Chapter 1
Linear Equations, Inequalities, and Applications

1.1 Linear Equations in One Variable

Classroom Examples, Now Try Exercises

1. **(a)** $9x+10=0$ is an *equation* because it contains an equals symbol.

(b) $9x+10$ is an *expression* because it does not contain an equals symbol.

N1. **(a)** $2x+17-3x$ is an *expression* because it does not contain an equals symbol.

(b) $2x+17=3x$ is an *equation* because it contains an equals symbol.

2.
$$\begin{aligned} 4x+8x &= -9+17x-1 \\ 12x &= 17x-10 && \text{Combine terms.} \\ 12x-17x &= 17x-10-17x && \text{Subtract } 17x. \\ -5x &= -10 && \text{Combine terms.} \\ \frac{-5x}{-5} &= \frac{-10}{-5} && \text{Divide by } -5. \\ x &= 2 \end{aligned}$$

Check by substituting 2 for x in the original equation.

$$\begin{aligned} 4x+8x &= -9+17x-1 \\ 4(2)+8(2) &\stackrel{?}{=} -9+17(2)-1 && \text{Let } x=2. \\ 8+16 &\stackrel{?}{=} -9+34-1 \\ 24 &\stackrel{?}{=} 25-1 \\ 24 &= 24 && \text{True} \end{aligned}$$

The solution set is $\{2\}$.

N2.
$$\begin{aligned} 5x+11 &= 2x-13-3x \\ 5x+11 &= -x-13 && \text{Combine terms.} \\ 5x+11+x &= -x-13+x && \text{Add } x. \\ 6x+11 &= -13 && \text{Combine terms.} \\ 6x+11-11 &= -13-11 && \text{Subtract } 11. \\ 6x &= -24 && \text{Combine terms.} \\ \frac{6x}{6} &= \frac{-24}{6} && \text{Divide by 6.} \\ x &= -4 \end{aligned}$$

Check by substituting -4 for x in the original equation.

$$\begin{aligned} 5x+11 &= 2x-13-3x \\ 5(-4)+11 &\stackrel{?}{=} 2(-4)-13-3(-4) && \text{Let } x=-4. \\ -20+11 &\stackrel{?}{=} -8-13+12 \\ -9 &\stackrel{?}{=} -21+12 \\ -9 &= -9 && \text{True} \end{aligned}$$

The solution set is $\{-4\}$.

3.
$$\begin{aligned} 3(4+x)-x &= 5x-3 \\ 12+3x-x &= 5x-3 && \text{Distributive prop.} \\ 12+2x &= 5x-3 && \text{Combine terms.} \\ 12+2x-2x &= 5x-3-2x && \text{Subtract } 2x. \\ 12 &= 3x-3 && \text{Combine terms.} \\ 12+3 &= 3x-3+3 && \text{Add 3.} \\ 15 &= 3x && \text{Combine terms.} \\ \frac{15}{3} &= \frac{3x}{3} && \text{Divide by 3.} \\ 5 &= x \end{aligned}$$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

Check $x=5$: $27-5=25-3$ True

The solution set is $\{5\}$.

N3.
$$\begin{aligned} 5(x-4)-12 &= 3-2x \\ 5x-20-12 &= 3-2x && \text{Distributive prop.} \\ 5x-32 &= 3-2x && \text{Combine terms.} \\ 5x-32+2x &= 3-2x+2x && \text{Add } 2x. \\ 7x-32 &= 3 && \text{Combine terms.} \\ 7x-32+32 &= 3+32 && \text{Add 32.} \\ 7x &= 35 && \text{Combine terms.} \\ \frac{7x}{7} &= \frac{35}{7} && \text{Divide by 7.} \\ x &= 5 \end{aligned}$$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

Check $x=5$: $5-12=3-10$ True

The solution set is $\{5\}$.

4. $6-(4+x)=8x-2(3x+5)$

$$\begin{array}{rll}
6-4-x=8x-6x-10 & & \text{Distributive prop.}\\
2-x=2x-10 & & \text{Combine terms.}\\
2-x+x=2x-10+x & & \text{Add } x.\\
2=3x-10 & & \text{Combine terms.}\\
2+10=3x-10+10 & & \text{Add 10.}\\
12=3x & & \text{Combine terms.}\\
\frac{12}{3}=\frac{3x}{3} & & \text{Divide by 3.}\\
4=x & &
\end{array}$$

Check $x=4$: $-2=32-34$ True

The solution set is $\{4\}$.

N4. $2-3(2+6x)=4(x+1)+36$

$$\begin{array}{rl}
2-6-18x=4x+4+36 & \text{Dist. prop.}\\
-4-18x=4x+40 & \text{Combine terms.}\\
-4-18x+18x=4x+40+18x & \text{Add } 18x.\\
-4=22x+40 & \text{Combine terms.}\\
-4-40=22x+40-40 & \text{Subtract 40.}\\
-44=22x & \text{Combine terms.}\\
\frac{-44}{22}=\frac{22x}{22} & \text{Divide by 22.}\\
-2=x &
\end{array}$$

Check $x=-2$: $2+30=-4+36$ True

The solution set is $\{-2\}$.

5. Multiply each side by the LCD, 4, and use the distributive property

$$\frac{x+1}{2}+\frac{x+3}{4}=\frac{1}{2}$$

$$4\left(\frac{x+1}{2}\right)+4\left(\frac{x+3}{4}\right)=4\left(\frac{1}{2}\right)$$

$$\begin{array}{rl}
2(x+1)+1(x+3)=2 & \\
2x+2+x+3=2 & \\
3x+5=2 & \\
3x=-3 & \text{Subtract 5.}\\
x=-1 & \text{Divide by 3.}
\end{array}$$

Check $x=-1$: $0+\frac{2}{4}=\frac{1}{2}$ True

The solution set is $\{-1\}$.

N5. Multiply each side by the LCD, 8, and use the distributive property.

$$\frac{x-4}{4}+\frac{2x+4}{8}=5$$

$$8\left(\frac{x-4}{4}\right)+8\left(\frac{2x+4}{8}\right)=8(5)$$

$$\begin{array}{rl}
2(x-4)+1(2x+4)=40 & \\
2x-8+2x+4=40 & \\
4x-4=40 & \\
4x=44 & \text{Add 4.}\\
x=11 & \text{Divide by 4.}
\end{array}$$

Check $x=11$: $\frac{7}{4}+\frac{13}{4}=5$ True

The solution set is $\{11\}$.

6. Multiply each term by 100.

$$\begin{aligned}
0.02(60)+0.04x&=0.03(50+x)\\
2(60)+4x&=3(50+x)\\
120+4x&=150+3x\\
4x&=30+3x\\
x&=30
\end{aligned}$$

Check $x=30$: $1.2+1.2=2.4$ True

The solution set is $\{30\}$.

N6. Multiply each term by 100.

$$\begin{aligned}
0.08x-0.12(x-4)&=0.03(x-5)\\
8x-12(x-4)&=3(x-5)\\
8x-12x+48&=3x-15\\
-4x+48&=3x-15\\
-4x+63&=3x\\
63&=7x\\
9&=x
\end{aligned}$$

Check $x=9$: $0.72-0.60=0.12$ True

The solution set is $\{9\}$.

7. **(a)** $5(x+2)-2(x+1)=3x+1$

$$\begin{array}{rl}
5x+10-2x-2=3x+1 & \\
3x+8=3x+1 & \\
3x+8-3x=3x+1-3x & \\
8=1 & \text{False}
\end{array}$$

Since the result, $8=1$, is *false*, the equation has no solution and is called a *contradiction*.
The solution set is $\varnothing$.

(b) Multiply each side by the LCD, 3, and use the distributive property.

$$\frac{x+1}{3}+\frac{2x}{3}=x+\frac{1}{3}$$
$$3\left(\frac{x+1}{3}\right)+3\left(\frac{2x}{3}\right)=3\left(x+\frac{1}{3}\right)$$
$$x+1+2x=3x+1$$
$$3x+1=3x+1$$

This is an *identity.* Any real number will make the equation true.
The solution set is {all real numbers}.

(c) $5(3x+1)=x+5$

$15x+5=x+5$

$14x+5=5$ Subtract x.

$14x=0$ Subtract 5.

$x=0$ Divide by 14.

This is a *conditional equation.*
Check $x=0$: $5(1)=0+5$ True
The solution set is $\{0\}$.

N7. **(a)** $9x-3(x+4)=6(x-2)$

$9x-3x-12=6x-12$

$6x-12=6x-12$

This is an *identity.* Any real number will make the equation true.
The solution set is {all real numbers}.

(b) $-3(2x-1)-2x=3+x$

$-6x+3-2x=3+x$

$-8x+3=3+x$

$-9x+3=3$ Subtract x.

$-9x=0$ Subtract 3.

$x=0$ Divide by -9.

This is a *conditional equation.*
Check $x=0$: $-3(-1)=3$ True
The solution set is $\{0\}$.

(c) $10x-21=2(x-5)+8x$

$10x-21=2x-10+8x$

$10x-21=10x-10$

$10x-21-10x=10x-10-10x$

$-21=-10$ False

Since the result, $-21=-10$, is *false*, the equation has no solution and is called a *contradiction.*
The solution set is $\varnothing$.

Exercises

1. A collection of numbers, variables, operation symbols, and grouping symbols, such as $2(8x-15)$, is an algebraic expression. While an equation *does* include an equality symbol, there *is not* an equality symbol in an algebraic expression.

2. A linear equation in one variable (here x) can be written in the form $Ax+B=C$, with $A\neq 0$. Another name for a linear equation is a first-degree equation, because the greatest power on the variable is *one*.

3. If we let $x=2$ in the linear equation $2x+5=9$, a *true* statement results. The number 2 is a solution of the equation, and $\{2\}$ is the solution set.

4. A linear equation with one solution in its solution set, such as the equation in Exercise 3, is a conditional equation.

5. A linear equation with an infinite number of solutions is an identity. Its solution set is {all real numbers}.

6. A linear equation with no solution is a contradiction. Its solution set is the empty set $\varnothing$.

7. $3x+x-2=0$ can be written as $4x=2$, so it is linear.
$9x-4=9$ is in linear form.
Choices A and C are linear.

8. $12=x^2$ is not a linear equation because the variable is squared.
$\frac{1}{8}x-\frac{1}{x}=0$ is not a linear equation because there is a variable in the denominator of the second term.
Choices B and D are nonlinear.

9. $3(x+4)=5x$ Original equation

$3(6+4)\stackrel{?}{=}5\cdot 6$ Let $x=6$.

$3(10)\stackrel{?}{=}30$ Add.

$30=30$ True

Since a true statement is obtained, 6 is a solution.

10. $5(x+4)-3(x+6)=9(x+1)$

$$
\begin{aligned}
5(-2+4)-3(-2+6) &\stackrel{?}{=} 9(-2+1) && \text{Let } x=-2.\\
5(2)-3(4) &\stackrel{?}{=} 9(-1) && \text{Add.}\\
10-12 &\stackrel{?}{=} -9 && \text{Multiply.}\\
-2 &= -9 && \text{False}
\end{aligned}
$$

Since a false statement is obtained, -2 is not a solution.

11. $-3x+2-4=x$ is an *equation* because it contains an equals symbol.

12. $-3x+2-4-x=4$ is an *equation* because it contains an equals symbol.

13. $4(x+3)-2(x+1)-10$ is an *expression* because it does not contain an equals symbol.

14. $4(x+3)-2(x+1)+10$ is an *expression* because it does not contain an equals symbol.

15. $-10x+12-4x=-3$ is an *equation* because it contains an equals symbol.

16. $-10x+12-4x+3=0$ is an *equation* because it contains an equals symbol.

17. A sign error was made when the distributive property was applied. The left side of the second line should be
$8x-4x+6.$

$$
\begin{aligned}
8x-2(2x-3) &= 3x+7\\
8x-4x+6 &= 3x+7 && \text{Distributive property}\\
4x+6 &= 3x+7 && \text{Combine like terms.}\\
x &= 1 && \text{Subtract } 3x \text{ and } 6.
\end{aligned}
$$

The correct solution is 1.

18. The negative sign $-$ represents -1.

$$
\begin{aligned}
&-5x-(2x-4)+5\\
&=-5x-2x+4+5 && \text{Distributive property}\\
&=-7x+9 && \text{Combine terms.}
\end{aligned}
$$

19.

$$
\begin{aligned}
7x+8 &= 1\\
7x+8-8 &= 1-8 && \text{Subtract 8.}\\
7x &= -7\\
\frac{7x}{7} &= \frac{-7}{7} && \text{Divide by 7.}\\
x &= -1
\end{aligned}
$$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

Check $x=-1$: $-7+8=1$ True

The solution set is $\{-1\}$.

20.

$$
\begin{aligned}
5x-4 &= 21\\
5x-4+4 &= 21+4 && \text{Add 4.}\\
5x &= 25\\
\frac{5x}{5} &= \frac{25}{5} && \text{Divide by 5.}\\
x &= 5
\end{aligned}
$$

Check $x=5$: $25-4=21$ True

The solution set is $\{5\}$.

21.

$$
\begin{aligned}
5x+2 &= 3x-6\\
5x+2-3x &= 3x-6-3x && \text{Subtract } 3x.\\
2x+2 &= -6\\
2x+2-2 &= -6-2 && \text{Subtract 2.}\\
2x &= -8\\
\frac{2x}{2} &= \frac{-8}{2} && \text{Divide by 2.}\\
x &= -4
\end{aligned}
$$

Check $x=-4$: $-20+2=-12-6$ True

The solution set is $\{-4\}$.

22.

$$
\begin{aligned}
9x+1 &= 7x-9\\
9x+1-7x &= 7x-9-7x && \text{Subtract } 7x.\\
2x+1 &= -9\\
2x+1-1 &= -9-1 && \text{Subtract 1.}\\
2x &= -10\\
\frac{2x}{2} &= \frac{-10}{2} && \text{Divide by 2.}\\
x &= -5
\end{aligned}
$$

Check $x=-5$: $-45+1=-35-9$ True

The solution set is $\{-5\}$.

23.

$$
\begin{aligned}
7x-5x+15 &= x+8\\
2x+15 &= x+8 && \text{Combine terms.}\\
2x &= x-7 && \text{Subtract 15.}\\
x &= -7 && \text{Subtract } x.
\end{aligned}
$$

Check $x=-7$: $-49+35+15=-7+8$ True

The solution set is $\{-7\}$.

24. $2x+4-x=4x-5$

$$\begin{aligned} x+4&=4x-5 && \text{Combine terms.}\\ -3x+4&=-5 && \text{Subtract } 4x.\\ -3x&=-9 && \text{Subtract 4.}\\ x&=3 && \text{Divide by} -3. \end{aligned}$$

Check $x=3$: $6+4-3=12-5$ True

The solution set is $\{3\}$.

25. $12w+15w-9+5=-3w+5-9$

$$\begin{aligned} 27w-4&=-3w-4 && \text{Combine terms.}\\ 30w-4&=-4. && \text{Add } 3w.\\ 30w&=0 && \text{Add 4.}\\ w&=0 && \text{Divide by 30.} \end{aligned}$$

Check $w=0$: $-9+5=5-9$ True

The solution set is $\{0\}$.

26. $-4x+5x-8+4=6x-4$

$$\begin{aligned} x-4&=6x-4 && \text{Combine terms.}\\ -5x-4&=-4 && \text{Subtract } 6x.\\ -5x&=0 && \text{Add 4.}\\ x&=0 && \text{Divide by} -5. \end{aligned}$$

Check $x=0$: $-8+4=-4$ True

The solution set is $\{0\}$.

27. $3(2t-4)=20-2t$

$$\begin{aligned} 6t-12&=20-2t && \text{Distributive property}\\ 8t-12&=20 && \text{Add } 2t.\\ 8t&=32 && \text{Add 12.}\\ t&=4 && \text{Divide by 8.} \end{aligned}$$

Check $t=4$: $3(4)=20-8$ True

The solution set is $\{4\}$.

28. $2(3-2x)=x-4$

$$\begin{aligned} 6-4x&=x-4 && \text{Distributive property}\\ 6-5x&=-4 && \text{Subtract } x.\\ -5x&=-10 && \text{Subtract 6.}\\ x&=2 && \text{Divide by} -5. \end{aligned}$$

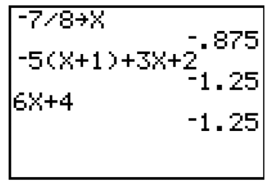
Check $x=2$: $2(-1)=2-4$ True

The solution set is $\{2\}$.

29. $-5(x+1)+3x+2=6x+4$

$$\begin{aligned} -5x-5+3x+2&=6x+4 && \text{Distributive prop.}\\ -2x-3&=6x+4 && \text{Combine terms.}\\ -3&=8x+4 && \text{Add } 2x.\\ -7&=8x && \text{Subtract 4.}\\ -\frac{7}{8}&=x && \text{Divide by 8.} \end{aligned}$$

Check: Substitute $-\frac{7}{8}$ for x and show that both sides equal -1.25. The screen shows a typical Check on a calculator.



The solution set is $\left\{-\frac{7}{8}\right\}$.

30. $5(x+3)+4x-5=4-2x$

$$\begin{aligned} 5x+15+4x-5&=4-2x && \text{Distributive prop.}\\ 9x+10&=4-2x && \text{Combine terms.}\\ 11x+10&=4 && \text{Add } 2x.\\ 11x&=-6 && \text{Subtract 10.}\\ x&=-\frac{6}{11} && \text{Divide by 11.} \end{aligned}$$

Check

$x=-\frac{6}{11}$:

$$\frac{135}{11}-\frac{24}{11}-\frac{55}{11}=\frac{44}{11}+\frac{12}{11} \quad \text{True}$$

The solution set is $\left\{-\frac{6}{11}\right\}$.

31. $-2x+5x-9=3(x-4)-5$

$$\begin{aligned} 3x-9&=3x-12-5\\ 3x-9&=3x-17\\ -9&=-17 && \text{False} \end{aligned}$$

The equation is a *contradiction.*
The solution set is $\varnothing$.

32. $-6x+2x-11=-2(2x-3)+4$

$$\begin{aligned} -4x-11&=-4x+6+4\\ -4x-11&=-4x+10\\ -11&=10 && \text{False} \end{aligned}$$

The equation is a *contradiction.*
The solution set is $\varnothing$.

33. $2(x+3) = -4(x+1)$

$$\begin{aligned} 2x+6 &= -4x-4 && \text{Remove parentheses.} \\ 6x+6 &= -4 && \text{Add 4x.} \\ 6x &= -10 && \text{Subtract 6.} \\ x &= \frac{-10}{6} = -\frac{5}{3} && \text{Divide by} -6. \end{aligned}$$

Check $x = -\frac{5}{3}$: $2\left(\frac{4}{3}\right) = -4\left(-\frac{2}{3}\right)$ True

The solution set is $\left\{-\frac{5}{3}\right\}$.

34. $4(x-9) = 8(x+3)$

$$\begin{aligned} 4x-36 &= 8x+24 && \text{Remove parentheses.} \\ -4x-36 &= 24 && \text{Subtract } 8x. \\ -4x &= 60 && \text{Add 36.} \\ x &= -15 && \text{Divide by} -4. \end{aligned}$$

Check $x = -15$: $4(-24) = 8(-12)$ True

The solution set is $\{-15\}$.

35. $3(2x+1) - 2(x-2) = 5$

$$\begin{aligned} 6x+3-2x+4 &= 5 \\ 4x+7 &= 5 \\ 4x &= -2 \\ x &= \frac{-2}{4} = -\frac{1}{2} \end{aligned}$$

Check $x = -\frac{1}{2}$: $3(0) - 2\left(-\frac{5}{2}\right) = 5$ True

The solution set is $\left\{-\frac{1}{2}\right\}$.

36. $4(x-2) + 2(x+3) = 6$

$$\begin{aligned} 4x-8+2x+6 &= 6 \\ 6x-2 &= 6 \\ 6x &= 8 \\ x &= \frac{8}{6} = \frac{4}{3} \end{aligned}$$

Check $x = \frac{4}{3}$: $4\left(-\frac{2}{3}\right) + 2\left(\frac{13}{3}\right) = 6$ True

The solution set is $\left\{\frac{4}{3}\right\}$.

37. $2x + 3(x-4) = 2(x-3)$

$$\begin{aligned} 2x+3x-12 &= 2x-6 \\ 5x-12 &= 2x-6 \\ 3x &= 6 \\ x &= \frac{6}{3} = 2 \end{aligned}$$

Check $x = 2$: $4 + 3(-2) = 2(-1)$ True

The solution set is $\{2\}$.

38. $6x - 3(5x+2) = 4(1-x)$

$$\begin{aligned} 6x-15x-6 &= 4-4x \\ -9x-6 &= 4-4x \\ -5x &= 10 \\ x &= \frac{10}{-5} = -2 \end{aligned}$$

Check $x = -2$: $-12 - 3(-8) = 4(3)$ True

The solution set is $\{-2\}$.

39. $6x - 4(3-2x) = 5(x-4) - 10$

$$\begin{aligned} 6x-12+8x &= 5x-20-10 \\ 14x-12 &= 5x-30 \\ 9x &= -18 \\ x &= -2 \end{aligned}$$

Check $x = -2$: $-12 - 4(7) = 5(-6) - 10$ True

The solution set is $\{-2\}$.

40. $-2x - 3(4-2x) = 2(x-3) + 2$

$$\begin{aligned} -2x-12+6x &= 2x-6+2 \\ 4x-12 &= 2x-4 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

Check $x = 4$: $-8 - 3(-4) = 2(1) + 2$ True

The solution set is $\{4\}$.

41. $-2(x+3) - x - 4 = -3(x+4) + 2$

$$\begin{aligned} -2x-6-x-4 &= -3x-12+2 \\ -3x-10 &= -3x-10 \end{aligned}$$

The equation is an *identity*.
The solution set is {all real numbers}.

42. $4(2x+7) = 2x + 25 + 3(2x+1)$

$$\begin{aligned} 8x+28 &= 2x+25+6x+3 \\ 8x+28 &= 8x+28 \end{aligned}$$

The equation is an *identity*.
The solution set is {all real numbers}.

43. $2[x-(2x+4)+3]=2(x+1)$

$$\begin{aligned}2[x-2x-4+3]&=2(x+1)\\ 2[-x-1]&=2(x+1)\\ -x-1&=x+1 &&\text{Divide by 2.}\\ -1&=2x+1 &&\text{Add } x.\\ -2&=2x &&\text{Subtract 1.}\\ -1&=x &&\text{Divide by 2.}\end{aligned}$$

Check $x=-1$: $2[-1-2+3]=0$ True

The solution set is $\{-1\}$.

44. $4[2x-(3-x)+5]=-(2+7x)$

$$\begin{aligned}4[2x-3+x+5]&=-(2+7x)\\ 4[3x+2]&=-(2+7x)\\ 12x+8&=-2-7x\\ 19x+8&=-2 &&\text{Add } 7x.\\ 19x&=-10 &&\text{Subtract 8.}\\ x&=-\frac{10}{19} &&\text{Divide by 19.}\end{aligned}$$

Check

$x=-\frac{10}{19}$:

$$4\left[-\frac{20}{19}-\frac{67}{19}+\frac{95}{19}\right]=-\left(-\frac{32}{19}\right) \quad \text{True}$$

The solution set is $\left\{-\frac{10}{19}\right\}$.

45. $-[2x-(5x+2)]=2+(2x+7)$

$$\begin{aligned}-[2x-5x-2]&=2+2x+7\\ -[-3x-2]&=2+2x+7\\ 3x+2&=2x+9\\ x&=7\end{aligned}$$

Check $x=7$: $-[14-37]=2+21$ True

The solution set is $\{7\}$.

46. $-[6x-(4x+8)]=9+(6x+3)$

$$\begin{aligned}-[6x-4x-8]&=9+6x+3\\ -(2x-8)&=6x+12\\ -2x+8&=6x+12\\ -8x&=4\\ x&=\frac{4}{-8}=-\frac{1}{2}\end{aligned}$$

Check $x=-\frac{1}{2}$: $-[-3-6]=9+0$ True

The solution set is $\left\{-\frac{1}{2}\right\}$.

47. $-3x+6-5(x-1)=-5x-(2x-4)+5$

$$\begin{aligned}-3x+6-5x+5&=-5x-2x+4+5\\ -8x+11&=-7x+9\\ -x&=-2\\ x&=2\end{aligned}$$

Check

$x=2$:

$$-6+6-5=-10-0+5 \quad \text{True}$$

The solution set is $\{2\}$.

48. $4(x+2)-8x-5=-3x+9-2(x+6)$

$$\begin{aligned}4x+8-8x-5&=-3x+9-2x-12\\ -4x+3&=-5x-3\\ x&=-6\end{aligned}$$

Check

$x=-6$:

$$-16+48-5=18+9-0 \quad \text{True}$$

The solution set is $\{-6\}$.

49. $7[2-(3+4x)]-2x=-9+2(1-15x)$

$$\begin{aligned}7[2-3-4x]-2x&=-9+2-30x\\ 7[-1-4x]-2x&=-7-30x\\ -7-28x-2x&=-7-30x\\ -7-30x&=-7-30x\end{aligned}$$

The equation is an *identity*.
The solution set is {all real numbers}.

50. $4[6-(1+2x)]+10x=2(10-3x)+8x$

$$\begin{aligned}4[6-1-2x]+10x&=20-6x+8x\\ 4(5-2x)+10x&=20+2x\\ 20-8x+10x&=20+2x\\ 20+2x&=20+2x\end{aligned}$$

The equation is an *identity*.
The solution set is {all real numbers}.

51. $-[3x-(2x+5)]=-4-[3(2x-4)-3x]$

$$\begin{aligned}-[3x-2x-5]&=-4-[6x-12-3x]\\ -[x-5]&=-4-[3x-12]\\ -x+5&=-4-3x+12\\ -x+5&=-3x+8\\ 2x&=3\\ x&=\frac{3}{2}\end{aligned}$$

Check

$x=\frac{3}{2}$:

$$-\left[\frac{9}{2}-8\right]=-4-\left[-3-\frac{9}{2}\right] \quad \text{True}$$

The solution set is $\left\{\frac{3}{2}\right\}$.

52. $2[-(x-1)+4]=5+[-(6x-7)+9x]$

$$\begin{aligned}2[-x+1+4]&=5+[-6x+7+9x]\\2[-x+5]&=5+[3x+7]\\-2x+10&=3x+12\\-5x&=2\\x&=-\frac{2}{5}\end{aligned}$$

Check $x=-\frac{2}{5}$:

$$2\left[\frac{7}{5}+4\right]=5+\left[\frac{47}{5}-\frac{18}{5}\right] \quad \text{True}$$

The solution set is $\left\{-\frac{2}{5}\right\}$.

53. The denominators of the fractions are 3, 4, 6, and 1. The least common denominator is 12 since it is the smallest number into which each denominator can divide without a remainder.

54. Yes, the coefficients will be larger, but you will get the correct solution. As long as you multiply both sides of the equation by the *same* nonzero number, the resulting equation is equivalent and the solution does not change.

55. **(a)** We need to make 0.04, 0.06, 0.03, and 1.46 integers. These numbers can be written as $\frac{4}{100}$, $\frac{6}{100}$, $\frac{3}{100}$, and $\frac{146}{100}$, respectively. Multiplying by 10^2, or 100, will eliminate the decimal points (the denominators) so that all the coefficients are integers.

(b) We need to make 0.006, 0.02, 0.03, 0.008, and 0.25 integers. These numbers can be written as $\frac{6}{1000}$, $\frac{2}{100}$, $\frac{3}{100}$, $\frac{8}{1000}$, and $\frac{25}{100}$, respectively. Multiplying by 10^3, or 1000, will eliminate the decimal points (the denominators) so that all the coefficients are integers.

56. $0.06(10-x)(100)$

$$\begin{aligned}&=0.06(100)(10-x)\\&=6(10-x)\\&=60-6x \quad \text{Choice B is correct.}\end{aligned}$$

57. $-\frac{5}{9}x=2$

$-5x=18$ Multiply by 9.

$x=\frac{18}{-5}=-\frac{18}{5}$ Divide by -5.

Check

$x=-\frac{18}{5}$: $\left(-\frac{5}{9}\right)\left(-\frac{18}{5}\right)=2$ True

The solution set is $\left\{-\frac{18}{5}\right\}$.

58. $\frac{3}{11}x=-5$

$3x=-55$ Multiply by 11.

$x=\frac{-55}{3}=-\frac{55}{3}$ Divide by 3.

Check $x=-\frac{55}{3}$: $\left(\frac{3}{11}\right)\left(-\frac{55}{3}\right)=-5$ True

The solution set is $\left\{-\frac{55}{3}\right\}$.

59. $\frac{6}{5}x=-1$

$6x=-5$ Multiply by 5.

$x=\frac{-5}{6}=-\frac{5}{6}$ Divide by 6.

Check

$x=-\frac{5}{6}$: $\left(\frac{6}{5}\right)\left(-\frac{5}{6}\right)=-1$ True

The solution set is $\left\{-\frac{5}{6}\right\}$.

60. $-\frac{7}{8}x=6$

$-7x=48$ Multiply by 8.

$x=\frac{48}{-7}=-\frac{48}{7}$ Divide by -7.

Check $x=-\frac{48}{7}$: $\left(-\frac{7}{8}\right)\left(-\frac{48}{7}\right)=6$ True

The solution set is $\left\{-\frac{48}{7}\right\}$.

61. Multiply both sides by the LCD, 6.

$$\frac{x}{2}+\frac{x}{3}=5$$
$$6\left(\frac{x}{2}+\frac{x}{3}\right)=6(5)$$
$$6\left(\frac{x}{2}\right)+6\left(\frac{x}{3}\right)=30 \quad \text{Distributive property}$$
$$3x+2x=30$$
$$5x=30 \quad \text{Add.}$$
$$x=6 \quad \text{Divide by 5.}$$

Check $x=6$: $3+2=5$ True

The solution set is $\{6\}$.

62. Multiply both sides by the LCD, 20.

$$\frac{x}{5}-\frac{x}{4}=1$$
$$20\left(\frac{x}{5}-\frac{x}{4}\right)=20(1)$$
$$20\left(\frac{x}{5}\right)-20\left(\frac{x}{4}\right)=20 \quad \text{Distributive property}$$
$$4x-5x=20$$
$$-x=20 \quad \text{Subtract.}$$
$$x=-20 \quad \text{Multiply by } -1.$$

Check $x=-20$: $-4+5=1$ True

The solution set is $\{-20\}$.

63. Multiply both sides by the LCD, 4.

$$\frac{3x}{4}+\frac{5x}{2}=13$$
$$4\left(\frac{3x}{4}+\frac{5x}{2}\right)=4(13)$$
$$4\left(\frac{3x}{4}\right)+4\left(\frac{5x}{2}\right)=4(13) \quad \text{Distributive prop.}$$
$$3x+10x=52$$
$$13x=52 \quad \text{Combine terms.}$$
$$x=4 \quad \text{Divide by 13.}$$

Check $x=4$: $3+10=13$ True

The solution set is $\{4\}$.

64. Multiply both sides by the LCD, 6.

$$\frac{8x}{3}-\frac{x}{2}=-13$$
$$6\left(\frac{8x}{3}-\frac{x}{2}\right)=6(-13)$$
$$6\left(\frac{8x}{3}\right)-6\left(\frac{x}{2}\right)=6(-13) \quad \text{Distributive prop.}$$
$$16x-3x=-78$$
$$13x=-78$$
$$x=-6 \quad \text{Divide by 26.}$$

Check $x=-6$: $-16+3=-13$ True

The solution set is $\{-6\}$.

65. Multiply both sides by the LCD, 6.

$$\frac{x-10}{5}+\frac{2}{5}=-\frac{x}{3}$$
$$15\left(\frac{x-10}{5}+\frac{2}{5}\right)=15\left(-\frac{x}{3}\right)$$
$$3(x-10)+3(2)=-5x$$
$$3x-30+6=-5x$$
$$8x=24$$
$$x=\frac{24}{8}=3$$

Check $x=3$: $-\frac{7}{5}+\frac{2}{5}=-1$ True

The solution set is $\{3\}$.

66. Multiply both sides by the LCD, 54.

$$\frac{5-x}{6}+\frac{5}{6}=\frac{x}{54}$$
$$54\left(\frac{5-x}{6}+\frac{5}{6}\right)=54\left(\frac{x}{54}\right)$$
$$9(5-x)+9(5)=x$$
$$45-9x+45=x$$
$$-9x+90=x$$
$$90=10x$$
$$x=\frac{90}{10}=9$$

Check $x=9$: $\frac{-36}{54}+\frac{45}{54}=\frac{9}{54}$ True

The solution set is $\{9\}$.

67. Multiply both sides by the LCD, 12.

$$\frac{3x-1}{4}+\frac{x+3}{6}=3$$
$$12\left(\frac{3x-1}{4}+\frac{x+3}{6}\right)=12(3)$$
$$3(3x-1)+2(x+3)=36$$
$$9x-3+2x+6=36$$
$$11x+3=36$$
$$11x=33$$
$$x=3$$

Check $x=3$: $2+1=3$ True

The solution set is $\{3\}$.

68. Multiply both sides by the LCD, 35.

$$\frac{3x+2}{7}-\frac{x+4}{5}=2$$
$$35\left(\frac{3x+2}{7}-\frac{x+4}{5}\right)=35(2)$$
$$5(3x+2)-7(x+4)=70$$
$$15x+10-7x-28=70$$
$$8x-18=70$$
$$8x=88$$
$$x=11$$

Check $x=11$: $5-3=2$ True

The solution set is $\{11\}$.

69. Multiply both sides by the LCD, 6.

$$\frac{4x+1}{3}=\frac{x+5}{6}+\frac{x-3}{6}$$
$$6\left(\frac{4x+1}{3}\right)=6\left(\frac{x+5}{6}+\frac{x-3}{6}\right)$$
$$2(4x+1)=(x+5)+(x+3)$$
$$8x+2=2x+2$$
$$6x=0$$
$$x=0$$

Check $x=0$: $\frac{1}{3}=\frac{5}{6}-\frac{3}{6}$ True

The solution set is $\{0\}$.

70. Multiply both sides by the LCD, 80.

$$\frac{2x+5}{5}=\frac{3x+1}{2}+\frac{-x+8}{16}$$
$$80\left(\frac{2x+5}{5}\right)=80\left(\frac{3x+1}{2}+\frac{-x+8}{16}\right)$$
$$16(2x+5)=40(3x+1)+5(-x+8)$$
$$32x+80=120x+40-5x+40$$
$$32x+80=115x+80$$
$$0=83x$$
$$\frac{0}{83}=x$$
$$0=x$$

Check $x=0$: $1=\frac{1}{2}+\frac{1}{2}$ True

The solution set is $\{0\}$.

71. Multiply each term by 100.

$$0.04x+0.06+0.03x=0.03x+1.46$$
$$4x+6+3x=3x+146$$
$$4x+6=146$$
$$4x=140$$
$$x=35$$

Check

$x=35$: $1.4+0.06+1.05=1.05+1.46$ True

The solution set is $\{35\}$.

72. Multiply each term by 100.

$$0.05x+0.08+0.06x=0.07x+0.68$$
$$5x+8+6x=7x+68$$
$$11x+8=7x+68$$
$$4x=60$$
$$x=15$$

Check

$x=15: 0.75+0.08+0.9=1.05+0.68$ True

The solution set is $\{15\}$.

73. Multiply each term by 1000.

$$0.006x-0.02x+0.03=0.008x+0.25$$
$$6x-20x+30=8x+250$$
$$-14x+30=8x+250$$
$$-22x=220$$
$$x=\frac{220}{-22}$$
$$x=-10$$

Check

$x=-10: -0.06+0.2+0.03=0.08+0.25$ True

The solution set is $\{-10\}$.

74. Multiply each term by 100.

$$\begin{aligned} 0.05x - 0.1x + 0.6 &= 0.04x + 2.22 \\ 5x - 10x + 60 &= 4x + 222 \\ -5x + 60 &= 4x + 222 \\ -9x &= 162 \\ x &= \frac{162}{-9} \\ x &= -18 \end{aligned}$$

Check
$x = -18: -0.9 + 1.8 + 0.6 = -0.72 + 2.22$ True

The solution set is $\{-18\}$.

75. Multiply each term by 100.

$$\begin{aligned} 0.05x + 0.12(x + 5000) &= 940 \\ 5x + 12(x + 5000) &= 100(940) \\ 5x + 12x + 60{,}000 &= 94{,}000 \\ 17x &= 34{,}000 \\ x &= 2000 \end{aligned}$$

Check $x = 2000$: $100 + 840 = 940$ True

The solution set is $\{2000\}$.

76. Multiply each term by 100.

$$\begin{aligned} 0.09x + 0.13(x + 300) &= 61 \\ 100[0.09x + 0.13(x + 300)] &= 100(61) \\ 100(0.09x) + 100(0.13)(x + 300) &= 6100 \\ 9x + 13(x + 300) &= 6100 \\ 9x + 13x + 3900 &= 6100 \\ 22x &= 2200 \\ x &= \frac{2200}{22} = 100 \end{aligned}$$

Check $x = 100$: $9 + 52 = 61$ True

The solution set is $\{100\}$.

77. Multiply each term by 100.

$$\begin{aligned} 0.02(50) + 0.08x &= 0.04(50 + x) \\ 2(50) + 8x &= 4(50 + x) \\ 100 + 8x &= 200 + 4x \\ 4x &= 100 \\ x &= 25 \end{aligned}$$

Check $x = 25$: $1 + 2 = 3$ True

The solution set is $\{25\}$.

78. Multiply each term by 100.

$$\begin{aligned} 0.04(90) + 0.12x &= 0.06(460 + x) \\ 100[0.04(90) + 0.12x] &= 100[0.06(460 + x)] \\ 4(90) + 12x &= 6(460 + x) \\ 360 + 12x &= 2760 + 6x \\ 360 + 6x &= 2760 \\ 6x &= 2400 \\ x &= 400 \end{aligned}$$

Check $x = 400$: $3.6 + 48 = 51.6$ True

The solution set is $\{400\}$.

79. Multiply each term by 100.

$$\begin{aligned} 0.05x + 0.10(200 - x) &= 0.45x \\ 5x + 10(200 - x) &= 45x \\ 5x + 2000 - 10x &= 45x \\ 2000 - 5x &= 45x \\ 2000 &= 50x \\ 40 &= x \end{aligned}$$

Check $x = 40$: $2 + 16 = 18$ True

The solution set is $\{40\}$.

80. Multiply each term by 100.

$$\begin{aligned} 0.08x + 0.12(260 - x) &= 0.48x \\ 8x + 12(260 - x) &= 48x \\ 8x + 3120 - 12x &= 48x \\ -4x + 3120 &= 48x \\ 3120 &= 52x \\ x &= \frac{3120}{52} = 60 \end{aligned}$$

Check $x = 60$: $4.8 + 24 = 28.8$ True

The solution set is $\{60\}$.

81. Multiply each term by 1000.

$$\begin{aligned} 0.006(x + 2) &= 0.007x + 0.009 \\ 6(x + 2) &= 7x + 9 \\ 6x + 12 &= 7x + 9 \\ 3 &= x \end{aligned}$$

Check $x = 3$: $0.03 = 0.021 + 0.009$ True

The solution set is $\{3\}$.

82. Multiply each term by 1000.

$$0.006(50-x)=0.272-0.004x$$
$$6(50-x)=272-4x$$
$$300-6x=272-4x$$
$$-2x=-28$$
$$x=14$$

Check $x=14$: $0.216=0.272-0.056$ True

The solution set is $\{14\}$.

1.2 Formulas and Percent

Classroom Examples, Now Try Exercises

1. To solve $d = rt$ for *r*, treat *r* as the only variable.

$d = rt$

$\frac{d}{t}=\frac{rt}{t}$ Divide by *t*.

$\frac{d}{t}=r,$ or $r=\frac{d}{t}$

N1. To solve $I = prt$ for *p*, treat *p* as the only variable.

$I = prt$

$I = p(rt)$ Associative property

$\frac{I}{rt}=\frac{p(rt)}{rt}$ Divide by *rt*.

$\frac{I}{rt}=p,$ or $p=\frac{I}{rt}$

2. Solve $P=2L+2W$ for *L*.

$P=2L+2W$

$P-2W=2L$ Subtract $2W$.

$\frac{P-2W}{2}=\frac{2L}{2}$ Divide by 2.

$\frac{P-2W}{2}=L,$ or $L=\frac{P}{2}-W$

N2. Solve $P=a+2b+c$ for *b*.

$P-a=2b+c$ Subtract *a*.

$P-a-c=2b$ Subtract *c*.

$\frac{P-a-c}{2}=\frac{2b}{2}$ Divide by 2.

