 - 39x^3 + 18x^4 &= 18x^4 - 39x^3 + 15x^2 \\ &= 3(6x^4 - 13x^3 + 5x^2) \\ &= 3x^2(6x^2 - 13x + 5) \\ &= 3x^2(2x - 1)(3x - 5) \end{aligned}$$
(because $-10x^2 - 3x^2 = -13x^2$)

53. $4x^{2n} + 13x^n - 12 = (4x^n - 3)(x^n + 4)$
(because $16x^n - 3x^n = 13x^n$)

54. $12B^{2n} + 19B^nH - 10H^2 = (12B^n - 5H)(B^n + 2H)$
(because $24HB^n - 5HB^n = 19HB^n$)

55. $$16t^2-80t+64=16\left(t^2-5t+4\right)$$
$$=16\left(t-4\right)\left(t-1\right)$$

56. $$9x^2-33Lx+30L^2=3\left(3x^2-11Lx+10L^2\right)$$
$$=3\left(3x-5L\right)\left(x-2L\right)$$

57. $$d^4-10d^2+16^2=(d^2-8)(d^2-2)$$

58. $$3e^2+18e-1560=3(e^2+6e-520)$$
$$=3(e+26)(e-20)$$

59. $$3Q^2+Q-30=(3Q+10)(Q-3)$$

60. $$bT^2-40bT+400b=b\left(T^2-40T+400\right)$$
$$=b\left(T-20\right)^2$$

61. $$V^2-2nBV+n^2B^2=\left(V-nB\right)^2$$

62. $$a^4+8a^2\pi^2f^2+16\pi^4f^4=\left(a^2+4\pi^2f^2\right)^2$$

63. $$wx^4-5wLx^3+6wL^2x^2=wx^2\left(x^2-5Lx+6L^2\right)$$
$$=wx^2\left(x-3L\right)\left(x-2L\right)$$

64. $$1-2r^2+r^4=\left(1-r^2\right)\left(1-r^2\right)$$
$$=\left(1+r\right)\left(1-r\right)\left(1+r\right)\left(1-r\right)$$
$$=\left(1+r\right)^2\left(1-r\right)^2$$

65. $$3Adu^2-4Aduv+Adv^2=Ad\left(3u^2-4uv+v^2\right)$$
$$=Ad\left(3u-v\right)\left(u-v\right)$$

66. $$k^2A^2-2k\lambda A+\lambda^2-\alpha^2=\left(k^2A^2+2k\lambda A+\lambda^2\right)-\alpha^2$$
$$=\left(kA+\lambda\right)^2-\alpha^2$$
$$=\left(kA+\lambda+\alpha\right)\left(kA+\lambda-\alpha\right)$$

67. Find two integer values of k that make $4x^2+kx+9$ a perfect square trinomial
If $4x^2+kx+9=(ax+b)^2$ then $a^2=4$ and $b^2=9$. The four possibilities are $(2x+3)^2=4x^2+12x+9$, $(2x-3)^2=4x^2-12x+9$, $(-2x+3)^2=4x^2-12x+9$ and $(-2x-3)^2=4x^2+12x+9$. The two possible values of k are $k=12$ or $k=-12$.

68. Find two integer values of k that make $16y^2 + ky + 25$ a perfect square trinomial
If $16y^2 + ky + 25 = (ay + b)^2$ then $a^2 = 16$ and $b^2 = 25$. The four possibilities are
$(4y+5)^2 = 16y^2 + 40y + 25$, $(4y-5)^2 = 16y^2 - 40y + 25$, $(-4y+5)^2 = 16y^2 - 40y + 25$ and
$(-4y-5)^2 = 16y^2 + 40y + 25$. The two possible values of k are $k = 40$ or $k = -40$.

69. Find six values of k such that $x^2 + kx + 18$ can be factored
Since $18 = (1)(18) = (2)(9) = (3)(6)$, we obtain
$(x+1)(x+18) = x^2 + 19x + 18$
$(x+2)(x+9) = x^2 + 11x + 18$
$(x+3)(x+6) = x^2 + 9x + 18$
Also, $18 = (-1)(-18) = (-2)(-9) = (-3)(-6)$ and so
$(x-1)(x-18) = x^2 - 19x + 18$
$(x-2)(x-9) = x^2 - 11x + 18$
$(x-3)(x-6) = x^2 - 9x + 18$
and so six values of k are -19, -11, -9, 9, 11, or 19.

70. $24x^2 - 23x - 12$ has many more middle term possibilities
since 24 and -12 have many possible factors each.
For $23x^2 - 18x - 5$, the numbers 23 and 5 are prime, and so the
factors are more easily determined.

71. If the height is given by $-16t^2 + V_0 t + S_0$ and $V_0 = 32$ and $S_0 = 128$, we have
$-16t^2 + 32t + 128 = -16(t^2 - 2t - 8) = -16(t-4)(t+2)$.

72. If the height is given by $-16t^2 + V_0 t + S_0$ and $V_0 = 24$ and $S_0 = 40$, we have
$-16t^2 + 24t + 40 = -8(2t^2 - 3t - 5) = -8(2t-5)(t+1)$.

6.3 The Sum and Difference of Cubes

1.
$$\begin{aligned} x^3 - 8 &= x^3 - 2^3 \\ &= (x-2)(x^2 + 2x + 2^2) \\ &= (x-2)(x^2 + 2x + 4) \end{aligned}$$

2.
$$\begin{aligned} ax^5 + a^4x^2 &= ax^2(x^3 + a^3) \\ &= ax^2(x+a)(x^2 - ax + a^2) \end{aligned}$$

3. $x^3 + 1 = (x+1)(x^2 - x + 1)$

4.
$$\begin{aligned} R^3 + 27 &= R^3 + 3^3 \\ &= (R+3)(R^2 - 3R + 9) \end{aligned}$$

5. $y^3 - 125 = y^3 - 5^3$
$= (y-5)(y^2 + 5y + 25)$

6. $z^3 - 8 = z^3 - 2^3$
$= (z-2)(z^2 + 2z + 4)$

7. $27 - t^3 = 3^3 - t^3$
$= (3-t)(9 + 3t + t^2)$

8. $8r^3 - 1 = (2r)^3 - 1^3$
$= (2r-1)(4r^2 + 2r + 1)$

9. $8a^3 - 27b^3 = (2a)^3 - (3b)^3$
$= (2a - 3b)(4a^2 + 6ab + 9b^2)$

10. $64x^4 + 125x = x\left((4x)^3 + 5^3\right)$
$= x(4x+5)(16x^2 - 20x + 25)$

11. $4x^3 + 32 = 4(x^3 + 8)$
$= 4(x+2)(x^2 - 2x + 4)$

12. $3y^3 - 81 = 3(y^3 - 27)$
$= 3(y-3)(y^2 + 3y + 9)$

13. $7n^5 - 7n^2 = 7n^2(n^3 - 1)$
$= 7n^2(n-1)(n^2 + n + 1)$

14. $64 - 8s^9 = 8\left(8 - (s^3)^3\right)$
$= 8\left(2^3 - (s^3)^3\right)$
$= 8(2 - s^3)(4 + 2s^3 + s^6)$

15. $54x^3y - 6x^3y^4 = 6x^3y(9 - y^3)$
which cannot be factored further.

16. $12a^3 + 96a^3b^3 = 12a^3(1 + 8b^3)$
$= 12a^3(1+2b)(1 - 2b + 4b^2)$

17. $x^6y^3 + x^3y^6 = x^3y^3\left(x^3 + y^3\right)$
$= x^3y^3(x+y)\left(x^2 - xy + y^2\right)$

18. $16r^3 - 432 = 16\left(r^3 - 27\right)$
$= 16(r-3)\left(r^2 + 3r + 9\right)$

19. $3a^6 - 3a^3 = 3a^3\left(a^3 - 1\right)$
$= 3a^3(a-1)\left(a^2 + 2a + 1\right)$

20. $81y^2 - x^6 = (9y + x^3)(9y - x^3)$

21. $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(R^3 - r^3)$
$= \frac{4}{3}\pi(R - r)(R^2 + Rr + r^2)$

22. $0.027x^3 + 0.125 = (0.3x)^3 + (0.5)^3$
$= (0.3x + 0.5)\left(0.09x^2 - 0.15x + 0.25\right)$

23. $27L^6 + 216L^3 = 27L^3\left(L^3 + 8\right)$
$= 27L^3(L+2)\left(L^2 - 2L + 4\right)$

24. $a^3s^5 - 8000a^3s^2 = a^3s^2\left(s^3 - 8000\right)$
$= a^3s^2(s-20)\left(s^2 + 20s + 400\right)$

25. $(a+b)^3 + 64 = (a+b)^3 + 4^3$
$= (a+b+4)\left((a+b)^2 - 4(a+b) + 4^2\right)$
$= (a+b+4)\left(a^2 + 2ab + b^2 - 4a - 4b + 16\right)$

26. $125 + (2x+y)^3 = 5^3 + (2x+y)^3$
$= \left[5 + (2x+y)\right]\left[25 - 5(2x+y) + (2x+y)^2\right]$
$= (2x + y + 5)\left(4x^2 + 4xy + y^2 - 10x - 5y + 25\right)$

27. $64 - x^6 = 4^3 - \left(x^2\right)^3$
$= \left(4 - x^2\right)\left(16 + 4x^2 + x^4\right)$
$= (2+x)(2-x)\left(16 + 4x^2 + x^4\right)$

28. $$\begin{aligned} 2a^6 - 54b^6 &= 2(a^6 - 27b^6) \\ &= 2((a^2)^3 - (3b^2)^3) \\ &= 2\left(a^2 - 3b^2\right)\left(\left(a^2\right)^2 + 3a^2b^2 + \left(3b^2\right)^2\right) \\ &= 2\left(a^2 - 3b^2\right)\left(a^4 + 3a^2b^2 + 9b^4\right) \end{aligned}$$

29. $$\begin{aligned} x^6 - 2x^3 + 1 &= (x^3 - 1)^2 \\ &= ((x-1)(x^2 + x + 1))^2 \\ &= (x-1)^2(x^2 + x + 1)^2 \end{aligned}$$

30. $n^6 + 4n^3 + 4 = (n^3 + 2)^2$
This factors into no smaller prime factors.

31. $$\begin{aligned} 32x - 4x^4 &= 4x(8 - x^3) \\ &= 4x(2 - x)(4 + 2x + x^2) \end{aligned}$$

32. $$\begin{aligned} kT^3 - kT_0^3 &= k\left(T^3 - T_0^3\right) \\ &= k\left(T - T_0\right)\left(T^2 + T \cdot T_0 + T_0^2\right) \end{aligned}$$

33. $$\begin{aligned} D^4 - d^3D &= D\left(D^3 - d^3\right) \\ &= D\left(D - d\right)\left(D^2 + Dd + d^2\right) \end{aligned}$$

34. $$\begin{aligned} \left(h + 2t\right)^3 - h^3 &= \left(h + 2t - h\right)\left(\left(h + 2t\right)^2 + \left(h + 2t\right)h + h^2\right) \\ &= 2t\left(h^2 + 4ht + 4t^2 + h^2 + 2ht + h^2\right) \\ &= 2t\left(3h^2 + 6ht + 4t^2\right) \end{aligned}$$

35. $$\begin{aligned} QH^4 + Q^4H &= QH\left(H^3 + Q^3\right) \\ &= QH\left(H + Q\right)\left(H^2 - HQ + Q^2\right) \end{aligned}$$

36. $$\begin{aligned} \left(\frac{s}{r}\right)^{12} - \left(\frac{s}{r}\right)^6 &= \left(\frac{s}{r}\right)^6\left(\left(\frac{s}{r}\right)^6 - 1\right) \\ &= \frac{s^6}{r^6}\left(\left(\left(\frac{s}{r}\right)^3\right)^2 - 1\right) \\ &= \frac{s^6}{r^6}\left(\left(\frac{s}{r}\right)^3 + 1\right)\left(\left(\frac{s}{r}\right)^3 - 1\right) \\ &= \frac{s^6}{r^6}\left(\frac{s}{r} + 1\right)\left(\frac{s^2}{r^2} - \frac{s}{r} + 1\right)\left(\frac{s}{r} - 1\right)\left(\frac{s^2}{r^2} + \frac{s}{r} + 1\right) \end{aligned}$$

37. $$\begin{aligned}(a+b)(a^2-ab+b^2) &= a(a^2-ab+b^2)+b(a^2-ab+b^2)\\ &= (a^3-a^2b+ab^2)+(a^2b-ab^2+b^3)\\ &= a^3+b^3\end{aligned}$$

38. $$\begin{aligned}(a-b)(a^2+ab+b^2) &= a(a^2+ab+b^2)-b(a^2+ab+b^2)\\ &= (a^3+a^2b+ab^2)-(a^2b+ab^2+b^3)\\ &= a^3-b^3\end{aligned}$$

39. $$\begin{aligned}x^6-y^6 &= (x^2)^3-(y^2)^3\\ &= (x^2-y^2)(x^4+x^2y^2+y^4)\end{aligned}$$

40. $$\begin{aligned}x^6-y^6 &= (x^3)^2-(y^3)^2\\ &= (x^3+y^3)(x^3-y^3)\end{aligned}$$

To show that this is the same as in exercise 39,

$$\begin{aligned}(x^3+y^3)(x^3-y^3) &= (x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)\\ &= (x+y)(x-y)(x^2+y^2+xy)(x^2+y^2-xy)\\ &= (x^2-y^2)((x^2+y^2)^2-(xy)^2)\\ &= (x^2-y^2)((x^4+2x^2y^2+y^4)-x^2y^2)\\ &= (x^2-y^2)(x^4+x^2y^2+y^4)\end{aligned}$$

41. $n^3+1=(n+1)(n^2-n+1)$

which, for $n>1$, is the product of two integers each greater than 1. Thus, this product cannot be prime.

42. $$\begin{aligned}1200v_2^3-1200v_1^3 &= 1200(v_2^3-v_1^3)\\ &= 1200(v_2-v_1)(v_2^2+v_1v_2+v_1^2)\end{aligned}$$

6.4 Equivalent Fractions

1. $$\begin{aligned}\frac{x^2-4x-12}{x^2-4} &= \frac{(x+2)(x-6)}{(x+2)(x-2)}\\ &= \frac{x-6}{x-2} \text{ where } x\neq 2, x\neq -2\end{aligned}$$

2. $$\begin{aligned}\frac{2x^4-32x^2}{20+7x-3x^2} &= \frac{2x^2\left(x^2-16\right)}{(4-x)(5+3x)}\\ &= \frac{2x^2(x+4)(x-4)}{-(x-4)(3x+5)}\\ &= -\frac{2x^2(x+4)}{3x+5} \text{ where } x\neq 4, x\neq -\tfrac{5}{3}\end{aligned}$$

3. $\dfrac{2}{3}=\dfrac{2}{3}\cdot\dfrac{7}{7}=\dfrac{14}{21}$

4. $\dfrac{7}{5}=\dfrac{7(9)}{5(9)}=\dfrac{63}{45}$

5. $\dfrac{ax}{y}=\dfrac{ax(3a)}{y(3a)}=\dfrac{3a^2x}{3ay}$

6. $\dfrac{2x^2y}{3n}=\dfrac{2x^2y\left(2xn^2\right)}{3n\left(2xn^2\right)}=\dfrac{4n^2x^3y}{6n^3x}$

7. $\dfrac{2}{x+3}=\dfrac{2(x-2)}{(x+3)(x-2)}=\dfrac{2x-4}{x^2+x-6}$

8. $\dfrac{7}{a-1}=\dfrac{7(a+2)}{(a-1)(a+2)}=\dfrac{7a+14}{a^2+a-2}$

9.
$$\begin{aligned}\frac{a(x-y)}{x-2y}&=\frac{a(x-y)(x+y)}{(x-2y)(x+y)}\\&=\frac{a\left(x^2-y^2\right)}{x^2-xy-2y^2}\\&=\frac{ax^2-ay^2}{x^2-xy-2y^2}\end{aligned}$$

10. $\dfrac{B-1}{B+1}=\dfrac{(B-1)(1-B)}{(B+1)(1-B)}=\dfrac{-(1-B)(1-B)}{(1+B)(1-B)}=\dfrac{-B^2+2B-1}{1-B^2}$

11. $\dfrac{28}{44}=\dfrac{\frac{28}{4}}{\frac{44}{4}}=\dfrac{7}{11}$

12. $\dfrac{25}{65}=\dfrac{\frac{25}{5}}{\frac{65}{5}}=\dfrac{5}{13}$

13. $\dfrac{4x^2y}{8xy^2}=\dfrac{\frac{4x^2y}{2x}}{\frac{8xy^2}{2x}}=\dfrac{2xy}{4y^2}$ where $x, y\neq 0$

14. $\dfrac{6a^3b^2}{9a^5b^4}=\dfrac{\frac{6a^3b^2}{3a^2b^2}}{\frac{9a^5b^4}{3a^2b^2}}=\dfrac{2a}{3a^3b^2}$ where $a, b\neq 0$

15. $\dfrac{4(R-2)}{(R-2)(R+2)}=\dfrac{\frac{4(R-2)}{(R-2)}}{\frac{(R-2)(R+2)}{(R-2)}}=\dfrac{4}{R+2}$ where $R\neq -2, 2$

16. $\dfrac{(x+5)(x-3)}{3(x+5)} = \dfrac{\frac{(x+5)(x-3)}{(x+5)}}{\frac{3(x+5)}{(x+5)}} = \dfrac{x-3}{3}$ where $x \neq -5$

17. $\dfrac{s^2-3s-10}{2s^2-3s-2} = \dfrac{\frac{(s+2)(s-5)}{(s+2)}}{\frac{(s+2)(2s-1)}{(s+2)}} = \dfrac{s-5}{2s-1}$ where $s \neq -2, \frac{1}{2}$

18. $\dfrac{6x^2+13x-5}{6x^3-2x^2} = \dfrac{\frac{(3x-1)(2x+5)}{(1-3x)}}{\frac{2x^2(3x-1)}{(1-3x)}} = \dfrac{-(2x+5)}{-2x^2}$ where $x \neq 0, \frac{1}{3}$

19.
$$\frac{A}{6y^2} = \frac{3x}{2y} \cdot \frac{3y}{3y}$$
$$\frac{A}{6y^2} = \frac{9xy}{6y^2}$$
$$A = 9xy$$

20.
$$\frac{2R^2T}{A} = \frac{2R}{R+T} \cdot \frac{RT}{RT}$$
$$\frac{2R^2T}{A} = \frac{2R^2T}{RT(R+T)}$$
$$A = RT(R+T)$$
$$A = R^2T + RT^2$$

21.
$$\frac{6a-24}{A} = \frac{6}{(a+4)} \cdot \frac{(a-4)}{(a-4)}$$
$$\frac{6a-24}{A} = \frac{6a-24}{a^2-16}$$
$$A = a^2-16$$

22.
$$\frac{A}{5a^3c-5a^2c} = \frac{a+1}{5a^2c} \cdot \frac{a-1}{a-1}$$
$$\frac{A}{5a^3c-5a^2c} = \frac{a^2-1}{5a^3c-5a^2c}$$
$$A = a^2-1$$

23.
$$\frac{A}{x^2-1} = \frac{2x^3+2x}{x^4-1}$$
$$\frac{A}{x^2-1} = \frac{2x(x^2+1)}{(x^2+1)(x^2-1)}$$
$$A = \frac{2x(x^2+1)}{(x^2+1)}$$
$$A = 2x$$

24. $\dfrac{A}{n^2-n+1}=\dfrac{n^2-1}{n^3+1}$

$\dfrac{A}{n^2-n+1}=\dfrac{(n+1)(n-1)}{(n+1)(n^2-n+1)}$

$A=\dfrac{(n+1)(n-1)}{(n+1)}$

$A=n-1$ where $n\neq -1$

25. $\dfrac{x+4b}{A}=\dfrac{x^2+3bx-4b^2}{x-b}$

$\dfrac{x+4b}{A}=\dfrac{(x+4b)(x-b)}{(x-b)}$

$\dfrac{x+4b}{A}=\dfrac{x+4b}{1}$ where $x\neq b$

$A=1$

26. $\dfrac{A}{2y+4}=\dfrac{4y^2-1}{4y^2+6y-4}$

$\dfrac{A}{2y+4}=\dfrac{(2y+1)(2y-1)}{(2y+4)(2y-1)}$

$\dfrac{A}{2y+4}=\dfrac{(2y+1)}{(2y+4)}$ where $y\neq \frac{1}{2}$

$A=2y+1$

27. $\dfrac{5a}{9a}=\dfrac{a\cdot 5}{a\cdot 9}=\dfrac{5}{9}$ where $a\neq 0$

28. $\dfrac{6x}{15x}=\dfrac{3x(2)}{3x(5)}=\dfrac{2}{5}$ where $x\neq 0$

29. $\dfrac{18x^2y}{24xy}=\dfrac{6xy(3x)}{6xy(4)}=\dfrac{3x}{4}$ where $x, y\neq 0$

30. $\dfrac{2a^2xy}{6axyz^2}=\dfrac{2axy(a)}{2axy(3z^2)}=\dfrac{a}{3z^2}$ where $a, x, y\neq 0$

31. $\dfrac{b+8}{5ab+40a}=\dfrac{1(b+8)}{5a(b+8)}=\dfrac{1}{5a}$ where $a\neq 0, b\neq -8$

32. $\dfrac{t-a}{t^2-a^2}=\dfrac{(t-a)}{(t-a)(t+a)}=\dfrac{1}{t+a}$ where $a\neq t, -t$

33. $\dfrac{4a-4b}{4a-2b}=\dfrac{4(a-b)}{2(2a-b)}=\dfrac{2(a-b)}{2a-b}$

34. $\dfrac{20s-5r}{10r-5s}=\dfrac{5(4s-r)}{5(2r-s)}=\dfrac{-r+4s}{2r-s}$

35. $\dfrac{4x^2+1}{4x^2-1}=\dfrac{4x^2+1}{(2x+1)(2x-1)}$

Since no cancellations can be made the fraction cannot be reduced further.

36. $\dfrac{x^2-y^2}{x^2+y^2}=\dfrac{(x+y)(x-y)}{(x^2+y^2)}$

No factors cancel, and the fraction cannot be reduced further.

37. $\dfrac{3x^2-6x}{x-2}=\dfrac{3x(x-2)}{(x-2)}=3x$ where $x\neq 2$

38. $\dfrac{10T^2+15T}{3+2T}=\dfrac{5T(2T+3)}{(2T+3)}=5T$ where $T\neq -\frac{3}{2}$

39. $\dfrac{3+2y}{4y^3+6y^2}=\dfrac{(2y+3)}{2y^2(2y+3)}=\dfrac{1}{2y^2}$ where $y\neq 0,-\frac{3}{2}$

40. $\dfrac{6-3t}{4t^3-8t^2}=\dfrac{-3(t-2)}{4t^2(t-2)}=\dfrac{-3}{4t^2}$ where $t\neq 0,2$

41. $\dfrac{x^2-10x+25}{x^2-25}=\dfrac{(x-5)(x-5)}{(x+5)(x-5)}=\dfrac{x-5}{x+5}$ where $x\neq -5,5$

42.
$$\frac{4a^2+12ab+9b^2}{4a^2+6ab}=\frac{(2a+3b)(2a+3b)}{2a(2a+3b)}$$
$$=\frac{2a+3b}{2a}\quad\text{where } 2a+3b\neq 0, a\neq 0$$

43. $\dfrac{2w^4+5w^2-3}{w^4+11w^2+24}=\dfrac{(2w^2-1)(w^2+3)}{(w^2+8)(w^2+3)}=\dfrac{2w^2-1}{w^2+8}$

44.
$$\frac{3y^3+7y^2+4y}{4+5y+y^2}=\frac{y(3y^2+7y+4)}{(4+y)(1+y)}$$
$$=\frac{y(3y+4)(y+1)}{(y+4)(y+1)}$$
$$=\frac{y(3y+4)}{(y+4)}\quad\text{where } y\neq -1,-4$$

45. $\dfrac{5x^2-6x-8}{x^3+x^2-6x}=\dfrac{(5x+4)(x-2)}{x(x^2+x-6)}$

$=\dfrac{(5x+4)(x-2)}{x(x+3)(x-2)}$

$=\dfrac{5x+4}{x(x+3)}$ where $x\neq -3, 0, 2$

46. $\dfrac{5s^2+8rs-4s^2}{6r^2-17rs+5s^2}=\dfrac{s^2+8rs}{(3r-s)(2r-5s)}$

$=\dfrac{s(s+8r)}{(3r-s)(2r-5s)}$

No factors cancel, so the fraction cannot be reduced further.

47. $\dfrac{N^4-16}{8N-16}=\dfrac{(N^2+4)(N^2-4)}{8(N-2)}$

$=\dfrac{(N^2+4)(N+2)(N-2)}{8(N-2)}$

$=\dfrac{(N^2+4)(N+2)}{8}$ where $N\neq 2$

48. $\dfrac{3+x(4+x)}{3+x}=\dfrac{3+4x+x^2}{3+x}$

$=\dfrac{(3+x)(1+x)}{(3+x)}$

$=1+x$ where $x\neq -3$

49. $\dfrac{t+4}{(2t+9)t+4}=\dfrac{t+4}{2t^2+9t+4}$

$=\dfrac{(t+4)}{(2t+1)(t+4)}$

$=\dfrac{1}{2t+1}$ where $t\neq -4, -\frac{1}{2}$

50. $\dfrac{2A^3+8A^4+8A^5}{4A+2}=\dfrac{8A^5+8A^4+2A^3}{2(2A+1)}$

$=\dfrac{2A^3(4A^2+4A+1)}{2(2A+1)}$

$=\dfrac{A^3(2A+1)(2A+1)}{(2A+1)}$

$=A^3(2A+1)$ where $A\neq -\frac{1}{2}$

51. $$\frac{(x-1)(3+x)}{(3-x)(1-x)}=\frac{(x-1)(3+x)}{-(3-x)(x-1)}$$
$$=\frac{3+x}{-(3-x)}$$
$$=\frac{x+3}{x-3} \text{ where } x \neq 1,3$$

52. $$\frac{(2x-1)(x+6)}{(x-3)(1-2x)}=\frac{(x+6)(2x-1)}{(x-3)(-1)(2x-1)}$$
$$=\frac{x+6}{-(3-x)}$$
$$=\frac{x+6}{x-3} \text{ where } x \neq \tfrac{1}{2},3$$

53. $$\frac{y^2-x^2}{2x-2y}=\frac{(y+x)(y-x)}{2(x-y)}$$
$$=\frac{(y-x)(y+x)}{-2(y-x)}$$
$$=-\frac{y+x}{2} \quad \text{where } x \neq y$$

54. $$\frac{x^2-y^2-4x+4y}{x^2-y^2+4x-4y}=\frac{(x+y)(x-y)-4(x-y)}{(x+y)(x-y)+4(x-y)}$$
$$=\frac{(x-y)(x+y-4)}{(x-y)(x+y+4)}$$
$$=\frac{x+y-4}{x+y+4} \quad \text{where } x \neq y, x+y+4 \neq 0$$

55. $$\frac{n^3+n^2-n-1}{n^3-n^2-n+1}=\frac{(n^3+n^2)-(n+1)}{(n^3-n^2)-(n-1)}$$
$$=\frac{n^2(n+1)-(n+1)}{n^2(n-1)-(n-1)}$$
$$=\frac{(n+1)(n^2-1)}{(n-1)(n^2-1)}$$
$$=\frac{n+1}{n-1} \quad \text{where } n \neq \pm 1$$

56. $$\frac{3a^2-13a-10}{5+4a-a^2}=\frac{(3a+2)(a-5)}{(5-a)(1+a)}$$
$$=\frac{(3a+2)(a-5)}{-(a-5)(a+1)}$$
$$=-\frac{3a+2}{a+1} \quad \text{where } a \neq 5$$

57. $\dfrac{(x+5)(x-2)(x+2)(3-x)}{(2-x)(5-x)(3+x)(2+x)}=\dfrac{-(x+5)(x-2)(x+2)(x-3)}{-(5-x)(x-2)(x+2)(x+3)}$

$=\dfrac{(x+5)(x-3)}{(5-x)(x+3)}$ where $x \neq \pm 2, -3, 5$

58. $\dfrac{(2x-3)(3-x)(x-7)(3x+1)}{(3x+2)(3-2x)(x-3)(7+x)}=\dfrac{-(2x-3)(x-3)(x-7)(3x+1)}{-(2x-3)(x-3)(x+7)(3x+2)}$

$=\dfrac{(x-7)(3x+1)}{(x+7)(3x+2)}$ where $x \neq -7, 3, -\frac{2}{3}, \frac{3}{2}$

59. $\dfrac{x^3+y^3}{2x+2y}=\dfrac{(x+y)(x^2-xy+y^2)}{2(x+y)}$

$=\dfrac{x^2-xy+y^2}{2}$ where $x \neq -y$

60. $\dfrac{w^3-8}{w^2+2w+4}=\dfrac{(w-2)(w^2+2w+4)}{(w^2+2w+4)}=w-2$

61. $\dfrac{6x^2+2x}{27x^3+1}=\dfrac{2x(3x+1)}{(3x+1)(9x^2-3x+1)}$

$=\dfrac{2x}{9x^2-3x+1}$ where $x \neq -\frac{1}{3}$

62. $\dfrac{24-3a^3}{a^2-4a+4}=\dfrac{-3(a^3-8)}{(a-2)(a-2)}$

$=\dfrac{-3(a-2)(a^2+2a+4)}{(a-2)(a-2)}$

$=\dfrac{-3(a^2+2a+4)}{a-2}$ where $a \neq 2$

63. (a) $\dfrac{x^2(x+2)}{x^2+4}$ will not reduce further since x^2+4 does not factor.

(b) $\dfrac{x^4+4x^2}{x^4-16}=\dfrac{x^2(x^2+4)}{(x^2+4)(x^2-4)}$

$=\dfrac{x^2}{x^2-4}$

$=\dfrac{x^2}{(x+2)(x-2)}$

will not reduce further since there are no more common factors.

64. **(a)** $\dfrac{2x+3}{2x+6}=\dfrac{2x+3}{2(x+3)}$

Numerator and denominator have no common factor.

(b) $\dfrac{2(x+6)}{2x+6}=\dfrac{2(x+6)}{2(x+3)}=\dfrac{x+6}{x+3}$

Numerator and denominator have no common factor.

65. **(a)** $\dfrac{x^2-x-2}{x^2-x}=\dfrac{(x-2)(x+1)}{x(x-1)}$

Numerator and denominator have no common factor.

(b) $\dfrac{x^2-x-2}{x^2+x}=\dfrac{(x-2)(x+1)}{x(x+1)}=\dfrac{x-2}{x}$ where $x\neq 0,-1$

Numerator and denominator have no common factor.

66. **(a)** $\dfrac{x^3-x}{1-x}=\dfrac{x(x^2-1)}{1-x}$

$=\dfrac{x(x+1)(x-1)}{-(x-1)}$

$=-x(x+1)$ where $x\neq 1$

(b) $\dfrac{2x^2+4x}{2x^2+4}=\dfrac{2x(x+2)}{2(x^2+2)}$

$=\dfrac{x(x+2)}{x^2+2}$

There are no common factors between the numerator and denominator.

67. $\dfrac{x^2-9}{3-x}=\dfrac{(x+3)(x-3)}{(-1)(x-3)}$

$=-(x+3)$

The statement $3-x<0$ guarantees $3<x$ or $6<x+3$ and so $-(x+3)<-6<0$.

We conclude that $\dfrac{x^2-9}{3-x}<0$, exactly the opposite inequality from that in the problem statement.

68. $\dfrac{x^2-16}{x^3+64}=\dfrac{(x+4)(x-4)}{(x+4)(x^2+4x+16)}$

$=\dfrac{x-4}{x^2+4x+16}$

The statement $x-4<0$ guarantees $\dfrac{x-4}{x^2+4x+16}<0$ because the denominator is

$x^2+4x+16=x^2+4x+4+12=(x+2)^2+12$ which is always at least 12,

hence, the denominator must be positive.

69. $$\frac{v^2-v_0^2}{vt-v_0t}=\frac{(v+v_0)(v-v_0)}{t(v-v_0)}$$
$$=\frac{v+v_0}{t} \text{ where } v\neq v_0, t\neq 0$$

70. $$\frac{a^3-b^3}{a^3-ab^2}=\frac{(a-b)(a^2+ab+b^2)}{a(a^2-b^2)}$$
$$=\frac{(a^2+ab+b^2)(a-b)}{a(a+b)(a-b)}$$
$$=\frac{a^2+ab+b^2}{a(a+b)} \text{ where } a\neq 0, b, -b$$

71. $$\frac{mu^2-mv^2}{mu-mv}=\frac{m(u^2-v^2)}{m(u-v)}$$
$$=\frac{(u+v)(u-v)}{(u-v)}$$
$$=u+v \quad \text{where } u\neq v$$

72. $$\frac{16(t^2-2tt_0+t_0^2)(t-t_0-3)}{3t-3t_0}=\frac{16(t-t_0)(t-t_0)(t-t_0-3)}{3(t-t_0)}$$
$$=\frac{16(t-t_0)(t-t_0-3)}{3} \quad \text{where } t\neq t_0$$

73. $$\frac{E^2R^2-E^2r^2}{(R^2+2Rr+r^2)^2}=\frac{E^2(R^2-r^2)}{\left[\left[(R+r)(R+r)\right]^2\right]^2}$$
$$=\frac{E^2(R+r)(R-r)}{(R+r)^4}$$
$$=\frac{E^2(R-r)}{(R+r)^3}$$
where $R\neq -r$ (shouldn't be an issue since resistance is always positive)

74. $$\frac{r_0^3-r_i^3}{r_0^2-r_i^2}=\frac{(r_0-r_i)(r_0^2+r_0r_i+r_i^2)}{(r_0+r_i)(r_0-r_i)}$$
$$=\frac{r_0^2+r_0r_i+r_i^2}{r_0+r_i}$$
where $r_0\neq r_i$ (shouldn't be an issue since inner and outer radii shouldn't be equivalent)

6.5 Multiplication and Division of Fractions

1. $$\frac{2x-4}{4x-10}\times\frac{2x^2+x-15}{3x-1}=\frac{2(x-2)(2x-5)(x+3)}{2(2x-5)(3x-1)}$$
$$=\frac{(x-2)(x+3)}{3x-1} \quad \text{where } x\neq \tfrac{5}{2},\tfrac{1}{3}$$

2. $$\frac{\frac{4-x^2}{x^2-3x+2}}{\frac{x+3}{x^2-9}}=\frac{4-x^2}{x^2-3x+2}\times\frac{x^2-9}{x+3}$$
$$=\frac{(2+x)(2-x)(x+3)(x-3)}{(x-2)(x-1)(x+3)}$$
$$=\frac{-(x-2)(x+2)(x-3)}{(x-2)(x-1)}$$
$$=-\frac{(x+2)(x-3)}{x-1} \quad \text{where } x\neq -3,1,2$$

3. $$\frac{3}{10}\times\frac{2}{9}=\frac{3}{5(2)}\times\frac{2}{3(3)}=\frac{1}{15}$$
(divide out common factors of 2 and 3)

4. $$11\times\frac{13}{33}=\frac{(11)(13)}{3(11)}=\frac{13}{3}$$
(divide out a common factor of 11)

5. $$\frac{4x}{3y}\times\frac{9y^2}{2}=\frac{2(2x)(3y)(3y)}{(3y)(2)}$$
$$=\frac{(2x)(3y)}{(1)(1)}$$
$$=6xy \quad \text{where } y\neq 0$$
(divide out a common factor of 2 and $3y$)

6. $$\frac{18sy^3}{ax^2}\times\frac{(ax)^2}{3s}=\frac{3s(6y^3)a^2x^2}{ax^2(3s)}$$
$$=\frac{6ay^3}{(1)(1)}$$
$$=6ay^3 \quad \text{where } a,s,x\neq 0$$
(divide out common factors of $3s$ and ax^2)

7. $$\frac{2}{9}\div\frac{4}{7}=\frac{2}{9}\times\frac{7}{4}=\frac{2(7)}{9(2)(2)}=\frac{7}{9(2)}=\frac{7}{18}$$
(divide out a common factor of 2)

8. $\frac{5}{16} \div \frac{25}{-13} = \frac{5}{16} \times \frac{-13}{25} = \frac{5(-13)}{16(5)(5)} = \frac{-13}{80} = -\frac{13}{80}$

(divide out a common factor of 5)

9. $\frac{yz}{az} \div \frac{bz}{ay} = \frac{yz}{az} \times \frac{ay}{bz} = \frac{yz(ay)}{az(bz)} = \frac{y^2}{bz}$ where $a, b, y, z \neq 0$

(divide out common factors of a and z)

10. $\frac{sr^2}{2t} \div \frac{st}{4} = \frac{sr^2}{2t} \times \frac{4}{st} = \frac{sr^2(2)(2)}{2t(st)} = \frac{2r^2}{t^2}$ where $s, t \neq 0$

(divide out common factors of 2 and s)

11. $\frac{4x+16}{5y} \times \frac{y^2}{2x+8} = \frac{4(x+4)y^2}{5y(2)(x+4)} = \frac{2y}{5}$ where $x \neq -4, y \neq 0$

(divide out common factors of $2y(x+4)$)

12. $\frac{2y^2+6y}{6z} \times \frac{z^3}{y^2-9} = \frac{2y(y+3)(z^3)}{2(3)z(y+3)(y-3)}$

$= \frac{yz^2}{3(y-3)}$ where $z \neq 0, y \neq -3, 3$

(divide out common factors of $2z$ and $y+3$)

13. $\frac{u^2-v^2}{u+2v}(3u+6v) = \frac{(u+v)(u-v)(3)(u+2v)}{u+2v}$

$= 3(u+v)(u-v)$ where $u \neq -2v$

(divide out a common factor of $(u+2v)$)

14. $(x-y)\frac{x+2y}{x^2-y^2} = \frac{(x-y)(x+2y)}{(x+y)(x-y)} = \frac{x+2y}{x+y}$ where $x \neq y, x \neq -y$

(divide out a common factor of $(x-y)$)

15. $\frac{2a+8}{15} \div \frac{16+8a+a^2}{125} = \frac{2(a+4)}{3\times 5} \times \frac{5\times 5\times 5}{(a+4)(a+4)}$

$= \frac{50}{3(a+4)}$ where $a \neq -4$

(divide out a common factor of $5(a+4)$)

16. $\dfrac{a^2-a}{3a+9} \div \dfrac{a^2-2a+1}{18-2a^2} = \dfrac{a^2-a}{3a+9} \times \dfrac{-2\left(a^2-9\right)}{a^2-2a+1}$

$$= \frac{-2a(a-1)(a+3)(a-3)}{3(a+3)(a-1)(a-1)}$$

$$= \frac{-2a(a-3)}{3(a-1)} \quad \text{where } a \neq 1, -3$$

(divide out common factors of $(a-1)$ and $(a+3)$)

17. $\dfrac{x^4-9}{x^2} \div \left(x^2+3\right)^2 = \dfrac{x^4-9}{x^2} \times \dfrac{1}{\left(x^2+3\right)^2}$

$$= \frac{\left(x^2+3\right)\left(x^2-3\right)}{x^2\left(x^2+3\right)^2}$$

$$= \frac{x^2-3}{x^2\left(x^2+3\right)} \quad \text{where } x \neq 0$$

(divide out a common factor of $\left(x^2+3\right)$)

18. $\dfrac{9B^2-16}{B+1} \div (4-3B) = \dfrac{9B^2-16}{B+1} \times \dfrac{1}{4-3B}$

$$= \frac{(3B+4)(3B-4)}{(B+1)(4-3B)}$$

$$= \frac{(3B+4)(3B-4)}{-(B+1)(3B-4)}$$

$$= -\frac{3B+4}{B+1} \quad \text{where } B \neq -1, \tfrac{4}{3}$$

(divide out a common factor of $(3B-4)$)

19. $\dfrac{3ax^2-9ax}{10x^2+5x} \times \dfrac{2x^2+x}{a^2x-3a^2} = \dfrac{3ax(x-3)}{5x(2x+1)} \times \dfrac{x(2x+1)}{a^2(x-3)}$

$$= \frac{3x}{5a} \quad \text{where } x \neq 0, 3, -\tfrac{1}{2} \quad \text{and } a \neq 0$$

(divide out a common factor of $ax(2x+1)(x-3)$)

20. $\dfrac{4R^2-36}{R^3-25R} \times \dfrac{7R-35}{3R^2+9R} = \dfrac{4\left(R^2-9\right) \times 7(R-5)}{R\left(R^2-25\right) \times 3R(R+3)}$

$$= \frac{28(R+3)(R-3)(R-5)}{3R^2(R+5)(R-5)(R+3)}$$

$$= \frac{28(R-3)}{3R^2(R+5)} \quad \text{where } R \neq -5, -3, 5$$

(divide out common factor of $(R+3)(R-5)$)

21. $\left(\dfrac{x^4-1}{8x+16}\right)\left(\dfrac{2x^2-8x}{x^3+x}\right)=\dfrac{(x^2-1)(x^2+1)(2x)(x-4)}{8(x+2)(x)(x^2+1)}$

$=\dfrac{2x(x+1)(x-1)(x-4)}{8x(x+2)}$

$=\dfrac{(x+1)(x-1)(x-4)}{4(x+2)}$ where $x\neq 0,-2$

(divide out common factors of 2, x, and (x^2+1))

22. $\left(\dfrac{2x^2-4x-6}{3x-x^2}\right)\left(\dfrac{x^3-4x^2}{4x^2-4x-8}\right)=\dfrac{(2x-6)(x+1)(x^2)(x-4)}{x(3-x)(4)(x^2-x-2)}$

$=\dfrac{2x^2(x-3)(x+1)(x-4)}{4x(-1)(x-3)(x-2)(x+1)}$

$=-\dfrac{x(x-4)}{2(x-2)}$ where $x\neq 0,3,-1,2$

(divide out common factors of 2, x, $(x-3)$, and $(x+1)$)

23. $\dfrac{\frac{x^2+ax}{2b-cx}}{\frac{a^2+2ax+x^2}{2bx-cx^2}}=\dfrac{x(x+a)}{(2b-cx)}\times\dfrac{x(2b-cx)}{a^2+2ax+x^2}$

$=\dfrac{x^2(a+x)(2b-cx)}{(2b-cx)(a+x)(a+x)}$

$=\dfrac{x^2}{a+x}$ where $x\neq -a$, $b\neq \frac{cx}{2}$

(divide out a common factor of $(a+x)(2b-cx)$)

24. $\dfrac{\frac{x^4-11x^2+28}{2x^2+6}}{\frac{4-x^2}{2x^2+3}}=\dfrac{(x^2-7)(x^2-4)}{2(x^2+3)}\times\dfrac{2x^2+3}{-(x^2-4)}$

$=-\dfrac{(x^2-7)(2x^2+3)}{2(x^2+3)}$ where $x\neq \pm 2$

(divide out a common factor of $-(x^2-4)$)

25. $\dfrac{35a+25}{12a+33}\div\dfrac{28a+20}{36a+99}=\dfrac{5(7a+5)}{3(4a+11)}\times\dfrac{9(4a+11)}{4(7a+5)}$

$=\dfrac{15}{4}$ where $a\neq -\frac{5}{7},-\frac{11}{4}$

(divide out common factors of 3, $(7a+5)$, and $(4a+11)$)

26. $\dfrac{2a^3+a^2}{2b^3+b^2}\div\dfrac{2ab+a}{2ab+b}=\dfrac{a^2(2a+1)}{b^2(2b+1)}\times\dfrac{b(2a+1)}{a(2b+1)}$

$$=\frac{a(2a+1)^2}{b(2b+1)^2}\quad\text{where } a,b\neq 0, b\neq -\tfrac{1}{2}$$

(divide out common factors of a and b)

27. $\dfrac{x^2-6x+5}{4x^2-5x-6}\times\dfrac{12x+9}{x^2-1}=\dfrac{(x-5)(x-1)(3)(4x+3)}{(4x+3)(x-2)(x+1)(x-1)}$

$$=\frac{3(x-5)}{(x-2)(x+1)}\quad\text{where } x\neq -1,1,2,-\tfrac{4}{3}$$

(divide out common factor $(x-1)(4x+3)$)

28. $\dfrac{n^2+5n}{3n^2+8n+4}\times\dfrac{2n^2-8}{n^3+3n^2-10n}=\dfrac{n(n+5)(2)(n^2-4)}{(3n+2)(n+2)n(n^2+3n-10)}$

$$=\frac{2n(n+5)(n+2)(n-2)}{n(3n+2)(n+2)(n+5)(n-2)}$$

$$=\frac{2}{3n+2}\quad\text{where } n\neq 0,-2,-5,2$$

(divide out common factors $n,(n+2,),(n+5),(n-2)$)

29. $\dfrac{\frac{6T^2-NT-N^2}{2V^2-9V-35}}{\frac{8T^2-2NT-N^2}{20V^2+26V-60}}=\dfrac{6T^2-NT-N^2}{2V^2-9V-35}\times\dfrac{20V^2+26V-60}{8T^2-2NT-N^2}$

$$=\frac{(2T-N)(3T+N)}{(2V+5)(V-7)}\times\frac{2(10V^2+13V-30)}{(4T+N)(2T-N)}$$

$$=\frac{2(2T-N)(3T+N)(5V-6)(2V+5)}{(2V+5)(V-7)(4T+N)(2T-N)}$$

$$=\frac{2(3T+N)(5V-6)}{(V-7)(4T+N)}\quad\text{where } T\neq\frac{N}{2}\ \text{ and }\ V\neq-\frac{5}{2}$$

(divide out a common factor of $(2V+5)$ and $(2T-N)$)

30. $\dfrac{\frac{4L^3-9L}{8L^2+10L-3}}{3L^2-2L^3}=\dfrac{4L^3-9L}{8L^2+10L-3}\times\dfrac{1}{3L^2-2L^3}$

$$=\frac{L(4L^2-9)}{(4L-1)(2L+3)L^2(3-2L)}$$

$$=\frac{L(2L+3)(2L-3)}{L^2(4L-1)(2L+3)(-1)(2L-3)}$$

$$=-\frac{1}{L(4L-1)}\quad\text{where } L\neq 0,\pm\frac{3}{2},\frac{1}{4}$$

(divide out common factors $L,(2L-3),(2L+3)$)

31. $$\frac{7x^2}{3a} \div \left(\frac{a}{x} \times \frac{a^2x}{x^2}\right) = \frac{7x^2}{3a} \times \left(\frac{x^3}{a^3x}\right)$$
$$= \frac{7x^5}{3a^4x}$$
$$= \frac{7x^4}{3a^4} \quad \text{where } a, x \neq 0$$
(divide out a common factor of x)

32. $$\left(\frac{3u}{8v^2} \div \frac{9u^2}{2w^2}\right) \times \frac{2u^4}{15vw} = \left(\frac{3u}{8v^2} \times \frac{2w^2}{9u^2}\right) \times \frac{2u^4}{15vw}$$
$$= \frac{12u^5w^2}{1080u^2v^3w}$$
$$= \frac{u^3w}{90v^3} \quad \text{where } u, v, w \neq 0$$
(divide out common factors of 12, u^2, and w)

33. $$\frac{4t^2-1}{t-5} \div \frac{2t+1}{2t} \times \frac{2t^2-50}{1+4t+4t^2} = \frac{4t^2-1}{t-5} \times \frac{2t}{2t+1} \times \frac{2t^2-50}{4t^2+4t+1}$$
$$= \frac{(2t+1)(2t-1)(2t)(2)(t^2-25)}{(t-5)(2t+1)(2t+1)(2t+1)}$$
$$= \frac{4t(2t+1)(2t-1)(t+5)(t-5)}{(t-5)(2t+1)(2t+1)(2t+1)}$$
$$= \frac{4t(2t-1)(t+5)}{(2t+1)^2} \quad \text{where } t \neq 5, 0, -\frac{1}{2}$$
(divide out common factors $(2t+1)$ and $(t-5)$)

34. $$\frac{2x^2-5x-3}{x-4} \div \left(\frac{x-3}{x^2-16} \times \frac{1}{3-x}\right) = \frac{2x^2-5x-3}{x-4} \times \left(\frac{x^2-16}{x-3} \times \frac{3-x}{1}\right)$$
$$= \frac{(2x+1)(x-3)(x+4)(x-4)(-1)(x-3)}{(x-4)(x-3)}$$
$$= -(2x+1)(x-3)(x+4) \quad \text{where } x \neq 3, \pm 4$$
(divide out common factors $(x-3)$ and $(x-4)$)

35. $$\frac{x^3-y^3}{2x^2-2y^2} \times \frac{y^2+2xy+x^2}{x^2+xy+y^2} = \frac{(x-y)(x^2+xy+y^2)(y+x)(y+x)}{2(x^2-y^2)(x^2+xy+y^2)}$$
$$= \frac{(x-y)(x^2+xy+y^2)(x+y)(x+y)}{2(x+y)(x-y)(x^2+xy+y^2)}$$
$$= \frac{x+y}{2}$$
(divide out common factors of $(x-y), (x+y), (x^2+xy+y^2)$)

so $x \neq \pm y$ and $x^2+xy+y^2 \neq 0$

36. $$\frac{2M^2+4M+2}{6M-6}\div\frac{5M+5}{M^2-1}=\frac{2(M+1)(M+1)}{6(M-1)}\times\frac{(M+1)(M-1)}{5(M+1)}$$
$$=\frac{(M+1)^2}{15}\quad\text{where } M\neq\pm1$$
(divide out common factors $2, (M-1)$, and $(M+1)$)

37. $$\left(\frac{ax+bx+ay+by}{p-q}\right)\left(\frac{3p^2+4pq-7q^2}{a+b}\right)=\frac{x(a+b)+y(a+b)}{p-q}\times\frac{(3p+7q)(p-q)}{a+b}$$
$$=\frac{(a+b)(x+y)(3p+7q)(p-q)}{(p-q)(a+b)}$$
$$=(x+y)(3p+7q)\quad\text{where } a\neq-b \text{ and } p\neq q$$
(divide out common factors of $(a+b)$ and $(p-q)$)

38. $$\frac{x^4+x^5-1-x}{x-1}\div\frac{x+1}{x}=\frac{x^4(1+x)-1(1+x)}{x-1}\times\frac{x}{x+1}$$
$$=\frac{(1+x)(x^4-1)x}{(x-1)(x+1)}$$
$$=\frac{x(x+1)(x^2+1)(x^2-1)}{(x-1)(x+1)}$$
$$=\frac{x(x+1)(x^2+1)(x+1)(x-1)}{(x-1)(x+1)}$$
$$=x(x+1)(x^2+1)\quad\text{where } x\neq0,\pm1$$
(divide out common factors $(x-1)$ and $(x+1)$)

39. $$\frac{x}{2x+4}\times\frac{x^2-4}{3x^2}=\frac{x(x+2)(x-2)}{2(x+2)(3x^2)}$$
$$=\frac{x-2}{6x}\quad\text{where } x\neq-2,0$$

40. $$\frac{4t^2-25}{4t^2}\div\frac{4t+10}{8}=\frac{4t^2-25}{4t^2}\times\frac{8}{4t+10}$$
$$=\frac{(2t+5)(2t-5)(8)}{4t^2(2)(2t+5)}$$
$$=\frac{2t-5}{t^2}\quad\text{where } x\neq0,-\frac{5}{2}$$

41. $$\frac{2x^2+3x-2}{2+3x-2x^2}\div\frac{5x+10}{4x+2}=\frac{2x^2+3x-2}{-(2x^2-3x-2)}\times\frac{4x+2}{5x+10}$$
$$=\frac{(2x-1)(x+2)}{-(2x+1)(x-2)}\times\frac{2(2x+1)}{5(x+2)}$$
$$=-\frac{2(2x-1)}{5(x-2)}\quad\text{where } x\neq-\frac{1}{2},\pm2$$

42. $$\frac{16x^2-8x+1}{9x}\times\frac{12x+3}{1-16x^2}=\frac{(4x-1)(4x-1)(3)(4x+1)}{-9x\left(16x^2-1\right)}$$
$$=\frac{(4x-1)(4x-1)(3)(4x+1)}{-9x(4x+1)(4x-1)}$$
$$=-\frac{4x-1}{3x}\quad \text{where } x\neq 0,\pm\frac{1}{4}$$

43. $$\frac{2\pi}{\lambda}\left(\frac{a+b}{2ab}\right)\left(\frac{ab\lambda}{2a+2b}\right)=\frac{2\pi(a+b)(ab\lambda)}{(2ab\lambda)(2)(a+b)}$$
$$=\frac{\pi}{2}\quad \text{where } a,b,\lambda\neq 0 \text{ and } a\neq -b$$

(divide out common factors 2, ab, $(a+b)$ and λ)

44. $$\frac{8\pi n^2eu}{mv^2-mvu^2}\left(\frac{mv^2}{2mv^2-2\pi ne^2}\right)=\frac{4(2)\pi n^2eumv^2}{mv(v-u^2)2(mv^2-\pi ne^2)}$$
$$=\frac{4\pi n^2euv}{(v-u^2)(mv^2-\pi ne^2)}\quad \text{where } m,v,v-u^2,mv^2-\pi ne^2\neq 0$$

(divide out common factors $m, 2,$ and v)

45. $$\frac{c\lambda^2-c\lambda_0^2}{\lambda_0^2}\div\frac{\lambda^2+\lambda_0^2}{\lambda_0^2}=\frac{c\left(\lambda^2-\lambda_0^2\right)}{\lambda_0^2}\times\frac{\lambda_0^2}{\lambda^2+\lambda_0^2}$$
$$=\frac{c(\lambda+\lambda_0)(\lambda-\lambda_0)}{\lambda_0^2}\times\frac{\lambda_0^2}{\lambda^2+\lambda_0^2}$$
$$=\frac{c(\lambda+\lambda_0)(\lambda-\lambda_0)}{\lambda^2+\lambda_0^2}\quad \text{where } \lambda_0\neq 0$$

46. $$(p_1-p_2)\div\left(\frac{\pi a^4p_1-\pi a^4p_2}{81u}\right)=(p_1-p_2)\times\frac{81u}{\pi a^4(p_1-p_2)}$$
$$=\frac{81u}{\pi a^4}\quad \text{where } p_1\neq p_2, a,u\neq 0$$

(divide out a common factor of (p_1-p_2))

47. $$\frac{GmM}{r^2}\div\frac{m}{r}=\frac{GmM}{r^2}\times\frac{r}{m}=\frac{GM}{r}\quad \text{where } m,r\neq 0$$

(divide out r, m)

48. $$\frac{28\left(\frac{\text{m}}{\text{s}}\right)^2}{2(0.90)\left(9.8\frac{\text{m}}{\text{s}^2}\right)}=\frac{28\text{ m}^2}{\text{s}^2}\times\frac{1\text{ s}^2}{2(0.90)(9.8)\text{ m}}=1.5873\text{ m which is rounded to 1.6 m.}$$

(divide out s^2, m)

6.6 Addition and Subtraction of Fractions

1.

$$\begin{aligned} 4a^2b &= 2\cdot 2\cdot a\cdot a\cdot b \\ 6ab^3 &= 2\cdot 3\cdot a\cdot b\cdot b\cdot b \\ 4a^2b^2 &= 2\cdot 2\cdot a\cdot a\cdot b\cdot b \\ L.C.D. &= 2^2\cdot 3\cdot a^2\cdot b^3 = 12a^2b^3 \end{aligned}$$

2.

$$\begin{aligned} \frac{a}{x-1}+\frac{a}{x+1}+\frac{2}{x^2-1} &= \frac{a}{(x-1)}+\frac{a}{(x+1)}+\frac{2}{(x+1)(x-1)} \\ &= \frac{a(x+1)+a(x-1)+2}{(x+1)(x-1)} \\ &= \frac{ax+a+ax-a+2}{(x+1)(x-1)} \\ &= \frac{2ax+2}{(x+1)(x-1)} \\ &= \frac{2(ax+1)}{(x+1)(x-1)} \end{aligned}$$

3.

$$\begin{aligned} \frac{1}{s}-\frac{1}{s+4}+\frac{8}{s^2+4s} &= \frac{1}{s}-\frac{1}{s+4}+\frac{8}{s(s+4)} \\ &= \frac{s+4}{s(s+4)}-\frac{s}{s(s+4)}+\frac{8}{s(s+4)} \\ &= \frac{(s+4)-s+8}{s(s+4)} \\ &= \frac{12}{s(s+4)} \end{aligned}$$

4.

$$\begin{aligned} \frac{\frac{2}{x^2}}{\frac{1}{x}+\frac{2}{x^2+4x}} &= \frac{\frac{2}{x^2}}{\frac{1}{x}+\frac{2}{x(x+4)}} \\ &= \frac{\frac{2}{x^2}}{\frac{x+4}{x(x+4)}+\frac{2}{x(x+4)}} \\ &= \frac{\frac{2}{x^2}}{\frac{x+6}{x(x+4)}} \\ &= \frac{2}{x^2}\times\frac{x(x+4)}{x+6} \\ &= \frac{2(x+4)}{x(x+6)} \text{ where } x \neq 0, -4, -6 \end{aligned}$$

5. $\frac{5}{6}+\frac{7}{6}=\frac{5+7}{6}=\frac{12}{6}=2$

6. $\frac{2}{13}+\frac{6}{13}=\frac{2+6}{13}=\frac{8}{13}$

7. $\frac{1}{x}+\frac{7}{x}=\frac{1+7}{x}=\frac{8}{x}$

8. $\frac{2}{a}+\frac{3}{a}=\frac{2+3}{a}=\frac{5}{a}$

9. $\frac{1}{2}+\frac{3}{4}=\frac{1(2)+3}{4}=\frac{2+3}{4}=\frac{5}{4}$

10. $\frac{5}{9}-\frac{1}{3}=\frac{5-1(3)}{9}=\frac{5-3}{9}=\frac{2}{9}$

11. $\frac{3}{4x}+\frac{x}{3}+\frac{1}{x}=\frac{3(3)+4x(x)+12(1)}{12x}=\frac{21+4x^2}{12x}$

12. $\frac{a+3}{a}-\frac{a}{2}=\frac{(a+3)(2)-a(a)}{2a}=\frac{2a+6-a^2}{2a}$

13. $\frac{a}{x}-\frac{b}{x^2}=\frac{a(x)-b}{x^2}=\frac{ax-b}{x^2}$

14. $\frac{3}{2s^2}+\frac{5}{4s}=\frac{3(2)+5(s)}{4s^2}=\frac{6+5s}{4s^2}$

15. $\frac{6}{5x^3}+\frac{a}{25x}=\frac{6(5)+a\left(x^2\right)}{25x^3}=\frac{30+ax^2}{25x^3}$

16. $\frac{a}{6y}-\frac{2b}{3y^4}=\frac{a\left(y^3\right)-2b(2)}{6y^4}=\frac{ay^3-4b}{6y^4}$

17. $\frac{2}{5a}+\frac{1}{a}-\frac{a}{10}=\frac{2(2)+1(10)-a(a)}{10a}=\frac{14-a^2}{10a}$

18. $\frac{1}{2A}-\frac{6}{B}-\frac{9}{4C}=\frac{1(2BC)-6(4AC)-9(AB)}{4ABC}=\frac{2BC-24AC-9AB}{4ABC}$

19.
$$\begin{aligned}\frac{x+1}{2x}-\frac{y-3}{4y}&=\frac{(x+1)(2y)-(y-3)(x)}{4xy}\\&=\frac{2xy+2y-xy+3x}{4xy}\\&=\frac{xy+2y+3x}{4xy}\end{aligned}$$

20. $$\begin{aligned}\frac{1-x}{6y}-\frac{3+x}{4y}&=\frac{(1-x)(2)-3(3+x)}{12y}\\&=\frac{2-2x-9-3x}{4y}\\&=\frac{-5x-7}{4y}\\&=-\frac{5x+7}{4y}\end{aligned}$$

21. $$\begin{aligned}\frac{y^2}{y+3}-\frac{2y+15}{y+3}&=\frac{y^2-2y-15}{y+3}\\&=\frac{(y+3)(y-5)}{y+3}\\&=y-5\end{aligned}$$

22. $$\begin{aligned}\frac{t^2+4}{t-4}-\frac{5t}{t-4}&=\frac{t^2-5t+4}{t-4}\\&=\frac{(t-4)(t-1)}{t-4}\\&=t-1\end{aligned}$$

23. $$\begin{aligned}\frac{x}{2x-2}+\frac{4}{3x-3}&=\frac{x}{2(x-1)}+\frac{4}{3(x-1)}\\&=\frac{3(x)+2(4)}{6(x-1)}\\&=\frac{3x+8}{6(x-1)}\end{aligned}$$

24. $$\begin{aligned}\frac{5}{6y+3}-\frac{a}{4+8y}&=\frac{5}{3(2y+1)}-\frac{a}{4(2y+1)}\\&=\frac{5(4)-a(3)}{12(2y+1)}\\&=\frac{20-3a}{12(2y+1)}\end{aligned}$$

25. $$\begin{aligned}\frac{4}{x(x+1)}-\frac{3}{2x}&=\frac{4(2)-3(x+1)}{2x(x+1)}\\&=\frac{8-3x-3}{2x(x+1)}\\&=\frac{5-3x}{2x(x+1)}\end{aligned}$$

26. $$\frac{3}{ax+ay}-\frac{1}{a^2}=\frac{3}{a(x+y)}-\frac{1}{a^2}$$
$$=\frac{3(a)-1(x+y)}{a^2(x+y)}$$
$$=\frac{3a-x-y}{a^2(x+y)}$$

27. $$\frac{s}{2s-6}+\frac{1}{4}-\frac{3s}{4s-12}=\frac{s}{2(s-3)}+\frac{1}{4}-\frac{3s}{4(s-3)}$$
$$=\frac{s(2)+1(s-3)-3s}{4(s-3)}$$
$$=\frac{2s+s-3-3s}{4(s-3)}$$
$$=\frac{-3}{4(s-3)}$$

28. $$\frac{2}{x+2}-\frac{3-x}{x^2+2x}+\frac{1}{x}=\frac{2}{x+2}-\frac{3-x}{x(x+2)}+\frac{1}{x}$$
$$=\frac{2(x)-(3-x)+1(x+2)}{x(x+2)}$$
$$=\frac{2x-3+x+x+2}{x(x+2)}$$
$$=\frac{4x-1}{x(x+2)}$$

29. $$\frac{3R}{R^2-9}-\frac{2}{3R+9}=\frac{3R}{(R+3)(R-3)}-\frac{2}{3(R+3)}$$
$$=\frac{3R(3)-2(R-3)}{3(R+3)(R-3)}$$
$$=\frac{9R-2R+6}{3(R+3)(R-3)}$$
$$=\frac{7R+6}{3(R+3)(R-3)}$$

30. $$\frac{2}{n^2+4n+4}-\frac{3}{4+2n}=\frac{2}{(n+2)^2}-\frac{3}{2(n+2)}$$
$$=\frac{2(2)-3(n+2)}{2(n+2)^2}$$
$$=\frac{4-3n-6}{2(n+2)^2}$$
$$=-\frac{3n+2}{2(n+2)^2}$$

31. $$\frac{3}{x^2-8x+16}-\frac{2}{4-x}=\frac{3}{(x-4)^2}+\frac{2}{x-4}$$
$$=\frac{3+2(x-4)}{(x-4)^2}$$
$$=\frac{3+2x-8}{(x-4)^2}$$
$$=\frac{2x-5}{(x-4)^2}$$

32. $$\frac{2a-b}{c-3d}-\frac{b-2a}{3d-c}=\frac{2a-b}{c-3d}+\frac{b-2a}{c-3d}$$
$$=\frac{2a-b+b-2a}{c-3d}$$
$$=0$$

33. $$\frac{v+4}{v^2+5v+4}-\frac{v-2}{v^2-5v+6}=\frac{v+4}{(v+4)(v+1)}-\frac{v-2}{(v-3)(v-2)}$$
$$=\frac{1}{v+1}-\frac{1}{v-3}\quad \text{where } v\neq -4, 2$$
$$=\frac{1(v-3)-1(v+1)}{(v+1)(v-3)}$$
$$=\frac{-4}{(v+1)(v-3)}$$

34. $$\frac{N-1}{2N^3-4N^2}-\frac{5}{2-N}==\frac{N-1}{2N^2(N-2)}+\frac{5}{(N-2)}$$
$$=\frac{N-1+5(2N^2)}{2N^2(N-2)}$$
$$=\frac{10N^2+N-1}{2N^2(N-2)}$$

35. $$\frac{x-1}{3x^2-13x+4}-\frac{3x+1}{4-x}=\frac{(x-1)}{(3x-1)(x-4)}+\frac{(3x+1)}{(x-4)}$$
$$=\frac{x-1+(3x+1)(3x-1)}{(3x-1)(x-4)}$$
$$=\frac{x-1+9x^2-1}{(3x-1)(x-4)}$$
$$=\frac{9x^2+x-2}{(3x-1)(x-4)}$$

36. $$\frac{x}{4x^2-12x+5}+\frac{2x-1}{4x^2-4x-15}=\frac{x}{(2x-5)(2x-1)}+\frac{2x-1}{(2x-5)(2x+3)}$$
$$=\frac{x(2x+3)+(2x-1)(2x-1)}{(2x-5)(2x-1)(2x+3)}$$
$$=\frac{2x^2+3x+4x^2-4x+1}{(2x+3)(2x-1)(2x-5)}$$
$$=\frac{6x^2-x+1}{(2x+3)(2x-1)(2x-5)}$$

37. $$\frac{t}{t^2-t-6}-\frac{2t}{t^2+6t+9}+\frac{t}{9-t^2}=\frac{t}{(t-3)(t+2)}-\frac{2t}{(t+3)(t+3)}-\frac{t}{(t+3)(t-3)}$$
$$=\frac{t(t+3)^2-2t(t-3)(t+2)-t(t+3)(t+2)}{(t-3)(t+2)(t+3)^2}$$
$$=\frac{t(t^2+6t+9)-2t(t^2-t-6)-t(t^2+5t+6)}{(t+3)^2(t-3)(t+2)}$$
$$=\frac{t^3+6t^2+9t-2t^3+2t^2+12t-t^3-5t^2-6t}{(t+3)^2(t-3)(t+2)}$$
$$=\frac{-2t^3+3t^2+15t}{(t+3)^2(t-3)(t+2)}$$
$$=\frac{-t(2t^2-3t-15)}{(t+3)^2(t-3)(t+2)}$$

38. $$\frac{5}{2x^3-3x^2+x}-\frac{x}{x^4-x^2}+\frac{2-x}{2x^2+x-1}=\frac{5}{x(2x^2-3x+1)}-\frac{x}{x^2(x^2-1)}+\frac{2-x}{(2x-1)(x+1)}$$
$$=\frac{5}{x(2x-1)(x-1)}-\frac{x}{x^2(x+1)(x-1)}+\frac{2-x}{(2x-1)(x+1)}$$
$$=\frac{5x(x+1)-x(2x-1)+(2-x)x^2(x-1)}{x^2(2x-1)(x-1)(x+1)}$$
$$=\frac{5x^2+5x-2x^2+x+x^2(2x-2-x^2+x)}{x^2(2x-1)(x-1)(x+1)}$$
$$=\frac{5x^2+5x-2x^2+x+3x^3-2x^2-x^4}{x^2(2x-1)(x-1)(x+1)}$$
$$=\frac{-x^4+3x^3+x^2+6x}{x^2(2x-1)(x-1)(x+1)}$$
$$=\frac{x(-x^3+3x^2+x+6)}{x^2(2x-1)(x-1)(x+1)}$$
$$=\frac{-x^3+3x^2+x+6}{x(2x-1)(x-1)(x+1)},\quad x\neq 0$$

39. $$\frac{1}{w^3+1}+\frac{1}{w+1}-2=\frac{1}{(w+1)(w^2-w+1)}+\frac{1}{w+1}-2$$
$$=\frac{1+1(w^2-w+1)-2(w+1)(w^2-w+1)}{(w+1)(w^2-w+1)}$$
$$=\frac{1+w^2-w+1-2(w^3+1)}{(w+1)(w^2-w+1)}$$
$$=\frac{w^2-w+2-2w^3-2}{(w+1)(w^2-w+1)}$$
$$=\frac{-2w^3+w^2-w}{(w+1)(w^2-w+1)}$$
$$=\frac{-w(2w^2-w+1)}{(w+1)(w^2-w+1)}$$

40. $$\frac{2}{8-x^3}+\frac{1}{x^2-x-2}=\frac{-2}{x^3-8}+\frac{1}{x^2-x-2}$$
$$=\frac{-2}{(x-2)(x^2+2x+4)}+\frac{1}{(x-2)(x+1)}$$
$$=\frac{-2(x+1)+1(x^2+2x+4)}{(x-2)(x^2+2x+4)(x+1)}$$
$$=\frac{-2x-2+x^2+2x+4}{(x-2)(x^2+2x+4)(x+1)}$$
$$=\frac{x^2+2}{(x-2)(x+1)(x^2+2x+4)}$$

41. $$\frac{\frac{3}{x}}{\frac{1}{x}-1}=\frac{\frac{3}{x}}{\frac{1-x}{x}}$$
$$=\frac{3}{x}\times\frac{x}{1-x}$$
$$=\frac{3}{1-x},\ x\neq 0,1$$

42. $\dfrac{a-\frac{1}{a}}{1-\frac{1}{a}}=\dfrac{\frac{a^2-1}{a}}{\frac{a-1}{a}}$

$=\dfrac{(a-1)(a+1)}{a}\times\dfrac{a}{a-1}$

$=a+1 \quad \text{where} \quad a\neq 0,1$

43. $\dfrac{\frac{x}{y}-\frac{y}{x}}{1+\frac{y}{x}}=\dfrac{\frac{x(x)-y(y)}{xy}}{\frac{x+y}{x}}$

$=\dfrac{x^2-y^2}{xy}\times\dfrac{x}{x+y}$

$=\dfrac{x(x+y)(x-y)}{xy(x+y)}$

$=\dfrac{x-y}{y} \quad \text{where} \quad x,y,x+y\neq 0$

44. $\dfrac{\frac{V^2-9}{V}}{\frac{1}{V}-\frac{1}{3}}=\dfrac{\frac{V^2-9}{V}}{\frac{3-V}{3V}}$

$=\dfrac{(V+3)(V-3)}{V}\times\dfrac{3V}{3-V}$

$=\dfrac{(V-3)(V+3)}{V}\times\dfrac{3V}{-(V-3)}$

$=-3(V+3) \quad \text{where} \quad V\neq 0,3$

45. $\dfrac{\frac{3}{x}+\frac{1}{x^2+x}}{\frac{1}{x+1}-\frac{1}{x-1}}=\dfrac{\frac{3}{x}+\frac{1}{x(x+1)}}{\frac{1}{x+1}-\frac{1}{x-1}}$

$=\dfrac{\frac{3(x+1)+1}{x(x+1)}}{\frac{1(x-1)-1(x+1)}{(x+1)(x-1)}}$

$=\dfrac{3x+3+1}{x(x+1)}\times\dfrac{(x+1)(x-1)}{x-1-x-1}$

$=\dfrac{3x+4}{x(x+1)}\times\dfrac{(x+1)(x-1)}{-2}$

$=-\dfrac{(3x+4)(x-1)}{2x} \quad , x\neq 0,1,-1$

46. $$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{x}}}=1+\cfrac{1}{1+\cfrac{1}{\frac{x+1}{x}}}$$
$$=1+\cfrac{1}{1+\cfrac{x}{x+1}}$$
$$=1+\cfrac{1}{\frac{x+1}{x+1}+\frac{x}{x+1}}$$
$$=1+\cfrac{1}{\frac{2x+1}{x+1}}$$
$$=1+\frac{x+1}{2x+1}$$
$$=\frac{2x+1}{2x+1}+\frac{x+1}{2x+1}$$
$$=\frac{3x+2}{2x+1}$$

47. $$f(x)=\frac{x}{x+1}$$
$$f(x+h)-f(x)=\frac{x+h}{x+h+1}-\frac{x}{x+1}$$
$$f(x+h)-f(x)=\frac{(x+h)(x+1)-x(x+h+1)}{(x+h+1)(x+1)}$$
$$f(x+h)-f(x)=\frac{x^2+x+hx+h-x^2-xh-x}{(x+1)(x+h+1)}$$
$$f(x+h)-f(x)=\frac{h}{(x+1)(x+h+1)}$$

48. $$f(x)=\frac{3}{1-2x}$$
$$f(x+h)-f(x)=\frac{3}{1-2(x+h)}-\frac{3}{1-2x}$$
$$=\frac{3}{1-2x-2h}-\frac{3}{1-2x}$$
$$=\frac{3(1-2x)-3(1-2x-2h)}{(1-2x)(1-2x-2h)}$$
$$=\frac{3-6x-3+6x+6h}{(1-2x)(1-2x-2h)}$$
$$=\frac{6h}{(1-2x)(1-2x-2h)}$$

49.
$$f(x)=\frac{1}{x^2}$$
$$\begin{aligned} f(x+h)-f(x)&=\frac{1}{(x+h)^2}-\frac{1}{x^2}\\ &=\frac{1(x^2)-1\left(x^2+2xh+h^2\right)}{x^2(x+h)^2}\\ &=\frac{x^2-x^2-2xh-h^2}{x^2(x+h)^2}\\ &=\frac{-2xh-h^2}{x^2(x+h)^2}\\ &=\frac{-h(2x+h)}{x^2(x+h)^2}\end{aligned}$$

50.
$$f(x)=\frac{2}{x^2+4}$$
$$\begin{aligned} f(x+h)-f(x)&=\frac{2}{(x+h)^2+4}-\frac{2}{x^2+4}\\ &=\frac{2}{x^2+2xh+h^2+4}-\frac{2}{x^2+4}\\ &=\frac{2\left(x^2+4\right)-2\left(x^2+2xh+h^2+4\right)}{\left(x^2+2xh+h^2+4\right)\left(x^2+4\right)}\\ &=\frac{2x^2+8-2x^2-4xh-2h^2-8}{\left(x^2+2xh+h^2+4\right)\left(x^2+4\right)}\\ &=\frac{-4xh-2h^2}{\left(x^2+2xh+h^2+4\right)\left(x^2+4\right)}\\ &=\frac{-2h(2x+h)}{\left(x^2+2xh+h^2+4\right)\left(x^2+4\right)}\end{aligned}$$

51.
$$\begin{aligned} (\tan\theta)(\cot\theta)+(\sin\theta)^2-\cos\theta&=\frac{y}{x}\cdot\frac{x}{y}+\left(\frac{y}{r}\right)^2-\frac{x}{r}\\ &=1+\frac{y^2}{r^2}-\frac{x}{r}\\ &=\frac{1(r^2)+y^2-x(r)}{r^2}\\ &=\frac{r^2+y^2-rx}{r^2}\end{aligned}$$

52. $\sec\theta-(\cot\theta)^2+\csc\theta=\dfrac{r}{x}-\left(\dfrac{x}{y}\right)^2+\dfrac{r}{y}$

$$=\frac{r}{x}-\frac{x^2}{y^2}+\frac{r}{y}$$

$$=\frac{r(y^2)-x^2(x)+r(xy)}{xy^2}$$

$$=\frac{ry^2-x^3+rxy}{xy^2}$$

53. $f(x)=2x-x^2$

$$f\left(\frac{1}{a}\right)=2\left(\frac{1}{a}\right)-\left(\frac{1}{a}\right)^2$$

$$f\left(\frac{1}{a}\right)=\frac{2}{a}-\frac{1}{a^2}$$

$$f\left(\frac{1}{a}\right)=\frac{2a-1}{a^2}$$

54. $f(x)=x^2+x$

$$f\left(a+\frac{1}{a}\right)=\left(a+\frac{1}{a}\right)^2+\left(a+\frac{1}{a}\right)$$

$$=\left(\frac{a^2+1}{a}\right)^2+\left(\frac{a^2+1}{a}\right)$$

$$=\frac{a^4+2a^2+1}{a^2}+\frac{a^2+1}{a}$$

$$=\frac{a^4+2a^2+1+(a^2+1)a}{a^2}$$

$$=\frac{a^4+2a^2+1+a^3+a}{a^2}$$

$$=\frac{a^4+a^3+2a^2+a+1}{a^2}$$

55. $f(x)=x-\dfrac{1}{x}$

$$f(a+1)=a+1-\frac{1}{a+1}$$

$$=\frac{(a+1)^2-1}{a+1}$$

$$=\frac{a^2+2a+1-1}{a+1}$$

$$=\frac{a^2+2a}{a+1}$$

56. $f(x) = 2x - 3$

$$f \ \frac{1}{f(x)} \ = f \ \frac{1}{2x-3}$$

$$f \ \frac{1}{f(x)} \ = 2 \ \frac{1}{2x-3} \ - 3$$

$$= \frac{2-3(2x-3)}{2x-3}$$

$$= \frac{2-6x+9}{2x-3}$$

$$= \frac{-6x+11}{2x-3}$$

57. $\dfrac{a+b}{\frac{1}{a}+\frac{1}{b}} = \dfrac{a+b}{\frac{b+a}{ab}}$

$$\frac{a+b}{\frac{1}{a}+\frac{1}{b}} = (a+b) \times \frac{ab}{(a+b)}$$

$$\frac{a+b}{\frac{1}{a}+\frac{1}{b}} = ab \quad \text{where } a \neq -b$$

58. $\dfrac{2x-9}{x^2-x-6} = \dfrac{A}{x-3} + \dfrac{B}{x+2}$

$$\frac{2x-9}{(x-3)(x+2)} = \frac{A(x+2)+B(x-3)}{(x-3)(x+2)}$$

$$2x-9 = A(x+2)+B(x-3)$$

$$2x-9 = Ax+2A+Bx-3B$$

$$2x-9 = Ax+Bx+2A-3B$$

$$2x-9 = (A+B)x+2A-3B$$

Comparing the x-term

$2 = A+B$ (Equation 1)

Comparing the constant term

$-9 = 2A-3B$ (Equation 2)

This is a system of equations (see Chapter 5)

Solving the first equation above

$B = 2 - A$

Substituting into the second equation above

$-9 = 2A - 3(2-A)$

$-9 = 2A - 6 + 3A$

$5A = -3$

$A = -\frac{3}{5}$

$B = 2 - -\frac{3}{5}$

$B = \frac{10+3}{5} = \frac{13}{5}$

59. $$\frac{y^2-x^2}{y^2+x^2}=\frac{\left(\frac{mn}{m-n}\right)^2-\left(\frac{mn}{m+n}\right)^2}{\left(\frac{mn}{m-n}\right)^2+\left(\frac{mn}{m+n}\right)^2}$$

$$=\frac{\frac{m^2n^2(m+n)^2-m^2n^2(m-n)^2}{(m-n)^2(m+n)^2}}{\frac{m^2n^2(m+n)^2+m^2n^2(m-n)^2}{(m-n)^2(m+n)^2}}$$

$$=\frac{m^2n^2(m+n)^2-m^2n^2(m-n)^2}{m^2n^2(m+n)^2+m^2n^2(m-n)^2}$$

$$=\frac{m^2n^2\left((m+n)^2-(m-n)^2\right)}{m^2n^2\left((m+n)^2+(m-n)^2\right)}$$

$$=\frac{m^2+2mn+n^2-(m^2-2mn+n^2)}{m^2+2mn+n^2+(m^2-2mn+n^2)}$$

$$=\frac{4mn}{2m^2+2n^2}$$

$$=\frac{2mn}{m^2+n^2}$$

60. Using the LCD gives simpler expressions, since you don't have to multiply out the larger factors present in any common denominator that isn't the lowest.

61. $$\frac{3}{4\pi}-\frac{3H_0}{4\pi H}=\frac{3(H)-3H_0}{4\pi H}=\frac{3(H-H_0)}{4\pi H}$$

62. $$1+\frac{9}{128T}-\frac{27p}{64T^3}=\frac{(1)128T^3+9(T^2)-27(2)p}{128T^3}$$

$$=\frac{128T^3+9T^2-54p}{128T^3}$$

63. $$\frac{2n^2-n-4}{2n^2+2n-4}+\frac{1}{n-1}=\frac{2n^2-n-4}{2(n^2+n-2)}+\frac{1}{n-1}$$

$$=\frac{2n^2-n-4}{2(n+2)(n-1)}+\frac{1}{n-1}$$

$$=\frac{2n^2-n-4+1(2)(n+2)}{2(n-1)(n+2)}$$

$$=\frac{2n^2-n-4+2n+4}{2(n-1)(n+2)}$$

$$=\frac{2n^2+n}{2(n-1)(n+2)}$$

$$=\frac{n(2n+1)}{2(n-1)(n+2)}$$

64. $$\frac{b}{x^2+y^2}-\frac{2bx^2}{x^4+2x^2y^2+y^4}=\frac{b}{x^2+y^2}-\frac{2bx^2}{(x^2+y^2)^2}$$
$$=\frac{b\left(x^2+y^2\right)-2bx^2}{\left(x^2+y^2\right)^2}$$
$$=\frac{b\left(x^2+y^2-2x^2\right)}{\left(x^2+y^2\right)^2}$$
$$=\frac{b\left(-x^2+y^2\right)}{\left(x^2+y^2\right)^2}$$

65. $$\left(\frac{3Px}{2L^2}\right)^2+\left(\frac{P}{2L}\right)^2=\frac{9P^2x^2}{4L^4}+\frac{P^2}{4L^2}$$
$$=\frac{9P^2x^2+P^2(L^2)}{4L^4}$$
$$=\frac{P^2\left(9x^2+L^2\right)}{4L^4}$$

66. $$\frac{a}{b^2h}+\frac{c}{bh^2}-\frac{1}{6bh}=\frac{a(6h)+c(6b)-1(bh)}{6b^2h^2}$$
$$=\frac{6ah+6bc-bh}{6b^2h^2}$$

67. $$\frac{\frac{L}{C}+\frac{R}{sC}}{sL+R+\frac{1}{sC}}=\frac{\frac{Ls+R}{sC}}{\frac{(sL+R)sC+1}{sC}}$$
$$=\frac{\frac{Ls+R}{sC}}{\frac{CLs^2+CRs+1}{sC}}$$
$$=\frac{Ls+R}{sC}\times\frac{sC}{CLs^2+CRs+1}$$
$$=\frac{Ls+R}{CLs^2+CRs+1}$$

68. $$\frac{\frac{m}{c}}{1-\frac{p^2}{c^2}}=\frac{\frac{m}{c}}{\frac{c^2-p^2}{c^2}}$$
$$=\frac{m}{c}\times\frac{c^2}{c^2-p^2}$$
$$=\frac{mc}{(c-p)(c+p)}$$

69. $$\frac{1}{R^2}+\left(\omega c-\frac{1}{\omega L}\right)^2=\frac{1}{R^2}+\left(\frac{\omega^2 Lc-1}{\omega L}\right)^2$$
$$=\frac{1}{R^2}+\frac{\omega^4 L^2c^2-2\omega^2 Lc+1}{\omega^2 L^2}$$
$$=\frac{\omega^2 L^2+\omega^4 L^2c^2R^2-2\omega^2 LcR^2+R^2}{R^2\omega^2 L^2}$$

70. $$\frac{\frac{x}{h_1}+\frac{x-L}{h_2}}{1+\frac{x(L-x)}{h_1h_2}}=\frac{\frac{xh_2+(x-L)h_1}{h_1h_2}}{\frac{h_1h_2+x(L-x)}{h_1h_2}}$$
$$=\frac{xh_2+(x-L)h_1}{h_1h_2}\times\frac{h_1h_2}{h_1h_2+x(L-x)}$$
$$=\frac{xh_2+(x-L)h_1}{h_1h_2+x(L-x)}$$
$$=\frac{xh_2+xh_1-Lh_1}{h_1h_2+xL-x^2}$$

71. The boat takes $\frac{5}{v-w}$ hours to travel upstream and $\frac{5}{v+w}$ hours to travel downstream. The total time is
$$\frac{5}{v-w}+\frac{5}{v+w}=\frac{5(v+w)}{(v-w)(v+w)}+\frac{5(v-w)}{(v-w)(v+w)}$$
$$=\frac{10v}{v^2-w^2}\text{ hours.}$$

72. The time for the first leg is $\frac{d}{v_1}$ and the time for the second leg is $\frac{d}{v_2}$.
The total distance traveled is $2d$. The average speed for the round trip is
$$\frac{2d}{\frac{d}{v_1}+\frac{d}{v_2}}=\frac{2d}{\frac{dv_2}{v_1v_2}+\frac{dv_1}{v_1v_2}}$$
$$=2d\div\frac{dv_2+dv_1}{v_1v_2}$$
$$=\frac{2d}{1}\times\frac{v_1v_2}{d(v_1+v_2)}$$
$$=\frac{2v_1v_2}{v_1+v_2}$$

6.7 Equations Involving Fractions

1.

$$\frac{x}{2}-\frac{1}{b}=\frac{x}{2b}$$
$$\frac{x(2b)}{2}-\frac{1(2b)}{b}=\frac{x(2b)}{2b}$$
$$xb-2=x$$
$$x(b-1)=2$$
$$x=\frac{2}{b-1}$$

2.

$$f=\frac{pq}{p+q}$$
$$f(p+q)=\frac{pq(p+q)}{p+q}$$
$$fp+fq=pq$$
$$fp-pq=-fq$$
$$p(f-q)=-fq$$
$$p=\frac{fq}{q-f}$$

3.

$$\frac{1}{2}v^2-\frac{GM}{r}=-\frac{GM}{2a}$$
$$\frac{1}{2}v^2(2ar)-\frac{GM}{r}(2ar)=-\frac{GM}{2a}(2ar)$$
$$arv^2-2aGM=-GMr$$
$$GMr-2aGM=-arv^2$$
$$G(Mr-2aM)=-arv^2$$
$$G=\frac{arv^2}{2aM-Mr}$$
$$G=\frac{arv^2}{M(2a-r)}$$

4.

$$\frac{2}{x+1}-\frac{1}{x}=-\frac{1}{x^2+x}$$
$$\frac{2}{x+1}-\frac{1}{x}=-\frac{1}{x(x+1)}$$
$$\frac{2x(x+1)}{x+1}-\frac{1x(x+1)}{x}=-\frac{1x(x+1)}{x(x+1)}$$
$$2x-(x+1)=-1$$
$$2x-x-1=-1$$
$$x=0$$

which gives division by zero in the second fraction.
Therefore, no solution.

5.
$$\begin{aligned}\frac{x}{2}+6&=2x\\ \frac{x(2)}{2}+6(2)&=2x(2)\\ x+12&=4x\\ -3x&=-12\\ x&=4\end{aligned}$$

6.
$$\begin{aligned}\frac{x}{5}+2&=\frac{15+x}{10}\\ \frac{x(10)}{5}+2(10)&=\frac{(15+x)(10)}{10}\\ 2x+20&=15+x\\ x&=-5\end{aligned}$$

7.
$$\begin{aligned}\frac{x}{6}-\frac{1}{2}&=\frac{x}{3}\\ \frac{x(6)}{6}-\frac{1(6)}{2}&=\frac{x(6)}{3}\\ x-3&=2x\\ -x&=3\\ x&=-3\end{aligned}$$

8.
$$\begin{aligned}\frac{3N}{8}-\frac{2}{4}&=\frac{N-4}{2}\\ \frac{3N(8)}{8}-\frac{2(8)}{4}&=\frac{(N-4)(8)}{2}\\ 3N-4&=4N-16\\ -N&=-10\\ N&=10\end{aligned}$$

9.
$$\begin{aligned}\frac{1}{4}-\frac{t-3}{8}&=\frac{1}{6}\\ \frac{1(24)}{4}-\frac{(t-3)(24)}{8}&=\frac{1(24)}{6}\\ 6-3(t-3)&=4\\ 6-3t+9&=4\\ -3t&=-11\\ t&=\frac{11}{3}\end{aligned}$$

10.
$$\frac{2x-7}{3}+5=\frac{1}{5}$$
$$\frac{(2x-7)(15)}{3}+5(15)=\frac{1(15)}{5}$$
$$10x-35+75=3$$
$$10x=-37$$
$$x=-\frac{37}{10}$$

11.
$$\frac{3x}{7}-\frac{5}{21}=\frac{2-x}{14}$$
$$\frac{3x(42)}{7}-\frac{5(42)}{21}=\frac{(2-x)(42)}{14}$$
$$18x-10=3(2-x)$$
$$18x-10=6-3x$$
$$21x=16$$
$$x=\frac{16}{21}$$

12.
$$\frac{F-3}{12}-\frac{2}{3}=\frac{1-3F}{2}$$
$$\frac{(F-3)(12)}{12}-\frac{2(12)}{3}=\frac{(1-3F)(12)}{2}$$
$$F-3-8=6(1-3F)$$
$$F-11=6-18F$$
$$19F=17$$
$$F=\frac{17}{19}$$

13.
$$\frac{3}{T}+2=\frac{5}{3}$$
$$\frac{3(3T)}{T}+2(3T)=\frac{5(3T)}{3}$$
$$9+6T=5T$$
$$T=-9$$

14.
$$\frac{1}{2y}-\frac{1}{2}=4$$
$$\frac{1(2y)}{2y}-\frac{1(2y)}{2}=4(2y)$$
$$1-y=8y$$
$$9y=1$$
$$y=\frac{1}{9}$$

15.
$$\frac{4}{a}+\frac{1}{5}=\frac{3}{a}$$
$$\frac{4(5a)}{a}+\frac{1(5a)}{5}=\frac{3(5a)}{a}$$
$$20+a=15$$
$$a=-5$$

16.
$$\frac{2}{3}+\frac{3}{y}=\frac{5}{2y}$$
$$\frac{2(6y)}{3}+\frac{3(6y)}{y}=\frac{5(6y)}{2y}$$
$$4y+18=15y$$
$$18=11y$$
$$y=\frac{18}{11}$$

17.
$$3-\frac{x-2}{5x}=\frac{1}{5}$$
$$3(5x)-\frac{(x-2)(5x)}{5x}=\frac{1(5x)}{5}$$
$$15x-x+2=x$$
$$13x=-2$$
$$x=-\frac{2}{13}$$

18.
$$\frac{1}{2R}=\frac{2}{3R}+\frac{1}{3}$$
$$\frac{1(6R)}{2R}=\frac{2(6R)}{3R}+\frac{1(6R)}{3}$$
$$3=4+2R$$
$$2R=-1$$
$$R=-\frac{1}{2}$$

19.
$$\frac{3a}{a-3}=5$$
$$\frac{3a(a-3)}{a-3}=5(a-3)$$
$$3a=5a-15$$
$$15=2a$$
$$a=\frac{15}{2}$$

20.

$$
\begin{aligned}
\frac{x}{2x-3} &= 4 \\
\frac{x(2x-3)}{2x-3} &= 4(2x-3) \\
x &= 8x-12 \\
-7x &= -12 \\
x &= \frac{12}{7}
\end{aligned}
$$

21.

$$
\begin{aligned}
\frac{2}{s} &= \frac{3}{s-1} \\
\frac{2s(s-1)}{s} &= \frac{3s(s-1)}{s-1} \\
2(s-1) &= 3s \\
2s-2 &= 3s \\
s &= -2
\end{aligned}
$$

22.

$$
\begin{aligned}
\frac{5}{2n+4} &= \frac{3}{4n} \\
\frac{5(4n)(n+2)}{2n+4} &= \frac{3(4n)(n+2)}{4n} \\
10n &= 3(n+2) \\
10n &= 3n+6 \\
7n &= 6 \\
n &= \frac{6}{7}
\end{aligned}
$$

23.

$$
\begin{aligned}
\frac{5}{2x+4} + \frac{3}{6x+12} &= 2 \\
\frac{5}{2(x+2)} + \frac{3}{6(x+2)} &= 2 \\
\frac{5(6)(x+2)}{2(x+2)} + \frac{3(6)(x+2)}{6(x+2)} &= 2(6)(x+2) \\
15+3 &= 12(x+2) \\
18 &= 12x+24 \\
-12x &= 6 \\
x &= -\frac{1}{2}
\end{aligned}
$$

24.
$$\frac{6}{4x-6}+\frac{2}{4}=\frac{6}{3-2x}$$
$$\frac{3}{2x-3}+\frac{1}{2}=\frac{-6}{2x-3}$$
$$\frac{3(2)(2x-3)}{2x-3}+\frac{1(2)(2x-3)}{2}=\frac{-6(2)(2x-3)}{2x-3}$$
$$6+2x-3=-12;$$
$$2x=-15$$
$$x=-\frac{15}{2}$$

25.
$$\frac{2}{z-5}-\frac{7}{10-2z}=3$$
$$\frac{2}{Z-5}+\frac{7}{2(Z-5)}=3$$
$$\frac{2(2)(z-5)}{z-5}+\frac{7(2)(z-5)}{2(z-5)}=3(2)(z-5)$$
$$4+7=6z-30$$
$$41=6z$$
$$z=\frac{41}{6}$$

26.
$$\frac{4}{4-x}+2=\frac{2}{12-3x}+\frac{1}{3}$$
$$\frac{4}{4-x}+2-\frac{2}{12-3x}=\frac{1}{3}$$
$$\frac{4}{4-x}+2-\frac{2}{3(4-x)}=\frac{1}{3}$$
$$\frac{4(3)(4-x)}{4-x}+2(3)(4-x)-\frac{2(3)(4-x)}{3(4-x)}=\frac{1(3)(4-x)}{3}$$
$$12+24-6x-2=4-x$$
$$-5x=-30$$
$$x=6$$

27.
$$\frac{1}{4x}+\frac{3}{2x}=\frac{2}{x+1}$$
$$\frac{1(4x)(x+1)}{4x}+\frac{3(4x)(x+1)}{2x}=\frac{2(4x)(x+1)}{x+1}$$
$$(x+1)+6(x+1)=8x$$
$$x+1+6x+6=8x$$
$$-x=-7$$
$$x=7$$

28.

$$\frac{3}{t+3}-\frac{1}{t}=\frac{5}{6+2t}$$
$$\frac{3}{t+3}-\frac{1}{t}=\frac{5}{2(t+3)}$$
$$\frac{3(2t)(t+3)}{t+3}-\frac{1(2t)(t+3)}{t}=\frac{5(2t)(t+3)}{2(t+3)}$$
$$6t-2(t+3)=5t$$
$$6t-2t-6=5t$$
$$-t=6$$
$$t=-6$$

29.

$$\frac{5}{y}=\frac{2}{y-3}+\frac{7}{2y^2-6y}$$
$$\frac{5}{y}=\frac{2}{y-3}+\frac{7}{2y(y-3)}$$
$$\frac{5(2y)(y-3)}{y}=\frac{2(2y)(y-3)}{y-3}+\frac{7(2y)(y-3)}{2y(y-3)}$$
$$10(y-3)=4y+7$$
$$10y-30=4y+7$$
$$6y=37$$
$$y=\frac{37}{6}$$

30.

$$\frac{1}{2x+3}=\frac{5}{2x}-\frac{4}{2x^2+3x}$$
$$\frac{1}{2x+3}=\frac{5}{2x}-\frac{4}{x(2x+3)}$$
$$\frac{1(2x)(2x+3)}{2x+3}=\frac{5(2x)(2x+3)}{2x}-\frac{4(2x)(2x+3)}{x(2x+3)}$$
$$2x=5(2x+3)-8$$
$$2x=10x+15-8$$
$$-8x=7$$
$$x=-\frac{7}{8}$$

31.

$$\frac{1}{x^2-x}-\frac{1}{x}+\frac{1}{1-x}=0$$

$$\frac{1}{x(x-1)}-\frac{1}{x}-\frac{1}{x-1}=0$$

$$\frac{1(x)(x-1)}{x(x-1)}-\frac{1(x)(x-1)}{x}-\frac{1(x)(x-1)}{x-1}=0$$

$$1-(x-1)-x=0$$

$$2-2x=0$$

$$-2x=-2$$

$$x=1$$

Substitution reveals a zero in the denominator for the first and third fractions in the original equation. Therefore no solution exists.

32.

$$\frac{2}{x^2-1}-\frac{2}{x+1}=\frac{1}{x-1}$$

$$\frac{2}{(x+1)(x-1)}-\frac{2}{x+1}=\frac{1}{x-1}$$

$$\frac{2(x+1)(x-1)}{(x+1)(x-1)}-\frac{2(x+1)(x-1)}{x+1}=\frac{1(x+1)(x-1)}{x-1}$$

$$2-2(x-1)=1(x+1)$$

$$2-2x+2=x+1$$

$$-3x=-3$$

$$x=1$$

Checking: Substituting $x=1$ into the first and third terms yields a zero in the denominator, which is undefined. Therefore, this equation does not have a solution.

