APPENDIX A BASIC ALGEBRA REVIEW

EXERCISE A-1

2. $7 + x$ **4.** $(xy)z$ **6.** $9m$

8. T; Commutative property of multiplication

10. T; Distributive property **12.** T; Multiplicative inverse property

14. F; Let $x = y = 1$, then $\frac{x}{3y} \div \frac{5y}{x} = \frac{1}{3} \div \frac{5}{1} = \frac{1}{3} \cdot \frac{1}{5} = \frac{1}{15}, \frac{15y^2}{x^2} = 15.$

16. T; Property of negatives

18. T; Property of additive inverse **20.** T; Property of negatives

22. F; Let $k = 2$, $b = 1$, then $\frac{k}{k+b} = \frac{2}{3}, \frac{1}{1+b} = \frac{1}{2}.$

24. T; Distributive property

26. T; Zero property

28. Yes. If neither u nor v are zero, then $uv \neq 0$.

30. (A) True

(B) False. For example, 4 is a real number, but is not irrational.

(C) True

32. $\frac{3}{5}$ and –1.43 are two examples of infinitely many.

34. (A) $-3 \in Z, Q, R$ (B) $3.14 \in Q, R$

(C) $\pi \in R$ (D) $\frac{2}{3} \in Q, R$

36. (A) True. This is the commutative property of addition.

(B) False. For example, $5 - 3 \neq 3 - 5$.

(C) True. This is the commutative property of multiplication.

(D) False. For example, $9 \div 3 \neq 3 \div 9$.

38.

$$\begin{aligned} C &= 0.181818\ldots \\ 100C &= 18.1818\ldots \\ 100C - C &= (18.1818\ldots) - (0.181818\ldots) \\ 99C &= 18 \\ C &= \frac{18}{99} = \frac{2}{11} \end{aligned}$$

40. (A) 0.888888888… (B) 0.27272727… (C) 2.23606797… (D) 1.375000000…

42. (A) $0.5 < \frac{9}{17} < 1;\ 1$ (B) $-\frac{12}{25} = -0.48 \approx 0;\ 0$

44. (A) $\frac{5}{6}-\frac{18}{19}\approx 1-1=0;\quad 0$ (B) $\frac{13}{5}+\frac{44}{21}\approx 3+2=5;\quad 5$

46. Tax rate: $\frac{29.86}{533.19}\approx 0.56;\quad 5.6\%$

48. $(0.04)4.30=.172;$ price of gas now: \$4.30 + \$0.17 = \$4.47.

EXERCISE A-2

2. The term of highest degree in $2x - 3$ is $2x$ and the degree of this term is 1.

4. $(2x - 3) + (2x^2 - x + 2) = 2x^2 + x - 1$

6. $(2x^2 - x + 2) - (2x - 3) = 2x^2 - 3x + 5$

8. $(2x - 3)(x^3 + 2x^2 - x + 3) = 2x^4 + x^3 - 8x^2 + 9x - 9$

10. $2(x - 1) + 3(2x - 3) - (4x - 5) = 2x - 2 + 6x - 9 - 4x + 5 = 4x - 6$

12. $2y - 3y[4 - 2(y - 1)] = 2y - 3y[4 - 2y + 2] = 2y - 3y[6 - 2y] = 2y - 18y + 6y^2 = 6y^2 - 16y$

14. $(m - n)(m + n) = m^2 - n^2$ (Special product)

16. $(4t - 3)(t - 2) = 4t^2 - 8t - 3t + 6 = 4t^2 - 11t + 6$

18. $(3x + 2y)(x - 3y) = 3x^2 - 9xy + 2xy - 6y^2 = 3x^2 - 7xy - 6y^2$

20. $(2m - 7)(2m + 7) = (2m)^2 - (7)^2 = 4m^2 - 49$ (Special product)

22. $-(5 - 3x)^2 = -(25 - 30x + 9x^2) = -25 + 30x - 9x^2$

24. $(3x - 2y)(3x + 2y) = (3x)^2 - (2y)^2 = 9x^2 - 4y^2$ (Special product)

26. $(4x - y)^2 = 16x^2 - 8xy + y^2$

28. $(a + b)(a^2 - ab + b^2) = a(a^2 - ab + b^2) + b(a^2 - ab + b^2) = a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$
$= a^3 + b^3$

30. $[a - (2b - c)][a + (2b - c)] = a^2 - (2b - c)^2 = a^2 - (4b^2 - 4bc + c^2) = a^2 - 4b^2 + 4bc - c^2$

32. $2x - 3\{x + 2[x - (x + 5)] + 1\} = 2x - 3\{x + 2[x - x - 5] + 1\} = 2x - 3\{x + 2(-5) + 1\} = 2x - 3\{x - 10 + 1\}$
$= 2x - 3\{x - 9\} = 2x - 3x + 27 = -x + 27$

34. $(3x - 2y)^2(2x + 5y) = (9x^2 - 12xy + 4y^2)(2x + 5y) = 18x^3 + 45x^2y - 24x^2y - 60xy^2 + 8xy^2 + 20y^3$
$= 18x^3 + 21x^2y - 52xy^2 + 20y^3$

36. $(2x - 1)^2 - (3x + 2)(3x - 2) = (2x - 1)^2 - ((3x)^2 - (2)^2) = (4x^2 - 4x + 1) - (9x^2 - 4) = 4x^2 - 4x + 1 - 9x^2 +$
4
$= -5x^2 - 4x + 5$

38. $(x - 3)(x + 3) - (x - 3)^2 = x^2 - 9 - (x^2 - 6x + 9) = x^2 - 9 - x^2 + 6x - 9 = 6x - 18$

40. $(3m + n)(m - 3n) - (m + 3n)(3m - n) = 3m^2 - 9mn + mn - 3n^2 - (3m^2 - mn + 9mn - 3n^2)$
$= 3m^2 - 9mn + mn - 3n^2 - 3m^2 + mn - 9mn + 3n^2 = -16mn$

42. $(x - y)^3 = (x - y)(x - y)^2 = (x - y)(x^2 - 2xy + y^2) = x^3 - 2x^2y + xy^2 - x^2y + 2xy^2 - y^3 = x^3 - 3x^2y + 3xy^2 - y^3$

44. $(2m - n)^3 = (2m - n)(2m - n)^2 = (2m - n)(4m^2 - 4mn + n^2) = 8m^3 - 8m^2n + 2mn^2 - 4m^2n + 4mn^2 - n^3$
$= 8m^3 - 12m^2n + 6mn^2 - n^3$

46. $\{(3m^2 - 3m - 2) + (m^3 + m^2 + 2)\} - \{(3m^2 - 2m + 5) + (4m^2 - m)\}$
$= \{3m^2 - 3m - 2 + m^3 + m^2 + 2\} - \{3m^2 - 2m + 5 + 4m^2 - m\} = \{m^3 + 4m^2 - 3m\} - \{7m^2 - 3m + 5\}$
$= m^3 + 4m^2 - 3m - 7m^2 + 3m - 5 = m^3 - 3m^2 - 5$