$\frac{P-a-c}{2}=b,$ or $b=\frac{P-a-c}{2}$

3. Solve $y=\frac{1}{2}(x+3)$ for *x*.

$2y=x+3$ Multiply by 2.

$2y-3=x,$ or $x=2y-3$ Subtract 3.

N3. Solve $P=2(L+W)$ for *L*.

$\frac{P}{2}=\frac{2(L+W)}{2}$ Divide by 2.

$\frac{P}{2}=L+W$

$\frac{P}{2}-W=L$ Subtract W.

$L=\frac{P}{2}-W,$ or $L=\frac{P-2W}{2}$

4. Solve $2x+7y=7$ for *y*.

$2x+7y-2x=7-2x$ Subtract $2x$.

$7y=7-2x$

$\frac{7y}{7}=\frac{7-2x}{7}$ Divide by 7.

$y=\frac{7-2x}{7},$ or $y=-\frac{2}{7}x+1$

N4. Solve $5x-6y=12$ for *y*.

$-6y=12-5x$ Subtract $5x$.

$y=\frac{12-5x}{-6},$ Divide by -6.

or $y=\frac{5}{6}x-2$

5. Use $d = rt$. Solve for *r*.

$\frac{d}{t}=\frac{rt}{t}$ Divide by *t*.

$\frac{d}{t}=r,$ or $r=\frac{d}{t}$

Now substitute $d=15$ and $t=\frac{1}{3}$.

$$r=\frac{15}{\frac{1}{3}}=15\times\frac{3}{1}=45$$

His average rate is 45 miles per hour.

N5. Use $d = rt$. Solve for r.

$\frac{d}{t} = \frac{rt}{t}$ Divide by t.

$\frac{d}{t} = r$, or $r = \frac{d}{t}$

Now substitute $d = 21$ and $t = \frac{1}{2}$.

$$r = \frac{21}{\frac{1}{2}} = 21 \times \frac{2}{1} = 42$$

Her average rate is 42 miles per hour.

6. **(a)** The given amount of mixture is 20 oz. The part that is oil is 1 oz. Thus, the percent of oil is

$$\frac{\text{partial amount}}{\text{whole amount}} = \frac{1}{20} = 0.05 = 5\%.$$

(b) Let x represent the amount of commission earned.

$\frac{x}{12,000} = 0.08$ $\frac{\text{partial}}{\text{whole}} = \text{percent}$

$x = 0.08(12,000)$ Multiply by 12,000.

$x = 960$

The salesman earns \$960.

N6. **(a)** The given amount of mixture is 5 L. The part that is antifreeze is 2 L. Thus, the percent of antifreeze is

$$\frac{\text{partial amount}}{\text{whole amount}} = \frac{2}{5} = 0.40 = 40\%.$$

(b) Let x represent the amount of interest earned.

$\frac{x}{7500} = 0.025$ $\frac{\text{partial}}{\text{whole}} = \text{percent}$

$x = 0.025(7500)$ Multiply by 7500.

$x = 187.50$

The interest earned is \$187.50.

7. Let x represent the amount spent on pet supplies/medicine.

$\frac{x}{53.3} = 0.238$ $23.8\% = 0.238$

$x = 0.238(53.3)$ Multiply by 53.3.

$x = 12.6854$

Therefore, about \$12.7 billion was spent on pet supplies/medicine.

N7. Let x represent the amount spent on vet care.

$\frac{x}{53.3} = 0.256$ $25.6\% = 0.256$

$x = 0.256(53.3)$ Multiply by 53.3.

$x = 13.6448$

Therefore, about \$13.6 billion was spent on vet care.

8. **(a)** Let x = the percent increase (as a decimal).

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

$$x = \frac{1352 - 1300}{1300}$$

$$x = \frac{52}{1300}$$

$$x = 0.04$$

The percent increase was 4%.

(b) Let x = the percent decrease (as a decimal).

$$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$

$$x = \frac{54.00 - 51.30}{54.00}$$

$$x = \frac{2.70}{54.00}$$

$$x = 0.05$$

The percent decrease was 5%.

N8. **(a)** Let x = the percent decrease (as a decimal).

$$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$

$$x = \frac{80 - 56}{80}$$

$$x = \frac{24}{80}$$

$$x = 0.3$$

The percent markdown was 30%.

(b) Let x = the percent increase (as a decimal).

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

$$x = \frac{689 - 650}{650}$$

$$x = \frac{39}{650}$$

$$x = 0.06$$

The percent increase was 6%.

9. $S = \dfrac{4k+7}{k+90}$

$S = \dfrac{4(9.072)+7}{(9.072)+90}$ Substitute $k = 9.072$.

$S = 0.44$

The child has a body surface area of 0.44 m^2.

N9. $S = \dfrac{4k+7}{k+90}$

$S = \dfrac{4(14.515)+7}{(14.515)+90}$ Substitute $k = 14.515$.

$S = 0.62$

The child has a body surface area of 0.62 m^2.

10. The surface area of the child is 0.44 m^2. The adult dosage is 500 mg.

$C = \dfrac{\text{body surface area in m}^2}{1.7} \times D$

$C = \dfrac{0.44}{1.7} \times 500$

$C = 129$

The child dosage is 129 mg.

N10. The surface area of the child is 0.62 m^2. The adult dosage is 100 mg.

$C = \dfrac{\text{body surface area in m}^2}{1.7} \times D$

$C = \dfrac{0.62}{1.7} \times 100$

$C = 36$

The child dosage is 36 mg.

Exercises

1. A <u>*formula*</u> is an equation in which variables are used to describe a relationship.

2. To solve a formula for a specified variable, treat that <u>variable</u> as if it were the only one and treat all other variables like <u>constants</u> (numbers).

3. **(a)** $0.35 = \dfrac{35}{100} = 35\%$

(b) $0.18 = \dfrac{18}{100} = 18\%$

(c) $0.02 = \dfrac{2}{100} = 2\%$

(d) $0.075 = \dfrac{7.5}{100} = 7.5\%$

(e) $1.5 = \dfrac{150}{100} = 150\%$

4. **(a)** $60\% = \dfrac{60}{100} = 0.6$

(b) $37\% = \dfrac{37}{100} = 0.37$

(c) $8\% = \dfrac{8}{100} = 0.08$

(d) $3.5\% = \dfrac{3.5}{100} = 0.035$

(e) $210\% = \dfrac{210}{100} = 2.1$

5. Solve $I = prt$ for r.

$I = prt$

$\dfrac{I}{pt} = \dfrac{prt}{pt}$

$\dfrac{I}{pt} = r$, or $r = \dfrac{I}{pt}$

6. Solve $d = rt$ for t.

$\dfrac{d}{r} = \dfrac{rt}{r}$ Divide by r.

$\dfrac{d}{r} = t$, or $t = \dfrac{d}{r}$

7. **(a)** Solve $A = bh$ for b.

$\dfrac{A}{h} = b$ Divide by h.

(b) Solve $A = bh$ for h.

$\dfrac{A}{b} = h$ Divide by b.

8. **(a)** Solve $A = LW$ for W.

$\dfrac{A}{L} = W$ Divide by L.

(b) Solve $A = LW$ for L.

$\dfrac{A}{W} = L$ Divide by W.

9. Solve $P = 2L + 2W$ for L.

$$P = 2L + 2W$$
$$P - 2W = 2L$$
$$\frac{P-2W}{2} = \frac{2L}{2}$$
$$\frac{P-2W}{2} = L, \quad \text{or} \quad L = \frac{P}{2} - W$$

10. **(a)** Solve $P = a + b + c$ for b.

$$P - (a+c) = a + b + c - (a+c)$$
$$P - a - c = b$$

(b) Solve $P = a + b + c$ for c.

$$P - (a+b) = a + b + c - (a+b)$$
$$P - a - b = c$$

11. **(a)** Solve for $V = LWH$ for W.

$$V = LWH$$
$$\frac{V}{LH} = \frac{LWH}{LH}$$
$$\frac{V}{LH} = W, \quad \text{or} \quad W = \frac{V}{LH}$$

(b) Solve for $V = LWH$ for H.

$$V = LWH$$
$$\frac{V}{LW} = \frac{LWH}{LW}$$
$$\frac{V}{LW} = H, \quad \text{or} \quad H = \frac{V}{LW}$$

12. **(a)** Solve $A = \frac{1}{2}bh$ for h.

$$\frac{2A}{b} = \frac{2}{b}\left(\frac{1}{2}bh\right)$$
$$\frac{2A}{b} = h$$

(b) Solve $A = \frac{1}{2}bh$ for b.

$$\frac{2A}{h} = \frac{2}{h}\left(\frac{1}{2}bh\right)$$
$$\frac{2A}{h} = b$$

13. Solve $C = 2\pi r$ for r.

$$C = 2\pi r$$
$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi} \quad \text{Divide by } 2\pi.$$
$$\frac{C}{2\pi} = r$$

14. Solve $V = \pi r^2 h$ for h.

$$V = \pi r^2 h$$
$$\frac{V}{\pi r^2} = h \qquad \text{Divide by } \pi r^2.$$

15. **(a)** Solve $A = \frac{1}{2}h(b+B)$ for b.

$$2A = h(b+B) \qquad \text{Multiply by 2.}$$
$$\frac{2A}{h} = b + B \qquad \text{Divide by } h.$$
$$\frac{2A}{h} - B = b, \qquad \text{Subtract } B.$$
$$\text{or} \quad b = \frac{2A - hB}{h}$$

(b) Solve $A = \frac{1}{2}h(b+B)$ for B.

$$2A = h(b+B) \text{ Multiply by 2.}$$
$$\frac{2A}{h} = b + B \qquad \text{Divide by } h.$$
$$\frac{2A}{h} - b = B \qquad \text{Subtract } b.$$

Another method to solve for B is the following.

$$2A = h(b+B) \quad \text{Multiply by 2.}$$
$$2A = hb + hB \quad \text{Distributive prop.}$$
$$2A - hb = hB \qquad \text{Subtract } hb.$$
$$\frac{2A - hb}{h} = B \qquad \text{Divide by } h.$$

16. Solve $V = \frac{1}{3}\pi r^2 h$ for h.

$$V = \frac{1}{3}\pi r^2 h$$
$$\frac{3V}{\pi r^2} = h \qquad \text{Divide by } \frac{1}{3}\pi r^2.$$

17. Solve $F = \frac{9}{5}C + 32$ for C.

$$F - 32 = \frac{9}{5}C$$
$$\frac{5}{9}(F - 32) = \frac{5}{9}\left(\frac{9}{5}C\right)$$
$$\frac{5}{9}(F - 32) = C$$

18. Solve $C=\frac{5}{9}(F-32)$ for F.

$$\frac{9}{5}C=\frac{9}{5}\cdot\frac{5}{9}(F-32) \quad \text{Multiply by } \frac{9}{5}.$$
$$\frac{9}{5}C=F-32$$
$$\frac{9}{5}C+32=F \qquad \text{Add 32.}$$

19. **(a)** Solve $Ax+B=C$ for x.

$$Ax+B=C$$
$$Ax=C-B$$
$$x=\frac{C-B}{A}$$

(b) Solve $Ax+B=C$ for A.

$$Ax+B=C$$
$$Ax=C-B$$
$$A=\frac{C-B}{x}$$

20. **(a)** Solve $y=mx+b$ for x.

$$y=mx+b$$
$$y-b=mx$$
$$\frac{y-b}{m}=x$$

(b) Solve $y=mx+b$ for m.

$$y=mx+b$$
$$y-b=mx$$
$$\frac{y-b}{x}=m$$

21. Solve $A=P(1+rt)$ for t.

$$A=P(1+rt)$$
$$\frac{A}{P}=1+rt$$
$$\frac{A}{P}-1=rt$$
$$\frac{\frac{A}{P}-1}{r}=t$$

This can also be written as $t=\frac{A-P}{Pr}$.

22. Solve $M=C(1+r)$ for r.

$$M=C(1+r)$$
$$\frac{M}{C}=1+r$$
$$\frac{M}{C}-1=r$$

This can also be written as $r=\frac{M-C}{C}$.

23. $A=\frac{1}{2}bh$

$$2A=2\left(\frac{1}{2}bh\right) \quad \text{Multiply by 2.}$$
$$2A=bh$$
$$\frac{2A}{b}=\frac{bh}{b} \qquad \text{Divide by } b.$$
$$\frac{2A}{b}=h$$
$$\frac{2A}{b}=\frac{2}{1}\cdot\frac{A}{b}$$
$$=2\left(\frac{A}{b}\right) \qquad \text{This choice is A.}$$
$$=2A\left(\frac{1}{b}\right) \qquad \text{This is choice B.}$$

To get choice C, divide $A=\frac{1}{2}bh$ by $\frac{1}{2}b$.

$\frac{A}{\frac{1}{2}b}=\frac{\frac{1}{2}bh}{\frac{1}{2}b}$ gives us $h=\frac{A}{\frac{1}{2}b}$.

Choice D, $h=\frac{\frac{1}{2}A}{b}$, can be multiplied by $\frac{2}{2}$ on the right side to get $h=\frac{A}{2b}$, so it is *not* equivalent to $h=\frac{2A}{b}$. Therefore, the correct answer is D.

24. $C=\frac{5}{9}(F-32)$

$C=\frac{5}{9}F-\frac{160}{9}$ This is choice A.

$C=\frac{5}{9}\cdot\frac{F}{1}-\frac{160}{9}$

$C=\frac{5F}{9}-\frac{160}{9}$ This is choice B.

$C=\frac{5F-160}{9}$ This is choice C.

Therefore, the correct answer is D.

25. $4x+y=1$

$y=-4x+1$

26. $3x+y=9$

$y=-3x+9$

27. $x-2y=-6$

$-2y=-6-x$

$y=\frac{-6-x}{-2}$

Simplified, this is $y=\frac{1}{2}x+3$.

28. $x-5y=-20$

$-5y=-20-x$

$y=\frac{-20-x}{-5}$

Simplified, this is $y=\frac{1}{5}x+4$.

29. $4x+9y=11$

$4x+9y-4x=11-4x$ Subtract $4x$.

$9y=-4x+11$

$\frac{9y}{9}=\frac{-4x+11}{9}$ Divide by 9.

$y=\frac{-4x+11}{9}$

$y=\frac{-4x}{9}+\frac{11}{9}$ $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$

Simplified, this is $y=-\frac{4}{9}x+\frac{11}{9}$.

30. $7x+8y=11$

$7x+8y-7x=11-7x$ Subtract $7x$.

$8y=-7x+11$

$\frac{8y}{8}=\frac{-7x+11}{8}$ Divide by 8.

$y=\frac{-7x+11}{8}$

$y=\frac{-7x}{8}+\frac{11}{8}$ $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$

Simplified, this is $y=-\frac{7}{8}x+\frac{11}{8}$.

31. $-3x+2y=5$

$2y=3x+5$ Add $3x$.

$y=\frac{3x+5}{2}$ Divide by 2.

Simplified, this is $y=\frac{3}{2}x+\frac{5}{2}$.

32. $-5x+3y=12$

$3y=5x+12$ Add $5x$.

$y=\frac{5x+12}{3}$ Divide by 3.

Simplified, this is $y=\frac{5}{3}x+4$.

33. $6x-5y=7$

$-5y=-6x+7$ Subtract $6x$.

$y=\frac{-6x+7}{-5}$ Divide by -5.

Simplified, this is $y=\frac{6}{5}x-\frac{7}{5}$.

34. $8x-3y=4$

$-3y=-8x+4$ Subtract $8x$.

$y=\frac{-8x+4}{-3}$ Divide by -3.

Simplified, this is $y=\frac{8}{3}x-\frac{4}{3}$.

35. $\frac{1}{2}x-\frac{1}{3}y=1$

$3x-2y=6$ Multiply by 6.

$-2y=6-3x$

$y=\frac{6-3x}{-2}$

Simplified, this is $y=\frac{3}{2}x-3$.

36. $\frac{2}{3}x-\frac{2}{5}y=2$

$\frac{1}{3}x-\frac{1}{5}y=1$ Divide by 2.

$5x-3y=15$ Multiply by 15.

$-3y=15-5x$

$y=\frac{15-5x}{-3}$

Simplified, this is $y=\frac{5}{3}x-5.$

37. Use the formula $d=rt$ to find the time.

$d=rt$

$\frac{d}{r}=t$

$\frac{500}{159.250}=t$

$3.140=t$

Therefore, Jimmie Johnson won the 2013 Daytona 500 in approximately 3.140 hours.

38. Solve $d=rt$ for t.

$t=\frac{d}{r}$

Replace d by 415 and r by 151.774.

$t=\frac{415}{151.774}\approx 2.734$

His time was about 2.734 hours.

39. Solve $d=rt$ for r.

$r=\frac{d}{t}$

$r=\frac{520}{10}=52$ Let $d=520, t=10.$

Her rate was 52 mph.

40. Solve $d=rt$ for t.

$t=\frac{d}{r}$

$t=\frac{10,500}{500}=21$ Let $d=10,500,\ r=500.$

The travel time is 21 hr.

41. Use the formula $F=\frac{9}{5}C+32.$

$F=\frac{9}{5}(45)+32$ Let $C=45.$

$=81+32$

$=113$

The corresponding temperature is 113°F.

42. Use the formula $C=\frac{5}{9}(F-32).$

$C=\frac{5}{9}(-58-32)$ Let $F=-58.$

$=\frac{5}{9}(-90)$

$=-50$

The corresponding temperature is about -50°C.

43. Solve $P=4s$ for s.

$s=\frac{P}{4}$

To find s, substitute 920 for P.

$s=\frac{920}{4}=230$

The length of each side is 230 m.

44. Use $V=\pi r^2h.$

Replace r by $\frac{35}{2}=17.5$ and h by 588.

$V=\pi(17.5)^2(588)$

$\approx 565,722.3$

To the nearest whole number, the volume is $565,722\text{ ft}^3$.

45. Use the formula $C=2\pi r.$

(a) $480\pi=2\pi r$ Let $C=480\pi.$

$\frac{480\pi}{2\pi}=r$ Divide by $2\pi.$

The radius of the circle is 240 inches.

(b) The diameter is twice the radius.

$d=2r$

$d=2(240)$

$d=480$

The diameter of the circle is 480 inches.

46. **(a)** $d=2r=2(2.5)=5$

The diameter is 5 inches.

(b) Use the formula $C=2\pi r.$

$C=2\pi r$

$C=2\pi(2.5)$

$C=5\pi$

47. Use $V = LWH$.

Let $V = 187$, $L = 11$, and $W = 8.5$.

$187 = 11(8.5)H$

$187 = 93.5H$

$2 = H$ Divide by 93.5.

The ream is 2 inches thick.

48. Use $V = LWH$.

Let $V = 238$, $W = 8.5$, and $H = 2$.

$238 = L(8.5)(2)$

$238 = L(17)$

$14 = L$ Divide by 17.

The length of a legal sheet of paper is 14 inches.

49. The mixture is 36 oz and that part which is alcohol is 9 oz. Thus, the percent of alcohol is

$\frac{9}{36} = \frac{1}{4} = \frac{25}{100} = 25\%.$

The percent of water is $100\% - 25\% = 75\%$.

50. Let x = the amount of pure acid in the mixture. Then x can be found by multiplying the total amount of the mixture by the percent of acid given as a decimal (0.35).

$x = 40(0.35) = 14$

There are 14 L of pure acid. Since there are 40 L altogether, there are $40 - 14$, or 26 L of pure water in the mixture.

51. Find what percent \$6300 is of \$210,000.

$\frac{6300}{210{,}000} = 0.03 = 3\%$

The agent received a 3% rate of commission.

52. Solve $I = prt$ for r.

$r = \frac{I}{pt}$

$r = \frac{25.50}{3400(1)}$

$= 0.0075 = 0.75\%$

The interest rate on this deposit is 0.75%.

53. **(a)** Texas:

$$\text{Pct.} = \frac{W}{W+L} = \frac{91}{91+72} = \frac{91}{163} \approx .558$$

(b) L.A. Angels:

$$\text{Pct.} = \frac{W}{W+L} = \frac{78}{78+84} = \frac{78}{162} \approx .481$$

(c) Seattle:

$$\text{Pct.} = \frac{W}{W+L} = \frac{71}{71+91} = \frac{71}{162} \approx .438$$

(d) Houston:

$$\text{Pct.} = \frac{W}{W+L} = \frac{51}{51+111} = \frac{51}{162} \approx .315$$

54. **(a)** Pittsburgh:

$$\text{Pct.} = \frac{W}{W+L} = \frac{94}{94+68} = \frac{94}{162} \approx .580$$

(b) Cincinatti:

$$\text{Pct.} = \frac{W}{W+L} = \frac{90}{90+72} = \frac{90}{162} \approx .556$$

(c) Milwaukee:

$$\text{Pct.} = \frac{W}{W+L} = \frac{74}{74+88} = \frac{74}{162} \approx .457$$

(d) Chicago Cubs:

$$\text{Pct.} = \frac{W}{W+L} = \frac{66}{66+96} = \frac{66}{162} \approx .407$$

55. $\frac{38.4 \text{ million}}{114.2 \text{ million}} \approx 0.34$

In 2013, about 34% of the U.S. households that owned at least one TV set owned at least 4 TV sets.

56. $\frac{94.2 \text{ million}}{114.2 \text{ million}} \approx 0.82$

In 2013, about 82% of the U.S. households that owned at least one TV set had a DVD player.

57. $0.905(114.2) = 103.351$

In 2013, about 103.4 million U.S. households that owned at least one TV set received basic cable.

58. $0.476(114.2) = 54.3592$

In 2013, about 54.4 million U.S. households that owned at least one TV set received premium cable.

59. $0.30(241{,}080) = 72{,}324$

To the nearest dollar, \$72,324 will be spent to provide housing.

60. $0.18(241{,}080) = 43{,}394.4$

To the nearest dollar, \$43,394 will be spent to provide child care and education.

61. Since 6% is one third of 18% divide 43,394 by 3.

$\frac{\$43,394}{3} \approx 14,464.67$

To the nearest dollar, $14,465 will be spent on clothing.

62. $\frac{\$38,500}{\$241,080} \approx 0.1597$

So the food cost is about 16%, which agrees with the percent shown in the graph.

63. Let x = the percent increase (as a decimal).

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$
$$x = \frac{11.76 - 10.50}{10.50}$$
$$x = \frac{1.26}{10.50}$$
$$x = 0.12$$

The percent increase was 12%.

64. Let x = the percent decrease (as a decimal).

$$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$
$$x = \frac{70.00 - 56.00}{70.00}$$
$$x = \frac{14.00}{70.00}$$
$$x = 0.20$$

The percent discount was 20%.

65. Let x = the percent decrease (as a decimal).

$$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$
$$x = \frac{484,674 - 369,250}{484,674}$$
$$x = \frac{115,424}{484,674}$$
$$x \approx 0.238$$

The percent decrease was 23.8%.

66. Let x = the percent increase (as a decimal).

$$\text{percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$
$$x = \frac{143,684 - 128,358}{128,358}$$
$$x = \frac{15,326}{128,358}$$
$$x \approx 0.119$$

The percent increase was 11.9%.

67. $$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$
$$= \frac{39.00 - 19.52}{39.00}$$
$$= \frac{19.48}{39.00} \approx 0.499$$

The percent discount was 49.9%.

68. $$\text{percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$
$$= \frac{52.99 - 31.99}{52.99}$$
$$= \frac{21.00}{52.99} \approx 0.396$$

The percent discount was 39.6%.

69. **(a)** $$S = \frac{4k + 7}{k + 90}$$
$$S = \frac{4(20) + 7}{(20) + 90}$$
$$S \approx 0.79$$

A 20 kg child has a body surface area of approximately 0.79 m^2.

(b) $$S = \frac{4k + 7}{k + 90}$$
$$S = \frac{4(26) + 7}{(26) + 90}$$
$$S \approx 0.96$$

A 26 kg child has a body surface area of approximately 0.96 m^2.

70. **(a)** $$S = \frac{4k + 7}{k + 90}$$
$$S = \frac{4(13.608) + 7}{(13.608) + 90}$$
$$S \approx 0.59$$

A 30 lb child (13.608 kg) has a body surface area of approximately 0.59 m^2.

(b) $$S = \frac{4k + 7}{k + 90}$$
$$S = \frac{4(11.793) + 7}{(11.793) + 90}$$
$$S \approx 0.53$$

A 26 lb child (11.793 kg) has a body surface area of approximately 0.53 m^2.

71. **(a)** Recall from exercise 69, part a, that a 20 kg child has a body surface area of approximately 0.79 m^2.

$$C = \frac{\text{body surface area in m}^2}{1.7} \times D$$
$$C = \frac{0.79}{1.7} \times 250$$
$$C \approx 116$$

A 20 kg child requires a dose of approximately 116 mg.

(b) Recall from exercise 69, part b, that a 26 kg child has a body surface area of approximately 0.96 m^2.

$$C = \frac{\text{body surface area in m}^2}{1.7} \times D$$
$$C = \frac{0.96}{1.7} \times 250$$
$$C \approx 141$$

A 26 kg child requires a dose of approximately 141 mg.

72. **(a)** Recall from exercise 70, part a, that a 30 lb child has a body surface area of approximately 0.59 m^2.

$$C = \frac{\text{body surface area in m}^2}{1.7} \times D$$
$$C = \frac{0.59}{1.7} \times 500$$
$$C \approx 174$$

A 30 lb child requires a dose of approximately 174 mg.

(b) Recall from exercise 70, part b, that a 26 lb child has a body surface area of approximately 0.53 m^2.

$$C = \frac{\text{body surface area in m}^2}{1.7} \times D$$
$$C = \frac{0.53}{1.7} \times 500$$
$$C \approx 156$$

A 26 lb child requires a dose of approximately 156 mg.

1.3 Applications of Linear Equations

Classroom Examples, Now Try Exercises

1. **(a)**
$$\begin{array}{ccc}
\text{The sum of a number and 6} & \text{is} & 28. \\
\downarrow & \downarrow & \downarrow \\
x+6 & = & 28
\end{array}$$

An equation is $x+6=28.$

(b)
$$\begin{array}{ccccc}
\text{The product of a number and 7} & \text{is} & \text{twice the number} & \text{plus} & 12. \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
7x & = & 2x & + & 12
\end{array}$$

An equation is $7x = 2x + 12.$

(c)
$$\begin{array}{ccccc}
\text{The quotient of a number and 6,} & \text{added to} & \text{twice the number,} & \text{is} & 7. \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\dfrac{x}{6} & + & 2x & = & 7
\end{array}$$

An equation is $\frac{x}{6} + 2x = 7,$ or equivalently,

$2x + \frac{x}{6} = 7.$

N1. **(a)**
$$\begin{array}{ccc}
\text{The quotient of a number and 10} & \text{is} & \text{twice the number.} \\
\downarrow & \downarrow & \downarrow \\
\dfrac{x}{10} & = & 2x
\end{array}$$

An equation is $\frac{x}{10} = 2x.$

(b)
$$\begin{array}{ccccc}
\text{The product of a number and 5,} & \text{decreased by} & 7, & \text{is} & \text{zero.} \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
5x & - & 7 & = & 0
\end{array}$$

An equation is $5x - 7 = 0.$

2. **(a)** $5x-3(x+2)=7$ is an *equation* because it has an equals symbol.

$$\begin{aligned} 5x-3(x+2)&=7 & \\ 5x-3x-6&=7 & \text{Distributive property} \\ 2x-6&=7 & \text{Combine like terms.} \\ 2x&=13 & \text{Add 6.} \\ x&=\frac{13}{2} & \text{Divide by 2.} \end{aligned}$$

The solution set is $\left\{\frac{13}{2}\right\}$.

(b) $5x-3(x+2)$ is an *expression* because there is no equals symbol.

$$\begin{aligned} &5x-3(x+2) & \\ &=5x-3x-6 & \text{Distributive property} \\ &=2x-6 & \text{Combine like terms.} \end{aligned}$$

N2. **(a)** $3(x-5)+2x-1$ is an *expression* because there is no equals symbol.

$$\begin{aligned} &3(x-5)+2x-1 & \\ &=3x-15+2x-1 & \text{Distributive property} \\ &=5x-16 & \text{Combine like terms.} \end{aligned}$$

(b) $3(x-5)+2x=1$ is an *equation* because it has an equals symbol.

$$\begin{aligned} 3(x-5)+2x&=1 & \\ 3x-15+2x&=1 & \text{Distributive property} \\ 5x-15&=1 & \text{Combine like terms.} \\ 5x&=16 & \text{Add 15.} \\ x&=\frac{16}{5} & \text{Divide by 5.} \end{aligned}$$

The solution set is $\left\{\frac{16}{5}\right\}$.

3. *Step 2*
The length and perimeter are given in terms of the width W. The length L is 5 cm more than the width, so
$L=W+5$.
The perimeter P is 5 times the width, so
$P=5W$.
Step 3
Use the formula for perimeter of a rectangle.

$$\begin{aligned} P&=2L+2W & \\ 5W&=2(W+5)+2W & P=5W;\ L=W+5 \end{aligned}$$

Step 4
Solve the equation.

$$\begin{aligned} 5W&=2W+10+2W & \text{Distributive property} \\ 5W&=4W+10 & \text{Combine terms.} \\ W&=10 & \text{Subtract } 4W. \end{aligned}$$

Step 5
The width is 10 and the length is
$L=W+5=10+5=15$.
The rectangle is 10 cm by 15 cm.
Step 6
15 is 5 more than 10 and
$P=2(10)+2(15)=50$ is five times 10.

N3. *Step 2*
The length and perimeter are given in terms of the width W. The length L is 2 ft more than twice the width, so
$L=2W+2$.
The perimeter P is 34, so
$P=34$.
Step 3
Use the formula for perimeter of a rectangle.

$$\begin{aligned} P&=2L+2W & \\ 34&=2(2W+2)+2W & P=34;\ L=2W+2 \end{aligned}$$

Step 4
Solve the equation.

$$\begin{aligned} 34&=4W+4+2W & \text{Distributive property} \\ 34&=6W+4 & \text{Combine terms.} \\ 30&=6W & \text{Subtract 4.} \\ 5&=W & \text{Divide by 6.} \end{aligned}$$

Step 5
The width is 5 and the length is
$L=2W+2=10+2=12$.
The rectangle is 5 ft by 12 ft.
Step 6
12 is 2 more than twice 5 and
$P=2(12)+2(5)=34$.

4. *Step 2*
Let $x=$ the number of home runs for Davis.
Then $x-9=$ the number of home runs for Cabrera.
Step 3
The sum of their home runs is 97, so an equation is
$x+(x-9)=97$.

Step 4
Solve the equation.
$2x-9=97$
$2x=106$ Add 9.
$x=53$ Divide by 2.
Step 5
Davis had 53 home runs and Cabrera had $53-9=44$ home runs.
Step 6
44 is 9 less than 53, and the sum of 53 and 44 is 97.

N4. *Step 2*
Let $x=$ the number of TDs for Brees.
Then $x+16=$ the number of TDs for Manning.
Step 3
The sum of their TDs is 94, so an equation is $x+(x+16)=94$.
Step 4
Solve the equation.
$2x+16=94$
$2x=78$ Subtract 16.
$x=39$ Divide by 2.
Step 5
Brees had 39 TDs and Manning had $39+16=55$ TDs.
Step 6
55 is 16 more than 39, and the sum of 39 and 55 is 94.

5. *Step 2*
Let $x=$ the Medicare spending in 2011. Since $4.8\%=4.8(0.01)=0.048$, $0.048x=$ the additional Medicare spending in 2012.
Step 3

The number in 2011	plus	the increase	is	572.5.
↓	↓	↓	↓	↓
x	$+$	$0.048x$	$=$	572.5

Step 4
Solve the equation.
$1x+0.048x=572.5$ Identity property
$1.048x=572.5$ Combine like terms.
$x=546.3$ Divide by 1.048.
Step 5
The approximate Medicare spending in 2011 was \$546.3 billion.
Step 6
Check that the increase, $572.5-546.3=26.2$, is about 4.8% of 546.3, $4.8\%\cdot 546.3=26.2$, as required.

N5. *Step 2*
Let $x=$ the number of Introductory Statistics students in the fall of 1992. Since $700\%=700(0.01)=7$, $7x=$ the number of additional students in 2015.
Step 3

The number in 1992	plus	the increase	is	96.
↓	↓	↓	↓	↓
x	$+$	$7x$	$=$	96

Step 4
Solve the equation.
$1x+7x=96$ Identify property
$8x=96$ Combine like terms.
$x=12$ Divide by 8.
Step 5
There were 12 students in the fall of 1992.
Step 6
Check that the increase, $96-12=84$, is 700% of 12, $700\%\cdot 12=700(0.01)(12)=84$, as required.

6. Let $x=$ the amount invested at 2.5%.
Then $34,000-x=$ the amount invested at 2%.
Use $I=prt$ with $t=1$.

Principal	**Rate (as a decimal)**	**Interest**
x	0.025	$0.025x$
$34,000-x$	0.02	$0.02(34,000-x)$
34,000	← Totals →	772.50

The last column gives the equation.
$0.025x+0.02(34,000-x)=772.50$
Solve the equation.
$0.025x+680-0.02x=772.50$ Dist. prop.
$0.005x+680=772.50$ Combine terms.
$0.005x=92.5$ Subtract 680.
$x=18,500$ Divide by 0.005.
The man invested \$18,500 at 2.5% and $\$34,000-\$18,500=\$15,500$ at 2%.

N6. Let $x =$ the amount invested at 3%.
Then $20,000 - x =$ the amount invested at 2.5%.
Use $I = prt$ with $t = 1$.
Make a table to organize the information.

Principal	Rate (as a decimal)	Interest
x	0.03	$0.03x$
$20,000 - x$	0.025	$0.025(20,000 - x)$
20,000	← Totals →	575

The last column gives the equation.
$0.03x + 0.025(20,000 - x) = 575$
Solve the equation.
$0.03x + 500 - 0.025x = 575$ Dist. prop.
$0.005x + 500 = 575$ Combine terms.
$0.005x = 75$ Subtract 500.
$x = 15,000$ Divide by 0.005.
Gary invested \$15,000 at 3% and $\$20,000 - \$15,000 = \$5,000$ at 2.5%.

7. Let $x =$ the amount of \$8 per lb candy. Then $x + 100 =$ the amount of \$7 per lb candy.
Make a table to organize the information.

Number of Pounds	Price per Pound	Value
x	\$8	$8x$
100	\$4	400
$x + 100$	\$7	$7(x + 100)$

The last column gives the equation.
$8x + 400 = 7(x + 100)$ Dist. prop.
$8x + 400 = 7x + 700$
$x + 400 = 700$ Subtract 7x.
$x = 300$ Divide by 400.
300 lb of candy worth \$8 per lb should be used.

N7. Let $x =$ the number of liters of 20% acid solution needed. Make a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
5	0.30	$0.30(5) = 1.5$
x	0.20	$0.20x$
$x + 5$	0.24	$0.24(x + 5)$

Acid in 30% solution + acid in 20% solution = acid in 24% solution.
$1.5 + 0.20x = 0.24(x + 5)$

Solve the equation.
$1.5 + 0.20x = 0.24x + 1.2$
$0.3 = 0.04x$
$x = 7.5$

$7\frac{1}{2}$ liters of the 20% solution are needed.

Check
30% of 5 is 1.5 and 20% of 7.5 is 1.5;
$1.5 + 1.5 = 3.0$, which is the same as 24% of $(5 + 7.5)$.

8. Let $x =$ the number of liters of water.

Number of Liters	Percent (as a decimal)	Liters of Pure Antifreeze
x	$0\% = 0$	0
20	$50\% = 0.50$	0.50(20)
$x + 20$	$40\% = 0.40$	$0.40(x + 20)$

The last column gives the equation.
$0 + 0.50(20) = 0.40(x + 20)$
$10 = 0.4x + 8$
$2 = 0.4x$ Subtract 8.
$5 = x$ Divide by 0.4.
5 L of water are needed.

N8. Let $x =$ the number of gallons of pure antifreeze.

Number of Liters	Percent (as a decimal)	Liters of Pure Antifreeze
x	$100\% = 1$	$1x$
3	$30\% = 0.30$	0.30(3)
$x + 3$	$40\% = 0.40$	$0.40(x + 3)$

The last column gives the equation.
$1x + 0.30(3) = 0.40(x + 3)$
$1.0x + 0.9 = 0.4x + 1.2$
$0.6x + 0.9 = 1.2$ Subtract $0.4x$.
$0.6x = 0.3$ Subtract 0.9.
$x = 0.5$ Divide by 0.6.

$\frac{1}{2}$ gallon of pure antifreeze is needed.

Exercises

1. **(a)** 15 more than a number $x + 15$

(b) 15 is more than a number. $15 > x$

2. **(a)** 5 greater than a number $x + 5$

(b) 5 is greater than a number. $5 > x$

3. **(a)** 8 less than a number $x-8$

(b) 8 is less than a number. $8<x$

4. **(a)** 6 less than a number $x-6$

(b) 6 is less than a number. $6<x$

5. 40% can be written as $0.40=0.4=\frac{40}{100}=\frac{4}{10}=\frac{2}{5}$, so "40% of a number" can be written as $0.40x$, $0.4x$, or $\frac{2x}{5}$.

We see that "40% of a number" cannot be written as $40x$, choice D.

6. $13-x$ is the translation of "x less than 13." The phrase "13 less than a number" is translated $x-13$.

7. Twice a number, decreased by 13
$2x-13$

8. Triple a number, decreased by 14
$3x-14$

9. 12 increased by four times a number
$12+4x$

10. 15 more than one-half of a number
$\frac{1}{2}x+15$

11. The product of 8 and 16 less than a number
$8(x-16)$

12. The product of 8 more than a number and 5 less than the number
$(x+8)(x-5)$

13. The quotient of three times a number and 10
$\frac{3x}{10}$

14. The quotient of 9 and five times a nonzero number $\frac{9}{5x}(x\neq 0)$

15. The sentence "the sum of a number and 6 is -31" can be translated as $x+6=-31$.
Solve for x.

$$x+6=-31$$
$$x=-37 \quad \text{Subtract 6.}$$

The number is -37.

16. The sentence "the sum of a number and -4 is 18" can be translated as $x+(-4)=18$.
Solve for x.

$$x+(-4)=18$$
$$x=22 \quad \text{Add 4.}$$

The number is 22.

17. The sentence "if the product of a number and -4 is subtracted from the number, the result is 9 more than the number" can be translated as $x-(-4x)=x+9$.
Solve for x.

$$x-(-4x)=x+9$$
$$x+4x=x+9$$
$$4x=9$$
$$x=\frac{9}{4}$$

The number is $\frac{9}{4}$.

18. The sentence "if the quotient of a number and 6 is added to twice the number, the result is 8 less than the number" can be translated as $2x+\frac{x}{6}=x-8$.
Solve for x.

$$2x+\frac{x}{6}=x-8$$
$$12x+x=6x-48 \quad \text{Multiply by 6.}$$
$$13x=6x-48$$
$$7x=-48 \quad \text{Subtract } 6x.$$
$$x=-\frac{48}{7} \quad \text{Divide by 7.}$$

The number is $-\frac{48}{7}$.

19. The sentence "when $\frac{2}{3}$ of a number is subtracted from 14, the result is 10" can be translated as $14-\frac{2}{3}x=10$.

Solve for x.

$$14-\frac{2}{3}x=10$$
$$42-2x=30 \quad \text{Multiply by 3.}$$
$$-2x=-12 \quad \text{Subtract 42.}$$
$$x=6 \quad \text{Divide by} -2.$$

The number is 6.

20. The sentence "when 75% of a number is added to 6, the result is 3 more than the number" can be translated as $6+0.75x=x+3$.
Solve for x.

$$\begin{aligned}6+0.75x&=x+3\\600+75x&=100x+300 &&\text{Multiply by 100.}\\600-25x&=300 &&\text{Subtract } 100x.\\-25x&=-300 &&\text{Subtract 600.}\\x&=12 &&\text{Divide by } -25.\end{aligned}$$

The number is 12.

21. $5(x+3)-8(2x-6)$ is an *expression* because there is no equals symbol.

$$\begin{aligned}&5(x+3)-8(2x-6)\\&=5x+15-16x+48 &&\text{Distributive property}\\&=-11x+63 &&\text{Combine like terms.}\end{aligned}$$

22. $-7(x+4)+13(x-6)$ has no equals symbol, so it is an *expression.*

$$\begin{aligned}&-7(x+4)+13(x-6)\\&=-7x-28+13x-78 &&\text{Distributive prop.}\\&=6x-106 &&\text{Combine like terms.}\end{aligned}$$

23. $5(x+3)-8(2x-6)=12$ has an equals symbol, so this represents an *equation.*

$$\begin{aligned}5(x+3)-8(2x-6)&=12\\5x+15-16x+48&=12 &&\text{Distributive prop.}\\-11x+63&=12 &&\text{Combine terms.}\\-11x&=-51 &&\text{Subtract 63.}\\x&=\frac{51}{11} &&\text{Divide by } -11.\end{aligned}$$

The solution set is $\left\{\frac{51}{11}\right\}$.