33.

$$\frac{2}{B^2-4}-\frac{1}{B-2}=\frac{1}{2B+4}$$

$$\frac{2}{(B+2)(B-2)}-\frac{1}{B-2}=\frac{1}{2(B+2)}$$

$$\frac{2(2)(B+2)(B-2)}{(B+2)(B-2)}-\frac{1(2)(B+2)(B-2)}{B-2}=\frac{1(2)(B+2)(B-2)}{2(B+2)}$$

$$4-2(B+2)=B-2$$

$$4-2B-4=B-2$$

$$-3B=-2$$

$$B=\frac{2}{3}$$

34.
$$
\begin{aligned}
\frac{2}{2x^2+5x-3}-\frac{1}{4x-2}+\frac{3}{2x+6}&=0\\
\frac{2}{(2x-1)(x+3)}-\frac{1}{2(2x-1)}+\frac{3}{2(x+3)}&=0\\
\frac{2(2)(2x-1)(x+3)}{(2x-1)(x+3)}-\frac{1(2)(2x-1)(x+3)}{2(2x-1)}+\frac{3(2)(2x-1)(x+3)}{2(x+3)}&=0\\
4-(x+3)+3(2x-1)&=0\\
4-x-3+6x-3&=0\\
5x&=2\\
x&=\frac{2}{5}
\end{aligned}
$$

35.
$$
\begin{aligned}
2-\frac{1}{b}+\frac{3}{c}&=0,\ \text{for } c\\
2bc-\frac{1bc}{b}+\frac{3bc}{c}&=0bc\\
2bc-c+3b&=0\\
c(2b-1)&=-3b\\
c&=\frac{3b}{1-2b}
\end{aligned}
$$

36.
$$
\begin{aligned}
\frac{2}{3}-\frac{h}{x}&=\frac{1}{6x},\quad \text{for } x\\
\frac{2(6x)}{3}-\frac{h(6x)}{x}&=\frac{1(6x)}{6x}\\
4x-6h&=1\\
4x&=1+6h\\
x&=\frac{1+6h}{4}
\end{aligned}
$$

37.
$$
\begin{aligned}
\frac{t-3}{b}-\frac{t}{2b-1}&=\frac{1}{2},\ \text{for } t\\
\frac{(t-3)(2b)(2b-1)}{b}-\frac{t(2b)(2b-1)}{2b-1}&=\frac{1(2b)(2b-1)}{2}\\
(t-3)(2b-1)(2)-t(2b)&=b(2b-1)\\
(t-3)(4b-2)-2bt&=2b^2-b\\
4bt-2t-12b+6-2bt&=2b^2-b\\
2bt-2t&=2b^2+11b-6\\
2t(b-1)&=(2b-1)(b+6)\\
t&=\frac{(2b-1)(b+6)}{2(b-1)}
\end{aligned}
$$

38.
$$\frac{1}{a^2+2a}-\frac{y}{2a}=\frac{2y}{a+2},\text{ for } y$$
$$\frac{1}{a(a+2)}-\frac{y}{2a}=\frac{2y}{a+2}$$
$$\frac{1(2a)(a+2)}{a(a+2)}-\frac{y(2a)(a+2)}{2a}=\frac{2y(2a)(a+2)}{a+2}$$
$$2-y(a+2)=2y(2a)$$
$$2-ay-2y=4ay$$
$$-5ay-2y=-2$$
$$-y(5a+2)=-2$$
$$y=\frac{2}{5a+2}$$

39.
$$\frac{s-s_\circ}{t}=\frac{v+v_\circ}{2},\text{ for } v$$
$$\frac{(s-s_\circ)(2t)}{t}=\frac{(v+v_\circ)(2t)}{2}$$
$$2(s-s_0)=t(v+v_0)$$
$$2(s-s_0)=tv+tv_0$$
$$2(s-s_0)-tv_0=tv$$
$$v=\frac{2(s-s_0)-tv_0}{t}$$

40.
$$S=\frac{P}{A}+\frac{Mc}{I}$$
$$SAI=\frac{PAI}{A}+\frac{McAI}{I}$$
$$SAI=PI+McA$$
$$SAI-McA=PI$$
$$A(SI-Mc)=PI$$
$$A=\frac{PI}{SI-Mc}$$

41.

$$V = 1.2\ \ 5.0 + \frac{8.0R}{8.0 + R}$$

$$V = 6.0 + \frac{9.6R}{8.0 + R}$$

$$V(8.0 + R) = 6.0(8.0 + R) + \frac{9.6R(8.0 + R)}{8.0 + R}$$

$$V\left(8.0 + R\right) = 48 + 6.0R + 9.6R$$

$$8.0V + RV = 48 + 15.6R$$

$$8.0V - 48 = R\left(15.6 - V\right)$$

$$R = \frac{8.0V - 48}{15.6 - V}$$

$$R = \frac{8.0(V - 6.0)}{15.6 - V}$$

42.

$$C = \frac{7p}{100 - p}$$

$$C\left(100 - p\right) = 7p$$

$$100C - pC = 7p$$

$$7p + pC = 100C$$

$$p\left(7 + C\right) = 100C$$

$$p = \frac{100C}{7 + C}$$

43.

$$z = \frac{1}{g_m} - \frac{jX}{g_m R}\ ,\ \text{for } R$$

$$zg_m R = \frac{1g_m R}{g_m} - \frac{jXg_m R}{g_m R}$$

$$g_m Rz = R - jX$$

$$g_m Rz - R = -jX$$

$$R\left(g_m z - 1\right) = -jX$$

$$R = \frac{jX}{1 - g_m z}$$

44.

$$A = \frac{1}{2}wp - \frac{1}{2}w^2 - \frac{\pi}{8}w^2$$

$$8A = \frac{1}{2}wp(8) - \frac{1}{2}w^2(8) - \frac{\pi}{8}w^2(8)$$

$$8A = 4wp - 4w^2 - \pi w^2$$

$$4wp = 8A + 4w^2 + \pi w^2$$

$$p = \frac{8A + 4w^2 + \pi w^2}{4w}$$

45.
$$p = \frac{RT}{V-b} - \frac{a}{V^2}$$
$$pV^2(V-b) = \frac{RTV^2(V-b)}{V-b} - \frac{aV^2(V-b)}{V^2}$$
$$pV^2\left(V-b\right) = RTV^2 - a\left(V-b\right)$$
$$pV^3 - pV^2b = RTV^2 - aV + ab$$
$$RTV^2 = pV^3 - pV^2b + aV - ab$$
$$T = \frac{pV^3 - pV^2b + aV - ab}{RV^2}$$

46.
$$\frac{1}{x} + \frac{1}{nx} = \frac{1}{f}$$
$$\frac{1nxf}{x} + \frac{1nxf}{nx} = \frac{1nxf}{f}$$
$$nf + f = nx$$
$$f\left(n+1\right) = nx$$
$$f = \frac{nx}{n+1}$$

47.
$$\frac{1}{C} = \frac{1}{C_2} + \frac{1}{C_1 + C_3}$$
$$\frac{1CC_2(C_1+C_3)}{C} = \frac{1CC_2(C_1+C_3)}{C_2} + \frac{1CC_2(C_1+C_3)}{C_1+C_3}$$
$$C_2\left(C_1 + C_3\right) = C\left(C_1 + C_3\right) + CC_2$$
$$C_1C_2 + C_2C_3 = CC_1 + CC_3 + CC_2$$
$$C_1\left(C_2 - C\right) = CC_3 + CC_2 - C_2C_3$$
$$C_1 = \frac{CC_3 + CC_2 - C_2C_3}{C_2 - C}$$

48.
$$D = \frac{wx^4}{24EI} - \frac{wLx^3}{6EI} + \frac{wL^2x^2}{4EI}$$
$$D(24EI) = \frac{wx^4(24EI)}{24EI} - \frac{wLx^3(24EI)}{6EI} + \frac{wL^2x^2(24EI)}{4EI}$$
$$24EID = wx^4 - 4wLx^3 + 6wL^2x^2$$
$$w\left(x^4 - 4Lx^3 + 6L^2x^2\right) = 24EID$$
$$w = \frac{24EID}{x^4 - 4Lx^3 + 6L^2x^2}$$
$$w = \frac{24EID}{x^2\left(x^2 - 4Lx + 6L^2\right)}$$

49.

$$f(n-1)=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

$$f(n-1)=\frac{1}{\frac{1R_2+1R_1}{R_1R_2}}$$

$$f(n-1)=\frac{R_1R_2}{R_1+R_2}$$

$$f(n-1)(R_1+R_2)=\frac{R_1R_2}{R_1+R_2}(R_1+R_2)$$

$$f(n-1)(R_1+R_2)=R_1R_2$$

$$f(n-1)(R_1)-R_1R_2=-f(n-1)(R_2)$$

$$R_1(nf-f-R_2)=f(1-n)(R_2)$$

$$R_1=\frac{f(1-n)(R_2)}{nf-f-R_2}$$

50.

$$P=\frac{\frac{1}{i+1}}{1-\frac{1}{1+i}}$$

$$P=\frac{\frac{1}{i+1}}{\frac{1+i-1}{1+i}}$$

$$P=\frac{\frac{1}{i+1}}{\frac{i}{i+1}}$$

$$P=\frac{1}{i+1}\times\frac{i+1}{i}$$

$$P=\frac{1}{i}$$

$$iP=\frac{1}{i}i$$

$$iP=1$$

$$i=\frac{1}{P}$$

51. If Pump 1 empties tank of volume V in 5.00 h, its rate of emptying is $\frac{V}{5.00}$.

If Pump 2 empties tank of volume V in 8.00 h, its rate of emptying is $\frac{V}{8.00}$.

If both are operating, then in time elapsed t, to empty the volume

$$\frac{V}{5.0}\times t+\frac{V}{8.0}\times t=V$$

Note how rate (in cubic metres per hour) multiplied by time (in hours) gives volume (in cubic metres). Now solving for t:

$$\frac{Vt(40)}{5.0}+\frac{Vt(40)}{8.0}=V(40)$$
$$8.0Vt+5.0Vt=40V$$
$$13Vt=40V$$
$$t=\frac{40}{13}$$
$$t=3.08\text{ h}$$

52. $\text{rate}=\dfrac{\text{work}}{\text{time}}$

work = rate × time

For crew 1: $r_1=\dfrac{1}{450}$

For crew 2: $r_2=\dfrac{1}{600}$

To complete the whole job

$$r_1t+r_2t=1\text{ complete job}$$
$$\frac{1}{450}t+\frac{1}{600}t=1$$
$$\frac{1800t}{450}+\frac{1800t}{600}=1800$$
$$4t+3t=1800$$
$$7t=1800$$
$$t=\frac{1800}{7}=260\text{ h}$$

53. $\text{rate}=\dfrac{\text{work}}{\text{time}}$

work = rate × time

For machine 1: $r_1=\dfrac{100}{12}$ boxes/min

For machine 2: $r_2=\dfrac{100}{10}$ boxes/min

For machine 3: $r_3=\dfrac{100}{8}$ boxes/min

To complete the whole job of 100 boxes

$$r_1t+r_2t+r_3t=100\text{ boxes (complete job)}$$
$$\frac{100}{12}t+\frac{100}{10}t+\frac{100}{8}t=100$$
$$\frac{100t(120)}{12}+\frac{100t(120)}{10}+\frac{100t(120)}{8}=100(120)$$
$$1000t+1200t+1500t=12000$$
$$3700t=12000$$
$$t=\frac{12000}{3700}=3.24\text{ min}$$

54. rate $= \dfrac{\text{work}}{\text{time}}$

work = rate × time

$$\text{For crew1: } r_1 = \frac{1}{12} \text{ jobs/h}$$

$$\text{For crews 1 and 2: } r_1 + r_2 = \frac{1}{7.2} \text{ jobs/h}$$

For crew 2 to complete one whole job

$$r_2 t = 1 \text{ (complete job)}$$

$$\left(\frac{1}{7.2} - r_1\right) t = 1 \text{ (from the combined job rate above)}$$

$$\left(\frac{1}{7.2} - \frac{1}{12}\right) t = 1 \text{ (substituting crew 1 rate above)}$$

$$\left(\frac{1}{7.2} - \frac{1}{12}\right) t(86.4) = 1(86.4)$$

$$12t - 7.2t = 86.4$$

$$4.8t = 86.4$$

$$t = \frac{86.4}{4.8} = 18.0 \text{ h}$$

55.

$$d = 2.00t_1 \text{ for trip up}$$

$$d = 2.20t_2 \text{ for trip down}$$

$$t_1 + t_2 + 90.0 = 5.00(60)$$

$$\frac{d}{2.00} + \frac{d}{2.20} + 90.0 = 300$$

$$\frac{d(4.4)}{2.0} + \frac{d(4.4)}{2.2} + 90(4.4) = 300(4.4)$$

$$2.2d + 2d + 396 = 1320$$

$$4.2d = 924$$

$$d = \frac{924}{4.2} = 220 \text{ m}$$

56.

$$d = rt;\ t = d / r$$

$$\text{Rate in air is } r_{\text{air}} = 3.00 \times 10^8 \text{ m/s}$$

$$\text{Rate in water is } r_{\text{water}} = 2.25 \times 10^8 \text{ m/s}$$

$$d_{\text{air}} + d_{\text{water}} = 4.50 \text{ m}$$

$$d_{\text{air}} = 4.50 - d_{\text{water}}$$

$$t_{\text{air}} + t_{\text{water}} = 1.67 \times 10^{-8} \text{ s}$$

$$\frac{d_{\text{air}}}{r_{\text{air}}} + \frac{d_{\text{water}}}{r_{\text{water}}} = 1.67 \times 10^{-8}$$

$$\frac{4.50 - d_{\text{water}}}{3.00 \times 10^8} + \frac{d_{\text{water}}}{2.25 \times 10^8} = 1.67 \times 10^{-8}$$

$$4.50 - d_{\text{water}} + 1.333\, d_{\text{water}} = 5.00$$

$$0.333\, d_{\text{water}} = 0.50$$

$$d_{\text{water}} = 1.50 \text{ m}$$

57.

$$\begin{aligned}
d &= rt;\ d = \text{amount of water}\\
\text{Fast rate is } r_{\text{fast}} &= 60.0\ \text{m}^3/\text{h}\\
\text{Slow rate is } r_{\text{slow}} &= 18.0\ \text{m}^3/\text{h}\\
d_{\text{fast}} + d_{\text{slow}} &= 276\ \text{m}^3\\
d_{\text{fast}} &= 276 - d_{\text{slow}}\\
t_{\text{fast}} + t_{\text{slow}} &= 4.83\ \text{h}\\
\frac{d_{\text{fast}}}{r_{\text{fast}}} + \frac{d_{\text{slow}}}{r_{\text{slow}}} &= 4.83\\
\frac{276 - d_{\text{slow}}}{60} + \frac{d_{\text{slow}}}{18} &= 4.83\\
828 - 3\,d_{\text{slow}} + 10\,d_{\text{slow}} &= 870\\
7\,d_{\text{slow}} &= 42\\
d_{\text{slow}} &= 6.00\ \text{m}^3
\end{aligned}$$

58.

$$\begin{aligned}
d &= vt;\ t = d/v\\
&\quad \text{Rate to work is } v_1\\
&\quad \text{Rate from work is } v_1 - 8.00\ \text{km/h}\\
d &= 36.0\ \text{km each way}\\
t_{\text{to}} + t_{\text{from}} &= 2.00\ \text{h}\\
\frac{36.0}{v_1} + \frac{36.0}{v_1 - 8.00} &= 2.00\\
\frac{36.0v_1(v_1 - 8.00)}{v_1} + \frac{36.0v_1(v_1 - 8.00)}{v_1 - 8.00} &= 2.00v_1(v_1 - 8.00)\\
36.0(v_1 - 8.00) + 36.0v_1 &= 2.00v_1^2 - 16.00v_1\\
v_1^2 - 44v_1 + 144 &= 0\\
v_1 &= 40.44\ \text{km/h by the quadratic formula}
\end{aligned}$$

59.

$$\begin{aligned}
d &= rt;\ t = d/r\\
\text{Rate in first part is } r_1 &= \text{Mach } 2\\
\text{Rate in second part is } r_2 &= \text{Mach } 1\\
d_1 &= 0.75\ \text{trip}\\
d_2 &= 0.25\ \text{trip}
\end{aligned}$$

The total time for the trip is

$$\begin{aligned}
\frac{d_1 + d_2}{r_{\text{average}}} &= \frac{d_1}{r_1} + \frac{d_2}{r_2}\\
\frac{1}{r_{\text{average}}} &= \frac{0.75}{2} + \frac{0.25}{1}\\
\frac{2r_{\text{average}}}{r_{\text{average}}} &= \frac{0.75(2r_{\text{average}})}{2} + \frac{0.25(2r_{\text{average}})}{1}\\
2 &= 0.75r_{\text{average}} + 0.5r_{\text{average}}\\
r_{\text{average}} &= \frac{2}{1.25} = 1.6
\end{aligned}$$

The average speed is Mach 1.6

60.

$$x = \text{distance between B and C}$$
$$x + 24 = \text{distance between A and B}$$
$$2x + 24 = \text{distance between A and C}$$

Assuming both trains make the trip from A to C in the same time t.

rate = distance / time, so

time = distance / rate

$$\frac{x+24}{60} + \frac{x}{30} = \frac{2x+24}{50}$$
$$\frac{x+24}{60}(300) + \frac{x}{30}(300) = \frac{2x+24}{50}(300)$$
$$5(x+24) + 10x = 6(2x+24)$$
$$5x + 120 + 10x = 12x + 144$$
$$3x = 24$$
$$x = 8.00 \text{ km}$$
$$2x + 24 = 40.0 \text{ km between A and C.}$$

61. Use $d = rt$

$$(450 + v) \times t = 2580 \text{ with wind}$$
$$(450 - v) \times t = 1800 \text{ against wind}$$
$$t = \frac{2580}{450 + v}$$
$$\text{and } t = \frac{1800}{450 - v}$$
$$\frac{2580}{450+v} = \frac{1800}{450-v}$$
$$\frac{2580(450+v)(450-v)}{450+v} = \frac{1800(450+v)(450-v)}{450-v}$$
$$2580(450 - v) = 1800(450 + v)$$
$$1\,161\,000 - 2580v = 810\,000 + 1800v$$
$$4380v = 351\,000$$
$$v = 80.1 \text{ km/h, wind speed}$$

62. Use $d = rt$ **:**

$$(28.0) \times t_1 = d \text{ on the ship}$$
$$140 \times t_2 = d \text{ on helicopter}$$

With a 6.00 h delay, and 15.0 h total trip,
the sum of the times are

$$t_1 + t_2 + 6 = 15.0$$
$$\frac{d}{28} + \frac{d}{140} + 6 = 15.0$$
$$\frac{140d}{28} + \frac{140d}{140} + 6(140) = 15.0(140)$$
$$5d + d + 840 = 2100$$
$$6d = 1260$$
$$d = 210 \text{ km}$$

63.

$$\frac{V}{R_1}+\frac{V}{R_2}=i$$

$$\frac{V}{2.7}+\frac{V}{6.0}=1.2$$

$$\frac{V}{2.7}(16.2)+\frac{V}{6.0}(16.2)=1.2(16.2)$$

$$6V+2.7V=19.44$$

$$8.7V=19.44$$

$$V=2.23\text{ V}$$

64.

Let x = distance fox covers in one leap

y = distance greyhound covers in one leap, then

$3y = 7x$

Let n = number of leaps fox takes before greyhound overtakes him.

m = number of leaps greyhound takes to overtake the fox, then

$6n = 9m$

Total distance travelled is equal, so

$$60x+nx=my$$

$$60\times\frac{3y}{7}+\frac{9m}{6}\times\frac{3y}{7}=my$$

$$\frac{180y}{7}+\frac{9my}{14}=my$$

$$(14)\frac{180y}{7}+(14)\frac{9my}{14}=(14)my$$

$$360y+9my=14my$$

$$360y=5my$$

$$m=\frac{360y}{5y}$$

$$m=72$$

65.

$$\frac{x-12}{x^2+x-6}=\frac{A}{x+3}+\frac{B}{x-2}$$

$$\frac{x-12}{(x+3)(x-2)}=\frac{A(x-2)+B(x+3)}{(x+3)(x-2)}$$

$$A(x-2)+B(x+3)=x-12$$

$$Ax-2A+Bx+3B=x-12$$

$$(A+B)x-2A+3B=x-12$$

Comparing the x-term

$1=A+B$ (Equation 1)

Comparing the constant term

$-12=-2A+3B$ (Equation 2)

This is a system of equations (see Chapter 5)

Solving the first equation above

$B = 1 - A$

Substituting into the second equation above

$$\begin{aligned}
-12 &= -2A + 3(1 - A) \\
-12 &= -2A + 3 - 3A \\
5A &= 15 \\
A &= 3 \\
B &= 1 - 3 \\
B &= -2
\end{aligned}$$

66. $$\frac{23 - x}{2x^2 + 7x - 4} = \frac{A}{2x - 1} - \frac{B}{x + 4}$$

$$\begin{aligned}
\frac{23 - x}{(2x - 1)(x + 4)} &= \frac{A(x + 4) - B(2x - 1)}{(2x - 1)(x + 4)} \\
A(x + 4) - B(2x - 1) &= 23 - x \\
Ax + 4A - 2Bx + B &= 23 - x \\
(A - 2B)x + 4A + B &= 23 - x
\end{aligned}$$

Comparing the x-term

$-1 = A - 2B$ (Equation 1)

Comparing the constant term

$23 = 4A + B$ (Equation 2)

This is a system of equations (see Chapter 5)

Solving the first equation above

$A = 2B - 1$

Substituting into the second equation above

$$\begin{aligned}
23 &= 4(2B - 1) + B \\
23 &= 8B - 4 + B \\
27 &= 9B \\
B &= 3 \\
A &= 2(3) - 1 \\
A &= 5
\end{aligned}$$

Review Exercises

1. False. $(2a - 3)^2 = 4a^2 - 12a + 9$

2. True

3. True

4. False. $(x + 2)^3 = x^3 + 6x^2 + 12x + 8$

5. False. $\dfrac{x^2+x-2}{x^2+3x-4}=\dfrac{(x+2)(x-1)}{(x+4)(x-1)}=\dfrac{x+2}{x+4}$. One cannot cancel the x^2 terms from the original numerator and denominator in order to simplify the fraction.

6. True. $\dfrac{x^2-y^2}{2x+1}\div\dfrac{x+y}{1}=\dfrac{(x+y)(x-y)}{2x+1}\times\dfrac{1}{x+y}=\dfrac{x-y}{2x+1}$

7. False. $\dfrac{x}{y}-\dfrac{x+1}{y+1}=\dfrac{x(y+1)}{y(y+1)}-\dfrac{y(x+1)}{y(y+1)}=\dfrac{x-y}{y^2+y}$

8. False. The value $x=-1$ makes several of the denominators zero which is forbidden for a solution to such equations.

9. $3a(4x+5a)=12ax+15a^2$

10. $-7xy(4x^2-7y)=-28x^3y+49xy^2$

11. $(3a+4b)(3a-4b)=9a^2-16b^2$

12. $(x-4z)(x+4z)=x^2-16z^2$

13. $(b-2)(b+8)=b^2+6b-16$

14. $(5-y)(7-y)=35-12y+y^2$

15. $5s+20t=5(s+4t)$

16. $7x-28y=7(x-4y)$

17. $a^2x^2+a^2=a^2(x^2+1)$

18. $3ax-6ax^4-9a=3a(x-2x^4-3)$

19. $$\begin{aligned}W^2b^{x+2}-144b^x&=b^x(W^2b^2-144)\\&=b^x(Wb+12)(Wb-12)\end{aligned}$$

20. $$\begin{aligned}900n^a-n^{a+4}&=n^a(900-n^4)\\&=n^a(30+n^2)(30-n^2)\end{aligned}$$

21. $$\begin{aligned}16(x+2)^2-t^4&=(4(x+2)+t^2)(4(x+2)-t^2)\\&=(4x+8+t^2)(4x+8-t^2)\end{aligned}$$

22. $25s^4 - 36t^2 = (5s^2 + 6t)(5s^2 - 6t)$

23. $$\begin{aligned} 36t^2 - 24t + 4 &= 4(9t^2 - 6t + 1) \\ &= 4(3t-1)(3t-1) \\ &= 4(3t-1)^2 \end{aligned}$$

24. $$\begin{aligned} 9 - 12x + 4x^2 &= 3^2 - 2(3)(2x) + (2x)^2 \\ &= (3-2x)^2 \end{aligned}$$

25. $$\begin{aligned} 25t^2 + 10t + 1 &= (5t)^2 + 2(5t)(1) + 1^2 \\ &= (5t+1)^2 \end{aligned}$$

26. $$\begin{aligned} 4c^2 + 36cd + 81d^2 &= (2c)^2 + 2(2c)(9d) + (9d)^2 \\ &= (2c+9d)^2 \end{aligned}$$

27. $x^2 + x - 42 = (x+7)(x-6)$

28. $x^2 - 4x - 45 = (x-9)(x+5)$

29. $$\begin{aligned} t^4 - 5t^2 - 36 &= (t^2 - 9)(t^2 + 4) \\ &= (t+3)(t-3)(t^2+4) \end{aligned}$$

30. $$\begin{aligned} 3N^4 - 33N^2 + 30 &= 3(N^4 - 11N^2 + 10) \\ &= 3(N^2 - 1)(N^2 - 10) \\ &= 3(N+1)(N-1)(N^2-10) \end{aligned}$$

31. $2k^2 - k - 36 = (2k-9)(k+4)$

32. $3 - 2x - 5x^2 = (3-5x)(1+x)$

33. $$\begin{aligned} 8x^2 - 8x - 70 &= 2(4x^2 - 4x - 35) \\ &= 2(2x+5)(2x-7) \end{aligned}$$

34. $$\begin{aligned} 27F^3 + 21F^2 - 48F &= 3F(9F^2 + 7F - 16) \\ &= 3F(9F+16)(F-1) \end{aligned}$$

35. $10b^2 + 23b - 5 = (5b-1)(2b+5)$

36. $12x^2 - 7xy - 12y^2 = (4x+3y)(3x-4y)$

37. $4x^2 - 16y^2 = 4\left(x^2 - 4y^2\right)$
$$= 4(x+2y)(x-2y)$$

38. $4a^2x^2 + 26a^2x + 36a^2 = 2a^2\left(2x^2 + 13x + 18\right)$
$$= 2a^2(2x+9)(x+2)$$

39. $250 - 16y^6 = 2\left(125 - 8y^6\right)$
$$= 2\left(5^3 - \left(2y^2\right)^3\right)$$
$$= 2\left(5 - 2y^2\right)\left(25 + 10y^2 + 4y^4\right)$$

40. $8a^6 + 64a^3 = 8a^3\left(a^3 + 8\right)$
$$= 8a^3\left(a^3 + 2^3\right)$$
$$= 8a^3(a+2)\left(a^2 - 2a + 4\right)$$

41. $8x^3 + 27 = (2x)^3 + 3^3$
$$= (2x+3)\left(4x^2 - 6x + 9\right)$$

42. $2a^6 + 4a^3 + 2 = 2((a^3)^2 + 2a^3 + 1)$
$$= 2(a^3 + 1)^2$$
$$= 2((a+1)(a^2 - a + 1))^2$$
$$= 2(a+1)^2(a^2 - a + 1)^2$$

43. $ab^2 - 3b^2 + a - 3 = b^2(a-3) + (a-3)$
$$= (a-3)\left(b^2 + 1\right)$$

44. $axy - ay + ax - a = ay(x-1) + a(x-1)$
$$= (x-1)(ay + a)$$
$$= a(x-1)(y+1)$$

45. $nx + 5n - x^2 + 25 = n(x+5) - \left(x^2 - 25\right)$
$$= n(x+5) - (x+5)(x-5)$$
$$= (x+5)(n - (x-5))$$
$$= (x+5)(n - x + 5)$$

46. $ty - 4t + y^2 - 16 = t(y-4) + (y+4)(y-4)$
$$= (y-4)(t + y + 4)$$

47. $\dfrac{48ax^3y^6}{9a^3xy^6}=\dfrac{3axy^6(16x^2)}{3axy^6(3a^2)}$

$=\dfrac{16x^2}{3a^2}$ where $a, x, y \neq 0$

48. $\dfrac{-39r^2s^4t^8}{52rs^5t}=\dfrac{13rs^4t\left(-3rt^7\right)}{13rs^4t(4s)}$

$=\dfrac{-3rt^7}{4s}$ where $r, s, t \neq 0$

49. $\dfrac{6x^2-7x-3}{2x^2-5x+3}=\dfrac{(3x+1)(2x-3)}{(2x-3)(x-1)}$

$=\dfrac{3x+1}{x-1}$ where $x \neq 1, \dfrac{3}{2}$

50. $\dfrac{p^4-4p^2-4}{12+p^2-p^4}=\dfrac{\left(p^4-4p^2-4\right)}{\left(4-p^2\right)\left(3+p^2\right)}=\dfrac{p^2-4p^2-4}{(2+p)(2-p)\left(p^2+3\right)}$

where $p \neq \pm 2$. No common factors; already in lowest terms.

51. $\dfrac{4x+4y}{35x^2}\times\dfrac{28x}{x^2-y^2}=\dfrac{4(x+y)}{7x(5x)}\times\dfrac{7x(4)}{(x+y)(x-y)}$

$=\dfrac{16}{5x(x-y)}$ where $x \neq 0, -y$

52. $\left(\dfrac{6x-3}{x^2}\right)\left(\dfrac{4x^2-12x}{6-12x}\right)=\dfrac{3(2x-1)}{x^2}\dfrac{4x(x-3)}{(-6)(2x-1)}$

$=\dfrac{12x(x-3)}{-6x^2}$

$=\dfrac{2(x-3)}{x}$ where $x \neq 0, \dfrac{1}{2}$

53. $\dfrac{18-6L}{L^2-6L+9}\div\dfrac{L^2-2L-15}{L^2-9}=\dfrac{18-6L}{L^2-6L+9}\times\dfrac{L^2-9}{L^2-2L-15}$

$=\dfrac{-6(L-3)}{(L-3)(L-3)}\times\dfrac{(L+3)(L-3)}{(L-5)(L+3)}$

$=\dfrac{-6}{L-5}$ where $L \neq \pm 3$

54. $$\frac{6x^2-xy-y^2}{2x^2+xy-y^2}\div\frac{16y^2-4x^2}{2x^2+6xy+4y^2}=\frac{6x^2-xy-y^2}{2x^2+xy-y^2}\times\frac{2x^2+6xy+4y^2}{16y^2-4x^2}$$
$$=\frac{(3x+y)(2x-y)}{(x+y)(2x-y)}\times\frac{2(x+y)(x+2y)}{(-4)(x+2y)(x-2y)}$$
$$=\frac{(3x+y)}{(-2)(x-2y)}$$
$$=-\frac{(3x+y)}{2(x-2y)}\quad\text{where } x\neq -y,\frac{y}{2},-2y$$

55. $$\frac{\frac{3x}{7x^2+13x-2}}{\frac{6x^2}{x^2+4x+4}}=\frac{3x}{7x^2+13x-2}\times\frac{x^2+4x+4}{6x^2}$$
$$=\frac{3x}{(7x-1)(x+2)}\times\frac{(x+2)(x+2)}{3x(2x)}$$
$$=\frac{x+2}{2x(7x-1)}\quad\text{where } x\neq 0,-2$$

56. $$\frac{\frac{3y-3x}{2x^2+3xy-2y^2}}{\frac{3x^2-3y^2}{x^2+4xy+4y^2}}=\frac{3y-3x}{2x^2+3xy-2y^2}\cdot\frac{x^2+4xy+4y^2}{3x^2-3y^2}$$
$$=\frac{-3(x-y)}{(2x-y)(x+2y)}\cdot\frac{(x+2y)(x+2y)}{3(x+y)(x-y)}$$
$$=-\frac{x+2y}{(2x-y)(x+y)}\quad\text{where } x\neq y,-y,-2y$$

57. $$\frac{x+\frac{1}{x}+1}{x^2-\frac{1}{x}}=\frac{\frac{x^2+x+1}{x}}{\frac{x^3-1}{x}}$$
$$=\frac{x^2+x+1}{x}\cdot\frac{x}{x^3-1}$$
$$=\frac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$$
$$=\frac{1}{x-1}\quad\text{where } x\neq 0,1$$

58. $$\frac{\frac{4}{y}-4y}{2-\frac{2}{y}}=\frac{\frac{4-4y^2}{y}}{\frac{2y-2}{y}}$$
$$=\frac{4-4y^2}{y}\cdot\frac{y}{2y-2}$$
$$=\frac{-4(y^2-1)y}{y\cdot 2(y-1)}$$
$$=\frac{-4y(y+1)(y-1)}{2y(y-1)}$$
$$=-2(y+1)\quad\text{where } y\neq 0,1$$

59. $$\frac{4}{9x}-\frac{5}{12x^2}=\frac{4(4x)-5(3)}{36x^2}$$
$$=\frac{16x-15}{36x^2}$$

60. $$\frac{3}{10a^2}+\frac{1}{4a^3}=\frac{3(2a)+1(5)}{20a^3}$$
$$=\frac{6a+5}{20a^3}$$

61. $$\frac{6}{x}-\frac{7}{2x}+\frac{3}{xy}=\frac{6(2y)-7(y)+3(2)}{2xy}$$
$$=\frac{12y-7y+6}{2xy}$$
$$=\frac{5y+6}{2xy}$$

62. $$\frac{T}{T^2+2}-\frac{1}{2T+T^3}=\frac{T}{\left(T^2+2\right)}-\frac{1}{T\left(T^2+2\right)}$$
$$=\frac{T(T)-1}{T\left(T^2+2\right)}$$
$$=\frac{T^2-1}{T\left(T^2+2\right)}$$
$$=\frac{(T+1)(T-1)}{T\left(T^2+2\right)}$$

63. $$\frac{a+1}{a}-\frac{a-3}{a+2}=\frac{(a+1)(a+2)-(a-3)a}{a(a+2)}$$
$$=\frac{a^2+3a+2-(a^2-3a)}{a(a+2)}$$
$$=\frac{6a+2}{a(a+2)}$$
$$=\frac{2(3a+1)}{a(a+2)}$$

64. $$\frac{y}{y+2}-\frac{1}{y^2+2y}=\frac{y}{(y+2)}-\frac{1}{y(y+2)}$$
$$=\frac{y(y)-1}{y(y+2)}$$
$$=\frac{y^2-1}{y(y+2)}$$
$$=\frac{(y+1)(y-1)}{y(y+2)}$$

65. $$\frac{2x}{x^2+2x-3}-\frac{1}{6x+2x^2}=\frac{2x}{(x+3)(x-1)}-\frac{1}{2x(x+3)}$$
$$=\frac{2x(2x)-1(x-1)}{2x(x+3)(x-1)}$$
$$=\frac{4x^2-x+1}{2x(x+3)(x-1)}$$

66. $$\frac{x}{4x^2+4x-3}+\frac{3}{9-4x^2}=\frac{x}{(2x+3)(2x-1)}+\frac{3}{(3+2x)(3-2x)}$$
$$=\frac{x}{(2x+3)(2x-1)}-\frac{3}{(3+2x)(2x-3)}$$
$$=\frac{x(2x-3)-3(2x-1)}{(2x+3)(2x-3)(2x-1)}$$
$$=\frac{2x^2-3x-6x+3}{(2x+3)(2x-3)(2x-1)}$$
$$=\frac{2x^2-9x+3}{(2x+3)(2x-1)(2x-3)}$$

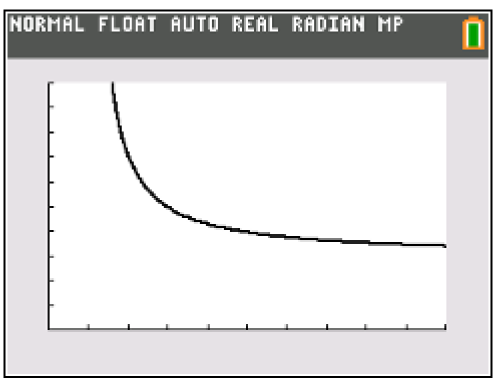
67. $$\frac{3x}{2x^2-2}-\frac{2}{4x^2-5x+1}=\frac{3x}{2(x+1)(x-1)}-\frac{2}{(4x-1)(x-1)}$$
$$=\frac{3x(4x-1)-2(2)(x+1)}{2(4x-1)(x+1)(x-1)}$$
$$=\frac{12x^2-3x-4x-4}{2(4x-1)(x+1)(x-1)}$$
$$=\frac{12x^2-7x-4}{2(4x-1)(x+1)(x-1)}$$

68. $$\frac{2n-1}{4-n}+\frac{n+2}{20-5n}=\frac{2n-1}{4-n}+\frac{n+2}{5(4-n)}$$
$$=\frac{(2n-1)(5)+(n+2)}{5(4-n)}$$
$$=\frac{10n-5+n+2}{5(4-n)}$$
$$=\frac{11n-3}{5(4-n)}$$

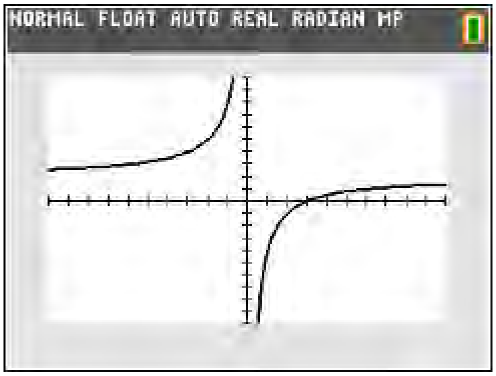
69.
$$\frac{3x}{x^2+2x-3}-\frac{2}{x^2+3x}+\frac{x}{x-1}=\frac{3x}{(x+3)(x-1)}-\frac{2}{x(x+3)}+\frac{x}{x-1}$$
$$=\frac{3x(x)-2(x-1)+x(x)(x+3)}{x(x+3)(x-1)}$$
$$=\frac{3x^2-2x+2+x^3+3x^2}{x(x+3)(x-1)}$$
$$=\frac{x^3+6x^2-2x+2}{x(x+3)(x-1)}$$

70.
$$\frac{3}{y^4-2y^3-8y^2}+\frac{y-1}{y^2+2y}-\frac{y-3}{y^2-4y}=\frac{3}{y^2(y-4)(y+2)}+\frac{y-1}{y(y+2)}-\frac{y-3}{y(y-4)}$$
$$=\frac{3+(y-1)(y)(y-4)-(y-3)(y)(y+2)}{y^2(y-4)(y+2)}$$
$$=\frac{3+y(y^2-5y+4)-y(y^2-y-6)}{y^2(y-4)(y+2)}$$
$$=\frac{3+y^3-5y^2+4y-y^3+y^2+6y}{y^2(y-4)(y+2)}$$
$$=\frac{-4y^2+10y+3}{y^2(y-4)(y+2)}$$

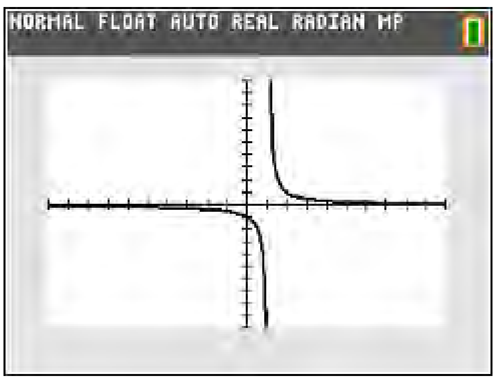
71. The two functions both have the same graph that looks like the following output.



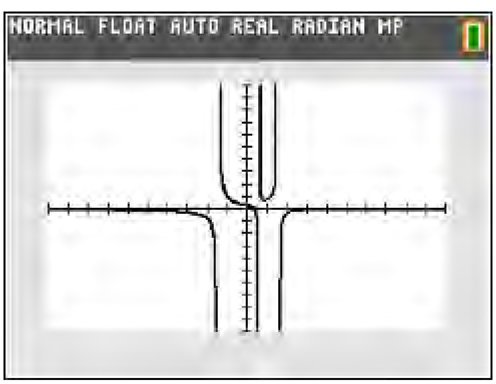
72. The two functions both have the same graph that looks like the following output.



73. The two functions both have the same graph that looks like the following output.



74. The two functions both have the same graph that looks like the following output.



75.
$$\frac{x}{2}-3=\frac{x-10}{4}$$
$$\frac{x}{2}-3\ (4)=\ \frac{x-10}{4}\ (4)$$
$$2x-12=x-10$$
$$x=2$$

76.
$$\frac{4x}{c}-\frac{2}{2c}=\frac{6}{c}-x$$
$$\left(\frac{4x}{c}-\frac{2}{2c}\right)\cdot 2c=\left(\frac{6}{c}-x\right)\cdot 2c$$
$$8x-2=12-2cx$$
$$8x+2cx=14$$
$$2x(4+c)=14$$
$$x=\frac{14}{2(4+c)}$$

77.
$$\frac{2}{t}-\frac{1}{at}=2+\frac{a}{t}$$
$$\frac{2}{t}-\frac{1}{at}\ \cdot at=\ 2+\frac{a}{t}\ \cdot at$$
$$2a-1=2at+a^2$$
$$2at=-a^2+2a-1$$
$$2at=-(a^2-2a+1)$$
$$2at=-(a-1)(a-1)$$
$$t=\frac{-(a-1)^2}{2a}$$

78.
$$\frac{3}{a^2y}-\frac{1}{ay}=\frac{9}{a}$$
$$\left(\frac{3}{a^2y}-\frac{1}{ay}\right)\cdot a^2y=\left(\frac{9}{a}\right)\cdot a^2y$$
$$3-a=9ay$$
$$y=\frac{3-a}{9a}$$

79.
$$\frac{2x}{2x^2-5x}-\frac{3}{x}=\frac{1}{4x-10}$$
$$\frac{2x}{x(2x-5)}-\frac{3}{x}=\frac{1}{2(2x-5)}$$
$$\frac{2x(2x)(2x-5)}{x(2x-5)}-\frac{3(2x)(2x-5)}{x}=\frac{1(2x)(2x-5)}{2(2x-5)}$$
$$4x-6(2x-5)=x$$
$$4x-12x+30=x$$
$$9x=30$$
$$x=\frac{10}{3}$$

80.
$$\frac{3}{x^2+3x}-\frac{1}{x}=\frac{1}{3+x}$$
$$\frac{3x(x+3)}{x(x+3)}-\frac{1x(x+3)}{x}=\frac{1x(x+3)}{x+3}$$
$$3-(x+3)=x$$
$$3-x-3=x$$
$$2x=0$$
$$x=0$$

which gives division by zero in the second term. No solution.

81.
$$f(x)=\frac{1}{x+2}$$
$$f(x+2)=\frac{1}{x+2+2}$$
$$2f(x)=\frac{2}{x+2}$$
But if $f(x+2)=2f(x)$

then $\frac{1}{x+2+2}=\frac{2}{x+2}$
$$\frac{1(x+2)(x+4)}{x+4}=\frac{2(x+2)(x+4)}{x+2}$$
$$x+2=2x+8$$
$$x=-6$$

82.
$$f(x)=\frac{x}{x+1}$$
$$3f(x)=\frac{3x}{x+1}$$
$$f\left(\frac{1}{x}\right)=\frac{\frac{1}{x}}{\frac{1}{x}+1}$$

But if $3f(x)+f\left(\frac{1}{x}\right)=2$

$$\text{then } \frac{3x}{x+1}+\frac{\frac{1}{x}}{\frac{1}{x}+1}=2$$

$$\frac{3x}{x+1}+\frac{\frac{1}{x}}{\frac{1+x}{x}}=2$$

$$\frac{3x}{x+1}+\frac{1}{x}\cdot\frac{x}{x+1}=2$$

$$\frac{3x}{x+1}+\frac{x}{x+1}=2$$

$$\frac{4x}{x+1}=2$$

$$\frac{4x(x+1)}{x+1}=2(x+1)$$

$$4x=2x+2$$

$$2x=2$$

$$x=1$$

83. **(a)** Changing an odd number of signs changes the sign of the fraction.

(b) Changing an even number of signs leaves the sign of the fraction unchanged.

84. $\frac{1}{\frac{1}{x}}=1\times\frac{x}{1}=x$

85.
$$xy=\frac{1}{4}\left[(x+y)^2-(x-y)^2\right]$$
$$xy=\frac{1}{4}\left[x^2+2xy+y^2-\left(x^2-2xy+y^2\right)\right]$$
$$xy=\frac{1}{4}\left[x^2+2xy+y^2-x^2+2xy-y^2\right]$$
$$xy=\frac{1}{4}[4xy]$$
$$xy=xy$$

86.
$$x^2+y^2=\frac{1}{2}\left[(x+y)^2+(x-y)^2\right]$$
$$x^2+y^2=\frac{1}{2}\left[x^2+2xy+y^2+x^2-2xy+y^2\right]$$
$$x^2+y^2=\frac{1}{2}\left[2x^2+2y^2\right]$$
$$x^2+y^2=x^2+y^2$$

87.
$$Pb(L+b)(L-b)=Pb(L^2-b^2)$$
$$=PL^2b-Pb^3$$

88. $kr(R-r) = krR - kr^2$

89. $$\begin{aligned}\left[2b+(n-1)\lambda\right]^2 &= 4b^2+4b(n-1)\lambda+(n-1)^2\lambda^2\\ &= 4b^2+4b\lambda(n-1)+\lambda^2(n^2-2n+1)\\ &= 4b^2+4bn\lambda-4b\lambda+n^2\lambda^2-2n\lambda^2+\lambda^2\end{aligned}$$

90. $$\begin{aligned}\pi r_1^2 l-\pi r_2^2 l &= \pi l\left(r_1^2-r_2^2\right)\\ &= \pi l\left(r_1+r_2\right)\left(r_1-r_2\right)\end{aligned}$$

91. $$\begin{aligned}cT_2-cT_1+RT_2-RT_1 &= c\left(T_2-T_1\right)+R\left(T_2-T_1\right)\\ &= \left(T_2-T_1\right)(c+R)\end{aligned}$$

92. $$\begin{aligned}9600t+8400t^2-1200t^3 &= 1200t\left(8+7t-t^2\right)\\ &= 1200t(8-t)(1+t)\end{aligned}$$

93. $$\begin{aligned}(2R-r)^2-\left(r^2+R^2\right) &= 4R^2-4Rr+r^2-r^2-R^2\\ &= 3R^2-4Rr\\ &= R(3R-4r)\end{aligned}$$

94. $$\begin{aligned}2R(R+r)-(R+r)^2 &= (R+r)\left(2R-(R+r)\right)\\ &= (R+r)(R-r)\end{aligned}$$

95. $$\begin{aligned}(n+1)^3(2n+1)^3 &= \left(n^3+3n^2+3n+1\right)\left(8n^3+12n^2+6n+1\right)\\ &= 8n^6+12n^5+6n^4+n^3+24n^5+36n^4+18n^3+3n^2+24n^4+36n^3+18n^2+3n+8n^3+12n^2+6n+1\\ &= 8n^6+36n^5+66n^4+63n^3+33n^2+9n+1\end{aligned}$$

96. $$\begin{aligned}2\left(e_1-e_2\right)^2+2\left(e_2-e_3\right)^2 &= 2\left(e_1^2-2e_1e_2+e_2^2+e_2^2-2e_2e_3+e_3^2\right)\\ &= 2e_1^2-4e_1e^2+4e_2^2-4e_2e_3+2e_3^2\end{aligned}$$

97. $$\begin{aligned}10a(T-t)+a(T-t)^2 &= 10aT-10at+a(T^2-2Tt+t^2)\\ &= 10aT-10at+aT^2-2aTt+at^2\end{aligned}$$

98. $$\begin{aligned}pa^2+(1-p)b^2-\quad pa+(1-p)b\quad^2 &= pa^2+(1-p)b^2-\quad p^2a^2+2pab(1-p)+(1-p)^2b^2\\ &= pa^2+(1-p)b^2-p^2a^2-2pab(1-p)-(1-p)^2b^2\\ &= pa^2-p^2a^2+(1-p)b^2-(1-p)2pab-(1-p)^2b^2\\ &= pa^2(1-p)+(1-p)\left(b^2-2pab-(1-p)b^2\right)\\ &= (1-p)\left(pa^2+b^2-2pab-b^2+pb^2\right)\\ &= (1-p)\left(pa^2-2pab+pb^2\right)\\ &= p(1-p)\left(a^2-2ab+b^2\right)\\ &= p(1-p)(a-b)(a-b)\\ &= p(1-p)(a-b)^2\end{aligned}$$

99. $$\begin{aligned} V_{\text{change}} &= V_2 - V_1 \\ &= (x+4)^3 - x^3 \\ &= x^3 + 12x^2 + 48x + 64 - x^3 \\ &= 12x^2 + 48x + 64 \\ &= 4\left(3x^2 + 12x + 16\right) \end{aligned}$$

100. $$\begin{aligned} V_{change} &= V_2 - V_1 \\ &= \frac{4}{3}\pi r^3 - \frac{4}{3}\pi(3)^3 \\ &= \frac{4}{3}\pi\left(r^3 - 3^3\right) \\ &= \frac{4}{3}\pi(r-3)\left(r^2 + 3r + 9\right) \end{aligned}$$

101. $$\begin{aligned} \frac{2wtv^2}{Dg} \quad \frac{b\pi^2 D^2}{n^2} \quad \frac{6}{bt^2} &= \frac{12wtv^2 b\pi^2 D^2}{Dgn^2bt^2} \\ &= \frac{bDt(12wv^2\pi^2 D)}{bDt(gn^2t)} \\ &= \frac{12wv^2\pi^2 D}{gn^2t} \end{aligned}$$

102. $$\begin{aligned} \frac{m}{c} \div \left[1 - \left(\frac{p}{c}\right)^2\right] &= \frac{m}{c} \div \left[\frac{c^2 - p^2}{c^2}\right] \\ &= \frac{m}{c} \cdot \left(\frac{c^2}{c^2 - p^2}\right) \\ &= \frac{mc}{c^2 - p^2} \\ &= \frac{mc}{(c+p)(c-p)} \end{aligned}$$

103. $$\begin{aligned} \frac{\frac{\pi ka}{2}\left(R^4 - r^4\right)}{\pi ka\left(R^2 - r^2\right)} &= \frac{\frac{\pi ka}{2}\left(R^2 + r^2\right)\left(R^2 - r^2\right)}{\pi ka\left(R^2 - r^2\right)} \\ &= \frac{\frac{1}{2}(R^2 + r^2)}{1} \\ &= \frac{(R^2 + r^2)}{2} \quad \text{where } R \neq r, -r \end{aligned}$$

104. $$\frac{V}{kp} - \frac{RT}{k^2p^2} = \frac{Vkp - RT}{k^2p^2}$$

105. $$1 - \frac{d^2}{2} + \frac{d^4}{24} - \frac{d^6}{120} = \frac{120 - 60d^2 + 5d^4 - d^6}{120}$$

106. $\dfrac{wx^2}{2T_0}+\dfrac{kx^4}{12T_0}=\dfrac{6wx^2+kx^4}{12T_0}$

$$=\frac{x^2(6w+kx^2)}{12T_0}$$

107. $\dfrac{4k-1}{4k-4}+\dfrac{1}{2k}=\dfrac{4k-1}{4(k-1)}+\dfrac{1}{2k}$

$$=\frac{(4k-1)k+1(2)(k-1)}{4k(k-1)}$$

$$=\frac{4k^2-k+2k-2}{4k(k-1)}$$

$$=\frac{4k^2+k-2}{4k(k-1)}$$

108. $\dfrac{Am}{k}-\dfrac{g}{2}\;\dfrac{m}{k}^{\;2}+\dfrac{AML}{k}=\dfrac{Am}{k}-\dfrac{gm^2}{2k^2}+\dfrac{AML}{k}$

$$=\frac{2kAm-gm^2+2kAML}{2k^2}$$

109. $1-\dfrac{3a}{4r}-\dfrac{a^3}{4r^3}=\dfrac{4r^3-3ar^2-a^3}{4r^3}$

110. $\dfrac{1}{F}+\dfrac{1}{f}-\dfrac{d}{fF}=\dfrac{f+F-d}{fF}$

111. $\dfrac{\frac{u^2}{2g}-x}{\frac{1}{2gc^2}-\frac{u^2}{2g}+x}=\dfrac{\frac{u^2}{2g}-x}{\frac{1}{2gc^2}-\frac{u^2}{2g}+x}\cdot\dfrac{2gc^2}{2gc^2}$

$$=\frac{u^2c^2-2gc^2x}{1-u^2c^2+2gc^2x}$$

$$=\frac{c^2(u^2-2gx)}{1-u^2c^2+2gc^2x}$$

112. $\dfrac{V}{\frac{1}{2R}+\frac{1}{2R+2}}=\dfrac{V}{\frac{1}{2R}+\frac{1}{2R+2}}\times\dfrac{2R(2R+2)}{2R(2R+2)}$

$$=\frac{2RV(2R+2)}{2R+2+2R}$$

$$=\frac{4RV(R+1)}{4R+2}$$

$$=\frac{4RV(R+1)}{2(2R+1)}$$

$$=\frac{2RV(R+1)}{2R+1}$$

113. $W = mgh_2 - mgh_1$

$W = mg(h_2 - h_1)$

$m = \dfrac{W}{g(h_2 - h_1)}$

114. $h(T_1 + T_2) = T_2$

$hT_1 + hT_2 = T_2$

$hT_1 = T_2 - hT_2$

$hT_1 = T_2(1-h)$

$T_1 = \dfrac{T_2(1-h)}{h}$

115. $R = \dfrac{wL}{H(w+L)}$

$RH(w+L) = wL$

$RHw + RHL = wL$

$wL - RHL = RHw$

$L(w - RH) = RHw$

$L = \dfrac{RHw}{w - RH}$

116. $\dfrac{J}{T} = \dfrac{t}{\omega_1 - \omega_2}$

$\dfrac{J}{T}T(\omega_1 - \omega_2) = \dfrac{t}{\omega_1 - \omega_2}T(\omega_1 - \omega_2)$

$J(\omega_1 - \omega_2) = Tt$

$J\omega_1 - J\omega_2 = Tt$

$J\omega_2 = J\omega_1 - Tt$

$\omega_2 = \dfrac{J\omega_1 - Tt}{J}$

$\omega_2 = \omega_1 - \dfrac{Tt}{J}$

117. $E = V_0 + \dfrac{(m+M)V^2}{2} + \dfrac{p^2}{2I}$

$E \cdot 2I = \left(V_0 + \dfrac{(m+M)V^2}{2} + \dfrac{p^2}{2I}\right) \cdot 2I$

$2EI = 2V_0 I + (m+M)IV^2 + p^2$

$2EI - 2V_0 I - (m+M)IV^2 = p^2$

$I\left(2E - 2V_0 - (m+M)V^2\right) = p^2$

$I = \dfrac{p^2}{2E - 2V_0 - (m+M)V^2}$

118.
$$\frac{q_2 - q_1}{d} = \frac{f + q_1}{D}$$
$$\frac{q_2 - q_1}{d} Dd = \frac{f + q_1}{D} Dd$$
$$D(q_2 - q_1) = d(f + q_1)$$
$$q_2 D - q_1 D = fd + q_1 d$$
$$q_1 d + q_1 D = q_2 D - fd$$
$$q_1 (d + D) = q_2 D - fd$$
$$q_1 = \frac{q_2 D - fd}{d + D}$$

119.
$$s^2 + \frac{cs}{m} + \frac{kL^2}{mb^2} = 0$$
$$mb^2 \left(s^2 + \frac{cs}{m} + \frac{kL^2}{mb^2} \right) = 0 \cdot mb^2$$
$$s^2 b^2 m + csb^2 + kL^2 = 0$$
$$csb^2 = -s^2 b^2 m - kL^2$$
$$c = -\frac{(s^2 b^2 m + kL^2)}{sb^2}$$

120.
$$I = \frac{A}{x^2} + \frac{B}{(10 - x)^2}$$
$$\frac{A}{x^2} = I - \frac{B}{(10 - x)^2}$$
$$A = Ix^2 - \frac{Bx^2}{(10 - x)^2}$$
$$A = x^2 \left(I - \frac{B}{(10 - x)^2} \right)$$

121.
$$F = \frac{m}{sC} + \frac{F_0}{s}$$
$$F \cdot sC = \left(\frac{m}{sC} + \frac{F_0}{s} \right) \cdot sC$$
$$FsC = m + F_0 C$$
$$FsC - F_0 C = m$$
$$C = \frac{m}{Fs - F_0}$$

122. $$C = \frac{k}{n(1-k)}$$

$$Cn(1-k) = k$$
$$Cn - Cnk = k$$
$$k + Cnk = Cn$$
$$k(1+Cn) = Cn$$
$$k = \frac{Cn}{1+Cn}$$

123. Let $\frac{1}{4}t$ = the amount of the car's battery depleted by the lights in 1 hour

Let $\frac{1}{24}t$ = the amount of the car's battery depleted by the radio in 1 hour

$$\frac{1}{4}t + \frac{1}{24}t = 1$$
$$\frac{6}{24}t + \frac{1}{24}t = 1$$
$$\frac{7}{24}t = 1$$
$$t = \frac{24}{7}$$
$$t = 3.43 \text{ h}$$

It will take 3.43 hours for the battery to go dead with both the lights and the radio on.

124. Let $\frac{5000}{20}t$ = the amount pumped by the first pump in gallons

Let $\frac{5000}{25}t$ = the amount pumped by the second pump in gallons

$$\frac{5000}{20}t + \frac{5000}{25}t = 5000$$
$$\frac{25\ 000}{100}t + \frac{20\ 000}{100}t = 5000$$
$$\frac{45\ 000}{100}t = 5000$$
$$t = 5000 \cdot \frac{100}{45\ 000}$$
$$t = 11.1 \text{ min}$$

Both pumps will take 11.1 minutes to pump 5000 gallons of water.

125. $$\frac{1}{m}=\frac{\frac{1}{x}+\frac{1}{y}}{2}$$
$$\frac{1}{m}=\frac{\frac{1}{400}+\frac{1}{1200}}{2}$$
$$\frac{1}{m}=\frac{\frac{1(3)+1}{1200}}{2}$$
$$\frac{1}{m}=\frac{\frac{4}{1200}}{2}$$
$$\frac{1}{m}=\frac{1}{300}\cdot\frac{1}{2}$$
$$\frac{1}{m}=\frac{1}{600}$$
$$m=600\text{ Hz}$$
The harmonic mean of the two musical notes is 600 Hz.

126. $$\text{rate}=\frac{\text{work}}{\text{time}}$$
$$\text{work}=\text{rate}\times\text{time}$$
For mechanic: $r_1=\dfrac{1}{3.00}$ jobs/h

For mechanic and assistant: $r_1+r_2=\dfrac{1}{2.10}$ jobs/h

For assistant to complete one whole job

$r_2t=1$ (complete job)

$\left(\dfrac{1}{2.1}-r_1\right)t=1$ (from the combined job rate above)

$\left(\dfrac{1}{2.1}-\dfrac{1}{3}\right)t=1$ (substituting crew 1 rate above)

$$\left(\frac{1}{2.1}-\frac{1}{3}\right)t(6.3)=1(6.3)$$
$$3t-2.1t=6.3$$
$$0.9t=6.3$$
$$t=\frac{6.3}{0.9}=7.00\text{ h}$$
It would take the apprentice 7.00 hours to do the job alone.

127. $sg = \dfrac{w_a}{w_a - w_w}$

$$sg = \frac{1.097w_w}{1.097w_w - w_w}$$

$$sg = \frac{w_w(1.097)}{w_w(1.097-1)}$$

$$sg = \frac{1.097}{(1.097-1)}$$

$$sg = \frac{1.097}{0.097}$$

$$sg = 11.3$$

The relative density of lead is 11.3.

128.

$$d = 80.0t_1 \text{ for half-trip (leg 1)}$$

$$d = 60.0t_2 \text{ for half-trip (leg 2)}$$

$$v_{avg} = \frac{2d}{t_1 + t_2}$$

$$2d = v_{\text{avg}}\left(t_1 + t_2\right)$$

$$2d = v_{\text{avg}} \quad \frac{1}{80}d + \frac{1}{60}d$$

$$2d \cdot (240) = v_{\text{avg}} \quad \frac{1}{80}d + \frac{1}{60}d \quad \cdot (240)$$

$$480d = v_{\text{avg}}\left(3d + 4d\right)$$

$$v_{\text{avg}} = \frac{480d}{7d}$$

$$v_{\text{avg}} = \frac{480}{7}$$

$$v_{\text{avg}} = 68.6 \text{ km/h}$$

The average speed of the car is 68.6 km/h

129. $\dfrac{1}{\mathrm{R}_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

$$\frac{1}{6} = \frac{1}{12} + \frac{1}{R} + \frac{1}{2R}$$

$$\frac{1}{6}12R = \left(\frac{1}{12} + \frac{1}{R} + \frac{1}{2R}\right) \cdot 12R$$

$$2R = \left(R + 12 + 6\right)$$

$$2R = R + 18$$

$$R = 18.0\ \Omega$$

130. $d = 36.0t_1$ to the accident

$d = 48.0t_2$ for trip back

$$t_1 + t_2 = \frac{35}{60}$$

$$\frac{d}{36} + \frac{d}{48} = \frac{7}{12}$$

$$144\left(\frac{d}{36} + \frac{d}{48}\right) = \frac{7}{12}(144)$$

$$4d + 3d = 84$$

$$7d = 84$$

$$d = 12.0 \text{ km}$$

The hospital was 12.0 km away from the accident.

131.
$$\frac{\left(1+\frac{1}{s}\right)\left(1+\frac{1}{s/2}\right)}{3+\frac{1}{s}+\frac{1}{s/2}} = \frac{\left(\frac{s+1}{s}\right)\left(1+\frac{2}{s}\right)}{\left(3+\frac{1}{s}+\frac{2}{s}\right)}$$

$$= \frac{\left(\frac{s+1}{s}\right)\left(\frac{s+2}{s}\right)}{\left(\frac{3s+1+2}{s}\right)}$$

$$= \left(\frac{s+1}{s}\right)\left(\frac{s+2}{s}\right) \times \left(\frac{s}{3s+3}\right)$$

$$= \frac{s(s+1)(s+2)}{3s^2(s+1)}$$

$$= \frac{s+2}{3s}$$

When you cancel, the basic operation being performed is division.

Chapter 7

Quadratic Equations

7.1 Quadratic Equations; Solution by Factoring

1. $3x^2+7x+2=0$

$(3x+1)(x+2)=0$ factor

$3x+1=0$ or $x+2=0$

$3x=-1 \qquad x=-2$

$x=-\frac{1}{3}$

The roots are $x=-\frac{1}{3}$ and $x=-2$.

Checking in the original equation:

$3\left(-\frac{1}{3}\right)^2+7\left(-\frac{1}{3}\right)+2=0 \qquad 3(-2)^2+7(-2)+2=0$

$\frac{1}{3}-\frac{7}{3}+2=0 \qquad 12-14+2=0$

$0=0 \qquad 0=0$

The roots are $-\frac{1}{3}, -2$.

2. $\frac{2}{x}+3=\frac{1}{x+2}$

$\frac{2(x+2)+3x(x+2)}{x(x+2)}=\frac{x}{x(x+2)}$ rearrrange fractions over LCD

$2x+4+3x^2+6x=x$

$3x^2+7x+4=0$

$(3x+4)(x+1)=0$ factor

$3x+4=0$ or $x+1=0$

$3x=-4 \qquad x=-1$

$x=-\frac{4}{3} \qquad x=-1$

The roots are $-\frac{4}{3},-1$

Checking in the original equation:

$\frac{2}{-4/3}+3=\frac{1}{-4/3+2} \qquad \frac{2}{-1}+3=\frac{1}{-1+2}$

$-\frac{3}{2}+3=\frac{3}{2} \qquad -2+3=\frac{1}{1}$

$\frac{3}{2}=\frac{3}{2} \qquad 1=1$

The roots are $-\frac{4}{3}, -1$.

3.
$$x(x-2)=4$$
$$x^2-2x-4=0$$
$$a=1,\ b=-2,\ c=-4$$

4.
$$(3x-2)^2=2$$
$$9x^2-12x+4=2$$
$$9x^2-12x+2=0$$
$$a=9,\ b=-12,\ c=2$$

5.
$$x^2=(x+2)^2$$
$$x^2=x^2+4x+4$$
$4x+4=0,$ no x^2 term so it is not quadratic

6.
$$x(2x^2+5)=7+2x^2$$
$$2x^3+5x=7+2x^2$$
Not quadratic since there is an x^3 term.

7.
$$n(n^2+n-1)=n^3$$
$$n^3+n^2-n=n^3$$
$$n^2-n=0$$
$$a=1,\ b=-1,\ c=0$$

8.
$$(T-7)^2=(2T+3)^2$$
$$T^2-14t+49=4T^2+12T+9$$
$$-3T^2-26T+40=0$$
$$3T^2+26T-40=0$$
$$a=3,\ b=26,\ c=-40$$

9.
$$y^2(y-2)=3(y-2)$$
$$y^3-2y^2=3y-6$$
Not quadratic since there is a y^3 term.

10.
$$z(z+4)=(z+1)(z+5)$$
$$z^2+4z=z^2+6z+5$$
$$0=2z+5$$
Not quadratic since there is no z^2 term.

11.
$$x^2-25=0$$
$$(x+5)(x-5)=0$$
$$x+5=0 \quad \text{or} \quad x-5=0$$
$$x=-5 \qquad\qquad x=5$$

12.
$$B^2 - 400 = 0$$
$$(B-20)(B+20) = 0$$
$$B - 20 = 0 \text{ or } B + 20 = 0$$
$$B = 20 \qquad B = -20$$

13.
$$4y^2 = 9$$
$$4y^2 - 9 = 0$$
$$(2y+3)(2y-3) = 0$$
$$2y + 3 = 0 \text{ or } 2y - 3 = 0$$
$$2y = -3 \qquad 2y = 3$$
$$y = -\frac{3}{2} \qquad y = \frac{3}{2}$$

14.
$$2x^2 = 0.32$$
$$2(x^2 - 0.16) = 0$$
$$2(x-0.4)(x+0.4) = 0$$
$$x - 0.4 = 0 \text{ or } x + 0.4 = 0$$
$$x = 0.4 \qquad x = -0.4$$

15.
$$x^2 - 5x - 14 = 0$$
$$(x-7)(x+2) = 0$$
$$x - 7 = 0 \text{ or } x + 2 = 0$$
$$x = 7 \qquad x = -2$$

16.
$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x + 3 = 0 \text{ or } x - 2 = 0$$
$$x = -3 \qquad x = 2$$

17.
$$R^2 + 12 = 7R$$
$$R^2 - 7R + 12 = 0$$
$$(R-4)(R-3) = 0$$
$$R - 4 = 0 \text{ or } R - 3 = 0$$
$$R = 4 \qquad R = 3$$

18.
$$x^2 + 30 = 11x$$
$$x^2 - 11x + 30 = 0$$
$$(x-6)(x-5) = 0$$
$$x - 6 = 0 \text{ or } x - 5 = 0$$
$$x = 6 \qquad x = 5$$

19.
$$
\begin{aligned}
40x - 16x^2 &= 0 \\
2x^2 - 5x &= 0 \\
x(2x-5) &= 0 \\
2x - 5 &= 0 \quad \text{or} \quad x = 0 \\
2x &= 5 \\
x &= \frac{5}{2}
\end{aligned}
$$

20.
$$
\begin{aligned}
15L &= 20L^2 \\
20L^2 - 15L &= 0 \\
5L(4L-3) &= 0 \\
4L - 3 &= 0 \quad \text{or} \quad 5L = 0 \\
4L &= 3 \qquad\qquad L = 0 \\
L &= \frac{3}{4}
\end{aligned}
$$

21.
$$
\begin{aligned}
12m^2 &= 3 \\
12m^2 - 3 &= 0 \\
3\left(4m^2 - 1\right) &= 0 \\
3(2m-1)(2m+1) &= 0 \\
2m - 1 &= 0 \quad \text{or} \quad 2m + 1 = 0 \\
2m &= 1 \qquad\qquad 2m = -1 \\
m &= \frac{1}{2} \qquad\qquad m = -\frac{1}{2}
\end{aligned}
$$

22.
$$
\begin{aligned}
9 &= a^2x^2 \\
a^2x^2 - 9 &= 0 \\
(ax+3)(ax-3) &= 0 \\
ax + 3 &= 0 \quad \text{or} \quad ax - 3 = 0 \\
x &= \frac{-3}{a} \qquad\qquad x = \frac{3}{a}
\end{aligned}
$$

23.
$$
\begin{aligned}
3x^2 - 13x + 4 &= 0 \\
(3x-1)(x-4) &= 0 \\
3x - 1 &= 0 \quad \text{or} \quad x - 4 = 0 \\
3x &= 1 \qquad\qquad x = 4 \\
x &= \frac{1}{3}
\end{aligned}
$$

24.
$$A^2 + 8A + 16 = 0$$
$$(A+4)(A+4) = 0$$
$$A + 4 = 0 \quad \text{or} \quad A + 4 = 0$$
$$A = -4 \qquad\qquad A = -4$$

25.
$$4x = 3 - 7x^2$$
$$7x^2 + 4x - 3 = 0$$
$$(7x-3)(x+1) = 0$$
$$7x - 3 = 0 \quad \text{or} \quad x + 1 = 0$$
$$7x = 3 \qquad\qquad x = -1$$
$$x = \frac{3}{7}$$

26.
$$4x^2 + 25 = 20x$$
$$4x^2 - 20x + 25 = 0$$
$$(2x-5)(2x-5) = 0$$
$$2x - 5 = 0$$
$$2x = 5$$
$$x = \frac{5}{2} \text{ (double root)}$$

27.
$$6x^2 = 13x - 6$$
$$6x^2 - 13x + 6 = 0$$
$$(3x-2)(2x-3) = 0$$
$$3x - 2 = 0 \quad \text{or} \quad 2x - 3 = 0$$
$$3x = 2 \qquad\qquad 2x = 3$$
$$x = \frac{2}{3} \qquad\qquad x = \frac{3}{2}$$

28.
$$6z^2 = 6 + 5z$$
$$6z^2 - 5z - 6 = 0$$
$$(3z+2)(2z-3) = 0$$
$$3z + 2 = 0 \quad \text{or} \quad 2z - 3 = 0$$
$$3z = -2 \qquad\qquad 2z = 3$$
$$z = -\frac{2}{3} \qquad\qquad z = \frac{3}{2}$$

29.

$$\begin{aligned}4x(x+1)&=3\\4x^2+4x-3&=0\\(2x-1)(2x+3)&=0\end{aligned}$$

$$\begin{aligned}2x-1&=0 &\quad\text{or}\quad 2x+3&=0\\2x&=1 & 2x&=-3\\x&=\frac{1}{2} & x&=-\frac{3}{2}\end{aligned}$$

30.

$$\begin{aligned}t(43+t)&=9-9t^2\\43t+t^2&=9-9t^2\\10t^2+43t-9&=0\\(5t-1)(2t+9)&=0\end{aligned}$$

$$\begin{aligned}5t-1&=0 &\quad\text{or}\quad 2t+9&=0\\5t&=1 & 2t&=-9\\t&=\frac{1}{5} & t&=-\frac{9}{2}\end{aligned}$$

31.

$$\begin{aligned}6y^2+by&=2b^2\\6y^2+by-2b^2&=0\\(2y-b)(3y+2b)&=0\end{aligned}$$

$$\begin{aligned}2y-b&=0 &\quad\text{or}\quad 3y+2b&=0\\y&=\frac{b}{2} & y&=\frac{-2b}{3}\end{aligned}$$

32.

$$\begin{aligned}2x^2-7ax+4a^2&=a^2\\2x^2-7ax+3a^2&=0\\(2x-a)(x-3a)&=0\end{aligned}$$

$$\begin{aligned}2x-a&=0 &\quad\text{or}\quad x-3a&=0\\x&=\frac{a}{2} & x&=3a\end{aligned}$$

33.

$$\begin{aligned}8s^2+16s&=90\\8s^2+16s-90&=0\\(4s-10)(2s+9)&=0\end{aligned}$$

$$\begin{aligned}4s-10&=0 &\quad\text{or}\quad 2s+9&=0\\4s&=10 & 2s&=-9\\s&=\frac{5}{2} & s&=-\frac{9}{2}\end{aligned}$$

34.
$$\begin{aligned} 18t^2 &= 48t - 32 \\ 18t^2 - 48t + 32 &= 0 \\ 2\left(9t^2 - 24t + 16\right) &= 0 \\ 2(3t-4)(3t-4) &= 0 \\ 3t - 4 &= 0 \\ 3t &= 4 \\ t &= \frac{4}{3} \text{ (double root)} \end{aligned}$$

35.
$$\begin{aligned} (x+2)^3 &= x^3 + 8 \\ x^3 + 6x^2 + 12x + 8 &= x^3 + 8 \\ 6x^2 + 12x &= 0 \\ 6x(x+2) &= 0 \\ 6x = 0 \quad &\text{or} \quad x + 2 = 0 \\ x = 0 \quad &\qquad x = -2 \end{aligned}$$

36.
$$\begin{aligned} V\left(V^2 - 4\right) &= V^2(V-1) \\ V^3 - 4V &= V^3 - V^2 \\ V^2 - 4V &= 0 \\ V(V-4) &= 0 \\ V - 4 = 0 \quad &\text{or} \quad V = 0 \\ V &= 4 \end{aligned}$$

37.
$$\begin{aligned} (x+a)^2 - b^2 &= 0 \\ (x+a-b)(x+a+b) &= 0 \\ x + a - b = 0 \quad &\text{or} \quad x + a + b = 0 \\ x = b - a \quad &\qquad x = -b - a \end{aligned}$$

38.
$$\begin{aligned} bx^2 - b &= x - b^2x \\ bx^2 - b &= (1-b^2)x \\ bx^2 + (b^2-1)x - b &= 0 \\ (bx-1)(x+b) &= 0 \\ bx - 1 = 0 \quad &\text{or} \quad x + b = 0 \\ x = \frac{1}{b} \quad &\qquad x = -b \end{aligned}$$

39.

$$\begin{aligned}
x^2 + 2ax &= b^2 - a^2 \\
x^2 + 2ax + a^2 - b^2 &= 0 \\
(x+a)^2 - b^2 &= 0 \\
\big((x+a)+b\big)\big((x+a)-b\big) &= 0 \\
x + a + b = 0 \quad &\text{or} \quad x + a - b = 0 \\
x = -a - b \quad & \quad\quad x = b - a
\end{aligned}$$

40.

$$\begin{aligned}
x^2\left(a^2 + 2ab + b^2\right) &= x\left(a+b\right) \\
x^2\left(a+b\right)^2 - x\left(a+b\right) &= 0 \\
x\left(a+b\right)\left[x\left(a+b\right) - 1\right] &= 0 \\
x\left(a+b\right) - 1 = 0 \quad &\text{or} \quad x = 0 \\
x\left(a+b\right) &= 1 \\
x &= \frac{1}{a+b}
\end{aligned}$$

41. For $a = 2$, $b = -7$, $c = 3$

Equation 7.1 ($ax^2 + bx + c = 0$) becomes

$$\begin{aligned}
2x^2 - 7x + 3 &= 0 \\
\left(2x-1\right)\left(x-3\right) &= 0 \\
x = \frac{1}{2} \;\text{ or }\; x &= 3
\end{aligned}$$

The sum of the roots is

$$\frac{1}{2} + 3 = \frac{7}{2} = -\frac{-7}{2} = -\frac{b}{a}$$

42. For $a = 2$, $b = -7$, $c = 3$

Equation 7.1 ($ax^2 + bx + c = 0$) becomes

$$\begin{aligned}
2x^2 - 7x + 3 &= 0 \\
\left(2x-1\right)\left(x-3\right) &= 0 \\
x = \frac{1}{2} \;\text{ or }\; x &= 3
\end{aligned}$$

The product of the roots is

$$\frac{1}{2} \times 3 = \frac{3}{2} = \frac{c}{a}$$

43.

$$\begin{aligned}
12x^2 - 64x + 64 &= 0 \\
4(3x-4)(x-4) &= 0 \\
3x - 4 = 0 \quad &\text{or} \quad x - 4 = 0 \\
x = \frac{4}{3} \quad & \quad\quad x = 4
\end{aligned}$$

We discard $x = \frac{4}{3}$ because of the constraint that $x > 2$, concluding $x = 4$.

44.
$$-16t^2 + 320t = 0$$
$$-16t(t-20) = 0$$
$$t - 20 = 0 \quad \text{or} \quad t = 0$$
$$t = 20$$
The height is 0 at $t = 0$ or $t = 20$ seconds.

45.
$$V = \alpha I + \beta I^2$$
$$2I + 0.5I^2 = 6$$
$$I^2 + 4I - 12 = 0$$
$$(I+6)(I-2) = 0$$
$$I + 6 = 0 \quad \text{or} \quad I - 2 = 0$$
$$I = -6 \qquad I = 2$$
The current is -6.00 A or 2.00 A.

46.
$$m = 135 - 6t - t^2$$
$$m = (9-t)(15+t)$$
$$9 - t = 0 \quad \text{or} \quad 15 + t = 0$$
$$t = 9 \text{ s} \qquad t = -15$$
Since $t > 0$, the booster will run out of fuel in 9.00 s.

47.
$$P = 4h^2 - 48h + 744$$
$$664 = 4h^2 - 48h + 774$$
$$4h^2 - 48h + 80 = 0$$
$$4(h^2 - 12h + 20) = 0$$
$$4(h-10)(h-2) = 0$$
$$h - 2 = 0 \quad \text{or} \quad h - 10 = 0$$
$$h = 2 \qquad h = 10$$
The power is 664 MW at 2:00 a.m. and 10:00 a.m

48.
$$s^2 - 16s = 3072$$
$$s^2 - 16s - 3072 = 0$$
$$(s+48)(s-64) = 0$$
$$s + 48 = 0 \quad \text{or} \quad s - 64 = 0$$
$$s = -48 \qquad s = 64$$
Since $s > 0$, the speed of the car is 64 km/h.

49. If the solutions are to be $x = 0.5$ and $x = 2$,
one such equation is
$$(x-0.5)(x-2) = 0$$
$$x^2 - 2.5x + 1 = 0$$

50. If the solutions are to be $x = a$ and $x = b$, one such equation is

$(x-a)(x-b) = 0$

$x^2 - (a+b)x + ab = 0$

51.

$$x^3 - x = 0$$
$$x(x^2 - 1) = 0$$
$$x(x+1)(x-1) = 0$$
$$x+1 = 0 \quad \text{or} \quad x-1=0 \quad \text{or} \quad x = 0$$
$$x = -1 \qquad\qquad x = 1$$

The three roots are $-1, 0, 1$.

52.

$$x^3 - 4x^2 - x + 4 = 0$$
$$x^2(x-4) - (x-4) = 0$$
$$(x-4)(x^2-1) = 0$$
$$(x-4)(x+1)(x-1) = 0$$
$$x-4=0 \quad \text{or} \quad x+1=0 \quad \text{or} \quad x-1=0$$
$$x = 4 \qquad x = -1 \qquad x = 1$$

The three roots are $-1, 1, 4$.

53.

$$\frac{1}{x-3} + \frac{4}{x} = 2$$
$$\frac{1(x)(x-3)}{x-3} + \frac{4x(x-3)}{x} = 2x(x-3) \quad \text{multiply by the LCD}$$
$$x + 4x - 12 = 2x^2 - 6x$$
$$-2x^2 + 11x - 12 = 0$$
$$2x^2 - 11x + 12 = 0$$
$$(x-4)(2x-3) = 0$$
$$2x - 3 = 0 \quad \text{or} \quad x - 4 = 0$$
$$2x = 3 \qquad\qquad x = 4$$
$$x = \frac{3}{2}$$

54.

$$2-\frac{1}{x}=\frac{3}{x+2}$$

$$2(x)(x+2)-\frac{1x(x+2)}{x}=\frac{3x(x+2)}{x+2} \qquad \text{multiply by the LCD}$$

$$2x^2+4x-x-2=3x$$

$$2x^2-2=0$$

$$2(x^2-1)=0$$

$$2(x+1)(x-1)=0$$

$$x+1=0 \quad \text{or} \quad x-1=0$$

$$x=-1 \qquad\qquad x=1$$

55.

$$\frac{1}{2x}-\frac{3}{4}=\frac{1}{2x+3}$$

$$\frac{1(4)(2x)(2x+3)}{2x}-\frac{3(4)(2x)(2x+3)}{4}=\frac{1(4)(2x)(2x+3)}{2x+3} \qquad \text{multiply by LCD}$$

$$8x+12-12x^2-18x=8x$$

$$-12x^2-18x+12=0$$

$$-6(2x-1)(x+2)=0$$

$$2x-1=0 \quad \text{or} \quad x+2=0$$

$$2x=1 \qquad\qquad x=-2$$

$$x=\frac{1}{2}$$

56.

$$\frac{x}{2}+\frac{1}{x-3}=3$$

$$\frac{x\cdot 2(x-3)}{2}+\frac{1(2)(x-3)}{x-3}=3\cdot 2(x-3) \qquad \text{multiply by LCD}$$

$$x^2-3x+2=6x-18$$

$$x^2-9x+20=0$$

$$(x-5)(x-4)=0$$

$$x-5=0 \quad \text{or} \quad x-4=0$$

$$x=5 \qquad\qquad x=4$$

57. $\frac{1}{k_c} = \frac{1}{k_1} + \frac{1}{k_2}$

Let k = the spring constant of the first spring in N/cm
Let $k + 3$ N/cm = the spring constant of the second spring in N/cm

$$\frac{1}{2} = \frac{1}{k} + \frac{1}{k+3}$$

$$\frac{1 \cdot 2k(k+3)}{2} = \frac{1 \cdot 2k(k+3)}{k} + \frac{1 \cdot 2k(k+3)}{k+3} \quad \text{multiply by LCD}$$

$$k^2 + 3k = 2k + 6 + 2k$$

$$k^2 - k - 6 = 0$$

$$(k-3)(k+2) = 0$$

$$k - 3 = 0 \quad \text{or} \quad k + 2 = 0$$

$$k = 3 \qquad k = -2 \quad \text{reject this solution since } k > 0$$

The one spring constant is 3 N/cm and the other spring constant is (3N/cm + 3 N/cm) = 6N/cm

58. Parallel:

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{3} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{3} = \frac{R_2 + R_1}{R_1 R_2}$$

$$\frac{R_1 R_2}{3} = R_2 + R_1$$

$$R_1 R_2 = 3R_2 + 3R_1$$

Series:

$$R_{\text{series}} = R_1 + R_2$$

$$16 = R_1 + R_2$$

$$R_1 = 16 - R_2$$

When we substitute R_1 from the series equation into R_1 in the parallel equation, we get

$$(16 - R_2)R_2 = 3R_2 + 3(16 - R_2)$$

$$16R_2 - R_2^2 = 3R_2 + 48 - 3R_2$$

$$-R_2^2 + 16R_2 - 48 = 0$$

$$R_2^2 - 16R_2 + 48 = 0$$

$$(R_2 - 12)(R_2 - 4) = 0$$

$$R_2 - 12 = 0 \quad \text{or} \quad R_2 - 4 = 0$$

$$R_2 = 12 \qquad R_2 = 4$$

$$R_1 = 16 - 12 = 4\ \Omega \qquad R_1 = 16 - 4 = 12\ \Omega$$

59. For 120 km round trip, each leg consists of 60 km

$v_1 = \dfrac{60}{t_1}$ going

$v_2 = \dfrac{60}{t_2}$ returning

The total time taken was

$t_1 + t_2 = 3.5$

$t_2 = 3.5 - t_1$

And we know that the first leg was 10 km/h slower than the return trip

$$\frac{60}{t_1} + 10 = \frac{60}{t_2}$$

$$\frac{60}{t_1} + 10 = \frac{60}{3.5 - t_1} \quad \text{multiply by LCD}$$

$$\frac{60t_1(3.5 - t_1)}{t_1} + 10t_1(3.5 - t_1) = \frac{60t_1(3.5 - t_1)}{3.5 - t_1}$$

$$210 - 60t_1 + 35t_1 - 10t_1^2 = 60t_1$$

$$-10t_1^2 - 85t_1 + 210 = 0$$

$$-5\left(2t_1^2 + 17t_1 - 42\right) = 0$$

$$(2t_1 + 21)(t_1 - 2) = 0$$

$t_1 - 2 = 0$ or $2t_1 + 21 = 0$

$t_1 = 2$ $\qquad t_1 = \dfrac{-21}{2}$ (ignore since $t > 0$)

$$v_1 = \frac{60}{2} = 30 \text{ km/h going}$$

$$v_2 = \frac{60}{3.5 - 2} = 40 \text{ km/h returning}$$

60. Let x = the amount of distance added to each dimension

$$A = bh = 20 \times 30 = 600$$

$$2A = 1200$$

$$1200 = (20 + x)(30 + x)$$

$$1200 = 600 + 50x + x^2$$

$$x^2 + 50x - 600 = 0$$

$$(x - 10)(x + 60) = 0$$

$x - 10 = 0$ or $x + 60 = 0$

$x = 10$ $\qquad x = -60$ (ignore, since $x > 0$)

10 cm is added to each side of the solar panel to end up with an area that is doubled.

7.2 Completing the Square

1.
$$x^2+6x-8=0$$
$$x^2+6x=8$$
$$x^2+6x+9=8+9$$
$$(x+3)^2=17$$
$$x+3=\pm\sqrt{17}$$
$$x=-3\pm\sqrt{17}$$

2.
$$2x^2+12x-9=0$$
$$2\left(x^2+6x-\frac{9}{2}\right)=0$$
$$x^2+6x+9=\frac{9}{2}+9$$
$$(x+3)^2=\frac{27}{2}$$
$$x+3=\pm\sqrt{\frac{27}{2}}$$
$$x=-3\pm\sqrt{\frac{27}{2}}$$
$$x=-3\pm\sqrt{\frac{3(9)}{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$
$$x=-3\pm\frac{3\sqrt{6}}{2}$$

3.
$$x^2=25$$
$$x=\pm\sqrt{25}$$
$$x=\pm 5$$

4.
$$x^2=100$$
$$\sqrt{x^2}=\pm\sqrt{100}$$
$$x=\pm 10$$

5.
$$x^2=7$$
$$x=\pm\sqrt{7}$$

6.
$$s^2=15$$
$$s=\pm\sqrt{15}$$

7. $2y^2 - 5 = 1$

$2y^2 = 6$

$y^2 = 3$

$y = \pm\sqrt{3}$

8. $4x^2 - 7 = 2$

$4x^2 = 9$

$x^2 = \dfrac{9}{4}$

$x = \pm\sqrt{\dfrac{9}{4}}$

$x = \pm\dfrac{3}{2}$

9. $(x-2)^2 = 25$

$x - 2 = \pm\sqrt{25}$

$x - 2 = \pm 5$

$x = 2 \pm 5$

$x = -3$ or $x = 7$

10. $(x+2)^2 = 10$

$x + 2 = \pm\sqrt{10}$

$x = -2 \pm \sqrt{10}$

11. $(y+3)^2 = 7$

$y + 3 = \pm\sqrt{7}$

$y = -3 \pm \sqrt{7}$

12. $\left(x - \dfrac{5}{2}\right)^2 = 100$

$x - \dfrac{5}{2} = \sqrt{100}$

$x - \dfrac{5}{2} = \pm 10$

$x = \dfrac{5}{2} \pm 10$

$x = \dfrac{25}{2}$ or $x = -\dfrac{15}{2}$

13.
$$\begin{aligned} x^2+2x-15&=0\\ x^2+2x&=15\\ x^2+2x+1&=15+1\\ (x+1)^2&=16\\ x+1&=\pm\sqrt{16}\\ x&=-1\pm 4\\ x&=3 \text{ or } x=-5 \end{aligned}$$

14.
$$\begin{aligned} x^2-8x-20&=0\\ x^2-8x&=20\\ x^2-8x+16&=20+16\\ (x-4)^2&=36\\ x-4&=\pm\sqrt{36}\\ x&=4\pm 6\\ x&=10 \text{ or } x=-2 \end{aligned}$$

15.
$$\begin{aligned} D^2+3D+2&=0\\ D^2+3D&=-2\\ D^2+3D+\frac{9}{4}&=-2+\frac{9}{4}\\ \left(D+\frac{3}{2}\right)^2&=\frac{1}{4}\\ D+\frac{3}{2}&=\pm\sqrt{\frac{1}{4}}\\ D&=-\frac{3}{2}\pm\frac{1}{2}\\ D&=-2 \text{ or } D=-1 \end{aligned}$$

16.
$$\begin{aligned} t^2+5t-6&=0\\ t^2+5t&=6\\ t^2+5t+\frac{25}{4}&=6+\frac{25}{4}\\ \left(t+\frac{5}{2}\right)^2&=\frac{49}{4}\\ t+\frac{5}{2}&=\pm\sqrt{\frac{49}{4}}\\ t&=-\frac{5}{2}\pm\frac{7}{2}\\ t&=-6 \text{ or } t=1 \end{aligned}$$

17.
$$\begin{aligned}
n^2 &= 6n-4\\
n^2-6n &= -4\\
n^2-6n+9 &= -4+9\\
(n-3)^2 &= 5\\
n-3 &= \pm\sqrt{5}\\
n &= 3\pm\sqrt{5}
\end{aligned}$$

18.
$$\begin{aligned}
(R+9)(R+1) &= 13\\
R^2+10R+9 &= 13\\
R^2+10R &= 4\\
R^2+10R+25 &= 4+25\\
(R+5)^2 &= 29\\
R+5 &= \pm\sqrt{29}\\
R &= -5\pm\sqrt{29}
\end{aligned}$$

19.
$$\begin{aligned}
v(v+4) &= 6\\
v^2+4v &= 6\\
v^2+4v+4 &= 6+4\\
(v+2)^2 &= 10\\
v+2 &= \pm\sqrt{10}\\
v &= -2\pm\sqrt{10}
\end{aligned}$$

20.
$$\begin{aligned}
12 &= 8Z-Z^2\\
Z^2-8Z &= -12\\
Z^2-8Z+16 &= -12+16\\
(Z-4)^2 &= 4\\
Z-4 &= \pm\sqrt{4}\\
Z &= 4\pm 2\\
Z &= 6 \text{ or } Z=2
\end{aligned}$$

21.
$$\begin{aligned}
2s^2+5s&=3\\
s^2+\frac{5}{2}s&=\frac{3}{2}\\
s^2+\frac{5}{2}s+\frac{25}{16}&=\frac{3}{2}+\frac{25}{16}\\
\left(s+\frac{5}{4}\right)^2&=\frac{49}{16}\\
s+\frac{5}{4}&=\pm\frac{7}{4}\\
s&=-\frac{5}{4}\pm\frac{7}{4}\\
s&=-3 \text{ or } s=\frac{1}{2}
\end{aligned}$$

22.
$$\begin{aligned}
8x^2+2x&=6\\
x^2+\frac{1}{4}x&=\frac{3}{4}\\
x^2+\frac{1}{4}x+\frac{1}{64}&=\frac{3}{4}+\frac{1}{64}\\
\left(x+\frac{1}{8}\right)^2&=\frac{49}{64}\\
x+\frac{1}{8}&=\pm\frac{7}{8}\\
x&=-\frac{1}{8}\pm\frac{7}{8}\\
x&=-1 \text{ or } x=\frac{3}{4}
\end{aligned}$$

23.
$$\begin{aligned}
3y^2&=3y+2\\
3y^2-3y&=2\\
y^2-y&=\frac{2}{3}\\
y^2-y+\frac{1}{4}&=\frac{2}{3}+\frac{1}{4}\\
\left(y-\frac{1}{2}\right)^2&=\frac{11}{12}\\
y-\frac{1}{2}&=\pm\sqrt{\frac{11}{12}}\\
y&=\frac{1}{2}\pm\sqrt{\frac{11}{4(3)}}\cdot\frac{\sqrt{3}}{\sqrt{3}}\\
y&=\frac{1}{2}\pm\frac{\sqrt{33}}{6}
\end{aligned}$$

24.
$$\begin{aligned}
3x^2 &= 3-4x\\
3x^2+4x &= 3\\
x^2+\frac{4}{3}x &= 1\\
x^2+\frac{4}{3}x+\frac{4}{9} &= 1+\frac{4}{9}\\
\left(x+\frac{2}{3}\right)^2 &= \frac{13}{9}\\
x+\frac{2}{3} &= \pm\sqrt{\frac{13}{9}}\\
x &= -\frac{2}{3}\pm\frac{\sqrt{13}}{3}
\end{aligned}$$

25.
$$\begin{aligned}
2y^2-y-2 &= 0\\
2y^2-y &= 2\\
y^2-\frac{1}{2}y+\frac{1}{16} &= 1+\frac{1}{16}\\
\left(y-\frac{1}{4}\right)^2 &= \frac{17}{16}\\
y-\frac{1}{4} &= \pm\frac{\sqrt{17}}{4}\\
y &= \frac{1}{4}\pm\frac{\sqrt{17}}{4}
\end{aligned}$$

26.
$$\begin{aligned}
2+6v &= 9v^2\\
9v^2-6v-2 &= 0\\
v^2-\frac{2}{3}v &= \frac{2}{9}\\
v^2-\frac{2}{3}v+\frac{1}{9} &= \frac{2}{9}+\frac{1}{9}\\
\left(v-\frac{1}{3}\right)^2 &= \frac{1}{3}\\
v-\frac{1}{3} &= \pm\sqrt{\frac{1}{3}}\\
v &= \frac{1}{3}\pm\sqrt{\frac{1}{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}\\
v &= \frac{1}{3}\pm\frac{\sqrt{3}}{3}
\end{aligned}$$

27.
$$\begin{aligned}
10T - 5T^2 &= 4 \\
5T^2 - 10T + 4 &= 0 \\
T^2 - 2T + \frac{4}{5} &= 0 \\
T^2 - 2T &= -\frac{4}{5} \\
T^2 - 2T + 1 &= -\frac{4}{5} + 1 \\
(T-1)^2 &= \frac{1}{5} \\
T - 1 &= \pm\sqrt{\frac{1}{5}} \\
T &= 1 \pm \sqrt{\frac{1}{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\
T &= 1 \pm \frac{\sqrt{5}}{5}
\end{aligned}$$

28.
$$\begin{aligned}
\pi^2 y^2 + 2\pi y &= 3 \\
\pi^2 y^2 + 2\pi y + 1 &= 3 + 1 \\
(\pi y + 1)^2 &= 4 \\
\pi y + 1 &= \pm\sqrt{4} \\
\pi y + 1 &= \pm 2 \\
\pi y &= 1 \text{ or } \pi y = -3 \\
y &= \frac{1}{\pi} \text{ or } y = -\frac{3}{\pi}
\end{aligned}$$

29.
$$\begin{aligned}
9x^2 + 6x + 1 &= 0 \\
9\left(x^2 + \frac{2}{3}x + \frac{1}{9}\right) &= 0 \\
\left(x + \frac{1}{3}\right)^2 &= 0 \\
x + \frac{1}{3} &= 0 \\
x &= -\frac{1}{3} \text{ (double root)}
\end{aligned}$$

30.
$$\begin{aligned}
2x^2 &= 3x - 2a \\
2x^2 - 3x + 2a &= 0 \\
2\left(x^2 - \frac{3}{2}x + a\right) &= 0 \\
x^2 - \frac{3}{2}x &= -a \\
x^2 - \frac{3}{2}x + \frac{9}{16} &= -a + \frac{9}{16} \\
\left(x - \frac{3}{4}\right)^2 &= \frac{9}{16} - a \\
x - \frac{3}{4} &= \pm\sqrt{\frac{9}{16} - a} \\
x &= \frac{3}{4} \pm \sqrt{\frac{9}{16} - \frac{16}{16}a} \\
x &= \frac{3}{4} \pm \frac{\sqrt{9-16a}}{\sqrt{16}} \cdot \frac{\sqrt{16}}{\sqrt{16}} \\
x &= \frac{3 \pm \sqrt{9-16a}}{4}
\end{aligned}$$

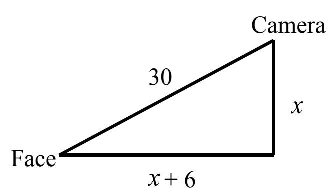
31.
$$\begin{aligned}
x^2 + 2bx + c &= 0 \\
x^2 + 2bx &= -c \\
x^2 + 2bx + b^2 &= b^2 - c \\
(x+b)^2 &= b^2 - c \\
x + b &= \pm\sqrt{b^2 - c} \\
x &= -b \pm \sqrt{b^2 - c}
\end{aligned}$$

32.
$$\begin{aligned}
px^2 + qx + r &= 0 \\
p\left(x^2 + \frac{qx}{p} + \frac{r}{p}\right) &= 0 \\
x^2 + \frac{qx}{p} &= -\frac{r}{p} \\
x^2 + \frac{qx}{p} + \frac{q^2}{4p^2} &= -\frac{r}{p} + \frac{q^2}{4p^2} \\
\left(x + \frac{q}{2p}\right)^2 &= \frac{q^2 - 4pr}{4p^2} \\
x + \frac{q}{2p} &= \pm\sqrt{\frac{q^2 - 4pr}{4p^2}} \\
x &= -\frac{q}{2p} \pm \frac{\sqrt{q^2 - 4pr}}{2p} \\
x &= \frac{-q \pm \sqrt{q^2 - 4pr}}{2p}
\end{aligned}$$

33.
$$\begin{aligned}
V = 4.0T - 0.2T^2 &= 15 \\
0.2T^2 - 4.0T + 15 &= 0 \\
0.2\left(T^2 - 20T + 75\right) &= 0 \\
T^2 - 20T &= -75 \\
T^2 - 20T + 100 &= -75 + 100 \\
(T-10)^2 &= 25 \\
T - 10 &= \pm 5 \\
T &= 10 \pm 5 \\
T = 5 \text{ or } T &= 15
\end{aligned}$$
The voltage is 15.0 V when the temperature is 5.0ºC or 15ºC.

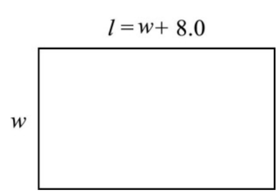
34.
$$\begin{aligned}
48 &= 64t - 16t^2 \\
t^2 - 4t + 3 &= 0 \\
(t-3)(t-1) &= 0 \\
t = 3 \text{ or } t &= 1
\end{aligned}$$
The flare is 48 ft above the ground at 3.0 s and 1.0 s .

35.



$$\begin{aligned}
c^2 &= a^2 + b^2 \\
30^2 &= (x+6)^2 + x^2 \\
900 &= x^2 + 12x + 36 + x^2 \\
2x^2 + 12x - 864 &= 0 \\
2\left(x^2 + 6x - 432\right) &= 0 \\
x^2 + 6x &= 432 \\
x^2 + 6x + 9 &= 432 + 9 \\
(x+3)^2 &= 441 \\
x + 3 &= \pm\sqrt{441} \\
x &= \pm 21 - 3 \\
x = 18 \text{ or } x &= -24
\end{aligned}$$
The camera is 18 in above her face.