48. $[5x(3x + 1) - 5(2x - 1)^2]^2 = [15x^2 + 5x - 5(4x^2 - 4x + 1)]^2 = [15x^2 + 5x - 20x^2 + 20x - 5]^2 = [-5x^2 + 25x - 5]^2$
$= 25x^4 + 625x^2 + 25 - 250x^3 + 50x^2 - 250x = 25x^4 - 250x^3 + 675x^2 - 250x + 25$

50. $-3x\{x[x - x(2 - x)] - (x + 2)(x^2 - 3)\} = -3x\{x[x - 2x + x^2] - [x^3 - 3x + 2x^2 - 6]\}$
$= -3x\{x[-x + x^2] - x^3 + 3x - 2x^2 + 6\} = -3x\{-x^2 + x^3 - x^3 + 3x - 2x^2 + 6\} = -3x\{-3x^2 + 3x + 6\}$
$= 9x^3 - 9x^2 - 18x$

52. m

54. Now the degree is less than or equal to m.

56. $(2 - 1)^2 \neq 2^2 - 1^2$; since
$(a - b)^2 = a^2 - 2ab + b^2$, $(a - b)^2 = a^2 - b^2$ only when
$a = b$ in which case $a^2 - 2ab + b^2 = a^2 - 2a^2 + a^2 = 0$ or
$b = 0$ in which case $a^2 - 2ab + b^2 = a^2$.

58. Let x = amount invested at 7%. Then $2x$ = amount invested at 9%, and $100{,}000 - 3x$ = amount invested at 11%.
$I = 0.07x + (0.09)(2x) + 0.11(100{,}000 - 3x) = 11{,}000 - 0.08x$

60. Let x = number of tickets at \$20. Then $6{,}000 - x$ = number of tickets at \$35.
The total receipts R are:

$R = 20x + 35(6{,}000 - x) = 210{,}000 - 15x$

62. Let x = number of ounces of food M used. Then $160 - x$ = number of ounces of food N used.
The total number of units of calcium, C, in the diet mix is given by:

$C = 8x + 5(160 - x) = 800 + 3x$

EXERCISE A-3

2. $2x^2$ is a common factor: $6x^4 - 8x^3 - 2x^2 = 2x^2(3x^2 - 4x - 1)$

4. $5xy$ is a common factor: $10x^3y + 20x^2y^2 - 15xy^3 = 5xy(2x^2 + 4xy - 3y^2)$

6. $(x + 1)$ is a common factor: $5x(x + 1) - 3(x + 1) = (x + 1)(5x - 3)$

8. $3(b-2c)$ is a common factor:
$12a(b-2c)-15b(b-2c)=3(b-2c)[4a-5b]=3(b-2c)(4a-5b)$

10. $x^2-3x+2x-6=(x^2-3x)+(2x-6)=x(x-3)+2(x-3)=(x-3)(x+2)$

12. $2x^2-x+6x-3=(2x^2-x)+(6x-3)=x(2x-1)+3(2x-1)=(2x-1)(x+3)$

14. $6x^2+9x-2x-3=(6x^2+9x)+(-2x-3)=3x(2x+3)-(2x+3)=(2x+3)(3x-1)$

16. $ac+ad+bc+bd=(ac+ad)+(bc+bd)=a(c+d)+b(c+d)=(a+b)(c+d)$

18. $ab+6+2a+3b=(ab+2a)+(6+3b)=a(b+2)+3(b+2)=(a+3)(b+2)$

20. $2x^2+5x-3$
$a=2,\ b=5,\ c=-3$

Step 1. Use the ac-test to test for factorability
$ac=(2)(-3)=-6$

$\underline{pq}$
$(1)(-6)$
$\boxed{(-1)(6)}$
$(2)(-3)$
$(-2)(3)$

Note that $-1+6=5=b$. Thus, $2x^2+5x-3$ has first-degree factors with integer coefficients.

Step 2. Split the middle term using $b=p+q$ and factor by grouping.
$5=-1+6$
$2x^2+5x-3=(2x^2-x)+(6x-3)=x(2x-1)+3(2x-1)=(2x-1)(x+3)$

22. $x^2-4xy-12y^2$; $a=1,\ b=-4,\ c=-12$

Step 1. Use the ac-test
$ac=1(-12)=-12$

$\underline{pq}$
$(1)(-12)$
$(-1)(12)$
$(-2)(6)$
$\boxed{(2),(-6)}$
$(-3)(4)$
$(3)(-4)$

Note that $2+(-6)=-4=b$. Thus $x^2-4xy-12y^2$ has first-degree factors with integer coefficients.

Step 2. Factor by grouping
$-4=2+(-6)$
$x^2+2xy-6xy-12y^2=(x^2+2xy)-(6xy+12y^2)=x(x+2y)-6y(x+2y)=(x+2y)(x-6y)$

24. $x^2 + x - 4$; $a = 1, b = 1, c = -4$

Step 1. Use the ac-test

$ac = (1)(-4) = -4$

pq
$(-1)(4)$
$(1)(-4)$
$(2)(-2)$
$(-2)(2)$

None of the factors add up to $1 = b$. Thus, this polynomial is *not factorable*.

26. $25m^2 - 16n^2 = (5m - 4n)(5m + 4n)$ (difference of squares)

28. $x^2 + 10xy + 25y^2 = (x + 5y)^2$ (perfect square)

30. $u^2 + 81$

$a = 1, b = 0, c = 81$

Step 1. Use the ac-test

$ac = (1)(81) = 81$

pq
$(1)(81)$
$(-1)(-81)$
$(3)(27)$
$(-3)(-27)$
$(9)(9)$
$(-9)(-9)$

None of the factors add up to $0 = b$. Thus this polynomial is *not factorable*.

32. $6x^2 + 48x + 72 = 6(x^2 + 8x + 12) = 6(x + 2)(x + 6)$

34. $2y^3 - 22y^2 + 48y = 2y(y^2 - 11y + 24) = 2y(y - 3)(y - 8)$

36. $16x^2y - 8xy + y = y(16x^2 - 8x + 1) = y(4x - 1)^2$

38. $6s^2 + 7st - 3t^2 = (3s - t)(2s + 3t)$

40. $x^3y - 9xy^3 = xy(x^2 - 9y^2) = xy(x - 3y)(x + 3y)$

42. $3m^3 - 6m^2 + 15m = 3m(m^2 - 2m + 5)$ [Note: $m^2 - 2m + 5$ is *not factorable*.]