24. $-7(x+4)+13(x-6)=18$ has an equals symbol, so it is an *equation.*

$$\begin{aligned}-7(x+4)+13(x-6)&=18\\-7x-28+13x-78&=18\\6x-106&=18\\6x&=124\\x&=\frac{124}{6}=\frac{62}{3}\end{aligned}$$

The solution set is $\left\{\frac{62}{3}\right\}$.

25. $\frac{1}{2}x-\frac{1}{6}x+\frac{3}{2}-8$ is an *expression* because there is no equals symbol.

$$\begin{aligned}&\frac{1}{2}x-\frac{1}{6}x+\frac{3}{2}-8\\&=\frac{3}{6}x-\frac{1}{6}x+\frac{3}{2}-\frac{16}{2} &&\text{Common denom.}\\&=\frac{2}{6}x-\frac{13}{2} &&\text{Combine like terms.}\\&=\frac{1}{3}x-\frac{13}{2} &&\text{Reduce.}\end{aligned}$$

26. $\frac{1}{3}x+\frac{1}{5}x-\frac{1}{2}+7$ is an *expression* because there is no equals symbol.

$$\begin{aligned}&\frac{1}{3}x+\frac{1}{5}x-\frac{1}{2}+7\\&=\frac{5}{15}x+\frac{3}{15}x-\frac{1}{2}+\frac{14}{2} &&\text{Common denom.}\\&=\frac{8}{15}x+\frac{13}{2} &&\text{Combine like terms.}\end{aligned}$$

27. *Step 1*
We are asked to find <u>the number of patents each corporation secured.</u>
Step 2
Let $x=$ the number of patents IBM secured.
Then $x-1414$ = the number of <u>patents that Samsung secured.</u>
Step 3
A total of 11,500 patents were secured, so
$$\underline{x}+\underline{x-1414}=11{,}500.$$
Step 4
$$\begin{aligned}2x-1414&=11{,}500\\2x&=12{,}914\\x&=\underline{6457}\end{aligned}$$
Step 5
IBM secured <u>6457</u> patents and Samsung secured $6457-1414=\underline{5043}$ patents.
Step 6
The number of Samsung patents was <u>1414</u> fewer than the number of <u>IBM patents</u>. The total number of patents was $6457+\underline{5043}=\underline{11{,}500}$.

28. *Step 1*
We are asked to find <u>the number of travelers to each country.</u>
Step 2
Let $x=$ the number of travelers to Mexico (in millions). Then $x-8.5=$ the number of <u>travelers to Canada.</u>

Step 3
A total of 31.7 million U.S. residents traveled to Mexico and Canada, so
$$\underline{x} + \underline{(x-8.5)} = 31.7.$$

Step 4
$$2x - 8.5 = 31.7$$
$$2x = 40.2$$
$$x = \underline{20.1}$$

Step 5
There were 20.1 million travelers to Mexico and $20.1 - 8.5 = \underline{11.6 \text{ million}}$ travelers to Canada.

Step 6
The number of travelers to Mexico was 8.5 million more than the number of travelers to Canada. The total number of these travelers was $20.1 + \underline{11.6} = \underline{31.7 \text{ million}}$.

29. *Step 2*
Let $W =$ the width of the base. Then $2W - 65$ is the length of the base.

Step 3
The perimeter of the base is 860 feet. Using $P = 2L + 2W$ gives us
$$2(2W - 65) + 2W = 860.$$

Step 4
$$4W - 130 + 2W = 860$$
$$6W - 130 = 860$$
$$6W = 990$$
$$W = \frac{990}{6} = 165$$

Step 5
The width of the base is 165 feet and the length of the base is $2(165) - 65 = 265$ feet.

Step 6
$2L + 2W = 2(265) + 2(165) = 530 + 330 = 860,$
which is the perimeter of the base.

30. *Step 2*
Let $L =$ the length of the top floor. Then $\frac{1}{2}L + 20$ is the width of the top floor.

Step 3
The perimeter of the top floor is 520 feet. Using $P = 2L + 2W$ gives us
$$2L + 2\left(\frac{1}{2}L + 20\right) = 520.$$

Step 4
$$2L + L + 40 = 520$$
$$3L + 40 = 520$$
$$3L = 480$$
$$L = 160$$

Step 5
The length of the top floor is 160 feet and the width of the top floor is
$\frac{1}{2}(160) + 20 = 100$ feet.

Step 6
$2L + 2W = 2(160) + 2(100) = 320 + 200 = 520,$
which is the perimeter of the top floor.

31. *Step 2*
Let x = the width of the painting. Then $x + 5.54$ = the height of the painting.

Step 3
The perimeter of the painting is 108.44 inches. Using $P = 2L + 2W$ gives us
$$2(x + 5.54) + 2x = 108.44.$$

Step 4
$$2x + 11.08 + 2x = 108.44$$
$$4x + 11.08 = 108.44$$
$$4x = 97.36$$
$$x = 24.34$$

Step 5
The width of the painting is 24.34 inches and the height is $24.34 + 5.54 = 29.88$ inches.

Step 6
29.88 is 5.54 more than 24.34 and
$2(29.88) + 2(24.34) = 108.44,$ as required.

32. *Step 2*
Let W = the width of the rectangle. Then $W + 12$ = the length of the rectangle.

Step 3
The perimeter, $2L + 2W$, is 16 times the width, so
$$2(W + 12) + 2W = 16(W).$$

Step 4
$$2W + 24 + 2W = 16W$$
$$4W + 24 = 16W$$
$$24 = 12W$$
$$2 = W$$

Step 5
The width of the rectangle is 2 centimeters. The length is $2 + 12 = 14$ centimeters.

Step 6
The length is 12 more than the width. The perimeter is $2(14)+2(2)=32$, which is 16 times the width, as required.

33. *Step 2*
Let x = the length of the middle side. Then the shortest side is $x-75$ and the longest side is $x+375$.
Step 3
The perimeter of the Bermuda Triangle is 3075 miles. Using $P=a+b+c$ gives us $x+(x-75)+(x+375)=3075$.
Step 4
$3x+300=3075$
$3x=2775$ Subtract 300.
$x=925$ Divide by 3.
Step 5
The length of the middle side is 925 miles. The length of the shortest side is $x-75=925-75=850$ miles. The length of the longest side is $x+375=925+375=1300$ miles.
Step 6
The answer checks since $925+850+1300=3075$ miles, which is the correct perimeter.

34. *Step 2*
Let x = the length of one of the sides of equal length.
Step 3
The perimeter of the triangle is 931.5 feet. Using $P=a+b+c$ gives us $x+x+438=931.5$.
Step 4
$2x+438=931.5$
$2x=493.5$ Subtract 438.
$x=246.75$ Divide by 2.
Step 5
The two walls are each 246.75 feet long.
Step 6
The answer checks since $246.75+246.75+438=931.5$, which is the correct perimeter.

35. *Step 2*
Let x = the amount of revenue for Wal-Mart. Then $x-19.3$ is the amount of revenue for Exxon Mobil (in billions).
Step 3
The total revenue was \$919.1 billion, so $x+(x-19.3)=919.1$.
Step 4
$2x-19.3=919.1$
$2x=938.4$ Add 19.3.
$x=469.2$ Divide by 2.
Step 5
The amount of revenue for Wal-Mart was \$469.2 billion. The amount of revenue for Exxon Mobil was $x-19.3=469.2-19.3=\$449.9$ billion.
Step 6
The answer checks since $469.2+449.9=\$919.1$ billion, which is the correct total revenue.

36. *Step 2*
Let x = the number of performances of *Cats*. Then $x-805=$ the number of performances of *Les Misérables.*
Step 3
There were 14,165 total performances, so $x+(x-805)=14{,}165$.
Step 4
$2x-805=14{,}165$
$2x=14{,}970$
$x=7485$
Step 5
There were 7485 performances of *Cats* and $7485-805=6680$ performances of *Les Misérables.*
Step 6
The total number of performances is 14,165, as required.

37. *Step 2*
Let $x=$ the height of the Eiffel Tower. Then $x-804=$ the height of the Leaning Tower of Pisa.
Step 3
Together these heights are 1164 ft, so $x+(x-804)=1164$.
Step 4 $2x-804=1164$
$2x=1968$
$x=984$
Step 5
The height of the Eiffel Tower is 984 feet and the height of the Leaning Tower of Pisa is $984-804=180$ feet.
Step 6
180 feet is 804 feet shorter than 984 feet and the sum of 180 feet and 984 feet is 1164 feet.

38. *Step 2*
Let $x =$ the Yankees' payroll (in millions). Then $x-0.1=$ the Dodgers' payroll (in millions).
Step 3
The two payrolls totaled \$473.9 million, so $x+(x-0.1)=473.9$

Step 4
$$\begin{aligned} 2x-0.1&=473.9\\ 2x&=474\\ x&=237 \end{aligned}$$
Step 5
In 2013, the Yankees' payroll was \$237.0 million and the Dodgers' payroll was $237.0-0.1=\$236.9$ million.
Step 6
\$236.9 million is \$0.1 million less than \$237.0 million and the sum of \$236.9 million and \$237.0 million is \$473.9 million.

39. *Step 2*
Let $x =$ number of electoral votes for Romney. Then $x+126=$ number of electoral votes for Obama.
Step 3
There were 538 total electoral votes, so $x+(x+126)=538$.
Step 4
$$\begin{aligned} 2x+126&=538\\ 2x&=412\\ x&=206 \end{aligned}$$
Step 5
Romney received 206 electoral votes, so Obama received $206+126=332$ electoral votes.
Step 6
332 is 126 more than 206 and the total is $206+332=538$.

40. *Step 2*
Let $x =$ the number of hits Williams got. Then $x+276=$ the number of hits Hornsby got.
Step 3
Their base hits totaled 5584, so $x+(x+276)=5584$.

Step 4
$$\begin{aligned} 2x+276&=5584\\ 2x&=5308\\ x&=2654 \end{aligned}$$
Step 5
Williams got 2654 base hits, and Hornsby got $2654+276=2930$ base hits.
Step 6
2930 is 276 more than 2654 and the total is $2654+2930=5584$.

41. Let $x =$ the percent increase.
$$\begin{aligned} x&=\frac{\text{amount of increase}}{\text{original amount}}\\ &=\frac{1,666,017-1,116,082}{1,116,082}=\frac{549,935}{1,116,082}\\ &\approx 0.493=49.3\% \end{aligned}$$
The percent increase was approximately 49.3%.

42. Let $x =$ the percent increase.
$$\begin{aligned} x&=\frac{\text{amount of increase}}{\text{original amount}}\\ &=\frac{21.1-20.8}{20.8}=\frac{0.3}{20.8}\\ &\approx 0.014=1.4\% \end{aligned}$$
The percent increase was approximately 1.4%.

43. Let $x =$ the approximate cost in 2013–2014. Since x is 240% more than the 1993–1994 cost, solve the equation below to find the cost in 2013–2014.
$$\begin{aligned} x&=2431+2.4(2431)\\ &=2431+5834.4\\ &=8265.4 \end{aligned}$$
To the nearest dollar, the cost was \$8265.

44. Let $x =$ the approximate cost in 2013–2014. Since x is 173.3% more than the 1993–1994 cost solve the equation below to find the cost in 2013–2014.
$$\begin{aligned} x&=9399+1.733(9399)\\ &=9399+16,288.467\\ &\approx 25,687.47 \end{aligned}$$
To the nearest dollar, the cost was \$25,687.

45. Let $x =$ the 2012 cost. Then solve the equation below to find the cost in 2012.
$$\begin{aligned} x-0.89\%(x)&=49.04\\ x-0.89(0.01)(x)&=49.04\\ 1x-0.0089x&=49.04\\ 0.9911x&=49.04\\ x&=\frac{49.04}{0.9911}\approx 49.48 \end{aligned}$$
The 2012 cost was \$49.48.

46. Let $x =$ the 1987 cost. Then solve the equation below to find the cost in 1987.

$$\begin{aligned} x + 83.3\%(x) &= 49.04 \\ x + 83.3(0.01)(x) &= 49.04 \\ 1x + 0.833x &= 49.04 \\ 1.833x &= 49.04 \\ x &= \frac{49.04}{1.833} \approx 26.75 \end{aligned}$$

The 1987 cost was \$26.75.

47. Let $x =$ the amount of the receipts excluding tax. Since the sales tax is 9% of x, solve the equation below to find the amount of the tax.

$$\begin{aligned} x + 0.09x &= 2725 \\ 1x + 0.09x &= 2725 \\ 1.09x &= 2725 \\ x + \frac{2725}{1.09} &= 2500 \end{aligned}$$

Thus, the tax was $0.09(2500) = \$225$.

48. Let $x =$ the amount of commission. Since x is 6% of the selling price,
$x = 0.06(159,000) = 9540.$
So after the agent was paid, he had
$159,000 - 9540 = \$149,460.$

49. Let $x =$ the amount invested at 3%. Then $12,000 - x =$ the amount invested at 4%. Complete the table. Use $I = prt$ with $t = 1$.

Principal	**Rate (as a decimal)**	**Interest**
x	0.03	$0.03x$
$12,000 - x$	0.04	$0.04(12,000 - x)$
12,000	← Totals →	440

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest} & & \text{interest} & & \text{total} \\ \text{at } 3\% & + & \text{at } 4\% & = & \text{interest.} \\ 0.03x & + & 0.04(12,000 - x) & = & 440 \end{array}$$

Multiply by 100.

$$\begin{aligned} 3x + 4(12,000 - x) &= 44,000 \\ 3x + 48,000 - 4x &= 44,000 \\ -x &= -4000 \\ x &= 4000 \end{aligned}$$

He should invest \$4000 at 3% and
$12,000 - 4000 = \$8000$ at 4%.
Check
\$4000 at 3% = \$120 and
\$8000 at 4% = \$320; \$120 + \$320 = \$440.

50. Let $x =$ the amount invested at 2%. Then $60,000 - x =$ the amount invested at 3%. Complete the table. Use $I = prt$ with $t = 1$.

Principal	**Rate (as a decimal)**	**Interest**
x	0.02	$0.02x$
$60,000 - x$	0.03	$0.03(60,000 - x)$
60,000	← Totals →	1600

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest} & & \text{interest} & & \text{total} \\ \text{at } 2\% & + & \text{at } 3\% & = & \text{interest.} \\ 0.02x & + & 0.03(60,000 - x) & = & 1600 \end{array}$$

Multiply by 100.

$$\begin{aligned} 2x + 3(60,000 - x) &= 160,000 \\ 2x + 180,000 - 3x &= 160,000 \\ -x &= -20,000 \\ x &= 20,000 \end{aligned}$$

She invested \$20,000 at 2% and
$60,000 - x = 60,000 - 20,000 = \$40,000$ at 3%.
Check
\$20,000 at 2% = \$400 and
\$40,000 at 3% = \$1200; \$400 + \$1200 = \$1600.

51. Let $x =$ the amount invested at 4.5%. Then $2x - 1000 =$ the amount invested at 3%. Use $I = prt$ with $t = 1$. Make a table.

Principal	**Rate (as a decimal)**	**Interest**
x	0.045	$0.045x$
$2x - 1000$	0.03	$0.03(2x - 1000)$
	Total →	1020

The last column gives the equation.

$$\begin{array}{ccccc} \text{Interest} & & \text{interest} & & \text{total} \\ \text{at } 4.5\% & + & \text{at } 3\% & = & \text{interest.} \\ 0.045x & + & 0.03(2x - 1000) & = & 1020 \end{array}$$

Multiply by 1000.

$$\begin{aligned} 45x + 30(2x - 1000) &= 1,020,000 \\ 45x + 60x - 30,000 &= 1,020,000 \\ 105x &= 1,050,000 \\ x &= \frac{1,050,000}{105} = 10,000 \end{aligned}$$

She invested \$10,000 at 4.5% and
$2x - 1000 = 2(10,000) - 1000 = \$19,000$ at 3%.

Check
\$19,000 is \$1000 less than two times \$10,000.
$\$10,000$ at $4.5\% = \$450$
$\$19,000$ at $3\% = \$570$; $\$450 + \$570 = \$1020$.

52. Let $x =$ the amount invested at 3.5%. Then $3x + 5000 =$ the amount invested at 4%. Use $I = prt$ with $t = 1$. Make a table.

Principal	Rate (as a decimal)	Interest
x	0.035	$0.035x$
$3x+5000$	0.04	$0.04(3x+5000)$
	Total →	1440

The last column gives the equation.

$$\underset{\text{at 3.5\%}}{\text{Interest}} + \underset{\text{at 4\%}}{\text{interest}} = 1440.$$

$$0.035x + 0.04(3x+5000) = 1440$$

Multiply by 1000.

$$35x + 40(3x+5000) = 1,440,000$$
$$35x + 120x + 200,000 = 1,440,000$$
$$155x = 1,240,000$$
$$x = \frac{1,240,000}{155} = 8000$$

He invested \$8000 at 3.5% and
$3x + 5000 = 3(8000) + 5000 = \$29,000$ at 4%.

Check
\$29,000 is \$5000 more than three times \$8000.
$\$8000$ at $3.5\% = \$280$ and
$\$29,000$ at $4\% = \$1160$; $\$280 + \$1160 = \$1440$.

53. Let $x =$ the amount of additional money to be invested at 3%.
Use $I = prt$ with $t = 1$. Make a table.
Use the fact that the total return on the two investments is 4%.

Principal	Rate (as a decimal)	Interest
12,000	0.06	0.06(12,000)
x	0.03	$0.03x$
$12,000 + x$	0.04	$0.04(12,000 + x)$

The last column gives the equation.

$$\underset{\text{at 6\%}}{\text{Interest}} + \underset{\text{at 3\%}}{\text{interest}} = \underset{\text{at 4\%.}}{\text{interest}}$$

$$0.06(12,000) + 0.03x = 0.04(12,000 + x)$$

Multiply by 100.

$$6(12,000) + 3x = 4(12,000 + x)$$
$$72,000 + 3x = 48,000 + 4x$$
$$24,000 = x$$

He should invest \$24,000 at 3%.
Check
$\$12,000$ at $6\% = \$720$ and
$\$24,000$ at $3\% = \$720$; $\$720 + \$720 = \$1440$,
which is the same as
$(\$12,000 + \$24,000)$ at 4%.

54. Let $x =$ the amount of additional money to be invested at 5%.
Use $I = prt$ with $t = 1$. Make a table. Use the fact that the total return on the two investments is 6%.

Principal	Rate (as a decimal)	Interest
17,000	0.065	0.065(17,000)
x	0.05	$0.05x$
$17,000 + x$	0.06	$0.06(17,000 + x)$

The last column gives the equation.

$$\underset{\text{at 6.5\%}}{\text{Interest}} + \underset{\text{at 5\%}}{\text{interest}} = \underset{\text{at 6\%}}{\text{interest}}$$

$$0.065(17,000) + 0.05x = 0.06(17,000 + x)$$

Multiply by 1000.

$$65(17,000) + 50x = 60(17,000 + x)$$
$$1,105,000 + 50x = 1,020,000 + 60x$$
$$85,000 = 10x$$
$$8500 = x$$

She should invest \$8500 at 6%.
Check
$\$17,000$ at $6.5\% = \$1105$ and
$\$8500$ at $5\% = \$425$;
$\$1105 + \$425 = \$1530$, which is the same as
$(\$17,000 + \$8500)$ at 6%.

55. Let $x =$ the number of liters of 10% acid solution needed. Make a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
10	0.04	$0.04(10) = 0.4$
x	0.10	$0.10x$
$x + 10$	0.06	$0.06(x+10)$

$$\underset{\text{solution}}{\text{Acid in 4\%}} + \underset{\text{solution}}{\text{acid in 10\%}} = \underset{\text{solution.}}{\text{acid in 6\%}}$$

$$0.4 + 0.10x = 0.06(x+10)$$

Solve the equation.

$0.4+0.10x=0.06x+0.6$ Distributive prop.
$0.4+0.04x=0.6$ Subtract $0.06x$.
$0.04x=0.2$ Subtract 0.4.
$x=5$ Divide by 0.04.

Five liters of the 10% solution are needed.
Check
4% of 10 is 0.4 and 10% of 5 is 0.5;
$0.4+0.5=0.9$, which is the same as 6% of $(10+5)$.

56. Let $x=$ the number of liters of 14% alcohol solution needed. Make a chart.

Liters of Solution	Percent (as a decimal)	Liters of Pure Alcohol
x	0.14	$0.14x$
20	0.50	$0.50(20)=10$
$x+20$	0.30	$0.30(x+20)$

Alcohol in 14% solution + alcohol in 50% solution = alcohol in 30% solution.
$0.14x + 10 = 0.30(x+20)$

Solve the equation.

$14x+1000=30(x+20)$ Multiply by 100.
$14x+1000=30x+600$ Distributive property
$-16x=-400$ Subtract 30x and 1000.
$x=25$ Divide by -16.

25 L of 14% solution must be added.
Check
14% of 25 is 3.5 and 50% of 20 is 10;
$3.5+10=13.5$, which is the same as 30% of $(25+20)$.

57. Let $x=$ the number of liters of the 20% alcohol solution. Make a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Alcohol
12	0.12	$0.12(12)=1.44$
x	0.20	$0.20x$
$x+12$	0.14	$0.14(x+12)$

Alcohol in 12% solution + alcohol in 20% solution = alcohol in 14% solution.
$1.44 + 0.20x = 0.14(x+12)$

Solve the equation.

$144+20x=14(x+12)$ Multiply by 100.
$144+20x=14x+168$ Distributive property
$6x=24$ Subtract $14x$ and 144.
$x=4$ Divide by 6.

4L of 20% alcohol solution are needed.
Check
12% of 12 is 1.44 and 20% of 4 is 0.8;
$1.44+0.8=2.24$, which is the same as 14% of $(12+4)$.

58. Let $x=$ the number of liters of 10% alcohol solution. Make a chart.

Liters of Solution	Percent (as a decimal)	Liters of Pure Alcohol
x	0.10	$0.10x$
40	0.50	$0.50(40)=20$
$x+40$	0.40	$0.40(x+40)$

Alcohol in 10% solution + alcohol in 50% solution = alcohol in 40% solution.
$0.10x + 20 = 0.40(x+40)$

Solve the equation.

$1x+200=4(x+40)$ Multiply by 10.
$x+200=4x+160$ Distributive property
$-3x=-40$ Subtract $4x$ and 200.
$x=\frac{40}{3}$ or $13\frac{1}{3}$ Divide by -3.

$13\frac{1}{3}$ L of 10% solution should be added.
Check
50% of 40 is 20 and 10% of $\frac{40}{3}$ is $\frac{4}{3}$;
$20+\frac{4}{3}=21\frac{1}{3}$, which is the same as 40% of $\left(\frac{40}{3}+40\right)$.

59. Let $x=$ the amount of pure dye used (pure dye is 100% dye). Make a table.

Gallons of Solution	Percent (as a decimal)	Gallons of Pure Dye
x	1	$1x=x$
4	0.25	$0.25(4)=1$
$x+4$	0.40	$0.40(x+4)$

Write the equation from the last column in the table.
$x+1=0.4(x+4)$

Solve the equation.

$x+1=0.4(x+4)$

$x+1=0.4x+1.6$ Distributive property

$0.6x=0.6$ Subtract 0.4x and 1.

$x=1$ Divide by 0.6.

One gallon of pure (100%) dye is needed.

Check

100% of 1 is 1 and 25% of 4 is 1; $1+1=2$, which is the same as 40% of $(1+4)$.

60. Let $x =$ the number of gallons of water. Make a chart.

Gallons of Solution	Percent (as a decimal)	Gallons of Pure Insecticide
x	0	$0(x)=0$
6	0.04	$0.04(6)=0.24$
$x+6$	0.03	$0.03(x+6)$

Insecticide in water + insecticide in 4% solution = insecticide in 3% solution.

$0 + 0.24 = 0.03(x+6)$

Solve the equation.

$0+24=3(x+6)$ Multiply by 100.

$24=3x+18$ Distributive property

$6=3x$ Subtract 18.

$2=x$ Divide by 3.

2 gallons of water should be added.

Check

4% of 6 is 0.24, which is the same as 3% of $(2+6)$.

61. Let $x =$ the amount of \$6 per lb nuts. Make a table.

Pounds of Nuts	Cost per lb	Total Cost
50	\$2	$2(50)=100$
x	\$6	$6x$
$x+50$	\$5	$5(x+50)$

The total value of the \$2 per lb nuts and the \$6 per lb nuts must equal the value of the \$5 per lb nuts.

$100+6x=5(x+50)$

Solve the equation.

$100+6x=5x+250$

$x=150$

He should use 150 lb of \$6 nuts.

Check

50 pounds of the \$2 per lb nuts are worth \$100 and 150 pounds of the \$6 per lb nuts are worth \$900; $\$100+\$900=\$1000$, which is the same as $(50+150)$ pounds worth \$5 per lb.

62. Let $x =$ the number of ounces of 2¢ per oz tea. Make a table.

Ounces of Tea	Cost per oz	Total Cost
x	2¢ or 0.02	$0.02x$
100	5¢ or 0.05	$0.05(100)=5$
$x+100$	3¢ or 0.03	$0.03(x+100)$

Cost of 2¢ tea + cost of 5¢ tea = cost of 3¢ tea.

$0.02x + 5 = 0.03(x+100)$

Solve the equation.

$2x+500=3(x+100)$ Multiply by 100.

$2x+500=3x+300$ Distributive property

$200=x$ Subtract $2x$ and 300.

200 oz of 2¢ per oz tea should be used.

Check

200 oz of 2¢ per oz tea is worth \$4 and 100 oz of 50 per oz tea is worth \$5; $\$4+\$5=\$9$, which is the same value as $(200+100)$ oz of 3¢ per oz tea.

63. We cannot expect the final mixture to be worth more than the more expensive of the two ingredients. Answers will vary.

64. Let $x =$ the number of liters of 30% acid solution. Make a chart.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
x	0.30	$0.30x$
15	0.50	$0.50(15)=7.5$
$x+15$	0.60	$0.60(x+15)$

Acid in 30% solution + acid in 50% solution = acid in 60% solution.

$0.30x + 7.5 = 0.60(x+15)$

Solve the equation.

$3x+75=6x+90$ Multiply by 10.

$-3x=15$ Subtract $6x$ and 75.

$x=-5$ Divide by -3.

The solution, -5, is impossible since the number of liters of 30% acid solution cannot be negative. Therefore, this problem has no solution.

65. **(a)** Let $x =$ the amount invested at 5%.
$800 - x =$ the amount invested at 10%.

(b) Let $y =$ the amount of 5% acid used.
$800 - y =$ the amount of 10% acid used.

66. **(a)** Organize the information in a table.

Principal	Percent (as a decimal)	Interest
x	0.05	$0.05x$
$800 - x$	0.10	$0.10(800 - x)$
800	0.0875	0.0875(800)

The amount of interest earned at 5% and 10% is found in the last column of the table, $0.05x$ and $0.10(800 - x)$.

(b) Organize the information in a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
y	0.05	$0.05y$
$800 - y$	0.10	$0.10(800 - y)$
800	0.0875	0.0875(800)

The amount of pure acid in the 5% and 10% mixtures is found in the last column of the table, $0.05y$ and $0.10(800 - y)$.

67. Refer to the tables for Exercise 66. In each case, the last column gives the equation.

(a) $0.05x + 0.10(800 - x) = 0.0875(800)$

(b) $0.05y + 0.10(800 - y) = 0.0875(800)$

68. In both cases, multiply by 10,000 to clear the decimals.

(a)
$$\begin{aligned} 0.05x + 0.10(800 - x) &= 0.0875(800) \\ 500x + 1000(800 - x) &= 875(800) \\ 500x + 800{,}000 - 1000x &= 700{,}000 \\ -500x &= -100{,}000 \\ x &= 200 \end{aligned}$$

Jack invested \$200 at 5% and $800 - x = 800 - 200 = \$600$ at 10%.

(b)
$$\begin{aligned} 0.05y + 0.10(800 - y) &= 0.0875(800) \\ 500y + 1000(800 - y) &= 875(800) \\ 500y + 800{,}000 - 1000y &= 700{,}000 \\ -500y &= -100{,}000 \\ y &= 200 \end{aligned}$$

Jill used 200 L of 5% acid solution and $800 - y = 800 - 200 = 600$ L of 10% acid solution.

(c) The processes used to solve Problems A and B were virtually the same. Aside from the variables chosen, the problem information was organized in similar tables and the equations solved were the same.

1.4 Further Applications of Linear Equations

Classroom Examples, Now Try Exercises

1. Let $x =$ the number of dimes.
Then $26 - x =$ the number of half-dollars.

Number of Coins	Denomination	Value
x	0.10	$0.10x$
$26 - x$	0.50	$0.50(26 - x)$
	Total →	8.60

Multiply the number of coins by the denominations, and add the results to get 8.60.

$$\begin{aligned} 0.10x + 0.50(26 - x) &= 8.60 \\ 1x + 5(26 - x) &= 86 \quad \text{Multiply by 10.} \\ 1x + 130 - 5x &= 86 \\ -4x &= -44 \\ x &= 11 \end{aligned}$$

He has 11 dimes and $26 - 11 = 15$ half-dollars.
Check
The number of coins is $11 + 15 = 26$ and the value of the coins is
$\$0.10(11) + \$0.50(15) = \$8.60$, as required.

N1. Let $x =$ the number of dimes.
Then $52 - x =$ the number of nickels.

Number of Coins	Denomination	Value
x	0.10	$0.10x$
$52 - x$	0.05	$0.05(52 - x)$
	Total →	3.70

Multiply the number of coins by the denominations, and add the results to get 3.70.

$$\begin{aligned} 0.10x + 0.05(52 - x) &= 3.70 \\ 10x + 5(52 - x) &= 370 \quad \text{Multiply by 100.} \\ 10x + 260 - 5x &= 370 \\ 5x &= 110 \\ x &= 22 \end{aligned}$$

He has 22 dimes and $52 - 22 = 30$ nickels.

Check
The number of coins is $22 + 30 = 52$ and the value of the coins is
$\$0.10(22) + \$0.05(30) = \$3.70$, as required.

2. Let $x =$ the amount of time needed for the cars to be 420 mi apart.
Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
Northbound Car	60	x	$60x$
Southbound Car	45	x	$45x$
Total			420

The total distance traveled is the sum of the distances traveled by each car, since they are traveling in opposite directions. This total is 420 mi.

$$60x + 45x = 420$$
$$105x = 420$$
$$x = \frac{420}{105} = 4$$

The cars will be 420 mi apart in 4 hr.
Check
The northbound car traveled $60(4) = 240$ miles. The southbound car traveled $45(4) = 180$ miles, for a total of $240 + 180 = 420$, as required.

N2. Let $x =$ the amount of time needed for the trains to be 387.5 km apart.
Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
First Train	80	x	$80x$
Second Train	75	x	$75x$
Total			387.5

The total distance traveled is the sum of the distances traveled by each train, since they are traveling in opposite directions. This total is 387.5 km.

$$80x + 75x = 387.5$$
$$155x = 387.5$$
$$x = \frac{387.5}{155} = 2.5$$

The trains will be 387.5 km apart in 2.5, or $2\frac{1}{2}$ hr.

Check
The first train traveled $80(2.5) = 200$ km. The second train traveled $75(2.5) = 187.5$ km, for a total of $200 + 187.5 = 387.5$, as required.

3. Let $x =$ the driving rate.
Then $x - 12 =$ the bus rate.
Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
Car	x	$\frac{1}{2}$	$\frac{1}{2}x$
Bus	$x - 12$	$\frac{3}{4}$	$\frac{3}{4}(x-12)$

The distances are equal.

$$\frac{1}{2}x = \frac{3}{4}(x-12)$$
$$2x = 3(x-12) \quad \text{Multiply by 4.}$$
$$2x = 3x - 36$$
$$36 = x$$

The distance he travels to work is
$\frac{1}{2}x = \frac{1}{2}(36) = 18$ miles.
Check
The distance he travels to work by bus is
$\frac{3}{4}(x-12) = \frac{3}{4}(36-12) = \frac{3}{4}(24) = 18$ miles,
which is the same as we found above (by car).

N3. Let $x =$ the driving rate.
Then $x - 30 =$ the bicycling rate.
Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
Car	x	$\frac{1}{2}$	$\frac{1}{2}x$
Bike	$x - 30$	$1\frac{1}{2} = \frac{3}{2}$	$\frac{3}{2}(x-30)$

The distances are equal.

$$\frac{1}{2}x = \frac{3}{2}(x-30)$$
$$1x = 3(x-30) \quad \text{Multiply by 2.}$$
$$x = 3x - 90$$
$$90 = 2x$$
$$45 = x$$

The distance he travels to work is
$\frac{1}{2}x = \frac{1}{2}(45) = 22.5$ miles.

Check
The distance he travels to work by bike is
$\frac{3}{2}(x-30)=\frac{3}{2}(45-30)=\frac{3}{2}(15)=22.5$ miles,
which is the same as we found above (by car).

4. The sum of the three measures must equal 180°.

$$\begin{aligned} x+(x+61)+(2x+7)&=180\\ 4x+68&=180\\ 4x&=112\\ x&=28\end{aligned}$$

The angles measure $28°$, $28+61=89°$, and $2(28)+7=63°$.
Check
Since $28°+89°+63°=180°$, the answers are correct.

N4. The sum of the three measures must equal 180°.

$$\begin{aligned} x+(x+11)+(3x-36)&=180\\ 5x-25&=180\\ 5x&=205\\ x&=41\end{aligned}$$

The angles measure $41°$, $41+11=52°$, and $3(41)-36=87°$.
Check
Since $41°+52°+87°=180°$, the answers are correct.

5. $x=$ the first of the three consecutive integers
$x+1=$ the second integer
$x+2=$ the third integer
Write the sum of the first and third numbers, $x+(x+2)$, and set it equal to 49 less than 3 times the second.

$$\begin{aligned} x+(x+2)&=3(x+1)-49\\ 2x+2&=3x+3-49\\ 2x+2&=3x-46\\ 2&=x-46\\ 48&=x\end{aligned}$$

The three consecutive numbers are 48, 49, and 50.
Check
The sum of the first and third consecutive integers is $48+50=98$.
49 less than 3 times the second, yields $3(49)-49=98$, as desired.

N5. $x=$ the first of the three consecutive integers
$x+1=$ the second integer
$x+2=$ the third integer
Write the sum of the first and second numbers, $x+(x+1)$, and set it equal to 43 less than 3 times the third.

$$\begin{aligned} x+(x+1)&=3(x+2)-43\\ 2x+1&=3x+6-43\\ 2x+1&=3x-37\\ 1&=x-37\\ 38&=x\end{aligned}$$

The three consecutive numbers are 38, 39, and 40.
Check
The sum of the first and second consecutive integers is $38+39=77$.
43 less than 3 times the third, yields $3(40)-43=77$, as desired.

Exercises

1. The total amount is

$$\begin{aligned} 12(0.10)+18(0.25)&=1.20+4.50\\ &=\$5.70.\end{aligned}$$

2. Use $d=rt$, or $t=\frac{d}{r}$.
Substitute 7700 for d and 550 for r.

$$t=\frac{7700}{550}=14$$

Its travel time is 14 hours.

3. Use $d=rt$, or $r=\frac{d}{t}$. Substitute 300 for d and 10 for t.

$$r=\frac{300}{10}=30$$

His rate was 30 mph.

4. Use $P=4s$ or $s=\frac{P}{4}$.
Substitute 160 for P.

$$s=\frac{160}{4}=40$$

The length of each side of the square is 40 in. This is also the length of each side of the equilateral triangle. To find the perimeter of the equilateral triangle, use $P=3s$.
Substitute 40 for s.

$$P=3(40)=120$$

The perimeter would be 120 inches.

5. The problem asks for the distance Geoff travels to the workplace, so we must multiply the rate, 10 mph, by the time, $\frac{3}{4}$ hr, to get the distance, 7.5 mi.

6. Begin by subtracting 36° from 180°. Then find half of this difference to get the measure of each of the equal angles.

7. No, the answers must be whole numbers because they represent the number of coins.

8. Since the distance d is in miles and the rate r is x miles per hour, we must have the time t in hours. $10 \text{ minutes} = \frac{10}{60} = \frac{1}{6} \text{ hour}$, so $d = rt = x\left(\frac{1}{6}\right) \text{ or } \frac{1}{6}x.$

9. Let $x =$ the number of pennies. Then x is also the number of dimes, and $44 - 2x$ is the number of quarters.

Number of Coins	Denomination	Value
x	0.01	$0.01x$
x	0.10	$0.10x$
$44-2x$	0.25	$0.25(44-2x)$
44	← Totals →	4.37

The sum of the values must equal the total value.

$$0.01x + 0.10x + 0.25(44-2x) = 4.37$$
$$x + 10x + 25(44-2x) = 437$$

Multiply both sides by 100.

$$x + 10x + 1100 - 50x = 437$$
$$-39x + 1100 = 437$$
$$-39x = -663$$
$$x = 17$$

There are 17 pennies, 17 dimes, and $44 - 2(17) = 10$ quarters.

Check

The number of coins is $17 + 17 + 10 = 44$ and the value of the coins is $\$0.01(17) + \$0.10(17) + \$0.25(10) = \$4.37,$ as required.

10. Let $x =$ the number of nickels and the number of quarters. Then $2x$ is the number of half-dollars.

Number of Coins	Denomination	Value
x	0.05	$0.05x$
x	0.25	$0.25x$
$2x$	0.50	$0.50(2x)$
	Total →	2.60

The sum of the values must equal the total value.

$$0.05x + 0.25x + 0.50(2x) = 2.60$$
$$5x + 25x + 50(2x) = 260$$

Multiply both sides by 100.

$$5x + 25x + 100x = 260$$
$$130x = 260$$
$$x = 2$$

She found 2 nickels, 2 quarters, and $2(2) = 4$ half-dollars.

Check

The number of coins is $2 + 2 + 4 = 8$ and the value of the coins is $\$0.05(2) + \$0.25(2) + \$0.50(4) = \$2.60,$ as required.

11. Let $x =$ the number of loonies. Then $37 - x$ is the number of toonies.

Number of Coins	Denomination	Value
x	1	$1x$
$37-x$	2	$2(37-x)$
37	← Totals →	51

The sum of the values must equal the total value.

$$1x + 2(37-x) = 51$$
$$x + 74 - 2x = 51$$
$$-x + 74 = 51$$
$$23 = x$$

She has 23 loonies and $37 - 23 = 14$ toonies.

Check

The number of coins is $23 + 14 = 37$ and the value of the coins is $\$1(23) + \$2(14) = \$51,$ as required.

12. Let $x =$ the number of \$1 bills. Then $119 - x$ is the number of \$5 bills.

Number of Bills	Denomination	Value
x	1	$1x$
$119 - x$	5	$5(119 - x)$
119	← Totals →	347

The sum of the values must equal the total value.

$$\begin{aligned} 1x + 5(119 - x) &= 347 \\ x + 595 - 5x &= 347 \\ -4x &= -248 \\ x &= 62 \end{aligned}$$

He has 62 \$1 bills and $119 - 62 = 57$ \$5 bills.
Check
The number of bills is $62 + 57 = 119$ and the value of the bills is
$\$1(62) + \$5(57) = \$62 + \$285 = \$347,$ as required.

13. Let $x =$ the number of \$10 coins. Then $41 - x$ is the number of \$20 coins.

Number of Coins	Denomination	Value
x	10	$10x$
$41 - x$	20	$20(41 - x)$
41	← Totals →	540

The sum of the values must equal the total value.

$$\begin{aligned} 10x + 20(41 - x) &= 540 \\ 10x + 820 - 20x &= 540 \\ -10x &= -280 \\ x &= 28 \end{aligned}$$

He has 28 \$10 coins and $41 - 28 = 13$ \$20 coins.
Check
The number of coins is $28 + 13 = 41$ and the value of the coins is $\$10(28) + \$20(13) = \$540,$ as required.

14. Let $x =$ the number of two-cent pieces. Then $3x$ is the number of three-cent pieces.

Number of Coins	Denomination	Value
x	0.02	$0.02x$
$3x$	0.03	$0.03(3x)$
	Total →	2.42

The sum of the values must equal the total value.

$$\begin{aligned} 0.02x + 0.03(3x) &= 2.42 \\ 2x + 3(3x) &= 242 \end{aligned}$$

Multiply both sides by 100.

$$\begin{aligned} 2x + 9x &= 242 \\ 11x &= 242 \\ x &= 22 \end{aligned}$$

She has 22 two-cent pieces and 3(22) = 66 three-cent pieces.
Check
66 is three times 22 and the value of the coins is $\$0.02(22) + \$0.03(66) = \$2.42,$ as required.