36.



$$\begin{aligned}A &= lw = 28\\ w(w+8) &= 28\\ w^2+8w &= 28\\ w^2+8w+16 &= 28+16\\ (w+4)^2 &= 44\\ w+4 &= \pm\sqrt{44}\\ w &= -4\pm\sqrt{44}\\ w &= 2.633249581 \text{ or } w = -10.63324958\end{aligned}$$

The width of the rectangle is 2.63 m, and the length is (2.63 m + 8.00 m) = 10.6 m.

7.3 The Quadratic Formula

1. $x^2+5x+6=0$; $a=1$, $b=5$, $c=6$

$$x=\frac{-5\pm\sqrt{5^2-4(1)(6)}}{2(1)}$$

$$x=\frac{-5\pm\sqrt{1}}{2}$$

$$x=\frac{-5\pm 1}{2}$$

$$x=\frac{-5+1}{2} \text{ or } x=\frac{-5-1}{2}$$

$$x=-2 \qquad x=-3$$

2. $3x^2-7x-5=0$; $a=3$, $b=-7$, $c=-5$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(3)(-5)}}{2(3)}$$

$$x=\frac{7\pm\sqrt{49-(-60)}}{6}$$

$$x=\frac{7\pm\sqrt{109}}{6}$$

3. $x^2+2x-15=0$; $a=1$, $b=2$, $c=-15$

$$x=\frac{-2\pm\sqrt{(2)^2-4(1)(-15)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{4-(-60)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{64}}{2}$$

$$x=\frac{-2\pm 8}{2}$$

$$x=3 \text{ or } x=-5$$

4. $x^2-8x-20=0;\ a=1,\ b=-8,\ c=-20$

$$x=\frac{-(-8)\pm\sqrt{(8)^2-4(1)(-20)}}{2(1)}$$

$$x=\frac{8\pm\sqrt{64-(-80)}}{2}$$

$$x=\frac{8\pm\sqrt{144}}{2}$$

$$x=\frac{8\pm 12}{2}$$

$$x=-2 \text{ or } x=10$$

5. $D^2+3D+2=0;\ a=1,\ b=3,\ c=2$

$$D=\frac{-3\pm\sqrt{(3)^2-4(1)(2)}}{2(1)}$$

$$D=\frac{-3\pm\sqrt{9-8}}{2}$$

$$D=\frac{-3\pm\sqrt{1}}{2}$$

$$D=\frac{-3\pm 1}{2}$$

$$D=-2 \text{ or } D=-1$$

6. $t^2+5t-6=0;\ a=1,\ b=5,\ c=-6$

$$t=\frac{-5\pm\sqrt{(5)^2-4(1)(-6)}}{2(1)}$$

$$t=\frac{-5\pm\sqrt{25-(-24)}}{2}$$

$$t=\frac{-5\pm\sqrt{49}}{2}$$

$$t=\frac{-5\pm 7}{2}$$

$$t=-6 \text{ or } t=1$$

7. $x^2-5x+3=0;\ a=1,\ b=-5,\ c=3$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(3)}}{(2)}$$

$$x=\frac{5\pm\sqrt{25-12}}{2}$$

$$x=\frac{5\pm\sqrt{13}}{2}$$

8. $x^2 + 10x - 4 = 0$; $a = 1$, $b = 10$, $c = -4$

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{100 - (-16)}}{2}$$

$$x = \frac{-10 \pm \sqrt{116}}{2}$$

$$x = \frac{-10 \pm 2\sqrt{29}}{2}$$

$$x = -5 \pm \sqrt{29}$$

9. $v^2 = 15 - 2v$

$v^2 + 2v - 15 = 0$; $a = 1$, $b = 2$, $c = -15$

$$v = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-15)}}{2(1)}$$

$$v = \frac{-2 \pm \sqrt{4 - (-60)}}{2}$$

$$v = \frac{-2 \pm \sqrt{64}}{2}$$

$$v = \frac{-2 \pm 8}{2} =$$

$v = -5$ or $v = 3$

10. $16V - 24 = 2V^2$

$2V^2 - 16V + 24 = 0$

$V^2 - 8V + 12 = 0$; $a = 1$, $b = -8$, $c = 12$

$$V = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(12)}}{2(1)}$$

$$V = \frac{8 \pm \sqrt{64 - 48}}{2}$$

$$V = \frac{8 \pm \sqrt{16}}{2}$$

$$V = \frac{8 \pm 4}{2}$$

$V = 6$ or $V = 2$

11. $$8s^2 + 20s = 12$$
$$2s^2 + 5s = 3$$
$$2s^2 + 5s - 3 = 0;\ a = 2,\ b = 5,\ c = -3$$
$$s = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-3)}}{2(2)}$$
$$s = \frac{-5 \pm \sqrt{25 - (-24)}}{4}$$
$$s = \frac{-5 \pm \sqrt{49}}{4}$$
$$s = \frac{-5 \pm 7}{4}$$
$$s = -3 \text{ or } s = \frac{1}{2}$$

12. $$4x^2 + x = 3;$$
$$4x^2 + x - 3 = 0;\ a = 4,\ b = 1,\ c = -3$$
$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(4)(-3)}}{2(4)}$$
$$x = \frac{-1 \pm \sqrt{1 - (-48)}}{8}$$
$$x = \frac{-1 \pm \sqrt{49}}{8}$$
$$x = \frac{-1 \pm 7}{8}$$
$$x = -1 \text{ or } x = \frac{3}{4}$$

13. $$3y^2 = 3y + 2$$
$$3y^2 - 3y - 2 = 0;\ a = 3,\ b = -3,\ c = -2$$
$$y = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-2)}}{2(3)}$$
$$y = \frac{3 \pm \sqrt{9 - (-24)}}{6}$$
$$y = \frac{3 \pm \sqrt{33}}{6}$$
$$y = \frac{1}{6}\left(3 \pm \sqrt{33}\right)$$

14. $3x^2 = 3 - 4x$

$3x^2 + 4x - 3 = 0;\ a = 3,\ b = 4,\ c = -3$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(-3)}}{2(3)}$$

$$x = \frac{-4 \pm \sqrt{16 - (-36)}}{6}$$

$$x = \frac{-4 \pm \sqrt{52}}{6}$$

$$x = \frac{-4 \pm 2\sqrt{13}}{6}$$

$$x = \frac{1}{3}\left(-2 \pm \sqrt{13}\right)$$

15. $z + 2 = 2z^2$

$2z^2 - z - 2 = 0;\ a = 2,\ b = -1,\ c = -2$

$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-2)}}{2(2)}$$

$$z = \frac{1 \pm \sqrt{1 - (-16)}}{4}$$

$$z = \frac{1 \pm \sqrt{17}}{4}$$

16. $2 + 6v = 9v^2$

$9v^2 - 6v - 2 = 0;\ a = 9,\ b = -6,\ c = -2$

$$v = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(-2)}}{2(9)}$$

$$v = \frac{6 \pm \sqrt{36 - (-72)}}{18}$$

$$v = \frac{6 \pm \sqrt{108}}{18}$$

$$v = \frac{6 \pm \sqrt{(36)(3)}}{18}$$

$$v = \frac{6 \pm 6\sqrt{3}}{18}$$

$$v = \frac{1}{3}\left(1 \pm \sqrt{3}\right)$$

17. $30y^2+23y-40=0;\ a=30,\ b=23,\ c=-40$

$$y=\frac{-23\pm\sqrt{(23)^2-4(30)(-40)}}{2(30)}$$

$$y=\frac{-23\pm\sqrt{529-(-4800)}}{60}$$

$$y=\frac{-23\pm\sqrt{5329}}{60}$$

$$y=\frac{-23\pm 73}{60}$$

$$y=-\frac{8}{5}\text{ or }y=\frac{5}{6}$$

18. $62x+63=40x^2$

$40x^2-62x-63=0;\ a=40,\ b=-62,\ c=-63$

$$x=\frac{-(-62)\pm\sqrt{(-62)^2-4(40)(-63)}}{2(40)}$$

$$x=\frac{62\pm\sqrt{3844-(-10\ 080)}}{80}$$

$$x=\frac{62\pm\sqrt{13\ 924}}{80}$$

$$x=\frac{62\pm 118}{80}$$

$$x=-\frac{7}{10}\text{ or }x=\frac{9}{4}$$

19. $5t^2+3=7t$

$5t^2-7t+3=0;\ a=5,\ b=-7,\ c=3$

$$t=\frac{-(-7)\pm\sqrt{(-7)^2-4(5)(3)}}{2(5)}$$

$$t=\frac{7\pm\sqrt{49-60}}{10}$$

$$t=\frac{7\pm\sqrt{-11}}{10}\text{ (imaginary roots)}$$

20. $2d(d-2)=-7$

$2d^2-4d+7=0;\ a=2,\ b=-4,\ c=7$

$$d=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(7)}}{2(2)}$$

$$d=\frac{4\pm\sqrt{16-56}}{4}$$

$$d=\frac{4\pm\sqrt{-40}}{4}$$

$$d=\frac{4\pm 2\sqrt{-10}}{4}$$

$$d=\frac{1}{2}\left(2\pm\sqrt{-10}\right)\text{ (imaginary roots)}$$

21. $s^2=9+s(1-2s)$

$s^2=9+s-2s^2$

$3s^2-s-9=0;\ a=3,\ b=-1,\ c=-9$

$$s=\frac{-(-1)\pm\sqrt{(-1)^2-4(3)(-9)}}{2(3)}$$

$$s=\frac{1\pm\sqrt{1-(-108)}}{6}$$

$$s=\frac{1\pm\sqrt{109}}{6}$$

$$s=\frac{1}{6}\left(1\pm\sqrt{109}\right)$$

22. $20r^2=20r+1$

$20r^2-20r-1=0;\ a=20,\ b=-20,\ c=-1$

$$r=\frac{-(-20)\pm\sqrt{(-20)^2-4(20)(-1)}}{2(20)}$$

$$r=\frac{20\pm\sqrt{400-(-80)}}{40}$$

$$r=\frac{20\pm\sqrt{480}}{40}$$

$$r=\frac{20\pm 4\sqrt{30}}{40}$$

$$r=\frac{5\pm\sqrt{30}}{10}$$

23. $25y^2 = 81$

$25y^2 - 81 = 0;\ a = 25,\ b = 0,\ c = -81$

$$y = \frac{-0 \pm \sqrt{(0)^2 - 4(25)(-81)}}{2(25)}$$

$$y = \frac{-0 \pm \sqrt{0 - (-8100)}}{50}$$

$$y = \frac{\pm\sqrt{8100}}{50}$$

$$y = \frac{\pm 90}{50}$$

$$y = \frac{9}{5} \text{ or } y = -\frac{9}{5}$$

24. $37T = T^2$

$T^2 - 37T = 0;\ a = 1,\ b = -37,\ c = 0$

$$T = \frac{-(-37) \pm \sqrt{(-37)^2 - 4(1)(0)}}{2(1)}$$

$$T = \frac{37 \pm \sqrt{1369 - 0}}{2}$$

$$T = \frac{37 \pm 37}{2}$$

$$T = 0 \text{ or } T = 37$$

25. $15 + 4z = 32z^2$

$32z^2 - 4z - 15 = 0;\ a = 32,\ b = -4,\ c = -15$

$$z = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(32)(-15)}}{2(32)}$$

$$z = \frac{4 \pm \sqrt{16 - (-1920)}}{64}$$

$$z = \frac{4 \pm \sqrt{1936}}{64}$$

$$z = \frac{4 \pm 44}{64}$$

$$z = \frac{3}{4} \text{ or } z = -\frac{5}{8}$$

26. $4x^2 - 12x = 7$

$4x^2 - 12x - 7 = 0$; $a = 4$, $b = -12$, $c = -7$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(-7)}}{2(4)}$$

$$x = \frac{12 \pm \sqrt{144 - (-112)}}{8}$$

$$x = \frac{12 \pm \sqrt{256}}{8}$$

$$x = \frac{12 \pm 16}{8}$$

$$x = -\frac{1}{2} \text{ or } x = \frac{7}{2}$$

27. $x^2 - 0.20x - 0.40 = 0$; $a = 1$, $b = -0.20$, $c = -0.40$

$$x = \frac{-(-0.20)0 \pm \sqrt{(-0.20)^2 - 4(1)(-0.40)}}{2(1)}$$

$$x = \frac{0.20 \pm \sqrt{0.040 - (-1.6)}}{2}$$

$$x = \frac{0.20 \pm \sqrt{1.64}}{2}$$

$$x = -0.54 \text{ or } x = 0.74$$

28. $3.2x^2 = 2.5x + 7.6$

$3.2x^2 - 2.5x - 7.6 = 0$; $a = 3.2$, $b = -2.5$, $c = -7.6$

$$x = \frac{-(-2.5)0 \pm \sqrt{(-2.5)^2 - 4(3.2)(-7.6)}}{2(3.2)}$$

$$x = \frac{2.5 \pm \sqrt{6.25 - (-97.28)}}{6.4}$$

$$x = \frac{2.5 \pm \sqrt{103.53}}{6.4}$$

$$x = 1.98 \text{ or } x = -1.199$$

$$x = 2.0 \text{ or } x = -1.2$$

29. $0.29Z^2 - 0.18 = 0.63Z$

$0.29Z^2 - 0.63Z - 0.18 = 0$; $a = 0.29$, $b = -0.63$, $c = -0.18$

$$Z = \frac{-(-0.63) \pm \sqrt{(-0.63)^2 - 4(0.29)(-0.18)}}{2(0.29)}$$

$$Z = \frac{0.63 \pm \sqrt{0.3969 - (-0.2088)}}{0.58}$$

$$Z = \frac{0.63 \pm \sqrt{0.6057}}{0.58}$$

$$Z = -0.256 \text{ or } Z = 2.43$$

30.
$$13.2x = 15.5 - 12.5x^2$$
$$12.5x^2 + 13.2x = 15.5$$
$$12.5x^2 + 13.2x - 15.5 = 0;\ a = 12.5,\ b = 13.2,\ c = -15.5$$
$$x = \frac{-13.2 \pm \sqrt{(13.2)^2 - 4(12.5)(-15.5)}}{2(12.5)}$$
$$x = \frac{-13.2 \pm \sqrt{174.24 - (-775)}}{25}$$
$$x = \frac{-13.2 \pm \sqrt{949.24}}{25}$$
$$x = -1.76 \quad \text{or} \quad x = 0.704$$

31. $x^2 + 2cx - 1 = 0;\ a = 1,\ b = 2c,\ c = -1$
$$x = \frac{-(2c) \pm \sqrt{(2c)^2 - 4(1)(-1)}}{2(1)}$$
$$x = \frac{-2c \pm \sqrt{4c^2 - (-4)}}{2}$$
$$x = \frac{-2c \pm 2\sqrt{c^2 + 1}}{2}$$
$$x = -c \pm \sqrt{c^2 + 1}$$

32. $x^2 - 7x + (6 + a) = 0;\ a = 1,\ b = -7,\ c = 6 + a$
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(6 + a)}}{2(1)}$$
$$x = \frac{7 \pm \sqrt{49 - 24 - 4a}}{2}$$
$$x = \frac{7 \pm \sqrt{25 - 4a}}{2}$$
$$x = \frac{1}{2}\left(7 \pm \sqrt{25 - 4a}\right)$$

33.
$$b^2x^2 + 1 - a = (b + 1)x$$
$$b^2x^2 - (b + 1)x + (1 - a) = 0;\ a = b^2,\ b = -(b + 1),\ c = 1 - a$$
$$x = \frac{-[-(b+1)] \pm \sqrt{[-(b+1)]^2 - 4(b^2)(1 - a)}}{2(b^2)}$$
$$x = \frac{b + 1 \pm \sqrt{b^2 + 2b + 1 - 4b^2 + 4ab^2}}{2b^2}$$
$$x = \frac{b + 1 \pm \sqrt{4ab^2 - 3b^2 + 2b + 1}}{2b^2}$$

34. $c^2x^2 - x - 1 = x^2$

$c^2x^2 - x^2 - x - 1 = 0;$

$(c^2 - 1)x^2 - x - 1 = 0;\ a = c^2 - 1,\ b = -1,\ c = -1$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(c^2 - 1)(-1)}}{2(c^2 - 1)}$$

$$x = \frac{1 \pm \sqrt{1 + 4c^2 - 4}}{2c^2 - 2}$$

$$x = \frac{1 \pm \sqrt{4c^2 - 3}}{2c^2 - 2}$$

35. $2x^2 - 7x = -8$

$2x^2 - 7x + 8 = 0;\ a = 2;\ b = -7;\ c = 8$

$D = \sqrt{(-7)^2 - 4(2)(8)} = \sqrt{-15},$

unequal imaginary roots

36. $3x^2 = 14 - 19x$

$3x^2 + 19x - 14 = 0$

$b^2 - 4ac = 19^2 - 4(3)(-14) = 529 = 23^2$

Since $b^2 - 4ac > 0$ and a perfect square

$\sqrt{529} = 23,$ so the roots are real, rational, and unequal.

37. $3.6t^2 + 2.1 = 7.7t$

$b^2 - 4ac = (-7.7)^2 - 4(3.6)(2.1) = 29.05$

Since $b^2 - 4ac > 0$ and not a perfect square, the roots are real, irrational, and unequal.

38. $0.45s^2 + 0.33 = 0.12s$

$b^2 - 4ac = (0.12)^2 - 4(0.45)(0.33) = -0.5796$

Since $b^2 - 4ac < 0,$ the roots contain imaginary numbers and are unequal.

39. $x^2 + 4x + k = 0$ will have a double root if $b^2 - 4ac = 0$

$4^2 - 4(1)(k) = 0$

$k = 4$

40. $x^2 + 3x + k = 0$ will have imaginary roots if $b^2 - 4ac < 0$

$3^2 - 4(1)k < 0$

$9 < 4k$

$4k > 9$

$k > \frac{9}{4} = 2.25$

3 is the smallest positive integer value of k for which the roots are imaginary.

41.
$$x^4 - 5x^2 + 4 = 0$$
$$\left(x^2\right)^2 - 5x^2 + 4 = 0$$
$$\left(x^2 - 4\right)\left(x^2 - 1\right) = 0$$
$$x^2 - 4 = 0 \quad \text{or} \quad x^2 - 1 = 0$$
$$x^2 = 4 \qquad x^2 = 1$$
$$x = \pm 2 \qquad x = \pm 1$$

42. If $b^2 - 4ac > 0,$ there are two intercepts, two real roots;
if $b^2 - 4ac = 0,$ there is one intercept (a double root);
if $b^2 - 4ac < 0,$ there are no intercepts, all roots are imaginary.

43. $a = 90, b = -123, c = 40$
$b^2 - 4ac = (-123)^2 - 4(90)(40)$
$= 729$
$= 27^2$
Since the discriminant is a perfect square, the roots will be rational and so the quadratic equation $90x^2 - 123x + 40 = 0$ can be solved by factoring.

44. **(a)**
$$6x^2 - x = 15$$
$$6x^2 - x - 15 = 0$$
$$(3x - 5)(2x + 3) = 0$$
$$3x - 5 = 0 \text{ or } 2x + 3 = 0$$
$$x = \frac{5}{3} \text{ or } x = -\frac{3}{2}$$

(b)
$$6x^2 - x = 15$$
$$x^2 - \frac{1}{6}x = \frac{5}{2}$$
$$x^2 - \frac{1}{6}x + \frac{1}{144} = \frac{5}{2} + \frac{1}{144}$$
$$\left(x - \frac{1}{12}\right)^2 = \frac{361}{144}$$
$$x - \frac{1}{12} = \pm\frac{19}{12}$$
$$x = \frac{5}{3} \text{ or } x = -\frac{3}{2}$$

(c) $6x^2 - x - 15 = 0; a = 6, b = -1, c = -15$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-15)}}{2a}$$

$$= \frac{1 \pm \sqrt{361}}{12}$$

$$= \frac{1 \pm 19}{12}$$

$$x = \frac{5}{3} \text{ or } x = -\frac{3}{2}$$

45. For $D = 3.625$

$$D_0^2 - DD_0 - 0.250D^2 = 0$$

$$D_0^2 - 3.625D_0 - 0.25(3.625)^2 = 0$$

$$D_0^2 - 3.625D_0 - 3.28515625 = 0$$

$$a = 1,\ b = -3.625,\ c = -3.28515625$$

$$D_0 = \frac{-(-3.625) \pm \sqrt{(-3.625)^2 - 4(1)(-3.28515625)}}{2}$$

$D_0 = 4.38$ cm or $D_0 = -0.751$ cm, reject since $D_0 > 0$.

46. (a) $s = 300 + 500t - 16t^2$, and if $s = 0$

$$16t^2 - 500t - 300 = 0$$

$$a = 16,\ b = -500,\ c = -300$$

$$t = \frac{-(-500) \pm \sqrt{500^2 - 4(16)(-300)}}{2(16)}$$

$t = 31.839$ s or $t = -0.589$ s (discard since $t > 0$)

$t = 32$ s

(b) $s = -16t^2 + 500t + 300$ and if $s = 1000$

$$1000 = -16t^2 + 500t + 300$$

$$16t^2 - 500t + 700 = 0$$

$$a = 16,\ b = -500,\ c = 700$$

$$t = \frac{-(-500) \pm \sqrt{(-500)^2 - 4(16)(700)}}{2(16)}$$

$t = 1.47$ s or $t = 29.78$ s

47. $8x^2 - 15Lx + 6L^2 = 0$

$$x = \frac{-(-15L) \pm \sqrt{(-15L)^2 - 4(8)(6L^2)}}{2(8)}$$

$$= \frac{15L \pm \sqrt{225L^2 - 192L^2}}{16}$$

$$= \frac{15 \pm \sqrt{33}}{16} L$$

In order for $x < L$, we must discard the root that results from addition in the numerator, and so

$$x = \frac{15 - \sqrt{33}}{16} L$$

48. Let w be the width and l be the length. Then $l = w + 2.0$ and $A = lw = (w + 2.0)w = w^2 + 2.0w$. Thus,

$20.0 = w^2 + 2.0w$

$w^2 + 2.0w - 20.0 = 0$

$$w = \frac{-2.0 \pm \sqrt{2.0^2 - 4(1)(-20.0)}}{2(1)}$$

$$w = \frac{-2.0 \pm \sqrt{84.0}}{2}$$

$w = 3.5826$ or $w = -5.5826$ (negative width discarded)

The patio is 3.6 m wide and 5.6 m long.

49. The cars have traveled x km and $x + 2.0$ km, forming the legs of a right triangle. The hypotenuse is 6.0 km. Using the Pythagorean theorem,

$6.0^2 = x^2 + (x + 2.0)^2$

$36.0 = 2x^2 + 4.0x + 4.0$

$2x^2 + 4.0x - 32.0 = 0$

$$x = \frac{-4.0 \pm \sqrt{(4.0)^2 - 4(2)(-32.0)}}{2(2)}$$

$x = 3.1231$ or $x = -5.1231$ (negative distance discarded)

The first car traveled 3.1 km and the second car traveled 5.1 km.

50. Let v be the speed going to college and w be the speed returning from college. We have $w = v - 3.0$. We use

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

obtaining

$$\frac{4.0}{v-3.0} = 0.25 + \frac{4.0}{v}$$

or, after clearing denominators,

$$4.0v = 0.25v(v-3.0) + 4.0(v-3.0)$$

$$4.0v = 0.25v^2 - 0.75v + 4.0v - 12.0$$

$$0.25v^2 - 0.75v - 12.0 = 0$$

$$v^2 - 3.0v - 48.0 = 0$$

$v = 8.5887$ or $v = -5.5887$ (negative speed discarded)

The speed to college was 8.6 km/hr and from college was 5.6 km/hr.

51.

$$\frac{l}{w} = \frac{l+w}{l}$$

$$l^2 = lw + w^2$$

$$l^2 - wl - w^2 = 0;\ a = 1,\ b = -w,\ c = -w^2$$

$$l = \frac{-(-w) \pm \sqrt{(-w)^2 - 4(1)(-w^2)}}{2(1)}$$

$l = \dfrac{w \pm w\sqrt{5}}{2}$ The (-) solution yields $l < 0$, so choose + to make $l > 0$

$$l = w\left(\frac{1 \pm \sqrt{5}}{2}\right)$$

$$\frac{l}{w} = \frac{1+\sqrt{5}}{2} = 1.618$$

52.

$$r = \frac{f^2}{p-f}$$

$$f^2 = rp - rf$$

$$f^2 + rf - rp = 0;\ a = 1,\ b = r,\ c = -rp$$

$$f = \frac{-r \pm \sqrt{r^2 - 4(1)(-rp)}}{2(1)}$$

$f = \dfrac{-r + \sqrt{r^2 + 4rp}}{2}$ assuming $f > 0$, choose + solution

53. $Lm^2 + Rm + \dfrac{1}{C} = 0;\ a = L,\ b = R,\ c = \dfrac{1}{C}$

$$m = \frac{-R \pm \sqrt{R^2 - 4(L)\left(\frac{1}{C}\right)}}{2(L)}$$

$$m = \frac{-R \pm \sqrt{R^2 - \frac{4L}{C}}}{2L}$$

54.

$$r = \frac{b^2 + 4h^2}{8h}$$

$$8hr = b^2 + 4h^2$$

$$4h^2 - 8hr + b^2 = 0$$

$$a = 4,\ b = -8r,\ c = b^2$$

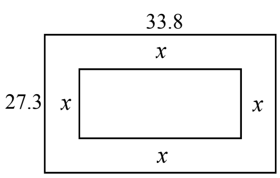
$$h = \frac{8r \pm \sqrt{64r^2 - 4(4)(b^2)}}{2(4)}$$

$$h = \frac{8r \pm \sqrt{64r^2 - 16b^2}}{8}$$

$$h = \frac{8r \pm 4\sqrt{4r^2 - b^2}}{8}$$

$$h = \frac{2r \pm \sqrt{4r^2 - b^2}}{2}$$

55.



$$80\% \text{ of area} = w \cdot l$$

$$0.8(33.8)(27.3) = (33.8 - 2x)(27.3 - 2x)$$

$$738.192 = 922.74 - 122.2x + 4x^2$$

$$4x^2 - 122.2x + 184.578 = 0$$

$$a = 4,\ b = -122.2,\ c = 184.578$$

$$x = \frac{-(-122.2) \pm \sqrt{(-122.2)^2 - 4(4)(184.578)}}{2(4)}$$

$x = 1.5936$ cm or $x = 28.9564$ cm (discard second one, too wide to fit on screen)

56. Let r = interest rate

After 1 year, the interest accrued is r * Principal, so amount at end is $(P + rP) = P(1 + r)$

$$2000(1+r)^2 + 3000(1+r) = 5319.05$$

$$2000r^2 + 7000r - 319.05 = 0$$

$$a = 2000,\ b = 7000,\ c = -319.05$$

$$r = \frac{-7000 \pm \sqrt{7000^2 - 4(2000)(-319.05)}}{2(2000)}$$

$$r = -3.545 \quad \text{or} \quad r = 0.045$$

The rate is 4.50%.

57. Let w be the added width to each dimension

$$(12+w)\times(16+w) = (12\times 16) + 80 = 272$$

$$w^2 + 28w + 192 = 272$$

$$w^2 + 28w - 80 = 0$$

$$a = 1,\ b = 28,\ c = -80$$

$$r = \frac{-28 \pm \sqrt{28^2 - 4(1)(-80)}}{2(1)}$$

$$w = 2.61324 \quad \text{or} \quad w = -30.61324$$

$$w = 2.6 \text{ ft}$$

58. t = time for pipe 1 to drain whole volume V

$t + 2$ = time for pipe 2 drain whole volume V

Q_1 = flow rate for pipe 1

Q_2 = flow rate for pipe 2

Together they drain whole tank V in 6 hours

$$(Q_1 + Q_2)\cdot 6 = V$$

$$Q_1 + Q_2 = \frac{V}{6} \quad \text{(Equation 1)}$$

Pipe 1 drains whole tank in unknown time t

$$Q_1 \cdot t = V$$

$$Q_1 = \frac{V}{t}$$

Pipe 2 drains whole tank in unknown time $t + 2$

$$Q_2 \cdot (t+2) = V$$

$$Q_2 = \frac{V}{t+2}$$

Plug rates into Equation 1

$$\frac{V}{t} + \frac{V}{t+2} = \frac{V}{6}$$

$$\frac{1}{t+2} + \frac{1}{t} = \frac{1}{6}$$

$6t+6(t+2)=t(t+2)$ multiplied by LCD

$6t+6t+12=t^2+2t$

$t^2-10t-12=0$

$a=1,\ b=-10,\ c=-12$

$$t=\frac{10\pm\sqrt{100-4(1)(-12)}}{2(1)}$$

$$t=\frac{10\pm\sqrt{148}}{2}$$

$t=11.1$ h or -1.08 h (ignore negative time)

$t=11.1$ h for pipe 1

$t+2.00=13.1$ h for pipe 2

59. The area 53.5 cm^2 can be obtained by subtracting the cut-away area from the full area:

$(15.6)(10.4)-(15.6-x)(10.4-x)=53.5$

$-x^2+26.0x=53.5$

$x^2-26.0x+53.5=0$

$$x=\frac{-(-26.0)\pm\sqrt{26.0^2-4(1)(53.5)}}{2(1)}$$

$$x=\frac{26.0\pm\sqrt{462}}{2}$$

$x=2.2529$ or $x=23.7471$ (discarded for being too much to cut away)

The required thickness is 2.25 cm.

60. We have $p=q+5.0,\ f=4.0$ and

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

so

$$\frac{1}{q+5.0}+\frac{1}{q}=\frac{1}{4.0}$$

After clearing denominators,

$4.0q+4.0(q+5.0)=q(q+5.0)$

$8.0q+20.0=q^2+5.0q$

$q^2-3.0q-20.0=0$

$$q=\frac{-(-3.0)\pm\sqrt{(-3.0)^2-4(1)(-20.0)}}{2(1)}$$

$q=6.217$ or $q=-3.217$ (discarded since $q>0$)

and so $q=6.2,\ p=q+5=11.2.$

61. We have $l = w + 12.8$, $A = lw$ and so

$262 = (w + 12.8)w$

$262 = w^2 + 12.8w$

$w^2 + 12.8w - 262 = 0$

$$w = \frac{-12.8 \pm \sqrt{12.8^2 - 4(1)(-262)}}{2(1)}$$

$w = 11.0$ or $w = -23.8$ (negative width discarded)

Therefore, the tennis court is 11.0 m wide and 23.8 m long.

62. If one spill has radius R, the other spill is centred 800 m away, so the two radii sum to 800 m.

$R =$ first radius

$800 - R =$ second radius

Total area $A = \pi R^2 + \pi(800 - R)^2 = 1.02 \times 10^6$

$\pi R^2 + 640\ 000 - 1600\pi R + \pi R^2 = 1.02 \times 10^6$

$2\pi R^2 - 1600\pi R + \left(640\ 000\pi - 1.02 \cdot 10^6\right) = 0$

$a = 2\pi,\ b = -1600\pi,\ c = 640\ 000\pi - 1.02 \cdot 10^6$

$$R = \frac{1600\pi \pm \sqrt{(1600\pi)^2 - 4(2\pi)\left(640000\pi - 1.02 \cdot 10^6\right)}}{4\pi}$$

$R = 352$ m or $R = 448$ m

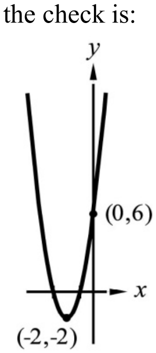
7.4 The Graph of the Quadratic Function

1. $y = 2x^2 + 8x + 6;\ a = 2,\ b = 8,\ c = 6$

$$x\text{-coordinate of vertex} = \frac{-b}{2a} = \frac{-8}{2(2)} = -2$$

$$y\text{-coordinate of vertex} = 2(-2)^2 + 8(-2) + 6 = -2$$

The vertex is $(-2, -2)$ and since $a > 0$, it is a minimum. Since $c = 6$, the y-intercept is (0, 6) and the check is:

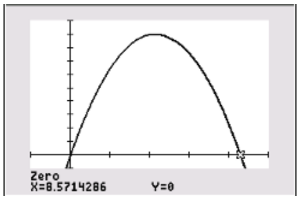


2. $s = -4.9t^2 + 42t$

Graph $y_1 = -4.9x^2 + 42x$

(1) Use the zero feature to find $y_1 = 0$ when $x = 8.5714286$.

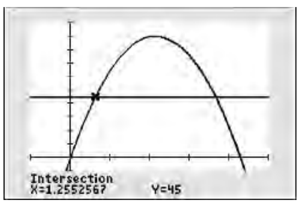
The projectile hits the ground at $t = 8.6$ seconds.

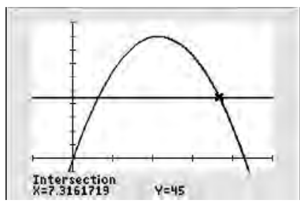


(2) Use the intersect feature to find $y_1 = 45$ when $x = 1.2552567$

and when $x = 7.3161719$

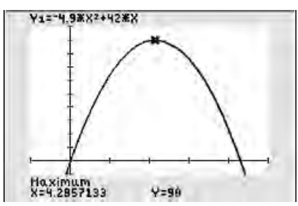
The projectile is at a height of 45 meters at $t = 1.3$ seconds and again at $t = 7.3$ seconds.





(3) Use the maximum feature to find $y_1 = 90$ when $x = 4.2857133$

The projectile is at a maximum height of 90.0 meters at $t = 4.3$ seconds.

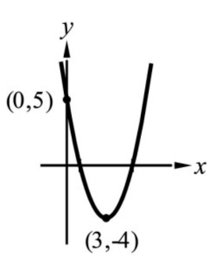


3. $y = x^2 - 6x + 5,$ which has $a = 1,\ b = -6,\ c = 5$

The x-coordinate of the extreme point is $\dfrac{-b}{2a} = \dfrac{-(-6)}{2(1)} = 3$

and the y-coordinate is $y = 3^2 - 6(3) + 5 = -4$

The extreme point is $(3, -4)$. Since $a > 0$ it is a minimum point.



Since $c = 5$, the y-intercept is (0, 5).
Use the minimum point $(3, -4)$ and the y-intercept (0, 5), and the fact that the graph is a parabola, to sketch the graph.

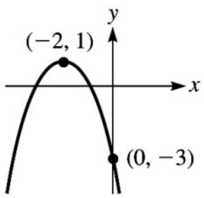
4. $y = -x^2 - 4x - 3$, with $a = -1$, $b = -4$, $c = -3$.
This means that the x-coordinate of the extreme is

$$\frac{-b}{2a} = \frac{-(-4)}{2(-1)} = -2,$$

and the y-coordinate is

$$y = -(-2)^2 - 4(-2) - 3 = 1.$$

Thus the extreme point is $(-2, 1)$. Since $a < 0$, it is a maximum point.



Since $c = -3$, the y-intercept is $(0, -3)$. Use the maximum point $(-2, 1)$ and the y-intercept $(0, -3)$, and the fact that the graph is a parabola, to sketch the graph.

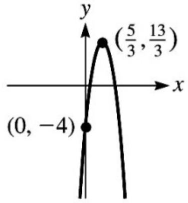
5. $y = -3x^2 + 10x - 4$, with $a = -3$, $b = 10$, $c = -4$.
This means that the x-coordinate of the extreme is

$$\frac{-b}{2a} = \frac{-10}{2(-3)} = \frac{10}{6} = \frac{5}{3}$$

and the y-coordinate is

$$y = -3\left(\frac{5}{3}\right)^2 + 10\left(\frac{5}{3}\right) - 4 = \frac{13}{3}.$$

Thus the extreme point is $\left(\frac{5}{3}, \frac{13}{3}\right)$.



Since $a < 0,$ it is a maximum point.
Since $c = -4,$ the y-intercept is $(0, -4)$. Use the maximum point $\left(\frac{5}{3}, \frac{13}{3}\right)$, and the y-intercept $(0, -4)$, and the fact that the graph is a parabola, to sketch the graph.

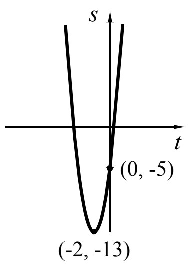
6. $s = 2t^2 + 8t - 5,$ with $a = 2,\ b = 8,\ c = -5.$
This means that the t-coordinate of the extreme is

$$\frac{-b}{2a} = \frac{-8}{2(2)} = -2,$$

and the s-coordinate is

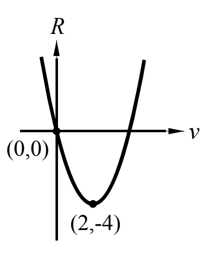
$$s = 2(-2)^2 + 8(-2) - 5 = -13.$$

Thus the extreme point is $(-2, -13)$.



Since $a > 0,$ it is a minimum point.
Since $c = -5,$ the s-intercept is $(0, -5)$. Use the minimum point $(-2, -13)$, and the s-intercept $(0, -5)$, and the fact that the graph is a parabola, to get an approximate sketch of the graph.

7. $R = v^2 - 4v + 0,$ with $a = 1,\ b = -4,\ c = 0$
The v-coordinate of the extreme point is $\dfrac{-b}{2a} = \dfrac{-(-4)}{2(1)} = 2,$
and the R-coordinate is $R = 2^2 - 4(2) = -4$
The extreme point is $(2, -4)$. Since $a > 0$ it is a minimum point.



Since $c = 0,$ the R-intercept is $(0, 0)$. Use the minimum point $(2, -4)$ and the R-intercept $(0, 0)$, and the fact that the graph is a parabola, to sketch the graph.

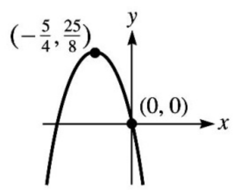
8. $y = -2x^2 - 5x$, with $a = -2$, $b = -5$, $c = 0$.
This means that the x-coordinate of the extreme is

$$\frac{-b}{2a} = \frac{-(-5)}{2(-2)} = -\frac{5}{4},$$

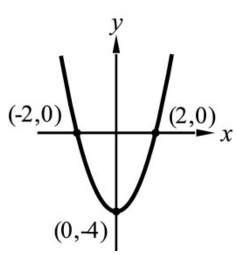
and the y-coordinate is

$$y = -2\left(-\frac{5}{4}\right)^2 - 5\left(-\frac{5}{4}\right) = \frac{25}{8},$$

Thus the extreme point is $\left(-\frac{5}{4}, \frac{25}{8}\right)$.
Since $a < 0$, it is a maximum point. Since $c = 0$, the y-intercept is (0, 0). Use the maximum point $\left(-\frac{5}{4}, \frac{25}{8}\right)$, and the y-intercept (0, 0), and the fact that the graph is a parabola, to sketch the graph.



9. $y = x^2 - 4 = x^2 + 0x - 4$; $a = 1$, $b = 0$, $c = -4$
The x-coordinate of the extreme point is
$\frac{-b}{2a} = \frac{-0}{2(1)} = 0$, and the y-coordinate is
$y = 0^2 - 4 = -4$.
The extreme point is $(0, -4)$.



Since $a > 0$, it is a minimum point.
Since $c = -4$, the y-intercept is $(0, -4)$.
$x^2 - 4 = 0$, $x^2 = 4$, $x = \pm 2$ are the x-intercepts.
Use the minimum points and intercepts to sketch the graph.

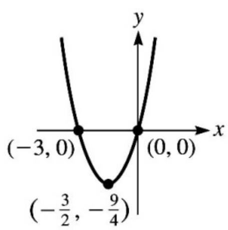
10. $y = x^2 + 3x;\ a = 1,\ b = 3,\ c = 0.$
This means that the x-coordinate of the extreme is

$$\frac{-b}{2a} = \frac{-3}{2(1)} = -\frac{3}{2},$$

and the y-coordinate is

$$y = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) = -\frac{9}{4}.$$

Thus the extreme point is $\left(-\frac{3}{2}, -\frac{9}{4}\right)$.



Since $a > 0,$ it is a minimum point.
Since $c = 0,$ the y-intercept is (0, 0). The x-intercepts are found by setting $y = 0;$
$0 = x^2 + 3x;\ x(x+3) = 0;\ x = 0,\ x = -3.$
Therefore the x-intercepts are (0, 0) and $(-3, 0)$. Use the minimum point $\left(-\frac{3}{2}, -\frac{9}{4}\right)$, and the x-intercepts (0,0), and $(-3, 0)$ to sketch the graph.

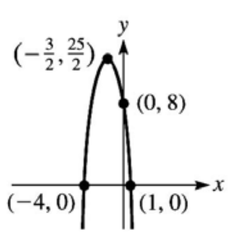
11. $y = -2x^2 - 6x + 8;\ a = -2,\ b = -6,\ c = 8.$
This means that the x-coordinate of the extreme is

$$\frac{-b}{2a} = \frac{-(-6)}{2(-2)} = -\frac{6}{4} = -\frac{3}{2},$$

and the y-coordinate is

$$y = -2\left(-\frac{3}{2}\right)^2 - 6\left(-\frac{3}{2}\right) + 8 = -\frac{25}{2}.$$

Thus the extreme point is $\left(-\frac{3}{2}, \frac{25}{2}\right)$.



Since $a < 0,$ it is a maximum point. Since $c = 8,$ the y-intercept is (0, 8). To find the x-intercepts, set $y = 0.$

$-2x^2 - 6x + 8 = 0$

Use the quadratic formula

$x = \dfrac{6 \pm \sqrt{36 - 4(-2)(8)}}{2(-2)} = \dfrac{6 \pm 10}{-4} = 4, 1.$

Therefore the x-intercepts are $(-4, 0)$ and $(1, 0)$.

Use the maximum point $\left(-\frac{3}{2}, \frac{25}{2}\right)$, the y-intercept (0, 8), and the x-intercepts $(-4, 0)$ and (1, 0) to sketch the graph.

12. $u = -3v^2 + 12v - 9;\ a = -3,\ b = 12,\ c = -9.$

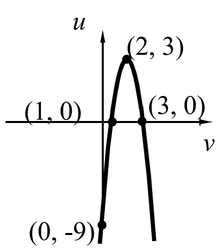
This means that the v-coordinate of the extreme is

$$\frac{-b}{2a} = \frac{-12}{2(-3)} = 2,$$

and the u-coordinate is

$$u = -3(2)^2 + 12(2) - 9 = 3.$$

Thus the extreme point is $(2, 3)$.



Since $a < 0,$ it is a maximum point. Since $c = -9,$ the u-intercept is $(0,\ -9)$. To find the v-intercepts, set $u = 0.$

$-3v^2 + 12v - 9 = 0$

which can be factored:

$-3(v-1)(v-3) = 0$

and so the v-intercepts are at $(1,0)$ and $(3,0)$.

Use the maximum point $(2, 3)$, the u-intercept $(0,\ -9)$, and the v-intercepts $(1, 0)$ and (3, 0) to sketch the graph.

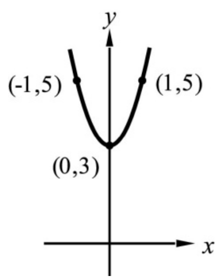
13. $y = 2x^2 + 3 = 2x^2 + 0x + 3$; $a = 2$, $b = 0$, $c = 3$

The x-coordinate of the extreme point is

$\frac{-b}{2a} = \frac{-0}{2(2)} = 0$, and the y-coordinate is

$y = 2(0)^2 + 3 = 3.$

The extreme point is $(0, 3)$. Since $a > 0$ it is a minimum point.



Since $c = 3$, the y-intercept is (0, 3) there are no x-intercepts, $b^2 - 4ac = -24$. $(-1, 5)$ and $(1, 5)$ are on the graph. Use the three points to sketch the graph.

14. $s = t^2 + 2t + 2$; $a = 1$, $b = 2$, $c = 2$.

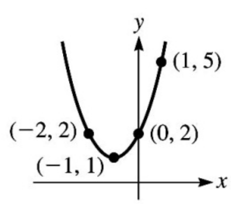
This means that the t-coordinate of the extreme is

$$\frac{-b}{2a} = \frac{-2}{2(1)} = -1,$$

and the s-coordinate is

$$s = (-1)^2 + 2(-1) + 2 = 1.$$

Thus the extreme point is $(-1, 1)$.



Since $a > 0$, it is a minimum point. Since $c = 2$, the s-intercept is (0, 2). Two other points which may be used are: If $t = -2$, then

$$s = (1)^2 + 2(1) + 2 = 5.$$

Therefore, (1, 5) is a point on the parabola. Sketch the graph using these points.

15. $y = -2x^2 - 2x - 6$; $a = -2$, $b = -2$, $c = -6$.

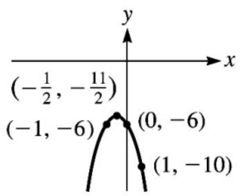
This means that the x-coordinate of the extreme is

$$\frac{-b}{2a} = \frac{-2}{4} = -\frac{1}{2},$$

and the y-coordinate is

$$y = -2\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) - 6 = -\frac{11}{2}.$$

Thus the extreme point is $\left(-\frac{1}{2}, -\frac{11}{2}\right)$.



Since $a < 0$, it is a maximum point. Since $c = -6$, the y-intercept is $(0, -6)$. Two other points which may be used are: If $x = -1$.

$$y = -2(-1)^2 - 2(-1) - 6 = -6.$$

Therefore, $(-1, -6)$ is a point on the parabola.

If $x = 1$.

$$y = -2(1)^2 - 2(1) - 6 = -10.$$

Therefore, $(1, -10)$ is a point on the parabola.

Sketch the graph using these points.

16. $y = -3x^2 - x$; $a = -3$, $b = -1$, $c = 0$.

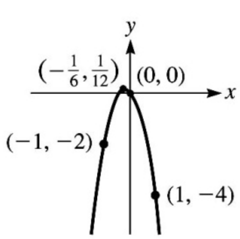
This means that the x-coordinate of the extreme is

$$\frac{-b}{2a} = -\frac{1}{6},$$

and the y-coordinate is

$$y = -3\left(-\frac{1}{6}\right)^2 - \left(-\frac{1}{6}\right) = \frac{1}{12}.$$

Thus the extreme point is $\left(-\frac{1}{6}, -\frac{1}{12}\right)$.



Since $a < 0$, it is a maximum point. Since $c = 0$, the y-intercept is (0, 0). Two other points which may be used are: If $x = -1$.

$$y = -3(-1)^2 - (-1) = -2.$$

Therefore, $(-1, -2)$ is a point on the parabola.

If $x = 1$.

$$y = -3(1)^2 - (1) = -4.$$

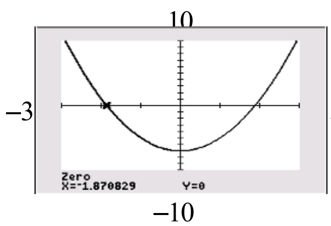
Therefore, $(1, -4)$ is a point on the parabola.

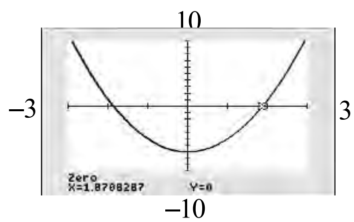
Sketch the graph using these points.

17. $2x^2 - 7 = 0.$

Graph $y = 2x^2 - 7$ and use the zero feature to find the roots.

$x = -1.87$ and $x = 1.87$.

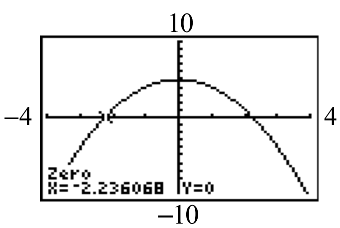


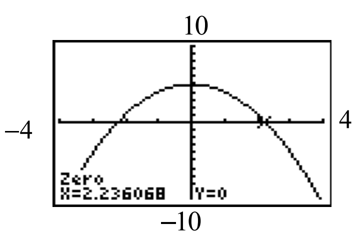


18. $5 - x^2 = 0$

Graph $y_1 = 5 - x^2$ and use the zero feature.

$x = -2.24$ and $x = 2.24$.

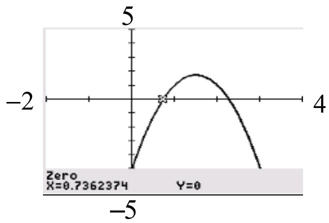


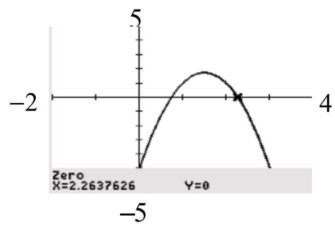


19. $-3x^2+9x-5=0$

Graph $y_1=-3x^2+9x-5$ and use the zero feature.

$x=0.736$ and $x=2.264$.



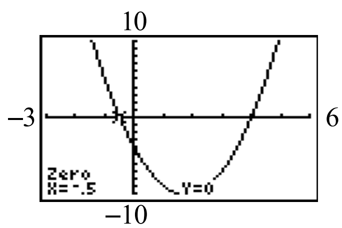


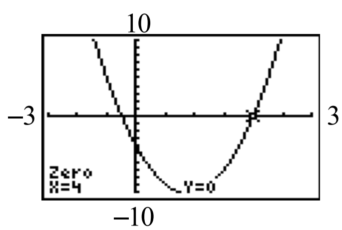
20. $2t^2=7t+4$

$2t^2-7t-4=0$

Graph $y_1=2x^2-7x-4$ and use the zero feature.

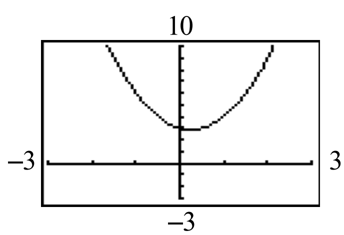
$x=-0.500$ and $x=4.00$.





21. $x(2x-1)=-3$

Graph $y_1=x(2x-1)+3$ and use the zero feature.



As the graph shows, there are no real solutions.

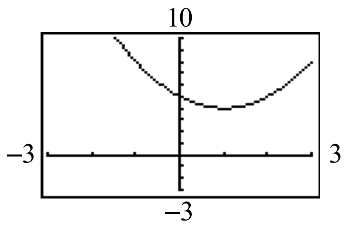
22. $6w-15=3w^2$

$3w^2-6w+15=0$ (divide both sides by 3)

$w^2-6w+5=0$

Graph $y_1=x^2-2x+5$ and use the zero feature.

As the graph shows there are no real solutions.

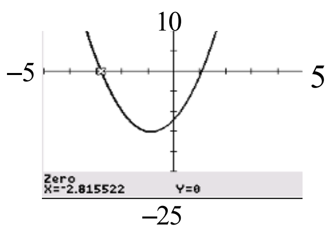


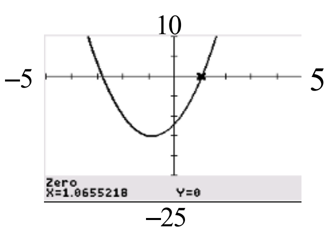
23. $4R^2=12-7R$

$4R^2+7R-12=0$

Graph $y_1=4x^2+7x-12$ and use the zero feature.

$R=-2.82$ and $R=1.07$.



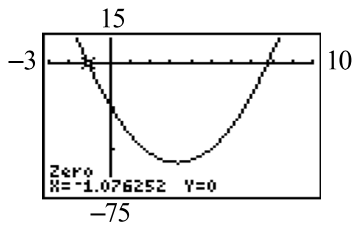


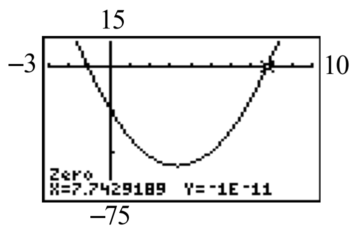
24. $3x^2 - 25 = 20x$

$3x^2 - 20x - 25 = 0$

Graph $y_1 = 3x^2 - 20x - 25$ and use the zero feature.

$x = -1.08$ and $x = 7.74$.

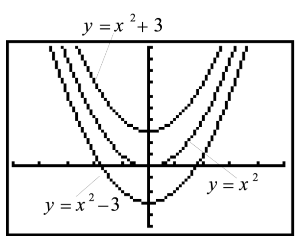




25. **(a)** $y = x^2$ **(b)** $y = x^2 + 3$ **(c)** $y = x^2 - 3$

The parabola $y = x^2 + 3$ is shifted up +3 units (minimum point $(0,\ 3)$).

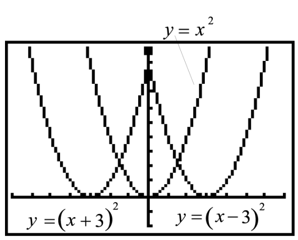
The parabola $y = x^2 - 3$ is shifted down -3 units (minimum point $(0, -3)$).



26. **(a)** $y = x^2$ **(b)** $y = (x-3)^2$ **(c)** $y = (x+3)^2$

The parabola $y = (x-3)^2$ is shifted over +3 units to the right (minimum point $(3,\ 0)$).

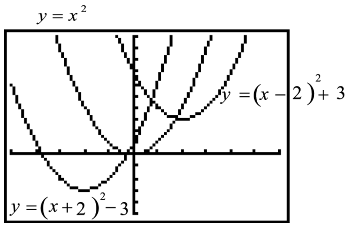
The parabola $y = (x+3)^2$ is shifted over -3 units to the left (minimum point $(-3,\ 0)$).



27. **(a)** $y = x^2$ **(b)** $y = (x-2)^2 + 3$ **(c)** $y = (x+2)^2 - 3$

$y = (x-2)^2 + 3$ is $y = x^2$ shifted right 2 and up 3.

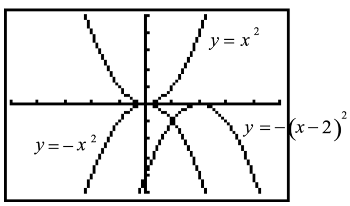
$y = (x+2)^2 - 3$ is $y = x^2$ shifted left 2 and down 3.



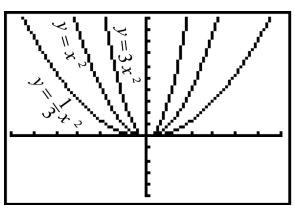
28. **(a)** $y = x^2$ **(b)** $y = -x^2$ **(c)** $y = -(x-2)^2$

$y = -x^2$ is $y = x^2$ reflected in the x-axis.

$y = -(x-2)^2$ is $y = -x^2$ shifted right 2 units.



29. **(a)** $y = x^2$ **(b)** $y = 3x^2$ **(c)** $y = \dfrac{1}{3}x^2$



The graph of $y = 3x^2$ is the graph of $y = x^2$ narrowed. The graph of $y = \frac{1}{3}x^2$ is the graph of $y = x^2$ broadened.

30. **(a)** $y = x^2$ **(b)** $y = -3(x-2)^2$ **(c)** $y = \dfrac{1}{3}(x+2)^2$

$y = -3(x-2)^2$ is $y = x^2$ reflected in the x-axis, shifted right 2 and narrowed.

$y = \dfrac{1}{3}(x+2)^2$ is $y = -x^2$ widened and shifted left 2.

31. The t-coordinate of the vertex is at

$$-\frac{b}{2a}=-\frac{25}{2(-9.8)}=\frac{25}{19.6}=1.2755102$$

and the s-coordinate is at

$$-9.8\left(\frac{25}{19.6}\right)^2+25\left(\frac{25}{19.6}\right)+4=19.943878$$

The vertex is at (1.28,19.94).

32. The vertex has t-coordinate $-\frac{b}{2a}=-\frac{64}{(-32)}=2$

and s-coordinate $-16(2)^2+64(2)+6=70$. This is a maximum since $a<0$ and so the range is $s\le 70$.

33. Since the vertex has x-coordinate equal to 0, $b=0$. Since it also has y-coordinate equal to 0, $c=0$ as well. It follows that the equation takes the form $y=ax^2$. By substituting $x=25$ and $y=125$, we find $125=a(25)^2$, or $a=\frac{125}{(25)^2}=\frac{1}{5}$.

The equation is $y=\frac{1}{5}x^2$.

34. Since the vertex has x-coordinate equal to 0, $b=0$. Since it also has y-coordinate equal to 0, $c=0$ as well. It follows that the equation takes the form $y=ax^2$. We have $x=36.0/2=18.0$ and $y=8.40$ from the information in the problem statement. Therefore,

$$8.40=a(18.0)^2$$

$$a=\frac{8.40}{(18.0)^2}=0.0259$$

and so the desired equation is

$$y=0.0259x^2$$

35. The quadratic equation $y=2x^2-4x-c$ will have two real roots if

$$b^2-4ac>0$$

$$(-4)^2-4(2)(-c)\ge 0$$

$$16+8c\ge 0$$

$$c\ge -2$$

-2 is the smallest integral value of c such that $y=2x^2-4x-c$ has two real roots.

36. The quadratic equation $y = 3x^2 - 12x + c$ will have no real roots if

$$\begin{aligned} b^2 - 4ac &< 0 \\ (-12)^2 - 4(3)(c) &< 0 \\ 144 - 12c &< 0 \\ 144 &< 12c \\ 12 &< c \end{aligned}$$

and so 13 is the smallest integer such that

$y = 3x^2 - 12x + c$ has no real roots.

37. Assume the equation is $y = ax^2 + bx + c$. Since $(0,-3)$ is on the parabola, we know $c = -3$.

Substituting $x = 2, y = 5$ yields $5 = 4a + 2b - 3$, or $4a + 2b = 8$ and so $b = 4 - 2a$.

Substituting $x = -2, y = -3$ yields $-3 = 4a - 2b - 3$, or $4a - 2b = 0$ and so $b = 2a$. Thus, $2a = 4 - 2a$, implying $a = 1$ and then $b = 2$.

The desired equation is $y = x^2 + 2x - 3$.

38. Assume the equation is $y = ax^2 + bx + c$. Since $(-1,14)$ is on the parabola, we know $a - b + c = 14$ and so $a + c = 14 + b$

Since $(1,9)$ is on the parabola, $a + b + c = 9$ and so $a + c = 9 - b$. Therefore,

$14 + b = 9 - b$, or $b = -\frac{5}{2}$. Also, $a + c = \frac{23}{2}$.

Since $(2,8)$ is on the parabola, $4a + 2b + c = 8$ and so $4a + c = 8 - 2b = 13$.

Therefore, $3a = (4a + c) - (a + c) = 13 - \frac{23}{2} = \frac{3}{2}$ and so $a = \frac{1}{2}$.

Finally, $c = \frac{23}{2} - \frac{1}{2} = 11$.

The desired equation is

$$y = \frac{1}{2}x^2 - \frac{5}{2}x + 11.$$

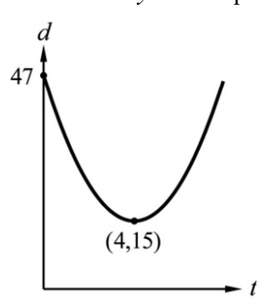
39.

$$d = 2t^2 - 16t + 47;\ a = 2,\ b = -16,\ c = 47$$

x-coordinate of vertex $= \frac{-b}{2a} = \frac{-(-16)}{2(2)} = 4$

y-coordinate of vertex $= 2(4)^2 - 16(4) + 47 = 15$

y-intercept $= (0,\ 47)$



40.

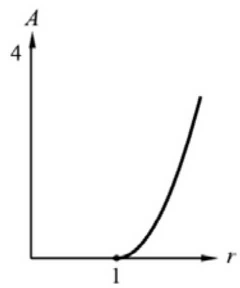
$$A = \pi r^2 - 2\pi r + \pi$$
$$a = \pi,\ b = -2\pi,\ c = \pi$$

r-coordinate of vertex $= \dfrac{-b}{2a} = \dfrac{-(-2\pi)}{2(\pi)} = 1$

A-coordinate of vertex $= \pi(1)^2 - 2\pi(1) + \pi = 0$

y-intercept $= (0, \pi)$

Note that the radius of the pipe cannot be less than 1 mm,so only the right half of the parabola is required.



41. If the Arch is modeled by $y = 192 - 0.0208x^2$, then the range must be $0 \le y \le 192$ (we disregard negative values of y.) Thus, the Arch must be 192 meters high.

We determine its width by finding the x-intercepts, solving

$0 = 192 - 0.0208x^2$

$x^2 = \dfrac{192}{0.0208}$

$x = \pm\sqrt{\dfrac{192}{0.0208}} = \pm 96.07689$

The distance between the two x-intercepts is 2(96.07689) = 192.154 and so the Arch is 192 meters wide.

42. $L = 0.0002q^2 + 0.005q$

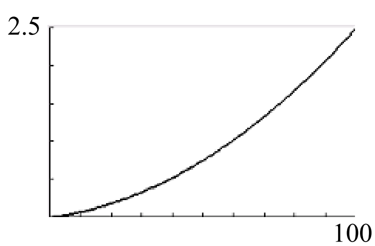
$L = 0.0002q[q + 25]$

q-intercepts occur at

$q = 0$ or $q = -25$

since

$a > 0$ so the curve opens upward. The minimum point is $(0, 0)$ assuming negative flow rate is prohibited.



43. $P = 50i - 3i^2$

$h_L = i[50 - 3i]$

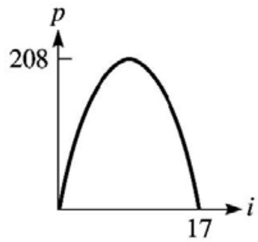
i-intercepts occur at

$i = 0$ or $i = \dfrac{50}{3}$

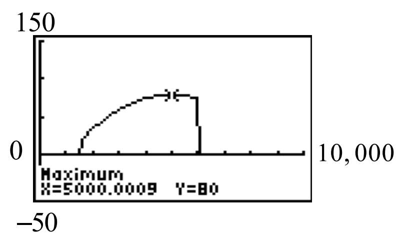
since

$a < 0$ so the curve opens downward. Vertex is halfway between the roots.

The maximum point is $\left(\dfrac{25}{3}, 208\right)$.

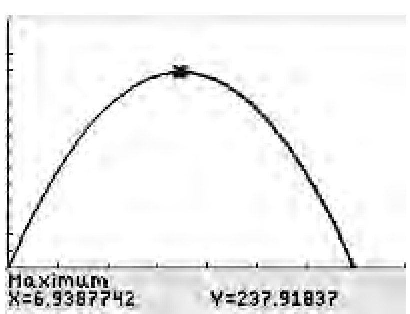


44. Graph $y_1 = -5.0 \times 10^{-6} x^2 + 0.05x - 45$,
$1500 < x < 6000$ and use the maximum feature.



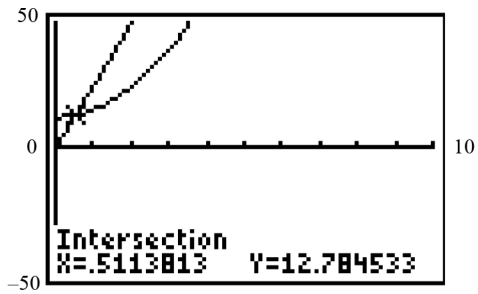
The maximum power is 80kw at 5000 rpm.

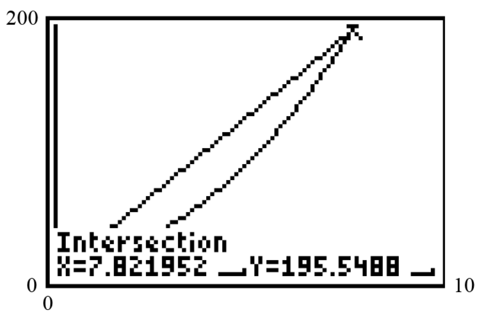
45. Graph $h = -4.9t^2 + 68t + 2$ and use the maximum feature,
finding the maximum occurs when $t = 6.9$ s.



46. Graph $y_1 = 25x$, $y_2 = 3\left(x^2 + 4\right)$.

Using the intersection feature, we find that the solutions are 0.5Ω and 7.8Ω.





47.

$$\begin{aligned}\pi(r+1)^2 &= 96.0\\ (r+1)^2 &= \frac{96.0}{\pi}\\ r+1 &= \pm\sqrt{\frac{96.0}{\pi}}\\ r &= \pm\sqrt{\frac{96.0}{\pi}}-1\\ r &= 4.53 \text{ cm}\end{aligned}$$

48.

$$(2x-3.00)^2 = x^2 + 15.0^2$$

$$4x^2 - 12.00x + 9.00 = x^2 + 225.0$$

$$3x^2 - 12.00x - 216.0 = 0$$

$$x^2 - 4.00x - 72.0 = 0$$

The quadratic formula gives

$$x = \frac{-(-4.00) \pm \sqrt{(-4.00)^2 - 4(1)(-72.0)}}{2(1)}$$

$$x = 10.72$$

$$\text{area} = 15.0(10.72) = 161 \text{ ft}^2$$

49. We let x represent the length of \$20/ft fence.

$$A = xy = 20000,$$

$$y = \frac{20000}{x}$$

$$\text{cost} = 20x + 30y = 7500$$

$$20x + 30\left(\frac{20000}{x}\right) = 7500$$

$$20x^2 - 7500x + 600\,000 = 0$$

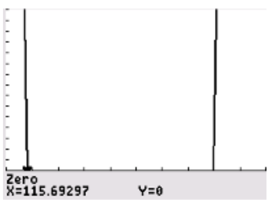
$$x^2 - 375x + 30000 = 0$$

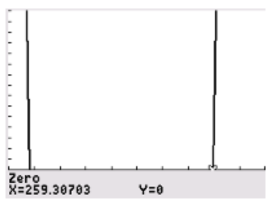
Using the zero finder, there are two solutions,

$x = 116$ ft. or $x = 259$ ft.

If $x = 116$ ft., then $y = 173$ ft.

If $x = 259$ ft., then $y = 77$ ft.





50. Let R = plane's speed,

$$vt = d$$

$$(R + 40)\left(\frac{630}{R} - \frac{1}{3}\right) = 630$$

$$630 + \frac{25200}{R} - \frac{1}{3}R - \frac{40}{3} = 630$$

$$-\frac{1}{3}R^2 - \frac{40}{3}R + 25200 = 0$$

$$R^2 + 40R - 75600 = 0$$

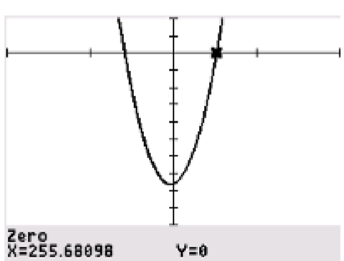
Graph $y_1 = x^2 + 40x - 75600$

and use the zero feature.

Find x for $y = 0$.

$R = 260$ mi/h

(The second solution is discarded since it is negative.)



Review Exercises

1. The solution $x = 2$ is not the only solution to $x^2 - 2x = 0$. The other is $x = 0$.

2. The first steps are to divide by the coefficient of x^2 and then to transfer the constant term to the right-hand side. It is then appropriate to halve the linear term, square the result and add this value to both sides of the equation.

3. This is the correct formula.

4. This is true.

5.
$$\begin{aligned} x^2 + 3x - 4 &= 0 \\ (x+4)(x-1) &= 0 \\ x + 4 = 0 \quad &\text{or} \quad x - 1 = 0 \\ x = -4 \quad &\quad\quad x = 1 \end{aligned}$$

6.
$$\begin{aligned} x^2 + 3x - 10 &= 0 \\ (x+5)(x-2) &= 0 \\ x + 5 = 0 \quad &\text{or} \quad x - 2 = 0 \\ x = -5 \quad &\quad\quad x = 2 \end{aligned}$$

7.
$$\begin{aligned} x^2 - 10x + 21 &= 0 \\ (x-3)(x-7) &= 0 \\ x - 3 = 0 \quad &\text{or} \quad x - 7 = 0 \\ x = 3 \quad &\quad\quad x = 7 \end{aligned}$$

8.
$$\begin{aligned} P^2 - 27 &= 6P \\ P^2 - 6P - 27 &= 0 \\ (P+3)(P-9) &= 0 \\ P + 3 = 0 \quad &\text{or} \quad P - 9 = 0 \\ P = -3 \quad &\quad\quad P = 9 \end{aligned}$$

9.
$$\begin{aligned} 3x^2 + 11x &= 4 \\ 3x^2 + 11x - 4 &= 0 \\ (3x-1)(x+4) &= 0 \\ 3x - 1 = 0 \quad &\text{or} \quad x + 4 = 0 \\ 3x = 1 \quad &\quad\quad x = -4 \\ x = \frac{1}{3} \quad & \end{aligned}$$

10.
$$11y = 6y^2 + 3$$
$$6y^2 - 11y + 3 = 0$$
$$(3y-1)(2y-3) = 0$$
$$3y - 1 = 0 \quad \text{or} \quad 2y - 3 = 0$$
$$y = \frac{1}{3} \qquad\qquad y = \frac{3}{2}$$

11.
$$6t^2 = 13t - 5$$
$$6t^2 - 13t + 5 = 0$$
$$(2t-1)(3t-5) = 0$$
$$2t - 1 = 0 \quad \text{or} \quad 3t - 5 = 0$$
$$t = \frac{1}{2} \qquad\qquad t = \frac{5}{3}$$

12.
$$3x^2 + 5x + 2 = 0$$
$$(x+1)(3x+2) = 0$$
$$x + 1 = 0 \quad \text{or} \quad 3x + 2 = 0$$
$$x = -1 \qquad\qquad x = -\frac{2}{3}$$

13.
$$4s^2 = 18s$$
$$4s^2 - 18s = 0$$
$$s(4s-18) = 0$$
$$4s - 18 = 0 \quad \text{or} \quad s = 0$$
$$4s = 18$$
$$s = \frac{9}{2}$$

14.
$$23n + 35 = 6n^2$$
$$6n^2 - 23n - 35 = 0$$
$$(6n+7)(n-5) = 0$$
$$6n + 7 = 0 \quad \text{or} \quad n - 5 = 0$$
$$n = -\frac{7}{6} \qquad\qquad n = 5$$

15.
$$4B^2 = 8B + 21$$
$$4B^2 - 8B - 21 = 0$$
$$(2B+3)(2B-7) = 0$$
$$2B + 3 = 0 \quad \text{or} \quad 2B - 7 = 0$$
$$B = -\frac{3}{2} \qquad\qquad B = \frac{7}{2}$$

16.
$$6\pi^2x^2 = 8 - 47\pi x$$
$$6\pi^2x^2 + 47\pi x - 8 = 0$$
$$(\pi x + 8)(6\pi x - 1) = 0$$
$$\pi x + 8 = 0 \quad \text{or} \quad 6\pi x - 1 = 0$$
$$x = -\frac{8}{\pi} \qquad\qquad x = \frac{1}{6\pi}$$

17. $x^2 - 4x - 96 = 0;\ a = 1;\ b = -4;\ c = -96$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-96)}}{2(1)}$$
$$x = \frac{4 \pm \sqrt{16 - (-384)}}{2}$$
$$x = \frac{4 \pm \sqrt{400}}{2}$$
$$x = \frac{4 \pm 20}{2}$$
$$x = -8 \quad \text{or} \quad x = 12$$

18. $x^2 + 3x - 18 = 0;\ a = 1;\ b = 3;\ c = -18$
$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-18)}}{2(1)}$$
$$x = \frac{-3 \pm \sqrt{9 - (-72)}}{2}$$
$$x = \frac{-3 \pm \sqrt{81}}{2}$$
$$x = \frac{-3 \pm 9}{2}$$
$$x = -6 \quad \text{or} \quad x = 3$$

19.
$$m^2 + 2m = 6$$
$$m^2 + 2m - 6 = 0;\ a = 1;\ b = 2;\ c = -6$$
$$m = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-6)}}{2(1)}$$
$$m = \frac{-2 \pm \sqrt{4 - (-24)}}{2}$$
$$m = \frac{-2 \pm \sqrt{28}}{2}$$
$$m = \frac{-2 \pm \sqrt{4 \times 7}}{2}$$
$$m = \frac{-2 \pm 2\sqrt{7}}{2}$$
$$m = -1 \pm \sqrt{7}$$

20. $1+7D=D^2$

$D^2-7D-1=0;\ a=1;\ b=-7;\ c=-1$

$$D=\frac{-(-7)\pm\sqrt{(-7)^2-4(1)(-1)}}{2(1)}$$

$$D=\frac{7\pm\sqrt{49-(-4)}}{2}$$

$$D=\frac{7\pm\sqrt{53}}{2}$$

21. $2x^2-x=36$

$2x^2-x-36=0;\ a=2;\ b=-1;\ c=-36$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(2)(-36)}}{2(2)}$$

$$x=\frac{1\pm\sqrt{1-(-288)}}{4}$$

$$x=\frac{1\pm\sqrt{289}}{4}$$

$$x=\frac{1\pm 17}{4}$$

$$x=\frac{9}{2}\quad\text{or}\quad x=-4$$

22. $6x^2=28-2x$

$6x^2+2x-28=0;\ a=6,\ b=2,\ c=-28$

$$x=\frac{-2\pm\sqrt{(2)^2-4(6)(-28)}}{2(6)}$$

$$x=\frac{-2\pm\sqrt{4-(-672)}}{12}$$

$$x=\frac{-2\pm\sqrt{676}}{12}$$

$$x=\frac{-2\pm 26}{6}$$

$$x=-\frac{7}{3}\quad\text{or}\quad x=2$$

23. $18s+12=24s^2$ (divide by common factor of 6)

$3s+2=4s^2$

$4s^2-3s-2=0;\ a=4;\ b=-3;\ c=-2$

$$s=\frac{-(-3)\pm\sqrt{(-3)^2-4(4)(-2)}}{2(4)}$$

$$s=\frac{3\pm\sqrt{9-(-32)}}{8}$$

$$s=\frac{3\pm\sqrt{41}}{8}$$

24. $2-7x=5x^2$

$5x^2+7x-2=0;\ a=5;\ b=7;\ c=-2$

$$x=\frac{-7\pm\sqrt{(7)^2-4(5)(-2)}}{2(5)}$$

$$x=\frac{-7\pm\sqrt{49-(-40)}}{10}$$

$$x=\frac{-7\pm\sqrt{89}}{10}$$

25. $2.1x^2+2.3x+5.5=0;\ a=2.1;\ b=2.3;\ c=5.5$

$$x=\frac{-2.3\pm\sqrt{(2.3)^2-4(2.1)(5.5)}}{2(2.1)}$$

$$x=\frac{-2.3\pm\sqrt{5.29-46.2}}{4.2}$$

$$x=\frac{-2.3\pm\sqrt{-40.91}}{4.2}\ \text{(imaginary roots)}$$

26. $0.30R^2-0.42R=0.15$

$0.30R^2-0.42R-0.15=0;\ a=0.30;\ b=-0.42;\ c=-0.15$

$$R=\frac{-(0.42)\pm\sqrt{(0.42)^2-4(0.30)(-0.15)}}{2(0.30)}$$

$$R=\frac{-0.42\pm\sqrt{0.1764-(-0.18)}}{0.60}$$

$$R=\frac{-0.42\pm\sqrt{0.3564}}{0.60}$$

$R=-0.29$ or $R=-1.7$

27.
$$4x = 9 - 6x^2$$
$$6x^2 + 4x - 9 = 0;\ a = 6;\ b = 4;\ c = -9$$
$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(6)(-9)}}{2(6)}$$
$$x = \frac{-4 \pm \sqrt{16 - (-216)}}{12}$$
$$x = \frac{-4 \pm \sqrt{232}}{12}$$
$$x = \frac{-4 \pm \sqrt{4 \times 58}}{12}$$
$$x = \frac{-4 \pm 2\sqrt{58}}{12}$$
$$x = \frac{-2 \pm \sqrt{58}}{6}$$

28.
$$25t = 24t^2 - 20$$
$$24t^2 - 25t - 20 = 0;\ a = 24;\ b = -25;\ c = -20$$
$$t = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(24)(-20)}}{2(24)}$$
$$t = \frac{25 \pm \sqrt{625 - (-1920)}}{48}$$
$$t = \frac{25 \pm \sqrt{2545}}{48}$$

29.
$$4x^2 - 5 = 15$$
$$4x^2 = 20$$
$$x^2 = 5$$
$$x = \pm\sqrt{5}$$

30.
$$12y^2 = 20y$$
$$12y^2 - 20y = 0$$
$$4y(3y - 5) = 0$$
$$4y = 0 \quad \text{or} \quad 3y - 5 = 0$$
$$y = 0 \qquad y = \frac{5}{3}$$

31. $x^2 + 4x - 4 = 0;\ a = 1;\ b = 4;\ c = -4$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-4)}}{2(1)}$$
$$x = \frac{-4 \pm \sqrt{16 - (-16)}}{2}$$
$$x = \frac{-4 \pm \sqrt{32}}{2}$$
$$x = \frac{-4 \pm \sqrt{16 \times 2}}{2}$$
$$x = \frac{-4 \pm 4\sqrt{2}}{2}$$
$$x = -2 \pm 2\sqrt{2}$$

32. $x^2 + 3x + 1 = 0;\ a = 1;\ b = 3;\ c = 1$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)}$$
$$x = \frac{-3 \pm \sqrt{9 - 4}}{2}$$
$$x = \frac{-3 \pm \sqrt{5}}{2}$$

33. $3x^2 + 8x + 2 = 0;\ a = 3;\ b = 8;\ c = 2$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)}$$
$$x = \frac{-8 \pm \sqrt{64 - 24}}{6}$$
$$x = \frac{-8 \pm \sqrt{40}}{6}$$
$$x = \frac{-8 \pm \sqrt{4 \times 10}}{6}$$
$$x = \frac{-8 \pm 2\sqrt{10}}{6}$$
$$x = \frac{-4 \pm \sqrt{10}}{3}$$

34. $3p^2 = 28 - 5p$

$$3p^2 + 5p - 28 = 0$$
$$(p + 4)(3p - 7) = 0$$
$$p + 4 = 0 \quad \text{or} \quad 3p - 7 = 0$$
$$p = -4 \qquad\qquad p = \frac{7}{3}$$

35.

$$\begin{aligned} 4v^2 + v &= 3 \\ 4v^2 + v - 3 &= 0 \\ (4v-3)(v+1) &= 0 \\ 4v - 3 &= 0 \quad \text{or} \quad v + 1 = 0 \\ 4v &= 3 \qquad\qquad v = -1 \\ v &= \frac{3}{4} \end{aligned}$$

36.

$$3n - 6 = 18n^2 \quad \text{(divide by common factor 3)}$$

$$n - 2 = 6n^2$$

$$6n^2 - n + 2 = 0;\ a = 6;\ b = -1;\ c = 2$$

$$n = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(2)}}{2(6)}$$

$$n = \frac{1 \pm \sqrt{1 - 48}}{12}$$

$$n = \frac{1 \pm \sqrt{-47}}{12} \text{ (imaginary roots)}$$

37.

$$7 + 3C = -2C^2$$

$$2C^2 + 3C + 7 = 0;\ a = 2;\ b = 3;\ c = 7$$

$$C = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(7)}}{2(2)}$$

$$C = \frac{-3 \pm \sqrt{9 - 56}}{4}$$

$$C = \frac{-3 \pm \sqrt{-47}}{4} \text{ (imaginary roots)}$$

38.

$$5y = 4y^2 - 8$$

$$4y^2 - 5y - 8 = 0;\ a = 4;\ b = -5;\ c = -8$$

$$y = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-8)}}{2(4)}$$

$$y = \frac{5 \pm \sqrt{25 - (-128)}}{8}$$

$$y = \frac{5 \pm \sqrt{153}}{8}$$

39. $a^2x^2 + 2ax + 2 = 0;\ a = a^2;\ b = 2a;\ c = 2$

$$x = \frac{-(2a) \pm \sqrt{(2a)^2 - 4\left(a^2\right)(2)}}{2\left(a^2\right)}$$

$$x = \frac{-2a \pm \sqrt{4a^2 - 8a^2}}{2a^2}$$

$$x = \frac{-2a \pm \sqrt{-4a^2}}{2a^2}$$

$$x = \frac{-2a \pm 2a\sqrt{-1}}{2a^2}$$

$$x = \frac{-1 \pm \sqrt{-1}}{a}\ (a \neq 0, \text{imaginary roots})$$

40.

$$16r^2 = 8r - 1$$

$$16r^2 - 8r + 1 = 0$$

$$(4r - 1)(4r - 1) = 0$$

$$r = \frac{1}{4}\ \text{(double root)}$$

41.

$$ay^2 = a - 3y$$

$ay^2 + 3y - a = 0;\ a = a;\ b = 3;\ c = -a$

$$y = \frac{-3 \pm \sqrt{(3)^2 - 4(a)(-a)}}{2a}$$

$$y = \frac{-3 \pm \sqrt{9 + 4a^2}}{2a}$$

42.

$$2bx = x^2 - 3b$$

$x^2 - 2bx - 3b = 0;\ a = 1;\ b = -2b;\ c = -3b$

$$x = \frac{-(-2b) \pm \sqrt{(-2b)^2 - 4(1)(-3b)}}{2(1)}$$

$$x = \frac{2b \pm \sqrt{4b^2 - (-12b)}}{2}$$

$$x = \frac{2b \pm \sqrt{4(b^2 + 3b)}}{2}$$

$$x = \frac{2b \pm 2\sqrt{b^2 + 3b}}{2}$$

$$x = b \pm \sqrt{b^2 + 3b}$$

43.
$$\begin{aligned}
x^2 - x - 30 &= 0\\
x^2 - x &= 30\\
x^2 - x + \frac{1}{4} &= 30 + \frac{1}{4}\\
\left(x - \frac{1}{2}\right)^2 &= \frac{121}{4}\\
x - \frac{1}{2} &= \pm\sqrt{\frac{121}{4}}\\
x - \frac{1}{2} &= \pm\frac{11}{2}\\
x &= \frac{1}{2} \pm \frac{11}{2}\\
x &= -5 \text{ or } x = 6
\end{aligned}$$

44.
$$\begin{aligned}
x^2 &= 2x + 5\\
x^2 - 2x &= 5\\
x^2 - 2x + 1 &= 5 + 1\\
(x-1)^2 &= 6\\
x - 1 &= \pm\sqrt{6}\\
x &= 1 \pm \sqrt{6}
\end{aligned}$$

45.
$$\begin{aligned}
2t^2 &= t + 4\\
2t^2 - t &= 4\\
t^2 - \frac{1}{2}t &= 2\\
t^2 - \frac{1}{2}t + \frac{1}{16} &= 2 + \frac{1}{16}\\
\left(t - \frac{1}{4}\right)^2 &= \frac{33}{16}\\
t - \frac{1}{4} &= \pm\sqrt{\frac{33}{16}}\\
t - \frac{1}{4} &= \pm\frac{\sqrt{33}}{4}\\
t &= \frac{1 \pm \sqrt{33}}{4}
\end{aligned}$$

46.

$$
\begin{aligned}
4x^2 - 8x &= 3 \\
x^2 - 2x &= \frac{3}{4} \\
x^2 - 2x + 1 &= \frac{3}{4} + 1 \\
(x-2)^2 &= \frac{7}{4} \\
x - 1 &= \pm\sqrt{\frac{7}{4}} \\
x &= 1 \pm \frac{\sqrt{7}}{2} \\
x &= \frac{2 \pm \sqrt{7}}{2}
\end{aligned}
$$

47.

$$
\begin{aligned}
\frac{x-4}{x-1} &= \frac{2}{x},\ (x \neq 1, 0) \\
x(x-4) &= 2(x-1) \\
x^2 - 4x &= 2x - 2 \\
x^2 - 6x + 2 &= 0;\ a = 1;\ b = -6;\ c = 2 \\
x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)} \\
x &= \frac{6 \pm \sqrt{36-8}}{2} \\
x &= \frac{6 \pm \sqrt{28}}{2} \\
x &= \frac{6 \pm \sqrt{4 \times 7}}{2} \\
x &= \frac{6 \pm 2\sqrt{7}}{2} \\
x &= 3 \pm \sqrt{7}
\end{aligned}
$$

48.

$$
\begin{aligned}
\frac{V-1}{3} &= \frac{5}{V} + 1,\ (V \neq 0) \\
\frac{V-1}{3} &= \frac{5+V}{V} \\
V(V-1) &= 3(5+V) \\
V^2 - V &= 15 + 3V \\
V^2 - 4V - 15 &= 0;\ a = 1;\ b = -4;\ c = -15
\end{aligned}
$$

$$V=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-15)}}{2(1)}$$
$$V=\frac{4\pm\sqrt{16-(-60)}}{2}$$
$$V=\frac{4\pm\sqrt{76}}{2}$$
$$V=\frac{4\pm\sqrt{4\times 19}}{2}$$
$$V=\frac{4\pm 2\sqrt{19}}{2}$$
$$V=2\pm\sqrt{19}$$

49. $\dfrac{x^2-3x}{x-3}=\dfrac{x^2}{x+2},\ (x\neq -2,\ 3)$

$$\frac{x\cancel{(x-3)}}{\cancel{(x-3)}}=\frac{x^2}{x+2}$$
$$x(x+2)=x^2$$
$$x^2+2x=x^2$$
$$2x=0$$
$$x=0$$

50. $\dfrac{x-2}{x-5}=\dfrac{15}{x^2-5x}$

$$\frac{x-2}{x-5}=\frac{15}{x(x-5)},\ (x\neq 0,\ 5)$$
$$x(x-2)(x-5)=15(x-5)$$
$$x(x-2)=15$$
$$x^2-2x-15=0$$
$$(x+3)(x-5)=0$$
$x+3=0$ or $x-5=0$

$x=-3$ $\quad$ $x=5$, (not a solution)

51. $y=2x^2-x-1;\ a=2,\ b=-1,\ c=-1$

$c=-1$

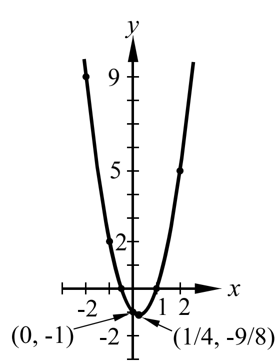
y-intercept $=-1$

$2x^2-x-1=0$

$(x-1)(2x+1)=0$

$x=1$ and $x=-\dfrac{1}{2}$ are the x-intercepts

$$x\text{ vertex}=\frac{-b}{2a}=\frac{-(-1)}{2(2)}=\frac{1}{4}$$
$$y\text{ vertex}=2\ \frac{1}{4}^{\ 2}-\ \frac{1}{4}\ -1=-\frac{9}{8}$$



52. $y=-4x^2-1;$ so $a=-4,\ b=0,\ c=-1$

$y\text{ - int}=(0,\ -1)$

$-4x^2-1=0$

$-4\quad x^2+\frac{1}{4}\quad =0$

$x^2=-\frac{1}{4}$ (no real solutions, so no x-intercepts)

$x\text{ vertex}=\frac{-b}{2a}=\frac{-0}{2(-4)}=0$

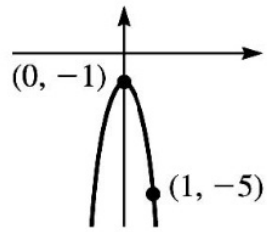
$y\text{ vertex}=-4(0)^2-1=-1$

Another Point

$x=1$

$y=-4(1)-1=-5$

$(1,-5)$



53. $y=x-3x^2;a=-3,\ b=1,\ c=0$

$y\text{- int}=(0,\ 0)$

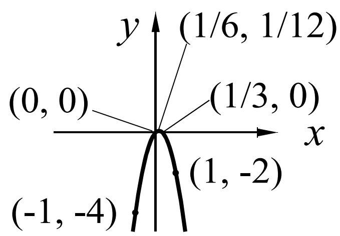
$-3x^2+x=0$

$x(-3x+1)=0$

$x=0$ or $x\ =-\frac{1}{3}$ are the x-intercepts

$x\text{ vertex}=\frac{-b}{2a}=\frac{-1}{2(-3)}=\frac{1}{6}$

$y\text{ vertex}=\frac{1}{6}-3\quad\frac{1}{6}^{\ 2}\ =\frac{1}{12}$



54. $y=2x^2+8x-10;\ a=2,b=8,c=-10$

$y\text{ - int}=(0,\ 10)$

$2x^2+8x-10=0$

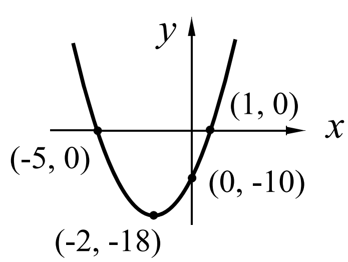
$2(x^2+4x-5)=0$

$2(x+5)(x-1)=0$

$x=-5$ or $x=1$ are the x-intercepts

$x\text{ vertex}=\frac{-b}{2a}=\frac{-8}{2(2)}=-2$

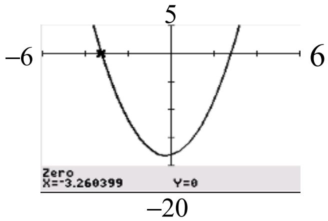
$y\text{ vertex}=2(-2)^2+8(-2)-10=-18$

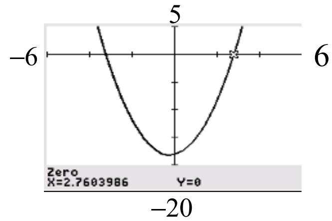


55. Graph $y_1 = 2x^2 + x - 18$ and use the zero feature.

$x = -3.26$ and

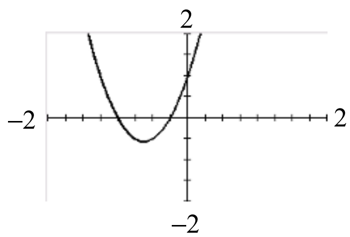
$x = 2.76$





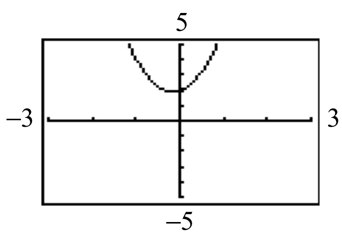
56. Graph $y_1 = 4x^2 + 5x + 1$ and use the zero feature to solve.

By inspection, the roots are $x = -1, x = -\dfrac{1}{4}$.



57. Graph $y_1 = 3x^2 + x + 2$ and use the zero feature to solve.

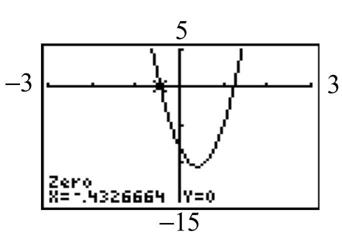
No real roots.

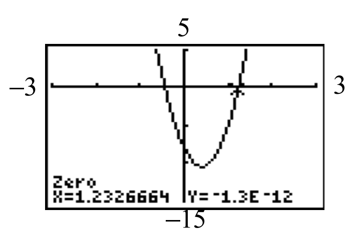


58. Graph $y_1 = x(15x - 12) - 8$ and use the zero feature to solve.

$x = -0.433$ or

$x = 1.23$





59. The roots are equally spaced on either side of

$$x = -1$$

$$\frac{x_1 + x_2}{2} = -1$$

$$x_1 + x_2 = -2$$

$$2 + x_2 = -2$$

$x_2 = -4$ is the other solution.

60.

$$y = 2x^2 + 16x + c$$

$b^2 - 4ac = 0$ for double root.

From above, $a = 2,\ b = 16$

$$16^2 - 4(2)c = 0$$

$$8c = 256$$

$$c = 32$$

61. $M = 0.5wLx - 0.5wx^2$

$M = 0.5wx(L - x)$

$M = 0$ for $x = 0$ and $x = L$

62. $I^2 - 17I - 12 = 0;\quad a = 1, b = -17, c = -12$

$$I = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$I = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(1)(-12)}}{2(1)}$$

$$I = \frac{17 \pm \sqrt{337}}{2}$$

$I = 17.7$ A, -0.679 A

Since $I > 0$, $I = 17.7$ A is the current.

63. $12x^2 - 80x + 96 = 0$

$3x^2 - 20x + 24 = 0, a = 3, b = -20, c = 24$

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(3)(24)}}{2(3)}$$

$$x = \frac{20 \pm \sqrt{400 - 288}}{6}$$

$$x = \frac{20 \pm \sqrt{112}}{6}$$

$x = 1.57$ or $x = 5.10$ (discarded since $x < 4$)

64. $h = 1.5 + 7.2x - 1.2x^2$

The maximum occurs at the vertex since $a = -1.2 < 0$ and so the parabola opens down.

The vertex is at $x = -\dfrac{b}{2a} = -\dfrac{7.2}{2(-1.2)} = 3$

and so the maximum height is $h = 1.5 + 7.2(3) - 1.2(3)^2 = 12.3$ m.

65. $0.1x^2 + 0.8x + 7 = 50$

$0.1x^2 + 0.8x - 43 = 0;\ a = 0.1, b = 0.8, c = -43$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0.8) \pm \sqrt{(0.8)^2 - 4(0.1)(-43)}}{2(0.1)}$$

$$x = \frac{-0.8 \pm \sqrt{17.84}}{0.2}$$

$x = 17.1$ units or -25.1 units

17 units can be made for \$50

66. $R^2 - 2.67R + 1.00 = 0.500$

$R^2 - 2.67R + 0.500 = 0;\ \ a = 1, b = -2.67, c = 0.500$

$$R = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$R = \frac{-(2.67) \pm \sqrt{(2.67)^2 - 4(1)(0.5)}}{2(1)}$$

$$R = \frac{-2.67 \pm \sqrt{5.1289}}{2}$$

$R = 0.203,\ 2.47$

but if it supposed to be a sudden enlargement

$R > 1$, so $R = 2.47$ is the solution.

67. $T^2 + 520T - 5300 = 0$

$a = 1, b = 520, c = -5300$

$$T = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T = \frac{-(520) \pm \sqrt{(520)^2 - 4(1)(-5300)}}{2(1)}$$

$$T = \frac{-520 \pm \sqrt{291600}}{2}$$

$T = 10$ or $T = -530$ (discarded from physical considerations)

Therefore, the boiling point will be approximately.

$212.0 - 10.0 = 202^\circ$ F

68.

$$v = -x^2 + 5.20x$$
$$4.80 = -x^2 + 5.20x$$
$$x^2 - 5.20x + 4.80 = 0$$
$$(x - 1.20)(x - 4.00) = 0$$
$$x = 1.20 \text{ cm} \ \text{ or } \ x = 4.00 \text{ cm}$$

This velocity is reached at two positions in the pipe.

69.

$$h = vt \sin\theta - 16t^2$$
$$18.00 = 44.0t \sin 65.0^\circ - 16t^2$$

$16t^2 - 39.87754t + 18.00 = 0$ from which

$$a = 16, b = -39.87754, c = 18.00$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-(-39.87754) \pm \sqrt{(-39.87754)^2 - 4(16)(18.00)}}{2(16)}$$
$$t = \frac{39.87754 \pm \sqrt{438.2182}}{32}$$
$$t = 0.59 \quad \text{or} \quad t = 1.90$$

The height $h = 18.0$ ft is reached when $t = 0.59$ s and 1.90 s.

It reaches that height twice (once on the way up, and once on the way down).

70. $\sin^2 A - 4 \sin A + 1 = 0$

$$a = 1, b = -4, c = 1$$
$$\sin A = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\sin A = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$
$$\sin A = \frac{4 \pm \sqrt{12}}{2}$$
$$\sin A = \begin{array}{l} 0.2679491924 \\ 3.7332050808, \text{ reject (out of domain)} \end{array}$$
$$A = \sin^{-1}(0.2679491924)$$
$$A = 15.5^\circ$$

71.

$$\frac{n^2}{500\ 000} = 144 - \frac{n}{500} \quad \text{(multiply equation by 500 000)}$$
$$n^2 + 1000n - 72\ 000\ 000 = 0$$
$$(n + 9000)(n - 8000) = 0$$
$$n = 8000 \quad \text{or} \quad n = -9000 \ \text{ reject since } n > 0$$

The company should produce 8000 components.

72. $$\frac{20}{R}+\frac{20}{R+10}=\frac{1}{5} \quad \text{(multiply by LCD)}$$

$$\frac{20\cdot 5R(R+10)}{R}+\frac{20\cdot 5R(R+10)}{R+10}=\frac{1\cdot 5R(R+10)}{5}$$

$$100R+1000+100R=R^2+10R$$

$$R^2-190R-1000=0$$

$$a=1, b=-190, c=-1000$$

$$R=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$R=\frac{-(-190)\pm\sqrt{(-190)^2-4(1)(-1000)}}{2(1)}$$

$$R=\frac{190\pm\sqrt{40100}}{2}$$

$$R=195.12 \quad \text{or} \quad R=-5.12 \text{ (reject since } R>0)$$

$$R=195\ \Omega$$

73. $$A=2\pi r^2+2\pi rh$$

$$2\pi r^2+2\pi hr-A=0$$

$$a=2\pi, b=2\pi h, c=-A$$

$$r=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$r=\frac{-2\pi h\pm\sqrt{(2\pi h)^2-4(2\pi)(-A)}}{2(2\pi)}$$

$$r=\frac{-2\pi h+\sqrt{4\pi^2h^2+8\pi A}}{4\pi}$$

$$r=\frac{-2\pi h+\sqrt{4\left(\pi^2h^2+2\pi A\right)}}{4\pi}$$

$$r=\frac{-2\pi h+2\sqrt{\pi^2h^2+2\pi A}}{4\pi}$$

$$r=\frac{-\pi h+\sqrt{\pi^2h^2+2\pi A}}{2\pi}$$

74. $$b=kr(R-r)$$

$$b=krR-kr^2$$

$$kr^2-kRr+b=0$$

$$a=k, b=-kR, c=b$$

$$r=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$r=\frac{-(-kR)\pm\sqrt{(-kR)^2-4(k)(b)}}{2k}$$

$$r=\frac{kR\pm\sqrt{k^2R^2-4kb}}{2k}$$

75.