44. $5x^3 + 40y^3 = 5(x^3 + 8y^3) = 5[x^3 + (2y)^3] = 5(x + 2y)(x^2 - 2xy + 4y^2)$ (sum of cubes)

46. $8a^3 - 1 = (2a)^3 - (1)^3 = (2a - 1)(4a^2 + 2a + 1)$ (difference of cubes)

48. $(a - b)^2 - 4(c - d)^2 = [(a - b) - 2(c - d)][(a - b) + 2(c - d)]$

50. $3x^2 - 2xy - 4y^2$ is *not factorable*

52. $4(A + B)^2 - 5(A + B) - 6$

Let $x = A + B$, then $4x^2 - 5x - 6$ can be written as

$4x^2 - 5x - 6 = (2x + 3)(x - 2)$. Now, replace x with $A + B$ to obtain:

$4(A + B)^2 - 5(A + B) - 6 = [2(A + B) + 3][(A + B) - 2]$

54. $m^4 - n^4 = (m^2 - n^2)(m^2 + n^2) = (m - n)(m + n)(m^2 + n^2)$ [Note: $m^2 + n^2$ is not factorable.]

56. $15x^2$ and $(3x - 1)^3$ are common factors:
$15x^2(3x - 1)^4 + 60x^3(3x - 1)^3 = 15x^2(3x - 1)^3[(3x - 1) + 4x] = 15x^2(3x - 1)^3[3x - 1 + 4x] = 15x^2(3x - 1)^3(7x - 1)$

58. False. Here is a counterexample. Consider $u^4 - v^2$, $m = 4$, $n = 2$, $m \neq n$, but $u^4 - v^2 = (u^2 - v)(u^2 + v)$.

60. True. $u^{2k+1} + v^{2k+1} = (u + v)(u^{2k} - u^{2k-1}v + u^{2k-2}v^2 - \ldots + v^{2k})$. For example, if $k = 2$, then as you know $u^3 + v^3 = (u + v)(u^2 - uv + v^2)$.

EXERCISE A-4

2. $\dfrac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} = \dfrac{\overset{5}{\cancel{10}}\cdot \overset{3}{\cancel{9}}\cdot 8}{\cancel{3}\cdot \cancel{2}\cdot 1} = 120$

4. $\dfrac{\cancel{15}\cdot \cancel{10}\cdot 5}{20\cdot \cancel{15}\cdot \cancel{10}} = \dfrac{1}{4}$

6. $\left(\dfrac{d^5}{3a} \div \dfrac{d^2}{6a^2}\right)\cdot \dfrac{a}{4d^3} = \left(\dfrac{d^5}{3a}\cdot \dfrac{6a^2}{d^2}\right)\cdot \dfrac{a}{4d^3} = (2ad^3)\cdot \dfrac{a}{4d^3} = \dfrac{a^2}{2}$

8. $\dfrac{2y}{18} - \dfrac{-1}{28} - \dfrac{y}{42} = \dfrac{28y}{252} - \dfrac{-9}{252} - \dfrac{6y}{252}$
$= \dfrac{28y + 9 - 6y}{252} = \dfrac{22y + 9}{252}$

We find the LCD of 18, 28, 42:
$18 = 2\cdot 3^2$, $28 = 2^2\cdot 7$, $42 = 2\cdot 3\cdot 7$.
Thus, LCD $= 2^2\cdot 3^2\cdot 7 = 252$.

10. $\dfrac{3x + 8}{4x^2} - \dfrac{2x - 1}{x^3} - \dfrac{5}{8x} = \dfrac{2x(3x + 8)}{8x^3} - \dfrac{8(2x - 1)}{8x^3} - \dfrac{5x^2}{8x^3}$
$= \dfrac{6x^2 + 16x - 16x + 8 - 5x^2}{8x^3} = \dfrac{x^2 + 8}{8x^3}$

Find the LCD of $4x^2$, x^3, $8x$:
$4x^2 = 2^2x^2$, $x^3 = x^3$,
$8x = 2^3\cdot x$.
Thus, LCD $= 8x^3$.

12. $\dfrac{2x^2 + 7x + 3}{4x^2 - 1} \div (x + 3) = \dfrac{(2x^2 + 7x + 3)}{(4x^2 - 1)(x + 3)} = \dfrac{(2x + 1)(x + 3)}{(2x - 1)(2x + 1)(x + 3)} = \dfrac{1}{2x - 1}$

14. $\dfrac{5}{m - 2} - \dfrac{3}{2m + 1} = \dfrac{5(2m + 1) - 3(m - 2)}{(m - 2)(2m + 1)} = \dfrac{10m + 5 - 3m + 6}{(m - 2)(2m + 1)} = \dfrac{7m + 11}{(m - 2)(2m + 1)}$

16. $\dfrac{3}{x^2 - 5x + 6} - \dfrac{5}{(x - 2)^2} = \dfrac{3}{(x - 2)(x - 3)} - \dfrac{5}{(x - 2)^2}$ [(LCD $= (x - 2)^2(x - 3)$)]
$= \dfrac{3(x - 2) - 5(x - 3)}{(x - 2)^2(x - 3)} = \dfrac{3x - 6 - 5x + 15}{(x - 2)^2(x - 3)} = \dfrac{-2x + 9}{(x - 2)^2(x - 3)}$

18. $m - 3 - \dfrac{m - 1}{m - 2} = \dfrac{(m - 3)(m - 2)}{m - 2} - \dfrac{m - 1}{m - 2}$ [LCD $= m - 2$]
$= \dfrac{(m - 3)(m - 2) - (m - 1)}{(m - 2)} = \dfrac{m^2 - 2m - 3m + 6 - m + 1}{m - 2} = \dfrac{m^2 - 6m + 7}{m - 2}$

20. $\dfrac{5}{x-3}-\dfrac{2}{3-x} = \dfrac{5}{(x-3)}-\dfrac{-2}{(x-3)} = \dfrac{5-(-2)}{x-3}=\dfrac{7}{x-3}$

(property of negatives)

22. $\dfrac{m+2}{m^2-2m}-\dfrac{m}{m^2-4} = \dfrac{m+2}{m(m-2)}-\dfrac{m}{(m-2)(m+2)}$ [LCD = $m(m-2)(m+2)$]

$= \dfrac{(m+2)^2}{m(m-2)(m+2)} - \dfrac{m^2}{m(m-2)(m+2)} = \dfrac{(m+2)^2-m^2}{m(m-2)(m+2)} = \dfrac{m^2+4m+4-m^2}{m(m-2)(m+2)}$

$= \dfrac{4(m+1)}{m(m-2)(m+2)}$

24. $\dfrac{y}{y^2-y-2}-\dfrac{1}{y^2+5y-14} - \dfrac{2}{y^2+8y+7} = \dfrac{y}{(y-2)(y+1)} - \dfrac{1}{(y+7)(y-2)}+\dfrac{2}{(y+1)(y+7)}$

$= \dfrac{y(y+7)}{(y-2)(y+1)(y+7)} - \dfrac{(y+1)}{(y-2)(y+1)(y+7)} - \dfrac{2(y-2)}{(y-2)(y+1)(y+7)} = \dfrac{y(y+7)-(y+1)-2(y-2)}{(y-2)(y+1)(y+7)}$