15. Let $x =$ the number of adult tickets sold. Then $1460 - x =$ the number of children and senior tickets sold.

Cost of Ticket	Number Sold	Amount Collected
\$23	x	$23x$
\$17	$1460 - x$	$17(1460 - x)$
Totals	1460	\$30,052

Write the equation from the last column of the table.

$$\begin{aligned} 23x + 17(1460 - x) &= 30{,}052 \\ 23x + 24{,}820 - 17x &= 30{,}052 \\ 6x &= 5232 \\ x &= 872 \end{aligned}$$

872 adult tickets were sold; $1460 - 872 = 588$ children and senior tickets were sold.
Check
The number of tickets sold was
$872 + 588 = 1460$
and the amount collected was

$$\begin{aligned} \$23(872) + \$17(588) &= \$20{,}056 + \$9996 \\ &= \$30{,}052, \end{aligned}$$

as required.

16. Let $x =$ the number of student tickets sold. Then $480 - x =$ the number of nonstudent tickets sold.

Cost of Ticket	Number Sold	Amount Collected
\$5	x	$5x$
\$8	$480 - x$	$8(480 - x)$
Totals	480	\$2895

Write the equation from the last column of the table.

$$\begin{aligned} 5x + 8(480 - x) &= 2895 \\ 5x + 3840 - 8x &= 2895 \\ -3x &= -945 \\ x &= 315 \end{aligned}$$

315 student tickets were sold; $480 - 315 = 165$ nonstudent tickets were sold.

Check
The number of tickets sold was
$315 + 165 = 480$ and the amount collected was
$\$5(315) + \$8(165) = \$1575 + \$1320 = \$2895,$ as required.

17. $d = rt,$ so

$$r = \frac{d}{t} = \frac{100}{12.35} \approx 8.10$$

Her rate was about 8.10 m/sec.

18. $d = rt,$ so

$$r = \frac{d}{t} = \frac{400}{52.70} \approx 7.59$$

Her rate was about 7.59 m/sec.

19. $d = rt,$ so

$$r = \frac{d}{t} = \frac{200}{19.32} \approx 10.35$$

His rate was about 10.35 m/sec.

20. $d = rt,$ so

$$r = \frac{d}{t} = \frac{400}{43.94} \approx 9.10$$

His rate was about 9.10 m/sec.

21. Let $t =$ the time until they are 110 mi apart. Use the formula $d = rt.$ Complete the table.

	Rate	Time	Distance
First Steamer	22	t	$22t$
Second Steamer	22	t	$22t$
			110

The total distance traveled is the sum of the distances traveled by each steamer, since they are traveling in opposite directions. This total is 110 mi.

$$22t + 22t = 110$$
$$44t = 110$$
$$t = \frac{110}{44} = \frac{5}{2} \quad \text{or} \quad 2\frac{1}{2}$$

It will take them $2\frac{1}{2}$ hr.

Check
Each steamer traveled $22(2.5) = 55$ miles for a total of
$2(55) = 110$ miles, as required.

22. Let $t =$ the time it takes for the trains to be 315 km apart. Use the formula $d = rt.$ Complete the table.

	Rate	Time	Distance
First Train	85	t	$85t$
Second Train	95	t	$95t$
			315

The total distance traveled is the sum of the distances traveled by each train, since they are traveling in opposite directions. This total is 315 km.

$$85t + 95t = 315$$
$$180t = 315$$
$$t = \frac{315}{180} = \frac{7}{4} \quad \text{or} \quad 1\frac{3}{4}$$

It will take the trains $1\frac{3}{4}$ hr before they are 315 km apart.

Check
The first train traveled $85(1.75) = 148.75$ km and the second train traveled $95(1.75) = 166.25.$ The sum is
315 km, as required.

23. Let $t =$ Mulder's time. Then $t - \frac{1}{2} =$ Scully's time.

	Rate	Time	Distance
Mulder	65	t	$65t$
Scully	68	$t - \frac{1}{2}$	$68\left(t - \frac{1}{2}\right)$

The distances are equal.

$$65t = 68\left(t - \frac{1}{2}\right)$$
$$65t = 68t - 34$$
$$-3t = -34$$
$$t = \frac{34}{3} \quad \text{or} \quad 11\frac{1}{3}$$

Mulder's time will be $11\frac{1}{3}$ hr. Since he left at 8:30 A.M., $11\frac{1}{3}$ hr or 11 hr 20 min later is 7:50 P.M.

Check
Mulder's distance was
$65\left(\frac{34}{3}\right)=736\frac{2}{3}$ miles. Scully's distance was
$68\left(\frac{34}{3}-\frac{1}{2}\right)=68\left(\frac{65}{6}\right)=736\frac{2}{3},$ as required.

24. Let $x=$ Lois' travel time.
Since Clark leaves 15 minutes after Lois, and $\frac{15}{60}=\frac{1}{4}$ hr, $x-\frac{1}{4}=$ time for Clark.
Complete the table using the formula $rt=d$.

	Rate	Time	Distance
Lois	35	x	$35x$
Clark	40	$x-\frac{1}{4}$	$40\left(x-\frac{1}{4}\right)$

Since Lois and Clark are going in opposite directions, we add their distances to get 140 mi.

$$35x+40\left(x-\frac{1}{4}\right)=140$$
$$35x+40x-10=140$$
$$75x=150$$
$$x=2$$

They will be 140 mi apart at 8 A.M. $+\ 2$ hr $=10$ A.M.
Check
Lois' distance was $35(2)=70.$ Clark's distance was $40\left(2-\frac{1}{4}\right)=40\left(\frac{7}{4}\right)=70.$ The sum is 140 miles, as required.

25. Let $x=$ her average rate on Sunday. Then $x+5=$ her average rate on Saturday.

	Rate	Time	Distance
Saturday	$x+5$	3.6	$3.6(x+5)$
Sunday	x	4	$4x$

The distances are equal.

$$3.6(x+5)=4x$$
$$3.6x+18=4x$$
$$18=0.4x \qquad \text{Subtract } 3.6x.$$
$$x=\frac{18}{0.4}=45$$

Her average rate on Sunday was 45 mph.
Check
On Sunday, 4 hours at 45 mph $=180$ miles. On Saturday, 3.6 hours at 50 mph $=180$ miles. The distances are equal.

26. Let $x=$ her biking rate.
Then $x-7=$ her walking rate.

	Rate	Time	Distance
Walking	$x-7$	$\frac{40}{60}=\frac{2}{3}$ hr	$\frac{2}{3}(x-7)$
Biking	x	$\frac{12}{60}=\frac{1}{5}$ hr	$\frac{1}{5}x$

The distances are equal.

$$\frac{2}{3}(x-7)=\frac{1}{5}x$$
$$10(x-7)=3x \qquad \text{Multiply by 15.}$$
$$10x-70=3x$$
$$7x=70$$
$$x=10$$

The distance from her house to the train station is $\frac{1}{5}x=\frac{1}{5}(10)=2$ miles.
Check
The distance walking is $(3\text{ mph})\left(\frac{2}{3}\text{hr}\right)=2$ mi.
The distance biking is $(10\text{ mph})\left(\frac{1}{5}\text{hr}\right)=2$ mi.
The distances are equal.

27. Let $x=$ Anne's time.
Then $x+\frac{1}{2}=$ Johnny's time.

	Rate	Time	Distance
Anne	60	x	$60x$
Johnny	50	$x+\frac{1}{2}$	$50\left(x+\frac{1}{2}\right)$

The total distance is 80.

$$60x+50\left(x+\frac{1}{2}\right)=80$$
$$60x+50x+25=80$$
$$110x=55$$
$$x=\frac{55}{110}=\frac{1}{2}$$

They will meet $\frac{1}{2}$ hr after Anne leaves.

Check
Anne travels $60\left(\frac{1}{2}\right) = 30$ miles. Johnny travels $50\left(\frac{1}{2}+\frac{1}{2}\right) = 50$ miles. The sum of the distances is 80 miles, as required.

28. Let $x =$ her rate (speed) during the first part of the trip. Then $x-25 =$ her rate during rush-hour traffic. Make a table using the formula $rt = d$.

	Rate	**Time**	**Distance**
First Part	x	2	$2x$
Rush-Hour	$x-25$	$\frac{1}{2}$	$\frac{1}{2}(x-25)$

The total distance was 125 miles.

$$2x+\frac{1}{2}(x-25) = 125$$
$$4x + x - 25 = 250 \quad \text{Multiply by 2.}$$
$$5x = 275$$
$$x = 55$$

The rate during the first part of the trip was 55 mph.
Check
The distance traveled during the first part of the trip was $55(2) = 110$ miles. The distance traveled during the second part of the trip was $(55-25)(0.5) = 15$ miles. The sum of the distances is 125 miles, as required.

29. The sum of the measures of the three angles of a triangle is 180°.

$$(x-30)+(2x-120)+\left(\frac{1}{2}x+15\right) = 180$$
$$\frac{7}{2}x - 135 = 180$$
$$7x - 270 = 360$$
$$7x = 630$$
$$x = 90$$

With $x = 90,$ the three angle measures become
$(90-30)° = 60°,$
$[2(90)-120]° = 60°,$
and $\left[\frac{1}{2}(90)+15\right]^{\circ} = 60°.$

30. The sum of the measures of the three angles of a triangle is 180°.

$$(x+15)+(10x-20)+(x+5) = 180$$
$$12x = 180$$
$$x = 15$$

With $x = 15,$ the three angle measures become
$(15+15)° = 30°,$
$(10\cdot 15-20)° = 130°,$
and $(15+5)° = 20°.$

31. The sum of the measures of the three angles of a triangle is 180°.

$$(3x+7)+(9x-4)+(4x+1) = 180$$
$$16x+4 = 180$$
$$16x = 176$$
$$x = 11$$

With $x = 11,$ the three angle measures become
$(3\cdot 11+7)° = 40°,$
$(9\cdot 11-4)° = 95°,$
and $(4\cdot 11+1)° = 45°.$

32. The sum of the measures of the three angles of a triangle is 180°.

$$(2x+7)+(x+61)+x = 180$$
$$4x+68 = 180$$
$$4x = 112$$
$$x = 28$$

With $x = 28,$ the three angle measures become
$(2\cdot 28+7)° = 63°,$
$(28+61)° = 89°,$ and 28°.

33. Vertical angles have equal measure.

$$8x+2 = 7x+17$$
$$x = 15$$

$8\cdot 15+2 = 122$ and $7\cdot 15+17 = 122.$
The angles are both 122°.

34. Vertical angles have equal measure.

$$9-5x = 25-3x$$
$$9 = 25+2x$$
$$-16 = 2x$$
$$-8 = x$$

$9-5(-8) = 49$ and $25-3(-8) = 49.$
The angles are both 49°.

35. The sum of the two angles is 90°.

$$
\begin{aligned}
(5x-1)+2x &= 90 \\
7x-1 &= 90 \\
7x &= 91 \\
x &= 13
\end{aligned}
$$

The measures of the two angles are $[5(13)-1]° = 64°$ and $[2(13)]° = 26°$.

36. The sum of the two angles is 90°.

$$
\begin{aligned}
(3x-9)+6x &= 90 \\
9x-9 &= 90 \\
9x &= 99 \\
x &= 11
\end{aligned}
$$

The measures of the two angles are $[3(11)-9]° = 24°$ and $[6(11)]° = 66°$.

37. Supplementary angles have an angle measure sum of 180°.

$$
\begin{aligned}
(3x+5)+(5x+15) &= 180 \\
8x+20 &= 180 \\
8x &= 160 \\
x &= 20
\end{aligned}
$$

With $x = 20$, the two angle measures become $(3\cdot 20+5)° = 65°$ and $(5\cdot 20+15)° = 115°$.

38. Supplementary angles have an angle measure sum of 180°.

$$
\begin{aligned}
(3x+1)+(7x+49) &= 180 \\
10x+50 &= 180 \\
10x &= 130 \\
x &= 13
\end{aligned}
$$

With $x = 13$, the two angle measures become $(3\cdot 13+1)° = 40°$ and $(7\cdot 13+49)° = 140°$.

39. Let $x =$ the first consecutive integer. Then $x+1$ will be the second consecutive integer, and $x+2$ will be the third consecutive integer. The sum of the first and twice the second is 17 more than twice the third.

$$
\begin{aligned}
x+2(x+1) &= 2(x+2)+17 \\
x+2x+2 &= 2x+4+17 \\
3x+2 &= 2x+21 \\
x &= 19
\end{aligned}
$$

Since $x = 19$, $x+1 = 20$, and $x+2 = 21$. The three consecutive integers are 19, 20, and 21.

40. Let $x =$ the first consecutive integer. Then $x+1$ will be the second consecutive integer, and $x+2$ will be the third consecutive integer. The sum of the first and twice the third is 39 more than twice the second.

$$
\begin{aligned}
x+2(x+2) &= 2(x+1)+39 \\
x+2x+4 &= 2x+2+39 \\
3x+4 &= 2x+41 \\
x &= 37
\end{aligned}
$$

Since $x = 37$, $x+1 = 38$, and $x+2 = 39$. The three consecutive integers are 37, 38, and 39.

41. Let $x =$ the first integer. Then $x+1$, $x+2$, and $x+3$ are the next three consecutive integers. The sum of the first three integers is 54 more than the fourth.

$$
\begin{aligned}
x+(x+1)+(x+2) &= (x+3)+54 \\
3x+3 &= x+57 \\
2x &= 54 \\
x &= 27
\end{aligned}
$$

The four consecutive integers are 27, 28, 29, and 30.

42. Let $x =$ the first integer. Then $x+1$, $x+2$, and $x+3$ are the next three consecutive integers. The sum of the last three integers is 86 more than the first.

$$
\begin{aligned}
(x+1)+(x+2)+(x+3) &= x+86 \\
3x+6 &= x+86 \\
2x &= 80 \\
x &= 40
\end{aligned}
$$

The four consecutive integers are 40, 41, 42, and 43.

43. Let $x =$ my current age. Then $x+1$ will be my age next year. The sum of these ages will be 103 years.

$$
\begin{aligned}
x+(x+1) &= 103 \\
2x+1 &= 103 \\
2x &= 102 \\
x &= 51
\end{aligned}
$$

If my current age is 51, in 10 years I will be $51+10 = 61$ years old.

44. Let $x =$ my current age. Then $x+1$ will be my age next year. The sum of these ages will be 129 years.

$$
\begin{aligned}
x+(x+1) &= 129 \\
2x+1 &= 129 \\
2x &= 128 \\
x &= 64
\end{aligned}
$$

If my current age is 64, in 5 years I will be $64+5=69$ years old.

45. Let $x=$ the first even integer. Then $x+2$ and $x+4$ are the next two consecutive even integers.
The sum of the least integer and the middle integer is 26 more than the greatest integer.

$$\begin{aligned} x+(x+2) &= (x+4)+26 \\ 2x+2 &= x+30 \\ x+2 &= 30 \\ x &= 28 \end{aligned}$$

The three even integers are 28, 30 and 32.

46. Let $x=$ the first even integer. Then $x+2$ and $x+4$ are the next two consecutive even integers.
The sum of the least integer and the greatest integer is 12 more than the middle integer.

$$\begin{aligned} x+(x+4) &= (x+2)+12 \\ 2x+4 &= x+14 \\ x+4 &= 14 \\ x &= 10 \end{aligned}$$

The three even integers are 10, 12, and 14.

47. Let $x=$ the first odd integer. Then $x+2$ and $x+4$ are the next two consecutive odd integers.
The sum of the least integer and the middle integer is 19 more than the greatest integer.

$$\begin{aligned} x+(x+2) &= (x+4)+19 \\ 2x+2 &= x+23 \\ x+2 &= 23 \\ x &= 21 \end{aligned}$$

The three even integers are 21, 23, and 25.

48. Let $x=$ the first odd integer. Then $x+2$ and $x+4$ are the next two consecutive odd integers.
The sum of the least integer and the greatest integer is 13 more than the middle integer.

$$\begin{aligned} x+(x+4) &= (x+2)+13 \\ 2x+4 &= x+15 \\ x+4 &= 15 \\ x &= 11 \end{aligned}$$

The three even integers are 11, 13, and 15.

49. The sum of the measures of the angles of a triangle is 180°.

$$\begin{aligned} x+2x+60 &= 180 \\ 3x+60 &= 180 \\ 3x &= 120 \\ x &= 40 \end{aligned}$$

The measures of the unknown angles are 40° and $2x=80°$.

50. Two angles which form a straight line add to 180°, so $180°-60°=120°$. The measure of the unknown angle is 120°.

51. The sum of the measures of the unknown angles in Exercise 49 is $40°+80°=120°$. This is equal to the measure of the angle in Exercise 50.

52. The sum of the measures of angles ① and ② is equal to the measure of angle ③.

Summary Exercises Applying Problem-Solving Techniques

1. Let $x=$ the width of the rectangle. Then $x+3$ is the length of the rectangle.
If the length were decreased by 2 inches and the width were increased by 1 inch, the perimeter would be 24 inches. Use the formula $P=2L+2W$, and substitute 24 for P, $(x+3)-2$ or $x+1$ for L, and $x+1$ for W.

$$\begin{aligned} P &= 2L+2W \\ 24 &= 2(x+1)+2(x+1) \\ 24 &= 2x+2+2x+2 \\ 24 &= 4x+4 \\ 20 &= 4x \\ 5 &= x \end{aligned}$$

The width of the rectangle is 5 inches, and the length is $5+3=8$ inches.

2. Let $x=$ the width. Then $2x$ is the length (twice the width).
Use $P=2L+2W$ and add one more width to cut the area into two parts. Thus, an equation is shown below. Solve the equation.

$$\begin{aligned} 2L+2W+W &= 210 \\ 2(2x)+2x+x &= 210 \\ 7x &= 210 \\ x &= 30 \end{aligned}$$

The width is 30 meters, and the length is $2(30)=60$ meters.

3. Let $x =$ the regular price of the item.
The sale price after a 20.5% (or 0.205) discount was \$317.99, so an equation is
$x - 0.205x = 317.99.$
Solve the equation

$$\begin{aligned} x - 0.205x &= 317.99 \\ 0.795x &= 317.99 \\ x &= \frac{317.99}{0.795} \approx 399.99 \end{aligned}$$

To the nearest cent, the regular price was \$399.99.

4. Let $x =$ the regular price of the HD television.
The sale price after a discount of 40% (or 0.40) was \$255, so an equation is $x - 0.40x = 255$.
Solve the equation.

$$\begin{aligned} x - 0.40x &= 255 \\ 0.60x &= 255 \\ x &= 425 \end{aligned}$$

The regular price of the HD television was \$425.

5. Let $x =$ the amount invested at 4%.
Then $2x$ is the amount invested at 5%.
Use $I = prt$ with $t = 1$ yr. Make a table.

Principal	**Rate (as a decimal)**	**Interest**
x	0.04	$0.04x$
$2x$	0.05	$0.05(2x) = 0.10x$
	Total $\rightarrow$	112

The last column gives the equation.
Interest at 4% + interest at 5% = total interest.

$$0.04x \quad + \quad 0.10x \quad = \quad 112$$

Multiply by 100.

$$\begin{aligned} 4x + 10x &= 11,200 \\ 14x &= 11,200 \\ x &= 800 \end{aligned}$$

\$800 is invested at 4% and $2(\$800) = \$1600$ at 5%.
Check
$\$800$ at $4\% = \$32$ and
$\$1600$ at $5\% = \$80; \$32 + \$80 = \$112$

6. Let $x =$ the amount invested at 3%.
Then $x + 2000$ is the amount invested at 4%.
Use $I = prt$ with $t = 1$ yr. Make a table.

Principal	**Rate (as a decimal)**	**Interest**
x	0.03	$0.03x$
$x + 2000$	0.04	$0.04(x + 2000)$
	Total $\rightarrow$	920

The last column gives the equation.
Interest at 3% + interest at 4% = total interest.

$$0.03x \quad + 0.04(x + 2000) = \quad 920$$

Multiply by 100.

$$\begin{aligned} 3x + 4(x + 2000) &= 92,000 \\ 3x + 4x + 8000 &= 92,000 \\ 7x &= 84,000 \\ x &= 12,000 \end{aligned}$$

\$12,000 is invested at 3% and \$14,000 is invested at 4%.
Check
$\$12,000$ at $3\% = \$360$ and
$\$14,000$ at $4\% = \$560; \$360 + \$560 = \$920$

7. Let $x =$ the number of points scored by Durant in the 2012–2013 season. Then $x - 147 =$ the number of points scored by Bryant in the 2012–2013 season. The total number of points scored by both was 4413.

$$\begin{aligned} x + (x - 147) &= 4413 \\ 2x - 147 &= 4413 \\ 2x &= 4560 \\ x &= 2280 \end{aligned}$$

Durant scored 2280 points and Bryant scored $2280 - 147 = 2133$ points.
Check
2133 is 147 fewer points than 2280 points, and $2280 + 2133 = 4413$.

8. Let $x =$ the amount grossed by *The Dark Knight*. Then $x + 67.5 =$ the amount grossed by *Titanic* (in millions). Together they grossed \$1134.1 million.

$$\begin{aligned} x + (x + 67.5) &= 1134.1 \\ 2x + 67.5 &= 1134.1 \\ 2x &= 1066.6 \\ x &= 533.3 \end{aligned}$$

The Dark Knight grossed \$533.3 million and *Titanic* grossed $533.3 + 67.5 = \$600.8$ million.

Check
\$600.8 million is \$67.5 million greater than \$533.3 million, and $600.8+533.3=1134.1$.

9. Let $t =$ the time it will take until John and Pat meet. Use $d = rt$ and make a table.

	Rate	Time	Distance
John	60	t	$60t$
Pat	28	t	$28t$

The total distance is 440 miles.

$$60t+28t=440$$
$$88t=440$$
$$t=5$$

It will take 5 hours for John and Pat to meet.
Check
John traveled $60(5)=300$ miles and Pat traveled $28(5)=140$ miles; $300+140=440$, as required.

10. Let $x =$ the length of the side of the square that Joshua is cutting out of his 12 by 16 sheet of tin. The width of the tin decreases by x at the left and right ends of the tin because small squares have been cut out at each corner. Thus, $W=12-2x$ is the width of the box made by cutting out the small squares. Similarly, $L=16-2x$ is the length of the box.
The length L of the box is twice its width W minus 5.

$$L=2W-5$$
$$16-2x=2(12-2x)-5$$

Solve for x.

$$16-2x=2(12-2x)-5$$
$$16-2x=24-4x-5$$
$$16-2x=19-4x$$
$$2x=3$$
$$x=1\frac{1}{2}$$

The length of a side of the square is $x=1\frac{1}{2}$ cm.

Check
The length of the box will be

$$16-2x=16-2\left(1\frac{1}{2}\right)=16-3=13.$$

Its width will be

$$12-2x=12-2\left(1\frac{1}{2}\right)=12-3=9.$$

Twice the width minus 5, $2W-5=2\cdot 9-5=13$, is equal to the length of the box as desired.

11. Let $x =$ the number of liters of the 5% drug solution.

Liters of Solution	Percent (as a decimal)	Liters of Pure Drug
20	0.10	$20(0.10)=2$
x	0.05	$0.05x$
$20+x$	0.08	$0.08(20+x)$

Drug in 10% + drug in 5% = drug in 8%.

$$2 + 0.05x = 0.08(20+x)$$

Multiply by 100.

$$200+5x=8(20+x)$$
$$200+5x=160+8x$$
$$40=3x$$
$$x=\frac{40}{3} \quad \text{or} \quad 13\frac{1}{3}$$

The pharmacist should add $13\frac{1}{3}$ L.

Check
10% of 20 is 2 and 5% of $\frac{40}{3}$ is $\frac{2}{3}$; $2+\frac{2}{3}=\frac{8}{3}$, which is the same as 8% of $\left(20+\frac{40}{3}\right)$.

12. Let $x =$ the number of kilograms of the metal that is 20% tin.

Kilograms of Metal	Percent Tin (as a decimal)	Kilograms of Pure Tin
80	0.70	$80(0.70)=56$
x	0.20	$0.20x$
$80+x$	0.50	$0.50(80+x)$

Tin in 70% + tin in 20% = tin in 50%.

$$56 + 0.20x = 0.50(80+x)$$

Multiply by 100.

$$560+2x=5(80+x)$$
$$560+2x=400+5x$$
$$160=3x$$
$$x=\frac{160}{3} \quad \text{or} \quad 53\frac{1}{3}$$

$53\frac{1}{3}$ kilograms should be added.

Check

70% of 80 is 56 and 20% of $\frac{160}{3}$ is $\frac{32}{3}$;

$56+\frac{32}{3}=66\frac{2}{3}$, which is the same as 50% of $\left(80+\frac{160}{3}\right)$.

13. Let $x=$ the number of \$5 bills. Then $126-x$ is the number of \$10 bills.

Number of Bills	Denomination	Value
x	5	$5x$
$126-x$	10	$10(126-x)$
126	← Totals →	840

The sum of the values must equal the total value.

$$\begin{aligned}5x+10(126-x)&=840\\5x+1260-10x&=840\\-5x&=-420\\x&=84\end{aligned}$$

There are 84 \$5 bills and $126-84=42$ \$10 bills.

Check

The number of bills is $84+42=126$ and the value of the bills is $\$5(84)+\$10(42)=\$840,$ as required.

14. Let $x=$ the number of \$7 tickets sold. Then $2460-x=$ the number of \$9 tickets sold.

Number Sold	Cost of Ticket	Amount Collected
x	\$7	$7x$
$2460-x$	\$9	$9(2460-x)$
2460	← Totals →	\$20,520

Write the equation from the last column of the table.

$$\begin{aligned}7x+9(2460-x)&=20,520\\7x+22,140-9x&=20,520\\-2x&=-1620\\x&=810\end{aligned}$$

810 \$7 tickets were sold and $2460-810=1650$ \$9 tickets were sold.

Check

The number of tickets sold was $810+1650=2460.$

The amount collected was

$$\begin{aligned}\$7(810)+\$9(1650)&=\$5670+\$14,850\\&=\$20,520\end{aligned}$$

as required.

15. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned}x+(6x-50)+(x-10)&=180\\8x-60&=180\\8x&=240\\x&=30\end{aligned}$$

With $x=30,$ the three angle measures become $(6\cdot30-50)°=130°,$ $(30-10)°=20°,$ and 30°.

16. In the figure, the two angles are supplementary, so their sum is 180°.

$$\begin{aligned}(10x+7)+(7x+3)&=180\\17x+10&=180\\17x&=170\\x&=10\end{aligned}$$

The two angle measures are $10(10)+7=107°$ and $7(10)+3=73°.$

17. Let $x=$ the least integer. Then $x+1$ is the middle integer and $x+2$ is the greatest integer.

"The sum of the least and greatest of three consecutive integers is 32 more than the middle integer" translates to

$$\begin{aligned}x+(x+2)&=32+(x+1).\\2x+2&=x+33\\x&=31\end{aligned}$$

The three consecutive integers are 31, 32, and 33.

Check

The sum of the least and greatest integers is $31+33=64,$ which is the same as 32 more than the middle integer.

18. Let $x=$ the first odd integer. Then $x+2$ is the next odd integer.

"If the lesser of two consecutive odd integers is doubled, the result is 7 more than the greater of the two integers" translates to

$$\begin{aligned}2(x)&=7+(x+2).\\2x&=x+9\\x&=9\end{aligned}$$

The two consecutive odd integers are 9 and 11.

Check

Doubling the lesser gives us $2(9)=18,$ which is equal to 7 more than 11.

19. Let $x =$ the length of the shortest side. Then $2x$ is the length of the middle side and $3x-2$ is the length of the longest side.
The perimeter is 34 inches. Using $P=a+b+c$ gives us

$$x+2x+(3x-2)=34.$$
$$6x-2=34$$
$$6x=36$$
$$x=6$$

The lengths of the three sides are 6 inches, $2(6)=12$ inches, and $3(6)-2=16$ inches.
Check
The sum of the lengths of the three sides is $6+12+16=34$ inches, as required.

20. Let $x =$ the length of the rectangle. Then the perimeter is $43+x.$

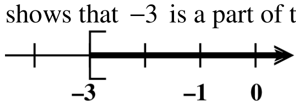
$$P=2L+2W$$
$$43+x=2x+2(10) \quad \textit{Let } W = 10.$$
$$43+x=2x+20$$
$$23=x$$

The length of the rectangle is 23 inches.
Check
$P=2L+2W=2(23)+2(10)=66$ inches, which is 43 inches more than the length.

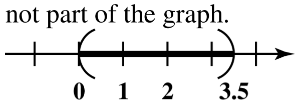
1.5 Linear Inequalities in One Variable

Classroom Examples, Now Try Exercises

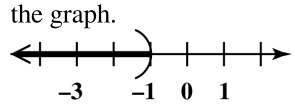
1. **(a)** The statement $x \ge -3$ says that x can represent any number greater than or equal to -3. This interval is written with a square bracket at -3 as $[-3, \infty)$.
Graph it by placing a bracket at -3 and drawing an arrow to the right. The bracket shows that -3 is a part of the graph.



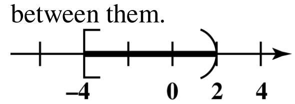
(b) The statement $0 < x < 3.5$ says that x can represent any number between 0 and 3.5 with both 0 and 3.5 excluded. This interval is written with parentheses as $(0, 3.5)$.
Graph it by placing parentheses at both 0 and 3.5. Fill in the points in between them. The parentheses show that 0 and -3 are not part of the graph.



N1. **(a)** The statement $x < -1$ says that x can represent any number less than -1, but x cannot equal -1. This interval is written with a parenthesis at -1 as $(-\infty, -1)$.
Graph it by placing a right parenthesis at -1 and drawing an arrow to the left. The parenthesis shows that -1 is not a part of the graph.



(b) The statement $-4 \le x < 2$ says that x can represent any number between -4 and 2 with -4 included and 2 excluded. This interval is written as $[-4, 2)$.
Graph it by placing a left bracket at -4 and a right parenthesis at 2. Fill in the points in between them.



2. $x-5>1$
$x>6$ Add 5.
Check
Substitute 6 for x in $x-5=1$.

$$6-5 \stackrel{?}{=} 1$$
$$1=1 \quad \text{True}$$

This shows that 6 is the boundary point. Choose 0 and 7 as test points.
$x-5>1$

Let $x=0$.	Let $x=7$.
$0-5 \stackrel{?}{>} 1$	$7-5 \stackrel{?}{>} 1$
$-5>1$ False	$2>1$ True
0 is not in the solution set.	7 is in the solution set.

The check confirms that $(6, \infty)$ is the solution set.

N2. $x-10>-7$
$x>3$ Add 10.
Check
Substitute 3 for x in $x-10=-7$.

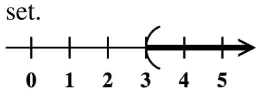
$$3-10 \stackrel{?}{=} -7$$
$$-7=-7 \quad \text{True}$$

This shows that 3 is the boundary point. Choose 0 and 7 as test points.

$$x-10>-7$$

Let $x=0$.	Let $x=7$.
$0-10\overset{?}{>}-7$	$7-10\overset{?}{>}-7$
$-10>-7$ False	$-3>-7$ True
0 is not in the solution set.	7 is in the solution set.

The check confirms that $(3, \infty)$ is the solution set.



3. $5x+3\geq 4x-1$

$x+3\geq -1$ Subtract $4x$.

$x\geq -4$ Subtract -3.

Check

Substitute -4 for x in $5x+3=4x-1$.

$5(-4)+3\overset{?}{>}4(-4)-1$

$-17=-17$ True

So -4 satisfies the equality part of $\geq$. Choose -5 and 0 as test points.

$5x+3\geq 4x-1$

$5(-5)+3\overset{?}{>}4(-5)-1$ Let $x=-5$.

$-22\geq -21$ False

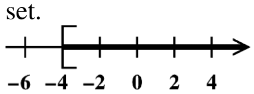
-5 is not in the solution set.

$5(0)+3\overset{?}{\geq}4(0)-1$ Let $x=0$.

$3\geq -1$ True

0 is in the solution set.

The check confirms that $[-4, \infty)$ is the solution set.



N3. $4x+1\geq 5x$

$1\geq x$ Subtract $4x$.

$x\leq 1$ Equivalent inequality.

Check

Substitute 1 for x in $4x+1=5x$.

$4(1)+1\overset{?}{=}5(1)$

$5=5$ True

So 1 satisfies the equality part of $\geq$. Choose 0 and 2 as test points.

$4x+1\geq 5x$

$4(0)+1\overset{?}{>}5(0)$ Let $x=0$.

$1\geq 0$ True

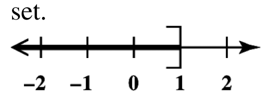
0 is in the solution set.

$4(2)+1\overset{?}{>}5(2)$ Let $x=2$.

$9\geq 10$ False

2 is not in the solution set.

The check confirms that $(-\infty, 1]$ is the solution set.

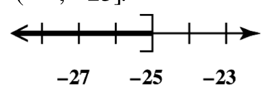


4. (a) $4x\leq -100$

$\frac{4x}{4}\leq\frac{-100}{4}$ Divide by $4>0$; do not reverse the symbol.

$x\leq -25$

Check that the solution set is the interval $(-\infty, -25]$.

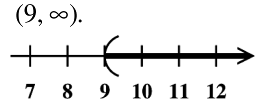


(b) $-9x<-81$

$\frac{-81}{-9}>\frac{-81}{-9}$ Divide by $-9<0$; reverse the symbol.

$x>9$

Check that the solution set is the interval $(9, \infty)$.

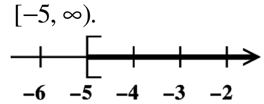


N4. (a) $8x\geq -40$

$\frac{8x}{8}\geq\frac{-40}{8}$ Divide by $8>0$; do not reverse the symbol.

$x\geq -5$

Check that the solution set is the interval $[-5, \infty)$.

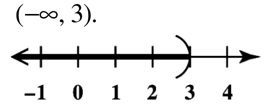


(b) $-20x>-60$

$\frac{-20x}{-20}<\frac{-60}{-20}$ Divide by $-20<0$; reverse the symbol.

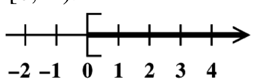
$x<3$

Check that the solution set is the interval $(-\infty, 3)$.



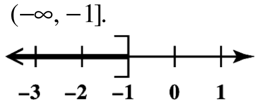
5. $$\begin{aligned} 6(x-1)+3x &\ge -x-3(x+2) \\ 6x-6+3x &\ge -x-3x-6 \\ 9x-6 &\ge -4x-6 \\ 13x-6 &\ge -6 \\ 13x &\ge 0 \\ \frac{13x}{13} &\ge \frac{0}{13} \\ x &\ge 0 \end{aligned}$$

Check that the solution set is the interval $[0, \infty)$.



N5. $$\begin{aligned} 5-2(x-4) &\le 11-4x \\ 5-2x+8 &\le 11-4x \\ -2x+13 &\le 11-4x \\ 2x+13 &\le 11 \\ 2x &\le -2 \\ \frac{2x}{2} &\le \frac{-2}{2} \\ x &\le -1 \end{aligned}$$

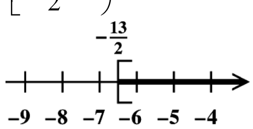
Check that the solution set is the interval $(-\infty, -1]$.



6. Solve the following inequality.

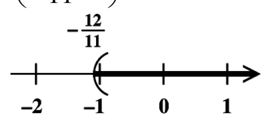
$$\begin{aligned} \frac{1}{4}(x+3)+2 &\le \frac{3}{4}(x+8) \\ 4\left[\frac{1}{4}(x+3)+2\right] &\le 4\left[\frac{3}{4}(x+8)\right] && \text{Multiply by 4.} \\ (x+3)+8 &\le 3(x+8) \\ x+11 &\le 3x+24 \\ -2x+11 &\le 24 && \text{Subtract } 3x. \\ -2x &\le 13 && \text{Subtract } 11. \\ x &\ge -\frac{13}{2} && \text{Divide by } -2; \text{ reverse symbol.} \end{aligned}$$

Check that the solution set is the interval $\left[-\frac{13}{2}, \infty\right)$.



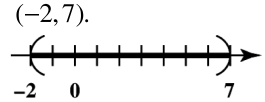
N6. $$\begin{aligned} \frac{3}{4}(x-2)+\frac{1}{2} &> \frac{1}{5}(x-8) \\ 20\left[\frac{3}{4}(x-2)+\frac{1}{2}\right] &> 20\left[\frac{1}{5}(x-8)\right] && \text{Mult. by 20.} \\ 15(x-2)+10 &> 4(x-8) \\ 15x-30+10 &> 4x-32 \\ 15x-20 &> 4x-32 \\ 11x-20 &> -32 && \text{Subtract } 4x. \\ 11x &> -12 && \text{Add 20.} \\ x &> -\frac{12}{11} && \text{Divide by } 11. \end{aligned}$$

Check that the solution set is the interval $\left(-\frac{12}{11}, \infty\right)$.



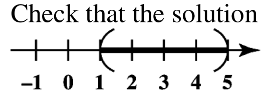
7. $$\begin{aligned} -4 < \ & x-2 < 5 \\ -2 < \ & x < 7 && \text{Add 2 to each part.} \end{aligned}$$

Check that the solution set is the interval $(-2, 7)$.



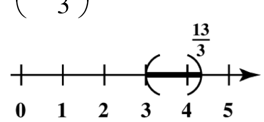
N7. $$\begin{aligned} -1 < \ & x-2 < 3 \\ 1 < \ & x < 5 && \text{Add 2 to each part.} \end{aligned}$$

Check that the solution set is the interval (1, 5).



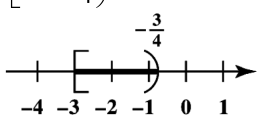
8. $$\begin{aligned} 5 < \ & 3x-4 < 9 \\ 9 < \ & 3x < 13 && \text{Add 4 to each part.} \\ \frac{9}{3} < \ & \frac{3x}{3} < \frac{13}{3} && \text{Divide each part by 3.} \\ 3 < \ & x < \frac{13}{3} \end{aligned}$$

Check that the solution set is the interval $\left(3, \frac{13}{3}\right)$.



N8.
$$\begin{array}{llll} -2 < & -4x-5 & \le 7 & \\ 3 < & -4x & \le 12 & \text{Add 5 to each part.} \\ \frac{3}{-4} > & \frac{-4x}{-4} & \ge \frac{12}{-4} & \text{Divide by } -4. \text{ Reverse inequalities.} \\ -\frac{3}{4} > & x & \ge -3 & \text{Reduce.} \\ -3 \le & x & < -\frac{3}{4} & \text{Equivalent inequality.} \end{array}$$

Check that the solution set is the interval $\left[-3, -\frac{3}{4}\right)$.



9. *Step 2*
Let h = the number of hours she can rent the leaf blower.
Step 3
She must pay \$5, plus \$1.75h, to rent the leaf blower for h hours, and this amount must be *no more than* \$26.

$$\begin{array}{ccc} \underbrace{\text{Cost of renting}} & \underbrace{\text{is no more than}} & \underbrace{\text{26 dollars.}} \\ \downarrow & \downarrow & \downarrow \\ 5+1.75h & \le & 26 \end{array}$$

Step 4
$$\begin{array}{rl} 1.75h \le 21 & \text{Subtract 5.} \\ h \le 12 & \text{Divide by } 1.75. \end{array}$$
Step 5
She can use the leaf blower for a maximum of 12 hr. (She may use it for less time, as indicated by the inequality $h \le 12$.)
Step 6
If Marge uses the leaf blower for 12 hr, she will spend $5+1.75(12)=26$, the maximum amount.

N9. *Step 2*
Let x = the number of months she belongs to the health club.
Step 3
She must pay \$40, plus \$35x, to belong to the health club for x months, and this amount must be *no more than* \$355.

$$\begin{array}{ccc} \underbrace{\text{Cost of belonging}} & \underbrace{\text{is no more than}} & \underbrace{\text{355 dollars.}} \\ \downarrow & \downarrow & \downarrow \\ 40+35x & \le & 355 \end{array}$$

Step 4
$$\begin{array}{rl} 35x \le 315 & \text{Subtract 40.} \\ x \le 9 & \text{Divide by 35.} \end{array}$$
Step 5
She can belong to the health club for a maximum of 9 months. (She may belong for less time, as indicated by the inequality $x \le 9$.)
Step 6
If Sara belongs for 9 months, she will spend $40+35(9)=355$, the maximum amount.

10. Let x = the grade she must make on the fourth test.
To find the average of the four tests, add them and divide by 4. This average must be at least 90, that is, greater than or equal to 90.
$$\frac{92+90+84+x}{4} \ge 90$$
$$\frac{266+x}{4} \ge 90$$
$$\begin{array}{rl} 266+x \ge 360 & \text{Multiply by 4.} \\ x \ge 94 & \text{Subtract 266.} \end{array}$$
Abby must score at least 94 on the fourth test.
Check $\frac{92+90+84+94}{4} = \frac{360}{4} = 90$
A score of 94 or more will give an average of at least 90, as required.