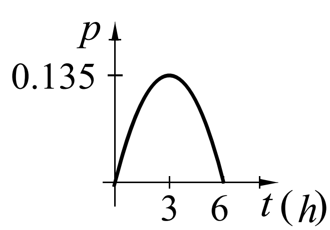
$$p_2 = p_1 + rp_1(1 - p_1)$$
$$p_2 = p_1 + rp_1 - rp_1^{\,2}$$
$$p_2 = p_1(1 + r) - rp_1^{\,2}$$
$$rp_1^2 - (r+1)p_1 + p_2 = 0$$
$$a = r,\ b = -(r+1), c = p_2$$
$$p_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$p_1 = \frac{-\left[-(r+1)\right] \pm \sqrt{\left[-(r+1)\right]^2 - 4rp_2}}{2r}$$
$$p_1 = \frac{r + 1 \pm \sqrt{(r+1)^2 - 4rp_2}}{2r}$$

76.

$$v^2 = k^2\left(\frac{L}{C} + \frac{C}{L}\right)$$
$$v^2 = k^2\left(\frac{L^2 + C^2}{LC}\right)$$
$$v^2CL = k^2L^2 + k^2C^2$$
$$k^2L^2 - v^2CL + k^2C^2 = 0$$
$$a = k^2,\ b = -v^2C, c = k^2C^2$$
$$L = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$L = \frac{-(-v^2C) \pm \sqrt{(-v^2C)^2 - 4k^2(k^2C^2)}}{2k^2}$$
$$L = \frac{v^2C \pm \sqrt{v^4C^2 - 4k^4C^2}}{2k^2}$$
$$L = \frac{v^2C \pm \sqrt{C^2\left(v^4 - 4k^4\right)}}{2k^2}$$
$$L = \frac{v^2C \pm C\sqrt{v^4 - 4k^4}}{2k^2}$$

77.

$$p = 0.090t - 0.015t^2$$
$$p = -0.015t^2 + 0.090t$$
$$a = -0.015,\ b = 0.090,\ c = 0$$
$$y\text{-int} = (0,\ 0)$$
$$-0.015t^2 + 0.090t = 0$$
$$0.015t(-t + 6) = 0$$
$t = 0$ or $t = 6$ are the x-intercepts
$$t \text{ vertex} = \frac{-b}{2a} = \frac{-0.090}{2(-0.015)} = 3$$
$$p \text{ vertex} = 0.090(3) - 0.015(3)^2 = 0.135$$



78. Let x represent the edge of one cube. Then $8-x$ represents the edge of the second cube. We then have

$$x^3+(8-x)^3=152$$
$$x^3+512-192x+24x^2-x^3=152$$
$$24x^2-192x+360=0$$

Divide by 24:

$$x^2-8x+15=0$$
$$(x-3)(x-5)=0$$

The cubes have edges of length 3 ft and 5 ft.

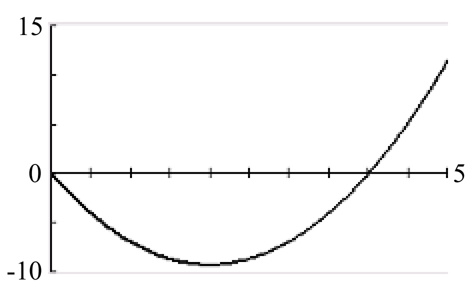
79. Original volume

$$V=x^3$$

New volume

$$V-29=(x-0.1)^3$$
$$V-29=x^3+3x^2(-0.1)+3x(-0.1)^2+(-0.1)^3$$
$$x^3-29=x^3-0.3x^2+0.03x-0.001$$
$$0.3x^2-0.03x-28.999=0$$
$$a=0.3,\ b=-0.03,\ c=-28.999$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-0.03)\pm\sqrt{(-0.03)^2-4(0.3)(-28.999)}}{2(0.3)}$$
$$x=\frac{0.03\pm\sqrt{34.7997}}{0.6}$$
$$x=9.88 \quad \text{or} \quad x=-9.78 \text{ (reject since } x>0)$$
$$x=9.88 \text{ cm}$$

80. The graph of $f(t)=2.3t^2-9.2t$, or $f(t)=2.3t(t-4)$ has t-intercepts at $t=0$ and $t=4$ and passes through the origin. The vertex is at $t=2$ and $p=f(2)=-9.2$. The parabola opens upward.



81.

$$
\begin{aligned}
\text{Original area } A &= l \cdot w \\
A &= (100 - x) \cdot (80 - x) \\
\text{New area } A_{new} &= (100) \cdot (80) = 8000 \\
\text{Difference in area } A_{new} - A &= 3000 \\
8000 - (100 - x) \cdot (80 - x) &= 3000 \\
8000 - 8000 + 100x + 80x - x^2 &= 3000 \\
x^2 - 180x + 3000 &= 0
\end{aligned}
$$

$$
\begin{aligned}
&a = 1,\ b = -180,\ c = 3000 \\
&x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&x = \frac{-(-180) \pm \sqrt{(-180)^2 - 4(1)(3000)}}{2(1)} \\
&x = \frac{180 \pm \sqrt{20400}}{2} \\
&x = 18.59 \quad \text{or} \quad x = 161.41 \text{ (reject since } x < 80) \\
&x = 18.6 \text{ m}
\end{aligned}
$$

The original dimensions are

$l = 100 - 18.6 = 81.4$ m and

$w = 80 - 18.6 = 61.4$ m

82.

$$
\begin{aligned}
d &= v \cdot t \\
1200 &= (v + 50)t_1 \\
t_1 &= \frac{1200}{v + 50} \\
570 &= (v + 20)t_2 \\
t_2 &= \frac{570}{v + 20} \\
t_1 + t_2 &= 3 \\
\frac{1200}{v + 50} + \frac{570}{v + 20} &= 3 \quad \text{Multiply by LCD} \\
1200(v + 20) + 570(v + 50) &= 3(v + 50)(v + 20) \\
1200v + 24000 + 570v + 28500 &= 3(v^2 + 70v + 1000) \\
1770v + 52500 &= 3v^2 + 210v + 3000 \\
3v^2 - 1560v - 49500 &= 0 \quad \text{(divide by 3)} \\
v^2 - 520v - 16500 &= 0
\end{aligned}
$$

$$
\begin{aligned}
&a = 1,\ b = -520,\ c = -16500 \\
&v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&v = \frac{-(-520) \pm \sqrt{(-520)^2 - 4(1)(-16500)}}{2(1)} \\
&v = \frac{520 \pm \sqrt{336400}}{2} \\
&v = 550 \quad \text{or} \quad v = -30 \text{ (reject since } v > 0) \\
&v = 550 \text{ mi/h}
\end{aligned}
$$

83. The original volume was $4(16)(12) = 768$.

The new volume is $4(16-x)(12-x) = 691.2$.

We solve this for x:

$768 - 112x + 4x^2 = 691.2$

$4x^2 - 112x + 76.8 = 0; a = 4, b = -112, c = 76.8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

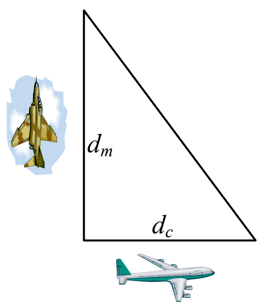
$$x = \frac{-(-112) \pm \sqrt{(-112)^2 - 4(4)(76.8)}}{2(4)}$$

$$x = \frac{112 \pm \sqrt{11315.2}}{8}$$

$x = 0.7034$ or $x = 27.297$ (discarded since $x < 12$ is required.)

The new dimensions are 4.00 cm thick, 15.3 cm long, and 11.3 cm wide.

84.



$d_c = vt$

$d_m = (4v + 200)t$

After 1.00 h they are 2050 mi apart.

$d_c{}^2 + d_m{}^2 = 2050^2$

$[v(1)]^2 + [(4v + 200)(1)]^2 = 2050^2$

$v^2 + 16v^2 + 1600v + 40000 = 4202500$

$17v^2 + 1600v - 4162500 = 0$

$a = 17, b = 1600, c = -4162500$

$$v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v = \frac{-1600 \pm \sqrt{1600^2 - 4(17)(-4162500)}}{2(17)}$$

$$v = \frac{-1600 \pm 16900}{34}$$

$v = 450$ or $v = -544$ (reject since $v > 0$)

$v = 450$ mi/hr, commercial jet

$4v + 200 = 2000$ mi/hr, military jet

85.
$$(h+22.9)^2+h^2=60.0^2$$
$$h^2+2(22.9)h+22.9^2+h^2=60.0^2$$
$$2h^2+45.8h-3075.59=0$$
$$a=2,\ b=45.8,\ c=-3075.59$$
$$h=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$h=\frac{-(45.8)\pm\sqrt{(45.8)^2-4(2)(-3075.59)}}{2(2)}$$
$$h=\frac{-45.8\pm\sqrt{26702.36}}{4}$$
$$h=29.4 \quad \text{or} \quad h=-52{,}3 \text{ (reject since } h>0\text{)}$$
$$h=29.4 \text{ in}$$
$$h+22.9=52.3 \text{ in}$$
The dimensions of the screen are 29.4 in $\times$ 52.3 in.

86.
$$x=2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cdots}}}$$
$x=2+\frac{1}{x}$ since the denominator of the fraction is the same endless ratio as x.
$$x^2=2x+1$$
$$x^2-2x-1=0$$
$$a=1,\ b=-2,\ c=-1$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$$
$$x=\frac{2\pm\sqrt{8}}{2}$$
$$x=\frac{2\pm2\sqrt{2}}{2}$$
$$x=1\pm\sqrt{2}$$
$x=1-\sqrt{2}$ is rejected since $x>0$

Solution is $x=1+\sqrt{2}$

87. Suppose n poles are placed along the road for a distance of 1 km and x is the distance, in km, between the poles, then $n\times x=1$. Increasing the distance between the poles to $x+0.01$ and decreasing the number of poles to $n-5$ gives $(n-5)(x+0.01)=1$. Substitution gives

$$(n-5)\left(\frac{1}{n}+0.01\right)=1$$
$$1+0.01n-\frac{5}{n}-0.05=1$$
$$0.01n^2-0.05n-5=0$$
$$n^2-5n-500=0$$
$$(n+20)(n-25)=0$$
$$n+20=0 \quad \text{or} \quad n-25=0$$
$$n=-20 \qquad\qquad n=25$$

There are 25 poles being placed each kilometre.

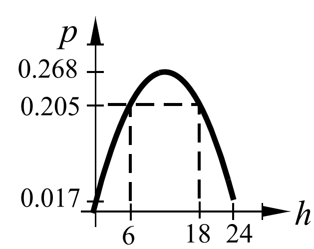
88. $$A_{\text{semicircle}}+A_{\text{rectangle}}=A$$
$$\frac{1}{2}(\pi r^2)+4.00(2r)=16.0$$
$$\frac{1}{2}\pi r^2+8.00r-16.0=0$$
$$a=\frac{\pi}{2},\ b=8.00,\ c=-16.0$$
$$r=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$r=\frac{-(8.00)\pm\sqrt{(8.00)^2-4\ \frac{\pi}{2}\ (-16.0)}}{2\ \frac{\pi}{2}}$$
$$r=\frac{-8.00\pm\sqrt{64.00+32\pi}}{\pi}$$

$r=1.54$ ft or $r=-6.63$ ft is rejected since $r>0$

So $r=1.54$ ft

89. $$p=0.001\,74(10+24h-h^2)$$
$$p=-0.00174h^2+0.04176h+0.0174$$

If $p=0.205$ ppm
$$0.205=-0.00174h^2+0.04176h+0.0174$$
$$0=-0.00174h^2+0.04176h-0.1876$$
$$a=-0.00174,\ b=0.04176,\ c=-0.1876$$
$$h=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$h=\frac{-0.04176\pm\sqrt{0.04176^2-4(-0.00174)(-0.1876)}}{2(-0.00174)}$$
$$h=5.98 \text{ and } h=18.0$$



From the graph $p=0.205$ at 6 h and 18 h.

90.

$$r = \text{radius of the hole}$$
$$\pi r^2 = 0.0136\left(\pi(r+53.0)^2\right)$$
$$\pi r^2 = 0.0136\pi\left(r^2+106r+2809\right)$$
$$r^2 = 0.0136r^2+1.4416r+38.2024$$
$$0.9864r^2-1.4416r-38.2024=0$$
$$a = 0.9864,\ b=-1.4416\ c=-38.2024$$
$$r=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$r=\frac{-(-1.4416)\pm\sqrt{(-1.4416)^2-4(0.9864)(-38.2024)}}{2(0.9864)}$$
$$r = 7.00 \text{ mm} \quad \text{or} \quad r=-5.54 \text{ mm is rejected since } r>0$$

So $r = 7.00$ mm

91. $\frac{1}{2}=\frac{1}{R}+\frac{1}{R+1}$

Multiply both sides by the LCD $2R(R+1)$ to obtain

$R(R+1)=2(R+1)+2R$

then simplify and set quadratic to zero to solve

$R^2+R=2R+2+2R$

$R^2-3R-2=0$

Solve using quadratic formula, which gives two solutions

$R = -0.562$ which is rejected since $R>0$ and

$R = 3.56$

Chapter 8

Trigonometric Functions of Any Angle

8.1 Signs of the Trigonometric Functions

1. **(a)** $\sin\left(150^\circ + 90^\circ\right) = \sin 240^\circ$ which is in Quandrant III is $-$

$\cos\left(290^\circ + 90^\circ\right) = \cos 380^\circ$ which is in Quandrant I is $+$

$\tan\left(190^\circ + 90^\circ\right) = \tan 280^\circ$ which is in Quandrant IV is $-$

$\cot\left(260^\circ + 90^\circ\right) = \cot 350^\circ$ which is in Quandrant IV is $-$

$\sec\left(350^\circ + 90^\circ\right) = \sec 440^\circ$ which is in Quandrant I is $+$

$\csc\left(100^\circ + 90^\circ\right) = \csc 190^\circ$ which is in Quandrant III is $-$

(b) $\sin\left(300^\circ + 90^\circ\right) = \sin 390^\circ$ which is in Quandrant I is $+$

$\cos\left(150^\circ + 90^\circ\right) = \cos 240^\circ$ which is in Quandrant III is $-$

$\tan\left(100^\circ + 90^\circ\right) = \tan 190^\circ$ which is in Quandrant III is $+$

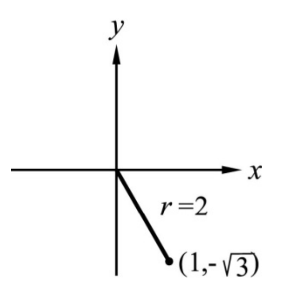
$\cot\left(300^\circ + 90^\circ\right) = \cot 390^\circ$ which is in Quandrant I is $+$

$\sec\left(200^\circ + 90^\circ\right) = \sec 290^\circ$ which is in Quandrant IV is $+$

$\csc\left(250^\circ + 90^\circ\right) = \csc 340^\circ$ which is in Quandrant IV is $-$

2. Point $\left(1, -\sqrt{3}\right)$, $x = 1$, $y = -\sqrt{3}$

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{1^2 + \left(-\sqrt{3}\right)^2}$$
$$r = \sqrt{1+3}$$
$$r = 2$$
$$\sin\theta = \frac{y}{r} = \frac{-\sqrt{3}}{2} = -0.866$$
$$\cos\theta = \frac{x}{r} = \frac{1}{2} = 0.500$$
$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{1} = -1.73$$
$$\cot\theta = \frac{x}{y} = \frac{1}{-\sqrt{3}} = -0.577$$
$$\sec\theta = \frac{r}{x} = \frac{2}{1} = 2.00$$
$$\csc\theta = \frac{r}{y} = \frac{2}{-\sqrt{3}} = -1.16$$



3. $\tan 135^\circ$ is negative since 135° is in Quadrant II where $\tan\theta$ is negative.
$\sec 50^\circ$ is positive since 50° is in Quadrant I where $\sec\theta$ is positive.

4. $\sin 240^\circ$ is negative since 240° is in Quadrant III where $\sin\theta$ is negative.
$\cos 300^\circ$ is positive since 300° is in Quadrant IV where $\cos\theta$ is positive.

5. $\sin 290^\circ$ is negative since 290° is in Quadrant IV where $\sin\theta$ is negative.
$\cos 200^\circ$ is negative since 200° is in Quadrant III where $\cos\theta$ is negative.

6. $\tan 320^\circ$ is negative since 320° is in Quadrant IV, where $\tan\theta$ is negative.
$\sec 185^\circ$ is negative since 185° is in Quadrant III, where $\sec\theta$ is negative.

7. $\csc 98^\circ$ is positive since 98° is in Quadrant II, where $\csc\theta$ is positive.
$\cot 82^\circ$ is positive since 82° is in Quadrant I, where $\cot\theta$ is positive.

8. $\cos 260^\circ$ is negative since 260° is in Quadrant III, where $\cos\theta$ is negative.
$\csc 290^\circ$ is negative since 290° is in Quadrant IV, where $\csc\theta$ is negative.

9. $\sec 150^\circ$ is negative since 150° is in Quadrant II, where $\sec\theta$ is negative.
$\tan 220^\circ$ is positive since 220° is in Quadrant III, where $\tan\theta$ is positive.

10. $\sin 335^\circ$ is negative since 335° is in Quadrant IV, where $\sin\theta$ is negative.
$\cot 265^\circ$ is positive since 265° is in Quadrant III, where $\cot\theta$ is positive.

11. $\cos 348^\circ$ is positive since 348° is in Quadrant IV, where $\cos\theta$ is positive.
$\csc 238^\circ$ is negative since 238° is in Quadrant III, where $\csc\theta$ is negative.

12. $\cot 110^\circ$ is negative since 110° is in Quadrant II, where $\cot\theta$ is negative.
$\sec 309^\circ$ is positive since 309° is in Quadrant IV, where $\sec\theta$ is positive.

13. $\tan 460^\circ$ is negative since 460° is in Quadrant II, where $\tan\theta$ is negative.
$\sin\left(-185^\circ\right)$ is negative since -185° is in Quadrant III, where $\sin\theta$ is negative.

14. $\csc\left(-200^\circ\right)$ is positive since -200° is in Quadrant II, where $\csc\theta$ is positive.
$\cos 550^\circ$ is negative since 550° is coterminal with $550^\circ - 360^\circ = 190^\circ$ which is in Quadrant III, where $\cos\theta$ is negative.

15. $\cot\left(-95^\circ\right)$ is positive since -95° is in Quadrant III, where $\cot\theta$ is positive.
$\cos 710^\circ$ is positive since 710° is coterminal with $710^\circ - 360^\circ = 350^\circ$ which is in Quadrant IV, where $\cos\theta$ is positive.

16. $\sin 539^\circ$ is positive since 539° is coterminal with $539^\circ - 360^\circ = 179^\circ$ which is in Quadrant II, where $\sin\theta$ is positive.
$\tan\left(-480^\circ\right)$ is positive since -480° is coterminal with $-480^\circ + 2(360^\circ) = 240^\circ$ which is in Quadrant III, where $\tan\theta$ is positive.

17. Point $(2, 1)$, $x = 2,\ y = 1$

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{2^2 + 1^2}$$
$$r = \sqrt{5}$$
$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{5}}$$
$$\cos\theta = \frac{x}{r} = \frac{2}{\sqrt{5}}$$
$$\tan\theta = \frac{y}{x} = \frac{1}{2}$$
$$\csc\theta = \frac{r}{y} = \sqrt{5}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{2}$$
$$\cot\theta = \frac{x}{y} = 2$$

18. Point $(-5, 5)$, $x = -5,\ y = 5$

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{(-5)^2 + 5^2}$$
$$r = \sqrt{50}$$
$$\sin\theta = \frac{y}{r} = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}$$
$$\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{50}} = -\frac{1}{\sqrt{2}}$$
$$\tan\theta = \frac{y}{x} = \frac{5}{-5} = -1$$
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{50}}{5} = \sqrt{2}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{50}}{-5} = -\sqrt{2}$$
$$\cot\theta = \frac{x}{y} = \frac{-5}{5} = -1$$

19. Point $(-2, -3)$, $x = -2$, $y = -3$

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{(-2)^2 + (-3)^2}$$
$$r = \sqrt{13}$$
$$\sin\theta = \frac{y}{r} = \frac{-3}{\sqrt{13}}$$
$$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{13}}$$
$$\tan\theta = \frac{y}{x} = \frac{-3}{-2} = \frac{3}{2}$$
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{-3} = -\frac{\sqrt{13}}{3}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{-2} = -\frac{\sqrt{13}}{2}$$
$$\cot\theta = \frac{x}{y} = \frac{-2}{-3} = \frac{2}{3}$$

20. Point $(16, -12)$, $x = 16$, $y = -12$

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{4^2 + 3^2}$$
$$r = 20$$
$$\sin\theta = \frac{y}{r} = \frac{-12}{20} = -\frac{3}{5}$$
$$\cos\theta = \frac{x}{r} = \frac{16}{20} = \frac{4}{5}$$
$$\tan\theta = \frac{y}{x} = \frac{-12}{16} = -\frac{3}{4}$$
$$\csc\theta = \frac{r}{y} = \frac{20}{-12} = -\frac{5}{3}$$
$$\sec\theta = \frac{r}{x} = \frac{20}{16} = \frac{5}{4}$$
$$\cot\theta = \frac{x}{y} = \frac{16}{-12} = -\frac{4}{3}$$

21. Point $(-0.5, 1.2)$, $x=-0.5$, $y=1.2$

$$r=\sqrt{x^2+y^2}$$
$$r=\sqrt{(-0.5)^2+(1.2)^2}$$
$$r=1.3$$
$$\sin\theta=\frac{y}{r}=\frac{1.2}{1.3}=\frac{12}{13}$$
$$\cos\theta=\frac{x}{r}=\frac{-0.5}{1.3}=-\frac{5}{13}$$
$$\tan\theta=\frac{y}{x}=\frac{1.2}{-0.5}=-\frac{12}{5}$$
$$\csc\theta=\frac{r}{y}=\frac{1.3}{1.2}=\frac{13}{12}$$
$$\sec\theta=\frac{r}{x}=\frac{1.3}{-0.5}=-\frac{13}{5}$$
$$\cot\theta=\frac{x}{y}=\frac{-0.5}{1.2}=-\frac{5}{12}$$

22. Point $(-39, -80)$, $x=-39$, $y=-80$

$$r=\sqrt{x^2+y^2}$$
$$r=\sqrt{(-39)^2+(-80)^2}$$
$$r=89$$
$$\sin\theta=\frac{y}{r}=\frac{-80}{89}=-\frac{80}{89}$$
$$\cos\theta=\frac{x}{r}=\frac{-39}{89}=-\frac{39}{89}$$
$$\tan\theta=\frac{y}{x}=\frac{-80}{-39}=\frac{80}{39}$$
$$\csc\theta=\frac{r}{y}=\frac{89}{-80}=-\frac{89}{80}$$
$$\sec\theta=\frac{r}{x}=\frac{89}{-39}=-\frac{89}{39}$$
$$\cot\theta=\frac{x}{y}=\frac{-39}{-80}=\frac{39}{80}$$

23. Point $(20, -8)$, $x = 20,\ y = -8$

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{20^2 + (-8)^2}$$
$$r = \sqrt{464} = \sqrt{16(29)}$$
$$r = 4\sqrt{29}$$
$$\sin\theta = \frac{y}{r} = \frac{-8}{4\sqrt{29}} = \frac{-2}{\sqrt{29}}$$
$$\cos\theta = \frac{x}{r} = \frac{20}{4\sqrt{29}} = \frac{5}{\sqrt{29}}$$
$$\tan\theta = \frac{y}{x} = \frac{-8}{20} = -\frac{2}{5}$$
$$\csc\theta = \frac{r}{y} = \frac{4\sqrt{29}}{-8} = -\frac{\sqrt{29}}{2}$$
$$\sec\theta = \frac{r}{x} = \frac{4\sqrt{29}}{20} = \frac{\sqrt{29}}{5}$$
$$\cot\theta = \frac{x}{y} = \frac{20}{-8} = -\frac{5}{2}$$

24. Point $(0.9, 4)$, $x = 0.9,\ y = 4$

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{0.9^2 + 4^2}$$
$$r = 4.1$$
$$\sin\theta = \frac{y}{r} = \frac{4}{4.1} = \frac{40}{41}$$
$$\cos\theta = \frac{x}{r} = \frac{0.9}{4.1} = \frac{9}{41}$$
$$\tan\theta = \frac{y}{x} = \frac{4}{0.9} = \frac{40}{9}$$
$$\csc\theta = \frac{r}{y} = \frac{4.1}{4} = \frac{41}{40}$$
$$\sec\theta = \frac{r}{x} = \frac{4.1}{0.9} = \frac{41}{9}$$
$$\cot\theta = \frac{x}{y} = \frac{0.9}{4} = \frac{9}{40}$$

25. $\sin\theta = 0.500$

$\sin\theta = \frac{y}{r}$. Since $\sin\theta$ is positive, y must be positive and the terminal side of the angle must lie in either Quadrant I or Quadrant II.

26. $\cos\theta = 0.866$

$\cos\theta = \dfrac{x}{r}$. Since $\cos\theta$ is positive, x must be positive and the terminal side of the angle must lie in either Quadrant I or Quadrant IV.

27. $\tan\theta = -2.50$

$\tan\theta = \dfrac{y}{x}$. Since $\tan\theta$ is negative, x and y must have different signs, and the terminal side of the angle must lie in either Quadrant II or Quadrant IV.

28. $\sin\theta = -0.866$

$\sin\theta = \dfrac{y}{r}$. Since $\sin\theta$ is negative, y must be negative, and the terminal side of the angle must lie in either Quadrant III or Quadrant IV.

29. $\cos\theta = -0.500$

$\cos\theta = \dfrac{x}{r}$. Since $\cos\theta$ is negative, x must be negative, and the terminal side of the angle must lie in either Quadrant II or Quadrant III.

30. $\tan\theta = 0.4270$

$\tan\theta = \dfrac{y}{x}$. Since $\tan\theta$ is positive, x and y must have the same sign, and the terminal side of the angle must lie in either Quadrant I or Quadrant III.

31. $\sin\theta$ is positive and $\cos\theta$ is negative
$\sin\theta$ is positive in Quadrant I and Quadrant II.
$\cos\theta$ is negative in Quadrant II and Quadrant III.
The terminal side of θ must lie in Quadrant II to meet both conditions.

32. $\tan\theta$ is positive and $\cos\theta$ is negative
$\tan\theta$ is positive in Quadrant I and Quadrant III.
$\cos\theta$ is negative in Quadrant II and Quadrant III.
The terminal side of θ must lie in Quadrant III to meet both conditions.

33. $\sec\theta$ is negative and $\cot\theta$ is negative
$\sec\theta$ is negative in Quadrant II and Quadrant III.
$\cot\theta$ is negative in Quadrant II and Quadrant IV.
The terminal side of θ must lie in Quadrant II to meet both conditions.

34. $\cos\theta$ is positive and $\csc\theta$ is negative
$\cos\theta$ is positive in Quadrant I and Quadrant IV.
$\csc\theta$ is negative in Quadrant III and Quadrant IV.
The terminal side of θ must lie in Quadrant IV to meet both conditions.

35. $\csc\theta$ is negative and $\tan\theta$ is negative
$\csc\theta$ is negative in Quadrant III and Quadrant IV.
$\tan\theta$ is negative in Quadrant II and Quadrant IV.
The terminal side of θ must lie in Quadrant IV to meet both conditions.

36. $\tan\theta$ is negative and $\cos\theta$ is positive
$\tan\theta$ is negative in Quadrant II and Quadrant IV.
$\cos\theta$ is positive in Quadrant I and Quadrant IV.
The terminal side of θ must lie in Quadrant IV to meet both conditions.

37. $\sin\theta$ is positive and $\tan\theta$ is positive
$\sin\theta$ is positive in Quadrant I and Quadrant II.
$\tan\theta$ is positive in Quadrant I and Quadrant III.
The terminal side of θ must lie in Quadrant I to meet both conditions.

38. $\sec\theta$ is positive and $\csc\theta$ is negative
$\sec\theta$ is positive in Quadrant I and Quadrant IV.
$\csc\theta$ is negative in Quadrant III and Quadrant IV.
The terminal side of θ must lie in Quadrant IV to meet both conditions.

39. $\sin\theta$ is positive and $\cot\theta$ is negative
$\sin\theta$ is positive in Quadrant I and Quadrant II.
$\cot\theta$ is negative in Quadrant II and Quadrant IV.
The terminal side of θ must lie in Quadrant II to meet both conditions.

40. $\tan\theta$ is positive and $\csc\theta$ is negative
$\tan\theta$ is positive in Quadrant I and Quadrant III.
$\csc\theta$ is negative in Quadrant III and Quadrant IV.
The terminal side of θ must lie in Quadrant III to meet both conditions.

41. For (x, y) in Quadrant III,
x is $(-)$ and y is $(-)$
$$\frac{x}{r} = \frac{(-)}{(+)} = (-)$$

42. For (x, y) in Quadrant II,
x is $(-)$ and y is $(+)$
$$\frac{y}{r} = \frac{(+)}{(+)} = (+)$$

43. For $(x,\ y)$ in Quadrant IV,

x is $(+)$ and y is $(-)$

$$\frac{y}{x}=\frac{(-)}{(+)}=(-)$$

44. For $(x,\ y)$ in Quadrant III,

x is $(-)$ and y is $(-)$

$$\frac{y}{x}=\frac{(-)}{(-)}=(+)$$

8.2 Trigonometric Functions of Any Angle

1. $\sin 200^\circ = -\sin\left(200^\circ - 180^\circ\right) = -\sin 20^\circ = -0.342$

$\tan 150^\circ = -\tan\left(180^\circ - 150^\circ\right) = -\tan 30^\circ = -0.577$

$\cos 265^\circ = -\cos\left(265^\circ - 180^\circ\right) = -\cos 85^\circ = -0.0872$

$\cot 300^\circ = -\cot\left(360^\circ - 300^\circ\right) = -\cot 60^\circ = -\dfrac{1}{\tan 60^\circ} = -0.577$

$\sec 344^\circ = \sec\left(360^\circ - 344^\circ\right) = \sec 16^\circ = \dfrac{1}{\cos 16^\circ} = 1.04$

$\sin 397^\circ = \sin\left(397^\circ - 360^\circ\right) = \sin 37^\circ = 0.602$

2. $\cos\theta = 0.1298$

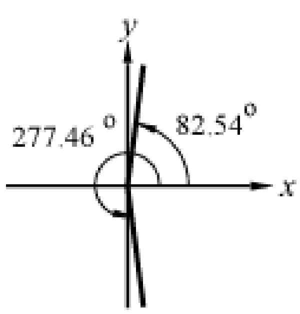
$\theta = \cos^{-1}(0.1298)$

$\theta_1 = 82.54^\circ$

$\cos\theta$ is also positive in Quadrant IV.

$\theta_4 = 360^\circ - 82.54^\circ$

$\theta_4 = 277.46^\circ$



3. $\sin 155^\circ = \sin\left(180^\circ - 155^\circ\right) = \sin 25^\circ$

$\cos 220^\circ = -\cos\left(220^\circ - 180^\circ\right) = -\cos 40^\circ$

4. $\tan 91^\circ = -\tan\left(180^\circ - 91^\circ\right) = -\tan 89^\circ$

$\sec 345^\circ = \sec\left(360^\circ - 345^\circ\right) = \sec 15^\circ$

5. $\tan 105^\circ = -\tan\left(180^\circ - 105^\circ\right) = -\tan 75^\circ$

$\csc 328^\circ = -\csc\left(360^\circ - 328^\circ\right) = -\csc 32^\circ$

6. $\cos 190^\circ = -\cos\left(190^\circ - 180^\circ\right) = -\cos 10^\circ$

$\tan 290^\circ = -\tan\left(360^\circ - 290^\circ\right) = -\tan 70^\circ$

7. $\sec 425^\circ = \sec\left(425^\circ - 360^\circ\right) = \sec 65^\circ$

$$\begin{aligned}\sin\left(-520^\circ\right) &= \sin\left(-520^\circ + 2\left(360^\circ\right)\right) = \sin 200^\circ \\ &= -\sin\left(200^\circ - 180^\circ\right) = -\sin 20^\circ\end{aligned}$$

8.

$$\begin{aligned}\tan 920^\circ &= \tan\left(920^\circ - 2\left(360^\circ\right)\right) \\ &= \tan 200^\circ \\ &= \tan\left(200^\circ - 180^\circ\right) \\ &= \tan 20^\circ\end{aligned}$$

$$\begin{aligned}\csc\left(-550^\circ\right) &= \csc\left(-550^\circ + 2\left(360^\circ\right)\right) \\ &= \csc 170^\circ \\ &= \csc\left(180^\circ - 170^\circ\right) \\ &= \csc 10^\circ\end{aligned}$$

9. $\sin 195^\circ = -\sin\left(195^\circ - 180^\circ\right) = -\sin 15^\circ = -0.259$

10. $\tan 311^\circ = -\tan\left(360^\circ - 311^\circ\right) = -\tan 49^\circ = -1.15$

11. $\cos 106.3^\circ = -\cos\left(180^\circ - 106.3^\circ\right) = -\cos\left(73.7^\circ\right) = -0.2807$

12. $\sin 93.4^\circ = \sin\left(180^\circ - 93.4^\circ\right) = \sin 86.6^\circ = 0.9982$

13. $\sec 328.33^\circ = \sec\left(360^\circ - 328.33^\circ\right) = \sec 31.67^\circ = 1.1750$

14. $\cot 516.53^\circ = \cot\left(516.53^\circ - 360^\circ\right)$
$= \cot\left(156.53^\circ\right)$
$= -\cot\left(180^\circ - 156.53^\circ\right)$
$= -\cot 23.47^\circ$
$= -2.3031$

15. $\tan\left(-109.1^\circ\right) = \tan\left(-109.1^\circ + 180^\circ\right) = \tan 70.9^\circ = 2.8878$

16. $\csc\left(-108.4^\circ\right) = \csc\left(-108.4^\circ + 360^\circ\right)$
$= \csc\left(251.6^\circ\right)$
$= -\csc\left(251.6^\circ - 180^\circ\right)$
$= -\csc 71.6^\circ$
$= -1.054$

17. $\cos\ \left(-62.7\right)^\circ = 0.4586$

18. $\cos\ 141.4^\circ = -0.7815$

19. $\sin\ 310.36^\circ = -0.76199$

20. $\tan\ 242.68^\circ = 1.9358$

21. $\csc\ 194.82^\circ = -3.9096$

22. $\sec\ 441.08^\circ = 6.4493$

23. $\tan 148.25^\circ = -0.6188$

24. $\sin\left(-215.5^\circ\right) = 0.5807$

25. $\sin\theta = -0.8480$
$\theta_{\text{ref}} = \sin^{-1} 0.8480$
$\theta_{\text{ref}} = 57.99^\circ$
Since $\sin\theta$ is negative, θ must lie in Quadrant III or Quadrant IV.
Therefore, $\theta_3 = 180^\circ + 57.99^\circ$
$\theta_3 = 237.99^\circ$
or $\theta_4 = 360^\circ - 57.99^\circ$
$\theta_4 = 302.01^\circ$

26. $\tan\theta = -1.830$

$\theta_{\text{ref}} = \tan^{-1} 1.830$

$\theta_{\text{ref}} = 61.35^\circ$

Since $\tan\theta$ is negative, θ must lie in Quadrant II or Quadrant IV.

Therefore, $\theta_2 = 180^\circ - 61.35^\circ$

$\theta_2 = 118.65^\circ$

or $\theta_4 = 360^\circ - 61.35^\circ$

$\theta_4 = 298.65^\circ$

27. $\cos\theta = 0.4003$

$\theta_{\text{ref}} = \cos^{-1} 0.4003$

$\theta_{\text{ref}} = 66.40^\circ$

Since $\cos\theta$ is positive, θ must lie in Quadrant I or Quadrant IV.

Therefore, $\theta_1 = 66.40^\circ$

or $\theta_4 = 360^\circ - 66.40^\circ$

$\theta_4 = 293.60^\circ$

28. $\sec\theta = -1.637$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-1.637} = -0.61087$$

$\theta_{\text{ref}} = \cos^{-1} 0.61087$

$\theta_{\text{ref}} = 52.35^\circ$

Since $\cos\theta$ is negative, θ must lie in Quadrant II or Quadrant III.

Therefore, $\theta_2 = 180^\circ - 52.35^\circ = 127.65^\circ$

and $\theta_3 = 180^\circ + 52.35^\circ = 232.35$

29. $\cot\theta = -0.0122$

$\theta_{\text{ref}} = \tan^{-1}(1/0.0122)$

$\theta_{\text{ref}} = 89.3^\circ$

Since $\cot\theta$ is negative, θ must lie in Quadrant II or Quadrant IV.

Therefore, $\theta_2 = 180^\circ - 89.3^\circ$

$\theta_2 = 90.7^\circ$

or $\theta_4 = 360^\circ - 89.3^\circ$

$\theta_4 = 270.7^\circ$

30. $\csc\theta = -8.09$

$\theta_{\text{ref}} = \sin^{-1}(1/8.09)$

$\theta_{\text{ref}} = 7.10^\circ$

Since $\csc\theta$ is negative, θ must lie in Quadrant III or Quadrant IV.

Therefore, $\theta_3 = 180^\circ + 7.10^\circ$

$\theta_3 = 187.10^\circ$

or $\theta_4 = 360^\circ - 7.10^\circ$

$\theta_4 = 352.90^\circ$

31. $\sin\theta = 0.870$

$\theta_{\text{ref}} = \sin^{-1} 0.870$

$\theta_{\text{ref}} = 60.5^\circ$

Since $\sin\theta$ is positive, θ must lie in Quadrant I or Quadrant II.

If $\cos\theta$ is negative, θ must lie in Quadrant II or Quadrant III.

To satisfy both conditions, θ must lie in Quadrant II.

$\theta_2 = 180^\circ - 60.5^\circ$

$\theta_2 = 119.5^\circ$

32. $\tan\theta = 0.932$

$\theta_{\text{ref}} = \tan^{-1} 0.932$

$\theta_{\text{ref}} = 43.0^\circ$

Since $\tan\theta$ is positive, θ must lie in Quadrant I or Quadrant III.

If $\sin\theta$ is negative, θ must lie in Quadrant III or Quadrant IV.

To satisfy both conditions, θ must lie in Quadrant III.

$\theta_3 = 180^\circ + 43.0^\circ$

$\theta_3 = 223.0^\circ$

33. $\cos\theta = -0.12$

$\theta_{\text{ref}} = \cos^{-1}(0.12)$

$\theta_{\text{ref}} = 83^\circ$

Since $\cos\theta$ is negative, θ must lie in Quadrant II or Quadrant III.

If $\tan\theta$ is positive, θ must lie in Quadrant I or Quadrant III.

To satisfy both conditions, θ must lie in Quadrant III.

$\theta_3 = 180^\circ + 83^\circ$

$\theta_3 = 263^\circ$

34. $\sin\theta = -0.192$

$\theta_{\text{ref}} = \sin^{-1} 0.192$

$\theta_{\text{ref}} = 11.1^\circ$

Since $\sin\theta$ is negative, θ must lie in Quadrant III or Quadrant IV.

If $\tan\theta$ is negative, θ must lie in Quadrant II or Quadrant IV.

To satisfy both conditions, θ must lie in Quadrant IV.

$\theta_4 = 360^\circ - 11.1^\circ$

$\theta_4 = 348.9^\circ$

35. $\csc\theta = -1.366$

$$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-1.366} = -0.732$$

$\theta_{\text{ref}} = \sin^{-1} 0.732$

$\theta_{\text{ref}} = 47.05^\circ$

Since $\sin\theta$ is negative, θ must lie in Quadrant III or Quadrant IV.

If $\cos\theta$ is positive, θ must lie in Quadrant I or Quadrant IV.

To satisfy both conditions, θ must lie in Quadrant IV.

$\theta_4 = 360^\circ - 47.05^\circ$

$\theta_4 = 312.95^\circ$

36. $\cos\theta = 0.0726$

$\theta_{\text{ref}} = \cos^{-1} 0.0726$

$\theta_{\text{ref}} = 85.8^\circ$

Since $\cos\theta$ is positive, θ must lie in Quadrant I or Quadrant IV.

If $\sin\theta$ is negative, θ must lie in Quadrant III or Quadrant IV.

To satisfy both conditions, θ must lie in Quadrant IV.

$\theta_4 = 360^\circ - 85.8^\circ$

$\theta_4 = 274.2^\circ$

37. $\sec\theta = 2.047$

$\theta_{\text{ref}} = \cos^{-1}(1/2.047)$

$\theta_{\text{ref}} = 60.76^\circ$

Since $\sec\theta$ is positive, θ must lie in Quadrant I or Quadrant IV.

If $\cot\theta$ is negative, θ must lie in Quadrant II or Quadrant IV.

To satisfy both conditions, θ must lie in Quadrant IV.

$\theta_4 = 360^\circ - 60.76^\circ$

$\theta_4 = 299.24^\circ$

38. $\cot\theta = -0.3256$

$\theta_{\text{ref}} = \tan^{-1}(1/0.3256)$

$\theta_{\text{ref}} = 71.96^\circ$

Since $\cot\theta$ is negative, θ must lie in Quadrant II or Quadrant IV.

If $\csc\theta$ is positive, θ must lie in Quadrant I or Quadrant II.

To satisfy both conditions, θ must lie in Quadrant II.

$\theta_2 = 180^\circ - 71.96^\circ$

$\theta_2 = 108.04^\circ$

39.
$$\begin{aligned}\cos 60^\circ + \cos 70^\circ + \cos 110^\circ &= \cos 60^\circ + \cos 70^\circ - \cos\left(180^\circ - 110^\circ\right)\\ &= \cos 60^\circ + \cos 70^\circ - \cos 70^\circ\\ &= \cos 60^\circ\\ &= 0.5000\end{aligned}$$

40.
$$\begin{aligned}\sin 200^\circ - \sin 150^\circ + \sin 160^\circ &= -\sin\left(200^\circ - 180^\circ\right) - \sin\left(180^\circ - 150^\circ\right) + \sin\left(180^\circ - 160^\circ\right)\\ &= -\sin 20^\circ - \sin 30^\circ + \sin 20^\circ\\ &= -\sin 30^\circ\\ &= -0.5000\end{aligned}$$

41.
$$\begin{aligned}\tan 40^\circ + \tan 135^\circ - \tan 220^\circ &= \tan 40^\circ - \tan\left(180^\circ - 135^\circ\right) - \tan\left(220^\circ - 180^\circ\right)\\ &= \tan 40^\circ - \tan 45^\circ - \tan 40^\circ\\ &= -\tan 45^\circ\\ &= -1.000\end{aligned}$$

42.
$$\begin{aligned}\sec 130^\circ - \sec 230^\circ + \sec 300^\circ &= -\sec\left(180^\circ - 130^\circ\right) + \sec\left(230^\circ - 180^\circ\right) + \sec\left(360^\circ - 300^\circ\right)\\ &= -\sec 50^\circ + \sec 50^\circ + \sec 60^\circ\\ &= \sec 60^\circ\\ &= 2.000\end{aligned}$$

43. $\sin\theta = -0.5736$

$\theta_{\text{ref}} = 35.00^\circ$

Since $\sin\theta$ is negative, θ must lie in Quadrant III or Quadrant IV.

If $\cos\theta$ is positive, θ must lie in Quadrant I or Quadrant IV.

To satisfy both conditions, θ must lie in Quadrant IV.

$\theta_4 = 360^\circ - 35.00^\circ$

$\theta_4 = 325.00^\circ$

$\theta_{\text{ref}} = \sin^{-1} 0.5736$

In general, the angle could be any solution

$\theta = 325.00^\circ + k \times 360^\circ$ where $k = 0, \pm 1, \pm 2, \cdots$

but all evaluations of the trigonometric functions will be identical for any integer number of rotations from the Quadrant IV solution.

$\tan\theta = \tan 325.00^\circ$

$\tan\theta = -0.7002$

44. $\cos\theta = 0.422$

$\theta_{\text{ref}} = \cos^{-1} 0.422$

$\theta_{\text{ref}} = 65.0^\circ$

Since $\cos\theta$ is positive, θ must lie in Quadrant I or Quadrant IV.

If $\tan\theta$ is negative, θ must lie in Quadrant II or Quadrant IV.

To satisfy both conditions, θ must lie in Quadrant IV.

$\theta_4 = 360^\circ - 65.0^\circ$

$\theta_4 = 295.0^\circ$

In general, the angle could be any solution

$\theta = 295.0^\circ + k \times 360^\circ$ where $k = 0, \pm 1, \pm 2, \cdots$

but all evaluations of the trigonometric functions will be identical for any integer number of rotations from the Quadrant IV solution.

$\sin\theta = \sin 295.0^\circ$

$\sin\theta = -0.906$

45. $\tan\theta = -0.809$

$\theta_{\text{ref}} = \tan^{-1} 0.809$

$\theta_{\text{ref}} = 39.0^\circ$

Since $\tan\theta$ is negative, θ must lie in Quadrant II or Quadrant IV.

If $\csc\theta$ is positive, θ must lie in Quadrant I or Quadrant II.

To satisfy both conditions, θ must lie in Quadrant II.

$\theta_2 = 180^\circ - 39.0^\circ$

$\theta_2 = 141.0^\circ$

In general, the angle could be any solution

$\theta = 141.0^\circ + k \times 360^\circ$ where $k = 0, \pm 1, \pm 2, \cdots$

but all evaluations of the trigonometric functions will be identical for any integer number of rotations from the Quadrant II solution.

$\cos\theta = \cos 141.0^\circ$

$\cos\theta = -0.777$

46. $\sec\theta = 6.122$

$\theta_{\text{ref}} = \cos^{-1}(1/6.122)$

$\theta_{\text{ref}} = 80.60^\circ$

Since $\sec\theta$ is positive, θ must lie in Quadrant I or Quadrant IV.
If $\sin\theta$ is negative, θ must lie in Quadrant III or Quadrant IV.
To satisfy both conditions, θ must lie in Quadrant IV.

$\theta_4 = 360^\circ - 80.60^\circ$

$\theta_4 = 279.40^\circ$

In general, the angle could be any solution

$\theta = 279.40^\circ + k \times 360^\circ$ where $k = 0, \pm 1, \pm 2, \cdots$

but all evaluations of the trigonometric functions will be identical for any integer number of rotations from the Quadrant IV solution.

$\cot\theta = \cot 279.40^\circ$

$\cot\theta = -0.1655$

47. $\sin 90^\circ = 1$, and

$2\sin 45^\circ = 2 \times \frac{\sqrt{2}}{2} = \sqrt{2}$

$1 < \sqrt{2}$

$\sin 90^\circ < 2\sin 45^\circ$

48. $\cos 360^\circ = 1$, and

$2\cos 180^\circ = -2$

$1 > -2$

$\cos 360^\circ > 2\ \cos 180^\circ$

49. $\tan 180^\circ = 0$, and

$\tan 0^\circ = 0$

$0 = 0$

$\tan 180^\circ = \tan 0^\circ$

50. $\sin 270^\circ = -1$, and

$3\sin 90^\circ = 3$

$-1 < 3$

$\sin 270^\circ < 3\sin 90^\circ$

51. $\theta = 195^\circ$ has $\theta_{\text{ref}} = 15^\circ$

$\cos 195^\circ = -\cos\left(195^\circ - 180^\circ\right)$

$\cos 195^\circ = -\cos 15^\circ$

15° and 75° are complementary, so their cofunctions are equivalent

$\cos 195^\circ = -\sin 75^\circ$

$\cos 195^\circ = -0.9659$

52. $\theta = 290^\circ$ has $\theta_{\text{ref}} = 70^\circ$

$\tan 290^\circ = -\tan\left(360^\circ - 290^\circ\right)$

$\tan 290^\circ = -\tan 70^\circ$

70° and 20° are complementary, so their cofunctions are equivalent

$\tan 290^\circ = -\cot 20^\circ$

$\tan 290^\circ = -2.747$

53. For $0^\circ < \theta < 90^\circ$, $270^\circ - \theta$ is in Quadrant III, where tangent is positive.

$\tan\left(270^\circ - \theta\right) = +\tan\left(270^\circ - \theta - 180^\circ\right)$

$\tan\left(270^\circ - \theta\right) = \tan\left(90^\circ - \theta\right)$

$90^\circ - \theta$ and θ are complementary, so their cofunctions are equivalent

$\tan\left(270^\circ - \theta\right) = \cot\ \theta$

54. For $0^\circ < \theta < 90^\circ$, $90^\circ + \theta$ is in Quadrant II, where cosine is negative.

$\cos\left(90^\circ + \theta\right) = -\cos\left(180^\circ - (90^\circ + \theta)\right)$

$\cos\left(90^\circ + \theta\right) = -\cos\left(90^\circ - \theta\right)$

$90^\circ - \theta$ and θ are complementary, so their cofunctions are equivalent

$\cos\left(90^\circ + \theta\right) = -\sin\theta$

55. In a triangle with angles A, B, C

we have $B + C = 180^\circ - A$

and so

$$\begin{aligned} \tan A + \tan(B + C) &= \tan A + \tan\left(180^\circ - A\right) \\ &= \tan A - \tan A \\ &= 0 \end{aligned}$$

56. **(a)** $\sin 180^\circ = 0;\ 2\sin 90^\circ = 2$
and so they are not equal.

(b) $\sin 360^\circ = 0;\ 2\sin 180^\circ = 0$
and so they are equal.

57. $i = i_m \sin\theta$

$i = \left(0.0259\text{ A}\right)\sin 495.2^\circ$

$i = 0.0183\text{ A}$

58. $F = \left(F_x\right)\left(\sec\theta\right)$

$F = \left(-365\text{ N}\right)\left(\sec 127.0^\circ\right)$

$F = 606\text{ N}$

59. $y\sin\alpha = x\sin\beta$; $x = 6.78$ in, $\alpha = 31.3^\circ$, $\beta = 104.7^\circ$

$$y = \frac{x\sin\beta}{\sin\alpha}$$

$$y = \frac{(6.78\text{ in})\sin 104.7^\circ}{\sin 31.3^\circ}$$

$$y = 12.6\text{ in}$$

60. $2ab\cos\theta = a^2 + b^2 - c^2$; $a = 12.9$ cm, $b = 15.3$ cm, $c = 24.5$ cm

$$\cos\theta = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos\theta = \frac{12.9^2 + 15.3^2 - 24.5^2}{2(12.9)(15.3)}$$

$$\cos\theta = -0.506029$$

$$\theta = \cos^{-1}(-0.506029)$$

$$\theta = 120.4^\circ$$

8.3 Radians

1. $2.80 = (2.80)\left(\frac{180^\circ}{\pi}\right) = 160^\circ$

2. $\sin\theta = 0.8829$, $0 \le \theta < 2\pi$

$$\theta = \sin^{-1} 0.8829$$

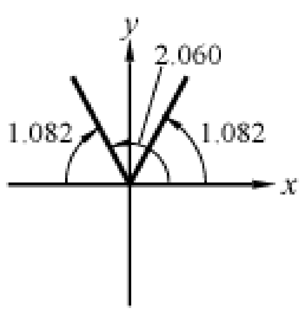
$$\theta = 1.082$$

Since $\sin\theta$ is positive, angle θ must lie in Quadrant I or Quadarant II.

$$\theta_1 = 1.082$$

$$\theta_2 = \pi - 1.082$$

$$\theta_2 = 2.060$$



3. $15^\circ = 15^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{12}$

$120^\circ = 120^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{2\pi}{3}$

4. $12^\circ = 12^\circ \quad \frac{\pi}{180^\circ} \quad = \frac{\pi}{15}$

$225^\circ = 225^\circ \quad \frac{\pi}{180^\circ} \quad = \frac{5\pi}{4}$

5. $75^\circ = 75^\circ \quad \frac{\pi}{180^\circ} \quad = \frac{5\pi}{12}$

$330^\circ = 330^\circ \quad \frac{\pi}{180^\circ} \quad = \frac{11\pi}{6}$

6. $36^\circ = 36^\circ \quad \frac{\pi}{180^\circ} \quad = \frac{\pi}{5}$

$315^\circ = 315^\circ \quad \frac{\pi}{180^\circ} \quad = \frac{7\pi}{4}$

7. $210^\circ = 210^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{7\pi}{6}$

$99^\circ = 99^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{11\pi}{20}$

8. $5^\circ = 5^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{36}$

$300^\circ = 300^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{5\pi}{3}$

9. $720^\circ = 720^\circ\left(\frac{\pi}{180^\circ}\right) = 4\pi$

$-9^\circ = -9^\circ\left(\frac{\pi}{180^\circ}\right) = -\frac{\pi}{20}$

10. $-66^\circ = -66^\circ\left(\frac{\pi}{180^\circ}\right) = -\frac{11\pi}{30}$

$540^\circ = 540^\circ\left(\frac{\pi}{180^\circ}\right) = 3\pi$

11. $\frac{3\pi}{5} = \frac{3\pi}{5}\left(\frac{180^\circ}{\pi}\right) = 108^\circ$

$\frac{3\pi}{2} = \frac{3\pi}{2}\left(\frac{180^\circ}{\pi}\right) = 270^\circ$

12. $\frac{3\pi}{10} = \frac{3\pi}{10}\left(\frac{180^\circ}{\pi}\right) = 54^\circ$

$\frac{11\pi}{6} = \frac{11\pi}{6}\left(\frac{180^\circ}{\pi}\right) = 330^\circ$

13. $\frac{5\pi}{9} = \frac{5\pi}{9}\left(\frac{180^\circ}{\pi}\right) = 100^\circ$

$\frac{7\pi}{4} = \frac{7\pi}{4}\left(\frac{180^\circ}{\pi}\right) = 315^\circ$

14. $\frac{8\pi}{15} = \frac{8\pi}{15}\ \frac{180^\circ}{\pi} \ = 96^\circ$

$\frac{4\pi}{3} = \frac{4\pi}{3}\ \frac{180^\circ}{\pi} \ = 240^\circ$

15. $\frac{7\pi}{18} = \frac{7\pi}{18}\ \frac{180^\circ}{\pi} \ = 70^\circ$

$\frac{5\pi}{6} = \frac{5\pi}{6}\ \frac{180^\circ}{\pi} \ = 150^\circ$

16. $\frac{\pi}{40} = \frac{\pi}{40}\left(\frac{180^\circ}{\pi}\right) = 4.5^\circ$

$\frac{5\pi}{4} = \frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right) = 225^\circ$

17. $-\frac{\pi}{15} = -\frac{\pi}{15}\left(\frac{180^\circ}{\pi}\right) = -12^\circ$

$\frac{3\pi}{20} = \frac{3\pi}{20}\left(\frac{180^\circ}{\pi}\right) = 27^\circ$

18. $\frac{9\pi}{2} = \frac{9\pi}{2}\left(\frac{180^\circ}{\pi}\right) = 810^\circ$

$-\frac{4\pi}{15} = -\frac{4\pi}{15}\left(\frac{180^\circ}{\pi}\right) = -48^\circ$

19. $84.0^\circ = 84.0^\circ\left(\frac{\pi \text{ rad}}{180^\circ}\right) = 1.47 \text{ rad}$

20. $54.3^\circ = 54.3^\circ\left(\frac{\pi \text{ rad}}{180^\circ}\right) = 0.948 \text{ rad}$

21. $252^\circ = 252^\circ\ \frac{\pi \text{ rad}}{180^\circ} \ = 4.40 \text{ rad}$

22. $104^\circ = 104^\circ\left(\frac{\pi \text{ rad}}{180^\circ}\right) = 1.82 \text{ rad}$

23. $-333.5^\circ = -333.5^\circ\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = -5.821 \text{ rad}$

24. $268.7^\circ = 268.7^\circ\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = 4.690 \text{ rad}$

25. $478.5^\circ = 478.5^\circ\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = 8.351 \text{ rad}$

26. $-86.1^\circ = -86.1^\circ\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = -1.50 \text{ rad}$

27. $0.750 = 0.750\left(\dfrac{180^\circ}{\pi}\right) = 43.0^\circ$

28. $0.240 = 0.240\left(\dfrac{180^\circ}{\pi}\right) = 13.8^\circ$

29. $3.407 = 3.407\left(\dfrac{180^\circ}{\pi}\right) = 195.2^\circ$

30. $1.703 = 1.703\left(\dfrac{180^\circ}{\pi}\right) = 97.57^\circ$

31. $12.4 = 12.4\left(\dfrac{180^\circ}{\pi}\right) = 710^\circ$

32. $-34.4 = -34.4\left(\dfrac{180^\circ}{\pi}\right) = -1970^\circ$

33. $-16.42 = -16.42\left(\dfrac{180^\circ}{\pi}\right) = -940.8^\circ$

34. $100.00 = 100.00\left(\dfrac{180^\circ}{\pi}\right) = 5729.6^\circ$

35. $\sin\dfrac{\pi}{4} = \sin\left[\left(\dfrac{\pi}{4}\right)\left(\dfrac{180^\circ}{\pi}\right)\right] = \sin 45^\circ = 0.7071$

36. $\cos\dfrac{\pi}{6} = \cos\left[\left(\dfrac{\pi}{6}\right)\left(\dfrac{180^\circ}{\pi}\right)\right] = \cos 30^\circ = 0.8660$

37. $\tan\frac{5\pi}{12} = \tan\left[\left(\frac{5\pi}{12}\right)\left(\frac{180^\circ}{\pi}\right)\right] = \tan 75^\circ = 3.732$

38. $\sin\frac{71\pi}{36} = \sin\left[\left(\frac{71\pi}{36}\right)\left(\frac{180^\circ}{\pi}\right)\right] = \sin 355^\circ = -0.08716$

39. $\cos\frac{5\pi}{6} = \cos\left[\left(\frac{5\pi}{6}\right)\left(\frac{180^\circ}{\pi}\right)\right] = \cos 150^\circ = -0.8660$

40. $\tan\left(-\frac{7\pi}{3}\right) = \tan\left[\left(-\frac{7\pi}{3}\right)\left(\frac{180^\circ}{\pi}\right)\right] = \tan -420^\circ = -1.732$

41.
$$\begin{aligned}\sec 4.5920 &= \sec\ \ 4.5920\ \ \frac{180^\circ}{\pi}\\ &= \sec 263.10^\circ\\ &= \frac{1}{\cos 263.10^\circ}\\ &= -8.3265\end{aligned}$$

42.
$$\begin{aligned}\cot 3.2732 &= \cot\ \ 3.2732\ \ \frac{180^\circ}{\pi}\\ &= \cot 187.54^\circ\\ &= \frac{1}{\tan 187.54^\circ}\\ &= 7.5544\end{aligned}$$

43. $\tan 0.7359 = 0.9056$

44. $\cos 1.4308 = 0.140$

45. $\sin 4.24 = -0.890$

46. $\tan 3.47 = 0.341$

47. $\sec 2.07 = \frac{1}{\cos 2.07} = -2.09$

48. $\sin(-1.34) = -0.973$

49. $\cot(-4.86) = \frac{1}{\tan(-4.86)} = 0.149$

50. $\csc 6.19 = \frac{1}{\sin 6.19} = -10.7$

51. $\sin\theta = 0.3090$

$\theta_{ref} = \sin^{-1} 0.3090$

$\theta_{ref} = 0.3141$

Since $\sin\theta$ is positive, θ must lie in Quadrant I or Quadrant II.

$\theta_1 = 0.3141$

$\theta_2 = \pi - 0.3141$

$\theta_2 = 2.827$

52. $\cos\theta = -0.9135$

$\theta_{ref} = \cos^{-1} 0.9135$

$\theta_{ref} = 0.4190$

Since $\cos\theta$ is negative, θ must lie in Quadrant II or Quadrant III.

$\theta_2 = \pi - 0.4190$

$\theta_2 = 2.723$

$\theta_3 = \pi + 0.4190$

$\theta_3 = 3.561$

53. $\tan\theta = -0.2126$

$\theta_{ref} = \tan^{-1} 0.2126$

$\theta_{ref} = 0.2095$

Since $\tan\theta$ is negative, θ must lie in Quadrant II or Quadrant IV.

$\theta_2 = \pi - 0.2095$

$\theta_2 = 2.932$

$\theta_4 = 2\pi - 0.2095$

$\theta_4 = 6.074$

54. $\sin\theta = -0.0436$

$\theta_{ref} = \sin^{-1} 0.0436$

$\theta_{ref} = 0.0436$

Since $\sin\theta$ is negative, θ must lie in Quadrant III or Quadrant IV.

$\theta_3 = \pi + 0.0436$

$\theta_3 = 3.185$

$\theta_4 = 2\pi - 0.0436$

$\theta_4 = 6.240$

55. $\cos\theta = 0.6742$

$\theta_{ref} = \cos^{-1} 0.6742$

$\theta_{ref} = 0.8309$

Since $\cos\theta$ is positive, θ must lie in Quadrant I or Quadrant IV.

$\theta_1 = 0.8309$

$\theta_4 = 2\pi - 0.8309$

$\theta_4 = 5.452$

56. $\cot\theta = 1.860$

$\theta_{ref} = \tan^{-1}(1/1.860)$

$\theta_{ref} = 0.4933$

Since $\cot\theta$ is positive, θ must lie in Quadrant I or Quadrant III.

$\theta_1 = 0.4933$

$\theta_3 = \pi + 0.4933$

$\theta_3 = 3.635$

57. $\sec\theta = -1.307$

$\theta_{ref} = \cos^{-1}(1/1.307)$

$\theta_{ref} = 0.6996$

Since $\sec\theta$ is negative, θ must lie in Quadrant II or Quadrant III.

$\theta_2 = \pi - 0.6966$

$\theta_2 = 2.442$

$\theta_3 = \pi + 0.6966$

$\theta_3 = 3.841$

58. $\csc\theta = 3.940$

$\theta_{ref} = \sin^{-1}(1/3.940)$

$\theta_{ref} = 0.2566$

Since $\csc\theta$ is positive, θ must lie in Quadrant I or Quadrant II.

$\theta_1 = 0.2566$

$\theta_2 = \pi - 0.2566$

$\theta_2 = 2.885$

59. The radian measure is $\dfrac{\text{arc length}}{\text{radius}} = \dfrac{15}{12} = 1.25.$

60. The arc length is radians $\times$ radius $= 3 \times 10\text{in} = 30$ in.

61. $\dfrac{5\pi}{8}$ has $\theta_{\text{ref}} = \pi - \dfrac{5\pi}{8} = \dfrac{3\pi}{8}$

$\dfrac{5\pi}{8}$ is in Quadrant II where $\cos\theta$ is negative.

$$\cos\frac{5\pi}{8} = -\cos\frac{3\pi}{8}$$

$\dfrac{3\pi}{8}$ and $\dfrac{\pi}{2} - \dfrac{3\pi}{8}$ are complementary, so their cofunctions are equivalent.

$$\cos\frac{5\pi}{8} = -\sin\ \frac{\pi}{2} - \frac{3\pi}{8}$$

$$\cos\frac{5\pi}{8} = -\sin\frac{\pi}{8}$$

$$\cos\frac{5\pi}{8} = -0.3827$$

62. $\frac{5\pi}{3}$ has $\theta_{\text{ref}} = 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$

$\frac{5\pi}{3}$ is in Quadrant IV where $\cot\theta$ is negative.

$$\cot\frac{5\pi}{3} = -\cot\frac{\pi}{3}$$

$\frac{\pi}{3}$ and $\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$ are complementary, so their cofunctions are equivalent.

$$\cot\frac{5\pi}{3} = -\tan\frac{\pi}{6}$$

$$\cot\frac{5\pi}{3} = -0.5774$$

63. For $0 < \theta < \frac{\pi}{2}$

The angle $\frac{\pi}{2}+\theta$ is in Quadrant II where $\tan\theta$ is negative.

Therefore $\frac{\pi}{2}+\theta$ has

$$\theta_{\text{ref}} = \pi - \left(\frac{\pi}{2}+\theta\right)$$

$$\theta_{\text{ref}} = \frac{\pi}{2}-\theta$$

$$\tan\left(\frac{\pi}{2}+\theta\right) = -\tan\left(\frac{\pi}{2}-\theta\right)$$

But θ and $\frac{\pi}{2}-\theta$ are complementary, so their cofunctions are equivalent.

$$\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$$

64. For $0 < \theta < \frac{\pi}{2}$

The angle $\frac{3\pi}{2}+\theta$ is in Quadrant IV where $\cos\theta$ is positive.

Therefore $\frac{3\pi}{2}+\theta$ has

$$\theta_{\text{ref}} = 2\pi - \left(\frac{3\pi}{2}+\theta\right)$$

$$\theta_{\text{ref}} = \frac{\pi}{2}-\theta$$

$$\cos\left(\frac{3\pi}{2}+\theta\right) = \cos\left(\frac{\pi}{2}-\theta\right)$$

But θ and $\frac{\pi}{2}-\theta$ are complementary, so their cofunctions are equivalent.

$$\cos\left(\frac{3\pi}{2}+\theta\right) = \sin\theta$$

65. $34.4^\circ = 34.4^\circ\left(\frac{1\text{ circumference}}{360^\circ}\right)\left(\frac{1\text{ mil}}{\frac{1}{6400}\text{ circumference}}\right) = 612\text{ mil}$

66. $25\text{ min} = 25\text{ min}\left(\frac{1\text{ revolution}}{60\text{ min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ revolution}}\right) = \frac{5\pi}{6}\text{ rad} = 2.6\text{ rad}$

67. $1.60\text{ revolutions} = 1.60\text{ revolutions}\left(\frac{2\pi\text{ rad}}{\text{revolution}}\right) = 10.1\text{ rad}$

68. A difference of 24 capsules (25th - 1st) revolves past, so $\frac{24}{32} = \frac{3}{4}$ of a revolution occurs. This is equivalent to

$\frac{3}{4}\text{ revolution}\left(2\pi\frac{\text{rad}}{\text{revolution}}\right) = \frac{3\pi}{2}\text{rad} = 4.71\text{ rad.}$

69. $V = \frac{1}{2}Wb\theta^2$

$V = \frac{1}{2}(8.75\text{ lb})(0.75\text{ ft})\left(5.5^\circ\frac{\pi}{180^\circ}\right)^2$

$V = 0.030\text{ ft}\cdot\text{lb}$

70. $q = A\sin\omega t$

The argument of the sine function must be in radians.
Therefore if ωt is measured in radians, and if square brackets represent "units of quantity", and time is in seconds (s)

$[\omega t] = \text{rad}$

$[\omega]\cdot\text{s} = \text{rad}$

$[\omega] = \text{rad/s}$

71. $h(t) = 1200\tan\frac{5t}{3t+10}$ Let t = 8.0 s

$h(8.0\text{ s}) = 1200\tan\frac{5(8.0)}{3(8.0)+10}$

$h(8.0\text{ s}) = 1200\tan\frac{40.0}{34.0}$

$h(8.0\text{ s}) = 2900\text{ m}$

72. $I = 0.023\cos^2(\pi\sin\theta)$, where $\theta = 40.0^\circ$

First put the calculator in degree mode to find

$\sin 40.0^\circ = 0.642788.$

Then put the calculator in radian mode to solve the rest of the problem.

$I = 0.023\cos^2(0.642788\pi)$

$I = 0.0043\text{ W/m}^2$

8.4 Applications of Radian Measure

1. $s = \theta r$

$s = \left(\frac{\pi}{4}\right)(3.00 \text{ cm})$

$s = 2.36 \text{ cm}$

2. $A = \frac{1}{2}\theta r^2$

18.5% of 360° is 66.6°

$A = \frac{1}{2}\left(66.6^\circ\right)\left(\frac{\pi}{180^\circ}\right)(8.50 \text{ cm})^2$

$A = 42.0 \text{ cm}^2$

3. $v = (0.125 \text{ rad/s})(115 \text{ m}) = 14.4 \text{ m/s}$

4. The velocity of the belt is

$v = \frac{10 \text{ ft}}{2.5 \text{ sec}} = 4.0 \text{ ft/sec.}$

Using $v = \omega r$, we have

$4.0 \text{ ft/sec} = \omega(0.50 \text{ ft})$

or

$\omega = 8.0 \text{ rad/s}$

$8.0 \frac{\text{rad}}{\text{s}} = \frac{8.0 \text{ rad}}{1 \text{ s}} \times \frac{1.0 \text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{1 \text{ min}} = 76.4$ revolutions per minute.

5. $s = \theta r = \left(\frac{\pi}{4}\right)(5.70 \text{ in}) = 4.48 \text{ in}$

6. $s = \theta r = 2.65(21.2 \text{ cm}) = 56.2 \text{ cm}$

7. $\theta = 136.0^\circ\left(\frac{\pi \text{ rad}}{180^\circ}\right) = 2.374 \text{ rad}$

$r = \frac{s}{\theta} = \frac{915 \text{ mm}}{2.374} = 385 \text{ mm}$

8. $\theta = 73.61^\circ\left(\frac{\pi}{180^\circ}\right) = 1.285 \text{ rad}$

$r = \frac{s}{\theta} = \frac{0.3456 \text{ ft}}{1.285} = 0.2690 \text{ ft}$

$A = \frac{1}{2}\theta r^2 = \frac{1}{2}(1.285)(0.2690 \text{ ft})^2 = 0.04648 \text{ ft}^2$

9. $\theta = \dfrac{s}{r} = \dfrac{0.3913 \text{ mi}}{0.9449 \text{ mi}} = 0.4141 \text{ rad}$

$A = \dfrac{1}{2}\theta r^2 = \dfrac{1}{2}(0.4141)(0.9449 \text{ mi})^2 = 0.1849 \text{ mi}^2$

10. $\theta = \dfrac{s}{r} = \dfrac{319 \text{ m}}{229 \text{ m}} = 1.39 \text{ rad}$

11. $A = \dfrac{1}{2}\theta r^2 = \dfrac{1}{2}(6.7)(3.8 \text{ cm})^2 = 48 \text{ cm}^2$

12. $A = \dfrac{1}{2}\theta r^2 = \dfrac{1}{2}\left(\dfrac{2\pi}{5}\right)(46.3 \text{ in})^2 = 1350 \text{ in}^2$

13. $\theta = 326.0^\circ\left(\dfrac{\pi}{180^\circ}\right) = 5.690 \text{ rad}$

$A = \dfrac{1}{2}\theta r^2$

$r = \sqrt{\dfrac{2A}{\theta}}$

$r = \sqrt{\dfrac{2\left(0.0119 \text{ ft}^2\right)}{5.690}}$

$r = 0.0647 \text{ ft}$

14. $\theta = 17^\circ \quad \dfrac{\pi}{180^\circ} \quad = 0.296706 \text{ rad (unrounded)}$

$A = \dfrac{1}{2}\theta r^2$

$r = \sqrt{\dfrac{2A}{\theta}}$

$r = \sqrt{\dfrac{2\left(1200 \text{ mm}^2\right)}{0.296706}}$

$r = 89.9378 \text{ mm (unrounded)}$

$s = \theta r = (0.296706)(89.9378 \text{ mm}) = 26.6851 \text{ mm}$

$s = 27 \text{ mm (rounded)}$

15. $A = \dfrac{1}{2}\theta r^2$

$\theta = \dfrac{2A}{r^2} = \dfrac{2\left(165 \text{ m}^2\right)}{(40.2 \text{ m})^2} = 2.04 \text{ rad}$

$s = \theta r = (2.04)(40.2 \text{ m}) = 82.1 \text{ m}$

16. $A = \frac{1}{2}\theta r^2$

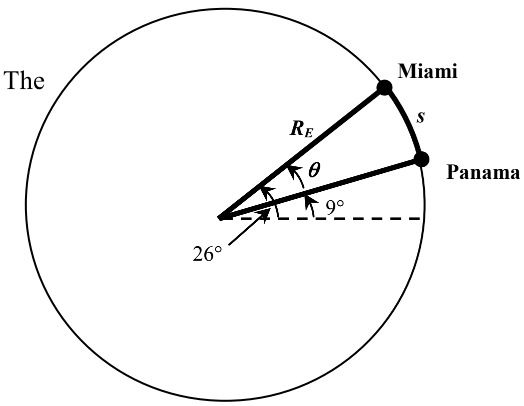
$\theta = \frac{2A}{r^2} = \frac{2\left(67.8 \text{ mi}^2\right)}{\left(67.8 \text{ mi}\right)^2} = 0.0295 \text{ rad}$

17. $r = \frac{s}{\theta} = \frac{0.203 \text{ mi}}{\frac{3}{4}\left(2\pi\right)}$

$r = 0.0431 \text{ mi} \quad \frac{5280 \text{ ft}}{1 \text{ mi}}$

$r = 228 \text{ ft}$

18. Since both cities are at the same longitude, Miami is directly north of the Panama Canal, on the same great circle for Earth. The difference in their latitudes will be the angle between the cities as measured from the centre of Earth.



$\theta = \left(26^\circ - 9^\circ\right) \quad \frac{\pi \text{ rad}}{180^\circ} \quad = 0.2967 \text{ rad}$

$s = \theta R_E = 0.2967(3960 \text{ mi}) = 1175 \text{ mi}$

19. Arc length is the product of radian measure and radius:

$s = 213^\circ \times \frac{\pi}{180^\circ} \times 4.50 \text{ in} = 16.7 \text{ in}$

20. Each piece is a sector with angle $\theta = \frac{2\pi}{8} = \frac{\pi}{4}$.

$A = \frac{1}{2}r^2\theta$

$r^2 = \frac{2A}{\theta}$

$d = 2r = 2\sqrt{\frac{2A}{\theta}}$

$d = 2\sqrt{\frac{2(88 \text{ cm}^2)}{\pi / 4}} = 29.9 \text{ cm}$

21. If time t is measured in hours from noon,
The hour hand rotates at the rate of a full revolution (2π rad) every 12 hours,

$$\text{Hour hand } \omega = \frac{2\pi \text{ rad}}{12 \text{ h}}$$

$$\theta_H = \omega t$$

$$\theta_H = \frac{2\pi}{12} t$$

The minute hand rotates at the rate of a full revolution (2π rad) every 1 hour,

$$\text{Minute hand } \omega = \frac{2\pi \text{ rad}}{1 \text{ h}}$$

$$\theta_M = 2\pi t$$

If the angle between the minute hand and hour hand is π rad,

$$\theta_H + \pi = \theta_M$$

$$\frac{2\pi}{12} t + \pi = 2\pi t$$

$$\pi = \left(\frac{12\pi}{6} - \frac{\pi}{6}\right) t$$

$$t = \frac{\pi}{11\pi / 6}$$

$$t = \frac{6}{11} \text{ h} \left(\frac{60 \text{ min}}{1 \text{ h}}\right) = 32.727 \text{ min}$$

$$t = 32 \text{ min} + 0.727 \text{ min} \left(\frac{60 \text{ s}}{1 \text{ min}}\right)$$

$$t = 32 \text{ min} + 44 \text{ s}$$

The clock will read 12:32:44 when the hour and minute hands will be at 180° apart.

22. $\theta = 165.58^\circ \left(\frac{\pi}{180^\circ}\right) = 2.8899 \text{ rad}$

$$p = s + 2r$$

$$p = \theta r + 2r$$

$$p = (2.8899)(1.875 \text{ in}) + 2(1.875 \text{ in})$$

$$p = 9.169 \text{ in}$$

23. $\theta = 115.0^\circ \left(\frac{\pi}{180^\circ}\right) = 2.007 \text{ rad}$

$$A = \frac{1}{2}\theta r^2 = \frac{1}{2}(2.007)(65.0 \text{ ft})^2 = 4240 \text{ ft}^2$$

24. $\theta = 75.0^\circ \quad \dfrac{\pi}{180^\circ} \quad = 1.31 \text{ rad}$

$A = \dfrac{1}{2}\theta r^2 = \dfrac{1}{2}(1.31 \text{ rad})(250 \text{ ft})^2 = 40900 \text{ ft}^2$

25. $\omega = \dfrac{\theta}{t} = \dfrac{\pi \text{ rad}}{6.0 \text{ s}} = 0.52 \text{ rad/s}$

26. $v = \omega r = \dfrac{8.5}{\text{s}} \times 61.0 \text{ cm} = 518.5 \ \dfrac{\text{cm}}{\text{s}}$

27. $s = \theta r$

$\theta = \dfrac{s}{r} = \dfrac{7.535 \text{ m}}{8.250 \text{ m}} = 0.9133 \text{ rad}$

$A = \dfrac{1}{2}\theta r^2$

$A_1 = \dfrac{1}{2}(0.9133)(8.250 \text{ m})^2 = 31.08 \text{ m}^2$

$A_2 = \dfrac{1}{2}(0.9133)(8.250 + 3.755)^2 = 65.81 \text{ m}^2$

$A_{hall} = A_2 - A_1$

$A_{hall} = 65.81 \text{ m}^2 - 31.08 \text{ m}^2 = 34.73 \text{ m}^2$

28. $\theta = 110.0^\circ \left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = 1.920 \text{ rad}$

$r_1 = 12.75 \text{ in} - \dfrac{1}{2}(15.00 \text{ in}) = 5.25 \text{ in}$

$r_2 = 12.75 \text{ in} + \dfrac{1}{2}(15.00 \text{ in}) = 20.25 \text{ in}$

$A = \dfrac{1}{2}\theta r^2$

$A_{swept} = A_2 - A_1$

$A_{swept} = \dfrac{1}{2}1.920\left(20.25^2 - 5.25^2\right)\text{in}^2$

$A_{swept} = 367.2 \text{ in}^2$

29. $\theta = 28.0^\circ \left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = 0.489 \text{ rad}$

From $s = \theta r$

$s_1 = 0.489(93.67 \text{ ft}) = 45.80 \text{ ft}$

$s_2 = 0.489(93.67 \text{ ft} + 4.71 \text{ ft}) = 48.11 \text{ ft}$

$s_2 - s_1 = 2.31 \text{ ft}$

Outer rail is 2.31 ft longer.

30. $v = \sqrt{2gh} = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(4.80\text{ m}\right)} = 9.70\text{ m/s}$

$\omega = \dfrac{v}{r} = \dfrac{9.70\text{ m/s}}{13.8\text{ m}} = 0.703\text{ rad/s}$

31. $\theta = 75.5^\circ \quad \dfrac{\pi}{180^\circ} \quad = 1.32\text{ rad}$

$A = \dfrac{1}{2}\theta r^2$

$r_1 = \dfrac{11.2\text{ m}}{2} = 5.60\text{ m}$

$r_2 = \dfrac{11.2\text{ m}}{2} + 2.50\text{ m} = 8.10\text{ m}$

$A_{between} = A_2 - A_1$

$A_{between} = \dfrac{1}{2}(1.32)\left(8.10^2 - 5.60^2\right)\text{m}^2$

$A_{between} = 22.6\text{ m}^2$

32. $r = \dfrac{d}{2} = \dfrac{2.38\text{ ft}}{2} = 1.19\text{ ft}$

$\theta = \dfrac{s}{r} = \dfrac{18.5\text{ ft}}{1.19\text{ ft}} = 15.5\text{ rad}$

33. $\theta = 15.6^\circ \quad \dfrac{\pi}{180^\circ} \quad = 0.272\text{ rad}$

$A = \dfrac{1}{2}\theta r^2$

$r_1 = 285.0\text{ m}$

$r_2 = 285.0\text{ m} + 15.2\text{ m} = 300.2\text{ m}$

$A_{road} = A_2 - A_1$

$A_{road} = \dfrac{1}{2}(0.272)\left(300.2^2 - 285.0^2\right)\text{m}^2$

$A_{road} = 1210\text{ m}^2$

Volume = Area $\times$ thickness

$V = At$

$V = 1210\text{ m}^2\,(0.305\text{ m})$

$V = 369\text{ m}^3$

34. $v = \omega r = (130\text{ rad/s})(22.5\text{ cm}) = 2925\text{ cm/s}$

35. $$\sin\frac{\theta}{2}=\frac{(185.0/2)\text{ km}}{r}$$

$$r=\frac{92.50\text{ km}}{\sin 6.4^\circ}=830\text{ km}$$

$$\theta=12.8^\circ\ \frac{\pi\text{ rad}}{180^\circ}=0.223\text{ rad}$$

$$s=\theta r$$

$$s=0.223(830\text{ km})=185.4\text{ km (from unrounded values)}$$

Extra distance flew $=185.4-185.0=0.4$ km

36. $$\theta=79.4^\circ\left(\frac{\pi\text{ rad}}{180^\circ}\right)=1.39\text{ rad}$$

$$r=\frac{s}{\theta}=\frac{330\text{ m}}{1.39}=237\text{ m}$$

37. $$\omega=20.0\text{ r/min}\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)=2.09\text{ rad/s}$$

$$v=\omega r=(2.09\text{ rad/s})(8.50\text{ ft})=17.8\text{ ft/s}$$

38. $$\omega=\frac{1\text{ r}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)=0.1047\text{ rad/s}$$

$$v=\omega r$$

$$v=(0.1047\text{ rad/s})(15.0\text{ mm})=1.57\text{ mm/s}$$

39. $$r=\frac{4.75\text{ in}}{2}=2.375\text{ in}$$

$$\omega=\frac{360.0\text{ r}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)=12.00\pi\text{ rad/s}$$

$$v=\omega r$$

$$v=(12.00\pi\text{ rad/s})(2.375\text{ in})=89.5\text{ in/s}$$

40. $$\omega=\frac{1\text{ r}}{37.00\text{ min}}\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)=0.00283\text{ rad/s}$$

$$v=\omega r=(0.00283\text{ rad/s})(150\text{ m})\left(\frac{100\text{ cm}}{1\text{ m}}\right)=42.5\text{ cm/s}$$

41. $$\omega=\frac{2\text{ r}}{1\text{ d}}\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ d}}{24\text{ h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)=0.0001454\text{ rad/s}$$

$$v=\omega r=(0.0001454\text{ rad/s})(26600\text{ km})=3.87\text{ km/s}$$

42. $$\omega=\frac{1\text{ r}}{365.25\text{ d}}\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ d}}{24\text{ h}}\right)=7.1677\times10^{-4}\text{ rad/h}$$

$$v=\omega r=(7.1677\times10^{-4}\text{ rad/h})(93000000\text{ mi})=66660\text{ mi/h}$$

43. $v_{wheel} = 15.0\ \frac{\text{mi}}{\text{h}}\ \frac{5280\text{ ft}}{1\text{ mi}}\ \frac{12\text{ in}}{1\text{ ft}}\ \frac{1\text{ h}}{60\text{ min}} = 15840\text{ in/min}$

$v = r\omega$

$\omega_{wheel} = \frac{v_{wheel}}{r_{wheel}}$

$\omega_{wheel} = \frac{15840\text{ in/min}}{14.00\text{ in}}$

$\omega_{wheel} = 1131\text{ rad/min}$

Since the wheel sprocket and the wheel are mounted on the same shaft they have the same rotational motion.

$\omega_{sprocket} = 1131\ \text{rad/min}$

The linear velocity of the edge of the sprocket is the same as the linear velocity of the chain, and of the edge of the pedal sprocket.

$v_{sprocket} = \omega_{sprocket} r_{sprocket}$

$v_{sprocket} = 1131\text{ rad/min}(2\text{ in})$

$v_{sprocket} = 2262\text{ in/min}$

The angular velocity of the pedal sprocket can now be found by dividing the velocity of its edge by its radius.

$\omega_{pedal} = \frac{v_{sprocket}}{r_{pedal}}$

$\omega_{pedal} = \frac{2262\text{ in/min}}{5\text{ in}} = 452.6\text{ rad/min}$

$\omega_{pedal} = 452.6\text{ rad/min}\ \frac{1\text{ r}}{2\pi\text{ rad}}$

$\omega_{pedal} = 72.03\text{ r/min}$

44. $r = \frac{0.36\text{ m}}{2} = 0.18\text{ m}$

$\omega = 750\text{ r/min}\ \frac{2\pi\text{ rad}}{1\text{ r}}\ \frac{1\text{ min}}{60\text{ s}} = 78.5398\text{ rad/s (unrounded)}$

$\theta = \omega t = 78.5398\text{ rad/s}(2.00\text{ s}) = 157.0796\text{ rad}$

$s = \theta r = 157.0796\text{ rad}(0.18\text{ m})$

$s = 28\text{ m}$

45. $\theta = 82.0^\circ\ \frac{\pi\text{ rad}}{180^\circ} = 1.43\text{ rad}$

$s = \theta r$

$s = 1.43(15.0\text{ ft})$

$s = 21.5\text{ ft}$

46. $\theta_{\max} = 52.00^\circ\left(\frac{1.500}{0.2500}\right) = 312.0^\circ\left(\frac{\pi\text{ rad}}{180^\circ}\right) = 5.445\text{ rad}$

$s = \theta r$

$s = 5.445(3.750\text{ in}) = 20.42\text{ in}$

47. $\omega = 1200\dfrac{\text{r}}{\text{min}}\left(\dfrac{2\pi \text{ rad}}{1\text{ r}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = 40\pi \text{ rad/s}$

$v = \omega r = 40\pi \text{ rad/s}(0.375\text{ in}) = 47.1\text{ in/s}$

48. $\omega_f = 420\dfrac{\text{r}}{\text{min}}\left(\dfrac{2\pi \text{ rad}}{1\text{ r}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = 14\pi \text{ rad/s}$

$v = \omega_f r = (14\pi \text{ rad/s})(2.75\text{ m}) = 121\text{ m/s}$

49. At 2.25 cm from the center, 1590 r/min corresponds to a linear velocity of

$v = \omega r = \dfrac{1590\text{ r}}{1\text{ min}} \times \dfrac{2\pi \text{ rad}}{1\text{ r}} \times \dfrac{2.25\text{ cm}}{1\text{ rad}} = 22478\text{ cm/min}$

At this linear velocity, the angular velocity at 5.51 cm is

$\omega = \dfrac{v}{r} = \dfrac{22478\text{ cm}}{1\text{ min}} \times \dfrac{1\text{ rad}}{5.51\text{ cm}} \times \dfrac{1\text{ r}}{2\pi \text{ rad}} = 649\text{ r/min}$

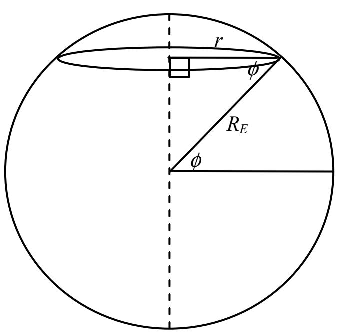
50. $\omega = \dfrac{1\text{ r}}{24.0\text{ h}}\left(\dfrac{2\pi \text{ rad}}{1\text{ r}}\right) = 0.262\text{ rad/h}$

$v = \omega r = 0.262\text{ rad/h}(3960\text{ mi}) = 1040\text{ mi/h}$

51. $\omega = \dfrac{40\text{ r}}{1\text{ min}}\left(\dfrac{2\pi \text{ rad}}{1\text{ r}}\right) = 251.33\text{ rad/min}$

$v = \omega r = 251.33\text{ rad/min}(35\text{ ft}) = 8800\text{ ft/min}$

52.



$\phi = 32^\circ 46' = 32.7667^\circ$

$\cos\phi = \dfrac{r}{R_E}$

$r = 3960\text{ mi} \times \cos 32.7667^\circ$

$r = 3330\text{ mi}$

$\omega = \dfrac{1\text{ r}}{24\text{ h}}\left(\dfrac{2\pi \text{ rad}}{1\text{ r}}\right) = \dfrac{\pi}{12}\text{ rad/h}$

$v = \omega r$

$v = \left(\dfrac{\pi}{12}\text{ rad/h}\right)(3330\text{ mi}) = 872\text{ mi/h}$

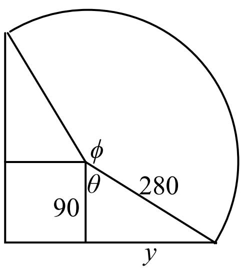
53. $\omega = 2400\frac{\text{r}}{\text{min}} \quad \frac{2\pi \text{ rad}}{1 \text{ r}} \quad \frac{1 \text{ min}}{60 \text{ s}} \quad = 80\pi \text{ rad/s}$

$\theta = \omega t = (80\pi \text{ rad/s})(1.0 \text{ s}) = 250 \text{ rad}$

54. $y = \sqrt{280^2 - 90^2} = \sqrt{70300} = 265 \text{ ft}$

$\cos\theta = \frac{90}{280}$

$\theta = \cos^{-1} \quad \frac{90}{280} \quad = 71.25^{\circ}$



Interior angle in the circular sector is ϕ

$360^{\circ} = 90^{\circ} + 2\theta + \phi$

$\phi = 360^{\circ} - 90^{\circ} - 2(71.25^{\circ}) = 127.5^{\circ}$

$\phi = 127.6^{\circ}\left(\frac{\pi \text{ rad}}{180^{\circ}}\right) = 2.23 \text{ rad}$

$A = A_{\text{infield}} + 2A_{\text{triangle}} + A_{\text{sector}}$

$A = x^2 + 2\left(\frac{1}{2}bh\right) + \frac{1}{2}\theta r^2$

$A = 90^2 + 2\left(\frac{1}{2}\right)(90)(265) + \frac{1}{2}(2.23)(280)^2$

$A = 119366 \text{ ft}^2 = 1.19 \times 10^5 \text{ ft}^2$

55. $r = \frac{d}{2} = \frac{1.2 \text{ m}}{2} = 0.60 \text{ m}$

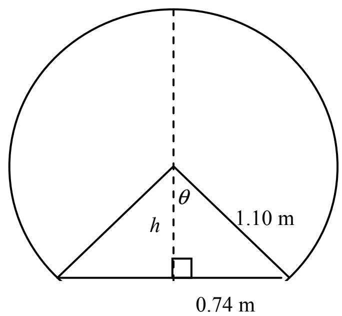
$\omega r = \frac{250 \text{ r}}{1 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ r}} \times \frac{0.60 \text{ m}}{1 \text{ rad}} = 940 \text{ m/s}$

56. $\theta = 160.0^{\circ} \quad \frac{\pi \text{ rad}}{180^{\circ}} \quad = 2.793 \text{ rad}$

$r = \frac{s}{\theta} = \frac{11.6 \text{ m}}{2.793} = 4.154 \text{ m}$

$A = \frac{1}{2}\theta r^2 = \frac{1}{2}(2.793)(4.154 \text{ m})^2 = 24.09 \text{ m}^2$

57.



$$h = \sqrt{1.10^2 - 0.74^2} = 0.8138 \text{ m};$$

$$\theta = \sin^{-1}\frac{0.74}{1.10} = 42.278^\circ$$

$$2\theta = 84.556^\circ \frac{\pi \text{ rad}}{180^\circ} = 1.476 \text{ rad}$$

$$A_{\text{sector}} = \frac{1}{2}(2\theta)r^2 = \frac{1}{2}(1.476)(1.10)^2 = 0.8929 \text{ m}^2$$

$$A_{\text{triangle}} = \frac{1}{2}bh = \frac{1}{2}(1.48)(0.8138) = 0.6023 \text{ m}^2$$

$$A_{\text{segment}} = 0.8929 - 0.6023 = 0.2906 \text{ m}^2$$

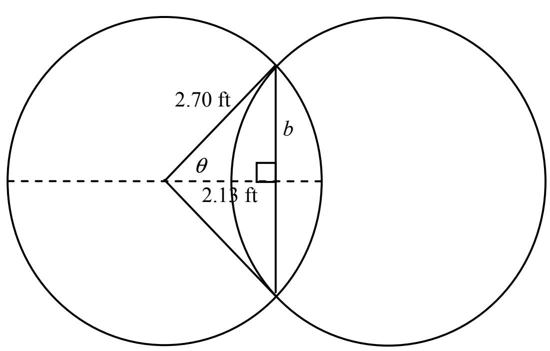
$$A_{end} = A_{circle} - A_{\text{segment}}$$

$$A_{end} = \pi(1.10 \text{ m})^2 - 0.2906 \text{ m}^2 = 3.5107 \text{ m}^2$$

$$V_{\text{tank}} = A_{end}L$$

$$V_{\text{tank}} = 3.5107 \text{ m}^2(4.25 \text{ m}) = 14.9 \text{ m}^3$$

58.



The area illuminated by both beams can be found by subtracting twice the area of the right triangle from the area of the sector of the each circle subtended by the angle 2θ. This will give half the total area illuminated by both beams.

$$b = \sqrt{2.70^2 - 2.13^2} = \sqrt{2.7531} = 1.65925 \text{ ft}$$

$$\cos\theta = \frac{2.13}{2.70}$$

$$\theta = \cos^{-1} \frac{2.13}{2.70}$$

$$\theta = 37.9^\circ$$

$$2\theta = 2(37.918^\circ) \frac{\pi \text{ rad}}{180^\circ} = 1.3236 \text{ rad}$$

$$A_{\text{sector}} = \frac{1}{2}(2\theta)r^2 = \frac{1}{2}(1.3236)(2.70)^2 = 4.8245 \text{ ft}^2$$

$$2A_{\text{triangle}} = 2 \; \frac{1}{2}bh \; = (1.65925)(2.13) = 3.5342 \text{ ft}^2$$

$$A_{\text{segment}} = 4.8245 - 3.5342 = 1.29 \text{ ft}^2$$

$$A_{\text{illuminated}} = 1.29(2) = 2.58 \text{ ft}^2$$

59.

θ	$\sin\theta/\theta$	$\tan\theta/\theta$
0.0001	0.999 999 998 3	1.000 000 003
0.001	0.999 999 833 3	1.000 000 333
0.01	0.999 983 333 4	1.000 033 335
0.1	0.998 334 166 5	1.003 346 721

The sequences both converge on 1.

For small θ in rad, $\theta \approx \sin\theta \approx \tan\theta$.

60. $$\theta = 0.001^\circ \frac{\pi \text{ rad}}{180^\circ} = 1.745 \times 10^{-5} \text{ rad}$$

From Eq. (8.17)

$\tan\theta = \theta$ for small angles, so

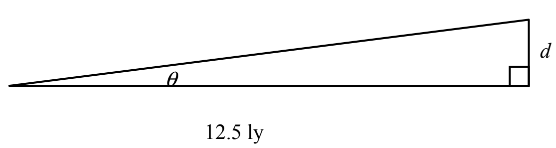
$$\tan 0.001^\circ = \tan 1.745 \times 10^{-5} = 1.745 \times 10^{-5}$$

Using a calculator,

$$\tan 0.001^\circ = 1.745 \times 10^{-5}$$

The results are the same.

61.



$$\theta = 0.2'' \frac{1^\circ}{3600''} \frac{\pi \text{ rad}}{180^\circ} = 9.696 \times 10^{-7} \text{ rad}$$

$$12.5 \text{ light years} = (12.5)(9.46 \times 10^{15}) \text{m}$$

$$\tan\theta = \frac{d}{12.5 \text{ ly}}$$

Using Eq. (8.17), $\tan\theta = \theta$ for small angles, so

$$\theta = \frac{d}{12.5 \text{ ly}}$$
$$d = (12.5 \text{ ly})(9.696\times10^{-7})$$
$$d = (1.212\times10^{-7} \text{ ly})\ \frac{9.46\times10^{12} \text{ km}}{1 \text{ ly}}$$
$$d = 1.15\times10^{8} \text{ km}$$

62. $$\theta = 0.05^\circ\ \frac{\pi \text{ rad}}{180^\circ} = 8.727\times10^{-4} \text{ rad}$$

$$\tan\theta = \frac{x}{136.0 \text{ m}}$$

Using Eq. (8.17), $\tan\theta = \theta$ for small angles, so

$$\theta = \frac{x}{136.0 \text{ m}}$$
$$x = (136.0 \text{ m})(8.727\times10^{-4})$$
$$x = 0.119 \text{ m}$$

Review Exercises

1. This is true. If θ is an angle in quadrant I or quadrant IV, then $\cos\theta$ is positive.

2. This is true. For any angle θ, $\sin\theta = -\sin(\theta + 180^\circ)$.

3. This is false. If $\tan\theta < 0$ then $\sin\theta$ and $\cos\theta$ have opposite signs. Given $\sin\theta > 0$ this forces $\cos\theta < 0$. Finally, since $\sec\theta = \frac{1}{\cos\theta}$, we have $\sec\theta < 0$.

4. This is true.

5. This is false. The correct arc length formula is $s = \theta r$.

6. This is true. The angle θ terminates in quadrant III precisely when both $\sin\theta < 0$ and $\cos\theta < 0$.

7. Point (6, 8), $x = 6,\ y = 8$

$$r = \sqrt{6^2 + 8^2} = 10$$

$$\sin\theta = \frac{y}{r} = \frac{8}{10} = \frac{4}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{6}{10} = \frac{3}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{8}{6} = \frac{4}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{4}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{3}{4}$$

8. Point $(-12,\ 5)$, $x = -12,\ y = 5$

$$r = \sqrt{(-12)^2 + 5^2} = 13$$

$$\sin\theta = \frac{y}{r} = \frac{5}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$$

$$\tan\theta = \frac{y}{x} = \frac{5}{-12} = -\frac{5}{12}$$

$$\csc\theta = \frac{r}{y} = \frac{13}{5}$$

$$\sec\theta = \frac{r}{x} = -\frac{13}{12}$$

$$\cot\theta = \frac{x}{y} = -\frac{12}{5}$$

9. Point $(42, -12)$, $x = 42,\ y = -12$

$$r = \sqrt{42^2 + (-12)^2} = \sqrt{1908} = \sqrt{36(53)} = 6\sqrt{53}$$

$$\sin\theta = \frac{y}{r} = \frac{-12}{6\sqrt{53}} = -\frac{2}{\sqrt{53}}$$

$$\cos\theta = \frac{x}{r} = \frac{42}{6\sqrt{53}} = \frac{7}{\sqrt{53}}$$

$$\tan\theta = \frac{y}{x} = \frac{-12}{42} = -\frac{2}{7}$$

$$\csc\theta = \frac{r}{y} = -\frac{\sqrt{53}}{2}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{53}}{7}$$

$$\cot\theta = \frac{x}{y} = -\frac{7}{2}$$

10. Point $(-0.2, -0.3)$, $x = -0.2$, $y = -0.3$

$$r = \sqrt{(-0.2)^2 + (-0.3)^2} = \sqrt{0.13}$$

$$\sin\theta = \frac{y}{r} = \frac{-0.3}{\sqrt{0.13}} = -\frac{3}{\sqrt{13}}$$

$$\cos\theta = \frac{x}{r} = \frac{-0.2}{\sqrt{0.13}} = -\frac{2}{\sqrt{13}}$$

$$\tan\theta = \frac{y}{x} = \frac{-0.3}{-0.2} = \frac{3}{2}$$

$$\csc\theta = \frac{r}{y} = -\frac{\sqrt{0.13}}{0.3} = -\frac{\sqrt{13}}{3}$$

$$\sec\theta = \frac{r}{x} = -\frac{\sqrt{0.13}}{0.2} = -\frac{\sqrt{13}}{2}$$

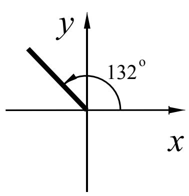
$$\cot\theta = \frac{x}{y} = \frac{-0.2}{-0.3} = \frac{2}{3}$$

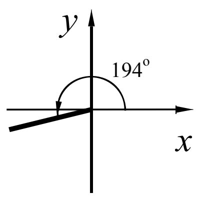
11. 132° is in Quadrant II, where $\cos\theta$ is negative.