$= \dfrac{y^2+7y-y-1-2y+4}{(y-2)(y+1)(y+7)} = \dfrac{y^2+4y+3}{(y-2)(y+1)(y+7)} = \dfrac{(y+1)(y+3)}{(y-2)(y+1)(y+7)} = \dfrac{y+3}{(y-2)(y+7)}$

26. $\dfrac{2}{5-\dfrac{3}{4x+1}} = \dfrac{2}{\dfrac{5(4x+1)-3}{4x+1}} = \dfrac{2}{\dfrac{20x+5-3}{4x+1}} = \dfrac{2}{20x+2}\cdot\dfrac{4x+1}{1} = \dfrac{1}{10x+1}\cdot\dfrac{4x+1}{1} = \dfrac{4x+1}{10x+1}$

28. $\dfrac{x+7}{ax-bx}+\dfrac{y+9}{by-ay} = \dfrac{x+7}{x(a-b)}+\dfrac{y+9}{-y(a-b)}$ [LCD = $xy(a-b)$]

$= \dfrac{y(x+7)}{xy(a-b)}-\dfrac{x(y+9)}{xy(a-b)} = \dfrac{y(x+7)-x(y+9)}{xy(a-b)} = \dfrac{xy+7y-xy-9x}{xy(a-b)} = \dfrac{7y-9x}{xy(a-b)}$

30. $\dfrac{1-\dfrac{y^2}{x^2}}{1-\dfrac{y}{x}} = \dfrac{\dfrac{x^2-y^2}{x^2}}{\dfrac{x-y}{x}} = \dfrac{x^2-y^2}{x^2}\cdot\dfrac{x}{x-y} = \dfrac{(x-y)(x+y)}{x^2}\cdot\dfrac{x}{x-y} = \dfrac{x(x-y)(x+y)}{x^2(x-y)} = \dfrac{x+y}{x}$

32. $\dfrac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h} = \dfrac{\dfrac{x}{x(x+h)}-\dfrac{x+h}{x(x+h)}}{h} = \dfrac{\dfrac{x-(x+h)}{x(x+h)}}{h} = \dfrac{\dfrac{-h}{x(x+h)}}{h} = -\dfrac{h}{x(x+h)}\cdot\dfrac{1}{h} = -\dfrac{1}{x(x+h)}$

34. $\dfrac{1+\dfrac{2}{x}-\dfrac{15}{x^2}}{1+\dfrac{4}{x}-\dfrac{5}{x^2}} = \dfrac{\dfrac{x^2}{x^2}+\dfrac{2x}{x^2}-\dfrac{15}{x^2}}{\dfrac{x^2}{x^2}+\dfrac{4x}{x^2}-\dfrac{5}{x^2}} = \dfrac{\dfrac{x^2+2x-15}{x^2}}{\dfrac{x^2+4x-5}{x^2}} = \dfrac{x^2+2x-15}{x^2}\cdot\dfrac{x^2}{x^2+4x-5} = \dfrac{(x+5)(x-3)}{x^2}\cdot\dfrac{x^2}{(x+5)(x-1)}$

$= \dfrac{x^2(x+5)(x-3)}{x^2(x+5)(x-1)} = \dfrac{x-3}{x-1}$

36. (A) $\dfrac{x^2-3x-4}{x-4} = x - 3$: Incorrect

(B) $\dfrac{x^2-3x-4}{x-4} = \dfrac{(x-4)(x+1)}{x-4} = x + 1$: Correct

38. (A) $\dfrac{(x+h)^3-x^3}{h} = 3x^2 + 3x + 1$: Incorrect

(B) $\dfrac{(x+h)^3-x^3}{h} = \dfrac{((x+h)-x)((x+h)^2+x(x+h)+x^2)}{h} = \dfrac{(x+h-x)(x^2+2hx+h^2+x^2+xh+x^2)}{h}$

$= \dfrac{h(3x^2+3hx+h^2)}{h} = 3x^2 + 3hx + h^2$

40. (A) $\dfrac{2}{x-1} - \dfrac{x+3}{x^2-1} = \dfrac{1}{x+1}$: Correct

42. (A) $x + \dfrac{x-2}{x^2-3x+2} = \dfrac{2}{x-2}$: Incorrect

(B) $x + \dfrac{x-2}{x^2-3x+2} = x + \dfrac{x-2}{(x-2)(x-1)} = x + \dfrac{1}{x-1}$ [LCD = $x - 1$]

$= \dfrac{x(x-1)}{x-1} + \dfrac{1}{x-1} = \dfrac{x(x-1)+1}{x-1} = \dfrac{x^2-x+1}{x-1}$

44. $\dfrac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h} = \left[\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}\right] \div \dfrac{h}{1} = \dfrac{x^2-(x+h)^2}{x^2(x+h)^2} \cdot \dfrac{1}{h} = \dfrac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2h} = \dfrac{x^2-x^2-2xh-h^2}{x^2(x+h)^2h}$

$= \dfrac{-2xh-h^2}{x^2(x+h)^2h} = \dfrac{-h(2x+h)}{x^2(x+h)^2h} = -\dfrac{2x+h}{x^2(x+h)^2}$

46. $2 - \dfrac{1}{1-\dfrac{2}{a+2}} = 2 - \dfrac{1}{\dfrac{a+2-2}{a+2}} = 2 - \dfrac{1}{\dfrac{a}{a+2}} = 2 - \dfrac{1}{1} \cdot \dfrac{a+2}{a} = 2 - \dfrac{a+2}{a} = \dfrac{2a-(a+2)}{a} = \dfrac{2a-a-2}{a} =$
$\dfrac{a-2}{a}$

EXERCISE A-5

2. $3y^{-5} = \dfrac{3}{y^5}$

4. $\dfrac{5}{4x^{-9}} = \dfrac{5x^9}{4}$

6. $3c^{-9}c^4 = 3c^{-9+4} = 3c^{-5} = \dfrac{3}{c^5}$

8. $\dfrac{m^{-11}}{m^{-5}} = m^{-11}m^5 = m^{-11+5} = m^{-6} = \dfrac{1}{m^6}$

10. $7d^{-4}d^4 = 7d^{-4+4} = 7d^0 = 7$

12. $(5b^{-2})^2 = \left(\dfrac{5}{b^2}\right)^2 = \dfrac{25}{b^4}$

14. $(a^{-3}b^4)^{-3} = a^{(-3)(-3)}b^{4(-3)} = a^9b^{-12} = \dfrac{a^9}{b^{12}}$

16. $5{,}380{,}000 = 5.38 \times 10^{6}$

18. $0.019 = 1.9 \times 10^{-2}$

20. $0.000\,000\,007\,832 = 7.832 \times 10^{-9}$

22. $9 \times 10^{6} = 9{,}000{,}000$

24. $2 \times 10^{-5} = 0.00002$

26. $3.044 \times 10^{3} = 3{,}044$

28. $1.13 \times 10^{-2} = 0.0113$

30. $(2x^3y^4)^0 = 1$

32. $\dfrac{10^{-17} \cdot 10^{-5}}{10^{-3} \cdot 10^{-14}} = \dfrac{10^{-22}}{10^{-17}} = 10^{-22} \cdot 10^{17} = 10^{-5} = \dfrac{1}{10^5}$