N10. Let x = the grade he must make on the fourth test.
To find the average of the four tests, add them and divide by 4. This average must be at least 90, that is, greater than or equal to 90.
$$\frac{82+97+93+x}{4} \ge 90$$
$$\frac{272+x}{4} \ge 90$$
$$\begin{array}{rl} 272+x \ge 360 & \text{Multiply by 4.} \\ x \ge 88 & \text{Subtract 272.} \end{array}$$
Joel must score at least 88 on the fourth test.
Check $\frac{82+97+93+88}{4} = \frac{360}{4} = 90$
A score of 88 or more will give an average of at least 90, as required.

Exercises

1. $x \le 3$
In interval notation, this inequality is written $(-\infty, 3]$. The bracket indicates that 3 is included. The answer is choice D.

2. $x > 3$
In interval notation, this inequality is written $(3, \infty)$. The parenthesis indicates that 3 is not included. The answer is choice C.

3. $x < 3$
In interval notation, this inequality is written $(-\infty, 3)$. The parenthesis indicates that 3 is not included. The graph of this inequality is shown in choice B.

4. $x \geq 3$
In interval notation, this inequality is written $[3, \infty)$. The bracket indicates that 3 is included. The graph of this inequality is shown in choice A.

5. $-3 \leq x \leq 3$
In interval notation, this inequality is written $[-3, 3]$. The brackets indicates that -3 and 3 are included. The answer is choice F.

6. $-3 < x < 3$
In interval notation, this inequality is written $(-3, 3)$. The parentheses indicate that neither -3 nor 3 is included. The answer is choice E.

7. **(a)** The optimal category corresponds to an LDL number of "less than 100."

$$\underbrace{\begin{matrix}\text{LDL}\\ \text{number}\end{matrix}}_{\begin{matrix}\downarrow\\ x\end{matrix}} \quad \underbrace{\begin{matrix}\text{is less}\\ \text{than}\end{matrix}}_{\begin{matrix}\downarrow\\ <\end{matrix}} \quad \underbrace{100.}_{\begin{matrix}\downarrow\\ 100\end{matrix}}$$

(b) The near optimal/above optimal category corresponds to an LDL number between "100–129." This is represented by the inequality below.

$$\underbrace{\begin{matrix}\text{LDL}\\ \text{number}\end{matrix}}_{\begin{matrix}\downarrow\\ 100 \leq x \leq 129\end{matrix}}$$

(c) The borderline high category corresponds to an LDL number between "130–159." This is represented by the inequality below.

$$\underbrace{\begin{matrix}\text{LDL}\\ \text{number}\end{matrix}}_{\begin{matrix}\downarrow\\ 130 \leq x \leq 159\end{matrix}}$$

(d) The high category corresponds to an LDL number between "160–189." This is represented by the inequality below.

$$\underbrace{\begin{matrix}\text{LDL}\\ \text{number}\end{matrix}}_{\begin{matrix}\downarrow\\ 160 \leq x \leq 189\end{matrix}}$$

(e) The very high category corresponds to an LDL number of "190 and above." This can also be written as "greater than or equal to 190."

$$\underbrace{\begin{matrix}\text{LDL}\\ \text{number}\end{matrix}}_{\begin{matrix}\downarrow\\ x\end{matrix}} \quad \underbrace{\begin{matrix}\text{is greater than}\\ \text{or equal to}\end{matrix}}_{\begin{matrix}\downarrow\\ \geq\end{matrix}} \quad \underbrace{190.}_{\begin{matrix}\downarrow\\ 190\end{matrix}}$$

8. **(a)** The normal category corresponds to a triglycerides number of "less than 100."

$$\underbrace{\begin{matrix}\text{Triglycerides}\\ \text{number}\end{matrix}}_{\begin{matrix}\downarrow\\ x\end{matrix}} \quad \underbrace{\begin{matrix}\text{is less}\\ \text{than}\end{matrix}}_{\begin{matrix}\downarrow\\ <\end{matrix}} \quad \underbrace{100.}_{\begin{matrix}\downarrow\\ 100\end{matrix}}$$

(b) The middle high category corresponds to a triglycerides number between "100–199." This is represented by the inequality below.

$$\underbrace{\begin{matrix}\text{triglycerides}\\ \text{number}\end{matrix}}_{\begin{matrix}\downarrow\\ 100 \leq x \leq 199\end{matrix}}$$

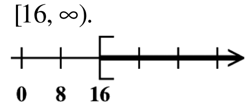
(c) The high category corresponds to a triglycerides number of "200–499." This is represented by the inequality below.

$$\underbrace{\begin{matrix}\text{triglycerides}\\ \text{number}\end{matrix}}_{\begin{matrix}\downarrow\\ 200 \leq x \leq 499\end{matrix}}$$

(d) The very high category corresponds to a triglycerides number of "500 or higher." This can also be written as "greater than or equal to 500."

$$\underbrace{\begin{matrix}\text{Triglycerides}\\ \text{number}\end{matrix}}_{\begin{matrix}\downarrow\\ x\end{matrix}} \quad \underbrace{\begin{matrix}\text{is greater than}\\ \text{or equal to}\end{matrix}}_{\begin{matrix}\downarrow\\ \geq\end{matrix}} \quad \underbrace{500.}_{\begin{matrix}\downarrow\\ 500\end{matrix}}$$

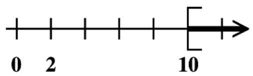
9. $x - 4 \geq 12$
$x \geq 16$ Add 4.
Check that the solution set is the interval $[16, \infty)$.



10. $x - 3 \geq 7$

$x \geq 10$ Add 3.

Check that the solution set is the interval $[10, \infty)$.

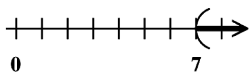


11. $3k + 1 > 22$

$3k > 21$ Subtract 1.

$k > 7$ Divide by 3.

Check that the solution set is the interval $(7, \infty)$.

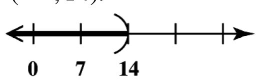


12. $5x + 6 < 76$

$5x < 70$ Subtract 6.

$x < 14$ Divide by 5.

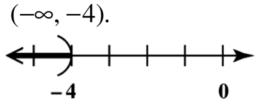
Check that the solution set is the interval $(-\infty, 14)$.



13. $4x < -16$

$x < -4$ Divide by 4.

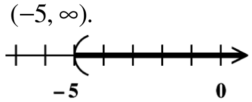
Check that the solution set is the interval $(-\infty, -4)$.



14. $2x > -10$

$x > -5$ Divide by 2.

Check that the solution set is the interval $(-5, \infty)$.



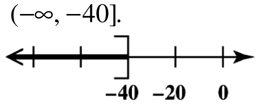
15. $-\frac{3}{4}x \geq 30$

Multiply both sides by $-\frac{4}{3}$ and reverse the inequality symbol.

$$-\frac{4}{3}\left(-\frac{3}{4}x\right) \leq -\frac{4}{3}(30)$$

$$x \leq -40$$

Check that the solution set is the interval $(-\infty, -40]$.



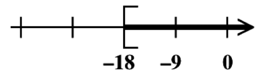
16. $-\frac{2}{3}x \leq 12$

Multiply both sides by $-\frac{3}{2}$ and reverse the inequality symbol.

$$-\frac{3}{2}\left(-\frac{2}{3}x\right) \geq -\frac{3}{2}(12)$$

$$x \geq -18$$

Check that the solution set is the interval $[-18, \infty)$.



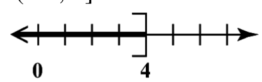
17. $-1.3x \geq -5.2$

Divide both sides by -1.3, and reverse the inequality symbol.

$$\frac{-1.3x}{-1.3} \leq \frac{-5.2}{-1.3}$$

$$x \leq 4$$

Check that the solution set is the interval $(-\infty, 4]$.



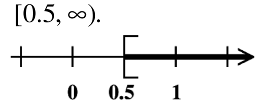
18. $-2.5x \leq -1.25$

Divide both sides by -2.5, and reverse the inequality symbol.

$$\frac{-2.5x}{-2.5} \geq \frac{-1.25}{-2.5}$$

$$x \geq 0.5$$

Check that the solution set is the interval $[0.5, \infty)$.

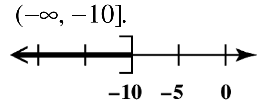


19. $5x + 2 \leq -48$

$5x \leq -50$ Subtract 2.

$x \leq -10$ Divide by 5.

Check that the solution set is the interval $(-\infty, -10]$.

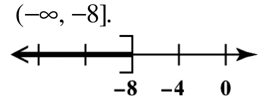


20. $4x + 1 \leq -31$

$4x \leq -32$ Subtract 1.

$x \leq -8$ Divide by 4.

Check that the solution set is the interval $(-\infty, -8]$.



21. $\dfrac{5x-6}{8}<8$

$8\left(\dfrac{5x-6}{8}\right)<8\cdot 8$ Multiply by 8.

$5x-6<64$

$5x<70$ Add 6.

$x<14$ Divide by 5.

Check Let $x=14$ in the *equation* $\dfrac{5x-6}{8}=8.$

$\dfrac{5(14)-6}{8}\stackrel{?}{=}8$

$\dfrac{64}{8}\stackrel{?}{=}8$

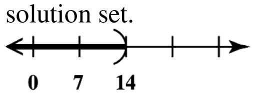
$8=8$ True

This shows that 14 is the boundary point. Now test a number on each side of 14. We choose 0 and 20.

$\dfrac{5x-6}{8}<8$

Let $x=0$.	Let $x=20$.
$\dfrac{5(0)-6}{8}\stackrel{?}{<}8$	$\dfrac{5(20)-6}{8}\stackrel{?}{<}8$
$-\dfrac{6}{8}<8$ True	$\dfrac{94}{8}\left(\text{or } 11\dfrac{6}{8}\right)<8$ False
0 is in the solution set.	20 is not in the solution set.

The check confirms that $(-\infty, 14)$ is the solution set.



22. $\dfrac{3x-1}{4}>5$

$4\left(\dfrac{3x-1}{4}\right)>4(5)$ Multiply by 4.

$3x-1>20$

$3x>21$ Add 1.

$x>7$ Divide by 3.

Check Let $x=7$ in the *equation* $\dfrac{3x-1}{4}=5.$

$\dfrac{3(7)-1}{4}\stackrel{?}{=}5$

$\dfrac{20}{4}\stackrel{?}{=}5$

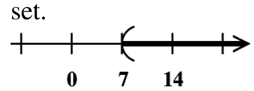
$5=5$ True

This shows that 7 is the boundary point. Now test a number on each side of 7. We choose 0 and 10.

$\dfrac{3x-1}{4}>5$

Let $x=0$.	Let $x=10$.
$\dfrac{3(0)-1}{4}\stackrel{?}{>}5$	$\dfrac{3(10)-1}{4}\stackrel{?}{>}5$
$\dfrac{-1}{4}>5$ False	$\dfrac{29}{4}>5$ True
0 is not in the solution set.	10 is in the solution set.

The check confirms that $(7,\infty)$ is the solution set.



23. Multiply both sides by $-4,$ and reverse the inequality symbol.

$\dfrac{2x-5}{-4}>5$

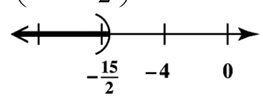
$-4\left(\dfrac{2x-5}{-4}\right)<-4(5)$

$2x-5<-20$

$2x<-15$ Add 5.

$x<-\dfrac{15}{2}$ Divide by 2.

Check that the solution set is the interval $\left(-\infty,-\dfrac{15}{2}\right).$



24. Multiply both sides by $-5,$ and reverse the inequality symbol.

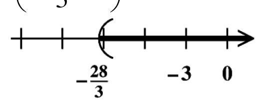
$\dfrac{3x-2}{-5}<6$

$3x-2>-30$

$3x>-28$ Add 2.

$x>-\dfrac{28}{3}$ Divide by 3.

Check that the solution set is the interval $\left(-\dfrac{28}{3},\infty\right).$



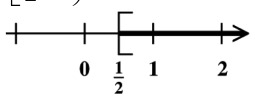
25. $6x - 4 \geq -2x$

$8x - 4 \geq 0$ Add $2x$.

$8x \geq 4$ Add 4.

$x \geq \frac{4}{8} = \frac{1}{2}$ Divide by 8.

Check that the solution set is the interval $\left[\frac{1}{2}, \infty\right)$.



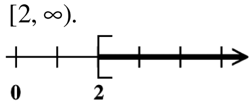
26. $2x - 8 \geq -2x$

$4x - 8 \geq 0$ Add $2x$.

$4x \geq 8$ Add 8.

$x \geq \frac{8}{4} = 2$ Divide by 4.

Check that the solution set is the interval $[2, \infty)$.



27. $x - 2(x - 4) \leq 3x$

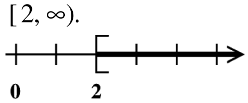
$x - 2x + 8 \leq 3x$

$-x + 8 \leq 3x$

$8 \leq 4x$ Add x.

$2 \leq x$, or $x \geq 2$

Check that the solution set is the interval $[2, \infty)$.



28. $x - 3(x + 1) \leq 4x$

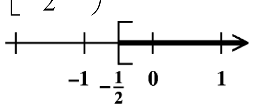
$x - 3x - 3 \leq 4x$

$-2x - 3 \leq 4x$

$-3 \leq 6x$ Add $2x$.

$-\frac{1}{2} \leq x$, or $x \geq -\frac{1}{2}$

Check that the solution set is the interval $\left[-\frac{1}{2}, \infty\right)$.



29. $-(4 + r) + 2 - 3r < -14$

$-4 - r + 2 - 3r < -14$

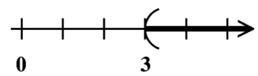
$-4r - 2 < -14$

$-4r < -12$

Divide by -4, and reverse the inequality symbol.

$r > 3$

Check that the solution set is the interval $(3, \infty)$.



30. $-(9 + x) - 5 + 4x \geq 4$

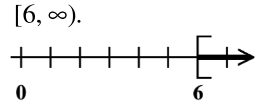
$-9 - x - 5 + 4x \geq 4$

$-14 + 3x \geq 4$

$3x \geq 18$

$x \geq 6$

Check that the solution set is the interval $[6, \infty)$.



31. $-3(x - 6) > 2x - 2$

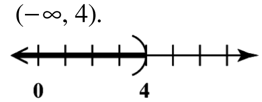
$-3x + 18 > 2x - 2$ Distributive property

$-5x > -20$ Subtract $2x$ and 18.

Divide by -5, and reverse the inequality symbol.

$x < 4$

Check that the solution set is the interval $(-\infty, 4)$.



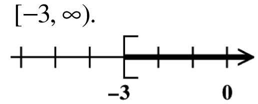
32. $-2(x + 4) \leq 6x + 16$

$-2x - 8 \leq 6x + 16$

$-8x \leq 24$

$x \geq -3$ Reverse symbol.

Check that the solution set is the interval $[-3, \infty)$.



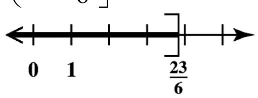
33. Multiply both sides by 6 to clear the fractions.

$$\frac{2}{3}(3x-1) \ge \frac{3}{2}(2x-3)$$
$$6 \cdot \frac{2}{3}(3x-1) \ge 6 \cdot \frac{3}{2}(2x-3)$$
$$4(3x-1) \ge 9(2x-3)$$
$$12x-4 \ge 18x-27$$
$$-6x \ge -23$$

Divide by -6, and reverse the inequality symbol.

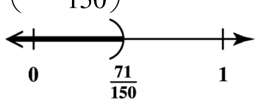
$$x \le \frac{23}{6}$$

Check that the solution set is the interval $\left(-\infty, \frac{23}{6}\right]$.



34.
$$\frac{7}{5}(10x-1) < \frac{2}{3}(6x+5)$$
$$15 \cdot \frac{7}{5}(10x-1) < 15 \cdot \frac{2}{3}(6x+5)$$
$$21(10x-1) < 10(6x+5)$$
$$210x-21 < 60x+50$$
$$150x < 71$$
$$x < \frac{71}{150}$$

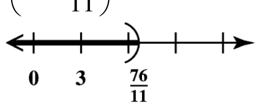
Check that the solution set is the interval $\left(-\infty, \frac{71}{150}\right)$.



35. Multiply each term by 4 to clear the fractions.

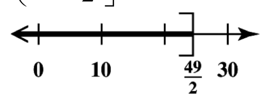
$$-\frac{1}{4}(p+6) + \frac{3}{2}(2p-5) < 10$$
$$-1(p+6) + 6(2p-5) < 40$$
$$-p-6+12p-30 < 40$$
$$11p-36 < 40$$
$$11p < 76$$
$$p < \frac{76}{11}$$

Check that the solution set is the interval $\left(-\infty, \frac{76}{11}\right)$.



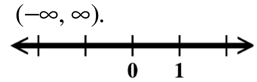
36.
$$\frac{3}{5}(t-2) - \frac{1}{4}(2t-7) \le 3$$
$$20 \cdot \frac{3}{5}(t-2) - 20 \cdot \frac{1}{4}(2t-7) \le 20 \cdot 3$$
$$12(t-2) - 5(2t-7) \le 60$$
$$12t-24-10t+35 \le 60$$
$$2t \le 49$$
$$t \le \frac{49}{2}$$

Check that the solution set is the interval $\left(-\infty, \frac{49}{2}\right]$.



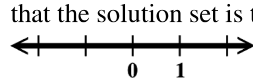
37.
$$3(2x-4)-4x < 2x+3$$
$$6x-12-4x < 2x+3$$
$$2x-12 < 2x+3$$
$$-12 < 3 \qquad \text{True}$$

The statement is true for all values of x. Therefore, the original inequality is true for any real number.
Check that the solution set is the interval $(-\infty, \infty)$.



38.
$$7(4-x)+5x < 2(16-x)$$
$$28-7x+5x < 32-2x$$
$$28-2x < 32-2x$$
$$28 < 32 \qquad \text{True}$$

This statement is true for all values of x. Check that the solution set is the interval $(-\infty, \infty)$.



39.
$$8\left(\frac{1}{2}x+3\right) < 8\left(\frac{1}{2}x-1\right)$$
$$4x+24 < 4x-8$$
$$24 < -8 \qquad \text{False}$$

This is a false statement, so the inequality is a contradiction.
Check that the solution set is $\varnothing$.

40. $10\left(\frac{1}{5}x+2\right)<10\left(\frac{1}{5}x+1\right)$

$$2x+20<2x+10$$

$$20<10 \quad \text{False}$$

This is a false statement, so the inequality is a contradiction.
Check that the solution set is $\varnothing$.

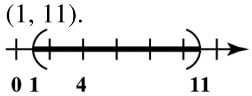
41. $-2<x$ is the same as $x>-2$. This inequality represents all real numbers greater than -2. Its graph is shown in choice A.
$-x>2$ is the same as $x<-2$. This inequality represents all real numbers less than -2. Its graph is shown in choice B.

42. Since $4>0$, the student should not have reversed the direction of the inequality symbol when dividing by 4. We reverse the inequality symbol only when multiplying or dividing by a *negative* number. The solution set is $[-16,\infty)$.

43. The goal is to isolate the variable x.

$$\begin{aligned} -4 <{}& x-5 < 6 \\ -4+5 <{}& x-5+5 < 6+5 \quad \text{Add 5.} \\ 1 <{}& x < 11 \end{aligned}$$

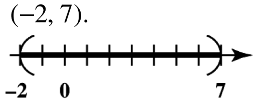
Check that the solution set is the interval (1, 11).



44. The goal is to isolate the variable x.

$$\begin{aligned} -1 <{}& x+1 < 8 \\ -1-1 <{}& x+1-1 < 8-1 \quad \text{Subtract 1.} \\ -2 <{}& x < 7 \end{aligned}$$

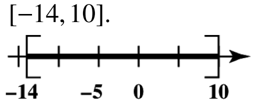
Check that the solution set is the interval $(-2, 7)$.



45.

$$\begin{aligned} -9 \le{}& x+5 \le 15 \\ -9-5 \le{}& x+5-5 \le 15-5 \quad \text{Subtract 5.} \\ -14 \le{}& x \le 10 \end{aligned}$$

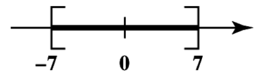
Check that the solution set is the interval $[-14, 10]$.



46.

$$\begin{aligned} -4 \le{}& x+3 \le 10 \\ -4-3 \le{}& x+3-3 \le 10-3 \quad \text{Subtract 3.} \\ -7 \le{}& x \le 7 \end{aligned}$$

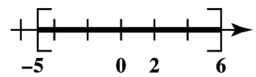
Check that the solution set is the interval $[-7, 7]$.



47.

$$\begin{aligned} -6 \le{}& 2x+4 \le 16 \\ -10 \le{}& 2x \le 12 \quad \text{Subtract 4.} \\ -5 \le{}& x \le 6 \quad \text{Divide by 2.} \end{aligned}$$

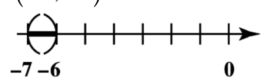
Check that the solution set is the interval $[-5, 6]$.



48.

$$\begin{aligned} -15 <{}& 3x+6 < -12 \\ -21 <{}& 3x < -18 \quad \text{Subtract 6.} \\ -7 <{}& x < -6 \quad \text{Divide by 3.} \end{aligned}$$

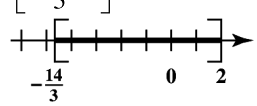
Check that the solution set is the interval $(-7,-6)$.



49.

$$\begin{aligned} -19 \le{}& 3x-5 \le 1 \\ -14 \le{}& 3x < 6 \quad \text{Add 5.} \\ -\frac{14}{3} \le{}& x \le 2 \quad \text{Divide by 3.} \end{aligned}$$

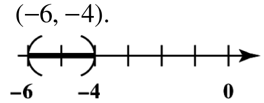
Check that the solution set is the interval $\left[-\frac{14}{3},2\right]$.



50.

$$\begin{aligned} -16 <{}& 3x+2 < -10 \\ -18 <{}& 3x < -12 \quad \text{Subtract 2.} \\ -6 <{}& x < -4 \quad \text{Divide by 3.} \end{aligned}$$

Check that the solution set is the interval $(-6, -4)$.

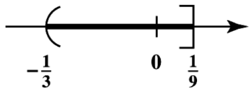


51.

$$\begin{aligned} 4 \le{}& -9x+5 < 8 \\ -1 \le{}& -9x < 3 \quad \text{Subtract 5.} \\ \frac{1}{9} \ge{}& x > -\frac{1}{3} \quad \text{Divide by } -9. \text{ Reverse inequalities.} \end{aligned}$$

The last inequality may be written as $-\frac{1}{3}<x\le\frac{1}{9}$.

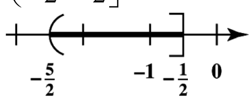
Check that the solution set is the interval $\left(-\frac{1}{3}, \frac{1}{9}\right]$.



52.
$$\begin{array}{rcll} 4 \le & -2x+3 & < 8 & \\ 1 \le & -2x & < 5 & \text{Subtract 3.} \\ -\frac{1}{2} \ge & x & > -\frac{5}{2} & \text{Divide by } -2. \text{ Reverse inequalities.} \end{array}$$

The last inequality may be written as $-\frac{5}{2} < x \le -\frac{1}{2}$.

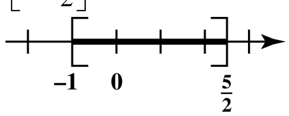
Check that the solution set is the interval $\left(-\frac{5}{2}, -\frac{1}{2}\right]$.



53.
$$\begin{array}{rcll} -8 \le & -4x+2 & \le 6 & \\ -10 \le & -4x & \le 4 & \text{Subtract 2.} \\ \frac{5}{2} \ge & x & \ge -1 & \text{Divide by } -4. \text{ Reverse inequalities.} \end{array}$$

The last inequality may be written as $-1 \le x \le \frac{5}{2}$.

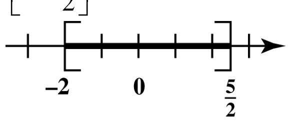
Check that the solution set is the interval $\left[-1, \frac{5}{2}\right]$.



54.
$$\begin{array}{rcll} -12 \le & -6x+3 & \le 15 & \\ -15 \le & -6x & \le 12 & \text{Subtract 3.} \\ \frac{5}{2} \ge & x & \ge -2 & \text{Divide by } -6. \text{ Reverse inequalities.} \end{array}$$

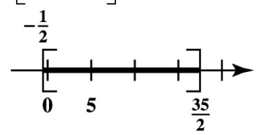
The last inequality may be written as $-2 \le x \le \frac{5}{2}$.

Check that the solution set is the interval $\left[-2, \frac{5}{2}\right]$.



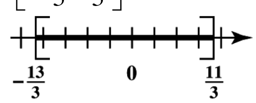
55.
$$\begin{array}{rcll} -1 \le & \frac{2x-5}{6} & \le 5 & \\ -6 \le & 2x-5 & \le 30 & \text{Multiply by 6.} \\ -1 \le & 2x & \le 35 & \text{Add 5.} \\ -\frac{1}{2} \le & x & \le \frac{35}{2} & \text{Divide by 2.} \end{array}$$

Check that the solution set is the interval $\left[-\frac{1}{2}, \frac{35}{2}\right]$.



56.
$$\begin{array}{rcll} -3 \le & \frac{3x+1}{4} & \le 3 & \\ -12 \le & 3x+1 & \le 12 & \text{Multiply by 4.} \\ -13 \le & 3x & \le 11 & \text{Subtract 1.} \\ -\frac{13}{3} \le & x & \le \frac{11}{3} & \text{Divide by 3.} \end{array}$$

Check that the solution set is the interval $\left[-\frac{13}{3}, \frac{11}{3}\right]$.



57. A number is between 0 and 1.
$0 < x < 1$
In interval notation, this is $(0,1)$.

58. A number is between -3 and -2.
$-3 < x < -2$
In interval notation, this is $(-3,-2)$.

59. Six times a number is between -12 and 12.
$$\begin{array}{rcll} -12 < & 6x & < 12 & \\ -2 < & x & < 2 & \text{Divide by 6.} \end{array}$$
This is the set of all numbers between -12 and 2—that is, $(-2, 2)$.

60. Half a number is between -3 and 2.
$$\begin{array}{rcll} -3 < & \frac{1}{2}x & < 2 & \\ -6 < & x & < 4 & \text{Multiply by 2.} \end{array}$$
This is the set of all numbers between -6 and 4—that is, $(-6, 4)$.

61. When 1 is added to twice a number, the result is greater than or equal to 7.

$2x+1\ge 7$

$2x\ge 6$ Subtract 1.

$x\ge 3$ Divide by 2.

This is the set of all numbers greater than or equal to 3—that is, $[3,\infty)$.

62. If 8 is subtracted from a number, then the result is at least 5.

$x-8\ge 5$

$x\ge 13$ Add 8.

This is the set of all numbers greater than or equal to 13—that is, $[13,\infty)$.

63. One third of a number is added to 6, giving a result of at least 3.

$6+\frac{1}{3}x\ge 3$

$\frac{1}{3}x\ge -3$ Subtract 6.

$x\ge -9$ Multiply by 3.

This is the set of all numbers greater than or equal to -9—that is, $[-9,\infty)$.

64. Three times a number, minus 5, is no more than 7.

$3x-5x\le 7$

$3x\le 12$ Add 5.

$x\le 4$ Divide by 3.

This is the set of all numbers less than or equal to 4—that is, $(-\infty,4]$.

65. Let x = her score on the third test.
Her average must be at least 84 (≥ 84). To find the average of three numbers, add them and divide by 3.

$\frac{90+82+x}{3}\ge 84$

$\frac{172+x}{3}\ge 84$ Add.

$172+x\ge 252$ Multiply by 3.

$x\ge 80$ Subtract 172.

She must score at least 80 on her third test.

66. Let x = his score on the third test.
His average must be at least 90.

$\frac{92+96+x}{3}\ge 90$

$92+96+x\ge 270$ Multiply by 3.

$188+x\ge 270$ Add.

$x\ge 82$ Subtract 188.

He must score at least 82 on his third test.

67. Let x = the number of months. The cost of Plan A is $54.99x$ and the cost of Plan B is $49.99x+129$. To determine the number of months that would be needed to make Plan B less expensive, solve the following inequality.

Plan B (cost) < Plan A (cost)

$49.99x+129<54.99x$

$129<5x$ Subtract $49.99x$.

$5x>129$ Equivalent

$x>\frac{129}{5}$ [= 25.8] Divide by 5.

It will take 26 months for Plan B to be the better deal.

68. Let x = the number of miles driven. The cost of renting from Budget is \$34.95 plus the mileage cost of \$0.25x, while the cost of renting from U-Haul is \$29.95 plus the mileage cost of \$0.28x.

Budget (cost) < U-Haul (cost)

$34.95+0.25x<29.95+0.28x$

$5+0.25x<0.28x$

$5<0.03x$

$0.03x>5$

$x>\frac{5}{0.03}\left[=\frac{500}{3}\text{ or }166\frac{2}{3}\right]$

The Budget rental will be a better deal than the U-Haul rental after 167 miles.

69. Let x = inches of rainfall in December. The average must be greater than 3.47 inches.

$\frac{2.88+3.13+x}{3}>3.47$

$\frac{6.01+x}{3}>3.47$ Add.

$6.01+x>10.41$ Multiply by 3.

$x>4.4$ Subtract 6.01.

In December, there must be more than 4.4 inches of rainfall in order for the average monthly rainfall to exceed 3.47 inches.

70. Let x = inches of rainfall in December. The average must be greater than 3.11 inches.

$$\begin{aligned} \frac{2.98+3.05+x}{3} &> 3.11 \\ \frac{6.03+x}{3} &> 3.11 && \text{Add.} \\ 6.03+x &> 9.33 && \text{Multiply by 3.} \\ x &> 3.3 && \text{Subtract 6.03.} \end{aligned}$$

In December, there must be more than 3.3 inches of rainfall in order for the average monthly rainfall to exceed 3.11 inches.

71. $$\text{BMI} = \frac{704 \times (\text{weight in pounds})}{(\text{height in inches})^2}$$

(a) Let the height equal 72.

$$\begin{aligned} 19 \le &\quad \text{BMI} &&\le 25 \\ 19 \le &\quad \frac{704w}{72^2} &&\le 25 \\ 19(72)^2 \le &\quad 704w &&\le 25(72)^2 \\ \frac{19(72)^2}{704} \le &\quad w &&\le \frac{25(72)^2}{704} \\ (\approx 139.91) \le &\quad w &&\le (\approx 184.09) \end{aligned}$$

According to the BMI formula, the healthy weight range (rounded to the nearest pound) for a person who is 72 inches tall is 140 to 184 pounds.

(b) Let the height equal 63.

$$\begin{aligned} 19 \le &\quad \text{BMI} &&\le 25 \\ 19 \le &\quad \frac{704w}{63^2} &&\le 25 \\ 19(63)^2 \le &\quad 704w &&\le 25(63)^2 \\ \frac{19(63)^2}{704} \le &\quad w &&\le \frac{25(63)^2}{704} \\ (\approx 107.12) \le &\quad w &&\le (\approx 140.94) \end{aligned}$$

According to the BMI formula, the healthy weight range (rounded to the nearest pound) for a person who is 63 inches tall is 107 to 141 pounds.

(c) Answers will vary.

72. **(a)** Let $A = 35$.

$$\begin{aligned} 0.7(220-A) \le &\quad \text{THR} &&\le 0.85(220-A) \\ 0.7(220-35) \le &\quad \text{THR} &&\le 0.85(220-35) \\ 0.7(185) \le &\quad \text{THR} &&\le 0.85(185) \\ 129.5 \le &\quad \text{THR} &&\le 157.25 \end{aligned}$$

The range is about 130 to 157 beats per minute.

(b) Let $A = 55$.

$$\begin{aligned} 0.7(220-A) \le &\quad \text{THR} &&\le 0.85(220-A) \\ 0.7(220-55) \le &\quad \text{THR} &&\le 0.85(220-55) \\ 0.7(165) \le &\quad \text{THR} &&\le 0.85(165) \\ 115.5 \le &\quad \text{THR} &&\le 140.25 \end{aligned}$$

The range is about 116 to 140 beats per minute.

(c) Answers will vary.

73. Cost $C = 20x + 100$; Revenue $R = 24x$

The business will show a profit only when $R > C$. Substitute the given expressions for R and C.

$$\begin{aligned} R &> C \\ 24x &> 20x + 100 \\ 4x &> 100 \\ x &> 25 \end{aligned}$$

The company will show a profit upon selling 26 DVDs.

74. $C = 3x + 2300$, $R = 5.50x$

To make a profit, $R > C$.

$$\begin{aligned} 5.50x &> 3x + 2300 \\ 2.5x &> 2300 \\ x &> 920 \end{aligned}$$

The company will show a profit after making 921 deliveries.

1.6 Set Operations and Compound Inequalities

Classroom Examples, Now Try Exercises

1. Let $A = \{3, 4, 5, 6\}$ and $B = \{5, 6, 7\}$.

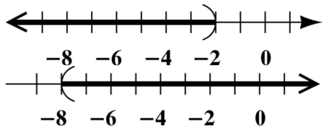
The set $A \cap B$, the intersection of A and B, contains those elements that belong to both A *and* B; that is, the numbers 5 and 6. Therefore, $A \cap B = \{5, 6\}$.

N1. Let $A = \{2, 4, 6, 8\}$ and $B = \{0, 2, 6, 8\}$.

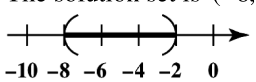
The set $A \cap B$, the intersection of A and B, contains those elements that belong to both A *and* B; that is, the numbers 2, 6, and 8. Therefore, $A \cap B = \{2, 6, 8\}$.

2. $x+3<1$ and $x-4>-12$
Solve each inequality.

$$x+3<1 \quad \text{and} \quad x-4>-12$$
$$x+3-3<1-3 \quad \text{and} \quad x-4+4>-12+4$$
$$x<-2 \quad \text{and} \quad x>-8$$

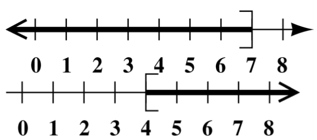


The values that satisfy both inequalities are the numbers between -8 and -2, excluding -8 and -2.
The solution set is $(-8,-2)$.

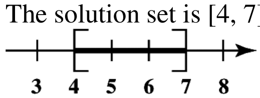


N2. $x-2\le 5$ and $x+5\ge 9$
Solve each inequality.

$$x-2\le 5 \quad \text{and} \quad x+5\ge 9$$
$$x-2+2\le 5+2 \quad \text{and} \quad x+5-5\ge 9-5$$
$$x\le 7 \quad \text{and} \quad x\ge 4$$

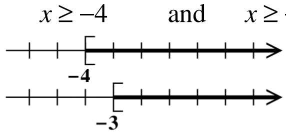


The values that satisfy both inequalities are the numbers between 4 and 7, including 4 and 7.
The solution set is [4, 7].

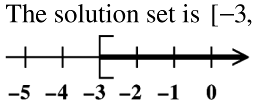


3. $2x\le 4x+8$ and $3x\ge -9$

$$-2x\le 8$$
$$x\ge -4 \quad \text{and} \quad x\ge -3$$

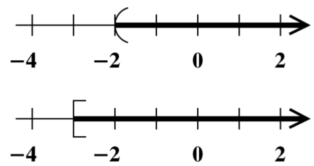


The overlap of the two graphs consists of the numbers that are greater than or equal to -4 and are also greater than or equal to -3; that is, the numbers greater than or equal to -3.
The solution set is $[-3,\infty)$.

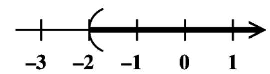


N3. $-4x-1<7$ and $3x+4\ge -5$

$$-4x<8 \quad \text{and} \quad 3x\ge -9$$
$$x>-2 \quad \text{and} \quad x\ge -3$$

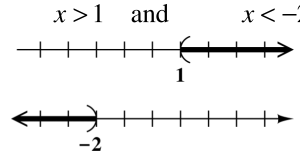


The overlap of the two graphs consists of the numbers that are greater than -2 and are also greater than or equal to -3; that is, the numbers greater than -2.
The solution set is $(-2,\infty)$.



4. $x+2>3$ and $2x+1<-3$

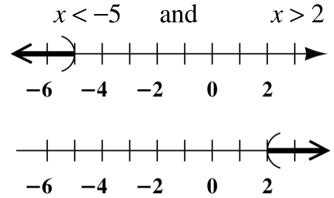
$$2x<-4$$
$$x>1 \quad \text{and} \quad x<-2$$



The two graphs do not overlap. Therefore, there is no number that is both greater than 1 *and* less than -2, so the given compound inequality has no solution. The solution set is $\varnothing$.

N4. $x-7<-12$ and $2x+1>5$

$$2x>4$$
$$x<-5 \quad \text{and} \quad x>2$$

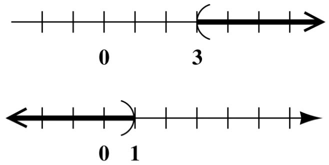


The two graphs do not overlap. Therefore, there is no number that is both greater than 2 *and* less than -5, so the given compound inequality has no solution. The solution set is $\varnothing$.

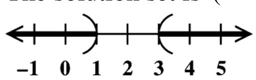
5. Let $A=\{3,4,5,6\}$ and $B=\{5,6,7\}$.
The set $A\cup B$, the union of A and B, consists of all elements in either A *or* B (or both). Start by listing the elements of set A: 3, 4, 5, 6. Then list any additional elements from set B. In this case, the elements 5 and 6 are already listed, so the only additional element is 7.
Therefore, $A\cup B=\{3,4,5,6,7\}$.

N5. Let $A = \{5, 10, 15, 20\}$ and $B = \{5, 15, 25\}$.
The set $A \cup B$, the union of A and B, consists of all elements in either A *or* B (or both). Start by listing the elements of set A: 5, 10, 15, 20. Then list any additional elements from set B. In this case, the elements 5 and 15 are already listed, so the only additional element is 25. Therefore, $A \cup B = \{5, 10, 15, 20, 25\}$.

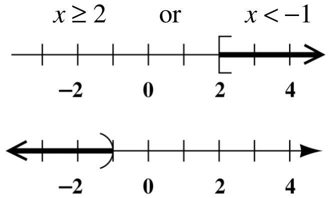
6. $x - 1 > 2$ or $3x + 5 < 2x + 6$
$x > 3$ or $x < 1$



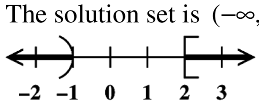
The graph of the solution set consists of all numbers greater than 3 *or* less than 1.
The solution set is $(-\infty, 1) \cup (3, \infty)$.



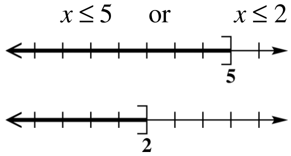
N6. $-12x \le -24$ or $x + 9 < 8$
$x \ge 2$ or $x < -1$



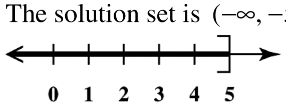
The graph of the solution set consists of all numbers greater than or equal to 2 *or* less than -1.
The solution set is $(-\infty, -1) \cup [2, \infty)$.



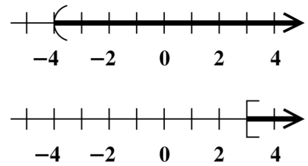
7. $3x - 2 \le 13$ or $x + 5 \le 7$
$3x \le 15$
$x \le 5$ or $x \le 2$



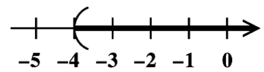
The solution set is all numbers that are either less than or equal to 5 *or* less than or equal to 2. All real numbers less than or equal to 5 are included.
The solution set is $(-\infty, -5]$.



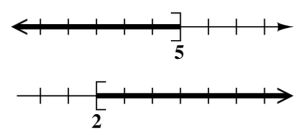
N7. $-x + 2 < 6$ or $6x - 8 \ge 10$
$-x < 4$ or $6x \ge 18$
$x > -4$ or $x \ge 3$



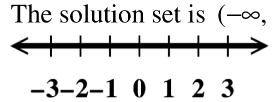
The solution set is all numbers that are either greater than -4 *or* greater than or equal to 3. All real numbers greater than -4 are included.
The solution set is $(-4, \infty)$.



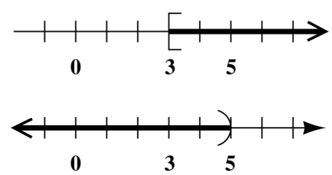
8. $3x - 2 \le 13$ or $x + 5 \ge 7$
$3x \le 15$
$x \le 5$ or $x \ge 2$



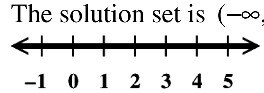
The solution set is all numbers that are either less than or equal to 5 *or* greater than or equal to 2. All real numbers are included.
The solution set is $(-\infty, \infty)$.