$\cos 132^\circ = -\cos\left(180^\circ - 132^\circ\right) = -\cos 48^\circ$

194° is in Quadrant III, where $\tan\theta$ is positive.

$\tan 194^\circ = +\tan\left(194^\circ - 180^\circ\right) = \tan 14^\circ$



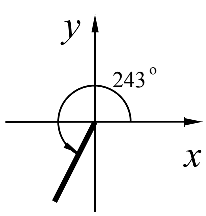


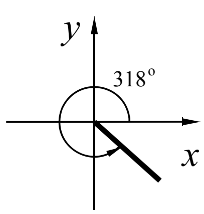
12. 243° is in Quadrant III, where $\sin\theta$ is negative.

$\sin 243^\circ = -\sin\left(243^\circ - 180^\circ\right) = -\sin 63^\circ$

318° is in Quadrant IV, where $\cot\theta$ is negative.

$\cot 318^\circ = -\cot\left(360^\circ - 318^\circ\right) = -\cot 42^\circ$



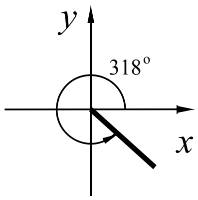


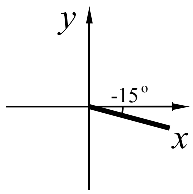
13. 289° is in Quadrant IV, where $\sin\theta$ is negative.

$\sin 289^\circ = -\sin\left(360^\circ - 289^\circ\right) = -\sin 71^\circ$

-15° is in Quadrant IV, where $\sec\theta$ is positive.

$\sec\left(-15^\circ\right) = \sec 15^\circ$



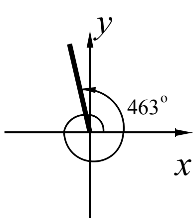


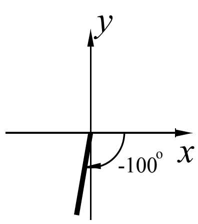
14. 463° is coterminal with 103° which is in Quadrant II, where $\cos\theta$ is negative.

$\cos 463^\circ = \cos\left(463^\circ - 360^\circ\right) = \cos 103^\circ = -\cos\left(180^\circ - 103^\circ\right) = -\cos 77^\circ$

-100° is coterminal with 260° which is in Quadrant III, where $\csc\theta$ is negative.

$\csc\left(-100^\circ\right) = \csc\left(-100^\circ + 360^\circ\right) = \csc 260^\circ = -\csc\left(260^\circ - 180^\circ\right) = -\csc 80^\circ$





15. $40^\circ = 40^\circ \ \dfrac{\pi}{180^\circ} = \dfrac{2\pi}{9}$

$153^\circ = 153^\circ \ \dfrac{\pi}{180^\circ} = \dfrac{17\pi}{20}$

16. $22.5^\circ = 22.5^\circ \ \dfrac{\pi}{180^\circ} = \dfrac{\pi}{8}$

$324^\circ = 324^\circ \ \dfrac{\pi}{180^\circ} = \dfrac{9\pi}{5}$

17. $408^\circ = 408^\circ \ \dfrac{\pi}{180^\circ} = \dfrac{34\pi}{15}$

$202.5^\circ = 202.5^\circ \ \dfrac{\pi}{180^\circ} = \dfrac{9\pi}{8}$

18. $12^\circ = 12^\circ \ \dfrac{\pi}{180^\circ} = \dfrac{\pi}{15}$

$-162^\circ = -162^\circ \ \dfrac{\pi}{180^\circ} = -\dfrac{9\pi}{10}$

19. $\frac{7\pi}{5} = \frac{7\pi}{5}\left(\frac{180^\circ}{\pi}\right) = 252^\circ$

$\frac{13\pi}{18} = \frac{13\pi}{18}\left(\frac{180^\circ}{\pi}\right) = 130^\circ$

20. $\frac{3\pi}{8} = \frac{3\pi}{8}\left(\frac{180^\circ}{\pi}\right) = 67.5^\circ$

$\frac{7\pi}{20} = \frac{7\pi}{20}\left(\frac{180^\circ}{\pi}\right) = 63^\circ$

21. $\frac{\pi}{15} = \frac{\pi}{15}\left(\frac{180^\circ}{\pi}\right) = 12^\circ$

$\frac{11\pi}{6} = \frac{11\pi}{6}\left(\frac{180^\circ}{\pi}\right) = 330^\circ$

22. $\frac{27\pi}{10} = \frac{27\pi}{10}\left(\frac{180^\circ}{\pi}\right) = 486^\circ$

$\frac{5\pi}{4} = \frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right) = 225^\circ$

23. $0.560 = 0.560\left(\frac{180^\circ}{\pi}\right) = 32.1^\circ$

24. $-1.354 = -1.354\left(\frac{180^\circ}{\pi}\right) = -77.58^\circ$

25. $-36.07 = -36.07\left(\frac{180^\circ}{\pi}\right) = -2067^\circ$

26. $14.5 = 14.5\left(\frac{180^\circ}{\pi}\right) = 831^\circ$

27. $102^\circ = 102^\circ\left(\frac{\pi \text{ rad}}{180^\circ}\right) = 1.78 \text{ rad}$

28. $305^\circ = 305^\circ\left(\frac{\pi \text{ rad}}{180^\circ}\right) = 5.32 \text{ rad}$

29. $20.25^\circ = 20.25^\circ\left(\frac{\pi \text{ rad}}{180^\circ}\right) = 0.3534 \text{ rad}$

30. $148.38^\circ = 148.38^\circ\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = 2.5897 \text{ rad}$

31. $-636.2^\circ = -636.2^\circ\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = -11.10 \text{ rad}$

32. $385.4^\circ = 385.4^\circ\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = 6.726 \text{ rad}$

33. $270^\circ = 270^\circ\left(\dfrac{\pi}{180^\circ}\right) = \dfrac{3}{2}\pi$

34. $210^\circ = 210^\circ\left(\dfrac{\pi}{180^\circ}\right) = \dfrac{7}{6}\pi$

35. $-300^\circ = -300^\circ\left(\dfrac{\pi}{180^\circ}\right) = -\dfrac{5}{3}\pi$

36. $75^\circ = 75^\circ\left(\dfrac{\pi}{180^\circ}\right) = \dfrac{5}{12}\pi$

37. $\cos 237.4^\circ = -0.5388$

38. $\sin 141.3^\circ = 0.6252$

39. $\cot 295^\circ = -0.466$

40. $\tan 184^\circ = 0.0699$

41. $\csc 247.82^\circ = -1.0799$

42. $\sec 96.17^\circ = -9.304$

43. $\sin 542.8^\circ = -0.04885$

44. $\cos 326.72^\circ = 0.83600$

45. $\tan 301.4^\circ = -1.638$

46. $\sin 703.9^\circ = -0.2773$

47. $\tan 436.42^\circ = 4.1398$

48. $\cos\left(-162.32^\circ\right) = -0.95277$

49. $\sin\dfrac{9\pi}{5} = -0.5878$

50. $\sec\frac{5\pi}{8} = -2.613$

51. $\cos\left(-\frac{7\pi}{6}\right) = -0.8660$

52. $\tan\frac{23\pi}{12} = -0.2679$

53. $\sin 0.5906 = 0.5569$

54. $\tan 0.8035 = 1.037$

55. $\csc 2.153 = 1.197$

56. $\cos(-7.190) = 0.6163$

57. $\tan\theta = 0.1817,\ 0 \le \theta < 360^\circ$

$\theta_{ref} = \tan^{-1}(0.1817) = 10.30^\circ$

Since $\tan\theta$ is positive, θ must lie in Quadrant I or Quadrant III.

$\theta_1 = 10.30^\circ$

$\theta_3 = 180^\circ + 10.30^\circ = 190.30^\circ$

58. $\sin\theta = -0.9323,\ 0 \le \theta < 360^\circ$

$\theta_{ref} = \sin^{-1}(0.9323) = 68.80^\circ$

Since $\sin\theta$ is negative, θ must lie in Quadrant III or Quadrant IV.

$\theta_3 = 180^\circ + 68.80^\circ = 248.80^\circ$

$\theta_4 = 360^\circ - 68.80^\circ = 291.20^\circ$

59. $\cos\theta = -0.4730,\ 0 \le \theta < 360^\circ$

$\theta_{ref} = \cos^{-1}(0.4730) = 61.77^\circ$

Since $\cos\theta$ is negative, θ must lie in Quadrant II or Quadrant III.

$\theta_2 = 180^\circ - 61.77^\circ = 118.23^\circ$

$\theta_3 = 180^\circ + 61.77^\circ = 241.77^\circ$

60. $\cot\theta = 1.196,\ 0 \le \theta < 360^\circ$

$\theta_{ref} = \tan^{-1}(1/1.196) = 39.90^\circ$

Since $\cot\theta$ is positive, θ must lie in Quadrant I or Quadrant III.

$\theta_1 = 39.90^\circ$

$\theta_3 = 180^\circ + 39.90^\circ = 219.90^\circ$

61. $\cos\theta = 0.8387,\ 0 \le \theta < 2\pi$

$\theta_{ref} = \cos^{-1}(0.8387) = 0.5759$

Since $\cos\theta$ is positive, θ must lie in Quadrant I or Quadrant IV.

$\theta_1 = 0.5759$

$\theta_4 = 2\pi - 0.5759 = 5.707$

62. $9.569 = \csc\theta = \dfrac{1}{\sin\theta},\ 0 \le \theta < 2\pi$

$\sin\theta = \dfrac{1}{9.569} = 0.1045$

$\theta_{ref} = \sin^{-1}(0.1045) = 0.1047$

Since $\sin\theta$ is positive, θ must lie in Quadrant I or Quadrant II.

$\theta_1 = 0.1047$

$\theta_2 = \pi - 0.1047 = 3.037$

63. $\sin\theta = -0.8650,\ 0 \le \theta < 2\pi$

$\theta_{ref} = \sin^{-1}(0.8650) = 1.045$

Since $\sin\theta$ is negative, θ must lie in Quadrant III or Quadrant IV.

$\theta_3 = \pi + 1.045 = 4.187$

$\theta_4 = 2\pi - 1.045 = 5.238$

64. $\tan\theta = 8.480,\ 0 \le \theta < 2\pi$

$\theta_{ref} = \tan^{-1}(8.480) = 1.453$

Since $\tan\theta$ is positive, θ must lie in Quadrant I or Quadrant III.

$\theta_1 = 1.453$

$\theta_3 = \pi + 1.453 = 4.595$

65. $\cos\theta = -0.672,\ 0^\circ \le \theta < 360^\circ$

$\theta_{ref} = \cos^{-1}(0.672) = 47.78^\circ$

Since $\cos\theta$ is negative, θ must lie in Quadrant II or Quadrant III.

Since $\sin\theta$ is negative, θ must lie in Quadrant III or Quadrant IV.

To satisfy both conditions, θ must lie in Quadrant III.

$\theta_3 = 180^\circ + 47.78^\circ = 227.78^\circ$

66. $\tan\theta = -1.683,\ 0^\circ \le \theta < 360^\circ$

$\theta_{ref} = \tan^{-1}(1.683) = 59.28^\circ$

Since $\tan\theta$ is negative, θ must lie in Quadrant II or Quadrant IV.

Since $\cos\theta$ is negative, θ must lie in Quadrant II or Quadrant III.

To satisfy both conditions, θ must lie in Quadrant II.

$\theta_2 = 180^\circ - 59.28^\circ = 120.72^\circ$

67. $\cot\theta = 0.4291,\ 0^\circ \le \theta < 360^\circ$

$\theta_{ref} = \tan^{-1}(1/0.4291) = 66.78^\circ$

Since $\cot\theta$ is positive, θ must lie in Quadrant I or Quadrant III.

Since $\cos\theta$ is negative, θ must lie in Quadrant II or Quadrant III.

To satisfy both conditions, θ must lie in Quadrant III.

$\theta_3 = 180^\circ + 66.78^\circ = 246.78^\circ$

68. $\sin\theta = 0.2626,\ 0^\circ \le \theta < 360^\circ$

$\theta_{ref} = \sin^{-1}(0.2626) = 15.22^\circ$

Since $\sin\theta$ is positive, θ must lie in Quadrant I or Quadrant II.

Since $\tan\theta$ is negative, θ must lie in Quadrant II or Quadrant IV.

To satisfy both conditions, θ must lie in Quadrant II.

$\theta_2 = 180^\circ - 15.22^\circ = 164.78^\circ$

69. $\theta = 107.5^\circ\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = 1.876 \text{ rad}$

$$r = \frac{s}{\theta} = \frac{20.3 \text{ in}}{1.876} = 10.8 \text{ in}$$

70. $\theta = \dfrac{s}{r} = \dfrac{5840 \text{ ft}}{1060 \text{ ft}} = 5.51 \text{ rad}$

71. $A = \dfrac{1}{2}\theta r^2$

$$\theta = \frac{2A}{r^2} = \frac{2\left(265 \text{ mm}^2\right)}{(12.8 \text{ mm})^2} = 3.23 \text{ rad}$$

72. $\theta = 234.5^\circ\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = 4.093 \text{ rad}$

$$A = \frac{1}{2}\theta r^2$$

$$r = \sqrt{\frac{2A}{\theta}} = \sqrt{\frac{2\left(0.908 \text{ km}^2\right)}{4.093}} = 0.666 \text{ km}$$

73. $A = \dfrac{1}{2}\theta r^2$

$$\theta = \frac{2A}{r^2} = \frac{2\left(32.8 \text{ m}^2\right)}{(4.62 \text{ m})^2} = 3.07 \text{ rad}$$

$$s = \theta r = 3.07(4.62 \text{ m}) = 14.2 \text{ m}$$

74. $\theta = 98.5^\circ\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = 1.72 \text{ rad}$

$$A = \frac{1}{2}\theta r^2$$

$$r = \sqrt{\frac{2A}{\theta}} = \sqrt{\frac{2\left(0.493 \text{ ft}^2\right)}{1.72}} = 0.757 \text{ ft}$$

$$s = \theta r = 1.72(0.757 \text{ ft}) = 1.30 \text{ ft}$$

75. $\theta = 0.85^\circ\left(\dfrac{\pi \text{ rad}}{180^\circ}\right) = 0.015 \text{ rad}$

$$s = \theta r$$

$$r = \frac{s}{\theta} = \frac{7.94 \text{ in}}{0.015} = 540 \text{ in}$$

$$A = \frac{1}{2}\theta r^2$$

$$A = \frac{1}{2}(0.015)(540 \text{ in})^2 = 2100 \text{ in}^2$$

76. $s = \theta r$

$$\theta = \frac{s}{r} = \frac{7.61 \text{ cm}}{254 \text{ cm}} = 0.0300 \text{ rad}$$

$$A = \frac{1}{2}\theta r^2 = \frac{1}{2}(0.0300)(254 \text{ cm})^2 = 966 \text{ cm}^2$$

77. We wish to compute $\tan 200^\circ + 2\cot 110^\circ + \tan(-160^\circ)$.

We note $\tan 200^\circ = \tan(-160^\circ) = \tan 20^\circ$ and

$$\cot 110^\circ = \frac{\cos 110^\circ}{\sin 110^\circ} = \frac{-\sin 20^\circ}{\cos 20^\circ} = -\tan 20^\circ.$$

Therefore,

$$\begin{aligned}\tan 200^\circ + 2\cot 110^\circ + \tan(-160^\circ) &= \tan 20^\circ - 2\tan 20^\circ + \tan 20^\circ \\ &= 0\end{aligned}$$

78. We wish to compute $2\cos 40^\circ + \cos 140^\circ + \sin 230^\circ$.

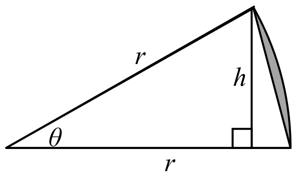
We note $\cos 140^\circ = -\cos 40^\circ$ and

$\sin 230^\circ = -\sin 50^\circ = -\cos 40^\circ$.

Therefore,

$$\begin{aligned}2\cos 40^\circ + \cos 140^\circ + \sin 230^\circ &= 2\cos 40^\circ - \cos 40^\circ - \cos 40^\circ \\ &= 0\end{aligned}$$

79.

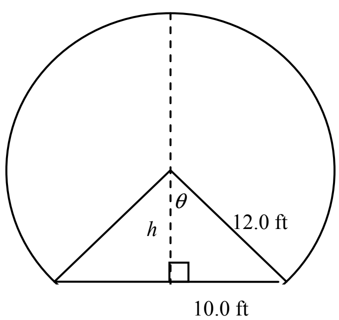


$$\sin\theta = \frac{h}{r}$$
$$h = r\sin\theta$$
$$A_{\text{segment}} = A_{\text{sector}} - A_{\text{triangle}}$$
$$A_{\text{segment}} = \frac{1}{2}\theta r^2 - \frac{1}{2}bh$$
$$A_{\text{segment}} = \frac{1}{2}\theta r^2 - \frac{1}{2}r\left(r\sin\theta\right)$$
$$A_{\text{segment}} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$
$$A_{\text{segment}} = \frac{1}{2}r^2\left(\theta - \sin\theta\right)$$

When $r = 4.00$ cm and $\theta = 1.45$,

$$A_{\text{segment}} = \frac{1}{2}(4.00\text{ cm})^2\left(1.45 - \sin 1.45\right)$$
$$= \frac{1}{2}\left(16.0\text{ cm}^2\right)\left(0.457287\right)$$
$$= 3.66\text{ cm}^2$$

80.



$$h = \sqrt{12.0^2 - 10.0^2} = \sqrt{44} = 6.63\text{ ft}$$
$$\theta = \sin^{-1}\frac{10.0}{12.0} = 56.4^\circ$$
$$2\theta = 112.9^\circ\ \frac{\pi\text{ rad}}{180^\circ} = 1.97\text{ rad}$$
$$A_{\text{sector}} = \frac{1}{2}(2\theta)r^2 = \frac{1}{2}\left(1.97\right)\left(12\right)^2 = 141.86\text{ ft}^2$$
$$A_{\text{triangle}} = \frac{1}{2}bh = \frac{1}{2}\left(20\right)\left(6.63\right) = 66.33\text{ ft}^2$$
$$A_{\text{segment}} = 141.86 - 66.33 = 75.52\text{ ft}^2$$
$$A_{tunnel} = A_{circle} - A_{\text{segment}}$$
$$A_{tunnel} = \pi\left(12.0\text{ ft}\right)^2 - 75.52\text{ ft}^2 = 377\text{ ft}^2$$

81. **(a)** $\theta = 20.0^\circ \dfrac{\pi \text{ rad}}{180^\circ} = \dfrac{\pi}{9.00} \text{ rad}$

$$r = \sqrt{40.0^2 + 30.0^2} = 50.0 \text{ m}$$

$$A = A_{\text{triangle}} + A_{\text{sector}}$$

$$A = \frac{1}{2}bh + \frac{1}{2}\theta r^2$$

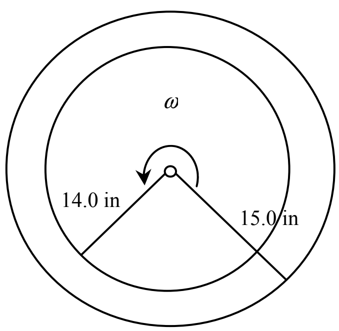
$$A = \frac{1}{2}(40.0 \text{ m})(30.0 \text{ m}) + \frac{1}{2}\ \frac{\pi}{9.00}\ (50.0 \text{ m})^2$$

$$A = 1040 \text{ m}^2$$

(b) The perimeter is the total of the lengths of the two legs, a radius of the circle, and the length of the circular arc.

$$P = 30.0 + 40.0 + 50.0 + 50.0 \cdot 20^\circ \cdot \frac{\pi}{180^\circ}$$
$$= 120.0 + 17.4533$$
$$= 137 \text{ m}$$

82.



$$v = 55.0 \text{ mi/h}\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{12 \text{ in}}{1 \text{ ft}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 968 \text{ in/s}$$

If the speedometer is calibrated to the smaller-diameter wheel, then a wheel of that size would be rotating at the rate

$$v = \omega r$$

$$\omega = \frac{v}{r} = \frac{968 \text{ in/s}}{14.0 \text{ in}} = 69.143 \text{ rad/s}$$

Since the larger-radius wheel is mounted on the same shaft, it will have the same rotational motion. Its edge velocity will be the new velocity of the car.

$$v = \omega r = (69.143 \text{ rad/s})(15.0 \text{ in}) = 1037 \text{ in/s}$$

$$v = 1037 \text{ in/s}\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(\frac{1 \text{ mi}}{5280 \text{ ft}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 58.9 \text{ mi/h}$$

83. $P = P_m \sin^2 377t$

$P = (0.120\text{ W}) \sin^2 (377 \cdot 2 \times 10^{-3})$

$P = 0.0562\text{ W}$

84. $x = a(\theta + \sin\theta)$

$x = (45.0\text{ cm})(0.175 + \sin 0.175)$

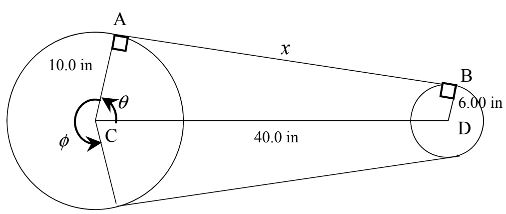
$x = 15.7\text{ cm}$

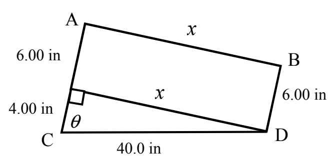
85. $s = \theta r$

$$\theta = \frac{s}{r} = \frac{6.60\text{ in}}{8.25\text{ in}} = 0.800\text{ rad}$$

$$\theta = 0.800\text{ rad } \frac{180^\circ}{\pi\text{ rad}} = 45.8^\circ$$

86.





Analyzing the right triangle,

$x = \sqrt{40.0^2 - 4.00^2}$

$x = \sqrt{1584}$

$x = 39.8\text{ in}$

$$\cos\theta = \frac{4.00}{40.0}$$

$$\theta = \cos^{-1}\frac{4.00}{40.0} = 1.47\text{ rad}$$

The interior angle for the belt on the small pulley is 2θ

(same as the angle that doesn't carry the belt on the large pulley).

The interior angle for the belt on the large pulley is ϕ.

$\phi = 2\pi - 2\theta = 2\pi - 2(1.47) = 3.34\text{ rad}$

Since $s = \theta r$

$L = \phi r_{\text{large}} + 2\theta r_{\text{small}} + 2x$

$L = 3.34(10.0\text{ in}) + 2(1.47)(6.0\text{ in}) + 2(39.8\text{ in})$

$L = 131\text{ in}$

87. $v = 3.5 \text{ mi/h} \left(\dfrac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\dfrac{1 \text{ h}}{60 \text{ min}}\right) = 308 \text{ ft/min}$

$$\omega = \frac{v}{r} = \frac{308 \text{ ft/min}}{\frac{1}{2}(4.8 \text{ ft})} = 128 \text{ rad/min}$$

$$= \frac{128 \text{ rad}}{1 \text{ min}} \times \frac{1 \text{ r}}{2\pi \text{ rad}} = 20.4 \text{ r/min}$$

88. $\omega = \dfrac{1 \text{ r}}{24.0 \text{ min}}\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right) = 0.2618 \text{ rad/min}$

$$v = \omega r = (0.2618 \text{ rad/min})\left(\frac{32.5 \text{ m}}{2}\right) = 4.25 \text{ m/min}$$

89. $\omega = \dfrac{1 \text{ r}}{28 \text{ d}}\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ d}}{24 \text{ h}}\right) = 0.0093 \text{ rad/h}$

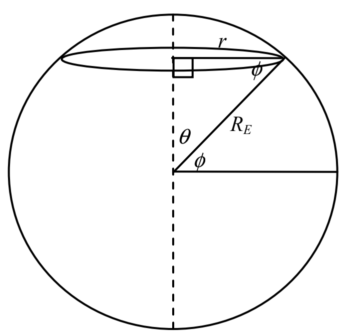
$$v = \omega r = (0.0093 \text{ rad/h})(240000 \text{ mi})$$

$$v = 2230 \text{ mi/h}$$

90. $\theta = \dfrac{s}{r} = \dfrac{1.22 \text{ m}}{1.06 \text{ m}} = 1.15 \text{ rad}$

$$\theta = 1.15 \text{ rad}\left(\frac{180^\circ}{\pi \text{ rad}}\right) = 65.9^\circ$$

91.



Latitude $\phi = 60^\circ$

(a) The angular distance from each city to the pole is $\theta = 90^\circ - \phi = 30^\circ$, making the over-pole angular distance $2\theta = 60^\circ$ or $\pi/3$ radians.

$$s = (2\theta)R_E = \frac{\pi}{3}(3960 \text{ mi}) = 4147 \text{ mi}$$

(b) The radius of the circle of latitude is found through

$$\cos\phi = \frac{r}{R_E}$$

At a latitude of 60°, this gives

$$r = 3960 \text{ mi } \cos 60^\circ$$
$$r = 1980 \text{ mi}$$

The net interior angle along the circle of latitude between 135°W and 30°E is 165° or 2.88 radians

$$s = 2.88(1980 \text{ mi}) = 5700 \text{ mi}$$

The results show that the distance over the north pole is shorter.

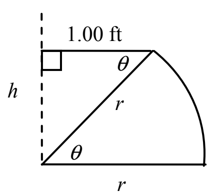
92. $$\theta = 220^\circ \ \frac{\pi \text{ rad}}{180^\circ} = \frac{11\pi}{9} \text{ rad}$$

$$A = \frac{1}{2}\theta r^2 = \frac{1}{2} \ \frac{11\pi}{9} \ \frac{15.0 \text{ cm}}{2}^{\ 2} = 108 \text{ cm}^2$$

93. $$\omega = 60.0 \text{ r/s} \ \frac{2\pi \text{ rad}}{1 \text{ r}} = 120.0\pi \text{ rad/s}$$

$$v = \omega r = (120.0\pi \text{ rad/s}) \ \frac{0.250 \text{ m}}{2} = 47.1 \text{ m/s}$$

94.



$$r = \frac{3.75 \text{ ft}}{2} = 1.875 \text{ ft}$$

$$\cos\theta = \frac{1.00}{1.875}$$

$$\theta = \cos^{-1}\frac{1.00}{1.875} = 1.01 \text{ rad}$$

$$\tan\theta = \frac{h}{1.00 \text{ ft}}$$

$$h = (1.00 \text{ ft})\tan(1.01)$$

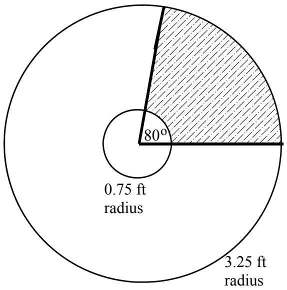
$$h = 1.59 \text{ ft}$$

$$A = 2\left(\frac{1}{2}\theta r^2\right) + \frac{1}{2}bh$$

$$A = (1.01)(1.875 \text{ ft})^2 + \frac{1}{2}(2.00 \text{ ft})(1.59 \text{ ft})$$

$$A = 5.14 \text{ ft}^2$$

95.



$$\theta = 80^\circ \left(\frac{\pi \text{ rad}}{180^\circ}\right) = \frac{4\pi}{9} \text{ rad}$$

$$A_{\text{circle}} = \pi \cdot 3.25^2$$

$$A_{\text{hole}} = \pi \cdot 0.75^2$$

$$A_{\text{sector(hole removed)}} = \frac{1}{2}\left(\frac{4\pi}{9}\right)\left(3.25^2 - 0.75^2\right)$$

$$A_{\text{hood}} = A_{\text{circle}} - A_{\text{hole}} - A_{\text{sector(hole removed)}}$$

$$A_{\text{hood}} = \pi \cdot 3.25^2 - \pi \cdot 0.75^2 - \frac{1}{2}\left(\frac{4\pi}{9}\right)\left(3.25^2 - 0.75^2\right)$$

$$A_{\text{hood}} = 24.4 \text{ ft}^2$$

96. Velocity of chain is the edge velocity of the sprocket :

$$v = \frac{s}{t} = \frac{108 \text{ cm}}{0.250 \text{ s}} = 432 \text{ cm/s}$$

$$v = r\omega$$

$$\omega = \frac{v}{r} = \frac{432 \text{ cm/s}}{(3.75 \text{ cm})} = 115 \text{ rad/s}$$

$$\omega = 115 \text{ rad/s} \quad \frac{1 \text{ r}}{2\pi \text{ rad}} \quad = 18.3 \text{ r/s}$$

97. $\omega = 80\ 000 \text{ r/min} \quad \dfrac{2\pi \text{ rad}}{1 \text{ r}} \quad \dfrac{1 \text{ min}}{60 \text{ s}} \quad = 8377.6 \text{ rad/s}$

$$v = r\omega = (3.60 \text{ cm})(8377.6 \text{ rad/s}) = 30159 \text{ cm/s}$$

$$v = 30159 \text{ cm/s} \quad \frac{1 \text{ m}}{100 \text{ cm}} \quad = 302 \text{ m/s}$$

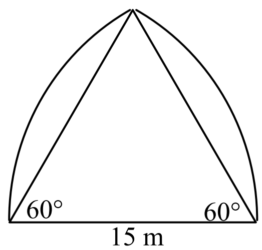
98. $p = 2.44 + 2.44 + s$

$$s = p - 4.88 = 7.32 - 4.88 = 2.44 \text{ cm}$$

$$\theta = \frac{s}{r} = \frac{2.44 \text{ cm}}{2.44 \text{ cm}} = 1.00 \text{ rad}$$

$$\theta = 1.00 \text{ rad} \quad \frac{180^\circ}{\pi \text{ rad}} \quad = 57.3^\circ$$

99.



Area in the arch will be the area of one sector, plus the segment area of one side. This segment area is the difference between the sector area and the equilateral triangle in the sector. Since the interior triangle has all sides 15.0 m long, it is equilateral, all internal angles are 60°.

$$\theta = 60^\circ = \frac{\pi}{3}\text{ rad}$$

$$h = (15.0\text{ m})\sin 60^\circ = 15.0\text{ m}\frac{\sqrt{3}}{2}$$

$$A = A_{\text{sector}} + A_{\text{segment}}$$

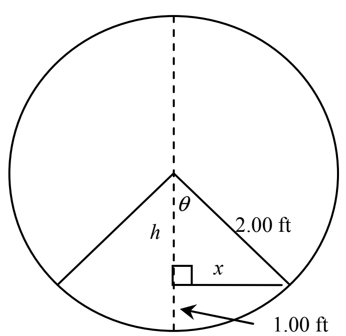
$$A = \frac{1}{2}\theta r^2 + \left(\frac{1}{2}\theta r^2 - \frac{1}{2}bh\right)$$

$$A = \theta r^2 - \frac{1}{2}bh$$

$$A = \frac{\pi}{3}(15.0\text{ m})^2 - \frac{1}{2}(15.0\text{ m})\left(15.0\text{ m}\frac{\sqrt{3}}{2}\right)$$

$$A = 138\text{ m}^2$$

100.



$$h = 2.00\text{ ft} - 1.00\text{ ft} = 1.00\text{ ft}$$

$$x = \sqrt{2.00^2 - h^2} = \sqrt{2.00^2 - 1.00^2} = 1.732\text{ ft}$$

$$\cos\theta = \frac{1.00\text{ ft}}{2.00\text{ ft}}$$

$$\theta = \cos^{-1}\frac{1.00\text{ ft}}{2.00\text{ ft}} = 60.0^\circ$$

$$2\theta = 120.0^\circ\ \frac{\pi\text{ rad}}{180^\circ} = \frac{2\pi}{3}\text{ rad}$$

$$A_{\text{sector}} = \frac{1}{2}(2\theta)r^2 = \frac{1}{2}\left(\frac{2\pi}{3}\right)(2.00\text{ ft})^2 = 4.18879\text{ ft}^2$$

$$A_{\text{triangle}} = \frac{1}{2}(2x)h = (1.732)(1.00) = 1.732\text{ ft}^2$$

$$A_{\text{oil}} = 4.18879 - 1.732 = 2.457\text{ ft}^2$$

$$V_{oil} = A_{oil}L$$

$$V_{oil} = 2.457\text{ ft}^2(40\text{ ft}) = 98.3\text{ ft}^3$$

101. $\theta = \left(\frac{0.0008^\circ}{2}\right)\left(\frac{\pi\text{ rad}}{180^\circ}\right) = 6.9813\times10^{-6}\text{ rad}$

$$\tan\theta = \frac{y}{x}$$

$$x = \frac{y}{\tan\theta}$$

Using Eq. (8.17),

$\tan\theta = \theta$ for small θ.

$$x = \frac{y}{\theta} = \frac{2.50\text{ km}}{6.9813\times10^{-6}}$$

$$x = 3.58\times10^5\text{ km}$$

102. For the radius of Venus, the angle is $\frac{1}{2}(15'')$, or $7.5''$

$$\theta = (7.5'')\left(\frac{1^\circ}{3600''}\right)\left(\frac{\pi\text{ rad}}{180^\circ}\right) = 3.6361\times10^{-5}\text{ rad}$$

$$\tan\theta = \frac{R_{Venus}}{x}$$

$$R_{Venus} = x\tan\theta$$

Using Eq. (8.17),

$\tan\theta = \theta$ for small θ.

$$R_{Venus} = x\theta = (1.04\times10^8\text{ mi})(3.6361\times10^{-5}\text{ rad})$$

$$R_{Venus} = 3780\text{ mi}$$

$$d_{Venus} = 7560\text{ mi}$$

103. Convert the angular velocity of 1 revolution every 1.95 h into rad/h.
Then find the total radius of the orbit by adding the lunar radius to the altitude.
Use the equation $v = \omega r$ to solve for the velocity.

$$\omega = \left(\frac{1\text{ r}}{1.95\text{ h}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right) = 3.222\text{ rad/h}$$

$$r = 1080\text{ mi} + 70\text{ mi} = 1150\text{ mi}$$

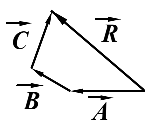
$$v = \omega r = (3.222\text{ rad/h})(1150\text{ mi}) = 3705\text{ mi/h}$$

Chapter 9

Vectors and Oblique Triangles

9.1 Introduction to Vectors

1.



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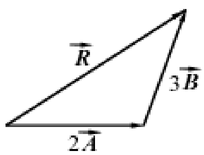
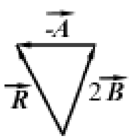
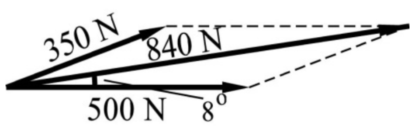


3.



4.



5. **(a)** scalar, no direction given

(b) vector, magnitude and direction both given

6. **(a)** 25 mi/h is scalar, it has magnitude but no direction.

(b) 25 mi/h from N is a vector, it has magnitude and direction.

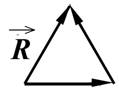
7. **(a)** 10 lb down is a vector, it has magnitude and direction.

(b) 10 lb is scalar; it has magnitude but not direction.

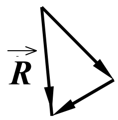
8. **(a)** 400 ft/s is scalar; it has magnitude but no direction.

(b) 400 ft/s perpendicular is a vector, it has magnitude and direction.

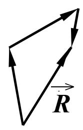
9.



10.



11.



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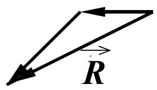

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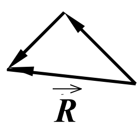

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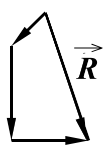
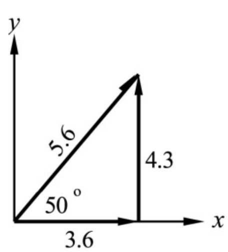
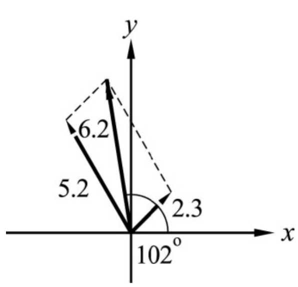


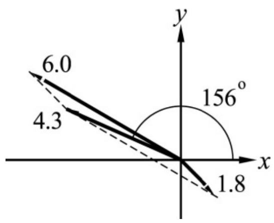
15. 5.6 cm, 50°



16. 6.2 cm, 102°



17. 4.3 cm, 156°



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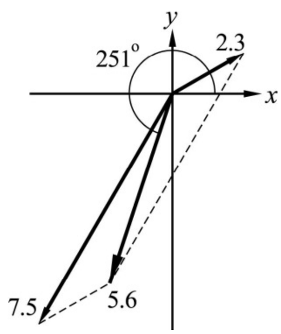

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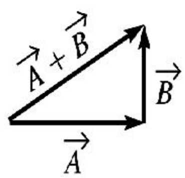

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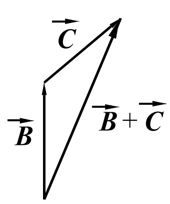

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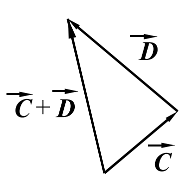

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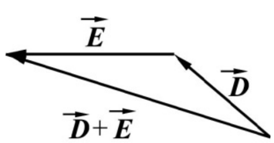

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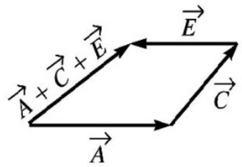

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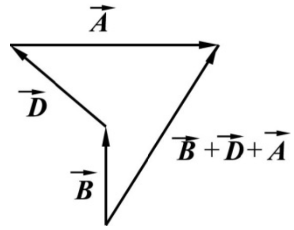

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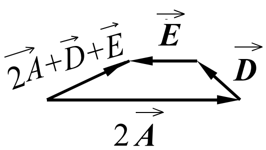

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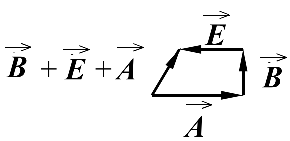

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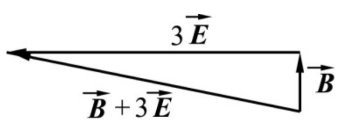

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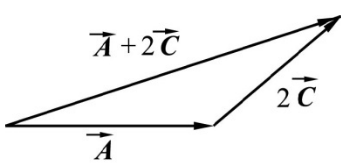

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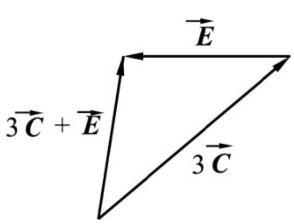

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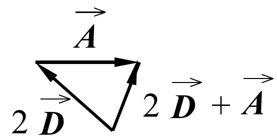

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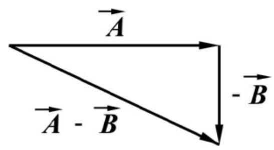

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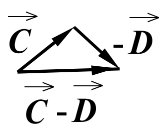

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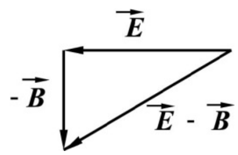

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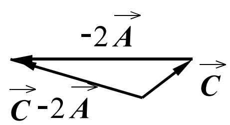

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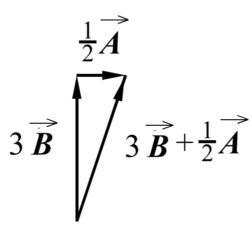

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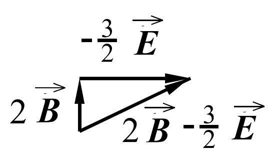

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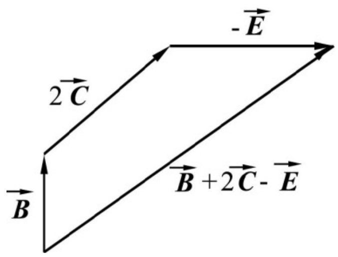
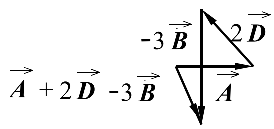


38.



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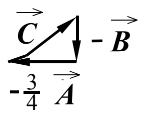

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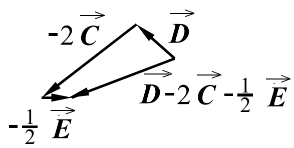
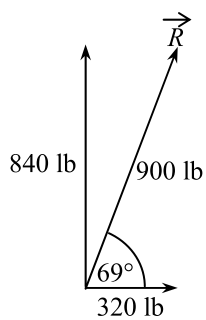
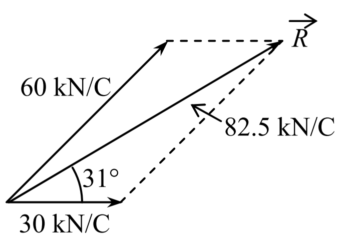


41.

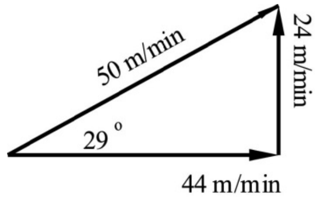


From the drawing, $\vec{R}$ is approximately 900 lb at 69°.

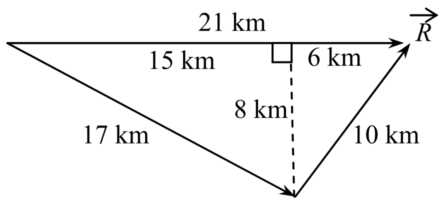
42. From the scale drawing, we find that the resultant electric field intensity is about 82.5 kN/C and acts at an angle of 31°.



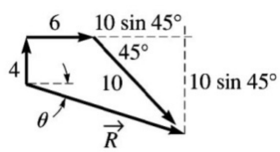
43.



44. From the scale drawing, we find that the resultant displacement is 21 km due east and at the same altitude of the jet's original position.

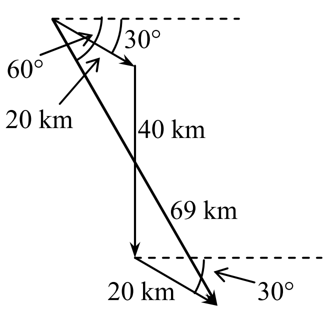


45.

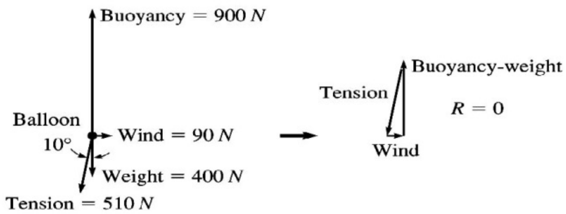


From the drawing,
$R = 13$ km
$\theta = 13^\circ$

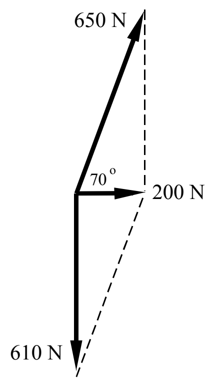
46. From the scale drawing, we see that the ship's displacement is 69 km and acts at an angle of 60° south of east.



47. From the scale drawing, we see that the resultant force is 0.



48.

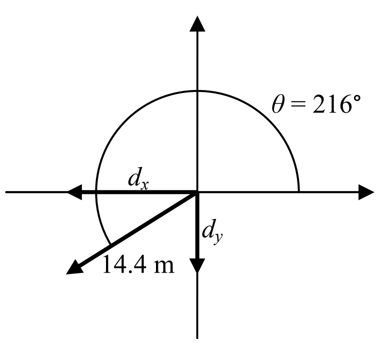


Since the total force is zero, the three vectors form a closed triangle. Measurements from the drawing indicate the horizontal force of the rope is approximately 200 N to the left.

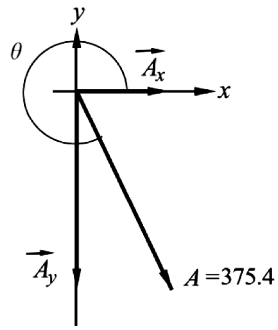
9.2 Components of Vectors

1.

$$d_x = d\cos\theta$$
$$d_x = 14.4\cos 216^\circ = -11.6 \text{ m}$$
$$d_y = d\sin\theta$$
$$d_y = 14.4\sin 216^\circ = -8.46 \text{ m}$$

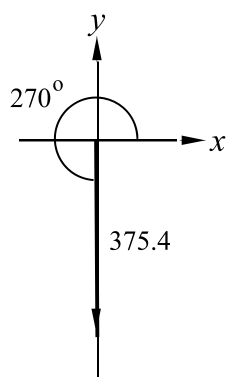


2.



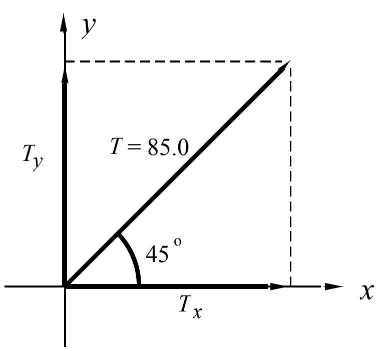
$$A_x = A\cos\theta$$
$$A_x = 375.4\cos 295.32^\circ$$
$$= 160.5$$
$$A_y = A\sin\theta$$
$$A_y = 375.4\sin 295.32^\circ$$
$$= -339.3$$

3.



$$A_x = 0$$
$$A_y = -375.4$$

4.



$$T_x = 85.0\cos 45^\circ = 60.1 \text{ lb}$$
$$T_y = 85.0\sin 45^\circ = 60.1 \text{ lb}$$

5. $V_x = 750\cos 28^\circ = 662$
$V_y = 750\sin 28^\circ = 352$

6. $V_x = 750\cos 105^\circ = -750\cos 75^\circ = -194$
$V_y = 750\sin 105^\circ = 750\sin 75^\circ = 724$

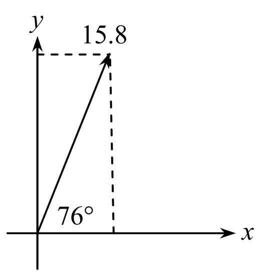
7. $V_x = -750\cos 62.3^\circ = 750\cos 242.3^\circ = -349$
$V_y = -750\sin 62.3^\circ = 750\sin 242.3^\circ = -664$

8. $V_x = 750\cos 52.4^\circ = 750\cos 307.6^\circ = 458$
$V_y = -750\sin 52.4^\circ = 750\sin 307.6^\circ = -594$

9. $V_x = -750$
$V_y = 0$

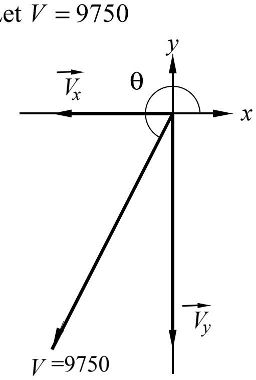
10. $V_x = 0$
$V_y = -750$

11. Let $V = 15.8$



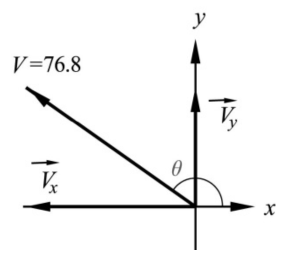
$V_x = 15.8\cos 76.0^\circ = 3.82$ lb
$V_y = 15.8\sin 76.0^\circ = 15.3$ lb

12. Let $V = 9750$



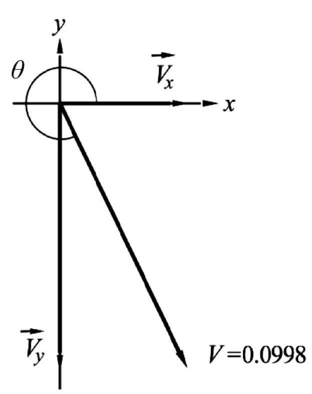
$V_x = V\cos 243.0^\circ = 9750(-0.4540) = -4430$ N
$V_y = V\sin 243.0^\circ = 9750(-0.8910) = -8690$ N

13. Let $V = 76.8$



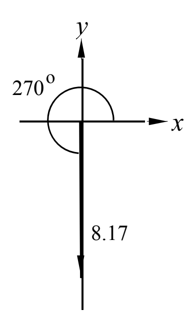
$$V_x = V\cos 145.0^\circ = 76.8(-0.819) = -62.9 \text{ m/s}$$
$$V_y = V\sin 145.0^\circ = 76.8(0.574) = 44.1 \text{ m/s}$$

14. Let $V = 0.0998$



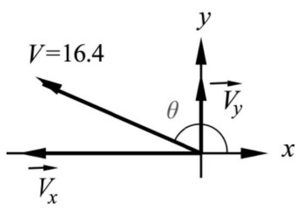
$$V_x = V\cos 296.0^\circ = 0.0998(0.438) = 0.0437 \text{ dm/s}$$
$$V_y = V\sin 296.0^\circ = 0.0998(-0.899) = -0.0897 \text{ dm/s}$$

15. Let $V = 8.17$, $\theta = 270^\circ$



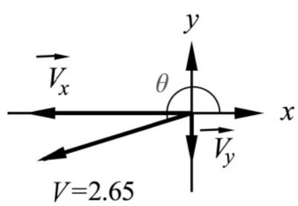
$$V_x = 8.17\cos 270^\circ = 0 \text{ ft/s}^2$$
$$V_y = 8.17\sin 270^\circ = -8.17 \text{ ft/s}^2$$

16. Let $V = 16.4$



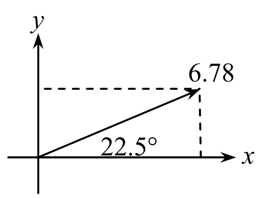
$$V_x = V\cos 156.5^\circ = 16.4(-0.917) = -15.0 \text{ cm/s}^2$$
$$V_y = V\sin 156.5^\circ = 16.4(0.399) = 6.54 \text{ cm/s}^2$$

17. Let $V = 2.65$



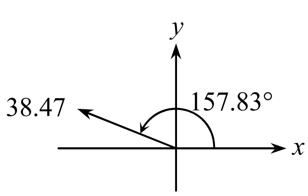
$$V_x = V\cos 197.3^\circ = 2.65(-0.955) = -2.53 \text{ mN}$$
$$V_y = V\sin 197.3^\circ = 2.65(-0.297) = -0.788 \text{ m/N}$$

18. Let $V = 6.78$



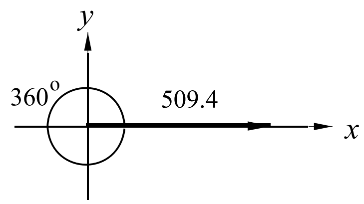
$$V_x = V\cos 22.5^\circ = 6.78(0.924) = 6.26 \text{ N}$$
$$V_y = V\sin 22.5^\circ = 6.78(0.383) = 2.59 \text{ N}$$

19. Let $V = 38.47$



$$V_x = 38.47\cos 157.83^\circ$$
$$= -35.63 \text{ ft}$$
$$V_y = 38.47\sin 157.83^\circ$$
$$= 14.52 \text{ ft}$$

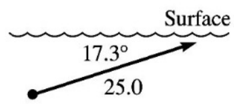
20. Let $V = 509.4,\ \theta = 360^\circ$



$V_x = V\cos 360^\circ = 509.4$ m

$V_y = V\sin 360^\circ = 0$ m

21.



$V_x = 25.0\cos 17.3^\circ = 23.9$ km/h

$V_y = 25.0\sin 17.3^\circ = 7.43$ km/h

22. $V_x = V\cos 66.4^\circ = 18.0(0.400) = 7.2$ ft/s

$V_y = -V\sin 66.4^\circ = -18.0(0.916) = -16.5$ ft/s

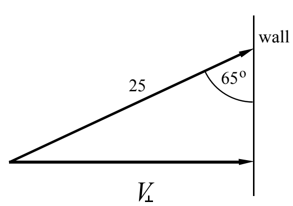
The component toward the highway is 16.5 ft/s.

23. $F_x = F\cos 3.5^\circ = 2790(0.998) = 2784$ lb

$F_y = F\sin 3.5^\circ = 2790(0.0610) = 170$ lb

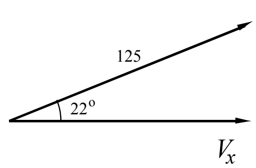
$W = 2784$ lb and $T = 170$ lb.

24.



$V_\perp = 25\sin 65^\circ = 23$ mi/h

25.



$V_x = 125\cos 22.0^\circ = 116$ km/h

26. $F_x = F\cos 78.6^\circ = 3.50(0.198) = 0.693$ ft

$F_y = F\sin 78.6^\circ = 3.50(0.980) = 3.43$ ft

27. The y-components must have the same magnitude and opposite directions.

$820\sin\theta = 760\sin 8^\circ$

$\sin\theta = \frac{760}{820}(0.139173) = 0.1289897$

$\theta = \sin^{-1}(0.1289897) = 7.4^\circ$

28. The angle of the rope to the surface is

$\theta = \sin^{-1} \quad \frac{12.0}{36.0} \quad = 19.47122^\circ$

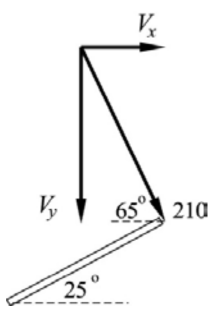
The vertical component is

$55 \sin \theta = 18.33$ lb

The horizontal component is

$55 \cos \theta = 51.85$ lb

29.



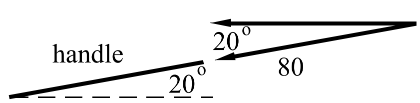
$V_x = 210\cos 65^\circ = 89$ N

$V_y = -210\sin 65^\circ = -190$ N

30. $28\sin \quad \tan^{-1}\frac{2.5}{9.5} \quad = 7.1$

The skier is rising 7.1 ft/s when leaving the ramp.

31. $80\cos 20^\circ = 75$ lb



32. $F_1 = 60.5$ lb

$F_{1y} = F_1 \sin 82.4^\circ = 60.0$ lb

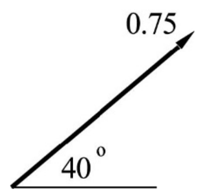
$F_2 = 37.2$ lb

$F_{2y} = F_2 \sin 31.9^\circ = 19.6$ lb

Total upward force $= F_{1y} + F_{2y} = 60.0 + 19.6$

$= 79.6$ lb

33.



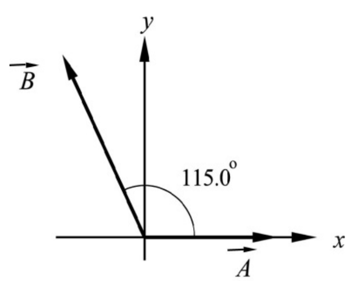
horizontal component $= 0.75\cos 40^\circ$
$= 0.57$ (km/h)/m
vertical component $= 0.75\sin 40^\circ$
$= 0.48$ (km/h)/m

34.

$$V_x = V\cos 2.55^\circ = 18550(0.9990)$$
$$= 18530 \text{ mi/h}$$
$$V_y = -V\sin 2.55^\circ = -18550(0.04449)$$
$$= -825 \text{ mi/h}$$

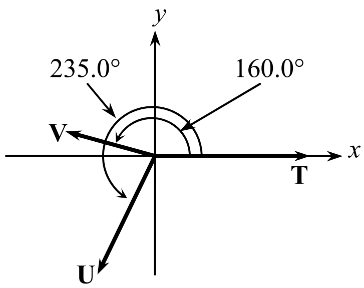
9.3 Vector Addition by Components

1. $A = 1200 = A_x$, $B = 1750$
$Ay = 0$



$$R_x = A_x + B_x = 1200 + 1750\cos 115^\circ = 460.4$$
$$R_y = A_y + B_y = 0 + 1750\sin 115^\circ = 1586$$
$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{460.4^2 + 1586^2} = 1650$$
$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{1586}{460.4} = 73.8^\circ$$

2.



$$\begin{aligned} R_x &= T_x + U_x + V_x \\ &= 422\cos 0^\circ + 405\cos 235^\circ + 210\cos 160^\circ \\ &= -7.6339 \\ R_y &= T_y + U_y + V_y \\ &= 422\sin 0^\circ + 405\sin 235^\circ + 210\sin 160^\circ \\ &= -259.9323 \end{aligned}$$

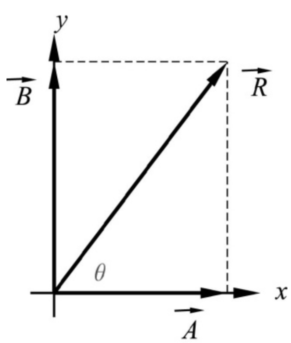
$$R = \sqrt{(-7.6339)^2 + (-259.9323)^2} = 260$$

$$\theta_{\text{ref}} = \tan^{-1}\left(\frac{-259.9323}{-7.6339}\right) = 88.31778^\circ$$

Since θ is in quadrant III because both components are negative, we have

$$\theta = \theta_{\text{ref}} + 180^\circ = 268^\circ$$

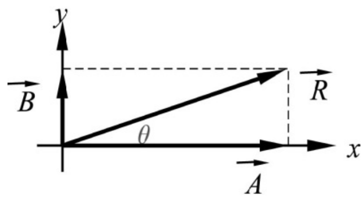
3.



$$R = \sqrt{14.7^2 + 19.2^\circ} = 24.2$$

$$\theta = \tan^{-1}\frac{19.2}{14.7} = 52.6^\circ$$

4.

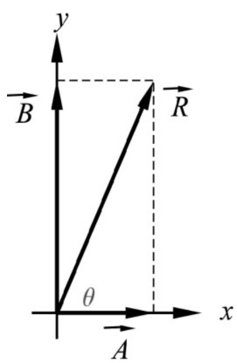


$R = \sqrt{A^2 + B^2}$; $A = 592, B = 195$
$R = \sqrt{592^2 + 195^2} = \sqrt{388\ 489} = 623$

$$\tan\theta = \frac{B}{A} = \frac{195}{592} = 0.329$$

$\theta = 18.2°$ (from $\vec{A}$)

5.

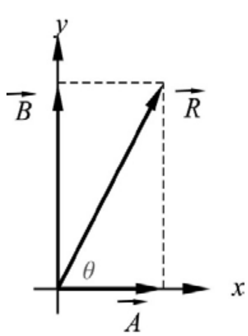


$R = \sqrt{3.086^2 + 7.143^2} = \sqrt{60.54} = 7.781$

$$\tan\theta = \frac{7.143}{3.086} = 2.315$$

$\theta = 66.63°$ (with $\vec{A}$)

6.

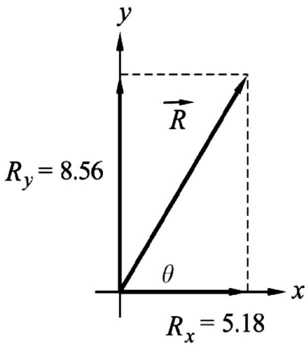


$R = \sqrt{1734^2 + 3297^2} = \sqrt{13\ 876\ 965} = 3725$

$$\tan\theta = \frac{B}{A} = \frac{3297}{1734} = 1.901$$

$\theta = 62.26°$ (from $\vec{A}$)

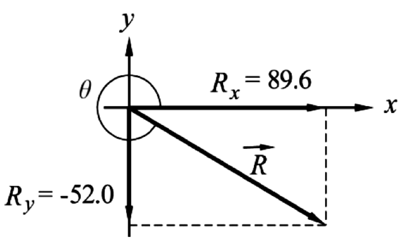
7.



$R = \sqrt{5.18^2 + 8.56^2} = 10.0$

$\theta = \tan^{-1} \dfrac{8.56}{5.18} = 58.8°$

8.



$R_x = 89.6, R_y = -52.0$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{89.6^2 + (-52.0)^2} = 104$

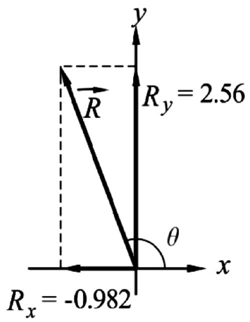
$\tan \theta_{\text{ref}} = \left| \dfrac{-52.0}{89.6} \right| = 0.580$

$\theta_{\text{ref}} = 30.1°$

$\theta = 360° - 30.1° = 329.9°$

(θ is in Quadrant IV since R_x is positive and R_y is negative)

9.



$R_x = -0.982, R_y = 2.56$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-0.982)^2 + 2.56^2} = 2.74$

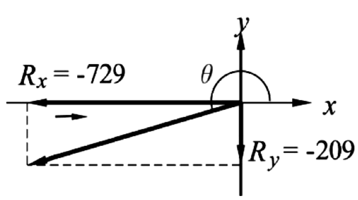
$\tan\theta_{\text{ref}} = \left|\dfrac{2.56}{-0.982}\right| = 2.61$

$\theta_{\text{ref}} = 69.0°$

$\theta = 180° - 69.0° = 111.0°$

(θ is in Quad II since R_x is negative and R_y is positive)

10.



$R_x = -729, R_y = -209$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-729)^2 + (-209)^2} = 758$

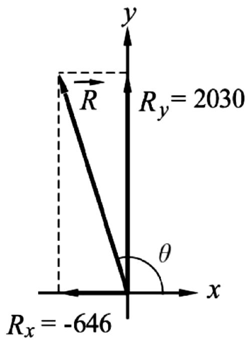
$\tan\theta_{\text{ref}} = \left|\dfrac{-209}{-729}\right| = 0.287$

$\theta_{\text{ref}} = 16.0°$

$\theta = 180° + 16.0° = 196.0°$

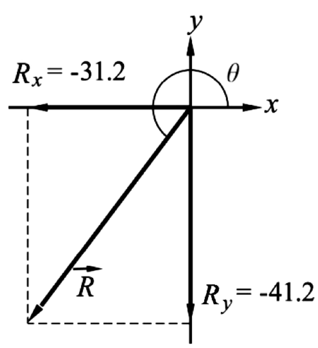
(θ is in Quad III since R_x is negative and R_y is negative)

11.



$$R = \sqrt{R_x^2 + R_y^2}$$
$$= \sqrt{(-646)^2 + (2030)^2} = 2130$$
$$\tan\theta_{\text{ref}} = \frac{|2030|}{|-646|} = 3.142$$
$$\theta_{\text{ref}} = 72.3^\circ$$
$$\theta = 107.7^\circ$$
(θ is in Quad II since R_x is negative and R_y is positive.)

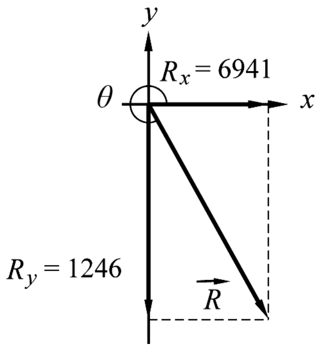
12.



$R_x = -31.2, R_y = -41.2$
$$R = \sqrt{(-31.2)^2 + (-41.2)^2} = 51.7$$
$$\tan\theta_{\text{ref}} = \left|\frac{-41.2}{-31.2}\right| = 1.321$$
$$\theta_{\text{ref}} = 52.9^\circ$$
$$\theta = 180^\circ + 52.9^\circ = 232.9^\circ$$

(θ is in Quad III since R_x is negative and R_y is negative)

13.



$R_x = 6941,\ R_y = -1246$

$R = \sqrt{6941^2 + (-1246)^2} = 7052$

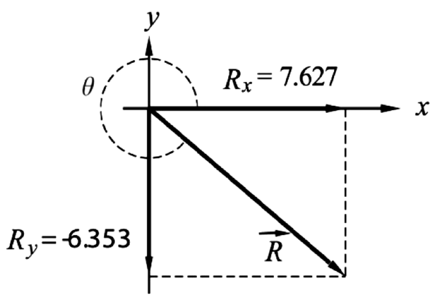
$\tan\theta_{\text{ref}} = \left|\dfrac{-1246}{6941}\right| = 0.1795$

$\theta_{\text{ref}} = 10.18^\circ$

$\theta = 360^\circ - 10.18^\circ = 349.82^\circ$

(θ is in QIV since R_x is positive and R_y is negative)

14.



$R_x = 7.627, R_y = -6.353$

$R = \sqrt{7.627^2 + (-6.353)^2} = 9.926$

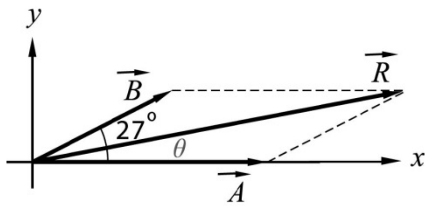
$\tan\theta_{\text{ref}} = \left|\dfrac{-6.353}{7.627}\right| = 0.8330$

$\theta_{\text{ref}} = 39.79^\circ$

$\theta = 360^\circ - 39.79^\circ = 320.21^\circ$

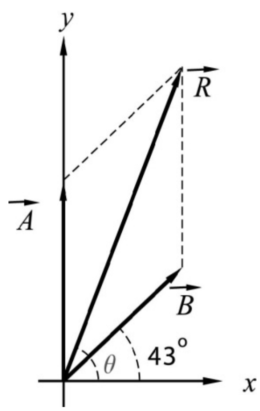
(θ is in Quad IV since R_x is positive and R_y is negative)

15.



$$R_x = 18 + 12\cos 27°$$
$$R_y = 0 + 12\sin 27°$$
$$R = \sqrt{R_x^2 + R_y^2} = 29.2$$
$$\theta = \tan^{-1}\frac{R_y}{R_x} = 10.8°$$

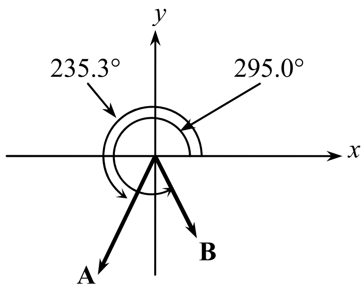
16.



$$A = 154, \theta_A = 90.0°;\ A_x = 154\cos 90.0° = 0$$
$$A_y = 154\sin 90.0° = 154$$
$$B = 128, \theta_B = 43.0°;\ B_x = 128\cos 43.0° = 93.6$$
$$B_y = 128\sin 43.0° = 87.3$$
$$R_x = A_x + B_x = 0 + 93.6 = 93.6$$
$$R_y = A_y + B_y = 154 + 87.3 = 241.3$$
$$R = \sqrt{93.6^2 + 241.3^2} = 259$$

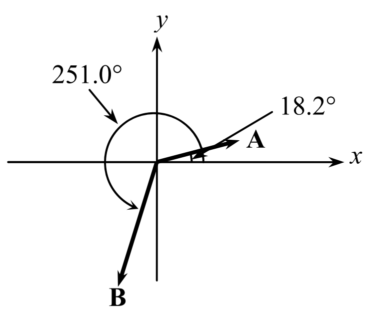
$$\tan\theta = \frac{R_y}{R_x} = \frac{241.3}{93.6} = 2.578, \theta = 68.8°$$

17.



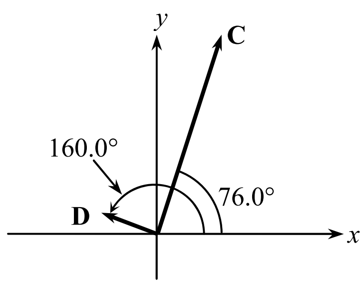
$$A_x = 368\cos 235.3^\circ = -209.4948646$$
$$A_y = 368\sin 235.3^\circ = -302.5490071$$
$$B_x = 227\cos 295.0^\circ = 95.93434542$$
$$B_y = 227\sin 295.0^\circ = -205.7318677$$
$$R_x = A_x + B_x = -113.5605192$$
$$R_y = A_y + B_y = -508.2808748$$
$$R = \sqrt{R_x^{\,2} + R_y^{\,2}}$$
$$= 521$$
$$\theta_{\text{ref}} = \tan^{-1}\left(\frac{R_y}{R_x}\right) = 77.40576722^\circ$$
$$\theta = 180^\circ + 77.4^\circ = 257.4^\circ$$

18.



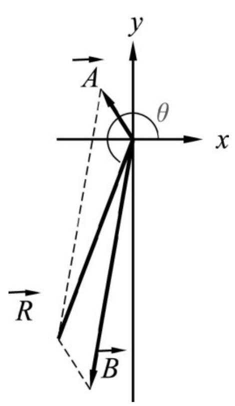
$$A_x = 30.7\cos 18.2^\circ = 29.164142$$
$$A_y = 30.7\sin 18.2^\circ = 9.588682$$
$$B_x = 45.2\cos 251.0^\circ = -14.715681$$
$$B_y = 45.2\sin 251.0^\circ = -42.737440$$
$$R_x = A_x + B_x = 14.448461$$
$$R_y = A_y + B_y = -33.148758$$
$$R = \sqrt{R_x^{\,2} + R_y^{\,2}}$$
$$= 36.160727$$
$$\theta_{\text{ref}} = \tan^{-1}\left(\frac{R_y}{R_x}\right) = -66.449185^\circ$$
$$\theta = 360^\circ - 66.4^\circ = 293.6^\circ$$

19.



$$C_x = 5650\cos 76.0^\circ = 1366.86$$
$$C_y = 5650\sin 76.0^\circ = 5482.17$$
$$D_x = 1280\cos 160.0^\circ = -1202.81$$
$$D_y = 1280\sin 160.0^\circ = -437.79$$
$$R_x = C_x + D_x = 164.05$$
$$R_y = C_y + D_y = 5044.38$$
$$R = \sqrt{R_x^{\ 2} + R_y^{\ 2}}$$
$$= 5047.05$$
$$\theta = \theta_{\text{ref}} = \tan^{-1}\left(\frac{R_y}{R_x}\right) = 88.14^\circ$$

20.

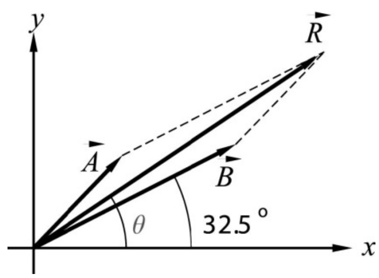


$$A = 6.89, \theta_A = 123.0^\circ$$
$$A_x = 6.89\cos 123.0^\circ = -3.75$$
$$A_y = 6.89\sin 123.0^\circ = 5.78$$
$$B = 29.0, \theta_B = 260.0^\circ$$
$$B_x = 29.0\cos 260.0^\circ = -5.04$$
$$B_y = 29.0\sin 260.0^\circ = -28.6$$
$$R_x = -3.75\sin 260.0^\circ = -8.79$$
$$R_y = 5.78 - 28.6 = -22.8$$
$$R = \sqrt{(-8.79)^2 + (-22.8)^2} = 24.4$$

$$\tan\theta_{\text{ref}} = \left|\frac{-22.8}{-8.79}\right| = 25.9$$

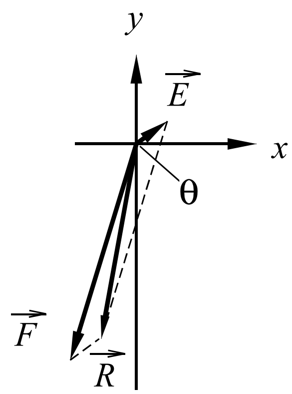
$$\theta_{\text{ref}} = 68.9^\circ$$
$$\theta = 180^\circ + 68.9^\circ = 248.9^\circ$$

21.



$$\begin{aligned} R_x &= A_x + B_x \\ &= 9.821\cos 34.27° + 17.45\cos 752.5° \\ R_y &= 9.821\sin 34.27° + 17.45\sin 752.5° \\ R &= \sqrt{R_x^2 + R_y^2} = 27.27 \\ \theta &= \tan^{-1}\frac{R_y}{R_x} = 33.14° \end{aligned}$$

22.



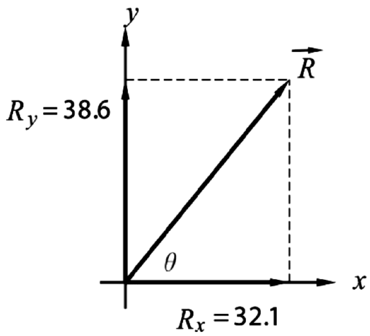
$$\begin{aligned} E &= 1653,\ \theta_E = 36.37^{\circ} \\ E_x &= 1653\cos 36.37^{\circ} = 1331 \\ E_y &= 1653\sin 36.37^{\circ} = 980 \\ F &= 9807,\ \theta_F = 253.06^{\circ} \\ F_x &= 9807\cos 253.06^{\circ} = -2857 \\ F_y &= 9807\sin 253.06^{\circ} = 9381 \\ R_x &= 1331 - 2857 = -1526 \\ R_y &= 980 - 9381 = -8401 \\ R &= \sqrt{(-1526)^2 + (-8401)^2} = 8538 \end{aligned}$$

$$\tan\theta_{\text{ref}} = \left|\frac{-8401}{-1526}\right|$$

$$\theta_{\text{ref}} = 79.70^{\circ}$$

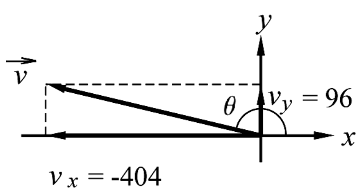
$$\theta = 180^{\circ} + \theta_{\text{ref}} = 259.70^{\circ}$$

23.



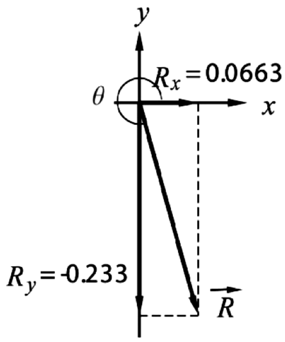
$$R_x = A_x + B_x + C_x$$
$$R_x = 21.9\cos 236.2° + 96.7\cos 11.5° + 62.9\cos 143.4°$$
$$R_y = A_y + B_y + C_y$$
$$R_y = 21.9\sin 236.2° + 96.7\sin 11.5° + 62.9\sin 143.4°$$
$$R = \sqrt{R_x^2 + R_y^2} = 50.2$$
$$\theta = \tan^{-1}\frac{R_y}{R_x} = 50.3°$$

24.



$$R = 630,\ \theta_R = 189.6°$$
$$R_x = 630\cos 189.6° = -621$$
$$R_y = 630\sin 189.6° = -105$$
$$F = 176,\ \theta_F = 320.1°$$
$$F_x = 176\cos 320.1° = 135$$
$$F_y = 176\sin 320.1° = -113$$
$$T = 324,\ \theta_T = 75.4°$$
$$T_x = 324\cos 75.4° = 81.67$$
$$T_y = 324\sin 75.4° = 314$$
$$U_x = -621 + 135 + 81.6 = -404$$
$$U_y = -105 - 113 + 314 = 96$$
$$U = \sqrt{(-404)^2 + 96^2} = 415$$
$$\tan\theta_{\text{ref}} = \left|\frac{96}{-404}\right|$$
$$\theta_{\text{ref}} = 13.40°$$
$$\theta = 180° - 13.4° = 166.6°$$
(θ is in Quad II since R_x is negative and R_y is positive)

25.



$U = 0.364, \theta_U = 175.7^\circ$

$U_x = 0.364\cos 175.7^\circ = -0.363$

$U_y = 0.364\sin 175.7^\circ = 0.0273$

$V = 0.596, \theta_V = 319.5^\circ$

$V_x = 0.596\cos 319.5^\circ = 0.453$

$V_y = 0.596\sin 319.5^\circ = -0.387$

$W = 0.129, \theta_W = 100.6$

$W_x = 0.129\cos 100.6^\circ = -0.0237$

$W_y = 0.139\sin 100.6^\circ = 0.137$

Using stored values (without rounding),

$R_x = 0.0665$

$R_y = -0.223$

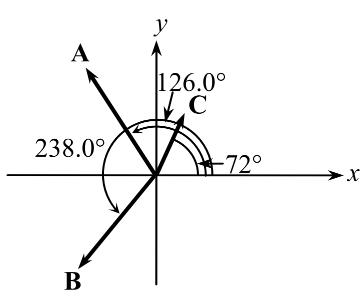
$R = \sqrt{R_x^2 + R_y^2} = 0.242$

$\theta_{\text{ref}} = \tan^{-1}\dfrac{|-0.223|}{|0.0665|} = 74.1^\circ$

$\theta = 360^\circ - 74.1^\circ = 285.9^\circ$

(θ is in Quad IV since R_x is positive and R_y is negative.)

26.



$R_x = A_x + B_x + C_x$

$= 64\cos 126^\circ + 59\cos 238^\circ + 32\cos 72^\circ$

$= -58.99$

$$R_y = A_y + B_y + C_y$$
$$= 64\sin 126^\circ + 59\sin 238^\circ + 32\sin 72^\circ$$
$$= 32.18$$
$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-58.99)^2 + 32.18^2} = 67.20$$
$$\theta_{\text{ref}} = \tan^{-1}\frac{32.18}{58.99} = 28.61^\circ$$
$$\theta = 180^\circ - \theta_{\text{ref}} = 151.39^\circ$$

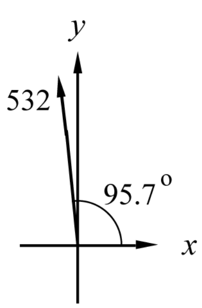
The vector R terminates in quadrant II.

27.

Vector	Magnitude	Ref. Angle
A	318	67.5°
B	245	73.7°

	x-component	y-component
	$-318\cos 67.5^\circ = -121.7$	$318\sin 67.5^\circ = 293.8$
	$245\cos 73.7^\circ = 68.76$	$245\sin 73.7^\circ = 235.2$
R	-52.9	529

$$\theta_{\text{ref}} = \tan^{-1}\left|\frac{529}{-52.9}\right| = 84.3^\circ$$
$$\theta = 180^\circ - 84.3^\circ = 95.7^\circ$$
$$R = \sqrt{(-52.9)^2 + 529^2} = 532$$



28.

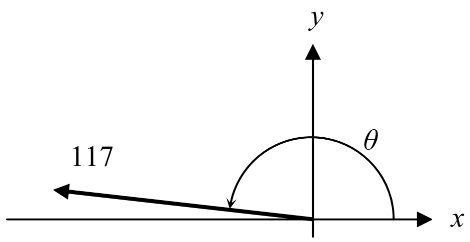
Vector	Magnitude	Ref. Angle
A	86.5	27.2°
B	62.0	49.5°

	x-component	y-component
	$-86.5\cos 27.2^\circ = -76.93$	$-86.5\sin 27.2^\circ = -39.54$
	$-62.0\cos 49.5^\circ = -40.26$	$62.0\sin 40.5^\circ = 47.14$
R	-117.2	7.64

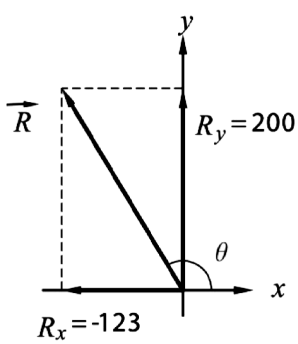
$$\theta_{\text{ref}} = \tan^{-1}\left|\frac{7.64}{-117.2}\right| = 3.73^\circ$$

$$\theta = 180^\circ - \theta_{\text{ref}} = 176.27^\circ$$

$$R = \sqrt{(-117.2)^2 + (7.64)^2} = 117$$

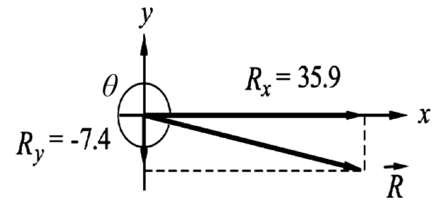


29.



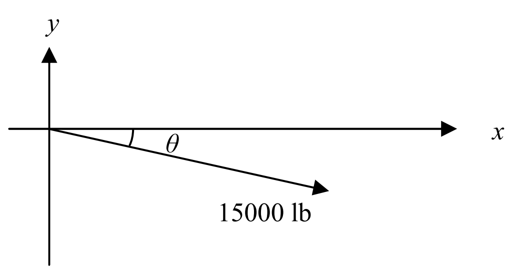
$$R_x = 302\cos(180^\circ - 45.4^\circ) + 155\cos(180^\circ + 53.0^\circ)$$
$$+ 212\cos 30.8^\circ = -123$$
$$R_y = 302\sin(180^\circ - 45.4^\circ) + 155\sin(180^\circ + 53.0^\circ)$$
$$+ 212\sin 30.8^\circ = 200$$
$$R = \sqrt{R_x{}^2 + R_y{}^2} = 235$$
$$\theta_{\text{ref}} = \tan^{-1}\frac{|R_y|}{|R_x|} = 58.4^\circ$$
$$\theta = 180^\circ - 58.4^\circ = 121.6^\circ$$

30.



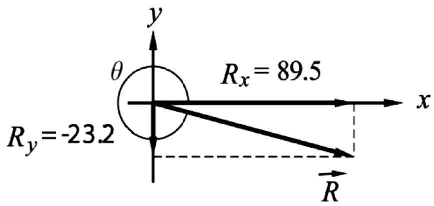
$$A = 41.5,\ \theta_A = 90^\circ - 25^\circ = 65^\circ$$
$$A_x = 41.5\cos 65^\circ = 17.5$$
$$A_y = 41.5\sin 65^\circ = 37.6$$
$$B = 22.3,\ \theta_B = 270^\circ - 62.6^\circ = 207.4^\circ$$
$$B_x = 22.3\cos 207.4^\circ = -19.8$$
$$B_y = 22.3\sin 207.4^\circ = -10.3$$
$$C = 51.6,\ \theta_C = 360^\circ - 42.2^\circ = 317.8^\circ$$
$$C_x = 51.6\cos 317.8^\circ = 38.2$$
$$C_y = 51.6\sin 317.8^\circ = -34.7$$
$$R_x = 17.5 - 19.8 + 38.2 = 35.9$$
$$R_y = 37.6 - 10.3 - 34.7 = -7.4$$
$$R = \sqrt{35.9^2 + (-7.4)^2} = 36.7$$
$$\theta_{\text{ref}} = \tan^{-1}\left|\frac{-7.4}{35.9}\right| = 11.6^\circ$$
$$\theta = 360^\circ - 11.6^\circ = 348.4^\circ$$

31.



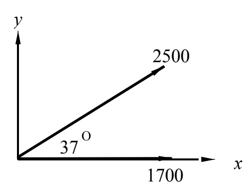
$$R_x = 5500 + 6500\cos(-15.5^\circ) + 3500\cos(-37.7^\circ)$$
$$= 14500$$
$$R_y = 6500\sin(-15.5^\circ) + 3500\sin(-37.7^\circ)$$
$$= -3880$$
$$R = \sqrt{14500^2 + (-3880)^2} = 15000 \text{ lb}$$
$$\theta_{\text{ref}} = \tan^{-1}\left|\frac{-3880}{14500}\right| = 14.98^\circ$$
$$\theta = 345^\circ$$

32.



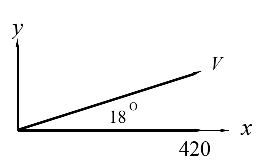
$$A = 54.0, \theta_A = 90^\circ + 18.7^\circ = 108.7^\circ$$
$$A_x = 54\cos 108.7^\circ = -17.3$$
$$A_y = 54\sin 108.7^\circ = 51.1$$
$$B = 64.5,\ \theta_B = 360^\circ - 15.6^\circ = 344.4^\circ$$
$$B_x = 64.5\cos 344.4^\circ = 62.1$$
$$B_y = 64.5\sin 344.4^\circ = -17.3$$
$$C = 72.4,\ \theta_C = 270^\circ + 38.1^\circ = 308.1^\circ$$
$$C_x = 72.4\cos 308.1^\circ = 44.7$$
$$C_y = 72.4\sin 308.1^\circ = -57.0$$
$$R_x = -17.3 + 62.1 + 44.7 = 89.5$$
$$R_y = 51.1 - 17.3 - 57.0 = -23.2$$
$$R = \sqrt{89.5^2 + (-23.2)^2} = 92.4$$
$$\theta_{\text{ref}} = \tan^{-1}\left|\frac{-23.2}{89.5}\right| = 14.5^\circ$$
$$\theta = 360^\circ - 14.5^\circ = 345.5^\circ$$

33.



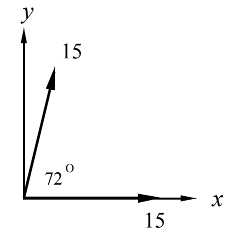
$$R = \sqrt{R_x{}^2 + R_y{}^2}$$
$$R = \sqrt{\left(1700 + 2500\cos 37^\circ\right)^2 + \left(2500\sin 37^\circ\right)^2}$$
$$R = 4000 \text{ N}$$

34.



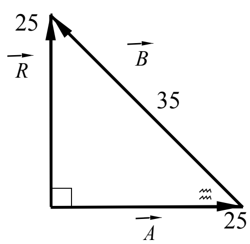
$$\cos 18^\circ = \frac{420}{V}$$
$$V = 442 \text{ mi/h}$$

35.



$$R = \sqrt{R_x^2 + R_y^2}$$
$$= \sqrt{\left(15\cos 72^\circ + 15\right)^2 + \left(15\sin 72^\circ\right)^2}$$
$$= 24.3 \text{ kg} \cdot \text{m/s}$$

36. Since $\vec{A}$ and $\vec{B}$ are displacements the vector triangle may be drawn as

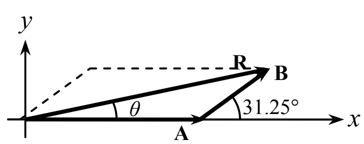


with $\vec{R} \perp$ to $\vec{A}$ from which $\tan\theta = \dfrac{25}{25}$, $\theta = 45^\circ$.

Note: $25^2 + 25^2 \neq 35^2$ because values have been rounded to two significant digits.

9.4 Applications of Vectors

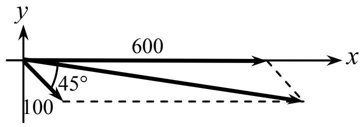
1.



$$R_x = A_x + B_x$$
$$= 32.50 + 16.18\cos 31.25^\circ$$
$$= 46.33$$
$$R_y = 16.18\sin 31.25^\circ$$
$$= 8.394$$
$$R = \sqrt{46.33^2 + 8.394^2}$$
$$= 47.08 \text{ km}$$
$$\theta = \tan^{-1}\frac{8.394}{46.33} = 10.27^\circ$$

The ship is 47.08 km from start in direction 10.27° N of E.

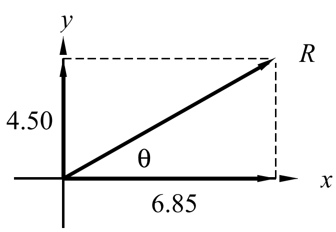
2.



$$v_{px} = v_{pa} + v_{wx}$$
$$= 600 + 100\cos(-45.0^\circ) = 671$$
$$v_{py} = v_{wy}$$
$$= 100\sin(-45.0^\circ) = -70.7$$
$$v = \sqrt{671^2 + 70.7^2} = 675 \text{ km/h}$$
$$\theta = \tan^{-1}\frac{v_{py}}{v_{px}}$$
$$= \tan^{-1}\frac{-70.7}{671} = -6.01^\circ$$

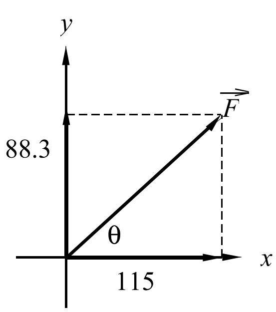
The plane is traveling 675 km/h in direction 6.01° S of E.

3.



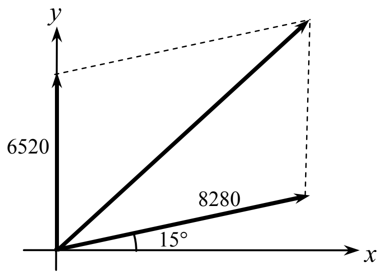
$$R = \sqrt{6.85^2 + 4.50^2} = 8.196 \text{ lb}$$
$$\theta = \tan^{-1}\frac{4.50}{6.85} = 33.3^\circ$$

4.



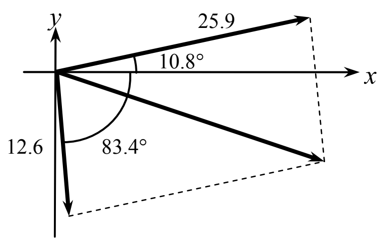
$$F_x = 88.3$$
$$F_y = 115$$
$$F = \sqrt{88.3^2 + 115^2} = 145 \text{ mi}$$
$$\tan\theta = \frac{88.3}{115}$$
$$\theta = 37.5^\circ \text{ North of East}$$

5.



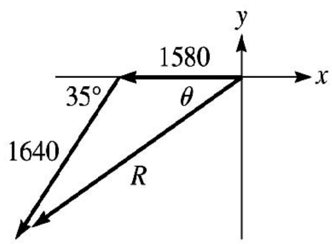
$$R_x = 8280\cos 15.0^\circ$$
$$= 7998 \text{ N}$$
$$R_y = 6520 + 8280\sin 15.0^\circ$$
$$= 8663 \text{ N}$$
$$R = \sqrt{7998^2 + 8663^2}$$
$$= 11800 \text{ N}$$
$$\theta = \tan^{-1}\frac{8663}{7998} = 47.3^\circ$$

6.



$$R_x = 25.9\cos 10.8^\circ + 12.6\cos(-83.4^\circ)$$
$$= 26.8894 \text{ kN/C}$$
$$R_y = 25.9\sin 10.8^\circ + 12.6\sin(-83.4^\circ)$$
$$= -7.6633 \text{ kN/C}$$
$$R = \sqrt{26.8894^2 + (-7.6633)^2}$$
$$= 27.96 \text{ kN/C}$$
$$\theta_{\text{ref}} = \tan^{-1}\frac{7.6633}{26.8894} = 15.9^\circ$$
$$\theta = 360^\circ - 15.9^\circ = 344.1^\circ$$

7.



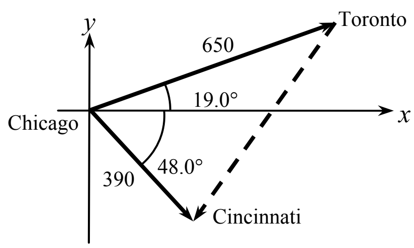
$R_x = -1580 - 1640\cos 35.0^\circ$

$R_y = -1640\sin 35.0^\circ$

$R = \sqrt{R_x^2 + R_y^2} = 3070 \text{ ft}$

$\theta = \tan^{-1}\dfrac{R_y}{R_x} = 17.8^\circ$ S of W

8.



$390\cos\left(-48^\circ\right) = 650\cos 19^\circ + R_x$

$R_x = 390\cos\left(-48^\circ\right) - 650\cos 19^\circ = -353.6261$

$390\sin\left(-48^\circ\right) = 650\sin 19^\circ + R_y$

$R_y = 390\sin\left(-48^\circ\right) - 650\sin 19^\circ = -501.4458$

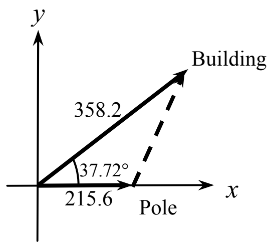
$R = \sqrt{R_x^2 + R_y^2} = 610 \text{ km}$

$\theta_{\text{ref}} = \tan^{-1}\dfrac{501.4458}{353.6261} = 54.8^\circ$

$\theta = \theta_{\text{ref}} + 180^\circ = 234.8^\circ$

Cincinnati is 610 km away at a bearing 54.8° S of W of Toronto.

9.



$358.2\cos(37.72^\circ) = 215.6 + R_x$

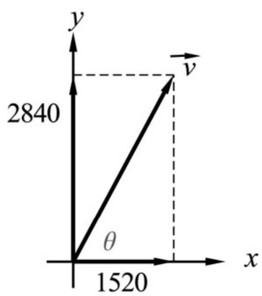
$R_x = 358.2\cos(37.72^\circ) - 215.6 = 67.74$

$R_y = 358.2\sin(37.72^\circ) = 219.15$

$R = \sqrt{R_x^2 + R_y^2} = 229.4 \text{ ft}$

$\theta = \tan^{-1}\dfrac{219.15}{67.74} = 72.82^\circ$

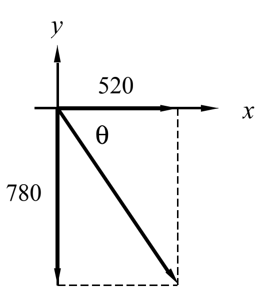
10.



$v = \sqrt{v_x^2 + v_y^2} = \sqrt{1520^2 + 2840^2} = 3220 \text{ km/h}$

$\tan\theta = \dfrac{2840}{1520} = 1.87,\ \theta = 61.8^\circ$

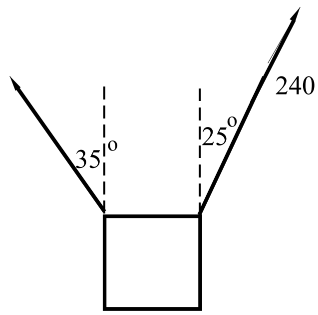
11.



$R = \sqrt{520^2 + 780^2} = 940 \text{ N}$

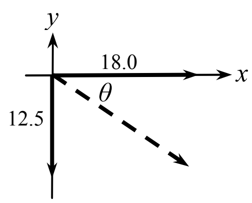
$\theta = \tan^{-1}\dfrac{780}{520} = 56.3^\circ$

12.



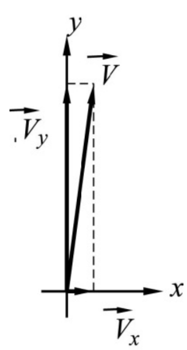
$$F \sin 35^\circ = 240 \sin 25^\circ$$
$$F = 177 \text{ lb}$$

13.



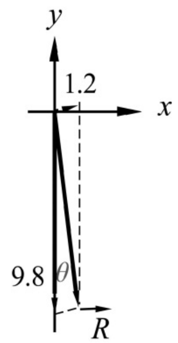
$$R = \sqrt{18.0^2 + 12.5^2} = 21.9 \text{ km/h}$$
$$\theta = \tan^{-1}\frac{12.5}{18.0} = 34.8^\circ \text{ south of east}$$

14.



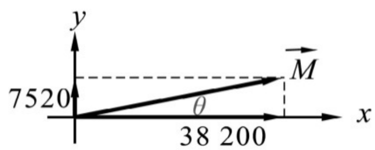
$$V = 3200$$
$$V_x = 420$$
$$V_y = \sqrt{V^2 - V_x^2}$$
$$V_y = \sqrt{3200^2 - 420^2} = 3170 \text{ lb}$$

15.



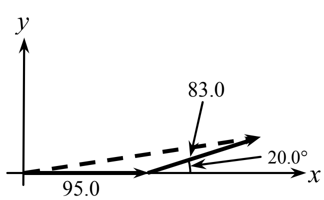
$$R_x = 1.2\cos 15° = 1.2$$
$$R_y = 1.2\sin 15° - 9.8 = -9.5$$
$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{1.2^2 + (-9.5)^2}$$
$$R = 9.6 \text{ m/s}^2$$
$$\theta = \tan^{-1}\frac{1.2}{9.5} = 7.0°$$

16.



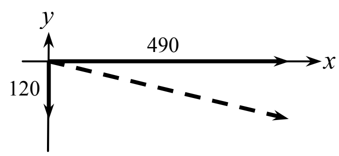
$$M_x = 22\ 100 + 17\ 800\cos 25.0° = 38\ 200 \text{ kg}\cdot\text{m/s}$$
$$M_y = 17\ 800\sin 25.0° = 7520 \text{ kg}\cdot\text{m/s}$$
$$M = \sqrt{38\ 200^2 + 7520^2} = 38\ 900 \text{ kg}\cdot\text{m/s}$$
$$\theta = \tan^{-1}\frac{7520}{38\ 200} = 11.1° \text{ from direction of truck.}$$

17.



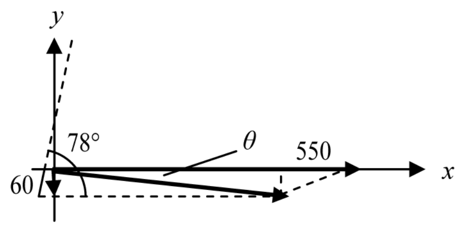
$$F_x = 95.0 + 83.0\cos 20.0° = 173$$
$$F_y = 83.0\sin 20.0° = 28.4$$
$$F = \sqrt{173^2 + 28.4^2} = 175 \text{ lb}$$
$$\tan\theta = \frac{28.4}{173}$$
$$\theta = 9.32°, \text{ above horizontal and to the right.}$$

18.