34. $(2m^{-3}n^2)^{-3} = \dfrac{1}{(2m^{-3}n^2)^3} = \dfrac{1}{8m^{-9}n^6} = \dfrac{m^9}{8n^6}$

36. $\left(\dfrac{2a}{3b^2}\right)^{-3} = \dfrac{1}{\left(\dfrac{2a}{3b^2}\right)^3} = \dfrac{1}{\dfrac{8a^3}{27b^6}} = \dfrac{27b^6}{8a^3}$

38. $\dfrac{9m^{-4}n^3}{12m^{-1}n^{-1}} = \dfrac{3}{4}m^{-4}n^3mn = \dfrac{3}{4}m^{-4+1}n^{3+1} = \dfrac{3}{4}m^{-3}n^4 = \dfrac{3n^4}{4m^3}$

40. $\dfrac{5x^3-2}{3x^2} = \dfrac{5x^3}{3x^2} - \dfrac{2}{3x^2} = \dfrac{5}{3}x - \dfrac{2}{3}x^{-2}$

42. $\dfrac{2x^3-3x^2+x}{2x^2} = \dfrac{2x^3}{2x^2} - \dfrac{3x^2}{2x^2} + \dfrac{x}{2x^2} = x - \dfrac{3}{2} + \dfrac{1}{2}x^{-1}$

44. $\dfrac{5x^4(x+3)^2 - 2x^5(x+3)}{(x+3)^4} = \dfrac{x^4(x+3)[5(x+3)-2x]}{(x+3)^4} = \dfrac{x^4(x+3)[5x+15-2x]}{(x+3)^4} = \dfrac{x^4(x+3)(3x+15)}{(x+3)^4}$

$= \dfrac{3x^4(x+3)(x+5)}{(x+3)^4} = \dfrac{3x^4(x+5)}{(x+3)^3}$

46. $2x(x+3)^{-1} - x^2(x+3)^{-2} = \dfrac{2x}{x+3} - \dfrac{x^2}{(x+3)^2} = \dfrac{2x(x+3)-x^2}{(x+3)^2} = \dfrac{2x^2+6x-x^2}{(x+3)^2} = \dfrac{x^2+6x}{(x+3)^2} = \dfrac{x(x+6)}{(x+3)^2}$

48. $\dfrac{(60{,}000)(0.000003)}{(0.0004)(1{,}500{,}000)} = \dfrac{(6\times10^4)(3\times10^{-6})}{(4\times10^{-4})(1.5\times10^6)} = \dfrac{18\times10^{-2}}{6\times10^2} = 3\times10^{-4}$; 0.0003

50. $\dfrac{(0.00000082)(230{,}000)}{(625{,}000)(0.0082)} = \dfrac{(8.2\times10^{-7})(2.3\times10^5)}{(6.25\times10^5)(8.2\times10^{-3})} = \dfrac{2.3\times10^{-2}}{6.25\times10^2} = 0.368 \times 10^{-4} = 3.68\times10^{-5}$;
0.000 0368

52. $2^{(3^2)} = 2^9 = 512$ while $(2^3)^2 = 8^2 = 64$ which is the calculator result.

54. Because $a^{-n}a^n = 1$, we can divide both side by a^n $(a \neq 0)$ which gives $a^{-n} = \dfrac{1}{a^n}$.

56. $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} = \dfrac{\dfrac{1}{x^2}-\dfrac{1}{y^2}}{\dfrac{1}{x}+\dfrac{1}{y}} = \dfrac{\dfrac{y^2-x^2}{x^2y^2}}{\dfrac{y+x}{xy}} = \dfrac{y^2-x^2}{x^2y^2}\cdot\dfrac{xy}{y+x} = \dfrac{(y-x)(y+x)xy}{x^2y^2(y+x)} = \dfrac{y-x}{xy}$

58. $\dfrac{xy^{-2}-yx^{-2}}{y^{-1}-x^{-1}} = [xy^{-2}-yx^{-2}] \div [y^{-1}-x^{-1}] = \left[\dfrac{x}{y^2}-\dfrac{y}{x^2}\right] \div \left[\dfrac{1}{y}-\dfrac{1}{x}\right] = \dfrac{x^3-y^3}{x^2y^2} \div \dfrac{x-y}{xy}$

$= \dfrac{(x-y)(x^2+xy+y^2)}{x^2y^2}\cdot\dfrac{xy}{x-y} = \dfrac{x^2+xy+y^2}{xy}$

60. (A) $\dfrac{5.674\times10^{12}}{2.81\times10^{8}} \approx 2.0192\times10^{4}$; $20,192

(B) $\dfrac{3.62\times10^{11}}{2.81\times10^{8}} \approx 1.288\times10^{3}$; $1,288

(C) $\dfrac{3.62\times10^{11}}{5.674\times10^{12}} \approx 0.638\times10^{-1} = 6.38\times10^{-2}$; 6.38%

62. (A) $\dfrac{0.03}{1{,}000{,}000} = \dfrac{3\times10^{-2}}{10^{6}} = 3\times10^{-8}$

(B) $\dfrac{0.03}{1{,}000{,}000} = 0.00000003$

(C) $\dfrac{0.03}{1{,}000{,}000} = 0.000003\%$

64. Population density: $\dfrac{323{,}000{,}000}{3{,}539{,}000} = \dfrac{3.23\times10^{8}}{3.539\times10^{6}} \approx 0.913\times10^{2} = 91.3$ people per square mile.

EXERCISE A-6

2. $7y^{2/5} = 7\sqrt[5]{y^2}$

4. $(7x^2y)^{5/7} = \sqrt[7]{(7x^2y)^5}$

6. $x^{1/2}+y^{1/2} = \sqrt{x}+\sqrt{y}$

8. $7m\sqrt[5]{n^2} = 7mn^{2/5}$

10. $\sqrt[7]{(8x^4y)^3} = ((8x^4y)^3)^{1/7} = (8^3x^{12}y^3)^{1/7} = (8x^4y)^{3/7}$

12. $\sqrt[3]{x^2+y^3} = (x^2+y^3)^{1/3}$

14. $64^{1/3} = (4^3)^{1/3} = 4$

16. $16^{3/4} = (16^{1/4})^3 = ((2^4)^{1/4})^3 = 2^3 = 8$

18. $(-49)^{1/2}$ is not a real number.