N8. $8x - 4 \ge 20$ or $-2x + 1 > -9$
$8x \ge 24$ or $-2x > -10$
$x \ge 3$ or $x < 5$



The solution set is all numbers that are either greater than or equal to 3 *or* less than 5. All real numbers are included.
The solution set is $(-\infty, \infty)$.



9. **(a)** Only Canada and Mexico received U.S. exports greater than \$150,000 million but neither of these countries imported less than \$150,000 million to the U.S. Therefore the set of countries is the empty set, $\varnothing$.

(b) Only China and Japan received U.S. exports between \$50,000 and \$150,000. Only Canada and China imported greater than \$300,000 million to the U.S. The word "or" represents the union of these two sets.

$\{\text{China, Japan}\} \cup \{\text{Canada, China}\}$
$= \{\text{Canada, China, Japan}\}$

N9. **(a)** Only Canada, China, and Mexico received U.S. exports greater than \$100,000 million. Of these countries, only Canada and Mexico imported less than \$400,000 million to the U.S. Therefore, the set of countries is $\{\text{Canada, Mexico}\}$.

(b) Only China, Japan, and Germany received U.S. exports less than \$200,000 million. Only Canada, China, and Mexico imported greater than \$250,000 million to the U.S. The word "or" represents the union of these two sets.

$\{\text{China, Japan, Germany}\}$
$\cup \{\text{Canada, China, Mexico}\}$
$= \{\text{Canada, China, Mexico, Japan, Germany}\}$

Exercises

1. This statement is *true*. The solution set of $x+1=6$ is $\{5\}$. The solution set of $x+1>6$ is $(5, \infty)$. The solution set of $x+1<6$ is $(-\infty, 5)$. Taken together we have the set of real numbers.

2. This statement is *false*. The intersection is $\{9\}$ since 9 is the element common to both sets.

3. This statement is *false*. The union is $(-\infty, 7) \cup (7, \infty)$. The only real number that is *not* in the union is 7.

4. This statement is *true* since 7 is the only element common to both sets.

5. This statement is *false* since 0 is a rational number but not an irrational number. The sets of rational numbers and irrational numbers have no common elements so their intersection is $\varnothing$.

6. This statement is *true*. The set of rational numbers together with the set of irrational numbers makes up the set of real numbers.

7. The intersection of sets B and A contains only those elements in both sets B and A.
   $B \cap A = \{1, 3, 5\}$ or set B

8. The intersection of sets A and B contains only those elements in both sets A and B.
   $A \cap B = \{1, 3, 5\}$ or set B
   Note that $A \cap B = B \cap A$.

9. The intersection of sets A and D is the set of all elements in both set A and D. Therefore, $A \cap D = \{4\}$ or set D.

10. 1 is the only element in both sets B and C, so $B \cap C = \{1\}$.

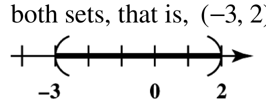
11. The intersection of set B and the set of no elements (empty set), $B \cap \varnothing$, is the set of no elements or $\varnothing$.

12. No element is common to both sets, so $A \cap \varnothing = \varnothing$, the empty set.

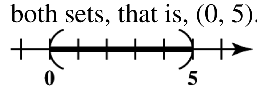
13. The union of sets A and B is the set of all elements that are in either set A or set B or both sets A and B. Since all numbers in set B are also in set A, the set $A \cup B$ will be the same as set A.
    $A \cup B = \{1, 2, 3, 4, 5, 6\}$ or set A

14. A union of sets contains all elements that belong to either set.
    $B \cup D = \{1, 3, 4, 5\}$

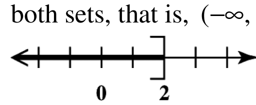
15. The first graph represents the set $(-\infty, 2)$. The second graph represents the set $(-3, \infty)$. The intersection includes the elements common to both sets, that is, $(-3, 2)$.



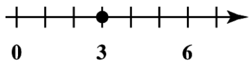
16. The first graph represents the set $(-\infty, 5)$. The second graph represents the set $(0, \infty)$. The intersection includes the elements common to both sets, that is, $(0, 5)$.



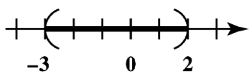
17. The first graph represents the set $(-\infty, 5]$. The second graph represents the set $(-\infty, 2]$. The intersection includes the elements common to both sets, that is, $(-\infty, 2]$.



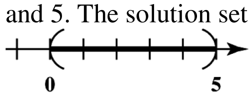
18. The first graph represents the set $[3, \infty)$. The second graph represents the set $(-\infty, 3]$. The intersection includes the elements common to both sets, that is, $\{3\}$.



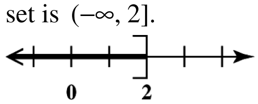
19. $x < 2$ and $x > -3$
The graph of the solution set will be all numbers that are both less than 2 and greater than -3. The solution set is $(-3, 2)$.



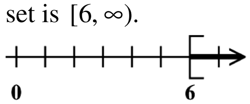
20. $x < 5$ and $x > 0$
And means intersection. The graph of the solution set will be all numbers that are both less than 5 and greater than 0. The overlap is the numbers between 0 and 5, not including 0 and 5. The solution set is (0, 5).



21. $x \le 2$ and $x \le 5$
The graph of the solution set will be all numbers that are both less than or equal to 2 and less than or equal to 5. The overlap is the numbers less than or equal to 2. The solution set is $(-\infty, 2]$.



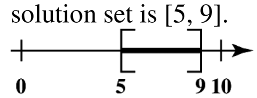
22. $x \ge 3$ and $x \ge 6$
The graph of the solution set will be all numbers that are both greater than or equal to 3 and greater than or equal to 6. This will be all numbers greater than or equal to 6. The solution set is $[6, \infty)$.



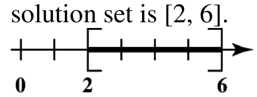
23. $x \le 3$ and $x \ge 6$
The graph of the solution set will be all numbers that are both less than or equal to 3 and greater than or equal to 6. There are no such numbers. The solution set is $\varnothing$.

24. $x \le -1$ and $x \ge 3$
The graph of the solution set will be all numbers that are both less than or equal to -1 and greater than or equal to 3. There are no such numbers. The solution set is $\varnothing$.

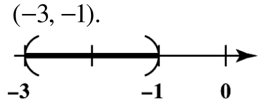
25. $x - 3 \le 6$ and $x + 2 \ge 7$
$x \le 9$ and $x \ge 5$
The graph of the solution set is all numbers that are both less than or equal to 9 and greater than or equal to 5. This is the intersection. The elements common to both sets are the numbers between 5 and 9, including the endpoints. The solution set is [5, 9].



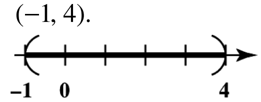
26. $x + 5 \le 11$ and $x - 3 \ge -1$
$x \le 6$ and $x \ge 2$
The graph of the solution set is all numbers that are both less than or equal to 6 and greater than or equal to 2. This is the intersection. The elements common to both sets are the numbers between 2 and 6, including the endpoints. The solution set is [2, 6].



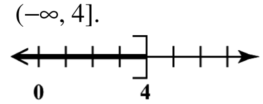
27. $-3x > 3$ and $x + 3 > 0$
$x < -1$ and $x > -3$
The graph of the solution set is all numbers that are both less than -1 and greater than -3. This is the intersection. The elements common to both sets are the numbers between -3 and -1, not including the endpoints. The solution set is $(-3, -1)$.



28. $-3x < 3$ and $x + 2 < 6$
$x > -1$ and $x < 4$
The graph of the solution set is all numbers that are both less than 4 and greater than -1. This is the intersection. The elements common to both sets are the numbers between -1 and 4, not including the endpoints. The solution set is $(-1, 4)$.



29. $3x - 4 \le 8$ and $-4x + 1 \ge -15$
$3x \le 12$ and $-4x \ge -16$
$x \le 4$ and $x \le 4$
Since both inequalities are identical, the graph of the solution set is the same as the graph of one of the inequalities. The solution set is $(-\infty, 4]$.

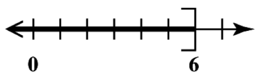


30. $7x+6 \le 48$ and $-4x \ge -24$

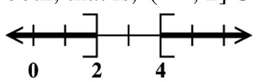
$7x \le 42$

$x \le 6$ and $x \le 6$

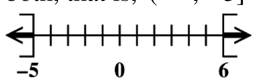
Since both inequalities are identical, the graph of the solution set is the same as the graph of one of the inequalities. The solution set is $(-\infty, 6]$.



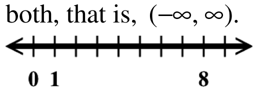
31. The first graph represents the set $(-\infty, 2]$. The second graph represents the set $[4, \infty)$. The union includes all elements in either set, or in both, that is, $(-\infty, 2] \cup [4, \infty)$.



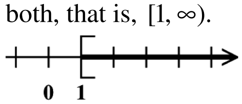
32. The first graph represents the set $(-\infty, -5]$. The second graph represents the set $[6, \infty)$. The union includes all elements in either set, or in both, that is, $(-\infty, -5] \cup [6, \infty)$.



33. The first graph represents the set $[1, \infty)$. The second graph represents the set $(-\infty, 8]$. The union includes all elements in either set, or in both, that is, $(-\infty, \infty)$.

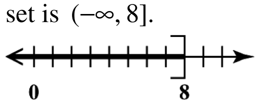


34. The first graph represents the set $[1, \infty)$. The second graph represents the set $[8, \infty)$. The union includes all elements in either set, or in both, that is, $[1, \infty)$.



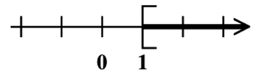
35. $x \le 1$ or $x \le 8$

The word "or" means to take the union of both sets. The graph of the solution set is all numbers that are either less than or equal to 1 *or* less than or equal to 8, or both. This is all numbers less than or equal to 8. The solution set is $(-\infty, 8]$.



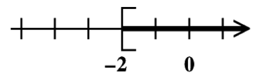
36. $x \ge 1$ or $x \ge 8$

The graph of the solution set will be all numbers that are either greater than or equal to 1 or greater than or equal to 8. The solution set is $[1, \infty)$.



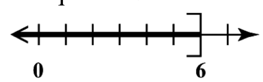
37. $x \ge -2$ or $x \ge 5$

The graph of the solution set will be all numbers that are either greater than or equal to -2 or greater than or equal to 5. The solution set is $[-2, \infty)$.



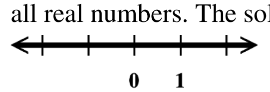
38. $x \le -2$ or $x \le 6$

The graph of the solution set will be all numbers that are either less than or equal to -2 or less than or equal to 6. The solution set is $(-\infty, 6]$.



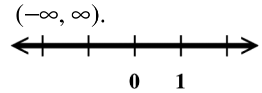
39. $x \ge -2$ or $x \le 4$

The graph of the solution set will be all numbers that are either greater than or equal to -2 or less than or equal to 4. This is the set of all real numbers. The solution set is $(-\infty, \infty)$.



40. $x \ge 5$ or $x \le 7$

The graph of the solution set will be all numbers that are either greater than or equal to 5 or less than or equal to 7. The solution set is $(-\infty, \infty)$.

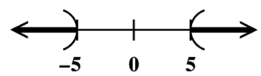


41. $x+2 > 7$ or $1-x > 6$

$-x > 5$

$x > 5$ or $x < -5$

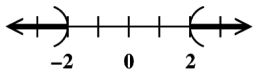
The graph of the solution set is all numbers either greater than 5 or less than -5. This is the union. The solution set is $(-\infty, -5) \cup (5, \infty)$.



42. $x+1>3$ or $x+4<2$

$x>2$ or $x<-2$

The graph of the solution set is all numbers either greater than 2 or less than -2. This is the union. The solution set is $(-\infty,-2)\cup(2,\infty)$.

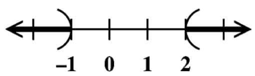


43. $x+1>3$ or $-4x+1>5$

$-4x>4$

$x>2$ or $x<-1$

The graph of the solution set is all numbers either less than -1 or greater than 2. This is the union. The solution set is $(-\infty,-1)\cup(2,\infty)$.

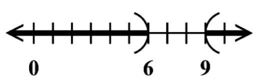


44. $3x<x+12$ or $x+1>10$

$2x<12$

$x<6$ or $x>9$

The graph of the solution set is all numbers either less than 6 or greater than 9. This is the union. The solution set is $(-\infty,6)\cup(9,\infty)$.

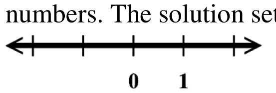


45. $4x+1\ge-7$ or $-2x+3\ge5$

$4x\ge-8$ or $-2x\ge2$

$x\ge-2$ or $x\le-1$

The graph of the solution set is all numbers either greater than or equal to -2 or less than or equal to -1. This is the set of all real numbers. The solution set is $(-\infty,\infty)$.

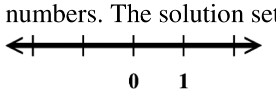


46. $3x+2\le-7$ or $-2x+1\le9$

$3x\le-9$ or $-2x\le8$

$x\le-3$ or $x\ge-4$

The graph of the solution set is all numbers either less than or equal to -3 or greater than or equal to -4. This is the set of all real numbers. The solution set is $(-\infty,\infty)$.



47. $(-\infty,-1]\cap[-4,\infty)$

The intersection is the set of numbers less than or equal to -1 and greater than or equal to -4. The numbers common to both original sets are between, and including, -4 and -1. The simplest interval form is $[-4,-1]$.

48. $[-1,\infty)\cap(-\infty,9]=(-\infty,9]\cap[-1,\infty)$

The intersection is the set of numbers less than or equal to 9 and greater than or equal to -1. The numbers common to both original sets are between, and including, -1 and 9. The simplest interval form is $[-1,9]$.

49. $(-\infty,-6]\cap[-9,\infty)$

The intersection is the set of numbers less than or equal to -6 and greater than or equal to -9. The numbers common to both original sets are between, and including, -9 and -6. The simplest interval form is $[-9,-6]$.

50. $(5,11]\cap[6,\infty)$

The intersection is the set of numbers between 5 and 11, including 11 but not 5, and greater than or equal to 6. The numbers common to both original sets are between, and including, 6 and 11. The simplest interval form is [6, 11].

51. $(-\infty,3)\cup(-\infty,-2)$

The union is the set of numbers that are either less than 3 or less than -2, or both. This is all numbers less than 3. The simplest interval form is $(-\infty,3)$.

52. $[-9,1]\cup(-\infty,-3)$

The union is the set of numbers between -9 and 1, including both, or less than -3. This is all numbers less than, and including, 1. The simplest interval form is $(-\infty,1]$.

53. $[3,6]\cup(4,9)$

The union is the set of numbers between, and including, 3 and 6, or between, but not including, 4 and 9. This is the set of numbers greater than or equal to 3 and less than 9. The simplest interval form is [3, 9).

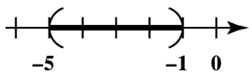
54. $[-1,2]\cup(0,5)$

The union is the set of numbers between, and including, -1 and 2, or between, but not including, 0 and 5. This is the set of numbers greater than or equal to -1 and less than 5. The simplest interval form is $[-1,5)$.

55. $x < -1$ and $x > -5$

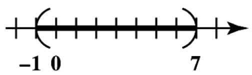
The word "and" means to take the intersection of both sets. $x < -1$ and $x > -5$ is true only when $-5 < x < -1$.

The graph of the solution set is all numbers greater than -5 *and* less than -1. This is all numbers between -5 and -1, not including -5 or -1. The solution set is $(-5, -1)$.



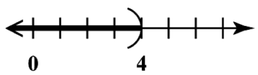
56. $x > -1$ and $x < 7$

This is an intersection. The solution set is $(-1, 7)$.



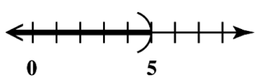
57. $x < 4$ or $x < -2$

The word "or" means to take the union of both sets. The graph of the solution set is all numbers that are either less than 4 *or* less than -2, or both. This is all numbers less than 4. The solution set is $(-\infty, 4)$.



58. $x < 5$ or $x < -3$

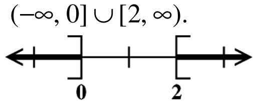
This is a union. The solution set is $(-\infty, 5)$.



59. $-3x \le -6$ or $-3x \ge 0$

$x \ge 2$ or $x \le 0$

The word "or" means to take the union of both sets. The graph of the solution set is all numbers that are either greater than or equal to 2 *or* less than or equal to 0. The solution set is $(-\infty, 0] \cup [2, \infty)$.

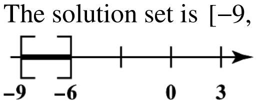


60. $2x - 6 \le -18$ and $2x \ge -18$

$2x \le -12$

$x \le -6$ and $x \ge -9$

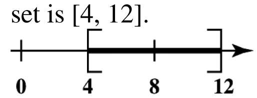
This is an intersection.
The solution set is $[-9, -6]$.



61. $x + 1 \ge 5$ and $x - 2 \le 10$

$x \ge 4$ and $x \le 12$

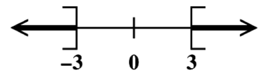
The word "and" means to take the intersection of both sets. The graph of the solution set is all numbers that are both greater than or equal to 4 *and* less than or equal to 12. This is all numbers between, and including, 4 and 12. The solution set is [4, 12].



62. $-8x \le -24$ or $-5x \ge 15$

$x \ge 3$ or $x \le -3$

This is a union.
The solution set is $(-\infty, -3] \cup [3, \infty)$.



63. The set of expenses that are less than \$8000 for public schools *and* are greater than \$15,000 for private schools is {Tuition and fees}.

64. The set of expenses that are greater than \$4000 for public schools *and* are less than \$5000 for private schools is {Board rates}.

65. The set of expenses that are less than \$8000 for public schools *or* are greater than \$15,000 for private schools is {Tuition and fees, Board rates, Dormitory charges}.

66. The set of expenses that are greater than \$15,000 *or* are between \$7000 and \$8000 is {Tuition and fees}.

67. Find "the yard can be fenced *and* the yard can be sodded."
A yard that can be fenced has $P \le 150$. Maria and Joe qualify.
A yard that can be sodded has $A \le 1400$. Again, Maria and Joe qualify.
Find the intersection. Maria's and Joe's yards are common to both sets, so Maria and Joe can have their yards both fenced and sodded.

68. Find "the yard can be fenced *and* the yard cannot be sodded."
A yard that can be fenced has $P \le 150$. Maria and Joe qualify.
A yard that cannot be sodded has $A > 1400$. Luigi and Than qualify.
Find the intersection. There are no yards common to both sets, so none of them qualify.

69. Find "the yard cannot be fenced *and* the yard can be sodded."
A yard that cannot be fenced has $P > 150$. Luigi and Than qualify.
A yard that can be sodded has $A \le 1400$. Maria and Joe qualify.
Find the intersection. There are no yards common to both sets, so none of them qualify.

70. Find "the yard cannot be fenced *and* the yard cannot be sodded."
A yard that cannot be fenced has $P > 150$. Luigi and Than qualify.
A yard that cannot be sodded has $A > 1400$. Again, Luigi and Than qualify.
Find the intersection. Luigi's and Than's yards are common to both sets, so Luigi and Than qualify.

71. Find "the yard can be fenced *or* the yard can be sodded." From Exercise 67, Maria's and Joe's yards qualify for both conditions, so the union is Maria and Joe.

72. Find "the yard cannot be fenced *or* the yard can be sodded." From Exercise 69, Luigi's and Than's yards cannot be fenced, and Maria's and Joe's yards can be sodded. The union includes all of them.

1.7 Absolute Value Equations and Inequalities

Classroom Examples, Now Try Exercises

1. $|3x-4|=11$

$3x-4=11$ or $3x-4=-11$
$3x=15$ or $3x=-7$
$x=5$ or $x=-\frac{7}{3}$

Check $x=5$: $|11|=11$ True

Check $x=-\frac{7}{3}$: $|-11|=11$ True

The solution set is $\left\{-\frac{7}{3}, 5\right\}$.

N1. $|4x-1|=11$

$4x-1=11$ or $4x-1=-11$
$4x=12$ or $4x=-10$
$x=3$ or $x=-\frac{5}{2}$

Check $x=3$: $|11|=11$ True

Check $x=-\frac{5}{2}$: $|-11|=11$ True

The solution set is $\left\{-\frac{5}{2}, 3\right\}$.

2. $|3x-4|\ge 11$

$3x-4\ge 11$ or $3x-4\le -11$
$3x\ge 15$ or $3x\le -7$
$x\ge 5$ or $x\le -\frac{7}{3}$

Check $x=-3, 0,$ and 6 in $|3x-4|\ge 11$.
Check $x=-3$: $|-13|\ge 11$ True
Check $x=0$: $|-4|\ge 11$ False
Check $x=6$: $|14|\ge 11$ True

The solution set is $\left(-\infty, -\frac{7}{3}\right] \cup [5, \infty)$.

N2. $|4x-1|>11$

$4x-1>11$ or $4x-1<-11$
$4x>12$ or $4x<-10$
$x>3$ or $x<-\frac{5}{2}$

Check $x=-3, 0,$ and 6 in $|4x-1|>11$.
Check $x=-3$: $|-13|>11$ True
Check $x=0$: $|-1|>11$ False
Check $x=6$: $|23|>11$ True

The solution set is $\left(-\infty, -\frac{5}{2}\right) \cup (3, \infty)$.

3. $|3x-4|\le 11$

$-11\le 3x-4 \le 11$
$-7\le 3x \le 15$
$-\frac{7}{3}\le x \le 5$

Check $x=-3, 0,$ and 6 in $|3x-4|\le 11$.
Check $x=-3$: $|-13|\le 11$ False
Check $x=0$: $|-4|\le 11$ True
Check $x=6$: $|14|\le 11$ False

The solution set is $\left[-\frac{7}{3}, 5\right]$.

N3. $|4x-1|<11$

$-11< 4x-1 < 11$
$-10< 4x < 12$
$-\frac{5}{2}< x < 3$

Check $x=-5, 0,$ and 5 in $|4x-1|<11$.
Check $x=-5$: $|-21|<11$ False
Check $x=0$: $|-1|<11$ True

Check $x = 5$: $|19| < 11$ False

The solution set is $\left(-\frac{5}{2}, 3\right)$.

4. $|20 - 2x| \geq 20$

$20 - 2x \geq 20$ or $20 - 2x \leq -20$

$-2x \geq 0$ or $-2x \leq -40$

$x \leq 0$ or $x \geq 20$

Check $x = -1, 10,$ and 21 in $|20 - 2x| \geq 20$

Check $x = -1$: $|22| \geq 20$ True

Check $x = 10$: $|0| \geq 20$ False

Check $x = 21$: $|-22| \geq 20$ True

The solution set is $(-\infty, 0] \cup [20, \infty)$.

N4. $|7 - 4x| \geq 7$

$7 - 4x \geq 7$ or $7 - 4x \leq -7$

$-4x \geq 0$ or $-4x \leq -14$

$x \leq 0$ or $x \geq \frac{7}{2}$ or $3\frac{1}{2}$

Check $x = -1, 3,$ and 4 in $|7 - 4x| \geq 7$

Check $x = -1$: $|11| \geq 7$ True

Check $x = 3$: $|-5| \geq 7$ False

Check $x = 4$: $|-9| \geq 7$ True

The solution set is $(-\infty, 0] \cup \left[\frac{7}{2}, \infty\right)$.

5. $|3x + 2| + 4 = 15$

We first *isolate* the absolute value expression, that is, rewrite the equation so that the absolute value expression is alone on one side of the equals symbol.

$|3x + 2| = 11$

$3x + 2 = 11$ or $3x + 2 = -11$

$3x = 9$ or $3x = -13$

$x = 3$ or $x = -\frac{13}{3}$

Check $x = 3$: $|11| + 4 = 15$ True

Check $x = -\frac{13}{3}$: $|-11| + 4 = 15$ True

The solution set is $\left\{-\frac{13}{3}, 3\right\}$.

N5. $|10x - 2| - 2 = 12$

We first *isolate* the absolute value expression, that is, rewrite the equation so that the absolute value expression is alone on one side of the equals symbol.

$|10x - 2| = 14$

$10x - 2 = 14$ or $10x - 2 = -14$

$10x = 16$ or $10x = -12$

$x = \frac{8}{5}$ or $x = -\frac{6}{5}$

Check $x = \frac{8}{5}$: $|14| - 2 = 12$ True

Check $x = -\frac{6}{5}$: $|-14| - 2 = 12$ True

The solution set is $\left\{-\frac{6}{5}, \frac{8}{5}\right\}$.

6. (a) $|x + 2| - 3 > 2$

$|x + 2| > 5$ Isolate.

$x + 2 > 5$ or $x + 2 < -5$

$x > 3$ or $x < -7$

The solution set is $(-\infty, -7) \cup (3, \infty)$.

(b) $|x + 2| - 3 < 2$

$|x + 2| < 5$ Isolate.

$-5 < x + 2 < 5$

$-7 < x < 3$

The solution set is $(-7, 3)$.

N6. (a) $|x - 1| - 4 \leq 2$

$|x - 1| \leq 6$ Isolate.

$-6 \leq x - 1 \leq 6$

$-5 \leq x \leq 7$

The solution set is $[-5, 7]$.

(b) $|x - 1| - 4 \geq 2$

$|x - 1| \geq 6$ Isolate.

$x - 1 \geq 6$ or $x - 1 \leq -6$

$x \geq 7$ or $x \leq -5$

The solution set is $(-\infty, -5] \cup [7, \infty)$.

7. $|4x - 1| = |3x + 5|$

$4x - 1 = 3x + 5$ or $4x - 1 = -(3x + 5)$

$x = 6$ or $4x - 1 = -3x - 5$

$7x = -4$

$x = -\frac{4}{7}$

Check $x = 6$: $|23| = |23|$ True

Check $x = -\frac{4}{7}$: $\left|-\frac{23}{7}\right| = \left|\frac{23}{7}\right|$ True

The solution set is $\left\{-\frac{4}{7}, 6\right\}$.

N7. $|3x-4|=|5x+12|$

$3x-4=5x+12$ or $3x-4=-(5x+12)$

$-2x=16$ or $3x-4=-5x-12$

$x=-8$ $\quad 8x=-8$

$x=-1$

Check $x=-8$: $|-28|=|-28|$ True

Check $x=-1$: $|-7|=|7|$ True

The solution set is $\{-8, -1\}$.

8. **(a)** $|6x+7|=-5$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.
The solution set is $\varnothing$.

(b) $\left|\frac{1}{4}x-3\right|=0$

The expression $\frac{1}{4}x-3$ will equal 0 *only* if $\frac{1}{4}x-3=0$.

$x-12=0$ Multiply by 4.

$x=12$

The solution set is $\{12\}$.

N8. **(a)** $|3x-8|=-2$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.
The solution set is $\varnothing$.

(b) $|7x+12|=0$

The expression $7x+12$ will equal 0 *only* if $7x+12=0$.

$7x=-12$ Subtract 12.

$x=-\frac{12}{7}$ Divide by 7.

The solution set is $\left\{-\frac{12}{7}\right\}$.

9. **(a)** $|x|>-1$

The absolute value of a number is always greater than or equal to 0. Therefore, the inequality is true for all real numbers.
The solution set is $(-\infty, \infty)$.

(b) $|x-10|-2\le -3$

$|x-10|\le -1$

There is no number whose absolute value is less than a negative number, so this inequality has no solution.
The solution set is $\varnothing$.

(c) $|x+2|\le 0$

The value of $|x+2|$ will never be less than 0. However, $|x+2|$ will equal 0 when $x=-2$.
The solution set is $\{-2\}$.

N9. **(a)** $|x|>-10$

The absolute value of a number is always greater than or equal to 0. Therefore, the inequality is true for all real numbers.
The solution set is $(-\infty, \infty)$.

(b) $|4x+1|+5<4$

$|4x+1|<-1$

There is no number whose absolute value is less than a negative number, so this inequality has no solution.
The solution set is $\varnothing$.

(c) $|x-2|-3\le -3$

$|x-2|\le 0$ Isolate abs. value.

The value of $|x-2|$ will never be less than 0.
However, $|x-2|$ will equal 0 when $x=2$.
The solution set is $\{2\}$.

10. Let x be the possible amount of orange juice in the carton. If the relative error is no greater than 0.035 oz, then the following inequality shows the possible ounces of orange juice in a filled carton. Note that there are 32 oz/qt.

$$\left|\frac{32-x}{32}\right|\le 0.035$$

$$-0.035\le \frac{32-x}{32} \le 0.035$$

$$-1.12\le 32-x \le 1.12$$

$$30.88\le x \le 33.12$$

The carton will contain between 30.88 oz and 33.12 oz.

N10. Let x be the possible amount of milk in the carton. If the relative error is no greater than 0.032 oz, then the following inequality shows the possible ounces of orange juice in a filled carton. Note that there are 32 oz/qt.

$$\left|\frac{32-x}{32}\right|\le 0.032$$

$$\begin{aligned} -0.032 \le & \frac{32-x}{32} \le 0.032 \\ -1.024 \le & \ 32-x \le 1.024 \\ 30.976 \le & \ x \le 33.024 \end{aligned}$$

The carton will contain between 30.976 oz and 33.024 oz.

Exercises

1. $|x| = 5$ has two solutions, $x = 5$ or $x = -5$. The graph is Choice E.

$|x| < 5$ is written $-5 < x < 5$. Notice that -5 and 5 are not included. The graph is Choice C, which uses parentheses.

$|x| > 5$ is written $x < -5$ or $x > 5$. The graph is Choice D, which uses parentheses.

$|x| \le 5$ is written $-5 \le x \le 5$. This time -5 and 5 are included. The graph is Choice B, which uses brackets.

$|x| \ge 5$ is written $x \le -5$ or $x \ge 5$. The graph is Choice A, which uses brackets.

2. $|x| = 9$ has two solutions, $x = 9$ and $x = -9$. The graph is Choice E.

$|x| > 9$ is written $x < -9$ or $x > 9$. Notice that -9 and 9 are not included. The graph is Choice D, which uses parentheses.

$|x| \ge 9$ is written $x \le -9$ or $x \ge 9$. This time -9 and 9 are included. The graph is Choice A, which uses brackets.

$|x| < 9$ is written $-9 < x < 9$. The graph is Choice C, which uses parentheses.

$|x| \le 9$ is written $-9 \le x \le 9$. The graph is Choice B, which uses brackets.

3. **(a)** $|ax+b| = k, k = 0$

This means the distance from $ax+b$ to 0 is 0, so $ax+b = 0$, which has one solution.

(b) $|ax+b| = k, k > 0$

This means the distance from $ax+b$ to 0 is a positive number, so $ax+b = k$ or $ax+b = -k$. There are two solutions.

(c) $|ax+b| = k, k < 0$

This means the distance from $ax+b$ to 0 is a negative number, which is impossible because distance is always positive. There are no solutions.

4. When solving an absolute value equation or inequality of the form $|ax+b| = k$, $|ax+b| < k$, or $|ax+b| > k$, where k is a positive number, use *or* for the equality statement and the > statement. Use *and* for the < statement.

5. $|x| = 12$

$x = 12$ or $x = -12$

The solution set is $\{-12, 12\}$.

6. $|x| = 14$

$x = 14$ or $x = -14$

The solution set is $\{-14, 14\}$.

7. $|4x| = 20$

$4x = 20$ or $4x = -20$

$x = 5$ or $x = -5$

The solution set is $\{-5, 5\}$.

8. $|5x| = 30$

$5x = 30$ or $5x = -30$

$x = 6$ or $x = -6$

The solution set is $\{-6, 6\}$.

9. $|x-3| = 9$

$x-3 = 9$ or $x-3 = -9$

$x = 12$ or $x = -6$

The solution set is $\{-6, 12\}$.

10. $|x-5| = 13$

$x-5 = 13$ or $x-5 = -13$

$x = 18$ or $x = -8$

The solution set is $\{-8, 18\}$.

11. $|2x-1| = 11$

$2x-1 = 11$ or $2x-1 = -11$

$2x = 12$ $\quad$ $2x = -10$

$x = 6$ or $x = -5$

The solution set is $\{-5, 6\}$.

12. $|2x+3| = 19$

$2x+3 = 19$ or $2x+3 = -19$

$2x = 16$ $\quad$ $2x = -22$

$x = 8$ or $x = -11$

The solution set is $\{-11, 8\}$.

13. $|4x-5|=17$

$4x-5=17$ or $4x-5=-17$

$4x=22$ $\quad$ $4x=-12$

$x=\dfrac{22}{4}=\dfrac{11}{2}$ or $x=-3$

The solution set is $\left\{-3, \dfrac{11}{2}\right\}$.

14. $|5x-1|=21$

$5x-1=21$ or $5x-1=-21$

$5x=22$ $\quad$ $5x=-20$

$x=\dfrac{22}{5}$ or $x=-4$

The solution set is $\left\{-4, \dfrac{22}{5}\right\}$.

15. $|2x+5|=14$

$2x+5=14$ or $2x+5=-14$

$2x=9$ $\quad$ $2x=-19$

$x=\dfrac{9}{2}$ or $x=-\dfrac{19}{2}$

The solution set is $\left\{-\dfrac{19}{2}, \dfrac{9}{2}\right\}$.

16. $|2x-9|=18$

$2x-9=18$ or $2x-9=-18$

$2x=27$ $\quad$ $2x=-9$

$x=\dfrac{27}{2}$ or $x=-\dfrac{9}{2}$

The solution set is $\left\{-\dfrac{9}{2}, \dfrac{27}{2}\right\}$.

17. $|-3x+8|=1$

$-3x+8=1$ or $-3x+8=-1$

$-3x=-7$ $\quad$ $-3x=-9$

$x=\dfrac{7}{3}$ or $x=3$

The solution set is $\left\{\dfrac{7}{3}, 3\right\}$.

18. $|-6x+5|=4$

$-6x+5=4$ or $-6x+5=-4$

$-6x=-1$ $\quad$ $-6x=-9$

$x=\dfrac{1}{6}$ or $x=\dfrac{3}{2}$

The solution set is $\left\{\dfrac{1}{6}, \dfrac{3}{2}\right\}$.

19. $\left|12-\dfrac{1}{2}x\right|=6$

$12-\dfrac{1}{2}x=6$ or $12-\dfrac{1}{2}x=-6$

$-\dfrac{1}{2}x=-6$ $\quad$ $-\dfrac{1}{2}x=-18$

$x=12$ or $x=36$

The solution set is $\{12, 36\}$.

20. $\left|14-\dfrac{1}{3}x\right|=8$

$14-\dfrac{1}{3}x=8$ or $14-\dfrac{1}{3}x=-8$

$-\dfrac{1}{3}x=-6$ $\quad$ $-\dfrac{1}{3}x=-22$

$x=18$ or $x=66$

The solution set is $\{18, 66\}$.

21. $|0.5x|=6$

$0.5x=6$ or $0.5x=-6$

$x=12$ or $x=-12$

The solution set is $\{-12, 12\}$.

22. $|0.3x|=9$

$0.3x=9$ or $0.3x=-9$

$x=30$ or $x=-30$

The solution set is $\{-30, 30\}$.

23. $\left|\dfrac{1}{2}x+3\right|=2$

$\dfrac{1}{2}x+3=2$ or $\dfrac{1}{2}x+3=-2$

$\dfrac{1}{2}x=-1$ $\quad$ $\dfrac{1}{2}x=-5$

$x=-2$ or $x=-10$

The solution set is $\{-10, -2\}$.

24. $\left|\dfrac{2}{3}x-1\right|=5$

$\dfrac{2}{3}x-1=5$ or $\dfrac{2}{3}x-1=-5$

$\dfrac{2}{3}x=6$ $\quad$ $\dfrac{2}{3}x=-4$

$x=\dfrac{3}{2}(6)$ $\quad$ $x=\dfrac{3}{2}(-4)$

$x=9$ or $x=-6$

The solution set is $\{-6, 9\}$.

25. $\left|1+\frac{3}{4}x\right|=7$

$1+\frac{3}{4}x=7$ or $1+\frac{3}{4}x=-7$

Multiply each side by 4.

$4+3x=28$ or $4+3x=-28$

$3x=24$ $\quad 3x=-32$

$x=8$ or $x=\frac{-32}{3}$

The solution set is $\left\{-\frac{32}{3}, 8\right\}$.

26. $\left|2-\frac{5}{2}x\right|=14$

$2-\frac{5}{2}x=14$ or $2-\frac{5}{2}x=-14$

$-\frac{5}{2}x=12$ $\quad -\frac{5}{2}x=-16$

$x=\left(-\frac{2}{5}\right)(12)$ $\quad x=\left(-\frac{2}{5}\right)(-16)$

$x=-\frac{24}{5}$ or $x=\frac{32}{5}$

The solution set is $\left\{-\frac{24}{5}, \frac{32}{5}\right\}$.

27. $|0.02x-1|=2.50$

$0.02x-1=2.50$ or $0.02x-1=-2.50$

$0.02x=3.50$ $\quad 0.02x=-1.50$

$x=50(3.5)$ $\quad x=50(-1.5)$

$x=175$ or $x=-75$

The solution set is $\{-75, 175\}$.

28. $|0.04x-3|=5.96$

$0.04x-3=5.96$ or $0.04x-3=-5.96$

$0.04x=8.96$ $\quad 0.04x=-2.96$

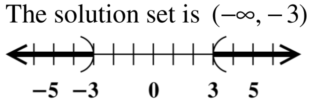
$x=25(8.96)$ $\quad x=25(-2.96)$

$x=224$ or $x=-74$

The solution set is $\{-74, 224\}$.

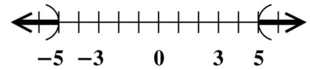
29. $|x|>3 \iff x>3$ or $x<-3$

The solution set is $(-\infty, -3)\cup(3, \infty)$.



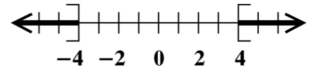
30. $|x|>5 \iff x>5$ or $x<-5$

The solution set is $(-\infty, -5)\cup(5, \infty)$.



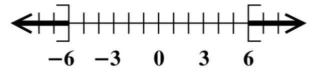
31. $|x|\ge 4 \iff x\ge 4$ or $x\le -4$

The solution set is $(-\infty, -4]\cup[4, \infty)$.



32. $|x|\ge 6 \iff x\ge 6$ or $x\le -6$

The solution set is $(-\infty, -6]\cup[6, \infty)$.

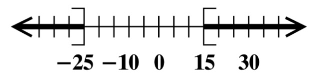


33. $|r+5|\ge 20$

$r+5\le -20$ or $r+5\ge 20$

$r\le -25$ or $r\ge 15$

The solution set is $(-\infty, -25]\cup[15, \infty)$.

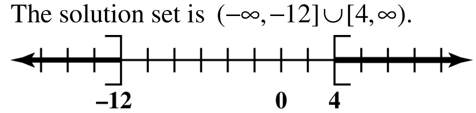


34. $|x+4|\ge 8$

$x+4\ge 8$ or $x+4\le -8$

$x\ge 4$ or $x\le -12$

The solution set is $(-\infty,-12]\cup[4,\infty)$.



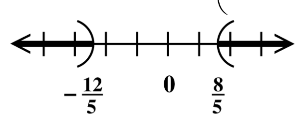
35. $|5x+2|>10$

$5x+2>10$ or $5x+2<-10$

$5x>8$ $\quad 5x<-12$

$x>\frac{8}{5}$ or $x<-\frac{12}{5}$

The solution set is $\left(-\infty, -\frac{12}{5}\right)\cup\left(\frac{8}{5}, \infty\right)$.



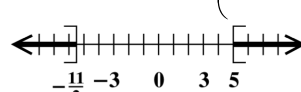
36. $|4x+1|\ge 21$

$4x+1\ge 21$ or $4x+1\le -21$

$4x\ge 20$ $\quad 4x\le -22$

$x\ge 5$ or $x\le -\frac{11}{2}$

The solution set is $\left(-\infty, -\frac{11}{2}\right]\cup[5, \infty)$.



37. $|3-x|>5$

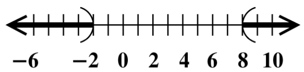
$3-x>5$ or $3-x<-5$

$-x>2$ or $-x<-8$

Multiply by -1, and reverse the inequality symbols.

$x<-2$ or $x>8$

The solution set is $(-\infty,-2)\cup(8,\infty)$.



38. $|5-x|>3$

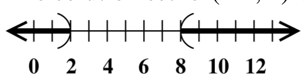
$5-x>3$ or $5-x<-3$

$-x>-2$ $\quad -x<-8$

Multiply by -1, and reverse the inequality symbols.