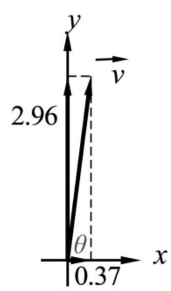
$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{490^2 + (-120)^2} = 504 \text{ lb}$$

19.



$$R = \sqrt{R_x^2 + R_y^2}$$
$$R = \sqrt{(550 - 60\cos 78°)^2 + (60\sin 78°)^2}$$
$$R = 540 \text{ km/h}$$
$$\theta = \tan^{-1}\frac{60\sin 78°}{550 - 60\cos 78°}$$
$$\theta = 6°$$

20.



$$v_{x\text{-assumed}} = 17.5\cos 26.3^\circ = 15.69$$
$$v_{y\text{-assumed}} = 17.5\sin 26.3^\circ = 7.75$$
$$v_{x\text{-actual}} = 19.3\cos 33.7^\circ = 16.06$$
$$v_{y\text{-actual}} = 19.3\sin 33.7^\circ = 10.71$$
$$v_x = v_{x\text{-actual}} - v_{x\text{-assumed}} = 16.06 - 15.69 = 0.37$$
$$v_y = v_{y\text{-actual}} - v_{y\text{-assumed}} = 10.71 - 7.75 = 2.96$$
$$v = \sqrt{0.37^2 + 2.96^2} = 3.0 \text{ km/h}$$
$$\tan\theta = \frac{2.96}{0.37} = 8.00$$
$$\theta = 82.9^\circ, \text{ N of E.}$$

21. The vertical component of T_2 matches that of the vector in Quadrant I:

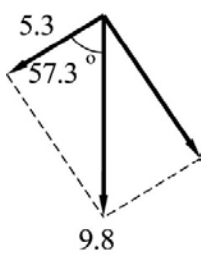
$T_2 \sin 37.2^\circ = 255 \sin 58.3^\circ$

$T_2 = \dfrac{255 \sin 58.3^\circ}{\sin 37.2^\circ} = 359\text{ N}$

In order for the horizontal components to be in equilibrium,

$T_1 = 255 \cos 58.3^\circ + 359 \cos 37.2^\circ = 420\text{ N}$

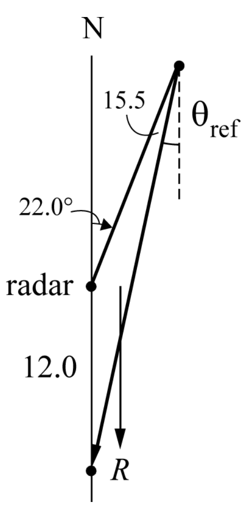
22.



$\cos\left(90^\circ - 32.7^\circ\right) = \dfrac{5.3}{g}$

$$g = \frac{5.3}{\cos 57.3^\circ} = 9.8\text{ m/s}^2$$

23.



Let **A** be the displacement of the first plane with respect to the radar and let **B** be the displacement of the second plane with respect to the radar. The displacemnt of the second plane from the first is the vector **R** = **B** − **A**.

$A_x = 15.5 \cos 68^\circ = 5.81$

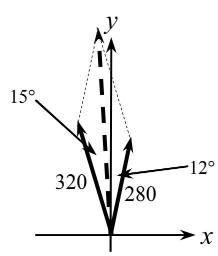
$A_y = 15.5 \sin 68^\circ = 14.37$

$B_x = 0$

$B_y = -12$

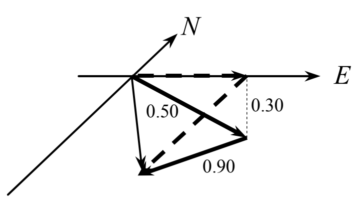
$R_x = -5.81$
$R_y = -12 - 14.37 = -26.37$
$$R = \sqrt{R_x^2 + R_y^2}$$
$$= \sqrt{(-5.81)^2 + (-26.37)^2}$$
$$= 27.0$$
$$\theta_{\text{ref}} = \tan^{-1}\left|\frac{-5.81}{-26.37}\right| = 12.4^\circ$$
$$\text{Bearing} = 180^\circ + 12.4^\circ$$
$$= 192.4^\circ$$
Displacement of second plane from first = 27.0 km at a bearing of 192.4°.

24.



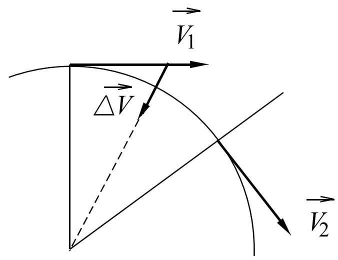
$F_x = 280\cos 78^\circ + 320\cos 105^\circ = -24.6$
$F_y = 280\sin 78^\circ + 320\sin 105^\circ = 583$
$F = \sqrt{(-24.6)^2 + 583^2} = 584\text{ N}$

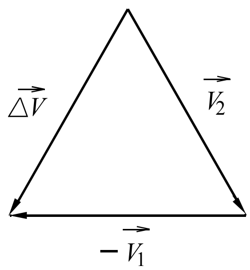
25.



The component along the surface for the eastward portion of the journey is $\sqrt{0.50^2 - 0.30^2} = 0.40$ km.
The component along the surface for the southward portion of the journey is $\sqrt{0.90^2 - 0.30^2} = 0.85$ km.
The displacement on the surface is
$$s = \sqrt{0.40^2 + (-0.85)^2} = 1.06\text{ km}$$
with an angle
$$\theta = \tan^{-1}\left(\frac{-0.85}{0.40}\right) = -64.8^\circ$$
and so the submarine is 1.06 km away from its origin at an angle of 64.8° south of east.

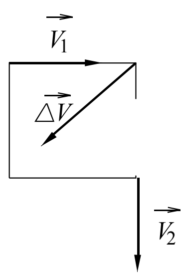
26. a)

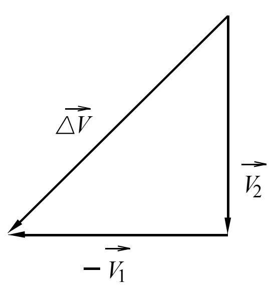




$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

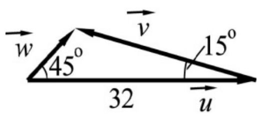
(b)





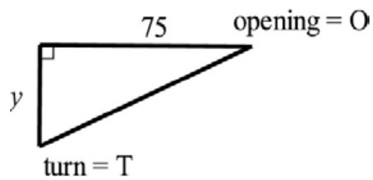
The direction of the change in velocity points toward the center of the earth in both cases.

27. Assume that as the smoke pours out of the funnel it immediately takes up the velocity of the wind. Let $\vec{w}$ = velocity of wind, $\vec{u}$ = velocity of boat, $\vec{v}$ = velocity of smoke as seen by passenger.



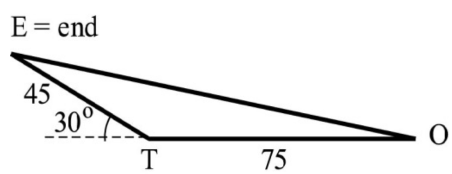
$$w\cos 45^\circ + v\cos 15^\circ = 32$$
$$w\sin 45^\circ = v\sin 15^\circ$$
$$v = \frac{w\sin 45^\circ}{\sin 15^\circ}$$
$$w\cos 45^\circ + \frac{w\sin 45^\circ}{\sin 15^\circ}\cos 15^\circ = 32$$
$$w = 9.3 \text{ km/h}$$

28. Assume 75 m is measured along horizontal surface. In cross-section view,



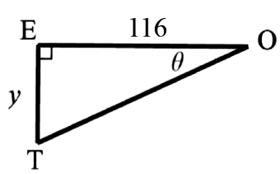
Cross-section view

At turn, shaft is $y = 75\tan 25^\circ$ m below surface.



Top view

$$OE = \sqrt{\left(75 + 45\cos 30^\circ\right)^2 + \left(45\sin 30^\circ\right)^2} = 116 \text{ m}$$

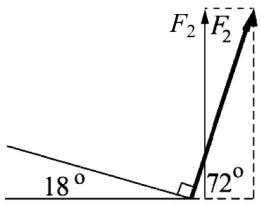


Cross-section view

$$\theta = \tan^{-1}\frac{y}{OE}$$

The displacement of the end of tunnel from opening is $OE = 116$ m measured along horizontal surface at an angle $\theta = 17^\circ$ below surface.

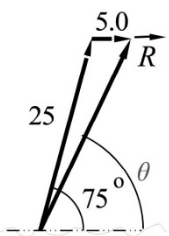
29.



Using the fact that $1.2F_1 = 0.3F_2$, we have

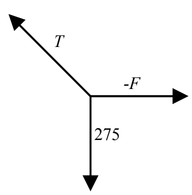
$$F_2 = \frac{1.2F_1}{0.3}$$
$$F_{2v} = F_2 \sin 72°$$
$$= \frac{1.2(240)}{0.3}\sin 72°$$
$$F_{2v} = 910 \text{ N}$$

30.



$$R = \sqrt{(25\cos 75° + 5.0)^2 + (25\sin 75°)^2}$$
$$R = 27 \text{ m/min}$$
$$\theta = \tan^{-1}\frac{25\sin 75°}{25\cos 75° + 5.0}$$
$$\theta = 65°$$

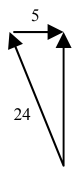
31.



The vector T that is at a 45° angle has equal size horizontal and vertical components, i.e., $T_y = -T_x$. The vertical component T_y must be 275 to balance out the weight of the sign. The horizontal component must be -275. The magnitude of T is $T = \sqrt{275^2 + (-275)^2} = 389$ lb.

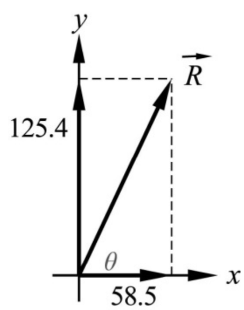
The force exerted by the horizontal bar must balance out the horizontal component and so this force is 275 lb to the right. The force F exerted on the horizontal bar must be 275 lb to the left.

32.



$v = \sqrt{24.00^2 - 5.00^2} = 23.5$ km/h, perpendicular to bank

33.



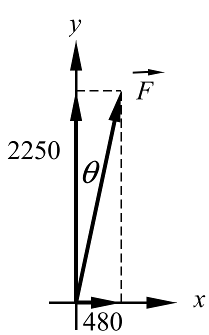
$R_x = 75.0 + 75.0\cos 65.0^\circ + 75.0\cos 130.0^\circ = 58.5$

$R_y = 75.0\sin 65.0^\circ + 75.0\sin 130.0^\circ = 125.4$

$R = \sqrt{58.5^2 + 125.4^2} = 138$ km

$\theta = \tan^{-1}\dfrac{125.4}{58.5} = 65.0^\circ$ N of E

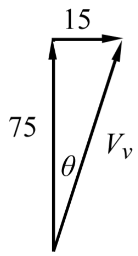
34.



$F = \sqrt{F_x^2 + F_y^2} = \sqrt{480^2 + 2250^2} = 2300$ lb

$\theta = \tan^{-1}\dfrac{480}{2250} = 12^\circ$

35. top view of the plane:

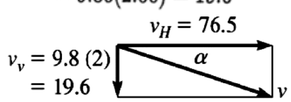


Let v_H be the horizontal component of the package's velocity. It is given by the sum of the velocity of the plane and the ejection velocity.

$$v_H = \sqrt{75.0^2 + 15.0^2} = 76.5$$

$$\theta = \tan^{-1}\frac{15.0}{75.0} = 11.3°$$

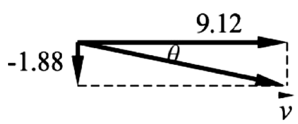
$$v_v = 9.80(2.00) = 19.6$$



$$v = \sqrt{76.5^2 + 19.6^2} = 79.0 \text{ m/s}$$

$$\alpha = \tan^{-1}\frac{19.6}{76.5} = 14.4°, 75.6° \text{ from vertical}$$

36.



Angle of travel within the barge $= \tan^{-1}\dfrac{20}{48} = 22.6°$

We fix the y-axis parallel to the stream.

$v_x = 4.5 + 5.0\cos 22.6° = 9.12$ km/h

$v_y = 5.0\sin 22.6° - 3.8 = -1.88$ km/h

$v = \sqrt{9.12^2 + (-1.88)^2} = 9.3$ km/h

$\theta = \tan^{-1}\left|\dfrac{-1.88}{9.12}\right| = 11.6°$ downstream from

where the barge is heading

37. The horizontal components satisfy

$T_1 \cos 51.0^\circ + T_2 \cos 119.6^\circ = 0$

$T_2 = -\dfrac{\cos 51.0^\circ}{\cos 119.6^\circ} T_1 = 1.274\ T_1$

The vertical components satisfy

$T_1 \sin 51.0^\circ + T_2 \sin 119.6^\circ = 975$

or

$0.777146 T_1 + 0.869495(1.274\ T_1) = 975$

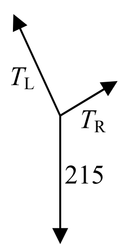
or

$T_1 = \dfrac{975}{0.777146 + 0.869495(1.274)} = 517 \text{ lb.}$

and

$T_2 = 1.274\ T_1 = 659 \text{ lb.}$

38.



The horizontal components satisfy

$T_L \cos 120.0^\circ + T_R \cos 37.0^\circ = 0$

$T_L = -\dfrac{\cos 37.0^\circ}{\cos 120.0^\circ} T_R = 1.597271\ T_R$

The vertical components satisfy

$T_L \sin 120.0^\circ + T_R \sin 37.0^\circ = 215 \text{ N}$

or

$0.866025\left(1.597271\ T_R\right) + 0.601815 T_R = 215$

or

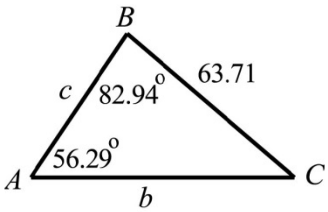
$T_R = \dfrac{215}{0.866025\left(1.597271\right) + 0.601815} = 108 \text{ N}$

and

$T_L = 1.597271\ T_R = 173 \text{ N}$

9.5 Oblique Triangles, the Law of Sines

1.



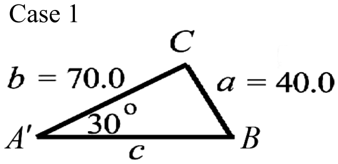
$$C = 180° - (56.29° - 82.94°) = 40.77°$$

$$\frac{b}{\sin 82.94°} = \frac{63.71}{\sin 56.29°} = \frac{c}{\sin 40.77°}$$

$$b = \frac{63.71 \sin 82.94°}{\sin 56.29°} = 76.01$$

$$c = \frac{63.71 \sin 40.77°}{\sin 56.29°} = 50.01$$

2. Case 1



Law of sines

$$\frac{70.0}{\sin B} = \frac{40.0}{\sin 30°}$$

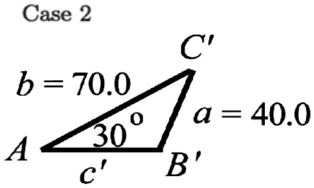
$$B = 61.0°$$

$$C = 180° - (30.0° + 61.0°) = 89.0°$$

$$\frac{c}{\sin 89.0°} = \frac{40.0}{\sin 30°}$$

$$c = 80.0$$

Case 2



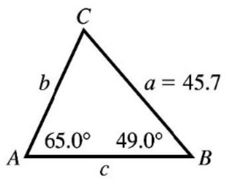
$$B' = 180° - 61.0° = 119.0°$$

$$C' = 180° - (30° + 119.0°) = 31.0°$$

$$\frac{c'}{\sin 31.0°} = \frac{40.0}{\sin 30°}$$

$$c' = 41.2$$

3.



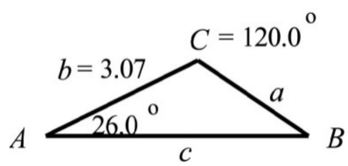
$$C = 180° - 65.0° - 49.0° = 66.0°$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{45.7}{\sin 65.0°} = \frac{b}{\sin 49.0°} \Rightarrow b = 38.1$$

$$\frac{45.7}{\sin 65.0°} = \frac{c}{\sin 66.0°} \Rightarrow c = 46.1$$

4.



$b = 3.07$, $A = 26.0°$, $C = 120.0°$

$B = 180.0° - (120.0° + 26.0°) = 34.0°$

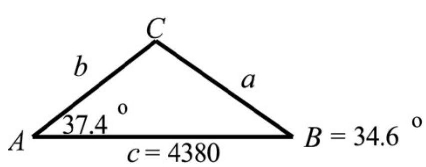
$$\frac{a}{\sin A} = \frac{b}{\sin B};\ \frac{a}{\sin 26.0°} = \frac{3.07}{\sin 34.0°}$$

$$a = \frac{3.07 \sin 26.0°}{\sin 34.0°} = 2.41$$

$$\frac{b}{\sin B} = \frac{c}{\sin C};\ \frac{3.07}{\sin 34.0°} = \frac{c}{\sin 120.0°}$$

$$c = \frac{3.07 \sin 120.0°}{\sin 34.0°} = 4.75$$

5.



$c = 4380$, $A = 37.4°$, $B = 34.6°$

$C = 180.0° - (37.4° + 34.6°) = 108.0°$

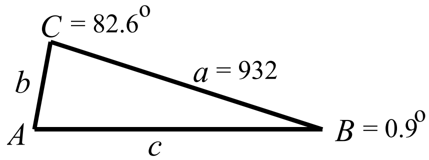
$$\frac{b}{\sin B} = \frac{c}{\sin C};\ \frac{b}{\sin 34.6°} = \frac{4380}{\sin 108.0°}$$

$$b = \frac{4380 \sin 34.6°}{\sin 108.0°} = 2620$$

$$\frac{a}{\sin A} = \frac{c}{\sin C};\ \frac{a}{\sin 37.4°} = \frac{4380}{\sin 108.0°}$$

$$a = \frac{4380 \sin 37.4°}{\sin 108.0°} = 2800$$

6.



$a = 932,\ B = 0.9^\circ,\ C = 82.6^\circ$

$A = 180^\circ - \left(0.9^\circ + 82.6^\circ\right) = 96.5^\circ$

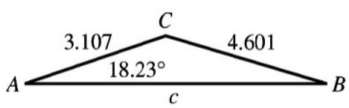
$$\frac{a}{\sin A} = \frac{b}{\sin B};\ \frac{932}{\sin 96.5^\circ} = \frac{b}{\sin 80.9^\circ}$$

$$b = \frac{932 \sin 0.9^\circ}{\sin 96.5} = 14.7$$

$$\frac{a}{\sin A} = \frac{c}{\sin C};\ \frac{932}{\sin 96.5} = \frac{c}{\sin 82.6^\circ}$$

$$c = \frac{932 \sin 82.6^\circ}{\sin 96.5^\circ} = 930$$

7.



Since 4.601 > 3.107, the longer side is opposite the known angle, and we have one solution.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{C}{\sin C}$$

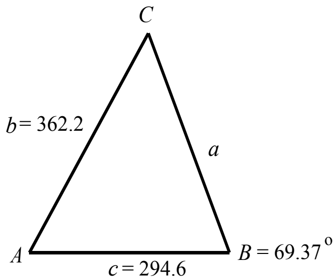
$$\frac{4.601}{\sin 18.23^\circ} = \frac{3.107}{\sin B} = \frac{C}{\sin C}$$

$$\sin B = \frac{3.107 \sin 18.23^\circ}{4.601}$$

$$B = 12.20^\circ,\ C = 180 - 18.23 - 12.20 = 149.57^\circ$$

$$c = \frac{4.601 \sin 149.57^\circ}{\sin 18.23^\circ} = 7.448$$

8.



Since 362.2 > 294.6, the longer side is opposite the known angle, and we have one solution.

$$b = 362.2,\ c = 294.6,\ B = 69.37^\circ$$

$$\frac{b}{\sin B} = \frac{c}{\sin C};\ \frac{362.2}{\sin 69.37^\circ} = \frac{294.6}{\sin C}$$

$$\sin C = \frac{294.6 \sin 69.37^\circ}{362.2}$$

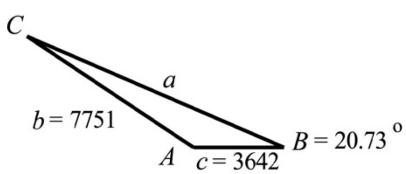
$$C = 49.57^\circ$$

$$A = 180.0^\circ - \left(69.37^\circ + 49.57^\circ\right) = 61.06^\circ$$

$$\frac{a}{\sin A} = \frac{b}{\sin B};\ \frac{a}{\sin 61.06^\circ} = \frac{362.2}{\sin 69.37^\circ}$$

$$a = \frac{362.2 \sin 61.06^\circ}{\sin 69.37^\circ} = 338.7$$

9.



Since 7751 > 3642, the longer side is opposite the known angle, and we have one solution.

$$b = 7751,\ c = 3642,\ B = 20.73^\circ$$

$$\frac{b}{\sin B} = \frac{c}{\sin C};\ \frac{7751}{\sin 20.73^\circ} = \frac{3642}{\sin C}$$

$$\sin C = \frac{3642 \sin 20.73^\circ}{7751} = 0.1663$$

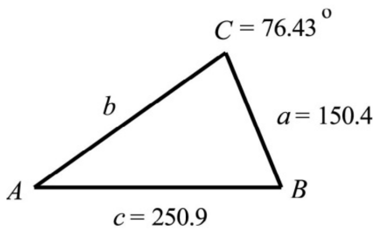
$$C = 9.574^\circ$$

$$A = 180.0^\circ - (20.73^\circ + 9.574^\circ) = 149.7^\circ$$

$$\frac{a}{\sin A} = \frac{b}{\sin B};\ \frac{a}{\sin 149.7^\circ} = \frac{7751}{\sin 20.73^\circ}$$

$$a = \frac{7751 \sin 149.7^\circ}{\sin 20.73^\circ} = 11\,050$$

10.



Since 250.9 > 150.4, the longer side is opposite the known angle, and we have one solution.

$a = 150.4,\ c = 250.9,\ C = 76.43°$

$$\frac{a}{\sin A} = \frac{c}{\sin C};\ \frac{150.4}{\sin A} = \frac{250.9}{\sin 76.43°}$$

$$\sin A = \frac{150.4 \sin 76.43°}{250.9} = 0.5827$$

$$A = 35.64°$$

$$B = 180.0° - (35.64° - 76.43°) = 67.93°$$

$$\frac{b}{\sin B} = \frac{c}{\sin C};\ \frac{b}{\sin 67.93°} = \frac{250.9}{\sin 76.43°}$$

$$b = \frac{250.9 \sin 67.93°}{\sin 76.43°} = 239.2$$

11.

$a = 63.8, B = 58.4°, C = 22.2°$

$A = 180.0° - 58.4° - 22.2° = 99.4°$

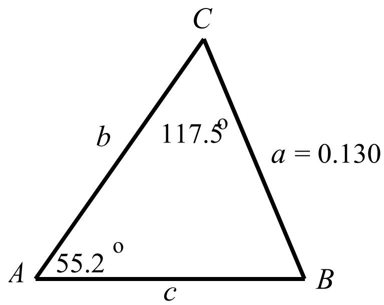
$$\frac{a}{\sin A} = \frac{b}{\sin B};\ \frac{63.8}{\sin 99.4°} = \frac{b}{\sin 58.4°}$$

$$b = \frac{63.8 \sin 58.4°}{\sin 99.4°} = 55.1$$

$$\frac{a}{\sin A} = \frac{c}{\sin C};\ \frac{63.8}{\sin 99.4°} = \frac{c}{\sin 22.2°}$$

$$c = \frac{63.8 \sin 22.2°}{\sin 99.4°} = 24.4$$

12.



$$a = 0.130,\ A = 55.2^\circ,\ C = 117.5^\circ$$

$$B = 180^\circ - 55.2^\circ - 117.5^\circ = 7.3^\circ$$

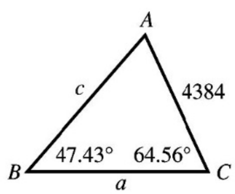
$$\frac{a}{\sin A} = \frac{b}{\sin B};\ \frac{0.130}{\sin 55.2^\circ} = \frac{b}{\sin 7.3^\circ}$$

$$b = \frac{0.130 \sin 7.3^\circ}{\sin 55.2^\circ} = 0.0201$$

$$\frac{a}{\sin A} = \frac{c}{\sin C};\ \frac{0.130}{\sin 55.2^\circ} = \frac{c}{\sin 117.5^\circ}$$

$$c = \frac{0.130 \sin 117.5^\circ}{\sin 55.2^\circ} = 0.140$$

13.



$$A = 180^\circ - 47.43^\circ - 64.56^\circ$$
$$= 68.01^\circ$$

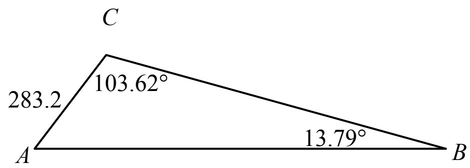
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 68.01^\circ} = \frac{4384}{\sin 47.43^\circ} = \frac{c}{\sin 64.56^\circ}$$

$$a = \frac{4384 \sin 68.01^\circ}{\sin 47.43^\circ} = 5520$$

$$c = \frac{4384 \sin 64.56^\circ}{\sin 47.43^\circ} = 5376$$

14.



$b = 283.2,\ B = 13.79^\circ,\ C = 103.62^\circ$

$A = 180^\circ - 13.79^\circ - 103.62^\circ = 62.59^\circ$

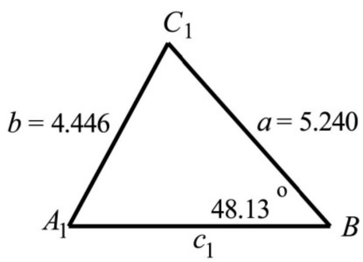
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B};$

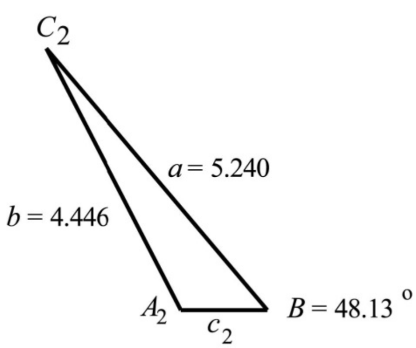
$a = \dfrac{283.2 \sin 62.59^\circ}{\sin 13.79^\circ} = 1054.7$

$\dfrac{c}{\sin C} = \dfrac{b}{\sin B};$

$c = \dfrac{283.2 \sin 103.62^\circ}{\sin 13.79^\circ} = 1154.7$

15.





Since 4.446 < 5.240, the shorter side is opposite the known angle, and we have two solutions.

$a = 5.240,\ b = 4.446,\ B = 48.13^\circ$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B};\ \dfrac{5.240}{\sin A} = \dfrac{4.446}{\sin 48.13^\circ}$

$\sin A = \dfrac{5.240 \sin 48.13^\circ}{4.446} = 0.8776$

$A_1 = 61.36^\circ$

$C_1 = 180.0^\circ - 48.13^\circ - 61.36^\circ = 70.51^\circ$

or

$A_2 = 180.0^\circ - 61.36^\circ = 118.64^\circ$

and

$C_2 = 180.0^\circ - 48.13^\circ - 118.64^\circ = 13.23^\circ$

$\frac{b}{\sin B} = \frac{c_1}{\sin C_1}; \frac{4.446}{\sin 48.13^\circ} = \frac{c}{\sin 70.51^\circ}$

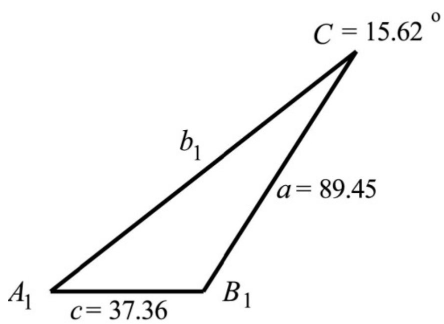
$c_1 = \frac{4.446 \sin 70.51^\circ}{\sin 48.13^\circ} = 5.628$

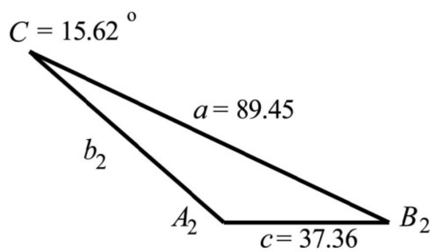
or

$\frac{b}{\sin B} = \frac{c_2}{\sin C_2}; \frac{4.446}{\sin 48.13^\circ} = \frac{c_2}{\sin 13.23^\circ}$

$c_2 = \frac{4.446 \sin 13.23^\circ}{\sin 48.13^\circ} = 1.366$

16.





Since 37.36 < 89.45, the shorter side is opposite the known angle, and we have two solutions.

$a = 89.45,\ c = 37.36,\ C = 15.62^\circ$

$\frac{a}{\sin A} = \frac{c}{\sin C}; \frac{89.45}{\sin A} = \frac{37.36}{\sin 15.62^\circ}$

$\sin A = \frac{89.45 \sin 15.62^\circ}{37.36} = 0.6447$

$A_1 = 40.14^\circ$

$B_1 = 180.0^\circ - 15.62^\circ - 40.14^\circ = 124.24^\circ$

$\frac{b_1}{\sin B_1} = \frac{c}{\sin C}; \frac{b}{\sin 124.24^\circ} = \frac{37.36}{\sin 15.62^\circ}$

$b_1 = \frac{37.36 \sin 124.24^\circ}{\sin 15.62^\circ} = 114.7$

or

$A_2 = 180.0^\circ - 40.14^\circ = 139.86^\circ$

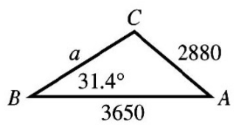
and

$B_2 = 180.0^\circ - 139.86^\circ - 15.62^\circ = 24.52^\circ$

$\frac{b_2}{\sin B_2} = \frac{c}{\sin C}; \frac{b_2}{\sin 24.52^\circ} = \frac{37.36}{\sin 15.62^\circ}$

$b_2 = \frac{37.36 \sin 24.52^\circ}{\sin 15.62^\circ} = 57.58$

17.



Since 2880 < 3650, the shorter side is opposite the known angle, so there are two solutions.

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin A} = \frac{2880}{\sin 31.4^\circ} = \frac{3650}{\sin C}$

$\sin C = \frac{3650 \sin 31.4^\circ}{2880}$

$C = 41.3^\circ$ or 138.7°

Case I.

$C = 41.3^\circ,$

$A = 180^\circ - 31.4^\circ - 41.3^\circ = 107.3^\circ$

$\frac{a}{\sin 107.3^\circ} = \frac{2880}{\sin 31.4^\circ}$

$a = 5280$

Case II.

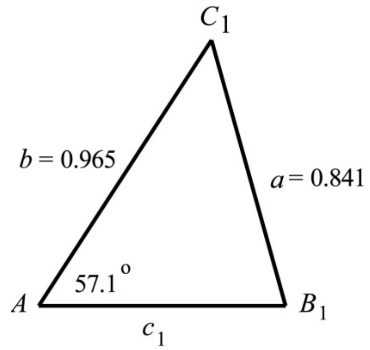
$C = 138.7^\circ,$

$A = 180^\circ - 31.4^\circ - 138.7^\circ = 9.9^\circ$

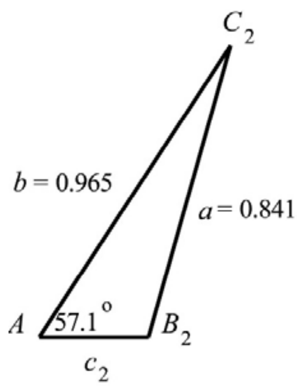
$\frac{a}{\sin 9.9^\circ} = \frac{2880}{\sin 31.4^\circ}$

$a = 950$

18.



Since 0.841 < 0.965, the shorter side is opposite the known angle, so there are two solutions.



$a = 0.841, b = 0.965, A = 57.1°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}; \frac{0.841}{\sin 57.1°} = \frac{0.965}{\sin B}$$

$$\sin B = \frac{0.965 \sin 57.1°}{0.841} = 0.963$$

$$B_1 = 74.5°$$

$$C_1 = 180.0° - 57.1° - 74.5° = 48.4°$$

$$\frac{a}{\sin A} = \frac{c_1}{\sin C_1}; \frac{0.841}{\sin 57.1°} = \frac{c_1}{\sin 48.4°}$$

$$c_1 = \frac{0.841 \sin 48.4°}{\sin 57.1°} = 0.750$$

or

$$B_2 = 180.0° - 74.5° = 105.5°$$

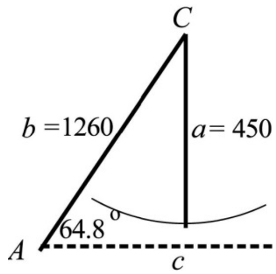
and

$$C_2 = 180.0° - 105.5° - 57.1° = 17.4°$$

$$\frac{a}{\sin A} = \frac{c_2}{\sin C_2}; \frac{0.841}{\sin 57.1°} = \frac{c_2}{\sin 17.4°}$$

$$c_2 = \frac{0.841 \sin 17.4°}{\sin 57.1°} = 0.299$$

19.



$a = 450,\ b = 1260,\ A = 64.8^\circ$

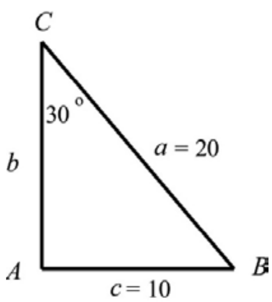
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B};\ \dfrac{450}{\sin 64.8^\circ} = \dfrac{1260}{\sin B}$

$\sin B = \dfrac{1260 \sin 64.8^\circ}{450} = 2.53$ (not ≤ 1)

Therefore, no solution.

Indeed, $450 < 1260 \sin 64.8^\circ = 1140$,
so there is no solution.

20.

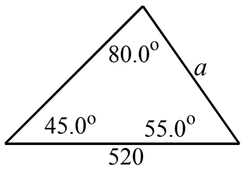


Since 10 = 20 sin(30º), there is one right triangle solution.

$a = 20, c = 10, C = 30^\circ$

$$\frac{a}{\sin A} = \frac{c}{\sin C};\ \frac{20}{\sin A} = \frac{10}{\sin 30}$$
$$\sin A = \frac{20 \sin 30^\circ}{10} = 1$$
$$A = 90.0^\circ$$
$$B = 180.0^\circ - 90.0^\circ - 30^\circ = 60.0^\circ$$
$$\frac{b}{\sin B} = \frac{c}{\sin C};\ \frac{b}{\sin 60.0^\circ} = \frac{10}{\sin 30^\circ}$$
$$b = \frac{10 \sin 60.0^\circ}{\sin 30.0^\circ} = 17$$

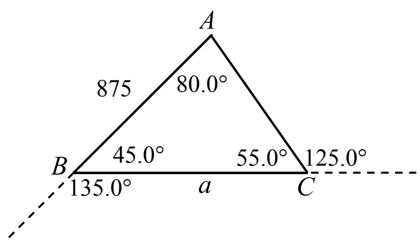
21. The third angle is $180^\circ - \left(45.0^\circ + 55.0^\circ\right) = 80.0^\circ$



The shortest side is opposite the 45.0° angle.
We use the law of sines with the other side we know.

$$\frac{520}{\sin 80.0^\circ} = \frac{a}{\sin 45.0^\circ}$$
$$a = 373 \text{ m}$$

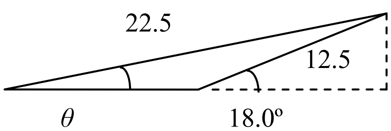
22.



We use the law of sines with the other side we know.

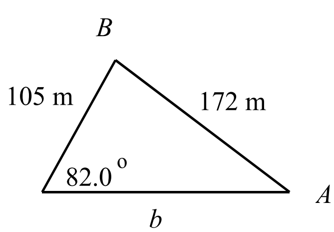
$$\frac{875}{\sin 55.0^\circ} = \frac{a}{\sin 80.0^\circ}$$
$$a = 1052 \text{ m}$$

23.



$$\frac{22.5}{\sin 162.0^\circ} = \frac{12.5}{\sin \theta}$$
$$\theta = \sin^{-1} \frac{12.5 \sin 162.0^\circ}{22.5}$$
$$= 9.885^\circ$$

24.



$$\frac{172}{\sin 82.0^\circ} = \frac{b}{\sin B} = \frac{105}{\sin A}$$

$$\sin A = \frac{105 \sin 82.0^\circ}{172}$$

$$A = 37.2^\circ$$

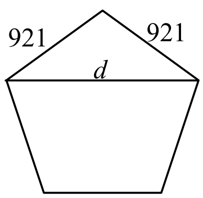
$$82.0^\circ + 37.2^\circ + B = 180^\circ$$

$$B = 60.8^\circ$$

$$\frac{b}{\sin 60.8^\circ} = \frac{172}{\sin 82.0^\circ}$$

$$b = 152 \text{ m}$$

25.



The angles of a regular pentagon are the same and sum to 540°, implying each angle is 540°/5=108°. The remaining angles of the triangle formed by two sides and a diagonal have measure

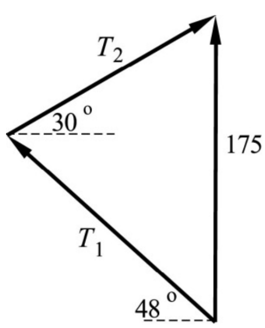
$$\frac{180^\circ - 108^\circ}{2} = 36^\circ$$

Using the law of sines, the diagonal d satisfies

$$\frac{921}{\sin 36^\circ} = \frac{d}{\sin 108^\circ}$$

$$d = \frac{921 \sin 108^\circ}{\sin 36^\circ} = 1490 \text{ ft}$$

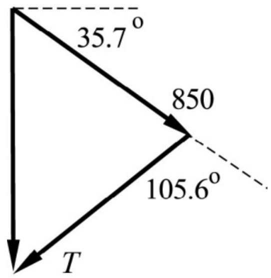
26.



$$\frac{T_1}{\sin 60.0^\circ} = \frac{175}{\sin 72.0^\circ};\ T_1 = \frac{175 \sin 60.0^\circ}{\sin 72.0^\circ} = 159 \text{ N}$$

$$\frac{T_2}{\sin 48.0^\circ} = \frac{175}{\sin 72.0^\circ};\ T_2 = \frac{175 \sin 48.0^\circ}{\sin 72.0^\circ} = 137 \text{ N}$$

27.



$$\frac{T}{\sin 54.3^\circ} = \frac{850}{\sin 51.3^\circ};\ T = \frac{850 \sin 54.3^\circ}{\sin 51.3^\circ} = 880 \text{ N}$$

28. $\dfrac{640}{\sin 136^\circ} = \dfrac{330}{\sin B}$

$$\sin B = \frac{330 \sin 136^\circ}{640} = 0.358183$$

$$B = 21^\circ$$

$$C = 180^\circ - \left(136^\circ + 21^\circ\right) = 23^\circ$$

$$\frac{640}{\sin 136^\circ} = \frac{c}{\sin 23^\circ}$$

$$c = 359.987$$

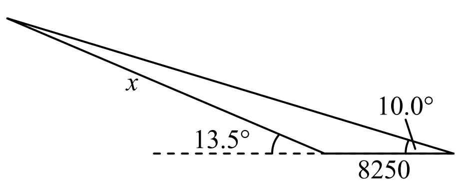
The distance from Atlanta to Raleigh is 360 mi.

29. $C = 180^\circ - \left(29.0^\circ + 35.0^\circ\right) = 114^\circ$

$$\frac{32.5}{\sin 114^\circ} = \frac{c}{\sin 35.0^\circ}$$

$$c = \frac{32.5 \sin 35.0^\circ}{\sin 114^\circ} = 20.4 \text{ m}$$

30.



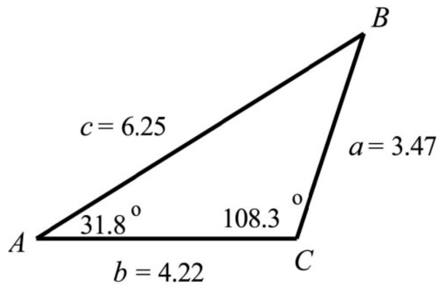
The obtuse angle is 166.5° and the remaining angle is $180° - (166.5° + 10.0°) = 3.5°$

$$\frac{x}{\sin 10.0^\circ} = \frac{8250}{\sin 3.5^\circ}$$

$$x = \frac{8250 \sin 10.0^\circ}{\sin 3.5^\circ}$$

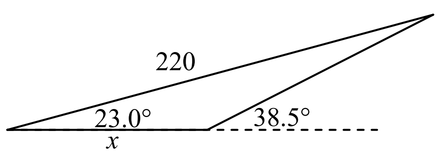
$$= 23500 \text{ ft}$$

31.



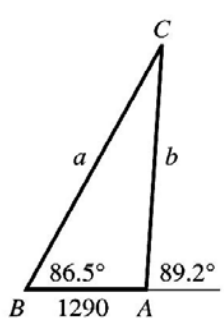
$$\frac{6.25}{\sin 108.3°} = \frac{a}{\sin 31.8°}$$
$$a = \frac{6.25 \sin 31.8°}{\sin 108.3°} = 3.47 \text{ cm}$$
$$B = 180° - 108.3° - 31.8° = 39.9°$$
$$\frac{6.25}{\sin 108.3°} = \frac{b}{\sin 39.9°}$$
$$b = \frac{6.25 \sin 39.9°}{\sin 108.3°} = 4.22 \text{ cm}$$
$$\text{Perimeter} = 6.25 + 3.47 + 4.22 = 13.94 \text{ cm}$$

32.



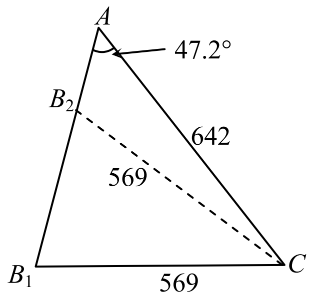
$$\frac{x}{\sin 15.5°} = \frac{220}{\sin 141.5°}$$
$$x = \frac{220 \sin 15.5°}{\sin 141.5°} = 94.3 \text{ ft}$$

33. $A = 180° - 89.2° = 90.8°$
$C = 180° - 86.5 - 90.8° = 2.7°$



$$\frac{b}{\sin 86.5°} = \frac{1290}{\sin 2.7°}$$
$$b = 27\,300 \text{ km}$$

34.



Since 642 sin 47.2° = 471 < 569 < 642, there are two possible lengths for the third side. Using the law of sines,

$$\frac{569}{\sin 47.2°} = \frac{642}{\sin B_1} = \frac{642}{\sin B_2}$$

$$\sin B_1 = \sin B_2 = \frac{642 \sin 47.2°}{569} = 0.827864$$

$$B_1 = \sin^{-1}(0.827864) = 55.9°$$

$$B_2 = 180° - B_1 = 124.1°$$

$$C_1 = 180° - 47.2° - B_1 = 76.9°$$

$$C_2 = 180° - 47.2° - B_2 = 8.7°$$

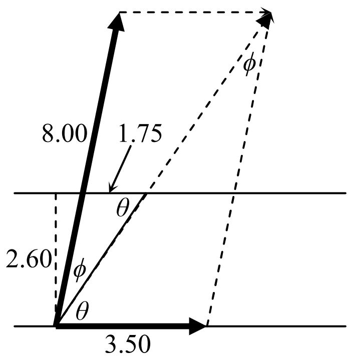
$$\frac{c_1}{\sin 76.9°} = \frac{569}{\sin 47.2°}$$

$$c_1 = \frac{569 \sin 76.9°}{\sin 47.2°} = 755 \text{ m}$$

$$\frac{c_2}{\sin 8.7°} = \frac{569}{\sin 47.2°}$$

$$c_2 = \frac{569 \sin 8.7°}{\sin 47.2°} = 117 \text{ m}$$

35.



The boat's heading will be $\theta + \phi$.

The angle θ satisfies

$$\tan\theta = \frac{2.60}{1.75}; \theta = \tan^{-1}\frac{2.60}{1.75} = 56.1°$$

From the law of sines,

$$\frac{8.00}{\sin 56.1°} = \frac{3.50}{\sin\phi}$$

$$\sin\phi = \frac{3.50 \sin 56.1°}{8.00} = 0.36313$$

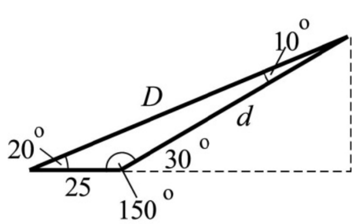
$$\phi = 21.3°$$

Therefore, the boat's heading is $\theta + \phi = 56.1° + 21.3° = 77.4°$.

36. distance = rate (time)

=(75 km/h) (20 min) (h/60 min)

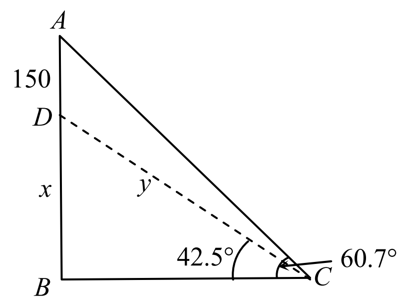
= 25 km



$$\frac{D}{\sin 150^{\circ}} = \frac{25}{\sin 10^{\circ}} = \frac{d}{\sin 20^{\circ}}$$

$$D - d = \frac{25\sin 150^{\circ}}{\sin 10^{\circ}} - \frac{25\sin 20^{\circ}}{\sin 10^{\circ}} = 23 \text{ km}$$

37.



We let x represent the height of the elevator and y represent the distance from the observer to the elevator at the first instant.

We have from the right triangle BDC

$x = y\sin 42.5°$

Also, we know angle A has measure 29.3° and in triangle ACD, angle C has measure 18.2°.

From the law of sines,

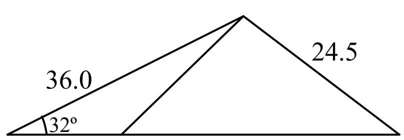
$$\frac{y}{\sin 29.3°} = \frac{150}{\sin 18.2°}$$

$$y = \frac{150\sin 29.3°}{\sin 18.2°} = 235.03 \text{ ft.}$$

$$x = y\sin 42.5° = 158.78 \text{ ft.}$$

and so the elevator is $x + 150 = 309$ ft. high at the second instant

38. We are given two sides, with the shorter side opposite the known angle. The position of P will fluctuate between the two solutions of the triangle when $\theta = 32°$.



$$\frac{24.5}{\sin 32.0^\circ} = \frac{36.0}{\sin B} = \frac{c}{\sin C}$$

$$B = \sin^{-1}\frac{36.0\sin 32.0^\circ}{24.5}$$

$$= 51.1^\circ$$

$$C = 180^\circ - (32.0^\circ + 51.1^\circ)$$

$$= 96.9^\circ$$

$$\frac{36.0}{\sin 51.1^\circ} = \frac{c}{\sin 96.9^\circ}$$

$$c = \frac{36.0\sin 96.9^\circ}{\sin 51.1^\circ}$$

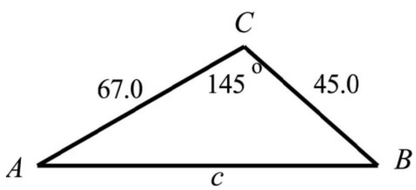
$$= 45.9 \text{ cm}$$

Or
$$B = 180^\circ - 51.1^\circ$$
$$= 128.9^\circ$$
$$C = 180^\circ - (32.0^\circ + 128.9^\circ)$$
$$= 19.1^\circ$$
$$\frac{36.0}{\sin 128.9^\circ} = \frac{c}{\sin 19.1^\circ}$$
$$c = \frac{36.0 \sin 19.1^\circ}{\sin 128.9^\circ}$$
$$= 15.1 \text{ cm}$$

Therefore, the distance between extreme positions of P is 30.8 cm.
The maximum possible value of the angle θ is attained when the triangle formed is a right triangle, so that
$$\theta = \sin^{-1} \frac{24.5}{36.0}$$
$$= 42.9^\circ$$

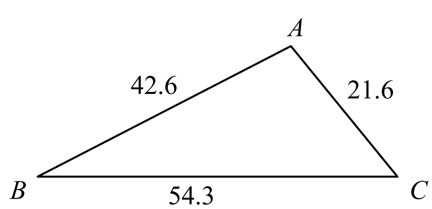
9.6 The Law of Cosines

1.



$$c = \sqrt{45.0^2 + 67.0^2 - 2(45.0)(67.0)\cos 145^\circ}$$
$$= 107$$
$$\frac{45.0}{\sin A} = \frac{c}{\sin 145^\circ} = \frac{67.0}{\sin B}$$
$$A = \sin^{-1} \frac{45.0 \sin 145^\circ}{c} = 14.0^\circ$$
$$B = \sin^{-1} \frac{67.0 \sin 145^\circ}{c} = 21.0^\circ$$

2.



$$\cos A = \frac{42.6^2 + 21.6^2 - 54.3^2}{2(42.6)(21.6)} = -0.362529$$

$A = \cos^{-1}(-0.362529) = 111.26°$

$$\frac{\sin B}{21.6} = \frac{\sin C}{42.6} = \frac{\sin 111.26°}{54.3} = 0.0171634$$

$\sin B = (21.6)(0.0171634) = 0.370729$

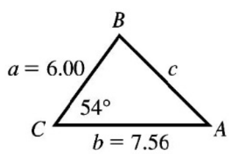
$B = 21.76°$

$\sin C = (42.6)(0.0171634) = 0.731161$

$C = 46.98°$

Alternatively, $C = 180° - A - B = 46.98°$.

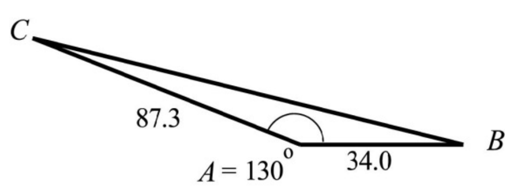
3.



$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 6.00^2 + 7.56^2 - 2(6.00)(7.56)\cos 54.0°$$
$$c = 6.31$$
$$b^2 + c^2 - 2bc\cos A = a^2$$
$$7.56^2 + 6.31^2 - 2(7.56)(6.31)\cos A = 6^2$$
$$\cos A = 0.639$$
$$A = 50.3°$$

$$B = 180° - 54.0° - 50.3°$$
$$B = 75.7°$$

4.



$b = 87.3, c = 34.0, A = 130.0°$

$$a = \sqrt{87.3^2 + 34.0^2 - 2(87.3)(34.0)(\cos 130.0°)}$$
$$= 112$$

$$\frac{\sin a}{\sin A} = \frac{b}{\sin B}; \frac{112}{\sin 130.0°} = \frac{87.3}{\sin B}$$

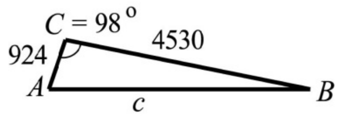
$$\sin B = \frac{87.3 \sin 130.0°}{112} = 0.597$$

0.596

$B = 36.6°$

$C = 180° - 130.0° - 36.6° = 13.4°$

5.

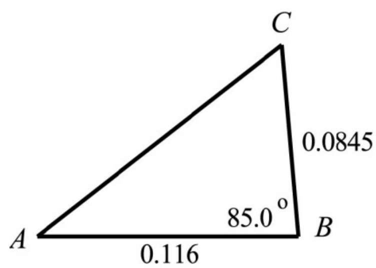


$a = 4530, b = 924, C = 98.0°$

$$c = \sqrt{4530^2 + 924^2 - 2(4530)(924)(\cos 98.0°)}$$
$$= 4750$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}; \frac{4750}{\sin 98.0°} = \frac{924}{\sin B}$$
$$\sin B = \frac{924 \sin 98.0°}{4750} = -0.193$$
$$B = 11.1°$$
$$A = 180° - 98.0° - 11.1° = 70.9°$$

6.

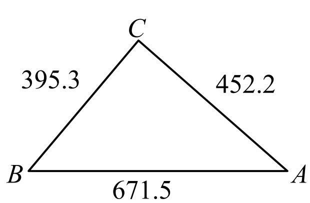


$a = 0.0845, c = 0.116, B = 85.0°$

$$b = \sqrt{0.0845^2 + 0.116^2 - 2(0.0845)(0.116)(\cos 85.0°)}$$
$$= 0.137$$

$$\frac{b}{\sin B} = \frac{a}{\sin A}; \frac{0.137}{\sin 85.0} = \frac{0.0845}{\sin A}$$
$$\sin A = \frac{0.0845 \sin 85.0°}{0.137} = 0.613$$
$$A = 37.8°$$
$$C = 180° - 37.9° - 85.0° = 57.2°$$

7.



$$\cos C = \frac{395.3^2 + 452.2^2 - 671.5^2}{2(395.3)(452.2)} = -0.252204$$

$C = \cos^{-1}(-0.252204) = 104.61°$

$$\frac{\sin A}{395.3} = \frac{\sin B}{452.2} = \frac{\sin 104.61°}{671.5}$$

$$\sin A = (395.3)\left(\frac{\sin 104.61°}{671.5}\right) = 0.569647$$

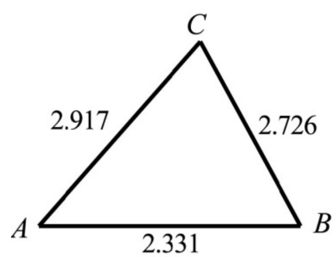
$A = 34.73°$

$$\sin B = (452.2)\left(\frac{\sin 104.61°}{671.5}\right) = 0.651643$$

$B = 40.66°$

As a check, $A + B + C = 34.73° + 40.66° + 104.61° = 180°$.

8.



$a = 2.331, b = 2.726, c = 2.917$

$$\cos A = \frac{2.726^2 + 2.917^2 - 2.331^2}{2(2.726)(2.917)} = 0.6606$$

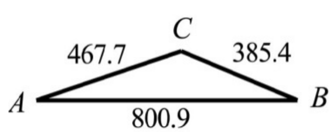
$A = 48.65°$

$$\cos B = \frac{2.331^2 + 2.917^2 - 2.726^2}{2(2.331)(2.917)} = 0.4788$$

$B = 61.39°$

$C = 180° - 48.65° - 61.39° = 69.96°$

9.



$a = 385.4, b = 467.7, c = 800.9$

$$\cos A = \frac{467.7^2 + 800.9^2 - 385.4^2}{2(467.7)(800.9)} = 0.9499$$

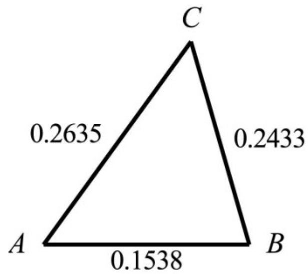
$A = 18.21°$

$$\cos B = \frac{385.4^2 + 800.9^2 - 467.7^2}{2(385.4)(800.9)} = 0.9253$$

$B = 22.28°$

$C = 180° - 18.21° - 22.28° = 139.51°$

10.



$a = 0.2433, b = 0.2635, c = 0.1538$

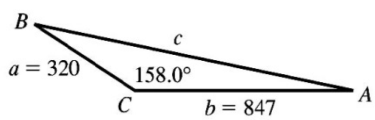
$$\cos A = \frac{0.2635^2 + 0.1538^2 - 0.2433^2}{2(0.2635)(0.1538)} = 0.4181$$

$$A = 65.28°$$

$$\cos B = \frac{0.2433^2 + 0.1538^2 - 0.2635^2}{2(0.2433)(0.1538)} = 0.1793$$

$$B = 79.67°$$
$$C = 180° - 65.28° - 79.67° = 35.05°$$

11.

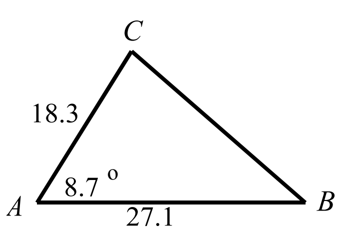


$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ &= 320^2 + 847^2 - 2(320)(847)\cos 158.0° \\ c &= 1150 \end{aligned}$$

$$b^2 + c^2 - 2bc\cos A = a^2$$
$$847^2 + 1150^2 - 2(847)(1150)\cos A = 320^2$$
$$\cos A = 0.9946$$
$$A = 6.0°$$

$$a^2 + c^2 - 2ac\cos B = b^2$$
$$320^2 + 1150^2 - 2(320)(1150)\cos B = 847^2$$
$$\cos B = 0.9612$$
$$B = 16.0°$$

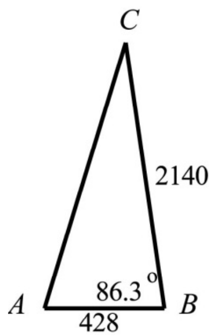
12.



$b = 18.3,\ c = 27.1,\ A = 8.7°$

$$a = \sqrt{18.3^2 + 27.1^2 - 2(18.3)(27.1)(\cos 8.7^\circ)}$$
$$= 9.43$$
$$\frac{a}{\sin A} = \frac{b}{\sin B}; \frac{9.43}{\sin 8.7^\circ} = \frac{18.3}{\sin B}$$
$$\sin B = \frac{18.3 \sin 8.7^\circ}{9.43}$$
$$B = 17.1^\circ$$
$$C = 180^\circ - 17.1^\circ - 8.7^\circ = 154.2^\circ$$

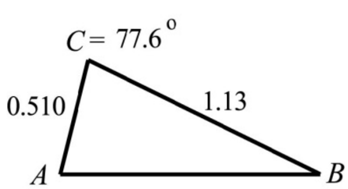
13.



$a = 2140, c = 428, B = 86.3°$

$$b = \sqrt{2140^2 + 428^2 - 2(2140)(428)(\cos 86.3^\circ)}$$
$$= 2160$$
$$\frac{b}{\sin B} = \frac{c}{\sin C}; \frac{2160}{\sin 86.3^\circ} = \frac{428}{\sin C}$$
$$\sin C = \frac{428 \sin 86.3^\circ}{2160} = 0.198$$
$$C = 11.4^\circ$$
$$A = 180^\circ - 86.3^\circ - 11.4^\circ = 82.3^\circ$$

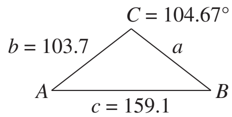
14.



$a = 1.13, b = 0.510, C = 77.6^\circ$

$$c = \sqrt{1.13^2 + 0.510^2 - 2(1.13)(0.510)(\cos 77.6^\circ)}$$
$$= 1.14$$
$$\frac{c}{\sin C} = \frac{b}{\sin B}; \frac{1.14}{\sin 77.6^\circ} = \frac{0.510}{\sin B}$$
$$\sin B = \frac{0.510 \sin 77.6^\circ}{1.14} = 0.437$$
$$B = 25.9^\circ$$
$$A = 180^\circ - 25.9^\circ - 77.6^\circ = 76.5^\circ$$

15.



$b = 103.7, c = 159.1, C = 104.67^\circ$

Case 2: Use law of sines.

$$\frac{a}{\sin A} = \frac{103.7}{\sin B} = \frac{159.1}{\sin 104.67^\circ}$$

$$\sin B = \frac{103.7 \sin 104.67^\circ}{159.1}$$

$$B = 39.09^\circ$$

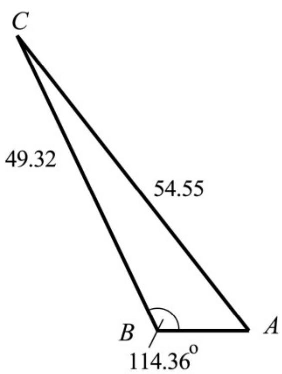
$$A = 180^\circ - (104.67^\circ + 39.09^\circ)$$

$$= 36.24^\circ$$

$$\frac{a}{\sin 36.24^\circ} = \frac{103.7}{\sin 39.09}$$

$$a = 97.22$$

16.



Case 2: Use law of sines:

$a = 49.32, b = 54.55, B = 114.36°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}; \frac{49.32}{\sin A} = \frac{54.55}{\sin 114.36°}$$

$$\sin A = \frac{49.32 \sin 114.36°}{54.55} = 0.8236$$

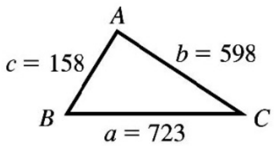
$A = 55.45°$

$C = 180° - 55.45° - 114.36° = 10.19°$

$$c = \sqrt{49.32^2 + 54.55^2 - 2(49.32)(54.55)(\cos 10.19°)}$$

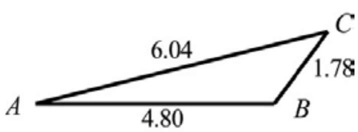
$$= 10.59$$

17.



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$723^2 = 598^2 + 158^2 - 2(598)(158)\cos A$$
$$\cos A = -0.7417$$
$$A = 138°$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$598^2 = 723^2 + 158^2 - 2(723)(158)\cos B$$
$$\cos B = 0.8320$$
$$B = 33.7°$$
$$C = 180° - A - B$$
$$= 180° - 138° - 33.7° = 8.3°$$

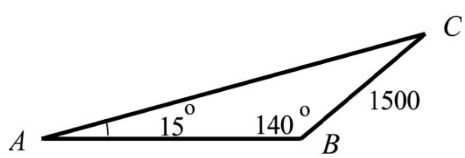
18.



$a = 1.78, b = 6.04, c = 4.80$

$$\cos C = \frac{1.78^2 + 6.04^2 - 4.80^2}{2(1.78)(6.04)} = 0.772$$
$$C = 39.4°$$
$$\cos A = \frac{6.04^2 + 4.80^2 - 1.78^2}{2(6.04)(4.80)} = 0.972$$
$$A = 13.6°$$
$$B = 180° - 13.6° - 39.4° = 127°$$

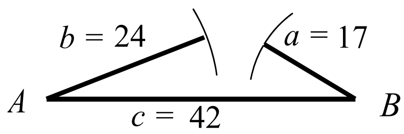
19.



$a = 1500, A = 15°, B = 140°$

$$\frac{a}{\sin A} = \frac{b}{\sin B};\ b = \frac{1500\sin 140°}{\sin 15°} = 3700$$
$$C = 180° - 15° - 140° = 25°$$
$$c = \sqrt{1500^2 + 3700^2 - 2(1500)(3700)(\cos 25°)}$$
$$= 2400$$

20.



Since $a+b<c$

$17+24<42$

$41<42,$

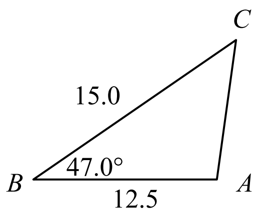
a, b, c do not determine a triangle.

We can also see that there is no solution if we use the law of cosines:

$$\cos C=\frac{17^2+24^2-42^2}{2(17)(24)}=-1.10$$

There is no such angle C.

21.



By the law of cosines,

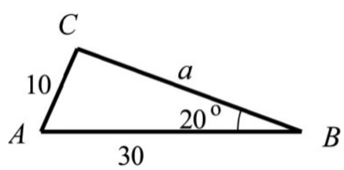
$c^2=a^2+b^2-2ab\cos C$

$c^2=12.5^2+15.0^2-2(12.5)(15.0)\cos 47.0°$

$c^2=125.5$

$c=\sqrt{125.5}=11.2$ ft

22.



Using the law of cosines, a may be found from the quadratic equation.

$a^2+30^2-2(a)(30)\cos 20°-10^2=0$ which may then be used to find A and C as follows.

$a^2=10^2+30^2-2(10)(30)\cos A$

$$A=\cos^{-1}\frac{10^2+30^2-a^2}{2(10)(30)}$$

$C=180°-(A+20°)$

The law of sines $\dfrac{a}{\sin A}=\dfrac{10}{\sin 20°}=\dfrac{30}{\sin C}$ is easier since it gives C directly from which a and A may be found.

23. By the law of cosines,

$c^2 = a^2 + b^2 - 2ab\cos C$

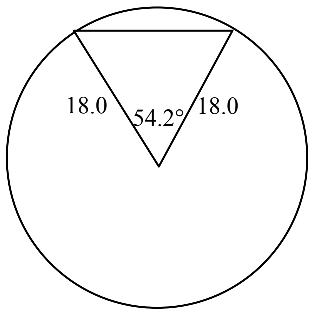
If $C = 90°$ then

$c^2 = a^2 + b^2 - 2ab\cos 90°$

$c^2 = a^2 + b^2$

which is the Pythagorean theorem.

24.



By the law of cosines,

$c^2 = a^2 + b^2 - 2ab\cos C$

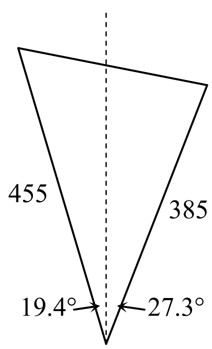
where $a = b = 18.0, C = 54.2°$

$c^2 = 18.0^2 + 18.0^2 - 2(18.0)(18.0)\cos 54.2°$

$c^2 = 268.947427$

$c = 16.4$

25.



By the law of cosines,

$c^2 = a^2 + b^2 - 2ab\cos C$

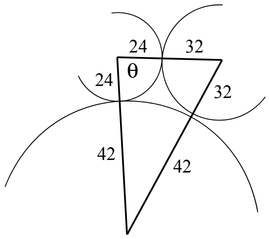
where $a = 385, b = 455, C = 19.4° + 27.3° = 46.7°$

$c^2 = 385^2 + 455^2 - 2(385)(455)\cos 46.7°$

$c^2 = 114973$

$c = 339$ mi

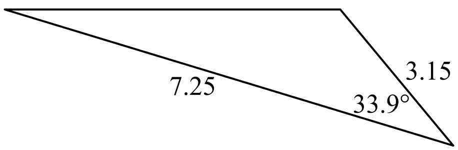
26.



$74^2 = 56^2 + 66^2 - 2(56)(66)\cos\theta$ from which

$\theta = 74^\circ$

27.

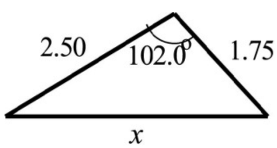


$c^2 = 7.25^2 + 3.15^2 - 2(7.25)(3.15)\cos 33.9°$

$c^2 = 24.574$

$c = 4.96$ km

28.



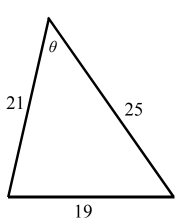
$x = \sqrt{1.75^2 + 2.50^2 - 2(2.50)(1.75)(\cos 102.0^\circ)}$

$= 3.34$ m

29. $19^2 = 21^2 + 25^2 - 2(21)(25)\cos\theta$

$\cos\theta = \dfrac{21^2 + 25^2 - 19^2}{2(21)(25)} = 0.6714286$

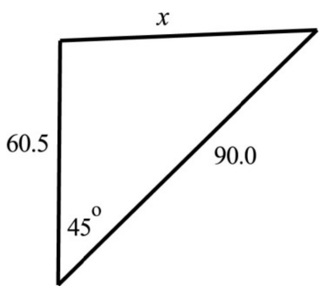
$\theta = 48^0$



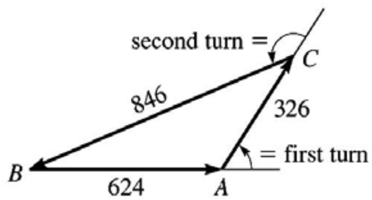
30. $x=\sqrt{60.5^2+90.0^2-2(60.5)(90.0)(\cos 45^\circ)}$

$=63.7\text{ ft}\left(\dfrac{0.3048\text{ m}}{1\text{ft}}\right)$

$=19.4\text{ m}$



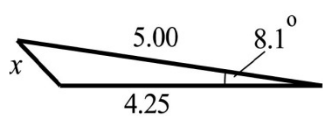
31.



$$\begin{aligned}846^2 &= 624^2+326^2-2(624)(326)\cos A\\ \cos A &= -0.5409\\ A &= 122.7^\circ\\ \text{first turn} &= 180^\circ-122.7^\circ=57.3^\circ\\ 624^2 &= 846^2+326^2-2(846)(326)\cos C\\ \cos C &= 0.7843\\ C &= 38.3^\circ\\ \text{second turn} &= 180^\circ-38.3^\circ=141.7^\circ\end{aligned}$$

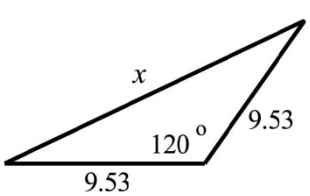
32. $x=\sqrt{5.00^2+4.25^2-2(5.00)(4.25)(\cos 8.1^\circ)}$

$=0.99\text{ in.}$

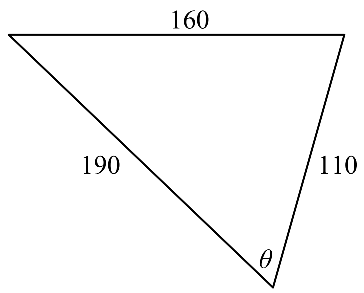


33. $x=\sqrt{9.53^2+9.53^2-2(9.53)(9.53)(\cos 120^\circ)}$

$=16.5\text{ mm}$

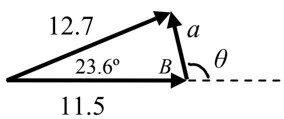


34.



$$\cos\theta = \frac{190^2 + 110^2 - 160^2}{2(190)(110)} = 0.54067,\ \theta = 57.3^\circ$$

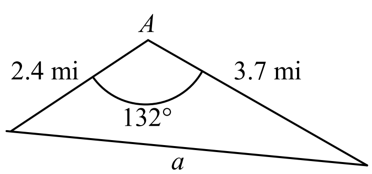
35.



$$a^2 = 12.7^2 + 11.5^2 - 2(12.7)(11.5)\cos 23.6^\circ$$
$$a = 5.09 \text{ km/h}$$
$$\frac{12.7}{\sin B} = \frac{5.09}{\sin 23.6^\circ}$$
$$B = \sin^{-1}\frac{12.7\sin 23.6^\circ}{5.09}$$
$$= 88.4^\circ$$
$$\theta = 180^\circ - 88.4^\circ$$
$$= 91.6^\circ$$

The current is moving at 5.09 km/h at an angle of $\theta = 92.7^\circ$ with respect to the land.

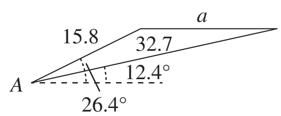
36.



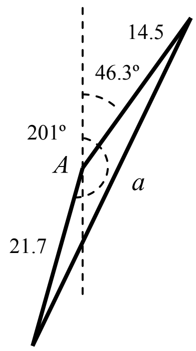
$$a^2 = 2.4^2 + 3.7^2 - 2(2.4)(3.7)\cos 132^\circ$$
$$= 31.334$$
$$a = 5.6 \text{ mi}$$

37. $A = 26.4^\circ - 12.4^\circ = 14.0^\circ$

$$a = \sqrt{15.8^2 + 32.7^2 - 2(15.8)(32.7)\cos 14.0^\circ}$$
$$= 17.8 \text{ mi}$$



38.

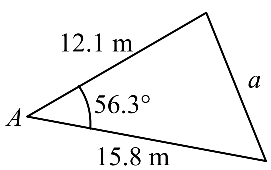


$A = 201^\circ - 46.3^\circ = 154.7^\circ$

$a^2 = 14.5^2 + 21.7^2 - 2(14.5)(21.7)\cos 154.7^\circ$

$a = 35.4 \text{ km}$

39.



$$a = \sqrt{15.8^2 + 12.1^2 - 2(15.8)(12.1)\cos 56.3^\circ}$$
$$= 13.6 \text{ m}$$

40. **(a)** If we know three sides, then we use the law of cosines (Case 4).

(b) If we know two sides and the angle opposite one of them, then we use the law of sines (Case 2). Since the longer side is opposite the known angle, the solution will be unique.

(c) If we know two sides and the angle between them, then we use the law of cosines (Case 3).

Review Exercises

1. True

2. False. The magnitude of the x-component is $A_x = A\cos\theta$.

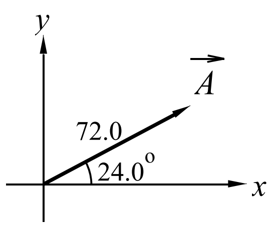
3. False. The direction of the resultant is also required.

4. False. In order to use the law of sines, at least one side and an opposite angle are needed.

5. False. One should use the law of sines to determine a second angle. It is possible for there to be two solutions. This is Case 2 from the Summary of Solving Oblique Triangles.

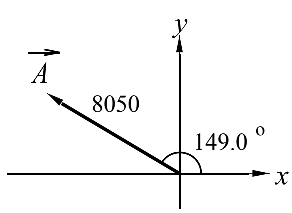
6. True. This is Case 2 from the Summary of Solving Oblique Triangles.

7.



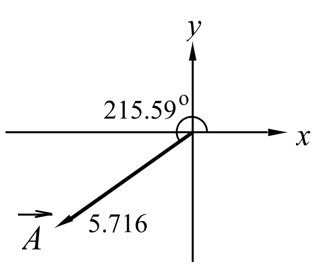
$$A_x = 72.0\cos 24.0^\circ = 65.8$$
$$A_y = 72.0\sin 24.0^\circ = 29.3$$

8.



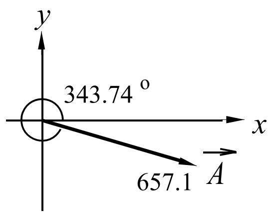
$$A_x = A\cos\theta_A = 8050\cos 149.0^\circ = -6900$$
$$A_y = A\sin\theta_A = 8050\sin 149.0^\circ = 4150$$

9.



$$A_x = A\cos\theta_A = 5.716\cos 215.59^\circ = -4.648$$
$$A_y = A\sin\theta_A = 5.716\sin 215.59^\circ = -3.327$$

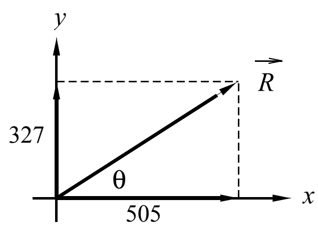
10.



$A_x = A\cos\theta_A = 657.1\cos 343.74^\circ = 630.8$

$A_y = A\sin\theta_A = 657.1\sin 343.74^\circ = -184.0$

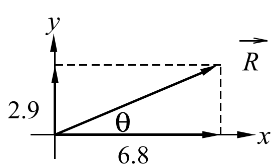
11.



$R = \sqrt{327^2 + 505^2} = 602$

$\theta = \tan^{-1}\dfrac{327}{505} = 32.9^\circ$

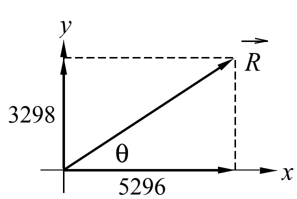
12.



$R = \sqrt{6.8^2 + 2.9^2} = 7.4$

$\theta = \tan^{-1}\dfrac{2.9}{6.8} = 23^\circ$

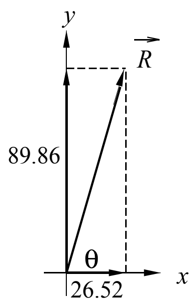
13.



$R = \sqrt{5296^2 + 3298^2} = 6239$

$\theta = \tan^{-1}\dfrac{3298}{5296} = 31.91^\circ$

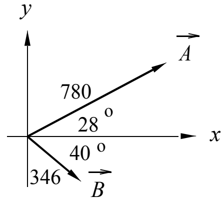
14.

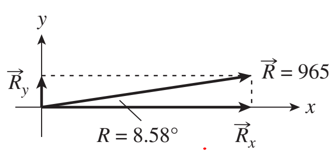


$$R = \sqrt{26.52^2 + 89.86^2} = 93.69$$

$$\theta = \tan^{-1}\frac{89.86}{26.52} = 73.56^\circ$$

15.





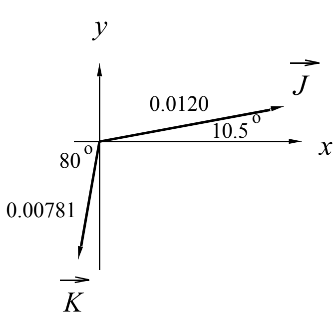
$$R_x = 780\cos 28.0^\circ + 346\cos 40.0^\circ$$

$$R_y = 780\sin 28.0^\circ - 346\sin 40.0^\circ$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{954^2 + 144^2} = 965$$

$$\theta_R = \tan^{-1}\frac{144}{954} = 8.58^\circ$$

16.



$J_x = 0.0120\cos 370.5^\circ = 0.0118$

$K_x = 0.00781\cos 260.0^\circ = -0.00136$

$R_x = J_x + K_x = 0.0104$

$J_y = 0.0120\sin 370.5^\circ = 0.00219$

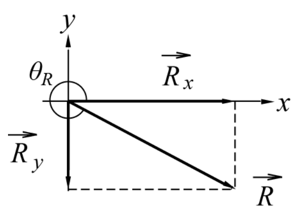
$K_y = 0.00781\sin 260.0^\circ = -0.00769$

$R_y = J_y + K_y = -0.0055$

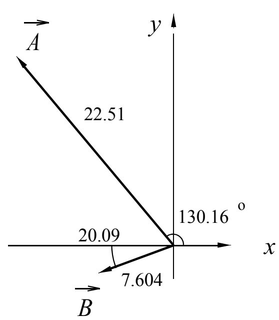
$R = \sqrt{R_x^2 + R_y^2} = 0.0118$

$\theta_{\text{ref}} = \tan^{-1}\left|\frac{R_y}{R_x}\right| = 27.79^\circ$

$\theta_R = 360^\circ - 27.79^\circ = 332.21^\circ$



17.



$A_x = 22.51\cos 130.16^\circ = -14.52$

$B_x = 7.604\cos 200.09^\circ = -7.141$

$R_x = A_x + B_x = -21.66$

$A_y = 22.51\sin 130.16^\circ = 17.20$

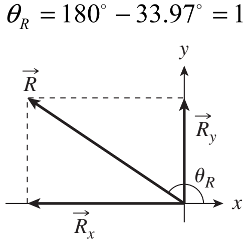
$B_y = 7.604\sin 200.09^\circ = -2.612$

$R_y = A_y + B_y = 14.59$

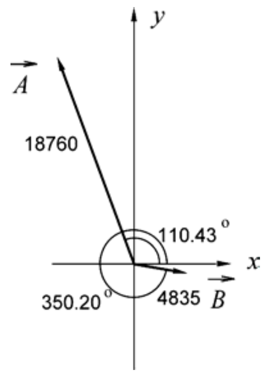
$R = \sqrt{R_x^2 + R_y^2} = 26.12$

$\theta_{\text{ref}} = \tan^{-1}\left|\frac{R_y}{R_x}\right| = 33.97^\circ$

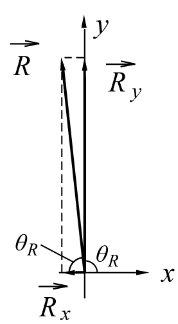
$\theta_R = 180^\circ - 33.97^\circ = 146.03^\circ$



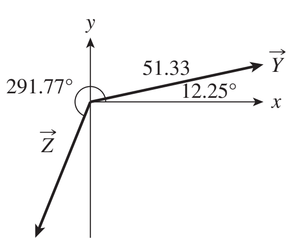
18.



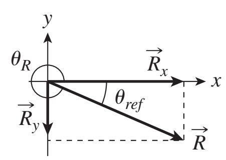
$$A_x = 18760\cos 110.43^\circ = -6548$$
$$B_x = 4835\cos 350.20^\circ = 4764$$
$$R_x = -1784$$
$$A_y = 18760\sin 110.43^\circ = 17580$$
$$B_y = 4835\sin 350.20^\circ = -823.0$$
$$R_y = 16760$$
$$R = \sqrt{R_x^2 + R_y^2} = 16850$$
$$\theta_{\text{ref}} = \tan^{-1}\left|\frac{R_y}{R_x}\right| = 83.92^\circ$$
$$\theta = 180^\circ - 83.92^\circ = 96.08^\circ$$



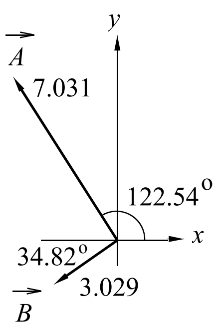
19.



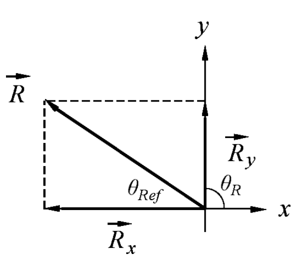
$$Y_x = 51.33\cos 12.25^\circ = 50.16$$
$$Y_y = 51.33\sin 12.25^\circ = 10.89$$
$$Z_x = 42.61\cos 291.77^\circ = 15.80$$
$$Z_y = 42.61\sin 291.77^\circ = -39.57$$
$$R_x = 50.16 + 15.80 = 65.96$$
$$R_y = 10.89 - 39.57 = -28.68$$
$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{65.96^2 + (-28.68)^2} = 71.94$$
$$\theta_{ref} = \tan^{-1}\left|\frac{-28.68}{65.96}\right| = 23.50^\circ$$
$$\theta_R = 360^\circ - 23.50^\circ = 336.5^\circ$$



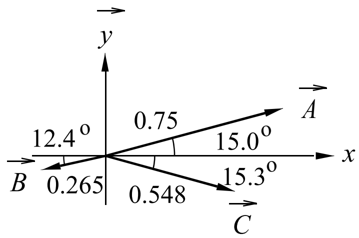
20.



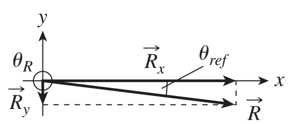
$$A_x = 7.031\cos 122.54^\circ = -3.782$$
$$B_x = 3.029\cos 214.82^\circ = -2.487$$
$$R_x = -6.269$$
$$A_y = 7.031\sin 122.54^\circ = 5.927$$
$$B_y = 3.029\sin 214.82^\circ = -1.730$$
$$R_y = 4.197$$
$$R = \sqrt{R_x^2 + R_y^2} = 7.544$$
$$\theta_{\text{ref}} = \tan^{-1}\left|\frac{R_y}{R_x}\right| = 33.80^\circ$$
$$\theta_R = 180^\circ - 33.80^\circ = 146.20^\circ$$



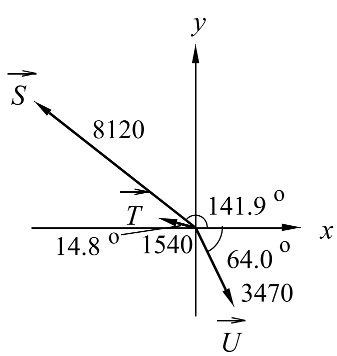
21.



$$A_x = 0.750\cos 15.0^\circ = 0.724$$
$$B_x = 0.265\cos 192.4^\circ = -0.259$$
$$C_x = 0.548\cos 344.7^\circ = 0.529$$
$$R_x = 0.994$$
$$A_y = 0.750\sin 15.0^\circ = 0.1941$$
$$B_y = 0.265\sin 192.4^\circ = -0.0569$$
$$C_y = 0.548\sin 344.7^\circ = -0.1446$$
$$R_y = -0.00740$$
$$R = \sqrt{R_x^2 + R_y^2} = 0.994$$
$$\theta_{ref} = \tan^{-1}\left|\frac{R_y}{R_x}\right| = 0.426^\circ$$
$$\theta_R = 360^\circ - 0.426^\circ = 359.6^\circ$$

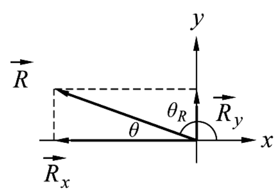


22.

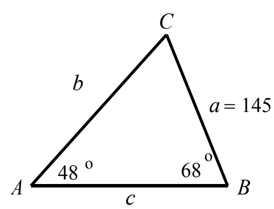


$$S_x = 8120\cos 141.9^\circ = -6390$$
$$T_x = 1540\cos 165.2^\circ = -1490$$
$$U_x = 3470\cos 296.0^\circ = 1520$$
$$R_x = -6360$$
$$S_y = 8120\sin 141.9 = 5010$$

$$T_y = 1540\sin 165.2 = 393$$
$$U_y = 3470\sin 296.0 = -3120$$
$$R_y = 2280$$
$$R = \sqrt{R_x^2 + R_y^2} = 6760$$
$$\theta_{\text{ref}} = \tan^{-1}\left|\frac{R_y}{R_x}\right| = 19.7^\circ$$
$$\theta_R = 180^\circ - 19.7^\circ = 160.3^\circ$$



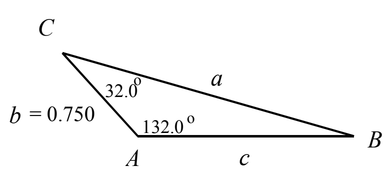
23.



Given: two angles and one side (Case 1).

$$C = 180^\circ - 48.0^\circ - 68.0^\circ = 64.0^\circ$$
$$\frac{145}{\sin 48.0^\circ} = \frac{b}{\sin 68.0^\circ} = \frac{c}{\sin 64.0^\circ}$$
$$b = \frac{145\sin 68.0^\circ}{\sin 48.0^\circ} = 181$$
$$c = \frac{145\sin 64.0^\circ}{\sin 48.0^\circ} = 175$$

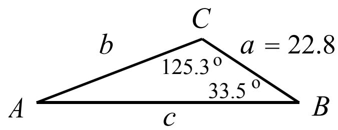
24.



Given: two angles and one side (Case 1).

$$B = 180.0^\circ - \left(32.0^\circ + 132.0^\circ\right) = 16.0^\circ$$
$$\frac{a}{\sin 132.0^\circ} = \frac{0.750}{\sin 16.0^\circ} = \frac{c}{\sin 32.0^\circ}$$
$$a = \frac{0.750\sin 132.0^\circ}{\sin 16.0^\circ} = 2.02$$
$$c = \frac{0.750\sin 32.0^\circ}{\sin 16.0^\circ} = 1.44$$

25.



Given: two angles and one side (Case 1).

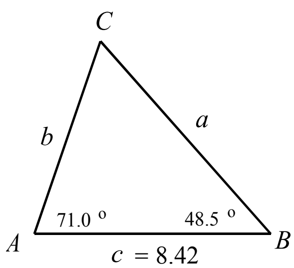
$A = 180.0^\circ - (125.3^\circ + 33.5^\circ) = 21.2^\circ$

$$\frac{22.8}{\sin 21.2^\circ} = \frac{b}{\sin 33.5^\circ} = \frac{c}{\sin 125.3^\circ}$$

$$b = \frac{22.8 \sin 33.5^\circ}{\sin 21.2^\circ} = 34.8$$

$$c = \frac{22.8 \sin 125.3^\circ}{\sin 21.2^\circ} = 51.5$$

26.



Given: two angles and one side (Case 1).

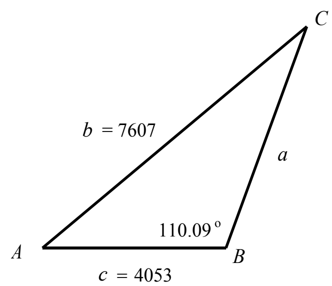
$C = 180.0^\circ - (71.0^\circ + 48.5^\circ) = 60.5^\circ$

$$\frac{a}{\sin 71.0^\circ} = \frac{b}{\sin 48.5^\circ} = \frac{8.42}{\sin 60.5^\circ}$$

$$a = \frac{8.42 \sin 71.0^\circ}{\sin 60.5^\circ} = 9.15$$

$$b = \frac{8.42 \sin 48.5^\circ}{\sin 60.5^\circ} = 7.25$$

27.



Given: two sides and the angle opposite to the longer side (Case 2, unique solution.)

$$\frac{7607}{\sin 110.09^\circ} = \frac{4053}{\sin C}$$

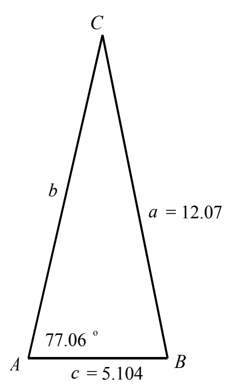
$$C = \sin^{-1}\frac{4053\sin 110.09^\circ}{7607} = 30.02^\circ$$

$$A = 180.0^\circ - \left(110.09^\circ + 30.02^\circ\right) = 39.89^\circ$$

$$\frac{a}{\sin 39.89^\circ} = \frac{7607}{\sin 110.09^\circ}$$

$$a = \frac{7607\sin 39.89^\circ}{\sin 110.09^\circ} = 5195$$

28.



Given: two sides and the angle opposite the longer side (Case 2, unique solution).

$$\frac{5.104}{\sin C} = \frac{12.07}{\sin 77.06^\circ}$$

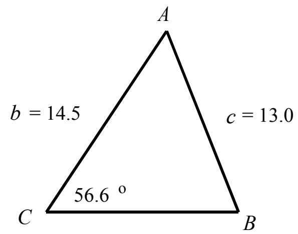
$$C = \sin^{-1}\frac{5.104\sin 77.06^\circ}{12.07} = 24.34^\circ$$

$$B = 180.0^\circ - \left(77.06^\circ + 24.34^\circ\right) = 78.60^\circ$$

$$\frac{12.07}{\sin 77.06^\circ} = \frac{b}{\sin 78.60^\circ}$$

$$b = \frac{12.07\sin 78.60^\circ}{\sin 77.06^\circ} = 12.14$$

29.



Given: two sides and the angle opposite the shorter side (Case 2, two solutions).

$$\frac{a}{\sin A} = \frac{14.5}{\sin B} = \frac{13.0}{\sin 56.6^\circ}$$

$$\sin B = \frac{14.5 \sin 56.6^\circ}{13.0}$$

$B = 68.6^\circ$ or 111.4°

Solution 1:

$B = 68.6^\circ$

$A = 180^\circ - 68.6^\circ - 56.6^\circ = 54.8^\circ$

$$\frac{a}{\sin 54.8^\circ} = \frac{13.0}{\sin 56.6^\circ}$$

$$a = \frac{13 \sin 54.8^\circ}{\sin 56.6^\circ} = 12.7$$

Solution 2:

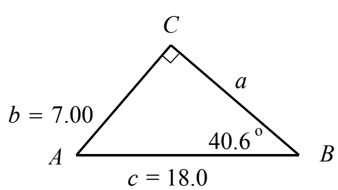
$B = 111.4^\circ$

$A = 180^\circ - 111.4^\circ - 56.6^\circ = 12.0^\circ$

$$\frac{a}{\sin 12.0^\circ} = \frac{13.0}{\sin 56.6^\circ}$$

$$a = \frac{13 \sin 12.0^\circ}{\sin 56.6^\circ} = 3.24$$

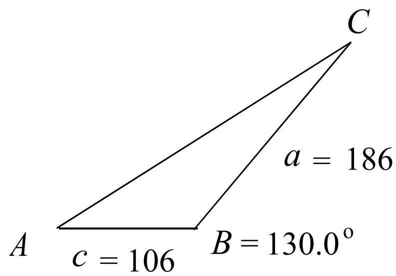
30.



Given: two sides and the angle opposite the shorter side (Case 2).

But $7 < 18 \sin 40.6^\circ = 11.7,$ so there is no solution.

31.



Given: two sides and the included angle (Case 3).

$b^2 = 106^2 + 186^2 - 2(106)(186)\cos 130.0^\circ$

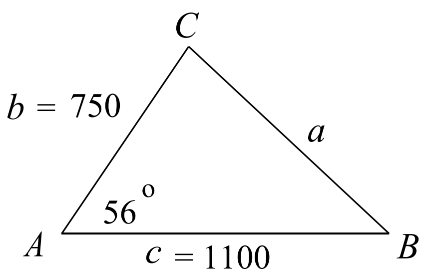
$b = 267$

$\dfrac{186}{\sin A} = \dfrac{267}{\sin 130.0^\circ}$

$A = \sin^{-1}\dfrac{186\sin 130.0^\circ}{267} = 32.3^\circ$

$C = 180^\circ - (130.0^\circ + 32.3^\circ) = 17.7^\circ$

32.



Given: two sides and the included angle (Case 3).

$a^2 = 750^2 + 1100^2 - 2(750)(1100)\cos 56^\circ$

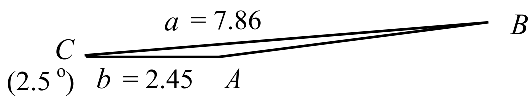
$a = 920$

$\dfrac{920}{\sin 56^\circ} = \dfrac{750}{\sin B}$

$B = \sin^{-1}\dfrac{750\sin 56^\circ}{920} = 43^\circ$

$C = 180^\circ - (56^\circ + 43^\circ) = 81^\circ$

33.



Given: two sides and the included angle (Case 3).

$c^2 = 7.86^2 + 2.45^2 - 2(7.86)(2.45)\cos 2.5^\circ$

$c = 5.413386814$

$c = 5.41$

$7.86^2 = 2.45^2 + c^2 - 2(2.45)(c)\cos A$

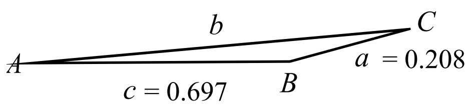
(We use c without rounding for more precision since the angle is so close to 180°).

$A = 176.4^\circ$

$B = 180^\circ - 2.5^\circ - 176.4^\circ$

$\quad = 1.1^\circ$

34.



Given: two sides and the included angle (Case 3).

$b^2 = 0.697^2 + 0.208^2 - 2(0.697)(0.208)\cos 165.4^\circ$

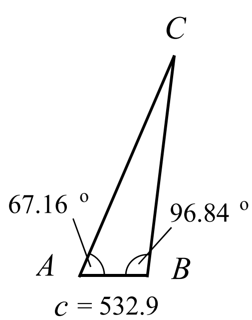
$b = 0.900$

$$\frac{0.208}{\sin A} = \frac{0.900}{\sin 165.4^\circ}$$

$$A = \sin^{-1}\frac{.208\sin 165.4^\circ}{0.900} = 3.34^\circ$$

$C = 180^\circ - (165.4^\circ + 3.34^\circ) = 11.3^\circ$

35.



Given: two angles and one side (Case 1).

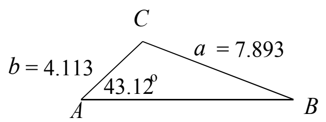
$C = 180^\circ - (67.16^\circ + 96.84^\circ) = 16.00^\circ$

$$\frac{a}{\sin 67.16^\circ} = \frac{b}{\sin 96.84^\circ} = \frac{532.9}{\sin 16.00^\circ}$$

$$a = \frac{532.9\sin 67.16^\circ}{\sin 16.00^\circ} = 1782$$

$$b = \frac{532.9\sin 96.84^\circ}{\sin 16.00^\circ} = 1920$$

36.



Given: two sides and the angle opposite the larger side (Case 2, unique solution).

$$\frac{7.893}{\sin 43.12^\circ} = \frac{4.113}{\sin B}$$

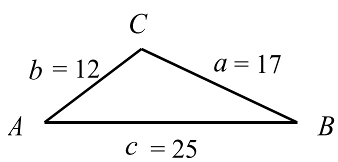
$$B = \sin^{-1}\frac{4.113\sin 43.12^\circ}{7.893} = 20.87^\circ$$

$$C = 180^\circ - \left(43.12^\circ + 20.87^\circ\right) = 116.0^\circ$$

$$\frac{7.893}{\sin 43.12^\circ} = \frac{c}{\sin 116.0^\circ}$$

$$c = \frac{7.893\sin 116.0^\circ}{\sin 43.12^\circ} = 10.38$$

37.



Given: three sides (Case 4).

$$17^2 = 12^2 + 25^2 - 2(12)(25)\cos A$$

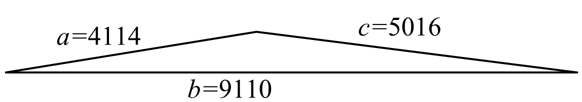
$$A = 37^\circ$$

$$12^2 = 17^2 + 25^2 - 2(17)(25)\cos B$$

$$B = 25^\circ$$

$$C = 180^\circ - 37^\circ - 25^\circ = 118^\circ$$

38.



Given: three sides (Case 4).

$$5016^2 = 9110^2 + 4114^2 - 2(9110)(4114)\cos C$$

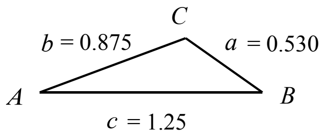
$$C = 4.19^\circ$$

$$9110^2 = 5016^2 + 4114^2 - 2(5016)(4114)\cos B$$

$$B = 172.38^\circ$$

$$A = 180^\circ - \left(4.19^\circ + 172.38^\circ\right) = 3.43^\circ$$

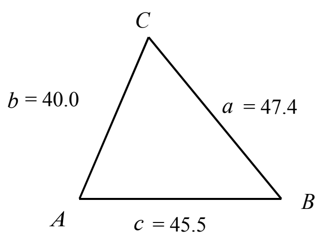
39.



Given: three sides (Case 4).

$$1.25^2 = 0.875^2 + 0.530^2 - 2(0.875)(0.530)\cos C$$
$$C = 124^\circ$$
$$0.530^2 = 0.875^2 + 1.25^2 - 2(0.875)(1.25)\cos A$$
$$A = 20.6^\circ$$
$$B = 180^\circ - (20.6^\circ + 124^\circ) = 35.4^\circ$$

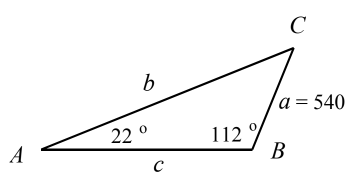
40.



Given: three sides (Case 4).

$$45.5^2 = 40.0^2 + 47.4^2 - 2(40.0)(47.4)\cos C$$
$$C = 62.1^\circ$$
$$47.4^2 = 40.0^2 + 45.5^2 - 2(40.0)(45.5)\cos A$$
$$A = 67.0^\circ$$
$$B = 180^\circ - (67.0^\circ + 62.1^\circ) = 50.9^\circ$$

41.



$$C = 180^\circ - (22^\circ + 112^\circ) = 46^\circ$$

The shortest side is opposite the smallest angle.