20. $(-64)^{2/3} = (-2^6)^{2/3} = ((-2^6)^2)^{1/3} = (2^{12})^{1/3} = 2^{12/3} = 2^4 = 16$

22. $\left(\frac{8}{27}\right)^{2/3} = \left(\frac{2^3}{3^3}\right)^{2/3} = \frac{2^2}{3^2} = \frac{4}{9}$

24. $8^{-2/3} = (8^{1/3})^{-2} = ((2^3)^{1/3})^{-2} = (2)^{-2} = \frac{1}{4}$

26. $y^{-3/7}y^{4/7} = y^{(-3/7)\ +(4/7)} = y^{1/7}$

28. $\frac{x^{1/4}}{x^{3/4}} = \frac{1}{x^{3/4}x^{-1/4}} = \frac{1}{x^{(3/4)-(1/4)}} = \frac{1}{x^{1/2}}$

30. $(4u^{-2}v^4)^{1/2} = 4^{1/2}\,u^{-2\times(1/2)}\,v^{4\times(1/2)} = 2u^{-1}v^2 = \frac{2v^2}{u}$

32. $\left(\frac{w^4}{9x^{-2}}\right)^{-1/2} = \frac{w^{4(-1/2)}}{(9x^{-2})^{-1/2}} = \frac{w^{-2}}{9^{-1/2}x} = \frac{3}{xw^2}$

34. $\frac{6a^{3/4}}{15a^{-1/3}} = \frac{2}{5}a^{(3/4)\ +(1/3)} = \frac{2}{5}a^{13/12}$

36. $\sqrt[3]{(7+2y)^3} = (7+2y)^{3/3} = 7+2y$

38. $\sqrt[5]{16a^4}\,\sqrt[5]{4a^2}\,\sqrt[5]{8a^3} = \sqrt[5]{(16a^4)(4a^2)(8a^3)} = \sqrt[5]{16\cdot 4\cdot 8\,a^9} = \sqrt[5]{(2^5a^5)\cdot 16a^4} = 2a\sqrt[5]{16a^4}$

40. $\frac{\sqrt{8}\sqrt{12y}}{\sqrt{6y}} = \frac{\sqrt{96y}}{\sqrt{6y}} = \sqrt{\frac{96y}{6y}} = \sqrt{16} = 4$

42. $2m^{1/3}(3m^{2/3} - m^6) = 6m^{(1/3)+(2/3)} - 2m^{(1/3)+6} = 6m - 2m^{19/3}$

44. $(a^{1/2} + 2b^{1/2})(a^{1/2} - 3b^{1/2}) = a^{(1/2)+(1/2)} - 3a^{1/2}b^{1/2} + 2a^{1/2}b^{1/2} - 6b^{(1/2)+(1/2)}$
$= a - (ab)^{1/2} - 6b$

46. $(2x - 3y^{1/3})(2x^{1/3} + 1) = 4x^{4/3} + 2x - 6x^{1/3}y^{1/3} - 3y^{1/3}$

48. $(x^{1/2} + 2y^{1/2})^2 = (x^{1/2})^2 + 4(x^{1/2})(y^{1/2}) + (2y^{1/2})^2 = x + 4x^{1/2}y^{1/2} + 4y$

50. $\frac{12\sqrt{x}-3}{4\sqrt{x}} = \frac{12x^{1/2}-3}{4x^{1/2}} = \frac{12x^{1/2}}{4x^{1/2}} - \frac{3}{4x^{1/2}} = 3 - \frac{3}{4}x^{-1/2}$

52. $\frac{3\sqrt[3]{x^2}+\sqrt{x}}{5x} = \frac{3x^{2/3}+x^{1/2}}{5x} = \frac{3x^{2/3}}{5x} + \frac{x^{1/2}}{5x} = \frac{3}{5}x^{2/3}x^{-1} + \frac{1}{5}x^{1/2}x^{-1} = \frac{3}{5}x^{-1/3} + \frac{1}{5}x^{-1/2}$

54. $\frac{x^2-4\sqrt{x}}{2\sqrt[3]{x}} = \frac{x^2-4x^{1/2}}{2x^{1/3}} = \frac{x^2}{2x^{1/3}} - \frac{4x^{1/2}}{2x^{1/3}} = \frac{1}{2}x^2x^{-1/3} - 2x^{1/2}x^{-1/3} = \frac{1}{2}x^{5/3} - 2x^{1/6}$

56. $\frac{14x^2}{\sqrt{7x}} = \frac{14x^2}{\sqrt{7x}}\cdot\frac{\sqrt{7x}}{\sqrt{7x}} = \frac{14x^2\sqrt{7x}}{7x} = 2x\sqrt{7x}$

58. $\dfrac{3(x+1)}{\sqrt{x+4}} = \dfrac{3(x+1)}{\sqrt{x+4}} \cdot \dfrac{\sqrt{x+4}}{\sqrt{x+4}} = \dfrac{3(x+1)\sqrt{x+4}}{x+4}$

60. $\dfrac{3a-3b}{\sqrt{a}+\sqrt{b}} = \dfrac{3(a-b)}{\sqrt{a}+\sqrt{b}} \cdot \dfrac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \dfrac{3(a-b)(\sqrt{a}-\sqrt{b})}{(\sqrt{a})^2-(\sqrt{b})^2} = \dfrac{3(a-b)(\sqrt{a}-\sqrt{b})}{a-b} = 3(\sqrt{a} - \sqrt{b})$

62. $\dfrac{\sqrt{3mn}}{3mn} = \dfrac{\sqrt{3mn}}{3mn} \cdot \dfrac{\sqrt{3mn}}{\sqrt{3mn}} = \dfrac{3mn}{3mn\sqrt{3mn}} = \dfrac{1}{\sqrt{3mn}}$

64. $\dfrac{\sqrt{2(a+h)}-\sqrt{2a}}{h} = \dfrac{\sqrt{2(a+h)}-\sqrt{2a}}{h} \cdot \dfrac{\sqrt{2(a+h)}+\sqrt{2a}}{\sqrt{2(a+h)}+\sqrt{2a}} = \dfrac{2(a+h)-2a}{h[\sqrt{2(a+h)}+\sqrt{2a}]}$

$= \dfrac{2a+2h-2a}{h[\sqrt{2(a+h)}+\sqrt{2a}]} = \dfrac{2}{\sqrt{2(a+h)}+\sqrt{2a}}$

66. $\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} \cdot \dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \dfrac{x-y}{(\sqrt{x}+\sqrt{y})^2} = \dfrac{x-y}{x+2\sqrt{xy}+y}$

68. $(x^3+y^3)^{1/3} \stackrel{?}{=} x+y$
Let $x = y = 1$, then $(x^3+y^3)^{1/3} = (1+1)^{1/3} = 2^{1/3}$
$1 + 1 = 2$; $2^{1/3} \neq 2$

70. $(x+y)^{-1/2} \stackrel{?}{=} \dfrac{1}{(x+y)^2}$

Let $x = y = 1$, then $(x+y)^{-1/2} = (2)^{-1/2} = \dfrac{1}{\sqrt{2}}$,

$\dfrac{1}{(x+y)^2} = \dfrac{1}{(1+1)^2} = \dfrac{1}{4}$; $\dfrac{1}{\sqrt{2}} \neq \dfrac{1}{4}$

72. True; $\sqrt{x^2} = \sqrt{|x|^2} = |x|$

74. True; $\sqrt[3]{x^3} = (x^3)^{1/3} = x^{3/3} = x$

76. True; negative numbers do not have square roots

78. False; every real number has exactly one real cube root. Roe example, the only real cube root of 8 is 2.

80. False; $r = 2\sqrt{6} - 5 < 0$ and negative numbers do not have square roots.

82. True; $(1-\sqrt{2})^3 = -.0710678119$ and $7-5\sqrt{2} = -.0710678119$. Therefore, $1-\sqrt{2}$ is a cube root of $7-5\sqrt{2}$.