$x<2$ or $x>8$

The solution set is $(-\infty,2)\cup(8,\infty)$.



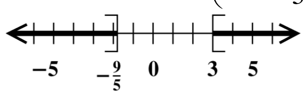
39. $|-5x+3|\ge 12$

$-5x+3\ge 12$ or $-5x+3\le -12$

$-5x\ge 9$ $\quad -5x\le -15$

$x\le -\frac{9}{5}$ or $x\ge 3$

The solution set is $\left(-\infty,-\frac{9}{5}\right]\cup[3,\infty)$.



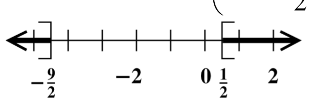
40. $|-2x-4|\ge 5$

$-2x-4\ge 5$ or $-2x-4\le -5$

$-2x\ge 9$ $\quad -2x\le -1$

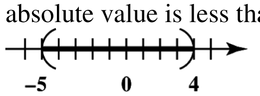
$x\le -\frac{9}{2}$ or $x\ge \frac{1}{2}$

The solution set is $\left(-\infty,-\frac{9}{2}\right]\cup\left[\frac{1}{2},\infty\right)$.



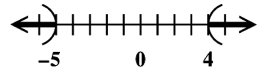
41. **(a)** $|2x+1|<9$

The graph of the solution set will be all numbers between -5 and 4, since the absolute value is less than 9.



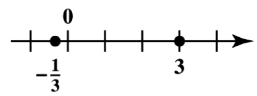
(b) $|2x+1|>9$

The graph of the solution set will be all numbers less than -5 or greater than 4, since the absolute value is greater than 9.



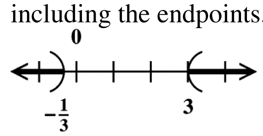
42. **(a)** $|3x-4|=5$

The solutions are the numbers at the endpoints in the given graph, $-\frac{1}{3}$ and 3.



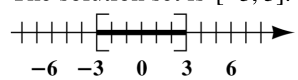
(b) $|3x-4|>5$

The solution set is composed of the numbers not in the given graph, not including the endpoints.



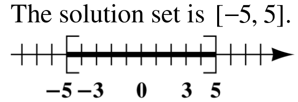
43. $|x|\le 3 \Leftrightarrow -3\le x\le 3$

The solution set is $[-3,3]$.



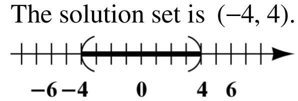
44. $|x|\le 5 \Leftrightarrow -5\le x\le 5$

The solution set is $[-5,5]$.



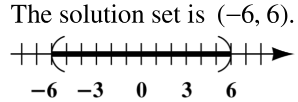
45. $|x|<4 \Leftrightarrow -4<x<4$

The solution set is $(-4,4)$.



46. $|x|<6 \Leftrightarrow -6<x<6$

The solution set is $(-6,6)$.

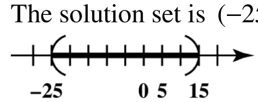


47. $|r+5|<20$

$-20< r+5 <20$

$-25< r <15$ Subtract 5.

The solution set is $(-25,15)$.

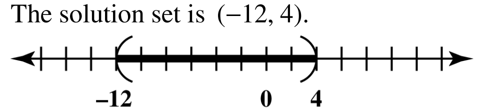


48. $|x+4|<8$

$-8< x+4 <8$

$-12< x <4$

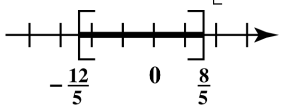
The solution set is $(-12,4)$.



49. $|5x+2| \le 10$

$$\begin{aligned} -10 \le \; & 5x+2 \le 10 \\ -12 \le \; & 5x \le 8 \\ -\frac{12}{5} \le \; & x \le \frac{8}{5} \end{aligned}$$

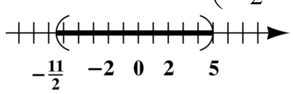
The solution set is $\left[-\frac{12}{5}, \frac{8}{5}\right]$.



50. $|4x+1| < 21$

$$\begin{aligned} -21 < \; & 4x+1 < 21 \\ -22 < \; & 4x < 20 \\ -\frac{11}{2} < \; & x < 5 \end{aligned}$$

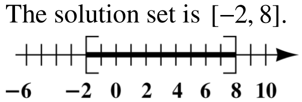
The solution set is $\left(-\frac{11}{2}, 5\right)$.



51. $|3-x| \le 5$

$$\begin{aligned} -5 \le \; & 3-x \le 5 \\ -8 \le \; & -x \le 2 \\ 8 \ge \; & x \ge -2 && \text{Multiply by } -1, \text{ reverse inequalities.} \\ -2 \le \; & x \le 8 && \text{Equivalent inequality} \end{aligned}$$

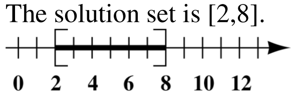
The solution set is $[-2, 8]$.



52. $|5-x| \le 3$

$$\begin{aligned} -3 \le \; & 5-x \le 3 \\ -8 \le \; & -x \le -2 \\ 8 \ge \; & x \ge 2 && \text{Multiply by } -1, \text{ reverse inequalities.} \\ 2 \le \; & x \le 8 && \text{Equivalent inequality} \end{aligned}$$

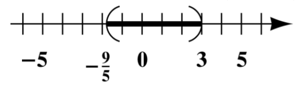
The solution set is [2,8].



53. $|-5x+3| < 12$

$$\begin{aligned} -12 < \; & -5x+3 < 12 \\ -15 < \; & -5x < 9 \\ 3 > \; & x > -\frac{9}{5} && \text{Divide by } -5, \text{ reverse inequalities.} \\ -\frac{9}{5} < \; & x < 3 && \text{Equivalent inequality} \end{aligned}$$

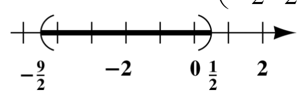
The solution set is $\left(-\frac{9}{5}, 3\right)$.



54. $|-2x-4| < 5$

$$\begin{aligned} -5 < \; & -2x-4 < 5 \\ -1 < \; & -2x < 9 \\ \frac{1}{2} > \; & x > -\frac{9}{2} && \text{Divide by } -2, \text{ reverse inequalities.} \\ -\frac{9}{2} < \; & x < \frac{1}{2} && \text{Equivalent inequality} \end{aligned}$$

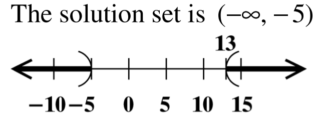
The solution set is $\left(-\frac{9}{2}, \frac{1}{2}\right)$.



55. $|-4+x| > 9$

$$\begin{aligned} -4+x > 9 \quad & \text{or} \quad -4+x < -9 \\ x > 13 \quad & \text{or} \quad x < -5 \end{aligned}$$

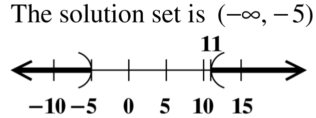
The solution set is $(-\infty, -5) \cup (13, \infty)$.



56. $|-3+x| > 8$

$$\begin{aligned} -3+x > 8 \quad & \text{or} \quad -3+x < -8 \\ x > 11 \quad & \text{or} \quad x < -5 \end{aligned}$$

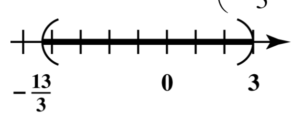
The solution set is $(-\infty, -5) \cup (11, \infty)$.



57. $|3x+2| < 11$

$$\begin{aligned} -11 < \; & 3x+2 < 11 \\ -13 < \; & 3x < 9 \\ -\frac{13}{3} < \; & x < 3 \end{aligned}$$

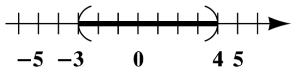
The solution set is $\left(-\frac{13}{3}, 3\right)$.



58. $|2x-1|<7$

$$\begin{aligned} -7 < {} & 2x-1 < 7 \\ -6 < {} & 2x < 8 \\ -3 < {} & x < 4 \end{aligned}$$

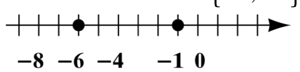
The solution set is $(-3, 4)$.



59. $|7+2x|=5$

$$\begin{array}{lcl} 7+2x=5 & \text{or} & 7+2x=-5 \\ 2x=-2 & & 2x=-12 \\ x=-1 & \text{or} & x=-6 \end{array}$$

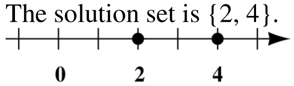
The solution set is $\{-6, -1\}$.



60. $|9-3x|=3$

$$\begin{array}{lcl} 9-3x=3 & \text{or} & 9-3x=-3 \\ -3x=-6 & & -3x=-12 \\ x=2 & \text{or} & x=4 \end{array}$$

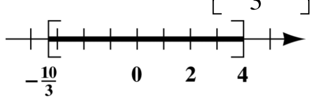
The solution set is $\{2, 4\}$.



61. $|3x-1|\le 11$

$$\begin{aligned} -11 \le {} & 3x-1 \le 11 \\ -10 \le {} & 3x \le 12 \\ -\frac{10}{3} \le {} & x \le 4 \end{aligned}$$

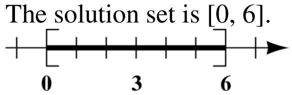
The solution set is $\left[-\frac{10}{3}, 4\right]$.



62. $|2x-6|\le 6$

$$\begin{aligned} -6 \le {} & 2x-6 \le 6 \\ 0 \le {} & 2x \le 12 \\ 0 \le {} & x \le 6 \end{aligned}$$

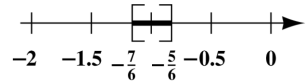
The solution set is $[0, 6]$.



63. $|-6x-6|\le 1$

$$\begin{aligned} -1 \le {} & -6x-6 \le 1 \\ 5 \le {} & -6x \le 7 \\ -\frac{5}{6} \ge {} & x \ge -\frac{7}{6} && \text{Divide by } -6. \text{ Reverse inequalities.} \\ -\frac{7}{6} \le {} & x \le -\frac{5}{6} && \text{Equivalent inequality} \end{aligned}$$

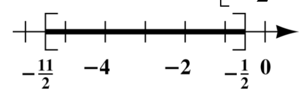
The solution set is $\left[-\frac{7}{6}, -\frac{5}{6}\right]$.



64. $|-2x-6|\le 5$

$$\begin{aligned} -5 \le {} & -2x-6 \le 5 \\ 1 \le {} & -2x \le 11 \\ -\frac{1}{2} \ge {} & x \ge -\frac{11}{2} && \text{Divide by } -2. \text{ Reverse inequalities.} \\ -\frac{11}{2} \le {} & x \le -\frac{1}{2} && \text{Equivalent inequality} \end{aligned}$$

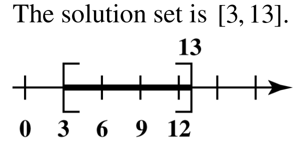
The solution set is $\left[-\frac{11}{2}, -\frac{1}{2}\right]$.



65. $|-8+x|\le 5$

$$\begin{aligned} -5 \le {} & -8+x \le 5 \\ 3 \le {} & x \le 13 \end{aligned}$$

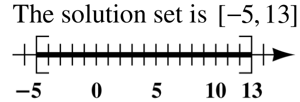
The solution set is $[3, 13]$.



66. $|-4+x|\le 9$

$$\begin{aligned} -9 \le {} & -4+x \le 9 \\ -5 \le {} & x \le 13 \end{aligned}$$

The solution set is $[-5, 13]$.



67. $|10-12x|\ge 4$

$$\begin{array}{lcl} 10-12x\ge 4 & \text{or} & 10-12x\le -4 \\ -12x\ge -6 & & -12x\le -14 \\ x\le \frac{1}{2} & \text{or} & x\ge \frac{7}{6} \end{array}$$

The solution set is $\left(-\infty, \frac{1}{2}\right]\cup\left[\frac{7}{6}, \infty\right)$.

68. $|8-10x|\ge 2$

$$\begin{array}{lcl} 8-10x\ge 2 & \text{or} & 8-10x\le -2 \\ -10x\ge -6 & & -10x\le -10 \\ x\le \frac{3}{5} & \text{or} & x\ge 1 \end{array}$$

The solution set is $\left(-\infty, \frac{3}{5}\right]\cup[1, \infty)$.

69. $|3(x-1)|=8$

$$\begin{aligned} 3(x-1)&=8 &\text{ or }\quad 3(x-1)&=-8\\ x-1&=\frac{8}{3} & x-1&=-\frac{8}{3}\\ x&=\frac{11}{3} &\text{ or }\quad x&=-\frac{5}{3}\end{aligned}$$

The solution set is $\left\{-\frac{5}{3}, \frac{11}{3}\right\}$.

70. $|7(x-2)|=4$

$$\begin{aligned} 7(x-2)&=4 &\text{ or }\quad 7(x-2)&=-4\\ x-2&=\frac{4}{7} & x-2&=-\frac{4}{7}\\ x&=\frac{18}{7} &\text{ or }\quad x&=\frac{10}{7}\end{aligned}$$

The solution set is $\left\{\frac{10}{7}, \frac{18}{7}\right\}$.

71. $|0.1x-1|>3$

$$\begin{aligned} 0.1x-1&>3 &\text{ or }\quad 0.1x-1&<-3\\ 0.1x&>4 & 0.1x&<-2\\ x&>40 &\text{ or }\quad x&<-20\end{aligned}$$

The solution set is $(\infty,-20)\cup(40,\infty)$.

72. $|0.1x+1|>2$

$$\begin{aligned} 0.1x+1&>2 &\text{ or }\quad 0.1x+1&<-2\\ 0.1x&>1 & 0.1x&<-3\\ x&>10 &\text{ or }\quad x&<-30\end{aligned}$$

The solution set is $(\infty,-30)\cup(10,\infty)$.

73. $|x+2|=5-2$

$$\begin{aligned} |x+2|&=3\\ x+2&=3 &\text{ or }\quad x+2&=-3\\ x&=1 &\text{ or }\quad x&=-5\end{aligned}$$

The solution set is $\{-5, 1\}$.

74. $|x+3|=12-2$

$$\begin{aligned} |x+3|&=10\\ x+3&=10 &\text{ or }\quad x+3&=-10\\ x&=7 &\text{ or }\quad x&=-13\end{aligned}$$

The solution set is $\{-13, 7\}$.

75. $3|x-6|=9$

$$\begin{aligned} |x-6|&=3\\ x-6&=3 &\text{ or }\quad x-6&=-3\\ x&=9 &\text{ or }\quad x&=3\end{aligned}$$

The solution set is $\{3, 9\}$.

76. $5|x-4|=5$

$$\begin{aligned} |x-4|&=1\\ x-4&=1 &\text{ or }\quad x-4&=-1\\ x&=5 &\text{ or }\quad x&=3\end{aligned}$$

The solution set is $\{3, 5\}$.

77. $|2-0.2x|=2$

$$\begin{aligned} 2-0.2x&=2 &\text{ or }\quad 2-0.2x&=-2\\ -0.2x&=0 & -0.2x&=-4\\ x&=0 &\text{ or }\quad x&=20\end{aligned}$$

The solution set is $\{0, 20\}$.

78. $|5-0.5x|=4$

$$\begin{aligned} 5-0.5x&=4 &\text{ or }\quad 5-0.5x&=-4\\ -0.5x&=-1 & -0.5x&=-9\\ x&=2 &\text{ or }\quad x&=18\end{aligned}$$

The solution set is $\{2, 18\}$.

79. $|x|-1=4$

$$\begin{aligned} |x|&=5\\ x=5 \quad&\text{or}\quad x=-5\end{aligned}$$

The solution set is $\{-5, 5\}$.

80. $|x|+3=10$

$$\begin{aligned} |x|&=7\\ x=7 \quad&\text{or}\quad x=-7\end{aligned}$$

The solution set is $\{-7, 7\}$.

81. $|x+4|+1=2$

$$\begin{aligned} |x+4|&=1\\ x+4&=1 &\text{ or }\quad x+4&=-1\\ x&=-3 &\text{ or }\quad x&=-5\end{aligned}$$

The solution set is $\{-5, -3\}$.

82. $|x+5|-2=12$

$$\begin{aligned} |x+5|&=14\\ x+5&=14 &\text{ or }\quad x+5&=-14\\ x&=9 &\text{ or }\quad x&=-19\end{aligned}$$

The solution set is $\{-19, 9\}$.

83. $|2x+1|+3>8$

$$\begin{aligned} |2x+1|&>5\\ 2x+1&>5 &\text{ or }\quad 2x+1&<-5\\ 2x&>4 & 2x&<-6\\ x&>2 &\text{ or }\quad x&<-3\end{aligned}$$

The solution set is $(-\infty, -3)\cup(2, \infty)$.

84. $|6x-1|-2>6$

$|6x-1|>8$

$6x-1>8$ or $6x-1<-8$

$6x>9$ $\quad$ $6x<-7$

$x>\frac{3}{2}$ or $x<-\frac{7}{6}$

The solution set is $\left(-\infty,-\frac{7}{6}\right)\cup\left(\frac{3}{2},\infty\right)$.

85. $|x+5|-6\le -1$

$|x+5|\le 5$

$-5\le x+5\le 5$

$-10\le x\le 0$

The solution set is $[-10, 0]$.

86. $|x-2|-3\le 4$

$|x-2|\le 7$

$-7\le x-2\le 7$

$-5\le x\le 9$

The solution set is $[-5, 9]$.

87. $|0.1x-2.5|+0.3\ge 0.8$

$|0.1x-2.5|\ge 0.5$

$0.1x-2.5\ge 0.5$ or $0.1x-2.5\le -0.5$

$0.1x\ge 3$ or $0.1x\le 2$

$x\ge 30$ or $x\le 20$

The solution set is $(-\infty, 20]\cup[30,\infty)$.

88. $|0.5x-3.5|+0.2\ge 0.6$

$|0.5x-3.5|\ge 0.4$

$0.5x-3.5\ge 0.4$ or $0.5x-3.5\le -0.4$

$0.5x\ge 3.9$ or $0.5x\le 3.1$

$x\ge 7.8$ or $x\le 6.2$

$x\ge\frac{39}{5}$ or $x\le\frac{31}{5}$

The solution set is $\left(-\infty,\frac{31}{5}\right]\cup\left[\frac{39}{5},\infty\right)$.

89. $\left|\frac{1}{2}x+\frac{1}{3}\right|+\frac{1}{4}=\frac{3}{4}$

$\left|\frac{1}{2}x+\frac{1}{3}\right|=\frac{1}{2}$

$\frac{1}{2}x+\frac{1}{3}=\frac{1}{2}$ or $\frac{1}{2}x+\frac{1}{3}=-\frac{1}{2}$

$6\left(\frac{1}{2}x+\frac{1}{3}\right)=6\left(\frac{1}{2}\right)$ $\quad$ $6\left(\frac{1}{2}x+\frac{1}{3}\right)=6\left(-\frac{1}{2}\right)$

$3x+2=3$ $\quad$ $3x+2=-3$

$3x=1$ $\quad$ $3x=-5$

$x=\frac{1}{3}$ or $x=-\frac{5}{3}$

The solution set is $\left\{-\frac{5}{3},\frac{1}{3}\right\}$.

90. $\left|\frac{2}{3}x+\frac{1}{6}\right|+\frac{1}{2}=\frac{5}{2}$

$\left|\frac{2}{3}x+\frac{1}{6}\right|=2$

$\frac{2}{3}x+\frac{1}{6}=2$ or $\frac{2}{3}x+\frac{1}{6}=-2$

$6\left(\frac{2}{3}x+\frac{1}{6}\right)=6(2)$ $\quad$ $6\left(\frac{2}{3}x+\frac{1}{6}\right)=6(-2)$

$4x+1=12$ $\quad$ $4x+1=-12$

$4x=11$ $\quad$ $4x=-13$

$x=\frac{11}{4}$ or $x=-\frac{13}{4}$

The solution set is $\left\{-\frac{13}{4},\frac{11}{4}\right\}$.

91. $|3x+1|=|2x+4|$

$3x+1=2x+4$ or $3x+1=-(2x+4)$

$3x+1=-2x-4$

$5x=-5$

$x=3$ or $x=-1$

The solution set is $\{-1, 3\}$.

92. $|7x+12|=|x-8|$

$7x+12=x-8$ or $7x+12=-(x-8)$

$6x=-20$ $\quad$ $7x+12=-x+8$

$8x=-4$

$x=-\frac{10}{3}$ or $x=-\frac{1}{2}$

The solution set is $\left\{-\frac{10}{3},-\frac{1}{2}\right\}$.

93. $\left|x-\frac{1}{2}\right|=\left|\frac{1}{2}x-2\right|$

$x-\frac{1}{2}=\frac{1}{2}x-2$ or $x-\frac{1}{2}=-\left(\frac{1}{2}x-2\right)$

$x-\frac{1}{2}=-\frac{1}{2}x+2$

$2x-1=x-4$ or $2x-1=-x+4$

$3x=5$

$x=-3$ or $x=\frac{5}{3}$

The solution set is $\left\{-3, \frac{5}{3}\right\}$.

94. $\left|\frac{2}{3}x-2\right|=\left|\frac{1}{3}x+3\right|$

$\frac{2}{3}x-2=\frac{1}{3}x+3$ or $\frac{2}{3}x-2=-\left(\frac{1}{3}x+3\right)$

$\frac{1}{3}x=5$ $\quad$ $\frac{2}{3}x-2=-\frac{1}{3}x-3$

$x=15$ or $x=-1$

The solution set is $\{-1,15\}$.

95. $|6x|=|9x+1|$

$6x=9x+1$ or $6x=-(9x+1)$

$-3x=1$ $\quad$ $6x=-9x-1$

$15x=-1$

$x=-\frac{1}{3}$ or $x=-\frac{1}{15}$

The solution set is $\left\{-\frac{1}{3}, -\frac{1}{15}\right\}$.

96. $|13x|=|2x+1|$

$13x=2x+1$ or $13x=-(2x+1)$

$11x=1$ $\quad$ $13x=-2x-1$

$15x=-1$

$x=\frac{1}{11}$ or $x=-\frac{1}{15}$

The solution set is $\left\{-\frac{1}{15}, \frac{1}{11}\right\}$.

97. $|2x-6|=|2x+11|$

$2x-6=2x+11$ or $2x-6=-(2x+11)$

$-6=11$ False $\quad$ $2x-6=-2x-11$

$4x=-5$

No solution or $x=-\frac{5}{4}$

The solution set is $\left\{-\frac{5}{4}\right\}$.

98. $|3x-1|=|3x+9|$

$3x-1=3x+9$ or $3x-1=-(3x+9)$

$-1=9$ False $\quad$ $3x-1=-3x-9$

$6x=-8$

No solution or $x=-\frac{4}{3}$

The solution set is $\left\{-\frac{4}{3}\right\}$.

99. $|x|\ge -10$

The absolute value of a number is always greater than or equal to 0. Therefore, the inequality is true for all real numbers.
The solution set is $(-\infty, \infty)$.

100. $|x|\ge -15$

The absolute value of a number is always greater than or equal to 0. Therefore, the inequality is true for all real numbers.
The solution set is $(-\infty, \infty)$.

101. $|12t-3|=-8$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.
The solution set is $\varnothing$.

102. $|13x+1|=-3$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.
The solution set is $\varnothing$.

103. $|4x+1|=0$

The expression $4x+1$ will equal 0 *only* for the solution of the equation
$4x+1=0.$

$4x=-1$

$x=\frac{-1}{4}$ or $-\frac{1}{4}$

The solution set is $\left\{-\frac{1}{4}\right\}$.

104. $|6x-2|=0$

The expression $6x-2$ will equal 0 *only* for the solution of the equation

$$6x-2=0$$
$$6x=2$$
$$x=\frac{2}{6}=\frac{1}{3}$$

The solution set is $\left\{\frac{1}{3}\right\}$.

105. $|2x-1|=-6$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.
The solution set is $\varnothing$.

106. $|8x+4|=-4$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.
The solution set is $\varnothing$.

107. $|x+5|>-9$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is true for any real number *x*.
The solution set is $(-\infty, \infty)$.

108. $|x+9|>-3$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is true for any real number *x*.
The solution set is $(-\infty, \infty)$.

109. $|7x+3|\le 0$

The absolute value of an expression is always nonnegative (positive or zero), so this inequality is true only when

$$7x+3=0$$
$$7x=-3$$
$$x=-\frac{3}{7}.$$

The solution set is $\left\{-\frac{3}{7}\right\}$.

110. $|4x-1|\le 0$

The absolute value of an expression is always nonnegative (positive or zero), so this inequality is true only when

$$4x-1=0$$
$$4x=1$$
$$x=\frac{1}{4}.$$

The solution set is $\left\{\frac{1}{4}\right\}$.

111. $|5x-2|=0$

The expression $5x-2$ will equal 0 *only* for the solution of the equation

$$5x-2=0.$$
$$5x=2$$
$$x=\frac{2}{5}$$

The solution set is $\left\{\frac{2}{5}\right\}$.

112. $|7x+4|=0$

The expression $7x+4$ will equal 0 *only* for the solution of the equation

$$7x+4=0.$$
$$7x=-4$$
$$x=-\frac{4}{7}$$

The solution set is $\left\{-\frac{4}{7}\right\}$.

113. $|x-2|+3\ge 2$

$$|x-2|\ge -1$$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is true for any real number *x*.
The solution set is $(-\infty,\infty)$.

114. $|x-4|+5\ge 4$

$$|x-4|\ge -1$$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is true for any real number *x*.
The solution set is $(-\infty, \infty)$.

115. $|10x+7|+3<1$

$$|10x+7|<-2$$

There is no number whose absolute value is less than -2, so this inequality has no solution.
The solution set is $\varnothing$.

116. $|4x+1|-2<-5$
$|4x+1|<-3$
There is no number whose absolute value is less than -3, so this inequality has no solution. The solution set is $\varnothing$.

117. $\left|\dfrac{32-x}{32}\right|\le 0.04$

$$\begin{aligned} -0.04 \le &\ \frac{32-x}{32} \le 0.04 \\ -1.28 \le &\ 32-x \le 1.28 \\ -33.28 \le &\ -x \le -30.72 \\ 33.28 \ge &\ x \ge 30.72 \end{aligned}$$

The carton may contain between 30.72 and 33.28 oz.

118. $\left|\dfrac{32-x}{32}\right|\le 0.03$

$$\begin{aligned} -0.03 \le &\ \frac{32-x}{32} \le 0.03 \\ -0.96 \le &\ 32-x \le 0.96 \\ -32.96 \le &\ -x \le -31.04 \\ 31.04 \ge &\ x \ge 32.96 \end{aligned}$$

The carton may contain between 31.04 and 32.96 oz.

119. $\left|\dfrac{32-x}{32}\right|\le 0.025$

$$\begin{aligned} -0.025 \le &\ \frac{32-x}{32} \le 0.025 \\ -0.8 \le &\ 32-x \le 0.8 \\ -32.8 \le &\ -x \le -31.2 \\ 31.2 \ge &\ x \ge 32.8 \end{aligned}$$

The carton may contain between 31.2 and 32.8 oz.

120. $\left|\dfrac{32-x}{32}\right|\le 0.015$

$$\begin{aligned} -0.015 \le &\ \frac{32-x}{32} \le 0.015 \\ -0.48 \le &\ 32-x \le 0.48 \\ -32.48 \le &\ -x \le -31.52 \\ 31.52 \ge &\ x \ge 32.48 \end{aligned}$$

The carton may contain between 31.52 and 32.48 oz.

121. $|y-1|<0.1$

$$\begin{aligned} -0.1 < &\ y-1 < 0.1 \\ 0.9 < &\ y < 1.1 \\ 0.9 < &\ 2x+1 < 1.1 \\ -0.1 < &\ 2x < 0.1 \\ -0.05 < &\ x < 0.05 \end{aligned}$$

Therefore, x must lie in the interval $(-0.05, 0.05)$.

122. $|y-2|<0.02$

$$\begin{aligned} -0.02 < &\ y-2 < 0.02 \\ 1.98 < &\ y < 2.02 \\ 1.98 < &\ 4x-6 < 2.02 \\ 7.98 < &\ 4x < 8.02 \\ 1.995 < &\ x < 2.005 \end{aligned}$$

Therefore, x must lie in the interval $(1.995, 2.005)$.

123. $|y-3|<0.001$

$$\begin{aligned} -0.001 < &\ y-3 < 0.001 \\ 2.999 < &\ y < 3.001 \\ 2.999 < &\ 4x-8 < 3.001 \\ 10.999 < &\ 4x < 11.001 \\ 2.74975 < &\ x < 2.75025 \end{aligned}$$

Therefore, x must lie in the interval $(2.74975, 2.75025)$.

124. $|y-4|<0.0001$

$$\begin{aligned} -0.0001 < &\ y-4 < 0.0001 \\ 3.9999 < &\ y < 4.0001 \\ 3.9999 < &\ 5x+12 < 4.0001 \\ -8.0001 < &\ 5x < -7.9999 \\ -1.60002 < &\ x < -1.59998 \end{aligned}$$

Therefore, x must lie in the interval $(-1.60002, -1.59998)$.

125.

$$\begin{aligned} &\ |y-8.3| < 1.5 \\ -1.5 < &\ y-8.3 < 1.5 \\ 6.8 < &\ y < 9.8 \end{aligned}$$

Therefore, 99% of the babies weighed between 6.8 and 9.8 lb.

126.
$$\begin{aligned} |y+85| &\le 55 \\ -55 \le y+85 &\le 55 \\ -140 \le y &\le -30 \end{aligned}$$
Therefore, the temperature on Mars ranges from $-140°C$ to $-30°C$.

127. Let x represent the calcium intake for a specific female. For x to be within 100 mg of 1000 mg, we must have the following.
$$|x-100| \le 100$$
$$-100 \le x-1000 \le 100$$
$$900 \le x \le 1100$$

128. Let x represent the clotting time for an individual. For x to be within 3.6 seconds of 7.45 seconds, we must have the following.
$$|x-7.45| \le 3.6$$
$$-3.6 \le x-7.45 \le 3.6$$
$$3.85 \le x \le 11.05$$

129. Add the given heights with a calculator to get 8105. There are 10 numbers, so divide the sum by 10.
$$\frac{8105}{10} = 810.5$$
The average height is 810.5 ft.

130. $|x-k| < 50$
Substitute 810.5 for k and solve the inequality.
$$|x-810.5| < 50$$
$$-50 < x-810.5 < 50$$
$$760.5 < x < 860.5$$
The buildings with heights between 760.5 ft and 860.5 ft are Bank of America Center and Texaco Heritage Plaza.

131. $|x-k| < 95$
Substitute 810.5 for k and solve the inequality.
$$|x-810.5| < 95$$
$$-95 < x-810.5 < 95$$
$$715.5 < x < 905.5$$
The buildings with heights between 715.5 ft and 905.5 ft are Williams Tower, Bank of America Center, Texaco Heritage Plaza, Enterprise Plaza, Centerpoint Energy Plaza, Continental Center I, and Fulbright Tower.

132. **(a)** This would be the opposite of the inequality in Exercise 109, that is, $|x-810.5| \ge 95$.

(b) $|x-810.5| \ge 95$
$$x-810.5 \ge 95 \quad \text{or} \quad x-810.5 \le -95$$
$$x \ge 905.5 \quad \text{or} \quad x \le 715.5$$

(c) The buildings that are not within 95 ft of the average have height less than or equal to 715.5 or greater than or equal to 905.5. They are JPMorgan Chase Tower, Wells Fargo Plaza, and One Shell Plaza.

(d) The answer makes sense because it includes all the buildings *not* listed earlier which had heights within 95 ft of the average.

Summary Exercises Solving Linear and Absolute Value Equations and Inequalities

1. $4x+1=49$
$$4x=48$$
$$x=12$$
The solution set is $\{12\}$.

2. $|x-1|=6$
$$x-1=6 \quad \text{or} \quad x-1=-6$$
$$x=7 \quad \text{or} \quad x=-5$$
The solution set is $\{-5, 7\}$.

3. $6x-9=12+3x$
$$3x=21$$
$$x=7$$
The solution set is $\{7\}$.

4. $3x+7=9+8x$
$$-5x=2$$
$$x=-\frac{2}{5}$$
The solution set is $\left\{-\frac{2}{5}\right\}$.

5. $|x+3|=-4$
Since the absolute value of an expression is always nonnegative, there is no number that makes this statement true. Therefore, the solution set is $\varnothing$.

6. $2x+1 \le x$
$$x \le -1$$
The solution set is $(-\infty, -1]$.

7. $8x+2 \ge 5x$
$$3x \ge -2$$
$$x \ge -\frac{2}{3}$$
The solution set is $\left[-\frac{2}{3}, \infty\right)$.

8. $4(x-11)+3x = 20x-31$
$$4x-44+3x = 20x-31$$
$$7x-44 = 20x-31$$
$$-13x = 13$$
$$x = -1$$
The solution set is $\{-1\}$.

9. $2x-1=-7$
$$2x=-6$$
$$x=-3$$
The solution set is $\{-3\}$.

10. $|3x-7|-4=0$
$$|3x-7|=4$$
$$3x-7=-4 \quad \text{or} \quad 3x-7=4$$
$$3x=3 \qquad\qquad 3x=11$$
$$x=1 \quad \text{or} \quad x=\frac{11}{3}$$
The solution set is $\left\{1, \frac{11}{3}\right\}$.

11. $6x-5 \le 3x+10$
$$3x \le 15$$
$$x \le 5$$
The solution set is $(-\infty, 5]$.

12. $|5x-8|+9 \ge 7$
$$|5x-8| \ge -2$$
The absolute value of an expression is always nonnegative, so the inequality is true for any real number x.
The solution set is $(-\infty, \infty)$.

13. $9x-3(x+1)=8x-7$
$$9x-3x-3=8x-7$$
$$6x-3=8x-7$$
$$4=2x$$
$$2=x$$
The solution set is $\{2\}$.

14. $|x| \ge 8 \Leftrightarrow \quad x \ge 8 \quad \text{or} \quad x \le -8$
The solution set is $(-\infty, -8] \cup [8, \infty)$.

15. $9x-5 \ge 9x+3$
$$-5 \ge 3 \quad \text{False}$$
This is a false statement, so the inequality is a contradiction.
The solution set is $\varnothing$.

16. $13x-5 > 13x-8$
$$-5 > -8$$
This inequality is true for every value of x. The solution set is $(-\infty, \infty)$.

17. $|x| < 5.5$
$$-5.5 < x < 5.5$$
The solution set is $(-5.5, 5.5)$.

18. $4x-1=12+x$
$$3x=13$$
$$x=\frac{13}{3}$$
The solution set is $\left\{\frac{13}{3}\right\}$.

19. $\frac{2}{3}x+8=\frac{1}{4}x$
$$8x+96=3x \qquad \text{Multiply by 12.}$$
$$5x=-96$$
$$x=-\frac{96}{5}$$
The solution set is $\left\{-\frac{96}{5}\right\}$.

20. $-\frac{5}{8}x \ge -20$
$$-\frac{8}{5}\left(-\frac{5}{8}x\right) \le -\frac{8}{5}(-20)$$
$$x \le 32$$
The solution set is $(-\infty, 32]$.

21. $\frac{1}{4}x < -6$
$$4\left(\frac{1}{4}x\right) < 4(-6)$$
$$x \le -24$$
The solution set is $(-\infty, -24)$.

22. $\frac{1}{2} \le \frac{2}{3}x \le \frac{5}{4}$

$6 \le 8x \le 15$ Multiply by 12.

$\frac{6}{8} \le x \le \frac{15}{8}$

$\frac{3}{4} \le x \le \frac{15}{8}$

The solution set is $\left[\frac{3}{4}, \frac{15}{8}\right]$.

23. $\frac{3}{5}x - \frac{1}{10} = 2$

$6x - 1 = 20$ Multiply by 10.

$6x = 21$

$x = \frac{21}{6} = \frac{7}{2}$

The solution set is $\left\{\frac{7}{2}\right\}$.

24. $\frac{x}{6} - \frac{3x}{5} = x - 86$

$5x - 18x = 30x - 2580$ Multiply by 30.

$-43x = -2580$

$x = \frac{-2580}{-43} = 60$

The solution set is $\{60\}$.

25. $x + 9 + 7x = 4(3 + 2x) - 3$

$8x + 9 = 12 + 8x - 3$

$8x + 9 = 8x + 9$

$0 = 0$ True

The last statement is true for any real number x.
The solution set is {all real numbers}.

26. $6 - 3(2 - x) < 2(1 + x) + 3$

$6 - 6 + 3x < 2 + 2x + 3$

$3x < 5 + 2x$

$x < 5$

The solution set is $(-\infty, 5)$.

27. $-6 \le \frac{3}{2} - x \le 6$

$-\frac{15}{2} \le -x \le \frac{9}{2}$ Subtract $\frac{3}{2}$.

$\frac{15}{2} \ge x \ge -\frac{9}{2}$ Multiply by -1. Reverse inequalities.

$-\frac{9}{2} \le x \le \frac{15}{2}$ Equivalent inequality

The solution set is $\left[-\frac{9}{2}, \frac{15}{2}\right]$.

28. $\frac{x}{4} - \frac{2x}{3} = -10$

$3x - 8x = -120$ Multiply by 12.

$-5x = -120$

$x = 24$

The solution set is $\{24\}$.

29. $|5x + 1| \le 0$

The expression $|5x + 1|$ is never less than 0 since an absolute value expression must be nonnegative. However, $|5x + 1| = 0$ if

$5x + 1 = 0$

$5x = -1$

$x = \frac{-1}{5} = -\frac{1}{5}$.

The solution set is $\left\{-\frac{1}{5}\right\}$.

30. $5x - (3 + x) \ge 2(3x + 1)$

$5x - 3 - x \ge 6x + 2$

$4x - 3 \ge 6x + 2$

$-2x \ge 5$

$x \le -\frac{5}{2}$

The solution set is $\left(-\infty, -\frac{5}{2}\right]$.

31. $-2 \le 3x - 1 \le 8$

$-1 \le 3x \le 9$

$-\frac{1}{3} \le x \le 3$

The solution set is $\left[-\frac{1}{3}, 3\right]$.

32.
$$\begin{aligned} -1 \le\ & 6 - x \le 5 \\ -7 \le\ & -x \le -1 \\ 7 \ge\ & x \ge 1 \\ 1 \le\ & x \le 7 \end{aligned}$$
The solution set is [1, 7].

33. $|7x-1| = |5x+3|$
$$\begin{array}{lcl} 7x-1=5x+3 & \text{or} & 7x-1=-(5x+3) \\ 2x=4 & & 7x-1=-5x-3 \\ & & 12x=-2 \\ x=2 & \text{or} & x=\dfrac{-2}{12}=-\dfrac{1}{6} \end{array}$$
The solution set is $\left\{-\frac{1}{6}, 2\right\}$.

34. $|x+2| = |x+4|$
$$\begin{array}{lcl} x+2=x+4 & \text{or} & x+2=-(x+4) \\ 2=4 \ \text{False} & & x+2=-x-4 \\ & & 2x=-6 \\ \text{No solution} & \text{or} & x=-3 \end{array}$$
The solution set is $\{-3\}$.

35. $|1-3x| \ge 4$
$$\begin{array}{lcl} 1-3x \ge 4 & \text{or} & 1-3x \le -4 \\ -3x \ge 3 & & -3x \le -5 \\ x \le -1 & \text{or} & x \ge \dfrac{5}{3} \end{array}$$
The solution set is $(-\infty, -1] \cup \left[\frac{5}{3}, \infty\right)$.

36.
$$\begin{aligned} 7x-3+2x &= 9x-8x \\ 9x-3 &= x \\ 8x &= 3 \\ x &= \frac{3}{8} \end{aligned}$$
The solution set is $\left\{\frac{3}{8}\right\}$.

37.
$$\begin{aligned} -(x+4)+2 &= 3x+8 \\ -x-4+2 &= 3x+8 \\ -x-2 &= 3x+8 \\ -10 &\le 4x \\ x &= \frac{-10}{4} = -\frac{5}{2} \end{aligned}$$
The solution set is $\left\{-\frac{5}{2}\right\}$.

38. $|x-1| < 7$
$$\begin{aligned} -7 <\ & x-1 < 7 \\ -6 <\ & x < 8 \end{aligned}$$
The solution set is $(-6, 8)$.

39. $|2x-3| > 11$
$$\begin{array}{lcl} 2x-3>11 & \text{or} & 2x-3<-11 \\ 2x>14 & & 2x<-8 \\ x>7 & \text{or} & x<-4 \end{array}$$
The solution set is $(-\infty, -4) \cup (7, \infty)$.