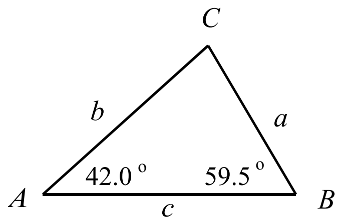
$a = 540$ m

The longest side is opposite the largest angle.

$b =$ longest side

$$\frac{b}{\sin 112^\circ} = \frac{540}{\sin 22^\circ}$$
$$b = \frac{54\sin 112^\circ}{\sin 22^\circ}$$
$$= 1300 \text{ m}$$

42.



$C = 180^\circ - \left(42.0^\circ + 59.5^\circ\right) = 78.5^\circ$, the largest angle

Therefore, C is the longest side and a is the shortest side and $c = a + 5.00$

$$\frac{a}{\sin 42.0^\circ} = \frac{b}{\sin 59.5^\circ} = \frac{c}{\sin 78.5^\circ}$$

$$\frac{a}{\sin 42.0^\circ} = \frac{a+5.00}{\sin 78.5^\circ}$$

$$a(\sin 78.5^\circ - \sin 42.0^\circ) = 5\sin 42.0^\circ$$

$$a = 10.8$$

$$c = a + 5.0$$

$$c = 15.8$$

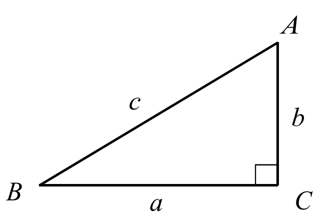
$$\frac{10.8}{\sin 42.0^\circ} = \frac{b}{\sin 59.5^\circ}$$

$$b = \frac{10.8\sin 59.5^\circ}{\sin 42.0^\circ}$$

$$= 13.9$$

$$p = a + b + c = 40.5 \text{ cm}$$

43.

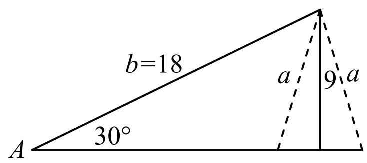


$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow b = \frac{a\sin B}{\sin A}$$

$$A_t = \frac{1}{2}ab = \frac{1}{2}\cdot a \cdot \frac{a\sin B}{\sin A}$$

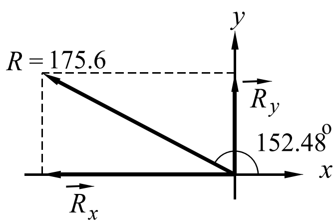
$$A_t = \frac{a^2 \sin B}{2\sin A}$$

44.



If $9 = b\cos A < a < 18$ then there are two possible solutions.

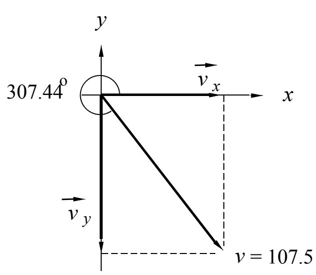
45.



$F_x = 175.6\cos 152.48^\circ = -155.7$ lb

$F_y = 175.6\sin 152.48^\circ = 81.14$ lb

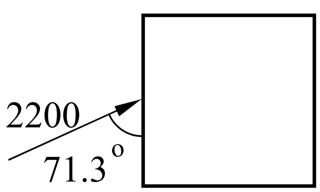
46.



$v_x = 170.5\cos 307.44^\circ = 103.7$ km/h

$v_y = 170.5\sin 307.44^\circ = -135.4$ km/h

47.



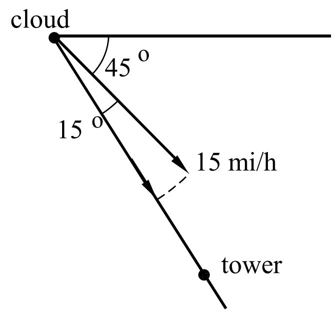
$v_\perp = 2200\sin 71.3^\circ$
$= 2084$ ft/s

We can also obtain $v_\perp$ if we find its direction in standard position, so that $\theta = 90^\circ - 71.3^\circ = 18.7^\circ$.

Then

$v_\perp = 2200\cos 18.7^\circ$
$= 2084$ ft/s

48.



$$\text{component toward tower} = 15\cos 15^\circ = 14 \text{ mi/h}$$

49. The horizontal components must sum to zero:

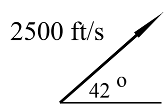
$$T_1 \cos 22.6° + 427\cos 162.5° = 0$$

$$T_1 = -\frac{427\cos 162.5°}{\cos 22.6°} = 441 \text{ lb}$$

In order for the vertical components to balance,

$$T_2 = T_1 \sin 22.6° + 427\sin 162.5° = 298 \text{ lb}$$

50.



$$v_x = 2500\cos 42.0^\circ = 1860 \text{ ft/s}$$

$$v_y = 2500\sin 42.0^\circ = 1670 \text{ ft/s}$$

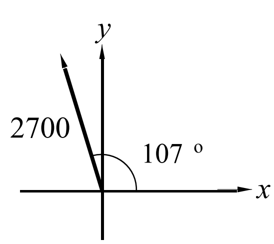
51.

vector	x-component	y-component
1300	$-1300\cos 54^\circ$	$1300\sin 54^\circ$
3200	$3200\sin 32^\circ$	$3200\cos 32^\circ$
2100	$-2100\cos 35^\circ$	$-2100\sin 35^\circ$
	-788.6	2561

$$R = \sqrt{(-788.6)^2 + (2561)^2} = 2700 \text{ lb}$$

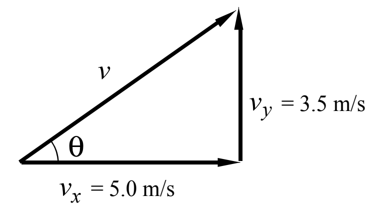
$$\theta_{\text{ref}} = \tan^{-1}\left|\frac{2561}{-788.6}\right| = 73^\circ$$

$$\theta = 180^\circ - \theta_{\text{ref}} = 107^\circ$$



52. $F_v = 15.0\cos 6.0^\circ = 14.9 \text{ mN}$

53.

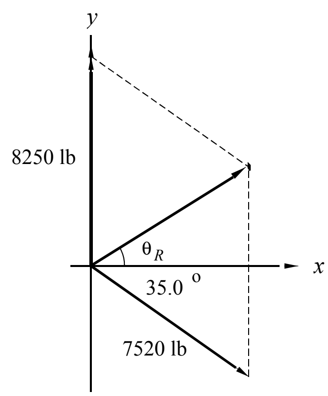


$$v = \sqrt{5.0^2 + 3.5^2} = 6.1$$

$$\theta = \tan^{-1}\frac{3.5}{5.0} = 35^\circ$$

resultant velocity = 6.1 m/s at 35° above horizontal

54.



$$R_x = 7520\cos 35.0^\circ$$

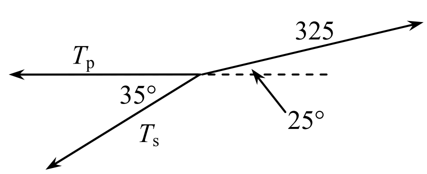
$$R_y = 8250 - 7520\sin 35.0^\circ$$

$$R = \sqrt{R_x^2 + R_y^2} = 7310 \text{ lb}$$

$$\theta_R = \tan^{-1}\frac{R_y}{R_x} = 32.6^\circ$$

55. upward force $= 0.15\sin 22.5^\circ + 0.20\sin 15.0^\circ$
$= 0.11$ N

56.



Vertical components:

$$T_s \sin 215° + 325\sin 25° = 0$$

$$T_s = 239 \text{ N}$$

Horizontal components:

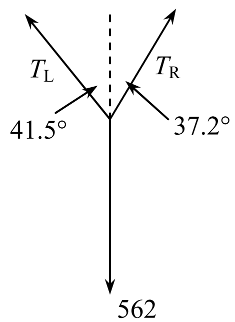
$$T_p = 325\cos 25° + T_s\cos 215°$$

$$T_p = 99 \text{ N}$$

57. $d^2 = 0.96^2 + 0.96^2 - 2(0.96)(0.96)\cos 105^\circ$

$d = 1.5 \text{ pm}$

58.



Horizontal components:

$$T_L \sin 41.5^\circ = T_R \sin 37.2^\circ$$
$$T_L = T_R \frac{\sin 37.2^\circ}{\sin 41.5^\circ}$$
$$T_L = 0.91244\ T_R$$

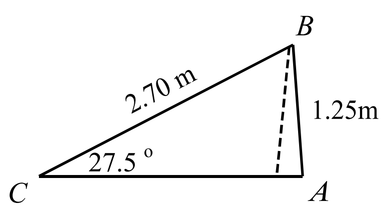
Vertical components:

$$562 = T_L \cos 41.5^\circ + T_R \cos 37.2^\circ$$
$$562 = 0.91244\ T_R\left(\cos 41.5^\circ\right) + T_R \cos 37.2^\circ$$
$$T_R = \frac{562}{0.91244\cos 41.5^\circ + \cos 37.2^\circ}$$
$$T_R = 380 \text{ lb}$$

To find T_L:

$$T_L = 0.91244\ T_R$$
$$T_L = 347 \text{ lb}$$

59.



$\dfrac{2.7}{\sin A} = \dfrac{1.25}{\sin 27.5} \qquad A_1 \quad 85.85^\circ$

$A_2 = 94.15^\circ$

$B_1 = 180^\circ - \left(27.5^\circ + A_1\right) = 66.65^\circ$

$B_2 = 180^\circ - \left(27.5^\circ + A_2\right) = 58.35^\circ$

short length

$= \sqrt{2.70^2 + 1.25^2 - 2(2.70)(1.25)\cos 58.35^\circ}$

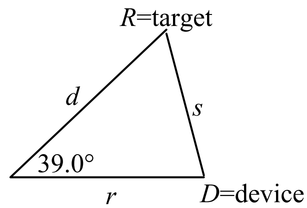
$= 2.30 \text{ m}$

long length

$= \sqrt{2.70^2 + 1.25^2 - 2(2.70)(1.25)\cos 66.651^\circ}$

$= 2.49 \text{ m}$

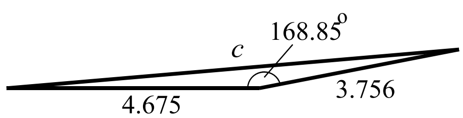
60.



$$d = 2200(0.20) = 440.0$$
$$s = 1130(0.32) = 361.6$$
$$\frac{361.6}{\sin 39^\circ} = \frac{440}{\sin D} = \frac{r}{\sin R}$$
$$D = \sin^{-1}\frac{440\sin 39.0^\circ}{361.6}$$
$$= 50.0^\circ \text{ or } 130.0^\circ$$
$$R = 180^\circ - (39.0^\circ + 50.0) = 91.0^\circ$$
$$r = \frac{361.6\sin 91.0^\circ}{\sin 39.0^\circ} = 574.5 \text{ ft}$$
or
$$R = 180^\circ - (39^\circ + 130^\circ) = 11^\circ$$
$$r = \frac{361.6\sin 11^\circ}{\sin 39^\circ} = 109.6 \text{ ft}$$

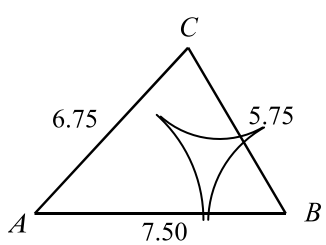
The recorder is on the ground either 109.6 ft or 574.5 ft from where the shot took place.

61.



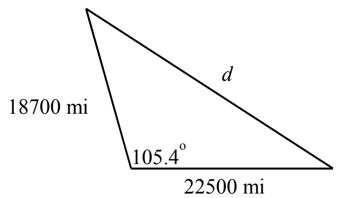
extra pipeline
$$= 3.756 + 4.675 - c$$
$$= 3.756 + 4.675$$
$$- \sqrt{3.756^2 + 4.675^2 - 2(3.756)(4.675)\cdot\cos 168.85^\circ}$$
$$= 0.039\,40 \text{ km}$$

62.



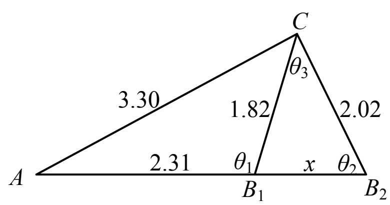
$$5.75^2 = 6.75^2 + 7.50^2 - 2(6.75)(7.50)\cos A$$
$$A = 47.2^\circ$$
$$6.75^2 = 5.75^2 + 7.50^2 - 2(5.75)(7.50)\cos B$$
$$B = 59.5^\circ$$
$$C = 180^\circ - (47.2^\circ + 59.5^\circ) = 73.3^\circ$$

63.



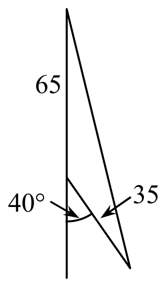
$$d = \sqrt{18700^2 + 22500^2 - 2(18700)(22500)\cos 105.4^\circ}$$
$$= 32900 \text{ mi}$$

64.



$$3.30^2 = 1.82^2 + 2.31^2 - 2(1.82)(2.31)\cos\theta_1$$
$$\theta_1 = 105.46^\circ$$
$$180^\circ - \theta_1 = 74.54^\circ$$
$$\frac{3.30}{\sin 105.46^\circ} = \frac{1.82}{\sin A}$$
$$A = \sin^{-1}\frac{1.82\sin 105.46^\circ}{3.30} = 32.11^\circ$$
$$\frac{3.30}{\sin\theta_2} = \frac{2.02}{\sin A}$$
$$\theta_2 = \sin^{-1}\frac{3.30\sin A}{2.02} = 60.27^\circ$$
$$\theta_3 = 180^\circ - 74.54^\circ - 60.27^\circ = 45.19^\circ$$
$$\frac{x}{\sin 45.19^\circ} = \frac{2.02}{\sin 74.54^\circ}$$
$$x = \frac{2.02\sin 45.19^\circ}{\sin 74.54^\circ} = 1.49 \text{ ft}$$

65.



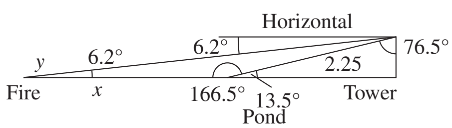
Components:

$$D_x = 35\sin 40° = 22.5 \text{ m}$$
$$D_y = -65 - 35\cos 40° = -91.8 \text{ m}$$
$$D = \sqrt{22.5^2 + (-91.8)^2} = 94.5 \text{ m}$$
$$\theta = \tan^{-1} \frac{-91.8}{22.5} = -76°$$

The skier is 94.5 m away at an angle of 14° east of south.

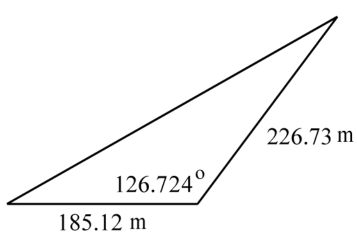
66. $\theta = \cos^{-1}\left(\dfrac{302^2 + 224^2 - 116^2}{2(302)(224)}\right) = 19.0°$

67.



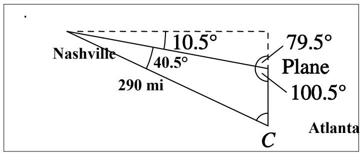
$$\frac{2.25}{\sin 6.2°} = \frac{x}{\sin 7.3°}$$
$$x = 2.65 \text{ km}$$

68.



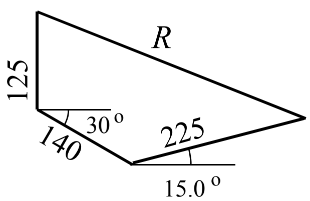
$$x^2 = 185.12^2 + 226.73^2 - 2(185.12)(226.73)\cos 126.724°$$
$$x = 368.61 \text{ m}$$

69.



$$C = 180^\circ - \left(\left(51.0^\circ - 10.5^\circ\right) - 100.5\right)$$
$$= 39^\circ$$
$$\frac{290}{\sin 100.5^\circ} = \frac{c}{\sin 39^\circ}$$
$$c = \frac{290 \sin 39^\circ}{\sin 100.5^\circ}$$
$$= 186 \text{ mi}$$

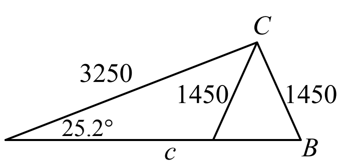
70.



$$R_x = 140\cos 30.0^\circ + 225\cos 15.0^\circ$$
$$R_y = -125 - 140\sin 30.0^\circ + 225\sin 15.0^\circ$$
$$R = \sqrt{R_x^2 + R_y^2} = 365$$
$$\theta_{ref} = \tan^{-1}\left|\frac{R_y}{R_x}\right| = 22.0^\circ$$
$$\theta_R = 338.0^\circ$$

displacement: 365 mi, 22.0° south of west

71.



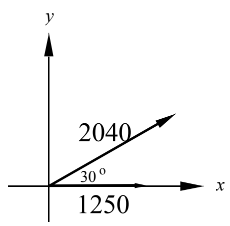
$$\frac{1450}{\sin 25.2^\circ} = \frac{3250}{\sin B} = \frac{c}{\sin C}$$
$$B = \sin^{-1}\frac{3250\sin 25.2^\circ}{1450}$$
$$= 72.6^\circ \text{ or } 107.4^\circ$$
$$C = 180^\circ - (25.2^\circ + 72.6^\circ)$$
$$= 82.2^\circ$$
$$c = \frac{1450\sin 82.2^\circ}{\sin 25.2^\circ}$$
$$= 3370 \text{ ft}$$

or

$$C = 180^\circ - (25.2^\circ + 107.4^\circ)$$
$$= 47.4^\circ$$
$$c = \frac{1450 \sin 47.4^\circ}{\sin 25.2^\circ}$$
$$= 2510 \text{ ft}$$

The observer is either 1680 m or 1270 m from the other end (ambiguous case).

72.



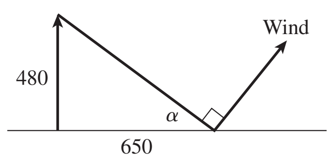
missle: $v_x = 1250 + 2040 \cos 30.0^\circ = 3016.69$

$v_y = 2040 \sin 30.0^\circ = 1020$

$v_v = -32.0t\big|_{t=10.0\text{ s}} = -320$

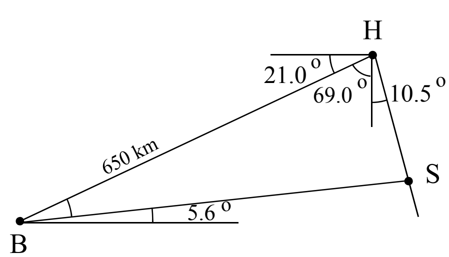
$v = \sqrt{v_x^2 + v_y^2 + v_v^2} = 3200 \text{ ft/s}$

73.



$$\tan\alpha = \frac{480}{650}$$
$$\alpha = 36^\circ \text{ N of E}$$
$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{650^2 + 480^2}$$
$$= 810 \text{ N}$$

74.

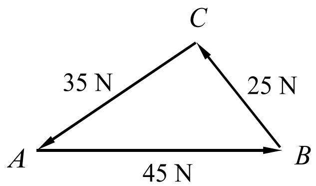


$$\frac{HS}{\sin 15.4^\circ} = \frac{BS}{\sin 79.5^\circ} = \frac{650}{\sin 85.1}$$

$BS = 641$ km from Boston to ship

$HS = 173$ km from Halifax to ship

75.



Since the resultant of the three forces is zero, the vectors form a closed triangle with sides of 45, 25, and 35. The angles between the forces may be found using the law of cosines.

$25^2 = 35^2 + 45^2 - 2(35)(45)\cos A$

$A = 34^\circ$

$35^2 = 25^2 + 45^2 - 2(25)(45)\cos B$

$B = 51^\circ$

$C = 180^\circ - (34^\circ - 51^\circ)$

$C = 95^\circ$

Chapter 10

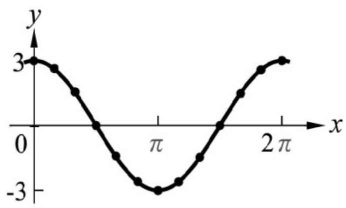
Graphs of The Trigonometric Functions

10.1 Graphs of $y = a \sin x$ and $y = a \cos x$

1. $y = 3 \cos x$

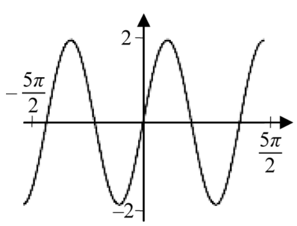
x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$
y	3	2.6	1.5	0	−1.5	−2.6

x	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
y	−3	−2.6	−1.5	0	1.5	2.6	3



2. $y = 2 \sin x$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	2	0	−2	0

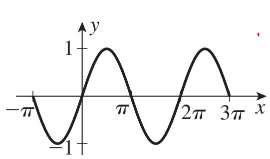


3.

$y = \sin x$

x	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	−0.7	−1	−0.7	0	0.7	1	0.7	0

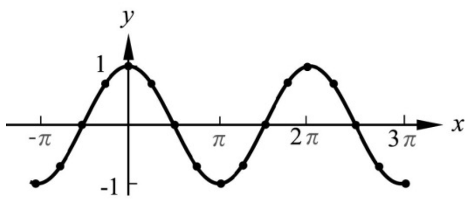
x	$\frac{5\pi}{4}$	$\frac{3\pi}{4}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5\pi}{2}$	$\frac{11\pi}{4}$	3π
y	−0.7	−1	−0.7	0	0.7	1	0.7	0



4. $y = \cos x$

x	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	–1	–0.7	0	0.7	1	0.7	0	–0.7	–1

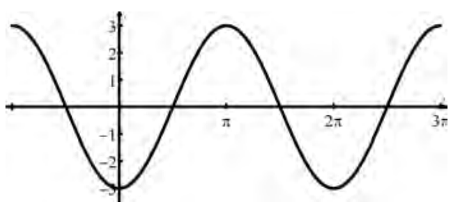
x	$\frac{5\pi}{4}$	$\frac{3\pi}{4}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5\pi}{2}$	$\frac{11\pi}{4}$	3π
y	–0.7	0	0.7	1	0.7	0	–0.7	–1



5. $y = -3 \cos x$

x	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	3	2.1	0	–2.1	–3	–2.1	0	2.1	3

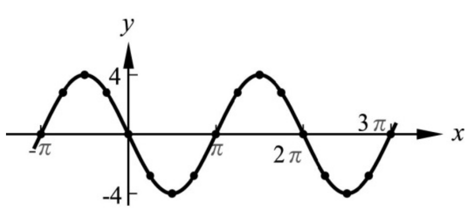
x	$\frac{5\pi}{4}$	$\frac{3\pi}{4}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5\pi}{2}$	$\frac{11\pi}{4}$	3π
y	2.1	0	–2.1	–3	–2.1	0	2.1	3



6. $y = -4 \sin x$

x	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	2.8	4	2.8	0	–2.8	–4	–2.8	0

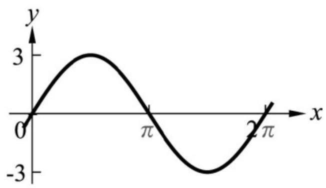
x	$\frac{5\pi}{4}$	$\frac{3\pi}{4}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5\pi}{2}$	$\frac{11\pi}{4}$	3π
y	2.8	4	2.8	0	–2.8	–4	–2.8	0



7. $y = 3 \sin x$ has amplitude 3.

The table for key values between 0 and 2π is

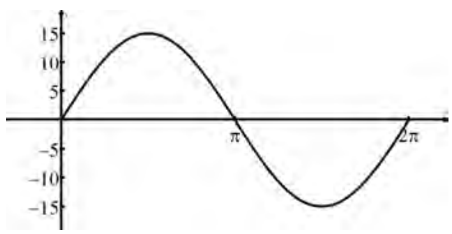
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	3	0	-3	0
		max.		min.	



8. $y = 15 \sin x$ has amplitude 15.

The table for key values between 0 and 2π is

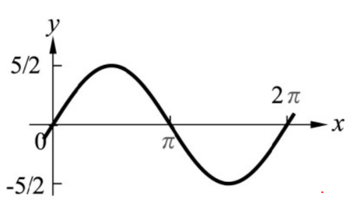
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0
$15 \sin x$	0	15	0	-15	0
		max.		min.	



9. $y = \frac{5}{2} \sin x$ has amplitude $\frac{5}{2}$.

The table for key values between 0 and 2π is

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	$\frac{5}{2}$	0	$-\frac{5}{2}$	0
		max		min	

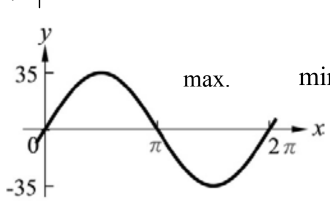


10. $y = 35 \sin x$ has amplitude 35.

The table for key values between 0 and 2π is

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	35	0	-35	0

max. min.

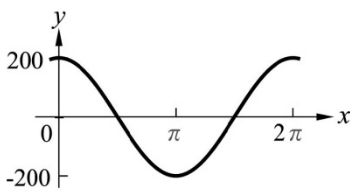


11. $y = 200 \cos x$ has amplitude 200

The table for key values between 0 and 2π is

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	200	0	–200	0	200

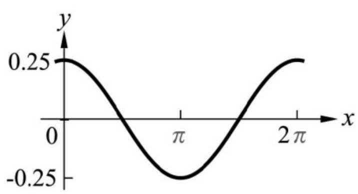
max min max



12. $y = 0.25 \cos x$ has amplitude 0.25.

The table for key values between 0 and 2π is

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0.25	0	–0.25	0	0.5

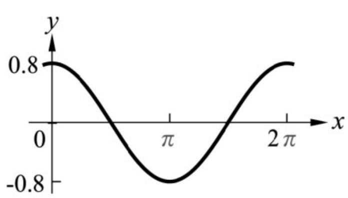


13. $y = 0.8 \cos x$ has amplitude 0.8.

The table for key values between 0 and 2π is

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0.8	0	–0.8	0	0.8

max min max

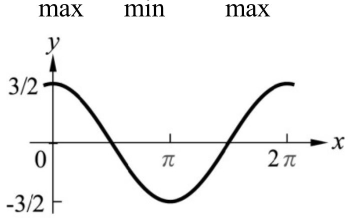


14. $y = \frac{3}{2} \cos x$ has amplitude $\frac{3}{2}$.

The table for key values between 0 and 2π is

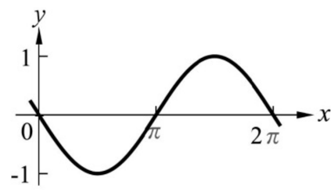
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	$\frac{3}{2}$	0	$-\frac{3}{2}$	0	$\frac{3}{2}$

max min max



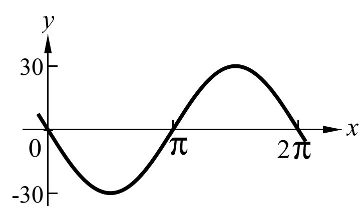
15. $y = -\sin x$ has amplitude 1.
The negative sign inverts the graph, so the table for key values between 0 and 2π is

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	-1	0	1	0
		min		max	



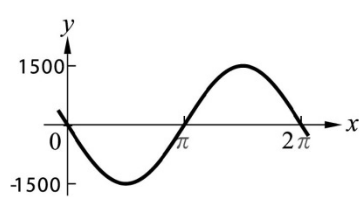
16. $y = -30 \sin x$ has amplitude 30. The negative sign inverts the graph, so the table of key values between 0 and 2π is

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	-30	0	30	0
		min		max	



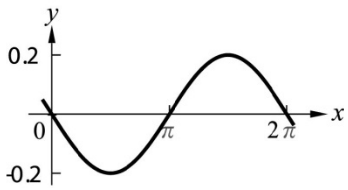
17. $y = -1500 \sin x$ has amplitude 1500.
The negative sign will invert the graph, so the table for key values between 0 and 2π is

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	-1500	0	1500	0
		min		max	



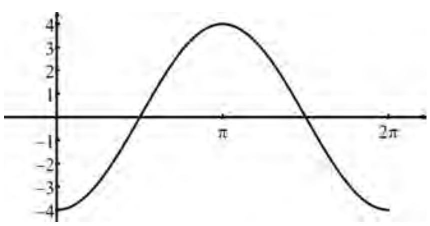
18. $y = -0.2 \sin x$ has amplitude 0.2.
The negative sign will invert the graph, so the table for key values between 0 and 2π is

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	–0.2	0	0.2	0
		min		max	



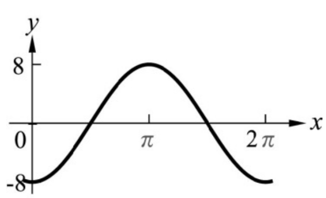
19. $y = -4 \cos x$ has amplitude 4.
The negative sign inverts the graph, so the table for key values between 0 and 2π is

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	–4	0	4	0	–4
	min		max		min



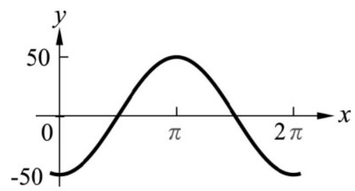
20. $y = -8 \cos x$ has amplitude 8.
The negative sign inverts the graph, so the table for key values between 0 and 2π is

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	–8	0	8	0	–8
	min		max		min



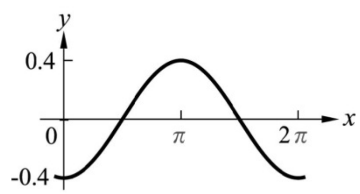
21. $y = -50 \cos x$ has amplitude 50.
The negative sign inverts the graph, so the table for key values between 0 and 2π is

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	-50	0	50	0	-50
	min		max		min



22. $y = -0.4 \cos x$ has amplitude 0.4.
The negative sign inverts the graph, so the table for key values between 0 and 2π is

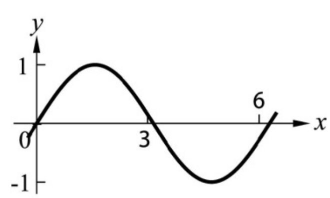
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	-0.4	0	0.4	0	-0.4



23. Sketch $y = \sin x$ for $x = 0, 1, 2, 3, 4, 5, 6, 7$

x	0	1	2	3	4
y	0	0.841	0.909	0.141	-0.757

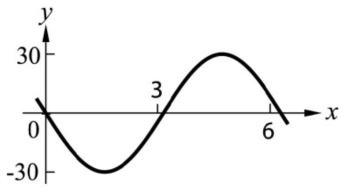
x	5	6	7
y	-0.959	-0.279	0.657



24. Sketch $y = -30 \sin x$ for $x = 0, 1, 2, 3, 4, 5, 6, 7$

x	0	1	2	3	4
$-30 \sin x$	0.0	–25.2	–27.3	–4.23	22.7

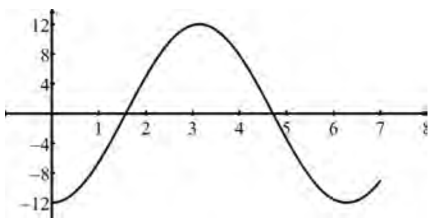
x	5	6	7
$-30 \sin x$	28.8	8.38	–19.7



25. Sketch $y = -12 \cos x$ for $x = 0, 1, 2, 3, 4, 5, 6, 7$

x	0	1	2	3	4
$-12 \cos x$	–12	–6.48	4.99	11.9	7.84

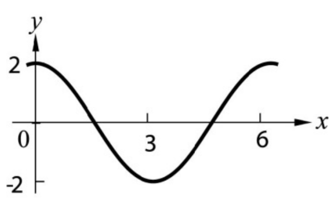
x	5	6	7
$-12 \cos x$	–3.40	–11.5	–9.05



26. Sketch $y = 2 \cos x$ for $x = 0, 1, 2, 3, 4, 5, 6, 7$

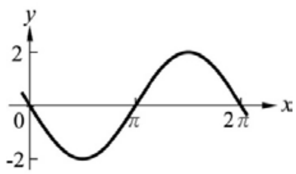
x	0	1	2	3	4
$2 \cos x$	2	1.08	–0.832	–1.98	–1.31

x	5	6	7
$2 \cos x$	0.567	1.92	1.51



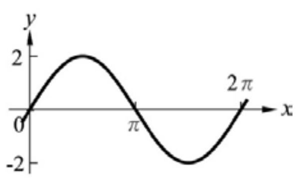
27. $y = a \sin x, \left(\dfrac{\pi}{2}, -2\right)$

$$-2 = a \sin \frac{\pi}{2}$$
$$-2 = a(1)$$
$$a = -2$$
$$y = -2 \sin x$$

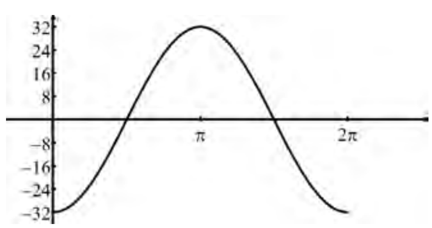


28. $y = a \sin x, \left(\dfrac{3\pi}{2}, -2\right)$

$$-2 = a \sin \frac{3\pi}{2}$$
$$-2 = a(-1)$$
$$a = 2$$
$$y = 2 \sin x$$

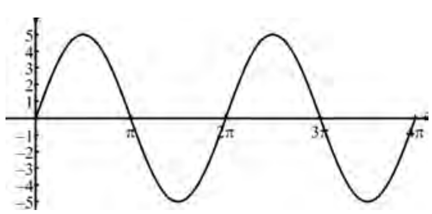


29. The graph of $h = -32\cos t$ passes through a complete cycle as t ranges from 0 to 2π.

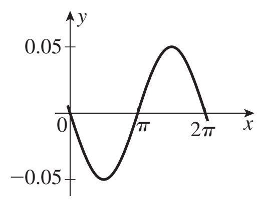


30.

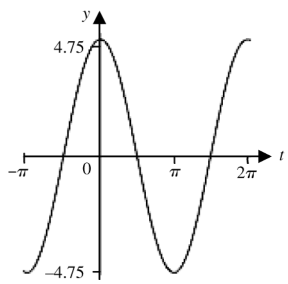
The graph of $d = 5\sin t$ passes through two complete cycles as t ranges from 0 to 4π.



31. $y = -0.05 \sin x$



32. $y = 4.75 \cos t$



33. The graph has zeros at $x = 0$, π, and, 2π, so it is a sine function. Its amplitude is 4, and it has not been inverted. Hence the function is $y = 4 \sin x$.

34. The graph has maxima at $x = 0$ and 2π and a minimum at $x = \pi$, so it is a cosine function. Its amplitude is 0.2, and it has not been inverted. Hence the function is $y = 0.2 \cos x$.

35. The graph has minima at $x = 0$ and 2π and a maximum at $x = \pi$, so it is a cosine function. Its amplitude is 1.5, and it has been inverted. Hence the function is $y = -1.5 \cos x$.

36. The graph has zeros at $x = 0$, π, 2π, so it is a sine function. Its amplitude is 6 and it has been inverted. Hence the function is $y = -6 \sin x$.

37. If amplitude is 2.50, the function has to be of the form $y = \pm 2.50 \sin x$ or $y = \pm 2.50 \cos x$. We evaluate each one at $x = 0.67$:

x	$\pm 2.50 \sin x$	$\pm 2.50 \cos x$
0.67	±1.55	±1.96

The function is $y = -2.50 \sin x$.

38. If amplitude is 2.50, the function has to be of the form $y = \pm 2.50 \sin x$ or $y = \pm 2.50 \cos x$. We evaluate each one at $x = -1.20$:

x	$\pm 2.50 \sin x$	$\pm 2.50 \cos x$
−1.20	∓ 2.33	±0.90

The function is $y = 2.50 \cos x$.

39. If amplitude is 2.50, the function has to be of the form $y = \pm 2.50 \sin x$ or $y = \pm 2.50 \cos x$. We evaluate each one at $x = 2.07$:

x	$\pm 2.50 \sin x$	$\pm 2.50 \cos x$
2.07	± 2.19	∓ 1.20

The function is $y = -2.50 \cos x$.

40. If amplitude is 2.50, the function has to be of the form $y = \pm\, 2.50 \sin x$ or $y = \pm\, 2.50 \cos x$. We evaluate each one at $x = -2.47$:

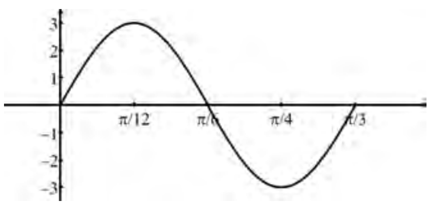
x	$\pm\, 2.50 \sin x$	$\pm\, 2.50 \cos x$
-2.47	∓ 1.56	∓ 1.96

The function is $y = 2.50 \sin x$.

10.2 Graphs of $y = a \sin bx$ and $y = a \cos bx$

1. $y = 3 \sin 6x$, amplitude = 3, period = $\frac{2\pi}{6} = \frac{\pi}{3}$, key values at multiples of $\frac{1}{4}\left(\frac{\pi}{3}\right) = \frac{\pi}{12}$.

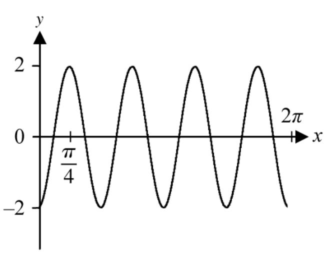
x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
y	0	3	0	−3	0	3	0	−3	0	3	0	−3	0



2. $y = -2 \cos 4x$, amplitude = 2, period = $\frac{2\pi}{4} = \frac{\pi}{2}$, key values at multiples of $\frac{1}{4}\left(\frac{\pi}{2}\right) = \frac{\pi}{8}$.

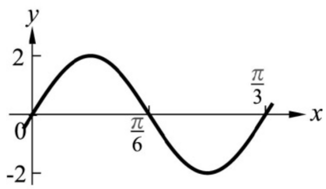
x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$	$\frac{5\pi}{8}$	$\frac{3\pi}{4}$	$\frac{7\pi}{8}$	π
y	−2	0	2	0	−2	0	2	0	−2

x	$\frac{9\pi}{8}$	$\frac{5\pi}{4}$	$\frac{11\pi}{8}$	$\frac{3\pi}{2}$	$\frac{13\pi}{8}$	$\frac{7\pi}{4}$	$\frac{15\pi}{8}$	2π
y	0	2	0	−2	0	2	0	−2



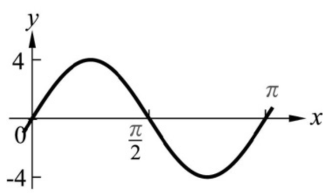
3. $y = 2 \sin 6x$ has amplitude of 2 and period $\dfrac{\pi}{3}$, with key values at multiples of $\dfrac{1}{4}\left(\dfrac{\pi}{3}\right) = \dfrac{\pi}{12}$

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	0	2	0	–2	0



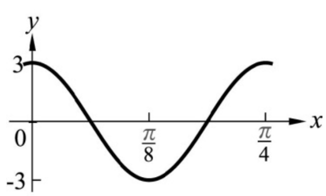
4. $y = 4 \sin 2x$ has amplitude of 4 and period π, with key values at multiples of $\pi/4$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$2x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin 2x$	0	1	0	–1	0
$4 \sin 2x$	0	4	0	–4	0



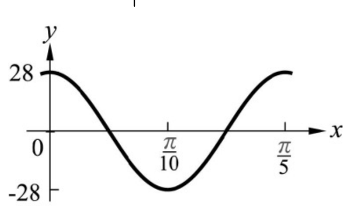
5. $y = 3 \cos 8x$ has amplitude of 3 and period $\dfrac{\pi}{4}$, with key values at multiples of $\dfrac{1}{4}\left(\dfrac{\pi}{4}\right) = \dfrac{\pi}{16}$.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	3	0	–3	0	3



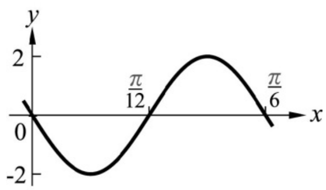
6. $y = 28 \cos 10x$ has amplitude of 28 and period of $\dfrac{\pi}{5}$, with key values at multiples of $\dfrac{1}{4}\left(\dfrac{\pi}{5}\right) = \dfrac{\pi}{20}$.

x	0	$\frac{\pi}{20}$	$\frac{\pi}{10}$	$\frac{3\pi}{20}$	$\frac{\pi}{5}$
$10x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$28\cos 10x$	28	0	–28	0	28



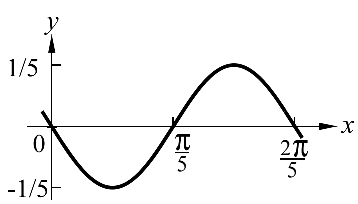
7. $y = -2 \sin 12x$ has amplitude of 2 and period of $\frac{\pi}{6}$, with key values at multiples of $\frac{1}{4}\left(\frac{\pi}{6}\right) = \frac{\pi}{24}$.

x	0	$\frac{\pi}{24}$	$\frac{\pi}{12}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$
y	0	-2	0	2	0



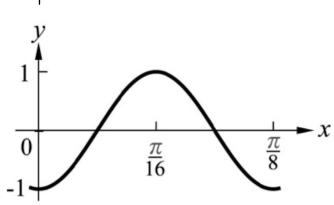
8. $y = -\frac{1}{5} \sin 5x$ has amplitude of $\left|-\frac{1}{5}\right| = \frac{1}{5}$, and period of $\frac{2\pi}{5}$, with key values at multiples of $\frac{1}{4}\left(\frac{2\pi}{5}\right) = \frac{\pi}{10}$.

x	0	$\frac{\pi}{10}$	$\frac{\pi}{5}$	$\frac{3\pi}{10}$	$\frac{2\pi}{5}$
$5x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\frac{1}{5}\sin 5x$	0	$\frac{1}{5}$	0	$-\frac{1}{5}$	0
$-\frac{1}{5}\sin 5x$	0	$-\frac{1}{5}$	0	$\frac{1}{5}$	0



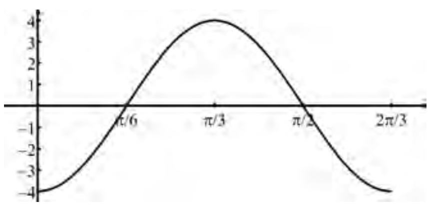
9. $y = -\cos 16x$ has amplitude of $|-1| = 1$, and period of $\frac{\pi}{8}$, with key values at multiples of $\frac{1}{4}\left(\frac{\pi}{8}\right) = \frac{\pi}{32}$.

x	0	$\frac{\pi}{32}$	$\frac{\pi}{16}$	$\frac{3\pi}{32}$	$\frac{\pi}{8}$
y	-1	0	1	0	-1



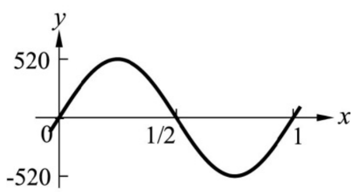
10. $y = -4\cos 3x$ has amplitude of $|-4| = 4,$ and period of $\frac{2\pi}{3}$, with key values at multiples of $\frac{1}{4}\left(\frac{2\pi}{3}\right) = \frac{\pi}{6}.$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$3x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos 3x$	1	0	-1	0	1
$-4\cos 3x$	-4	0	4	0	4



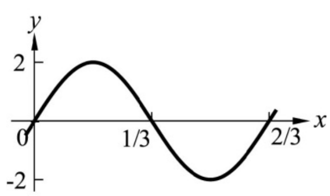
11. $y = 520\sin 2\pi x$ has amplitude of 520 and period of 1,
with key values at multiples of ¼ = 0.25

x	0	0.25	0.50	0.75	1.0
y	0	520	0	-520	0



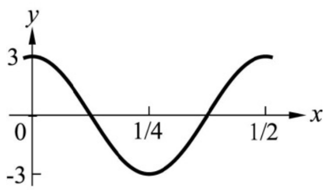
12. $y = 2\sin 3\pi x$ has amplitude of 2 and period of $\frac{2}{3}$, with key values at $\frac{1}{4}\left(\frac{2}{3}\right) = \frac{1}{6}.$

x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$
$3\pi x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin 3\pi x$	0	1	0	-1	0
$2\sin 3\pi x$	0	2	0	-2	0



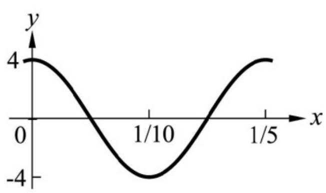
13. $y = 3\cos 4\pi x$ has amplitude of 3 and period of $\frac{1}{2}$, with key values at multiples of $\frac{1}{4}\left(\frac{1}{2}\right) = \frac{1}{8}$.

x	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$
y	3	0	–3	0	3



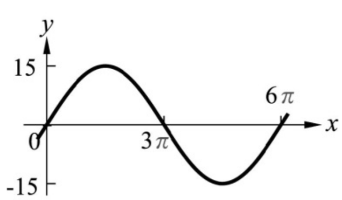
14. $y = 4\cos 10\pi x$ has amplitude of 4 and period of $\frac{1}{5}$, with key values at multiples of $\frac{1}{4}\left(\frac{1}{5}\right) = \frac{1}{20}$.

x	0	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{3}{20}$	$\frac{1}{5}$
$10\pi x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos 10\pi x$	1	0	–1	0	1
$4\cos 10\pi x$	4	0	–4	0	4



15. $y = 15\sin\frac{1}{3}x$ has amplitude 15 and period of 6π, with key values at multiples of $\frac{1}{4}\cdot 6\pi = \frac{3\pi}{2}$.

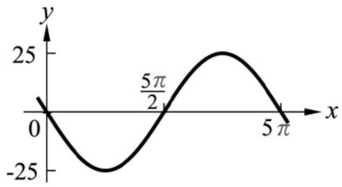
x	0	$\frac{3\pi}{2}$	3π	$\frac{9\pi}{2}$	6π
y	0	15	0	–15	0



16. $y = -25 \sin 0.4x$ or $y = -25 \sin \frac{2}{5}x$ has amplitude 25 and period of 5π,

with key values at multiples of $\frac{1}{4} \cdot 5\pi$

x	0	$\frac{5\pi}{4}$	$\frac{5\pi}{2}$	$\frac{15\pi}{4}$	5π
$\frac{2}{5}x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin \frac{2}{5}x$	0	1	0	-1	0
$-25 \sin \frac{2}{5}x$	0	-25	0	25	0

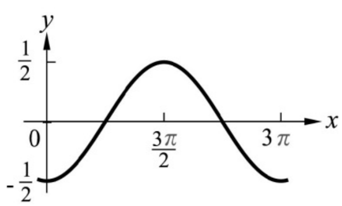


17. $y = -\frac{1}{2} \cos \frac{2}{3}x$ has amplitude of $\left|-\frac{1}{2}\right| = \frac{1}{2}$, and

period of 3π, with key values at multiples of

$\frac{1}{4} \cdot 3\pi = \frac{3\pi}{4}$.

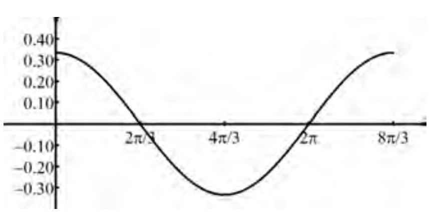
x	0	$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	$\frac{9\pi}{4}$	3π
y	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$



18. $y = \frac{1}{3} \cos 0.75x$ or $y = \frac{1}{3} \cos \frac{3}{4}x$ has amplitude $\frac{1}{3}$ and period of $\frac{8\pi}{3}$,

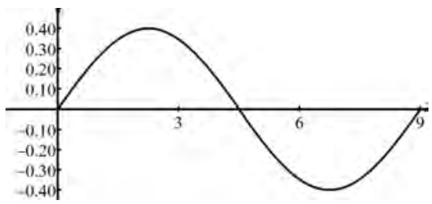
with key values at multiples of $\frac{1}{4} \cdot \frac{8\pi}{3} = \frac{2\pi}{3}$.

x	0	$\frac{2\pi}{3}$	$\frac{4\pi}{3}$	2π	$\frac{8\pi}{3}$
$\frac{3}{4}x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos \frac{3}{4}x$	1	0	-1	0	1
$\frac{1}{3} \cos \frac{3}{4}x$	$\frac{1}{3}$	0	$-\frac{1}{3}$	0	$\frac{1}{3}$



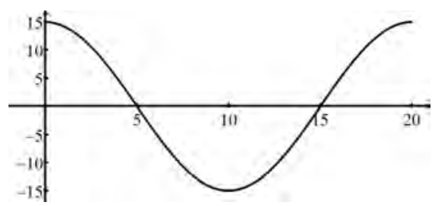
19. $y = 0.4 \sin \dfrac{2\pi x}{9}$ has amplitude 0.4 and period of 9, with key values at multiples of $\dfrac{9}{4}$.

x	0	2.25	4.50	6.75	9.0
y	0	0.4	0	−0.4	0



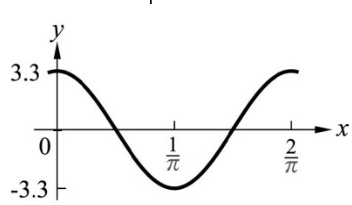
20. $y = 15 \cos \dfrac{\pi x}{10} x$ has amplitude of 15 and period of 20,with key values at multiples of $\dfrac{1}{4} \cdot 20 = 5$.

x	0	5	10	15	20
$\frac{\pi x}{10}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos \frac{\pi x}{10}$	1	0	−1	0	1
$15 \cos \frac{\pi x}{10}$	15	0	−15	0	15



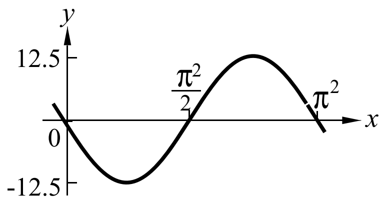
21. $y = 3.3 \cos \pi^2 x$ has amplitude of 3.3 and period of $\dfrac{2}{\pi}$, with key values at multiples of $\dfrac{1}{4}\left(\dfrac{2}{\pi}\right) = \dfrac{1}{2\pi}$.

x	0	$\frac{1}{2\pi}$	$\frac{1}{\pi}$	$\frac{3}{2\pi}$	$\frac{2}{\pi}$
$\pi^2 x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos \pi^2 x$	1	0	−1	0	1
$3.3 \cos \pi^2 x$	3.3	0	−3.3	0	3.3



22. $y = -12.5 \sin \dfrac{2x}{\pi}$ has amplitude of 12.5 and period of π^2, with key values at multiples of $\dfrac{\pi^2}{4}$.

x	0	$\frac{\pi^2}{4}$	$\frac{\pi^2}{2}$	$\frac{3\pi^2}{4}$	π^2
$\frac{2x}{\pi}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin \frac{2x}{\pi}$	0	1	0	-1	0
$-12.5 \sin \frac{2x}{\pi}$	0	-12.5	0	12.5	0



23. $b = \dfrac{2\pi}{\pi/3} = 6$; $y = \sin 6x$

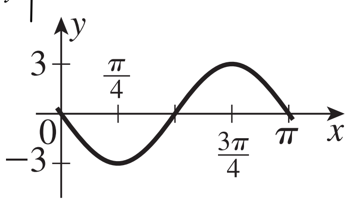
24. $b = \dfrac{2\pi}{5\pi/2} = \dfrac{4}{5}$; $y = \sin \dfrac{4}{5}x$

25. $b = \dfrac{2\pi}{1/3} = 6\pi$; $y = \sin 6\pi x$

26. $b = \dfrac{2\pi}{6} = \dfrac{\pi}{3}$; $y = \sin \dfrac{\pi}{3}x$

27. $y = 3 \sin(-2x) = -3 \sin 2x$, since $\sin(-x) = -\sin(x)$ (see Eq. 8.7). Therefore, the amplitude is 3, the period is $2\pi/2=\pi$, and the key values are at multiples of $\pi/4$.

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
y	0	-3	0	3	0

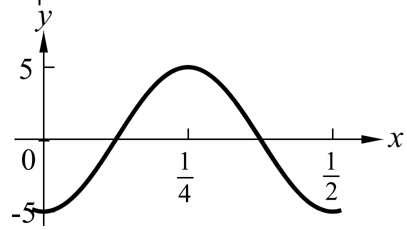


28. $y = -5\cos\left(-4\pi x\right) = -5\cos 4\pi x$, since

$\cos(-x) = \cos(x)$ (see Eq. 8.7). Therefore. the amplitude is 5 and the period is $\dfrac{2\pi}{4\pi} = \dfrac{1}{2}$

with key values at multiples of 1/8.

x	0	1/8	1/4	3/8	1/2
y	–5	0	5	0	–5



29. $y = 8\left|\cos \dfrac{\pi}{2}x\right|$

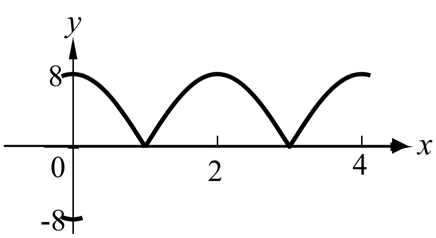
This function has amplitude 8. Moreover,

$\cos(\pi x/2)$ has period $\dfrac{2\pi}{\frac{\pi}{2}} = 4$ so that key

values are at multiples of 4/4=1.

Also, the absolute value makes the y values positive whenever they are negative and leaves positive values intact, so that the negative parts of the curve are reflected with respect to the x axis. Note that the function repeats itself every 2 units, so that the absolute value changes the period from 4 to 2.

x	0	1	2	3	4
y	8	0	8	0	8

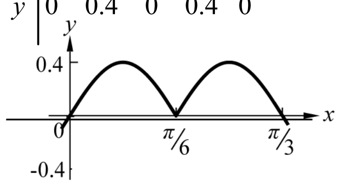


30. $y = 0.4\left|\sin 6x\right|$

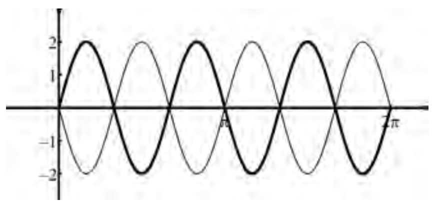
This function has amplitude 0.4. Moreover, $\sin 6x$ has period $2\pi/6=\pi/3$, with key values at multiples of $\dfrac{1}{4}\left(\dfrac{\pi}{3}\right)$.

Also, the absolute value makes the y values positive whenever they are negative and leaves positive values intact, so that the negative parts of the curve are reflected with respect to the x axis. Note that the function repeats itself every $\pi/6$ units, so that the absolute value changes the period from $\pi/3$ to $\pi/6$.

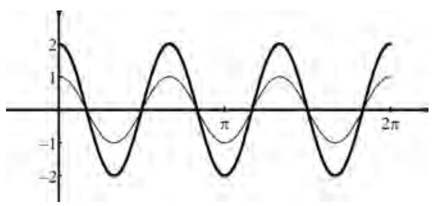
x	0	$\pi/12$	$\pi/6$	$\pi/4$	$\pi/3$
y	0	0.4	0	0.4	0



31. The two graphs are reflections of each other in the x-axis. The value of $\sin(-x)$ is negative the value of $\sin(x)$.



32. The graph of $2\cos(3x)$ has twice the amplitude and is on the same side of the x-axis as the graph of $\cos(-3x)$. This indicates that the values of $\cos(-x)$ are equal to those of $\cos x$.



33. $\sin\ 2x$ has period $\dfrac{2\pi}{2} = \pi$

$\sin 3x$ has period $\dfrac{2\pi}{3}$

The period of $\sin\ 2x + \sin\ 3x$ is the least common multiple of π, and $\dfrac{2\pi}{3}$, which is 2π.

34. $\cos\ \dfrac{1}{2}x$ has period $\dfrac{2\pi}{\frac{1}{2}} = 4\pi$

$\cos\ \dfrac{1}{3}x$ has period $\dfrac{2\pi}{\frac{1}{3}} = 6\pi$

The period of $\cos\ \dfrac{1}{2}x + \cos\ \dfrac{1}{3}x$ is the least common multiple of 4π and 6π, which is 12π.

35. $y = -2 \sin bx, \left(\dfrac{\pi}{4}, -2\right), b > 0$

$-2 = -2 \sin b \cdot \dfrac{\pi}{4}$

$\sin \dfrac{b\pi}{4} = 1$

$\dfrac{b\pi}{4} = \dfrac{\pi}{2} + 2\pi n$

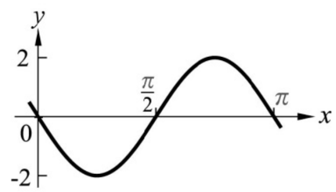
$b = 2 + 8n,$ of which

the smallest is $b = 2$

$y = -2 \sin 2x$ is the function.

It has amplitude 2, period $2\pi/2=\pi$, with key values at multiples of $\pi/4$

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
y	0	-2	0	2	0



36. $y = 2 \sin bx, \left(\dfrac{\pi}{6}, 2\right), b > 0$

$2 = 2 \sin \dfrac{b\pi}{6}$

$\dfrac{b\pi}{6} = \dfrac{\pi}{2} + 2\pi n$

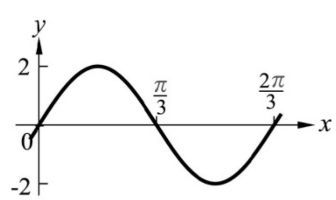
$b = 3 + 3\pi n$ of which

the smallest is $b = 3$

$y = 2 \sin 3x$ is the function.

It has amplitude 2, period $2\pi/3$, with key values at multiples of $\dfrac{1}{4}\left(\dfrac{2\pi}{3}\right) = \dfrac{\pi}{6}$.

x	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$
y	0	2	0	-2	0



37. The period is $\dfrac{2\pi}{6.60 \times 10^8}$ and so the frequency is

$\dfrac{6.60 \times 10^8}{2\pi} = 1.05 \times 10^8 \text{ Hz} = 1.05 \times 10^2 \text{ MHz}.$

38. The period is $\dfrac{2\pi}{120\pi} = \dfrac{1}{60}$ and so the frequency is 60.

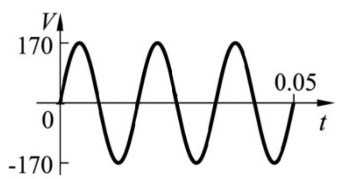
39. $V = 170 \sin 120\pi t$

V has amplitude 170 and period $\dfrac{2\pi}{120\pi} = \dfrac{1}{60}$

with key values at multiples of 1/240. Between 0 and 0.05, the function completes three cycles $\left(\dfrac{0.05}{\frac{1}{60}} = 3\right)$.

t	0	1/240	1/120	1/80	1/60	1/48	1/40
V	0	170	0	–170	0	170	0

t	7/240	1/30	3/80	1/24	11/240	1/20 = 0.05
V	–170	0	170	0	–170	0



40. $y = 3.2 \cos 880\pi t$

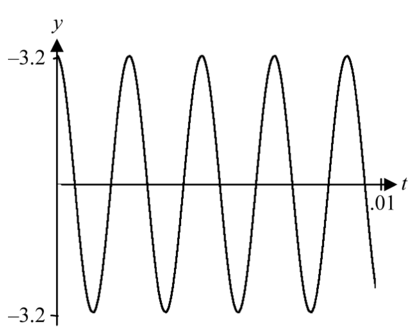
y has amplitude 3.2 and period $\dfrac{2\pi}{880\pi} = \dfrac{1}{440}$

with key values at multiples of 1/1760. Between 0 and 0.01, the function completes 4.4 cycles
$(.01 \div \frac{1}{440} = 4.4)$

t	0	$\frac{1}{1760}$	$\frac{1}{880}$	$\frac{3}{1760}$	$\frac{1}{440}$	$\frac{1}{352}$	$\frac{3}{880}$
y	3.2	0	–3.2	0	3.2	0	–3.2

t	$\frac{7}{1760}$	$\frac{1}{220}$	$\frac{9}{1760}$	$\frac{1}{176}$	$\frac{1}{160}$	$\frac{3}{440}$
y	0	3.2	0	–3.2	0	3.2

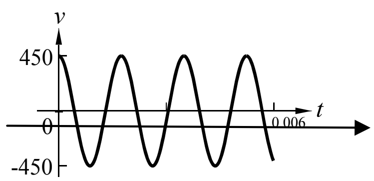
t	$\frac{13}{1760}$	$\frac{7}{880}$	$\frac{3}{352}$	$\frac{1}{110}$	$\frac{17}{1760}$	0.01
y	0	–3.2	0	3.2	0	–3.2



41. $v = 450 \cos 3600t$; amplitude is 450; period is $\frac{2\pi}{3600} = \frac{\pi}{1800}$, with key values at multiples of $\frac{1}{4}\ \frac{\pi}{1800} = \frac{\pi}{7200}$. Between 0 and 0.006, the function completes 3.4 cycles.

$$0.006 \div \frac{\pi}{1800} = 3.4 \text{ cycles}$$

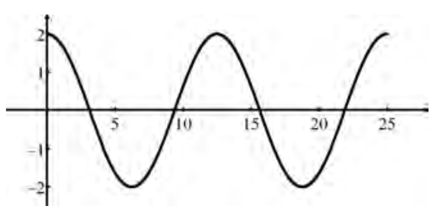
t	0	$\frac{\pi}{7200}$	$\frac{\pi}{3600}$	$\frac{\pi}{2400}$	$\frac{\pi}{1800}$
$3600t$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos 3600t$	1	0	−1	0	1
$450 \cos 3600t$	450	0	−450	0	450



42. The amplitude is $a = 4.0 / 2 = 2.0$.

The period is $12.5 = \frac{2\pi}{b}; b = \frac{2\pi}{12.5} = \frac{4\pi}{25}$.

The desired function is $y = 2.0 \cos \frac{4\pi}{25} x$.



43. The function has amplitude 0.5, period π, with a maximum at $x = 0$ (so it is a cosine function).
$b = 2\pi/\pi$
$b = 2$
Hence the function is $y = \frac{1}{2} \cos 2x$

44. The function has amplitude 8, period π/4, with a zero at $x = 0$ and a maximum after one-fourth of a period (so it is a sine function).
$b = (2\pi)/(\pi/4)$
$b = 8$
Hence the function is $y = 8 \sin 8x$

45. The function has amplitude 4, period 2, with a zero at $x = 0$ and a minimum after one-fourth of a period (so it is an inverted sine function).
$b = 2\pi/2$
$b = \pi$
Hence the function is $y = -4 \sin \pi x$

46. The function has amplitude 0.1, period 1/2, with a minimum $x = 0$ (so it is an inverted cosine function).
$b = (2\pi)/(1/2)b = 4\pi$
Hence the function is $y = -0.1 \cos 4\pi x$

10.3 Graphs of $y = a \sin(bx + c)$ and $y = a \cos(bx + c)$

1. $y = -\cos\left(2x - \frac{\pi}{6}\right)$

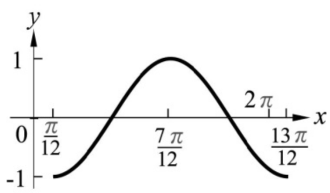
(1) the amplitude is 1

(2) the period is $\frac{2\pi}{2} = \pi$

(3) the displacement is $-\frac{\frac{-\pi}{6}}{2} = \frac{\pi}{12}$

One-fourth period is π/4, so key values for one full cycle start at π/12, end at 13π/12, and are found π/4 units apart. The table of key values is

x	$\frac{\pi}{12}$	$\frac{\pi}{3}$	$\frac{7\pi}{12}$	$\frac{5\pi}{6}$	$\frac{13\pi}{12}$
y	−1	0	1	0	−1



2. $y = 2\cos\left(\frac{1}{2}x + \frac{\pi}{6}\right)$

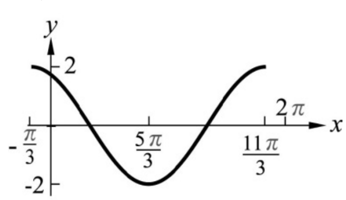
(1) the amplitude is 2

(2) the period is $\frac{2\pi}{\frac{1}{2}} = 4\pi$

(3) the displacement is $-\frac{\frac{\pi}{6}}{\frac{1}{2}} = -\frac{\pi}{3}$

One-fourth period is π, so key values for one full cycle start at –π/3, end at 11π/3, and are found π units apart. The table of key values is

x	$-\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{5\pi}{3}$	$\frac{8\pi}{3}$	$\frac{11\pi}{3}$
y	2	0	−2	0	2



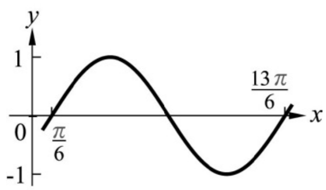
3. $y = \sin \quad x - \dfrac{\pi}{6}$; $a = 1,\ b = 1,\ c = -\dfrac{\pi}{6}$

Amplitude is $|a| = 1$; period is $\dfrac{2\pi}{b} = 2\pi$;

displacement is $-\dfrac{c}{b} = \dfrac{\pi}{6}$

One-fourth period is π/2, so key values for one full cycle start at π/6, end at 13π/6, and are found π/2 units apart. The table of key values is

x	$\frac{\pi}{6}$	$\frac{2\pi}{3}$	$\frac{7\pi}{6}$	$\frac{5\pi}{3}$	$\frac{13\pi}{6}$
y	0	1	0	–1	0



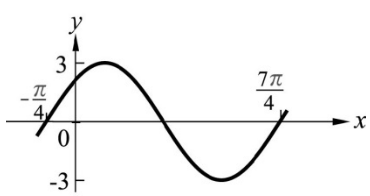
4. $y = 3 \sin \quad x + \dfrac{\pi}{4}$; $a = 3,\ b = 1,\ c = \dfrac{\pi}{4}$

Amplitude is $|a| = 3$; period is $\dfrac{2\pi}{b} = \dfrac{2\pi}{1} = 2\pi$;

displacement is $-\dfrac{c}{b} = -\dfrac{\pi/4}{1} = -\dfrac{\pi}{4}$.

One-fourth period is π/2, so key values for one full cycle start at –π/4, end at 7π/4, and are found π/2 units apart. The table of key values is

x	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$
y	0	1	0	–1	0



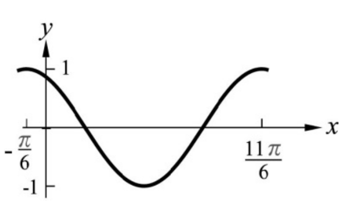
5. $y = \cos \quad x + \dfrac{\pi}{6}$; $a = 1,\ b = 1,\ c = \dfrac{\pi}{6}$

Amplitude is $|a| = 1$; period is $\dfrac{2\pi}{b} = 2\pi$;

displacement is $-\dfrac{c}{b} = -\dfrac{\pi}{6}$

One-fourth period is π/2, so key values for one full cycle start at –π/6, end at 11π/6, and are found π/2 units apart. The table of key values is

x	$-\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{4\pi}{3}$	$\frac{11\pi}{6}$
y	0	1	0	–1	0



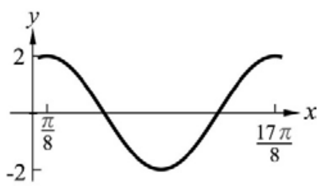
6. $y = 2\cos\left(x - \frac{\pi}{8}\right)$; $a = 2$, $b = 1$, $c = \frac{-\pi}{8}$

Amplitude is $|a| = 2$; period is $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$;

displacement is $-\frac{c}{b} = -\left(\frac{-\pi}{8}\right) = \frac{\pi}{8}$

One-fourth period is π/2, so key values for one full cycle start at π/8, end at 17π/8, and are found π/2 units apart. The table of key values is

x	$\frac{\pi}{8}$	$\frac{5\pi}{8}$	$\frac{9\pi}{8}$	$\frac{13\pi}{8}$	$\frac{17\pi}{8}$
y	2	0	–2	0	2



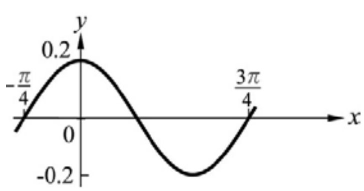
7. $y = 0.2\sin\left(2x + \frac{\pi}{2}\right)$; $a = 0.2$, $b = 2$, $c = \frac{\pi}{2}$

Amplitude is $|a| = 0.2$; period is $\frac{2\pi}{b} = \pi$;

displacement is $-\frac{c}{b} = -\frac{\pi}{4}$

One-fourth period is π/4, so key values for one full cycle start at –π/4, end at 3π/4, and are found π/4 units apart. The table of key values is

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
y	0	0.2	0	–0.2	0



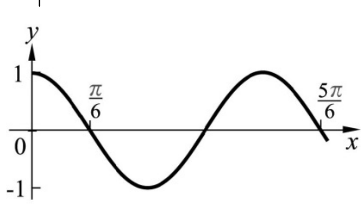
8. $y = -\sin\left(3x - \frac{\pi}{2}\right)$; $a = -1$, $b = 3$, $c = -\frac{\pi}{2}$

Amplitude is $|a| = 1$; period is $\frac{2\pi}{b} = \frac{2\pi}{3}$;

displacement is $-\frac{c}{b} = -\left(\frac{-\pi/2}{3}\right) = \frac{\pi}{6}$

One-fourth period is π/6, so key values for one full cycle start at π/6, end at 5π/6, and are found π/6 units apart. The table of key values is

x	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$
y	0	–1	0	1	0



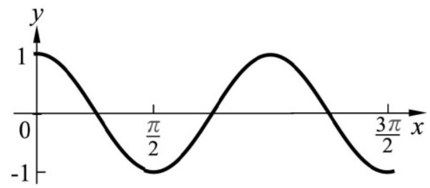
9. $y = -\cos(2x - \pi)$; $a = -1$, $b = 2$, $c = -\pi$

Amplitude is $|a| = 1$; period is $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$;

displacement is $-\frac{c}{b} = -\left(\frac{-\pi}{2}\right) = \frac{\pi}{2}$

One-fourth period is π/4, so key values for one full cycle start at π/2, end at 3π/2, and are found π/4 units apart. The table of key values is

x	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
y	−1	0	1	0	−1



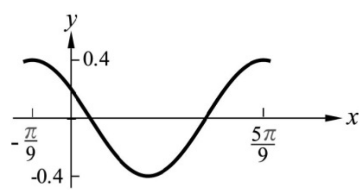
10. $y = 0.4\cos\left(3x + \frac{\pi}{3}\right)$; $a = 0.4$, $b = 3$, $c = \frac{\pi}{3}$

Amplitude is $|a| = 0.4$; period is $\frac{2\pi}{b} = \frac{2\pi}{3}$;

displacement is $-\frac{c}{b} = \frac{-\pi/3}{3} = \frac{-\pi}{9}$

One-fourth period is π/6, so key values for one full cycle start at –π/9, end at 5π/9, and are found π/6 units apart. The table of key values is

x	$-\frac{\pi}{9}$	$\frac{\pi}{18}$	$\frac{2\pi}{9}$	$\frac{7\pi}{18}$	$\frac{5\pi}{9}$
y	0.4	0	−0.4	0	0.4



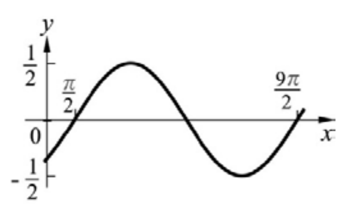
11. $y = \frac{1}{2}\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right)$; $a = \frac{1}{2}$, $b = \frac{1}{2}$, $c = -\frac{\pi}{4}$

Amplitude is $|a| = \frac{1}{2}$; period is $\frac{2\pi}{b} = 4\pi$;

displacement is $-\frac{c}{b} = \frac{\pi}{2}$

One-fourth period is π, so key values for one full cycle start at π/2, end at 9π/2, and are found π units apart. The table of key values is

x	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$\frac{5\pi}{2}$	$\frac{7\pi}{2}$	$\frac{9\pi}{2}$
y	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0



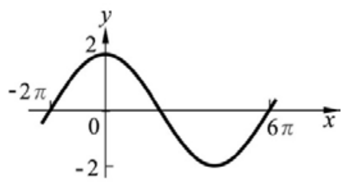
12. $y = 2\sin\left(\frac{1}{4}x + \frac{\pi}{2}\right)$; $a = 2,\ b = \frac{1}{4},\ c = \frac{\pi}{2}$

Amplitude is $|a| = 2$; period is $\frac{2\pi}{b} = \frac{2\pi}{1/4} = 8\pi$;

displacement is $-\frac{c}{b} = \frac{-\pi/2}{1/4} = -2\pi$

One-fourth period is 2π, so key values for one full cycle start at –2π, end at 6π, and are found 2π units apart. The table of key values is

x	-2π	0	2π	4π	6π
y	0	2	0	–2	0



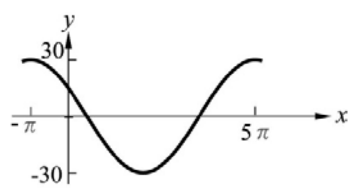
13. $y = 30\cos\left(\frac{1}{3}x + \frac{\pi}{3}\right)$; $a = 30,\ b = \frac{1}{3},\ c = \frac{\pi}{3}$

Amplitude is $|a| = 30$; period is $\frac{2\pi}{b} = \frac{2\pi}{1/3} = 6\pi$;

displacement is $-\frac{c}{b} = \frac{-\pi/3}{1/3} = -\pi$

One-fourth period is 3π/2, so key values for one full cycle start at –π, end at 5π, and are found 3π/2 units apart. The table of key values is

x	$-\pi$	$\frac{\pi}{2}$	2π	$\frac{7\pi}{2}$	5π
y	30	0	–30	0	30



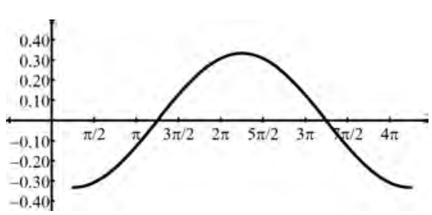
14. $y = -\frac{1}{3}\cos\left(\frac{1}{2}x - \frac{\pi}{8}\right)$; $a = \frac{1}{3},\ b = \frac{1}{2},\ c = -\frac{\pi}{8}$

Amplitude is $|a| = \frac{1}{3}$; period is $\frac{2\pi}{b} = \frac{2\pi}{1/2} = 4\pi$;

displacement is $-\frac{c}{b} = -\left(\frac{-\pi/8}{1/2}\right) = \frac{\pi}{4}$

One-fourth period is π, so key values for one full cycle start at π/4, end at 17π/4, and are found π units apart. The table of key values is

x	$\frac{\pi}{4}$	$\frac{5\pi}{4}$	$\frac{9\pi}{4}$	$\frac{13\pi}{4}$	$\frac{17\pi}{4}$
y	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	$-\frac{1}{3}$



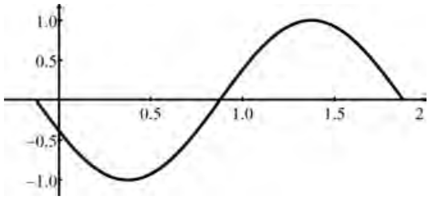
15. $y = -\sin\left(\pi x + \dfrac{\pi}{8}\right)$; $a = 1$, $b = \pi$, $c = \dfrac{\pi}{8}$

Amplitude is $|a| = 1$; period is $\dfrac{2\pi}{b} = 2$;

displacement is $-\dfrac{c}{b} = -\dfrac{1}{8}$

One-fourth period is 1/2, so key values for one full cycle start at –1/8, end at 15/8, and are found 1/2 units apart. The table of key values is

x	$-\frac{1}{8}$	$\frac{3}{8}$	$\frac{7}{8}$	$\frac{11}{8}$	$\frac{15}{8}$
y	0	–1	0	1	0



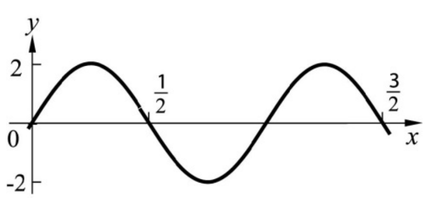
16. $y = -2\sin(2\pi x - \pi)$; $a = -2$, $b = 2\pi$, $c = -\pi$

Amplitude is $|a| = 2$; period is $\dfrac{2\pi}{b} = \dfrac{2\pi}{2\pi} = 1$;

displacement is $-\dfrac{c}{b} = -\ \dfrac{-\pi}{2\pi}\ = \dfrac{1}{2}$

One-fourth period is 1/4, so key values for one full cycle start at 1/2, end at 3/2, and are found 1/4 units apart. The table of key values is

x	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{3}{2}$
y	0	–2	0	2	0



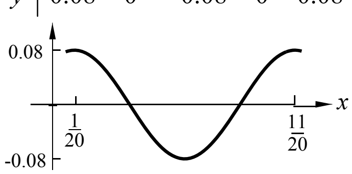
17. $y = 0.08\cos\ 4\pi x - \dfrac{\pi}{5}\ $; $a = 0.08$, $b = 4\pi$, $c = -\dfrac{\pi}{5}$

Amplitude is $|a| = 0.08$; period is $\dfrac{2\pi}{b} = \dfrac{2\pi}{4\pi} = \dfrac{1}{2}$;

displacement is $-\dfrac{c}{b} = -\ -\dfrac{\pi/5}{4\pi}\ = \dfrac{1}{20}$

One-fourth period is 1/8, so key values for one full cycle start at 1/20, end at 11/20, and are found 1/8 units apart. The table of key values is

x	$\frac{1}{20}$	$\frac{7}{40}$	$\frac{3}{10}$	$\frac{17}{40}$	$\frac{11}{20}$
y	0.08	0	–0.08	0	0.08



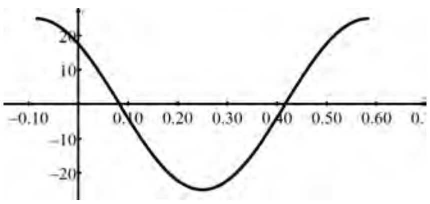
18. $y = 25\cos\ 3\pi x + \dfrac{\pi}{4}$; $a = 25,\ b = 3\pi,\ c = \dfrac{\pi}{4}$

Amplitude is $|a| = 25$; period is $\dfrac{2\pi}{b} = \dfrac{2\pi}{3\pi} = \dfrac{2}{3}$;

displacement is $-\dfrac{c}{b} = -\dfrac{\pi/4}{3\pi} = -\dfrac{1}{12}$

One-fourth period is 1/6, so key values for one full cycle start at –1/12, end at 7/12, and are found 1/6 units apart. The table of key values is

x	$-\frac{1}{12}$	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{5}{12}$	$\frac{7}{12}$
y	25	0	–25	0	25



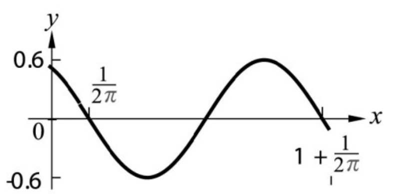
19. $y = -0.6\sin(2\pi x - 1)$; $a = -0.6,\ b = 2\pi,\ c = -1$

Amplitude is $|a| = 0.6$; period is $\dfrac{2\pi}{b} = 1$;

displacement is $-\dfrac{c}{b} = \dfrac{1}{2\pi}$

One-fourth period is 1/4, so key values for one full cycle start at 1/(2π), end at 1+1/(2π), and are found 1/4 units apart. The table of key values is

x	$\frac{1}{2\pi}$	$\frac{1}{4}+\frac{1}{2\pi}$	$\frac{1}{2}+\frac{1}{2\pi}$	$\frac{3}{4}+\frac{1}{2\pi}$	$1+\frac{1}{2\pi}$
y	0	–0.6	0	0.6	0



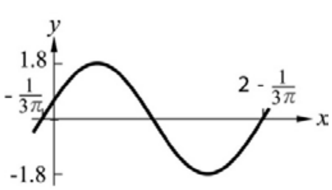
20. $y = 1.8\sin\ \pi x + \dfrac{1}{3}$; $a = 1.8,\ b = \pi,\ c = \dfrac{1}{3}$

Amplitude is $|a| = 1.8$; period is $\dfrac{2\pi}{b} = \dfrac{2\pi}{\pi} = 2$;

displacement is $-\dfrac{c}{b} = -\dfrac{1/3}{\pi} = -\dfrac{1}{3\pi}$

One-fourth period is 1/2, so key values for one full cycle start at –1/(3π), end at 2–1/(3π), and are found 1/2 units apart. The table of key values is

x	$-\frac{1}{3\pi}$	$\frac{1}{2}-\frac{1}{3\pi}$	$1-\frac{1}{3\pi}$	$\frac{3}{2}-\frac{1}{3\pi}$	$2-\frac{1}{3\pi}$
y	0	1.8	0	–1.8	0



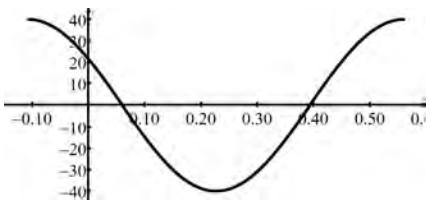
21. $y = 40\cos(3\pi x + 1)$; $a = 40$, $b = 3\pi$, $c = 1$

Amplitude is $|a| = 40$; period is $\frac{2\pi}{b} = \frac{2\pi}{3\pi} = \frac{2}{3}$;

displacement is $-\frac{1}{b} = -\frac{1}{3\pi}$

One-fourth period is 1/6, so key values for one full cycle start at –1/(3π), end at 2/3–1/(3π), and are found 1/6 units apart. The table of key values is

x	$-\frac{1}{3\pi}$	$\frac{1}{6}-\frac{1}{3\pi}$	$\frac{1}{3}-\frac{1}{3\pi}$	$\frac{1}{2}-\frac{1}{3\pi}$	$\frac{2}{3}-\frac{1}{3\pi}$
y	40	0	–40	0	40



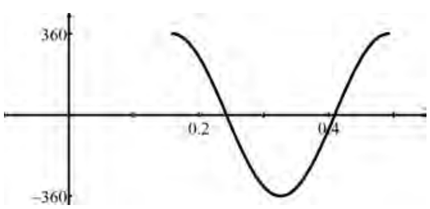
22. $y = 360\cos(6\pi x - 3)$; $a = 360$, $b = 6\pi$, $c = -3$

Amplitude is $|a| = 360$; period is $\frac{2\pi}{b} = \frac{2\pi}{6\pi} = \frac{1}{3}$;

displacement is $-\frac{c}{b} = -\frac{-3}{6\pi} = \frac{1}{2\pi}$

One-fourth period is 1/12, so key values for one full cycle start at 1/(2π), end at 1/3+1/(2π), and are found 1/12 units apart. The table of key values is

x	$\frac{1}{2\pi}$	$\frac{1}{12}+\frac{1}{2\pi}$	$\frac{1}{6}+\frac{1}{2\pi}$	$\frac{1}{4}+\frac{1}{2\pi}$	$\frac{1}{3}+\frac{1}{2\pi}$
y	360	0	–360	0	360



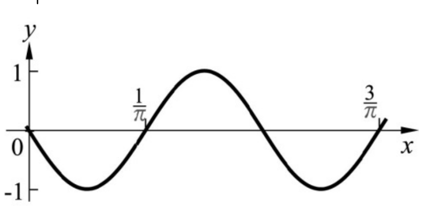
23. $y = \sin(\pi^2 x - \pi)$; $a = 1$, $b = \pi^2$, $c = -\pi$

Amplitude is $|a| = 1$; period is $\frac{2\pi}{b} = \frac{2}{\pi}$;

displacement is $-\frac{c}{b} = \frac{1}{\pi}$

One-fourth period is 1/(2π), so key values for one full cycle start at 1/π, end at 3/π, and are found 1/(2π) units apart. The table of key values is

x	$\frac{1}{\pi}$	$\frac{3}{2\pi}$	$\frac{2}{\pi}$	$\frac{5}{2\pi}$	$\frac{3}{\pi}$
y	0	1	0	–1	0



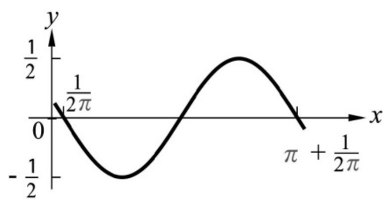
24. $y = -\frac{1}{2}\sin\left(2x - \frac{1}{\pi}\right)$; $a = -\frac{1}{2}$, $b = 2$, $c = -\frac{1}{\pi}$

Amplitude is $|a| = \frac{1}{2}$; period is $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$;

displacement is $-\frac{c}{b} = -\left(\frac{-1/\pi}{2}\right) = \frac{1}{2\pi}$

One-fourth period is π/4, so key values for one full cycle start at 1/(2π), end at π+1/(2π), and are found π/4 units apart. The table of key values is

x	$\frac{1}{2\pi}$	$\frac{\pi}{4}+\frac{1}{2\pi}$	$\frac{\pi}{2}+\frac{1}{2\pi}$	$\frac{3\pi}{4}+\frac{1}{2\pi}$	$\pi+\frac{1}{2\pi}$
y	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0



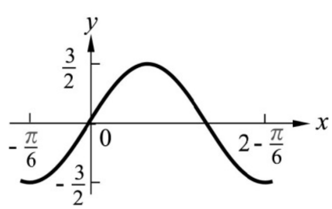
25. $y = -\frac{3}{2}\cos\left(\pi x + \frac{\pi^2}{6}\right)$; $a = -\frac{3}{2}$, $b = \pi$, $c = \frac{\pi^2}{6}$

Amplitude is $|a| = \frac{3}{2}$; period is $\frac{2\pi}{b} = \frac{2\pi}{\pi} = 2$;

displacement is $-\frac{c}{b} = -\frac{\pi^2/6}{\pi} = -\frac{\pi}{6}$

One-fourth period is 1/2, so key values for one full cycle start at –π/6, end at 2–π/6, and are found 1/2 units apart. The table of key values is

x	$-\frac{\pi}{6}$	$\frac{1}{2}-\frac{\pi}{6}$	$1-\frac{\pi}{6}$	$\frac{3}{2}-\frac{\pi}{6}$	$2-\frac{\pi}{6}$
y	$-\frac{3}{2}$	0	$\frac{3}{2}$	0	$-\frac{3}{2}$



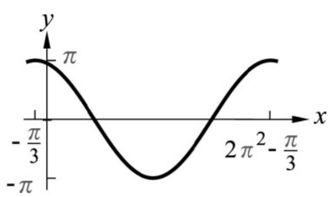
26. $y = \pi\cos\left(\frac{1}{\pi}x + \frac{1}{3}\right)$; $a = \pi$, $b = \frac{1}{\pi}$, $c = \frac{1}{3}$

Amplitude is $|a| = \pi$; period is $\frac{2\pi}{b} = \frac{2\pi}{1/\pi} = 2\pi^2$;

displacement is $-\frac{c}{b} = -\frac{1/3}{1/\pi} = -\frac{\pi}{3}$

One-fourth period is $\pi^2/2$, so key values for one full cycle start at $-\pi/3$, end at $2\pi^2 - \pi/3$, and are found π^2 units apart. The table of key values is

x	$-\frac{\pi}{3}$	$\frac{\pi^2}{2}-\frac{\pi}{3}$	$\pi^2-\frac{\pi}{3}$	$\frac{3\pi^2}{2}-\frac{\pi}{3}$	$2\pi^2-\frac{\pi}{3}$
y	π	0	$-\pi$	0	π



27.

$$\begin{aligned}
\text{amplitude:} \quad & a = 4; \\
\text{period:} \quad & \tfrac{2\pi}{b} = 3\pi \\
& b = \tfrac{2}{3}; \\
\text{displacement:} \quad & -\tfrac{c}{b} = -\tfrac{\pi}{4} \\
& -\tfrac{c}{\frac{2}{3}} = -\tfrac{\pi}{4} \\
& c = \tfrac{\pi}{6} \\
\text{Solution:} \quad & y = 4\sin\left(\tfrac{2}{3}x + \tfrac{\pi}{6}\right)
\end{aligned}$$

28. amplitude: $a = 8$;

period: $\frac{2\pi}{b} = \frac{2\pi}{3}$

$b = 3$

displacement: $-\frac{c}{b} = \frac{\pi}{3}$

$-\frac{c}{3} = \frac{\pi}{3}$

$c = -\pi$

Solution: $y = 8\cos(3x - \pi)$

29. amplitude: $a = 12;$

period: $\frac{2\pi}{b} = \frac{1}{2}$

$b = 4\pi$

displacement: $-\frac{c}{b} = \frac{1}{8}$

$-\frac{c}{4\pi} = \frac{1}{8}$

$c = -\frac{\pi}{2}$

Solution: $y = 12\cos \left(4\pi x - \frac{\pi}{2}\right)$

30. amplitude: $a = 18;$

period: $\frac{2\pi}{b} = 4$

$b = \frac{\pi}{2}$

displacement: $-\frac{c}{b} = -1$

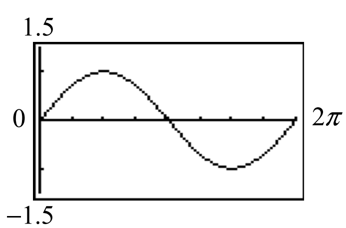
$-\frac{c}{\pi / 2} = -1$

$c = \frac{\pi}{2}$

Solution: $y = 18\sin \left(\frac{\pi}{2}x + \frac{\pi}{2}\right)$

31. The function sin x and the function cos x are the same, except that the cosine is shifted $\pi/2$ units to the left. If we shift it to the right $\pi/2$ units ($\cos(x-\pi/2)$), it coincides with sin x.

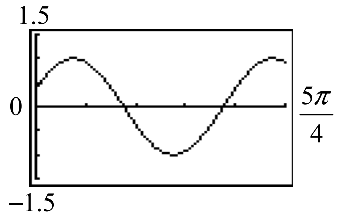
Graph $y_1 = \sin x,\ y_2 = \cos\left(x - \frac{\pi}{2}\right)$. Graphs are the same.



32. We know cos(–*x*) = cos(*x*). Since 2*x*–3π/8= –(3π/8 –2*x*), the two functions are the same.

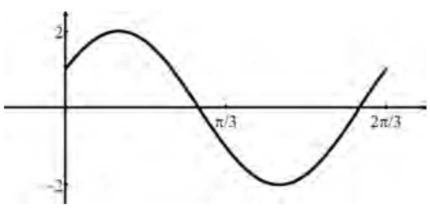
Graph $y_1 = \cos\left(2x - \dfrac{3\pi}{8}\right)$ and $y_2 = \cos\left(\dfrac{3\pi}{8} - 2x\right)$.

Graphs are the same.



33. Graph $y_1 = 2\sin\ \ 3x + \dfrac{\pi}{6}$ and $y_2 = -2\sin\ \ -\ \ 3x + \dfrac{\pi}{6}$.

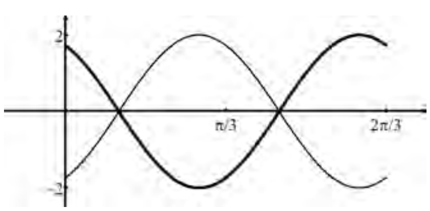
The graphs are the same.



34. Graph $y_1 = 2\cos\ \ 3x + \dfrac{\pi}{6}$ and $y_2 = -2\cos\ \ -\ \ 3x + \dfrac{\pi}{6}$.

The graph of $y_1 = 2\cos\ \ 3x + \dfrac{\pi}{6}$ is reflected in the *x*-axis

to obtain the graph of $y_2 = -2\cos\ \ -\ \ 3x + \dfrac{\pi}{6}$.



35. Given the current $i = 12\sin(120\pi t)$ and a lag of 60° for the voltage,
the resulting function is $v = 12\sin(120\pi t - 60°)$.
We convert 60° to $\frac{\pi}{3}$ radians, obtaining a displacement of
$-\dfrac{c}{b} = -\dfrac{-\pi/3}{120\pi} = \dfrac{1}{360}$.

36. Given the current $i = 8.50\sin(120\pi t)$ and a lead of 45° for the voltage, the resulting function is $v = 8.50\sin(120\pi t + 45°)$.

We convert 45° to $\frac{\pi}{4}$ radians, obtaining a displacement of

$$-\frac{c}{b} = -\frac{\pi/4}{120\pi} = -\frac{1}{480}.$$

37. $y = 2.00\sin 2\pi\left(\frac{t}{0.100} - \frac{5.00}{20.0}\right)$; $a = 2.00$,

$b = \frac{2\pi}{0.100}$, $c = \frac{-5.00(2\pi)}{20.0}$

Amplitude $= |a| = 2.00$,

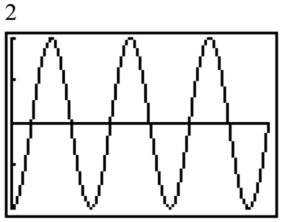
period $= \frac{2\pi}{b} = 0.100$,

displacement $= -\frac{c}{b} = 0.025$

One-fourth period is 0.025, so key values for one full cycle start at 0.025, end at 0.125, and are found 0.025 units apart. Three cycles end at 0.325. The table of key values in the first cycle is

t	0.025	0.05	0.075	0.1	0.125
i	0	2.00	0	–2.00	0

The curve repeats after that.



38. $i = 3.8\cos 2\pi(t + 0.20)$; $a = 3.8$, $b = 2\pi$, $c = 0.4\pi$

Amplitude $= |a| = 3.8$,

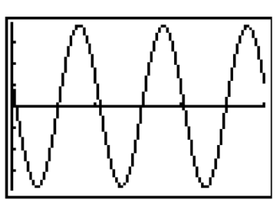
period $= \frac{2\pi}{b} = \frac{2\pi}{2\pi} = 1$,

displacement $= -\frac{c}{b} = -\frac{0.4\pi}{2\pi} = -0.2$

One-fourth period is 0.25, so key values for one full cycle start at –0.2, end at 0.8, and are found 0.25 units apart. Note that t is time, so we only consider $t \ge 0$, and we obtain key values for the cycle that starts at $t = -0.2 + 0.25 = 0.05$. The table of key values for that cycle is

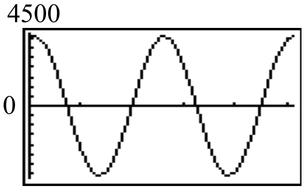
t	0	0.05	0.3	0.55	0.8	1.05
i	1.2	0	–3.8	0	3.8	0

The curve repeats after that and completes three cycles between $t = 0$ and $t = 3$.

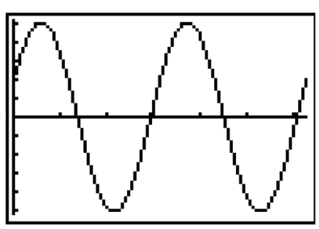


39. $y = 4500\cos(0.025t - 0.25)$; $a = 4500$, $b = 0.025$,
$c = -0.25$
Amplitude $= |a| = 4500$;
period $= \frac{2\pi}{b} = \frac{2\pi}{0.025} = 80\pi$;
displacement $= -\frac{c}{b} = -\ \frac{-0.25}{0.025}\ = 10$
One-fourth period is 20π, so key values for one full cycle start at 10, end at 10+80π, and are found 20π units apart. Two cycles end at 10+160π. The table of key values in the first cycle is

t	10	$10+20\pi$	$10+40\pi$	$10+60\pi$	$10+80\pi$
y	4500	0	–4500	0	4500

4500
0  $160\pi+10$
–4500

40. $I = A\sin(\omega t + \theta) = 5\sin(2\times 10^5 t + 0.4)$
Amplitude = 5;
period $= \frac{2\pi}{2\times 10^5} = 3.14\times 10^{-5}$;
displacement $= \frac{-0.4}{2\times 10^5} = -2\times 10^{-6}$
Note that t is time, so we only consider $t \ge 0$, so that two cycles end at 6.28×10^{-5}.

5
0  6.28×10^{-5}
–5

41. The maximum is at $y = 5$, so amplitude is 5. A full cycle takes place between –1 and 15, so the period is 16. The graph has a zero at $x = -1$, so displacement of the sine function is –1. We thus have

amplitude: $a = 5$;

period: $\frac{2\pi}{b} = 16$

$b = \frac{\pi}{8}$;

displacement: $-\frac{c}{b} = -1$

$-\frac{c}{\pi/8} = -1$

$c = \frac{\pi}{8}$;

Solution: $y = 5\sin\ \frac{\pi}{8}x + \frac{\pi}{8}$

42. The maximum is at $y = 5$, so amplitude is 5. A full cycle takes place between –1 and 15, so the period is 16. One-fourth period is 4, so the graph has a maximum at $x = 3$, and displacement of the sine function is +3. We thus have

amplitude: $a = 5;$

period: $\frac{2\pi}{b} = 16$

$b = \frac{\pi}{8};$

displacement: $-\frac{c}{b} = 3$

$-\frac{c}{\pi/8} = 3$

$c = -\frac{3\pi}{8};$

Solution: $y = 5\cos\left(\frac{\pi}{8}x - \frac{3\pi}{8}\right)$

43. The maximum is at $y = 0.8$, so amplitude is 0.8. A full cycle takes place between $\pi/4$ and $5\pi/4$, so the period is π. The graph has a minimum at 0, so there is no displacement of the cosine function, but it has been inverted (so $a < 0$). We thus have

amplitude: $a = -0.8;$

period: $\frac{2\pi}{b} = \pi$

$b = 2;$

displacement: $-\frac{c}{b} = 0$

$c = 0;$

Solution: $y = -0.8\cos 2x$

44. The maximum is at $y = 0.8$, so amplitude is 0.8. A full cycle takes place between $\pi/4$ and $5\pi/4$, so the period is π. The graph has a zero at $x = \pi/4$, so displacement of the sine function is $\pi/4$. We thus have

amplitude: $a = 0.8;$

period: $\frac{2\pi}{b} = \pi$

$b = 2;$

displacement: $-\frac{c}{b} = \frac{\pi}{4}$

$-\frac{c}{2} = \frac{\pi}{4}$

$c = -\frac{\pi}{2};$

Solution: $y = 0.8\sin\left(2x - \frac{\pi}{2}\right)$

10.4 Graphs of $y = \tan x$, $y = \cot x$, $y = \sec x$, $y = \csc x$

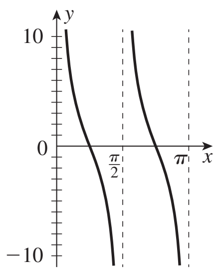
1. $y = 5 \cot 2x$.