84. $2(x-2)^{-1/2} - \frac{1}{2}(2x+3)(x-2)^{-3/2} = \frac{2}{(x-2)^{1/2}} - \frac{2x+3}{2(x-2)^{3/2}} = \frac{4(x-2)-(2x+3)}{2(x-2)^{3/2}}$

$= \frac{4x-8-2x-3}{2(x-2)^{3/2}} = \frac{2x-11}{2(x-2)^{3/2}}$

86. $\frac{(2x-1)^{1/2}-(x+2)\left(\frac{1}{2}\right)(2x-1)^{-1/2}(2)}{2x-1} = \frac{(2x-1)^{1/2}}{2x-1} - \frac{(x+2)(2x-1)^{-1/2}}{2x-1}$

$= \frac{1}{(2x-1)^{1/2}} - \frac{x+2}{(2x-1)^{3/2}} = \frac{(2x-1)-(x+2)}{(2x-1)^{3/2}} = \frac{2x-1-x-2}{(2x-1)^{3/2}} = \frac{x-3}{(2x-1)^{3/2}}$

88. $\frac{2(3x-1)^{1/3}-(2x+1)\left(\frac{1}{3}\right)(3x-1)^{-2/3}(3)}{(3x-1)^{2/3}} = \frac{2(3x-1)^{1/3}}{(3x-1)^{2/3}} - \frac{(2x+1)(3x-1)^{-2/3}}{(3x-1)^{2/3}}$

$= \frac{2}{(3x-1)^{1/3}} - \frac{2x+1}{(3x-1)^{4/3}} = \frac{2(3x-1)-(2x+1)}{(3x-1)^{4/3}} = \frac{6x-2-2x-1}{(3x-1)^{4/3}} = \frac{4x-3}{(3x-1)^{4/3}}$

90. $15^{5/4} = 15^{1.25} = 29.52$

92. $103^{-3/4} = \frac{1}{103^{3/4}} = \frac{1}{(103)^{0.75}} = 0.03093$

94. $2.876^{8/5} = (2.876)^{1.6} = 5.421$

96. (A) $2\sqrt[3]{2+\sqrt{5}} = 3.236$ (B) $\sqrt{8} = 2.828$ (C) $\sqrt{3} + \sqrt{7} = 4.378$

(D) $\sqrt{3+\sqrt{8}} + \sqrt{3-\sqrt{8}} = 2.828$ (E) $\sqrt{10+\sqrt{84}} = 4.378$

(F) $1 + \sqrt{5} = 3.236$

(A) and (F) have the same value:

$\left(2\sqrt[3]{2+\sqrt{5}}\right)^3 = 8(2 + \sqrt{5}) = 16 + 8\sqrt{5}$

$(1 + \sqrt{5})^3 = 1 + 3\sqrt{5} + 15 + 5\sqrt{5} = 16 + 8\sqrt{5}$

(B) and (D) have the same value:

$(\sqrt{8})^2 = 8$

$\left(\sqrt{3+\sqrt{8}} + \sqrt{3-\sqrt{8}}\right)^2 = 3 + \sqrt{8} + 3 - \sqrt{8} + 2\sqrt{(3+\sqrt{8})(3-\sqrt{8})} = 6 + 2\sqrt{9-8} = 6 + 2 = 8$

(C) and (E) have the same value:

$(\sqrt{3} + \sqrt{7})^2 = 3 + 7 + 2\sqrt{21} = 10 + 2\sqrt{21}$

$\left(\sqrt{10+\sqrt{84}}\right)^2 = 10 + \sqrt{84} = 10 + \sqrt{4\times 21} = 10 + 2\sqrt{21}$

EXERCISE A-7

2. $3m^2 - 21 = 0$
$3m^2 = 21$
$m^2 = 7$
$m = \pm\sqrt{7}$

4. $(2x+1)^2 = 16$
$2x + 1 = \pm 4$
$2x = \pm 4 - 1$
$x = \dfrac{\pm 4 - 1}{2}$
$x = -\dfrac{5}{2}$ and $x = \dfrac{3}{2}$

6. $3x^2 - 18x + 15 = 0$
$x^2 - 6x + 5 = 0$
$(x-1)(x-5) = 0$
$x - 1 = 0$ or $x - 5 = 0$
$x = 1$ or $x = 5$

8. $n^2 = 3n$
$n^2 - 3n = 0$
$n(n-3) = 0$
$n = 0$ or $n - 3 = 0$
$n = 0$ or $n = 3$

10. $m^2 + 8m + 3 = 0$
$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a = 1$, $b = 8$, $c = 3$
$= \dfrac{-8 \pm \sqrt{64-12}}{2} = \dfrac{-8 \pm \sqrt{52}}{2} = \dfrac{-8 \pm 2\sqrt{13}}{2} = -4 \pm \sqrt{13}$

12. $2x^2 - 20x - 6 = 0$
$x^2 - 10x - 3 = 0$
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a = 1$, $b = -10$, $c = -3$
$- = \dfrac{-(-10) \pm \sqrt{100+12}}{2} = \dfrac{10 \pm \sqrt{112}}{2} = \dfrac{10 \pm 4\sqrt{7}}{2} = 5 \pm 2\sqrt{7}$

14. $x^2 = -\dfrac{3}{4}x$
$x^2 + \dfrac{3}{4}x = 0$
$x\left(x + \dfrac{3}{4}\right) = 0$
$x = 0$ and $x = -\dfrac{3}{4}$

16. $9y^2 - 25 = 0$
$9y^2 = 25$
$y^2 = \dfrac{25}{9}$
$y = \pm\sqrt{\dfrac{25}{9}} = \pm\dfrac{5}{3}$

18. $9x^2 - 6 = 15x$
$3x^2 - 2 = 5x$
$3x^2 - 5x - 2 = 0$
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a = 3$, $b = -5$, $c = -2$
$= \dfrac{5 \pm \sqrt{25+24}}{6} = \dfrac{5 \pm \sqrt{49}}{6} = \dfrac{5 \pm 7}{6}$
$x = \dfrac{5-7}{6} = -\dfrac{1}{3}$, $x = \dfrac{5+7}{6} = 2$

20. $m^2 = 1 - 3m$

$m^2 + 3m - 1 = 0$

$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a = 1$, $b = 3$, $c = -1$

$= \dfrac{-3 \pm \sqrt{9+4}}{2} = \dfrac{-3 \pm \sqrt{13}}{2}$

22. $2x^2 = 4x - 1$

$2x^2 - 4x + 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a = 2$, $b = -4$, $c = 1$

$x = \dfrac{4 \pm \sqrt{16-8}}{4} = \dfrac{4 \pm \sqrt{8}}{4} = \dfrac{4 \pm 2\sqrt{2}}{4} = \dfrac{2 \pm \sqrt{2}}{2}$

24. $x^2 - 2x = -3$

$x^2 - 2x + 3 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a = 1$, $b = -2$, $c = 3$

$b^2 - 4ac = 4 - 12 = -8$

Since $b^2 - 4ac < 0$, there are no real solutions.