40. $|5-x| < 4$
$$\begin{array}{ll} -4 < 5-x < 4 & \\ -9 < -x < -1 & \text{Subtract 5.} \\ 9 > x > 1 & \text{Multiply by } -1. \text{ Reverse inequalities.} \\ 1 < x < 9 & \text{Equivalent inequality} \end{array}$$
The solution set is (1, 9).

41. $|x-1| \ge -6$
The absolute value of an expression is always nonnegative, so the inequality is true for any real number x.
The solution set is $(-\infty, \infty)$.

42. $|2x-5| = |x+4|$
$$\begin{array}{lcl} 2x-5=x+4 & \text{or} & 2x-5=-(x+4) \\ & & 2x-5=-x-4 \\ & & 3x=1 \\ x=9 & \text{or} & x=\dfrac{1}{3} \end{array}$$
The solution set is $\left\{\frac{1}{3}, 9\right\}$.

43.
$$\begin{aligned} 8x-(1-x) &= 3(1+3x)-4 \\ 8x-1+x &= 3+9x-4 \\ 9x-1 &= 9x-1 \quad \text{True} \end{aligned}$$
This is an identity.
The solution set is {all real numbers}.

44. $$\begin{aligned} 8x-(x+3) &= -(2x+1)-12 \\ 8x-x-3 &= -2x-1-12 \\ 7x-3 &= -2x-13 \\ 9x &= -10 \\ x &= -\frac{10}{9} \end{aligned}$$

The solution set is $\left\{-\frac{10}{9}\right\}$.

45. $|x-5| = |x+9|$

$$\begin{array}{lcl} x-5 = x+9 & \text{or} & x-5 = -(x+9) \\ -5 = 9 \quad \text{False} & & x-5 = -x-9 \\ & & 2x = -4 \\ \text{No solution} & \text{or} & x = -2 \end{array}$$

The solution set is $\{-2\}$.

46. $|x+2| < -3$

There are no numbers whose absolute value is negative, so this inequality has no solution. The solution set is $\varnothing$.

47. $$\begin{array}{lcl} 2x+1 > 5 & \text{or} & 3x+4 < 1 \\ 2x > 4 & & 3x < -3 \\ x > 2 & \text{or} & x < -1 \end{array}$$

The solution set is $(-\infty, -1) \cup (2, \infty)$.

48. $$\begin{array}{lcl} 1-2x \geq 5 & \text{and} & 7+3x \geq -2 \\ -2x \geq 4 & & 3x \geq -9 \\ x \leq -2 & \text{and} & x \geq -3 \end{array}$$

This is an intersection.
The solution set is $[-3, -2]$.

Chapter 1 Review Exercises

1. $$\begin{aligned} -(8+3x)+5 &= 2x+6 \\ -8-3x+5 &= 2x+6 \\ -3x-3 &= 2x+6 \\ -5x &= 9 \\ x &= -\frac{9}{5} \end{aligned}$$

The solution set is $\left\{-\frac{9}{5}\right\}$.

2. $$\begin{aligned} -\frac{3}{4}x &= -12 \\ -3x &= -48 \quad \text{Multiply by 4.} \\ x &= 16 \end{aligned}$$

The solution set is $\{16\}$.

3. $$\begin{aligned} \frac{2x+1}{3} - \frac{x-1}{4} &= 0 \\ 4(2x+1)-3(x-1) &= 0 \quad \text{Multiply by 12.} \\ 8x+4-3x+3 &= 0 \\ 5x+7 &= 0 \\ 5x &= -7 \\ x &= -\frac{7}{5} \end{aligned}$$

The solution set is $\left\{-\frac{7}{5}\right\}$.

4. $$\begin{aligned} 5(2x-3) &= 6(x-1)+4x \\ 10x-15 &= 6x-6+4x \\ 10x-15 &= 10x-6 \\ -15 &= -6 \quad \text{False} \end{aligned}$$

This is a false statement, so the equation is a contradiction.
The solution set is $\varnothing$.

5. $$\begin{aligned} 10x-3(2x-4) &= 4(x+3) \\ 10x-6x+12 &= 4x+12 \\ 4x+12 &= 4x+12 \quad \text{True} \end{aligned}$$

This equation is an *identity*.
The solution set is {all real numbers}.

6. $$\begin{aligned} 13x-(x+7) &= 12x-13 \\ 13x-x-7 &= 12x-13 \\ 12x-7 &= 12x-13 \\ -7 &= -13 \quad \text{False} \end{aligned}$$

This equation is a *contradiction*.
The solution set is $\varnothing$.

7. $$\begin{aligned} x+6(x-1)-(4-x) &= -10 \\ x+6x-6-4+x &= -10 \\ 8x-10 &= -10 \\ 8x &= 0 \\ x &= 0 \end{aligned}$$

This equation is a *conditional* equation.
The solution set is $\{0\}$.

8. $5+3(2x+6) = 5+3(2x+6)$ True

This equation is an *identity*.
The solution set is {all real numbers}.

9. Solve $V = LWH$ for L.

$$\frac{V}{WH} = \frac{LWH}{WH}$$

$$\frac{V}{WH} = L, \quad \text{or} \quad L = \frac{V}{WH}$$

10. Solve $A = \frac{1}{2}h(b+B)$ for b.

$2A = h(b+B)$ Multiply by 2.

$2A = hb + hB$ Distributive prop.

$2A - hB = hb$ Subtract hB.

$\frac{2A - hB}{h} = b$ Divide by h.

Another method for solving is the following.

$2A = h(b+B)$ Multiply by 2.

$\frac{2A}{h} = b + B$ Divide by h.

$\frac{2A}{h} - B = b,$ Subtract B.

11. Solve $P = a + b + c + B$ for c.

$P = a + b + c + B$

$c = P - a - b - B$ Subtract a, b, and B.

12. Solve $4x + 7y = 9$ for y.

$4x + 7y = 9$

$7y = 9 - 4x$ Subtract $4x$.

$y = \frac{9}{7} - \frac{4x}{7}$ Divide by 7.

13. Use the formula $V = LWH$ and substitute 180 for V, 6 for L, and 5 for W.

$180 = 6(5)H$

$180 = 30H$

$6 = H$

The height is 6 feet.

14. Divide the amount of decrease by the original amount (amounts in millions).

$\frac{20.1 - 19.5}{20.1} = \frac{0.6}{20.1} \approx 0.2985$

The percent decrease was about 3.0%.

15. Use the formula $I = prt$. Substitute \$7800 for I, \$30,000 for p, and 4 for t. Solve for r.

$I = prt$

$\$7800 = (\$30{,}000)r(4)$

$7800 = 120{,}000r$

$r = \frac{7800}{120{,}000} = 0.065$

The rate is 6.5%.

16. Use the formula $C = \frac{5}{9}(F - 32)$ and substitute 77 for F.

$C = \frac{5}{9}(77 - 32) = \frac{5}{9}(45) = 25$

The Celsius temperature is 25°.

17. The amount of money spent on Social Security in 2012 was about $0.219(\$3540 \text{ billion}) \approx \$775.3$ billion.

18. The amount of money spent on education and social services in 2012 was about $0.026(\$3540 \text{ billion}) \approx \$92.0$ billion.

19. "One-third of a number, subtracted from 9" is written $9 - \frac{1}{3}x$.

20. "The product of 4 and a number, divided by 9 more than the number" is written $\frac{4x}{x+9}$.

21. Let x = the width of the rectangle.
Then $2x - 3 =$ the length of the rectangle.
Use the formula $P = 2L + 2W$ with $P = 42$.

$42 = 2(2x - 3) + 2x$

$42 = 4x - 6 + 2x$

$48 = 6x$

$8 = x$

The width is 8 meters and the length is $2(8) - 3 = 13$ meters.

22. Let x = the length of each equal side. Then $2x - 15 =$ the length of the third side.
Use the formula $P = a + b + c$ with $P = 53$.

$53 = x + x + (2x - 15)$

$53 = 4x - 15$

$68 = 4x$

$17 = x$

The lengths of the three sides are 17 inches, 17 inches, and $2(17) - 15 = 19$ inches.

23. Let x = the number of kilograms of peanut clusters. Then $3x$ is the number of kilograms of chocolate creams.
The clerk has a total of 48 kg.

$x + 3x = 48$

$4x = 48$

$x = 12$

The clerk has 12 kilograms of peanut clusters.

24. Let x = the number of liters of the 20% solution. Make a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Chemical
x	0.20	$0.20x$
15	0.50	$0.50(15) = 7.5$
$x+15$	0.30	$0.30(x+15)$

The last column gives the equation.

$$0.20x + 7.5 = 0.30(x+15)$$
$$0.20x + 7.5 = 0.30x + 4.5$$
$$3 = 0.10x$$
$$30 = x$$

30 L of the 20% solution should be used.

25. Let x = the number of liters of water.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
30	0.40	$0.40(30) = 12$
x	0	$0(x) = 0$
$30 + x$	0.30	$0.30(30 + x)$

The last column gives the equation.

$$12 + 0 = 0.30(30 + x)$$
$$12 = 9 + 0.3x$$
$$3 = 0.3x$$
$$10 = x$$

10 L of water should be added.

26. Let x = the amount invested at 6%. Then $x - 4000 =$ the amount invested at 4%.

Principal	Rate (as a decimal)	Interest
x	0.06	$0.06x$
$x - 4000$	0.04	$0.04(x - 4000)$
	Total →	\$840

The last column gives the equation.

$$0.06x + 0.04(x - 4000) = 840$$
$$6x + 4(x - 4000) = 84,000$$
$$6x + 4x - 16,000 = 84,000$$
$$10 = 100,000$$
$$x = 10,000$$

Eric should invest \$10,000 at 6% and $\$10,000 - \$4000 = \$6000$ at 4%.

27. Let x = the number of quarters. Then $2x - 1$ is the number of dimes.

Number of Coins	Denomination	Value
x	0.25	$0.25x$
$2x - 1$	0.10	$0.10(2x - 1)$
	Total →	3.50

The sum of the values equals the total value.

$$0.25x + 0.10(2x - 1) = 3.50$$
$$25x + 10(2x - 1) = 350 \quad \text{Multiply by 100.}$$
$$25x + 20x - 10 = 350$$
$$45x = 360$$
$$x = 8$$

There are 8 quarters and $2(8) - 1 = 15$ dimes.

Check: $8(0.25) + 15(0.10) = 3.50$

28. Let x = the number of nickels.
Then $19 - x$ is the number of dimes.

Number of Coins	Denomination	Value
x	0.05	$0.05x$
$19 - x$	0.10	$0.10(19 - x)$
	Total →	1.55

The sum of the values equals the total value.

$$0.05x + 0.10(19 - x) = 1.55$$
$$5x + 10(19 - x) = 155 \quad \text{Multiply by 100.}$$
$$5x + 190 - 10x = 155$$
$$-5x = -35$$
$$x = 7$$

He had 7 nickels and $19 - 7 = 12$ dimes.
Check: $7(0.05) + 12(0.10) = 1.55$

29. Use the formula $d = rt$ or $r = \frac{d}{t}$. Here, d is about 400 mi and t is about 8 hr. Since $\frac{400}{8} = 50$, the best estimate is choice A.

30. Use the formula $d = rt$. Here, $r = 53$ mph and $t = 10$ hr.
$d = 53(10) = 530$
The distance is 530 miles.

31. Use the formula $d = rt$. Here, $r = 164$ mph and $t = 2$ hr.
$d = 164(2) = 328$
The distance is 328 miles.

32. Let x = the time it takes for the trains to be 297 mi apart.
Use the formula $d = rt$.

	Rate	Time	Distance
Passenger Train	60	x	$60x$
Freight Train	75	x	$75x$
			297

The total distance traveled is the sum of the distances traveled by each train.

$$60x + 75x = 297$$
$$135x = 297$$
$$x = 2.2$$

It will take the trains 2.2 hours before they are 297 miles apart.

33. Let x = the rate of the faster car and $x-15 =$ the rate of the slower car. Make a table.

	Rate	Time	Distance
Faster Car	x	2	$2x$
Slower Car	$x-15$	2	$2(x-15)$
			230

The total distance traveled is the sum of the distances traveled by each car.

$$2x + 2(x-15) = 230$$
$$2x + 2x - 30 = 230$$
$$4x = 260$$
$$x = 65$$

The faster car travels at 65 km/hr, while the slower car travels at $65-15 = 50$ km/hr.
Check: $2(65) + 2(50) = 230$

34. Let x = the average rate for the first hour. Then $x-7$ the average rate for the second hour. Using $d = rt$, the distance traveled for the first hour is $x(1)$, for the second hour is $(x-7)(1)$, and for the whole trip, 85.

$$x + (x-7) = 85$$
$$2x - 7 = 85$$
$$2x = 92$$
$$x = 46$$

The average rate for the first hour was 46 mph.
Check: 46 mph for 1 hour = 46 miles and $46-7 = 39$ mph for 1 hour = 39 miles; $46+39 = 85$.

35. The sum of the angles in a triangle is 180°.

$$(3x+7) + (4x+1) + (9x-4) = 180$$
$$16x + 4 = 180$$
$$16x = 176$$
$$x = 11$$

The first angle is $3(11)+7 = 40°$.
The second angle is $4(11)+1 = 45°$.
The third angle is $9(11)-4 = 95°$.

36. The marked angles are supplements which have a sum of 180°.

$$(15x+15) + (3x+3) = 180$$
$$18x + 18 = 180$$
$$18x = 162$$
$$x = 9$$

The angle measures are $15(9)+15 = 150°$ and $3(9)+3 = 30°$.

37. $-\frac{2}{3}x < 6$
$-2x < 18$ Multiply by 3.
Divide by -2; reverse the inequality symbol.
$x > -9$
The solution set is $(-9, \infty)$.

38. $-5x - 4 \geq 11$
$-5x \geq 15$
Divide by -5; reverse the inequality symbol.
$x \leq -3$
The solution set is $(-\infty, -3]$.

39. $\frac{6x+3}{-4} < -3$
Multiply by -4; reverse the inequality symbol.

$$6x + 3 > 12$$
$$6x > 9$$
$$x > \frac{9}{6} = \frac{3}{2}$$

The solution set is $\left(\frac{3}{2}, \infty\right)$.

40.

$$5 - (6 - 4x) \geq 2x - 7$$
$$5 - 6 + 4x \geq 2x - 7$$
$$4x - 1 \geq 2x - 7$$
$$2x \geq -6$$
$$x \geq -3$$

The solution set is $[-3, \infty)$.

41. $$\begin{aligned} 8 \le\ & 3x-1 < 14 \\ 9 \le\ & 3x < 15 \\ 3 \le\ & x < 5 \end{aligned}$$

The solution set is [3, 5).

42. $$\frac{5}{3}(x-2)+\frac{2}{5}(x+1) > 1$$

$$25(x-2)+6(x+1) > 15 \quad \text{Multiply by 15}$$

$$\begin{aligned} 25x-50+6x+6 &> 15 \\ 31x-44 &> 15 \\ 31x &> 59 \\ x &> \frac{59}{31} \end{aligned}$$

The solution set is $\left(\frac{59}{31}, \infty\right)$.

43. **(a)** A small frame corresponds to a wrist size that is less than 6.75 inches. Let x be the wrist size. Then the corresponding inequality is $x < 6.75$.

(b) A medium frame corresponds to a wrist size that is between 6.75 and 7.25 inches, inclusive. Let x be the wrist size. Then the corresponding inequality is $6.75 \le x \le 7.25$.

(c) A large frame corresponds to a wrist size that is greater than 7.25 inches. Let x be the wrist size. Then the corresponding inequality is $x > 7.25$.

44. Let x = the other dimension of the rectangle. One dimension of the rectangle is 22 and the perimeter can be no greater than 120.

$$\begin{aligned} P &\le 120 \\ 2L+2W &\le 120 \\ 2(x)+2(22) &\le 120 \\ 2x+44 &\le 120 \\ 2x &\le 76 \\ x &\le 38 \end{aligned}$$

The other dimension must be 38 meters or less.

45. Let x = the number of tickets that can be purchased. The total cost of the tickets is \$51 per ticket. Staying within the available \$1600, we have the following inequality.

$$51x \le 1600$$

Solve the inequality.

$$\begin{aligned} 51x &\le 1600 \\ x &\le \frac{1600}{51} \\ x &\le 31.37 \end{aligned}$$

The group can purchase 31 tickets or fewer (but at least 15).

46. Let x = the student's score on the fifth test. The average of the five test scores must be at least 70. The inequality is the following.

$$\frac{75+79+64+71+x}{5} \ge 70$$

$$\begin{aligned} 75+79+64+71+x &\ge 350 \\ 289+x &\ge 350 \\ x &\ge 61 \end{aligned}$$

The student will pass algebra if any score greater than or equal to 61 on the fifth test is achieved.

47. $$\begin{aligned} A \cap B &= \{\text{a, b, c, d}\} \cap \{\text{a, c, e, f}\} \\ &= \{\text{a, c}\} \end{aligned}$$

48. $$\begin{aligned} A \cap C &= \{\text{a, b, c, d}\} \cap \{\text{a, e, f, g}\} \\ &= \{\text{a}\} \end{aligned}$$

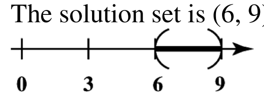
49. $$\begin{aligned} B \cup C &= \{\text{a, c, e, f}\} \cup \{\text{a, e,f g}\} \\ &= \{\text{a, c, e, f, g}\} \end{aligned}$$

50. $$\begin{aligned} A \cup C &= \{\text{a, b, c, d}\} \cup \{\text{a, e, f, g}\} \\ &= \{\text{a, b, c, d, e, f, g}\} \end{aligned}$$

51. $x > 6$ and $x < 9$

The graph of the solution set will be all numbers which are both greater than 6 and less than 9. The overlap is the numbers between 6 and 9, not including the endpoints.

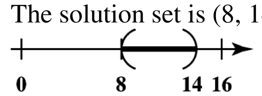
The solution set is (6, 9).



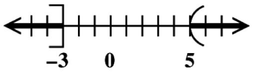
52. $$\begin{aligned} x+4 > 12 \quad &\text{and} \quad x-2 < 12 \\ x > 8 \quad &\text{and} \quad x < 14 \end{aligned}$$

The graph of the solution set will be all numbers between 8 and 14, not including the endpoints.

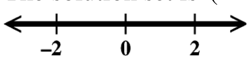
The solution set is (8, 14).



53. $x > 5$ or $x \le -3$
The graph of the solution set will be all numbers that are either greater than 5 or less than or equal to -3.
The solution set is $(-\infty, -3] \cup (5, \infty)$.

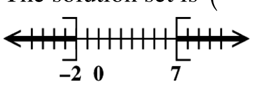


54. $x \ge -2$ or $x < 2$
The graph of the solution set will be all numbers that are either greater than or equal to -2 or less than 2. All real numbers satisfy these criteria.
The solution set is $(-\infty, \infty)$.



55. $x - 4 > 6$ and $x + 3 \le 10$
$x > 10$ and $x \le 7$
The graph of the solution set will be all numbers that are both greater than 10 and less than or equal to 7. There are no real numbers satisfying these criteria.
The solution set is $\varnothing$.

56. $-5x + 1 \ge 11$ or $3x + 5 \ge 26$
$-5x \ge 10$ $\quad$ $3x \ge 21$
$x \le -2$ or $x \ge 7$
The graph of the solution set will be all numbers that are either less than or equal to -2 or greater than or equal to 7.
The solution set is $(-\infty, -2] \cup [7, \infty)$.



57. $(-3, \infty) \cap (-\infty, 4)$
$(-3, \infty)$ includes all real numbers greater than -3. $(-\infty, 4)$ includes all real numbers less than 4. Find the intersection. The numbers common to both sets are greater than -3 and less than 4.
$-3 < x < 4$
The solution set is $(-3, 4)$.

58. $(-\infty, 6) \cap (-\infty, 2)$
$(-\infty, 6)$ includes all real numbers less than 6.
$(-\infty, 2)$ includes all real numbers less than 2.
Find the intersection. The numbers common to both sets are less than 2.
The solution set is $(-\infty, 2)$.

59. $(4, \infty) \cup (9, \infty)$
$(4, \infty)$ includes all real numbers greater than 4. $(9, \infty)$ includes all real numbers greater than 9. Find the union. The numbers in the first set, the second set, or in both sets are all the real numbers that are greater than 4.
The solution set is $(4, \infty)$.

60. $(1, 2) \cup (1, \infty)$
(1, 2) includes the real numbers between 1 and 2, not including 1 and 2.
$(1, \infty)$ includes all real numbers greater than 1.
Find the union. The numbers in the first set, the second set, or in both sets are all real numbers greater than 1.
The solution set is $(1, \infty)$.

61. $|x| = 7 \Leftrightarrow x = 7$ or $x = -7$
The solution set is $\{-7, 7\}$.

62. $|x + 2| = 9$
$x + 2 = 9$ or $x + 2 = -9$
$x = 7$ or $x = -11$
The solution set is $\{-11, 7\}$.

63. $|3x - 7| = 8$
$3x - 7 = 8$ or $3x - 7 = -8$
$3x = 15$ $\quad$ $3x = -1$
$x = 5$ or $x = -\frac{1}{3}$
The solution set is $\left\{-\frac{1}{3}, 5\right\}$.

64. $|x - 4| = -12$
Since the absolute value of an expression can never be negative, there are no solutions for this equation.
The solution set is $\varnothing$.

65. $|2x - 7| + 4 = 11$
$|2x - 7| = 7$
$2x - 7 = 7$ or $2x - 7 = -7$
$2x = 14$ $\quad$ $2x = 0$
$x = 7$ or $x = 0$
The solution set is $\{0, 7\}$.

66. $|4x+2|-7=-3$

$|4x+2|=4$

$4x+2=4$ or $4x+2=-4$

$4x=2 \qquad 4x=-6$

$x=\frac{2}{4} \qquad x=-\frac{6}{4}$

$x=\frac{1}{2}$ or $x=-\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2}, \frac{1}{2}\right\}$.

67. $|3x+1|=|x+2|$

$3x+1=x+2$ or $3x+1=-(x+2)$

$2x=1 \qquad 3x+1=-x-2$

$4x=-3$

$x=\frac{1}{2}$ or $x=-\frac{3}{4}$

The solution set is $\left\{-\frac{3}{4}, \frac{1}{2}\right\}$.

68. $|2x-1|=|2x+3|$

$2x-1=2x+3$ or $2x-1=-(2x+3)$

$-1=3$ False $\qquad 2x-1=-2x-3$

$4x=-2$

No solution or $x=-\frac{2}{4}=-\frac{1}{2}$

The solution set is $\left\{-\frac{1}{2}\right\}$.

69. $|x|<14 \Leftrightarrow -14<x<14$

The solution set is $(-14, 14)$.

70. $-x+6\le 7$

$-7\le -x+6 \le 7$

$-13\le -x \le 1$ Subtract 6.

$13\ge x \ge -1$ Multiply by -1. Reverse inequalities.

$-1\le x \le 13$ Equivalent inequality

The solution set is $[-1,13]$.

71. $|2x+5|\le 1$

$-1\le 2x+5 \le 1$

$-6\le 2x \le -4$

$-3\le x \le -2$

The solution set is $[-3, -2]$.

72. $|x+1|\ge -3$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is *true* for any real number x.
The solution set is $(-\infty, \infty)$.

73. $|3-4x|+7<-4$

$|3-4x|<-11$

The absolute value of any number cannot be less than 0. Therefore, the solution set is $\emptyset$.

74. $|-8-3x|-7>-8$

$|-8-3x|>-1$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is *true* for any real number x.
The solution set is $(-\infty, \infty)$.

Chapter 1 Mixed Review Exercises

1. $5-(6-4x)>2x-5$

$5-6+4x>2x-5$

$-1+4x>2x-5$

$2x>-4$

$x>-2$

The solution set is $(-2, \infty)$.

2. Solve $ak+bt=6r$ for k.

$ak=6r-bt$

$k=\frac{6r-bt}{a}$

3. $x+4<7$ and $x+5\ge 3$

$x<3 \qquad x\ge -2$

The real numbers that are common to both sets are the numbers greater than or equal to -2 and less than 3.
$-2\le x<3$
The solution set is $[-2, 3)$.

4. Clear fractions by multiplying by the LCD, 16.

$$\frac{4x+2}{4}+\frac{3x-1}{8}=\frac{x+6}{16}$$

$4(4x+2)+2(3x-1)=x+6$

$16x+8+6x-2=x+6$

$22x+6=x+6$

$21x=0$

$x=0$

The solution set is $\{0\}$.

5. $|3x+6| \geq 0$

The absolute value of an expression is always nonnegative, so the inequality is true for any real number *k*.
The solution set is $(-\infty, \infty)$.

6. $-5x \geq -10$

$x \leq 2$ Divide by -5.

The solution set is $(-\infty, 2]$.

7. $|3x+2|+4=9$

$$|3x+2|=5$$

$$3x+2=5 \quad \text{or} \quad 3x+2=-5$$

$$3x=3 \qquad 3x=-7$$

$$x=1 \quad \text{or} \quad x=-\frac{7}{3}$$

The solution set is $\left\{-\frac{7}{3}, 1\right\}$.

8. $0.05x+0.03(1200-x)=42$

$$5x+3(1200-x)=4200$$

$$5x+3600-3x=4200$$

$$2x=600$$

$$x=300$$

The solution set is $\{300\}$.

9. $|x+3| \leq 13$

$$-13 \leq x+3 \leq 13$$

$$-16 \leq x \leq 10$$

The solution set is $[-16, 10]$.

10. $\frac{3}{4}(x-2)-\frac{1}{3}(5-2x)<-2$

$$9(x-2)-4(5-2x)<-24$$

$$9x-18-20+8x<-24$$

$$17x-38<-24$$

$$17x<14$$

$$x<\frac{14}{17}$$

The solution set is $\left(-\infty, \frac{14}{17}\right)$.

11. $-4< 3-2x <9$

$-7< -2x <6$ Subtract 3.

$\frac{7}{2}> x >-3$ Divide by -2. Reverse inequalities.

$-3< x <\frac{7}{2}$ Equivalent inequality

The solution set is $\left(-3, \frac{7}{2}\right)$.

12. $-0.3x+2.1(x-4) \leq -6.6$

$$-3x+21(x-4) \leq -66$$

$$-3x+21x-84 \leq -66$$

$$18x-84 \leq -66$$

$$18x \leq 18$$

$$x \leq 1$$

The solution set is $(-\infty, 1]$.

13. $|5x-1|>14$

$$5x-1>14 \quad \text{or} \quad 5x-1<-14$$

$$5x>15 \qquad 5x<-13$$

$$x>3 \quad \text{or} \quad x<-\frac{13}{5}$$

The solution set is $\left(-\infty, -\frac{13}{5}\right) \cup (3, \infty)$.

14. $x \geq -2$ or $x<4$

The solution set includes all numbers either greater than or equal to -2 or all numbers less than 4. This is the union and is the set of all real numbers.
The solution set is $(-\infty, \infty)$.

15. $|x-1|=|2x+3|$

$$x-1=2x+3 \quad \text{or} \quad x-1=-(2x+3)$$

$$x-1=-2x+3$$

$$3x=-2$$

$$-4=x \quad \text{or} \quad x=-\frac{2}{3}$$

The solution set is $\left\{-4, -\frac{2}{3}\right\}$.

16. $\frac{3x}{5}-\frac{x}{2}=3$

$6x-5x=30$ Multiply by 10.

$$x=30$$

The solution set is $\{30\}$.

17. $|3x-7|=4$

$$3x-7=4 \quad \text{or} \quad 3x-7=-4$$
$$3x=11 \qquad 3x=3$$
$$x=\frac{11}{3} \quad \text{or} \quad x=1$$

The solution set is $\left\{1, \frac{11}{3}\right\}$.

18. $5(2x-7)=2(5x+3)$

$$10x-35=10x+6$$
$$-35=6 \quad \text{False}$$

This equation is a *contradiction*.
The solution set is $\varnothing$.

19. $-5x<-30$ and $-7x>-56$

$$x>6 \qquad x<8$$

The graph of the solution set is all numbers both greater than 6 *and* less than 8. This is the intersection. The elements common to both sets are the numbers between 6 and 8, not including the endpoints. The solution set is (6, 8).

20. $-5x+1\ge 11$ or $3x+5\ge 26$

$$-5x\ge 10 \qquad 3x\ge 21$$
$$x\le -2 \quad \text{or} \quad x\ge 7$$

The graph of the solution set is all numbers either less than or equal to -2 *or* greater than or equal to 7. This is the union. The solution set is $(-\infty, -2]\cup[7, \infty)$.

21. Use the formula $V=LWH$, and solve for H.

$$\frac{V}{LW}=\frac{LWH}{LW}$$
$$\frac{V}{LW}=H, \quad \text{or} \quad H=\frac{V}{LW}$$

Substitute 1.5 for W, 5 for L, and 75 for V.

$$H=\frac{75}{5(1.5)}=\frac{75}{7.5}=10$$

The height of the box is 10 ft.

22. Let x = the first consecutive integer. Then $x+1=$ the second consecutive integer and $x+2=$ the third consecutive integer.
The sum of the first and third integers is 47 more than the second integer, so an equation is $x+(x+2)=47+(x+1)$.

$$2x+2=48+x$$
$$x=46$$

Then $x+1=47$, and $x+2=48$.
The integers are 46, 47, and 48.

23. Let x = the employee's earnings during the fifth month. The average of the five months must be at least $1000.

$$\frac{900+1200+1040+760+x}{5}\ge 1000$$
$$900+1200+1040+760+x\ge 5000$$
$$3900+x\ge 5000$$
$$x\ge 1100$$

Any amount greater than or equal to $1100 will qualify the employee for the pension plan.

24. Let x = the number of liters of the 20% solution. Then $x+10$ is the number of liters of the resulting 40% solution.

Liters of Solution	Percent (as a decimal)	Liters of Mixture
x	0.20	$0.20x$
10	0.50	$0.50(10)=5$
$x+10$	0.40	$0.40(x+10)$

From the last column:

$$0.20x+5=0.40(x+10)$$
$$0.20x+5=0.40x+4$$
$$1=0.20x$$
$$5=x$$

5 L of the 20% solution should be used.

25. Let t be the time that the rate was 45 mph. Then $4-t$ is the time that the rate was 50 mph. Use the formula $d=rt$.

Time	Rate	Distance
t	45	$45t$
$4-t$	50	$50(4-t)$

The sum of both distances is 195 miles.

$$45t+50(4-t)=195$$
$$45t+200-50t=195$$
$$200-5t=195$$
$$-5t=-5$$
$$t=1$$

The automobile traveled at 45 mph for 1 hour.

Chapter 1 Test

1. $$\begin{aligned} 3(2x-2)-4(x+6) &= 3x+8+x \\ 6x-6-4x-24 &= 4x+8 \\ 2x-30 &= 4x+8 \\ -2x &= 38 \\ x &= -19 \end{aligned}$$

The solution set is $\{-19\}$.

2. $$\begin{aligned} 0.08x+0.06(x+9) &= 1.24 \\ 8x+6(x+9) &= 124 \\ 8x+6x+54 &= 124 \\ 14x+54 &= 124 \\ 14x &= 70 \\ x &= 5 \end{aligned}$$

The solution set is $\{5\}$.

3. **(a)** $$\begin{aligned} 3x-(2-x)+4x+2 &= 8x+3 \\ 3x-2+x+4x+2 &= 8x+3 \\ 8x &= 8x+3 \\ 0 &= 3 \quad \text{False} \end{aligned}$$

The false statement indicates that the equation is a *contradiction*.
The solution set is $\varnothing$.

(b) $\frac{x}{3}+7=\frac{5x}{6}-2-\frac{x}{2}+9$

Multiply each side by the LCD, 6.

$$\begin{aligned} 2x+42 &= 5x-12-3x+54 \\ 2x+42 &= 2x+42 \\ 0 &= 0 \quad \text{True} \end{aligned}$$

This equation is an *identity*.
The solution set is $\{\text{all real numbers}\}$.

(c) $$\begin{aligned} -4(2x-6) &= 5x+24-7x \\ -8x+24 &= -2x+24 \\ 24 &= 6x+24 \\ 0 &= 6x \\ 0 &= x \end{aligned}$$

This is a *conditional equation*.
The solution set is $\{0\}$.

(d) $$\begin{aligned} \frac{x+6}{10}+\frac{x-4}{15} &= \frac{x+2}{6} \\ 3(x+6)+2(x-4) &= 5(x+2) \\ 3x+18+2x-8 &= 5x+10 \\ 5x+10 &= 5x+10 \quad \text{True} \end{aligned}$$

This is an *identity*.
The solution set is $\{\text{all real numbers}\}$.

4. Solve $V=\frac{1}{3}bh$ for h.

$$\begin{aligned} V &= \frac{1}{3}bh \\ 3V &= bh \quad \text{Multiply by 3.} \\ \frac{3V}{b} &= h \quad \text{Divide by } b. \end{aligned}$$

5. Solve $-3x+2y=6$ for y.

$$\begin{aligned} 2y &= 3x+6 \quad \text{Add } 3x. \\ y &= \frac{3}{2}x+3 \quad \text{Divide by 2.} \end{aligned}$$

6. Solve $d=rt$ for t and substitute 500 for d and 187.433 for r.

$$t=\frac{d}{r}=\frac{500}{187.433}\approx 2.668$$

His time was about 2.668 hr.

7. Use $I=Prt$ and substitute \$456.25 for I, \$36,500 for P, and 1 for t.

$$\begin{aligned} 456.25 &= 36{,}500r(1) \\ r &= \frac{456.25}{36{,}500} = 0.0125 \end{aligned}$$

The rate of interest is 1.25%.

8. $\frac{68.7}{160.0}\approx 0.429$

42.9% of the items were first-class mail.

9. Let x = the amount invested at 1.5%. Then $28{,}000-x=$ the amount invested at 2.5%.

Principal	Rate (as a decimal)	Interest
x	0.015	$0.015x$
$28{,}000-x$	0.025	$0.025(28{,}000-x)$
\$28,000	← Totals →	\$620

From the last column:

$$\begin{aligned} 0.015x+0.025(28{,}000-x) &= 620 \\ 15x+25(28{,}000-x) &= 620{,}000 \\ 15x+700{,}000-25x &= 620{,}000 \\ -10x &= -80{,}000 \\ x &= 8000 \end{aligned}$$

He invested \$8000 at 1.5% and $\$28{,}000-\$8000=\$20{,}000$ at 2.5%.

10. Let $x =$ the rate of the slower car. Then $x + 15 =$ the rate of the faster car.
Use the formula $d = rt$.

	Rate	Time	Distance
Slower Car	x	6	$6x$
Faster Car	$x+15$	6	$6(x+15)$
			630

The total distance traveled is the sum of the distances traveled by each car.

$$\begin{aligned} 6x + 6(x+15) &= 630 \\ 6x + 6x + 90 &= 630 \\ 12x &= 540 \\ x &= 45 \end{aligned}$$

The slower car traveled at 45 mph, while the faster car traveled at $45 + 15 = 60$ mph.

11. The sum of the three angle measures is 180°

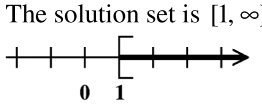
$$\begin{aligned} (2x+20) + x + x &= 180 \\ 4x + 20 &= 180 \\ 4x &= 160 \\ x &= 40 \end{aligned}$$

The three angle measures are 40°, 40°, and $(2 \cdot 40 + 20)° = 100°$.

12. $$\begin{aligned} 4 - 6(x+3) &\le -2 - 3(x+6) + 3x \\ 4 - 6x - 18 &\le -2 - 3x - 18 + 3x \\ -6x - 14 &\le -20 \\ -6x &\le -6 \end{aligned}$$

Divide by -6, and reverse the inequality symbol.
$x \ge 1$

The solution set is $[1, \infty)$.

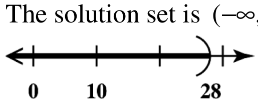


13. $-\dfrac{4}{7}x > -16$

$-4x > -112$ Multiply by 7.

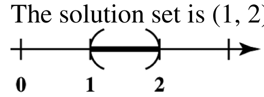
Divide by -4, and reverse the inequality symbol.
$x < 28$

The solution set is $(-\infty, 28)$.



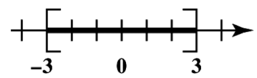
14. $$\begin{aligned} -1 < \ & 3x - 4 < 2 \\ 3 < \ & 3x < 6 \quad \text{Add 4.} \\ 1 < \ & x < 2 \quad \text{Divide by 3.} \end{aligned}$$

The solution set is $(1, 2)$.



15. $$\begin{aligned} -6 \le \ & \tfrac{4}{3}x - 2 \le 2 \\ -18 \le \ & 4x - 6 \le 6 \quad \text{Multiply by 3.} \\ -12 \le \ & 4x \le 12 \quad \text{Add 6.} \\ -3 \le \ & x \le 3 \quad \text{Divide by 4.} \end{aligned}$$

The solution set is $[-3, 3]$.



16. For each inequality, divide both sides by -3 and reverse the direction of the inequality symbol.

A. $-3x < 9$
$x > -3$

B. $-3x > -9$
$x < 3$

C. $-3x > 9$
$x < -3$

D. $-3x < -9$
$x > 3$

Thus, inequality C is equivalent to $x < -3$.

17. Let $x =$ the score on the fourth test.

$$\begin{aligned} \frac{83 + 76 + 79 + x}{4} &\ge 80 \\ \frac{238 + x}{4} &\ge 80 \\ 238 + x &\ge 320 \\ x &\ge 82 \end{aligned}$$

The minimum score must be 82 to guarantee a B.

18. **(a)** $A \cap B = \{1, 2, 5, 7\} \cap \{1, 5, 9, 12\}$
$= \{1, 5\}$

(b) $A \cup B = \{1, 2, 5, 7\} \cup \{1, 5, 9, 12\}$
$= \{1, 2, 5, 7, 9, 12\}$

19. $3x \ge 6$ and $x < 9$
$x \ge 2$

The solution set is all numbers both greater than or equal to 2 *and* less than 9. This is the intersection. The numbers common to both sets are between 2 and 9, including 2 but not 9. The solution set is [2, 9).

20. $-4x \le -24$ or $4x < 12$

$x \ge 6$ or $x < 3$

The solution set is all numbers less than 3 or greater than or equal to 6. This is the union. The solution set is $(-\infty, 3) \cup [6, \infty)$.

21. $|4x+3| \le 7$

$$-7 \le 4x+3 \le 7$$
$$-10 \le 4x \le 4$$
$$-\frac{10}{4} \le x \le \frac{4}{4}$$
$$-\frac{5}{2} \le x \le 1$$

The solution set is $\left[-\frac{5}{2}, 1\right]$.

22. $|5-6x| > 12$

$5-6x > 12$ or $5-6x < -12$

$-6x > 7$ $\quad -6x < -17$

$x < -\frac{7}{6}$ or $x > \frac{17}{6}$

The solution set is $\left(-\infty, -\frac{7}{6}\right) \cup \left(\frac{17}{6}, \infty\right)$.

23. $|3x-9| = 6$

$3x-9 = 6$ or $3x-9 = -6$

$3x = 15$ or $3x = 3$

$x = 5$ or $x = 1$

The solution set is $\{1, 5\}$.

24. $|-3x+4| - 4 < -1$

$$|-3x+4| < 3$$
$$-3 < -3x+4 < 3$$
$$-7 < -3x < -1$$
$$\frac{7}{3} > x > \frac{1}{3} \quad \text{Reverse inequalities.}$$
$$\frac{1}{3} < x < \frac{7}{3} \quad \text{Equivalent inequality}$$

The solution set is $\left(\frac{1}{3}, \frac{7}{3}\right)$.

25. $|7-x| \le -1$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is *false* for any real number *x*. The solution set is $\varnothing$.

26. $|3x-2| + 1 = 8$

$$|3x-2| = 7$$

$3x-2 = 7$ or $3x-2 = -7$

$3x = 9$ or $3x = -5$

$x = \frac{9}{3} = 3$ or $x = -\frac{5}{3}$

The solution set is $\left\{-\frac{5}{3}, 3\right\}$.

27. $|3-5x| = |2x+8|$

$3-5x = 2x+8$ or $3-5x = -(2x+8)$

$-7x = 5$ $\quad 3-5x = -2x-8$

$-3x = -11$

$x = -\frac{5}{7}$ or $x = \frac{11}{3}$

The solution set is $\left\{-\frac{5}{7}, \frac{11}{3}\right\}$.

28. **(a)** $|8x-5| < k$

If $k < 0$, then $|8x-5|$ would be less than a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is $\varnothing$.

(b) $|8x-5| > k$

If $k < 0$, then $|8x-5|$ would be greater than a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is the set of all real numbers, $(-\infty, \infty)$.

(c) $|8x-5| = k$

If $k < 0$, then $|8x-5|$ would be equal to a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is $\varnothing$.