Since the period of $y = \cot x$ is π, the period of this function is π/2. We have the following table of key values:

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	*	0	*	0	*

(* = asymptote)

Since $a = 5$, the function increases much faster than $y = \cot x$.

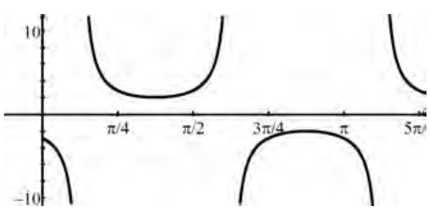


2. $y = 2\csc\left(2x - \frac{\pi}{4}\right)$. Graph $y_1 = \dfrac{2}{\sin\left(2x - \frac{\pi}{4}\right)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$	$\frac{5\pi}{8}$	$\frac{3\pi}{4}$
y	$-2\sqrt{2}$	*	$2\sqrt{2}$	2	$2\sqrt{2}$	*	$-2\sqrt{2}$

x	$\frac{7\pi}{8}$	π	$\frac{9\pi}{8}$	$\frac{5\pi}{4}$	$\frac{11\pi}{8}$	$\frac{3\pi}{2}$
y	-2	$-2\sqrt{2}$	*	$2\sqrt{2}$	2	$2\sqrt{2}$

(* = asymptote)



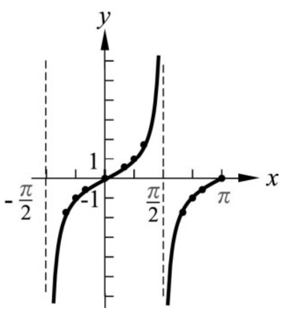
3.

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$
y	*	$-\sqrt{3}$	-1	$\frac{-1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1

x	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	$\sqrt{3}$	*	$-\sqrt{3}$	-1	$\frac{1}{-\sqrt{3}}$	0

(* = asymptote)

$y = \tan x$

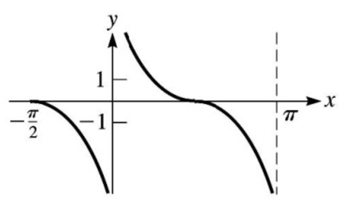


4.

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$
$\cot x$	0	–0.58	–1	–1.7	*	1.7	1

x	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cot x$	0.58	0	–0.58	–1	–1.7	*

(* = asymptote)

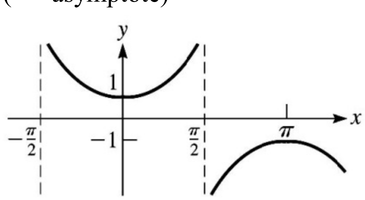


5.

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$
$\sec x$	*	2	1.4	1.2	1	1.2	1.4

x	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sec x$	2	*	–2	–1.4	–1.2	–1

(* = asymptote)

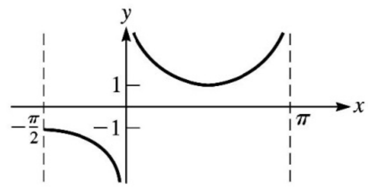


6.

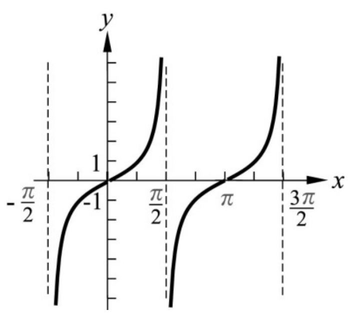
x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\csc x$	−1	−1.2	−1.4	−2	*	2	1.4	1.2

x	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\csc x$	1	1.2	1.4	2	*

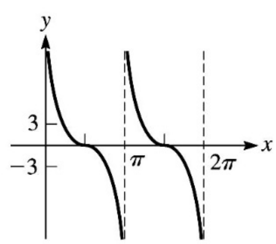
(* = asymptote)



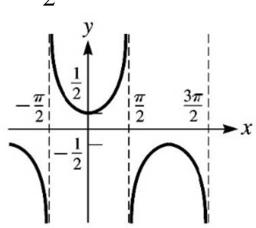
7. $y = 2\tan x$ is the graph of $y = \tan x$ stretched by a factor of 2.



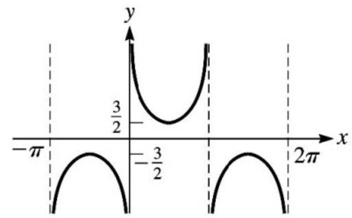
8. For $y = 3\cot x$, first sketch the graph of $y = \cot x$, then multiply the $y-$values of the cotangent function by 3 and graph.



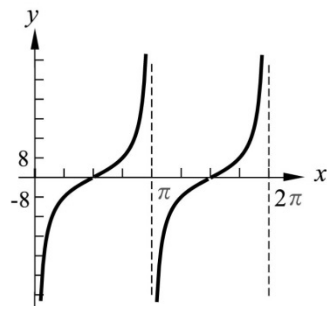
9. For $y = \frac{1}{2}\sec x$, first sketch the graph of $y = \sec x$, then multiply the $y-$values of the secant function by $\frac{1}{2}$ and graph.



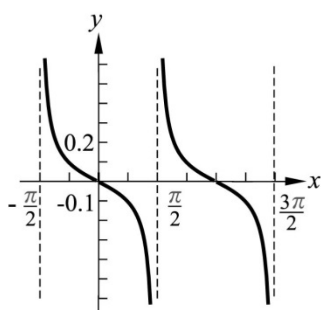
10. For $y = \frac{3}{2}\csc x$, first sketch the graph of $y = \csc x$, then multiply the $y -$ values of the cosecant function by $\frac{3}{2}$ and graph.



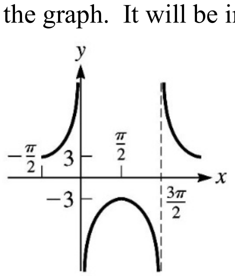
11. The graph of $y = -8\cot x$ is the graph of $y = \tan x$ reflected in the x-axis and stretched by a factor of 8.



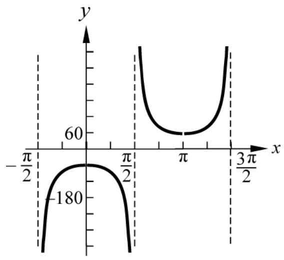
12. For $y = -0.1\tan x$, sketch the graph of $y = \tan x$, then multiply the $y -$ values by -0.1, and resketch the graph. It will be inverted.



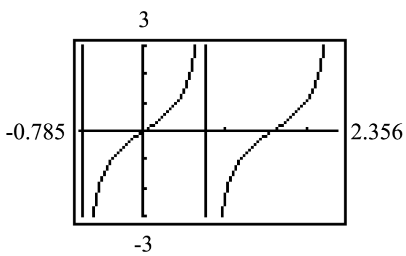
13. For $y = -3\csc x$, sketch the graph of $y = \csc x$, then multiply the $y -$ values by -3, and resketch the graph. It will be inverted.



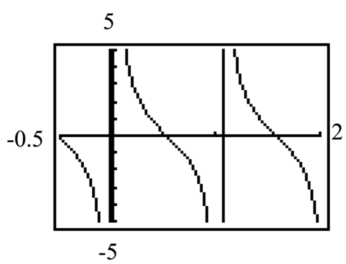
14. For $y = -60 \sec x$, sketch the graph of $y = \sec x$, then multiply the $y-$values by -60, and resketch the graph. It will be inverted.



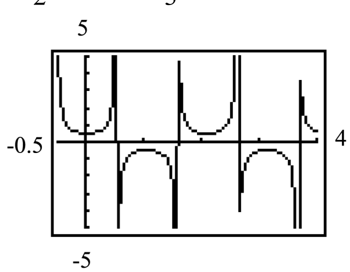
15. Since the period of $y = \tan x$ is π, the period of $y = \tan 2x$ is $\frac{\pi}{2}$. Graph $y_1 = \tan(2x)$.



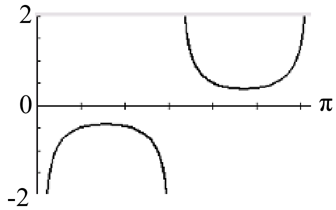
16. Since the period of $\cot x$ is π, the period of $y = 2\cot 3x$ is $\frac{\pi}{3}$. Graph $y_1 = 2(\tan 3x)^{-1}$.



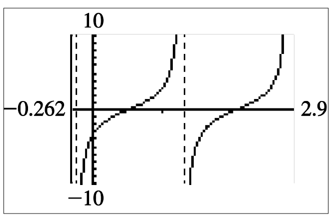
17. Since the period of $\sec x$ is 2π, the period of $y = \frac{1}{2}\sec 3x$ is $\frac{2\pi}{3}$. Graph $y_1 = 0.5(\cos 3x)^{-1}$.



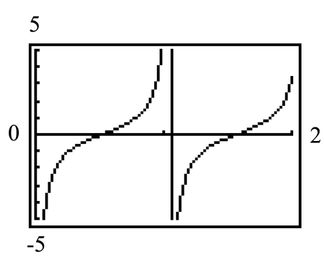
18. Since the period of $\csc x$ is 2π, the period of $y = -0.4\csc 2x$ is $\dfrac{2\pi}{2} = \pi$. Graph $y_1 = -0.4(\sin 2x)^{-1}$.



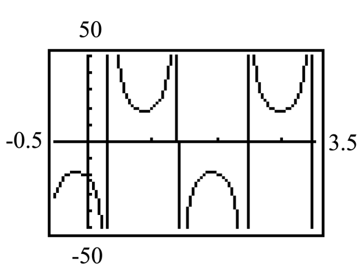
19. Since the period of $\cot x$ is π, the period of $y = -2\cot\left(2x + \dfrac{\pi}{6}\right)$ is $\dfrac{\pi}{2}$. The displacement is $-\dfrac{\pi/6}{2} = -\dfrac{\pi}{12}$. Graph $y_1 = -2/\tan(2x + \pi/6)$.



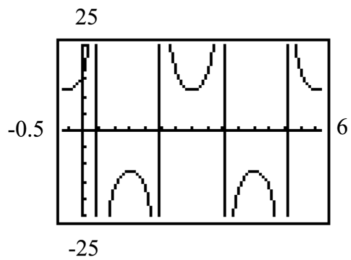
20. Since the period of $\tan x$ is π, the period of $y = \tan\ 3x - \dfrac{\pi}{2}$ is $\dfrac{\pi}{3}$. The displacement is $-\ -\dfrac{\pi/2}{3}\ = \dfrac{\pi}{6}$. Graph $y_1 = \tan(3x - \pi/2)$.



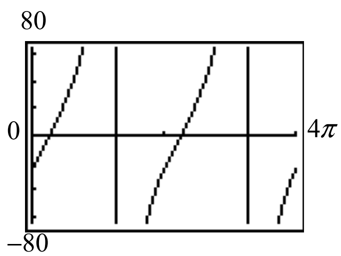
21. Since the period of $\csc x$ is 2π, the period of $y = 18\csc\left(3x - \dfrac{\pi}{3}\right)$ is $\dfrac{2\pi}{3}$. The displacement is $-\left(-\dfrac{\pi/3}{3}\right) = \dfrac{\pi}{9}$. Graph $y_1 = 18\left(\sin\left(3x - \dfrac{\pi}{3}\right)\right)^{-1}$.



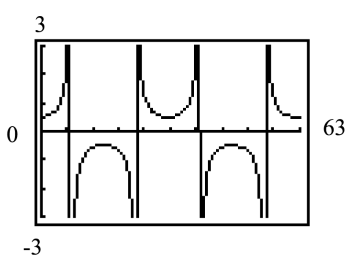
22. Since the period of sec x is 2π, the period of $y = 12\ \sec\left(2x + \frac{\pi}{4}\right)$ is $\frac{2\pi}{2}$. The displacement is $-\frac{\pi/4}{2} = -\frac{\pi}{8}$. Graph $y_1 = 12\left(\cos\left(2x + \frac{\pi}{4}\right)\right)^{-1}$.



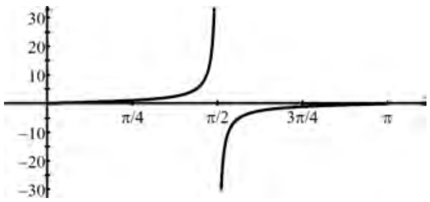
23. Since the period of tan x is π, the period of $y = 75\ \tan\left(0.5x - \frac{\pi}{16}\right)$ is $\frac{2\pi}{0.5}$. The displacement is $-\frac{-\pi/8}{0.5} = \frac{\pi}{8}$. Graph $y_1 = 75\ \tan(0.5x - \pi/16)$.



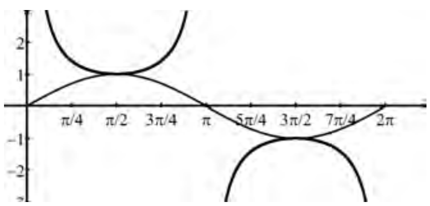
24. Since the period of sec x is 2π, the period of $y = 0.5\ \sec\left(0.2x + \frac{\pi}{25}\right)$ is $\frac{2\pi}{0.2}$. The displacement is $-\frac{\pi/25}{0.2} = -\frac{\pi}{5}$. Graph $y_1 =$ $0.5\left(\cos\left(0.2x + \frac{\pi}{25}\right)\right)^{-1}$.



25. Using the graph of $y = \tan x$, we see that $\tan x$ becomes very large and positive as x approaches $\frac{\pi}{2}$ from the left and $\tan x$ becomes very large and negative as x approaches $\frac{\pi}{2}$ from the right.



26. Using the graphs of $y = \sin x$ and $y = \csc x$, we see that $\csc x$ has a local minimum when $\sin x$ reaches a maximum and that $\csc x$ has a local maximum when $\sin x$ reaches a minimum.



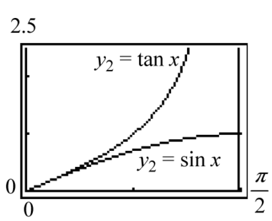
27. If displacement is zero, the secant function is of the form $y = a \sec(bx)$. Since the period is 4π, $2\pi/b = 4\pi$. Therefore, $b = 1/2$. Now we substitute $x = 0$ and $y = -3$ into $y = a \sec\left(\frac{x}{2}\right)$ to get

$-3 = a \sec(0)$

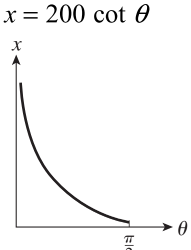
$-3 = a$

Solution: $y = -3\sec\left(\frac{x}{2}\right)$

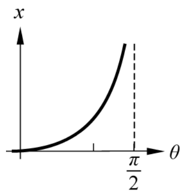
28. From the graph, $\sin x < \tan x$ for $0 < x < \pi/2$.



29. $x = 200 \cot \theta$



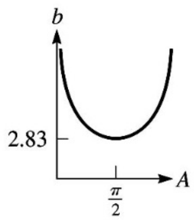
30. $x = a \tan \theta = 5.00 \tan \theta$



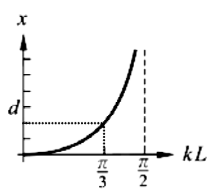
31. $b = (a \sin B) \csc A$

$= \left(4.00 \sin \dfrac{\pi}{4}\right) \csc A$

$= 2.83 \csc A$

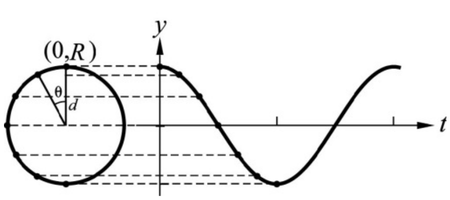


32. $x = d(\sec (kL) - 1)$



10.5 Applications of the Trigonometric Graphs

1. The displacement of the projection on the y-axis is d and is given by $d = R \cos \omega t$.



2. (a) $d = 2.5 \sin\left(\omega t - \dfrac{\pi}{4}\right)$

(b) $d = 2.5 \cos\left(\omega t - \dfrac{3\pi}{4}\right)$

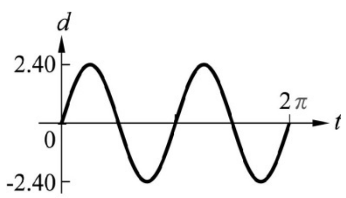
3. Since the frequency and period are reciprocals, if the frequency is 40 cycles/min, then the period is $\frac{1}{40}$ min.

4. Since the frequency and period are reciprocals, if the period is $\frac{1}{10}$ s then the frequency is 10 cycles/s.

5. If ω=40π rad/s, using $\omega = 2\pi f$, we have
$f = \frac{\omega}{2\pi} = \frac{40\pi}{2\pi} = 20$ cycles/s and the period is $\frac{1}{20}$ s.

6. If the period is $\frac{1}{5}$ s then the frequency is $f = 5$ cycles/s.
Then we have $\omega = 2\pi f = 10\pi$ rad/s.

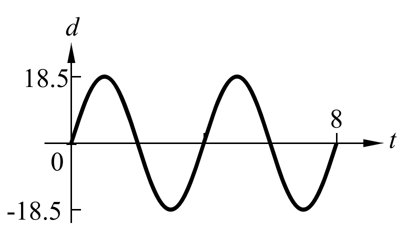
7. $d = R \sin \omega t = 2.40 \sin 2t$ has amplitude
$a = 2.40$ cm, period $= \frac{2\pi}{2} = \pi$ s, and
displacement = 0 s.



8. $y = R \sin \omega$t

$$= 18.5 \sin\left[(0.250)(2\pi)\right]t$$
$$= 18.5 \sin(0.500\pi t)$$

Amplitude is 1.80 m;
period is $\frac{1}{0.250} = 4.00$ s,
8.00 s for 2 cycles;
displacement is 0 s.



9. $y = R \cos \omega t$

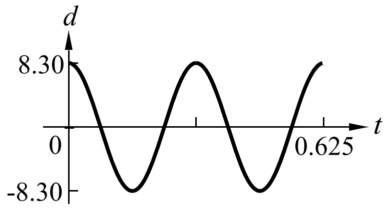
$= 8.30 \cos\left[(3.20)(2\pi)\right]t$

Amplitude is 8.30 cm;

period is $\dfrac{1}{3.20} = 0.3125$ s,

0.625 s for 2 cycles;

displacement is 0 s.



10. $y = R \cos \omega t$

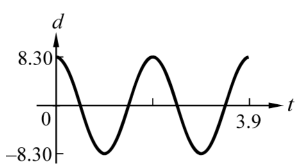
$= 8.30 \cos 3.20t$

Amplitude is 8.30 cm;

period is $\dfrac{2\pi}{3.20} = 1.96$ s,

3.92 s for 2 cycles;

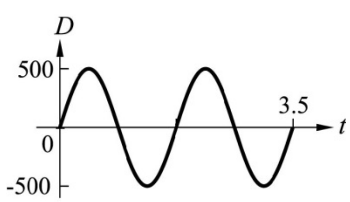
displacement is 0 s.



11. $D = A \sin(\omega t + \alpha) = 500 \sin(3.6t)$ has

amplitude $a = 500$ mi, period $= \dfrac{2\pi}{3.6} = \dfrac{5\pi}{9}$ h, and

displacement = 0 h.



12. $D = A\sin(\omega t + \alpha)$

$$= 850\sin\ \left(2\pi \times 1.6 \times 10^{-4}\right)t + \frac{\pi}{3}$$

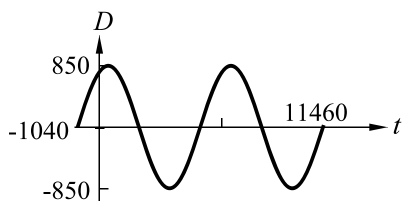
$$= 850\sin\ 0.0010t + \frac{\pi}{3}$$

Amplitude is 850 km,

period is $\dfrac{2\pi}{2\pi\left(1.6 \times 10^{-4}\right)} = 6250$ s,

12,500 for 2 cycles;

displacement is $\dfrac{-\pi/3}{2\pi\left(1.6 \times 10^{-4}\right)} = -1040$ s



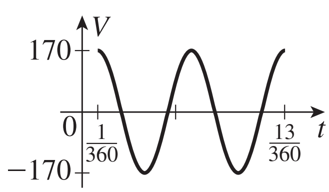
13. $V = E\cos(\omega t + \alpha)$

$$= 170\cos\ 2\pi(60.0)t - \frac{\pi}{3}$$

Amplitude is 170 V,

period is $\dfrac{2\pi}{2\pi(60.0)} = 0.016$ s, 0.033 s

for 2 cycles; displacement is $\dfrac{\pi/3}{2\pi(60.0)} = \dfrac{1}{360}$ s

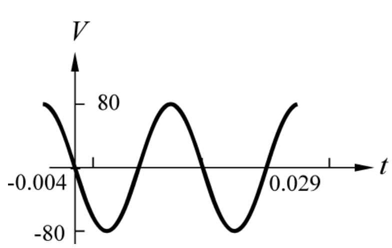


14. $V = 80\cos\ 377t + \dfrac{\pi}{2}$

Amplitude is 80 mV,

period is $\dfrac{2\pi}{377} = 0.016$ s, 0.033 s

for 2 cycles; displacement is $\dfrac{-\pi/2}{377} = -0.004$ s



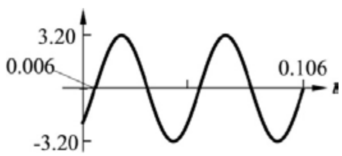
15. $y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$y = 3.20 \sin 2\pi\left(\frac{t}{0.050} - \frac{5.00}{40.0}\right)$ has amplitude

$a = 3.20$ mm,

period $= \frac{2\pi}{\frac{2\pi}{0.050}} = 0.050$ s, and displacement

$= -\frac{\frac{-2\pi(5.00)}{40.0}}{\frac{2\pi}{0.050}} = 0.00625$ s



16. $y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$= 0.350 \sin 2\pi\left(\frac{t}{0.250} - \frac{20.0}{24.0}\right)$

$= 0.350 \sin\left(\frac{2\pi t}{0.250} - \frac{40.0\pi}{24.0}\right)$

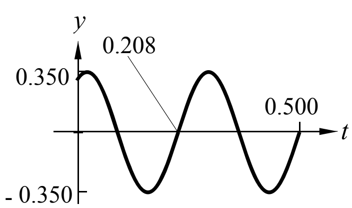
$= 0.350 \sin\left(8.00\pi t - \frac{5.00\pi}{3.00}\right)$

Amplitude is 0.350 in,

period is $\frac{2\pi}{8.00\pi} = 0.250$ s

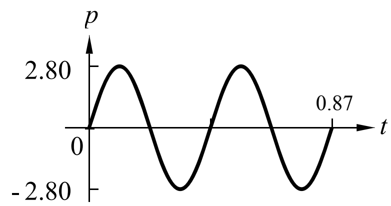
for 1 cycle, 0.500 s for 2 cycles;

displacement is $-\frac{-5.00\pi / 3.00}{8.00\pi} = 0.208$ s



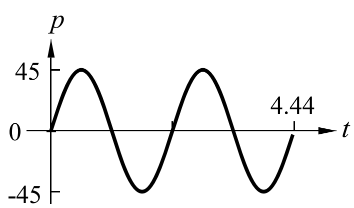
17. $p = p_0 \sin 2\pi ft$
$= 2.80 \sin\left[2\pi(2.30)\right]t$
$= 2.80 \sin 14.45t$

Amplitude is 2.80 lb/in^2, period is $\frac{2\pi}{14.45} = 0.435$ s for 1 cycle, 0.87 s for 2 cycles; displacement is 0 s

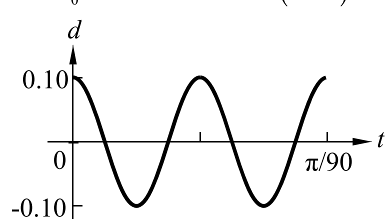


18. $p = p_0 \sin 2\pi ft$
$= 45.0 \sin\left[2\pi(0.450)\right]t$
$= 45.0 \sin 2.83t$

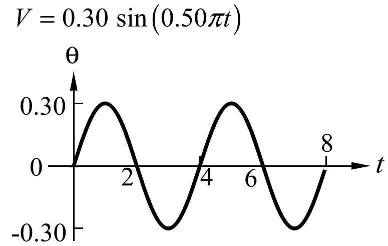
Amplitude is 45.0 kPa, period is $\frac{2\pi}{2.83} = 2.22$ s
4.44 s for 2 cycles; displacement is 0 s



19. $\theta = \theta_0 \cos \omega t = 0.10 \cos(360t)$



20. $V = 0.30 \sin(0.50\pi t)$



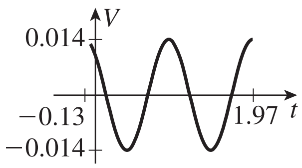
21. $V = 0.014\cos\left(2\pi ft + \dfrac{\pi}{4}\right)$

$= 0.014\cos\left[2\pi(0.950)t + \dfrac{\pi}{4}\right]$

Amplitude is 0.014 V,

period is $\dfrac{2\pi}{2\pi(0.950)} = 1.05$ s, 2.10 s

for 2 cycles; displacement is $\dfrac{-\pi/4}{2\pi(0.950)} = -0.13$ s



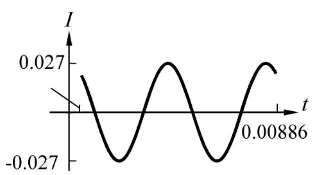
22. $I = A\cos(2\pi ft - \alpha)$

$= 0.027\cos[2\pi(240)t - 0.80]$

Amplitude is 0.027 W/m^2,

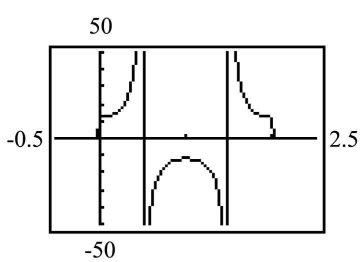
period is $\dfrac{2\pi}{2\pi(240)} = 0.0042$ s, 0.0083 s

for 2 cycles; displacement is $\dfrac{0.80}{2\pi(240)} = 5.3\times10^{-4}$ s



23. **(a)** $L = 12\sec \pi t$ has period = 2.0 s

Graph $y_1 = 12/\cos(\pi x)$, $0 \le x \le 2.0$.



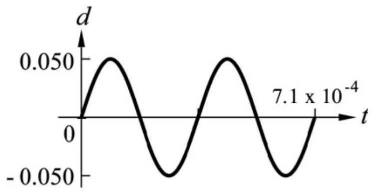
(b) Since L represents length, which is non-negative, only those parts of the graph where L is positive (above the x-axis) are meaningful.

24. $2a = 0.100$

$a = 0.050$

$\omega = 2800 \text{ r/min} \times \dfrac{2\pi}{r} = 5600\pi$

$d = 0.050 \sin(5600\pi t)$



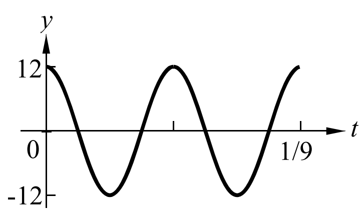
25. The frequency is 18 cycles/min and the amplitude is 12 ft, so

$y = 12 \sin(18 \cdot 2\pi t)$

$y = 12 \sin(36\pi t)$

with y in ft and t in min.

One cycles takes 1/18 min, so two cycles end at t=1/9 min.



26.

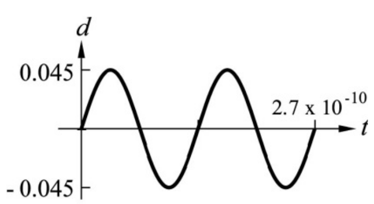
amplitude: $a = 0.045;$

angular velocity: $\omega = 2\pi f$

$= 2\pi(7.5 \times 10^9)$

$\omega = 1.5 \times 10^{10}\pi$ rad/s

Solution: $d = 0.045 \sin(1.5 \times 10^{10} \pi x)$

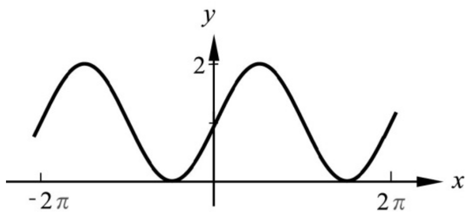


10.6 Composite Trigonometric Curves

1. $y = 1 + \sin x$

x	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	1	2	1	0	1	2	1	0	1

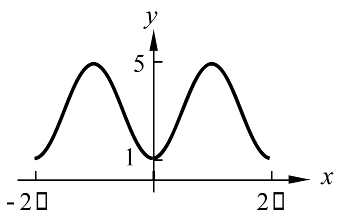
This is a vertical shift of 1 unit of the graph of $y = \sin x$.



2. $y = 3 - 2\cos x$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos x$	1	0	–1	0	1
$2\cos x$	2	0	–2	0	2
$3 - 2\cos x$	1	3	5	3	1

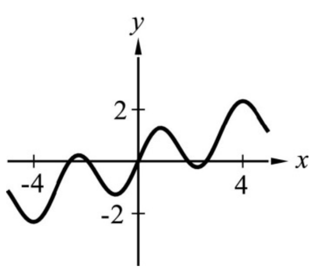
We have tabulated $y = -2\cos x$ rather than $y = 2\cos x$ because for addition or ordinates, it is easier to add graphic values than to subtract them. Note also that this is a vertical shift of 3 units of the function $y = -2\cos x$.



3. $y = \frac{1}{3}x + \sin 2x$

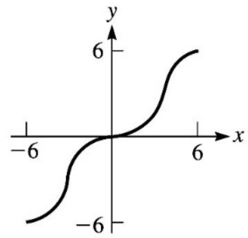
x	–4	–2.61	–2.27	–1.92	–0.87
y	–2.32	0	0.23	0	–1.28

x	0	0.87	1.92	2.27	2.61
y	0	1.28	0	–0.23	0



4. $y = x - \sin x$

x	−6	−4	−2	0	2	4	6
y	−6.3	−4.8	−1.1	0	1.1	4.8	6.3

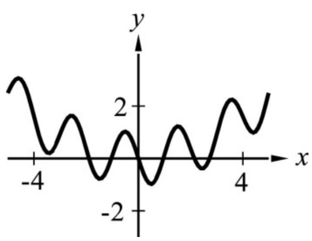


5. $y = \frac{1}{10}x^2 - \sin \pi x$

x	−4	−3.43	−2.55	−1.88	−1.47
y	1.60	0.20	1.64	0	−0.78

x	−1.03	−0.51	0	0.49	0.97	1.53
y	0	1.03	0	−0.98	0	1.23

x	2.15	2.45	2.73	0
y	0	−0.39	4	1.6

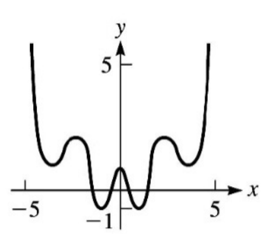


6. $y = \frac{1}{4}x^2 + \cos 3x$

x	−4	−3.57	−2.97	−2.55	−2.22
y	4.84	2.93	1.33	1.82	2.16

x	−1.4	−0.99	−0.55	0	0.55	0.99
y	0	−0.74	0	1	0	−0.74

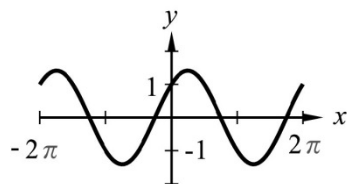
x	1.4	2.22	2.97	3.57	4
y	0	2.16	1.33	2.93	4.84



7. $y = \sin x + \cos x$

x	–5.5	–3.9	–2.4	–0.8
y	1.41	0	–1.41	0

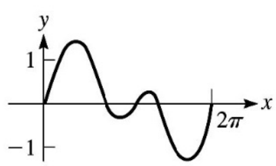
x	0	0.8	2.4	3.9	5.5
y	1	1.41	0	–1.41	0



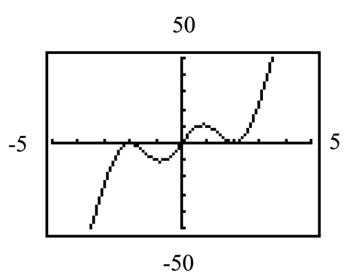
8. $y = \sin x + \sin 2x$

x	0	0.94	2.09	2.57	3.14
y	0	1.76	0	–0.369	0

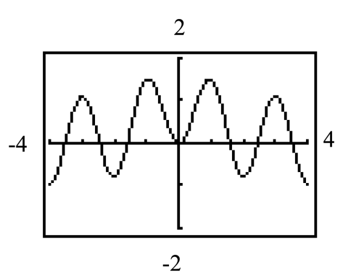
x	3.71	4.19	5.34	6.28
y	0.369	0	–1.76	0



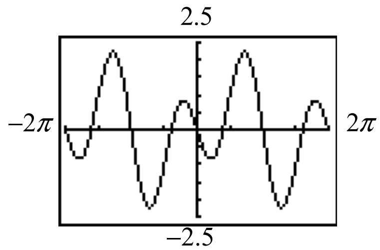
9. Graph $y_1 = x^3 + 10 \sin 2x$.



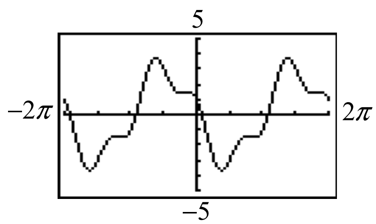
10. Graph $y_1 = \dfrac{1}{x^2+1} - \cos \pi x$.



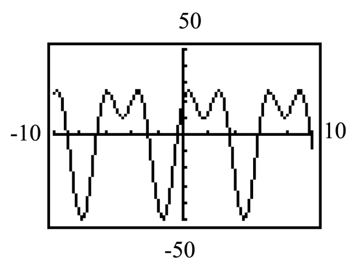
11. Graph $y_1 = \sin x - 1.5 \sin 2x.$



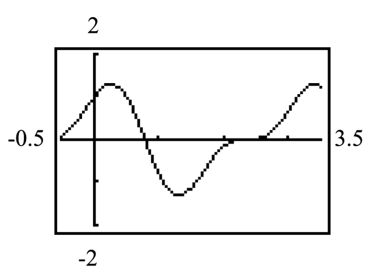
12. Graph $y_1 = \cos 3x - 3 \sin x.$



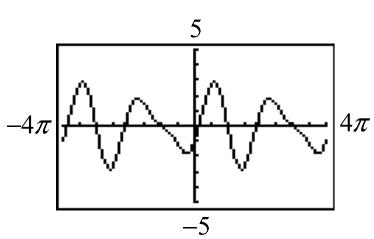
13. Graph $y_1 = 20 \cos 2x + 30 \sin x.$



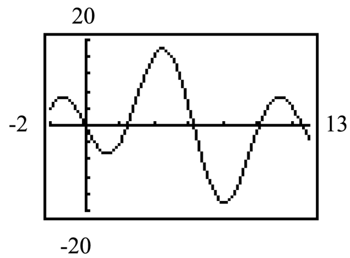
14. Graph $y_1 = \frac{1}{2} \sin 4x + \cos 2x.$



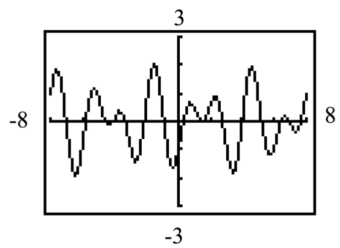
15. Graph $y_1 = 2 \sin x - \cos 1.5x.$



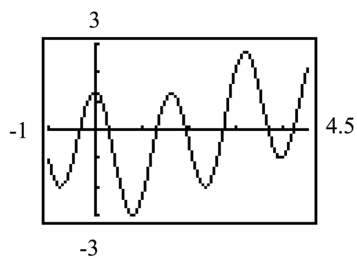
16. Graph $y_1 = 8 \sin \frac{x}{2} - 12 \sin x$.



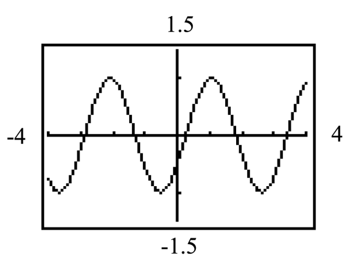
17. Graph $y_1 = \sin \pi x - \cos 2x$.



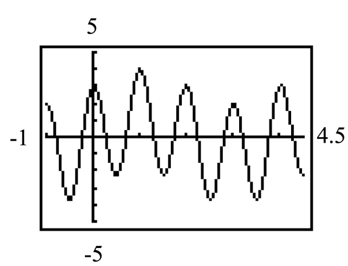
18. Graph $y_1 = 2 \cos 4x - \cos\left(x - \frac{\pi}{4}\right)$.



19. Graph $y_1 = 2 \sin\left(2x - \frac{\pi}{6}\right) + \cos\left(2x + \frac{\pi}{3}\right)$.

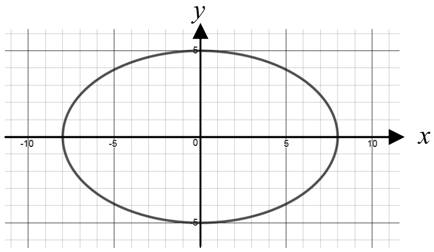


20. Graph $y_1 = 3 \cos 2\pi x + \sin \frac{\pi}{2} x$.



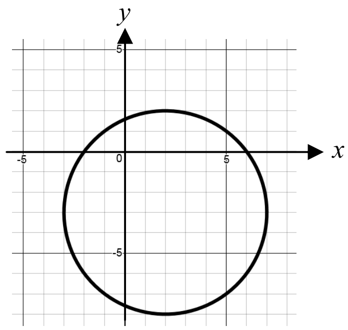
21. In parametric mode graph

$x_{1T} = 8\cos t,\ y_{1T} = 5\sin t$



22. In parametric mode graph

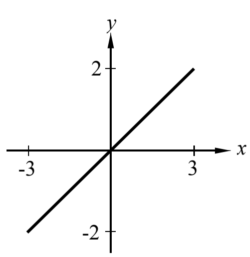
$x_{1T} = 5\cos t + 2,\ y_{1T} = 5\sin t - 3$



23. In parametric mode graph

$x_{1T} = 3\sin t,\ y_{1T} = 2\sin t$

t	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
x	−3	−2.12	0	2.12	3
y	−2	−1.41	0	1.41	2

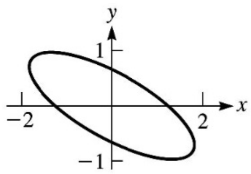


24. In parametric mode graph

$x_{1T} = 2\cos t,\ y_{1T} = \cos(t + 4)$

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
x	2	1.414	0	−1.414	−2
y	−0.65	0.073	0.757	0.997	0.65

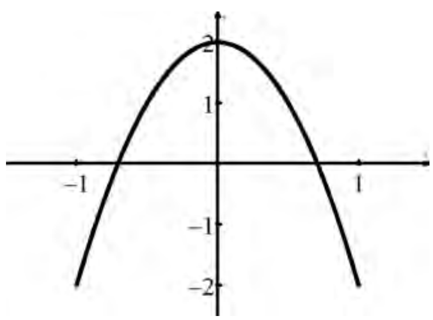
t	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
x	–1.414	0	1.414	2
y	–0.73	–0.757	–0.997	–0.65



25. In parametric mode graph
$x = \sin \pi t,\ y = 2\cos 2\pi t$

t	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
x	0	0.707	1	0.707	0
y	2	0	–2	0	2

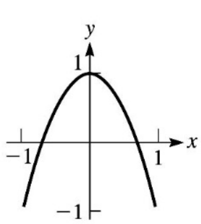
t	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
x	–0.707	–1	–0.707	0
y	0	–2	0	2



26. $x = \cos\left(t + \dfrac{\pi}{4}\right),\ y = \sin\ 2t$

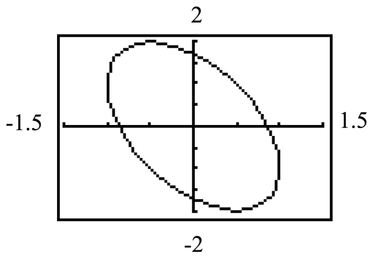
t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
x	0.707	0	–0.707	–1	–0.707
y	0	1	0	–1	0

t	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
x	0	0.707	1	0.707
y	1	0	–1	0



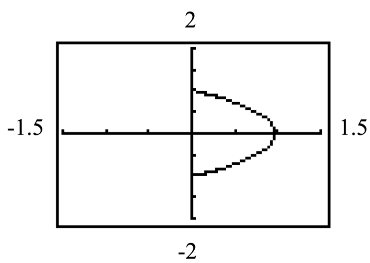
27. In parametric mode, graph

$x_{1T} = \cos \pi\left(t + \frac{1}{6}\right),\ y_{1T} = 2 \sin \pi t$



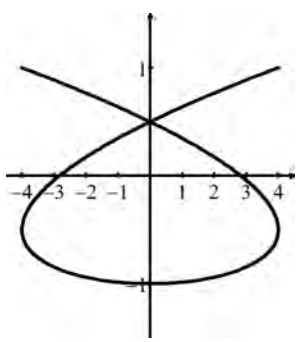
28. In parametric mode, graph

$x_{1T} = \sin^2 \pi t,\ y_{1T} = \cos \pi t.$



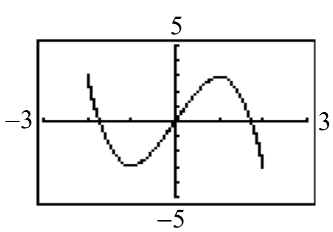
29. In parametric mode, graph

$x_{1T} = 4 \cos 3t,\ y_{1T} = \cos 2t.$

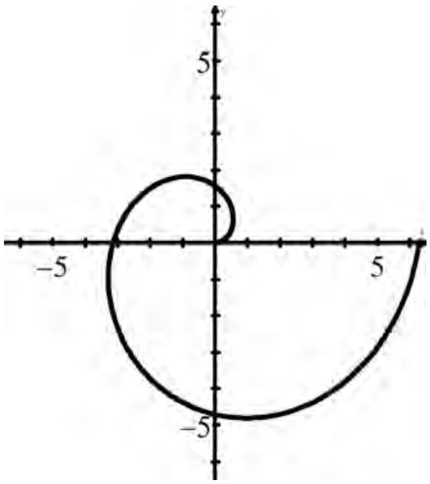


30. In parametric mode, graph

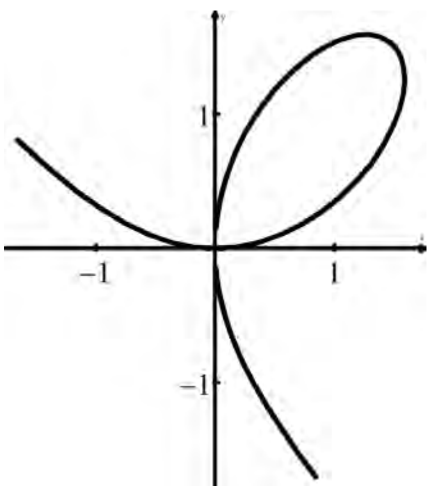
$x_{1T} = 2 \sin \pi t,\ y_{1T} = 3 \sin 3\pi t.$



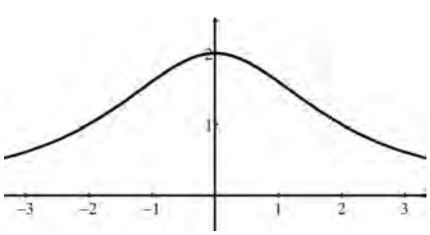
31. In parametric mode, graph
$x_{IT} = t\cos t,\ y_{IT} = t\sin t.$



32. In parametric mode, graph
$x_{IT} = 3t/(1+t^3),\ y_{IT} = 3t^2/(1+t^3).$

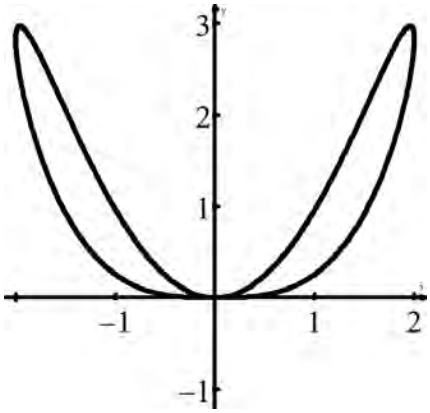


33. In parametric mode, graph
$x_{IT} = 2\cos t/\sin t,\ y_{IT} = 1-\cos(2t).$

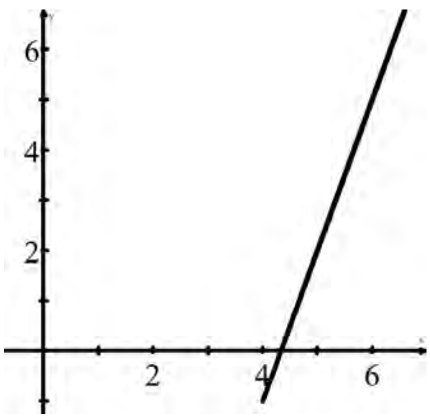


34. In parametric mode, graph

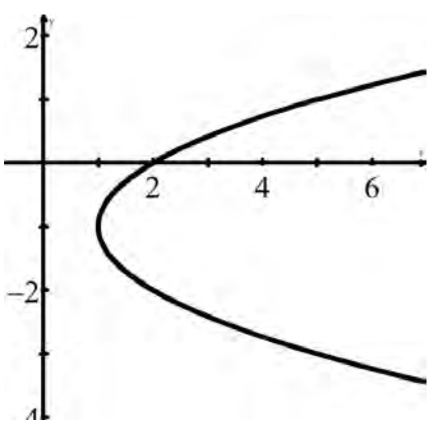
$x_{IT} = 2\sin(2t),\ y_{IT} = 2(\sin(2t))^3 / \cos t.$



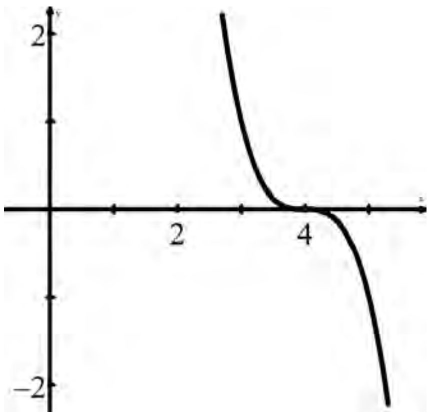
35. Graph $x = \sqrt{t} + 4, y = 3\sqrt{t} - 1; t \geq 0$



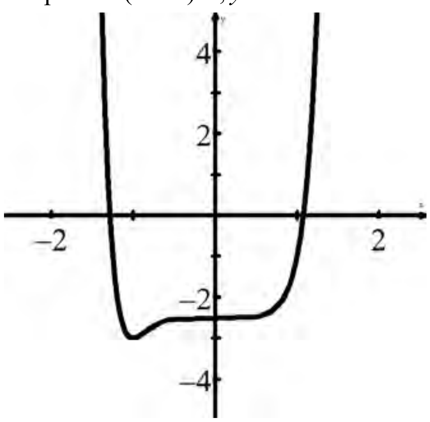
36. Graph $x = t^2 + 1, y = t - 1$



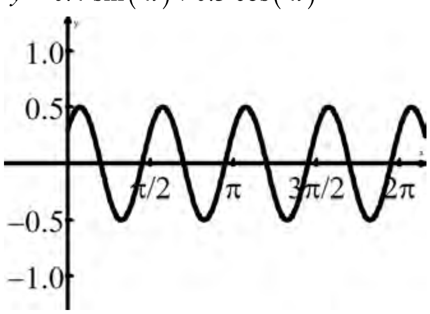
37. Graph $x = 4 - t, y = t^3$



38. Graph $x = (2t-1)^{1/5}, y = 2t^2 - 3$

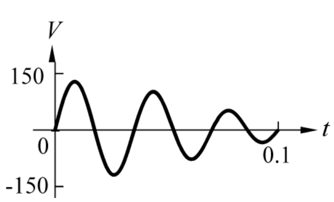


39. $y = 0.4\ \sin(4t) + 0.3\ \cos(4t)$



40. $e = 50\ \sin(50\pi t) + 80\ \sin(60\pi t)$

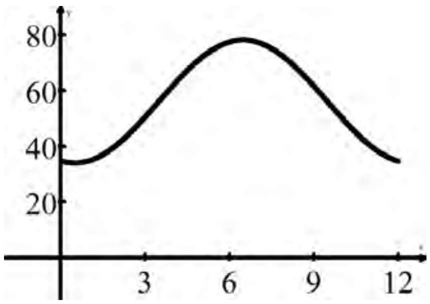
Graph $y_1 = 50\ \sin(50\pi x) + 80\sin(60\pi x)$



41. $T = 56 - 22\cos\left[\frac{\pi}{6}(x-0.5)\right]$

Graph $y_1 = 56 - 22\cos\left[\frac{\pi}{6}(x-0.5)\right]$

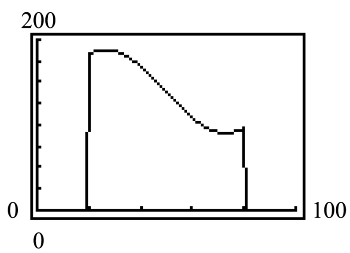
for x between 0 and 12



42. $d = 139.3 + 48.6\sin(0.0674x - 0.210)$

Graph $y_1 = 139.3 + 48.6\sin(0.0674x - 0.210)$

for $20 \le x \le 80.$



43. Time is in seconds so we have

$$60\frac{\text{beats}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} = 1\frac{\text{beat}}{\text{s}}$$

This means that the period is 1 s. The pressure varies between a maximum of 120 mmHg and a minimum of 80 mmHg, so the amplitude is half of this difference, or

$$\frac{120-80}{2} = 20\text{ mmHg.}$$

Pressure starts at 120 mmHg (a maximum), so we can use a cosine function with no displacement. Finally, instead of varying between -20 and 20, the function varies between 80 and 120, so it has been shifted up 100 mmHg. We have amplitude: $a = -20$;

period: $\frac{2\pi}{b} = 1$

$b = 2\pi$;

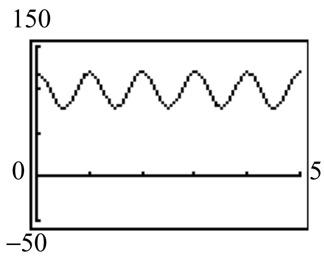
displacement: $-\frac{c}{b} = 0$

$c = 0$;

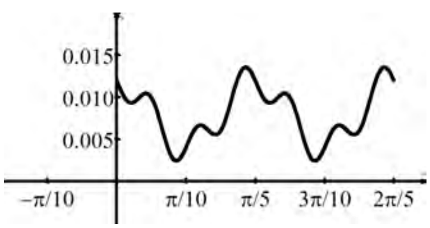
shift: $y = 100$;

Solution: $y = 100 + 20\cos 2\pi t$

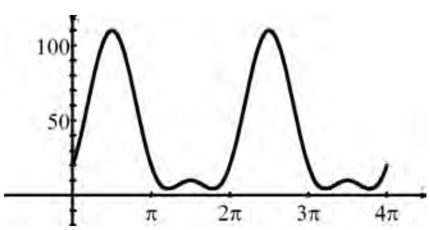
Graph $y_1 = 100 + 20\ \cos(2\pi x)$.



44. To graph $e = 0.0080 - 0.0020\sin 30t + 0.0040\cos 10t$,
graph $y_1 = 0.0080 - 0.0020\sin 30x + 0.0040\cos 10x$.



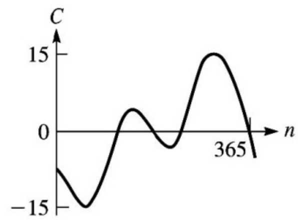
45. To graph $\mathrm{I} = 40 + 50\ \sin t - 20\ \cos\ 2t$,
graph $y_1 = 40 + 50\ \sin x - 20\ \cos\ 2x$.



46. $C = 10\sin\frac{1}{29}(n-80) - 7.5\cos\frac{1}{58}(n-80).$

Graph $y = 10\sin\frac{1}{29}(x-80) - 7.5\cos\frac{1}{58}(x-80)$

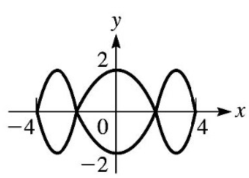
for x between 0 and 365.



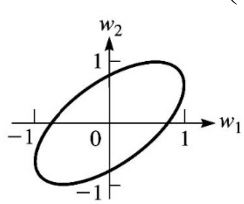
47. $x = 4\cos \pi t,\ y = 2\sin 3\pi t$

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
x	4	−3.12	0.88	1.75	−3.61
y	0	1.80	1.57	−0.43	−1.94

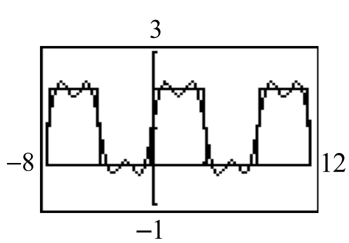
t	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
x	3.90	−2.48	−0.028	2.52
y	−1.27	0.84	2.0	0.91



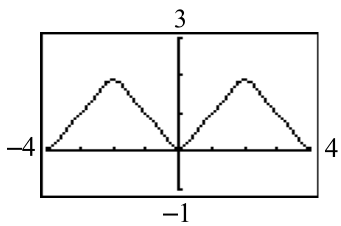
48. $w_1 = \sin \omega t,\ w_2 = \sin\left(\omega t + \frac{\pi}{4}\right).$



49. Graph $y_1 = 1 + \frac{4}{\pi}\sin\left(\frac{\pi x}{4}\right) + \frac{4}{3\pi}\sin\left(\frac{3\pi x}{4}\right).$



50. Graph $y_1 = 1 - \frac{8}{\pi^2}\left(\cos\frac{\pi x}{2} + \frac{1}{9}\cos\frac{3\pi x}{2}\right)$



Review Exercises

1. This is true. $-2 = 2\cos\pi$

2. This is true. For $b = \frac{1}{2}\pi$, period $= \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}\pi} = 4.$

3. This is false. For $b = 2$ and $c = -\frac{\pi}{4}$, displacement $= -\frac{c}{b} = -\frac{-\frac{\pi}{4}}{2} = \frac{\pi}{8}.$

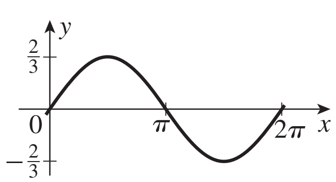
4. This is false. Amplitude is meaningless for the graphs of tangent functions.

5. This is false.
For $b = 60\pi$, period $= \frac{2\pi}{60\pi} = \frac{1}{30}$ s. The frequency is the reciprocal of the period, or 30 Hz.

6. This is true. $0 = \sin\pi - 2\cos\left(\frac{1}{2}\pi\right).$

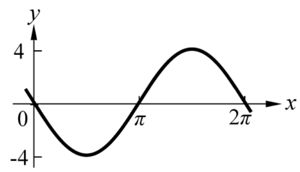
7. $y = \frac{2}{3}\sin x$ has amplitude 2/3. The table of key values between 0 and 2π is:

x	0	π/2	π	3π/2	2π
y	0	2/3	0	-2/3	0



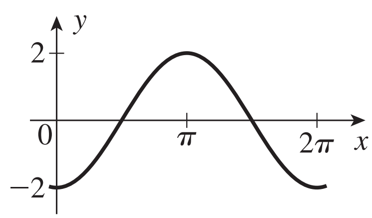
8. $y = -4 \sin x$ has amplitude 4 and has been inverted. The table of key values between 0 and 2π is:

x	0	$\pi/2$	π	$3\pi/2$	2π
y	0	-4	0	4	0



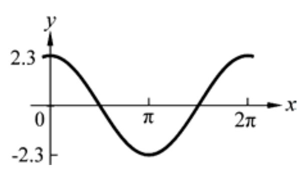
9. $y = -2 \cos x$ has amplitude 2 and has been inverted. The table of key values between 0 and 2π is:

x	0	$\pi/2$	π	$3\pi/2$	2π
y	-2	0	2	0	-2



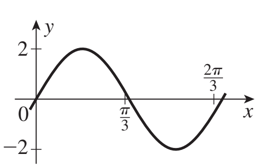
10. $y = 2.3 \cos(-x)$ is the same as $y = 2.3 \cos x$ (see Eq. (8.7)). It has amplitude 2.3. The table of key values between 0 and 2π is:

x	0	$\pi/2$	π	$3\pi/2$	2π
y	2.3	0	-2.3	0	2.3



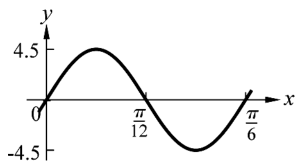
11. $y = 2 \sin 3x$ has amplitude 2 and period $2\pi/3$, with key values at multiples of $\frac{1}{4}\left(\frac{2\pi}{3}\right) = \frac{\pi}{6}$.
The table of key values between 0 and $2\pi/3$ is:

x	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$
y	0	2	0	-2	0



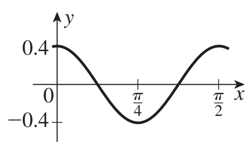
12. $y = 4.5 \sin 12x$ has amplitude 4.5 and period $2\pi/12 = \pi/6$, with key values at multiples of $\frac{1}{4}\left(\frac{\pi}{6}\right) = \frac{\pi}{24}$. The table of key values between 0 and $\pi/6$ is:

x	0	$\pi/24$	$\pi/12$	$\pi/8$	$\pi/6$
y	0	4.5	0	-4.5	0



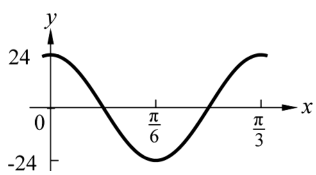
13. $y = 0.4 \cos 4x$ has amplitude 0.4 and period $2\pi/4 = \pi/2$, with key values at multiples of $\frac{1}{4}\left(\frac{\pi}{2}\right) = \frac{\pi}{8}$. The table of key values between 0 and $\pi/2$ is:

x	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$
y	0.4	0	-0.4	0	0.4



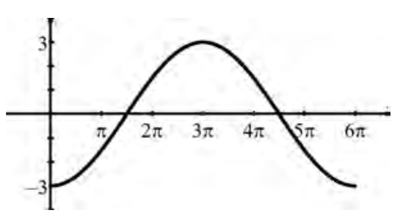
14. $y = 24 \cos 6x$ has amplitude 24 and period $2\pi/6 = \pi/3$, with key values at multiples of $\frac{1}{4}\left(\frac{\pi}{3}\right) = \frac{\pi}{12}$. The table of key values between 0 and $\pi/3$ is:

x	0	$\pi/12$	$\pi/6$	$\pi/4$	$\pi/3$
y	24	0	-24	0	24



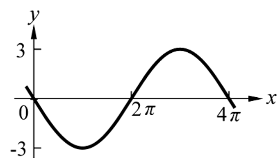
15. $y = -3 \cos \dfrac{1}{3}x$ has amplitude 3 and period $2\pi/(1/3) = 6\pi$, with key values at multiples of $\frac{1}{4}(6\pi) = \frac{3\pi}{2}$. The table of key values between 0 and 6π is:

x	0	$3\pi/2$	3π	$9\pi/2$	6π
y	-3	0	3	0	-3



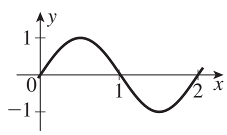
16. $y = 3\sin(-0.5x)$ is the same as $y = -3\sin 0.5x$ (see Eq. 8.7). It has amplitude 3 and period $2\pi/(1/2)=4\pi$, with key values at multiples of $\frac{1}{4}(4\pi) = \pi$, and it has been inverted. The table of key values between 0 and 4π is:

x	0	π	2π	3π	4π
y	0	-3	0	3	0



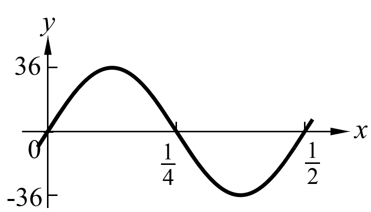
17. $y = \sin \pi x$ has amplitude 1 and period $2\pi/\pi=2$, with key values at multiples of $\frac{1}{4}(2) = \frac{1}{2}$. The table of key values between 0 and 2 is:

x	0	0.5	1	1.5	2
y	0	1	0	-1	0



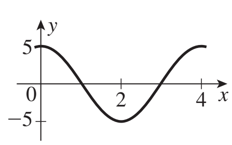
18. $y = 36\sin 4\pi x$ has amplitude 36 and period $2\pi/4\pi=1/2$, with key values at multiples of $\frac{1}{4}\left(\frac{1}{2}\right) = \frac{1}{8}$. The table of key values between 0 and 1/2 is:

x	0	1/8	¼	3/8	1/2
y	0	36	0	-36	0



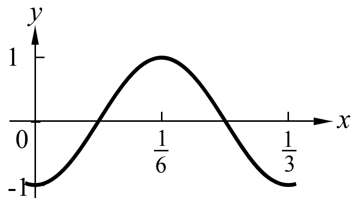
19. $y = 5\cos\left(\frac{\pi x}{2}\right)$ has amplitude 5 and period $2\pi/(\pi/2)=4$, with key values at multiples of $\frac{1}{4}(4) = 1$. The table of key values between 0 and 4 is:

x	0	1	2	3	4
y	5	0	-5	0	5



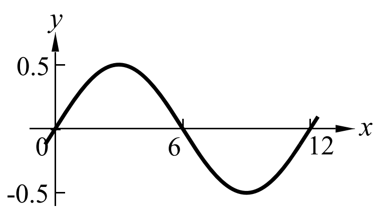
20. $y = -\cos 6\pi x$ has amplitude 1 and period 2π/(6π)=1/3, with key values at multiples of $\frac{1}{4}\left(\frac{1}{3}\right) = \frac{1}{12}$. The table of key values between 0 and 1/3 is:

x	0	1/12	1/6	1/4	1/3
y	-1	0	1	0	-1



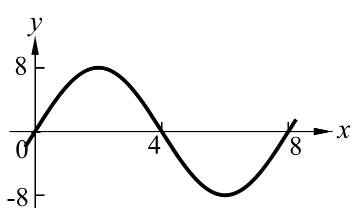
21. $y = -0.5 \sin\left(\frac{-\pi x}{6}\right)$ is the same as $y = 0.5\sin\left(\frac{\pi x}{6}\right)$ (see Eq. (8.7)). It has amplitude 0.5 and period 2π/(π/6)=12, with key values at multiples of $\frac{1}{4}(12) = 3$. The table of key values between 0 and 12 is:

x	0	3	6	9	12
y	0	0.5	0	-0.5	0



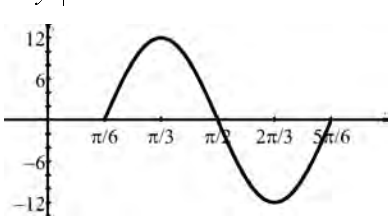
22. $y = 8 \sin\frac{\pi}{4}x$ has amplitude 8and period 2π/(π/4)=8, with key values at multiples of $\frac{1}{4}(8) = 2$. The table of key values between 0 and 8 is:

x	0	2	4	6	8
y	0	8	0	-8	0



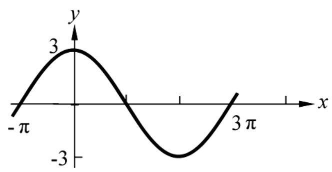
23. $y = 12 \sin\left(3x - \frac{\pi}{2}\right)$ has amplitude 12, period 2π/3, and displacement $-\left(-\frac{\pi}{2}\right)\cdot\frac{1}{3} = \frac{\pi}{6}$. One-fourth period is π/6, so key values for one full cycle start at π/6, end at 5π/6, and are found π/6 units apart. The table of key values is:

x	π/6	π /12	π /4	π /3	5 π /6
y	0	12	0	-12	0



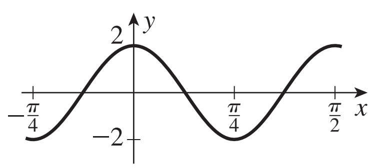
24. $y = 3\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)$ has amplitude 3, period 2π/(1/2)=4π, and displacement $-\left(-\frac{\pi}{2}\right)\cdot\frac{1}{1/2} = -\pi$. One-fourth period is π, so key values for one full cycle start at -π, end at 3π, and are found π units apart. The table of key values is:

x	-π	0	π	2π	3π
y	0	3	0	-3	0



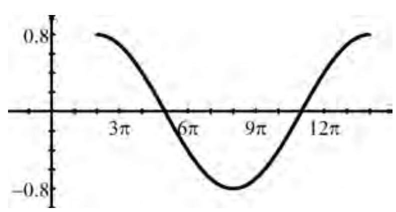
25. $y = -2\cos(4x+\pi)$ has amplitude 2 (inverted), period 2π/4=π/2, and displacement $-\frac{\pi}{4}$. One-fourth period is π/8, so key values for one full cycle start at –π/4, end at π/4, and are found π/8 units apart. The table of key values is:

x	-π/4	-π/8	0	π/8	π/4
y	-2	0	2	0	-2



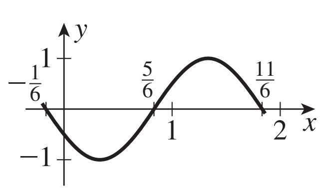
26. $y = 0.8\cos\left(\frac{x}{6}-\frac{\pi}{3}\right)$ has amplitude 0.8, period 2π/(1/6)=12π, and displacement $-\frac{c}{b} = -\frac{-\pi/3}{1/6} = 2\pi$ One-fourth period is 3π, so key values for one full cycle start at 2π, end at 14π, and are found 3π units apart. The table of key values is:

x	2π	5π	8π	11π	14π
y	0.8	0	-0.8	0	0.8



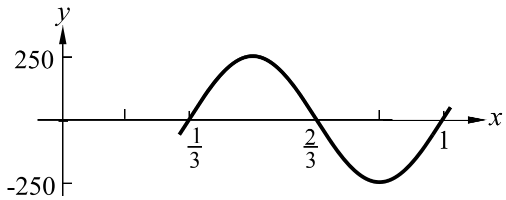
27. $y = -\sin\left(\pi x+\frac{\pi}{6}\right)$ has amplitude 1(inverted), period 2π/π=2, and displacement $-\left(\frac{\pi}{6}\right)\cdot\frac{1}{\pi} = -\frac{1}{6}$. One-fourth period is 1/2, so key values for one full cycle start at –1/6, end at 11/6, and are found 1/2 units apart. The table of key values is:

x	-1/6	1/3	5/6	4/3	11/6
y	0	-1	0	1	0



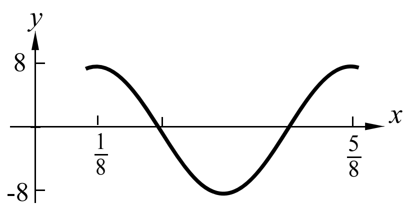
28. $y = 250\sin(3\pi x - \pi)$ has amplitude 250, period 2π/3π=2/3, and displacement $-(-\pi)\cdot\frac{1}{3\pi} = \frac{1}{3}$. One-fourth period is 1/6, so key values for one full cycle start at 1/3, end at 1, and are found 1/6 units apart. The table of key values is:

x	1/3	1/2	2/3	5/6	1
y	0	250	0	-250	0



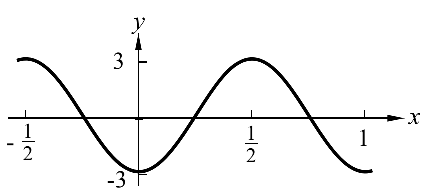
29. $y = 8\cos\left(4\pi x - \frac{\pi}{2}\right)$ has amplitude 8, period 2π/4π=1/2, and displacement $-\left(-\frac{\pi}{2}\right)\cdot\frac{1}{4\pi} = \frac{1}{8}$. One-fourth period is 1/8, so key values for one full cycle start at 1/8, end at 5/8, and are found 1/8 units apart. The table of key values is:

x	1/8	1/4	3/8	1/2	5/8
y	8	0	-8	0	8

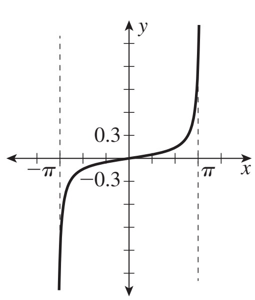


30. $y = 3\cos(2\pi x + \pi)$ has amplitude 3, period 2π/2π=1, and displacement $-(\pi)\cdot\frac{1}{2\pi} = -\frac{1}{2}$. One-fourth period is 1/4, so key values for one full cycle start at –1/2, end at 1/2, and are found 1/4 units apart. The table of key values is:

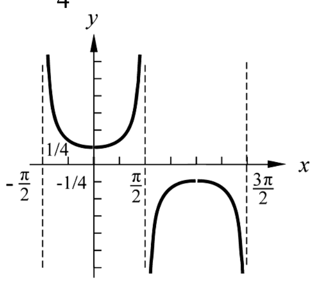
x	-1/2	-1/4	0	1/4	1/2
y	3	0	-3	0	3



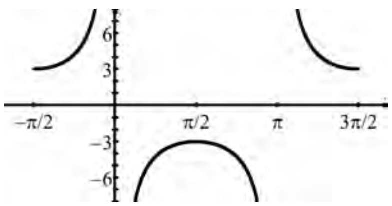
31. $y = 0.3\tan 0.5x$ has period 2π instead of period π. Key values occur every half period, with asymptotes at –π and π, and a zero at x=0.



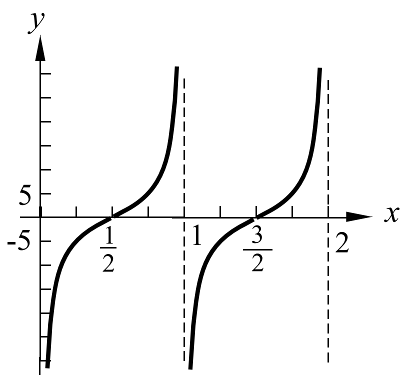
32. $y = \frac{1}{4}\sec x$ has the same period as the secant function. Values are multiplied by ¼.



33. $y = -3\csc x$ has the same period as the cosecant function, with values multiplied by 1/3 and inverted.

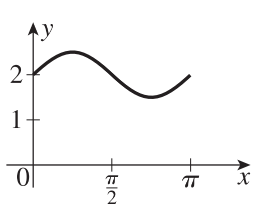


34. $y = -5\cot \pi x$ has period π/π=1. Every value is multiplied by 5 and then inverted. Asymptotes are at 0, 1, 2, etc, and zeros at 1/2, 3/2.

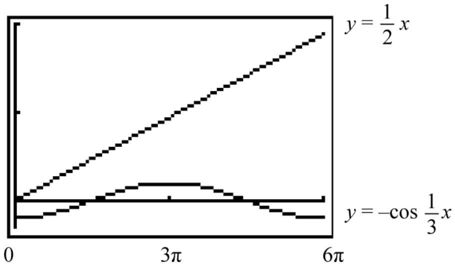


35. $y = 2 + \frac{1}{2}\sin 2x$ is the result of shifting the function $y = \frac{1}{2}\sin 2x$ vertically 2 units. This function has amplitude ½, period 2π/2=π, and no displacement. The table of key values is

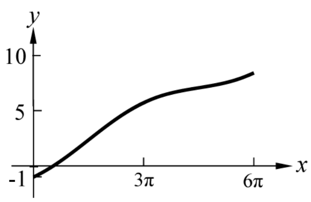
x	0	π/4	π/2	3π/4	π
$\frac{1}{2}\sin 2x$	0	1/2	0	-1/2	0
$\frac{1}{2}\sin 2x + 2$	2	5/2	2	3/2	0



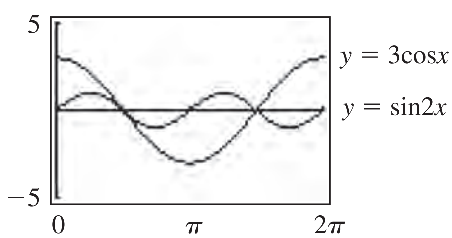
36. $y = \frac{1}{2}x - \cos\frac{1}{3}x$



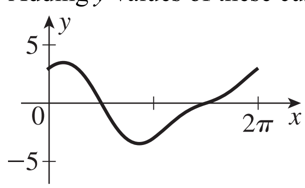
Adding y values of these curves gives



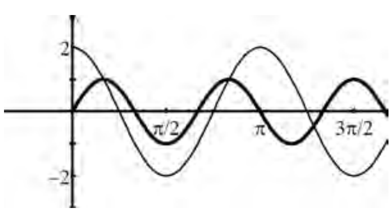
37. $y = \sin 2x + 3\cos x$



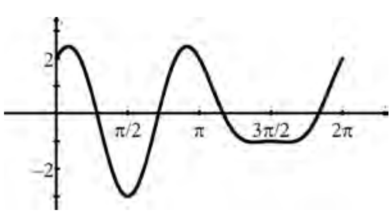
Adding y values of these curves gives



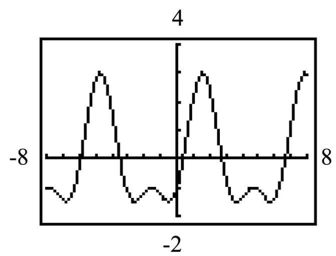
38. $y = \sin 3x + 2\cos 2x$



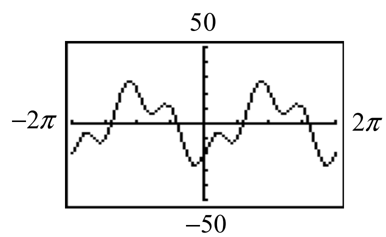
Adding y values of these curves gives



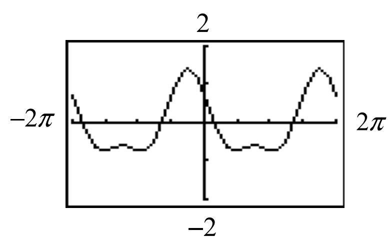
39. Graph $y_1 = 2\sin x - \cos 2x$.



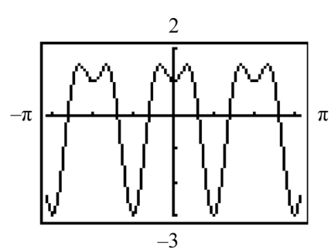
40. Graph $y_1 = 10\sin 3x - 20\cos x$.



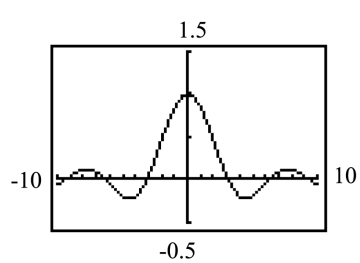
41. Graph $y_1 = \cos\left(x + \frac{\pi}{4}\right) - 0.4\sin 2x$.



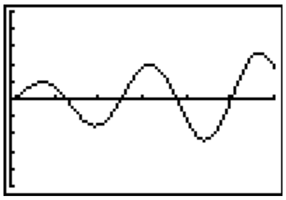
42. Graph $y_1 = 2\cos(\pi x) + \cos(2\pi x - \pi)$.



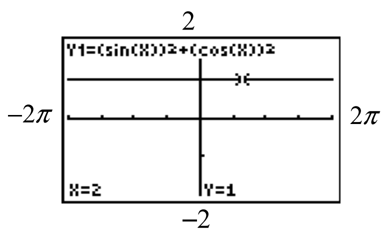
43. Graph $y_1 = \frac{\sin x}{x}$.



44. Graph $y_1 = \sqrt{x}\ \sin 0.5x$



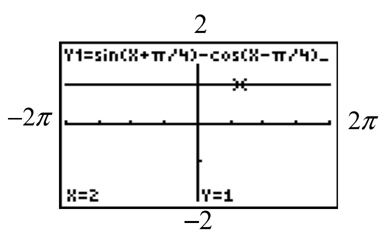
45. Graph $y_1 = \sin^2 x + \cos^2 x$. The graph is the horizontal line $y = 1$.



We conclude that the following identity holds:
$\sin^2 x + \cos^2 x = 1$
(We will discuss this identity in Chapter **20.**)

46. Graph $y_1 = \sin\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) + 1$.

The graph is the horizontal line $y = 1$.



We conclude that the following identity holds:

$\sin\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 0$

or

$\sin\left(x + \frac{\pi}{4}\right) = \cos\left(x - \frac{\pi}{4}\right)$

We know that this is true because the cosine function is the sine function shifted to the left π/2 units. If we shift the sine to the left π/4 units, and the cosine to the right π/4 units, the two graphs coincide.

47. amplitude: $a = 2$

period: $\frac{2\pi}{b} = \pi$

$b = 2;$

displacement: $-\frac{c}{b} = -\frac{\pi}{4}$

$-\frac{c}{2} = -\frac{\pi}{4}$

$c = \frac{\pi}{2}$

Solution: $y = 2\sin\left(2x + \frac{\pi}{2}\right)$

48. amplitude: $a = 2$

period: $\frac{2\pi}{b} = \pi$

$b = 2;$

displacement: $-\frac{c}{b} = 0$

$c = 0$

Solution: $y = 2\cos 2x$

49. amplitude: $a = 1$

period: $\frac{2\pi}{b} = 8$

$b = \frac{\pi}{4};$

displacement: $-\frac{c}{b} = 3$

$-\frac{c}{\pi/4} = 3$

$c = -\frac{3\pi}{4}$

Solution: $y = \cos\left(\frac{\pi}{4}x - \frac{3\pi}{4}\right)$

50. amplitude: $a = 1$

period: $\frac{2\pi}{b} = 8$

$b = \frac{\pi}{4};$

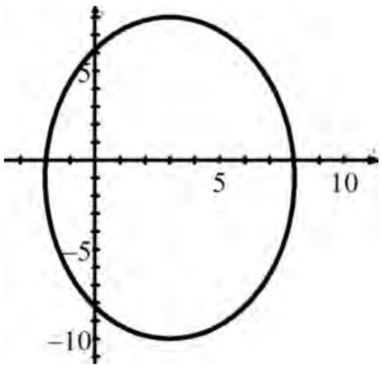
displacement: $-\frac{c}{b} = 1$

$-\frac{c}{\pi/4} = 1$

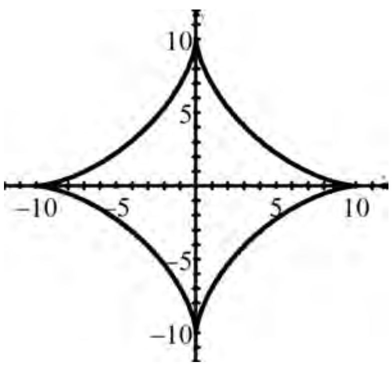
$c = -\frac{\pi}{4}$

Solution: $y = \sin\left(\frac{\pi}{4}x - \frac{\pi}{4}\right)$

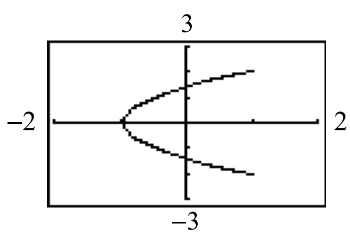
51. Graph $x_1 = 5\cos t + 3,\ y_1 = 9\sin t - 1$



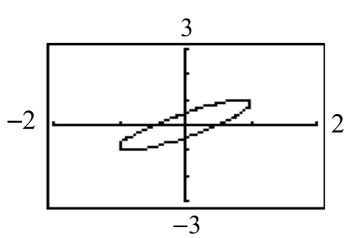
52. Graph $x_1 = 10(\cos t)^3,\ y_1 = 10(\sin t)^3$



53. In parametric mode graph
$x_1 = -\cos 2\pi t,\ y_1 = 2\sin \pi t$

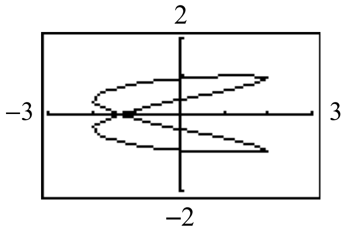


54. In parametric mode graph
$x_1 = \sin\left(t + \dfrac{\pi}{6}\right),\ y_1 = \sin t$



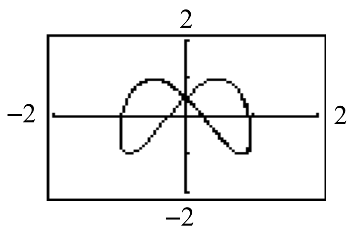
55. In parametric mode graph

$x_1 = 2\cos\left(2\pi t + \frac{\pi}{4}\right),\ y_1 = \cos \pi t$

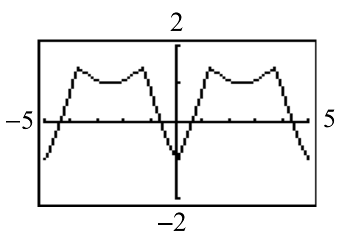


56. In parametric mode graph

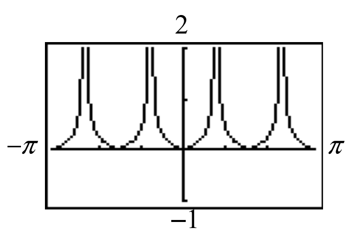
$x_1 = \cos\left(t - \frac{\pi}{6}\right),\ y_1 = \cos\left(2t + \frac{\pi}{3}\right)$



57. Graph $y_1 = 2|2\sin 0.2\pi x| - |\cos 0.4\pi x|$

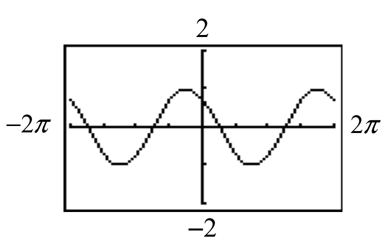


58. Graph $y_1 = 0.2|\tan 2x|$



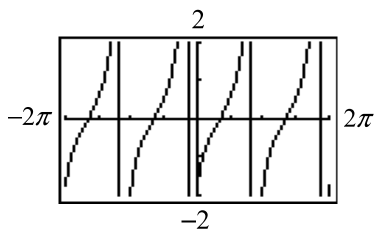
59. Graph $y_1 = \cos\left(x + \frac{\pi}{4}\right),\ y_2 = \sin\left(\frac{\pi}{4} - x\right)$.

The graphs are the same.



60. Graph $y_1 = \tan\left(x - \frac{\pi}{3}\right)$, $y_2 = -\tan\left(\frac{\pi}{3} - x\right)$.

The graphs are the same.



61. The period of 2 cos $0.5x$ is $\frac{2\pi}{0.5} = 4\pi$.

The period of sin $3x$ is $\frac{2\pi}{3}$.

The period of $y = 2\cos 0.5x + \sin 3x$ is the least common multiple of 4π and $\frac{2\pi}{3}$, which is 4π.

62. $y_1 = \sin \pi x$ has period $= \frac{2\pi}{\pi} = 2$

$y_2 = 3\sin 0.25\pi x$ has period $= \frac{2\pi}{0.25\pi} = 8$

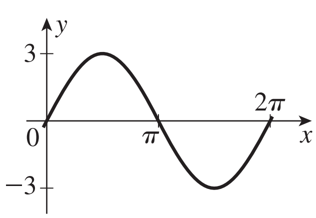
The period of $y = \sin \pi x + 3 \sin 0.25\pi x$ is the least common multiple of 2 and 8, which is 8.

63. Substitute $x = 5\pi/2$ and $y = 3$ into $y = a \sin x$ to get

$a \sin \frac{5\pi}{2} = 3$

$a = 3$

Solution: $y = 3 \sin x$

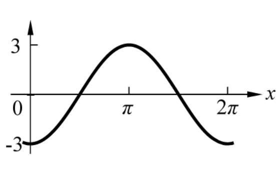


64. Substitute $x = 4\pi$ and $y = -3$ into $y = a \cos x$ to get

$a \cos 4\pi = 3$

$a = 3$

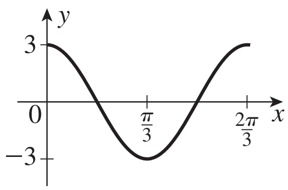
Solution: $y = -3 \cos x$



65. Substitute $x = \pi/3$ and $y = -3$ into $y = 3\cos bx$ to get

$$3\cos\tfrac{\pi}{3}b = -3$$
$$\tfrac{\pi}{3}b = \pi + 2\pi n$$

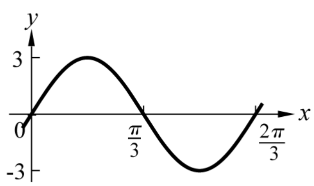
from where the smallest positive b is $b = 3$.
Solution: $y = 3\cos 3x$



66. Substitute $x = \pi/3$ and $y = 0$ into $y = 3\sin bx$ to get

$$3\sin\tfrac{\pi}{3}b = 0$$
$$\tfrac{\pi}{3}b = \pi n$$

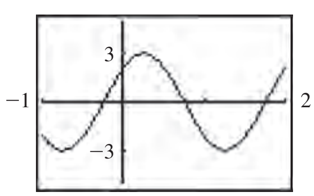
from where the smallest positive b is $b = 3$.
Solution: $y = 3\sin 3x$



67. Substitute $x = -0.25$ and $y = 0$ into $y = 3\sin(\pi x + c)$ to get

$$3\sin\left(\pi(-0.25)+c\right) = 0$$
$$-\frac{\pi}{4}c = n\pi$$
$$c = \frac{\pi}{4} + n\pi$$

from where the smallest positive c is $c = \pi/4$. Solution: $y = 3\sin\left(\pi x + \tfrac{\pi}{4}\right)$

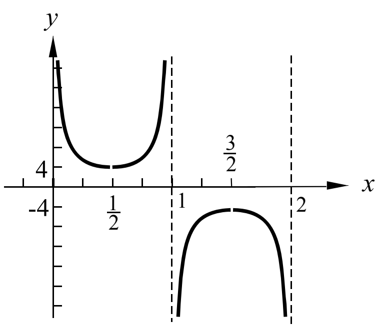


68. If displacement is zero, the cosecant function is of the form $y = a\csc(bx)$. Since the period is 2, $2\pi / b = 2$ and so $b = \pi$. We now substitute $x = 0.5$ and $y = 4$ into $y = a\csc(bx)$ to get

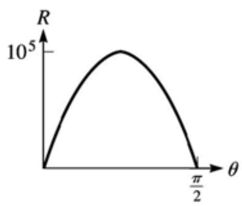
$4 = a\csc(0.5\pi)$

$4 = a$

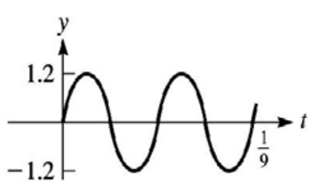
Solution: $y = 4\csc(\pi x)$.



69.

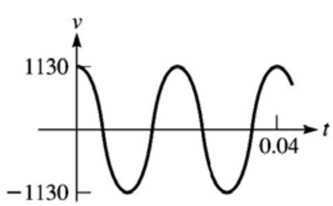


70. Note that θ varies between 0 and π/2.

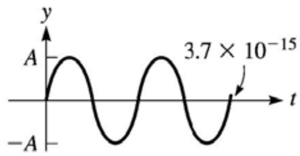


71. The frequency of 3000 r/min is the same as 50 r/s, so that $\omega = 2\pi(50) = 100\pi$ rad/s. Therefore

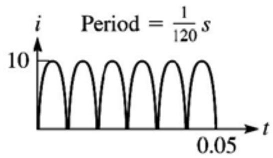
$v = 100\pi(3.6)\cos(100\pi t)$



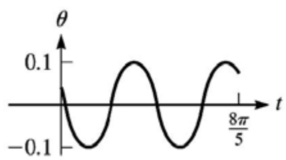
72. The period is $2\pi/(3.4\times10^{15}) = 1.8\times10^{-15}$. The amplitude is A, there is no displacement, so two cycles start at (0,0) and end at $(3.7\times10^{-15},0)$.



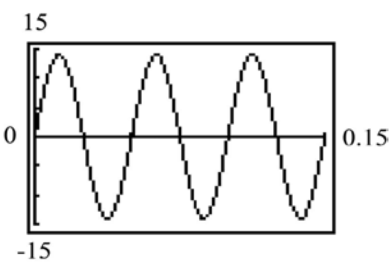
73. The period of $y = \sin 120\pi\, t$ is $2\pi/120\pi = 1/60$ s. With the absolute value, every negative part of the function is reflected with respect to the x axis to become positive, so the period is reduced by half. The period is 1/120 s. There are six complete cycles between 0 and 0.05 s.



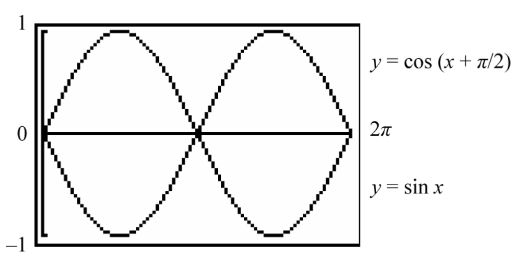
74. $y = 0.100 \cos(2.50\, t + \pi/4)$ has amplitude 0.100, period $2\pi/2.50 = 4\pi/5$ and displacement $-(\pi/4)/2.50 = 0.1\pi$. Two cycles extend to $t = 8\pi/5$.



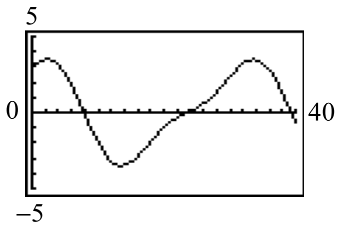
75. $y = 14.0 \sin 40.0\pi t$ has amplitude 14, period $2\pi/40.0\pi = 1/20$ and no displacement Two cycles extend to $t = 0.1$



76. The graph of $y = \sin x + \cos(x + \pi/2)$ is the constant 0, so it does represent destructive interference of two waves. If we graph the two waves separately, we can see that they are reflections of each other around the x axis, so that when adding ordinates we get zero at every x.



77. Graph $y = 3.0 \cos 0.2x + 1.0 \sin 0.4x$



78. The second hand takes 60 seconds to make a complete revolution, so the period is $60 = \frac{2\pi}{b}$ and so $b = \frac{\pi}{30}$. The amplitude will be 12mm and, since the y-coordinate is maximized at $t = 0$, it makes sense to use a cosine function. We can model the vertical projection with

$$y = 12\cos \frac{\pi}{30}t$$

79. A car in the *London Eye* takes 30 minutes to make a complete revolution, so the period is $30 = \frac{2\pi}{b}$ and so $b = \frac{\pi}{15}$. The amplitude will be $\frac{424}{2\pi}$ m and, since the y-coordinate is minimized at $t = 0$, it makes sense to use a negative cosine function. We shift up by $\frac{424}{2\pi}$ in order for the height to be 0 at $t = 0$. We can model the height of a car at time t with

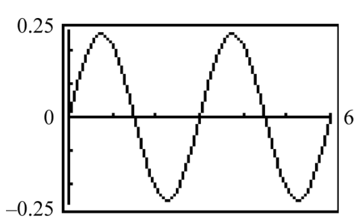
$$y = \frac{424}{2\pi} - \frac{424}{2\pi} \cos \frac{\pi}{15}t .$$

80. Amplitude: $R = 0.250$ m

period: $\frac{2\pi}{\omega} = 3.00$

$\omega = \frac{2\pi}{3}$ rad/s

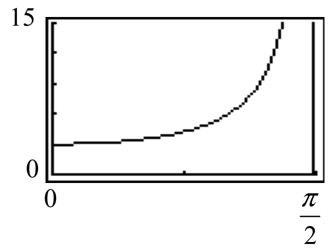
Solution: $d = 0.250\sin \frac{2\pi}{3}t$



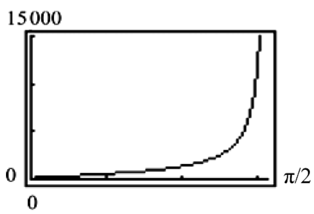
81. $d = a \sec \theta$, $a = 3.00$

$d = 3.00 \sec \theta$

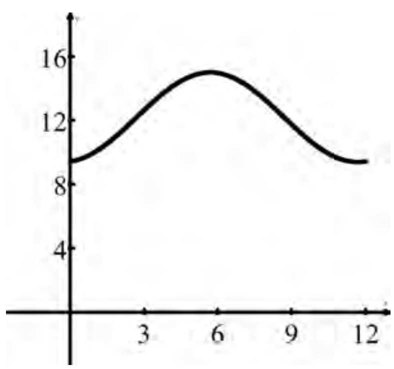
We graph the function from 0 to $\pi/2$.



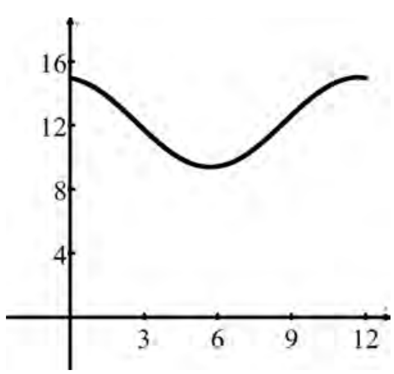
82. We graph the function between 0 and $\pi/2$.



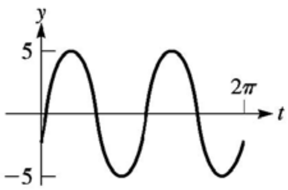
83. We graph the function between 0 and 12.



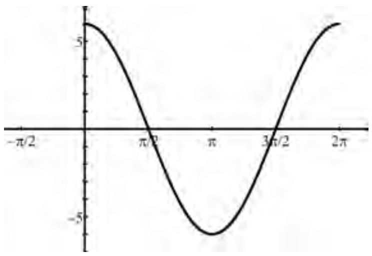
84. At the same latitude but in the southern hemisphere, the function is shifted six months to the left. We thus evaluate the function at $t + 6$ to get



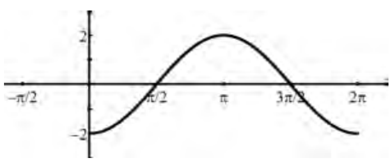
85. $y = 4\sin 2t - 2\cos 2t$



86. $y_1 + y_2 = 2\cos x + 4\sin(x + \pi/2)$

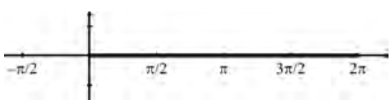


$y_1 + y_3 = 2\cos x + 4\sin(x + 3\pi/2)$

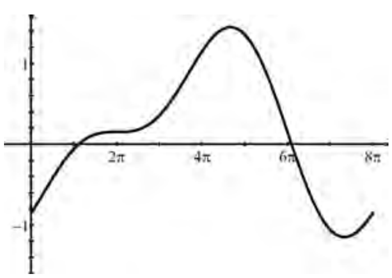


$y_2 + y_3 = 4\sin(x + \pi/2) + 4\sin(x + 3\pi/2)$

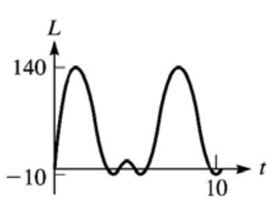
This graph is completely on the x-axis because $y_2 + y_3 = 0$



87.

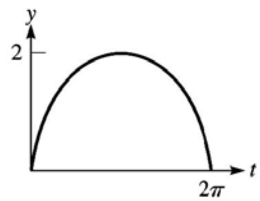


88.

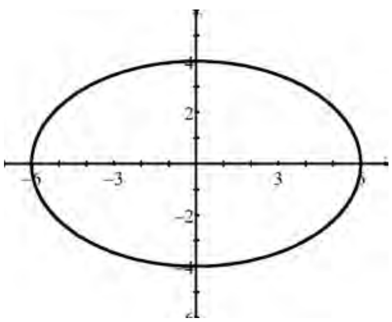


89.

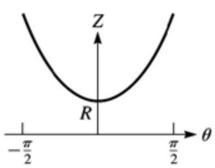
θ	0	$\pi/2$	π	$3\pi/2$	2π
x	0	$\pi/2-1$	π	$3\pi/2+1$	2π
y	0	1	2	1	0



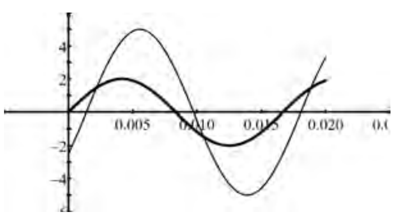
90.



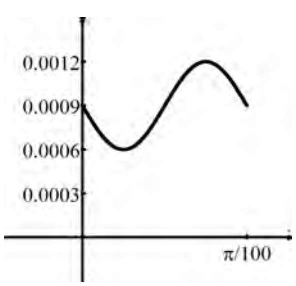
91. $Z = R\sec\phi,\ -\dfrac{\pi}{2} < \phi < \dfrac{\pi}{2}$. Graph shown is for an R value of 1. In general, the y-intercept would be R.



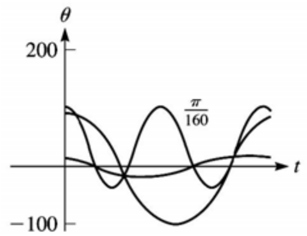
92.



93.



94.



95. **(a)** If a is doubled in $y = a\sin(bx + c)$ the amplitude will be doubled.

(b) If b is doubled in $y = a\sin(bx + c)$ the period will be reduced by one half.

(c) If c is doubled in $y = a\sin(bx + c)$ the displacement will be doubled.