26. $(5x - 2)^2 = 7$

$5x - 2 = \pm\sqrt{7}$

$5x = \pm\sqrt{7} + 2$

$x = \dfrac{\pm\sqrt{7} + 2}{5}$

$x = \dfrac{2 - \sqrt{7}}{5}$ and $x = \dfrac{2 + \sqrt{7}}{5}$

28. $x - \dfrac{7}{x} = 0$

Since $x \neq 0$, $\dfrac{x^2 - 7}{x} = 0$ implies $x^2 - 7 = 0$, and $x = \pm\sqrt{7}$.

30. $2 + \dfrac{5}{u} = \dfrac{3}{u^2}$

Multiply both sides by $u^2 \neq 0$.

$2u^2 + 5u = 3$

$2u^2 + 5u - 3 = 0$

$u = \dfrac{-5 \pm \sqrt{25+24}}{4} = \dfrac{-5 \pm \sqrt{49}}{4} = \dfrac{-5 \pm 7}{4}$

$u = \dfrac{-5-7}{4} = -3$ and $u = \dfrac{-5+7}{4} = \dfrac{1}{2}$

32. $x^2 - 28x - 128$

Step 1. Test for factorability

$$\sqrt{b^2 - 4ac} = \sqrt{(-28)^2 - 4(1)(-128)} = 36$$

Since the result is an integer, the polynomial has first-degree factors with integer coefficients.

Step 2. Use the factor theorem

$x^2 - 28x - 128 = 0$

$$x = \frac{28 \pm 36}{2} = -4,\ 32 \text{ (by the quadratic formula)}$$

Thus, $x^2 - 28x - 128 = [x - (-4)](x - 32) = (x + 4)(x - 32)$

34. $x^2 + 52x + 208$

Step 1. Test for factorability

$$\sqrt{b^2 - 4ac} = \sqrt{(52)^2 - 4(1)(208)} = \sqrt{1872} \approx 43.27$$

Since this is not an integer, the polynomial is not factorable.

36. $3x^2 - 32x - 140$

Step 1. Test for factorability

$$\sqrt{b^2 - 4ac} = \sqrt{(-32)^2 - 4(3)(-140)} = 52$$

Thus, the polynomial has first-degree factors with integer coefficients.

Step 2. Use the factor theorem

$3x^2 - 32x - 140 = 0$

$$x = \frac{32 \pm 52}{6} = -\frac{10}{3},\ 14$$

Thus, $3x^2 - 32x - 140 = 3\left[x - \left(-\frac{10}{3}\right)\right](x - 14)$

$$= 3 \cdot \frac{(3x + 10)}{3} \cdot (x - 14) = (3x + 10)(x - 14)$$

38. $6x^2 - 427x - 360$

Step 1. Test for factorability

$$\sqrt{b^2 - 4ac} = \sqrt{(-427)^2 - 4(6)(-360)} = 437$$

Thus, the polynomial has first-degree factors with integer coefficients.

Step 2. Use the factor theorem

$6x^2 - 427x - 360 = 0$

$$x = \frac{427 \pm 437}{12} = -\frac{5}{6},\ 72$$

Thus, $6x^2 - 427x - 360 = 6\left[x - \left(-\frac{5}{6}\right)\right](x - 72) = (6x + 5)(x - 72)$

40. $x^2 + 3mx - 3n = 0$

$$x = \frac{-3m \pm \sqrt{9m^2 + 12n}}{2}$$

$$x = \frac{-3m - \sqrt{9m^2 + 12n}}{2} \text{ and } x = \frac{-3m + \sqrt{9m^2 + 12n}}{2}$$

42. $x^2 - 2x + c = 0$

The discriminant is: $4 - 4c$

(A) If $4 - 4c > 0$, i.e., if $c < 1$, then the equation has two distinct real roots.

(B) If $4 - 4c = 0$, i.e., if $c = 1$, then the equation has one real double root.

(C) If $4 - 4c < 0$, i.e., if $c > 1$, then there are no real roots.

44. $x^3 - 8 = (x-2)(x^2 + 2x + 4) = 0; \quad x = 2$

46. $2x^3 + 250 = 2(x^3 + 125) = 2(x+5)(x^2 - 5x + 25) = 0; \quad x = -5$

48. $x^4 - 12x^2 + 32 = (x^2 - 4)(x^2 - 8) = (x-2)(x+2)(x - 2\sqrt{2})(x + 2\sqrt{2}) = 0; \quad x = 2, -2, 2\sqrt{2}, -2\sqrt{2}$

50. Setting the supply equation equal to the demand equation, we have

$$\frac{x}{6} + 9 = \frac{24,840}{x}$$

$$\frac{1}{6}x^2 + 9x = 24,840$$

$$x^2 + 54x - 149,040 = 0$$

$$x = \frac{-54 \pm \sqrt{(54)^2 - 4(1)(-149,040)}}{2}$$

$$= \frac{-54 \pm 774}{2} = 360 \text{ units}$$

Note, we discard the negative root since a negative number of units cannot be produced or sold. Substituting $x = 360$ into either equation (we use the demand equation), we get

$$p = \frac{24,840}{360} = 69$$

Supply equals demand at \$69 per unit.

52. $A = P(1 + r)^2 = P(1 + 2r + r^2)$

Let $P = \$1,000$, $A = \$1,210$. Then,

$$1,210 = 1,000(1 + 2r + r^2)$$

$$r^2 + 2r + 1 = \frac{1210}{1000} = 1.21$$

$$r^2 + 2r - .21 = 0$$

$$r = \frac{-2 \pm \sqrt{4 - 4(-.21)}}{2} = \frac{-2 \pm \sqrt{4 + .84}}{2} = \frac{-2 \pm \sqrt{4.84}}{2} = \frac{-2 \pm 2.2}{2}$$

$$= 0.10 \text{ or } 10\% \quad \text{(disregard the negative root)}$$

54. $d = 0.044v^2 + 1.1v$

For $d = 550$ we have

$$0.044v^2 + 1.1v - 550 = 0$$

$$44v^2 + 1100v - 550{,}000 = 0$$

$$v^2 + 25v - 12{,}500 = 0$$

$$v = \frac{-25 \pm \sqrt{625 + 50{,}000}}{2} = \frac{-25 \pm 225}{2} = \frac{200}{2} = 100 \text{ miles per hour} \quad \text{(disregard the negative root)}$$